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Investigation of 3D Shock

Jesse Little UNIVERSITY OF ARIZONA

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SBLI poses a	roadblock for t	the reliable ar	nd efficient operatio	n of high-spe	ed flight ve	hicles. A combined research approach,	
encompassin	g spatially and	temporally re	solved measureme	nts, highly ac	curate num	nerical simulations as well as local and	
global stabilit	y analyses, has	s been execut	ted for investigating	three-dimen	sional SBLI	. The 3D SBLI are generated by an	
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Final Report

Investigation of 3D Shock-Boundary Layer Interaction: A Combined Approach using Experiments, Numerical Simulations and Stability Analysis

Air Force Office of Scientific Research Contract number: FA9550-15-1-0430 Program manager: Dr. Ivett Leyva

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Abstract

Due to the renewed interest in sustained high-speed atmospheric flight, Shock Boundary Layer Interactions (SBLIs) are again receiving considerable attention. The incomplete understanding of the underlying physical mechanisms for both 2D and 3D SBLIs poses a roadblock for the reliable and efficient operation of high-speed flight vehicles. A combined research approach, encompassing spatially and temporally resolved measurements, highly accurate numerical simulations as well as local and global stability analyses, has been executed for investigating three-dimensional SBLIs. The 3D SBLIs are generated by an impinging oblique shock wave that is swept back with respect to the mean flow primarily at Mach 2.3. A specific area of focus is on SBLI-generated low-frequency unsteadiness due to its potential coupling with structural modes, influence on wall heat transfer and importance as an unresolved problem in high-speed aerodynamics research in general. Swept SBLIs generated purely by a shockinduced pressure rise have received surprisingly little attention in the literature despite their practical occurrence in scramjet inlets and isolators. Even a basic understanding of this SBLI was lacking at the award onset and (to our knowledge) we are the only research group examining this important flow. This report summarizes some of our contributions related to the understanding of swept impinging oblique SBLIs over the last three years. The findings are placed in the context of other more conventional 3D SBLIs (e.q. ramps and fins) wherever possible including those studied by other groups performing research in parallel to our own. For additional information, the reader is referred to our numerous publications on this topic that are listed herein.

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Nomenclature

Symbols

- Constant, $\Lambda = \sqrt{1 \left(\frac{M_{\infty}}{M}\right)^2 \frac{T_{\infty}}{T} \sin^2 \psi}$ Wave-number Δ Λ
- Λ
- Constant, $\Pi = \Lambda \frac{\cos \alpha_n}{\sin \psi} \frac{M}{M_{\infty}} \sqrt{\frac{T}{T_{\infty}}}$ Transformation matrix П
- Ω
- x_n -y plane streamline angle, deg α
- β Shock angle, deg
- Spanwise mode number β
- Ratio of specific heats, $\gamma = 1.4$ γ
- Boundary layer height, m δ_0
- δ^* Displacement thickness
- turbulent dissipation rate ε
- x-z plane flow deflection angle, deg η
- Spanwise deflection angle η
- θ x-y plane flow deflection angle, deg
- θ Wall-normal deflection angle, momentum thickness
- Sweep angle, Eigenvalue λ
- Dynamic viscosity μ
- Kinematic viscosity ν
- Fin inclusion angle, between \vec{z} and $\vec{y'}$, around \vec{x} , deg ξ
- Density ρ
- Shock angle σ
- τ Shear stress
- y-z deflection angle, deg ϕ
- ψ Angle of sweep, deg
- Spanwise vorticity ω_{z}
- Amplitude Α

$$C_p$$
 Pressure rise coefficient, $C_p = \frac{p - p_{\infty}}{q_{\infty} \cos^2 \psi}$

$$C_T$$
 Temperature rise coefficient, $C_T = \frac{T - T_{\infty}}{(T_0 - T_{\infty}) \cos^2 \psi}$

- Η Shape factor
- LLength scale, m
- L Length scale, streamwise domain extent
- \vec{N} Shock unit vector
- Q State vector in conservative variables
- QVortex identification criterion
- RSpecific gas constant, R = 287.1 J/kg/K
- S_{ij} Strain rate tensor
- TTemperature, K
- U Flow speed, m/s

