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Study and optimization of atmospheric propagation of partially coherent vortex beams

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14. ABSTRACT
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Study and Optimization of Atmospheric Propagation of Partially Coherent Vortex Beams

Final Performance Report

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1. Summary

The objective of this project was to investigate the behavior of partially coherent beams possessing orbital angular momentum (partially coherent vortex beams, or PCVBs) with the goal of applying and optimizing them for stable propagation through the atmosphere. In recent years, partially coherent beams have been shown to be more resistant to scintillations (intensity fluctuations) in the atmosphere, making them ideal for reducing the bit-error rate in free-space optical communications. Optical vortices, with their discrete topological properties and resistance to distortions, have been considered as an alternative carrier of information in free-space optical communications. Combining the two phenomena, with an eye towards collecting both of their benefits, was a natural – albeit challenging – path to explore.

Over the course of this project, we introduced and modeled analytically new classes of partially coherent vortex beams, including an infinite set based on the complete set of Laguerre-Gauss beams. We demonstrated properties of these beams that are of great significance to communications problems, including a previously-unappreciated self-healing property of certain classes of PCVBs. Furthermore, a new PCVB with two distinct types of orbital angular momentum was introduced.

A study of the orbital angular momentum (OAM) of partially coherent vortex beams led to the recognition of three fundamental classes of beams, based on the distribution of OAM in their cross-section. Beams can have a constant OAM density, like a coherent vortex beam, have a quadratic OAM density, like a twisted Gaussian Schell-model beam, or a mixture of the two, leading to a Rankine vortex behavior. But further research showed us that these are not the only OAM behaviors possible, and that coherence can be used to create complicated distributions of OAM, even for relatively simple beams.

We propagated a number of these PCVBs through simulated atmosphere, and noted some improvement in the robustness of the vortex structure, as compared to the coherent case. Though more work needs to be done to look at how to optimize the effects, these simulations show that PCVBs hold promise in atmospheric propagation.

Along the way, we looked at several schemes for the generation and detection of PCVBs. Early in the project, we published a paper on the use of a triangular aperture to detect vortex structures in a diffraction pattern. We have concluded that it is not the best way to analyze vortex beams, but can be used as a very simple and quick test method. In what amounts to the completion of work started in a previous project, we also studied the use of surface plasmons to modify the state of coherence of a light beam, and found a novel bandgap phenomenon that applies only to the coherence function.

Finally, we studied some other vortex-related phenomena of interest. We have developed a simple and intuitive way to mathematically generate beams with vortices placed in any position and with any order desired; this technique can also be used to create fields with superoscillations, oscillations of a wave faster than allowed by its bandlimit. This superoscilllation work has led to the development of a simple method for designing superoscillatory lenses, which may be used to do superresolved imaging in microscopy. Also, we extended some of the curious mathematics of fractional vortex fields to the case of fractional polarization fields, and showed a new Hilbert's Hotel-like behavior.

Each of these subjects is described in more detail below.

2. Partially coherent vortex beams

An optical vortex, which possesses orbital angular momentum, is entirely a phase structure; partially coherent beams, in contrast, do not possess a well-defined phase. To generate a partially coherent vortex field, then, the simplest solution is to create an ensemble of field realizations, all of which possess the vortex structure, and observe what happens to the vortex as the spatial coherence is decreased.

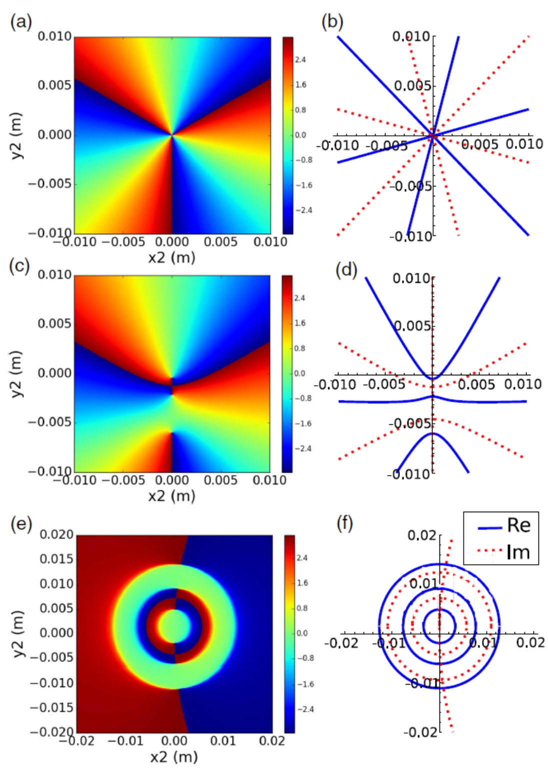

Figure 1. Phase of the cross-spectral density as the spatial coherence is decreased. Left show phase contours, right shows zeros of Re and Im parts; vortices are intersection points. After [Publication 3].

A specific strategy of this type was introduced long ago [1] and is known as the "beam wander" model of PCVBs. In this model, an ensemble of identical vortex beams are used, with the position of the central axis being the random variable. Though this model has been around since 2004, and produces analytic expressions for a partially coherent vortex beam in the source plane and on propagation, it has so far been limited to vortex beams of first or second order. To make such beams useful for communications, however, with different orders encoding different information, it is necessary to understand beams of arbitrary vortex order.

In [Publication 3], we evaluated the beam wander model to produce PCVBs of arbitrary azimuthal order *m* and radial order *n* = 0, and evaluated both the topological phase structure of the correlation function as well as its OAM characteristics. A sample of the beam behavior is shown in Fig. 1. As in the first-order case, when the spatial coherence of the beam is decreased, an equal and opposite topological charge approaches from infinity, making the net topological charge of the beam equal to zero near the beam center when the coherence is sufficiently low. This indicates, as expected, that there is a natural trade-off between partial coherence and OAM: as the spatial coherence is decreased, the OAM will also decrease. Thus any application that wants to take advantage of coherence and vortices will need to find a balance point which optimizes both characteristics. It was further noted, however, that although the net topological charge decreases as the coherence decreases for any order, higher order beams will have their charge last longer as coherence decreases.

The azimuthal order, which characterizes the OAM of the beam, is the most physically important characteristic for communications. However, the radial order, which is equal to the number of zero rings that appear in the transverse cross-section, is also of great importance, especially since it was shown not too long ago that beams of different radial orders could also be multiplexed and sorted [2]. Therefore it is of great interest to "complete" the set of PCVBs and derive analytic expressions for beams of any radial order *n* and azimuthal order *m*, a non-trivial task due to the presence of Laguerre polynomials in the integrals. In [Publication 10], however, this was finally achieved, and an example of the phase of such a PCVB is shown in Fig. 2.

