(U) Nyquist Theorem applied to Digital Radiography

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(U) Abstract: The detectability threshold of defects is crucial to radiographic testing. Military product specifications for critical and major defects requires precise measurement of relevant defect sizes. In this paper we introduce artificial defects in M1 shells and investigate the resulting line profiles and establish a Nyquist Criterion for detectability.

(U) Research Innovation and Objective(s): MATLAB® is used to introduce artificial cracks in shells. The Nyquist Theorem is established from a theoretical and experimental point of view. We take a small sample of empirical data using items of interest to the Department of Defense and apply this to our experimental MATLAB® implementation. The objective here is to elucidate inspection requirements and explain some of the aspects of a radiographic qualification.

(U) Impacts on Warfighter Mission: Ensuring accurate measurements are taken by radiographers across the industrial base is critical. Measuring the sizes of relevant indications is central to radiographic inspection. Military Specifications are stringent as to what categorizes a major or critical defect.

(U) Keywords: Nyquist Theorem, Relevant Indication, Line Profile, X-Ray, Non-Destructive Testing (NDT), MATLAB®, Crack detection

1. (U) Introduction

(U) Radiographic testing is a non-destructive testing technique which allows for the detection of subsurface flaws. It is applicable to most materials and can reveal fabrication and underlying assembly errors. [1] A critical question in radiographic inspection is the size of the smallest relevant indication that can be detected. There are several theoretical and practical considerations when determining the smallest detectable relevant indication.

2. (U) Method

2.1 (U) Theory

(U) Let SOD represent the source to object distance and let ODD represent the object to detector distance. Then, adding these two distances will yield the source to detector distance or, SDD.

$$SOD + ODD = SDD$$

The magnification, M, is defined as the ratio of the SDD to SOD.

$$M = \frac{SDD}{SOD}$$

Pixel pitch is defined as the distance from the midpoint of one pixel to the midpoint of the adjacent pixel. [2] Taking magnification into account, one can obtain an “effective” pixel pitch.

$$EPP = \frac{PP}{M}$$

The effective pixel pitch defines the scaling when taking measurements on a digital radiograph.

The well-known Nyquist Theorem, as it relates to images, states that to properly sample an image, the sampling frequency must be at least twice the maximum spatial frequency present. [3] This paper will elaborate on the radiographic use of the Nyquist Theorem. The application of Nyquist for x-ray inspection requires 3 pixels in order to reliably discern defects.

The problem of measuring crack width is dependent on the activation of certain pixels. This treatise will focus on the radiographic end result. That is, once the gray values appear on the screen, it is the radiographer’s job to interpret and classify the appearance of these gray values in a qualitative and quantitative way. We will not be concerned with the attenuation of the x-rays through the object and the subsequent activation of pixels on the detector.

Consider the following pixel orientation depicted below in Figure [1]. Assume a crack orientation that spans the width of two pixels. The first scenario we describe is one in which both pixels remain "on". That is, the gray values of these pixels is 65,535. The opposite activation is depicted in Figure [2].

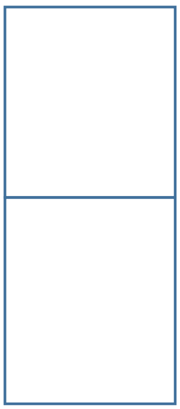

Figure [1] Depicted here are two pixels. Both pixels here are in the "on" state, and so in matrix notation both pixels would have a value of 65,535.

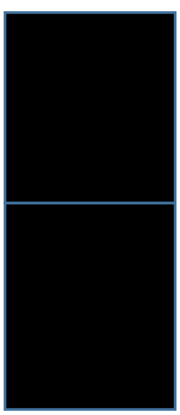

Figure [2] Depicted here are two pixels. Both pixels here are in the "off" state, and so in matrix notation, the pixel values would yield zero.

