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SHANNON'S THEOREM, (U)
MAY 67 G S INNIS NOBSR-95181
UNCLASSIFIED DRL-A-284 NL

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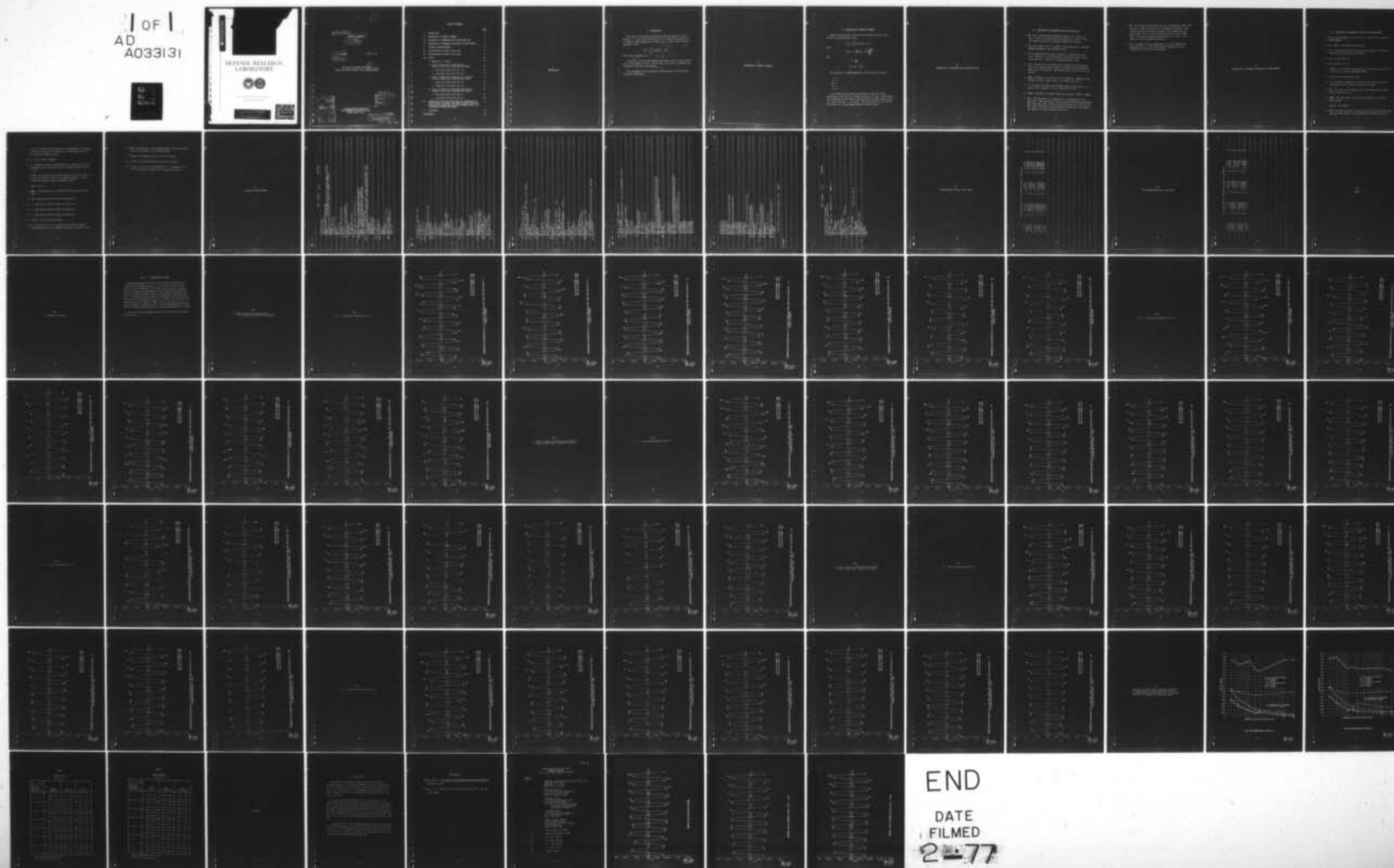

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DEFENSE RESEARCH LABORATORY

THE UNIVERSITY OF TEXAS
AUSTIN, TEXAS 78712

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SHANNON'S THEOREM

by

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DEFENSE RESEARCH LABORATORY
THE UNIVERSITY OF TEXAS AT AUSTIN
AUSTIN, TEXAS 78712

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TABLE OF CONTENTS

I.

INTRODUCTION

I. INTRODUCTION

This report is concerned with some of the practical aspects of Shannon's Theorem, i.e., that a band-limited time series is completely determined by its value at equally spaced time intervals, Δt, if $1/2\Delta t$ is greater than the bandwidth. Thus,

$$f(t) = \sum_{n} f(n\Delta t)\, h(t - n\Delta t) \quad ,$$

where $1/2\Delta t >$ bandwidth of f.

In practice, f is only approximately band limited and the series contains only a finite number of terms. In this report, the effects of bandwidth and truncation of the series are studied.

The programming and other assistance of Miss Cheryl A. Burr are hereby gratefully acknowledged.

II.

EXPLANATION OF SHANNON'S THEOREM

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Shannon's Theorem states that if $x(t)$ is a time series with center frequency ω_o and bandwidth B, then

$$x(t) = \sum x(n\tau)\, h(t - n\tau) \quad ,$$

where

$$h(t) = \cos \frac{\pi t}{2\tau} (2m - 1) \frac{\sin \frac{\pi t}{2\tau}}{\frac{\pi t}{2\tau}} \quad ,$$

with

$$\tau \leq \frac{1}{2B} \quad ,$$

$$\omega_o = (2m - 1) \frac{B}{2} \quad .$$

The parameters of PROGRAM RANDOM that occur above are as follows:

T = t
N = n
TAU = τ
PI = π
M = m .

It is evident from the form of the function f that the heaviest contributors to the sum will be those for which $n\tau$ is near t. This observation is employed in this study in that the series is always evaluated only for those n such that $t - n\tau$ is small. Thus, the values of $f(n\tau)$ used are those with $n\tau$ in an approximately symmetric interval about t.

III.

EXPLANATION OF PARAMETERS READ IN FROM DATA CARD

III. EXPLANATION OF PARAMETERS READ IN FROM DATA CARD

A. NN - used to indicate which plotting routines are to be used. If NN = 1, only the calculated graph will be plotted. If NN = 0, both the original graph and the calculated graph will be plotted.

B. NOY - the "number of Y's", or number of data points used. In PROGRAM RANDOM using Magnetic Tape 138, NOY = 3000.

C. LP - the division factor used to calculate or reconstruct the data curve. When LP = 3, every third point of the data is used to reconstruct the curve. When LP = 5, every fifth point is used.

D. ITT - the size of the data block used to reconstruct the data points. ITT/2 is the number of points taken on each side of the reconstructed point used for calculation. ITT has been equal to 10, 20, 30, 40, 100, and 200.

E. LSUM - the number of data points to be reconstructed. LSUM may be any integer less than or equal to NOY. It is usually set at 40.

F. M - an integer for which the band-limited signal is between (M - 1) · W and M · W Hz. When LP = 3, M = 2. When LP = 5, M = 3.

G. DELTA - the number of seconds between each data point. DELTA = 0.00005.

H. NNA - the first point of the data block to be reconstructed. Here NNA = 1330. When the data are unfiltered (analog filtered) the parameter NA is set equal to NNA. When the data are filtered, NA is set equal to NNA - (FPL - 1)/2 since the filtered data are moved over to the left by the number of the filter weights divided by 2.

I. NNZ - the last point of the data block to be reconstructed. NNZ = 1370 when LP = 5. NNZ = 1372 when LP = 3 for purposes of division. When the data are unfiltered (analog filtered), the parameter NZ is set equal to NNZ. When the data are filtered, NZ is set equal to NNZ - (FPL - 1)/2 since the filtered data are moved over to the left by the number of the filter weights divided by 2.

J. FPL - the number of filter weights used. In these programs, BPF1 (Band-pass Filter 1, 3 kHz-7 kHz) and BPF2 (Band-pass, Filter 2, 4 kHz-6 kHz) both have 201 filter weights.

IV.

EXPLANATION OF PARAMETERS CALCULATED IN PROGRAM RANDOM

IV. EXPLANATION OF PARAMETERS CALCULATED IN PROGRAM RANDOM

The following parameters are in the order in which they are calculated in PROGRAM RANDOM.

A. ND - number of data points used from tape.

B. TAU - self-explanatory; with varying data the parameter TAU equals the division factor (LP) multiplied by DELTA.

C. NA - (see Chap. III, H.).

D. NZ - (see Chap. III, I.).

E. S Array - Y is an integer array and must be converted to real and stored in the array S for use in SUBROUTINE FILTER.

F. XT Array - for saving storage space.

G. N - the number of weights put in as data. These must have a mirror image made in order to get the total array of weights.

H. NP1 - Since there are N weights read in, their subscripts will range from 1 to NP1 (or N + 1).

I. NPMP1 - the total number in the array of weights after having been mirror-imaged.

J. W Array - the weights.

K. NOX - the number of points in the array to be used for calculation which equals the total number of points divided by the division factor LP.

L. Z Array - the array consisting of every third point (LP = 3) or every fifth point (LP = 5) of the original data. The Z array is used for calculating the complete array.

M. T - the time (equals I*DELTA).

N. R - parameter that gives the subscript of the element in the Z array making that element equivalent to the corresponding element in the Y array.

O. LSTART - the subscript of the initial element used in the Z array in scanning ITT number of points for calculating one point. In this scanning the subscripts range from LSTART to LSTOP.

P. LSTOP - (see O.).

Q. ZMEAN - the mean that is to be subtracted from every point in the Z array.

R. TAR - taken directly from the formula (see Chap. II).

S. A - taken directly from the formula (see Chap. II.).

T. B - taken directly from the formula (see Chap. II.).

U. C - taken directly from the formula (see Chap. II.).

V. S Array - the newly calculated array.

W. N1 - the sup error, i.e., the greatest deviation between any two corresponding points in the original array and the calculated array.

X. RMS - the rms error, i.e., the average deviation over all the points from the original data to the calculated data.

Y. O Array - the original array to be used for plotting.

Z. C Array - the calculated array to be used for plotting.

AA. X Array - the value of the subscripts I, I = 1 through ND, to be used for plotting both against the O array and the C array.

V.

LISTING OF PROGRAM RANDOM

3200 FORTRAN (2.1) 27/09/66

```
      PROGRAM RANDOM
      DIMENSION Y(3000)
      COMMON XT(3000),W(200),XX(400)
      COMMON             X(43),S(3000),K(41),O(43),MI(41),MABS(4 ),Z(1000
     1),ZZ(1000),C(43),IC(52),SUPN(11),PRMSN(11),RMSN(11)  ,TP(28)
      INTEGER Y
      INTEGER FPL  ,TP
      EQUIVALENCE  (Y,XT)
      ND=3000
      NDX= 0
      DO 4 I=1,13
      BUFFER IN  (3,1)(Y(1),Y(2))
    4 CONTINUE
   51 BUFFER IN  (3,1)(Y(1),Y(ND))
   10 GO TO  (10,123,13,13)UNITSTF(3)
   13 BACKSPACE 3
      NDX= NDX + 1
      IF (NDX .EQ. 5)14,51
   14 PRINT 15
   15 FORMAT(22H PARITY ERROR ON LUN 3)
      GO TO 42
  123 READ 3,NN,NOY,LP,ITT,LSUM,M,DELTA,NNA,NNZ,FPL
    3 FORMAT  (I2,2XI5,4(3XI5), 3XF11.7 ,3(XI5))
      PRINT 3,NN,NOY,LP,ITT,LSUM,M,DELTA,NNA,NNZ,FPL
      PRINT 22222,LP,NNA,NNZ
22222 FORMAT (26H RANDOM CURVE USING EVERY ,I2,23H POINTS BETWEEN POINTS
     1 ,I5,5H AND,I5,47H FOR CALCULATION, FILTERED WITH BPF1, 3KC - 7KC)
      TAU=LP*DELTA
      NA=NNA-(FPL-1)/2
      NZ=NNZ-(FPL-1)/2
      DO 5555 I=1,ND
 5555 S(I)=Y(I)
      DO 5556 I=1,ND
 5556 XT(I)=S(I)
      N=100
      NP1=N+1
      NPMP1=NP1+N
      READ 8888, (W(I),I=1,NP1)
 8888 FORMAT (8(F10.5))
      CALL FILTER (ND,NP1,MN)
      PRINT 59
```

```
   59 FORMAT(15H  ORIGINAL Y(I))
      DO 6666 I=1,ND
 6666 S(I)=XT(I)
      DO 6667 I=1,ND
 6667 Y(I)=S(I)
      DO 57 I=1,ND
      IF(I.GE.NA.AND.I.LE.NZ) 61,57
   61 PRINT 58,Y(I)
   58 FORMAT(3XI10)
   57 CONTINUE
      DO 64 I=NA,NZ
      II=I-NA+1
      K(II)= Y(I)
   64 CONTINUE
      NSTART = NOY/2
      NOX = NOY/LP
      NFF = NSTART + LP*NOX/2
      IFF = 2*NSTART - NFF
      DO 2 I=1,NOX
      JFF = IFF + LP*I
      Z(I) = Y(JFF)
    2 CONTINUE
      DO 150 I = 1,ND
      Y(I) = 0
  150 CONTINUE
      PI = 3.141592654
      DO 20 I=NA,NZ
      T = I * DELTA
      S(I) = 0
      R=T/LP
      LSTART=R-ITT/2+.001
      LSTOP=R+ITT/2+.001
      SM=0.0
      DO 997 NO=LSTART,LSTOP
  997 SM=SM+Z(NO)
      ZMEAN=SM/(LSTOP-LSTART+1)
      IF (LSTOP.GT.NOX) 85,84
   85 LSTOP = NOX
   84 DO 19 N=LSTART,LSTOP
      ZZ(N)=Z(N)-ZMEAN
      TAR = T - (N * TAU)
      IF (TAR .EQ. 0)47,48
   47 A = 1
```

```
      B = 1
      GO TO 52
   48 A = (2 * TAU)/(PI * TAR)
      B = SINF((PI * TAR)/(2 * TAU))
   52 C = COSF(PI * ((2 * M) - 1) * TAR * (1/(2 * TAU)))
      S(I)=S(I)+ZZ(N)*A*B*C
   19 CONTINUE
      Y(I) = S(I)
      PRINT 6, T, Y(I)
    6 FORMAT(20X4H  X(,F10.7,6H) =    ,I10)
   20 CONTINUE
      MSUM = 0
      N1 = 0
      DO 62 I=NA,NZ
      II=I-NA+1
      MI(II)=IABS(Y(I) -K(II))
      MSUM = MSUM + (MI(II) ** 2)
      IF (MI(II) .GT. N1) 74,62
   74 N1 = MI(II)
   62 CONTINUE
      PRINT 65, N1
   65 FORMAT(//20X,13H SUPNORM =    ,I10)
      SUM = MSUM
      RMS = SQRTF (SUM/(NZ-NA+1))
      PRINT 66, RMS
   66 FORMAT(20X,13H RMSNORM =    ,F10.4)
      M=LP/2
      MM=M+1
      DO 222 L=1,MM
      SUPN(L)=0
  222 PRMSN(L)=0
      DO 11 I=NA,NZ
      II=I-NA+1
      J=(I+M)/LP
      L=IABS (LP*J-I)+1
      P=IABS (Y(I) - K(II))
      IF (P.GT.SUPN(L)) 110,11
  110 SUPN(L)=P
   11 PRMSN(L)=PRMSN(L) + P**2
      DO 420 L=1,MM
      NP = ((NZ-NA)/LP) *2
      IF (L.EQ.MM.AND.2 * (L-1).GT.(LP-1).OR.L.EQ.1) 589,420
  589 NP=(NZ-NA)/LP
```

```
420 RMSN(L) = SQRTF (PRMSN(L)/NP)
    DO 669 L=1,MM
 68 FORMAT (//20X,8HSUPNORM ,I1,3H = ,F10.4,10X,8HRMSNORM ,I1,3H = ,
   1F15.12)
669 PRINT 68,L,SUPN(L),L,RMSN(L)
    DO 69 I=NA,NZ
    II=I-NA+1
    O(II) = K(II)
    C(II) = Y(I)
    X(II)=I
 69 CONTINUE
    CALL PLOT(5.0,-12.0,-3)
    CALL PLOT(0.0,1.5,-3)
    IX = 4H I
    IO = 4HO(I)
    IF (NN.GT.0) 355,356
356 CONTINUE
    X(42) = NA
    X(43) = 5.0
    O(42)=-240.0
    O(43)=60.0
    CALL AXIS(0.0, 0.0,IO,4,8.0,90.0,O(42),O(43),0.1,1.0,1)
    CALL AXIS(0.0,4.0,IX,-4,8.0,0.0,X(42),X(43),0.1,1.0,1)
    CALL LINE(X,O,41,1,1,4,0.08,1,0.0)
    CALL PLOT(12.0,0.0,-3)
355 CONTINUE
    X(42) = NA
    X(43) = 5.0
    C(42)=-240.0
    C(43)=60.0
    ICT=4HC(I)
    CALL AXIS (0.0, 0.0,ICT,4,8.0,90.0,C(42),C(43),0.1,1.0,1)
    CALL AXIS(0.0,4.0,IX,-4,8.0,0.0,X(42),X(43),0.1,1.0,1)
    CALL LINE(X,C,41,1,1,4,0.08,1,0.0)
       CALL LINE (X,C,41,1,-LP,1,0.08,1,0.0)
    IC(1)=4HNO.
    IC(2)=4HOF P
    IC(3)=4HOINT
    IC(4)=4HS FR
    IC(5)=4HOM F
    IC(6)=4HACH
    IC(7)=4HSIDE
    IC(8)=4H USE
```

```
      IC(9)=4HD FO
      IC(10)=4HR CA
      IC(11)=4HLCUL
      IC(12)=4HATIO
      IC(13)=4HN -
      CALL SYMBOL (.5, 0.0,.14,IC(1),0.0,52)
      FPN=ITT/2
      CALL NUMBER (7.0, 0.0,.14,FPN,0.0,-1)
      TP(1)=4HFILT
      TP(2)=4HERED
      TP(3)=4H BY
      TP(4)=4HBPF)
      TP(5)=4H (3K
      TP(6)=4HC -
      TP(7)=4H7KC)
      CALL SYMBOL (2.5,-0.5,.14,TP(1),0.0,28)
      NDX= 0
   36 BUFFER OUT (4,1)(Y(1),Y(ND))
   31 GO TO (31,32,34,34)UNITSTF(4)
   34 BACKSPACE 4
      NDX= NDX + 1
      IF (NDX .EQ. 5)26,36
   32 NDX= 0
   33 BUFFER OUT (4,1)(Z(1),Z(NOX))
   41 GO TO (41,42,44,44)UNITSTF(4)
   44 BACKSPACE 4
      NDX= NDX + 1
      IF (NDX .EQ. 5)26,33
   26 PRINT 27
   27 FORMAT(22H PARITY ERROR ON LUN 4)
   42 CONTINUE
         END

        3200 FORTRAN DIAGNOSTIC RESULTS - FOR    RANDOM

NO ERRORS
```

3200 FORTRAN (2.1) 27/09/66

```
      SUBROUTINE FILTER (N,MX,MN)
      COMMON YT(3000),H(200),XX(400)
      MN = 2*MX -1
      JAX = 0     $NOX = MX     $XX(MX) = H(1)     $NOP = 1
      MMX=MX+1
      DO 1 I=MMX,MN
      NOP = NOP + 1
      XX(I) = H(NOP)
      NOX = NOX - 1
    1 XX(NOX) = H(NOP)
      NCHEK = N - 2*(MX - 1)
      K = MN - 1              $ KP = 0
    6 K = K+ 1
      IF(N.GT.K - 1) 4,5
    4 JAX = JAX + 1
      NOR = 0     $ SUM=0
      KP = KP + 1                        $ MO = KP + MN - 1
      DO 3 KX=KP,MO
      NOR = NOR + 1
    3 SUM=SUM+YT(KX)*XX(NOR)
      YT(JAX)=SUM
      GO TO 6
    5 CONTINUE
      END
```

VI.

FILTER WEIGHTS FOR BPF1 (3 kHz-7 kHz)

FILTER WEIGHTS FOR BAND-PASS FILTER 1 (3KC-7KC)

.30001	0	-0.25331	0	.14161	0	-0.02815	0
-0.03540	0	.04053	0	-0.01573	0	-0.00517	0
.00585	0	-0.00313	0	0	0	-0.00209	0
.00393	0	-0.00150	0	-0.00289	0	.00450	0
-0.00221	0	-0.00088	0	.00175	0	-0.00070	0
0	0	-0.00057	0	.00117	0	-0.00048	0
-0.00098	0	.00162	0	-0.00084	0	-0.00035	0
.00072	0	-0.00030	0	0	0	-0.00026	0
.00055	0	-0.00023	0	-0.00049	0	.00083	0
-0.00044	0	-0.00019	0	.00039	0	-0.00017	0
0	0	-0.00015	0	.00032	0	-0.00014	0
-0.00024	0	.00050	0	-0.00027	0	-0.00011	0
.00025	0	-0.00011	0	0			

VII.

FILTER WEIGHTS FOR BPF2 (4 kHz-6 kHz)

FILTER WEIGHTS FOR BAND-PASS FILTER 2 (4KC-6KC)

.1999[illegible]	0	-0.18597	0	.14779	0	-0.09561	0
.04246	0	0	0	-0.02495	0	.03174	0
-0.02504	0	.01216	0	0	0	-0.00719	0
.00859	0	-0.00606	0	.00244	0	0	0
-0.00049	0	-0.00035	0	.00123	0	-0.00117	0
0	0	.00164	0	-0.00280	0	.00282	0
-0.00169	0	0	0	.00144	0	-0.00205	0
.00173	0	-0.00086	0	0	0	.00044	0
-0.00039	0	.00009	0	.00011	0	0	0
-0.00037	0	.00075	0	-0.00086	0	.00057	0
0	0	-0.00059	0	.00091	0	-0.00084	0
.00046	0	0	0	-0.00032	0	.00039	0
-0.00026	0	.00008	0	0			

VIII.

GRAPHS

VIII.

A. Explanation of Graphs

VIII. A. EXPLANATION OF GRAPHS

The graphs bound within the text of this report are reproduced data calculated using Shannon's Theorem. In an envelope at the back of the report are three transparencies. The first of these shows the original data, which has been analog band-pass filtered (4-6 kHz) and digitized at 20 kHz. The graphs of Chapter VIII, B. are attempts at reproducing these data. As is easily seen from overlaying the graphs and from the peak and rms errors, these reproductions are not very accurate. The second and third transparencies and the graphs of Chapter VIII, C. and VIII, D. are the results of digitally filtering the data on the first transparency, using band-pass filters BPF1 and BPF2 respectively, and then applying Shannon's Theorem.

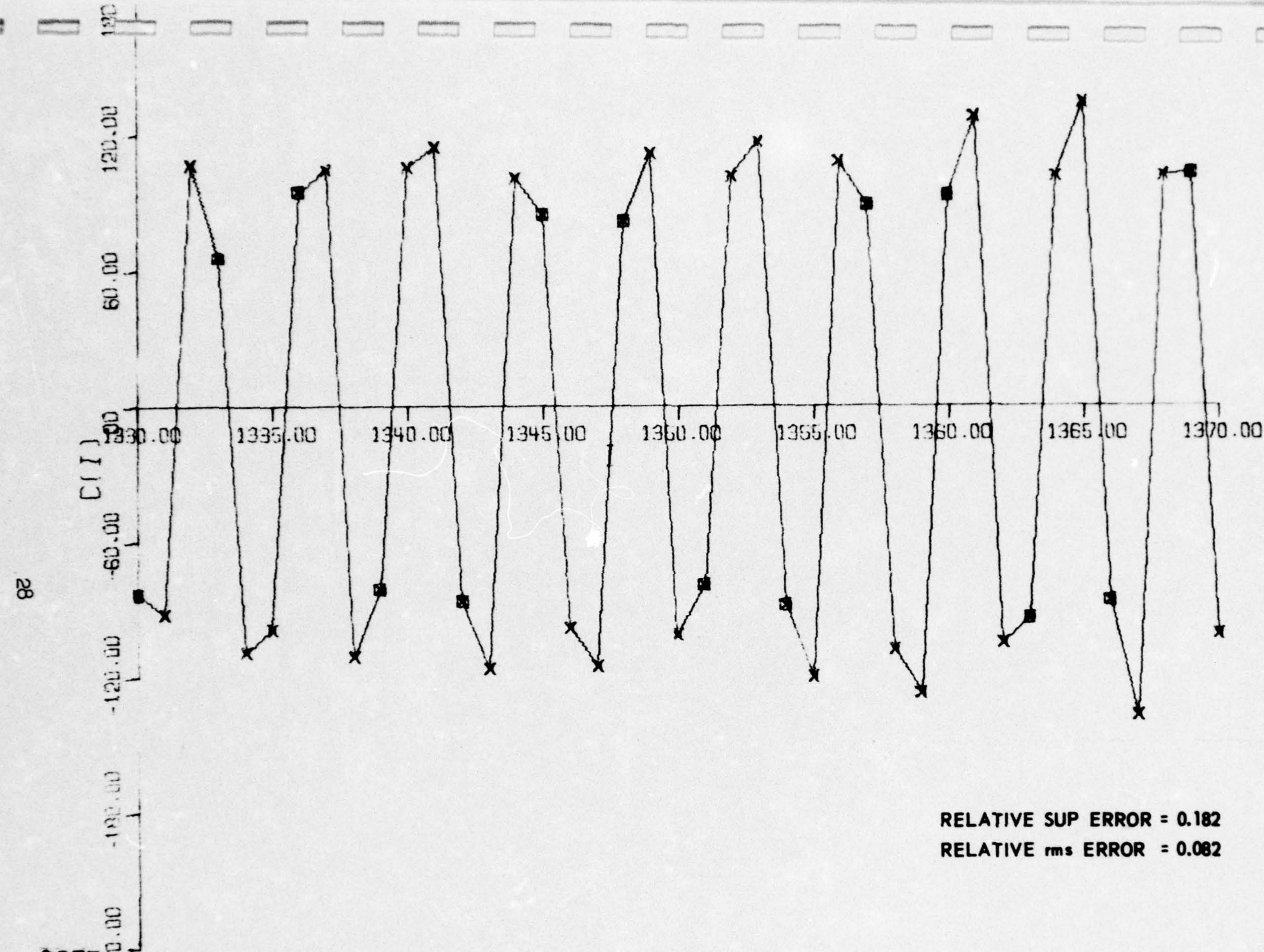
Data from outside the displayed interval were used in the calculations when necessary.