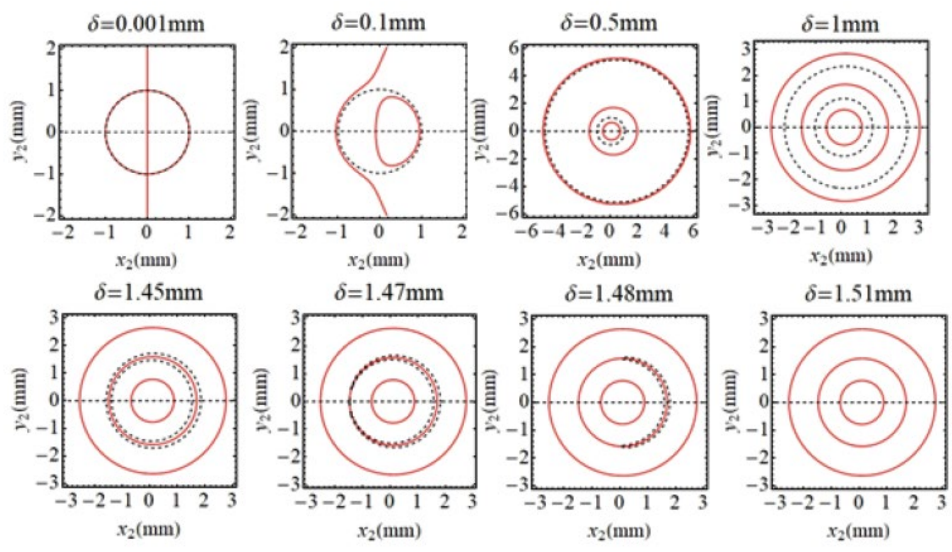

Figure 2. Zeros of Re and Im parts of cross-spectral density for a PCVB of radial order n = 1 and azimuthal order m = 1. After [Publication 10].

We found that every zero ring, being the perfect overlap of Re{W} = 0 and Im{W} = 0, immediately breaks into two vortices of equal and opposite sign. Furthermore, just as the central vortex has its equal and opposite number approach from infinity, the zero rings have their own counterparts approach from infinity, breaking into a pair of equal and opposite vortices themselves. When the spatial coherence of the field is sufficiently reduced, then, a PCVB of order (n,m) has $m+2n$ positive vortices and $m+2n$ negative vortices near the beam center.

This result suggests that beams with $n > 0$ will underperform beams with $n = 0$. As noted by the PI some time ago [3], the creation of vortex pairs tends to lead to vortex measurement errors, as one member of the pair can appear outside the detector. The presence of zero rings will thus result in additional measurement errors. It is, however, interesting to note that one could use the radial order as an error-checking mechanism of vortex beams. By interleaving orders – (n,m) = (0,1), (1,2), (0,3), etc. -- one could use the radial order to further distinguish the different azimuthal orders and reduce errors.

In studying the radial order PCVBs, we discovered one unexpected surprise that has somehow gone unnoticed in the past decade of investigating such beams. The propagation of a low-coherence beam of order $n = 0$, $m = 1$ is shown in Fig. 3. At the source plane, we can see the two equal and opposite vortices characteristic of a low-coherence PCVB, but as it propagates to the plane $z = 100$ m, we find that the second vortex has moved away again and we are left with what appears to be a fully coherent vortex beam again.

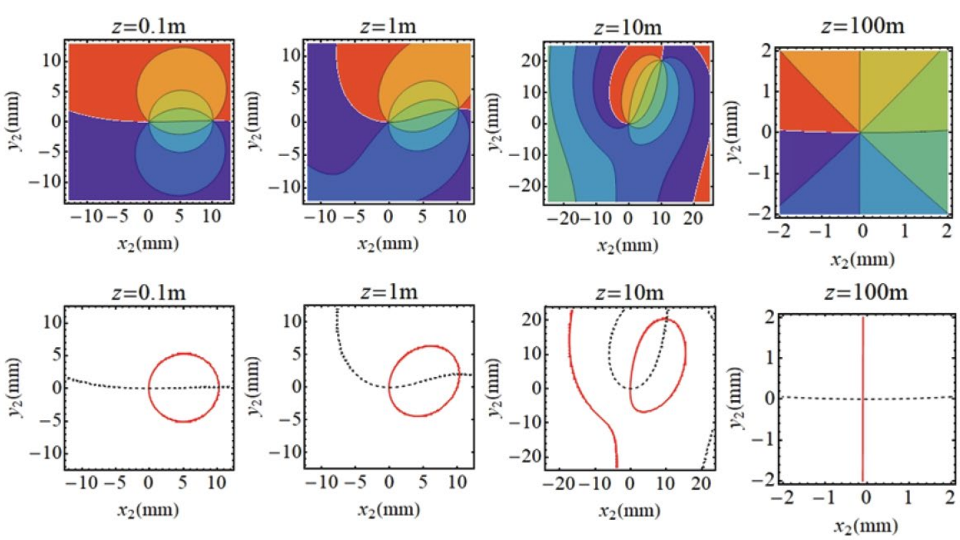

Figure 3. Evolution of a low-coherence PCVB on propagation. After [Publication 10].

This runs contrary to what one expects from a partially coherent beam. In a typical laboratory arrangement to create a PCVB, a coherent vortex beam is scrambled at a source plane. The vortex still exists at the source plane, but the beam eventually evolves into a Gaussian spot. Here, we have the opposite effect: a field with little vortex structure in the source plane evolves a well-defined coherence spot.

We have dubbed this effect a "van Cittert-Zernike (VCZ) effect for vortex beams." It arises from the specific nature of our beam wander model. The effective coherence of our PCVBs at the source is determined by the ratio of the beam width w to the beam wander δ, i.e. w/δ. A low value of this ratio is low coherence. But every member of our ensemble propagates parallel to the z-axis. As the beam propagates, w increases, but δ does not. On propagation, then, the ratio w/δ increases without bound, resulting in a field that looks increasingly like a fully coherent beam, with its vortex structure maintained.

In the typical laboratory arrangement mentioned above, this does not happen because the "scrambling" of the beam results in different members of the ensemble propagating at different angles, instead of positions. This suggests that there are two distinct classes of PCVBs, one with angular randomization, and one with positional randomization; the two forms can be transformed into each other by a simple lens [4], as shown in Fig. 4.

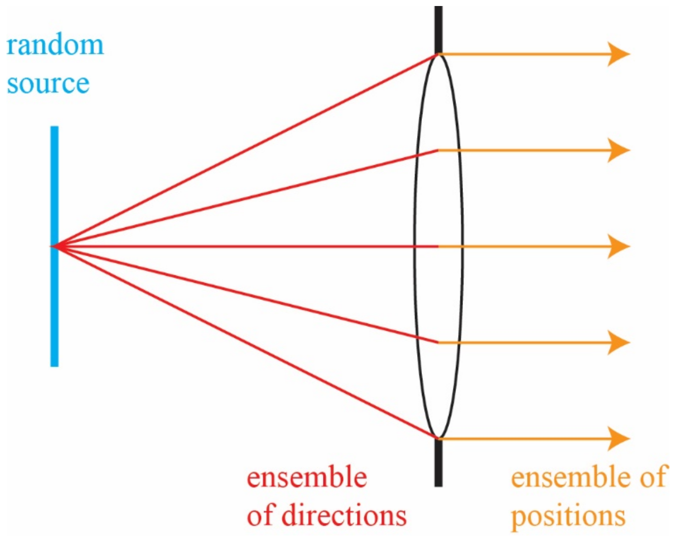

Figure 4. Transforming an ensemble of random directions into an ensemble of random positions.