- a_i Time coefficient
- \vec{a} Streamline unit vector, $\vec{a} = [u, v, w] / U$
- c_f Skin friction coefficient
- c_p Pressure coefficient, specific heat
- d_s Spanwise shift
- f Frequency
- h Height of shock generator from wall, m
- k Spanwise mode number
- p Pressure, N/m²
- q Dynamic pressure, $q = \frac{1}{2}\gamma p M^2 \text{ N/m}^2$
- **q** State vector in primitive variables
- *r* Radius from delta wing leading vertex, m
- t Time
- $u,\,v,\,w\quad {\rm Velocities}$
- x, y, z Coordinates

Subscripts

- 0 Stagnation conditions
- ∞ Freestream conditions
- T Turbulent
- *e* Angles defined in the effective 2D plane
- *i* Flow properties associated with the impinging shock
- i Incipient, mode number
- *int* Interaction
- n Component in $x_n y z_n$ reference plane
- r Flow properties associated with the reflected shock
- s Separation
- sg Shock generator geometries in xyz reference plane
- *ref* Reference conditions
- w Wall

Superscripts/Accents

- + In wall units
- * Dimensional quantity, density averaged
- ' Shock generator geometries in xy'z' reference plane
- ~ Horizontal shock generator geometries
- Geometries defined in downstream swept fin reference frame
- [•] Non-dimensional geometries

Dimensionless Numbers

M	Mach number	$M = U/\sqrt{\gamma RT}$
\Pr	Prandtl number	$\Pr = \nu / \alpha$
Re	Reynolds number	$\operatorname{Re} = \rho U L / \mu$
St	Strouhal number	$\mathrm{St} = fL/U$
Stk	Stokes number	$\mathrm{Stk} = \tau U/d$
Kn	Knudsen number	$\mathrm{Kn} = \lambda/L$

1. Introduction

1.1 SBLI Overview

After years of neglect, high-speed aerodynamics has once again become a focal point of intense research. In particular, interest in boundary layer transition and Shock Boundary Layer Interactions (SBLIs) is seeing a resurgence. For many technical applications, including engine inlets, body flaps, and missile fins, SBLI problems arise such as local heat peaks, unsteady aerodynamic loads, increase of drag, jet intake performance loss and flutter or intake buzz, to name only a few. The fundamentals of SBLI problems are discussed in text books such as Smits and Dussauge [1] and Babinsky and Harvey [2]. Recent progress on swept SBLIs is summarized in review papers such as Clemens and Narayanaswamy [3]. The introduction of a shock-wave causes a sharp pressure increase which can lead to boundary layer separation and the formation of a separation bubble. Such interactions were found to feature a low-frequency unsteadiness at separation and a high-frequency unsteadiness at reattachment [3]. The associated aerodynamic loads can be quite high and potentially destructive. This is especially the case for low-frequency unsteadiness which can possibly excite structural modes. According to Dolling [4] important quantities like peak heating in strong interactions and unsteady pressure peaks still cannot be predicted very accurately or even not at all, especially for complex geometries and flow fields.

Fundamental research into SBLI has sought to simplify the flow geometries into canonical non-dimensional forms. Three classical unswept examples are i) normal SBLIs [5, 6, 7], ii) compression ramp SBLIs [8, 9, 10, 11], and iii) shock reflection SBLIs [12, 13, 14, 15, 16] (see Figure 1.2 and Figure 1.1). Each of these relies on model geometry defined only in terms of angles, meaning the characteristic length scale of the incoming boundary layer is the only length to influence the interaction [17].



Figure 1.1: Unswept oblique impinging SBLI [18].

The focus of this work is primarily turbulent SBLIs, but laminar interactions can provide insight into the more complicated fully turbulent case. For a shock-induced separation bubble, Robinet [19] found a bifurcation of an initially 2D steady flow that, for increasing shock intensity, evolved into a 3D, stationary asymptotic state and eventually into a 3D unsteady state. A secondary recirculation within the primary separation bubble characterized the three-dimensionality of the interaction region along with a spanwise velocity component. A global analysis was performed to explain the physical origin of the three-dimensionality and 3D unstable global mode was found. Rist and co-workers [20, 21] examined the stability of hypersonic boundary layers over a compression ramp and a flat-plate with an impinging shock-wave. By comparing numerical simulations and experiments, they clearly showed that non-parallel effects led to increased growth rates of the disturbances. In laminar and turbulent interactions particular attention is focused on the unsteadiness of the shock motion at relatively low frequencies with respect to the characteristic frequency of the boundary layer. The characteristics



Figure 1.2: Schematic representation of an oblique shock wave impinging on a laminar boundary layer. Reproduced from Sansica et al. [29]

of unsteadiness related to the separation shock and separation bubble have been widely reported in the literature and strong similarities are seen between various configurations commonly studied in SBLI, such as compression ramps, blunt fins, and impinging shocks. Reasonable agreement has been reported between numerical simulations and experiments but the mechanisms that drive this unsteadiness remain unclear.

