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### ABSTRACT

The research program is concerned with the development of concepts and methodology for the analysis and design of control strategies for complex systems for use in aerospace avionics. This report highlights the results obtained during the grant period, which are fully documented in the journal articles, meeting papers, and reports cited in the list of publications. The results are categorized into the following headings: Control of systems containing random parameters, discrete-time dynamic games, control of mixed distributed and lumped parameter systems, nonlinear programming and optimal control, singular perturbation and order reduction, decomposition in linear regulator design, singularly perturbed stochastic control, decomposition of time scales in trajectory optimization, and sensitivity and controllability.

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# I. Faculty Employed on Grant During Part or All of Year

Jose B. Cruz, Jr., Professor, Principal Investigator William R. Perkins, Professor Petar V. Kokotovic, Professor 1

## II. Summary of Research Accomplishments

## A. Control of Systems Containing Random Parameters

The problem of controlling a stochastic system containing unknown parameters has been the subject of numerous studies as is shown by recent surveys. One general approach involves the estimation of the unknown parameters and the state of the system and the design of a control law that satisfies a given performance criterion. The accuracy of the estimators will be in general a function of the control, while the quality of the control will be a function of how accurately the system is known. Thus a controller must find a compromise between the estimation and control objectives. This is what is called a dual control problem. The exact solution of this problem is theoretically extremely difficult and computationally not feasible at the present time. An alternative to this situation has been obtained by subsuming the estimation objective to the control objective. This is achieved by postulating a fixed structure estimatorcontroller whose parameters can be adjusted to satisfy the control objective.

We have developed a procedure for controller design for discrete time stochastic systems with unknown but constant parameters [C1,D1].

A performance index that is quadratic in both the state and the control over N periods is considered. This performance index is minimized with respect to the feedback control gain matrix, the estimation dynamics matrix, and the filter gain matrix. The last two matrices are introduced for the computation of the estimate of the state. It is important to note that both the identification (parameter estimates) and the state estimation are performed so that a control objective is satisfied, in contrast with other approaches reported in the literature in which the parameters are estimated with a particular objective different from the objective of the control. The performance index is a mathematical expectation taken over the distribution in the noise disturbances, the initial state, and the parameter uncertainty. Minimization of this expectation has the interpretation of reducing the sensitivity of the standard criterion function with respect to fluctuations in parameter values.

An explicit expression for the total cost as a function of the control, estimation, and filter gain matrices has been obtained. To simplify computation, this expression was approximated in terms of first and second moments of the random plant matrices. Finally, necessary conditions for the minimization of the cost function were obtained. These conditions are in terms of gradient matrices which are used for the numerical optimization.

The controller design uses the apriori statistics of the unknown parameters instead of relying on an on-line parameter identification scheme during the optimization procedure. The procedure is particularly advantageous when the number of stages is small. For the long term, when several successive N-stage periods are considered, self-tuning properties can be obtained by monitoring long term trends in the statistics of the plant parameters and changing control design values to correspond to the new statistics. For more details see [C1,D1].

## B. Discrete-Time Dynamic Games

A dynamic game is a dynamic system in which there is more than one controller. Each controller tries to optimize a performance index which may be different from those of others. A differential game is described in terms of differential equations, and a discrete-time dynamic game is described in terms of difference equations. System which involve digital controls are characterized at the sampling instants by difference equations. Up to this time, little has been done on discrete-time games.

We have investigated Nash and Stackelberg strategies for discretetime games [E1]. Recursive formulas have been derived for linear games with general quadratic performance criteria. These formulas expand on our previous work on continuous-time games. Both dynamic programming and variational methods were used to derive necessary conditions for optimality. For details, see [E1].

# C. Control of Mixed Distributed and Lumped Parameter Systems

Two publications based on research on distributed parameter systems undertaken under Air Force Grant AFOSR 73-2570 during 1974-75 appeared during 1975-76.

Observability of a class of mixed distributed and lumped parameter systems with respect to physically realizable measurements was studied. Measurements were allowed only on the lumped subsystem or at certain interior points of the spatial domain, corresponding to practically acceptable sensor configurations. For both cases algebraic observability conditions were obtained. Details are given in [A2].

