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The IAU Resolutions on Astronomical Constants, Time Scales, and the Fundamental Reference Frame

Edited by
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Errata for USNO Circular 163

1. On page A8, the second version of the formula for the equinox corrrection should read:

$$E = 0^{s}.0775 + 0^{s}.085\,T$$

2. On page A7, the second term in the formulas for θ and μ should be omitted, as pointed out by Murray (1981, Monthly Notices Royal Astr. Soc., vol. 195, p. 639). That is, these formulas should read:

$$\theta = 0''.00407 \left(\frac{1 + \cos D}{\sin D} \right)$$

$$\mu \equiv \frac{\theta}{\sin D} = 0''.00407 \left(\frac{1}{1 - \cos D} \right)$$

I regret that this error has been propagated from previous works.

— *George Kaplan*

CONTENTS

The IAU Resolutions
on
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the Fundamental Reference Frame

This Circular presents the resolutions recently adopted by the International Astronomical Union (IAU) regarding constants, time scales, and the new fundamental astronomical reference frame, the FK5. The resolutions are intended to apply to the reduction of observations taken on or after 1 January 1984, and to be used in the preparation of ephemerides for the years 1984 and beyond. These resolutions will necessitate major revisions in reference data, procedures, and algorithms for all work related to fundamental astronomy, astrometry, timekeeping, celestial mechanics, and geodesy.

These resolutions were prepared by various working groups and were adopted by the General Assembly of the IAU at the 1976 and 1979 meetings. In their original form, the resolutions appear in the *Proceedings* of the Sixteenth and Seventeenth General Assemblies of the IAU. As presented here, however, related resolutions have been grouped together for convenience; the original numbering scheme has been discarded. In the few cases where a resolution has been superseded by a subsequent resolution, only the final version is given. Similarly, if a numerical value within a resolution has been affected by a later resolution, or if the original value was preliminary, the value as given in this Circular has been appropriately updated. Each group of resolutions is followed by a series of notes which offer brief explanations. Most of these notes were prepared by the working groups which drafted the resolutions. Further information can be found in the references cited in the notes.

In considering these resolutions, the following should be noted:

(1) The resolutions form a self-consistent and interrelated system of changes necessary to improve the astronomical reference frame. Thus, the resolutions cannot be applied individually or selectively -- they must be implemented simultaneously.

(2) In astronomical publications the use of the new standard epoch, J2000.0, as a reference epoch for astronomical data implies that all reductions and computations were performed in accordance with the resolutions. Futhermore, proper utilization of such data (for example, for the computation of *a priori* observables) requires adherence to the resolutions in any calculations or procedures.

Appendix A of this Circular lists a number of formulas which may be useful in implementing the resolutions. The list of formulas comprising Appendix A is not meant to be exhaustive; other expressions may be found in the various references.

A large volume of astronomical reference data currently exists which ideally should be transformed so as to be compatible with the new resolutions. However, because of the wide variety of such data, and the many different observing techniques and reduction procedures used in their preparation, no single all-inclusive conversion procedure exists. Appendix B provides conversion procedures for several common types of data. However, all such procedures are to some extent approximations; in particular, it must be emphasized that a rigorous transition to the FK5 system cannot be accomplished until the FK5 catalog is available.

These resolutions have necessitated a complete recomputation of the fundamental ephemerides of Solar System bodies. This work has been going on for the past several years at the Jet Propulsion Laboratory and the U. S. Naval Observatory. In the construction of the new ephemerides, advantage has been taken of recent high-accuracy radar, laser, and spacecraft data, along with the classical optical observations. In a related development, the IAU recently adopted a new set of rotational elements for the planets. The new planetary and lunar ephemerides and the new rotational elements will be used in the *Astronomical Almanac* for the years 1984 and beyond; see Appendix C for a description.

A number of efforts are currently in progress which will facilitate the implementation of the IAU resolutions described in this Circular. The FK5 catalog is now being prepared at the Astronomisches Rechen-Institut in Heidelberg. Work on the Solar System ephemerides is continuing at the Naval Observatory, the Naval Surface Weapons Center, and the Jet Propulsion Laboratory. Data reduction standards for project MERIT (the IAU-IUGG Earth rotation project), consistent with the IAU resolutions, are to be issued in 1982. Computer subroutines for mean to apparent place reductions for stars and planets are currently being tested at the Naval Observatory. Finally, a reference book to replace the *Explanatory Supplement to the Ephemeris* is being jointly planned by the Nautical Almanac Offices of the Naval Observatory and the Royal Greenwich Observatory.

G. H. Kaplan

Units:

The units meter (m), kilogram (kg), and second (s) are the units of length, mass, and time in the International System of Units (SI).

The astronomical unit of time is a time interval of one day (D) of 86400 seconds. An interval of 36525 days is one Julian century.

The astronomical unit of mass is the mass of the Sun (S).

The astronomical unit of length is that length (A) for which the Gaussian gravitational constant (k) takes the value 0.01720209895 when the units of measurement are the astronomical units of length, mass, and time. The dimensions of k^2 are those of the constant of gravitation (G), i.e., $L^3M^{-1}T^{-2}$. The term "unit distance" is also used for the length A.

Defining constant:

1.	Gaussian gravitational constant	k	= 0.01720209895

Primary constants:

2.	Speed of light	c	= 299792458 m s^{-1}
3.	Light-time for unit distance	τ_A	= 499.004782 s
4.	Equatorial radius for Earth	a_e	= 6378140 m
	[IUGG value	a_e	= 6378137 m]
5.	Dynamical form-factor for Earth	J_2	= 0.00108263
6.	Geocentric gravitational constant	GE	= 3.986005×10^{14} m^3s^{-2}
7.	Constant of gravitation	G	= 6.672×10^{-11} $m^3kg^{-1}s^{-2}$
8.	Ratio of mass of Moon to that of Earth	μ	= 0.01230002
9.	General precession in longitude, per Julian century, at standard epoch 2000	p	= 5029.″0966
10.	Obliquity of the ecliptic, at standard epoch 2000	ϵ	= 23° 26′ 21.″448

Derived constants:

11.	Constant of nutation, at standard epoch 2000	N	= 9.″2025
12.	Unit distance	$c\tau_A = A$	= $1.49597870 \times 10^{11}$ m
13.	Solar parallax	$\arcsin(a_e/A) = \pi_\odot$	= 8.″794148
14.	Constant of aberration, for standard epoch 2000	κ	= 20.″49552
15.	Flattening factor for the Earth	f	= 0.00335281
			= 1/298.257
16.	Heliocentric gravitational constant	$A^3k^2/D^2 = GS$	= $1.32712438 \times 10^{20}$ m^3s^{-2}
17.	Ratio of mass of Sun to that of the Earth	$(GS)/(GE) = S/E$	= 332946.0
18.	Ratio of mass of Sun to that of Earth + Moon	$(S/E)/(1+\mu)$	= 328900.5

19. Mass of the Sun $(GS) / G = S = 1.9891 \times 10^{30}$ kg

20. System of planetary masses

Ratios of mass of Sun to those of the planets

Mercury	6023600	Jupiter	1047.355
Venus	408523.5	Saturn	3498.5
Earth + Moon	328900.5	Uranus	22869
Mars	3098710	Neptune	19314
		Pluto	3000000 [130,000,000]

Other Quantities for Use in the Preparation of Ephemerides

It is recommended that the values given in the following list should normally be used in the preparation of new ephemerides.