The revelation that our beam wander PCVBs result in an improved vortex structure on propagation suggests that they would be better for communications applications; more investigations on this VCZ effect and its implications are ongoing.

It is to be noted that, although Laguerre-Gauss beams of different orders are orthogonal, the same is not true of the PCVBs constructed from Laguerre-Gauss beams. One interesting question going forward is whether it is possible to make an orthogonal basis set of PCVBs.

3. Orbital angular momentum in PCVBs

So far we have focused on the phase vortex structure of PCVBs, and have alluded to the fact that the phase structure is intimately connected to the OAM behavior. However, it has long been known [5] that the topological phase and the OAM are not directly related, except for exceedingly simple beams like Laguerre-Gauss beams. An understanding of the OAM properties of PCVBs is therefore essential if they are to be used in applications.

Early work by Swartzlander et al. [6] demonstrated that the topological charge of a PCVB is distributed in the beam's cross-section in the form of a Rankine vortex, with a quadratic increase in the beam's core, like a rigid body rotator, and a constant value in the outskirts, equivalent to a fluid rotator. Rankine vortices are best known in nature in the form of hurricanes, where the "eye" has the rigid body rotational form. Since Rankine vortices are associated with rotation, it was much more natural to study the OAM properties of the beam to see if they also manifested Rankine behavior. To do so, we introduced the normalized OAM flux per photon: the average OAM flux density of the beam divided by the average photon flux of the beam. The resulting quantity is the average OAM measured for a photon at a particular point in the beam's cross-section.

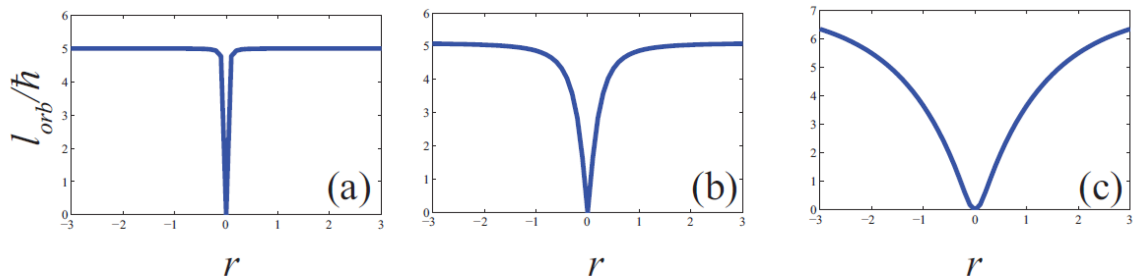

Figure 5. The normalized OAM flux density for a PCVB as the spatial coherence is decreased, i.e. the beam wander is increased. From [Publication 5]. The original beam had a topological charge of 5.

In fact, Rankine vortex behavior was observed, as illustrated in Fig. 5 from [Publication 5]. A decrease in spatial coherence increases the width of the quadratic region in the beam's core. But if PCVBs with a mixture of rigid body and fluid body rotation can exist, it would seem likely that PCVBs that act as pure rigid and pure fluid rotators must also exist. In fact, we have shown that two existing classes of PCVBs fit these conditions. Partially coherent vortex beams with a separable phase were introduced in 2003 [7] and act as pure fluid rotators, and the venerable twisted Gaussian Schell-model (tGSM) beams [8] act as pure rigid rotators.

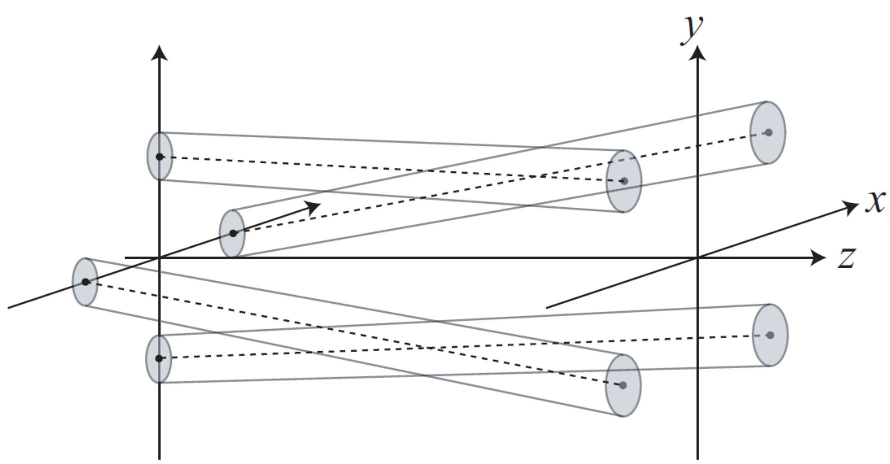

Figure 6. Construction model for a tGSM beam, as an ensemble of Gaussian beams that are all tilted to one side to produce a "handedness" without a vortex.

As shown by Ambrosini et al. [9], a tGSM can be considered an incoherent ensemble of tilted Gaussian beams, as shown in Fig. 6. But this model is similar to the beam wander model presented earlier, and suggested to us that it should be possible to combine a vortex phase with a partially coherent "twist." In [Publication 6], we introduced twisted vortex Gaussian Schell-model beams (tvGSM), which contain OAM from two sources: the underlying discrete phase twist of a vortex beam and a continuous tilt from tGSMs.

The combination of OAM from two distinct sources allows for unusual beam behaviors. For example, by tuning the ‘twist’ parameter α, it is possible to create a beam with a net OAM equal to zero but with nonzero local OAM in the beam’s cross-section, with regions of positive and negative counter-rotation, as seen in Fig. 7.