Some of the first, systematic experimental studies on laminar and turbulent boundary layers interacting with shock–waves were conducted by Ackeret et al. [22] and Liepmann [23]. Recent work in SBLI has mainly focused on interactions that are two–dimensional in the mean (2D SBLI) and typically generated in the laboratory by an impinging oblique shock–wave or compression corner. The SBLI generated in this manner share common features and generally exhibit both low– and high–frequency unsteadiness. In the recent work particular focus has been placed on identifying the source of the low–frequency unsteadiness. The frequency associated with the low–frequency unsteadiness is about one to three orders of magnitude lower than the dominant characteristic frequency of the incoming turbulent boundary layer. In several practical applications the low–frequency unsteadiness can couple with the structural dynamics of control surfaces and fins or affect the performance of air–breathing propulsion systems. In such instances, in–flight structural fatigue and catastrophic loss of the vehicle may occur.

1.2 SBLI Unsteadiness

A variety of reasons for the mechanisms causing the low-frequency behavior have been proposed in the literature for unswept SBLIs. Recent experimental studies have suggested that the low-frequency unsteadiness may be correlated to longitudinal coherent structures (streaks) in the incoming turbulent boundary layer (see Ganapathisubramani et al. [10, 24] and Humble et al. [25, 26]). Some other studies have suggested that the unsteadiness arises from the dynamics of the separation bubble itself [27]. Large-eddy simulations (LES) performed by Touber & Sandham [28] showed that the coherent structures in the incoming boundary layer are not a prerequisite for low-frequency unsteadiness. However, they suggest that the streaks could excite intrinsic SBLI dynamics resulting in the behavior observed by Ganapathisubramani et al. [10, 24] and Humble et al. [25, 26].

Although some of the earlier research points to the relevance of underlying hydrodynamic instabilities, the research findings are contradictory and therefore inconclusive. Thus, there is a clear need for a fundamental and unequivocal understanding of the hydrodynamic instability mechanisms that are relevant for SBLI. The incomplete understanding of the underlying physical mechanisms for SBLI (in particular with regard to the low–frequency oscillations) poses a roadblock for the reliable and efficient operation of high–speed atmospheric flight vehicles. This lack of understanding is also the reason why effective and efficient active flow control (AFC) techniques are so difficult to realize for SBLI.

CHAPTER 1. INTRODUCTION

Piponniau et al. [14] related the low-frequency unsteadiness to a breathing motion of the separation bubble which can be associated with a flapping of the mixing layer that develops downstream of the reflected shock. Souverein et al. [15] varied the shock angle and the Reynolds number. When the reverse flow in the interaction was similar, the unsteadiness did not depend on the Reynolds number. Instantaneous flow visualizations revealed the shedding of spanwise coherent structures. Ganapathisubramani et al. [10, 24] observed elongated regions (superstructures that are eight or more boundary layer thicknesses long) of low-velocity fluid in the log region upstream of a Mach 2 compression ramp that led to spanwise deformations of the separation line. They concluded that the temporal behavior of the separation line resulted from the superstructures as well as very-low-frequency oscillations of the boundary layer and downstream separation bubble effects. Similar findings were made by Humble et al. [25] who referred to the spanwise deformations as "rippling patterns". Experiments by Dussauge et al. [27] suggest that the three-dimensional (3-D) structures of the separation bubble may be at the origin of the low-frequency unsteadiness. Morgan et al. [30] performed a large-eddy simulation (LES) of a Mach 2.05 unswept interaction. Flow visualizations revealed a spanwise "undulation" of the reflected shock near the top of the separation bubble which appeared to correlate with large turbulent structures in the approach boundary layer. Blinde et al. [31] employed micro vortex generators for introducing spanwise perturbations into the boundary layer which lead to a spanwise deformation of the separation line. Interestingly, this stabilized the shock motion for a straight interaction. Similar observations were made by Babinsky et al. [32] and Giepman et al. [33] which may suggest that the low-frequency unsteadiness is related to the spanwise deformations or ripples. Priebe and Martin [11] investigated a Mach 2.9 compression ramp flow. The low-frequency shock motion was found to be well correlated with a breathing motion of the bubble and an associated flapping of the separated boundary layer but less so with upstream boundary layer events. The conjecture was made that an inherent instability of the downstream separated flow, similar to the global mode obtained by linear stability theory by Touber and Sandham [28], was responsible for the low-frequency unsteadiness. These findings are in contrast to Ganapathisubramani et al. [10, 24] and Humble et al. [25] who considered weak separations. Priebe and Martin [11] argue that both upstream and downstream effects are always present and that upstream perturbations become more important for weakly or incipiently separated interactions.