21. Masses of minor planets

Minor planet	Mass in solar mass
(1) Ceres	5.9×10^{-10}
(2) Pallas	1.1×10^{-10}
(4) Vesta	1.2×10^{-10}

22. Masses of satellites

Planet	Satellite	Satellite / Planet mass
Jupiter	Io	4.70×10^{-5}
	Europa	2.56×10^{-5}
	Ganymede	7.84×10^{-5}
	Callisto	5.6×10^{-5}
Saturn	Titan	2.41×10^{-4}
Neptune	Triton	2×10^{-3}

23. Equatorial radii in km

Mercury	2439	Jupiter	71398	Pluto	2500
Venus	6052	Saturn	60000		
Earth	6378.140	Uranus	25400	Moon	1738
Mars	3397.2	Neptune	24300	Sun	696000

24. Gravity fields of the planets

Planet	J_2	J_3	J_4
Earth	+0.00108263	-0.254×10^{-5}	-0.161×10^{-5}
Mars	+0.001964	$+0.36 \times 10^{-4}$	
Jupiter	+0.01475		-0.58×10^{-3}
Saturn	+0.01645		-0.10×10^{-2}
Uranus	+0.012		
Neptune	+0.004		

(Mars: $C_{22} = -0.000055$, $S_{22} = +0.000031$, $S_{31} = +0.000026$)

25. Gravity field of the Moon

$\gamma = (B - A) / C = 0.0002278$ $C / MR^2 = 0.392$

$\beta = (C - A) / B = 0.0006313$ $I = 5552''.7 = 1° \; 32' \; 32''.7$

$C_{20} = -0.0002027$	$C_{30} = -0.000006$	$C_{32} = +0.0000048$
$C_{22} = +0.0000223$	$C_{31} = +0.000029$	$S_{32} = +0.0000017$
	$S_{31} = +0.000004$	$C_{33} = +0.0000018$
		$S_{33} = -0.000001$

Units:

The constants of this revised system are generally expressed in terms of the SI units in order to ensure compatibility with the usage in related sciences. In astronomy it is, however, necessary to use the astronomical system of units of length, mass and time. The astronomical unit of time (day) is redefined in terms of the SI second, which was itself defined so as to be equal to the ephemeris second to within the error of the determination. Specifically it is the SI second at mean sea level.

Defining constant:

1. The Gaussian gravitational constant serves to define the astronomical unit of length when the corresponding astronomical units of time and mass are already defined. The value for k is that adopted by the IAU in 1938. The value of k is treated as exact, and defines the astronomical unit as the radius of a circular orbit in which a body of negligible mass, and free of perturbations, would revolve around the Sun (unit mass) in $2\pi / k$ days.

Primary constants:

2. The value for the speed of light is that recommended by the fifteenth General Conference on Weights and Measures in 1975. It is understood that this value will remain unchanged even if the meter is redefined in terms of a different wavelength from that now used.

3. The value for the light-time for unit distance (1 astronomical unit of length) is based on radar measurements of planetary distances. It is numerically equal to the number of light-seconds in 1 astronomical unit of length. Its reciprocal, $1 / \tau_A$, is equal to the speed of light in astronomical units of length per second. The speed of light in astronomical units per day is $86400 / \tau_A$.

4. The values for constants 4, 5, 6, and 15 are those recommended by the International Association of Geodesy (IAG) at its Sixteenth General Assembly at Grenoble in 1975 (ref. 17). The term "equatorial radius for Earth" refers to the equatorial radius of an ellipsoid of revolution that approximates the geoid. The value listed is the IAG (1975) value; however, the International Union of Geodesy and Geophysics (IUGG) recommends the value in brackets in the Geodetic Reference System 1980 (ref. 44). The two values differ by 3 meters, and the uncertainty of measurement is estimated to be 5 meters.

5. The term "dynamical form-factor for Earth" refers to the coefficient of the second zonal harmonic in the expression for the Earth's gravitational potential as defined in ref. 18. The value is equal to that of the Geodetic Reference System 1980 (ref. 44). See also notes 4 and 15.

6. The geocentric gravitational constant is appropriate for use for geocentric orbits when the units are the meter and the second; E denotes the mass of the Earth including its atmosphere. See also note 4 and ref. 44.

7. The value for the constant of gravitation is that given in the CODATA system of physical constants of 1973 (ref. 5).

8. The value for the mass ratio is based on recent data from lunar and planetary spacecraft. The reciprocal of the listed value for μ is (approximately) 81.3007.
9. The value listed for the general precession in longitude represents the first change in this fundamental quantity since the Paris Conference of 1896 agreed to adopt Newcomb's value. The change in the precession constant will have far-reaching effects on fundamental astronomy. The value listed has been derived on the basis of recent determinations of the corrections to Newcomb's value of lunisolar precession, and on a new value of planetary precession derived from the new planetary masses (refs. 10, 11, 12, 23). For the convenience of those making differential corrections the exact value $\Delta p_1 = +1\overset{''}{.}10$ has been adopted in computing the new value of p for epoch 2000. The four decimal places of p are required in order to secure consistency with computations based on the current value for epoch 1900 and the correction Δp_1. A set of expressions, complete with numerical coefficients, for the new precessional reductions is given in refs. 23 and 22.
10. The value of the obliquity of the ecliptic results from applying secular terms computed with the new values of the planetary masses to the current value for 1900. A discussion of this computation, and the new expression for obtaining the value of the obliquity at an arbitrary epoch, are given in ref. 23.

Derived constants:

The values of the derived constants 11 through 19 have been computed from the values of the defining and primary constants. All the values are consistent with those determined more directly from observations.
11. The value of the constant of nutation, which is the amplitude of the principal nutation term in obliquity, is from the 1980 IAU Theory of Nutation. Because this nutation theory is based on a non-rigid model of the Earth, the constant of nutation is no longer a scaling factor for the nutation series. The usefulness of this parameter is therefore limited. The 1980 IAU Theory of Nutation is described in refs. 20 and 38.
12. The number of meters in one astronomical unit of length is now treated as a derived constant, the more fundamental constant being the light-time for unit distance.
13. The solar parallax is the angle subtended by the equatorial radius of the Earth when viewed from a distance of one astronomical unit of length.
14. The constant of aberration is the ratio of the mean speed of the Earth in its orbit to the speed of light, and is conventionally expressed in seconds of arc. It is calculated in radians from the expression $F\, k\, \tau_A\, /\, 86400$ where F is the ratio of the mean speed of the Earth to the speed of a hypothetical planet of negligible mass moving around the Sun in a circular orbit of one astronomical unit radius. The value of F for epoch 2000 is 1.0001414, derived from $F\, k = n\, a\, (1 - e^2)^{-\frac{1}{2}}$ where n is the sidereal mean motion of the Sun in radians per day, a is the perturbed mean distance of the Sun in astronomical units, and e is the mean eccentricity of the Earth's orbit.
15. The flattening factor for the Earth is derived from the adopted values of the primary geodetic parameters using the condition that the corresponding ellipsoid of revolution shall be an equipotential surface. The value listed for f is that recommended by the IAG in 1975 (see note 4); the value was re-affirmed in the Geodetic Reference System 1980. (Refs. 16, 17, and 44)

16. The heliocentric gravitational constant is appropriate for use for heliocentric orbits when the units are the meter and the second.
17. – 20. The values given for the reciprocal masses of the planets (planet mass in solar masses = 1 / reciprocal mass) include the contributions from atmospheres and satellites. For Mercury, Venus, and Mars values close to the best spacecraft determinations are adopted. For the Earth the mass is that derived from the adopted values of A, GE and μ . For Jupiter, Uranus, and Neptune the modern determinations do not indicate the necessity to change the previous (Newcomb) values. The value for Saturn is the unweighted mean of the most reliable determinations. The value for Pluto is based on analyses of the orbit of Neptune; a more recent value, based on the motion of the satellite, is given in brackets. (Refs. 15, 1, 7, and 13)

The values given for the reciprocal masses are to be treated as exact, except that for Earth + Moon the gravitational constant should be calculated from $GE\ (1 + \mu)$, that is, from the exact values of the primary constants, if numerical consistency is required.

The mass of the Sun in kilograms is given to indicate the relationship between the astronomical and SI units of mass; it is known only to the low precision with which the constant of gravitation is known in SI units. The corresponding values of the masses of the planets in kilograms are obtained by dividing the mass of the Sun in kilograms by the planetary reciprocal masses listed as constants 20.

