

REPORT DOCUMENTATION PAGE			Form Approved OMB NO. 0704-0188		
The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.					
1. REPORT DATE (DD-MM-YYYY) 15-06-2015		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 15-May-2014 - 14-Feb-2015	
4. TITLE AND SUBTITLE Final Report: Stable Intrinsic Localized Modes in Microelectromechanical Cantilever Structures			5a. CONTRACT NUMBER W911NF-14-1-0202		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER 611102		
6. AUTHORS Surajit Sen			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES State University of New York (SUNY) at B The UB Commons 520 Lee Entrance, Suite 211 Amherst, NY 14228 -2567			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSOR/MONITOR'S ACRONYM(S) ARO		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) 65336-MS-II.10		
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for Public Release; Distribution Unlimited					
13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.					
14. ABSTRACT Protection of humans from large, finite time perturbations is an important area of research. Hence it is of interest to develop systems that are thin, light-weight and capable of absorbing large accelerations. An area of special interest concerns highly nonlinear systems which possess a rich variety of dynamical responses. This study focused on whether it is possible to convert any incident perturbation into localized excitations. The study explored the dynamics of a weakly nonlinear system with quadratic and quartic on-site and inter-site potentials where the linear pieces are significantly stronger than the nonlinear pieces that has been introduced by Sievers and coworkers. The					
15. SUBJECT TERMS Breathers, intrinsic localized modes, Sievers system, Fermi-Pasta-Ulam system					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Surajit Sen
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 716-645-2007

Report Title

Final Report: Stable Intrinsic Localized Modes in Microelectromechanical Cantilever Structures

ABSTRACT

Protection of humans from large, finite time perturbations is an important area of research. Hence it is of interest to develop systems that are thin, light-weight and capable of absorbing large accelerations. An area of special interest concerns highly nonlinear systems which possess a rich variety of dynamical responses. This study focused on whether it is possible to convert any incident perturbation into localized excitations. The study explored the dynamics of a weakly nonlinear system with quadratic and quartic on-site and inter-site potentials where the linear pieces are significantly stronger than the nonlinear pieces that has been introduced by Sievers and coworkers. The system is realized by a micromechanical cantilever structure. We showed that the Sievers system can trap excitations that are in a specific amplitude and frequency window. We have studied whether breathers form from noisy perturbations in the Fermi-Pasta-Ulam system which consists of masses interacting via a quadratic and a quartic potentials. Turns out that only weak breathers form from such perturbations. Our work suggests the emergence of highly localized excitations when synchronized or nearly synchronized perturbations from two ends meet in granular chains with soft centers. Similar physics is also seen in preliminary studies on Fermi-Pasta-Ulam chains.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

<u>Received</u>	<u>Paper</u>
06/10/2015	7 Ding Han, Matthew Westley, Surajit Sen. Mechanical energy fluctuations in granular chains: The possibility of rogue fluctuations or waves, PHYSICAL REVIEW E, (09 2014): 32904. doi:
06/10/2015	8 Yannan Shen, Panos G Kevrekidis, Surajit Sen, Aaron Hoffman. Characterizing traveling-wave collisions in granular chains starting from integrable limits: The case of the Korteweg-de Vries equation and the Toda lattice, PHYSICAL REVIEW E, (08 2014): 22905. doi:
06/10/2015	9 Mukesh Tiwari, T R Krishna Mohan, Surajit Sen. Drag-force regimes in granular impact, PHYSICAL REVIEW E, (12 2014): 62202. doi:
TOTAL:	3

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL: 4

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Yoichi Takato and Surajit Sen, Nanoparticle collisions, PASI Conference, La Plata, Argentina, August 2014 (Takato was awarded a NSF Fellowship to present this work at La Plata).

Number of Presentations: 1.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts	
<u>Received</u>	<u>Paper</u>
TOTAL:	
Number of Manuscripts:	

Books	
<u>Received</u>	<u>Book</u>
TOTAL:	
<u>Received</u>	<u>Book Chapter</u>
TOTAL:	

Patents Submitted
Patents Awarded
Awards

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Yoichi Takato	0.47	
Matthew Westley	0.20	
Rahul Kashyap	0.28	
FTE Equivalent:	0.95	
Total Number:	3	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Surajit Sen	0.25	
FTE Equivalent:	0.25	
Total Number:	1	

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Michael E Benson	0.00	BS (Mathematical Physics)
FTE Equivalent:	0.00	
Total Number:	1	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:..... 0.00

Names of Personnel receiving masters degrees

<u>NAME</u>
Total Number:

Names of personnel receiving PhDs

<u>NAME</u> William J Falls Yoichi Takato Total Number:	2
---	----------

Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Outcomes

William J Falls and Yoichi Takato defended their PhD theses during the period of the STIR Grant. Dr Falls is now an assistant professor at Erie Community College of the SUNY and Dr Takato is a Research Associate at the Okinawa Institute of Science and Technology in Okinawa, Japan. The papers published and under consideration during the grant period reflect a part of what was accomplished. The complete summary of the actual project work has been captured in this document. Published work included analyses of solitary waves and breathers in Fermi-Pasta-Ulam systems, of how solitary waves get temporarily localized near soft walls in Hertz systems, of possible interactions between nanoscale grains, of the nature of cratering in impacts on 3D beds with the most recent manuscript (under preparation) being on a pilot study of how granular chains with soft centers may help localize any incident energy. We have also carried out studies on how to localize energy in Fermi-Pasta-Ulam systems and a detailed paper is being completed now. The work has been geared to not just solve the problem at hand but do so in an encompassing way by exploring issues such as miniaturization, fundamentals such as the relationships between solitary waves and breathers and the formal theoretical underpinnings that tie the two, and even on how impacts damage a surface.