VIII.

B. Effect of Shannon's Theorem Used with Analog Filtered Data, Illustrated by Graphs

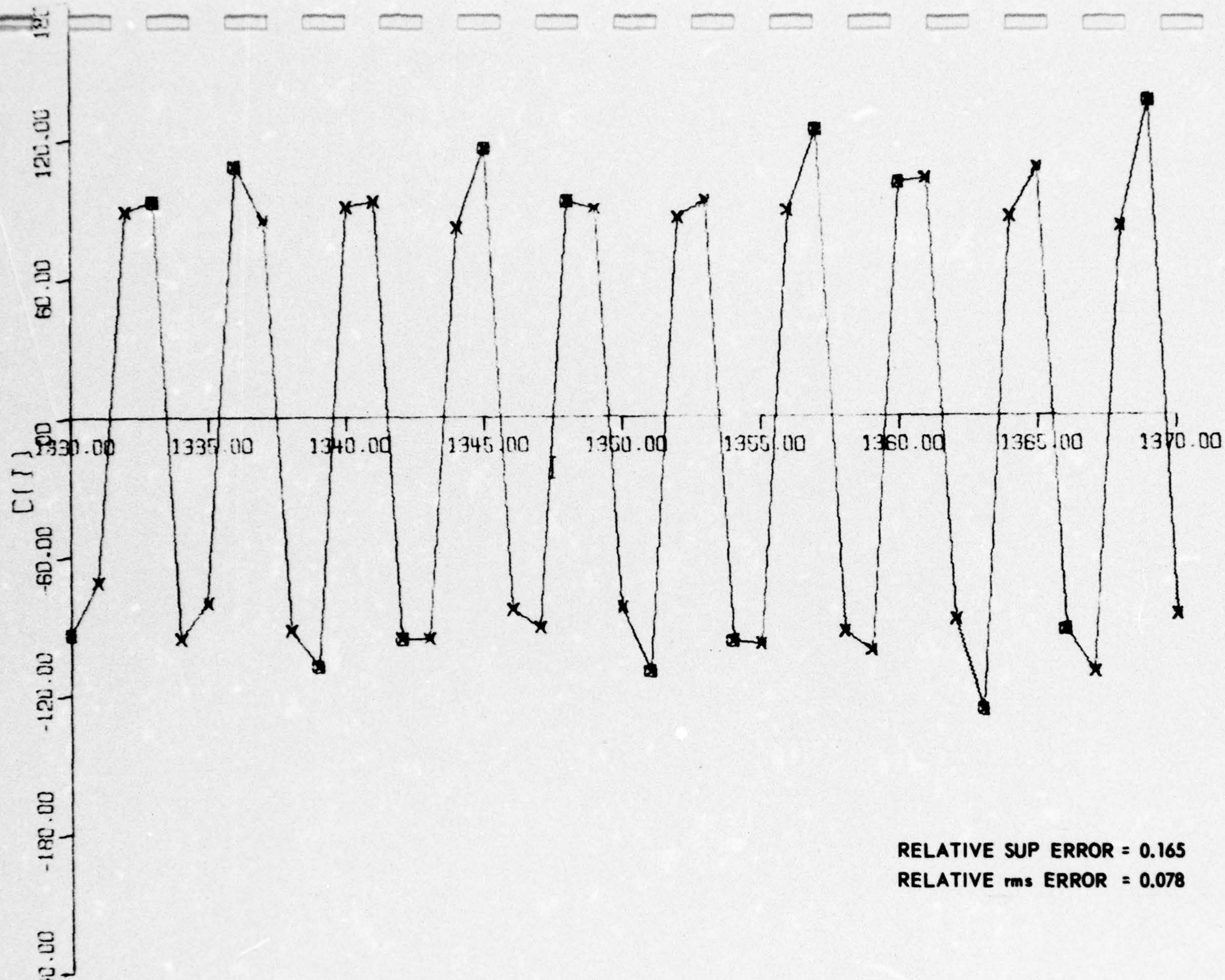
VIII.

B. 1. Using Every Third Point (LP = 3)



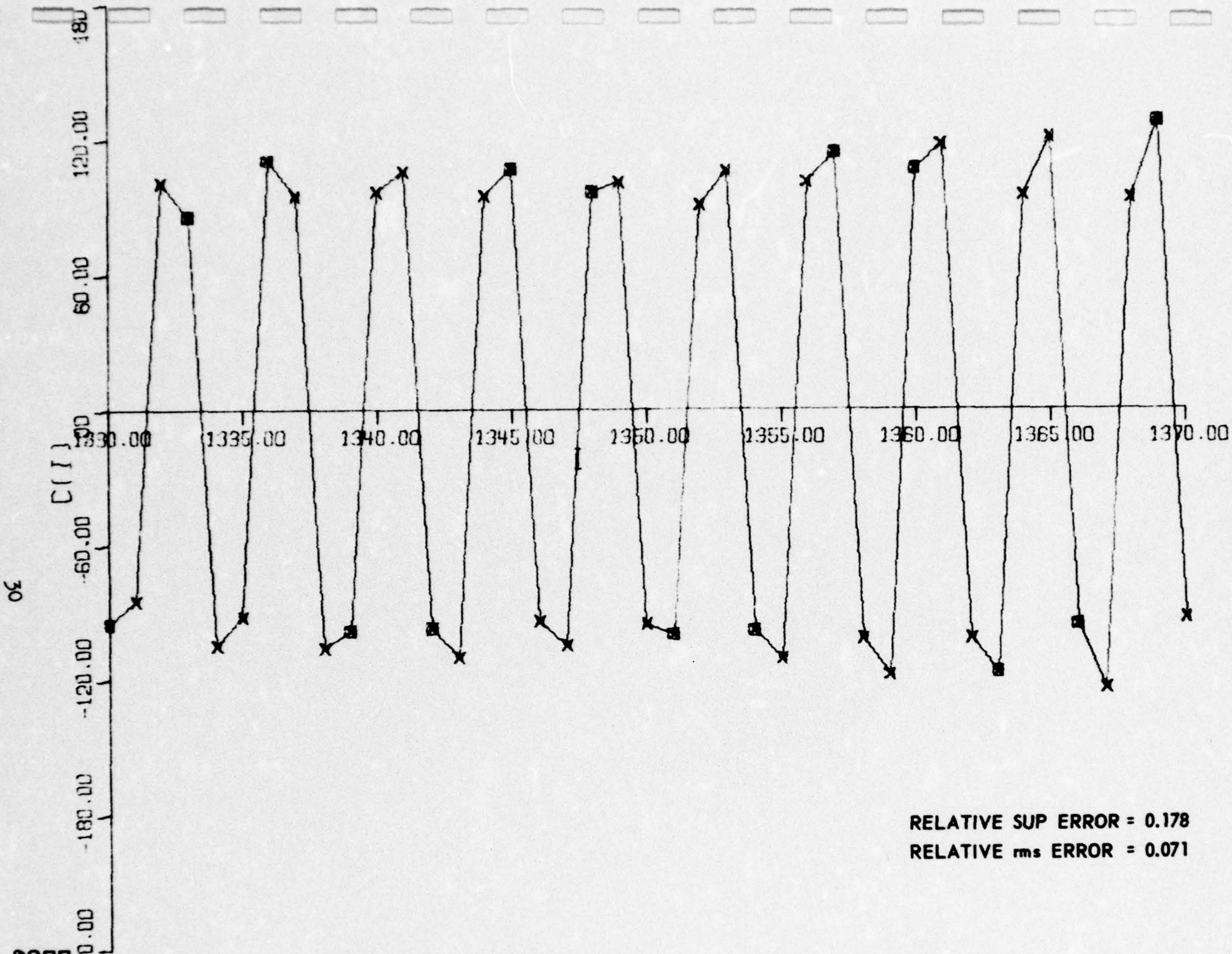
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 3
ANALOG FILTERED

DRL - UT
DWG AS-67-644
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATI^N – 5
ANALOG FILTERED

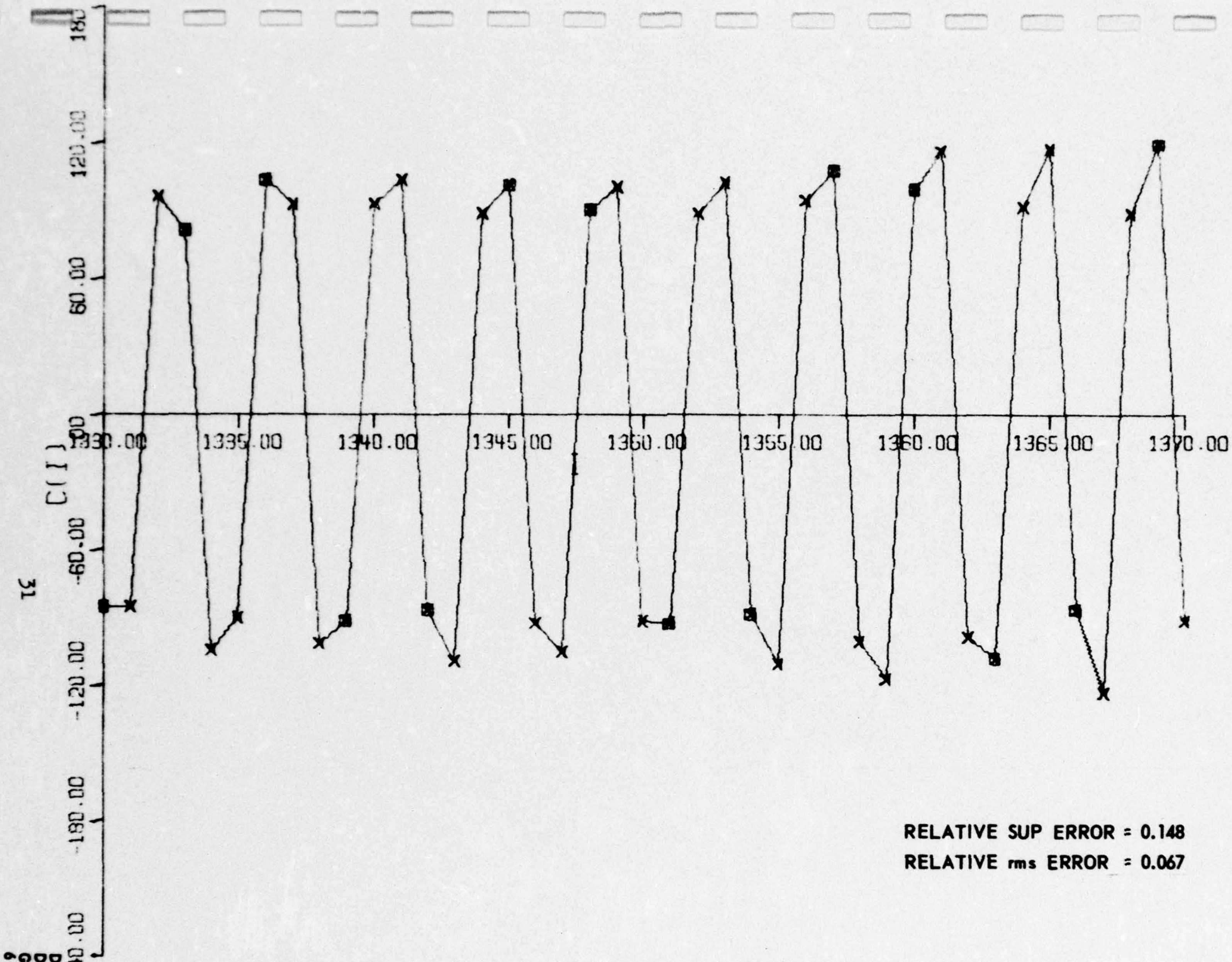
DRL - UT
DWG AS-67-645
GSI - EJW
6 - 20 - 67



RELATIVE SUP ERROR = 0.178
RELATIVE rms ERROR = 0.071

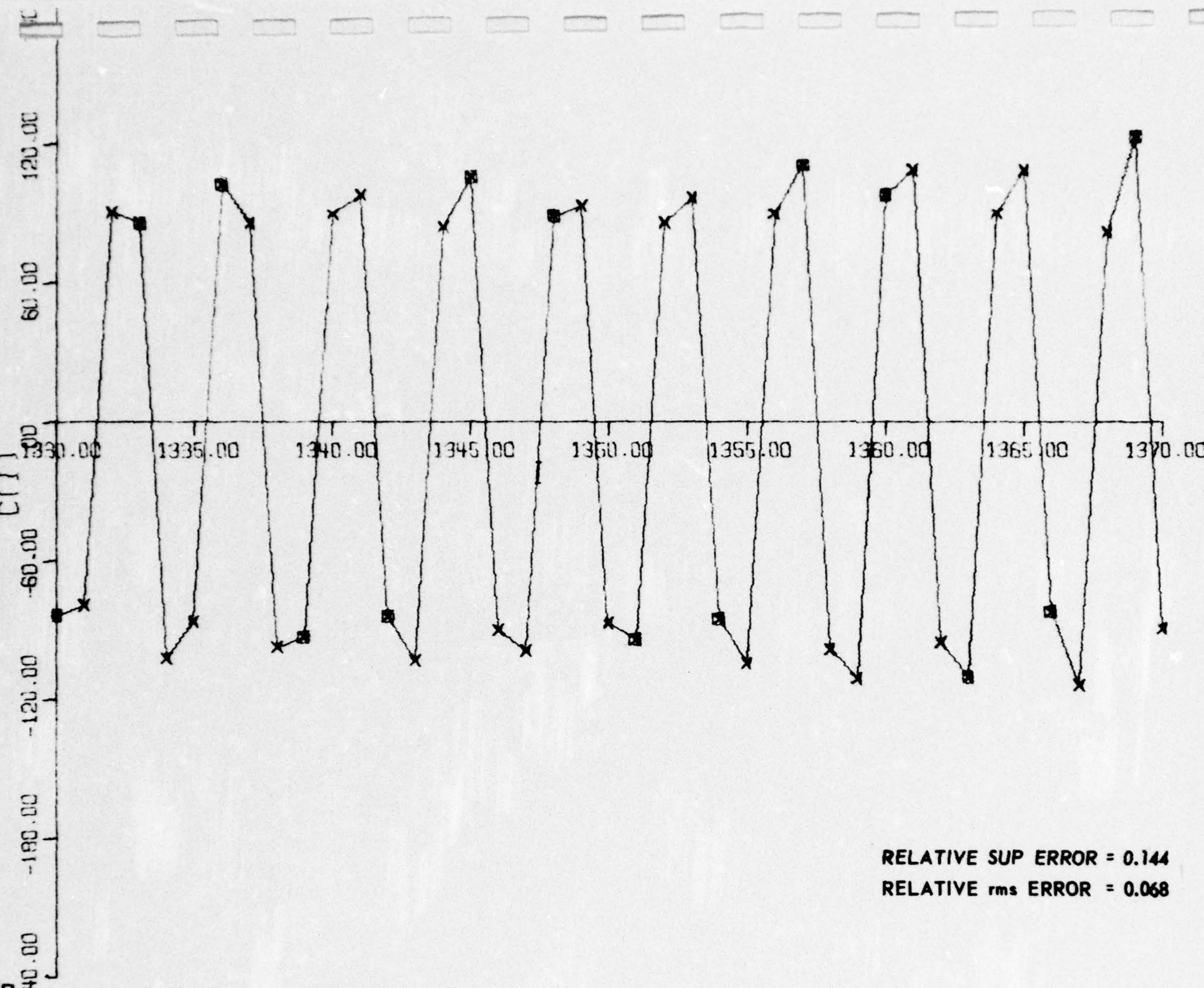
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 10
ANALOG FILTERED

DRL - UT
DWG AS-67-646
GSI - EJW
6 - 20 - 67



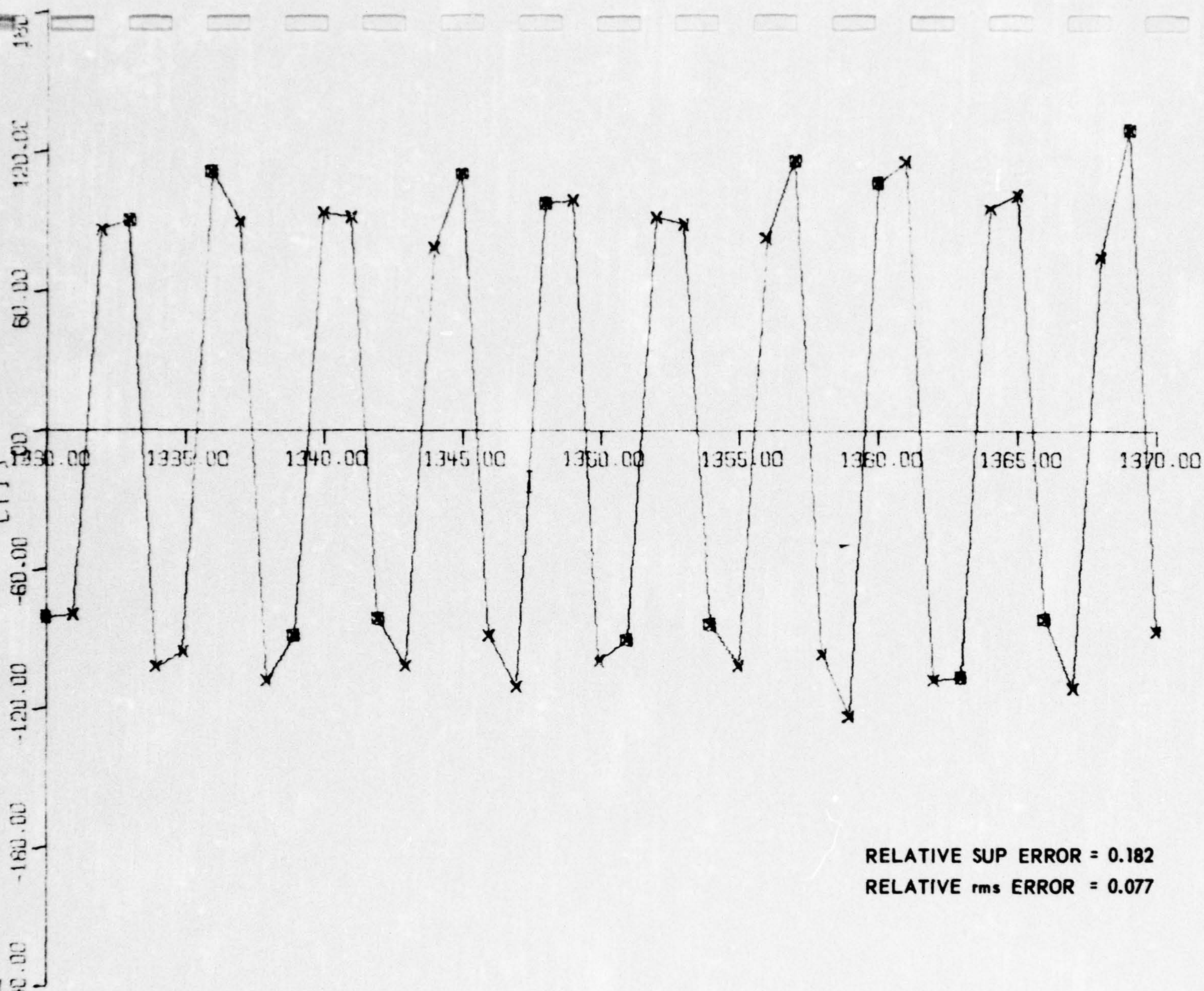
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATIO ' – 15
ANALOG FILTERED

DRL - UT
DWG AS-67-647
GSI - EJW
6 - 20 - 67



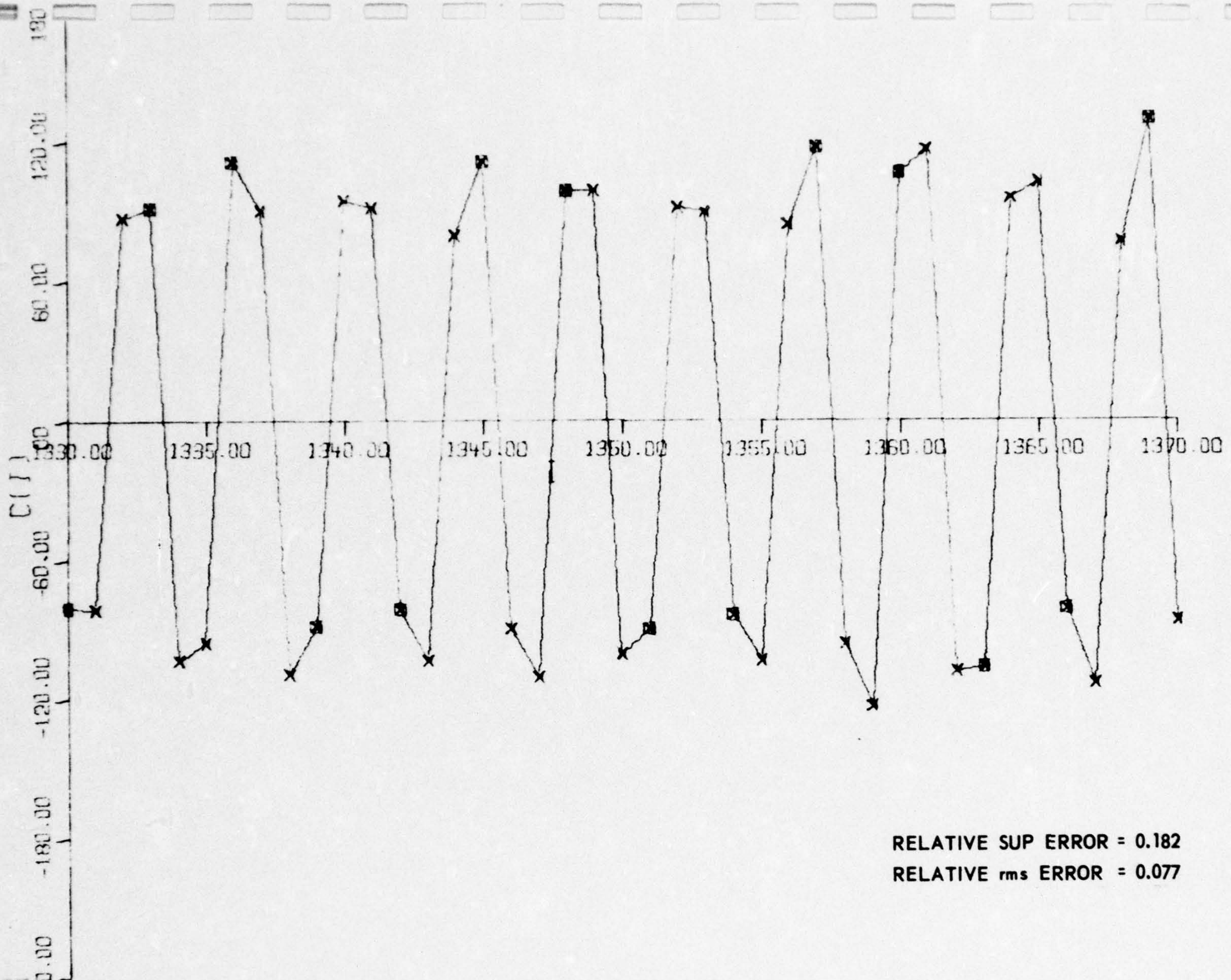
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 20
ANALOG FILTERED

DRL - UT
DWG AS-67-648
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 100
ANALOG FILTERED

DRL - UT
DWG AS-67-649
GSI - EJW
6 - 20 - 67

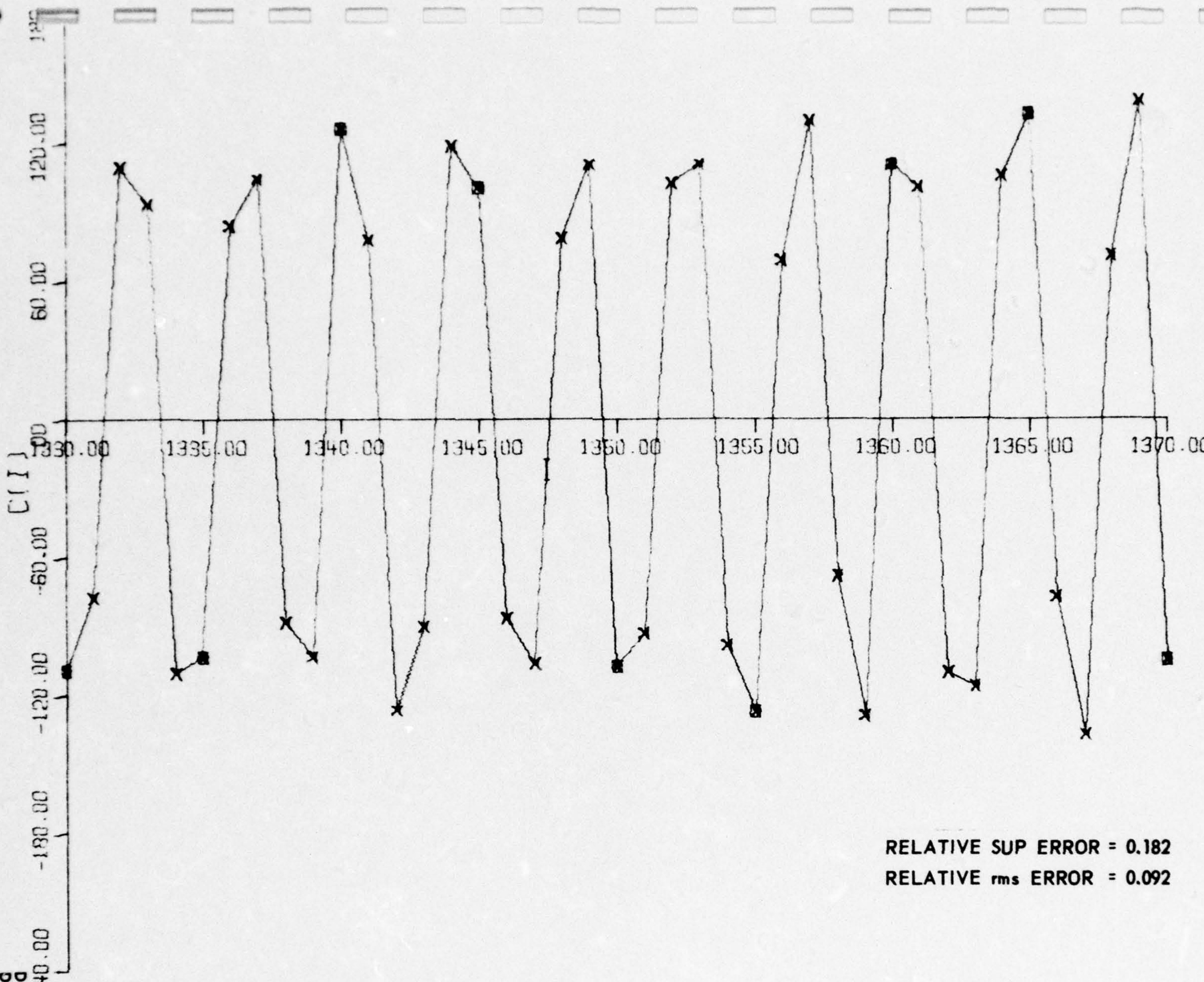


REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATI`N – 200
ANALOG FILTERED