Other quantitites:

21. There are not enough independent determinations of the masses of Ceres, Pallas and Vesta to derive ranges of uncertainty, but the internal standard errors are ± (0.3, 0.2, 0.1) x 10^{-10} solar masses, respectively (refs. 29, 30, and 14).
22. Masses of satellites of Jupiter are derived from Pioneer 10. Mass of Titan is derived from the motion of Iapetus. Mass of Triton is estimated from the motion of Neptune. (Refs. 3 and 6)
23. For Mercury, Venus, Earth, Mars, and the Moon the values refer to the planet's crust. A value for Venus including the height of the cloud layer is 6110 km. The radius of the Moon implicit in Watts' profile of the lunar limb is 1738.065 km. For Jupiter the value is based on determinations from Pioneer 10 and 11. For Saturn, Uranus, and Neptune the values are means of the best optical measures by double image micrometer and heliometer. The value for Pluto is a crude estimate. (Refs. 15, 17, 2, 1, and 28)
24. For notation see ref. 8. For the Earth, the coefficients are given for only three terms that have a significant effect on the orbital motion of the Moon; they should not be considered as defining the dynamical model of the Earth. The end figures are subject to change. The values for Mars are derived from Mars orbiter data. The coefficients given are the ones having a significant effect on the orbital motion of the satellites. The values for Jupiter are derived from Pioneer 10 and 11 results. Saturn and Neptune values are derived from motions of their nearby satellites. The Uranus value is based on optical measures of the flattening and is in reasonable agreement with the dynamical determination. (Refs. 17, 1, and 28)
25. The values are the best estimates based on lunar laser ranging data and spacecraft data. (Refs. 24, 33, and 39)

1. The new standard epoch and equinox. In a resolution adopted by the IAU in 1976 it was recommended that:

(a) the new standard epoch (designated J2000.0) shall be 2000 January 1.5, which is JD 2451545.0, and the new standard equinox shall correspond to this instant;

(b) the unit of time for use in the fundamental formulas for precession shall be the Julian century of 36525 days; and

(c) the epochs for the beginning of year shall differ from the standard epoch by multiples of the Julian year of 365.25 days.

2. Time scales for dynamical theories and ephemerides. In resolutions adopted by the IAU in 1976 and 1979, it was recommended that:

(a) at the instant 1977 January $01^d 00^h 00^m 00^s$ TAI, the value of the new time-scale for apparent geocentric ephemerides be 1977 January $1^{d}.0003725$ exactly;

(b) the unit of this time-scale be a day of 86400 SI seconds at mean sea level;

(c) the time scales for equations of motion referred to the barycenter of the Solar System be such that there be only periodic variations between these time-scales and that for the apparent geocentric ephemerides;

(d) no time step be introduced in International Atomic Time;

(e) the time scale for the equations of motion referred to the barycenter of the Solar System be designated Barycentric Dynamical Time (TDB);

(f) the time scale for apparent geocentric ephemerides be designated Terrestrial Dynamical Time (TDT).

3. The expression for Greenwich mean sidereal time in terms of UT1. A resolution adopted by the IAU in 1979 states:

In consideration that it is planned to introduce the IAU (1976) System of Astronomical Constants, the 1980 IAU Theory of Nutation, and the equinox of the FK5 on 1984 January 1, it is recommended:

(a) the relationship between mean sidereal time and UT1 be modified so that there is no change in either value or rate of UTl, due to a correction to the zero point of right ascension of the FK4 and a correction for the motion of the zero point, to be introduced in the FK5;

(b) the new expression for Greenwich mean sidereal time of 0^h UT1 be:

$$\text{GMST of } 0^h \text{ UT1} = 6^h 41^m 50^s.54841 + 8640184^s.812866\, T_U + 0^s.093104\, T_U^2 - 6^s.2 \times 10^{-6}\, T_U^3$$

where T_U is the number of Julian centuries of 36525 days of universal time elapsed since 2000 January 1, 12^h UT1 (JD 2451545.0).

1. The new standard epoch is exactly one Julian century after 1900 January $0^{d}.5$, which corresponds to the fundamental epoch of Newcomb's planetary theories. The new standard epoch is expressed in terms of dynamical time instead of universal time. Specifically, for precise planetary and lunar theories, it is expressed in terms of the time scale of the equations of motion with respect to the barycenter of the Solar System, Barycentric Dynamical Time (TDB).

In the new system a Julian epoch is given by

$$J2000.0 + (JD - 2451545.0) / 365.25,$$

where JD symbolizes the Julian date. If the Besselian epoch is still required, it is given by

$$B1900.0 + (JD - 2415020.31352) / 365.242198781.$$

The Besselian year is here fixed at the length of the tropical year ($365^{d}.242198781$) at B1900.0 (JD 2415020.31352).

The prefixes J and B are used to distinguish Julian and Besselian epochs; they may be omitted only where the context, or precision, makes them superfluous. (Ref. 22)

2. The time-like arguments of dynamical theories and ephemerides are referred to as dynamical time scales. While it is possible, and desirable, to base the unit of a dynamical time scale on the SI second (which is used in the IAU (1976) System of Astronomical Constants), it is necessary to recognize that in relativistic theories there will be periodic variations between the unit of time for an apparent geocentric ephemeris and the unit of time for the equations of motion, which may, for example, be referred to the barycenter of the Solar System. (In the terminology of the theory of general relativity such time scales may be considered to be *proper time* and *coordinate time*, respectively.) The time scales for an apparent geocentric ephemeris and for the equations of motion will be related by a transformation that depends on the system being modeled and on the relativistic theory being used (refs. 4 and 27). The arbitrary constants in the transformation can be chosen so that the time scales have only periodic variations with respect to each other. Thus, it is sufficient to specify the basis of a unique time scale to be used for new, precise, apparent geocentric ephemerides. This time scale is to be designated Terrestrial Dynamical Time (TDT).

The dynamical time scale for apparent geocentric ephemerides, Terrestrial Dynamical Time (TDT), is a unique time scale independent of any theory while the dynamical time scales referred to the barycenter of the Solar System are a family of time scales resulting from the transformations of various theories and metrics of relativistic theories. The barycentric time scales are to be designated Barycentric Dynamical Time (TDB).

This resolution specifies that Terrestrial Dynamical Time (TDT), to be used for apparent geocentric ephemerides, is for practical purposes equal to TAI + $32^{s}.184$. (There are formal differences arising from random and, possibly, systematic errors in the length of the TAI second and the method of forming TAI, but the accumulated effect of such errors is likely to be insignificant for astronomical purposes over long periods of time.) The scale is specified with respect to TAI in order to take advantage of the direct availability of UTC (which is based on the SI second and is an integral number of seconds offset from TAI), and to provide continuity with the current values and practice in the use of Ephemeris Time. Continuity is achieved since the chosen offset between TDT and TAI is the current estimate of the difference between ET and TAI, and since the SI second was defined so as to make it equal to the ephemeris second within the error of measurement. It will be possible to use most available ephemerides as if the arguments were on the new scale. The offset has been expressed in the resolution as an exact decimal fraction of a day since the arguments of theories and ephemerides are normally expressed in days.

In view of the desirability of maintaining the continuity of TAI and of avoiding the confusion that could arise if it were to be redefined retrospectively, no step in TAI is proposed. Although the recommendation is in terms of TAI, in practice astronomers will use UTC and convert directly to the dynamical time scales. (Ref. 40)

3. Universal time, UT1, which forms the basis for the worldwide system of civil time, is indirectly derived from the transit times of stars. UT1 is formally defined by an expression which relates it to mean sidereal time, the latter quantity being directly obtained from the apparent right ascensions of transiting stars. The star positions currently used are based on the FK4 system. On any given date, a star's computed apparent place, upon which its contribution to the UT1 determination depends, is a function not only of its catalog position and proper motion, but also of the adopted constants of precession, nutation, aberration, etc. Therefore, a change of reference system and astronomical constants can have complex and subtle effects on the resulting UT1 values. Recommendation 3(a) requires continuity in the UT1 determinations. The expression given in 3(b), which differs from the provisional expression originally listed with this resolution, will maintain the continuity of the UT1 determinations across the interface to the new (FK5-based) system. The expression is based on an equinox correction of $E = 0^{s}.035 + 0^{s}.085\,(y - 1950) / 100$, where y is the year, and E is in the sense FK5 − FK4. The expression is also consistent with the new precessional constant.

The expression given in 3(b) implies that the ratio of solar to sidereal time is 0.997269566329084 at epoch J2000.0, the inverse of which is 1.002737909350795. The expression also implies that the rotational angular velocity of the Earth is $7.2921151467 \times 10^{-5}$ radian sec^{-1}, consistent with the IAG value listed in ref. 17 and the value given in the Geodetic Reference System 1980, ref. 44. A derivation and discussion of this expression is given in ref. 25.