In the next stage, we are looking forward to solving this localization problem and that is precisely the focus of the newly submitted proposal.

Technology Transfer

Abstract

Protection of humans from large, finite time perturbations is an important area of research. Hence it is of interest to develop systems that are thin, light-weight and capable of absorbing large accelerations. An area of special interest concerns highly nonlinear systems which possess a rich variety of dynamical responses. This study focused on whether it is possible to convert any incident perturbation into localized excitations. The study explored the dynamics of a weakly nonlinear system with quadratic and quartic on-site and inter-site potentials where the linear pieces are significantly stronger than the nonlinear pieces that have been introduced by Sievers and coworkers. The system is realized by a micromechanical cantilever structure. We showed that the Sievers system can trap excitations that are in a specific amplitude and frequency window. We have studied whether breathers form from noisy perturbations in the Fermi-Pasta-Ulam system which consists of masses interacting via a quadratic and a quartic potentials. Turns out that only weak breathers form from such perturbations. Our work suggests the emergence of highly localized excitations when synchronized or nearly synchronized perturbations from two ends meet in granular chains with soft centers. Similar physics is also seen in preliminary studies on Fermi-Pasta-Ulam chains.

1. Introduction

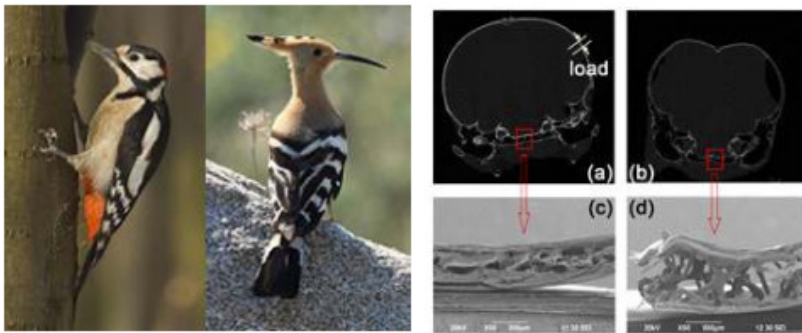


Fig. 1 (left): Great spotted woodpecker on the left and the Eurasian Hoopoe on the right; **Fig. 1 (right):** Woodpecker's head and skull bones on the left (a,c) and Hoopoe's head and skull bone on the right (b,d) [From L. Wang, *et al.* PLOS One 6(2011)e26490].

Protection of humans, equipment and machinery from high amplitude, short duration perturbations is an important area of basic and applied research. It is hence of significant interest to develop thin, strong and light mass materials for purposes of protection from impacts.

One can then ask whether it would be possible to make systems which would be very thin, very light, non-brittle and at the same time capable of damping sudden large changes in acceleration. While developing such materials has been a challenging journey, woodpeckers provide an inspiring example of what may someday be possible!

Typically, woodpeckers peck on wood about 20-22 times per second. Such rapid impacts can result in accelerations $\sim 1200g$ [1]. However, woodpeckers don't get brain damage! An acceleration change of $1200g$ is a good order of magnitude larger than the protection that the best helmets can provide [2,3]. Studies show that the woodpecker skulls are thin and lightweight and have almost no bulk fluids (Fig. 1) [1,4,5]. Their skulls also have dense plate-

like structures and have low porosity [5]. How exactly do these skulls absorb such large acceleration changes is presently unclear.

While one way to build highly impact absorbing structures may be to mimic the woodpecker skull itself, we feel there may be other and more scalable ways to do so. One possibility is by invoking the very special properties of *strongly nonlinear systems*, which allow pulse transformation/disintegration on very short distances unlike linear or weakly nonlinear systems [6-9].

Special features of nonlinear many body systems: Many body systems with strongly nonlinear interactions are special because they exhibit a rich variety of ways to propagate energy through them [6-11]. These systems can admit the propagation of non-dispersive bundles of energy as compression pulses or solitary waves [12] and dilation pulses or anti-solitary waves, localized excitations or breathers [10-11], and localized or collective harmonic or acoustic oscillations or phonons. The existence of any or all of these dynamical quantities depends upon the system properties, the boundary conditions and the nature of the perturbations (e.g., amplitude, duration) effected onto the

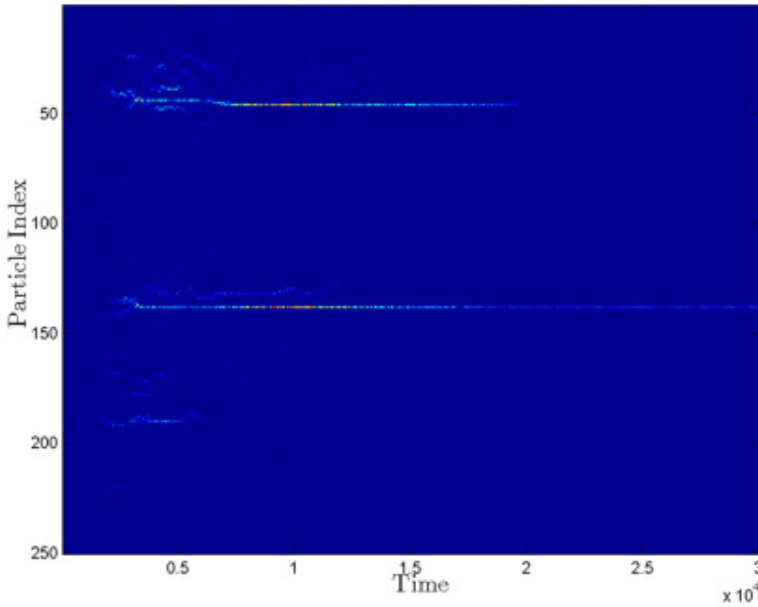


Fig. 3: Intrinsic localized modes precipitated in a driven-dissipative Sievers system (see Eq. 2 in Sec. 2), similar to what has been seen experimentally [24].