DRL - UT
DWG AS-67-650
GSI - EJW
6 - 20 - 67

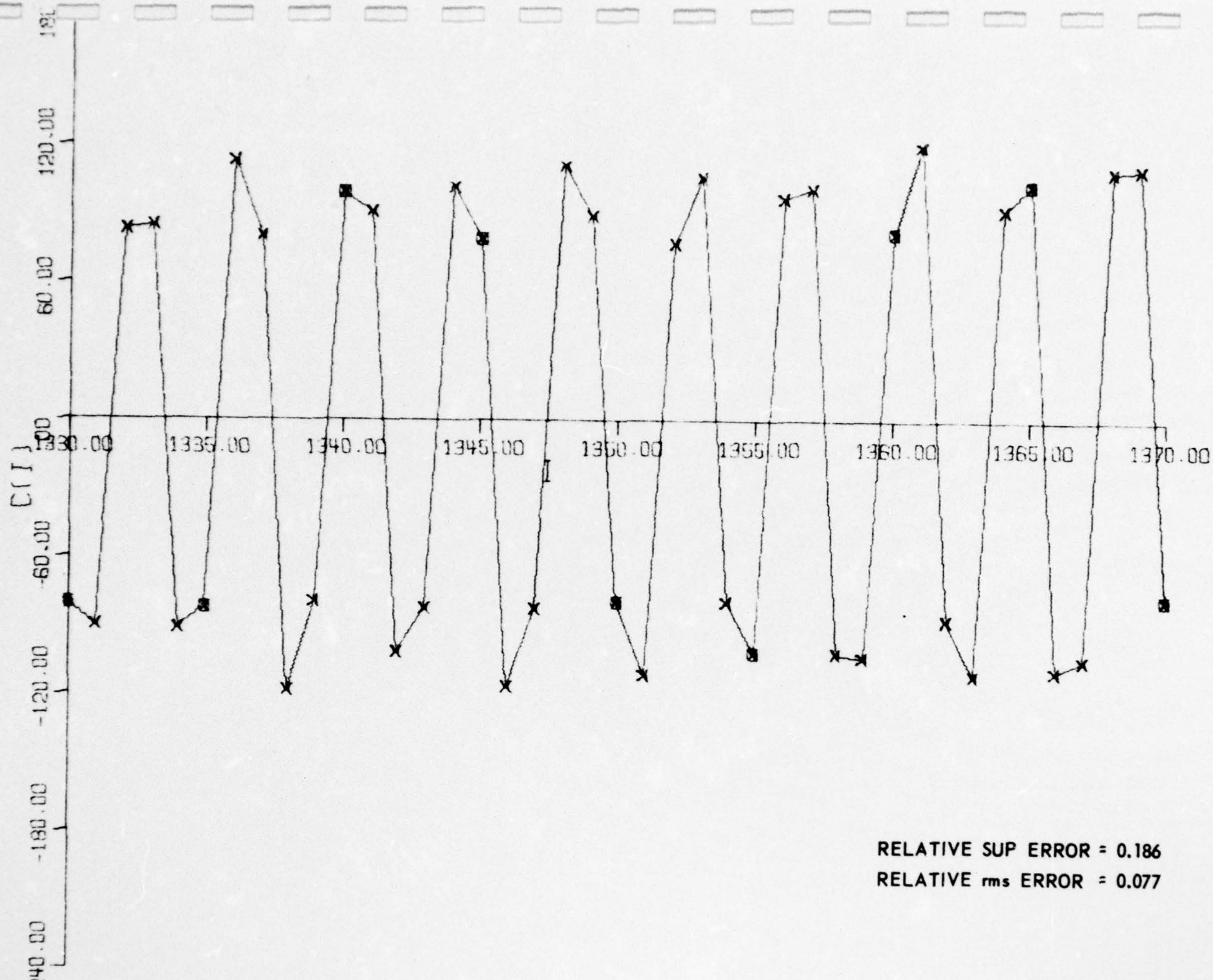
VIII.

B. 2. Using Every Fifth Point (LP = 5)



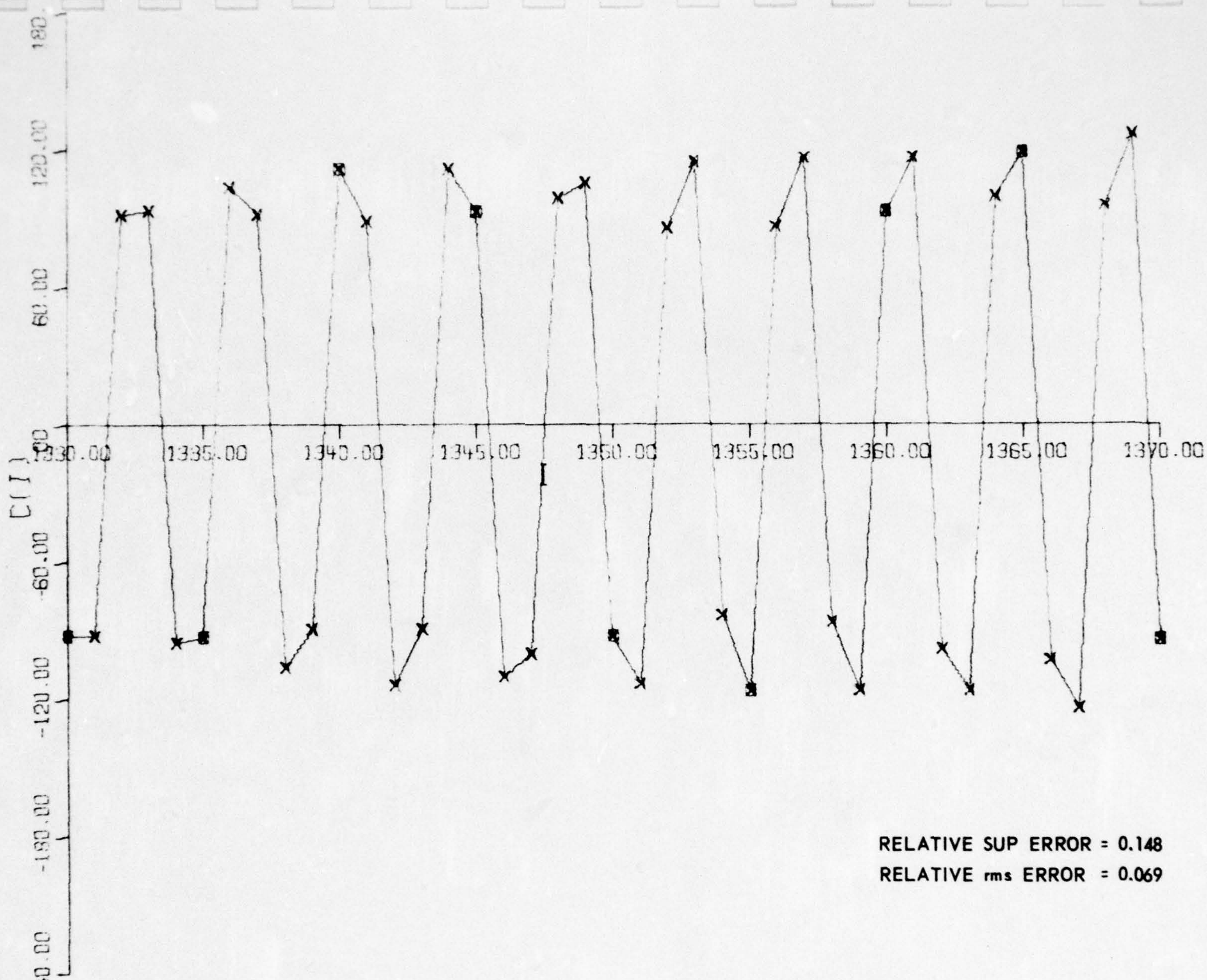
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 3
ANALOG FILTERED

DRL - UT
DWG AS-67-651
GSI - EJW
6 - 20 - 67



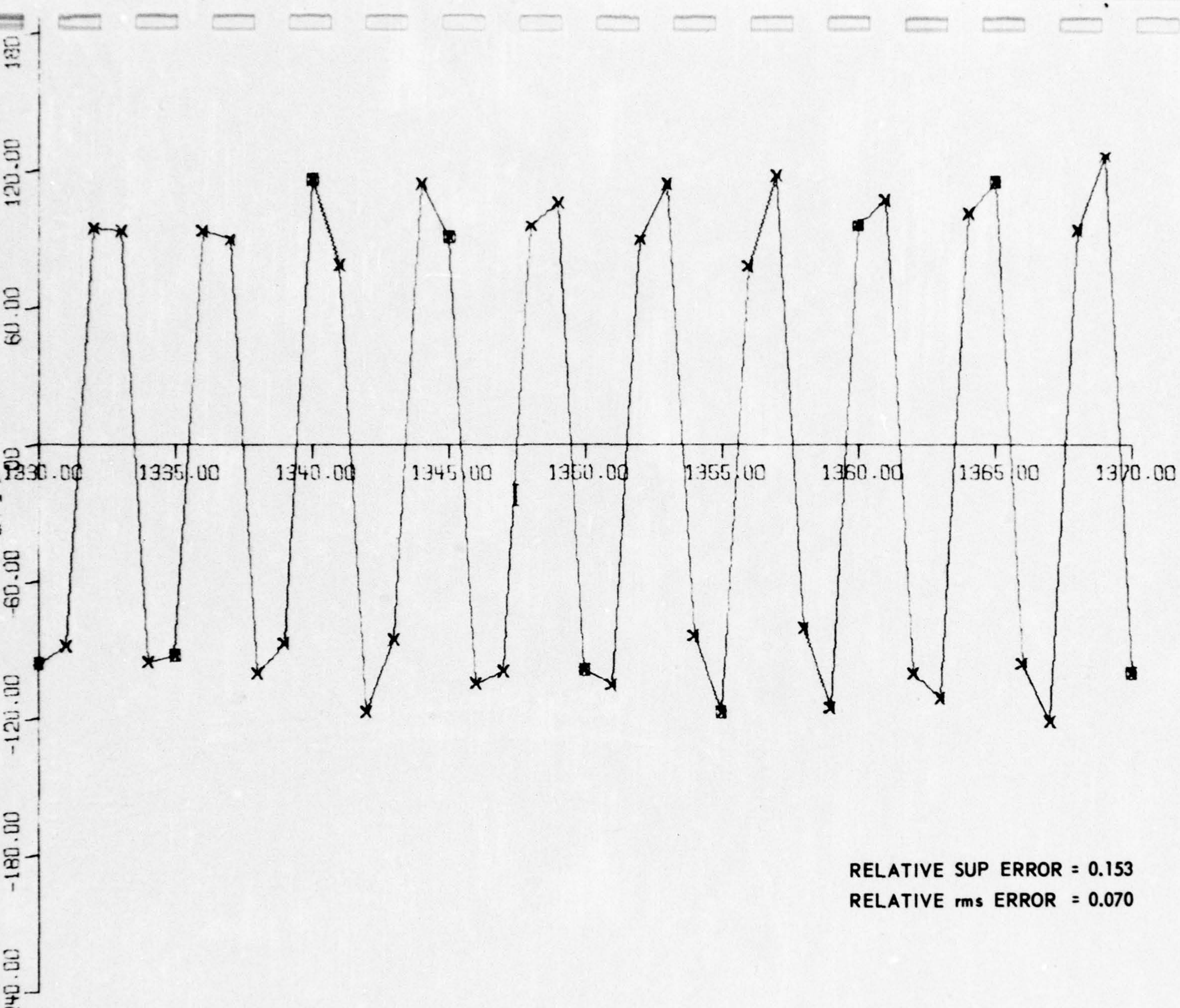
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULAT ON – 5
ANALOG FILTERED

DRL - UT
DWG AS-67-652
GSI - EJW
6 - 20 - 67



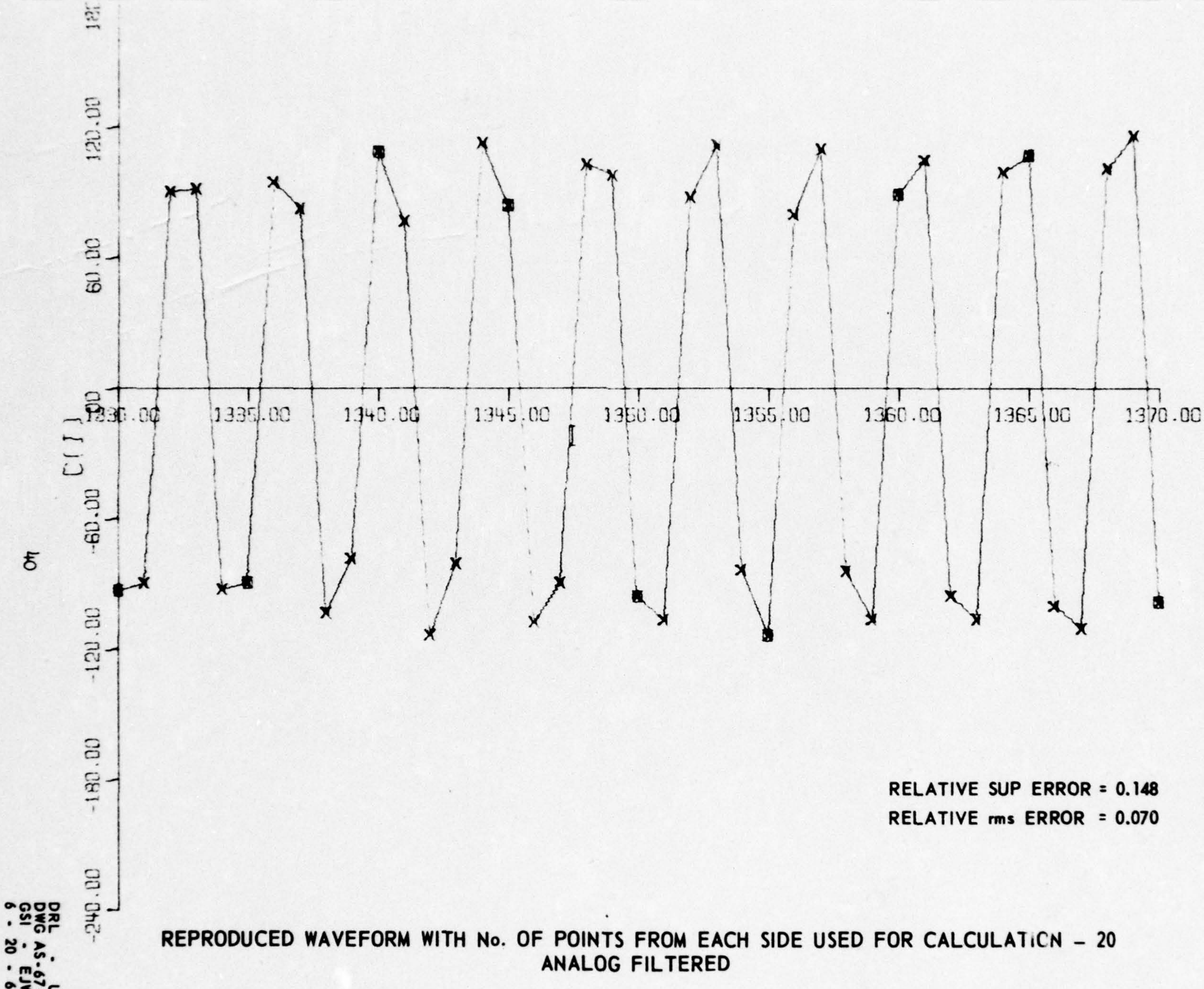
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 10
ANALOG FILTERED

DRL - UT
DWG AS-67-653
GSI - EJW
6 - 20 - 67

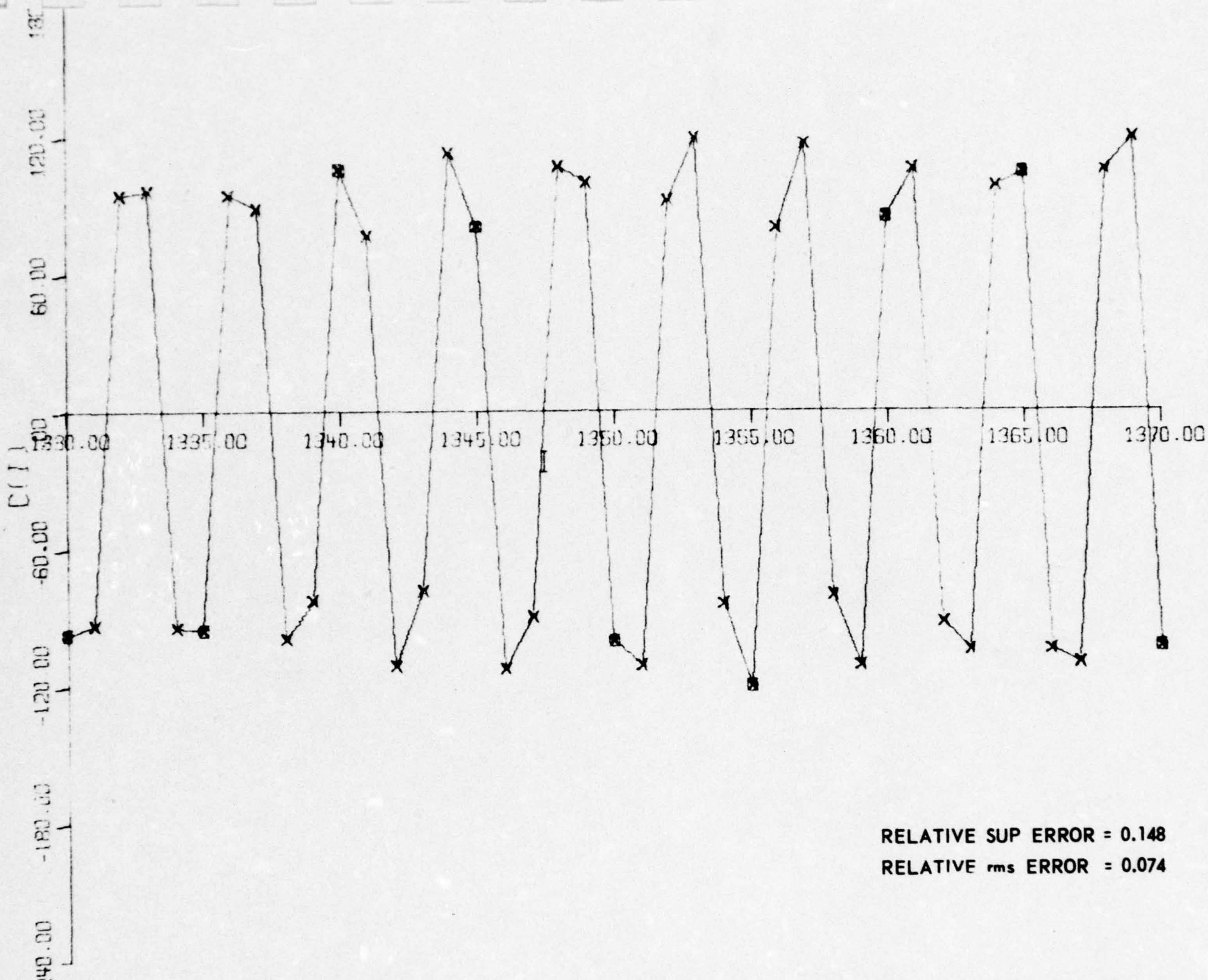


REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION - 15
ANALOG FILTERED

DRL - UT
DWG AS-67-654
GSI - EJW
6 - 20 - 67

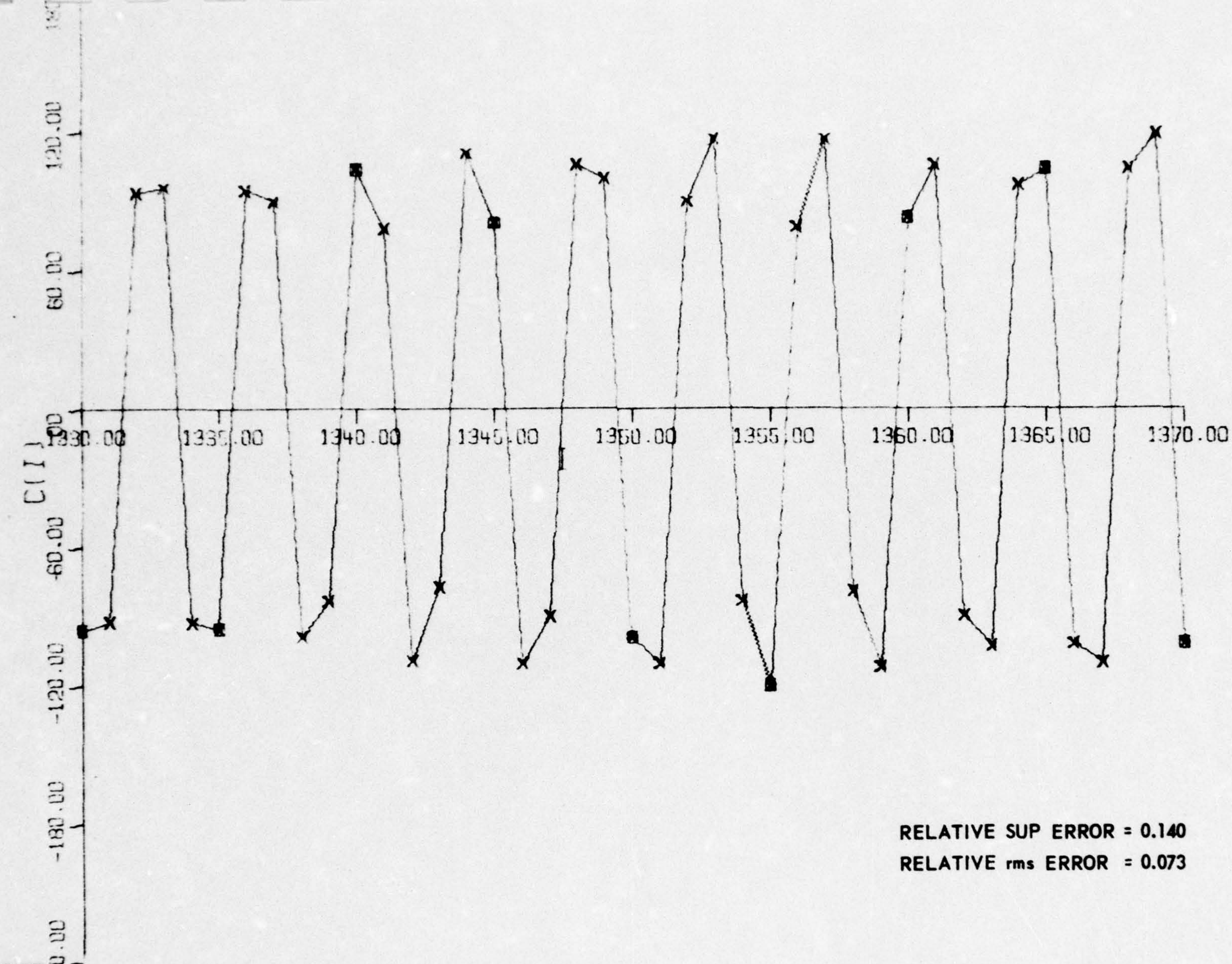


REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 20
ANALOG FILTERED



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 100
ANALOG FILTERED

DRL • UT
DWG AS-67-656
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 200
ANALOG FILTERED

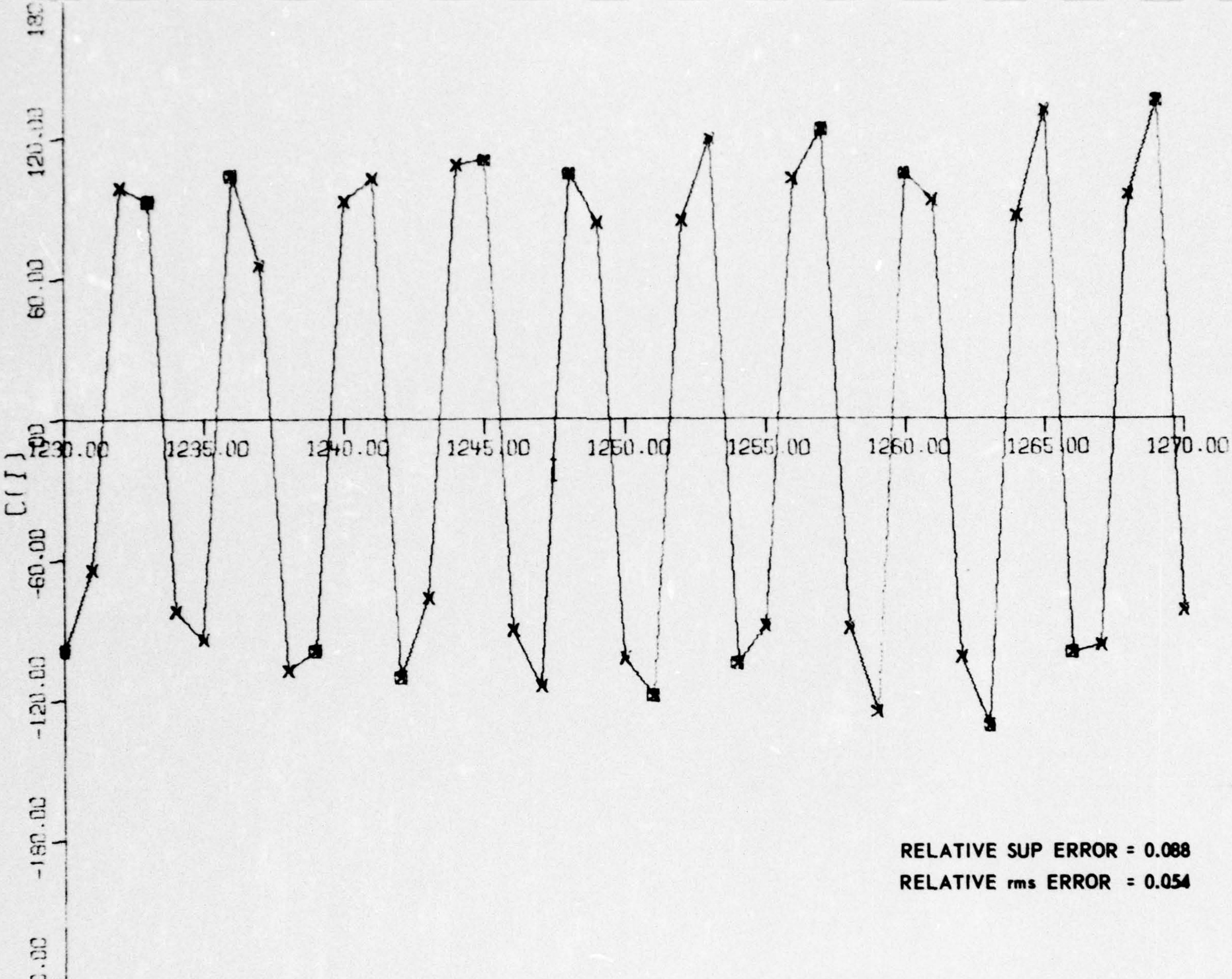
DRL - UT
DWG AS-67-657
GSI - EJW
6 - 20 - 67

VIII.