1. The fundamental reference frame. In a resolution adopted by the IAU in 1976, it was recommended that:

(a) the fundamental reference frame defined by the positions and centennial variations in the FK5 shall correspond as closely as possible to the dynamical reference frame;

(b) a correction to the zero point of right ascensions of the FK4 (equinox correction) and a correction to the motion of the equinox of the FK4 shall be derived from relevânt modern observations;

(c) the expression for Greenwich mean sidereal time at 0^{h}UT shall be amended by the same equinox correction and motion as adopted for the FK5 in order to avoid a discontinuity in UT.

2. The procedures for the computation of apparent places and the reduction of observations. In a resolution adopted by the IAU in 1976, it was recommened that:

(a) stellar aberration shall be computed from the total velocity of the Earth referred to the barycenter of the Solar System, and the mean places shall not contain E-terms;

(b) reductions to apparent place shall be computed rigorously and directly, without the intermediary of the mean place for the beginning of the year, whenever high precision is required.

3. The IAU nutation theory. IAU Commissions 4, 7, 8, 19, 24, and 31 adopted the following resolution in 1981:

The IAU adopts the 1980 IAU Theory of Nutation in replacement of the 1979 IAU Theory of Nutation, endorses the recommendations given in the Report of the Working Group on Nutation and recommends that they shall be used in the national and international ephemerides for the year 1984 onwards, and in all other relevant astronomical work.

1. The new fundamental reference frame will be defined by the positions and proper motions of the stars given in the FK5 catalog. This catalog is currently in preparation by W. Fricke and associates at the Astronomisches Rechen-Institut.

Failure to distinguish between the catalog equinox of the FK4 (its zero point of right ascension on its equator) and the dynamical equinox (the crossing point of the ecliptic on the equator) has been the cause of much difficulty. The FK4 equinox was based on determinations of the dynamical equinox before 1930.

Recent determinations of the equinox corrections have to be taken into account in the determination of the system of the FK5 such that the equinox error will be removed as far as currently possible together with the removal of any erroneous motion of the equinox. The corrections to equinox motion and precession must be applied together to avoid introducing an additional fictitious rotation into the stellar proper motions in right ascension. Recent determinations of the equinox correction indicate that the FK4 equinox is in error by an amount $E = 0^{s}.035 + 0^{s}.085 \times (y - 1950) / 100$ where y is the year; i.e., if E is added to the coordinates of stars in the FK4 system, the resulting reference frame will closely approximate the dynamical (inertial) reference frame. The correction E will be implicitly included in the positions and proper motions of the stars in the FK5. (Refs. 12, 34, and 41)

The expression for Greenwich mean sidereal time has been discussed previously in the notes on time scales (see page 10).

2. The elliptic component in the Earth's velocity has traditionally been omitted in the computation of the day numbers, and the so-called E-terms of aberration have remained imbedded in the mean places of celestial objects. This practice has caused much confusion and it is convenient to use the occasion of other changes to remove E-terms from mean places and to include them in the reduction from mean to apparent place so that the apparent places will not be changed. The E-terms are discussed in references 9 and 31. The mean places in the FK5 will not contain E-terms, that is, they will be the positions of the stars at epoch J2000.0 as viewed from the barycenter of the Solar System, in the coordinate system defined by the Earth's mean equator and equinox of J2000.0.

Reductions from mean place (as given in catalogs) to apparent place of date can be most rigorously and efficiently performed using a vector and matrix formulation (see, for example, ref. 32). Formulas and tabulations to facilitate such reductions will be made available in the international ephemerides. For users who do not require the highest precision, Besselian day numbers will still be provided.

An effect not previously taken into account in mean to apparent place reductions, the deflection of light in the Sun's gravitational field, will now be routinely included. This effect, which is wavelength independent, amounts to 0.011 arcseconds 45° from the Sun and 0.004 arcseconds 90° from the Sun. (Refs. 36 and 26)

3. The theory of nutation currently in use, developed by Woolard, is based on a rigid model of the Earth. The "constant of nutation", which serves as a scaling factor for Woolard's nutation series, is an empirical value not consistent with other adopted astronomical constants. It has been known for several decades that the non-rigidity of the Earth has observationally significant effects on nutation. With the increasing accuracy of observations of various types, the continued use of the Woolard rigid-Earth nutation series would degrade the determinations of accurate star and radio source positions, UT1, and polar motion. Therefore, in 1977, the president of IAU Commission 4 (Ephemerides) established the Working Group on Nutation. The Working Group presented a report to the IAU General Assembly in August 1979 recommending the adoption of a nutation series developed by H. Kinoshita based on an Earth model by M. Molodensky. At the IAU General Assembly, a resolution was passed adopting this series as the 1979 IAU Theory of Nutation.

In December 1979, the International Union of Geodesy and Geophysics (IUGG) adopted a resolution requesting that the IAU reconsider its choice of a nutation series. The IUGG considered the Molodensky Earth model inadequate on geophysical grounds.

After considerable correspondence and discussion at IAU Colloquium #56 in September 1980 in Warsaw, Poland, the Working Group on Nutation decided to recommend a change to the 1980 IAU Theory of Nutation, which is based on the theory developed by J. Wahr and H. Kinoshita.

With the approval of the Secretary General of the IAU, the issue was submitted to all the members of the relevant IAU commissions; the resolution listed on page 11 was adopted by mail vote in early 1981.

The Summary of the Final Report of the Working Group on Nutation describes the new theory as follows:

(a) A non-rigid model of the Earth without axial symmetry is used.

(b) The constants are consistent with the IAU (1976) System of Astronomical Constants and are in agreement with available observational data of various types.

(c) The reference pole is selected so that there are no diurnal or quasi-diurnal motions of this pole with respect to either a space-fixed or Earth-fixed coordinate system. The phenomenon of dynamical variation of latitude, otherwise known as forced diurnal polar motion, is included implicity in the new nutation theory. The new nutation theory thus includes all externally-forced motions of the Earth's rotation axis; no geophysical (internally induced) or free motions are included. The new reference pole shall be referred to as the "Celestial Ephemeris Pole" (CEP).

The new nutation theory, incorporating the above changes, shall be referred to as the "1980 IAU Theory of Nutation". This nutation theory was developed by J. Wahr, based on previous work by H. Kinoshita and F. Gilbert and A. Dziewonski.

The Final Report of the Working Group contains, as its Table 1, the two 106-term series which comprise the new theory, along with discussions of their derivation, evaluation, and use. (Refs. 20, 38, 37, and 21)

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Appendices

Appendix A
Useful Formulas

In this Appendix:

TDT is Terrestrial Dynamical Time.

TDB is Barycentric Dynamical Time.

J_s is the TDB Julian date of the starting (or only) epoch.

J_e is the TDB Julian date of the ending epoch.

T is the interval, measured in Julian centuries of TDB, between J2000.0 and the starting (or only) epoch J_s.

t is the interval, measured in Julian centuries of TDB, between the starting epoch J_s and the ending epoch J_e.

$T = (J_s - 2451545.0) / 36525$

$t = (J_e - J_s) / 36525$

Formulas relating time scales:

In the following formulas:

UTC is Coordinated Universal Time (broadcast).

UT1 is universal time, affected by irregularities in the Earth's rate of rotation, derived from worldwide observations.

GMST is Greenwich mean sidereal time, the Greenwich hour angle of the mean (FK5) equinox of date.

TAI is International Atomic Time.

g is the mean anomaly of the Earth in its orbit.

T_U is the number of centuries of 36525 days of universal time elapsed since 2000 January 1, 12^h UT1 (JD 2451545.0 UT1).

ΔAT is the difference TAI − UTC.

ΔUT1 is the difference UT1 − UTC.

For current values of ΔAT and ΔUT1, see the *Astronomical Almanac*, p. B5, Circular D of the BIH, or the USNO Time Service Announcements Series 7.

