AD-A037 118 MITRE CORP BEDFORD MASS F/G 17/2
HIGH ALTITUDE SATELLITE COMMUNICATIONS, WITH CROSSLINKS.(U)
JAN 77 P F CHRISTOPHER, E R EDELMAN F19628-76-C-0001
UNCLASSIFIED MTR-3161 ESD-TR-76-363 NL

1 OF 2
AD
A037 118

ESD-TR-76-363

MTR-3161

HIGH ALTITUDE SATELLITE COMMUNICATIONS, WITH CROSSLINKS

JANUARY 1977

Prepared for

DEPUTY FOR CONTROL AND COMMUNICATIONS SYSTEMS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Hanscom Air Force Base, Bedford, Massachusetts

Project No. 636B

Prepared by
THE MITRE CORPORATION
Bedford, Massachusetts

Contract No. F19628-76-C-0001

When U.S. Government drawings, specifications, or other data are used for any purpose other than a definitely related government procurement operation, the government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Do not return this copy. Retain or destroy.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved for publication.

JOSEPH P. CROCCO, Lt. Colonel, USAF
Chief, Development Branch
Satellite Communications SPO
Deputy for Control and
Communications System

PAUL F. ANTONELLI
Acting Director of Engineering
Satellite Communications SPO
Deputy for Control and
Communications System

JAMES E. BAKER, Colonel, USAF
System Program Director
Satellite Communications SPO
Deputy for Control and
Communications System

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: ESD-TR-76-363	2. GOVT ACCESSION NO.
	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): HIGH ALTITUDE SATELLITE COMMUNICATIONS, WITH CROSSLINKS	5. TYPE OF REPORT & PERIOD COVERED
	6. PERFORMING ORG. REPORT NUMBER: MTR-3161
7. AUTHOR(s): P. F. Christopher, S. M. Maciorowski, E. R. Edelman	8. CONTRACT OR GRANT NUMBER(s): F19628-76-C-0001
9. PERFORMING ORGANIZATION NAME AND ADDRESS: The MITRE Corporation, Box 208, Bedford, MA 01730	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Project No. 636B
11. CONTROLLING OFFICE NAME AND ADDRESS: Deputy for Control and Communications Systems, Electronic Systems Division, AFSC, Hanscom Air Force Base, Bedford, MA 01731	12. REPORT DATE: JAN[redacted]977
	13. NUMBER OF PAGES: 143
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

Technical rept.,

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ANGLES	KEPLERIAN ORBITS	UPLINKS
CROSSLINKS	LUNAR PERTURBATIONS	
DOPPLERS	RANGES	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Analysis and FORTRAN programs that quantitatively describe the uplink, crosslinks, and downlinks for a 10 satellite, two ground-station satellite communications system are described. The existence of each link (accounting for earth obstruction for crosslinks and coverage for up/downlinks), angles, ranges, and Dopplers are given as outputs of the programs listed in less than one minute central processing unit (CPU)

(over)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

235 050

20. Abstract (Continued)

time on a 370/158 computer. General Keplerian orbits are allowed (0 ≤ eccentricity ≤ 0.99). A new analysis is presented, which results in simple estimates of orbital stability as a function of lunar perturbations. Thus, very general satellite communication orbits (altitude range 3000 to 250,000 nautical miles) can be analyzed with little additional CPU time.

ACKNOWLEDGMENTS

Several individuals have contributed to the work presented herein. W.T. Brandon and A.L. Cohn have recognized many interesting features of crosslinks which served as a valuable background. H.B. Gershman provided the impetus for a convenient local satellite coordinate system which appears in program SATLUNAE (App. 6). He later used this coordinate system to study the effects of satellite obstructions on crosslink antenna pointing. E.E. Crampton's continuing interest in the development of the programs (App. 1-9) resulted in a short, efficient subroutine for the solution of Kepler's equation for high eccentricity. He also developed an alternate solution to ground station azimuth angle which is a valuable check for the program AZ1 (App. 8).

This report has been prepared by The MITRE Corporation under Project No. 636B. The contract is sponsored by the Electronic Systems Division, Air Force Systems Command, Hanscom Air Force Base, Massachusetts.

TABLE OF CONTENTS

TABLE OF CONTENTS - (cont.)

LIST OF FIGURES

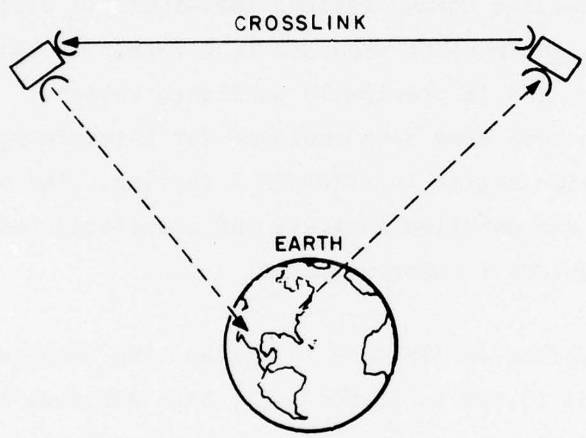
LIST OF TABLES

INTRODUCTION AND SUMMARY

This report gives a quantitative description of the uplinks, crosslinks, and downlinks of a general high altitude satellite system. The analysis was performed at intervals during 1973 and 1974, in support of the Electronic Systems Division (ESD) responsibility to define a high altitude, crosslinked system of satellites for the Air Force Satellite Communications (AFSATCOM) II program. Specific quantitative results provided as a result of this effort have been incorporated in previously published reports. The computer programs have also been employed for determining test sites having low elevation angles in AFSATCOM I testing. The present report documents the detailed analysis and associated computer programs, and provides further examples.

Efforts to define an AFSATCOM II system have required analysis of medium altitude (3,000 to 20,000 nmi), high altitude (20,000 to 250,000 nmi), and highly eccentric (eccentricity >0.7) orbits. Various system concepts also employed satellite to satellite links or crosslinks as shown in Figure 1. While there were some computer programs available for problems in each class of orbit, there were severe limitations on their utility in context of the AFSATCOM system problem. Close analysis of such a general satellite communication system (uplinks, crosslinks, and downlinks) has been going on only for the past few years.

Principal, significant results of the work described in this report are:



IA-45187

Figure 1 A GENERAL SATELLITE COMMUNICATIONS LINK INCLUDING CROSSLINK

- Analytic solution for high altitude lunar perturbations, which has allowed study of a wide range of orbit parameters because of the resulting programs are less costly to run than previously used numerical integration programs.

This led to a conclusion in a particular case that 12-day retrograde orbits have stability comparable to 10-day posigrade orbits.

- Computer programs, which offer the following extensions in comparison to other programs known to us:

    1. Unified uplink, crosslink, downlink analysis in a single program.

    2. Programs which handle high eccentricity orbits efficiently.

    3. Allowance for incorporation of vector antenna radiation patterns for precise received power at the ground terminals.

    4. Low central processing unit (CPU) time requirements to run the programs. For example, less than 30 seconds CPU time (IBM 370/158) is required for uplinks, crosslinks, and downlinks for two ground stations and 10 satellites.

- Coverage of multiple satellites.

Part I of the report includes a simple two-body analysis (satellite, earth), which is a good basis for the communications engineer in designing orbital links of 3000 to 20,000 nautical miles (nmi) altitude. General Keplerian orbits are analyzed with the aid of a notably efficient iterative solution to Kepler's equation.

Part II considers orbital altitudes between 20,000 to 250,000 nmi. Analysis of higher eccentricity orbits was achieved through use of a Taylor series expansion of eccentric anomaly, which allows both analysis of a higher eccentricity and lower CPU time than standard methods.

Analysis of high altitude orbits had been previously carried out using a numerical integration program.[1] For altitudes greater than approximately 20,000 nmi, lunar and solar perturbations significantly disturb the satellite orbit.* Results had shown that retrograde equatorial satellite orbits are more stable than posigrade orbits. A new approach was adopted of combining perturbed orbital elements to provide an analytic solution to orbit perturbation and resulting stability.

Derivation of the rate of change of semi-major axis has provided physical insight into the mechanism which causes satellites moving retrograde with respect to the moon's motion to be more stable than those moving posigrade. This result had been inexplicable previously. As noted above, the low running cost of the program has allowed more extensive analysis of orbits for stability. As in Part I, the uplink, crosslink, and downlink analysis is reduced to a program.

Examples of the use of Part I and II programs are given in Part III. Computer programs are listed and described in Appendices 1 through 9. The table of contents identifies the author of each program. A portion of the above programs was used to generate a succinct program for coverage in a multiple satellite system (Example 3 in Part III and Appendix 9).

The programs are intended for actual communications link analysis. The positions of all stations and satellites are retained in vector form in the programs so that antenna gain patterns can be added later for specific link calculations. It is expected that these programs will be useful in high altitude satellite communications planning. In addition, the low CPU time of the attached programs may allow a spacecraft with limited processing capability to autonomously calculate its own available communications links.

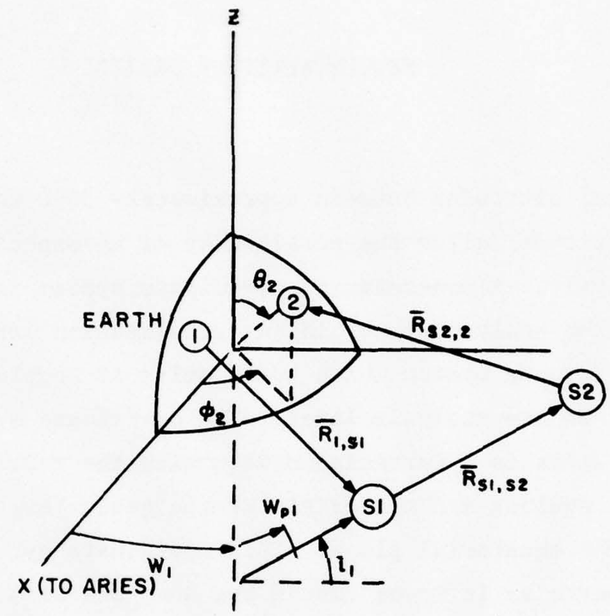
PART I

MEDIUM ALTITUDE ORBITS

Orbital altitudes between approximately 3000 to 20,000 nmi (medium altitude) allow the possibility of an especially simple orbit analysis. A non-rotating coordinate system is chosen as a basis for the analysis to avoid any acceleration problems inherent in the coordinate system which would arise if Doppler rates were added to the analysis later. The coordinate system is shown in Figure 2; it is a Cartesian system with the x axis pointing to the vernal equinox and the origin at the geocenter. The x - y plane is the equatorial plane. This coordinate system translates with the earth as it moves around the sun, but does not rotate. Aries (the direction of the x-axis and the vernal equinox) is so far away that movements of the earth around the sun cause insignificant changes in direction of the axes.

In Figure 2, note that all four sites are moving. The ground stations are rotating with the earth at 15°/hr, and the motion of the satellites is determined by Kepler's laws.

A convenient starting point for an orbital analysis (time = T = 0 hrs) can occur when earth coordinates equal celestial coordinates. This occurs at 12 noon Greenwich mean time (GMT) on March 21. Immediately after, the inertial longitude of an earth station will be greater than the earth coordinates.



ONE WAY TO LINK GROUND STATIONS 1 AND 2 IS SHOWN. THE COORDINATE SYSTEM IS STATIONARY, BUT ALL FOUR SITES ARE MOVING. THE ORBIT OF SATELLITE S1 CAN BE COMPLETELY DESCRIBED FOR KEPLERIAN ORBITS BY

- W_1 = RIGHT ASCENSION, DEGREES
- i_1 = INCLINATION WITH RESPECT TO THE EQUATORIAL PLANE, DEGREES
- W_{P_1} = ARGUMENT OF PERIGEE, DEGREES
- T_{P_1} = TIME OF PERIGEE, HOURS
- a = SEMIMAJOR AXIS, NAUTICAL MILES (OR, IN KM BY MULTIPLYING THE SEMIMAJOR AXIS BY 1.852)
- e = ECCENTRICITY, WHERE $0 \leq e < 1$. AT $e = 0$, THE ORBIT IS CIRCULAR

1A-45,619

Figure 2 GEOMETRY OF A TWO SATELLITE, TWO GROUND STATION SYSTEM

1.1 MOTION OF THE SATELLITE

Only elliptical orbits are considered here, because a circular orbit can be considered a degenerate ellipse. Kepler's laws of planetary motion can be abbreviated as:[2]

1. The orbit of a satellite is an ellipse, with the earth's center (geocenter) at one of the foci:

$$R = \frac{P}{1+e} \cos \theta \qquad (1\text{-}1)$$

The parameters are shown in Figure 3.

2. In the coordinate system of Figure 4 (called the prime system in the remainder of the discussion), the radius vector of each satellite sweeps through equal areas in equal times:

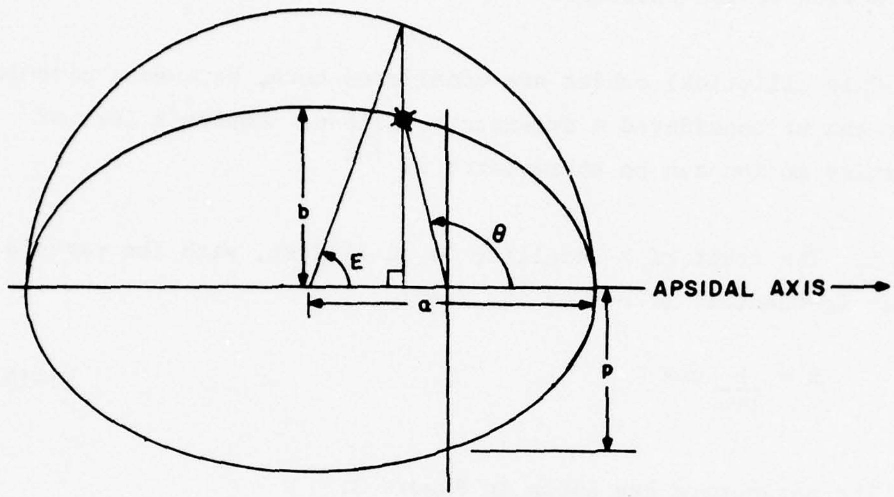
$$R^2 \dot{\theta} = \text{constant} = \sqrt{\mu P} \quad \text{where } \mu = Gm_e \qquad (1\text{-}2)$$

= earth's gravitational constant.

3. The squares of the periods of the satellites are to each other as the cubes of the semi-major axes of their respective orbits:

$$\frac{\tau_1^2}{a_1^3} = \frac{\tau_2^2}{a_2^3} \qquad (1\text{-}3)$$

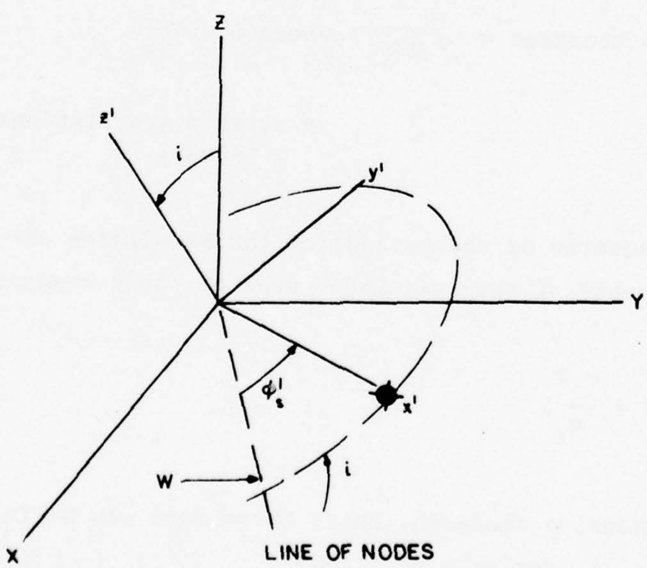
After extensive analysis, these three laws can be interpreted as a single equation (Kepler's equation):



1A-45,620

THE COORDINATE SYSTEM IS AT ONE FOCUS (GEOCENTER) OF THE ELLIPTICAL ORBIT

Figure 3 RELATION OF TRUE ANOMALY θ TO ECCENTRIC ANOMALY E



1A-45,622

Figure 4 ROTATIONS THROUGH THREE EULER ANGLES AND THEIR RELATION TO THE SATELLITE ON THE X' AXIS

$$M = E - e \sin E \qquad (1\text{-}4)$$

where M = mean eccentric anomaly = $n(t-tp)$

n = mean angular rate

t = time, hrs

tp = time at perigee

e = eccentricity

and E = eccentric anomaly.

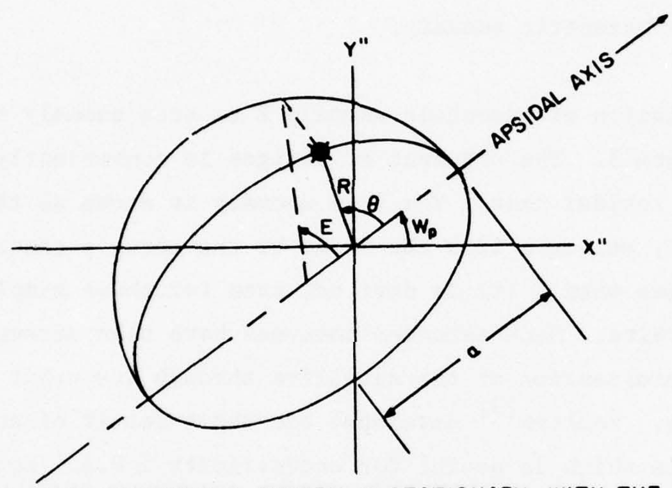
The relation of eccentric anomaly E to true anomaly θ can be seen in Figure 5. The argument of perigee is conveniently located here on the apsidal axis. The true anomaly is shown as the angle from perigee, measured from the focus at the earth's center. Serious problems arise when $\theta(t)$ is desired, even for these simple Keplerian orbits. Many extended analyses have been attempted to describe the progression of the satellite through its orbit as a function of time. Moulton[3] developed one short result of an important analysis which is useful for eccentricity $\lesssim 0.5$. Eccentricities greater than 0.5 were desired for this analysis and for most of these programs, however.

A direct attack on Kepler's equation (Equation (1-4) has been employed. Although the Kepler equation is transcendental, an initial guess at eccentric anomaly

$$E_1 \doteq M + e \sin M \qquad (1\text{-}5)$$

enables one to take a second, much more accurate estimate,

$$E_2 \doteq \frac{M + e\,(\sin E_1 - (e \cos E_1)\,E_1}{1 - e \cos (E_1)} \qquad (1\text{-}6)$$



1A-45,623

THE DOUBLE PRIME COORDINATE SYSTEM IS STATIONARY, WITH THE X" AXIS ON THE LINE OF NODES (EQUATORIAL PLANE AND ORBITAL PLANE)

θ = TRUE ANOMALY

R = GEOCENTRIC DISTANCE OF SATELLITE

W_p = ARGUMENT OF PERIGEE

E = ECCENTRIC ANOMALY

$\phi_s' = W + \theta$

Figure 5 MOTION IN THE ORBITAL PLANE

This estimate for E_2 is the result of linearizing Kepler's equation by means of a Taylor series expansion. All terms beyond the first power in E_1 are neglected. More estimates for eccentric anomaly can be similarly made (e.g., E_3 as a function of E_2). When differences of successive approximations are adequately small, E_3 can be accepted as the solution for E. The true anomaly θ is related[2] to the eccentric anomaly by

$$\theta = \cos^{-1} \left(\frac{\cos E - e}{1 - e \cos E} \right) \tag{1-7}$$

This true anomaly will be needed to accurately determine the time-varying position of the satellite on its prescribed ellipse. The range from the geocenter is given as

$$R = \frac{a\,(1 - e^2)}{1 + e \cos \theta} \tag{1-8}$$

These relations (Equations (1-5) to (1-8)) are implemented in subroutine ELLIP of the computer programs. The iterations for E are surprisingly fast; representatively, they are as fast as Moulton's esoteric development, which was meant expressly to free the early twentieth century analyst from laborious computations. The advantage over Moulton's development is that the programs can handle eccentricity from 0 to 0.99. The subroutine ELLIP can be forced into seven iterations at $e = 0.99$. This iterative technique requires less CPU time than a standard procedure.[4] In the subroutine, M becomes

$$z = \frac{2\pi(t-tp)}{T} = \text{mean angular rate, radians.}$$

Also, ϕ'_s = Wp + θ since θ is measured from the argument of perigee, but ϕ'_s is measured from the line of nodes where the orbital plane intersects the equatorial plane.

1.2 COORDINATE TRANSFORMATIONS

In the previous section, it was found convenient to describe angular motion (true anomaly θ) in the plane of the satellite. However, after a position (or a velocity) in the satellite plane has been found, it should be converted back into an inertial coordinate system, which is required for correct interstation vectors.

Fortunately, a coordinate transformation that compares directly to the needs here has been extensively analyzed.[5] This transformation makes use of the Euler angles. The first Euler angle occurs with a rotation about the z axis. See Figure 6, where the nomenclature of Goldstein is used. This first rotation through W will correspond to right ascension for this analysis.

The second rotation occurs about ξ; again, this rotation is counterclockwise. This rotation is an angle i, where i here corresponds to orbital inclination. Finally, a rotation (ϕ'_s) in the orbital plane can occur which is measured from the line of nodes. See Figure 4.

The three rotations define a new (PRIME) coordinate system (x',y',z'). For the purpose of this analysis, the x' axis is a radius vector from the geocenter to the satellite.

The transformation of a vector $\underline{x}'$ in the prime coordinate system to the inertial system can be expressed in matrix form as

$$\underline{x}' = \underline{A}\,\underline{x}$$

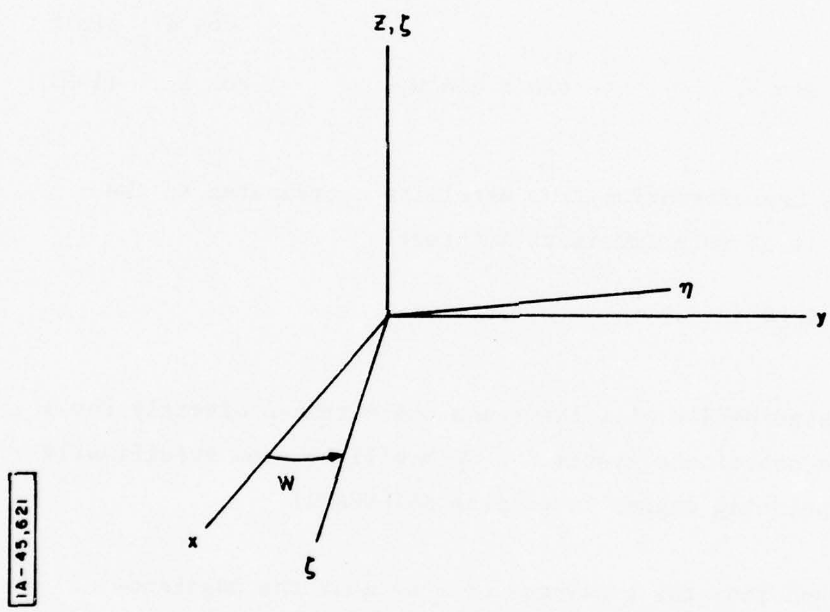


Figure 6 THE FIRST ROTATION OF EULER ANGLE W ABOUT THE Z AXIS

and $\underline{A}$ is the product of three separate matrices, $\underline{B}$, $\underline{C}$, and $\underline{D}$. Omit Goldstein's intermediate steps,

$$\underline{A} = \begin{pmatrix} \cos\phi'_s \cos W - \cos i \sin W \sin\phi'_s, & \cos\phi'_s \sin W + \cos i \cos W \sin\phi'_s, & + \sin W \sin i \\ -\sin\phi'_s \cos W - \cos i \sin W \cos\phi'_s & -\sin\phi'_s \sin W + \cos i \cos W \cos\phi'_s, & \cos\phi'_s \sin i \\ \sin i \sin W, & - \sin i \cos W, & \cos i \end{pmatrix} \tag{1-9}$$

and the inverse transformation from satellite coordinates to the inertial frame is of more immediate interest:

$$\underline{x} = A^{-1} x^1$$

(A subroutine UNPRIM will later use the matrix $\underline{A}$ directly for a local satellite coordinate system. UNPRIM will be used specifically for crosslink pointing angles in program SATLUNAE.)

$\underline{A^{-1}}$ is found from the transpose of $\underline{A}$ because the magnitude of $\underline{A}$ is unity. The elements of $\underline{A^{-1}}$ are then

$$a_{11} = \cos\phi'_s \cos W - \cos i \sin W \sin\phi'_s$$

$$a_{12} = -\sin\phi'_s \cos W - \cos i \sin W \cos\phi'_s$$

$$a_{13} = \sin i \sin W$$

$$a_{21} = \cos\phi'_s \sin W + \cos i \cos W \sin\phi'_s$$

$$a_{22} = -\sin \phi'_s \sin W + \cos i \cos W \cos \phi'_s$$

$$a_{23} = -\sin i \cos W$$

$$a_{31} = \sin i \sin \phi'_s$$

$$a_{32} = \sin i \cos \phi'_s$$

$$a_{33} = \cos i \qquad (1\text{-}10)$$

The subscripts refer to row and column.

All of these elements except a_{13}, a_{23} and a_{33} are inserted into the attached subroutine PRIME. They are omitted because there is no z component for satellite coordinate.

An example of the calling of subroutine PRIME can be useful. If the position of Satellite #8 in its own coordinate system is specified by x' = R8, y' = 0., z' = 0., this information must be sent to PRIME along with the three angles specifying the coordinate transformation. The angle of the satellite from the line of nodes is FSP, the right ascension is W8, and the inclination is x18 for the purpose of this example. The attached programs call PRIME in the following way.

```
CALL PRIME (FSP, W8, x18, R8, 0.,  xs, ys, zs).
                 input variables   output variables
```

The output variables xs, ys, and zs represent position in inertial space. The position will be useful in getting crosslink and downlink vectors.

A slightly different form of coordinate conversion will be required for range rate. When it is discussed later, it should not be confused with the prime coordinate system in which the x' axis is pointed through the satellite. It will use a stationary x" axis pointed along the line of nodes.

1.3 DOPPLER ANALYSIS

In a central force field such as that assumed for Keplerian orbits, the velocity of a satellite is dependent only on the magnitude of its radius vector once its initial conditions have been determined. Similarly, it is also determined by θ (its angle measured from perigee) since $R(\theta)$ is determined by the Keplerian ellipse. Expressions directly related to velocity[2] are

$$\dot{R} = \frac{(2\pi A\ e\ )}{\tau\ \sqrt{1-e^2}}\ \sin\theta \qquad (1\text{-}11)$$

and

$$\dot{\theta} = \frac{2\pi\ (1 + e\cos\theta)^2}{\tau\ (1-e^2)^{3/2}} \qquad (1\text{-}12)$$

The geometry is given by Figure 6,

where $\phi'_s = \theta + W_p$, W_p = argument of perigee

and $(\dot{\phi}'_s) = \dot{\theta}$.

The x" axis is stationary and lies in the equatorial plane.