The previous cases shown in Figures [1] and [2] are what we classify as "on the grid". As a result of acquisition, attenuation, and detector response, another configuration of pixel activation is possible.

Consider the case illustrated in Figure [3] below. Suppose we know there is a crack that spans the width of two pixels. Due to underlying physics, the pixel activation is not "on the grid" but rather "off the grid".

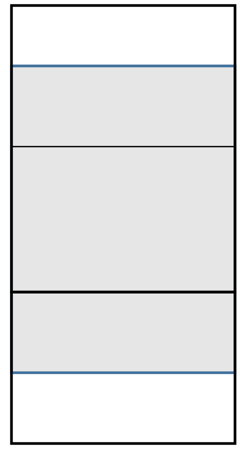

Figure [3] Width of two pixels in the off grid configuration.

The off grid configuration means that due to acquistion, the pixels at the endpoints are not entirely on or off.

2.1.1 (U) Program and Logic

To investigate the effects of the Nyquist Theorem on dectability we wrote a program to simulate the appearance of cracks in M1 shells. We then constructed line profiles across these artificial cracks to elucidate detectability.

For the "on grid" configurations that follow, the following MATLAB® logic was used. Each pixel in the simulated crack is set to 65,000. This corresponds to a white pixel.

For the "off grid" configurations that follow, the following MATLAB® logic was used. The pixels in the interior of the crack are set to 65,000, this being best case for detectability. The pixels at the endpoints are set to 32,000 for the first case study. We note that this half activation is the worst case in regards to detectability.

The program is provided in the appendix for the interested reader.

3. (U) Results and Discussion

We overlay the synthetic crack over actual M1 rounds and elucidate detectability using the line profile feature in MATLAB®. All Graphs that follow are magnified and included in the appendix under label, Figure XA.

3.1 (U) Synthetic Crack: Case Study 1

We begin with a synthetic crack that's two pixels wide. Figure [4] illustrates a synthetic crack of width two on an M1 shell. In this case, the crack is oriented in the "on grid" configuration. The crack is clearly visible to the naked eye. A line profile is drawn vertically, across the crack. A picture of the line profile, perpendicular to the crack is shown in Figure [5]. The graph of the line profile is shown in Figure [6].

Figure [4] Synthetic Crack of Width 2 snapped on the grid.

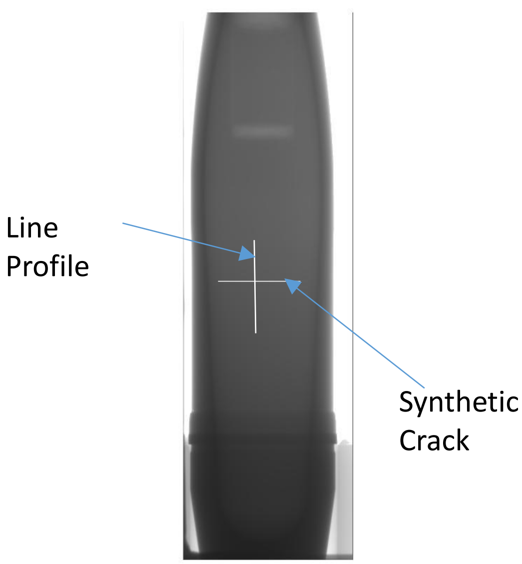

Figure [5] Synthetic Crack of Width 2 in the on grid configuration. A Line profile is shown perpendicular to the synthetic crack.

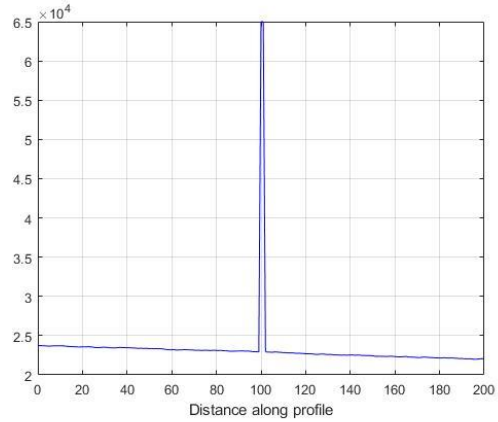

Figure [6] Line Profile of synthetic crack of width 2, in the "on grid" configuration. Gray value is plotted as a function of distance along the line profile.