$TAI = UTC + \Delta AT$

$TDT = TAI + 32\overset{s}{.}184$

$TDB \approx TDT + 0\overset{s}{.}001658 \sin(g + 0.0167 \sin g)$

$\quad + \text{ lunar and planetary terms of order } 10^{-5} \text{ sec}$

$\quad + \text{ daily terms of order } 10^{-6} \text{ sec}$

where $g = (357\overset{\circ}{.}528 + 35999\overset{\circ}{.}050\, T) \times 2\pi / 360°$

$UTC + \Delta UT1$

$$\text{GMST of } 0^h \text{ UT1} = 24110^s.54841 + 8640184^s.812866\, T_U + 0^s.093104\, T_U^2 - 6^s.2 \times 10^{-6}\, T_U^3$$

$$\text{GMST} = 67310^s.54841 + (876600^h + 8640184^s.812866)\, T_U + 0^s.093104\, T_U^2 - 6^s.2 \times 10^{-6}\, T_U^3$$

Given two events, e_1 and e_2,

$$r = \frac{\text{measure of UT1 between } e_1 \text{ and } e_2}{\text{measure of GMST between } e_1 \text{ and } e_2} = 0.997269566329084 - 5.8684 \times 10^{-11}\, T_U + 5.9 \times 10^{-15}\, T_U^2$$

$$\frac{1}{r} = \frac{\text{measure of GMST between } e_1 \text{ and } e_2}{\text{measure of UT1 between } e_1 \text{ and } e_2} = 1.002737909350795 + 5.9006 \times 10^{-11}\, T_U - 5.9 \times 10^{-15}\, T_U^2$$

(Refs. 27 and 25)

Formulas for precession:

In the following formulas:

ζ_A, z_A, θ_A are the equatorial precession parameters, the three angles which determine the rotations required to transform the equatorial reference system from one epoch to another. These angles correspond to the angles ζ_0, z, θ, respectively, in Newcomb's notation.

P_N is the accumulated general precession in longitude. Specifically, it is the arc of the fixed ecliptic westward from the fixed equinox to the point where the fixed ecliptic is intersected by the great circle through the mean equinox of date and the pole of the fixed ecliptic.

m is the rate of general precession in right ascension.

n is the rate of general precession in declination; the rate of change of the inclination of the equatorial plane

Equatorial precession parameters, for precession between J_s and J_e:

$$\zeta_A = (2306''.2181 + 1''.39656T - 0''.000139T^2)t + (0''.30188 - 0''.000344T)t^2 + 0''.017998t^3$$

$$z_A = (2306''.2181 + 1''.39656T - 0''.000139T^2)t + (1''.09468 + 0''.000066T)t^2 + 0''.018203t^3$$

$$\theta_A = (2004''.3109 - 0''.85330T - 0''.000217T^2)t - (0''.42665 + 0''.000217T)t^2 - 0''.041833t^3$$

Rates of precession, per Julian century:

$m = 4612''.4362 + 2''.79312\,T - 0''.000278\,T^2$

$n = 2004''.3109 - 0''.85330\,T - 0''.000217\,T^2$

General precession in longitude between J_s and J_e, referred to the fixed ecliptic and equinox of J_s:

$$P_N = (5029''.0966 + 2''.22226T - 0''.000042T^2)t + (1''.11161 - 0''.000127T)t^2 - 0''.000113t^3$$

(Refs. 23 and 22)

Formulas for nutation:

In the following formulas:

$\Delta\Psi$ is the nutation in longitude at J_s.
$\Delta\epsilon$ is the nutation in obliquity at J_s.
ϵ, ϵ' are the mean and true obliquity of the ecliptic, respectively, at J_s.
l is the mean anomaly of the Moon.
l' is the mean anomaly of the Sun (Earth).
Ω is the longitude of the ascending node of the Moon's mean orbit on the ecliptic, measured from the mean equinox of date.
D is the mean elongation of the Moon from the Sun.
F is the difference $L-\Omega$, where L is the mean longitude of the Moon.

$\epsilon = 84381''.448 - 46''.8150\,T - 0''.00059\,T^2 + 0''.001813\,T^3$

$\epsilon' = \epsilon + \Delta\epsilon$

Fundamental arguments at J_s:

$l = 485866''.733 + (1325^r + 715922''.633)T + 31''.310T^2 + 0''.064T^3$

$l' = 1287099''.804 + (99^r + 1292581''.224)T - 0''.577T^2 - 0''.012T^3$

$F = 335778''.877 + (1342^r + 295263''.137)T - 13''.257T^2 + 0''.011T^3$

$D = 1072261''.307 + (1236^r + 1105601''.328)T - 6''.891T^2 + 0''.019T^3$

$\Omega = 450160''.280 - (5^r + 482890''.539)T + 7''.455T^2 + 0''.008T^3$

where $1^r = 360° = 1296000''$

Table 1
1980 IAU Theory of Nutation

Series for nutation in longitude $\Delta\Psi$ and obliquity $\Delta\epsilon$, referred to the mean equator and equinox of date, with T measured in Julian centuries from epoch J2000.0.

	ARGUMENT l l' F D Ω	PERIOD (DAYS)	LONGITUDE (.0001″)		OBLIQUITY (.0001″)	
1	0 0 0 0 1	6798.4	−171996	−174.2T	92025	8.9T
2	0 0 0 0 2	3399.2	2062	.2T	−895	.5T
3	-2 0 2 0 1	1305.5	46	0.0T	−24	0.0T
4	2 0 -2 0 0	1095.2	11	0.0T	0	0.0T
5	-2 0 2 0 2	1615.7	−3	0.0T	1	0.0T
6	1 -1 0 -1 0	3232.9	−3	0.0T	0	0.0T
7	0 -2 2 -2 1	6786.3	−2	0.0T	1	0.0T
8	2 0 -2 0 1	943.2	1	0.0T	0	0.0T
9	0 0 2 -2 2	182.6	−13187	−1.6T	5736	−3.1T
10	0 1 0 0 0	365.3	1426	−3.4T	54	−.1T
11	0 1 2 -2 2	121.7	− 517	1.2T	224	−.6T
12	0 -1 2 -2 2	365.2	217	−.5T	−95	.3T
13	0 0 2 -2 1	177.8	129	.1T	−70	0.0T
14	2 0 0 -2 0	205.9	48	0.0T	1	0.0T
15	0 0 2 -2 0	173.3	−22	0.0T	0	0.0T
16	0 2 0 0 0	182.6	17	−.1T	0	0.0T
17	0 1 0 0 1	386.0	−15	0.0T	9	0.0T
18	0 2 2 -2 2	91.3	−16	0.1T	7	0.0T
19	0 -1 0 0 1	346.6	−12	0.0T	6	0.0T
20	-2 0 0 2 1	199.8	−6	0.0T	3	0.0T
21	0 -1 2 -2 1	346.6	−5	0.0T	3	0.0T
22	2 0 0 -2 1	212.3	4	0.0T	−2	0.0T
23	0 1 2 -2 1	119.6	4	0.0T	−2	0.0T
24	1 0 0 -1 0	411.8	−4	0.0T	0	0.0T
25	2 1 0 -2 0	131.7	1	0.0T	0	0.0T
26	0 0 -2 2 1	169.0	1	0.0T	0	0.0T
27	0 1 -2 2 0	329.8	−1	0.0T	0	0.0T
28	0 1 0 0 2	409.2	1	0.0T	0	0.0T
29	-1 0 0 1 1	388.3	1	0.0T	0	0.0T
30	0 1 2 -2 0	117.5	−1	0.0T	0	0.0T
31	0 0 2 0 2	13.7	−2274	−.2T	977	−.5T
32	1 0 0 0 0	27.6	712	.1T	−7	0.0T
33	0 0 2 0 1	13.6	−386	−.4T	200	0.0T
34	1 0 2 0 2	9.1	−301	0.0T	129	−.1T
35	1 0 0 -2 0	31.8	−158	0.0T	−1	0.0T
36	-1 0 2 0 2	27.1	123	0.0T	−53	0.0T
37	0 0 0 2 0	14.8	63	0.0T	−2	0.0T

(continued)