Further, the derivatives of the position vector (with the x" axis now lying along the line of nodes and the y" axis 90° ahead in the orbit plane) can give the x" component of velocity and the y" component of velocity. The symbol ϕ'_s is later changed to FSP so it can be more directly related to the FORTRAN programs.

$$x'' \text{ component} = \left(\frac{1}{3600}\right)\left(\dot{R} \cos (\phi'_s) - R\,\dot{\theta} \sin (\phi'_s)\right) \text{ nmi/sec} \quad (1\text{-}13)$$

and

$$y'' \text{ component} = \left(\frac{1}{3600}\right)\left(\dot{R} \sin (\phi'_s) + R\,\dot{\theta} \cos (\phi'_s)\right) \text{ nmi/sec} \quad (1\text{-}14)$$

To convert these velocity components to components in the inertial system, the general coodinate conversion (PRIME) can be called, but with $\phi'_s = 0$, because the x" axis does not rotate with the satellite in this case. Velocity components are found in the subroutine DOPE of the attached programs. This stationary x" axis was not required for range rate but will be desirable if $\ddot{R}$ is analyzed.

The velocities of the earth stations are straightforward.

$$\dot{x}_1 = \frac{-\omega}{3600} R_e \sin \theta_1 \sin \phi_1$$

$$\dot{y}_1 = \frac{\omega}{3600} R_e \sin \theta_1 \cos \phi_1$$

where

$$\omega = \frac{15°/\text{hr}}{57.296} \text{ earth rotation rate, rad/hr *}$$

*Note that the distinction between solar hours and sidereal hours is not retained here because only three place accuracy is desired in the answers.

$\dot{z}_1 = 0$ because rotation is about the z axis.

The background for velocitites of satellites and earth stations is now concluded. The relation of velocities to uplink and crosslink Doppler shift is of more interest to the communications engineer than velocities per se.

The relation of a Doppler shifted frequency to the transmitted frequency is shown by Jackson[6] to be

$$F' = \frac{F}{\sqrt{1 - \frac{v_R^2}{c^2}}} \quad (1 - \frac{v_R}{c} \cos \theta_j) \tag{1-15a}$$

Where v_R is relative velocity between transmitter and receiver and θ_j is the angle between the relative velocity vector and the pointing vector between transmitter and receiver.

This expression includes the (relativistic) transverse Doppler shift which may be of interest for crosslink Doppler. For $v_R \ll c$, Equation (1-15a) becomes

$$F' = F \left(1 + 1/2 \frac{v_R^2}{c^2}\right) \left(1 - \frac{v_R}{c} \cos \theta_j\right) \tag{1-15b}$$

or

$$\Delta F = \text{Doppler shift} = F \left(1/2 \frac{v_R^2}{c^2}\right) - \frac{F}{c} (v_R \cos \theta_j) \tag{1-15c}$$

The two components are called transverse Doppler shift and ordinary Doppler shift, respectively. At synchronous altitude, the transverse Doppler shift can be of the order of (first term of Equation (1-15c))

$$(35 \times 10^9 \text{ Hz}) \left(1/2\left(\frac{1.655 \text{ nmi/sec}}{1.6198 \times 10^5 \text{ nmi/sec}}\right)^2\right) = 1.82 \text{ Hz at K band.}$$

This is so small, even with a millimeter wave carrier, that tranverse Doppler is dropped for the remainder of the discussion and in the computer programs (Appendices 1-9).

The ordinary Doppler shift is

$$F = - \frac{F}{C} (v_R \cos \theta_j)$$

In order to get ($\cos \theta_j$), the relative velocity vector $\bar{v}$ is found by taking differences of velocity components in the inertial frame:

$$\bar{v}_R = (\dot{x}_2 - \dot{x}_1)\ \bar{i} + (\dot{y}_2 - \dot{y}_1)\ \bar{j} + (\dot{z}_2 - \dot{z}_1)\ \bar{k}$$

By vector analysis,

$$v_r (\cos \theta_j) = \bar{v}_R \cdot \overline{P_{12}}$$

where

$\overline{P_{12}}$ = unit vector which points from transmitter to receiver.

$$= \frac{(x_2 - x_1)\,\bar{i} + (y_2 - y_1)\,\bar{j} + (z_2 - z_1)\,\bar{k}}{R_{12}}$$

so,

$$V_R\ (\cos\theta_j) = \bar{v}_R \cdot \bar{P}_{12} = \frac{(\dot{x}_2-\dot{x}_1)\ (x_2-x_1) + (\dot{y}_2-\dot{y}_1)\ (y_2-y_1) + (\dot{z}_2-\dot{z}_1)\,(z_2-z_1)}{R_{12}}$$

(1-16)

and the one-way Doppler shift can be found by substituting this result, Equation (1-16), into

$$\Delta F = \frac{F}{C}\,(v_R \cos\theta_j).$$

This calculation is done in the attached programs after the subroutine DOPE is called.

1.4 ELEVATION ANGLE

The elevation angle for the ground station antenna is important to the communications engineer in many ways: (1) it serves as a check for satellite visibility; (2) it can give an estimate of mean atmospheric attenuation of signal strength and (3) its derivative is useful in an estimate of antenna slewing rate. Only the elevation angle is treated here, and not its derivative.

Elevation angle is found by noticing the following geometry, which is in the plane of the ground station position vector and the satellite position vector (Figure 7).

The geocentric angle A_{13} can be found by taking the dot product $\bar{R}_1 \cdot \bar{R}_{1S}$:

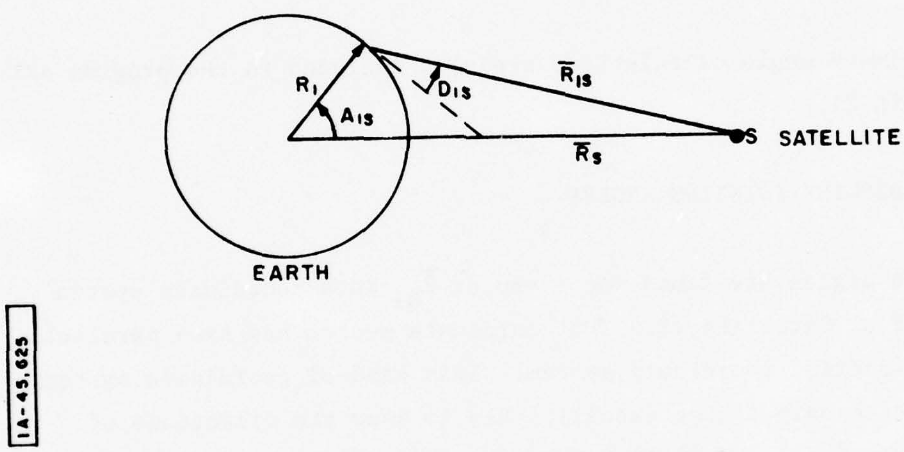


Figure 7 ELEVATION ANGLE D_{IS}

$$A_{1S} = \cos^{-1}\left(\frac{(x_1 x_s + y_1 y_s + z_1 z_s)}{R_e\ R_s}\right) \quad (1\text{-}17)$$

with the aid of the law of sines, the elevation angle

$$D_{1S} = \sin^{-1}\left((\sin A_{1S})\ \frac{R_s}{R_{1S}} - \pi/2\right) \quad (1\text{-}18)$$

is found. A sign check for D_{1S} is still necessary because the obtuse angle ($D_{1S} + \pi/2$) in Figure 7 is read as ($\pi/2 - D_1$) by the computer. It cannot distinguish between sines of the first and second quadrants.

Azimuth angle calculations are also included in the program AZ1 (Appendix 8).

1.5 CROSSLINK POINTING ANGLES

The angles are found for a vector $\bar{R}_{89}$ in a coordinate system centered at Satellite #8. The coordinate system has axes parallel to the inertial coordinate system. This kind of coordinate system makes sense only if the satellite has to know the directions of the inertial axes anyway. Otherwise, the coordinate system is a local satellite coordinate system (shown as angles TU12 and FU12 of the program SATLUNAE in Appendix 6).

The angles are found in terms of local longitude and colatitude.

$$\text{Colatitude} \quad \theta_{S1,S2} = \cos^{-1}\left(\frac{z_{S2} - z_{S1}}{R_{S1,\ S2}}\right) \quad (1\text{-}19)$$

$$\text{Longitude} \quad \phi_{S1,S2} = \tan^{-1}\left(\frac{y_{S2} - y_{S1}}{x_{S2} - x_{S1}}\right) \quad (1\text{-}20)$$

These angles are found near the end of the main part of the programs. Again, some sign tests must occur because the computer has trouble with correct quadrants. Another problem, which is merely inconvenient, remains: Longitudes less than -360° sometimes occur. If this is too inconvenient for the user, a two-line logical check can be done as on lines 662 and 664 of the program AZ1 (Appendix 8).

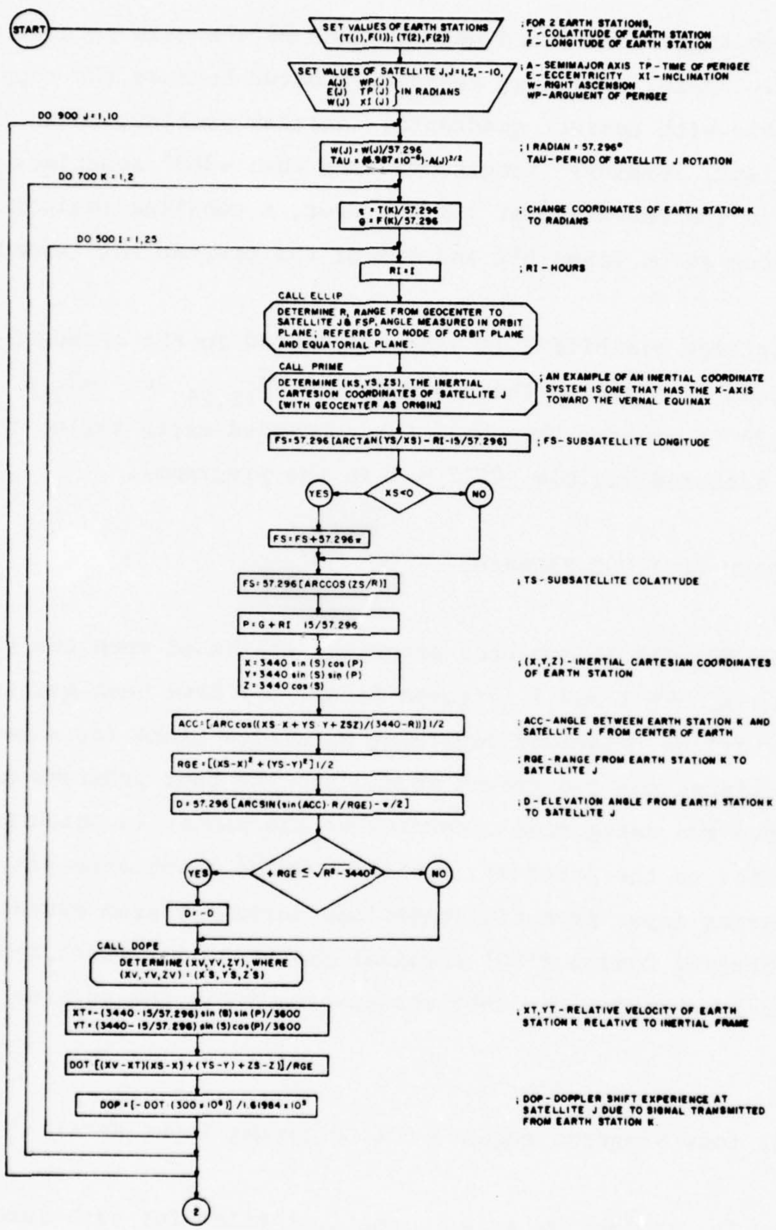
Crosslink visibility is also calculated in the attached programs. It is found by checking the angle between $\bar{R}_{1S,2S}$ and $(-\bar{R}_{1S})$; if this angle is greater than half the subtended earth angle, the cross-link is declared visible (CVIS = 1 in the programs).

1.6 MEDIUM ALTITUDE PROGRAMS

Four FORTRAN IV computer programs, all based upon the flowchart listed on Blocks 1 and 2 (Figures 8a and 8b) have been written to examine various relations regarding satellite links for a maximum of ten satellites and two ground stations. The four programs may be divided into two categories according to the manner in which input data is supplied to the program: those for which input data are supplied by answering input prompting questions during program execution at a Time Sharing Option (TSO) terminal and those for which input data are supplied by changing the data statements within the program before execution.

All four programs require the following input data:

1. The semi-major axis in nautical miles for each satellite.
2. The eccentricity for each satellite.
3. The right ascension in degrees for each satellite.
4. The argument of perigee in degrees for each satellite.
5. The time of perigee in hours for each satellite.



IC-45,726

Figure 8a FLOWCHART, BLOCK I

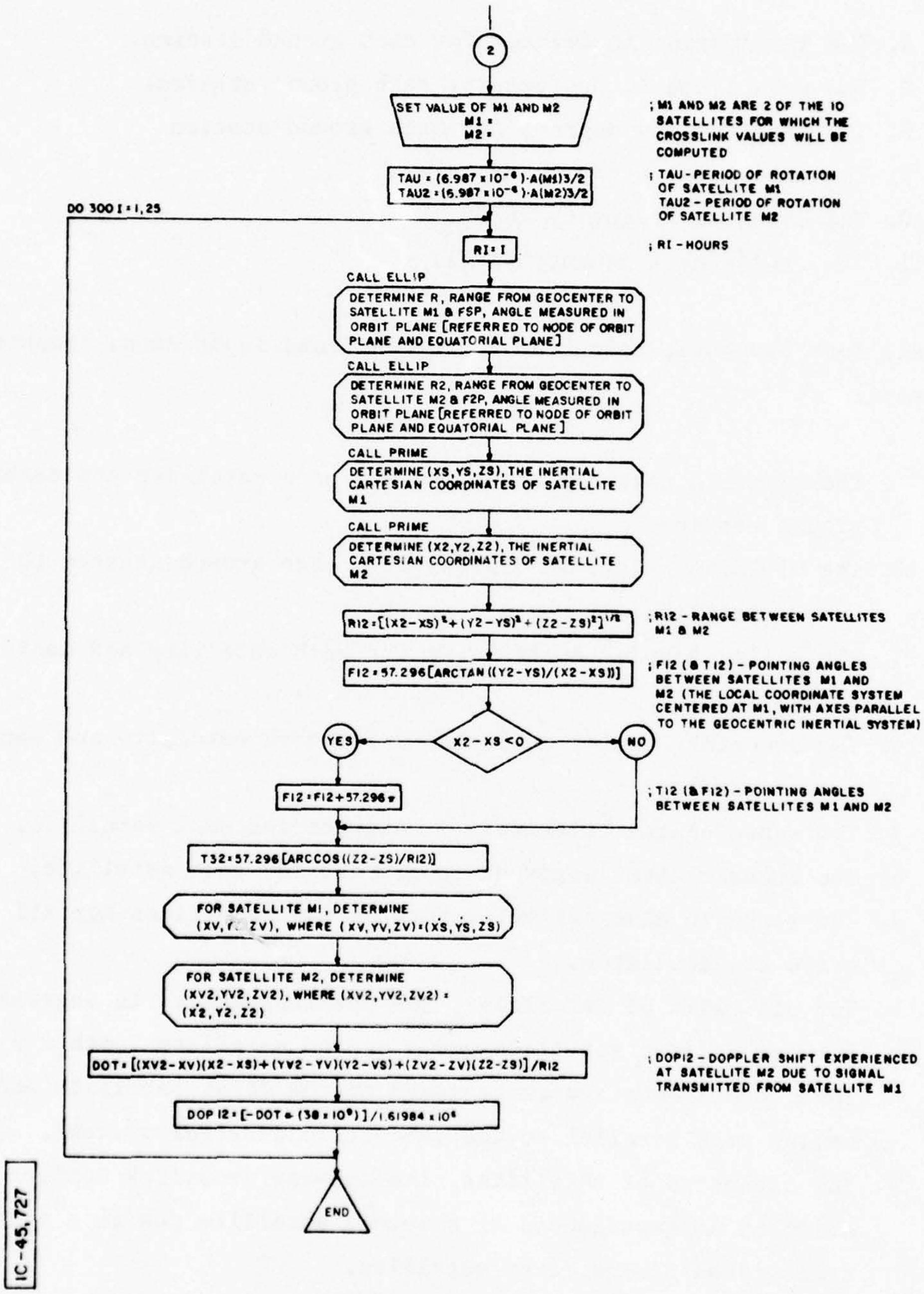


Figure 8b FLOWCHART, BLOCK 2

6. The inclination in degrees for each ground station.
7. The colatitude in degrees for each ground station.
8. The longitude in degrees for each ground station.
9. The uplink frequency in Hz.
10. The downlink frequency in Hz.
11. The crosslink frequency in Hz.

All four programs, using the above-mentioned input data, compute the following:

1. The range in nautical miles between each satellite and each ground station.
2. The elevation angle in degrees from each ground station to each satellite.
3. The uplink Doppler shift in Hz for each satellite and each ground station.
4. The downlink Doppler shift in Hz for each satellite and each ground station.
5. The subsatellite colatitude in degrees for each satellite.
6. The subsatellite longitude in degrees for each satellite.
7. The range in nautical miles between two satellites for all pairs of satellites.
8. For all pairs of satellites, the pointing angles in degrees between a first satellite and a second satellite (with the local coordinate system centered at the first satellite and having axes parallel to the geocentric inertial system).
9. For all pairs of satellites, the one-way crosslink Doppler shift in Hz experienced at a second satellite due to a signal transmitted from a first satellite.
10. The crosslink visibility between two satellites for all pairs of satellites.

1.6.1 SATE (Appendix 1)

The first program listed has been entered on TSO, and may be executed either in the foreground* with the output printed at the terminal or in the background with the output printed on the high speed printer. Input data is entered in this program by changing the data statements within the program before execution.

1.6.2 SATD (Appendix 2)

The second program listed is simply the double precision version of SATE; that is to say, SATD has the same attributes as SATE except for the fact that it has a precision of approximately 16 decimal digits as opposed to the precision of approximately seven decimal digits of SATE. Because of this increased accuracy, SATD has a running time longer than that of SATE.

1.6.3 SATVIZE (Appendix 3)

The third program listed has also been entered on TSO but, unlike the two above-mentioned programs, is on-line foreground executable only with output being printed at the terminal. Also, the necessary input data in this program are entered during program execution merely by answering the input prompting questions supplied by the program. For the casual user, default values and points for stopping various parts of program execution have been incorporated within the program.

*To run this program in the foreground, lines 10 to 40 inclusive must be deleted first.

1.6.4 SATVIZD (Appendix 4)

The fourth program listed is merely the double precision version of SATVIZE. Because of its increased precision, SATVIZD, like SATD, has a running time longer than that of its single precision counterpart.

PART II

HIGH ALTITUDE SATELLITES WITH LUNAR PERTURBATIONS

Conventional three-body (earth-moon-satellite) perturbation analysis can consume great amounts of computer time with concomitant expense. For example, the carefully planned and very extensive Lincoln Laboratory Planetary Ephemeris Program (PEP) can perform numerical integrations of satellite motion, including the perturbations of the satellite orbit by many non-terrestrial bodies. The implicit penalty of this program is its expense. The disadvantages of numerical integration become even more marked for a multiple satellite communications system. For these reasons, and to account for long term lunar perturbations, programs incorporate existing short analytic (i.e., closed-form) results. Dr. M. Ash's elegant and useful analytic results[1] for variation of right ascension, argument of perigee, and eccentricity are combined with a short approximate result for the time rate of change of a semi-major axis which we developed. A changing semi-major axis can account for the difference in stability of posigrade and retrograde orbits.

The programs discussed and listed in this section (Appendices 5-7) are the only ones known to us that can be used to estimate orbital stability with modest computation time, while noting the difference between posigrade and retrograde orbits.

2.1 ANALYSIS

One way to avoid extensive computations for the position of a satellite in a non-Keplerian orbit is to describe the fundamental motion at any time as Keplerian with defined orbital elements. However, these orbital elements can be allowed to change as a function of the perturbing forces. When the Keplerian motion is elliptical, the corresponding orbital elements for perturbed motion are called "osculating" elliptic orbital elements.

With a few changes to the nomenclature of Ash,

a = semi-major axis

e = eccentricity

I = inclination with respect to lunar plane

Ω = right ascension of ascending node on lunar plane

W_p = argument of perigee

M = mean anomaly

μ_m = gravitational constant times lunar mass

μ = gravitational constant times mass of earth

$n = \mu^{1/2} \; a^{-3/2}$ = mean motion, rad/hr

θ = true anomaly

p = semi-latus rectum

$r = \dfrac{p}{1 + e \cos \psi}$ = radius from geocenter

ρ = lunar radius

$\eta = \theta + W_p$

Further, if $\tilde{R}$, $\tilde{S}$, and $\tilde{W}$ are the three orthogonal perturbation force components ($\tilde{R}$ along radius vector from geocenter, $\tilde{S}$ close to the velocity vector, and $\tilde{W}$ completing the right handed coordinate system) for a lunar mass spread into a torus at lunar altitude, Ash derived the relations for $\tilde{R}$, $\tilde{S}$, $\tilde{W}$ given by Equations (2-1), (2-2) and (2-3).

$$\tilde{R} = \frac{\mu_m}{\rho^2} \sum_{\ell = 1}^{\infty} 2\ell \left(\frac{r}{\rho}\right)^{2\ell-1} \sum_{k = 0}^{\ell} (-1)^{k+\ell} \frac{1\cdot3\cdot5 \cdots\cdots (2k + 2\ell - 1)}{(\ell-k)!\ 2^{\ell+k}\ k!}$$

$$\cdot \sum_{m = 0}^{k} \frac{(\cos I)^{2k-2m}}{m!\ (k-m)!} (\cos \eta)^{2m} (\sin \eta)^{2k-2m} \qquad (2\text{-}1)$$

$$\tilde{S} = \frac{\mu_m}{\rho^2} \sum_{\ell = 1}^{\infty} \left(\frac{r}{\rho}\right)^{2\ell-1} \sum_{k = 1}^{\ell} (-1)^{k+\ell} \frac{1\cdot3\cdot5 \cdots\cdots (2k + 2\ell -1)}{(\ell-k)!\ 2^{\ell+k} k!}$$

$$\cdot \sum_{m = 0}^{k} \frac{(\cos I)^{2k-2m}}{m!(k-m)!} [(2k-2m) (\cos \eta)^{2m+1} (\sin \eta)^{2k-2m-1}$$

$$-2m(\cos \eta)^{2m-1} (\sin \eta)^{2k-2m+1}] \qquad (2\text{-}2)$$

$$\tilde{W} = \frac{-\mu_m \sin I}{\rho^2} \sum_{\ell = 1}^{\infty} \left(\frac{r}{\rho}\right)^{2\ell-1} \sum_{k = 1}^{\ell} (-1)^{k+\ell} \frac{1\cdot3\cdot5 \cdots\cdots (2k + 2\ell -1)}{(\ell-k)!\ 2^{\ell+k}\ k!}$$

$$\cdot \sum_{m = 0}^{k-1} \frac{(2k - 2m)}{m!\ (k-m)!} (\cos I)^{2k-2m-1} (\cos \eta)^{2m} \cdot (\sin \eta)^{2k-2m-1} \qquad (2\text{-}3)$$

When Equations (2-1), (2-2), and (2-3) are substituted into Gauss' form of the equation of the osculating elements and extensive operations are performed, Ash finds the change of right ascension, eccentricity, and argument of perigee. Change in semi-major axis and inclination were deemed negligible.

The changes in right ascension, eccentricity, and argument of perigee per orbit were found to be:

$$\Delta W = -2\pi \left(\frac{\mu_m}{\mu}\right) \left(\frac{a}{\rho}\right)^3 \cos I \left\{ \frac{3}{4} + \left(\frac{a}{\rho}\right)^2 \left[-\frac{135}{128} + \frac{315}{128} \cos^2 I \right] + \left(\frac{a}{\rho}\right)^4 \left[\frac{2625}{2048} - \frac{7875}{1024} \cos^2 I + \frac{17325}{2048} \cos^4 I \right] + \cdots \right\} \tag{2-4}$$

$$\Delta e = -\pi \left(\frac{\mu_m}{\mu}\right) \left(\frac{a}{\rho}\right)^3 e \sin(2W_p) \left\{ -\frac{15}{4} \sin^2 I + \left(\frac{a}{\rho}\right)^2 \left[\frac{315}{128} - \frac{315}{16} \cos^2 I + \frac{2205}{128} \cos^4 I \right] + \cdots \right\} \tag{2-5}$$

$$\Delta W_p = \pi \left(\frac{\mu_m}{\mu}\right) \left(\frac{a}{\rho}\right)^3 \left\{ 3 - \frac{15}{2} \sin^2 W_p \sin^2 I + \left(\frac{a}{\rho}\right)^2 \left[-\frac{45}{32} + \frac{315}{64} \sin^2 W_p + \left(\frac{225}{32} - \frac{315}{8} \sin^2 W_p\right) \cos^2 I + \frac{2205}{64} \sin^2 W_p \cos^4 I \right] + \cdots \right\} \tag{2-6}$$

The changes represented by Equations (2-4), (2-5) and (2-6) can be very useful for long term (secular) changes in orbits as a function of lunar perturbations. The computational order which should be followed is:

1. ΔW_p should be computed from Equation (2-6) from an initial value of argument of perigee W_o. A new value of W_p is then found from $(W_o + W_p)$. Sometimes, however, ΔW_p is identically zero and a stable argument of perigee exists; this can happen at high inclination angle (XI) and large semi-major axis (a). Program PERTP, Appendix 5, calculates the stability of argument of perigee in a subroutine ARGPER.

2. W_p is then substituted into Equation (2-5) to find Δe per orbit. This is done in PERTP on line 630 (App. 5).

3. The change of right ascension is calculated from Equation (2-4). This is done in PERTP on line 620 (App. 5).

PERTP is also more comprehensive; this will be shown in the following estimates of change in semi-major axis.

2.2 ESTIMATED CHANGE IN SEMI-MAJOR AXIS

A surprising difference in long term stability of posigrade compared to retrograde orbits was identified by Ash[2]. For example, 12-day retrograde orbits appear approximately as stable as 10-day posigrade orbits over a two-year interval. This difference in stability does not appear in Equations (2-1), (2-5), and (2-6) after the development of perturbations due to a torus of lunar matter.

Clearly, at least one major physical concern has been deleted from the development for Equations (2-4), (2-5), and (2-6). This is the interaction time between the satellite and the moon, which would show a difference between posigrade and retrograde orbits. Figure 9 gives a planar diagram of lunar perturbation.