1.3 3D SBLI Overview

Highly three-dimensional geometries associated with supersonic engine intakes and control surfaces can induce complex shock systems which promote SBLIs and potentially undesirable effects. These often swept interactions can significantly influence efficiency, unsteady loading and surface heating [4]. It is therefore of paramount importance to understand 3D SBLIs so that designs may be improved in future vehicles. Fundamental studies of various SBLIs have often sought to retain a non-dimensional form, whereby the only length scale is the boundary layer which offers a more generalized flow-field [17]. The introduction of sweep retains this form, enabling further analysis of fundamental SBLI physics. Investigation of swept SBLIs have been relatively limited compared to unswept research efforts. Non-dimensional means of inducing a swept shock have been restricted to sharp fins (straight [34, 35, 36, 37, 38, 39] and swept [40]), and swept compression ramps [41, 42, 43, 44, 45, 46, 47, 48, 49].

A new form of canonical interaction has been studied at University of Arizona (UA) and New Mexico State University (NMSU) featuring a swept impinging oblique shock [50, 51, 52, 53]. Until recently, only one study has been reported in literature and is limited to hypersonic flow [54]. Like it's unswept counterpart, this configuration offers a fundamental flow which may be experienced on a variety of high-speed vehicles and often related to inlets. Figure 1.3 shows a schematic of the expected quasi-infinite span inviscid shock structure [55]. Much like unswept flows, the impinging shock reflects off the floor to form a secondary reflected shock. The pressure rise across both shocks therefore contributes to the adverse pressure gradient and may induce flow separation. The inviscid shock structure does not directly correspond to a swept form of the unswept configuration, but can be analytically determined using thin-shock theory [55]. In this case, the addition of sweep acts to increase the strength of the shocks, causing them to steepen, moving the floor impingement location further upstream.

While a large body of research is concerned with unswept SBLI problems, swept interactions which are certainly not a rarity, have not been investigated in great detail. Different from unswept interactions, which can be thought of as infinite in the spanwise direction, swept interactions can have a virtual origin. For example, a shock-wave originating from a conical shock generator will have an origin that is related to the tip of the shock generator. Settles and Kimmel [36] proposed that depending on the boundary layer and shock generator properties, the separation can either exhibit cylindrical or conical similarity. Conical similarity implies that the separation opens up in the spanwise direction while cylindrical similarity implies that the streamwise extent of the separation



Figure 1.3: Equivalent swept shock-inducing model geometries [55]. The red plane indicates impinging shock, while the blue plane indicates reflected shock. This thick black line indicates the model leading edge that coincides with the incident shock plane (note, not necessarily with x-z plane).

remains constant away from the similarity plane. It may be argued that far enough downstream of the origin, the separation will always asymptotically approach cylindrical similarity. Experiments by Erengil and Dolling [49] and Reynolds-Averaged Navier-Stokes (RANS) calculations by Holst and Schmisseur [56] of swept compression ramps revealed conical similarity of the separated region for sweep angles between 10° and 50°. This agrees with highly-swept glancing interactions associated with sharp-fins that retain conical similarities and demonstrate a growth of length scales as the interaction develops in the direction of sweep [41, 40, 38]. The situation for lesser-swept interactions induced by compression ramps is more ambiguous. Early studies reported a cylindrical similarity [41]; however, it is not yet clear if this observation is influenced by the limited domain size associated with most supersonic wind tunnel tests [43]. Regardless, two isolated regimes of interaction development have been observed (cylindrical/conical). The mechanisms driving the appearance of conical similarity for swept SBLIs remain unclear, but have been associated with inviscid flow detachment [41].