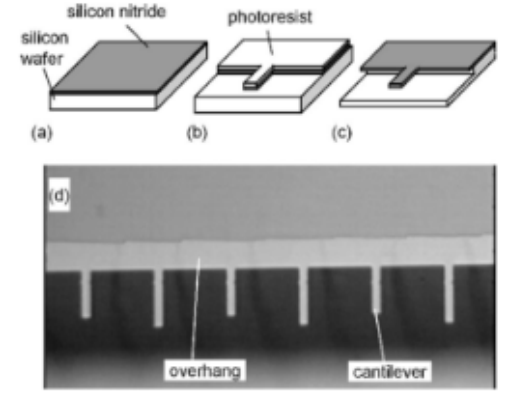


Fig. 2: The microelectromechanical (MEM) cantilever system in one of Sievers' studies with two different overhangs relating to a diatomic system (From [17-18]).

system [12,13]. The complex nature of the possible excitations also provides unique opportunities to control propagation and localization.

Why 1D matters: Ours and many other studies suggest that strongly nonlinear one dimensional systems are truly special [9,13]. Some of their behavior are translated into 2-D and 3-D systems [15,16], but many of the non-dispersive properties of the purely nonlinear entities such as solitary waves and breathers may not hold up very well in 2 or 3D systems except in special cases. It is hence desirable to place arrays of 1D chains to construct higher dimensional systems.

Using the geometry and the simplicity of the 1D systems would hence in principle be best to explore how to convert incident perturbations into energetic breathers. However, making breathers out of incident perturbations is a challenging task.

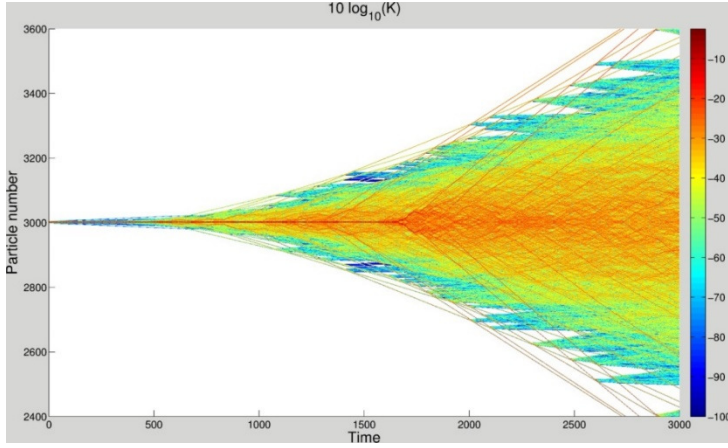


Fig. 4: A 6000 particle FPU system is studied where a breather has been seeded by stretching the center bond. We plot kinetic energy (in color) as functions of space and time. Kinetic energy is plotted on a log scale. Breathers decay here by simultaneously emitting solitary and anti-solitary waves, which in turn generate more such waves. In a smaller system wall effects lead to shorter breather lifetimes.

Much of Sievers' works have been done with small systems, typically the cantilever structures being $\sim 50\mu\text{m}$ in size [17]. The ILMs can form when the system is driven at sufficiently high frequencies (how high depends of course on the dimensions of the system) (see Fig. 3).

Seeded and Unseeded Breathers: Breather formation can be extremely robust in many body systems with quadratic and quartic inter-site interaction potentials (i.e., a form of the Fermi-Pasta-Ulam systems) provided the breathers are initially seeded by stretching one or more bonds (Fig. 4). However, breather formation is weak when initiated by any form of random perturbations in such systems. No significant spontaneous breather formation exists in driven-dissipative FPU systems [25]. Whatever breather formation we have seen are for limited ranges of the harmonic and quartic spring couplings and the breathers are very low energy and short lived. In short, something special is needed to reliably make energetic breathers out of arbitrary excitations. In addition to the above studies in Fermi-Pasta-Ulam [6-8] and so called Sievers systems [17-23], precompressed diatomic granular chains, when sufficiently excited, can also precipitate ILM formation in much the same way as in the works of Sievers and coworkers [26].

Intrinsic Localized Modes/Unseeded Breathers with High Frequency Driving: Thus far, robust breather-like structures [10,11], referred to in the literature as intrinsic localized modes or ILMs, have been seen by Sievers and by several other groups following a recipe pioneered by Sievers [11,17-23]. Their driven-dissipative system is the one described by a potential energy function which consists of an inter-site coupling between the masses, which has both quadratic and quartic springs along with an on-site potential well which also exhibits both quadratic and quartic terms. In experiments, these systems can be realized by cantilever structures (Fig. 2). In all of Sievers' works, the magnitudes of the quartic interactions, inter-site and on-site, are two orders of magnitude less than those of the corresponding harmonic terms. These systems hence are weakly nonlinear systems.