A line profile plots the gray value as a function of position on the M1 Shell. Consider a line profile of a synthetic crack of width 2 in the "off grid" configuration. This is shown in Figure [7] below. We refer the reader to figures [6A] and [7A] in the appendix for larger graphical representations.

A few comments are in order. Notice the sharp peak of the line profile in the "off grid" vs. "on grid" configurations. There is a relatively simple explanation for this phenomonon. When the crack is in the "off grid" configuration one has effectively decreased the width of the crack.

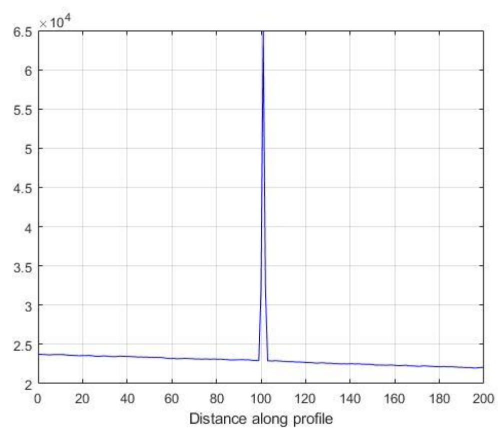

Figure (7) Line Profile of synthetic crack of width 2, in the off grid configuration.

Now consider a crack of width three. One can see both a quantitative and qualitative change in the line profile. Figure [8] and Figure [9] below provide the on and off grid arrangements repsectively.

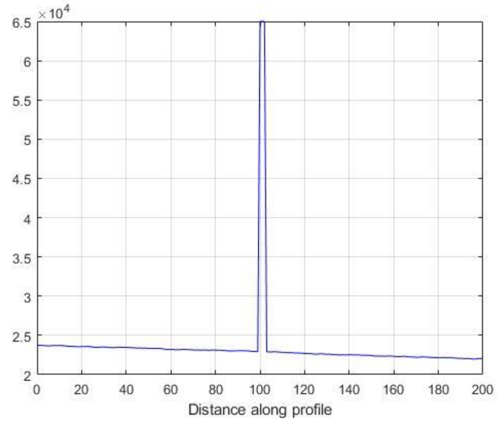

Figure [8]. Line Profile of synthetic crack of width 3, snapped on the grid.

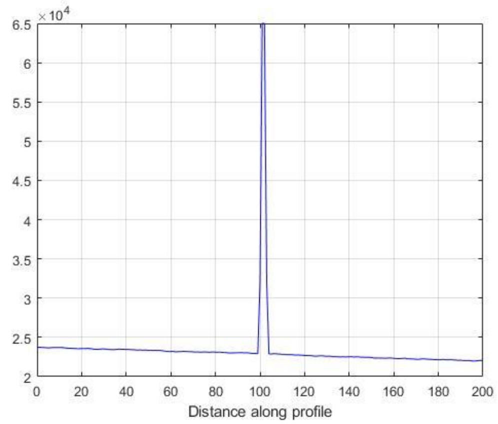

Figure [9] Line Profile of synthetic crack of width 3, snapped off the grid.

The sharp corner in the line profile of the synthetic crack of width 3, in the off grid configuration has been significatly reduced. One is effectively seeing an increase in crack width, now that an additional full pixel is allowed to become activated.

We now can return to the theoretical discussion introduced earlier in the paper. Once an effective pixel pitch has been obtained, one multiplies the effective pixel pitch by three, to obtain the smallest detectable flaw of a radiographic system. We refer to this as the Nyquist criterion.