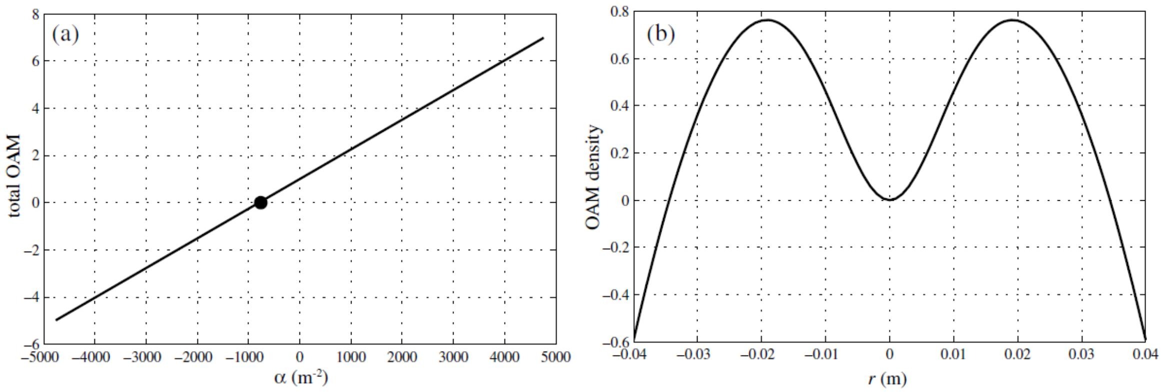

Figure 7. Creation of a beam with a total OAM of zero by (a) tuning the twist parameter, and (b) the normalized OAM density in cross-section that results, with positive and negative counter-rotating regions. After [Publication 6].

It is also possible to make beams with a “dead zone” of OAM in the middle, by canceling out the quadratic parts of the twist and vortex OAM components. Perhaps most important, and yet to be explored fully, we can imagine using the twist OAM to compensate for the loss of discrete OAM on atmospheric propagation.

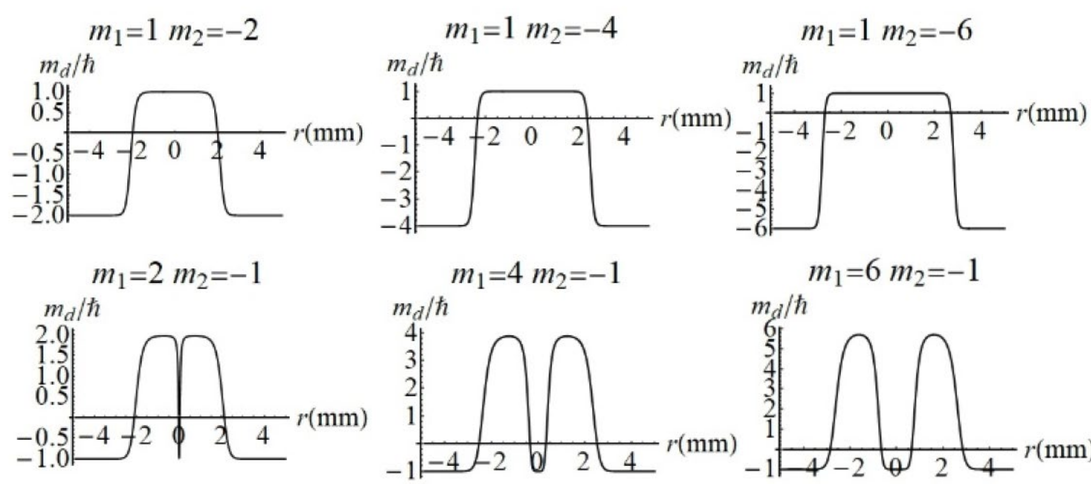

Figure 8. OAM distribution for a pair of beams, one with width 1 mm, the second with width 5 mm. After [Publication 9].

Even more extreme distributions of OAM can be generated with relatively simple beams, as we have demonstrated in [Publication 9]. By an incoherent superposition of a small number of beams with different azimuthal and radial orders and widths, one can create any number of counter-rotating OAM regions, even for a fixed value of the total topological charge. A characteristic example is shown in Fig. 8, where two equally-weighted modes with different charges produce different positive and negative OAM regions. The most immediate use of such observations is in microscopic optical trapping, where this freedom in manipulating OAM regions can be used for fine manipulation of particles, but the recognition that beams with the same total angular momentum can nevertheless have different spatial distributions is an insight which will prove helpful in atmospheric propagation, as well.

This last publication has led us to a very significant observation, which we are currently exploring in more detail. A coherent vortex beam in a source plane has a constant OAM density, like a fluid rotator. If we apply an isotropic Schell-model correlation to the beam, the OAM density will not change in the source plane, but it will change dramatically on propagation, and depend strongly on the correlation length and the specific form of the correlation function. We may view this effect as *correlation-induced OAM changes*, which are analogous to the well-known correlation-induced spectral changes introduced by Emil Wolf in the late 1980s, and the correlation-induced polarization changes recognized soon after. It is a previously unrecognized influence that the state of coherence can have on the propagation of light.

4. PCVBs in atmospheric turbulence

With a variety of partially coherent vortex beams available for study, our next step was to propagate a variety of them through simulated atmospheric turbulence to see the outcome. We focused on the topological properties in these studies, i.e. the preservation of the discrete vortex structure.

Of the variety of beam classes available, we considered the following: single vortex beams, coherent vortex arrays, incoherent vortex arrays, separable phase beams (after [7]), and our beam wander PCVBs. All of these results have been reported in the 2018 PhD thesis of Charlotte Stahl, and will be published as [Publication 11].

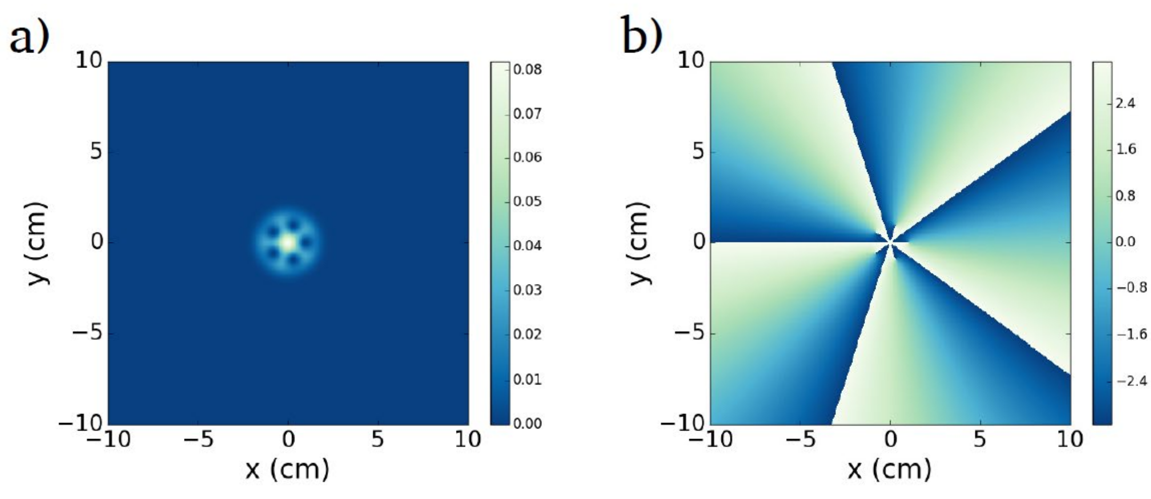

Figure 9. The (a) intensity and (b) phase of a 5th order coherent vortex array.