The effect of sweep on SBLI unsteadiness remains under investigation [42, 52, 49, 57, 58]. Experimental results show that as sweep is increased, the low-frequency motion of the separation shock reduces in strength and translates to higher frequencies [39] but the mechanism behind this observation is not established [59, 49]. Erengil and Dolling [49] found large-amplitude, low-frequency oscillations for cases with cylindrical similarity and more benign low-amplitude, high-frequency oscillations for cases with conical similarity. Holst and Schmisseur [56] argue that large portions of the recirculating separated flow are supersonic for cases with high sweep angles which prevents disturbances from traveling upstream. Adler and Gaitonde [42] simulated Mach 2 turbulent compression corner flows with 22.5° and 37.5° sweep angle and found no indications of a low-frequency unsteadiness. They argue that compared to unswept interactions (which they refer to as "closed" separations), for swept interactions (which they refer to as "closed" separations), for swept interactions (which they refer to as good indicator for the onset of instability since disturbances are convected outward in the spanwise direction.

The following sections summarize our combined approach for studying swept impinging oblique SBLIs over the last three years. The dearth of knowledge on this fundamental and practically relevant flow required us to start from the very basics. We have made contributions to the understanding of the inviscid behavior progressing all the way through highly resolved studies of the instantaneous flow structure in both experiments and simulations. From a practical perspective, our findings can be used to improve the design of scramjet inlets and isolators through better prediction of the mean flow topology as well as the low-frequency unsteadiness, wall heat transfer and their potential minimization.

2. Inviscid Analysis of Swept Oblique Shock Reflections

An inviscid flow model is presented to gain a basic understanding of the reflection of a swept oblique shock from a planar wall. The analytical model is constructed to describe the fundamental influence of sweep on this shock configuration which has been commonly studied as an unswept non-dimensional SBLI. Transformation of model parameters into a plane perpendicular to the sweep angle reduces the resultant flow to a two-parameter system. An equivalency between this configuration and others commonly assessed is presented with advisory notes on the definition of effective coordinate systems. Inviscid shock detachment has been associated with the onset of quasi-conical SBLI spanwise development [41]. Its occurrence for this SBLI configuration is determined for a range of conditions and compared to experimental observations of swept SBLIs claiming cylindrical/conical similarity scalings. Finally, influence of a zero-mass flux plane associated with typical experimental and numerical analyses is presented with an accompanying model for the shock structure. This serves as useful resource when designing swept impinging oblique SBLI studies, it also provides a vital benchmark for this complex configuration and helps to unify various SBLI configurations that are often analyzed in isolation.

2.1 Background

Before details of the SBLIs are assessed with regards to sweep, it is important to note that there appears to be a broad oversimplification in literature on the influence of sweep on the incident shock itself. A common misconception is that sweep simply adds an additional component of velocity running parallel to the shock which can be disregarded as it will not change across the shock. While it is true that this velocity component is present, it does not fully evaluate changes to the shock due to sweep. A feature often overlooked is that as sweep increases, the deflection angle normal to the sweep also increases, affecting shock strength. It is well-known that two-dimensional turbulent SBLIs scale with the inviscid pressure rise across an interaction [60, 61]. It is reasonable to assume that 3D SBLIs will follow suit particularly for moderate sweep. Subtle changes in the inviscid pressure rise due to sweep are expected to be important especially in laminar and transitional SBLIs. Thus, accurate understanding/prediction of the inviscid behavior is essential for the study 3D SBLIs.

For inviscid swept compression ramp flows, the shock is anchored to the ramp corner (when inducing an attached oblique shock). Variations of the shock (in terms of strength and angle) caused by moderate changes in sweep for a given x-y plane ramp deflection are often overlooked since the influence can be small. While such variance has been observed in inviscid flows [62, 51], their influence in viscous interactions has not been factored into characterization studies.