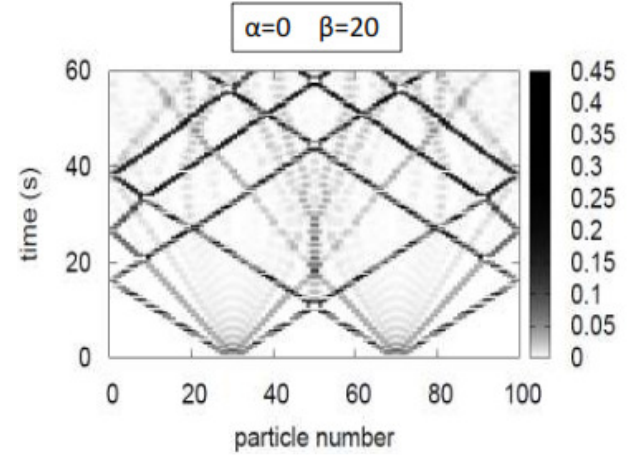


Fig. 5: A compression pulse moving right and a dilation pulse moving left collide at particle 50 in this space-time plot of energy propagation in an FPU chain to precipitate a metastable breather. Dark lines depict higher kinetic energy [27].

Breathers made to order?: Given the difficulty in spontaneously generating energetic breathers out of random

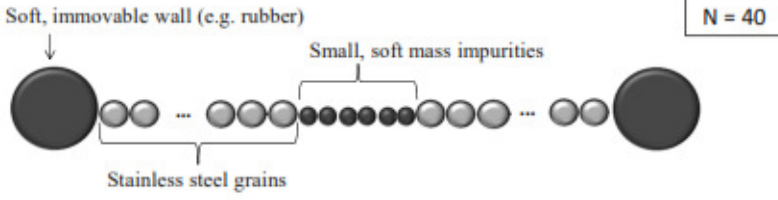


Fig. 6: Elastic grains placed between two soft end walls with 6 soft grains in the center.

anti-solitary waves collide such that the region ends up getting squeezed. Specifically, we see that breathers form in the Fermi Pasta Ulam systems when two opposite propagating waves, one a solitary wave or a compression pulse and one an anti-solitary wave or a dilation pulse, both carrying equal energies, collide (Fig. 5) [27].

Breather formation is more enhanced when two solitary waves collide in granular chains where the collision region has a few soft grains [28]. Such granular chain systems can be best realized via use of elastic spherical grain alignments where the centers can be made quite deformable (Figs. 6 and 7) (in this context see Refs. [14] and [29]). Similar behavior is also seen in Fermi Pasta Ulam chains when two opposite propagating solitary waves collide in the middle and the middle region carries nonlinear springs that are stiffer than that elsewhere [25]. Our pilot studies hence suggest that it may be possible to precipitate breathers out of a variety of different forms of excitations provided that the right forms of interactions, the right interplay of time scales and the right geometric criteria can be realized [25].

2. Results from the STIR Study

It is meaningful to consider the problem of breather formation for the simplest possible nonlinear many body systems. The first one is the classic system introduced by Fermi, Pasta and Ulam (FPU) in 1955 given by the Hamiltonian below [6-8],

$$H_{FPU} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \sum_{i=1}^N \left[\frac{\alpha_{i,i+1}}{2} (x_i - x_{i+1})^2 + \frac{\beta_{i,i+1}}{4} (x_i - x_{i+1})^4 \right], \quad (1)$$

where p_i , m_i , $\alpha_{i,i+1}$, $\beta_{i,i+1}$ represent the particle momenta, masses, and strengths of the quadratic and quartic inter-site couplings, respectively and N represents the number of particles in the system. Here and below x_i denotes the displacement from the equilibrium position. A version of the FPU-type system, but with added on-site potentials can be realized using cantilever structures and it was invoked by Sievers and coworkers, which we refer to as the Sievers system. The Sievers system is described as follows [17-18]

$$H_{Sievers} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \sum_{i=1}^N \left[\frac{k_{2,i}}{2} (x_i)^2 + \frac{k_{4,i}}{4} (x_i)^4 \right] + \sum_{i=1}^N \left[\frac{k_{2,i+1}}{2} (x_i - x_{i+1})^2 + \frac{k_{4,i+1}}{4} (x_i - x_{i+1})^4 \right], \quad (2)$$

perturbations in the various systems, it is important to see whether there are indirect ways of enhancing breather generation and thus increasing energy trapping. As we shall see below, our studies suggest that breathers tend to be initiated when a large amount of potential energy ends up at a site or a group of sites. A possible way to achieve this would be in regions where solitary waves or solitary and

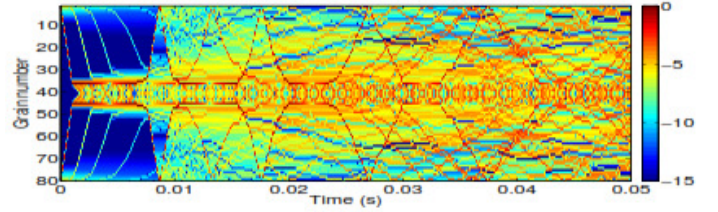


Fig. 7: Two colliding solitary waves make a central breather in a non-dissipative granular chain with soft central impurities. In a dissipative system, the center will act as an energy sink [28].

where the on-site ($k_{2,i}, k_{4,i}$) and inter-site ($k_{2,i,i+1}, k_{4,i,i+1}$) quadratic and quartic couplings are present. Thus, the Sievers system provides the capability to pin excitations at specific sites. Further, the parameter space available can be quite vast.