For a coherent vortex array, such as seen in Fig. 9, we used the method of [Publication 2] (to be discussed in Section 6) to create *m* first-order vortices in a ring around the center of a coherent Gaussian beam. Our hypothesis: a vortex is subject to beam wander in the atmosphere, and an *m*th order vortex will tend to have all the stacked vortices wander together, and more likely to leave the detector aperture at the same time, while a group of *m* spatially separated vortices would wander independently, and be more likely to leave some fraction of the total vortices in the detector region. This turned out to not be the case: the coherent vortex array lost its topological charge much more rapidly than a single *m*th order vortex. We speculate that this result arises from the fact that the array vortices are already closer to the edge of the aperture, giving them a 'head start' at being lost. Furthermore, interactions between the coherent field and the atmosphere may cause the vortices to repel one another, even though in free space they propagate in parallel to the detector.

We next tried an incoherent vortex array, in which *N* mutually incoherent vortex beams, each of order *m*, were arranged around the central axis and propagated to the detector. As seen in Fig. 10, the variance of the topological charge of the incoherent array (in red) appears to be better than that of a coherent vortex beam (in blue), and performs better as the size of the array increases. We may cautiously say that such incoherent arrays provide some improvement in topological detection.

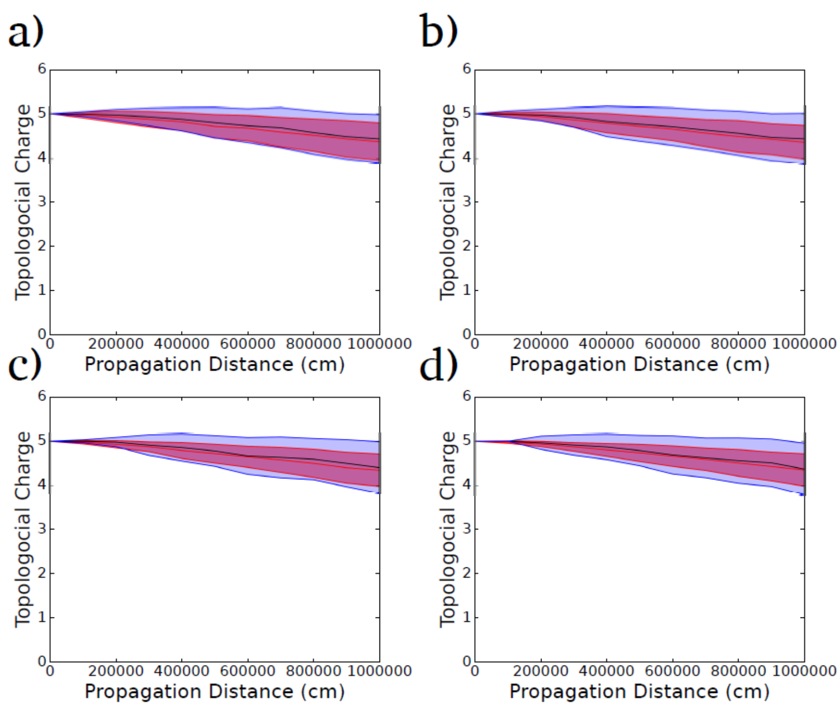

Figure 10. Topological charge mean and variance for incoherent array (red) and coherent beam (blue), for different numbers of beamlets (a) N = 3, (b) N = 5, (c) N = 10 and (d) N = 20. The beam width is taken to be 2.0 cm.

This improvement of the variance may appear relatively small, but can be a key feature for reducing the crosstalk in atmospheric communications. In Figure 11, we compare two coherent vortex beams with two incoherent array beams at different orders, and see that the variances of the coherent beams overlap at long distances, whereas the variances of the incoherent array beams do not. One is less likely to confuse the signals of the incoherent array beams, and the bit error rate will be smaller.

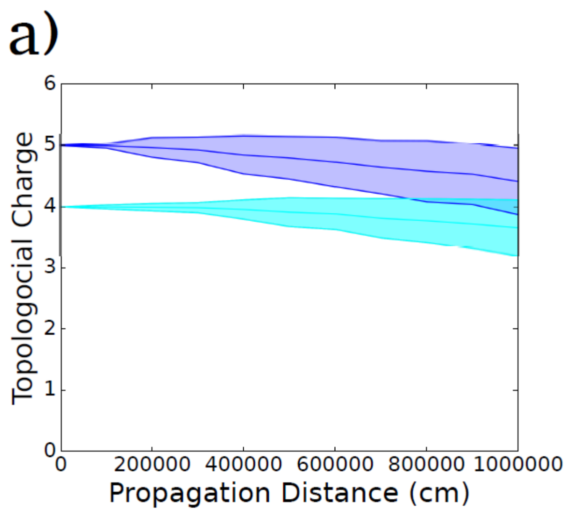

Figure 11. Comparison of (a) coherent beams of m = 4 and m = 5 with (b) a 20 beam incoherent array with m = 4 and m = 5.

One might hope that the separable phase beams of Ref. [7] would work particularly well, as they have a "pure" vortex state to begin with. However, as seen in Fig. 12, this is not the case. The azimuthal order of all incoherently superimposed modes is the same; as the number of combined radial orders is increased, there is a sharp decrease in the average topological charge, even though the variance decreases.

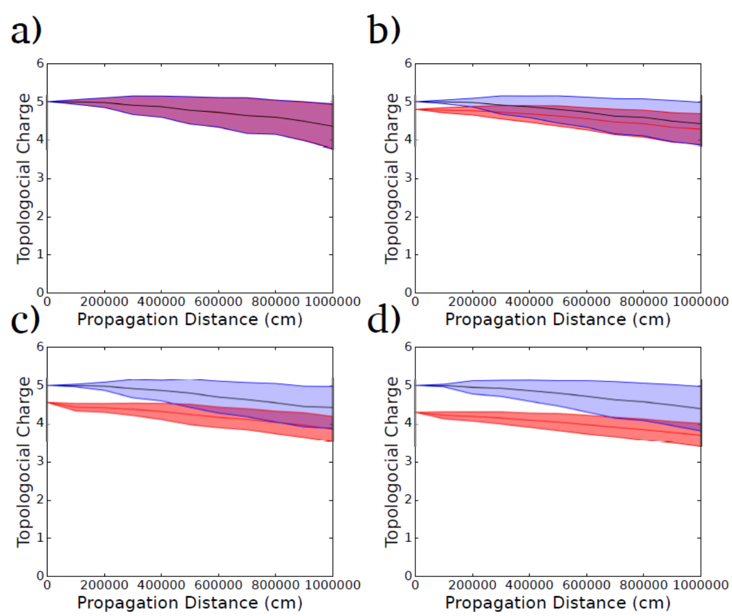

Figure 12. Topological properties of separable phase beams (red) with (a) 2 radial modes, (b) 10 radial modes, (c) 20 radial modes, and (d) 30 radial modes, compared to coherent beams (blue).