If the satellite is in a circular orbit about the earth, it becomes convenient to consider the tangential impulse and the radial impulse imparted to the satellite from the moon. By symmetry, it can be seen that the satellite loses about as much tangential impulse as it passes the moon as when it approaches. However, the impulse along the radial direction is always positive.

$$\text{Radial force per unit mass} = F_r = \frac{GM_m}{r_{ij}^2} (\cos \theta_m) \qquad \text{(2-7a)}$$

where

r_{ij} = distance between satellite and lunacenter

G = gravitational constant

$$\theta_m = \theta_e \cdot \frac{R_s}{(R_m - R_s)} \quad \text{for } \theta_e \gtrsim 0.15 \text{ rad} \qquad \text{(2-7b)}$$

M_m = lunar mass

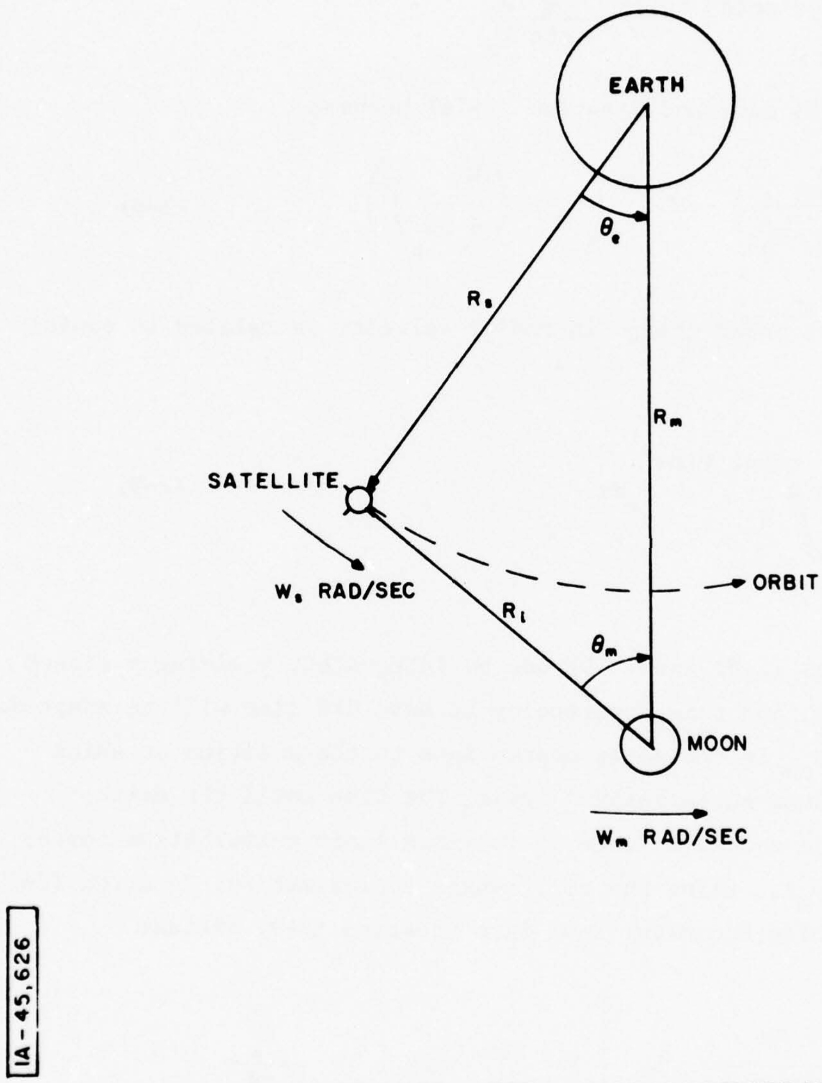


Figure 9 GEOMETRY FOR LUNAR PERTURBATIONS

Further,

r_{ij} can be noted to be $\dfrac{R_m - R_s}{\cos\theta_m}$

for $\theta_e \gtrsim 0.15$ rad, and Equation (2-7a) becomes

$$F_r \doteq \frac{GM_m}{(R_m - R_s)^2} \cos^3 \left(\theta_e \cdot \left(\frac{R_s}{R_m - R_s}\right)\right) . \qquad (2\text{-}8)$$

The first order change in radial velocity is related to radial impulse by

$$\Delta V_r = \int_o^{\text{total time}} F_r dt \qquad (2\text{-}9)$$

Equations (2-8) and (2-9) can be integrated, yielding a closed form solution, and this opportunity to save CPU time will be adopted. If an angle θ_{eo} is chosen as appropriate to the position at which noticeable lunar perturbations begin, the time until the earth, satellite, and moon are lined up (maximum lunar perturbation force) is $\theta_{eo}(\omega_s - \omega_m)$. Using the small angle approximation, Equation (2-7b), and substituting Equation (2-8) into Equation (2-9) yields:

$$\Delta V_r = \frac{1}{\omega_{rel}} \cdot \frac{2\ GM_m}{(R_m - R_s)} R_s \left[1/3 \sin\left(\theta_{eo} \cdot \frac{R_s}{R_m - R_s}\right) \cdot \left\{ \cos^2\left(\theta_{eo} \cdot \frac{R_s}{R_m - R_s}\right) + 2 \right\} \right] \qquad (2\text{-}10)$$

where ω_{rel} = relative angular velocity of satellite and moon. This is shown explicitly later in Equation (2-16).

Equation (2-10) has been implemented in subroutine MOON of PERTP (Appendix 5). The limit of integration, θ_{eo}, is entered as TE = 0.3 radians on line 1210. This is felt to give an underestimate of lunar perturbations, because a tidal displacement actually occurs during the radial impulse. The radial velocity of Equation (2-10) must be related to orbital energy, and hence to semi-major axis (a) as soon as the satellite leaves the influence of the moon.

The <u>vis viva</u> integral is the relation between orbital energy and semi-major axis for a simple central force field and has the form

$$\frac{V^2}{\mu} - \frac{2}{R} = -\frac{1}{a} \text{ for elliptic motion} \tag{2-11}$$

where

V = total velocity

$\mu = Gm_e$

m_e = mass of earth

R = geocentric distance.

When Equation (2-11) is rewritten

$$\frac{V^2}{2} - \frac{\mu}{r} = \frac{\mu}{2}\left(-\frac{1}{a}\right) ,$$

the left side is recognized as total orbital energy, or

$$-\frac{2}{\mu}\ (\text{total energy}) = \frac{1}{a}$$

After rearranging Equation (2-13) and differentiating the expression of (a), a function of (total energy), one finds

$$da = \left(\frac{\mu}{2}\right) \frac{d\ (\text{total energy})}{(\text{total energy})} \quad . \qquad (2\text{-}14)$$

Changing Equation (2-14) to an incremental form and substituting $\frac{\Delta V_r^{\ 2}}{2}$ for change in total energy gives

$$\Delta a = \frac{(\Delta V_r)^2\ a^2}{Gm_e} \quad . \qquad (2\text{-}15)$$

Since ΔV_r is proportional to interaction time, the square of ΔV_r in Equation (2-15) implies that the instability of the semi-major axis increases in a nonlinear way with the lunar interaction time. The actual interaction time depends on a vector difference of angular velocities, $(\omega_s - \omega_m)$. The relative angular velocity can be approximated by

$$\omega_{rel} = (\omega_s \cos I - \omega_m)^2 + (\omega_s \sin I)^2 \qquad (2\text{-}16)$$

where again I is the inclination angle of the satellite with respect to the lunar plane.

The remaining considerations which have gone into the subroutine MOON are:

1. The total region of possible interaction of the satellite with the moon is approximately limited (to ANG) if the inclination angle is large and,

2. Change of (a) per year is proportional to the number of times per year in which interaction happens, or $\Delta a/_{yr}$ $\alpha \omega_{rel}$.

Even with the second item included for completeness, however, a distinct difference between the stabilities of posigrade and retrograde orbits is seen in only a few seconds of CPU time. When the effects of subroutine MOON are included in PERTP and a variable to represent instability

$$V_5 = a\,(1 + e)/a_o(1 + e_o) \qquad (2\text{-}17)$$

is generated to represent normalized apogee distance, a 10-day orbit at inclination 0° has approximately the same value for V_5 (1.003) as a 12-day orbit at inclination 180°.

It appears that the gross secular results for orbital instability[1] have been qualitatively reproduced with very modest CPU requirements.

The reader will notice that a running change in units has occurred in these programs. At the time these lunar perturbation programs were generated, a new Federal interest was expressed in metric units. Semi-major axis (a) is expressed in the program of Example 2 and Appendices 5-7 in kilometers. The relation between nautical miles and kilometers is:

1.852 x (length in nautical miles) = length in kilometers.

2.3 PROGRAM PERTP (Appendix 5)

Program PERTP calculates a new semi-major axis (a, km), eccentricity (E), right ascension (W), and normalized apogee distance V_5. It does this from estimated lunar secular perturbations. Iterations in orbital elements are performed every 10 days for a total of 400 days.

CALCOMP plots are generated (e.g., V_5 vs time in days).

This program gives an estimate of orbital stability implicit in the variable V_5. When V_5 increases to approximately 1.01 in 400 days, the orbit's stability is very questionable. This stability has qualitative agreement with Ash's numerical results.

2.4 PROGRAM SATLUNAE (Appendix 6)

Program SATLUNAE combines a variation of SATE and a subroutine LUNA. To save CPU time for orbits which are to be perturbed by the moon for a long time (say, T > 3 years), integrated versions of Equations (2-4), (2-5), and (2-6) were prepared for subroutine LUNA. The equation for argument of perigee offered two choices: either (1) it kept rotating, or (2) it was assigned a stationary value immediately as in the ARGPER subroutine of PERTP. If it kept rotating, an average value of angular velocity was assigned by using $\sin_2 \omega = 1/2$.

The order of calculations in LUNA was as follows:

1. A linear estimate of growth in semi-major axis (a) was done, so that $\bar{A}$ (AB in the program) could be used for later calculations.

2. Argument of perigee was estimated as either stationary or rotating, which gave the possibility of two integrals for the eccentricity calculation.

3. If a stationary ω_p existed, a simple exponential growth in eccentricity resulted. This is similar to the result of the 1974 Ash report[2]. A less simple result occurred (on line 3020) if ω_p kept rotating. Lines 3020 and 3030 are deceptive in their simplicity, and to see the complexity one should substitute the various c's. Many of the c's are not constants, but are functions of the orbital elements.

Right ascension and normalized apogee distance are finally estimated. The results show a satisfying but not exact comparison to an iterated (not integrated) change in orbital elements. A program which iterates the orbital elements appears as SCOREE (Appendix 7).

PART III

EXAMPLES

3.1 EXAMPLE 1

Figure 10 gives an illustration of the 10 satellites chosen for this example; they are a running example throughout this section. These 10 satellites are examined here with program SATE. They appear again in Example 2, where lunar perturbations are examined, and in a coverage problem of Example 3. The 10 satellites give an idea of the capability of these programs and are not intended as a specifically useful satellite communications system. Satellites #1 and #2 are Aerospace Type A[7] satellites selected to give polar coverage. These 12-hour orbits differ in right ascension (W) by 90° in order to give the same ground trace. Satellites #3, #4, #5, and #6 are synchronous equatorial satellites. Satellites #7, #8, #9, and #10 are in 4-day polar orbits. The orbital elements are entered in program SATE on lines 160 to 240 (Table 1). The ground station coordinates are entered on line 250; Lexington, Massachusetts and San Diego, California, are entered as ground stations 1 and 2.

Lines 160 and 170 list the semi-major axes of the satellites in nautical miles, in order of increasing altitude. Line 180 gives the respective eccentricities. Lines 190 and 200 give the right ascensions in degrees. Line 210 gives arguments of perigee in degrees. Lines 220 and 230 give the time at perigee in hours, and line 240 gives orbital inclination in degrees. Line 250 gives colatitudes of Lexington, Massachusetts, and San Diego, California; then, the longitudes of the same sites.

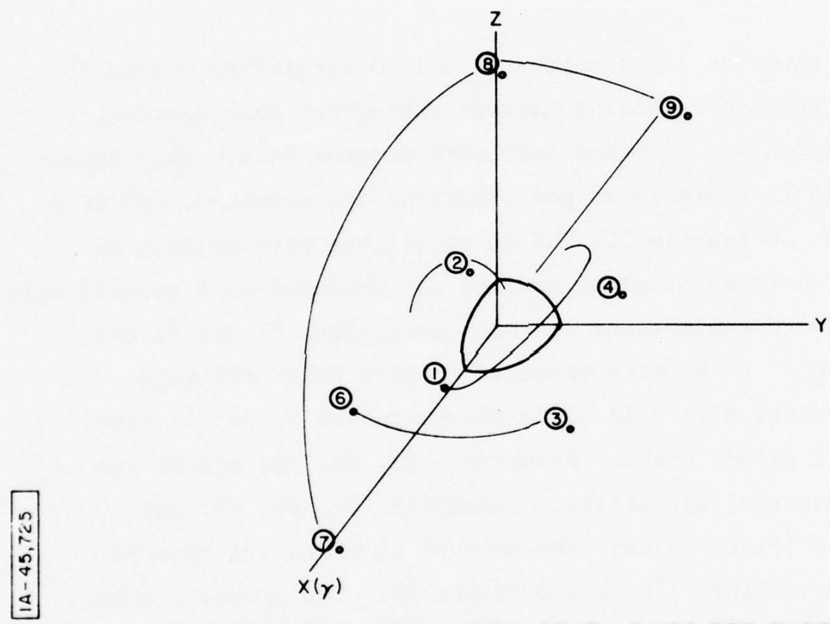


Figure 10 ARRAY OF SATELLITES AT T = 0 HRS FOR THREE EXAMPLES. TEN SATELLITES TOTAL. (NOT TO SCALE; 5 AND 10 NOT SHOWN)

TABLE 1

ORBITAL ELEMENTS ENTERED IN SATE

```
      Dimension A(10), E(10), W(10), WP(10), TP(10), XI(10), B(4)
      Data A/14342., 14342., 22767., 22767., 22767., 22767.,
     3 57369.2, 57369.2, 57369.2, 56369.2/
      Data E/.725, .725, 0., 0., 0., 0., 0., 0., 0., 0./
      Data W/o., 270., 0., 0., 0., 0.,
     4 0., 0., 0., 0./
      Data WP/-90., -90., 0., 0., 0., 0., 0., 0., 0., 0./
      Data TP/0., -6., -3., -9., -15., -21.,
     1 0., -24., -48., -72./
      Data xI/63.435, 63.435, 0., 0., 0., 0., 90., 90., 90., 90./
      Data B/47.54, 57.23, 288.73, 242.8/
```

As an important aside, it should be noted that the communications designer does not really care as much about orbital elements as he does about the subsatellite traces which he will require in order to get adequate satellite visibility. The designer can tell that Satellite #1 was at -90° longitude (90° West longitude) at T = 0 hrs because TP = 0, WP = -90°, and W = 0. The subsatellite "starting point" of a satellite with TP ≠ 0 requires some calculation. Satellite #2, with TP = -6 hrs, was at perigee 6 hours before the computation began. It is therefore halfway through its 12-hour orbit at T = 0 hrs, and is at apogee. Since the right ascension is 270° and the argument of perigee is -90° for Satellite #2, the perigee occurred at 180° longitude in celestial coordinates. However, perigee occurred 6 hours previously for Satellite #2; the earth advanced 1/4 revolution in that period, or perigee occurred over 270° East longitude on the earth. Since this is involved, a simple check for the communications engineer is available on the computer output which gives the subsatellite trace.

The computer output of SATE gives the time history of all combinations of links. It is not a graphical output, but a compensatory feature is its 28-second CPU time on a 370/158 for 25 hours of data. A sample link from Satellite #1 to Lexington, Massachusetts, is shown on Table 2. From left to right, the columns are satellite number, time in hours, range in nautical miles, elevation angle in degrees, uplink Doppler shift (Hz) for a 300 MHz signal, downlink Doppler shift (Hz) for a 245 MHz signal, subsatellite longitude, subsatellite colatitude, and ground station number. It is seen that range increases until a maximum is reached at 6 hours for the first apogee.

No earth harmonics for the potential field have been introduced for Table 2, so this near earth orbit should not be examined beyond two orbits or 24 hours.

The 55 possible crosslinks are also listed in the computer output. A crosslink from Satellite #1 to Satellite #3 is examined in Table 3. This represents a highly eccentric Aerospace Type A orbit crosslinking to a synchronous satellite. From left to right, time is given in hours, range in nautical miles, L12 and C12 are the longitude and colatitude of the crosslink pointing vector (in the inertial coordinate system), crosslink Doppler (Hz) is given for the 60 GHz crosslink[8], and CVIS gives a visibility check on the crosslink. CVIS = 1 means the crosslink exists; CVIS = 0 if the earth blocks the crosslink. The first maximum for crosslink range is seen to occur at 6 hours and 27822

TABLE 2

AN ECCENTRIC SATELLITE LINK TO LEXINGTON, MASSACHUSETTS

(Range in nautical miles, Angle = elevation angle, Up and Down Doppler in Hz.)

SAT	HRS	RANGE	ANGLE	UPDOPPLER	DNDOPPLER	SUBL	SUBC	STATION
1	1.0	9260.367	12.721	-2913.700	-2379.520	-0.850	63.945	1
1	2.0	14332.676	29.205	-2244.127	-1832.704	1.094	44.073	1
1	3.0	18017.570	35.326	-1568.874	-1281.247	0.369	35.092	1
1	4.0	20501.965	37.995	-1000.994	-817.478	-0.347	30.087	1
1	5.0	21946.953	38.956	-492.098	-401.880	-0.431	27.416	1
1	6.0	22429.563	39.012	-5.600	-4.573	0.000	26.565	1
1	7.0	21965.715	38.540	486.038	396.931	0.421	27.415	1
1	8.0	20519.180	37.624	1009.954	824.795	0.337	30.085	1
1	9.0	17999.438	35.788	1595.607	1303.079	-0.378	35.087	1
1	10.0	14262.535	30.786	2259.376	1845.157	-1.104	44.065	1
1	11.0	9336.133	11.419	2637.558	2154.005	0.836	63.923	1
1	12.0	7227.129	-77.318	-1064.329	-869.202	89.594	153.434	1
1	13.0	10861.879	-14.168	-2537.382	-2072.195	-180.864	63.967	1
1	14.0	15498.383	7.132	-2120.911	-1732.076	-178.915	44.081	1
1	15.0	19018.594	15.295	-1508.299	-1231.777	-179.640	35.096	1
1	16.0	21405.051	19.402	-958.151	-782.490	-180.356	30.090	1
1	17.0	22778.605	21.613	-461.964	-377.271	-180.441	27.417	1
1	18.0	23219.723	22.477	5.412	4.420	-180.000	26.565	1
1	19.0	22762.797	21.947	466.511	380.984	-179.588	27.413	1
1	20.0	21393.457	19.693	948.133	774.308	-179.673	30.082	1
1	21.0	19043.566	14.962	1481.625	1209.994	-180.388	35.083	1
1	22.0	15573.766	5.998	2106.593	1720.383	-181.114	44.057	1
1	23.0	10810.207	-13.120	2770.763	2262.790	-179.178	63.901	1
1	24.0	5944.375	-50.624	1303.522	1064.543	-90.810	153.433	1
1	25.0	9252.035	12.697	-2914.108	-2379.854	-0.877	63.988	1

TABLE 3

CROSSLINK BETWEEN AN ECCENTRIC SATELLITE AND A SYNCHRONOUS SATELLITE

(Range in nautical miles, Crosslink Doppler in Hz (60 GHz crosslink))[2]

HRS	RANGE	L12	C12	CRDOPPLER	CVIS	SAT1	SAT2
1.0	18135.488	82.850	104.826	-3149.510	1	1	3
2.0	20296.223	103.276	125.216	-327932.750	1	1	3
3.0	23453.848	119.355	134.814	-299587.875	1	1	3
4.0	25918.691	133.851	139.511	-203648.563	1	1	3
5.0	27382.699	148.046	141.850	-97484.000	1	1	3
6.0	27822.402	162.619	142.687	6063.633	1	1	3
7.0	27285.309	177.877	142.114	103529.188	1	1	3
8.0	25834.590	193.715	139.736	193547.375	1	1	3
9.0	23550.559	209.504	134.566	272253.875	1	1	3
10.0	20638.660	223.687	124.561	318659.688	1	1	3
11.0	17884.590	232.492	105.057	180939.813	1	1	3
12.0	21833.891	221.684	80.702	-1390808.000	1	1	3
13.0	30509.875	226.932	98.738	-489000.813	1	1	3
14.0	33991.332	240.728	110.134	-268575.625	1	1	3
15.0	36094.801	255.249	117.252	-173069.813	1	1	3
16.0	37454.848	-89.800	121.752	-109688.313	1	1	3
17.0	38259.199	-74.595	124.251	-56613.570	1	1	3
18.0	38558.977	-59.365	125.021	-4373.715	1	1	3
19.0	38330.488	-44.338	124.182	52297.195	1	1	3
20.0	37515.762	-29.592	121.703	116766.875	1	1	3
21.0	36030.969	-14.988	117.318	190206.625	1	1	3
22.0	33790.941	0.044	110.279	272366.438	1	1	3
23.0	30666.230	17.363	98.725	382652.000	1	1	3
24.0	24319.582	47.896	81.660	1249408.000	1	1	3
25.0	18133.688	82.804	104.791	-2092.753	1	1	3

nautical miles. Crosslink Doppler peaks at 1.39 MHz at 12 hours. A more convenient and intuitive crosslink pointing system will be given later in program SATLUNAE.

These results were given in single precision. Because elevation angle was required only within 0.3° and earth harmonic terms were omitted, greater accuracy was not felt to be justified here. Single precision can lead to a strange result for crosslink Doppler between two coplanar satellites in circular orbit; the required zero Doppler shift is calculated to be a fraction of a Hz because it is the result of a subtraction of two large numbers. Another embarrassment for single precision can occur in crosslink pointing angles. The tangent of angles near 90° may be read as tan (90°). If these inconveniences are to be avoided, the double precision program SATD (Appendix 2) can be used.

3.2 EXAMPLE 2

The orbital configuration of Figure 10 is used again in Example 2, but this time the stability of the orbits relative to lunar perturbations is to be checked. The orbital elements of Figure 10 are entered into program SATLUNAE (Appendix 6), but unlike the previous example the semi-major axis is entered in km. The initial semi-major axes are 26561 km, 42164 km, and 106247 km respectively for the 12-hr, 24-hr, and 4-day orbits.

The first part of the output of SATLUNAE is an estimate of new orbital elements after a long period of lunar perturbation. Although SATLUNAE is meant for very long periods, like five years, 365 days is the period used for this example. The estimate for the new orbital elements after one year is shown in Table 4a.

The semi-major axis for the 12-hour satellites is seen to be the same as the initial value, but the eccentricity has increased from 0.725 to 0.745. This jump in eccentricity is due to the entry of the fourth column, WPS. WPS is a stationary argument of perigee which was set immediately in the computer program SATLUNAE. This stationary argument of perigee allows the eccentricity from the program to start growing immediately, although real satellite eccentricity does not immediately grow. Therefore the new eccentricity E1 should be considered an upper bound. The new right ascension W is only slightly changed from the original 0° for Satellite #1. The normalized apogee distance V_5 has increased 1.2% in one year.

TABLE 4a

FUTURE SEMI-MAJOR AXIS, ECCENTRICITY, RIGHT ASCENSION, ARGUMENT OF PERIGEE, AND NORMALIZED APOGEE DISTANCE V_5

ABIG	E1	W	WPS	V_5
26561.0	0.745798409	-0.360600	44.963928	1.0120573
26561.0	0.745798409	269.639160	44.963928	1.0120573
42164.2	0.0	-1.655902	0.0	1.0000038
42164.2	0.0	-1.655902	0.0	1.0000038
42164.2	0.0	-1.655902	0.0	1.0000038
42164.2	0.0	-1.655902	0.0	1.0000038
106249.1	0.0	-0.000002	39.585815	1.0000191
106249.1	0.0	-0.000002	39.585815	1.0000191
106249.1	0.0	-0.000002	39.585815	1.0000191
106249.1	0.0	-0.000002	39.585815	1.0000191

Surprising stability appears in the V_5 column for Satellites #3 through #10. This is because the initial eccentricity was 0. A more reasonable estimate of stability can be found if a non-zero value for initial eccentricity is used. Table 4b lets the initial eccentricity value be 0.05.

The synchronous Satellites #3 through #6 are still stable since V_5 retains a value close to unity. However, it is seen that the 4-day polar satellites show a 1.4% increase in normalized apogee distance. Questionable stability is indicated.

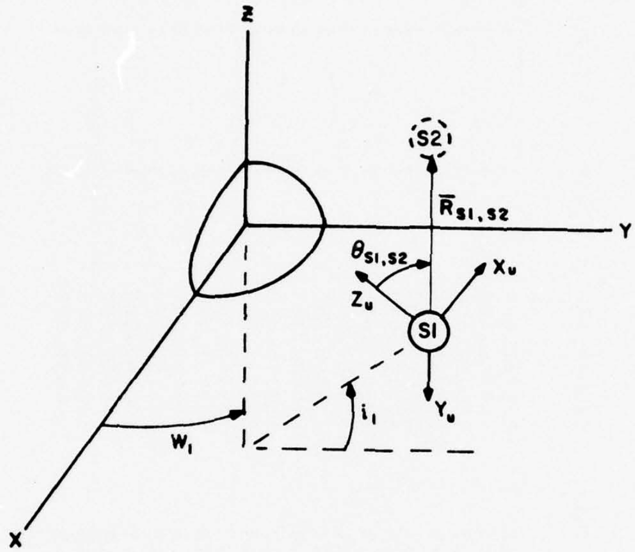
TABLE 4b

SATELLITES #3 THROUGH #10, INITIAL ECCENTRICITY = 0.05

ABIG	E1	W	WPS	V_5
26561.0	0.745798409	-0.360600	44.963928	1.0120573
26561.0	0.745798409	269.639160	44.963928	1.0120573
42164.2	0.050000001	-1.655902	0.0	1.0000038
42164.2	0.050000001	-1.655902	0.0	1.0000038
42164.2	0.050000001	-1.655902	0.0	1.0000038
42164.2	0.050000001	-1.655902	0.0	1.0000038
106249.1	0.065072298	-0.000002	39.585815	1.0143747
106249.1	0.065072298	-0.000002	39.585815	1.0143747
106249.1	0.065072298	-0.000002	39.585815	1.0143747
106249.1	0.065072298	-0.000002	39.585815	1.0143747

SATLUNAE then goes through link calculations with the new orbital elements. These are largely omitted here, since they are reminiscent of the output from SATE. One other crosslink calculation does occur here, however, which makes it worthwhile to include one crosslink. Crosslink angle in a local satellite coordinate system is shown in Figure 11 and it is included as TU12 and FU12 in Table 5; this table is part of one output table of SATLUNAE.

Except for the columns FU12 and TU12, Table 5 was discussed in Example 1. TU12 and FU12 are discussed in Figure 11. Again, TU12 is a local co-elevation angle and FU12 is a local azimuth angle.