C. Effect of Shannon's Theorem with Data Filtered By BPF1 (3 kHz-7 kHz), Illustrated by Graphs

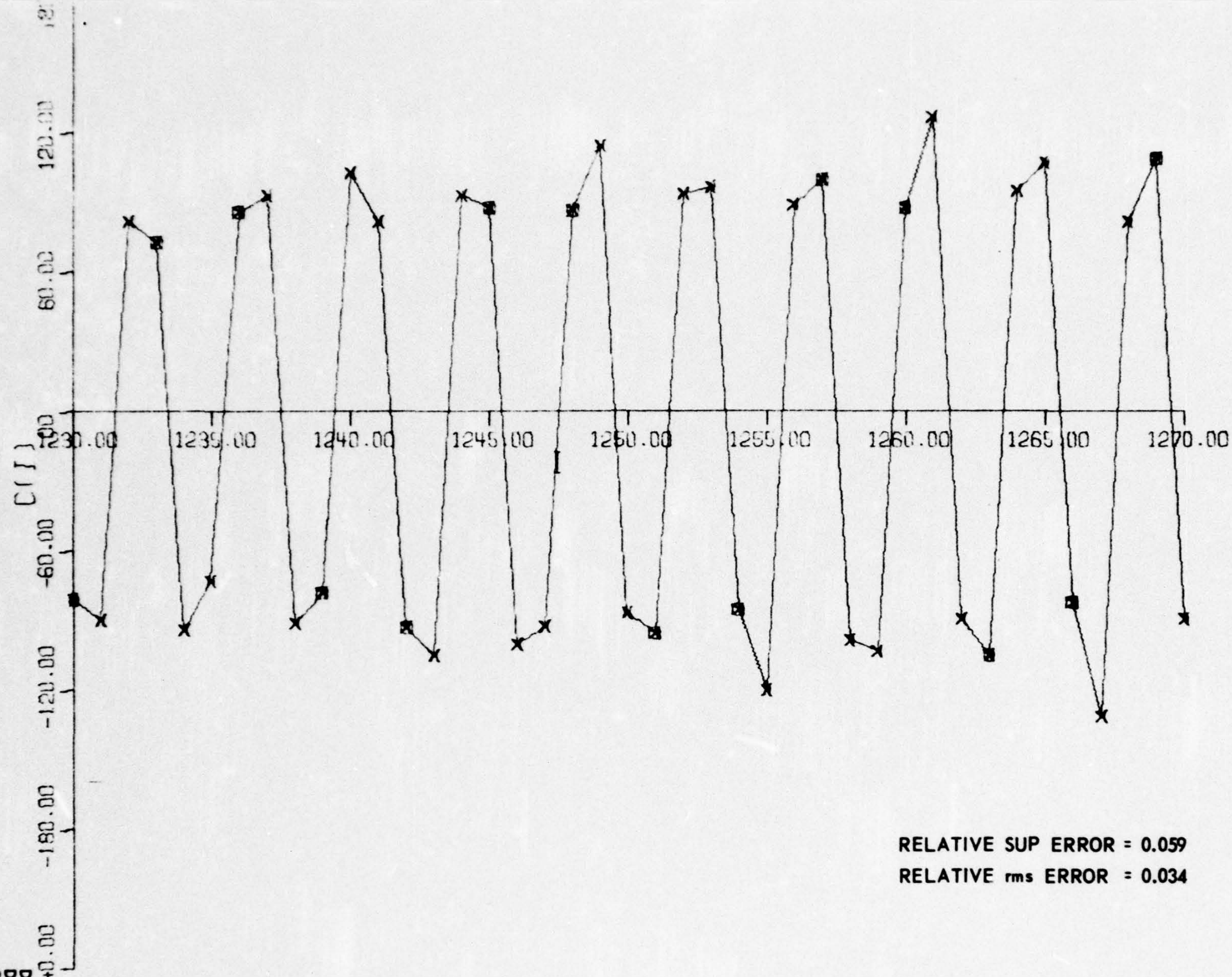
VIII.

C. 1. Using Every Third Point (LP = 3)



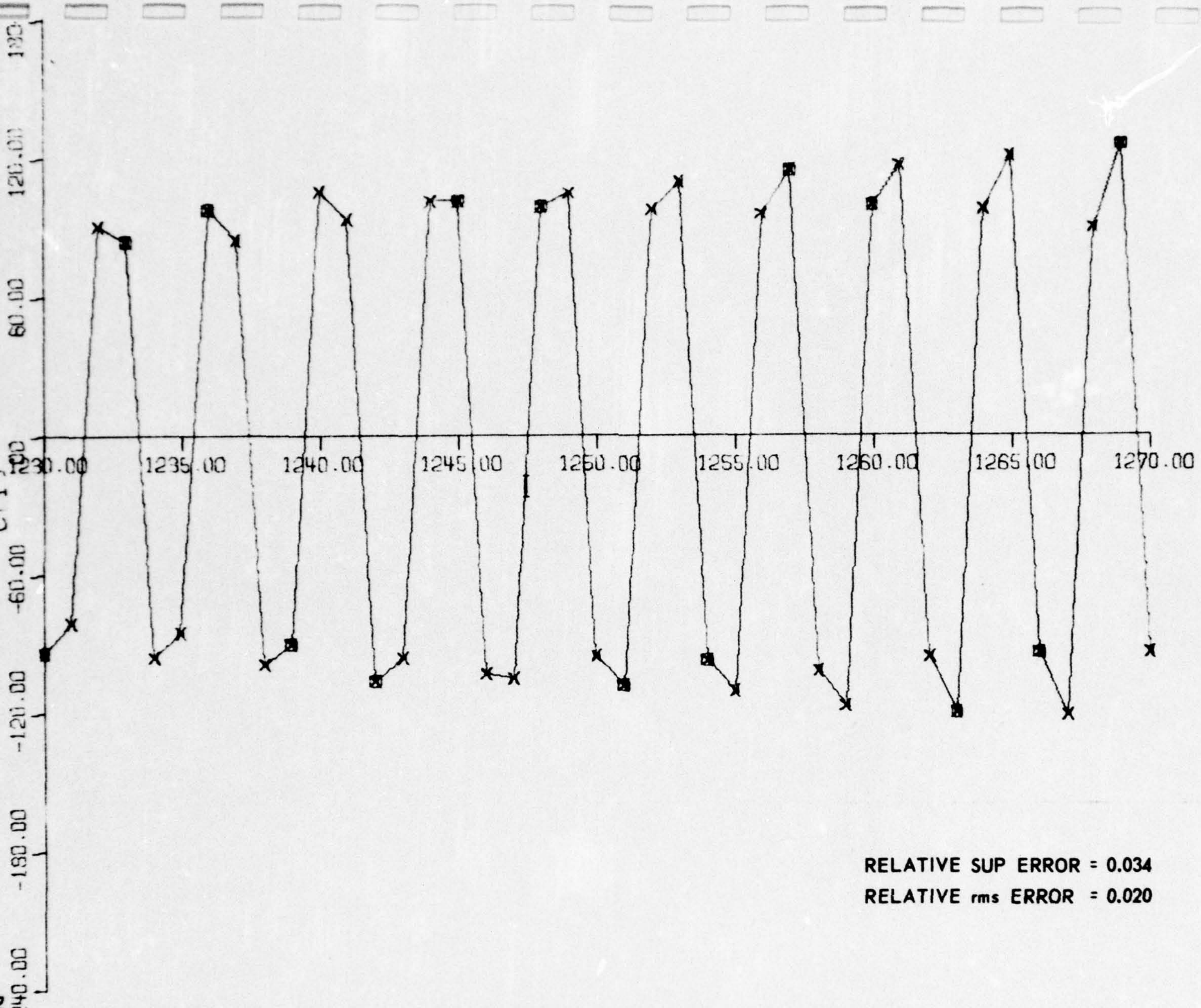
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 3
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-658
GSI - EJW
6 - 20 - 67



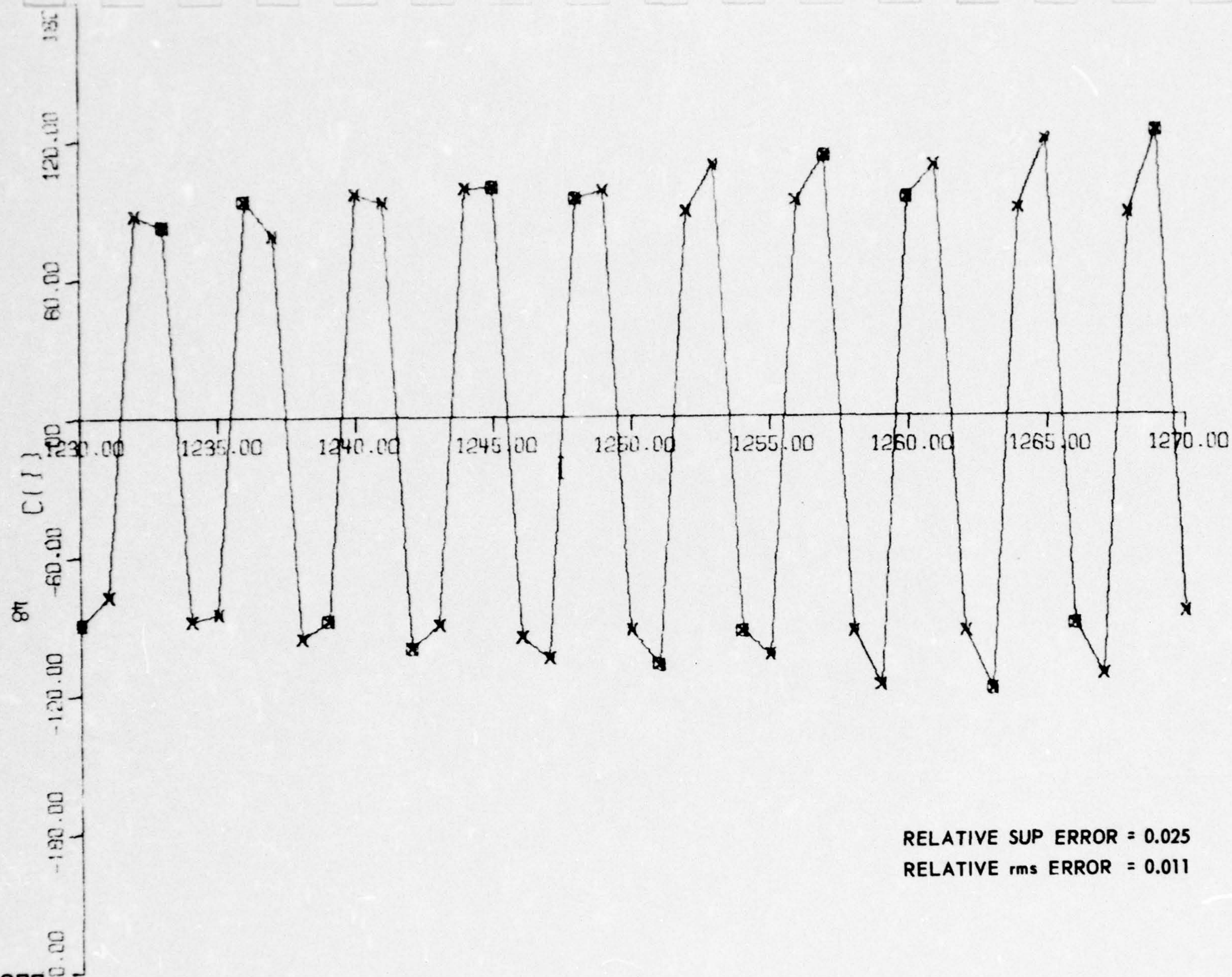
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 5
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-659
GSI - EJW
6 - 20 - 67



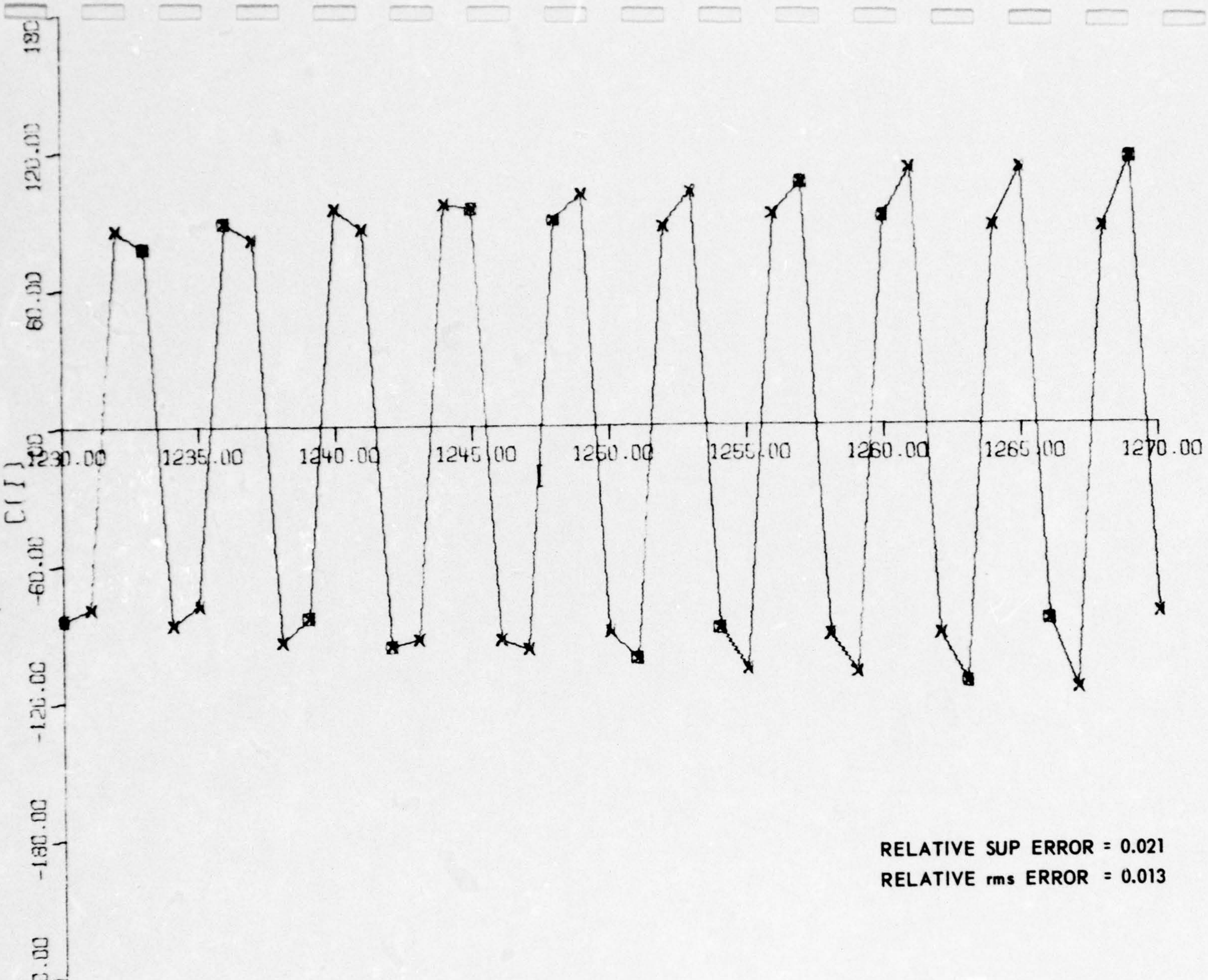
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION - 10
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-660
GSI - EJW
6 - 20 - 67



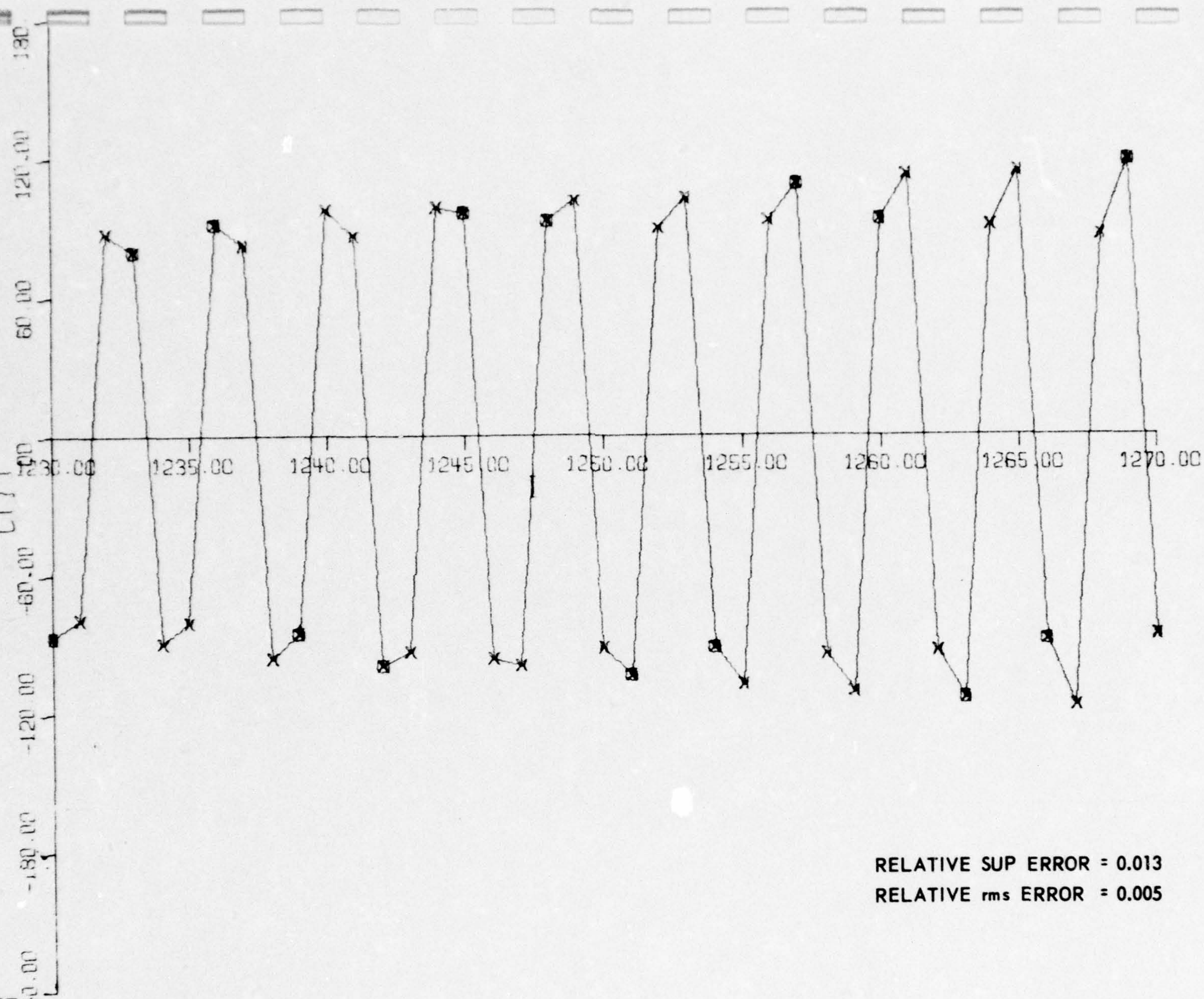
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 15
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-661
GSI - EJW
6 - 20 - 67



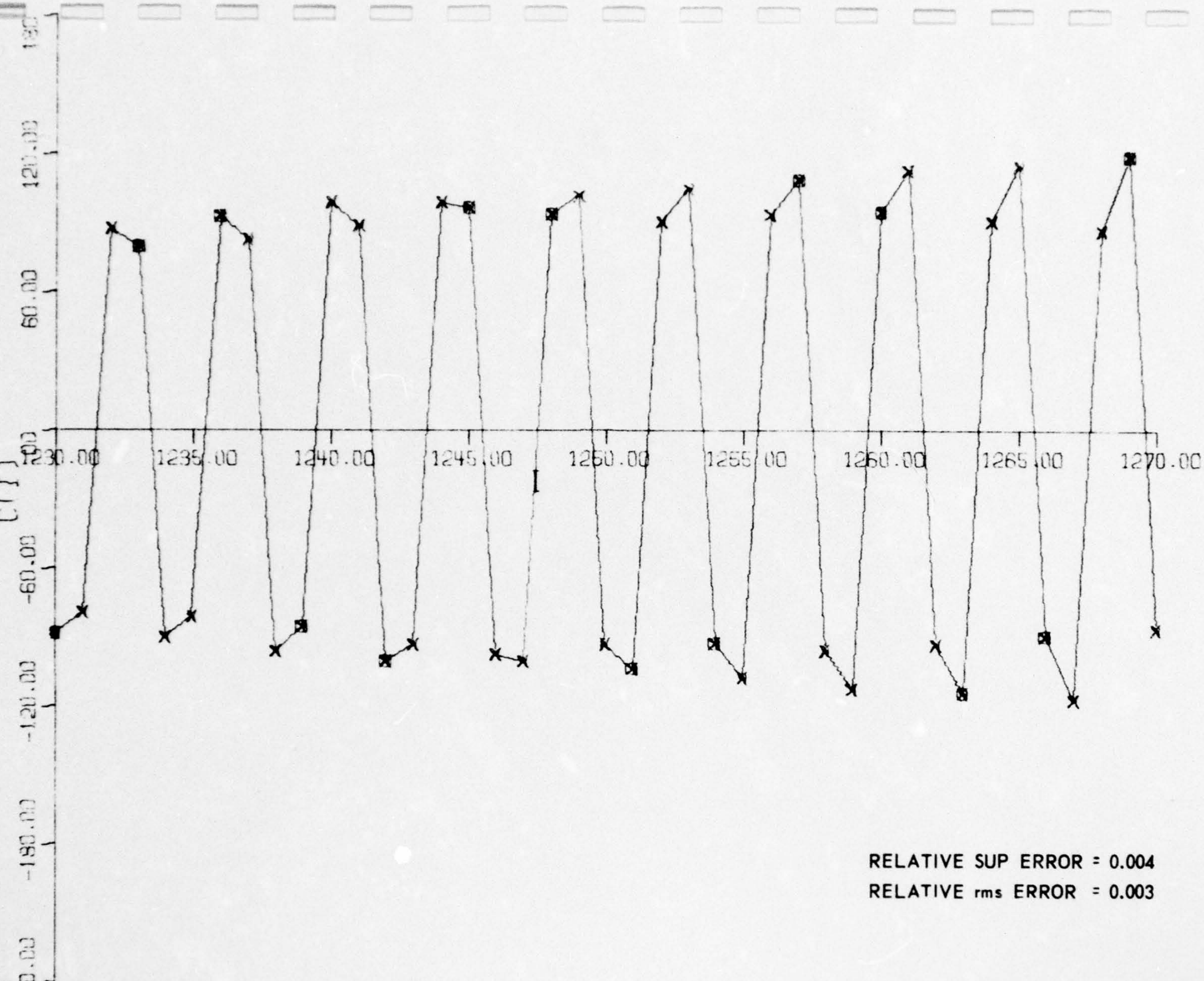
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 20
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-662
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 100
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-663
GSI - EJW
6 - 20 - 67