We trace the origin of this loss back to the discussion of Section 2. When a zero ring, such as that which appears in a Laguerre-Gauss mode of non-zero radial order, is distorted through a decrease in coherence it splits into a vortex pair. Our separable phase beams consist of modes with a large number of zero rings, and it seems that the vortex pairs from those rings are bringing down the average charge.

The loss of charge is not necessarily a detriment, however: we can imagine calibrating a communications system where we take into account the initial drop in average charge. This allows us to use the reduced variance to our advantage, albeit with a shift set of discrete topological levels.

A similar thing happens for our set of PCVBs produced by the beam wander model. As seen in Fig. 13, as the coherence is decreased both the variance and the average topological charge decrease. In the beam wander model, we can interpret the decrease in charge as being due to the vortex spending too much time "wandering" outside of the detector region. We again find a tradeoff between optimizing coherence properties and optimizing vortex properties. Calibrating a system to acknowledge the reduced topological charge, as mentioned above, will also work in this case.

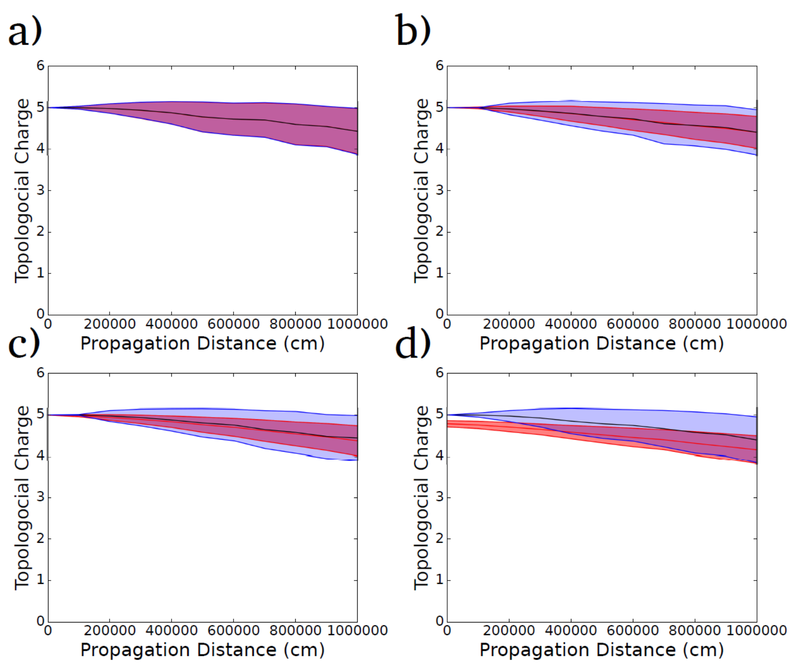

Figure 13. Mean and variance of topological charge for a coherent beam (blue) and a beam wander PCVB (red), for a beam wander of (a) 0.1 cm, (b) 0.5 cm, (c) 1.0 cm and (d) 2.0 cm.

There are many more possibilities to be explored, which we discuss in the conclusions.

5. Generation and detection

An important aspect of any system that takes advantage of vortices and coherence for communication and sensing is the ability to generate such beams and reliably detect their topological properties. Over the course of this project, we have touched upon a couple of these aspects, which we summarize briefly.

From the work on the previous grant's effort, we published a paper on the use of triangular apertures to detect the topological charge of optical vortices. As was first reported by Hickmann et al. [10], a vortex beam passing through a triangular aperture will produce a triangular diffraction pattern with a number of spots equal to the order of the vortex plus one. This is shown in Fig. 14, from [Publication 1].

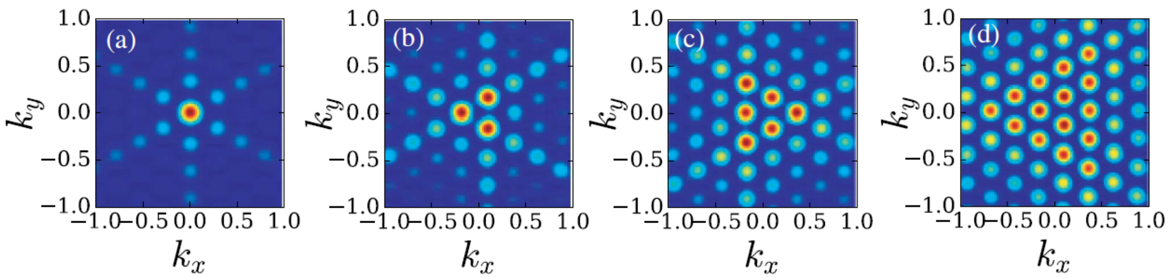

Figure 14. Diffraction pattern produced by a vortex beam passing through a triangular aperture, for (a) m = 0, (b) m = 1, (c) m = -2, (d) m = 4. After [Publication 1].

We developed an improved analytic model for this phenomenon which allowed us to study whether this pattern can be used for displaced vortex beams or multimode vortex beams, both effects that will appear in atmospheric propagation. Though the method worked well for displaced beams, the pattern of a multimode vortex beam tends to be dominated by the lowest-order mode. We feel that the triangular detection method will work well for quickly and easily testing a vortex system, but has already been supplanted by methods that use geometric transformations to demultiplex OAM beams [11].

Another significant challenge to using PCVBs in applications like optical communications is the need for a very rapidly fluctuating partially coherent source. In order to get the benefits of partial coherence, the source must have fluctuations much faster than the rate at which data is to be transmitted. However, in laboratory settings, partial coherence is still often generated by the use of the traditional rotating ground glass plate, a method much too slow – and bulky – for applications.

In a previous grant period, we investigated the use of surface plasmons to modulate the global state of coherence of a light wave [12]. In this method, an array of holes or structures on a plasmon-supporting metal plate is used to mix an incident light beam in a transverse plane, allowing for dramatic changes in the spatial coherence of light. Though many challenges need to be understood and overcome, this presents a possibility of making a fast, tunable partially coherent source. The system can only be studied computationally, however, and the complexity of the arrangement made it difficult to draw firm conclusions at that time.

Recently, we returned to the problem in the hopes of better understanding the interactions of plasmons and coherence, in [Publication 8]. We made two significant changes to our approach: we worked with a one-dimensional array of holes, rather than a two-dimensional array, and we introduced a quantity that roughly represents the global spatial coherence of the entire output beam.

Once this new quantity was used, we immediately recognized an intriguing new effect in plasmons and coherence: a coherence bandgap, a range of frequencies over which the output spatial coherence is suppressed to the incident level, as illustrated in Fig. 15. Though there are certain frequencies at which there is a significant increase in coherence, such as around 700-750 nm, there is also a region where the coherence is suppressed to the input level, around 650-700 nm. This is the coherence bandgap. It arises because the plasmonic waves, which need to propagate long distances in order to change the coherence, are suppressed by the periodic structure of the hole array. This is evidently the first time a bandgap effect has been recognized in a coherence function, rather than the transmission of light.