THE LOCAL ZENITH ANGLE $\theta_{S1,S2}$ IS COMPUTED IN SUBROUTINE UNPRIM AND PRINTED OUT AS TU_{12} IN PROGRAM SATLUNAE. THE LOCAL AZIMUTH ANGLE (PROJECTION IN THE X_uY_u PLANE, MEASURED FROM THE X_u AXIS) IS PRINTED AS FU_{12}

1A-45,624

Figure 11 LOCAL COORDINATE SYSTEM FOR SATELLITE CROSSLINK

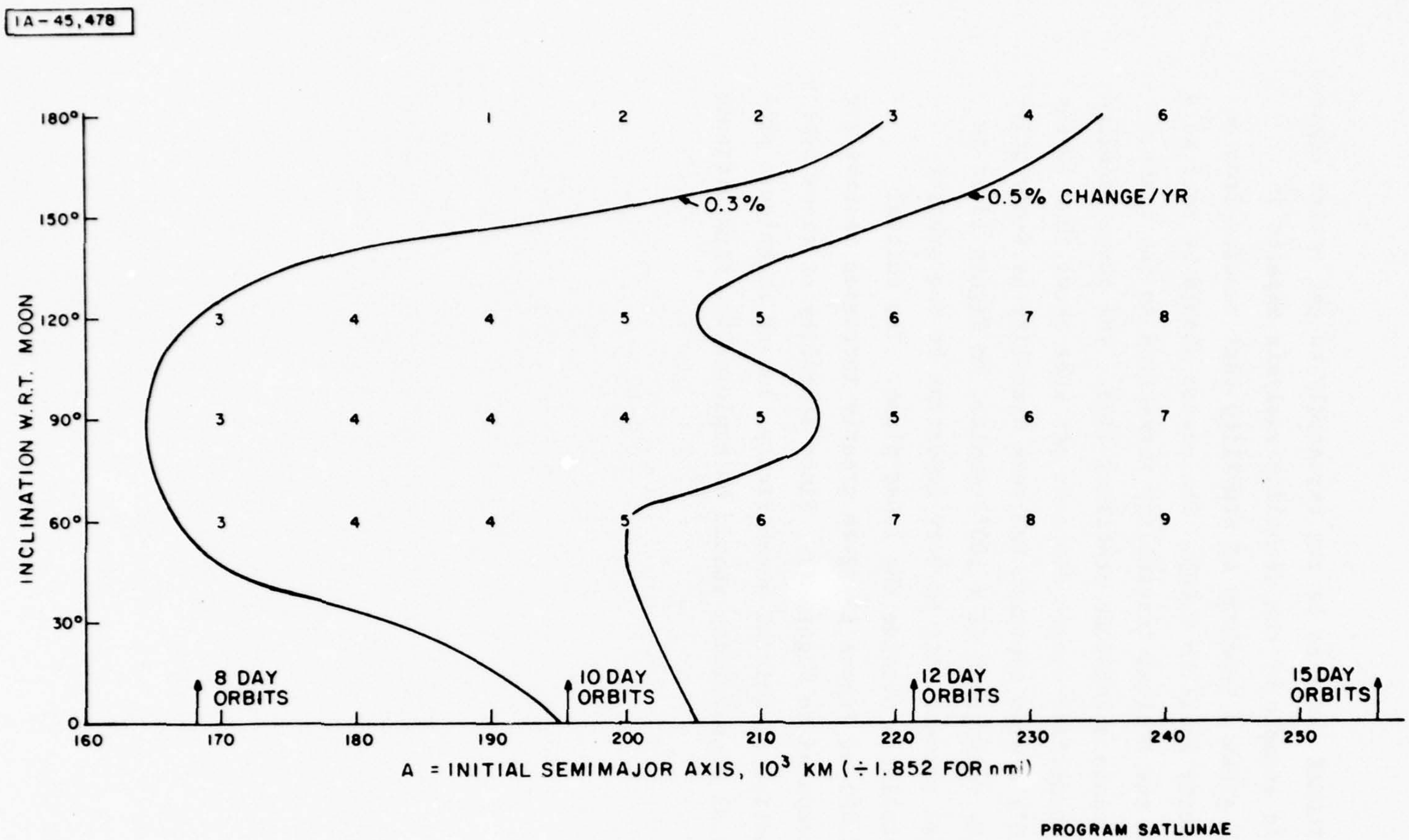
TABLE 5

CROSSLINK RELATIONS FROM PROGRAM SATLUNAE

(FU12 and TU12 are local satellite crosslink pointing angles)

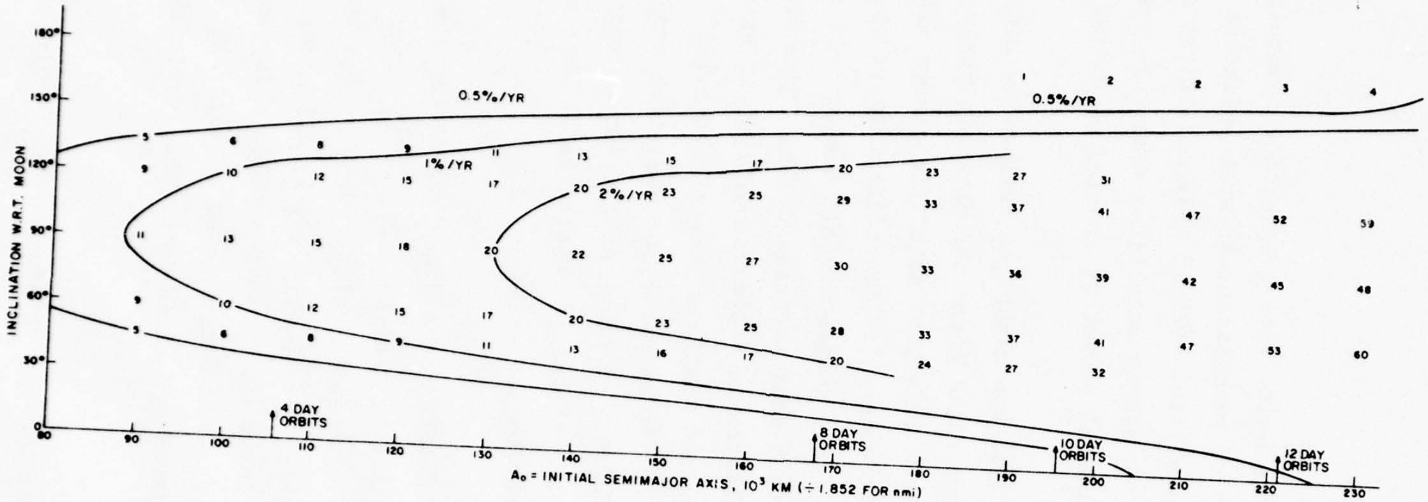
HRS	RANGE	L12	C12	FU12	TU12	CRDOPPLER	CVIS	SAT1	SAT2
3.0	66205.875	67.6	65.4	105.0	36.1	-125757.625	1	1	3
6.0	67722.688	111.1	53.5	139.0	37.8	43986.363	1	1	3
9.0	62309.625	160.3	58.2	179.0	41.3	160184.250	1	1	3
12.0	45816.824	224.8	97.9	-43.3	55.5	1145508.000	0	1	3
15.0	44591.199	-54.7	51.3	-75.0	61.1	-365303.750	1	1	3
18.0	56475.742	-15.7	44.5	-41.0	47.3	-52756.477	1	1	3
21.0	49892.930	23.8	48.9	-1.1	55.5	295032.813	1	1	3
24.0	39494.520	41.3	99.2	136.2	107.1	-1323487.000	1	1	3
27.0	66208.438	67.6	65.4	105.0	36.1	-126798.250	1	1	3
30.0	67727.313	111.1	53.5	139.0	37.8	44000.723	1	1	3
33.0	62314.855	160.3	58.2	178.9	41.3	160175.688	1	1	3
36.0	45827.313	224.8	97.9	-43.8	55.4	1145065.000	0	1	3
39.0	44583.375	-54.7	51.8	-75.1	61.1	-365848.563	1	1	3
42.0	56469.711	-15.7	44.5	-41.0	47.3	-52783.488	1	1	3
45.0	49891.406	23.7	48.9	-1.1	55.5	295005.250	1	1	3
48.0	39475.130	41.3	99.2	136.2	107.2	-1323150.000	1	1	3
51.0	66211.438	67.6	65.4	104.9	36.1	-125840.125	1	1	3
54.0	67732.000	111.1	53.5	139.0	37.8	44015.590	1	1	3
57.0	62319.609	160.3	58.2	178.9	41.3	160163.063	1	1	3
60.0	45835.004	224.7	97.9	-43.8	55.3	1144835.000	0	1	3

Program SATLUNAE can also be run repeatedly to get entire regions of stability. An example of the stability analysis appears in Figure 12, which gives a topology of stability that results from an initial eccentricity equal to 0.005. The curves should be read as a contour map with the orbital instability increasing to the right. Each curve represents a constant stability line. The curve labelled 0.3% change in normalized apogee distance per year shows that 10-day posigrade (I = 0°) orbits approach the same stability as 8-day polar orbits and 12-day retrograde (I = 180°) orbits. On Figure 13 it is seen that initial eccentricity is very important to the orbital stability of satellites outside the lunar plane. The initial eccentricity (0.05) of Figure 13 gives greatly increased instability at I = 90° as compared to Figure 12. Since stability of these orbits is a strong function of initial eccentricity, launch techniques producing low initial eccentricity should be employed for high altitude orbits.



(REGIONS TO THE LEFT OF THE CURVES ARE "STABLE")

Figure 12 ESTIMATED AVERAGE CHANGE IN NORMALIZED APOGEE DISTANCE $\left[\frac{a(1+e)}{a_o(1+e_o)}\right]$ PER YEAR (AVERAGED OVER 5 YEARS) AS A FUNCTION OF LUNAR PERTURBATIONS $\underline{e_0 = .005}$



(REGIONS TO THE LEFT OF THE CURVES ARE CALLED STABLE)

18-45,479

FROM PROGRAM SATLUNAE

Figure 13 ESTIMATED AVERAGE CHANGE IN NORMALIZED APOGEE DISTANCE $\left[\frac{a(1+e)}{a_0(1+e_0)}\right]$ PER YEAR (AVERAGED OVER 5 YEARS) AS A FUNCTION OF LUNAR PERTURBATIONS. $\underline{e_0 = .05}$

3.3 EXAMPLE 3

Two of the initial problems which face a satellite communications designer are: (1) Given a set of satellites, at what points on the earth can an observer see a satellite? (2) If many satellites are involved, at what spots on earth are no satellites visible? Preferably, the designer should see a graphical output of covered/uncovered areas.

In seeking an answer to (1), one might get swamped by information from the 180° x 260° projection which forms the Mercator projection. The really interesting locations in a good satellite system might be those which can see no satellite at all. Identification of these uncovered locations yields a fast, comprehensible output. It is with this idea of limited output that program NOLINKE (Appendix 9) was generated. With 10 satellites in Keplerian orbits, it examines a 12 x 24 grid of possible ground stations (covering the 180° x 360° Mercator projection in 15° increments) for elevation angle to all ten satellites. It performs a printout at a given coordinate only if elevation angle to each of the 10 satellites is less than some minimum acceptable elevation angle (EM).

The lines 10 to 70 of NOLINKE (Appendix 9) represent the orbital elements of 10 satellites. The four DO LOOPS, progressing from outside to inside, are time (RI), colatitude (T), longitude (F), and satellite number (L). So, at each (T, F) all satellites are considered. Elevation angle (D) to each satellite is generated at each location. A print statement is allowed only if all satellites have been considered and if no satellite is visible (SATNO = 0).

This limited output can be very helpful to the designer of a many satellite system.

The program NOLINKE runs in the foreground on TSO. The limited output allows this to be done conveniently. For example, the orbital elements implied by Figure 10 can be typed into lines 20 to 70, a minimum elevation angle = 35° typed into line 210, and the resultant output (at T = 0 hrs, 1 hr) is shown on Table 6. T1 and F1 are ground station colatitude and longitude at which no satellite is observed. No. is the number of satellites observed at these locations. T is time in hours.

TABLE 6

AN OUTPUT OF NOLINKE

T1	F1	NO.	T
60.000	90.000	0.0	0.0
60.000	270.000	0.0	0.0
120.000	90.000	0.0	0.0
120.000	270.000	0.0	0.0
60.000	90.000	0.0	1.000
60.000	270.000	0.0	1.000
120.000	90.000	0.0	1.000
120.000	270.000	0.0	1.000

Although only four locations appear at each hour, 12 x 24 = 288 calculations were done worldwide to see if elevation angle requirements were met. The locations and times of Table 6 were the only places which did not meet the required 35° elevation angle.

No output occurs for NOLINKE for a 6-hour period if a 25° elevation angle is required. Therefore, this 10-satellite system meets a 25° minimum elevation angle requirement.

CONCLUSIONS AND RECOMMENDATIONS

An analysis has been presented which gives fast, convenient computations for range, Doppler, crosslink data, and pointing angles for a two ground-station, 10-satellite system. With the programs listed, data on all ways to link one ground station with the other via the satellite are given on one computer output. The writers are not aware of the existence of other complete link solutions on one output. Three place accuracy is representatively given. Completely general Keplerian orbits are allowable with $0 \leq$ eccentricity ≤ 0.99.

The coverage of a 10-satellite system is quickly found in a program with very limited output.

In continuing the emphasis on low computation time, analytic results for orbital stability as a function of lunar perturbations were used to generate a very efficient program. It can be used as a first economical estimate of high altitude orbital parameters after many years of lunar perturbations. Figures 12 and 13 were also included to indicate stability regions.

None of the listed FORTRAN programs requires more than one minute CPU time on an IBM 370/158 computer.

In the future, it is recommended that real signal propagation be interposed between the stations. This could include ideal antenna gain patterns and regions of ionospheric disturbance to yield estimates of the received signal/noise ratio. A "point ahead" capability should also be added to the crosslinks; this can be done conveniently with the existing velocity calculations.

NOTATION

Symbol	Definition	Comment
a	semi-major axis; invariant for Keplerian orbits	Nautical miles, except in programs for lunar perturbations, where a is entered in km. 1 nmi = 1.852 km. Subscript 1 is used to denote satellite 1, etc.
$\bar{a}$	average semi-major axis	Average taken over a period of lunar perturbation.
a_o	initial semi-major axis (before lunar perturbations)	
$a_{11}-a_{33}$	elements of Euler rotation matrix	
ABIG	future semi-major axis, subject to lunar perturbations	
$\underline{A}$	Euler rotation matrix	
$\underline{A}^{-1}$	inverse of $\underline{A}$	
A_{1S}	geocentric angle between sub-satellite point and ground station 1	
AZ1	FORTRAN program which includes azimuth calculations (App. 8)	
b	semi-minor axis	Not used here.
c	velocity of light = 1.6187 x 10^5 nmi/sec	

D_{1S}	elevation angle from ground station to satellite, degrees	
Δ	denotes small changes in orbital elements (e.g. Δe=small change in eccentricity)	
e	eccentricity; for the elliptical Keplerian orbits considered here $0 \leq e < 1$.	The programs with the iterative subroutine ELLIP work for $0 \leq e \leq 0.99$
e_1	future eccentricity, subject to lunar perturbations	
E	eccentric anomaly	E_1, E_2, ... used for successive approximations to eccentric anomaly.
F	carrier frequency, Hz	
F'	Doppler shifted carrier frequency, Hz	
F_r	lunar radial force/unit mass of satellite (scalar)	
FSP	angle measured from the line of nodes in the satellite plane	
ϕ'_s	same as FSP	$\phi'_s = W_p + \theta$
G	gravitational constant = 6.6732 x 10^{-11} newton meter 2/kg^2	
i	orbital inclination angle with respect to equatorial plane	Becomes XI in FORTRAN programs. Numbered subscripts refer to satellite number.
$\bar{i}$, $\bar{j}$, $\bar{k}$	unit vectors in x, y, and z directions, respectively	
I	orbital inclination with respect to lunar plane	

M	mean anomaly	
M_e	mass of earth	
M_m	mass of moon	
μ	GM_e	
μ_m	GM_m	
nautical mile (nmi)	1.852 km (exact)	
n	mean angular rate, rad/hr	$M = n(t-t_p)$
NOLINKE	a coverage program of very limited output for ten satellites (App. 9)	
	angle in satellite plane measured from line of nodes in the lunar plane	
p	semi-latus rectum	
PERTP	FORTRAN program which calculates effects of lunar perturbations on orbital elements (App. 5)	
$\bar{P}_{12}$	unit vector from transmitter to receiver	
R	geocentric distance of satellite at a given instant of time (nmi for programs without lunar perturbations, km for those with)	
$\dot{R}$	$\frac{d}{dt}(R)$	
R_e	earth radius = 3440 nmi	
R_{12}	distance from transmitter to receiver	

R_{1S}	distance from ground transmitter 1 to satellite	
$R_{S1,\ S2}$	distance from Satellite #1 to Satellite #2	
RI	time index (hrs) in programs (T in analysis)	Note that solar hours are not distinguished from sidereal hours because only three-place accuracy is desired in results.
$\tilde{R}$, $\tilde{S}$, $\tilde{W}$	lunar perturbation components in Ash analysis	
SATD	FORTRAN program which calculates ground-satellite relations ($e \leq 0.5$) in double precision (App. 2)	
SATE	FORTRAN program which calculates ground-satellite relations ($0 \leq e \leq 0.99$) in single precision (App. 1)	
SATLUNAE	like SATE, but includes long term estimates ($\geq$5 years) of lunar perturbations (App. 6)	
SATVIZD	like SATD, but interactive with the user (App. 4)	
SATVIZE	like SATE, but interactive with the user (App. 3)	
SCOREE	like SATE, but includes effects of lunar perturbations on orbital elements for periods up to a few years (App. 7)	
T	time in hours	RI in programs
TP	time at perigee, hours	

τ	period for Keplerian orbit (subscripts are added for particular satellites)	
TU12, FU12	crosslink angles in a local satellite coordinate system	
θ	true anomaly	
$\dot{\theta}$	$\frac{d}{dt}$ ( )	
θ_j	angle between relative velocity vector and pointing vector	
θ_1, ϕ_2	spherical coordinates associated with ground station 1	
$\theta_{S1, S2}$;		
$\phi_{S1, S2}$	inertial spherical coordinates associated with crosslink vector (called L12, C12 in computer output)	TU12, FU12 are usually more convenient than $\theta_{S1, S2}$; $\phi_{S1, S2}$.
θ_e	angle of satellite with respect to earth-moon vector	
θ_{eo}	bounding angle for lunar perturbations (representatively, 0.3 radians)	
θ_m	angle of satellite with respect to moon-earth vector	
V_R	relative velocity between transmitter and receiver	
V_r	satellite radial velocity	
V	potential energy in _vis viva_ integral	
V_5	normalized apogee distance = final apogee/initial apogee	

vis viva integral	gives relation between potential and kinetic energy for Keplerian orbits	
W	right ascension for Keplerian orbit, degrees	Subscripts are used for particular satellites.
W_p	argument of perigee for Keplerian orbit, degrees	
WPS	stationary argument of perigee, subject to lunar perturbations	
ω	earth rotation rate - 15°/solar hour = 15.04°/sidereal hour	
ω_s	angular rotation rate of satellite about earth, rad/hr	
ω_m	angular rotation rate of moon about earth, rad/hr	
ω_{rel}	vector difference of ω_s, ω_m	ω_{rel} determines the time of interaction of satellite and moon.
$\widetilde{W}$	one of three lunar perturbation components used by Ash (others, $\widetilde{R}$, $\widetilde{S}$)	
X,Y,Z,	inertial Cartesian coordinate system, with x pointed toward Aries (X,Y,Z, associated with ground station 1)	
X',Y',Z',	a moving coordinate system in the satellite plane	

X",Y",Z",	a stationary coordinate system in the satellite plane	The double prime system will be convenient if Doppler rates are later added to the programs.
X_S, Y_S, Z_S	coordinates of satellite in inertial space axis of a related coordinate system which lies along Z axis	

REFERENCES AND NOTES

1. M. Ash, "Doubly Averaged Effect of the Moon and Sun on a High Altitude Earth Satellite Orbit", TN 1974-5, Lincoln Laboratory, 1 March 1974

2. J. Jensen, et al., Design Guide to Orbital Flight, McGraw-Hill Book Co. Inc., N.Y. 1962

   The title of this book suggests a very straightforward handbook presentation; this is deceptive, however, and the book is replete with subtle and extensive derivations.

3. F. R. Moulton, An Introduction to Celestial Mechanics, MacMillan Co., N.Y. 1914

   This book has been important for insight; the purpose of many derivations is to minimize computational tedium.

4. P. R. Escobal, Methods of Orbit Determination, John Wiley & Sons, N.Y., 1965

5. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Co., Reading, Mass., 1962

6. J. D. Jackson, Classical Electrodynamics, J. Wiley, N.Y., 1962

7. J. L. LeMay, et al., "HANS High Altitude Navigation Study", Aerospace Report No. TR-0073(3491)-1, Air Force Report No. SAMSO-TR-74-10, 29 June 1973

8. J. W. Dees, J. C. Wiltse, "An Overview of Millimeter Wave Systems", Microwave Journal 12, No. 11, November 1969, pp 42-49.

ADDITIONAL BIBLIOGRAPHY

S. Chandrasekhar, Principles of Stellar Dynamics, Dover, N.Y. 1942

Chandrasekhar developed the concept of "dynamical friction". An important part of dynamical friction is the integration time of the perturbations. This concept led to the integration here of the radial perturbation during a dynamic interaction of the satellite and moon. Differences between posigrade and retrograde orbits were seen in the results which are not present in static analyses.

P. Michaels, J. Crocco, "Behavior of Selected Satellite Orbits Near a Lunar Libration Point", USAFESD Working Paper 1974-75, 1 May 1974

APPENDIX 1

PROGRAM SATE

This background version of SATVIZE is usually more valuable than SATVIZE for more than two satellites or two ground stations. The high speed printer allows the extensive output to be more quickly and neatly finished than the terminal output of SATVIZE.

Orbital elements are entered on lines 160-240. The ground station latitudes are entered consecutively for the ground stations, then the longitudes (East). For example, the locations of Lincoln, Massachusetts (47.54° colatitude, 288.73° E. Long.) and San Diego, California (57.34° colatitude, 242.8° E. Long.) are entered as:

```
DATA B/47.54, 57.23, 288.73, 242.8/                240
```

```
//TS0420A JOB (6360,D91,DESK),'CHRISTOPHER P',NOTIFY=TS0420,
// TIME=1
// EXEC FORTGCG
//FORT.SYSIN DD *
C THIS SATELLITE VISIBILITY PROGRAM IS WRITTEN FOR A MAXIMUM OF
C TEN SATELLITES AND TWO GROUND STATIONS.
C THE ARRAYS CONTAIN THE FOLLOWING INFORMATION FOR EACH SATELLITE:
C A--SEMIMAJOR AXIS IN NAUTICAL MILES
C E--ECCENTRICITY
C W--RIGHT ASCENSION IN DEGREES
C WP--ARGUMENT OF PERIGEE IN DEGREES
C TP--TIME OF PERIGEE IN HOURS
C XI--INCLINATION IN DEGREES
C ARRAY B CONTAINS THE COLATITUDE AND LONGITUDE FOR EACH GROUND STATION.
            DIMENSION A(10),E(10),W(10),WP(10),TP(10),XI(10),B(4)
      DATA A/14342.,14342.,22767.,22767.,22767.,22767.,
     3 57369.2,57369.2,57369.2,57369.2/
      DATA E/.725,.725,0.,0.,0.,0.,0.,0.,0.,0./
        DATA W/0.,270.,243.733,243.733,0.,0.,
     4 0.,0.,0.,0./
      DATA WP/-90.,-90.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA TP/0.,-6.,0.,-6.,-15.,-21.,
     1 0.,-24.,-48.,-72./
      DATA XI/63.435,63.435,23.4,23.4,0.,0.,90.,90.,90.,90./
        DATA B/153.435,26.565,270.,0./
C NUMS IS THE NUMBER OF SATELLITES TO BE CONSIDERED.
C NUMG IS THE NUMBER OF GROUND STATIONS TO BE CONSIDERED.
      NUMS=4
      NUMG=2
            P=3.1415926
            RE=3440.
            RTD=57.29577951
            WRAD=15.0/RTD
C CL IS THE VELOCITY OF LIGHT IN NAUTICAL MILES PER SECOND.
            CL=1.61984*10.**5
C FR IS THE UPLINK FREQUENCY IN HZ.
C FR1 IS THE DOWNLINK FREQUENCY IN HZ.
            FR=300.0*10.**6
            FR1=245.0*10.**6
            CON=5.987*10.**(-6)
            DO 900 J=1,NUMS
            W(J)=W(J)/RTD
            WP(J)=WP(J)/RTD
            XI(J)=XI(J)/RTD
C TAU IS THE PERIOD OF ROTATION OF SATELLITE J.
            TAU=CON*A(J)**1.5
            DO 700 K=1,NUMG
            WRITE(6,10)
  10        FORMAT(1H ,'SAT',5X,'HRS',12X,'RANGE',9X,'ANGLE',8X,'UPDOPPLER
     1',8X,'DNDOPPLER',9X,'SUBL',9X,'SUBC',6X,'STATION',//)
            T=B(K)/RTD
            G=B(K+2)/RTD
C I IS THE HOUR.
            DO 500 I=1,25
            RI=I
C SUBROUTINE ELLIP COMPUTES THE RANGE FROM GEOCENTER TO SATELLITE J AND
C THE ANGLE MEASURED IN ORBIT PLANE;REFERRED TO NODE OF ORBIT PLANE
C AND EQUATORIAL PLANE.
            CALL ELLIP(RI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
C SUBROUTINE PRIME COMPUTES THE INERTIAL CARTESIAN COORDINATES OF THE
C SATELLITE J (WITH GEOCENTER AS THE ORIGIN).
```

```
      CALL PRIME(FSP,W(J),XI(J),R,0.,XS,YS,ZS)
C FS IS THE SUBSATELLITE LONGITUDE IN DEGREES.
      FS=(ATAN(YS/XS)-WRAD*RI)*RTD
      IF(XS.LT.0.) FS=FS+P*RTD
11    IF(FS.LT.-360.) FS=FS+360.
      IF(FS.LT.-360.)GO TO 11
C TS IS THE SUBSATELLITE COLATITUDE IN DEGREES.
      TS=RTD*ARCOS(ZS/R)
      F=G+WRAD*RI
C (X,Y,Z) ARE THE INERTIAL CARTESIAN COORDINATES OF GROUND STATION K.
      X=RE*SIN(T)*COS(F)
      Y=RE*SIN(T)*SIN(F)
      Z=RE*COS(T)
C ACC IS THE ANGLE BETWEEN GROUND STATION K AND SATELLITE J FROM THE
C CENTER OF THE EARTH.
C RGE IS THE RANGE FROM GROUND STATION K TO SATELLITE J IN NAUTICAL
C MILES.
      ACC=ARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
      RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
      ARG=SIN(ACC)*R/RGE
      IF(ARG.GT.1.0000000)ARG=1.0000000
C D IS THE ELEVATION ANGLE IN DEGREES FROM GROUND STATION K TO
C SATELLITE J.
      D=(ARSIN(ARG)-P/2.)*RTD
      RT=SQRT(R*R-RE*RE)
      IF(RGE.LE.RT) D=-D
C SUBROUTINE DOPE COMPUTES THE COMPONENTS OF SATELLITE VELOCITY.
      CALL DOPE(R,E(J),A(J),TAU,FSP,WP(J),W(J),XI(J),XV,YV,ZV)
C (XT,YT) ARE THE COMPONENTS OF THE RELATIVE VELOCITY OF GROUND
C STATION K.
      XT=-WRAD*RE*SIN(T)*SIN(F)/3600.
      YT=WRAD*RE*SIN(T)*COS(F)/3600.
      DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+ZV*(ZS-Z)
      DOT=-DOT/RGE
C UPDOP IS THE UPLINK DOPPLER SHIFT IN HZ.
C DNDOP IS THE DOWNLINK DOPPLER SHIFT IN HZ.
      UPDOP=DOT*FR/CL
      DNDOP=DOT*FR1/CL
      WRITE(6,20)J,RI,RGE,D,UPDOP,DNDOP,FS,TS,K
 20   FORMAT(1H ,I3,F8.1,F17.3,F14.3,F17.3,F17.3,F13.3,F13.3,I10)
 500  CONTINUE
      WRITE(6,25)
 25   FORMAT(1H ,//)
 700  CONTINUE
 900  CONTINUE
      WRITE(6,30)
 30   FORMAT(1H ,/////)
      L=NUMS-1
      IF(L.EQ.0)GO TO 350
C FR IS THE CROSSLINK FREQUENCY IN HZ.
    FR=60.*10.**9
C M1 IS THE FIRST SATELLITE.
C M2 IS THE SECOND SATELLITE.
      DO 350 M1=1,L
      TAU=CON*A(M1)**1.5
      N=M1+1
      DO 250 M2=N,NUMS
      WRITE(6,40)
 40   FORMAT(1H ,2X,'HRS',12X,'RANGE',11X,'L12',12X,'C12',11X,
    2 'CRDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
      TAU2=CON*A(M2)**1.5
```

```
      DO 300 I=1,25
      RI=I
C NOVIZ DETERMINES CROSSLINK VISIBILITY WHERE 0 MEANS NO VISIBILITY
C AND 1 MEANS VISIBILITY.
      NOVIZ=1
      CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
      CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
      CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
      CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
      TC1=X2-XS
      TC2=Y2-YS
      TC3=Z2-ZS
C R12 IS THE RANGE BETWEEN TWO SATELLITES IN NAUTICAL MILES.
C F12 AND T12 ARE THE POINTING ANGLES BETWEEN TWO SATELLITES IN DEGREES.
      R12=(TC1**2+TC2**2+TC3**2)**.5
      F12=RTD*ATAN(TC2/TC1)
      IF(TC1.LT.0.)F12=F12+P*RTD
      VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
      IF(VIZ.GT..99999999)VIZ=.99999999
      DEL=ARCOS(VIZ)
      DELM=ARSIN(RE/R)
      IF(DEL.LE.DELM)NOVIZ=0
      T12=ARCOS(TC3/R12)*RTD
      CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
     CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
      DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
      DOT=DOT/R12
C DOP12 IS THE CROSSLINK DOPPLER SHIFT IN HZ.
      DOP12=-DOT*FR/CL
      WRITE(6,50)RI,R12,F12,T12,DOP12,NOVIZ,M1,M2
 50   FORMAT(1H ,F5.1,F17.3,F14.3,F15.3,F20.3,I9,I10,I10)
 300  CONTINUE
      WRITE(6,60)
 60   FORMAT(1H ,//)
 250  CONTINUE
 350  CONTINUE
      END
      SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
      A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
      A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
      XS=A11*XPS+A12*YPS
      YS=A21*XPS+A22*YPS
      ZS=A31*XPS+A32*YPS
      RETURN
       END
      SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
      P=3.1415926
       Z=2.*P*(T-TP)/TAU
      P2=2.*P
2     IF(Z.GT.P2)Z=Z-P2
      IF(Z.GT.P2)GO TO 2
      E1=Z+E*SIN(Z)
      E2=(Z+E*(SIN(E1))-(E*COS(E1))*E1)/(1.-E*COS(E1))
      Q=0.
4     E3=(Z+E*(SIN(E2))-(E*COS(E2))*E2)/(1.-E*COS(E2))
      Q=Q+1.
       DE=E3-E2
```

```
DE2=DE**2
E2=E3
IF(DE2.GT..00000001)GO TO 4
TH=ARCOS((COS(E2)-E)/(1.-E*COS(E2)))
IF(Z.GT.P)TH=2.*P-TH
FSP=WP+TH
R=(A*(1.-E**2))/(1.+E*COS(FSP-WP))
RETURN
END
SUBROUTINE DOPE(R,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
P=3.1415926
T=FSP-WP
RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*SIN(T)
TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*COS(T))**2
XPC=RD*COS(FSP)-R*TD*SIN(FSP)
XPC=XPC/3600.
YPC=RD*SIN(FSP)+R*TD*COS(FSP)
YPC=YPC/3600.
CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
RETURN
END
```

```
*******************************************************************************
```