3.2 (U) Synthetic Crack: Case Study 2

Recognize that the gray values of 65,000, and 32,000, were chosen arbitrarly. Using actual data of cracks, one can improve upon Case Study 1 and obtain better synthetic data of cracks in M1 Shells.

The appendix contains a short table of data collected from radiographs of M1 shells.

For our application here, it is sufficient to set the mean gray value of our synthetic cracks to the mean gray value that appears in the table. We demonstrate the results of our program for a crack of width 4 with mean gray value of 30,000.

Displayed below is the line profile. Figure 10 demonstrates the on grid configuration. One can clearly observe the absensce of the sharp corner that was present in Figure [7].

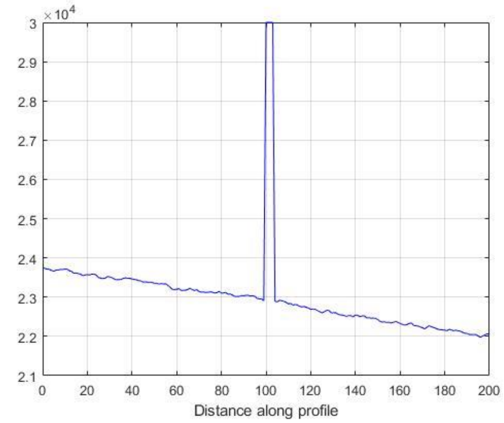

Figure [10] Line Profile of synthetic crack of width 4, snapped on the grid

In this example the Nyquist Theorem is satisfied and so the sharp corner in the line profile along the crack is absent.

4. (U) Conclusion

This paper successfully demonstrated the Nyquist Theorem as applied to Digital Radiography. We illustrated different types of phenomena that could occur and provided a qualitative and quantitative measure of reliable crack detection. We hope this paper illustrated some of the key concepts behind dectability. This paper also provided a first look into synthetic radiography. A bridge between theoretical and experimental radiography was established for the detection of defects and their physical representation in an image.

5. (U) Future Work and Acknowledgments

The issue of creating synthetic radiographs and synthetic flaws is an important area of research. As the laboratory moves towards applications in machine learning, increasing the volume of data will be increasingly improtant. This treatise was mainly geared towards the Nyquist Theorem and the detectability of cracks within shells. In a later paper, the issue of synthetic radiographs will be explored at greater depth.

Thank you to our colleague and friend Scott McClain for fruitful discussions and for proposing the idea of this project.

References

1. Rodriguez et al. X-Radiographic Parallax Reduction with 3D-Printed Fixturing. DTIC Technical Reports 2019
2. Digital Radiography, E. Seeram, Springer, Second Edition
3. Fundamentals of Electronic Image Processing, Columbus, OH: Personnel Training Publications, 2016

4. (U) Appendix

4.1 MATLAB® Code

The MATLAB® code is shown below so one can implement this program and or the principles therein.

```
cd 'C:\Users\Documents\Images\'

X = dicomread('MC00AL7B_2.dcm');
% Address:
["C:\Users\walter.s.rose5\Documents
\Images\Verified
tiffs\MC00C754_1.dcm"]

Y = X; %On to Grid

Z = X; %Off the Grid

n = 5;  %Set width of crack (n+1)

i = 1000; %Starting row

xline = [300 300];

yline = [900 1100];

%On Grid

for m = 0:n-1

        for j = 150:500

            Y(i+m,j) = 65000;

        end

end

%Off Grid

Z(i,150:500) = 32000;
Z(i+n,150:500) = 32000;

for m = 1:n-1

        for j = 150:500

            Z(i+m,j) = 65000;

        end

end

H = Y(999:1008,150:500);

I = Z(999:1008,150:500);

figure(1)
imshow(X);

figure(2)
imshow(Y);

figure(3)
imshow(Z)

figure (4)
improfile(Y,xline,yline), grid on;

figure (5)
improfile(Z,xline,yline), grid on;
```