Both systems can be experimentally realized in precompressed granular metamaterials or laminate systems made from plates and o-rings. Additional modifications corresponding to changes in the onsite potential can be easily added in these systems, such as those introducing strong nonlinearity and effective dissipation with a unique capability for tuning by the initial precompression. These mechanical systems can be tuned into linear elastic, weakly nonlinear, strongly nonlinear or purely nonlinear responses depending on the initial precompression and the amplitude of the disturbance [6]. The simplest of these is the Hertz system discussed in Sec. 2, which consists of an alignment of elastic spheres and is described below. For simplicity we describe the Hertz system below [see also, 30-32],

$$H_{Hertz} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \sum_{i=1}^N [a_{i,i+1} (2R - (x_i - x_{i+1}))^2]^{\frac{5}{2}}, \quad (3)$$

where $2R$ is the diameter of each spherical grain in the alignment and $a_{i,i+1}$ is a function of the Young's modulus and the Poisson ratio and refers to the prefactor associated with the Hertz potential which is the second term on the right hand side in the above equation [33]. Note that there is no interaction between the elastic spheres in the Hertz system when grain-grain contact is broken or $(x_i - x_{i+1} > 2R)$ and the interaction between the adjacent grains is intrinsically nonlinear. When the grains are pre-compressed in the Hertz system, the grains there may no longer lose contact so easily and have oscillations about some mean positions. Such a system carries the presence of the harmonic component as well as nonlinear terms, both coming from the expansion of the Hertzian potential about strain caused by initial compression [6,13]. In this case the granular chains or the metamaterials assembled from plates and o-rings are essentially weakly nonlinear FPU systems for relatively small amplitudes of the perturbations [6,30-32].

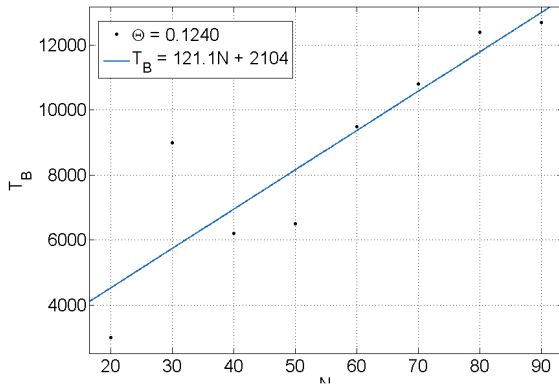


Fig. 8: The lifetime (T_B) of a seeded breather in the center of a FPU system is shown to grow as number of elements (N) increases.

In all dynamical many body systems at late enough times, on average, only a part of the total energy is kinetic and the rest is potential. Virial theorem dictates how much of the total energy is kinetic and how much is potential [13]. In systems with quadratic potentials, half the system is kinetic and half is potential. In nonlinear systems, the amount of kinetic energy exceeds the amount of potential energy. By how much kinetic energy exceeds the potential energy can depend on a number of parameters such as the strengths of the various terms in the potential energy function, the amplitude of the perturbation and the boundary conditions. If a lot of potential energy can somehow end up in a region of a strongly nonlinear system, it would be difficult for it to quickly disperse off that energy to satisfy the virial theorem

globally. Our studies reveal that fairly long lived (metastable) breathers emerge only when a significant amount of potential energy somehow ends up in a specific site [34,35,36].

As alluded to in Sec. 1, nowhere is the issue of excess potential energy leading to stable breathers better seen than in the case of a seeded breather for the FPU system. To seed a breather, one can stretch a bond or two adjacent bonds at some location in a linear chain, for simplicity's sake say at the center. All the system's energy cannot rapidly leak out of the high energy bond(s). In fact, in many of these systems bonds can only leak energy via emission of solitary and anti-solitary wave pairs (see Fig. 4 above). These waves eventually bounce off the boundaries and perturb the breather itself, ultimately destabilizing the breather and leading to its demise into an equilibrium-like phase [36]. In the presence of strongly harmonic terms, phonons also appear as part of the leakage process [34,35]. However, it is somehow not easy for a breather to quickly convert its energy into phonons [34,35]. The upshot is that the breather lives for a span of time. For systems with just the right conditions, the breathers can live for very long times. In general, the larger a system, farther the walls and longer the breather may be able to survive (see Fig. 4). Such breathers can also be initiated in the Sievers system and possibly in the compressed Hertz system though the compressed Hertz system (grain or o-ring based) has not been exhaustively investigated. However, it is not necessarily of great interest to seed a breather if one is to make use of breathers to trap incident energy where the incident energy can be from arbitrary sources and across arbitrary times.

Breathers can be excited in the Sievers-type discrete systems by driving the system at some specific high optical band frequency window and allowing the additional energy to dissipate at an appropriate rate such that there is long term energy balance in the system alluded to in Sec. 1 [18]. In the system's natural units, the quadratic couplings are significantly stronger than the nonlinear couplings, i.e., when excited, these systems have weakly nonlinear interactions and an abundance of phonons [25]. Sievers and coworkers have worked mostly with diatomic systems though some monatomic system work has been done as well [18]. The existence of some high frequency response in these systems is needed. When these systems are excited by driving at high enough frequencies for long enough intervals, enough energy ends up in high frequency vibrations, which in turn can get localized in space while surrounded by a phonon bath. The phonon bath presumably suppresses the leakage of the ILMs thereby giving the structure some stability. In experiments, and simulations driven dissipative systems are typically needed to sustain ILMs. We will specifically differentiate ILMs and breathers because as we shall see in Sec 2 below, ILMs may be a specific type of breathers.