APPENDIX 2

PROGRAM SATD

Unlike SATE, SATD runs in double precision. It is rarely used except to check crosslink Doppler for low-eccentricity satellites within a given constellation. Differences of large velocities can be found accurately with SATD, and accurate crosslink Dopplers result.

```
//TS0420A JOB (6360,D91,DESK),'CHRISTOPHER P',NOTIFY=TS0420,
// TIME=3
// EXEC FORTGCG
//FORT.SYSIN DD *
C THIS SATELLITE VISIBILITY PROGRAM IS WRITTEN FOR A MAXIMUM OF
C TEN SATELLITES AND TWO GROUND STATIONS.
C THE ARRAYS CONTAIN THE FOLLOWING INFORMATION FOR EACH SATELLITE:
C A--SEMIMAJOR AXIS IN NAUTICAL MILES
C E--ECCENTRICITY
C W--RIGHT ASCENSION IN DEGREES
C WP--ARGUMENT OF PERIGEE IN DEGREES
C TP--TIME OF PERIGEE IN HOURS
C XI--INCLINATION IN DEGREES
C ARRAY B CONTAINS THE COLATITUDE AND LONGITUDE FOR EACH GROUND STATION.
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(10),E(10),W(10),WP(10),TP(10),XI(10),B(4)
      DATA A/22767.,22767.,22767.,22767.,36140.36,36140.36,
     3 36140.36,36140.36,14342.3,14342.3/
      DATA E/.1,.1,.1,.1,.1,.1,.1,.1,.65,.65/
      DATA W/243.733,243.733,243.733,243.733,243.733,243.733,
     4 243.733,243.733,243.733,243.733/
      DATA WP/0.,0.,0.,0.,0.,0.,0.,0.,-90.0,-90.0/
      DATA TP/0.,-6.,-12.,-18.,0.,-12.,-24.,-36.,-.75,-6.75/
      DATA XI/23.4,23.4,23.4,23.4,113.4,113.4,113.4,113.4,63.4,63.4/
      DATA B/47.54,57.23,288.73,242.8/
C NUMS IS THE NUMBER OF SATELLITES TO BE CONSIDERED.
C NUMG IS THE NUMBER OF GROUND STATIONS TO BE CONSIDERED.
      NUMS=10
      NUMG=2
      P=3.1415926
      RE=3440.
      RTD=57.29577951
      WRAD=15.0/RTD
C CL IS THE VELOCITY OF LIGHT IN NAUTICAL MILES PER SECOND.
      CL=1.61984*10.**5
C FR IS THE UPLINK FREQUENCY IN HZ.
C FR1 IS THE DOWNLINK FREQUENCY IN HZ.
      FR=300.0*10.**6
      FR1=245.0*10.**6
      CON=6.987*10.**(-6)
      DO 900 J=1,NUMS
      W(J)=W(J)/RTD
      WP(J)=WP(J)/RTD
      XI(J)=XI(J)/RTD
C TAU IS THE PERIOD OF ROTATION OF SATELLITE J.
      TAU=CON*A(J)**1.5
      DO 700 K=1,NUMG
      WRITE(6,10)
   10 FORMAT(1H ,'SAT',5X,'HRS',12X,'RANGE',9X,'ANGLE',8X,'UPDOPPLER
     1',8X,'DNDOPPLER',9X,'SUBL',9X,'SUBC',6X,'STATION',//)
      T=B(K)/RTD
      G=B(K+2)/RTD
C I IS THE HOUR.
      DO 500 I=1,25
      RI=I
C SUBROUTINE ELLIP COMPUTES THE RANGE FROM GEOCENTER TO SATELLITE J AND
C THE ANGLE MEASURED IN ORBIT PLANE;REFERRED TO NODE OF ORBIT PLANE
C AND EQUATORIAL PLANE.
      CALL ELLIP(RI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
C SUBROUTINE PRIME COMPUTES THE INERTIAL CARTESIAN COORDINATES OF THE
C SATELLITE J (WITH GEOCENTER AS THE ORIGIN).
```

```
      CALL PRIME(ESP,W(J),XI(J),R,0.,XS,YS,ZS)
C FS IS THE SUBSATELLITE LONGITUDE IN DEGREES.
      FS=(DATAN(YS/XS)-WRAD*RI)*RTD
      IF(XS.LT.0.)FS=FS+P*RTD
C TS IS THE SUBSATELLITE COLATITUDE IN DEGREES.
      TS=RTD*DARCOS(ZS/R)
      F=G+WRAD*RI
C (X,Y,Z) ARE THE INERTIAL CARTESIAN COORDINATES OF GROUND STATION K.
      X=RE*DSIN(T)*DCOS(F)
      Y=RE*DSIN(T)*DSIN(F)
      Z=RE*DCOS(T)
C ACC IS THE ANGLE BETWEEN GROUND STATION K AND SATELLITE J FROM THE
C CENTER OF THE EARTH.
C RGE IS THE RANGE FROM GROUND STATION K TO SATELLITE J IN NAUTICAL
C MILES.
      ACC=DARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
      RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
      ARG=DSIN(ACC)*R/RGE
      IF(ARG.GT.1.0000000)ARG=1.0000000
C D IS THE ELEVATION ANGLE IN DEGREES FROM GROUND STATION K TO
C SATELLITE J.
      D=(DARSIN(ARG)-P/2.)*RTD
      RT=DSQRT(R*R-RE*RE)
      IF(RGE.LE.RT)D=-D
C SUBROUTINE DOPE COMPUTES THE COMPONENTS OF SATELLITE VELOCITY.
      CALL DOPE(R,E(J),A(J),TAU,ESP,WP(J),W(J),XI(J),XV,YV,ZV)
C (XT,YT) ARE THE COMPONENTS OF THE RELATIVE VELOCITY OF GROUND
C STATION K.
      XT=-WRAD*RE*DSIN(T)*DSIN(F)/3600.
      YT=WRAD*RE*DSIN(T)*DCOS(F)/3600.
      DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+ZV*(ZS-Z)
      DOT=-DOT/RGE
C UPDOP IS THE UPLINK DOPPLER SHIFT IN HZ.
C DNDOP IS THE DOWNLINK DOPPLER SHIFT IN HZ.
      UPDOP=DOT*FR/CL
      DNDOP=DOT*FR1/CL
      WRITE(6,20)J,RI,RGE,D,UPDOP,DNDOP,FS,TS,K
 20   FORMAT(1H ,I3,F9.1,F17.3,F14.3,F17.3,F17.3,F13.3,F13.3,I10)
 500  CONTINUE
      WRITE(6,25)
 25   FORMAT(1H ,//)
 700  CONTINUE
 900  CONTINUE
      WRITE(6,30)
 30   FORMAT(1H ,/////)
      L=NUMS-1
      IF(L.EQ.0)GO TO 350
C FR IS THE CROSSLINK FREQUENCY IN HZ.
   FR=2.83*10.**13
C M1 IS THE FIRST SATELLITE.
C M2 IS THE SECOND SATELLITE.
      DO 350 M1=1,L
      TAU=CON*A(M1)**1.5
      N=M1+1
      DO 250 M2=N,NUMS
      WRITE(6,40)
 40   FORMAT(1H ,2X,'HRS',12X,'RANGE',11X,'L12',12X,'C12',11X,
     2 'CRDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
      TAU2=CON*A(M2)**1.5
      DO 300 I=1,25
      RI=I
```

```
C NOVIZ DETERMINES CROSSLINK VISIBILITY WHERE 0 MEANS NO VISIBILITY
C AND 1 MEANS VISIBILITY.
            NOVIZ=1
            CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
            CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
            CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
            CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
            TC1=X2-XS
            TC2=Y2-YS
            TC3=Z2-ZS
C R12 IS THE RANGE BETWEEN TWO SATELLITES IN NAUTICAL MILES.
C F12 AND T12 ARE THE POINTING ANGLES BETWEEN TWO SATELLITES IN DEGREES.
            R12=(TC1**2+TC2**2+TC3**2)**.5
            F12=RTD*DATAN(TC2/TC1)
            IF(TC1.LT.0.) F12=F12+P*RTD
            VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
            IF(VIZ.GT..99999999)VIZ=.99999999
            DEL=DARCOS(VIZ)
            DELM=DARSIN(RE/R)
            IF(DEL.LE.DELM)NOVIZ=0
            T12=DARCOS(TC3/R12)*RTD
            CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
          CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
            DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
            DOT=DOT/R12
C DOP12 IS THE CROSSLINK DOPPLER SHIFT IN HZ.
            DOP12=-DOT*FR/CL
            WRITE(6,50)RI,R12,F12,T12,DOP12,NOVIZ,M1,M2
  50        FORMAT(1H ,F5.1,F17.3,F14.3,F15.3,F20.3,I9,I10,I10)
  300       CONTINUE
            WRITE(6,60)
  60        FORMAT(1H ,//)
  250       CONTINUE
  350       CONTINUE
            END
        SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
            IMPLICIT REAL*8(A-H,O-Z)
        A11=DCOS(FSP)*DCOS(WS)-DCOS(XIS)*DSIN(WS)*DSIN(FSP)
        A12=-DSIN(FSP)*DCOS(WS)-DCOS(XIS)*DSIN(WS)*DCOS(FSP)
        A21=DCOS(FSP)*DSIN(WS)+DCOS(XIS)*DCOS(WS)*DSIN(FSP)
        A22=-DSIN(FSP)*DSIN(WS)+DCOS(XIS)*DCOS(WS)*DCOS(FSP)
        A31=DSIN(XIS)*DSIN(FSP)
        A32=DSIN(XIS)*DCOS(FSP)
        XS=A11*XPS+A12*YPS
        YS=A21*XPS+A22*YPS
        ZS=A31*XPS+A32*YPS
        RETURN
         END
        SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
            IMPLICIT REAL*8(A-H,O-Z)
        P=3.1415926
        Z=2.*P*(T-TP)/TAU
        S5M=DSIN(5.*Z)
        S6M=DSIN(6.*Z)
        S7M=DSIN(7.*Z)
        C5M=DCOS(5.*Z)
        C6M=DCOS(6.*Z)
        C7M=DCOS(7.*Z)
        SM=DSIN(Z)
        S2M=DSIN(2.*Z)
        S3M=DSIN(3.*Z)
```

```
S4M=DSIN(4.*Z)
CM=DCOS(Z)
FSP=Z+2.*E*SM+1.25*(E**2)*S2M+((E**3)/12.)*(13.*S3M-3.*SM)
FSP=FSP+WP+((E**4)/96.)*(103.*S4M-44.*S2M)
F5=((E**5)/960.)*(1097.*S5M-645.*S3M+50.*SM)
F6=((E**6)/960.)*(1223.*S6M-902.*S4M+85.*S2M)
F7=((E**7)/32256.)*(47273.*S7M-41699.*S5M+5985.*S3M+749.*CM)
 FSP=FSP+F5+F6+F7
R=(A*(1.-E**2))/(1.+E*DCOS(FSP-WP))
RETURN
END
SUBROUTINE DOPE(R,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
    IMPLICIT REAL*8(A-H,O-Z)
P=3.1415926
T=FSP-WP
RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*DSIN(T)
TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*DCOS(T))**2
XPC=RD*DCOS(FSP)-R*TD*DSIN(FSP)
XPC=XPC/3600.
YPC=RD*DSIN(FSP)+R*TD*DCOS(FSP)
YPC=YPC/3600.
CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
RETURN
END
```

****

APPENDIX 3

PROGRAM SATVIZE

SATVIZE is an interactive FORTRAN program which queries the TSO user for the number of satellites, number of ground stations, orbital elements of each satellite, location of each ground station, and frequencies. Default values are included in the program to allow the undecided user a chance to study the form of the output.

Arbitrary Keplerian elements can be entered, with $0 \leq$ eccentricity ≤ 0.99.

```
      DIMENSION A(11),E(11),W(11),WP(11),TP(11),XI(11),C(3),RL(3)
      J1=0
      DATA A/0.,22767.,22767.,22767.,22767.,36140.36,36140.36,
     1 36140.36,36140.36,14342.3,14342.3/
      DATA E/0.,.1,.1,.1,.1,.1,.1,.1,.1,.1,.1/
      DATA W/243.733,243.733,243.733,243.733,243.733,243.733,243.733
     2,243.733,243.733,243.733,243.733/
      DATA WP/0.,0.,0.,0.,0.,0.,0.,0.,0.,-90.0,-90.0/
      DATA TP/0.,0.,-6.,-12.,-18.,0.,-12.,-24.,-36.,-.75,-6.75/
      DATA XI/23.4,23.4,23.4,23.4,23.4,113.4,113.4,113.4,113.4,63.4,
     163.4/
      DATA C /0.,47.54,57.23/
      DATA RL/0.,288.73,242.8/
      WRITE(6,110)
110   FORMAT(1H ,///)
      WRITE(6,115)
115   FORMAT(1H ,'THIS SATELLITE VISIBILITY PROGRAM IS WRITTEN FOR A
     1 MAXIMUM OF 10 SATELLITES AND 2 GROUND STATIONS.',//)
      WRITE(6,120)
120   FORMAT(1H ,'FOR EACH SATELLITE THE FOLLOWING DATA MUST BE SUPP
     1LIED:',/,' SEMIMAJOR AXIS',/,' ECCENTRICITY',/,' RIGHT ASCENSION',
     1/,' ARGUMENT OF PERIGEE',/,' TIME OF PERIGEE',/,' INCLINATION',//)
       WRITE(6,125)
125    FORMAT(1H ,'FOR EACH GROUND STATION THE FOLLOWING DATA MUST B
     1E SUPPLIED:',/,' COLATITUDE',/,' LONGITUDE',//)
      WRITE(6,142)
142   FORMAT(1H ,'ADDITIONAL DATA WHICH MUST BE SUPPLIED:',/,' UPLIN
     1K FREQUENCY',/,' DOWNLINK FREQUENCY',/,' CROSSLINK FREQUENCY',//)
      WRITE(6,130)
130   FORMAT(1H ,'THE USER IS ASKED TO ANSWER SEVERAL QUESTIONS.',/,
     1' IF THE USER DOES NOT KNOW THE VALUE OF AN INPUT VARIABLE,',/,' HO
     1E SHOULD ENTER THE NUMBER 400.0.  A DEFAULT VALUE WILL THEN BE USE
     1D.',/,' THE USER IS EXPECTED,HOWEVER,TO SPECIFY THE NUMBER OF SATE
     1LLITES AND GROUND STATIONS TO BE CONSIDERED.',////)
      WRITE(6,145)
145   FORMAT(1H ,'HOW MANY SATELLITES ARE TO BE CONSIDERED? ENTER IN
     1TEGER WITH RIGHTMOST DIGIT IN COLUMN 2.')
      READ 150, M
      WRITE(6,135)M
135   FORMAT(1H ,I3)
      WRITE(6,155)
155   FORMAT(1H ,'HOW MANY GROUND STATIONS ARE TO BE CONSIDERED? ENT
     1ER INTEGER IN COLUMN 2.')
      READ 150, N
      WRITE(6,135) N
150   FORMAT(I2)
      WRITE(6,25)
       WRITE(6,25)
      WRITE(6,95)
95    FORMAT(1H ,'ENTER FOLLOWING DATA IN DECIMAL FORM ANYWHERE IN T
     1HE FIRST 20 COLUMNS.',//)
      WRITE(6,140)
140   FORMAT(1H ,'WHAT IS THE SEMIMAJOR AXIS VALUE IN NAUTICAL MILES
     1 FOR SATELLITE 1?')
      DO 101 I=1,M
      K=I+1
      READ 170, A(I)
      IF(A(I).NE.400.0)GO TO 605
      A(I)=A(K)
      J1=1
605   WRITE(6,305) A(I)
```

```
 305     FORMAT(1H ,F20.8)
         IF(I.EQ.M)GO TO 101
         PRINT 310,K
 310     FORMAT(1H ,'FOR SATELLITE',I3,'?')
 101     CONTINUE
         WRITE(6,315)
 315     FORMAT(1H ,/,' WHAT IS THE ECCENTRICITY OF SATELLITE 1?')
         DO 103 I=1,M
         K=I+1
         READ 170, E(I)
         IF(E(I).NE.400.0)GO TO 610
        E(I)=E(K)
         J1=1
 610     WRITE(6,305)E(I)
         IF(I.EQ.M)GO TO 103
         PRINT 310, K
 103     CONTINUE
         WRITE(6,320)
 320     FORMAT(1H ,/,' WHAT IS THE RIGHT ASCENSION VALUE IN DEGREES FO
     1R SATELLITE 1?')
         DO 104 I=1,M
         K=I+1
         READ 170, W(I)
         IF(W(I).NE.400.0)GO TO 615
         W(I)=W(K)
         J1=1
 615     WRITE(6,305)W(I)
         IF(I.EQ.M)GO TO 104
         PRINT 310, K
 104     CONTINUE
         WRITE(6,325)
 325     FORMAT(1H ,/,' WHAT IS THE ARGUMENT OF PERIGEE VALUE IN DEGREE
     1S FOR SATELLITE 1?')
         DO 106 I=1,M
         K=I+1
         READ 170, WP(I)
         IF(WP(I).NE.400.0)GO TO 620
         WP(I)=WP(K)
         J1=1
 620     WRITE(6,305)WP(I)
         IF(I.EQ.M)GO TO 106
         PRINT 310, K
 106     CONTINUE
         WRITE(6,330)
 330     FORMAT(1H ,/,' WHAT IS THE TIME OF PERIGEE IN HOURS FOR SATELL
     1ITE 1?')
         DO 107 I=1,M
         K=I+1
         READ 170, TP(I)
         IF(TP(I).NE.400.0)GO TO 625
         TP(I)=TP(K)
         J1=1
 625     WRITE(6,305)TP(I)
         IF(I.EQ.M)GO TO 107
         PRINT 310, K
 107     CONTINUE
         WRITE(6,335)
 335     FORMAT(1H ,/,' WHAT IS THE INCLINATION VALUE IN DEGREES FOR SA
     1TELLITE 1?')
         DO 108 I=1,M
         K=I+1
```

```
      READ 170, XI(I)
      IF(XI(I).NE.400.0)GO TO 630
      XI(I)=XI(K)
      J1=1
630   WRITE(6,305)XI(I)
      IF(I.EQ.M)GO TO 108
      PRINT 310, K
108   CONTINUE
      WRITE(6,340)
340   FORMAT(1H ,//,' WHAT IS THE COLATITUDE IN DEGREES FOR GROUND S
     1TATION 1?')
      DO 102 I=1,N
       K=I+1
      READ 170, C(I)
      IF(C(I).NE.400.)GO TO 635
      C(I)=C(K)
      J1=1
635   WRITE(6,305)C(I)
      IF(I.EQ.N)GO TO 102
      PRINT 205, K
205   FORMAT(1H ,'FOR GROUND STATION',I3,'?')
102   CONTINUE
      WRITE(6,345)
345   FORMAT(1H ,/,' WHAT IS THE LONGITUDE IN DEGREES FOR GROUND STA
     1TION 1?')
     DO 109 I=1,N
      K=I+1
      READ 170, RL(I)
170   FORMAT(F20.8)
      IF(RL(I).NE.400.0)GO TO 640
      RL(I)=RL(K)
      J1=1
640   WRITE(6,305)RL(I)
      IF(I.EQ.N)GO TO 109
      PRINT 205, K
109   CONTINUE
      WRITE(6,840)
840   FORMAT(1H ,////,' ENTER THE FOLLOWING DATA IN EXPONENTIAL FORM
     2 (EG. 300.0*10.**6 IS WRITTEN AS 300.0E6)',/,' WITH THE RIGHTMOST
     2DIGIT APPEARING IN COLUMN 10.',//)
      WRITE(6,845)
845   FORMAT(1H ,'WHAT IS THE UPLINK FREQUENCY IN HZ?')
      READ 850, FR1
850   FORMAT(E10.2)
      IF(FR1.EQ.400.0)FR1=300.0E6
      WRITE(6,855)FR1
855   FORMAT(1H ,E20.5)
      WRITE(6,860)
860   FORMAT(1H ,/,' WHAT IS THE DOWNLINK FREQUENCY IN HZ?')
      READ 850, FR2
      IF(FR2.EQ.400.0)FR2=245.0E6
      WRITE(6,855)FR2
      WRITE(6,865)
865   FORMAT(1H ,/,' WHAT IS THE CROSSLINK FREQUENCY IN HZ?')
      READ 850, FR3
      IF(FR3.EQ.400.0)FR3=38.0E9
      WRITE(6,855)FR3
      IF(J1.EQ.0)GO TO 660
      WRITE(6,670)
670   FORMAT(1H ,//,' YOU HAVE NOT USED A COMPLETE DATA SET.',/,' TH
     1EREFORE, THE RESULTS ARE FOR DEMONSTRATION PURPOSES ONLY.')
```

```
      GO TO 675
660   WRITE(6,665)
665   FORMAT(1H ,//,' YOUR DATA SET IS COMPLETE.')
675   WRITE(6,105)
105   FORMAT(1H ,///)
      P=3.1415926
      RE=3440.
      RTD=57.29577951
      WRAD=15.0/RTD
      CL=1.61984*10.**5
      CON=6.987*10.**(-6)
      WRITE(6,190)
190   FORMAT(1H ,'RESULTS LISTED IN ORDER OF APPEARANCE ARE:',/,' RA
     2NGE BETWEEN GROUND STATION AND SATELLITE IN NAUTICAL MILES',/,' EL
     2EVATION ANGLE IN DEGREES FROM GROUND STATION TO SATELLITE',/,' UPL
     2INK DOPPLER SHIFT IN HZ',/,' DOWNLINK DOPPLER SHIFT IN HZ',/,' SUB
     2SATELLITE COLATITUDE IN DEGREES',/,' SUBSATELLITE LONGITUDE IN DEG
     2REES',///)
      DO 890 IN=1,M
      W(IN)=W(IN)/RTD
      WP(IN)=WP(IN)/RTD
      XI(IN)=XI(IN)/RTD
890   CONTINUE
      DO 900 J=1,M
      TAU=CON*A(J)**1.5
      DO 700 K=1,N
      WRITE(6,10)
10    FORMAT(1H ,'SAT',5X,'HRS',12X,'RANGE',9X,'ANGLE',8X,'UPDOPPLER
     1',8X,'DNDOPPLER',9X,'SUBC',9X,'SUBL',6X,'STATION',//)
      T=C(K)/RTD
      G=RL(K)/RTD
      DO 500 I=1,25
      RI=I
      CALL ELLIP(RI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
      CALL PRIME(FSP,W(J),XI(J),R,0.,XS,YS,ZS)
      FS=(ATAN(YS/XS)-WRAD*RI)*RTD
      IF(XS.LT.0.)FS=FS+P*RTD
      TS=RTD*ARCOS(ZS/R)
      F=G+WRAD*RI
      X=RE*SIN(T)*COS(F)
      Y=RE*SIN(T)*SIN(F)
      Z=RE*COS(T)
      ACC=ARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
      RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
      ARG=SIN(ACC)*R/RGE
      IF(ARG.GT.1.0000000)ARG=1.00000000
      D=(ARSIN(ARG)-P/2.)*RTD
      RT=SQRT(R*R-RE*RE)
      IF(RGE.LE.RT)D=-D
      CALL DOPE(R,E(J),A(J),TAU,FSP,WP(J),W(J),XI(J),XV,YV,ZV)
      XT=-WRAD*RE*SIN(T)*SIN(F)/3600.
      YT=WRAD*RE*SIN(T)*COS(F)/3600.
      DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+ZV*(ZS-Z)
      DOT=DOT/RGE
      UPDOP=-DOT*FR1/CL
      DNDOP=-DOT*FR2/CL
      WRITE(6,20)J,RI,RGE,D,UPDOP,DNDOP,TS,FS,K
20    FORMAT(1H ,I3,F8.1,F17.3,F14.3,F17.3,F17.3,F13.3,F13.3,I10)
500   CONTINUE
      WRITE(6,25)
```

```
25        FORMAT(1H ,//)
          IF(J1.EQ.0)GO TO 700
          WRITE(6,680)
680       FORMAT(1H ,'DO YOU WANT THIS PART OF THE PROGRAM TO CONTINUE?'
     1,/,' ENTER AN INTEGER 0 IF THE ANSWER IS NO OR AN INTEGER 1 IF THE
     1 ANSWER IS YES IN COLUMN 2.')
          READ 150, NO
          IF(NO.EQ.0)GO TO 685
          WRITE(6,25)
700       CONTINUE
900       CONTINUE
685       WRITE(6,30)
30        FORMAT(1H ,/////)
          L=M-1
          IF(L.EQ.0)GO TO 690
          WRITE(6,260)
260       FORMAT(1H ,'THIS PART OF THE PROGRAM COMPUTES CROSSLINK VALUES
     7 FOR ALL PAIRS OF SATELLITES.',///,' RESULTS LISTED IN ORDER OF AP
     1PEARANCE ARE:',/,' RANGE BETWEEN TWO SATELLITES IN NAUTICAL MILES'
     1,/,' POINTING ANGLES BETWEEN SATELLITES(C12&L12) IN DEGREES',/,' D
     2OPPLER SHIFT EXPERIENCED AT SECOND SATELLITE DUE TO SIGNAL TRANSMI
     2TTED FROM THE FIRST SATELLITE IN HZ.',/,' CROSSLINK VISIBILITY WHE
     2RE 0 MEANS THERE IS NO VISIBILITY BETWEEN SATELLITES AND 1 MEANS T
     2HERE IS VISIBILITY',///)
          DO 350 M1=1,L
          TAU=CON*A(M1)**1.5
          N1=M1+1
          DO 250 M2=N1,M
          WRITE(6,40)
40        FORMAT(1H ,2X,'HRS',12X,'RANGE',11X,'C12',12X,'L12',11X,
     2'CFDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
          TAU2=CON*A(M2)**1.5
          DO 300 I=1,25
          RI=I
          NOVIZ=1
          CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
          CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
          CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
          CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
          TC1=X2-XS
          TC2=Y2-YS
          TC3=Z2-ZS
          R12=(TC1**2+TC2**2+TC3**2)**.5
          F12=RTD*ATAN(TC2/TC1)
          IF(TC1.LT.0.) F12=F12+P*RTD
          T12=ARCOS(TC3/R12)*RTD
          VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
          IF(VIZ.GT..9999999) VIZ=.9999999
          DEL=ARCOS(VIZ)
          DELM=ARSIN(RE/R)
          IF(DEL.LE.DELM)NOVIZ=0
          CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
        CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
          DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
          DOT=DOT/R12
          DOP12=-DOT*FR3/CL
          WRITE(6,50)RI,R12,T12,F12,DOP12,NOVIZ,M1,M2
50        FORMAT(1H ,F5.1,F17.3,F14.3,F15.3,F20.3,I9,I10,I10)
300       CONTINUE
          WRITE(6,60)
```