4.2 Table of M1 Crack Statistics

Crack Width	Change in Gray Value	Mean Gray Value	Standard Deviation
7	700	23811	2.888742
4	80	15567	1.09559
8	200	24431	1.012248
4	120	22614	0.821074

4.3 Line Profiles

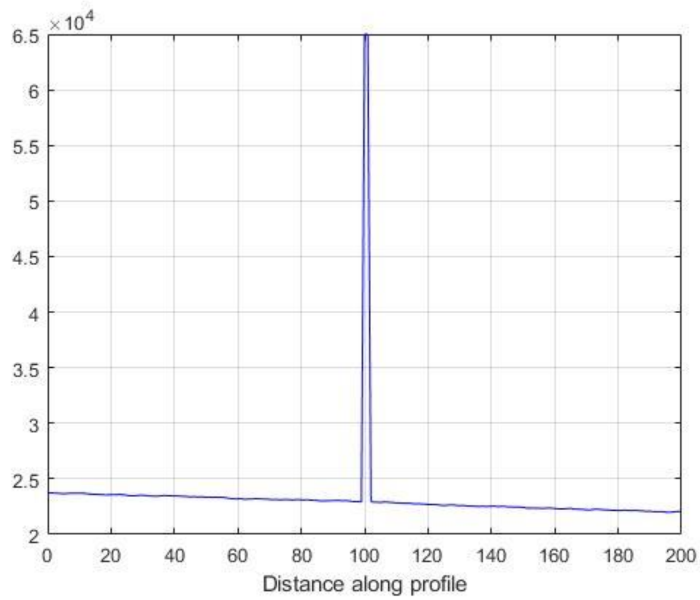

Figure (6A) Line Profile of synthetic crack of width 2, on grid configuration.

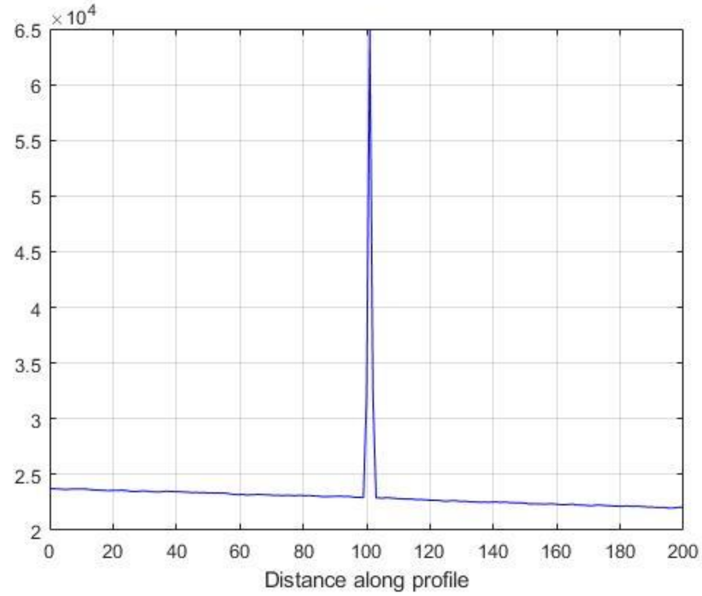

Figure (7A) Line Profile of synthetic crack of width 2, off grid configuration.