A second study involved controllability of linear time-delay systems having piecewise-constant inputs. The input restriction corresponds to the use of digital controllers with D/A converters of the "zero-order-hold" type, for example. Several kinds of controllability were studied, and algebraic conditions were obtained for special cases. Complete details are reported in [A1].

### D. Nonlinear Programming and Optimal Control

A report [E2] based on research undertaken under Air Force Grant AFOSR 73-2570 during 1974-75 has appeared during 1975-76. Specifically, this research involves optimal control with state-variable inequality constraints. The problem is formulated as a standard optimal control problem with one additional constraint of the form

# $S(x(t)) \leq 0.$

This constraint is assumed to be of p-th order where p is an integer and  $p \ge 1$ , that is, the p-th time derivative of the constraint is the first to contain the control variable explicitly. The constraint is added to the standard problem whenever the unconstrained optimal trajectory would drive the system through a physically undesirable state. Thus, the constraint serves to divide the state space into a feasible and an infeasible subspace and the optimization is forced to take place within a bounded region; i.e., the state trajectory must satisfy  $S(x(t)) \le 0$  at all time instants. The major contribution of this research is to prove that the optimal trajectory must also satisfy

$$\frac{dS}{dt} (x(t_i)) = 0 \qquad \text{if } p \ge 1$$

$$\frac{dS}{dt} (x(t_i - \Delta t), u(t_i - \Delta t)) \ge 0 \quad \text{and} \quad \frac{dS}{dt} (x(t_i + \Delta t), u(t_i + \Delta t)) \le 0 \quad \text{if } p = 1$$

where t, is any junction time defined by

$$S(x(t_i)) = 0 \text{ and} \begin{cases} S(x(t_i - \Delta t)) \neq 0 & i = 1, 2, \dots, k \\ \\ \text{or } S(x(t_i + \Delta t)) \neq 0 \end{cases}$$

and  $\Delta t$  is an arbitrarily small positive time increment.

Properties of the optimal trajectory are sought which may be inferred from various necessary conditions, given the new trajectory constraints. The resultant findings are conditioned upon the order of the constraint surface. For p>1, if the Hamiltonian is "regular," then the optimal control will be continuous at all junction times, not just those which lead onto or away from a boundary arc as was established previously. In addition, it is known that if the optimal control is continuous at some junction time t,, then the first (p-2) time derivatives of the control will also be continuous at that junction time. Moreover, the costate is allowed to be discontinuous at that time and the size of the discontinuity is proportional to the size of the discontinuity of the (p-1)-th time derivative of the control. If p=1 and the Hamiltonian is "regular" then the new trajectory constraint will force the costate to be continuous at all junction times and, consequently,  $\frac{dS}{dt}(x(t_i),u(t_i)) = 0$ . Regularity simply means that a unique value of the optimal control is known as a function of the state and the costate at any time.

Finally, it is demonstrated that the new trajectory constraints allow the constrained problem to be solved easily and with great accuracy using a multi-point boundary value approach whenever:

1. state dimension = constraint order = 1

and S(x) = 0 has a unique, time-invariant solution

2. state dimension = constraint order = 2

and S(x) = 0 and  $\tilde{S}(x) = 0$  together have a unique, time-invariant solution.

## E. Singular Perturbations and Order Reduction

Although many optimum control theory concepts are valid for any system order, their actual use is limited to low order models. In trajectory optimization and guidance problems the "curse of dimensionality" is not only in a formidable amount of computation, but also in the ill-conditioned initial and two point boundary value problems. The interaction of fast and slow phenomena in high-order systems results in "stiff" numerical problems which require expensive integration routines.

A singular perturbation method is being developed which alleviates both dimensionality and stiffness difficulties. The method lowers the model order by first neglecting the fast phenomena. It then improves the approximation by reintroducing their effect as "boundary layer" corrections calculated in separate time scales. Further improvements are possible by asymptotic expansion methods. In addition to being helpful in design procedures, the singular perturbation approach is an indispensable tool for analytical investigations of robustness of system properties, behavior of

optimal controls near singular arcs, and other effects of intentional or unintentional changes of system order.