AD-A037 118 MITRE CORP BEDFORD MASS F/G 17/2

HIGH ALTITUDE SATELLITE COMMUNICATIONS, WITH CROSSLINKS.(U)

JAN 77 P F CHRISTOPHER, E R EDELMAN F19628-76-C-0001

UNCLASSIFIED MTR-3161 ESD-TR-76-363 NL

2 OF 2

AD A037 118

END

DATE FILMED

4-77

```
60      FORMAT(1H ,//)
        IF(J1.EQ.0)GO TO 250
        WRITE(6,680)
        READ 150, NO
        IF(NO.EQ.0)GO TO 690
        WRITE(6,25)
250     CONTINUE
350     CONTINUE
690     CONTINUE
        END
      SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
      A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
      A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
      XS=A11*XPS+A12*YPS
      YS=A21*XPS+A22*YPS
      ZS=A31*XPS+A32*YPS
      RETURN
       END
      SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
      P=3.1415926
       Z=2.*P*(T-TP)/TAU
      P2=2.*P
2     IF(Z.GT.P2)Z=Z-P2
      IF(Z.GT.P2)GO TO 2
      E1=Z+E*SIN(Z)
      E2=(Z+E*(SIN(E1))-(E*COS(E1))*E1)/(1.-E*COS(E1))
      Q=0.
4     E3=(Z+E*(SIN(E2))-(E*COS(E2))*E2)/(1.-E*COS(E2))
      Q=Q+1.
       DE=E3-E2
      DE2=DE**2
      E2=E3
      IF(DE2.GT..00000001)GO TO 4
      TH=ARCOS((COS(E2)-E)/(1.-E*COS(E2)))
      IF(Z.GT.P)TH=2.*P-TH
      FSP=WP+TH
      R=(A*(1.-E**2))/(1.+E*COS(FSP-WP))
      RETURN
      END
      SUBROUTINE DOPE(R,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
      P=3.1415926
      T=FSP-WP
      RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*SIN(T)
      TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*COS(T))**2
      XPC=RD*COS(FSP)-R*TD*SIN(FSP)
      XPC=XPC/3600.
      YPC=RD*SIN(FSP)+R*TD*COS(FSP)
      YPC=YPC/3600.
      CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
      RETURN
      END
```

****

APPENDIX 4

PROGRAM SATVIZD

This program is similar to SATVIZE but it gives results in double precision. Its CPU time requirements can be more than 50% greater than SATVIZE, so its use has been infrequent and specialized.

The subroutine ELLIP listed here should not be confused with the ELLIP routines of the other appendices. It contains the expansion derived by F. R. Moulton[3] for true anomaly, and is accurate only for eccentricity ≤ 0.5.

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(11),E(11),W(11),WP(11),TP(11),XI(11),C(3),RL(3)
      J1=0
      DATA A/0.,22767.,22767.,22767.,22767.,36140.36,36140.36,
     1 36140.36,36140.36,14342.3,14342.3/
      DATA E/0.,.1,.1,.1,.1,.1,.1,.1,.1,.1,.1/
      DATA W/243.733,243.733,243.733,243.733,243.733,243.733,243.733
     2,243.733,243.733,243.733,243.733/
      DATA WP/0.,0.,0.,0.,0.,0.,0.,0.,0.,-90.0,-90.0/
      DATA TP/0.,0.,-6.,-12.,-18.,0.,-12.,-24.,-36.,-.75,-6.75/
      DATA XI/23.4,23.4,23.4,23.4,23.4,113.4,113.4,113.4,113.4,63.4,
     163.4/
      DATA C /0.,47.54,57.23/
      DATA RL/0.,288.73,242.8/
      WRITE(6,110)
110   FORMAT(1H ,///)
      WRITE(6,115)
115   FORMAT(1H ,'THIS SATELLITE VISIBILITY PROGRAM IS WRITTEN FOR A
     1 MAXIMUM OF 10 SATELLITES AND 2 GROUND STATIONS.',//)
      WRITE(6,120)
120   FORMAT(1H ,'FOR EACH SATELLITE THE FOLLOWING DATA MUST BE SUPP
     1LIED:',/,' SEMIMAJOR AXIS',/,' ECCENTRICITY',/,' RIGHT ASCENSION',
     1/,' ARGUMENT OF PERIGEE',/,' TIME OF PERIGEE',/,' INCLINATION',//)
       WRITE(6,125)
125    FORMAT(1H ,'FOR EACH GROUND STATION THE FOLLOWING DATA MUST B
     1E SUPPLIED:',/,' COLATITUDE',/,' LONGITUDE',//)
      WRITE(6,142)
142   FORMAT(1H ,'ADDITIONAL DATA WHICH MUST BE SUPPLIED:',/,' UPLIN
     1K FREQUENCY',/,' DOWNLINK FREQUENCY',/,' CROSSLINK FREQUENCY',//)
       WRITE(6,130)
130   FORMAT(1H ,'THE USER IS ASKED TO ANSWER SEVERAL QUESTIONS.',/,
     1' IF THE USER DOES NOT KNOW THE VALUE OF AN INPUT VARIABLE,',/,' H
     1E SHOULD ENTER THE NUMBER 400.0.  A DEFAULT VALUE WILL THEN BE USE
     1D.',/,' THE USER IS EXPECTED,HOWEVER,TO SPECIFY THE NUMBER OF SATE
     1LLITES AND GROUND STATIONS TO BE CONSIDERED.',////)
      WRITE(6,145)
145   FORMAT(1H ,'HOW MANY SATELLITES ARE TO BE CONSIDERED? ENTER IN
     1TEGER WITH RIGHTMOST DIGIT IN COLUMN 2.')
      READ 150, M
      WRITE(6,135)M
135   FORMAT(1H ,I3)
      WRITE(6,155)
155   FORMAT(1H ,'HOW MANY GROUND STATIONS ARE TO BE CONSIDERED? ENT
     1ER INTEGER IN COLUMN 2.')
      READ 150, N
      WRITE(6,135) N
150   FORMAT(I2)
      WRITE(6,25)
       WRITE(6,25)
      WRITE(6,95)
95    FORMAT(1H ,'ENTER FOLLOWING DATA IN DECIMAL FORM ANYWHERE IN T
     1HE FIRST 20 COLUMNS.',//)
      WRITE(6,140)
140   FORMAT(1H ,'WHAT IS THE SEMIMAJOR AXIS VALUE IN NAUTICAL MILES
     1 FOR SATELLITE 1?')
      DO 101 I=1,M
      K=I+1
      READ 170, A(I)
      IF(A(I).NE.400.0)GO TO 605
      A(I)=A(K)
      J1=1
```

```
605    WRITE(6,305) A(I)
305    FORMAT(1H ,F20.8)
       IF(I.EQ.M)GO TO 101
       PRINT 310,K
310    FORMAT(1H ,'FOR SATELLITE',I3,'?')
101    CONTINUE
       WRITE(6,315)
315    FORMAT(1H ,/,' WHAT IS THE ECCENTRICITY OF SATELLITE 1?')
       DO 103 I=1,M
       K=I+1
       READ 170, E(I)
       IF(E(I).NE.400.0)GO TO 610
       E(I)=E(K)
       J1=1
610    WRITE(6,305)E(I)
       IF(I.EQ.M)GO TO 103
       PRINT 310, K
103    CONTINUE
       WRITE(6,320)
320    FORMAT(1H ,/,' WHAT IS THE RIGHT ASCENSION VALUE IN DEGREES FO
      1R SATELLITE 1?')
       DO 104 I=1,M
       K=I+1
       READ 170, W(I)
       IF(W(I).NE.400.0)GO TO 615
       W(I)=W(K)
       J1=1
615    WRITE(6,305)W(I)
       IF(I.EQ.M)GO TO 104
       PRINT 310, K
104    CONTINUE
       WRITE(6,325)
325    FORMAT(1H ,/,' WHAT IS THE ARGUMENT OF PERIGEE VALUE IN DEGREE
      1S FOR SATELLITE 1?')
       DO 106 I=1,M
       K=I+1
       READ 170, WP(I)
       IF(WP(I).NE.400.0)GO TO 620
       WP(I)=WP(K)
       J1=1
620    WRITE(6,305)WP(I)
       IF(I.EQ.M)GO TO 106
       PRINT 310, K
106    CONTINUE
       WRITE(6,330)
330    FORMAT(1H ,/,' WHAT IS THE TIME OF PERIGEE IN HOURS FOR SATELL
      1ITE 1?')
       DO 107 I=1,M
       K=I+1
       READ 170, TP(I)
       IF(TP(I).NE.400.0)GO TO 625
       TP(I)=TP(K)
       J1=1
625    WRITE(6,305)TP(I)
       IF(I.EQ.M)GO TO 107
       PRINT 310, K
107    CONTINUE
       WRITE(6,335)
335    FORMAT(1H ,/,' WHAT IS THE INCLINATION VALUE IN DEGREES FOR SA
      1TELLITE 1?')
       DO 108 I=1,M
```

```
      K=I+1
      READ 170, XI(I)
      IF(XI(I).NE.400.0)GO TO 630
      XI(I)=XI(K)
      J1=1
630   WRITE(6,305)XI(I)
      IF(I.EQ.N)GO TO 108
      PRINT 310, K
108   CONTINUE
      WRITE(6,340)
340   FORMAT(1H ,//,' WHAT IS THE COLATITUDE IN DEGREES FOR GROUND S
     1TATION 1?')
      DO 102 I=1,N
       K=I+1
      READ 170, C(I)
      IF(C(I).NE.400.)GO TO 635
      C(I)=C(K)
      J1=1
635   WRITE(6,305)C(I)
      IF(I.EQ.N)GO TO 102
      PRINT 205, K
205   FORMAT(1H ,'FOR GROUND STATION',I3,'?')
102   CONTINUE
      WRITE(6,345)
345   FORMAT(1H ,/,' WHAT IS THE LONGITUDE IN DEGREES FOR GROUND STA
     1TION 1?')
     DO 109 I=1,N
      K=I+1
      READ 170, RL(I)
170   FORMAT(F20.8)
      IF(RL(I).NE.400.0)GO TO 640
      RL(I)=RL(K)
      J1=1
640   WRITE(6,305)RL(I)
      IF(I.EQ.N)GO TO 109
      PRINT 205, K
109   CONTINUE
      WRITE(6,840)
840   FORMAT(1H ,////,' ENTER THE FOLLOWING DATA IN EXPONENTIAL FORM
     2 (EG. 300.0*10.**6 IS WRITTEN AS 300.0D6)',/,' WITH THE RIGHTMOST
     2DIGIT APPEARING IN COLUMN 10.',//)
      WRITE(6,845)
845   FORMAT(1H ,'WHAT IS THE UPLINK FREQUENCY IN HZ?')
      READ 850, FR1
850   FORMAT(D10.2)
      IF(FR1.EQ.400.0)FR1=300.0D6
      WRITE(6,855)FR1
855   FORMAT(1H ,D20.5)
      WRITE(6,860)
860   FORMAT(1H ,/,' WHAT IS THE DOWNLINK FREQUENCY IN HZ?')
      READ 850, FR2
      IF(FR2.EQ.400.0)FR2=245.0D6
      WRITE(6,855)FR2
      WRITE(6,865)
865   FORMAT(1H ,/,' WHAT IS THE CROSSLINK FREQUENCY IN HZ?')
      READ 850, FR3
      IF(FR3.EQ.400.0)FR3=38.0D9
      WRITE(6,855)FR3
      IF(J1.EQ.0)GO TO 660
      WRITE(6,670)
670   FORMAT(1H ,//,' YOU HAVE NOT USED A COMPLETE DATA SET.',/,' TH
```

```
     'EREFORE, THE RESULTS ARE FOR DEMONSTRATION PURPOSES ONLY.')
      GO TO 675
660   WRITE(6,665)
665   FORMAT(1H ,//,' YOUR DATA SET IS COMPLETE.')
675   WRITE(6,105)
105   FORMAT(1H ,///)
      P=3.1415926
      RE=3440.
      RTD=57.29577951
      WRAD=15.0/RTD
      CL=1.61984*10.**5
      CON=6.987*10.**(-6)
      WRITE(6,190)
190   FORMAT(1H ,'RESULTS LISTED IN ORDER OF APPEARANCE ARE:',/,' RA
     2NGE BETWEEN GROUND STATION AND SATELLITE IN NAUTICAL MILES',/,' EL
     2EVATION ANGLE IN DEGREES FROM GROUND STATION TO SATELLITE',/,' UPL
     2INK DOPPLER SHIFT IN HZ',/,' DOWNLINK DOPPLER SHIFT IN HZ',/,' SUB
     2SATELLITE COLATITUDE IN DEGREES',/,' SUBSATELLITE LONGITUDE IN DEG
     2REES',///)
      DO 890 IN=1,M
      W(IN)=W(IN)/RTD
      WP(IN)=WP(IN)/RTD
      XI(IN)=XI(IN)/RTD
890   CONTINUE
      DO 900 J=1,M
      TAU=CON*A(J)**1.5
      DO 700 K=1,N
      WRITE(6,10)
10    FORMAT(1H ,'SAT',5X,'HPS',12X,'RANGE',9X,'ANGLE',8X,'UPDOPPLER
     1',8X,'DNDOPPLER',9X,'SUBC',9X,'SUBL',6X,'STATION',//)
      T=C(K)/RTD
      G=RL(K)/RTD
      DO 500 I=1,25
      RI=I
      CALL ELLIP(RI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
      CALL PRIME(FSP,W(J),XI(J),R,0.,XS,YS,ZS)
      FS=(DATAN(YS/XS)-WRAD*RI)*RTD
      IF(XS.LT.0.) FS=FS+P*RTD
      TS=RTD*DARCOS(ZS/R)
      F=G+WRAD*RI
      X=RE*DSIN(T)*DCOS(F)
      Y=RE*DSIN(T)*DSIN(F)
      Z=RE*DCOS(T)
      ACC=DARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
      RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
      ARG=DSIN(ACC)*R/RGE
      IF(ARG.GT.1.00000000) ARG=1.00000000
      D=(DARSIN(ARG)-P/2.)*RTD
      RT=DSQRT(R*R-RE*RE)
      IF(RGE.LE.RT) D=-D
      CALL DOPE(R,E(J),A(J),TAU,FSP,WP(J),W(J),XI(J),XV,YV,ZV)
      XT=-WRAD*RE*DSIN(T)*DSIN(F)/3600.
      YT=WRAD*RE*DSIN(T)*DCOS(F)/3600.
      DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+ZV*(ZS-Z)
      DOT=DOT/RGE
      UPDOP=-DOT*FR1/CL
      DNDOP=-DOT*FR2/CL
      WRITE(6,20)J,RI,RGE,D,UPDOP,DNDOP,TS,FS,K
20    FORMAT(1H ,I3,F8.1,F17.3,F14.3,F17.3,F17.3,F13.3,F13.3,I10)
500   CONTINUE
```

```
      WRITE(6,25)
25    FORMAT(1H ,//)
      IF(J1.EQ.0) GO TO 700
      WRITE(6,680)
680   FORMAT(1H ,'DO YOU WANT THIS PART OF THE PROGRAM TO CONTINUE?'
     1,/,' ENTER AN INTEGER 0 IF THE ANSWER IS NO OR AN INTEGER 1 IF THE
     1 ANSWER IS YES IN COLUMN 2.')
      READ 150, NO
      IF(NO.EQ.0) GO TO 685
      WRITE(6,25)
700   CONTINUE
900   CONTINUE
685   WRITE(6,30)
30    FORMAT(1H ,/////)
      L=M-1
      IF(L.EQ.0)GO TO 690
      WRITE(6,260)
260   FORMAT(1H ,'THIS PART OF THE PROGRAM COMPUTES CROSSLINK VALUES
     7 FOR ALL PAIRS OF SATELLITES.',///,' RESULTS LISTED IN ORDER OF AP
     1PEARANCE ARE:',/,' RANGE BETWEEN TWO SATELLITES IN NAUTICAL MILES'
     1,/,' POINTING ANGLES BETWEEN SATELLITES(C12&L12) IN DEGREES',/,' D
     2OPPLER SHIFT EXPERIENCED AT SECOND SATELLITE DUE TO SIGNAL TRANSMI
     2TTED FROM THE FIRST SATELLITE IN HZ.',/,' CROSSLINK VISIBILITY WHE
     2RE 0 MEANS THERE IS NO VISIBILITY BETWEEN SATELLITES AND 1 MEANS T
     2HERE IS VISIBILITY',///)
      DO 350 M1=1,L
      TAU=CON*A(M1)**1.5
      N1=M1+1
      DO 250 M2=N1,M
      WRITE(6,40)
40    FORMAT(1H ,2X,'HRS',12X,'RANGE',11X,'C12',12X,'L12',11X,
     2 'CRDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
      TAU2=CON*A(M2)**1.5
      DO 300 I=1,25
      RI=I
      NOVIZ=1
      CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
      CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
      CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
      CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
      TC1=X2-XS
      TC2=Y2-YS
      TC3=Z2-ZS
      R12=(TC1**2+TC2**2+TC3**2)**.5
      F12=RTD*DATAN(TC2/TC1)
      IF(TC1.LT.0.) F12=F12+P*RTD
      T12=DARCOS(TC3/R12)*RTD
      VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
      IF(VIZ.GT..9999999) VIZ=.9999999
      DEL=DARCOS(VIZ)
      DELM=DARSIN(RE/R)
      IF(DEL.LE.DELM) NOVIZ=0
      CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
    CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
      DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
      DOT=DOT/R12
      DOP12=-DOT*FR3/CL
      WRITE(6,50) RI,R12,T12,F12,DOP12,NOVIZ,M1,M2
50    FORMAT(1H ,F5.1,F17.3,F14.3,F15.3,F20.3,I9,I10,I10)
300   CONTINUE
```

```
      WRITE(6,60)
60    FORMAT(1H ,//)
      IF(J1.EQ.0)GO TO 250
      WRITE(6,680)
      READ 150, NO
      IF(NO.EQ.0)GO TO 690
      WRITE(6,25)
250   CONTINUE
350   CONTINUE
690   CONTINUE
      END
      SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
      IMPLICIT REAL*8(A-H,O-Z)
      A11=DCOS(FSP)*DCOS(WS)-DCOS(XIS)*DSIN(WS)*DSIN(FSP)
      A12=-DSIN(FSP)*DCOS(WS)-DCOS(XIS)*DSIN(WS)*DCOS(FSP)
      A21=DCOS(FSP)*DSIN(WS)+DCOS(XIS)*DCOS(WS)*DSIN(FSP)
      A22=-DSIN(FSP)*DSIN(WS)+DCOS(XIS)*DCOS(WS)*DCOS(FSP)
      A31=DSIN(XIS)*DSIN(FSP)
      A32=DSIN(XIS)*DCOS(FSP)
      XS=A11*XPS+A12*YPS
      YS=A21*XPS+A22*YPS
      ZS=A31*XPS+A32*YPS
      RETURN
       END
      SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
      IMPLICIT REAL*8(A-H,O-Z)
      P=3.1415926
      Z=2.*P*(T-TP)/TAU
      S5M=DSIN(5.*Z)
      S6M=DSIN(6.*Z)
      S7M=DSIN(7.*Z)
      C5M=DCOS(5.*Z)
      C6M=DCOS(6.*Z)
      C7M=DCOS(7.*Z)
      SM=DSIN(Z)
      S2M=DSIN(2.*Z)
      S3M=DSIN(3.*Z)
      S4M=DSIN(4.*Z)
      CM=DCOS(Z)
      FSP=Z+2.*E*SM+1.25*(E**2)*S2M+((E**3)/12.)*(13.*S3M-3.*SM)
      FSP=FSP+WP+((E**4)/96.)*(103.*S4M-44.*S2M)
      F5=((E**5)/960.)*(1097.*S5M-645.*S3M+50.*SM)
      F6=((E**6)/960.)*(1223.*S6M-902.*S4M+85.*S2M)
      F7=((E**7)/32256.)*(47273.*S7M-41699.*S5M+5985.*S3M+749.*CM)
       FSP=FSP+F5+F6+F7
      R=(A*(1.-E**2))/(1.+E*DCOS(FSP-WP))
      RETURN
      END
      SUBROUTINE DOPE(P,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
      IMPLICIT REAL*8(A-H,O-Z)
      P=3.1415926
      T=FSP-WP
      RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*DSIN(T)
      TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*DCOS(T))**2
      XPC=RD*DCOS(FSP)-R*TD*DSIN(FSP)
      XPC=XPC/3600.
      YPC=RD*DSIN(FSP)+R*TD*DCOS(FSP)
      YPC=YPC/3600.
      CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
      RETURN
```

END 00003580

****

APPENDIX 5

PROGRAM PERTP

PERTP calculates the secular changes in argument of perigee, right ascension, eccentricity, and semi-major axis as a function of lunar perturbations. A normalized apogee distance (V_5) is plotted on the CALCOMP plotter as a function of time.

Unlike Appendices 1 through 4 which require distances in nautical miles, PERTP requires distances in kilometers.