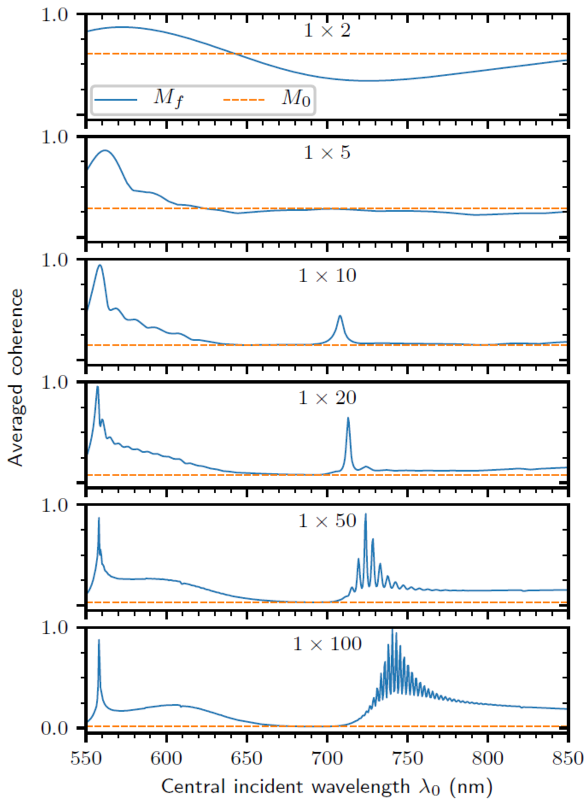

Figure 15. Illustration of the growth of a coherence bandgap as the number of holes in the array is increased. The bandgap runs from about 650 nm to 700 nm. After [Publication 8]. The input coherence is the gold dashed line.

We justified calling this a bandgap with two observations: one, by seeing it develop as the number of holes increases, as in Fig. 15, and two, by randomizing the hole positions and seeing the effect then destroyed.

There has long been a question of whether surface plasmons can be used to decrease spatial coherence as well as increase it. In our study, we found that the lowest value achieved in a large hole array was the value of the input coherence. In the design of a tunable coherence source, we have therefore concluded that one should start with a very low coherence input light source and use plasmons to increase it. Previously, we thought it would be possible to raise and lower coherence from a middle coherence value.

6. Other vortex phenomena

At the conclusion of the previous project period, we had demonstrated a new analytic technique for placing vortices in any location in a beam; this work appeared as [Publication 2]. This allowed us, for example, to design coherent vortex arrays for propagation through atmospheric turbulence, as discussed in Section 4. There is hypothetically no limit to our ability to place vortices in a beam, as illustrated in Fig. 16, as first-order vortices were used to create the letters "UNC," while the second "C" was created out of second-order vortices.

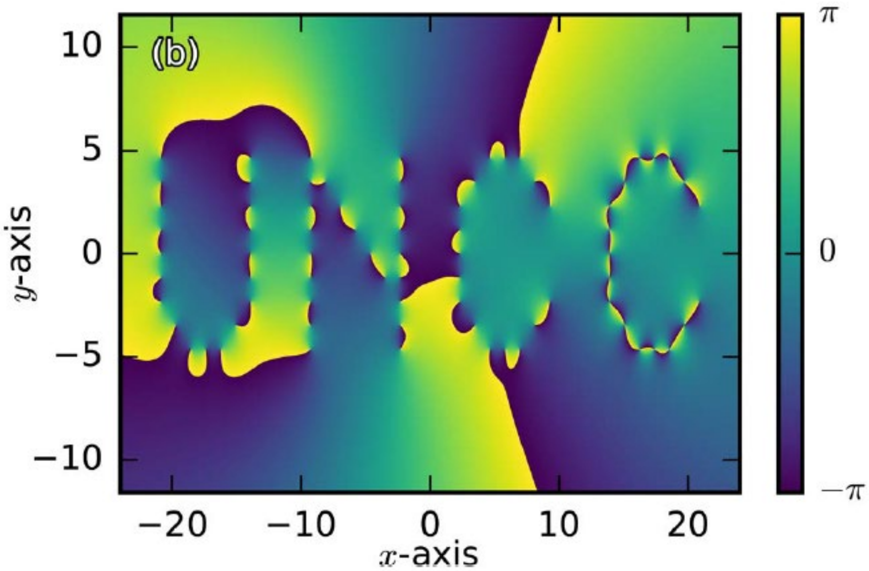

Figure 16. Designed arrangement of vortices in a wavefield using a superoscillation method. After [Publication 2].

The waves used in these simulations were taken to be bandlimited in spatial frequency, though there is no limit to how closely together the vortices may be spaced and consequently how rapidly the wave may oscillate. This phenomenon, in which a signal oscillates faster than its Fourier bandlimit would seem to allow, is known as a superoscillation.

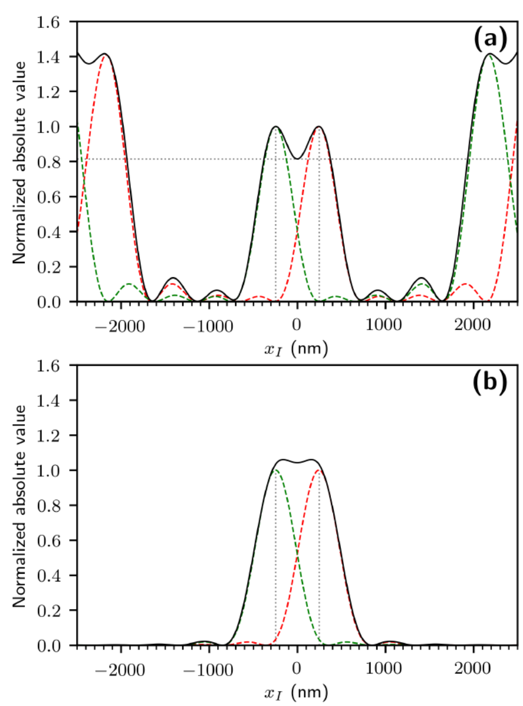

Figure 17. Resolving two closely spaced sources with (a) a superoscillatory lens and (b) a diffraction limited lens. The improved resolution of the two objects in case (a) is clear.

Our investigation of this phenomenon, which has become of great interest to the imaging community, led to the publication of a review article on the subject, which is [Publication 7]. While we were exploring our method for placing vortices anywhere in a beam, we also looked at making zero rings, instead of vortices, with an eye towards a simple method for designing superresolution lenses. An illustration of the advantages of such a lens are shown in Fig. 17, taken from the PhD dissertation of Matt Smith.