Infinite-span swept oblique inviscid shock structures have been modeled in an important contribution by [62], who extend the standard two-dimensional θ - β -M relations to a three-dimensional θ - β -M- ψ form to include the influence of sweep on oblique shocks. The model is formulated in terms of a cubic expression that can be solved to determine shock characteristics. However, the specific formulation is primarily applicable to single-shock structures (such as those from swept sharp-fin or compression ramps). Without significant manipulation, it is therefore of limited application to multiple-shock configurations with exception of validating the initial shock [51]. In addition, the implications of limited span oblique shocks that can be critical in moderate aspect ratio flows associated with engine inlets and highly-swept delta-wings are not modeled.

For interactions limited in the spanwise direction by a zero spanwise mass flux criteria (either through an experimental symmetry plane [40], simulation slip plane [42, 53], or an endplate/fence [51, 63]) the shock angle also changes at this location [40]. This effect is small for inviscid swept compression ramp flows since the shock foot is anchored to the ramp corner. However, it cannot be overlooked in swept impinging oblique SBLIs since changes in the shock angle alters the wall-impingement location, meaning the interaction is no longer anchored at a region on the wall that runs parallel to the imposed sweep. Such an influence is not encountered in unswept sharp-fin flows since the inviscid flow remains two-dimensional. However, it has a significant effect on swept



Figure 2.1: Inviscid shock schematic diagram. The incident shock is shown in magenta, the reflected shock is shown in blue. The incident shock-wall impingement line is shown in red.

sharp-fins since the inviscid shock location at the wall does not agree with simple 2D shock theory [40].

Since the incident swept oblique shock is inclined from normal in both the x-y and x-z planes, one would expect an element of spanwise crossflow in addition to the downward flow deflection induced by the shock generator. After encountering the reflected shock this effect would be amplified, creating a component of spanwise flow that would be experienced downstream of the interaction. Restriction of this crossflow through the use of a fence [43] or symmetry plane [40] would stifle this injection of mass into the swept SBLI, causing the shocks to decrease in strength to account for the downstream divergence (analogous to two-dimensional vs. axi-symmetric compression ramps). This will result in a spanwise pressure gradient within the interaction near the root plane, and potentially a curved shock-wall impingement line in the case of a swept oblique impinging SBLI. Such influences will distort the flow beyond 2D features present in a quasi-infinite span configuration.

2.2 Model description

The model geometry employed herein is defined in figure 2.1. The x-y-z reference frame is set by: x) the incoming flow vector, y) the wall-normal vector, and z) the resultant spanwise vector. Three-dimensional velocity components u-v-w are defined relative to the x-y-z reference frame, respectively. A swept shock generator is located above the wall and is defined according to the xyz-reference frame to retain a fundamental approach. The leading edge is located in the x-z plane and runs parallel to the wall. Sweep ψ is defined in the x-z plane as the rotation of the leading edge vector around the y axis, relative to the z axis. The angle of the shock generator surface relative to the incoming flow is defined in the x-y plane relative to the x axis. The downward flow deflection beneath the shock generator produces a swept oblique shock which impinges upon the wall. By imposing a slip-plane condition on the wall the incident shock is reflected and forms a second swept oblique shock. Spanwise influences are disregarded, resulting a quasi-2D flow field. Where dimensional values have been quoted, the gas medium is assumed to be an inviscid perfect gas, with $\gamma = 1.4$ and R = 287.1 J/kg/K to reflect dry air. Flow across shocks is considered to be adiabatic, as are conditions at the wall.

The application of continuity across an oblique shock mandates that the components of velocity running parallel to the shock are conserved. For convenience, the reference frame is transformed via rotation about the y axis using transformation matrix $\mathbf{\Omega}$ to be aligned with the incident shock-wall impingement line, such that the transformed z_n axis runs parallel to this line [64] (see figure 2.2 and equation 2.1).

$$\begin{bmatrix} u_n \\ v \\ w_n \end{bmatrix} = \mathbf{\Omega} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u\cos\psi - w\sin\psi \\ v \\ u\sin\psi + w\cos\psi \end{bmatrix}$$
(2.1)



Figure 2.2: Inviscid shock schematic diagram, rotated around the y-axis such that the side view in the x_n -y plane is aligned with the shock.

where:
$$\mathbf{\Omega} = \begin{bmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix}$$

Since the z_n -axis runs parallel to the wall-impingement line, it also runs parallel to both shocks. Therefore, the w_n velocity component is considered constant across the entire shock reflection. Equation (2.2) is used to determine velocity magnitude as a function of total temperature T_0 , with the sweep-aligned velocity component established using equation (2.3). By disregarding this velocity component, the equivalent incoming Mach number for a two-dimensional shock reflection in the x_n - y_n plane is identified using equation (2.4).