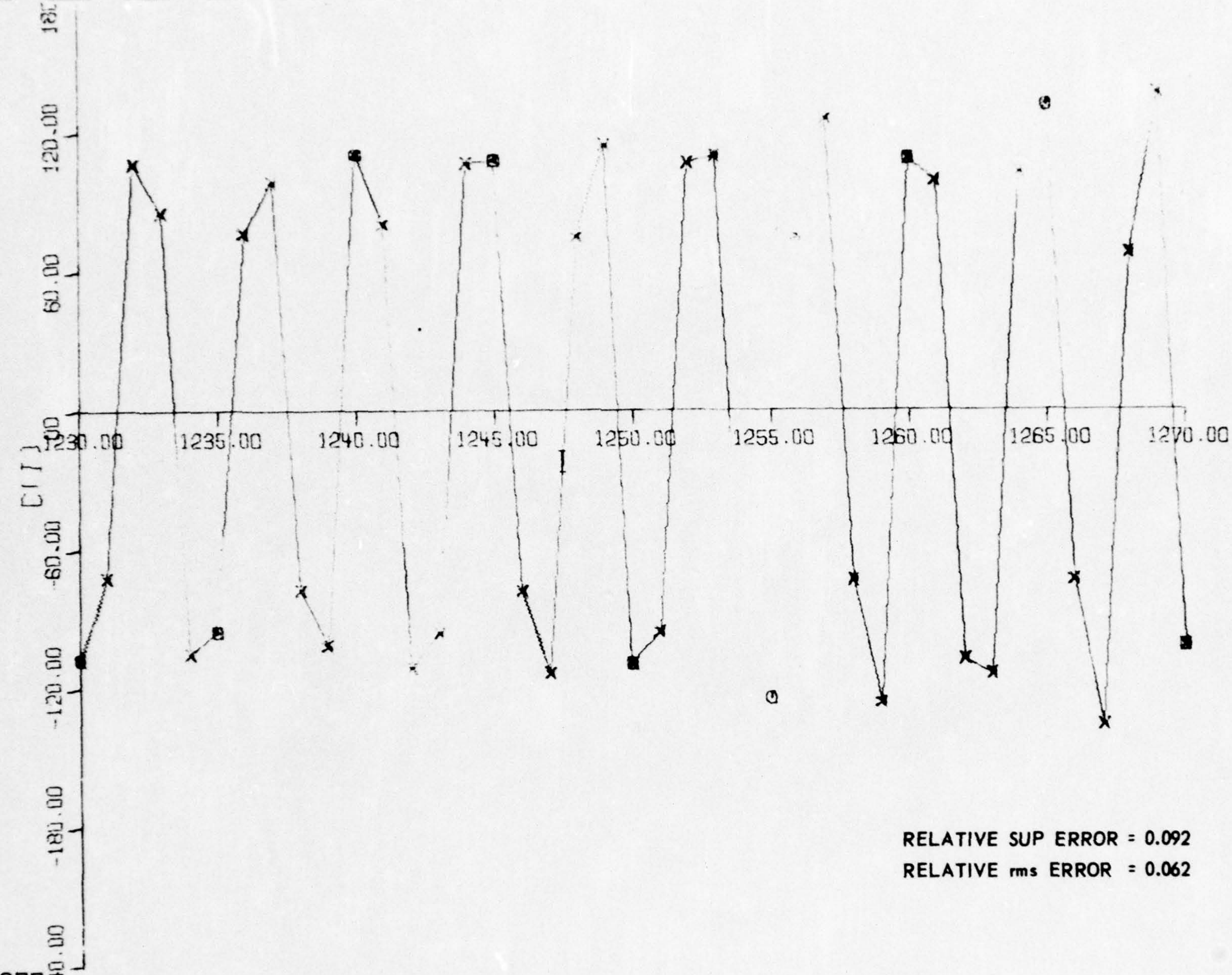


REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION - 200
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-664
GSI - EJW
6 - 20 - 67

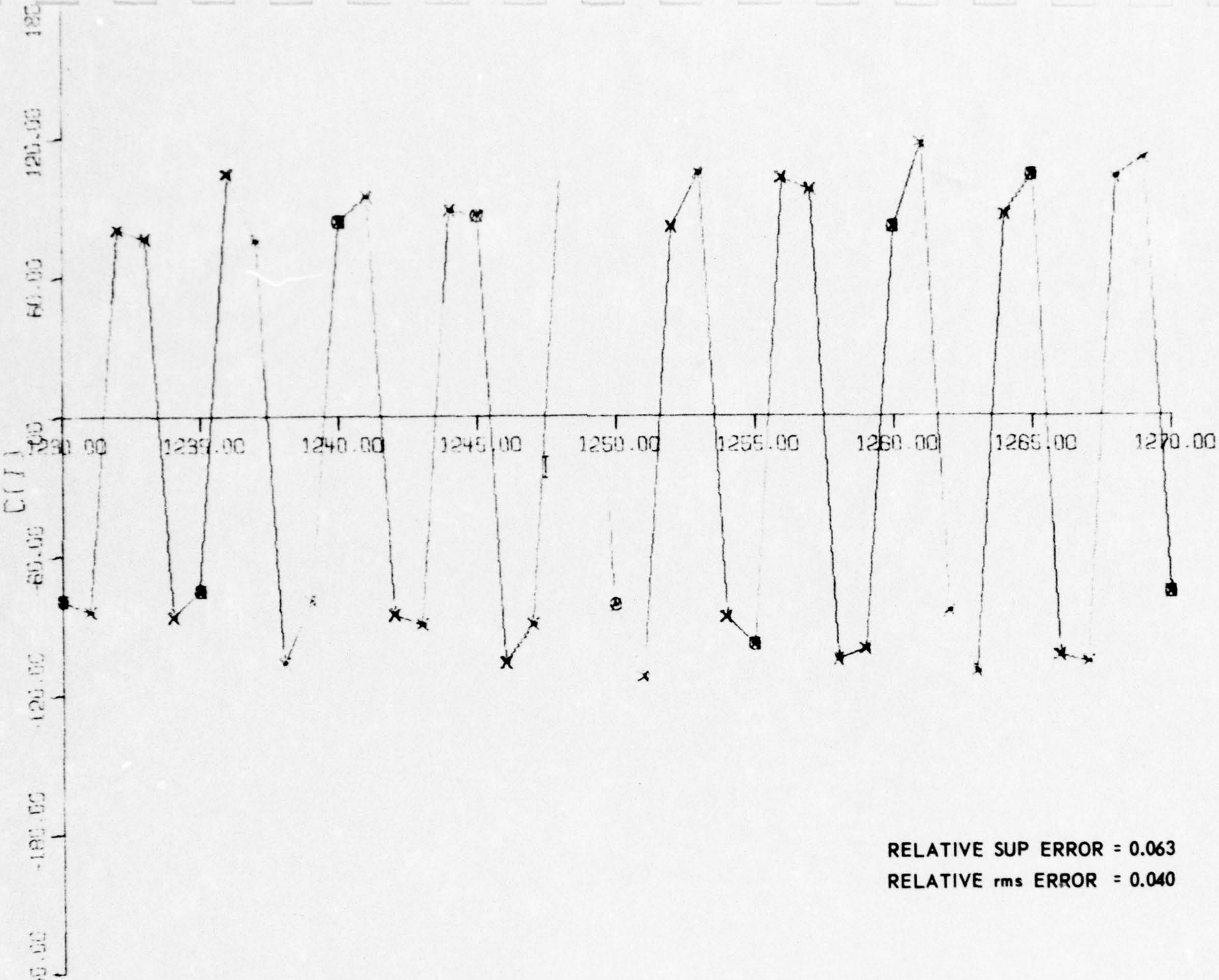
VIII.

C. 2. Using Every Fifth Point (LP = 5)



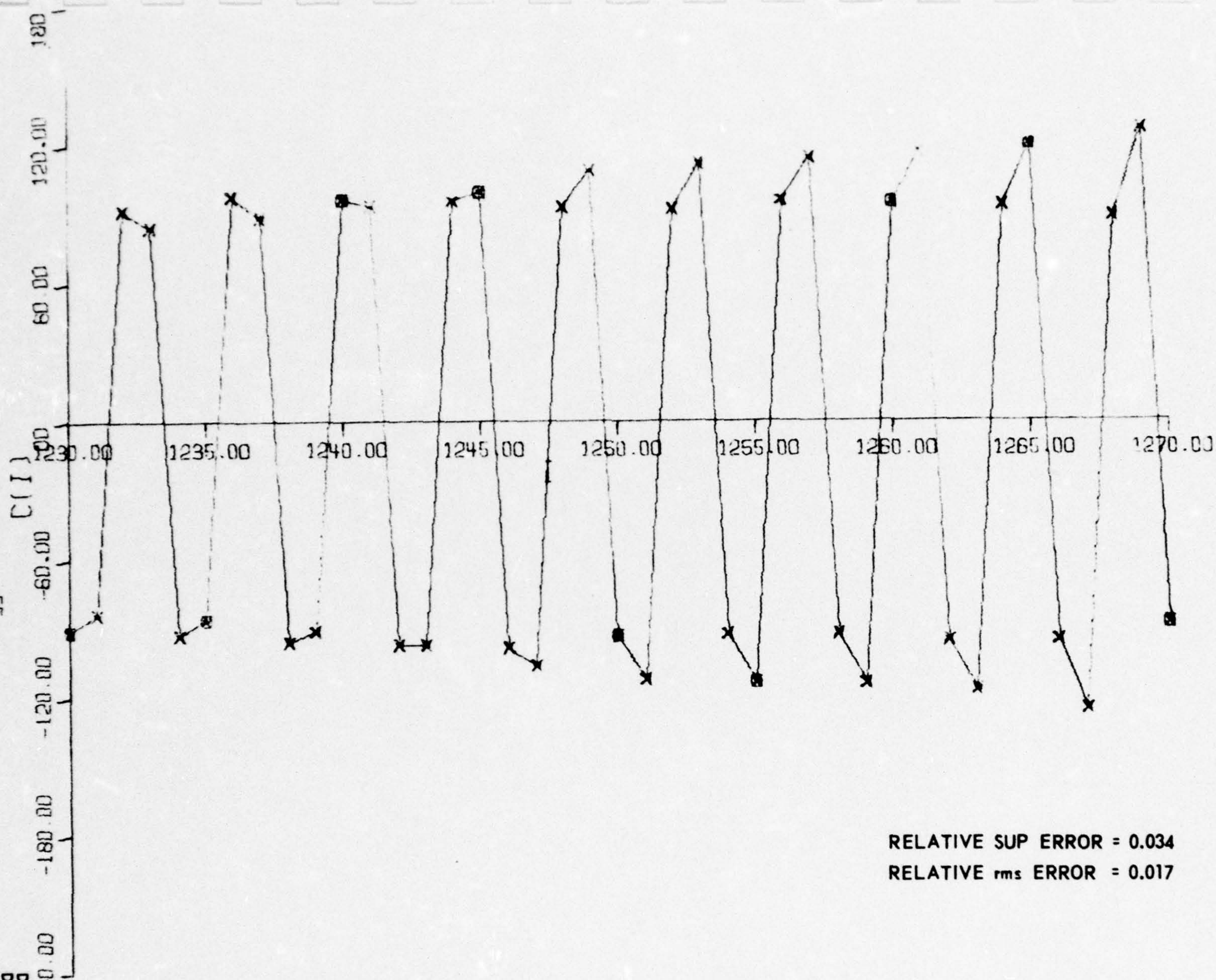
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 3
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-665
GSI - EJW
6 - 20 - 67



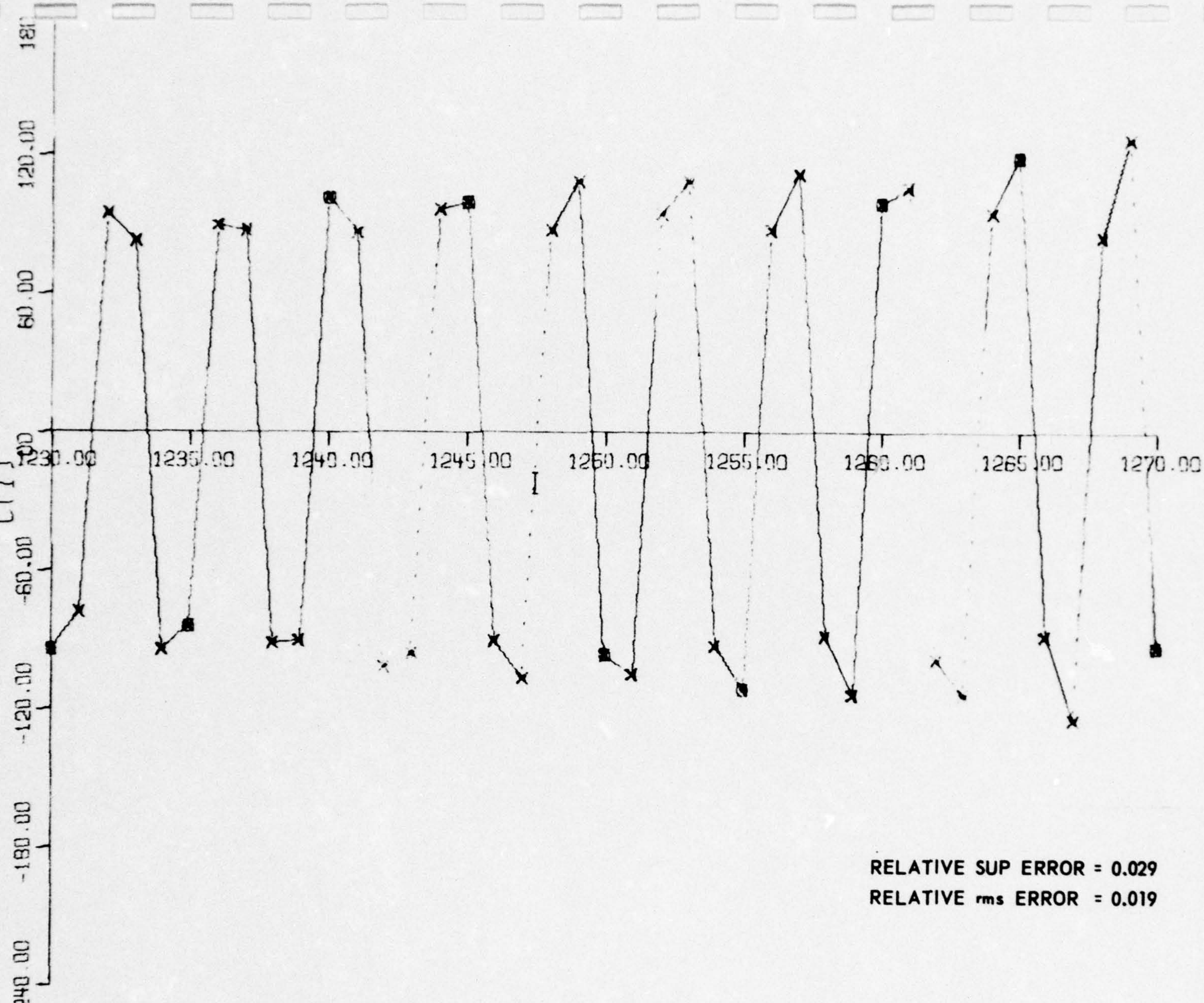
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 5
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL . UT
DWG AS-67-666
GSI - EJW
6 - 20 - 67



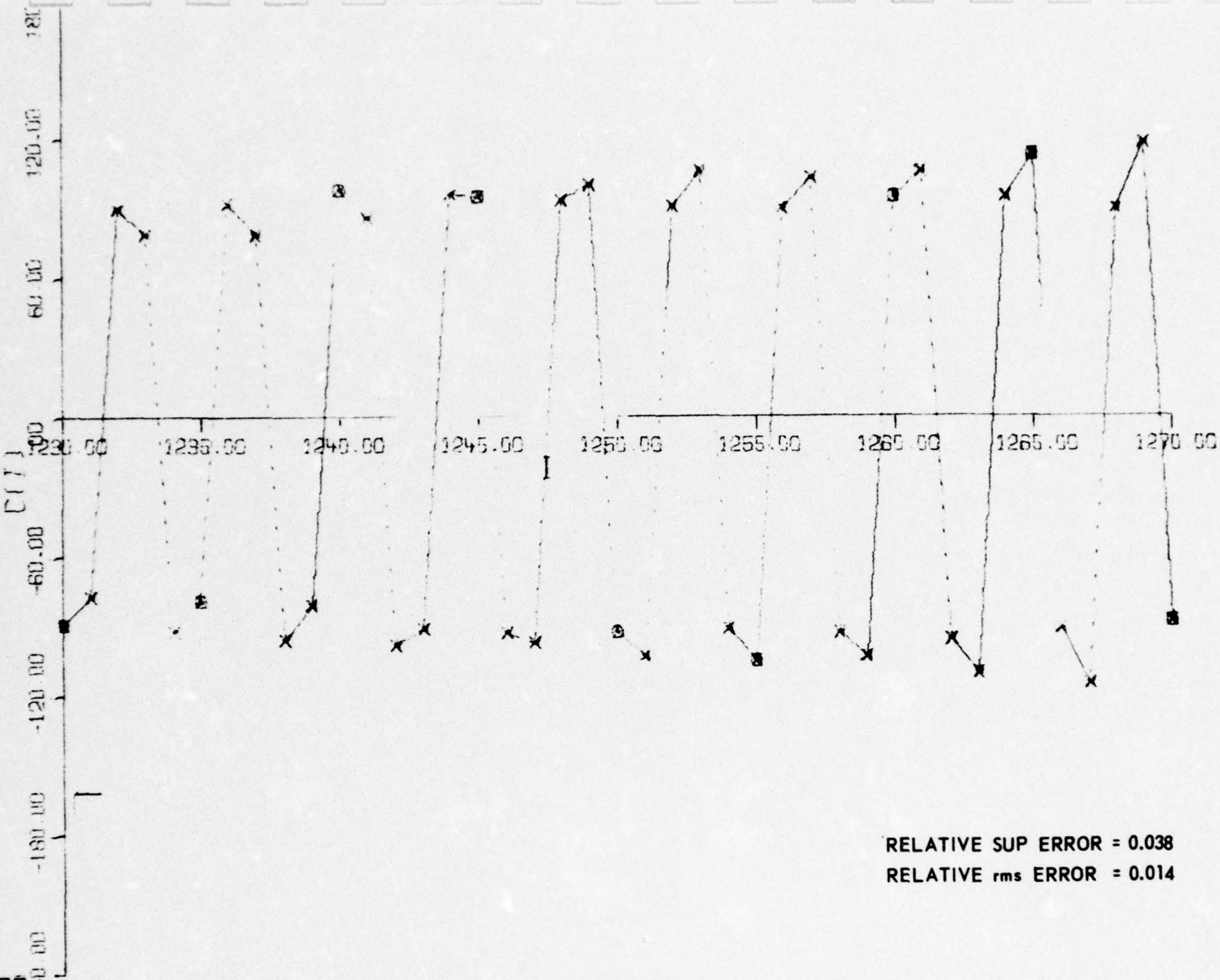
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 10
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-667
GSI - EJW
6 - 20 - 67



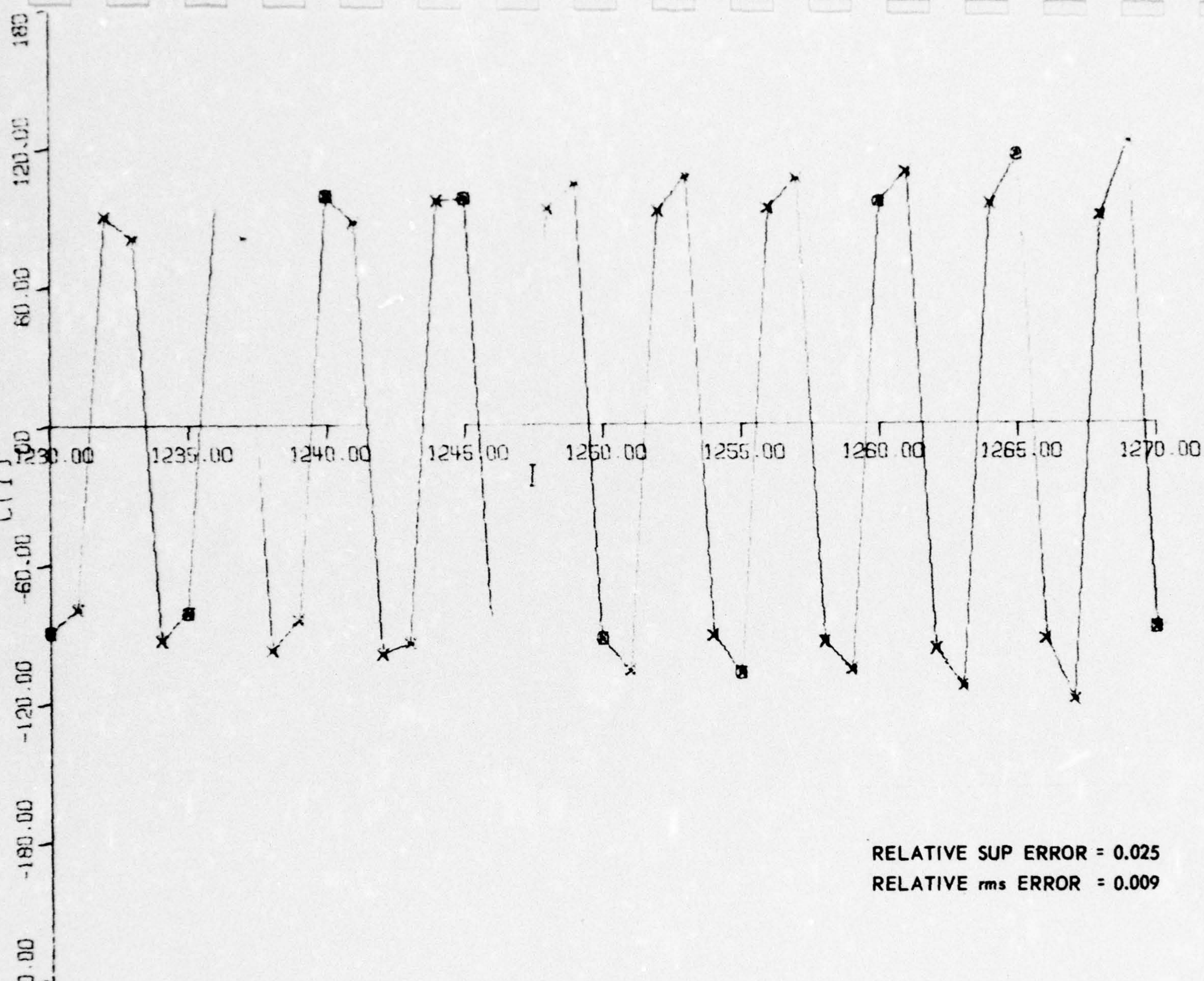
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 15
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-668
GSI - EJW
6 - 20 - 67



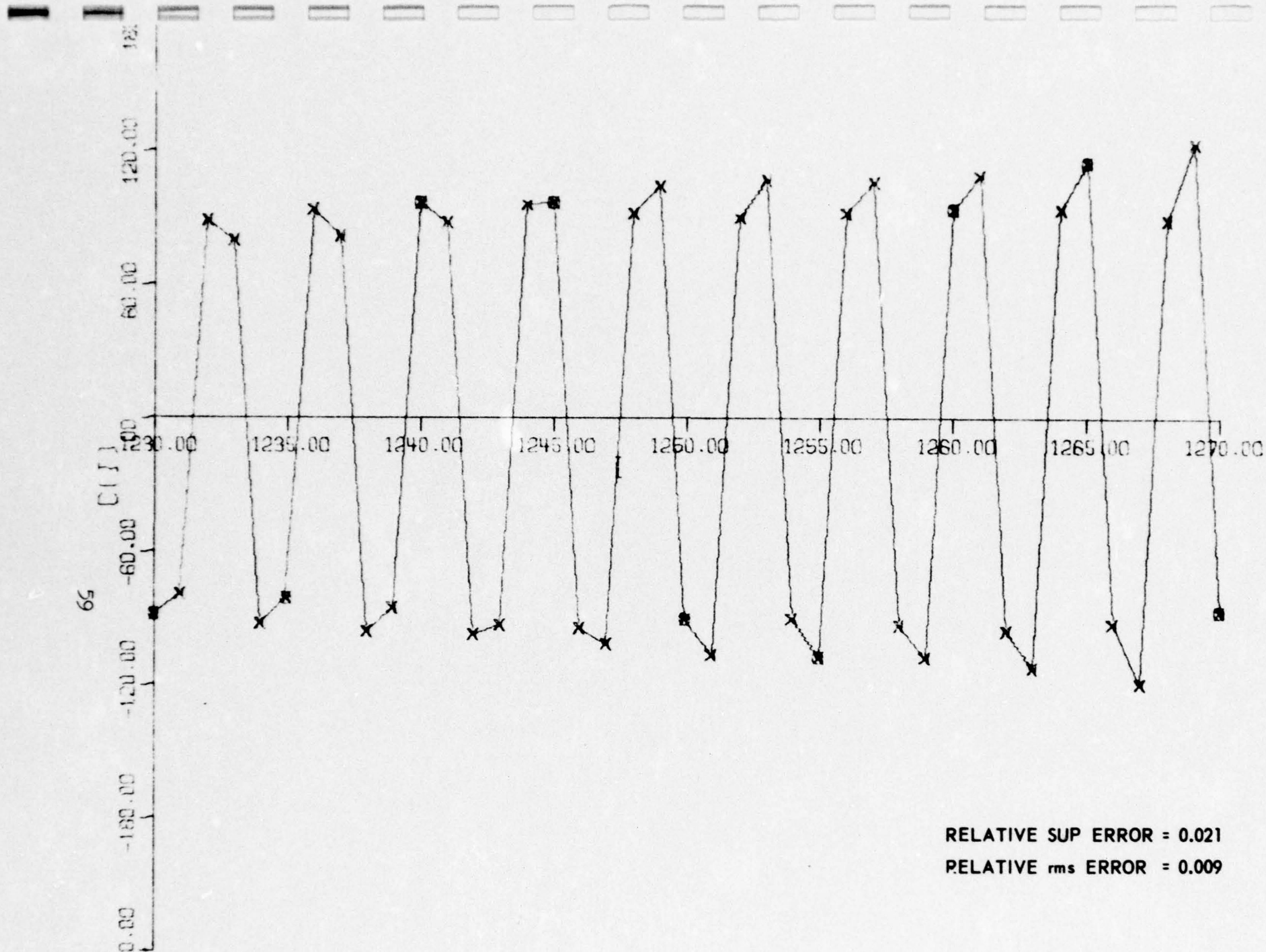
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 20
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-669
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 100
FILTERED BY BPF1 (3 kHz – 7 kHz)