However, even this approach is not of much practical use when it comes to trapping just about any form of the frequency spectrum of the incident energy. Thus, we are left with limited choices on how to directly initiate breather formation from any form of incident perturbation.

One hope when it comes to the Sievers and FPU-type systems is whether it is possible to precipitate breathers out of the right range of system constants and the right perturbations which are not necessarily high frequency and within a well-defined window. There indeed are parameter regimes and perturbation types where one finds breathers forming in some situations. However, in no case that we have thus far identified are we able to find direct

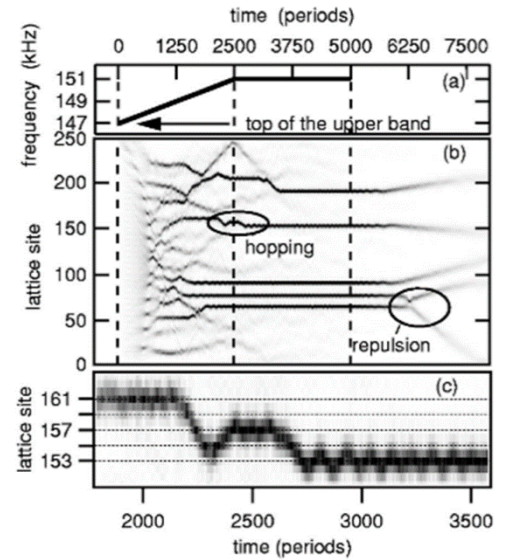


Fig. 9: The figure from Sievers' work shows the driving frequency behavior in time (upper panel), ILM (or breather) formation in real time and space (middle panel) and a typical drift of an ILM (lower panel).

precipitation of *energetic* breathers from random perturbations [25]. Ultimately, these studies suggest that there may not be any direct way to generate energetic breathers from arbitrary perturbations. So we turned our attention toward using the help of solitary and anti-solitary waves as an intermediate step to generate breathers out of random perturbations. And this, as we shall see below, turns out to be a promising avenue to explore.

Although the FPU model was introduced in 1955 [6], there is only a relatively modest amount of understanding of how solitary and anti-solitary waves form, propagate and collide in these systems [27]. We have performed extensive analyses on the formation and propagation of solitary and anti-solitary waves in the FPU systems (which was originally introduced strictly to explain mechanism of thermalization/heat conduction using weakly nonlinear interaction). We find that in the strongly nonlinear regime, head on collisions between solitary and anti-solitary waves can lead to the formation of breathers in the vicinity of the collision regime (Fig. 5) [27]. Nevertheless, while these breathers can be stronger than those that are precipitated out of random perturbations in the FPU and Sievers systems, *it may still not be easy to make breathers with significant amounts of energy*. However, the study of the collision of solitary and anti-solitary waves suggested a new avenue to pursue – is it conceivable then that one can use solitary wave collisions to generate breathers? If so, can one make those breathers stick? And can one precipitate breathers with significant energies?

Hertz systems are simpler in the sense that the grains interact only when in contact. However, when the grains gently touch one another, the repulsive force is intrinsically nonlinear in nature, even nonlinearizable, which makes their behavior dramatically different from classical weakly nonlinear FPU system. If the granular chain is kept in a

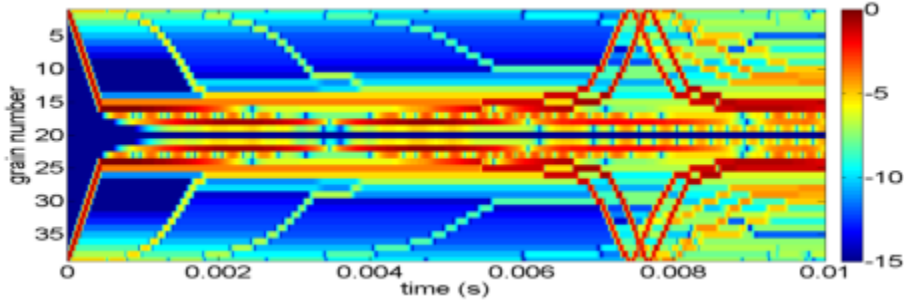


Fig. 10: A granular chain is shown where there are 3 central soft grains with a hard and a soft grain to each side of the central grains. Here we see the formation of two breathers in the center. These preliminary studies were not optimized to trap maximum energy. Also dissipative effects have been ignored. Observe the cold spot in the center (the blue line) [28].

compressed state with some preloading, then a harmonic force is invoked in addition to the nonlinear force component, both due to the Hertz interaction, making its behavior similar to FPU system. At zero or very low preloadings, any delta function perturbation initiated at the edge of the chain becomes a solitary wave. Further, our earlier work has shown that these solitary waves can be rather sticky when they interact with very soft walls [14].

These findings raise an intriguing possibility. What if we can initiate two solitary waves from the two ends of a granular chain and make them collide in a central region made up of soft grains and trap the energy for a while?