$$U^{2} = \frac{2\gamma RT_{0}M^{2}}{2 + (\gamma - 1)M^{2}}$$
(2.2)

$$w_n = U_\infty \sin \psi \tag{2.3}$$

$$M_{\infty n} = M_{\infty} \cos \psi \tag{2.4}$$

The flow deflection angle in the x_n -y plane will be steeper than that measured on the shock generator in the x-y plane due to the addition of sweep. This angle is calculated using equation (2.5). Determination of the remaining flow structure in the x_n -y plane follows a standard two-dimensional approach with well-established relations. Shock angle β_n and incoming flow angle α_n are defined relative to the x_n - z_n plane (all angles are defined with same rotation direction around the z_n axis for a given shock, such that β_n and θ_n are positive). Equation (2.6) is used to find the shock angle in the x_n -y plane, with the downstream x_n -y plane Mach number given by equation (2.7) (subscripts a and b indicate conditions upstream or downstream of the shock being analyzed, respectively).

$$\tan \theta_n = \frac{\tan \theta_{sg}}{\cos \psi} \tag{2.5}$$

$$\tan \theta_n = \frac{2}{\tan \left(\beta_n - \alpha_n\right)} \left\{ \frac{M_{a_n}^2 \sin^2 \left(\beta_n - \alpha\right) - 1}{M_{a_n}^2 \left[\gamma + \cos \left(2\beta_n - 2\alpha_n\right)\right] + 2} \right\}$$
(2.6)

$$M_{bn}^{2} \sin^{2} (\beta_{n} - \alpha_{n} - \theta_{n}) = \frac{2 + (\gamma - 1) M_{an}^{2} \sin^{2} (\beta_{n} - \alpha_{n})}{2\gamma M_{an}^{2} \sin^{2} (\beta_{n} - \alpha_{n}) - 2 (\gamma - 1)}$$
(2.7)

The inclination of the shock and the downstream flow vector are first found in the x_n -y plane, before applying the inverse transformation matrix from equation (2.1). The shock inclination is defined using a unit vector normal to the shock plane given by equation (2.8), where α_n is the angle of the incoming flow relative to the x_n - z_n plane.

$$\vec{N}_n = \begin{bmatrix} \sin \beta_n \\ \cos \beta_n \\ 0 \end{bmatrix}$$
(2.8)

The downstream flow is similarly defined using a unit vector, which requires recombining velocity components downstream of the shock. While the w_n component is already known from equation (2.3), definition of u_n and v components necessitates finding the downstream Mach number M_2 using equation (2.9), which is then converted back to flow speed U using equation (2.2). The velocity magnitude in the x_n -y plane is then given by equation (2.10), with the flow unit vector given by equation (2.11).

$$M^{2} = \frac{M_{n}^{2} + \frac{w_{n}^{2}}{\gamma R T_{0}}}{1 - (\gamma - 1) \frac{w_{n}^{2}}{2\gamma R T_{0}}} = \frac{M_{n}^{2} \left[2 + M_{\infty}^{2} \left(\gamma - 1\right)\right] + 2M_{\infty}^{2} \sin^{2} \psi}{2 + M_{\infty}^{2} (\gamma - 1) \left(1 - \sin^{2} \psi\right)}$$
(2.9)

$$U_n^2 = U^2 - w_n^2 \tag{2.10}$$

$$\vec{a}_n = \begin{bmatrix} u_n \\ v \\ w_n \end{bmatrix} = \frac{1}{U} \begin{bmatrix} U_n \cos \alpha_n \\ U_n \sin \alpha_n \\ U_\infty \sin \psi \end{bmatrix} = \frac{M_\infty}{M} \sqrt{\frac{T_\infty}{T}} \begin{bmatrix} \Lambda \cos \alpha_n \\ -\Lambda \sin \alpha_n \\ \sin \psi \end{bmatrix}$$
(2.11)

where:
$$\Lambda = \sqrt{\frac{M^2T}{M_{\infty}^2T_{\infty}} - \sin^2\psi}$$

Unit vectors describing shock inclination and flow direction in the x_n -y plane are then transformed, using the Ω' transformation matrix in equation (2.1), to return components in the xyz reference frame (see equation 2.12) and equation 2.13). The ratio T_{∞}/T is calculated using adiabatic relations (see equation 2.14).