DRL - UT
DWG AS-67-670
GSI - EJW
6 - 20 - 67



RELATIVE SUP ERROR = 0.021

RELATIVE rms ERROR = 0.009

REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION -- 200
FILTERED BY BPF1 (3 kHz – 7 kHz)

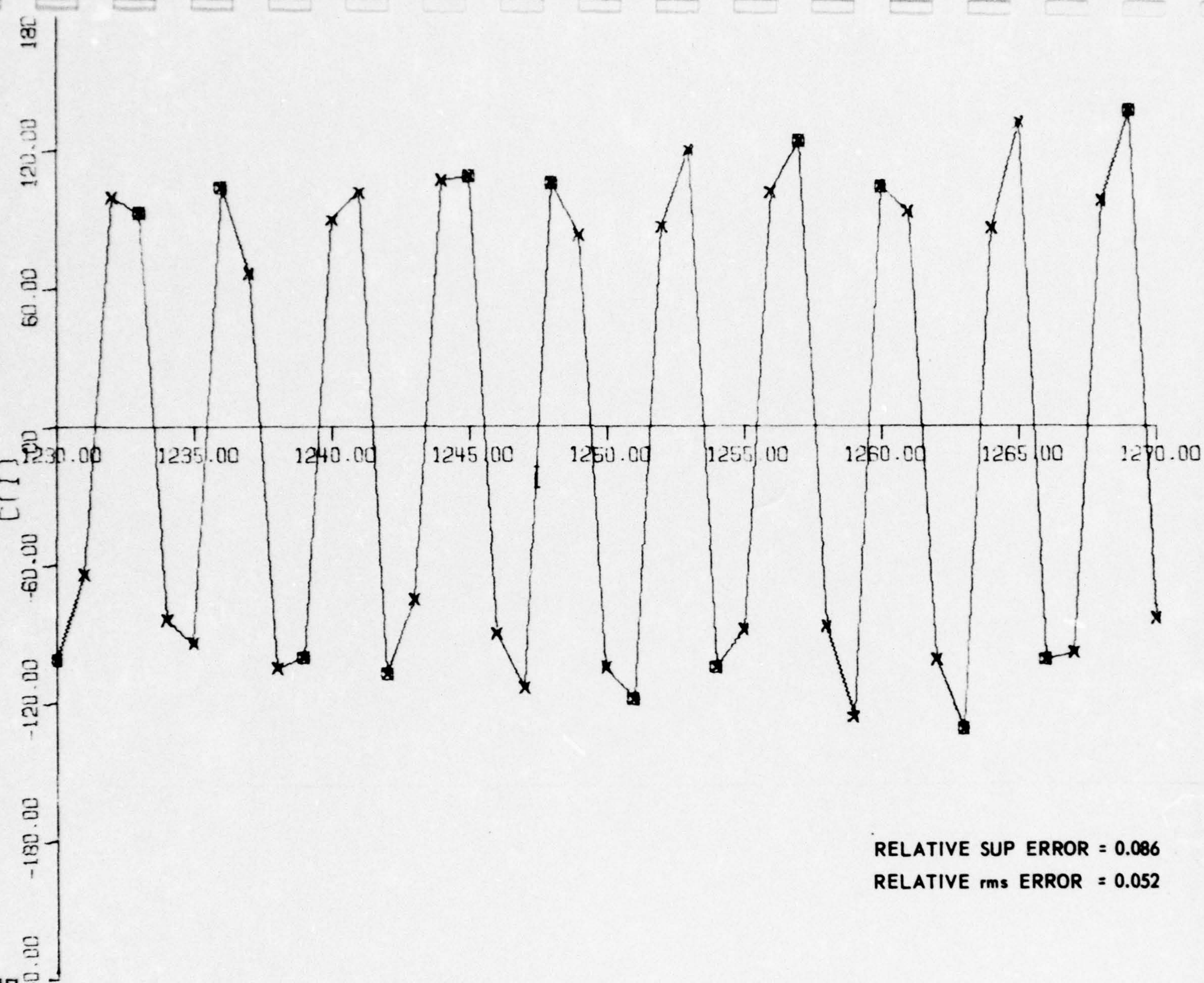
DRL - UT
DWG AS-67-671
GSI - EJW
6 - 20 - 67

VIII.

D. Effect of Shannon's Theorem with Data Filtered By BPF2 (4 kHz-6 kHz), Illustrated by Graphs

VIII.

D. 1. Using Every Third Point (LP = 3)

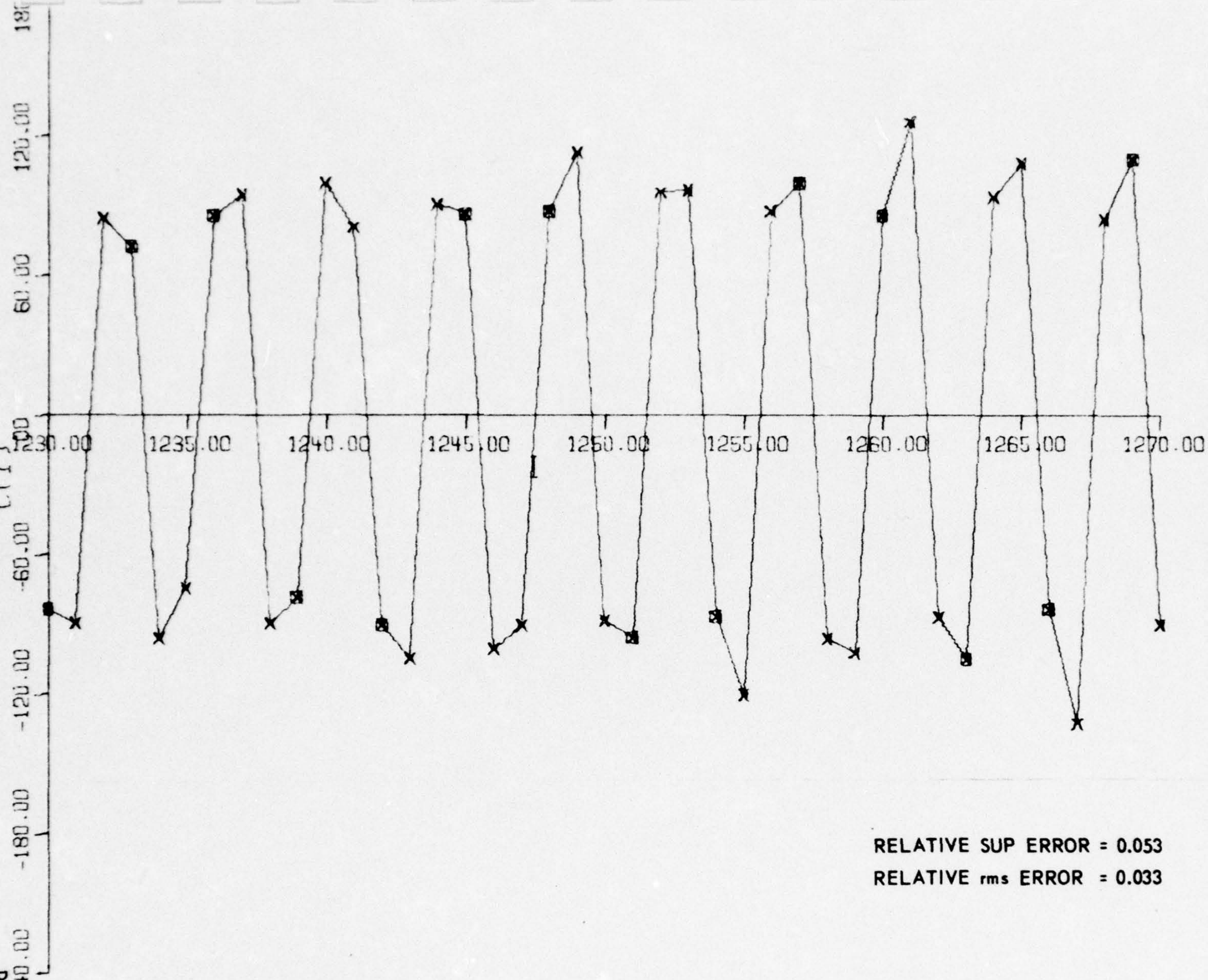


RELATIVE SUP ERROR = 0.086

RELATIVE rms ERROR = 0.052

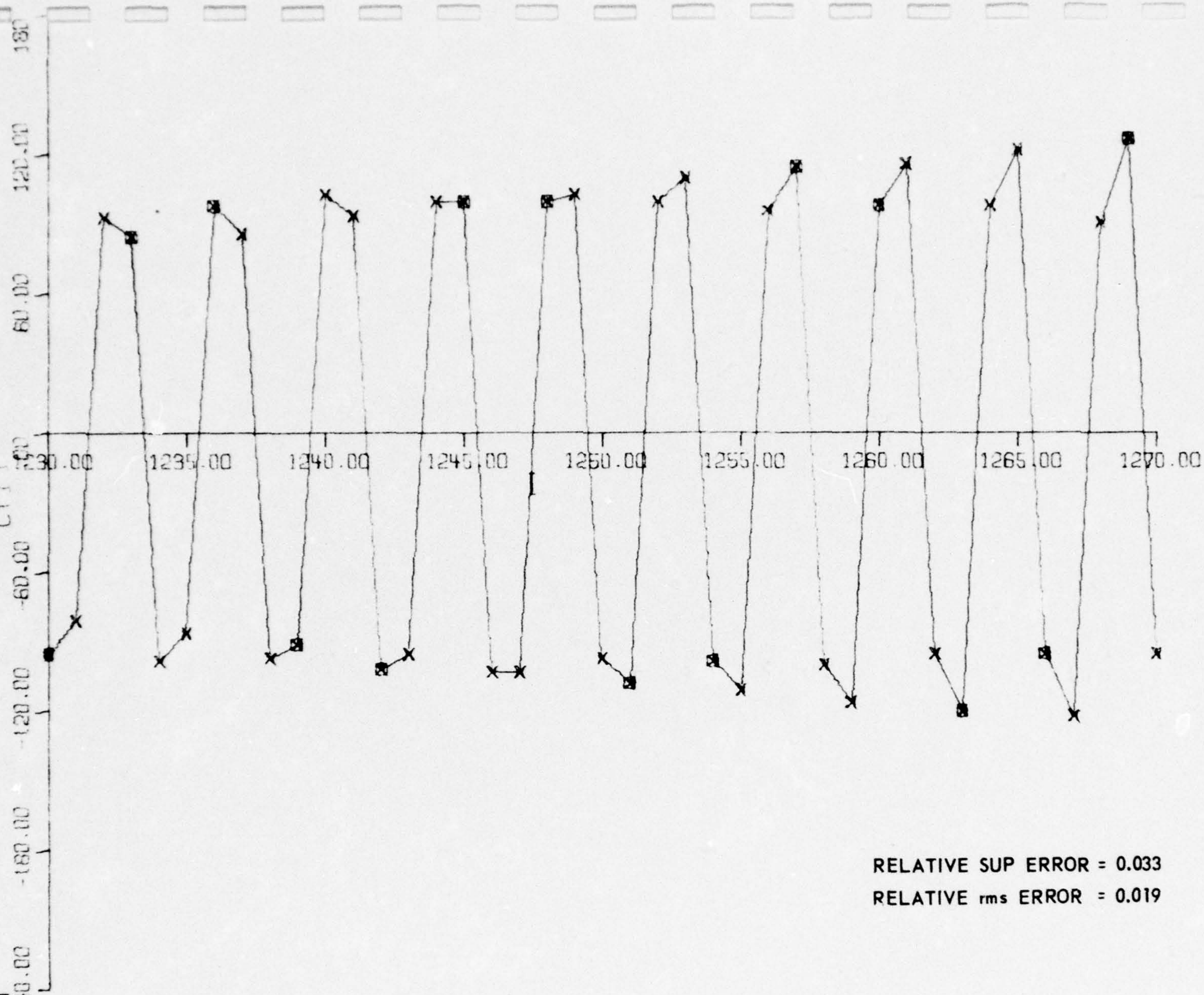
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 3
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-672
GSI - EJW
6 - 20 - 67



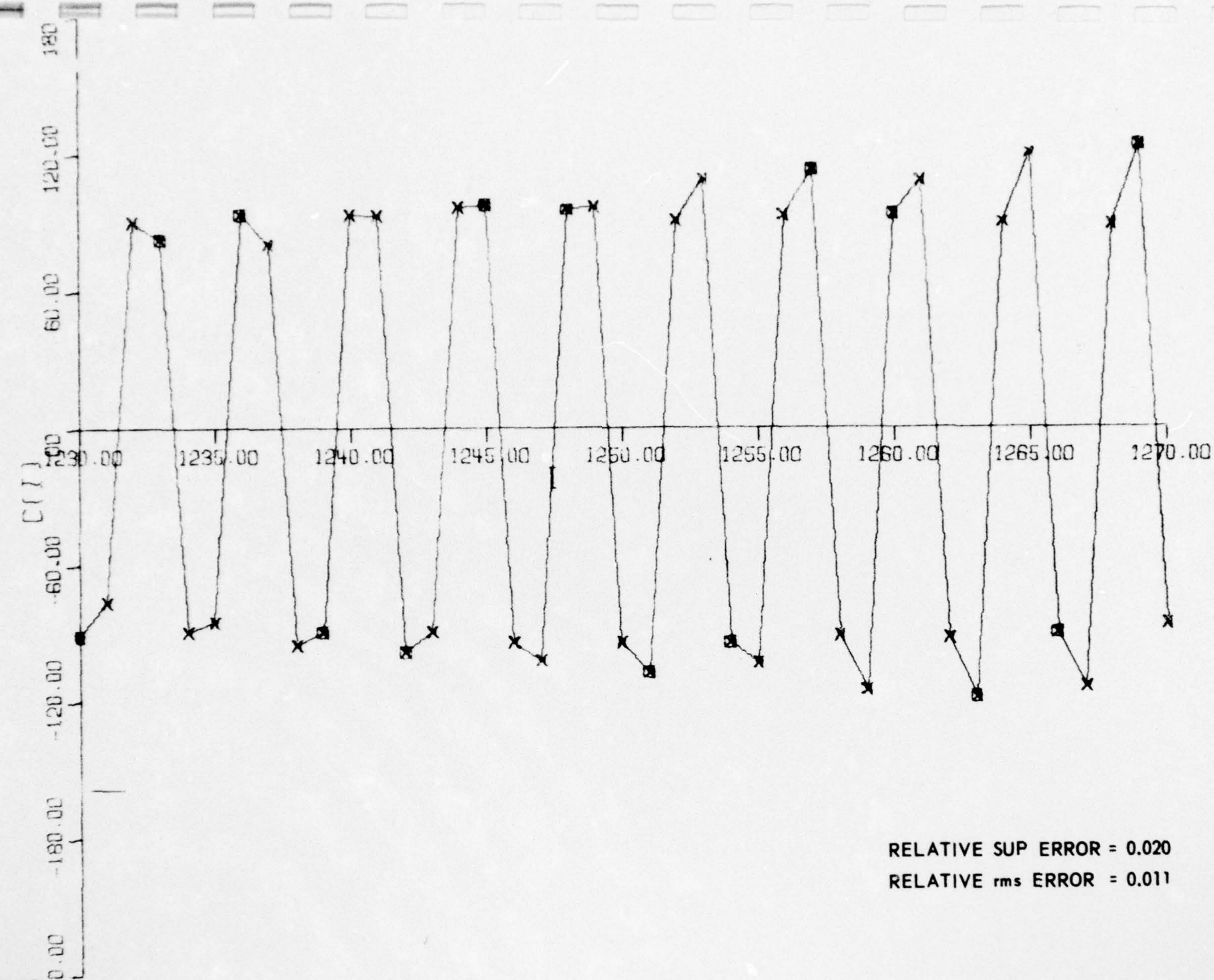
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 5
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-673
GSI - EJW
6 - 20 - 67



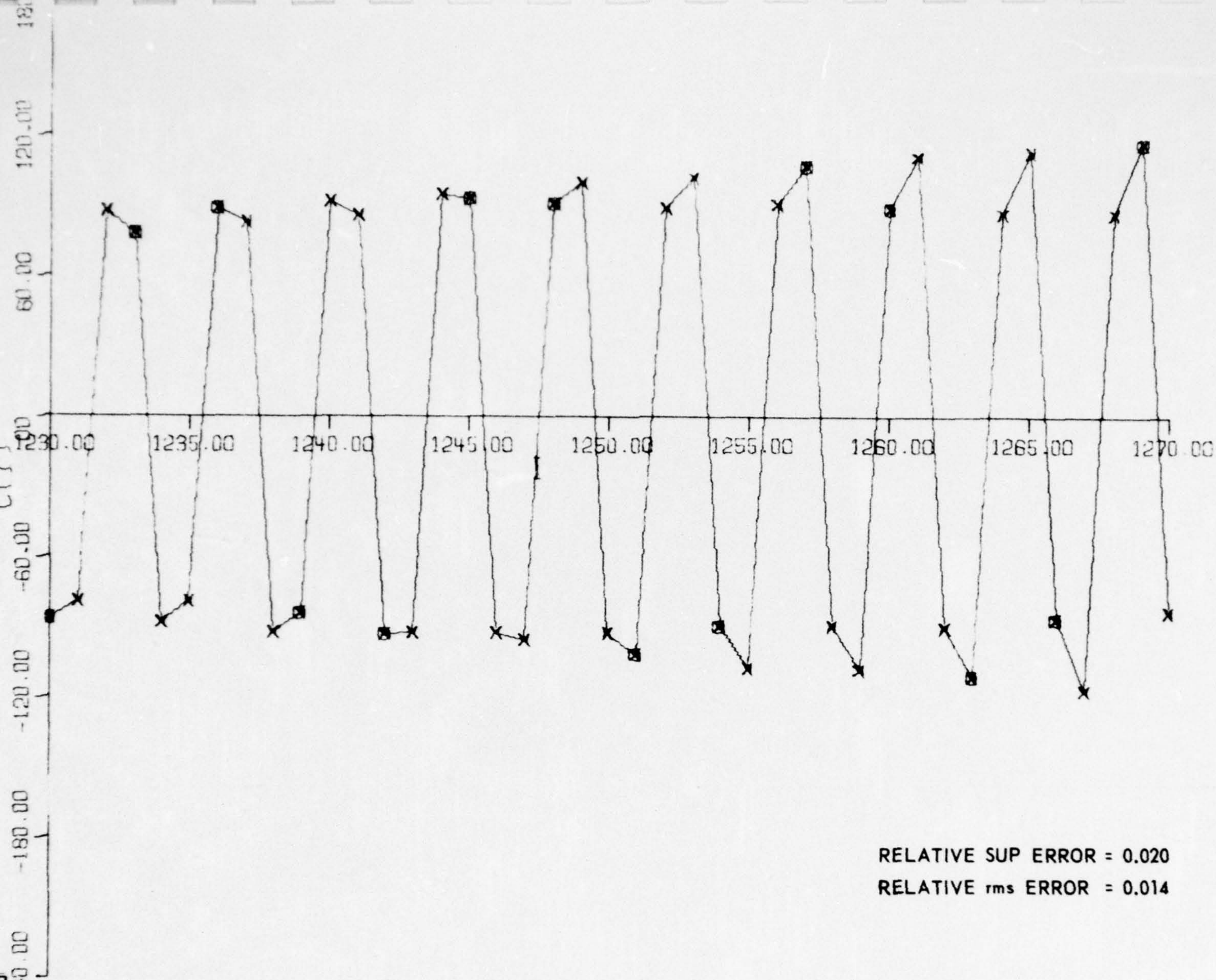
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 10
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-674
GSI - EJW
6 - 20 - 67



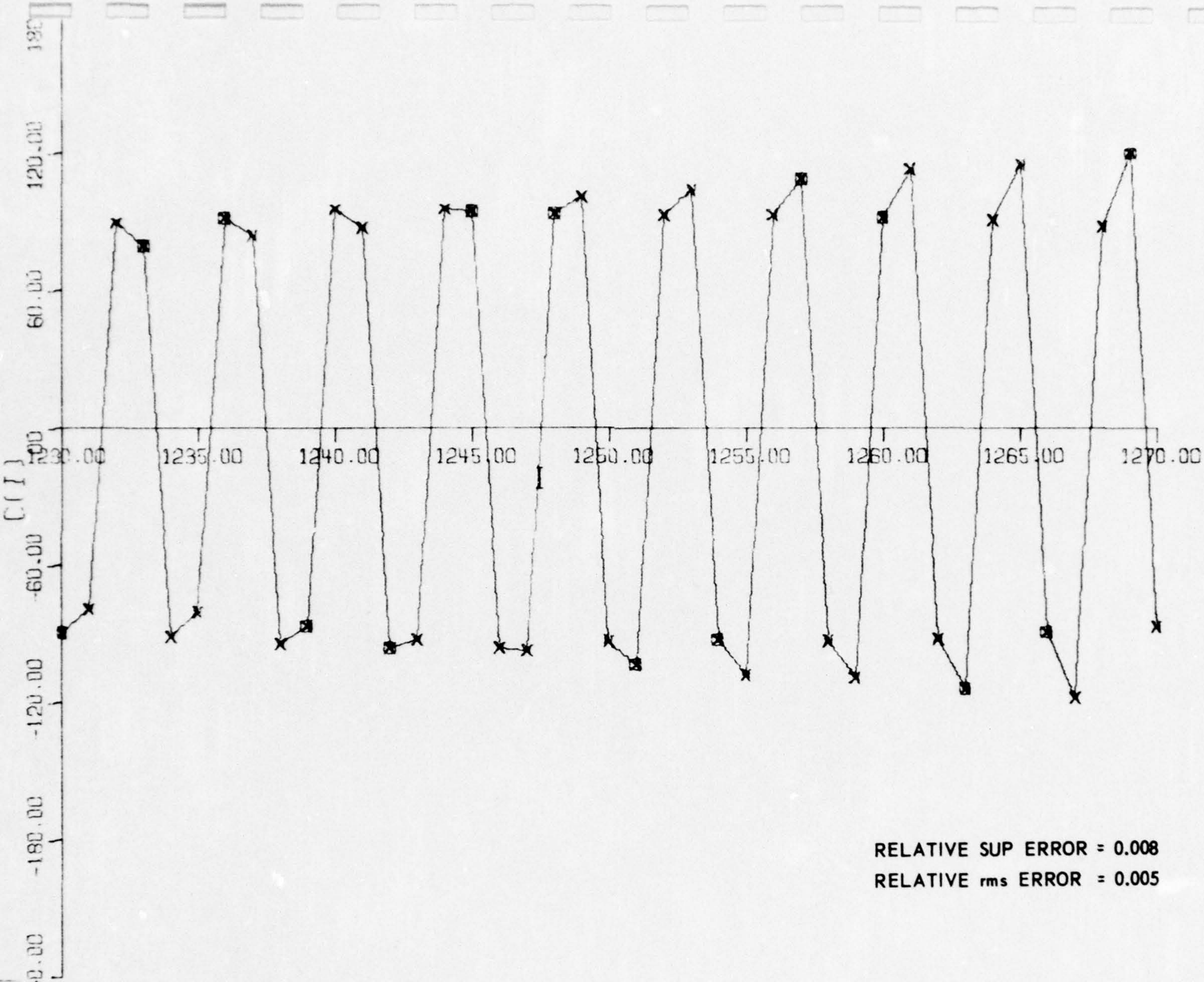
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 15
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-675
GSI - EJW
6 - 20 - 67



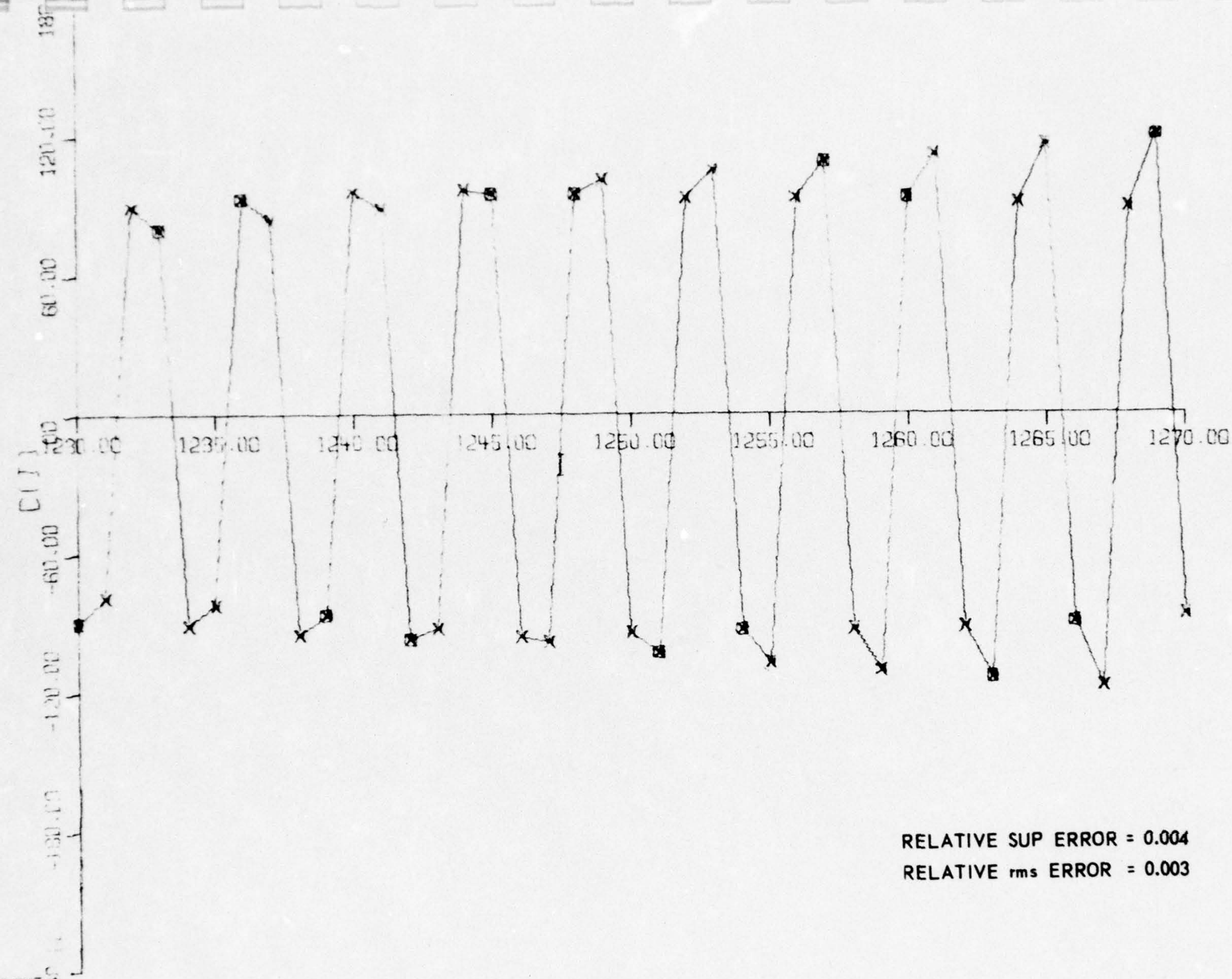
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION - 20
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-676
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 100
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-677
GSI - EJW
6 - 20 - 67