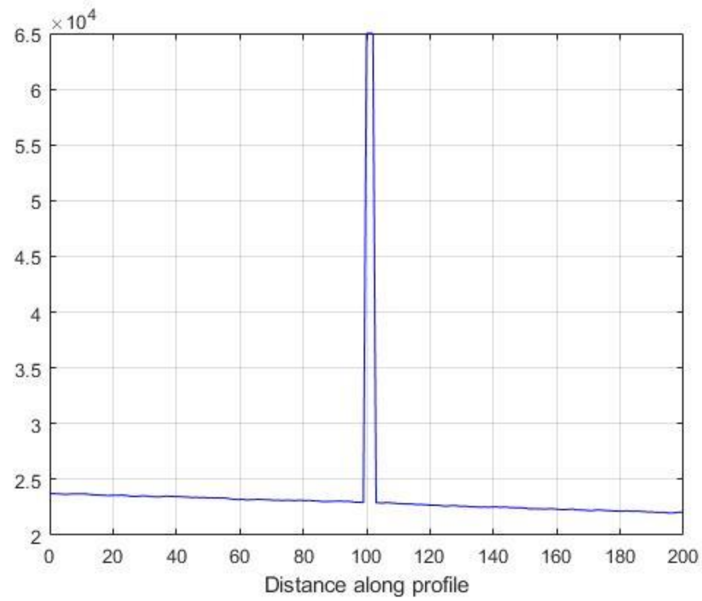

Figure 8A. Line Profile of synthetic crack of width 3, on grid configuration.

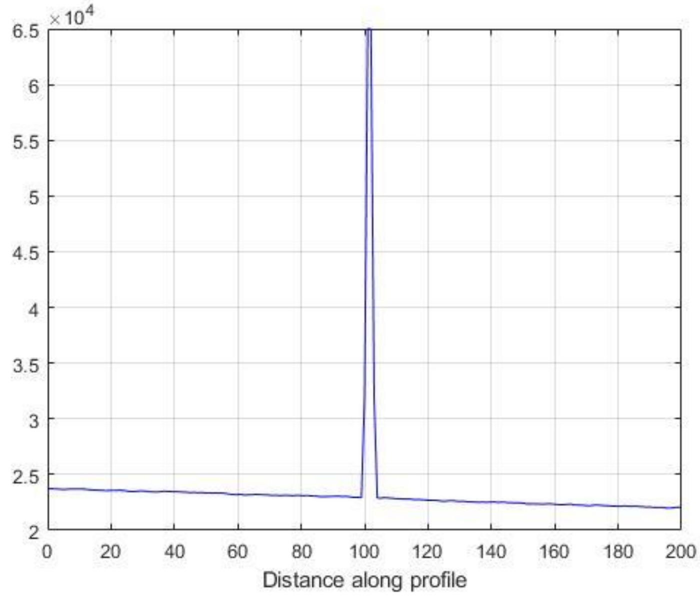

Figure (9A) Line Profile of synthetic crack of width 3, off grid configuration.

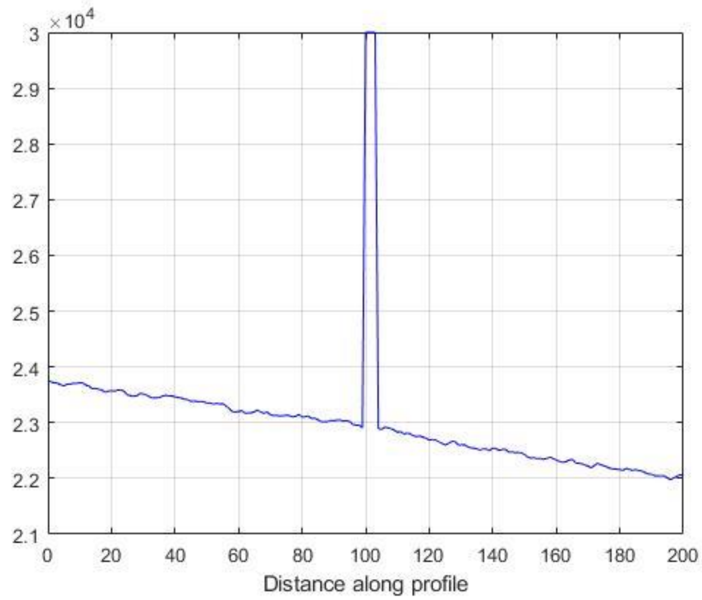

Figure (10A) Line Profile of synthetic crack of width 4, on grid configuration