Singular perturbation techniques applicable to control and optimization problems have been surveyed in [A4,B1,C4] and their relationship with other order reduction and multiple time scale methods has been discussed in [B2]. A procedure for representing stiff linear systems as singularly perturbed systems is developed in [C3] and applied to an eigenvalue placement problem in [B5].

# F. Decomposition in Linear Regulator Design

Earlier methods for two stage design of linear optimal regulators have been extended to allow a complete separation of slow and fast subsystem designs [B3,B5,C2,E3]. A fundamental property of this new design is its insensitivity with respect to singular perturbation parameters. For a second order near optimum performance these parameters need not be known, while a first order approximation is still achievable when neither the parameters, nor the exact model of the fast subsystem are known. Among potential applications of this procedure are recent control problems in advanced helicopter design with widespread interacting modes.

## G. Singularly Perturbed Stochastic Control

Properties of singularly perturbed systems with white noise input have been investigated in [A5,D5] and the optimal filter problem is solved in two time scales. The two filters yield estimates of slow-mode and fast mode states. The stochastic control problem is then formulated for linear singularly perturbed systems and preliminary results are obtained in [D2].

## I. Decomposition of Time Scales in Trajectory Optimization

Time scale separation properties of singularly perturbed systems have been exploited for a decomposition of trajectory optimization problems. First an iterative method has been developed for time-optimal control of linear systems with slow and fast modes [B4,C5,E4]. The method has reduced computational complexities and improved convergence properties to such an extent that fourth to sixth order systems can be optimized on microprocess in real time. The main principle of alternatively optimizing slow and fast subsystems is now being applied to nonlinear systems and a wider class of trajectory optimization problems and the results are highly encouraging [D5].

# J. On Sensitivity and Controllability

Singular perturbation analysis has led to new insights in controllability and sensitivity properties of linear systems. Singular perturbations can be viewed as resulting in or being caused by weak and strong coupling between the system modes and control inputs. A sensitivity measure of controllability is introduced in [C3] and used in allocating the control effort among the controllers [D3].

# III. Publications

## A. Journal Articles Published

- A. Thowsen and W. R. Perkins, "On the Controllability of Linear Time-Delay Systems with Piecewise-Constant Inputs," <u>Int. J. System Science</u>, Vol. 7, No. 3, 1976, pp. 347-360.
- A. Thowsen and W. R. Perkins, "Observability Conditions for a Class of Mixed Distributed and Lumped Systems," <u>Automatica</u>, Vol. 12, May 1976, pp. 273-275.

- P. Kokotovic, "A Riccati Equation for Block-Diagonalization of Ill-Conditioned Systems," <u>IEEE Trans. on Automatic Control</u>, Vol. AC-20, Dec. 1975, pp. 812-814.
- P. Kokotovic, R. O'Malley, and P. Sannuti, "Singular Perturbations and Order Reduction in Control Theory - An Overview," <u>Automatica</u>, Vol. 12, March 1976, pp. 123-132.
- 5. A. H. Haddad, "Linear Filtering of Singularly Perturbed Systems," <u>IEEE</u> Trans. on Automatic Control, Vol. AC-21, Aug. 1976, pp. 515-520.

#### B. Meeting Papers Published

- P. Kokotovic, R. O'Malley, Jr., and P. Sannuti, "Singular Perturbations and Order Reduction in Control Theory - An Overview," <u>Proc. Sixth World</u> <u>Congress of IFAC</u>, Aug. 1975, pp. 3/1 - 11.
- 2. P. Kokotovic, "Separation of Time Scales in Modeling and Control," <u>Proc.</u> <u>1975 IEEE Conf. on Decision and Control</u>, pp. 463-467.
- J. H. Chow, "Two Stage Design of Singularly Perturbed Linear Regulators," <u>Proc. of 13th Annual Allerton Conf. on Circuit and System Theory</u>, University of Illinois, Oct. 1975, pp. 48-57.
- S. H. Javid, "A Recursive Algorithm for Time-Optimal Control of Singularly Perturbed Systems," <u>Proc. 13th Annual Allerton Conf. on Circuit and</u> <u>System Theory</u>, Oct. 1975, pp. 839-847.
- 5. J. H. Chow and P. V. Kokotovic, "Eigenvalue Placement in Two-Time-Scale Systems," <u>Proc. of IFAC Symp. on Large Scale Systems</u>, Udine, Italy, June 1976, pp. 321-326.