One can see the improvement in the resolution that is achieved with the superoscillatory lens, though it comes with a price: there are extremely large sidelobes on either side of the objects to be imaged, which means that there is an extremely limited field of view for such a superresolved imaging system. This limitation is common to all superoscillatory systems.

One strong motivation for studying superoscillations in this project is looking at whether is it possible to have superoscillations in partially coherent fields. This would again be a synthesis of coherence and optical vortices, and might allow unusual effects such as making a field which is overall spatially coherent but locally highly incoherent, or vice versa. Such unusual beams may again prove useful in atmospheric propagation. However, correlation functions must satisfy strict mathematical conditions – they must be Hermitian and non-negative definite – and it is not clear if it is possible to make superoscillations under such conditions. Investigations are ongoing.

Finally, we briefly note a continuation of our work on the relationship between singularities in wavefields and transfinite mathematics, in the form of "Hilbert's Hotel." In [Publication 4], we extended our work on fractional vortex states to looking at fractional polarization states, in which the effective topological index of a nonuniformly polarized light beam is fractional. As both fractional beams and nonuniformly polarized beams have been found to have interesting and potentially advantageous properties on propagation, it is worthwhile to consider their synthesis. In our work, we noted that we can get discontinuous jumps in the topological index by changing a system parameter. These rapid switches mirror those of the vortex form of Hilbert's Hotel reported previously [13], but have topological features unique to the polarization of light. As in the vortex case, these results may provide insight into how to rapidly switch beams from one unusual polarization state to another.

7. Future work

Partially coherent beams possessing optical vortices and angular momentum have become a subject of intense interest in only a few short years. Here, we have analyzed a number of fundamental beam classes in the atmosphere, but more work needs to be done to understand how to use them best in applications. Future research will involve using analytic models to study such beams, and to tackle more sophisticated classes of partially coherent vortex beams, such as the tvGSM introduced here, fractional partially coherent beams, and vector fractional partially coherent beams. We would also like to a detailed comparison between beam classes, including the more familiar tGSM beams. One big question to be answered: why do some beams perform better than others?

Now that we have seen a van Cittert-Zernike effect for the beam wander class of beams, it is worthwhile to explore in more detail whether that revival of vortex structure provides any benefit for propagation. We would also like to compare the beam wander-style beams on propagation with the more familiar scrambled partially coherent vortex beams.

One of the limitations in finding optimal solutions for PCVBs, and partially coherent beams in general, for atmospheric propagation is the complexity of the beams themselves. In our next project, we would like to take a step back and start looking at the effectiveness of a small number of mutually incoherent modes, or partially correlated modes, such as were studied in [Publication 9]. The richness of the OAM structure of such beams suggests that they may provide a lot of flexibility in reducing deleterious atmospheric effects, with a significant benefit of being easier to understand theoretically.

REFERENCES

[1] G. Gbur, T.D. Visser and E. Wolf, “'Hidden' singularities in partially coherent wavefields,” J. Opt. A: Pure Appl. Opt. 6 (2004), S239.

[2] Y. Zhou, M. Mirhosseini, D. Fu, J. Zhao, S.M.H. Rafsanjani, A.E. Willner and R.W. Boyd, “Sorting photons by radial quantum number,” Phys. Rev. Lett. 119 (2017), 263602.

[3] G. Gbur and R.K. Tyson, “Vortex beam propagation through atmospheric turbulence and topological charge conservation,” J. Opt. Soc. Am. A 25 (2008), 225.

[4] Y. Gu and G. Gbur, “Topological reactions of correlation vortices,” Opt. Commun. 282 (2009), 709.

[5] M. Berry, “Paraxial beams of spinning light,” Proc. SPIE 3487 (1998), 6.

[6] G.A. Swartzlander Jr. and R.I. Hernandez-Aranda, “Optical Rankine vortex and anomalous circulation of light,” Phys. Rev. Lett. 99 (2007), 163901.

[7] G.V. Bogatyryova, C.V. Fel’de, P.V. Polyanskii, S.A. Ponomarenko, M.S. Soskin, and E. Wolf, “Partially coherent vortex beams with a separable phase,” Opt. Lett. 28 (2003), 878.

[8] R. Simon and N. Mukunda, “Twisted Gaussian Schell-model beams,” J. Opt. Soc. Am. A 10 (1993), 95.

[9] D. Ambrosini, V. Bagini, F. Gori, and M. Santarsiero, “Twisted Gaussian Schell-model beams: a superposition model,” J. Mod. Opt. 41 (1994), 1391.

[10] J. Hickmann, E. Fonesca, W. Soares, and S. Chávez-Cerda, “Unveiling a truncated optical lattice associated with a triangular aperture using light’s orbital angular momentum,” Phys. Rev. Lett. 105 (2010) , 053904.

[11] M. Mirhosseini, M. Malik, Z. Shi, and Robert W. Boyd, “Efficient separation of the orbital angular momentum eigenstates of light,” Nature Communications 4 (2013), 2781.

[12] C.H. Gan, Y. Gu, T.D. Visser and G. Gbur, “Coherence converting plasmonic hole arrays,” Plasmonics 7 (2012), 313.

[13] G. Gbur, “Fractional vortex Hilbert’s Hotel,” Optica 3 (2016), 222.

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1. C. Stahl and G. Gbur, "Analytic calculation of vortex diffraction by a triangular aperture," J. Opt. Soc. Am. A 33 (2016), 1175.

2. M.K. Smith and G.J. Gbur, "Construction of arbitrary vortex and superoscillatory fields," Opt. Lett. 41 (2016), 4979.

3. C.S.D. Stahl and G. Gbur, "Partially coherent vortex beams of arbitrary order," J. Opt. Soc. Am. A 34 (2017), 1793.

4. Y. Wang and G. Gbur, "Hilbert's Hotel in polarization singularities," Opt. Lett. 42 (2017), 5154.

5. G. Gbur, "Partially coherent vortex beams," Proc. SPIE 10549 (2018), 1054903.

6. C.S.D. Stahl and G. Gbur, "Twisted vortex Gaussian Schell-model beams," J. Opt. Soc. Am. A 35 (2018), 1899.

7. G. Gbur, "Using superoscillations for superresolved imaging and subwavelength focusing," Nanophotonics 8 (2019), 205.

8. M. Smith and G. Gbur, "Coherence resonances and band gaps in plasmonic hole arrays," Phys. Rev. A 99 (2019), 023812.

9. Y. Zhang, Y. Cai and G. Gbur, "Control of orbital angular momentum with partially coherent vortex beams," Optics Letters, in press.

10. Y. Zhang, Y. Cai and G. Gbur, "Partially coherent vortex beams of arbitrary radial order and a van Cittert-Zernike theorem for vortices," Physical Review A, submitted.

11. C.S.D Stahl and G. Gbur, "Partially coherent vortex beams in atmospheric turbulence," in preparation.

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Charlotte Stahl, defended early 2018

Matt Smith, defended early 2019