```
//PERTP JOB (6360,D91,DESK),'EDELMAN F',NOTIFY=TS0141,
// CLASS=F,TIME=2,TYPRUN=HOLD
/*SETUP    TAPE     PLXXXX
// EXEC FORTGCLG
//FORT.SYSIN DD *
C    THIS PROGRAM PRODUCES  THE SECULAR VARIANCE OF ,AN INITIALLY,
C  HIGH ALTITUDE ORBIT, AND AT THE SAME TIME GENERATES STABILITY OF
C  ECCENTRICITY, RIGHT ASCENSION AND ARGUMENT OF PERIGEE. IT ALSO
C  PRODUCES THE EVER CHANGING SEMI-MAJOR AXIS.
C  AND THEN,USING THE CALCOMP PLOTTER
C  PRODUCES PLOTS FOR DIFFERENT ANGLES
C  OF INCLINATION,GRAPHING V5 VS. DAYS.
C   ALL VALUES IN THIS PROGRAM HAVE BEEN
C  CONVERTED TO THE METRIC SYSTEM,
C  WHERE 1 NAUTICAL MILE=1.852 KM.
      DIMENSION IBUF(2000),XARRAY(42),YARRAY(42)
      CON=2.77218*10.**(-6)
      PI=3.1415926
      RTD=57.29577951
      RHO=383364.0000
      DO 200 J=45,135,45
      RJ=J
      XI=RJ
      AO=256452.9085
      TAU=CON*AO**1.5
      V=(24./TAU)*10.
      EO=.15
      V5O=AO*(1.+EO)
      XI=XI/RTD
      W=0.
      WP=45.
      W=W/RTD
      WP=WP/RTD
      CALL ARGPER(AO,XI,WP,WS,V1)
      IF(V1.GT.0.) WP=WS
      XID=XI*RTD
      WRITE(6,10)
 10   FORMAT(1H ,9X,'E',17X,'WD',15X,'WPD',17X,'DE',11X,'V5',15X,
     1 'DWPD',9X,'XI',6X,'DAYS')
      WRITE(6,15) XID
 15   FORMAT(1H ,105X,F6.2)
      DO 100 I=1,40
      RI=I
      IF(I.EQ.1) E=EO
      DAYS=RI*10.
      XARRAY(I)=DAYS
      IF(I.GT.1) AO=A
      CALL MOON(AO,XI,A)
      UPA3=PI*(1./81.)*((A/RHO)**3)
      A2=((A/RHO)**2)
      A4=((A/RHO)**4)
      CI2=(COS(XI))**2
      CI4=(COS(XI))**4
      SP2=(SIN(WP))**2
      SI2=(SIN(XI))**2
      PAR1=((-135./128.)+(315./128.)*CI2)
      PAR2=((2625./2048.)-(7875./1024.)*CI2+(17325./2048.)*CI4)
      PAR3=((315./128.)-(315./16.)*CI2+(2205./128.)*CI4)
      PAR4=(3.-(15./2.)*SP2*SI2)
      PAR5=((225./32.)-(315./8.)*SP2)
      PAR6=((-45./32.)+(315./64.)*SP2+PAR5*CI2+(2205./64.)*SP2*CI4)
```

```
      DW=-2*UPA3*COS(XI)*((3./4.)+A2*PAR1+(A4*PAR2))*V
      DE=-UPA3*E*SIN(2*WP)*((-15./4.)*SI2+A2*PAR3)*V
      DWP=UPA3*(PAR4+A2*PAR6)*V
      W=W+DW
      E=E+DE
      V5=(A*(1.+E))/V50
      WP=WP+DWP
      WD=W*RTD
      WPD=WP*RTD
      DWD=DW*RTD
      DWPD=DWP*RTD
      YARRAY(I)=V5
      WRITE(6,20)E,WD,WPD,DE,V5,DWPD,DAYS
 20   FORMAT(1H ,6E17.7,F17.2)
100   CONTINUE
      WRITE(6,25)
 25   FORMAT(1H ,//)
      CALL PLOTS(IBUF,2000,6)
      CALL PLOT(0.0,1.0,-3)
      CALL SCALE(XARRAY,10.0,40,1)
      CALL SCALE(YARRAY,8.0,40,1)
      CALL AXIS(0.0,0.0,12HTIME IN DAYS,-12,10.0
     1 ,0.0,XARRAY(41),XARRAY(42))
C   DELTA APOGEE, IS THE NEW SEMI-MAJOR AXIS
C  TIMES, THE NEW ECCENTRICITY PLUS ONE, DIVIDED
C  BY THE INITIAL VALUE OF THE SEMI-MAJOR AXIS
C  TIMES, THE INITIAL ECCENTRICITY PLUS ONE;
C  OR IN  EQUATION FORM,(A*(1.+E))/(AO*(1.+EO))
      CALL AXIS(0.0,0.0,12HDELTA APOGEE,+12,8.0
     1 ,90.0,YARRAY(41),YARRAY(42))
      CALL LINE(XARRAY,YARRAY,40,1,0,0)
      CALL PLOT(18.0,-30.0,-3)
200   CONTINUE
      CALL PLOT(12.0,0.0,999)
      RETURN
      END
      SUBROUTINE ARGPER(AO,XI,WP,WS,V1)
      PI=3.14159265
      RHO=383364.0000
      RTD=57.29577951
      V1=0.
      WS=0.
      A2=(AO/RHO)**2
      V4=XI
      IF(V4.EQ.0.)V4=.0001
      NUM=-3.+A2*((45./32.)-(225./32.)*(COS(V4))**2)
      DEN=-7.5*(SIN(V4))**2+A2*((315./64.)-(315./8.)*(COS(V4))**2
     1 +(2205./64.)*(COS(V4))**2)
      RAT2=NUM/DEN
      IF(RAT2.LE.0.)RETURN
      IF(RAT2.GT.1.)RETURN
      ARG=SQRT(RAT2)
      WS=ARSIN(ARG)
      V1=1.
      RETURN
      END
      SUBROUTINE MOON(AO,XI,A)
      PI=3.14159265
      P2=PI/2
      TE=.3
      MUE=.3986*10.**6
```

```
      RM=383364.0000
      C2=(4.8998*10.**3)/((RM-AO)**2)
      WS=630.7828*(AO**(-1.5))
      WM=2.66381*(10.**(-6))
      W=(WS*COS(XI)-WM)
      IF(W.LT.0.)W=-W
      WREL=((W)**2+(WS*SIN(XI))**2)
      WREL=SQRT(WREL)
      RAT=AO/(RM-AO)
      PAR=.33333333*SIN(TE*RAT)*((COS(TE*RAT))**2+2.)
      DVR=2.*C2*PAR/(WREL*RAT)
      DA=((DVR)**2)*(AO**2)/MUE
      V=SIN(XI)
      IF(V.EQ.0.)V=.000000001
      ANG=TE/V
      IF(ANG.GT.P2)ANG=P2
      DAD=86400.*(WREL*ANG/(PI**2))*DA
      DADY=365.*DAD
      RATA=DADY/AO
      A=AO+10.*DAD
      RETURN
      END
//LKED.SYSLIB DD DISP=SHR
//     DD DSN=SYS1.CALCOMP,DISP=SHR
//GO.PLOTTAPE DD DSN=PLOT,DISP=(,KEEP),
// UNIT=(TAPE7,,DEFER),DCB=DEN=1,
// VOL=SER=PLXXXX,LABEL=(,NL)
```

APPENDIX 6

PROGRAM SATLUNAE

SATLUNAE has the same purpose as SCOREE, (App. 7), but approximate integrations rather than the time iterations of SCOREE are used. Its utility, then, is for time >5 years or when very stringent CPU requirements are imposed.

The future time of interest for the perturbed elements is entered on line 350 in days.

SATLUNAE also includes a more convenient crosslink pointing angle coordinate system then SATE. See Figure 11. TU12 and FU12 are chosen in a local satellite coordinate system.

```
//TS0420A JOB (6360,D31,DESK),'CHRISTOPHER P',NOTIFY=TS042C,
// TIME=2
// EXEC FORTGCG
//FORT.SYSIN DD *
C THIS SATELLITE VISIBILITY PROGRAM IS WRITTEN FOR A MAXIMUM OF
C TEN SATELLITES AND TWO GROUND STATIONS.
C THE ARRAYS CONTAIN THE FOLLOWING INFORMATION FOR EACH SATELLITE:
C A--SEMIMAJOR AXIS IN KM
C E--ECCENTRICITY
C W--RIGHT ASCENSION IN DEGREES
C WP--ARGUMENT OF PERIGEE IN DEGREES
C TP--TIME OF PERIGEE IN HOURS
C XI--INCLINATION IN DEGREES
C ARRAY B CONTAINS THE COLATITUDE AND LONGITUDE FOR EACH GROUND STATION.
      DIMENSION A(10),E0(10),W0(10),WP0(10),TP(10),XI(10),B(4)
      DATA A/26561.,26561.,42164.,42164.,42164.,42164.,
     3 106247.,106247.,106247.,106247./
      DATA E0/.725,.725,0.,0.,0.,0.,0.,0.,0.,0./
      DATA W0/0.,270.,0.,0.,0.,0.,
     4 0.,0.,0.,0./
      DATA WP0/-90.,-90.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA TP/0.,-6.,-3.,-9.,-15.,-21.,
     1 0.,-24.,-48.,-72./
      DATA XI/63.435,63.435,0.,0.,0.,0.,90.,90.,90.,90./
      DATA B/47.54,57.23,288.73,242.8/
      WRITE(6,4)
 4     FORMAT(1H ,'ABIG',13X,'E1',13X,'W',13X,'WPS',13X,'V5',/)
      DIMENSION E(10),WP(10),ABIG(10),E1(10),W(10),WPS(10),V5(10)
      DIMENSION XM(10)
      RTD=57.29578
      DO 8 JR=1,10
      W0(JR)=W0(JR)/RTD
      WP0(JR)=WP0(JR)/RTD
      XM(JR)=(XI(JR)-23.4)/RTD
      DAYS=400.
      CALL LUNA(A(JR),E0(JR),W0(JR),WP0(JR),XM(JR),DAYS,ABIG(JR)
     1 ,E1(JR),W(JR),WPS(JR),V5(JR))
      WD=W(JR)*RTD
      WPSD=WPS(JR)*RTD
      WRITE(6,9)ABIG(JR),E1(JR),WD,WPSD,V5(JR)
9     FORMAT(1H ,1F14.1,1F14.9,1F14.6,1F14.6,1F14.7)
8     CONTINUE
C NUMS IS THE NUMBER OF SATELLITES TO BE CONSIDERED.
C NUMG IS THE NUMBER OF GROUND STATIONS TO BE CONSIDERED.
          NUMS=10
          NUMG=2
          P=3.1415926
      RE=6370.8
          RTD=57.29577951
          W3AD=15.0/RTD
C CL IS THE VELOCITY OF LIGHT IN MILES PER SECOND.
      CL=2.99793*10.**5
C FR IS THE UPLINK FREQUENCY IN HZ.
C FR1 IS THE DOWNLINK FREQUENCY IN HZ.
          FR=300.0*10.**6
          FR1=245.0*10.**6
      CON=2.77218*10.**(-6)
          DO 900 J=1,NUMS
          XI(J)=XI(J)/RTD
      E(J)=E1(J)
      A(J)=ABIG(J)
```

```
      WP(J)=WPS(J)
C TAU IS THE PERIOD OF ROTATION OF SATELLITE J.
         TAU=CON*A(J)**1.5
         DO 700 K=1,NUMG
         WRITE(6,10)
 10      FORMAT(1H ,'SAT',5X,'HRS',12X,'RANGE',9X,'ANGLE',8X,'UPDOPPLER
    1',8X,'DNDOPPLER',9X,'SUBL',9X,'SUBC',6X,'STATION',//)
         T=B(K)/RTD
         G=B(K+2)/RTD
C I IS THE HOUR.
      DO 500 I=3,51,3
         RI=I
C SUBROUTINE ELLIP COMPUTES THE RANGE FROM GEOCENTER TO SATELLITE J AND
C THE ANGLE MEASURED IN ORBIT PLANE;REFERRED TO NODE OF ORBIT PLANE
C AND EQUATORIAL PLANE.
         CALL ELLIP(RI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
C SUBROUTINE PRIME COMPUTES THE INERTIAL CARTESIAN COORDINATES OF THE
C SATELLITE J (WITH GEOCENTER AS THE ORIGIN).
         CALL PRIME(FSP,W(J),XI(J),R,0.,XS,YS,ZS)
C FS IS THE SUBSATELLITE LONGITUDE IN DEGREES.
         FS=(ATAN(YS/XS)-WRAD*RI)*RTD
         IF(XS.LT.0.)FS=FS+P*RTD
C TS IS THE SUBSATELLITE COLATITUDE IN DEGREES.
         TS=RTD*ARCOS(ZS/R)
         F=G+WRAD*RI
C (X,Y,Z) ARE THE INERTIAL CARTESIAN COORDINATES OF GROUND STATION K.
         X=RE*SIN(T)*COS(F)
         Y=RE*SIN(T)*SIN(F)
         Z=RE*COS(T)
C ACC IS THE ANGLE BETWEEN GROUND STATION K AND SATELLITE J FROM THE
C CENTER OF THE EARTH.
C RGE IS THE RANGE FROM GROUND STATION K TO SATELLITE J IN NAUTICAL
C MILES.
         ACC=ARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
         RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
         ARG=SIN(ACC)*R/RGE
         IF(ARG.GT.1.0000000)ARG=1.0000000
C D IS THE ELEVATION ANGLE IN DEGREES FROM GROUND STATION K TO
C SATELLITE J.
         D=(ARSIN(ARG)-P/2.)*RTD
         RT=SQRT(R*R-RE*RE)
         IF(RGE.LE.RT)D=-D
C SUBROUTINE DOPE COMPUTES THE COMPONENTS OF SATELLITE VELOCITY.
         CALL DOPE(R,E(J),A(J),TAU,FSP,WP(J),W(J),XI(J),XV,YV,ZV)
C (XT,YT) ARE THE COMPONENTS OF THE RELATIVE VELOCITY OF GROUND
C STATION K.
         XT=-WRAD*RE*SIN(T)*SIN(F)/3600.
         YT=WRAD*RE*SIN(T)*COS(F)/3600.
         DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+ZV*(ZS-Z)
         DOT=-DOT/RGE
C UPDOP IS THE UPLINK DOPPLER SHIFT IN HZ.
C DNDOP IS THE DOWNLINK DOPPLER SHIFT IN HZ.
         UPDOP=DOT*FR/CL
         DNDOP=DOT*FR1/CL
         WRITE(6,20)J,RI,RGE,D,UPDOP,DNDOP,FS,TS,K
 20      FORMAT(1H ,I3,F8.1,F17.3,F14.3,F17.3,F17.3,F13.3,F13.3,I10)
 500     CONTINUE
         WRITE(6,25)
 25      FORMAT(1H ,//)
 700     CONTINUE
 900     CONTINUE
```

```
      WRITE(6,30)
   30 FORMAT(1H ,/////)
      L=NUMS-1
      IF(L.EQ.0)GO TO 350
C FR IS THE CROSSLINK FREQUENCY IN HZ.
   FR=60.*10.**9
C M1 IS THE FIRST SATELLITE.
C M2 IS THE SECOND SATELLITE.
      DO 350 M1=1,L
      TAU=CON*A(M1)**1.5
      N=M1+1
      DO 250 M2=N,NUMS
      WRITE(6,40)
40   FORMAT(1H ,2X,'HRS',12X,'RANGE',5X,'L12',5X,'C12',3X,'FU12',
    2 8X,'TU12',4X,'CRDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
      TAU2=CON*A(M2)**1.5
    DO 300 I=3,150,3
      RI=I
C NOVIZ DETERMINES CROSSLINK VISIBILITY WHERE 0 MEANS NO VISIBILITY
C AND 1 MEANS VISIBILITY.
      NOVIZ=1
      CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
      CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
      CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
      CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
      TC1=X2-XS
      TC2=Y2-YS
      TC3=Z2-ZS
C R12 IS THE RANGE BETWEEN TWO SATELLITES IN KM.
C F12 AND T12 ARE THE POINTING ANGLES BETWEEN TWO SATELLITES IN DEGREES.
      R12=(TC1**2+TC2**2+TC3**2)**.5
    CALL UNPRIM(TC1,TC2,TC3,FSP,W(M1),XI(M1),R12,FU12,TU12)
      F12=RTD*ATAN(TC2/TC1)
      IF(TC1.LT.0.)F12=F12+P*RTD
      VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
      IF(VIZ.GT..99999999)VIZ=.99999999
    IF(VIZ.LT.-.99999999)VIZ=-.99999999
      DEL=ARCOS(VIZ)
      DELM=ARSIN(RE/R)
      IF(DEL.LE.DELM)NOVIZ=0
    TR3=TC3/R12
    IF(TR3.GE.1.)TR3=.99999999
    IF(TR3.LE.-1.)TR3=-.99999999
    T12=ARCOS(TR3)*RTD
      CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
     CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
      DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
      DOT=DOT/R12
C DOP12 IS THE CROSSLINK DOPPLER SHIFT IN HZ.
      DOP12=-DOT*FR/CL
    WRITE(6,50)RI,R12,F12,T12,FU12,TU12,DOP12,NOVIZ,M1,M2
50   FORMAT(1H ,F5.1,F17.3,F9.1,F7.1,F10.1,F6.1,F17.3,I9,I10,I10)
  300 CONTINUE
      WRITE(6,60)
   60 FORMAT(1H ,//)
  250 CONTINUE
  350 CONTINUE
      END
    SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
    A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
    A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
```

```
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
      XS=A11*XPS+A12*YPS
      YS=A21*XPS+A22*YPS
      ZS=A31*XPS+A32*YPS
      RETURN
       END
      SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
      P=3.1415926
       Z=2.*P*(T-TP)/TAU
      P2=2.*P
2     IF(Z.GT.P2)Z=Z-P2
      IF(Z.GT.P2)GO TO 2
      E1=Z+E*SIN(Z)
      E2=(Z+E*(SIN(E1))-(E*COS(E1))*E1)/(1.-E*COS(E1))
      Q=0.
4     E3=(Z+E*(SIN(E2))-(E*COS(E2))*E2)/(1.-E*COS(E2))
      Q=Q+1.
       DE=E3-E2
      DE2=DE**2
      E2=E3
      IF(DE2.GT..00000001)GO TO 4
      TH=ARCOS((COS(E2)-E)/(1.-E*COS(E2)))
      IF(Z.GT.P)TH=2.*P-TH
      FSP=WP+TH
      R=(A*(1.-E**2))/(1.+E*COS(FSP-WP))
      RETURN
      END
      SUBROUTINE DOPE(R,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
      P=3.1415926
      T=FSP-WP
      RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*SIN(T)
      TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*COS(T))**2
      XPC=RD*COS(FSP)-R*TD*SIN(FSP)
      XPC=XPC/3600.
      YPC=RD*SIN(FSP)+R*TD*COS(FSP)
      YPC=YPC/3600.
      CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
      RETURN
      END
      SUBROUTINE UNPRIM(TC1,TC2,TC3,FSP,WS,XIS,R12,FU12,TU12)
      A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
      A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
      A13=SIN(XIS)*SIN(WS)
      A23=-SIN(XIS)*COS(WS)
      A33=COS(XIS)
      Z1=A11*TC1+A21*TC2+A31*TC3
      X1=A12*TC1+A22*TC2+A32*TC3
      Y1=A13*TC1+A23*TC2+A33*TC3
      XU=X1
      YU=-Y1
      ZU=-Z1
      RTD=57.29577951
      P=3.14159265
```

```
      FU12=RTD*ATAN(YU/XU)
      IF(XU.LT.0.) FU12=FU12+P*RTD
      ZR=ZU/R12
      IF(ZR.GE.1.) ZR=.99999999
      IF(ZR.LE.-1.) ZR=-.99999999
      TU12=ARCOS(ZR)*RTD
      RETURN
      END
      SUBROUTINE LUNA(A,E0,W0,WP0,XI,T,ABIG,E1,W,WPS,V5)
      P=3.14159265
      T=24.*T
      P2=P/2.
      RTD=57.29578
      WM=2.6638*10.**(-6)
       WS=630.7828*A**(-1.5)
       RM=383368.8
       MUE=.3986*10.**6
      TE=.3
      IF(XI.LT..0001) XI=.0001
      IF(XI.EQ.P) XI=P-.0001
      RAT=A/(RM-A)
       PAR=.33333333*SIN(TE*RAT)*((COS(TE*RAT))**2+2.)
       WREL=SQRT((WS*COS(XI)-WM)**2+(WS*SIN(XI))**2)
       C2=(4.8999*10.**3)/((RM-A)**2)
       DVP=2.*C2*PAR/(WREL*RAT)
      V7=SIN(XI)
      IF(V7.EQ.0.) V7=.000000001
      ANG=TE/V7
       IF(ANG.GT.P2) ANG=P2
       DAH=3600.*(WREL*ANG/P**2)*((DVP**2)*A**2)/MUE
       C5=DAH
      DAY=8760.*DAH
       AB=A+(C5*T)/2.
      RHO=RM
       C6=P*(1./81.)*(1./RHO)**3
       C7=2.77218*10.**(-6)
       S2I=(SIN(XI))**2
       C2I=(COS(XI))**2
       C4I=C2I**2
       AR2=(AB/RHO)**2
      C9=3.+AR2*((-45./32.)+(225./32.)*C2I)
       C10=-7.5*S2I+AR2*((315./64.)-(315./8.)*C2I+(2205./64.)*C4I)
       RATIO=-C9/C10
       C11=-(15./4.)*S2I+AR2*((315./128.)-(315./16.)*C2I+(2205./128.)*C4
     1 I)
      WPS=0.
      IF(RATIO.LT.0.) GO TO 50
      IF(RATIO.GT.1.) GO TO 50
      WPS=ARSIN(SQRT(RATIO))
      E1=E0*EXP(-C6*(AB**1.5)*(SIN(2.*WPS))*(C11/C7)*T)
       GO TO 100
50     WPC=(P/4.)-.001
       C8=3.-(15./4.)*S2I+AR2*(1.054688-22.5*C2I+17.22656*C4I)
       WANG=C6*C8*(AB**1.5)/C7
      C21=2.*C6*(AB**1.5)*C8/C7
      C20=2.*WPC
      E1=E0*EXP(-C6*(AB**1.5)*(C11/C7)*(-(1./C21)*COS(C20)+(1./C21)*
     2 COS(C20+C21*T)))
100    DW=-2.*C6*(AB**3)*COS(XI)*(.75+AR2*(-(135./128.)+(315./128.)*C2I
     3 )+(AR2**2)*((2625./2048.)-(7875./1024.)*C2I+(17325./2048.)*C4I
```

```
4 ))
 DWH=DW/(C7*AB**1.5)
 W=W0+DWH*T
 ABIG=A+(C5*T)
 V50=A*(1.+E0)
 V5=ABIG*(1.+E1)/V50
 RETURN
 END
```

****

APPENDIX 7

PROGRAM SCOREE

The lunar perturbation results of PERTP are combined with the link program SATE to calculate new crosslink relations after a long period (e.g., a few years). The future time of interest is entered on line 260 as EPOCH, in days.

Semi-major axes are in kilometers.

```
//SCORE JOB (6360,D91,DESK),'EDELMAN E',NOTIFY=TS0141,
// TIME=2
// EXEC FORTGCG
//FORT.SYSIN DD *
C TEN SATELLITES AND TWO GROUND STATIONS.
C THE ARRAYS CONTAIN THE FOLLOWING INFORMATION FOR EACH SATELLITE:
C A--SEMIMAJOR AXIS IN NAUTICAL MILES
C E--ECCENTRICITY
C W--RIGHT ASCENSION IN DEGREES
C WP--ARGUMENT OF PERIGEE IN DEGREES
C TP--TIME OF PERIGEE IN HOURS
C XI--INCLINATION IN DEGREES
C ARRAY B CONTAINS THE COLATITUDE AND LONGITUDE FOR EACH GROUND STATION.
      DIMENSION AO(10),EO(10),WO(10),WPO(10),TP(10),XI(10),B(4)
      DATA AO/200000.,200000.,200000.,200000.,200000.,200000.,
     1 250000.,250000.,250000.,250000./
      DATA EO/.1,.1,.1,.1,.1,.1,.1,.1,.1,.1/
      DATA WO/0.,0.,0.,0.,0.,0.,
     4 0.,72.,144.,216./
      DATA WPO/0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA TP/0.,-80.,-160.,-40.,-120.,-200.,
     1 0.,-57.6,-115.2,-172.8/
      DATA XI/23.4,23.4,23.4,203.4,203.4,203.4,113.4,113.4,113.4,113.4/
      DATA B/47.54,57.23,288.73,242.8/
C EPOCH ,IS THE TIME IN DAYS.
      EPOCH=400.
C NUMS IS THE NUMBER OF SATELLITES TO BE CONSIDERED.
C NUMG IS THE NUMBER OF GROUND STATIONS TO BE CONSIDERED.
      NUMS=10
      NUMG=2
      P=3.1415926
      RE=6370.8800
      RTD=57.29577951
      WRAD=15.0/RTD
      WRITE(6,15)
 15   FORMAT(1H ,'2X,'V5',24X,'A',24X,'E',24X,'W',23X,'WP')
      DIMENSION A(10),E(10),W(10),WP(10),V5(10)
      DO 3 JP=1,10
      XI(JP)=XI(JP)/RTD
      WO(JP)=WO(JP)/RTD
      WPO(JP)=WPO(JP)/RTD
      CALL PERT(EPOCH,AO(JP),EO(JP),WO(JP),WPO(JP),XI(JP),
     1 A(JP),E(JP),W(JP),WP(JP),V5(JP))
      WD=W(JP)*RTD
      WPD=WP(JP)*RTD
      WRITE(6,25) V5(JP),A(JP),E(JP),WD,WPD
 25   FORMAT(1H ,5E24.9)
  3   CONTINUE
C CL IS THE VELOCITY OF LIGHT IN KILOMETERS PER SECOND.
      CL=2.99994*10.**5
C FR IS THE UPLINK FREQUENCY IN HZ.
C FR1 IS THE DOWNLINK FREQUENCY IN HZ.
      FR=300.0*10.**6
      FR1=245.0*10.**6
      CON=2.77218*10.**(-6)
      WRITE(6,27)
 27   FORMAT(1H ,//////)
      DO 900 J=1,NUMS
C TAU IS THE PERIOD OF ROTATION OF SATELLITE J.
      TAU=CON*A(J)**1.5
      DO 700 K=1,NUMG
```

```
      WRITE(6,10)
   10 FORMAT(1H ,'SAT',5X,'HRS',12X,'RANGE',9X,'ANGLE',8X,'UPDOPPLER
     1',8X,'DNDOPPLER',9X,'SUBL',9X,'SUBC',6X,'STATION',//)
      T=B(K)/RTD
      G=B(K+2)/RTD
C I IS THE HOUR.
      DO 500 I=3,150,3
      RI=I
C SUBROUTINE ELLIP COMPUTES THE RANGE FROM GEOCENTER TO SATELLITE J AND
C THE ANGLE MEASURED IN ORBIT PLANE;REFERRED TO NODE OF ORBIT PLANE
C AND EQUATORIAL PLANE.
      CALL ELLIP(RI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
C SUBROUTINE PRIME COMPUTES THE INERTIAL CARTESIAN COORDINATES OF THE
C SATELLITE J (WITH GEOCENTER AS THE ORIGIN).
      CALL PRIME(FSP,W(J),XI(J),R,0.,XS,YS,ZS)
C FS IS THE SUBSATELLITE LONGITUDE IN DEGREES.
      FS=(ATAN(YS/XS)-WRAD*RI)*RTD
      IF(XS.LT.0.) FS=FS+P*RTD
C TS IS THE SUBSATELLITE COLATITUDE IN DEGREES.
      TS=RTD*ARCOS(ZS/R)
      F=G+WRAD*RI
C (X,Y,Z) ARE THE INERTIAL CARTESIAN COORDINATES OF GROUND STATION K.
      X=RE*SIN(T)*COS(F)
      Y=RE*SIN(T)*SIN(F)
      Z=RE*COS(T)
C ACC IS THE ANGLE BETWEEN GROUND STATION K AND SATELLITE J FROM THE
C CENTER OF THE EARTH.
C RGE IS THE RANGE FROM GROUND STATION K TO SATELLITE J IN NAUTICAL
C MILES.
      ACC=ARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
      RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
      ARG=SIN(ACC)*R/RGE
      IF(ARG.GT.1.00000000)ARG=1.00000000
C D IS THE ELEVATION ANGLE IN DEGREES FROM GROUND STATION K TO
C SATELLITE J.
      D=(ARSIN(ARG)-P/2.)*RTD
      RT=SQRT(R*R-RE*RE)
      IF(RGE.LE.RT) D=-D
C SUBROUTINE DOPE COMPUTES THE COMPONENTS OF SATELLITE VELOCITY.
      CALL DOPE(R,E(J),A(J),TAU,FSP,WP(J),W(J),XI(J),XV,YV,ZV)
C (XT,YT) ARE THE COMPONENTS OF THE RELATIVE VELOCITY OF GROUND
C STATION K.
      XT=-WRAD*RE*SIN(T)*SIN(F)/3600.
      YT=WRAD*RE*SIN(T)*COS(F)/3600.
      DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+ZV*(ZS-Z)
      DOT=-DOT/RGE
C UPDOP IS THE UPLINK DOPPLER SHIFT IN HZ.
C DNDOP IS THE DOWNLINK DOPPLER SHIFT IN HZ.
      UPDOP=DOT*FR/CL
      DNDOP=DOT*FR1/CL
      WRITE(6,20) J,RI,RGE,D,UPDOP,DNDOP,FS,TS,K
   20 FORMAT(1H ,I3,F8.1,F17.3,F14.3,F17.3,F17.3,F13.3,F13.3,I10)
  500 CONTINUE
      WRITE(6,26)
   26 FORMAT(1H ,//)
  700 CONTINUE
  900 CONTINUE
      WRITE(6,30)
   30 FORMAT(1H ,/////)
      L=NUMS-1
      IF(L.EQ.0)GO TO 350
```