	ARGUMENT l l' F D Ω	PERIOD (DAYS)	LONGITUDE (.0001″)		OBLIQUITY (.0001″)	
38	1 0 0 0 1	27.7	63	.1T	−33	0.0T
39	-1 0 0 0 1	27.4	−58	−.1T	32	0.0T
40	-1 0 2 2 2	9.6	−59	0.0T	26	0.0T
41	1 0 2 0 1	9.1	−51	0.0T	27	0.0T
42	0 0 2 2 2	7.1	−38	0.0T	16	0.0T
43	2 0 0 0 0	13.8	29	0.0T	−1	0.0T
44	1 0 2 -2 2	23.9	29	0.0T	−12	0.0T
45	2 0 2 0 2	6.9	−31	0.0T	13	0.0T
46	0 0 2 0 0	13.6	26	0.0T	−1	0.0T
47	-1 0 2 0 1	27.0	21	0.0T	−10	0.0T
48	-1 0 0 2 1	32.0	16	0.0T	−8	0.0T
49	1 0 0 -2 1	31.7	−13	0.0T	7	0.0T
50	-1 0 2 2 1	9.5	−10	0.0T	5	0.0T
51	1 1 0 -2 0	34.8	−7	0.0T	0	0.0T
52	0 1 2 0 2	13.2	7	0.0T	−3	0.0T
53	0 -1 2 0 2	14.2	−7	0.0T	3	0.0T
54	1 0 2 2 2	5.6	−8	0.0T	3	0.0T
55	1 0 0 2 0	9.6	6	0.0T	0	0.0T
56	2 0 2 -2 2	12.8	6	0.0T	−3	0.0T
57	0 0 0 2 1	14.8	−6	0.0T	3	0.0T
58	0 0 2 2 1	7.1	−7	0.0T	3	0.0T
59	1 0 2 -2 1	23.9	6	0.0T	−3	0.0T
60	0 0 0 -2 1	14.7	−5	0.0T	3	0.0T
61	1 -1 0 0 0	29.8	5	0.0T	0	0.0T
62	2 0 2 0 1	6.9	−5	0.0T	3	0.0T
63	0 1 0 -2 0	15.4	−4	0.0T	0	0.0T
64	1 0 -2 0 0	26.9	4	0.0T	0	0.0T
65	0 0 0 1 0	29.5	−4	0.0T	0	0.0T
66	1 1 0 0 0	25.6	−3	0.0T	0	0.0T
67	1 0 2 0 0	9.1	3	0.0T	0	0.0T
68	1 -1 2 0 2	9.4	−3	0.0T	1	0.0T
69	-1 -1 2 2 2	9.8	−3	0.0T	1	0.0T
70	-2 0 0 0 1	13.7	−2	0.0T	1	0.0T
71	3 0 2 0 2	5.5	−3	0.0T	1	0.0T
72	0 -1 2 2 2	7.2	−3	0.0T	1	0.0T
73	1 1 2 0 2	8.9	2	0.0T	−1	0.0T
74	-1 0 2 -2 1	32.6	−2	0.0T	1	0.0T

(continued)

TABLE 1 (continued)

	ARGUMENT l l' F D Ω	PERIOD (DAYS)	LONGITUDE (.0001″)		OBLIQUITY (.0001″)	
75	2 0 0 0 1	13.8	2	0.0T	—1	0.0T
76	1 0 0 0 2	27.8	—2	0.0T	1	0.0T
77	3 0 0 0 0	9.2	2	0.0T	0	0.0T
78	0 0 2 1 2	9.3	2	0.0T	—1	0.0T
79	-1 0 0 0 2	27.3	1	0.0T	—1	0.0T
80	1 0 0 -4 0	10.1	—1	0.0T	0	0.0T
81	-2 0 2 2 2	14.6	1	0.0T	—1	0.0T
82	-1 0 2 4 2	5.8	—2	0.0T	1	0.0T
83	2 0 0 -4 0	15.9	—1	0.0T	0	0.0T
84	1 1 2 -2 2	22.5	1	0.0T	—1	0.0T
85	1 0 2 2 1	5.6	—1	0.0T	1	0.0T
86	-2 0 2 4 2	7.3	—1	0.0T	1	0.0T
87	-1 0 4 0 2	9.1	1	0.0T	0	0.0T
88	1 -1 0 -2 0	29.3	1	0.0T	0	0.0T
89	2 0 2 -2 1	12.8	1	0.0T	—1	0.0T
90	2 0 2 2 2	4.7	—1	0.0T	0	0.0T
91	1 0 0 2 1	9.6	—1	0.0T	0	0.0T
92	0 0 4 -2 2	12.7	1	0.0T	0	0.0T
93	3 0 2 -2 2	8.7	1	0.0T	0	0.0T
94	1 0 2 -2 0	23.8	—1	0.0T	0	0.0T
95	0 1 2 0 1	13.1	1	0.0T	0	0.0T
96	-1 -1 0 2 1	35.0	1	0.0T	0	0.0T
97	0 0 -2 0 1	13.6	—1	0.0T	0	0.0T
98	0 0 2 -1 2	25.4	—1	0.0T	0	0.0T
99	0 1 0 2 0	14.2	—1	0.0T	0	0.0T
100	1 0 -2 -2 0	9.5	—1	0.0T	0	0.0T
101	0 -1 2 0 1	14.2	—1	0.0T	0	0.0T
102	1 1 0 -2 1	34.7	—1	0.0T	0	0.0T
103	1 0 -2 2 0	32.8	—1	0.0T	0	0.0T
104	2 0 0 2 0	7.1	1	0.0T	0	0.0T
105	0 0 2 4 2	4.8	—1	0.0T	0	0.0T
106	0 1 0 1 0	27.3	1	0.0T	0	0.0T

$$\epsilon_{J2000} = 23^\circ 26' 21''.448$$

$$\sin \epsilon_{J2000} = .39777716$$

(Refs. 20 and 38)

Formulas for the gravitational deflection of light:

In the following formulas:

D	is the geocentric angular separation of the source from the Sun (D = 0° to 180°)
θ	is the apparent angle of deflection, always positive; the apparent deflection is radially outward from the Sun
α, δ	are the right ascension and declination, respectively, of the source
$\alpha_\odot$, $\delta_\odot$	are the right ascension and declination, respectively of the Sun
$\Delta\alpha$, $\Delta\delta$	are the increments to be added to α and δ, respectively, to obtain the apparent deflected coordinates of the source

$$\cos D = \sin\delta \sin\delta_\odot + \cos\delta \cos\delta_\odot \cos(\alpha - \alpha_\odot)$$

$$\sin D = \sqrt{1 - \cos^2 D}$$

$$\sin 2D = 2 \sin D \cos D$$

$$\theta = 0''.00407 \left(\frac{1 + \cos D}{\sin D} + \frac{1}{4} \sin 2D\right)$$

$$\text{Define } \mu \equiv \frac{\theta}{\sin D} = 0''.00407 \left(\frac{1}{1 - \cos D} + \frac{1}{2} \cos D\right)$$

$$\Delta\alpha = \mu \sec\delta \cos\delta_\odot \sin(\alpha - \alpha_\odot)$$

$$\Delta\delta = \mu \left[\sin\delta \cos\delta_\odot \cos(\alpha - \alpha_\odot) - \sin\delta_\odot \cos\delta\right]$$

The above formulas apply to sources outside of the Solar System. They may be used, with some small error, for the outer planets, but a more complex algorithm is needed for Mercury, Venus, the Moon, and Earth-crossing asteroids and comets.

(Ref. 36)

Miscellaneous formulas

Equinox correction for mean or apparent right ascensions of stars from the FK4 or FK4-based star catalogs, in the sense E = FK5 − FK4:

$$E = 0^{s}.035 + 0^{s}.085\,(y - 1950) / 100 \qquad \text{where y is the year}$$

$$= 0^{s}.0775 + 0^{s}.850\,T$$

E is to be added to the FK4 right ascensions at epoch y (T centuries from J2000.0) to refer them to the FK5 equinox.

(Ref. 41)

Mean obliquity of the ecliptic:

$$\epsilon = 23^{\circ}26'21''.448 - 46''.8150\,T - 0''.00059\,T^2 + 0''.001813\,T^3$$

(Ref. 23)

E-terms of aberration, to be added to existing 1950.0 catalog mean places of stars to obtain true mean places:

$$\Delta\alpha = 0^{s}.0227 \sin(\alpha + 11^{h}.25) \sec\delta$$

$$\Delta\delta = 0''.341 \cos(\alpha + 11^{h}.25) \sin\delta + 0''.029 \cos\delta$$

where α and δ are the existing catalog right ascension and declination, and $\alpha + \Delta\alpha$ and $\delta + \Delta\delta$ are the true mean right ascension and declination.