#### C. Journal Articles Accepted and Scheduled for Publication

- C. S. Padilla and J. B. Cruz, Jr., "A Linear Dynamic Feedback Controller for Stochastic Systems with Unknown Parameters," to appear in <u>IEEE Trans.</u> on Automatic Control, Vol. AC-22, February 1977.
- J. H. Chow and P. V. Kokotovic, "A Decomposition of Near-Optimum Regulators for Systems with Slow and Fast Modes," to appear in <u>IEEE</u> <u>Trans. on Automatic Control</u>, Vol. AC-21, Oct. 1976.
- 3. J. H. Chow, "Preservation of Controllability in Linear Time-Invariant Perturbed Systems," to appear in <u>Int. J. of Control</u>.

- 4. P. Kokotovic, "Singular Perturbations in Optimal Control," to appear in Rocky Mountain Journal of Mathematics.
- 5. S. H. Javid and P. V. Kokotovic, "A Decomposition of Time Scales for Iterative Computation of Time-Optimal Controls," to appear in <u>Journal</u> of Optimization Theory and <u>Applications</u>.

### D. Meeting Papers Accepted and Scheduled for Presentation

- C. S. Padilla and J. B. Cruz, Jr., "A Linear Dynamic Feedback Controller for Stochastic Systems with Unknown Parameters," <u>1976 IEEE Conf. on</u> <u>Decision and Control</u>, Dec. 1-3, 1976, Clearwater Beach, Florida.
- A. Haddad and P. Kokotovic, "Stochastic Optimal Control of Linear Singularly Perturbed Systems," to appear in Proc. 14th Allerton Conf. on Circuit and System Theory, University of Illinois, Oct. 1976.
- 3. J. H. Chow, "Pole Placement Design of Multiple Controller Systems via Weak and Strong Controllability," to appear in <u>Proc. 14th Annual Allerton</u> <u>Conf. on Circuit and System Theory</u>, University of Illinois, 1976.
- 4. S. H. Javid, "The Time-Optimal Control of a Class of Nonlinear Singularly Perturbed Systems," to appear in <u>Proc. 14th Annual Allerton Conf. on</u> Circuit and System Theory, University of Illinois, 1976.
- H. Khalil, "On Singularly Perturbed Systems with Stochastic Inputs," to appear in <u>Proc. 14th Annual Allerton Conf. on Circuit and System Theory</u>, University of Illinois, 1976.

### E. CSL Reports

- 1. B. F. Gardner, Jr., "Discrete-Time Dynamic Games," R-691, September 1975.
- 2. A. L. Hendricks, "On Optimal Control Problems with State-Variable Inequality Constraints," R-727, May 1976.
- 3. J. H. Chow, "Separation of Time Scales in Linear Time-Invariant Systems," R-688, September 1975.
- 4. S. H. Javid, "The Time-Optimal Control of a Class of Singularly Perturbed Systems," R-700, November 1975.

## IV. Other Activities

Professor Cruz was elected Vice-President for Financial and Administrative Activities of the IEEE Control Systems Society for 1976. He was recently reelected as Vice-President for 1977. He served as General Chairman for the 1975 IEEE Conference on Decision and Control, held in Houston, Texas, December 10-12, 1975. He continued to serve as a member of the Theory Committee of AACC.

Professor Perkins continued to serve as a member of the Theory Committee of AACC. He started his second year as Chairman of the Awards and Fellow Nominations Committee of the IEEE Control Systems Society. He was on the Program Committee of the 1975 IEEE Conference on Decision and Control.

Professor Kokotovic continues to serve as an elected member of the Administrative Committee of the IEEE Control Systems Society. He also continues to serve as Associate Editor of Automatica.