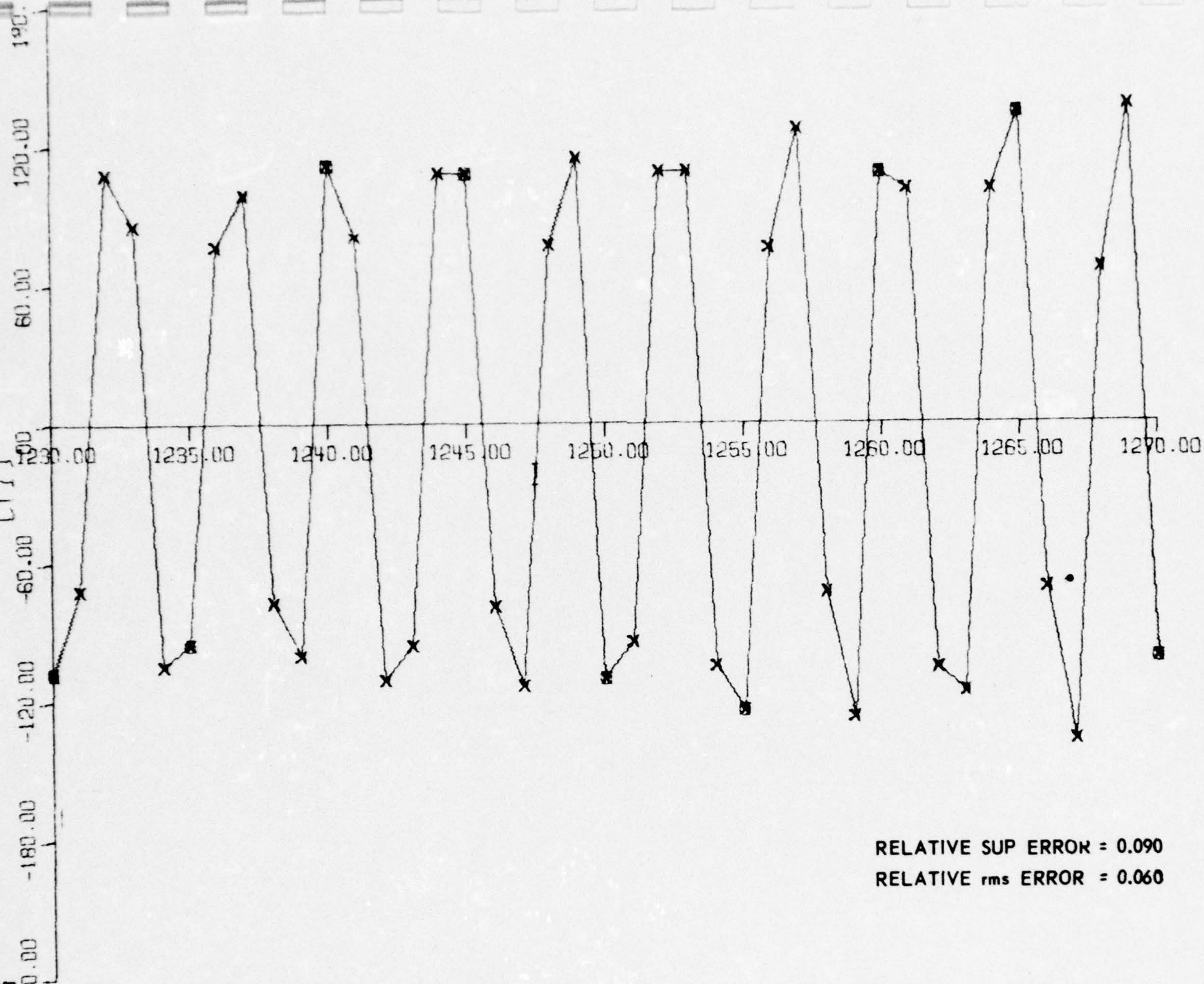


REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 200
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-678
GSI - EJW
6 - 20 - 67

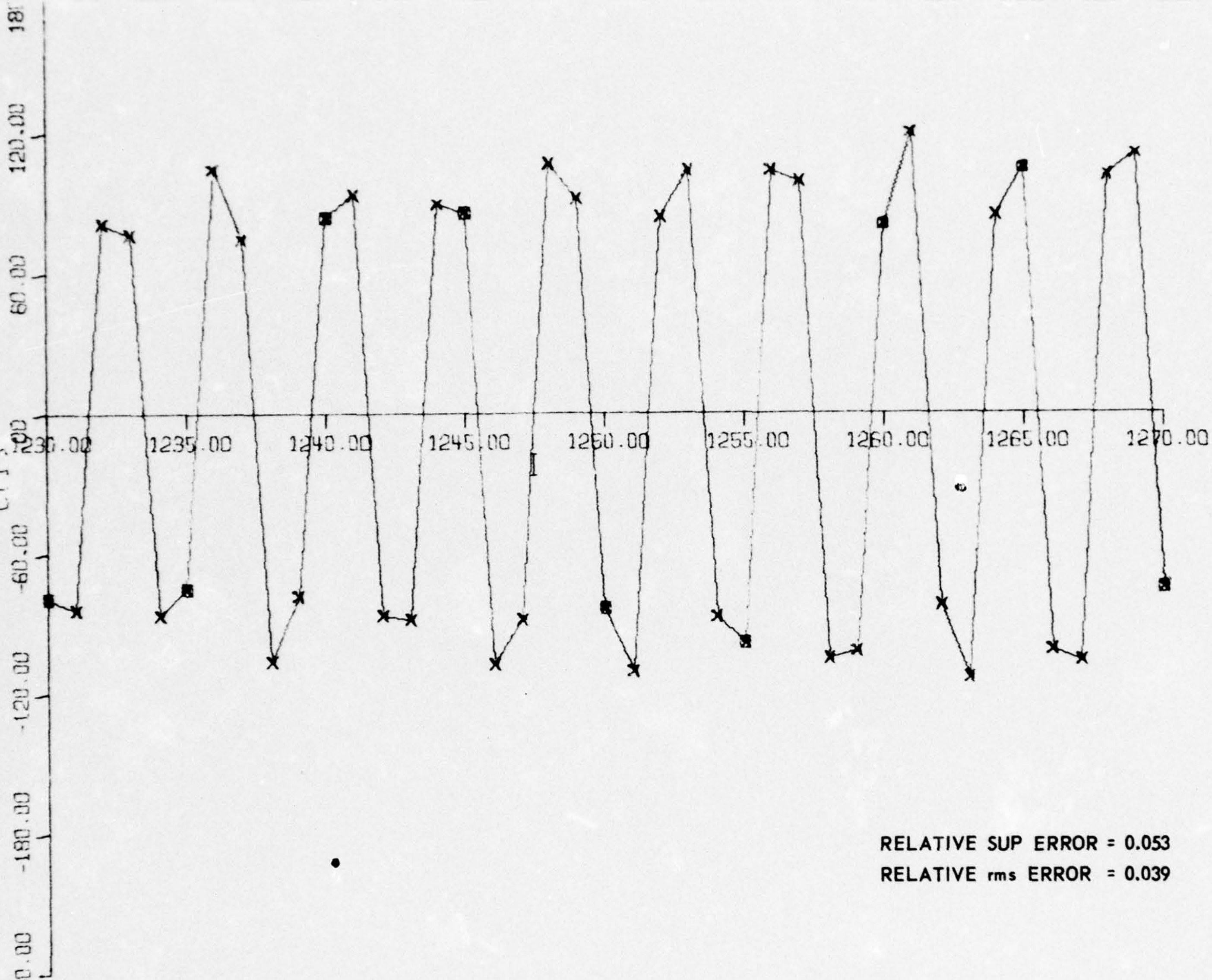
VIII.

D. 2. Using Every Fifth Point (LP = 5)



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 3
FILTERED BY BPF2 (4 kHz – 6 kHz)

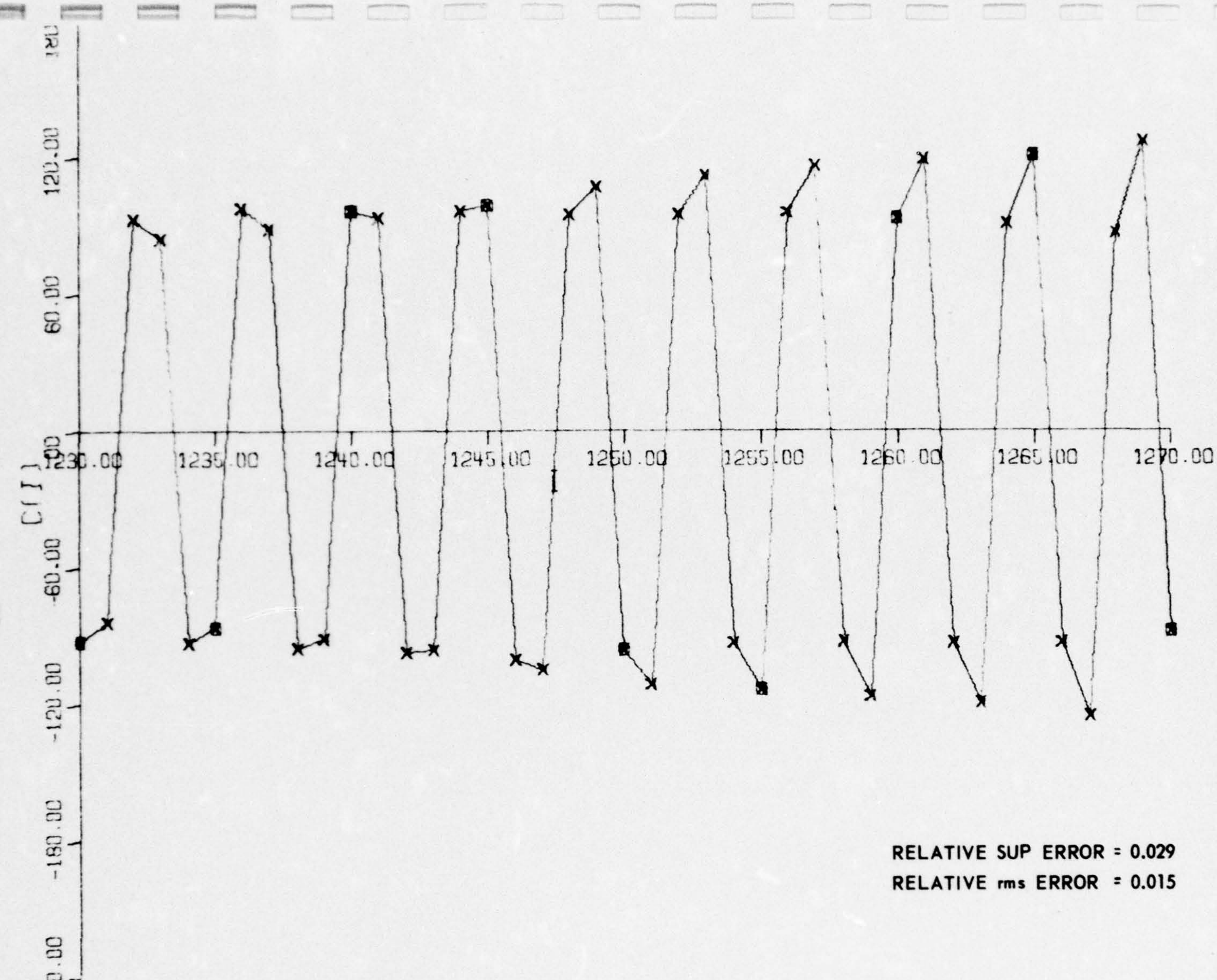
DRL - UT
DWG AS-67-679
GSI - EJW
6 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 5
FILTERED BY BPF2 (4 kHz – 6 kHz)

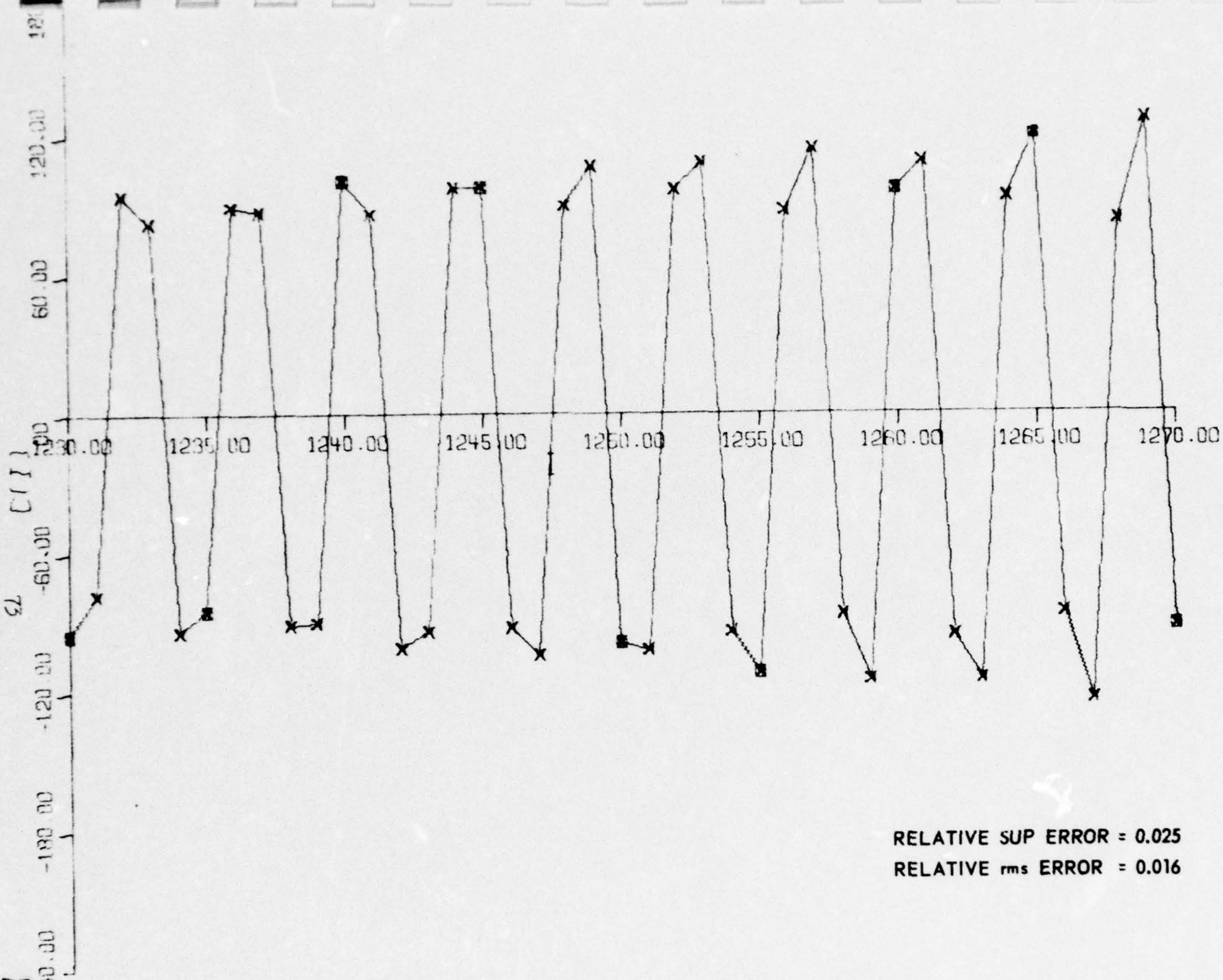
DRL - UT
DWG AS-67-680
GSI - EJW
6 - 20 - 67

24 May 1967



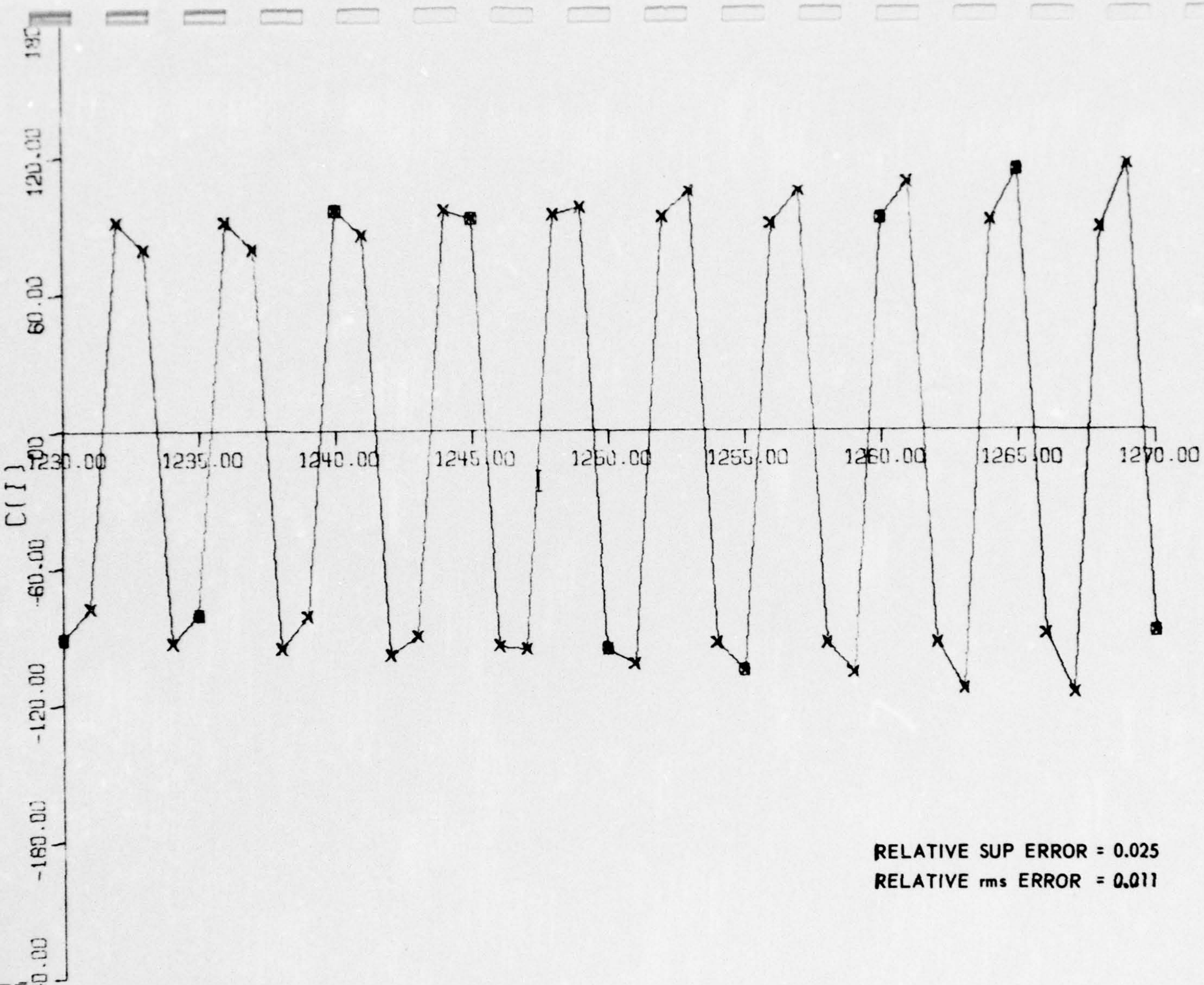
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 10
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-681
GSI - EJW
6 - 20 - 67



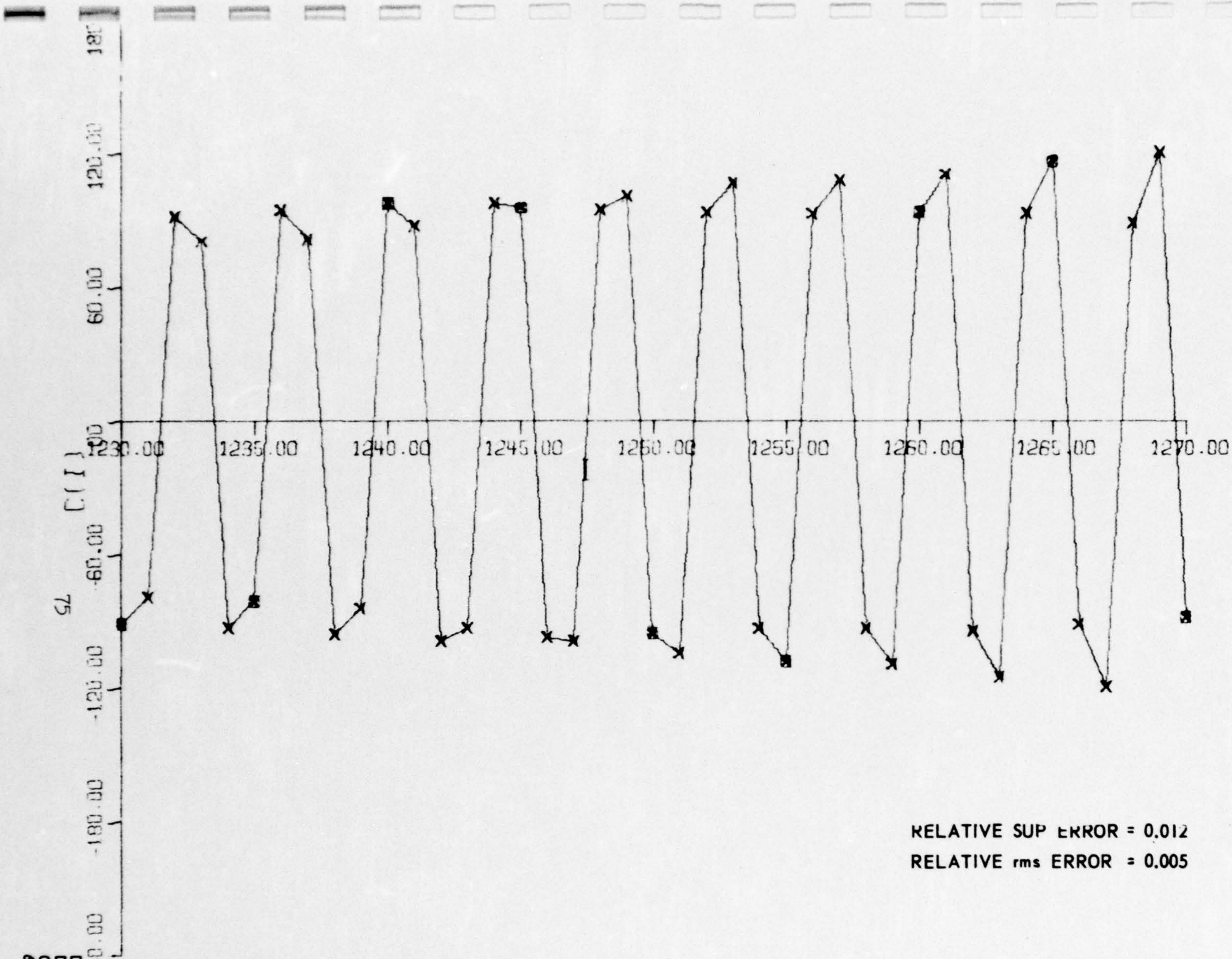
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 15
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-682
GSI - EJW
6 - 20 - 67