(Refs. 9 and 43)

Appendix B

Conversion of Existing Reference Data

This appendix describes the procedures for the conversion of several common types of existing astrometric reference data to the new standard epoch, J2000.0, so that the data may be used in accordance with the new IAU resolutions. It must be recognized that all such procedures are, to some extent, approximations. Rigorous conversion of reference data would require re-reduction of original observations. Even if re-reduction of observations were feasible, a rigorous conversion might not result, since most observations are differential measurements with respect to unconverted reference standards. Futhermore, since the FK5 catalog is not yet available, a complete and correct conversion to its system is not yet possible. The current astronomical constants and reference system are embedded in complex and subtle ways in all existing astrometric data. For work of the highest precision, the change of constants and reference system requires an unfortunate, but unavoidable, break with the past.

In the following, the symbols α and δ refer to the right ascension and declination, respectively, of a celestial object, and μ_α and μ_δ refer to its proper motion in right ascension and declination, respectively (if applicable). Subscript 1 refers to coordinates with respect to the mean equator and equinox of 1950.0 (old system), and subscript 2 refers to coordinates with respect to the mean equator and equinox of J2000.0 (new system). Primed or double-primed symbols refer to temporary quantities, i. e. intermediate results, in the computations.

Conversion of epoch 1950.0 FK4-based star position catalogs:

1. Compute the mean place of each star at JD 2451545.0 (1.5 January 2000, Besselian epoch 2000.001278) using the 1950 mean places and proper motions listed in the catalog, and the old (Newcomb) precession formulas. That is, compute α'_2 and δ'_2 from α_1, δ_1, $\mu_{\alpha 1}$, $\mu_{\delta 1}$, and the old precession formulas. Or, if provided in the catalog, the centennial variations for each star may be used in place of separate proper motion and precession computations.

2. For each star, transform the 1950 proper motions, $\mu_{\alpha 1}$ and $\mu_{\delta 1}$, to 2000 proper motions, $\mu'_{\alpha 2}$ and $\mu'_{\delta 2}$, as follows.

(a) Using the results from step 1, compute the average of each star's 1950 and 2000 mean places; the result will be, approximately, the star's mean place for an epoch near the beginning of 1975. That is, compute

$$\bar{\alpha} = (\alpha_1 + \alpha'_2 \ / 2 \quad \text{and} \quad \bar{\delta} = \ \delta_1 + \delta'_2) / 2$$

(b) Convert the proper motions using the following standard formulas, valid for the 50.001278 tropical year interval between 1950 and 2000:

$$\begin{aligned}
\mu'_{\alpha 2} = \mu_{\alpha 1} &+ 0.0048581\ (\mu_{\alpha 1} \cos\bar{\alpha} \tan\bar{\delta} + \mu_{\delta 1} \sin\bar{\alpha} \sec^2\bar{\delta}) \\
&+ 1.000026\ \ \mu_{\alpha 1}\mu_{\delta 1} \tan\bar{\delta} \\
&- 0.0001024\ \ \mu_{\alpha 1} V \pi
\end{aligned}$$

$$\begin{aligned}
\mu'_{\delta 2} \quad \mu_{\delta 1} &- 0.0048581\ (\mu_{\alpha 1} \sin\bar{\alpha}) \\
&- 0.500013\ \ \mu_{\alpha 1}^2 \sin\bar{\delta} \cos\bar{\delta} \\
&- 0.0001024\ \ \mu_{\delta 1} V \pi
\end{aligned}$$

In the above, the proper motions are expressed in radians per tropical century. The quantities V and π are the star's radial velocity in km/sec and parallax in arcseconds, respectively. If these quantities are unknown, use values of zero; the last terms in the above formulas, the so-called foreshortening terms, are significant for only a few nearby high proper motion stars.

(c) Convert the proper motions from tropical time units to Julian time units by multiplying the proper motions, expressed in angular measure per tropical century, by 1.00002136. Proper motions are now expressed in angular measure per Julian century.

3. Correct the 2000 mean places and proper motions for epoch 2000 distortions in the FK4 system (FK5—FK4 systematic differences), to the extent known.

4. Eliminate the E-terms of aberration from each star's 2000 mean place, by adding the following increments to the 2000 mean right ascension and declination:

$$\Delta\alpha = 0\overset{s}{.}0227 \sin(\alpha'_2 + 11\overset{h}{.}25) \sec\delta'_2$$

$$\Delta\delta = 0\overset{''}{.}341 \cos(\alpha'_2 + 11\overset{h}{.}25) \sin\delta'_2 + 0\overset{''}{.}029 \cos\delta'_2$$

5. Correct the 2000 mean right ascensions for the FK4 equinox error at epoch 2000: add $1\overset{''}{.}1625 = 0\overset{s}{.}0775$ to all right ascensions. There is no equator (declination) correction.

6. Correct the 2000 proper motions, μ''_{α_2} and μ''_{δ_2}, resulting from steps 2 and 3, for the FK4 equinox motion and the change in the precession constant:

$$\mu_{\alpha_2} = \mu''_{\alpha_2} - \Delta m - \Delta n \sin \alpha_2 \tan \delta_2 + \Delta e$$
$$\mu_{\delta_2} = \mu''_{\delta_2} - \Delta n \cos \alpha_2$$

where α_2 and δ_2 are the 2000 mean right ascension and declination, respectively, resulting from steps 1,3,4, and 5. The constants to be used are:

$\Delta m = 1''.037 = 0^s.6912$ per Julian century
$\Delta n = 0''.436 = 0^s.0291$ per Julian century
$\Delta e = 1''.275 = 0^s.0850$ per Julian century

The mean places and proper motions of the stars in the catalog have now been converted to equator, equinox, and epoch J2000.0, in such a way that the catalog can now be used in accordance with the new IAU resolutions. For example, in apparent place computations, the new expressions for precession and nutation should be used, and the new expression for sidereal time should be used when computing hour angles or transit times.

When the FK5 catalog becomes available, it will contain information on the differences between its system and that of the FK4. Until the FK5 is published, the above procedure can be used to obtain an approximation to the FK5 system.

Conversion of epoch 1950.0 radio source position catalogs:

1. Determine the (weighted) mean epoch of observations contributing to the catalog. If the observations of the various sources were made at widely different times, compute the mean epoch of observations for each source individually.

2. Compute the mean place of each source at its mean epoch of observations, as determined in step 1, using the 1950 mean places listed in the catalog, and the old (Newcomb) precession formulas (assuming the latter were used in the construction of the catalog). That is, compute α'_T and δ'_T from α_1, δ_1, and the old precession formulas, where the subscript T refers to the mean epoch of observations.

3. For each source, add the following correction to its right ascension, α'_T, at the mean epoch of observations:

$$\Delta\alpha = 0^s.0775 + 0^s.0851\,T + 0^s.0002\,T^2$$

where T represents the mean epoch of observations, expressed in Julian centuries from JD 2451545.0 (1.5 Jan 2000). Note that this correction will generally be of order $+0^s.06$ for sources in most modern astrometric radio source catalogs. This adjustment corrects for the difference in sidereal time expressions in the old and new systems, and guarantees that a given source will have the same hour angle at a given UT1 time in the old and new systems (aside from the small periodic differences due to the use of different nutation theories). This adjustment is analogous to the equinox correction for star catalogs.

4. Compute the mean place of each source at JD 2451545.0 (1.5 Jan 2000) using the mean place at the mean epoch of observations, resulting from steps 2 and 3, and the new precession formulas. That is, use the new precession expressions to precess the source positions from the mean epoch of observations to epoch 2000.

5. If the E-terms of aberration were embedded in the original catalog's 1950 mean places, eliminate the E-terms from each source's 2000 mean place, by adding the following increments to the 2000 mean right ascension and declination:

$$\Delta\alpha = 0\overset{s}{.}0227 \sin(\alpha_2' + 11\overset{h}{.}25) \sec \delta_2'$$

$$\Delta\delta = 0\overset{''}{.}341 \cos(\alpha_2' + 11\overset{h}{.}25) \sin \delta_2' + 0\overset{''}{.}029 \cos \delta_2'$$

where α_2' and δ_2' are the 2000 mean right ascension and declination, respectively, resulting from steps 2, 3, and 4. The final coordinates are therefore $\alpha_2 = \alpha_2' + \Delta\alpha$ and $\delta_2 = \delta_2' + \Delta\delta$.

The mean places of the sources in the catalog have now been converted to the equator and equinox of J2000.0, in such a way that the catalog can now be used in accordance with the new IAU resolutions. For example, in the computation of observables, the new expressions for precession, nutation, and sidereal time should be used.

This conversion procedure can also be used for catalogs of star positions derived from limited observational programs, where proper motions are not available.