Our preliminary studies show that indeed such solitary wave collisions can precipitate breathers in granular chains [28]. We have done some pilot studies on collision between solitary waves in FPU chains where the central region contains springs with large spring constants. The experimental and theoretical investigations of the co-PI strongly suggest that similar results are to be anticipated for the more versatile and tunable discrete metamaterial assembled system of o-ring and plate periodic alignments, with sufficiently rigid o-rings [25,30-32,40]. Our successes hence suggest that there is indeed a way to make breathers at will and possibly out of arbitrary perturbations provided the

system is designed right and this is what we will pursue in the proposed research section below. The simulational and phenomenological research will be largely pursued by the PI. In addition, experimental work on the highly tunable and strongly nonlinear o-ring based systems would be carried out by the co-PI.

3. Outcomes

William J Falls and Yoichi Takato defended their PhD theses during the period of the STIR Grant. Dr Falls is now an assistant professor at Erie Community College of the SUNY and Dr Takato is a Research Associate at the Okinawa Institute of Science and Technology in Okinawa, Japan. The papers published and under consideration during the grant period reflect a part of what was accomplished. The complete summary of the actual project work has been captured in this document. Published work included analyses of solitary waves and breathers in Fermi-Pasta-Ulam systems, of how solitary waves get temporarily localized near soft walls in Hertz systems, of possible interactions between nanoscale grains, of the nature of cratering in impacts on 3D beds with the most recent manuscript (under preparation) being on a pilot study of how granular chains with soft centers may help localize any incident energy. We have also carried out studies on how to localize energy in Fermi-Pasta-Ulam systems and a detailed paper is being completed now. The work has been geared to not just solve the problem at hand but do so in an encompassing way by exploring issues such as miniaturization, fundamentals such as the relationships between solitary waves and breathers and the formal theoretical underpinnings that tie the two, and even on how impacts damage a surface.

In the next stage, we are looking forward to solving this localization problem and that is precisely the focus of the newly submitted proposal.

Bibliography:

1. S.H. Yoon and S. Park, "A mechanical analysis of woodpecker drumming and its application to shock absorbing systems," *Bioinspiration & Biomimetics*, 6 016003 (2011), doi:10.1088/1748-3182/6/1/016003.
2. See for example in A. Makris, J. Nerenberg, J.P. Dionne, C.R. Bass and C. Chichester, "Reduction of blast induced head acceleration in the field of anti-personnel mine clearance," (Army CECOM, NVESD, Ft. Belvoir, 2006), available at <http://www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA458451>.
3. See for example a recent discussion on pressure changes on helmets by S. Ganpule, L. Gu, A. Alai and N. Chandra, "Role of helmet in the mechanics of shock wave propagation under blast loading conditions," *Computer Methods in Biomechanics and Biomedical Engineering* (2011) doi: 10.1080/10255842.2011.597353.
4. P.R.A. May, P. Newman, J.M Fuster, A. Hirschman, "Woodpeckers and head injury," *The Lancet*, 307 454-455 (1976).
5. L. Wang, J.T.M. Cheung, F. Pu, D. Li, M. Zhang, Y Fan, "Why Do Woodpeckers Resist Head Impact Injury: A Biomechanical Investigation," *PLOS One*, 6 e26490 (2011).
6. V.F. Nesterenko, *Dynamics of heterogeneous materials* (Springer, New York, 1991) and references therein, especially those corresponding to Chapter 1.
7. E. Fermi, J. R. Pasta, and S. M. Ulam, *Collected Works of E. Fermi*, University of Chicago Press, Chicago, 1955.
8. M. Toda, *Theory of nonlinear lattices* (Springer, Heidelberg, 1981).
9. E.A. Jackson, *Perspectives of nonlinear dynamics*, vols 1 and 2 (Cambridge University Press, Cambridge and New York, 1991).