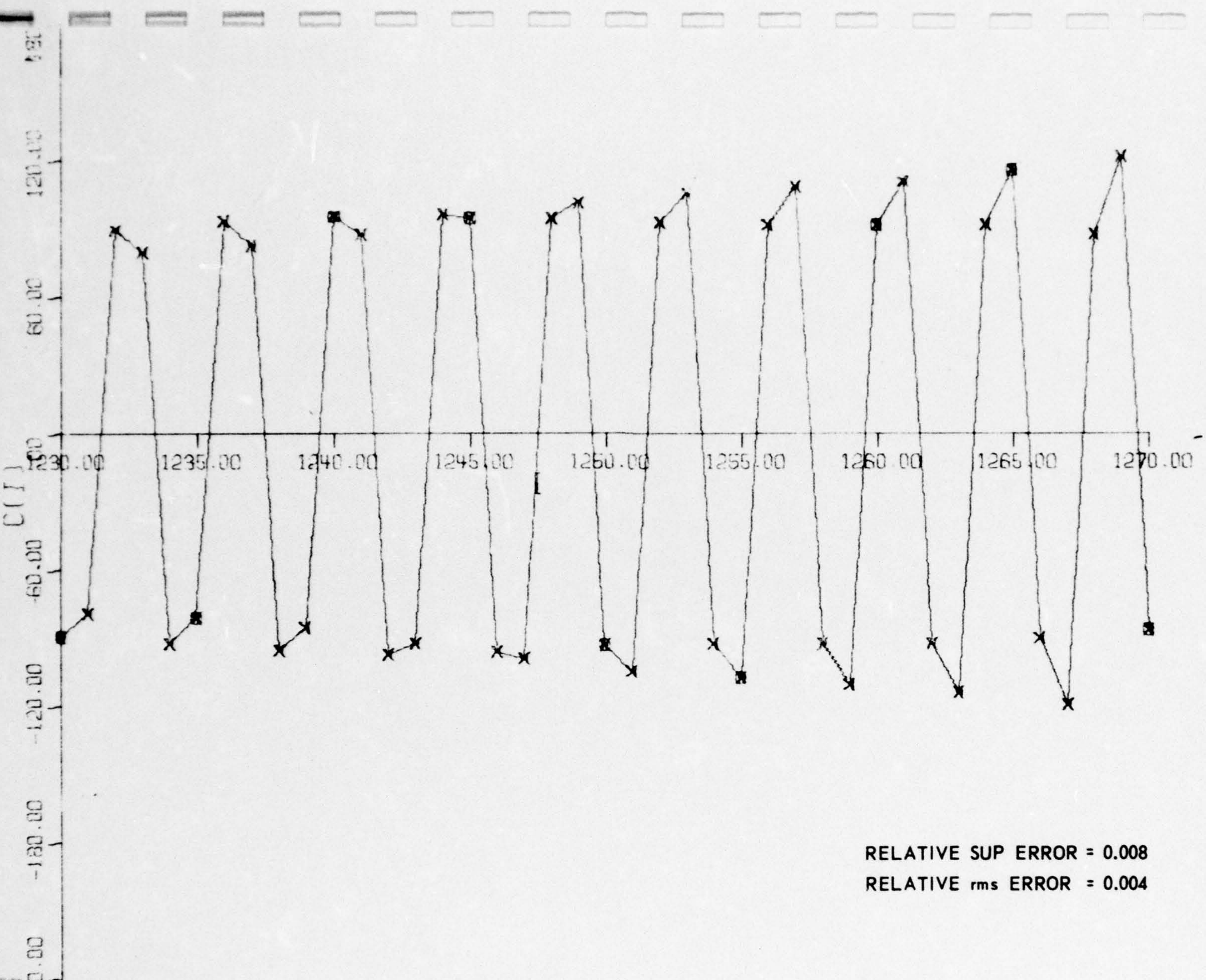
REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 20
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-683
GSI - EJW
5 - 20 - 67



REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION – 100
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-684
GSI - EJW
6 - 20 - 67

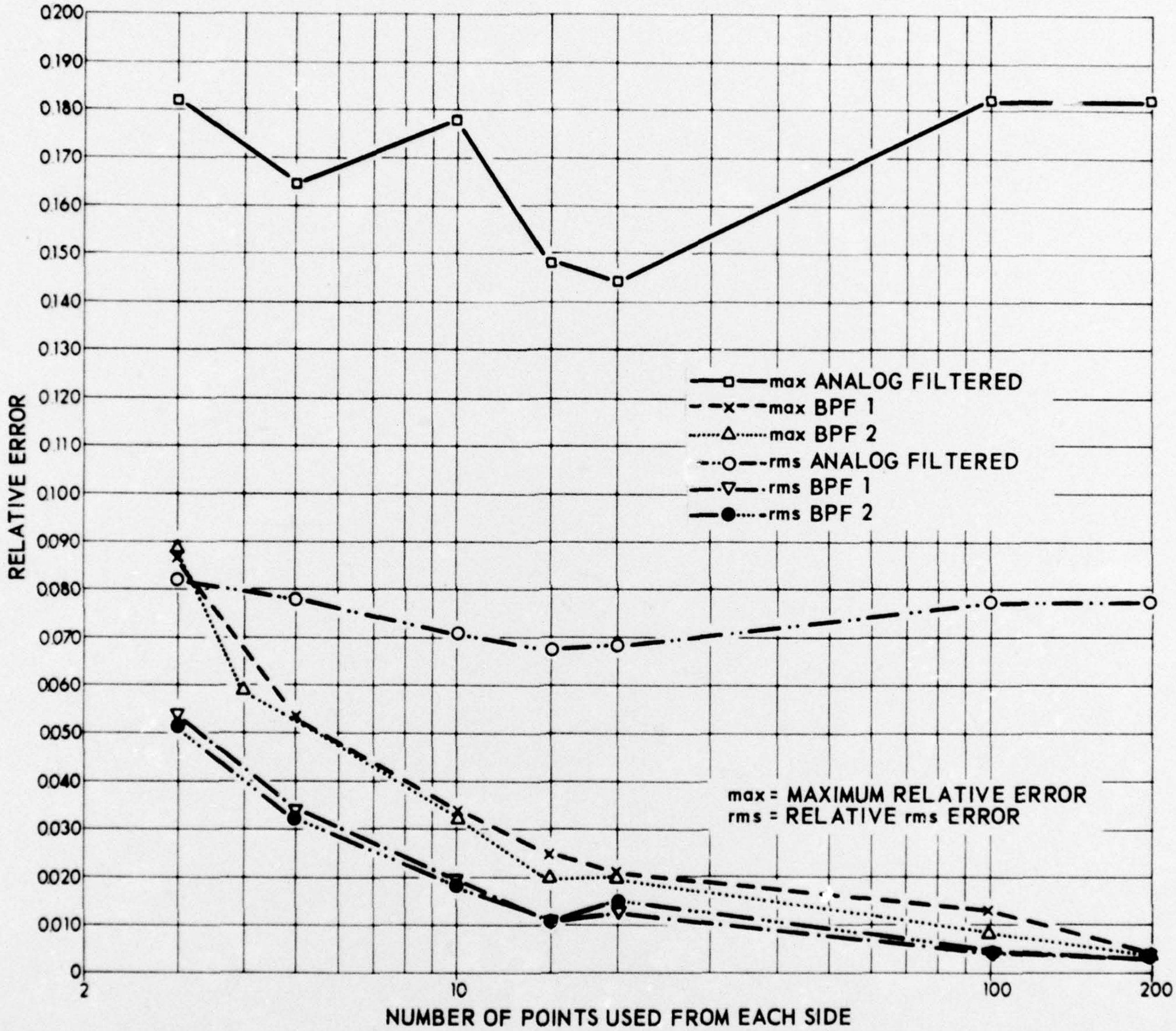


REPRODUCED WAVEFORM WITH No. OF POINTS FROM EACH SIDE USED FOR CALCULATION - 200
FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-685
GSI - EJW
6 - 20 - 67

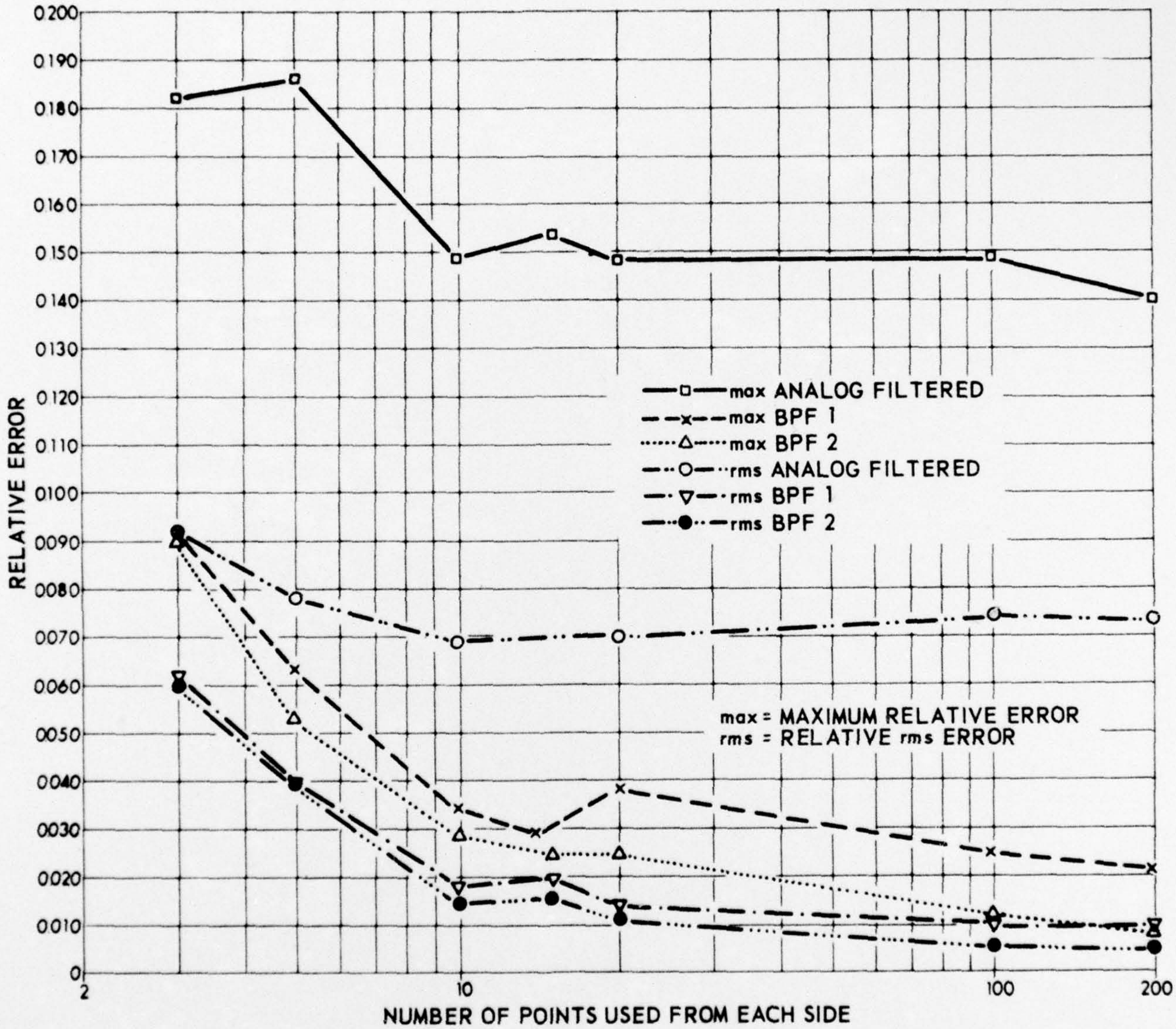
IX.

COMPARISON OF RELATIVE ERRORS INVOLVING COMBINATIONS OF VARIOUS TYPES OF FILTERS, THE NUMBER OF POINTS USED FROM EACH SIDE FOR CALCULATION, AND THE DIFFERENT VALUES OF LP, ILLUSTRATED BY GRAPHS AND TABLES



RELATIVE ERROR WHEN LP EQUALS 3

DRL - UT
DWG AS-67-686
CAB - BJB
6 - 3 - 67



RELATIVE ERROR WHEN LP EQUALS 5

DRL - UT
DWG AS-67-687
CAB - BJB
6 - 3 - 67

TABLE I

RELATIVE ERRORS

when LP = 3

FILTER / NO. OF PTS. FROM EACH SIDE USED	ANALOG FILTERED	BPF1 (3 kHz-7 kHz)	BPF2 (4 kHz-6 kHz)
3	max = 0.182 rms = 0.082	max = 0.088 rms = 0.054	max = 0.086 rms = 0.052
5	max = 0.165 rms = 0.078	max = 0.059 rms = 0.034	max = 0.053 rms = 0.033
10	max = 0.178 rms = 0.071	max = 0.034 rms = 0.020	max = 0.033 rms = 0.019
15	max = 0.148 rms = 0.067	max = 0.025 rms = 0.011	max = 0.020 rms = 0.011
20	max = 0.144 rms = 0.068	max = 0.021 rms = 0.013	max = 0.020 rms = 0.014
100	max = 0.182 rms = 0.077	max = 0.013 rms = 0.005	max = 0.008 rms = 0.005
200	max = 0.182 rms = 0.077	max = 0.004 rms = 0.003	max = 0.004 rms = 0.003

max is the maximum relative error.

rms is the relative rms error.

TABLE II

RELATIVE ERRORS

when LP = 5

NO. OF PTS. FROM EACH SIDE USED \ FILTER	ANALOG FILTERED	BPF1 (3 kHz-7 kHz)	BPF2 (4 kHz-6 kHz)
3	max = 0.182 rms = 0.092	max = 0.092 rms - 0.062	max = 0.090 rms = 0.060
5	max = 0.186 rms = 0.077	max = 0.063 rms = 0.040	max = 0.053 rms = 0.039
10	max = 0.148 rms = 0.069	max = 0.034 rms = 0.017	max = 0.029 rms = 0.015
15	max = 0.153 rms = 0.070	max = 0.029 rms = 0.019	max = 0.025 rms = 0.016
20	max = 0.148 rms = 0.070	max = 0.038 rms = 0.014	max = 0.025 rms = 0.011
100	max = 0.148 rms = 0.074	max = 0.025 rms = 0.009	max = 0.012 rms = 0.005
200	max = 0.140 rms = 0.073	max = 0.021 rms = 0.009	max = 0.008 rms = 0.004

max is the maximum relative error.

rms is the relative rms error.

X.

CONCLUSION

X. CONCLUSION

The results of this study are relatively consistent with the anticipated results. If the data are assumed to be in the 4-6 kHz band, LP = 5, the results are not as accurate as if the data are assumed to be in the 3.3-6.7 kHz band, LP = 3. Accordingly, as more data points are used to compute the value of the time series at a given point, the error tends to decrease.

The analog filtered data appear to have contained a dc bias and possibly some other noise outside the desired frequency band. This could have occurred after filtering, before digitization, or during digitization. The improvement produced by digitally filtering the data is marked, and to some extent, expected because of the low error level in digital operations. The close correspondence of the results using BPF1 and BPF2 indicates that most of the noise which entered the digitized data was outside the 3-7 kHz band.

In summation, there is a notable correlation between the original and the calculated data when only a few data points are used in the calculation of the time series. Moreover, there is a remarkable accuracy in locating the zeroes of the functions when straight line approximations for both the raw data and the reproduced function are used.

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Harman, Willis W., Principles of the Statistical Theory of Communication, McGraw-Hill, 1963.

Shannon, C. E., "Communication in the Presence of Noise," Proc. IRE, 37, 10-21 (1949).

24 May 1967

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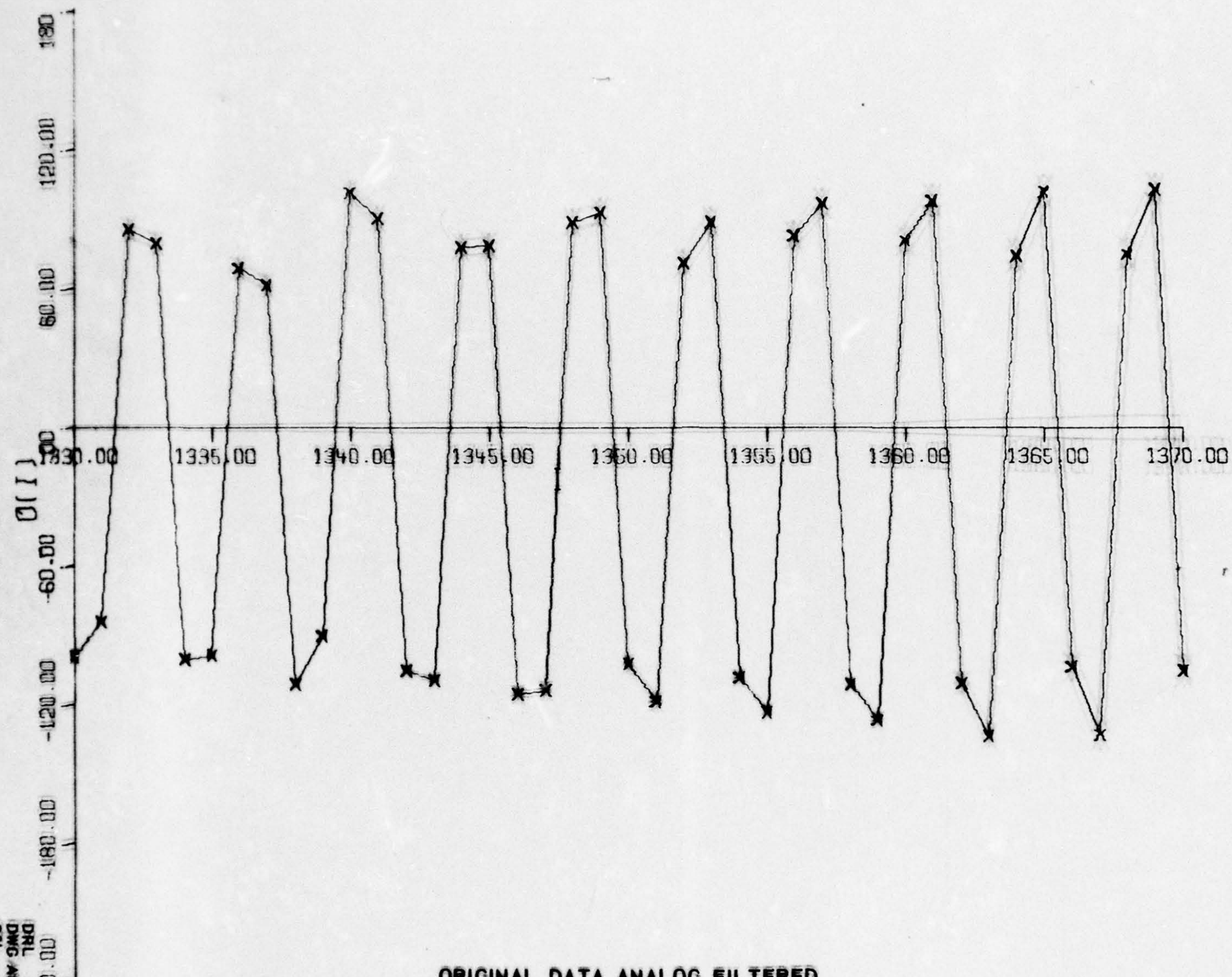
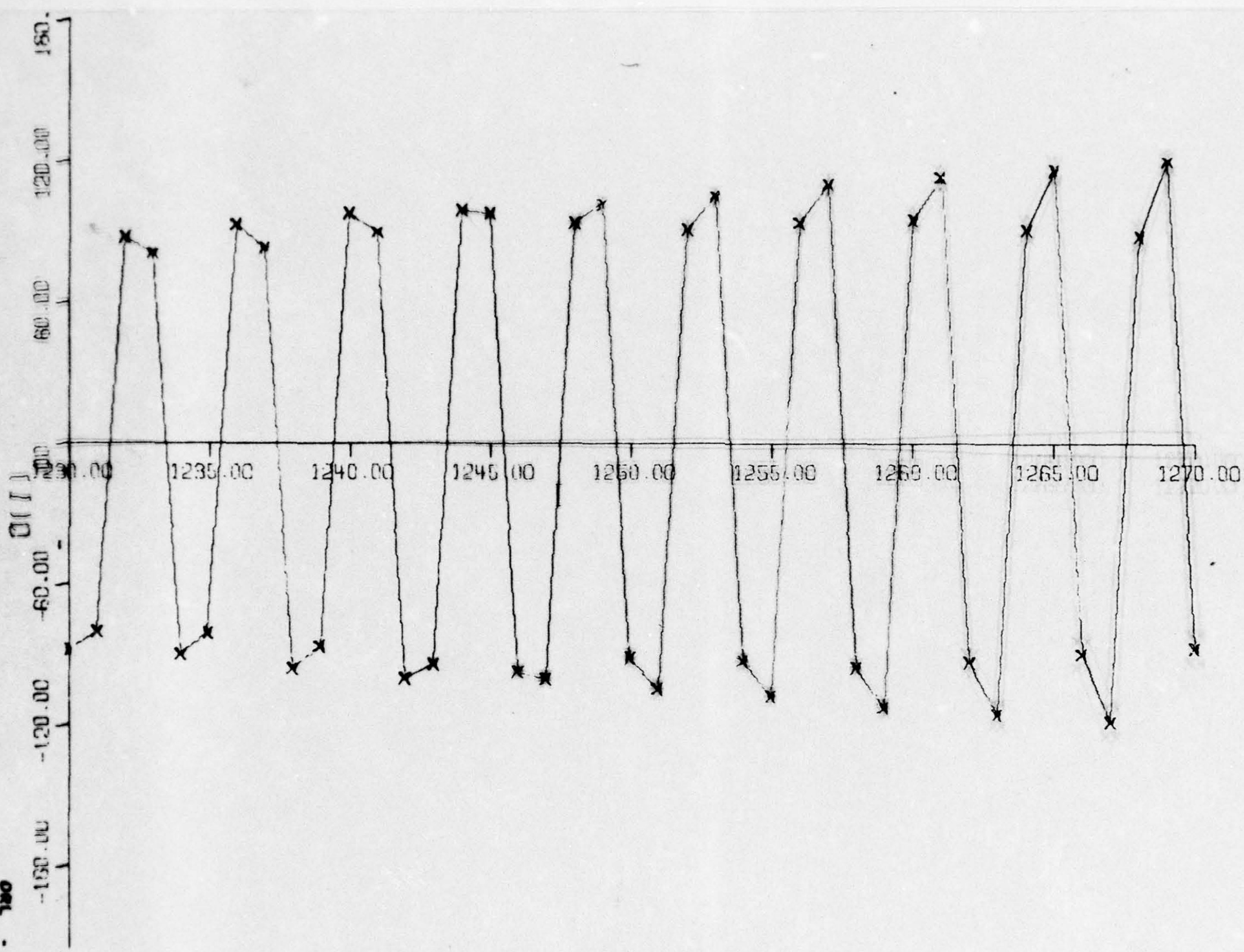


ORIGINAL DATA ANALOG FILTERED

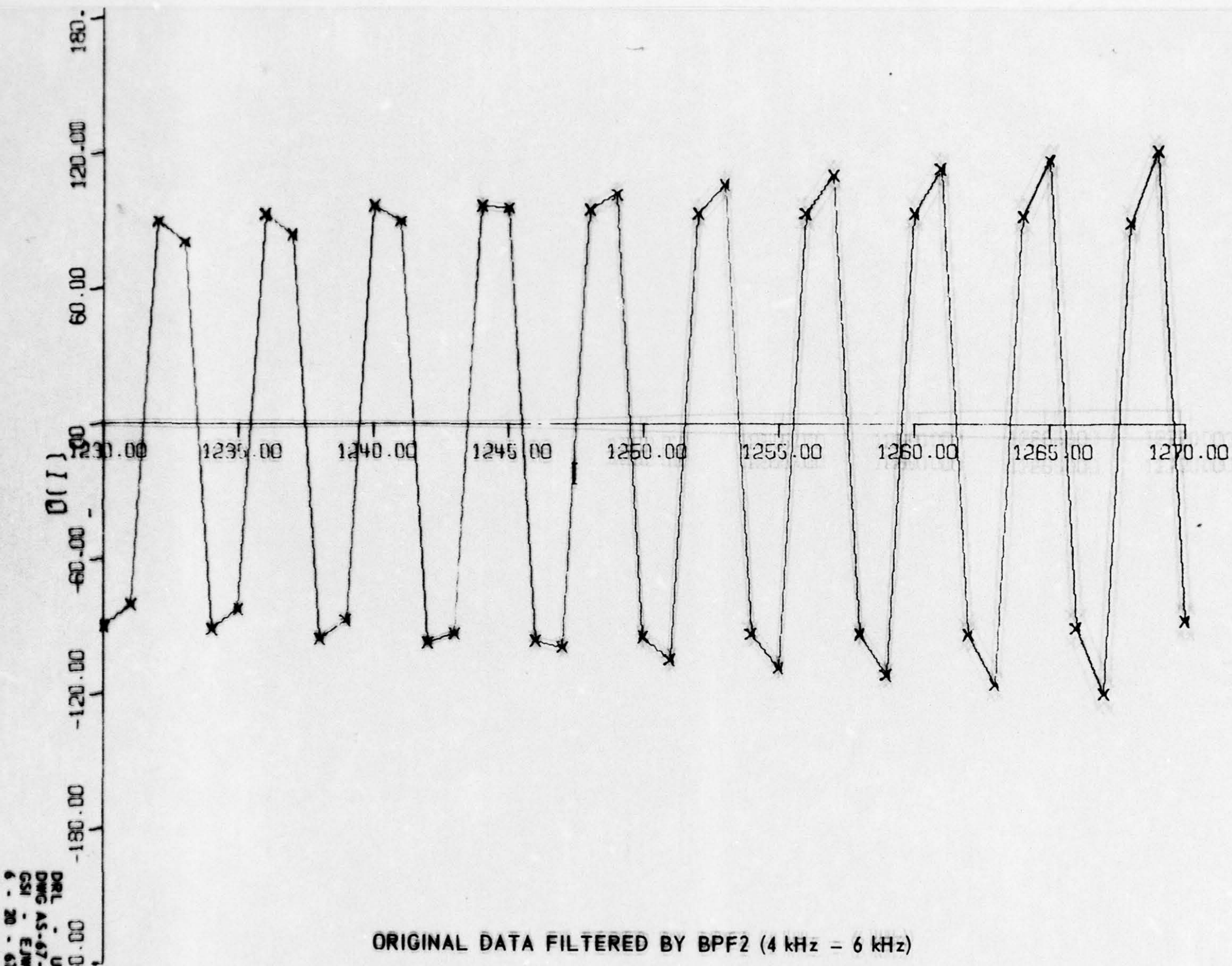
DRL - UT
DWG AS-67-688
GSI - EJW
6 - 20 - 67



ORIGINAL DATA FILTERED BY BPF1 (3 kHz – 7 kHz)

ORL - UT
DWG AS-67-689
GSI - EJW
6 - 20 - 67

GSI - EJW
6 - 20 - 67



ORIGINAL DATA FILTERED BY BPF2 (4 kHz – 6 kHz)

DRL - UT
DWG AS-67-690
GSI - EJW
6 - 20 - 67