```
C FR IS THE CROSSLINK FREQUENCY IN HZ.
      FR=60.*10.**9
C M1 IS THE FIRST SATELLITE.
C M2 IS THE SECOND SATELLITE.
      DO 350 M1=1,L
      TAU=CON*A(M1)**1.5
      N=M1+1
      DO 250 M2=N,NUMS
      WRITE(6,40)
 40   FORMAT(1H ,2X,'HRS',12X,'RANGE',5X,'L12',5X,'C12',23X,
     2 'CRDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
      TAU2=CON*A(M2)**1.5
      DO 300 I=3,150,3
      RI=I
C NOVIZ DETERMINES CROSSLINK VISIBILITY WHERE 0 MEANS NO VISIBILITY
C AND 1 MEANS VISIBILITY.
      NOVIZ=1
      CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
      CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
      CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
      CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
      TC1=X2-XS
      TC2=Y2-YS
      TC3=Z2-ZS
C R12 IS THE RANGE BETWEEN TWO SATELLITES IN NAUTICAL MILES.
C F12 AND T12 ARE THE POINTING ANGLES BETWEEN TWO SATELLITES IN DEGREES.
      R12=(TC1**2+TC2**2+TC3**2)**.5
      CALL UNPRIM(TC1,TC2,TC3,FSP,W(M1),XI(M1),P12,FU12,TU12)
      F12=RTD*ATAN(TC2/TC1)
      IF(TC1.LT.0.)F12=F12+P*RTD
      VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
      IF(VIZ.GT..99999999)VIZ=.99999999
      IF(VIZ.LT.-.99999999)VIZ=-.99999999
      DEL=ARCOS(VIZ)
      DELM=ARSIN(RE/R)
      IF(DEL.LE.DELM)NOVIZ=0
      TR3=TC3/R12
      IF(TR3.GE.1.)TR3=.99999999
      IF(TR3.LE.-1.)TR3=-.99999999
      T12=ARCOS(TR3)*RTD
      CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
      CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
      DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
      DOT=DOT/R12
C DOP12 IS THE CROSSLINK DOPPLER SHIFT IN HZ.
      DOP12=-DOT*FR/CL
      WRITE(6,50)RI,R12,F12,T12,FU12,TU12,DOP12,NOVIZ,M1,M2
 50   FORMAT(1H ,F5.1,F17.3,F9.1,F7.1,F10.1,F6.1,F17.3,I9,I10,I10)
300   CONTINUE
      WRITE(6,60)
 60   FORMAT(1H ,//)
250   CONTINUE
350   CONTINUE
      END
      SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
      A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
      A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
```

```
      XS=A11*XPS+A12*YPS
      YS=A21*XPS+A22*YPS
      ZS=A31*XPS+A32*YPS
      RETURN
      END
      SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
      P=3.1415926
       Z=2.*P*(T-TP)/TAU
      P2=2.*P
2     IF(Z.GT.P2) Z=Z-P2
      IF(Z.GT.P2)GO TO 2
      E1=Z+E*SIN(Z)
      E2=(Z+E*(SIN(E1))-(E*COS(E1))*E1)/(1.-E*COS(E1))
      Q=0.
4     E3=(Z+E*(SIN(E2))-(E*COS(E2))*E2)/(1.-E*COS(E2))
      Q=Q+1.
       DE=E3-E2
      DE2=DE**2
      E2=E3
      IF(DE2.GT..00000001)GO TO 4
      TH=ARCOS((COS(E2)-E)/(1.-E*COS(E2)))
      IF(Z.GT.P) TH=2.*P-TH
      FSP=WP+TH
      R=(A*(1.-E**2))/(1.+E*COS(FSP-WP))
      RETURN
      END
      SUBROUTINE DOPE(R,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
      P=3.1415926
      T=FSP-WP
      RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*SIN(T)
      TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*COS(T))**2
      XPC=RD*COS(FSP)-R*TD*SIN(FSP)
      XPC=XPC/3600.
      YPC=RD*SIN(FSP)+R*TD*COS(FSP)
      YPC=YPC/3600.
      CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
      RETURN
      END
      SUBROUTINE UNPRIM(TC1,TC2,TC3,FSP,WS,XIS,R12,FU12,TU12)
      A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
      A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
      A13=SIN(XIS)*SIN(WS)
      A23=-SIN(XIS)*COS(WS)
      A33=COS(XIS)
      Z1=A11*TC1+A21*TC2+A31*TC3
      X1=A12*TC1+A22*TC2+A32*TC3
      Y1=A13*TC1+A23*TC2+A33*TC3
      XU=X1
      YU=-Y1
      ZU=-Z1
      RTD=57.29577951
      P=3.14159265
      FU12=RTD*ATAN(YU/XU)
      IF(XU.LT.0.) FU12=FU12+P*RTD
      ZR=ZU/R12
      IF(ZR.GE.1.) ZR=.99999999
```

```
      IF(ZR.LE.-1.) ZR=-.9999999
      TU12=ARCOS(ZR)*RTD
      RETURN
      END
      SUBROUTINE PERT(EPOCH,AO,EO,WO,WPO,XI,
     1 A,E,W,WP,V5)
C   THIS SUBROUTINE PRODUCES THE SECULAR VARIANCE OF ,AN INITIALLY,
C  HIGH ALTITUDE ORBIT, AND AT THE SAME TIME GENERATES STABILITY OF
C  ECCENTRICITY, RIGHT ASCENSION AND ARGUMENT OF PERIGEE. IT ALSO
C  PRODUCES THE EVER CHANGING SEMI-MAJOR AXIS.
C   ALL VALUES IN THIS PROGRAM HAVE BEEN
C  CONVERTED TO THE METRIC SYSTEM,
C  WHERE 1 NAUTICAL MILE=1.852 KM.
      CON=2.77218*10.**(-6)
      PI=3.1415926
      RTD=57.29577951
      RHO=393364.0000
      W=WO
      WP=WPO
      IF(WP.EQ.0.) WP=.00001
      V6=EPOCH/400.
      DO 200 I=1,40
      TAU=CON*AO**1.5
      V=(24./TAU)*10.
      V=V*V6
      V50=AO*(1.+EO)
      CALL ARGPER(AO,XI,WS,V1)
      IF(V1.GT.0.) WP=WS
      IF(I.EQ.1) E=EO
C    MOON REQUIRES A NEW SEMI-MAJOR AXIS 'A'
C  AT EACH NEW ITERATION.  THEREFORE FOR EVERY ITERATION AFTER THE
C  FIRST, AO IS REALLY THE NEW'A' AS COMPUTED BY THE PREVIOUS ITERATION.
C  SO FOR THE SECOND ITERATION AO REALLY EQUALS A1,FOR THE THIRD
C  A2 AND SO ON.  THUS THE LOGIC STATEMENT IS REQUIRED.
      IF(I.GT.1) AO=A
      CALL MOON(AO,XI,A)
      UPA3=PI*(1./81.)*((A/RHO)**3)
      A2=((A/RHO)**2)
      A4=((A/RHO)**4)
      CI2=(COS(XI))**2
      CI4=(COS(XI))**4
      SP2=(SIN(WP))**2
      SI2=(SIN(XI))**2
      PAR1=((-135./128.)+(315./128.)*CI2)
      PAR2=((2625./2048.)-(7875./1024.)*CI2+(17325./2048.)*CI4)
      PAR3=((315./128.)-(315./16.)*CI2+(2205./128.)*CI4)
      PAR4=(3.-(15./2.)*SP2*SI2)
      PAR5=((225./32.)-(315./8.)*SP2)
      PAR6=((-45./32.)+(315./64.)*SP2+PAR5*CI2+(2205./64.)*SP2*CI4)
      DW=-2*UPA3*COS(XI)*((3./4.)+A2*PAR1+(A4*PAR2))*V
      DE=-UPA3*E*SIN(2*WP)*((-15./4.)*SI2+A2*PAR3)*V
      DWP=UPA3*(PAR4+A2*PAR6)*V
      W=W+DW
      E=E+DE
      V5=(A*(1.+E))/V50
      WP=WP+DWP
200   CONTINUE
C   DELTA APOGEE, IS THE NEW SEMI-MAJOR AXIS
C  TIMES, THE NEW ECCENTRICITY PLUS ONE, DIVIDED
C  BY THE INITIAL VALUE OF THE SEMI-MAJOR AXIS
```

```
C  TIMES, THE INITIAL ECCENTRICITY PLUS ONE;
C  OR IN  EQUATION FORM,(A*(1.+E))/(AO*(1.+EO))
      RETURN
      END
      SUBROUTINE ARGPER(AO,XI,WS,V1)
      PI=3.14159265
      RHO=383364.0000
      RTD=57.29577951
      V1=0.
      WS=0.
      A2=(AO/RHO)**2
      V4=XI
      IF(V4.EQ.0.)V4=.0001
      NUM=-3.+A2*((45./32.)-(225./32.)*(COS(V4))**2)
      DEN=-7.5*(SIN(V4))**2+A2*((315./64.)-(315./8.)*(COS(V4))**2
     1 +(2205./64.)*(COS(V4))**4)
      RAT2=NUM/DEN
      IF(RAT2.LE.0.)RETURN
      IF(RAT2.GT.1.)RETURN
      ARG=SQRT(RAT2)
      WS=ARSIN(ARG)
      V1=1.
      RETURN
      END
      SUBROUTINE MOON(AO,XI,A)
      PI=3.14159265
      P2=PI/2
      TE=.3
      MUE=.3986*10.**6
      RM=383364.0000
      C2=(4.8998*10.**3)/((RM-AO)**2)
      WS=630.7828*(AO**(-1.5))
      WM=2.66381*(10.**(-6))
      W=(WS*COS(XI)-WM)
      IF(W.LT.0.)W=-W
      WREL=((W)**2+(WS*SIN(XI))**2)
      WREL=SQRT(WREL)
      RAT=AO/(RM-AO)
      PAR=.33333333*SIN(TE*RAT)*((COS(TE*RAT))**2+2.)
      DVR=2.*C2*PAR/(WREL*RAT)
      DA=((DVR)**2)*(AO**2)/MUE
      V=SIN(XI)
      IF(V.EQ.0.)V=.000000001
      ANG=TE/V
      IF(ANG.GT.P2)ANG=P2
      DAD=86400.*(WREL*ANG/(PI**2))*DA
      DADY=365.*DAD
      RATA=DADY/AO
      A=AO+10.*DAD
      RETURN
      END
```

****

APPENDIX 8

PROGRAM AZ1

A ground station azimuth angle calcualtion is added (lines 870-1025) to program SATE.

```
//TS0420A JOB (6360,D91,DESK),'CHRISTOPHER P',
// TIME=1
// EXEC FORTGCG
//FORT.SYSIN DD *
C          AZ1 (APP 75) GIVES AZIMUTH ANGLE
C THIS SATELLITE VISIBILITY PROGRAM IS WRITTEN FOR A MAXIMUM OF
C TEN SATELLITES AND TWO GROUND STATIONS.
C THE ARRAYS CONTAIN THE FOLLOWING INFORMATION FOR EACH SATELLITE:
C A--SEMIMAJOR AXIS IN NAUTICAL MILES
C E--ECCENTRICITY
C W--RIGHT ASCENSION IN DEGREES
C WP--ARGUMENT OF PERIGEE IN DEGREES
C TP--TIME OF PERIGEE IN HOURS
C XI--INCLINATION IN DEGREES
C ARRAY B CONTAINS THE COLATITUDE AND LONGITUDE FOR EACH GROUND STATION.
          DIMENSION A(10),E(10),W(10),WP(10),TP(10),XI(10),B(4)
      DATA A/14342.,14342.,14342.,61421.,61421.,61421.,
     3 61421.,119402.,119402.,119402./
      DATA E/.725,.725,.725,.4,.4,.4,.4,.5,.5,.5/
          DATA W/0.0,270.0,63.435,63.435,63.435,63.435,
     4    243.733,243.733,243.733,243.733/
      DATA WP/-90.0,-90.0/
      DATA TP/0.,-6.0,-12.,0.,-26.587,-53.173,-79.76,0.,
     1 -96.08,-192.16/
      DATA XI/63.435,63.435,23.4,90.,90.,90.,90.,0.,0.,0./
      DATA B/45.0,0.0/
C NUMS IS THE NUMBER OF SATELLITES TO BE CONSIDERED.
C NUMG IS THE NUMBER OF GROUND STATIONS TO BE CONSIDERED.
          NUMS=2
          NUMG=1
          P=3.1415926
      P12=.5*P
      P32=1.5*P
      P2=2.*P
          RE=3440.
          RTD=57.29577951
          WRAD=15.0/RTD
C CL IS THE VELOCITY OF LIGHT IN NAUTICAL MILES PER SECOND.
          CL=1.61984*10.**5
C FR IS THE UPLINK FREQUENCY IN HZ.
C FR1 IS THE DOWNLINK FREQUENCY IN HZ.
          FR=300.0*10.**6
          FR1=245.0*10.**6
          CON=6.987*10.**(-6)
          DO 900 J=1,NUMS
          W(J)=W(J)/RTD
          WP(J)=WP(J)/RTD
          XI(J)=XI(J)/RTD
C TAU IS THE PERIOD OF ROTATION OF SATELLITE J.
          TAU=CON*A(J)**1.5
          DO 700 K=1,NUMG,2
          WRITE(6,10)
   10     FORMAT(1H ,'SAT',5X,'HRS',12X,'RANGE',9X,'ANGLE',8X,'AZIMUTH',
     18X,'UPDOPPLER',8X,'DNDOPPLER',9X,'SUBL',9X,'SUBC',6X,'STATION',//)
          T=B(K)/RTD
          K1=K+1
          G=B(K1)/RTD
C I IS THE HOUR.
          DO 500 I=1,25
          PI=I
C SUBROUTINE ELLIP COMPUTES THE RANGE FROM GEOCENTER TO SATELLITE J AND
```

```
C THE ANGLE MEASURED IN ORBIT PLANE;REFERRED TO NODE OF ORBIT PLANE
C AND EQUATORIAL PLANE.
        CALL ELLIP(PI,E(J),WP(J),TP(J),A(J),TAU,FSP,R)
C SUBROUTINE PRIME COMPUTES THE INERTIAL CARTESIAN COORDINATES OF THE
C SATELLITE J (WITH GEOCENTER AS THE ORIGIN).
        CALL PRIME(FSP,W(J),XI(J),R,0.,XS,YS,ZS)
C FS IS THE SUBSATELLITE LONGITUDE IN DEGREES.
        FS=(ATAN(YS/XS)-WRAD*RI)*RTD
        IF(XS.LT.0.)FS=FS+P*RTD
 11     IF(FS.LT.-360.)FS=FS+360.
     IF(FS.LT.-360.)GO TO 11
C TS IS THE SUBSATELLITE COLATITUDE IN DEGREES.
        TS=RTD*ARCOS(ZS/R)
        F=G+WRAD*RI
C (X,Y,Z) ARE THE INERTIAL CARTESIAN COORDINATES OF GROUND STATION K.
        X=RE*SIN(T)*COS(F)
        Y=RE*SIN(T)*SIN(F)
        Z=RE*COS(T)
C ACC IS THE ANGLE BETWEEN GROUND STATION K AND SATELLITE J FROM THE
C CENTER OF THE EARTH.
C RGE IS THE RANGE FROM GROUND STATION K TO SATELLITE J IN NAUTICAL
C MILES.
        ACC=ARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
        RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
        ARG=SIN(ACC)*R/RGE
        IF(ARG.GT.1.0000000)ARG=1.0000000
C D IS THE ELEVATION ANGLE IN DEGREES FROM GROUND STATION K TO
C SATELLITE J.
        D=(ARSIN(ARG)-P/2.)*RTD
        RT=SQRT(R*R-RE*RE)
        IF(RGE.LE.RT)D=-D
C  AZIMUT IS THE AZIMUTH ANGLE.
        SLAT=TS/RTD
        SLON=FS/RTD
        ARG=G-SLON
        ALPHA=(SLAT-T)/2.0
        BETA=(SLAT+T)/2.0
        GAMMA=ABS(ARG)/2.0
     IF(GAMMA.GT.P12.AND.GAMMA.LT.P)GAMMA=P-GAMMA
        SINA=SIN(ALPHA)
        SINB=SIN(BETA)
        COSA=COS(ALPHA)
        COSB=COS(BETA)
        COTG=COTAN(GAMMA)
        U1=SINA*COTG/SINB
        ALPHA=ATAN(U1)
        U2=COSA*COTG/COSB
        BETA=ATAN(U2)
     IF(BETA.GT.P32 .AND. BETA.LT.P2)BETA=BETA+P
     IF(BETA.GT.-P12 .AND. BETA.LT.0.)BETA=BETA+P
        AZIMUT=(ALPHA+BETA)*RTD
        ARG=SIN(ARG)
        IF(ARG.GT.0.0)AZIMUT=360.0-AZIMUT
C  SUBROUTINE DOPE COMPUTES THE COMPONENTS OF SATELLITE  VELOCITY.
        CALL DOPE(R,E(J),A(J),TAU,FSP,WP(J),W(J),XI(J),XV,YV,ZV)
C (XT,YT) ARE THE COMPONENTS OF THE RELATIVE VELOCITY OF GROUND
C STATION K.
        XT=-WRAD*RE*SIN(T)*SIN(F)/3600.
        YT=WRAD*RE*SIN(T)*COS(F)/3600.
     DOT=(XV-XT)*(XS-X)+(YV-YT)*(YS-Y)+(ZV)*(ZS-Z)
        DOT=-DOT/RGE
```

```
C UPDOP IS THE UPLINK DOPPLER SHIFT IN HZ.
C DNDOP IS THE DOWNLINK DOPPLER SHIFT IN HZ.
      UPDOP=DOT*FP/CL
      DNDOP=DOT*FR1/CL
      WRITE(6,20) J,RI,RGE,D,AZIMUT,UPDOP,DNDOP,FS,TS,K
 20   FORMAT(1H ,I3,F8.1,F17.3,F14.3,F15.3,F17.3,F17.3,F13.3,F13.3,
     1I10)
 500  CONTINUE
      WRITE(6,25)
 25   FORMAT(1H ,//)
 700  CONTINUE
 900  CONTINUE
      WRITE(6,30)
 30   FORMAT(1H ,/////)
      L=NUMS-1
      IF(L.EQ.0) GO TO 350
C FR IS THE CROSSLINK FREQUENCY IN HZ.
      FR=38.*10.**9
C M1 IS THE FIRST SATELLITE.
C M2 IS THE SECOND SATELLITE.
      DO 350 M1=1,L
      TAU=CON*A(M1)**1.5
      N=M1+1
      DO 250 M2=N,NUMS
      WRITE(6,40)
 40   FORMAT(1H ,2X,'HRS',12X,'RANGE',11X,'L12',12X,'C12',11X,
     2'CRDOPPLER',6X,'CVIS',6X,'SAT1',6X,'SAT2',//)
      TAU2=CON*A(M2)**1.5
      DO 300 I=1,25
      RI=I
C NOVIZ DETERMINES CROSSLINK VISIBILITY WHERE 0 MEANS NO VISIBILITY
C AND 1 MEANS VISIBILITY.
      NOVIZ=1
      CALL ELLIP(RI,E(M1),WP(M1),TP(M1),A(M1),TAU,FSP,R)
      CALL ELLIP(RI,E(M2),WP(M2),TP(M2),A(M2),TAU2,F2P,R2)
      CALL PRIME(FSP,W(M1),XI(M1),R,0.,XS,YS,ZS)
      CALL PRIME(F2P,W(M2),XI(M2),R2,0.,X2,Y2,Z2)
      TC1=X2-XS
      TC2=Y2-YS
      TC3=Z2-ZS
C R12 IS THE RANGE BETWEEN TWO SATELLITES IN NAUTICAL MILES.
C F12 AND T12 ARE THE POINTING ANGLES BETWEEN TWO SATELLITES IN DEGREES.
      R12=(TC1**2+TC2**2+TC3**2)**.5
      F12=RTD*ATAN(TC2/TC1)
      IF(TC1.LT.0.)F12=F12+P*RTD
      VIZ=(-XS*TC1-YS*TC2-ZS*TC3)/(R12*R)
      IF(VIZ.GT..99999999) VIZ=.99999999
      DEL=ARCOS(VIZ)
      DELM=ARSIN(RE/R)
      IF(DEL.LE.DELM) NOVIZ=0
      T12=ARCOS(TC3/R12)*RTD
      CALL DOPE(R,E(M1),A(M1),TAU,FSP,WP(M1),W(M1),XI(M1),XV,YV,ZV)
     CALL DOPE(R2,E(M2),A(M2),TAU2,F2P,WP(M2),W(M2),XI(M2),S2,U2,V2)
      DOT=(S2-XV)*TC1+(U2-YV)*TC2+(V2-ZV)*TC3
      DOT=DOT/R12
C DOP12 IS THE CROSSLINK DOPPLER SHIFT IN HZ.
      DOP12=-DOT*FR/CL
      WRITE(6,50) RI,R12,F12,T12,DOP12,NOVIZ,M1,M2
 50   FORMAT(1H ,F5.1,F17.3,F14.3,F15.3,F20.3,I9,I10,I10)
 300  CONTINUE
      WRITE(6,60)
```

```
60      FORMAT(1H ,//)
250     CONTINUE
350     CONTINUE
        END
        SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
        A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
        A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
        A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
        A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
        A31=SIN(XIS)*SIN(FSP)
        A32=SIN(XIS)*COS(FSP)
        XS=A11*XPS+A12*YPS
        YS=A21*XPS+A22*YPS
        ZS=A31*XPS+A32*YPS
        RETURN
        END
        SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
        P=3.1415926
        Z=2.*P*(T-TP)/TAU
        P2=2.*P
2       IF(Z.GT.P2)Z=Z-P2
        IF(Z.GT.P2)GO TO 2
        E1=Z+E*SIN(Z)
        E2=(Z+E*(SIN(E1))-(E*COS(E1))*E1)/(1.-E*COS(E1))
        Q=0.
4       E3=(Z+E*(SIN(E2))-(E*COS(E2))*E2)/(1.-E*COS(E2))
        Q=Q+1.
        DE=E3-E2
        DE2=DE**2
        E2=E3
        IF(DE2.GT..00000001)GO TO 4
        TH=ARCOS((COS(E2)-E)/(1.-E*COS(E2)))
        IF(Z.GT.P)TH=2.*P-TH
        FSP=WP+TH
        R=(A*(1.-E**2))/(1.+E*COS(FSP-WP))
        RETURN
        END
        SUBROUTINE DOPE(R,E,A,TAU,FSP,WP,WS,XIS,XD,YD,ZD)
        P=3.1415926
        T=FSP-WP
        RD=(A*E*2.*P/(TAU*(1.-E**2)**.5))*SIN(T)
        TD=(2.*P/TAU)*((1.-E**2)**(-1.5))*(1.+E*COS(T))**2
        XPC=RD*COS(FSP)-R*TD*SIN(FSP)
        XPC=XPC/3600.
        YPC=RD*SIN(FSP)+R*TD*COS(FSP)
        YPC=YPC/3600.
        CALL PRIME(0.,WS,XIS,XPC,YPC,XD,YD,ZD)
        RETURN
        END
```

****

APPENDIX 9

PROGRAM NOLINKE

NOLINKE searches for any visible satellite at regular intervals around the earth. If no satellite is visible, the coordinates of that unfortunate location are printed out in the foreground. A good 10-satellite system can require thousands of elevation angle calculations for very little printout. Care should be used with NOLINKE or large CPU time and expense will result.

The orbital elements (A in nautical miles) are entered on lines 20-70. Elevation angle requirements are entered on line 210 as EM, in degrees. Time intervals of one hour are examined (RI = time in hours on line 230).

```
      DIMENSION A(10),E(10),W(10),WP(10),TP(10),XI(10)
      DATA A/14342.,14342.,22767.,22767.,22767.,22767.,
     3 57369.,57369.,57369.,57369./
      DATA E/.725,.725,0.,0.,0.,0.,0.,0.,0.,0./
      DATA W/0.,270.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA WP/-90.,-90.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA TP/0.,-6.,-3.,-9.,-15.,-21.,
     1 0.,-24.,-48.,-72./
      DATA XI/63.435,63.435,0.,0.,0.,0.,90.,90.,90.,90./
      WRITE(6,7)
7      FORMAT(1H ,5X,'T1',8X,'F1',13X,'NO.',13X,'T',/)
       RTD=57.2957795
      DO 11 M=1,10
      W(M)=W(M)/RTD
      WP(M)=WP(M)/RTD
      XI(M)=XI(M)/RTD
11     CONTINUE
       P=3.1415926
      RE=3440.
      WRAD=15.0/RTD
      EM=35.
      DO 600 I=1,6
      RI=I-1.
      DO 500 J=1,13
      RJ=J
      T=(RJ-1.)*15./RTD
      DO 400 K=1,25
      RK=K
      FIN=((RK-1.)*15./RTD)+RI*WRAD
      F=FIN-RI*WRAD
      CON=6.987*10.**(-6)
      SATNO=0.
      DO 300 L=1,10
      RL=L
      TAU=CON*A(L)**1.5
      CALL ELLIP(RI,E(L),WP(L),TP(L),A(L),TAU,ESP,R)
      CALL PRIME(ESP,W(L),XI(L),R,0.,XS,YS,ZS)
      X=RE*SIN(T)*COS(FIN)
      Y=RE*SIN(T)*SIN(FIN)
      Z=RE*COS(T)
      ACC=ARCOS((X*XS+Y*YS+Z*ZS)/(RE*R))
      RGE=((XS-X)**2+(YS-Y)**2+(ZS-Z)**2)**.5
      ARG=SIN(ACC)*R/RGE
      IF(ARG.GT..9999999)ARG=.9999999
      D=(ARSIN(ARG)-P/2.)*RTD
      RT=SQRT(R*R-RE*RE)
      IF(RGE.LE.RT)D=-D
      IF(D.GT.EM)SATNO=SATNO+1.
      IF(SATNO.GT..1)GO TO 400
      IF(RL.LT.9.9)GO TO 300
      T1=T*RTD
      F1=F*RTD
      WRITE(6,20)T1,F1,SATNO,RI
20     FORMAT(1H ,4F12.3)
300    CONTINUE
       IF(RJ.LE.1.)GO TO 500
      IF(RJ.GT.12.)GO TO 500
400    CONTINUE
500    CONTINUE
600    CONTINUE
      END
```

```
      SUBROUTINE PRIME(FSP,WS,XIS,XPS,YPS,XS,YS,ZS)
      A11=COS(FSP)*COS(WS)-COS(XIS)*SIN(WS)*SIN(FSP)
      A12=-SIN(FSP)*COS(WS)-COS(XIS)*SIN(WS)*COS(FSP)
      A21=COS(FSP)*SIN(WS)+COS(XIS)*COS(WS)*SIN(FSP)
      A22=-SIN(FSP)*SIN(WS)+COS(XIS)*COS(WS)*COS(FSP)
      A31=SIN(XIS)*SIN(FSP)
      A32=SIN(XIS)*COS(FSP)
      XS=A11*XPS+A12*YPS
      YS=A21*XPS+A22*YPS
      ZS=A31*XPS+A32*YPS
      RETURN
       END
      SUBROUTINE ELLIP(T,E,WP,TP,A,TAU,FSP,R)
      P=3.1415926
       Z=2.*P*(T-TP)/TAU
      P2=2.*P
2     IF(Z.GT.P2) Z=Z-P2
      IF(Z.GT.P2) GO TO 2
      E1=Z+E*SIN(Z)
      E2=(Z+E*(SIN(E1))-(E*COS(E1))*E1)/(1.-E*COS(E1))
      Q=0.
4     E3=(Z+E*(SIN(E2))-(E*COS(E2))*E2)/(1.-E*COS(E2))
      Q=Q+1.
       DE=E3-E2
      DE2=DE**2
      E2=E3
      IF(DE2.GT..00000001)GO TO 4
      TH=ARCOS((COS(E2)-E)/(1.-E*COS(E2)))
      IF(Z.GT.P) TH=2.*P-TH
      FSP=WP+TH
      R=(A*(1.-E**2))/(1.+E*COS(FSP-WP))
      RETURN
      END
```