10. A.A. Ovchinnikov, "Localized long-lived vibrational states in molecular crystals," Sov. Phys. JETP, 30 147-150 (1970); Ibid, "Localized vibrational excitations in a system of nonlinear oscillators," engl translation of Teor. Mat. Fiz. 11 366-376 (1972).
11. See for example in A.J. Sievers and S. Takeno, "Intrinsic localized modes in anharmonic crystals," Phys. Rev. Lett. 61 970-973 (1988). It may be noted that there is extensive work on breathers in the literature, for a relatively recent review of the formal work see S. Flach and A.V. Gorbach, "Discrete breathers — Advances in theory and applications," Physics Repts, 467, 1-116 (2008)
12. N.J. Zabusky and M. D. Kruskal. "Interaction of solitons in a collisionless plasma and the recurrence of initial states." Phys. Rev. Lett 15, no. 6 240-243 (1965).
13. S. Sen, J. Hong, J. Bang, E. Avalos and R. Doney, "Solitary waves in granular chains" Physics Reports 462, pp 21-66 (2008).
14. See for example, E. Avalos and S. Sen, "Granular chain between asymmetric boundaries and the quasi-equilibrium state," Phys. Rev. E 89, 053202 (2014).
15. A.R. Bishop and T. Schneider, Solitons in condensed matter physics (Springer, Heidelberg, New York, 1978).
16. W. J Falls and S. Sen, "Solitary wave propagation through 2D tree-like structures," Physical Review E, 89, 023209 (2014).
17. M. Sato, B.E. Hubbard, A.J. Sievers, B. Ilic, D.A. Czaplewski and H.G. Craighead, "Observation of locked intrinsic localized vibrational modes in a micromechanical cantilever array," Phys. Rev. Lett. 90, 044102 (2003).
18. M. Sato, B.E. Hubbard and A.J. Sievers, "Colloquium: Nonlinear energy localization and its manipulation in micromechanical oscillator arrays," Revs. Mod. Phys. 78, 137-157 (2006).
19. A. Xie, L. van der Meer, W. Hoff and R.H. Austin, "Long-lived amide I vibrational modes in Myoglobin," Phys. Rev. Lett. 84 (2000) 5435-8.
20. M. Sato and A.J. Sievers, "Direct observation of the discrete character of intrinsic localized modes in an antiferromagnet," Nature 432 (2004) 486-488;
21. B. Swanson, J. Brozik, S. Love, G. Strouse, A.P. Shreve, A.R. Bishop, W-Z. Wang and M.I. Salkola, "Observation of intrinsically localized modes in a discrete low-dimensional material," Phys. Rev. Lett. 82 (1999) 3288-3291.
22. M.E. Manley, A.J. Sievers, J.W. Lynn, S.A. Kiselev, N.I. Abgladze, Y. Chen, A. Llobet, A. Alastas, "Intrinsic localized modes observed in the high-temperature vibrational spectrum of NaI," Phys. Rev. B 79 (2009) 134304.
23. M. Sato, B.E. Hubbard, A.J. Sievers, B. Ilic, D. Czaplewski and H.G. Craighead, "Observation of intrinsic localized vibrational modes in a micromechanical oscillator array," Phys. Rev. Lett. 90 (2003) 044102.
24. R. Kashyap, M. Westley and S. Sen, "Breather precipitation in Sievers' nonlinear chain," in preparation.
25. This result is a consequence of the research carried out by the PI and his students M. Westley, and R. Kashyap between 2013-2015. The studies have been supported by a US Army Research Office Short Term Innovative Research Fund. The extensive results from these studies are currently being prepared for publication.
26. N. Boechler, G. Theocharis, S. Job, P.G. Kevrekidis, M.A. Porter and C. Daraio, "Discrete breathers in one-dimensional diatomic granular crystals," Phys. Rev. Lett. 104, 244302 (2010).
27. W.J. Falls, PhD Dissertation, SUNY Buffalo, 2014.
28. M. Przedborski, T.A. Harroun and S. Sen, manuscript in preparation.

29. S. Job, F. Melo, A. Sokolow and S. Sen, "How Hertzian solitary waves interact with boundaries in a 1D granular medium," *Phys. Rev. Lett.* 94, 178002-1-178002-4 (2005).
30. E.B. Herbold and V.F. Nesterenko, "Solitary and shock waves in discrete strongly nonlinear double power law materials," *Appl. Phys. Lett.* 90, 261902 (2007).
31. I.L.D. Pinto, A. Rosas, A.H. Romero and K. Lindenberg, "Pulse propagation in a chain of o-rings with and without precompression," *Phys. Rev. E* 82, 031308 (2010).
32. Yichao Xu and Vitali F. Nesterenko, Propagation of short stress pulses in discrete strongly nonlinear tunable metamaterials, *Phil. Trans. R. Soc. A*, vol. 372, 20130186 (2014).
33. H. Hertz, "On the contact of elastic solids." *J. reine angew. Math* 92, 156-171 (1881).
34. S. Sen and T.R. Krishna Mohan, "Dynamics of Metastable Breathers in Nonlinear chains in acoustic vacuum," *Phys. Rev. E* 79, 036603(2009); <http://dx.doi.org/10.1103/PhysRevE.79.036603>
35. T.R. Krishna Mohan and S. Sen, "Linearity stabilizes discrete breathers, " *Pramana – J of Physics* 77 (5), 975-986 (2011); <http://dx.doi.org/10.1007%2Fs12043-011-0200-6>.
36. M. Westley, N. DeMeglio, R. Kashyap, W.J. Falls, T.R. Krishna Mohan, A. Datta and S. Sen, "Seeded breathers in the Fermi-Pasta-Ulam-Tsingou chain: On breather lifetimes," submitted for publication.
37. Chien-Wei Lee and Vitali F. Nesterenko, Path dependent high strain, strain-rate deformation of polymer toroidal elements, *Journal of Applied Physics*, vol. 116, 083512 (2014).
38. Chien-Wei Lee, and Vitali F. Nesterenko, Dynamic deformation of strongly nonlinear toroidal rubber elements, *Journal of Applied Physics*, vol. 114, 083509 (2013).
39. Si Yin Wang, Eric B. Herbold, Vitali F. Nesterenko, "Wave Propagation In Strongly Nonlinear Two-Mass Chains", in *IUTAM Proceedings on granular materials*, J.D. Goddard, J.T. Jenkins, P. Giovine (eds.), AIP Conference Proceedings, IUTAM-ISIMM Symposium on Mathematical Modeling and Physical Instances of Granular Flow, Reggio Calabria, September 14-18, 2009. American Institute of Physics, vol. 1227, Melville, New York, pp. 425-434, (2010).
40. Yichao Xu and Vitali F. Nesterenko, Attenuation of short stress pulses in strongly nonlinear dissipative metamaterial, *Journal of Applied Physics* (in press).
41. Y. Takato, S. Sen and J. Lechman, "Strong plastic deformation and softening of fast colliding nanoparticles," *Phys. Rev.*, 89, 033308 (2014).
42. Y. Takato, M.E. Benson and S. Sen, "Collision dynamics of soft and sticky nanoparticles," *Physical Review E* (submitted)
43. Y. Takato, Ph.D Thesis, SUNY Buffalo, 2015.