Conversion of heliocentric equatorial planetary coordinates:

Let x, y, and z represent the heliocentric equatorial rectangular coordinates of a planet (typically expressed in astronomical units). The following approximate conversion can be used to obtain coordinates with respect to the mean equator and equinox of 2000, given coordinates with respect to the mean equator and equinox of 1950:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_2 = \begin{pmatrix} 0.9999257080 & -0.0111789381 & -0.0048590038 \\ 0.0111789381 & 0.9999375133 & -0.0000271626 \\ 0.0048590038 & -0.0000271579 & 0.9999881946 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_1$$

It must be emphasized that this is a very approximate procedure. Because of the complexity of the computations involved in the construction of planetary ephemerides, no straightforward procedure exists for properly converting ephemerides based on one set of constants to those based on another set. Errors of at least one arcsecond in the heliocentric planetary positions may be expected if the above conversion is used.

Heliocentric planetary and lunar ephemerides on the new system have been computed and are available from the U. S. Naval Observatory and the Jet Propulsion Laboratory.

Conversion of geocentric apparent coordinates:

Geocentric apparent coordinates of stars or planets, measured with respect to the equator and (FK4-based) equinox of date, may be placed on the new system by simply adding the equinox correction to the right ascension. That is, add the following increment to the right ascension of date:

$$\Delta a = 0\overset{s}{.}0775 + 0\overset{s}{.}0850\ T$$

where T represents the date, expressed in Julian centuries from J2000.0. Strictly, this is correct only for coordinates measured with respect to the *mean* equator and equinox of date, since the small periodic differences due to the change of nutation theories are neglected. The latter are of order $0\overset{''}{.}05$ and may be important for very precise work.

New Planetary and Lunar Ephemerides

New planetary and lunar ephemerides have been computed based on the IAU (1976) System of Astronomical Constants and conforming to the applicable IAU resolutions presented in this Circular. The new ephemerides are the result of a large number of numerical integrations of the motions of the major planets and the Moon performed over the past several years at the Jet Propulsion Laboratory, the U. S. Naval Observatory, and the Naval Surface Weapons Center. A considerable amount of effort has gone into achieving the optimum fit to various types of observational data currently available: optical, radar, laser, and spacecraft. Different integration procedures were intercompared in an attempt to remove the possibility of errors in the algorithms and numerical procedures. The final ephemeris, designated DE200/LE200, was produced by an N-body numerical integration program developed at the Jet Propulsion Laboratory. The integration, which covers the years 1800 to 2050, was computed in a rectangular coordinate system aligned with the Earth equator and equinox of J2000.0. This ephemeris will form the basis of the *Astronomical Almanac* beginning with the 1984 edition.

The computed orbits of the inner planets and the Moon are tightly constrained by high-accuracy radar or laser ranging observations and spacecraft data. Farther out the observational constraints weaken and a number of uncertainties arise. For the planets beyond Saturn, only classical optical observations are available and the masses involved are not accurately known. For Neptune and Pluto, less than one full orbit has been observed. There are still long-period systematic trends in the observational residuals from the newly computed orbits of Uranus and Neptune. The origin of these systematic effects is at present unknown and will remain the subject of continued study.

New ephemerides of the largest minor planets and the satellites of the major planets are currently under development in a joint effort by the Naval Observatory, the Naval Surface Weapons Center, and the Jet Propulsion Laboratory.

At the 1979 meeting of the IAU, the Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites was adopted (see ref. 42). This report provides the definitions of planetocentric and planetographic coordinate systems for the major planets and satellites and gives expressions for the orientation of their poles and prime meridians as functions of time. The recommendations in this report will be incorporated into the physical ephemerides of the planets beginning with the 1984 edition of the *Astronomical Almanac.*

The following description of the DE200 / LE200 ephemeris has been provided by E. M. Standish. More extensive documentation will be forthcoming in the astronomical literature.

The new planetary and lunar ephemeris which will form the basis of the *Astronomical Almanac* starting in 1984 is Development Ephemeris Number DE200 / LE200. This ephemeris provides a dynamically consistent system whose reference frame has been accurately adjusted to the dynamical equinox of J2000.0. Furthermore, the relative positions and velocities of the four inner planets and the Moon are well-determined and their mean motions are accurately represented with respect to inertial apace. These statements are now discussed.

The ephemeris represents the results of numerical integrations of the equations of motion governing the motions of the major bodies in the Solar System. Two assumptions are made: (1) the equations of motion accurately represent our understanding of the physical laws of nature, at least to the presently observable accuracy; and (2) the numerical integration program is sufficiently accurate. These two assumptions alone provide a dynamically consistent system -- one which is actually physically possible in inertial space. Of course, it may not be an accurate representation of our own Solar System. This depends only on the choice of initial conditions and the accuracy of other relevant parameters which enter into the equations of motion, e. g. the planetary masses. The accuracy of the initial conditions and parameters, in turn, are a direct function of the fit to the observational data.

Very accurate mass values for the planets now exist out through Saturn. Also, accurate range measurements between the Earth and another planet, if appropriately distributed in time, determine the shapes and relative orientations of the two orbits in question. It may be shown that range measurements are sufficient by themselves to determine the mean motions of the Earth and the planet with respect to inertial space. For example, Earth-Mars spacecraft ranging from Mariner 9 and the Viking Landers yield inertial mean motions to an accuracy of $\pm 0''.02$ / century.

Lunar laser ranging data determine the orbit of the Moon. Furthermore, since these data are sensitive to both the Earth's instantaneous equator and the Earth's instantaneous orbital plane (the ecliptic), the obliquity of the ecliptic is well-determined (to an accuracy of $\pm 0''.01$) over the time span of these data (mean epoch of 1975).

With such an accurately determined system for the inner Solar System (positions, velocities, inertial mean motions, masses, and obliquity) it has been possible to locate the actual dynamical equinox of DE200 / LE200 at the origin of the reference system. This was done through a detailed analysis of the orbital motion of the Earth-Moon barycenter and a subsequent adjustment of the coordinate axis. One may note that the reference frame of the ephemerides will not strictly coincide with the origin of the FK5 catalog system. The difference will be the amount by which the origin of the FK5 itself fails to coincide with the dynamical equinox of J2000.0. In actuality, the adjustment angle used in DE200 / LE200 of $0''.531$ is surprisingly close to that of $0''.525$ which will be used for the FK5 (see ref. 41).

Associated with the ephemeris is the set of astronomical constants used in its creation. Some of the values do not agree exactly with those of the IAU (1976) System of Astronomical Constants. The differences have been necessary in order to achieve a "best fit" in the least squares sense to the observational data. In particular, the following constants used for DE200 / LE200 *differ* from the corresponding IAU values:

Constant	IAU (1976) Value	DE200 / LE200 Value
Light-time for unit distance τ_A	499.004782	499.0047837 . . .
Geocentric gravitational constant *GE*	3.986005×10^{14}	$3.98600448 \ldots \times 10^{14}$
Heliocentric gravitational constant *GS*	$1.32712438 \times 10^{20}$	$1.32712440 \ldots \times 10^{20}$
Ratio of mass of Sun to that of Earth (*GS*) / (*GE*)	332946.0	332946.038 . . .
Ratio of mass of Moon to that of Earth μ	0.01230002	0.012300034
Obliquity of the ecliptic at J2000.0 ϵ	23° 26′ 21.″448	23°26′21.″4119 . . .
Unit distance *A*	$1.49597870 \times 10^{11}$	$1.4959787066 \times 10^{11}$
Ratio of mass of Sun to that of Earth + Moon	328900.5	328900.55
Planetary reciprocal masses:		
Jupiter	1047.355	1047.350
Saturn	3498.5	3498.0
Uranus	22869	22960
Pluto	3×10^6	1.3×10^8
Pallas	9.09×10^9	9.247×10^9
Vesta	8.33×10^9	7.253×10^9

Copies of the DE200 / LE200 ephemeris, on magnetic computer tape, are available from:

E. Myles Standish
JPL 264-664
Jet Propulsion Laboratory
Pasadena, CA 91109

Copies are also available from:

Nautical Almanac Office
U. S. Naval Observatory
Washington, D. C. 20390

When requesting a copy of the ephemeris, please send a good-quality blank tape and specify the density and character set desired, and the type of computer which will be used to read the tape.