$$\vec{N} = \mathbf{\Omega}' \vec{N}_n = \sin \beta_n \begin{bmatrix} \cos \psi \\ 1/\tan \beta_n \\ -\sin \psi \end{bmatrix}$$
(2.12)

$$\vec{a} = \mathbf{\Omega}' \vec{a}_n = \begin{bmatrix} \sin^2 \psi + \Lambda \cos \alpha_n \cos \psi \\ -\Lambda \sin \alpha_n \\ \sin \psi \left(\cos \psi - \Lambda \cos \alpha_n \right) \end{bmatrix} \frac{M_\infty}{M} \sqrt{\frac{T_\infty}{T}}$$
(2.13)

$$\frac{T_{\infty}}{T} = \frac{2 + (\gamma - 1)M^2}{2 + (\gamma - 1)M_{\infty}^2}$$
(2.14)

Pertinent angles of the flow structure are determined from these vectors. The x-y plane shock angle is given by equation (2.15), the x-y plane flow deflection angle is given by equation (2.16), the x-z plane spanwise deflection angle is given by equation (2.17), and the y-z plane flow rotation angle, which defines the effective two-dimensional oblique shock plane [62], is given by equation (2.18).

$$\tan \beta = N_x / N_y = \cos \psi \cdot \tan \beta_n \tag{2.15}$$

$$\tan \theta = -\frac{a_y}{a_x} = \frac{-\Lambda \sin \alpha_n}{\sin^2 \psi + \Lambda \cos \alpha_n \cos \psi}$$
(2.16)

$$\tan \eta = a_z/a_x = \frac{\cos \psi - \Lambda \cos \alpha_n}{\sin \psi + \Lambda \frac{\cos \alpha_n}{\tan \psi}}$$
(2.17)

$$\tan \phi = -a_z/a_y = \sin \psi \left(\frac{1}{\tan \alpha_n} - \frac{\cos \psi}{\Lambda \sin \alpha_n} \right)$$
(2.18)

Finally, angles of flow deflection and shocks defined on the effective 2D plane are given by equation (2.19) and equation (2.20), respectively.

$$\tan \theta_e = \frac{\tan \theta_{sg} \cos \xi}{\cos (\phi - \xi)}$$
(2.19)

$$\tan \beta_e = \tan \beta \cos \phi \tag{2.20}$$

where: $\tan \xi = \tan \theta_{sg} \tan \psi$

2.3 Results

2.3.1 Summary of results

Implementation of the inviscid model above is shown in figure 2.3 for two example flows with upstream Mach numbers of 2.3 and 3.0, each encountering a moderately swept shock generator ($\psi = 22.5^{\circ}$ and $\theta = 12.5^{\circ}$) mounted parallel to a slip-wall floor below. Incoming flow encounters the incident shock and is deflected downwards and towards the angle of sweep. Since the inviscid flow downstream of the shock must remain parallel to the shock generator face, the resultant deflection may be considered as a combination of two components: i) x-y plane deflection that is below that of the shock generator (as flow passes the face towards the tip), and ii) the additional x-z plane deflection induced by the oblique sweep angle.

Both flows clearly demonstrate the key features of inviscid shock reflections, namely that flow downstream of the reflection is: i) parallel to the wall, ii) deflected towards the sweep angle, iii) negatively skewed in the x-y plane, and iv) positively skewed in the x-z plane. In addition, shock inclination changes with sweep leading to variable strength shocks affecting downstream pressure rise.

Figure 2.4 shows a corresponding shock structure plot extracted in an x-y plane, equivalent to commonly reported PIV domains. The effect of a sweep is clearly evident, increasing steepness of shocks and moving the wall-impingement location further upstream.



(b) Mach 5.0.

Figure 2.3: Inviscid flowfields with $\psi = 22.5^{\circ}$ and $\theta_{sg} = 12.5^{\circ}$. The shock generator is shown in grey. The incident shock is shown in magenta, the reflected shock is shown in blue. The incident shock-wall impingement line is shown in red. Streamlines are shown by thin green lines, and timelines are shown by broad green regions. The dashed black line indicates a cross section of the shock system in the x-y plane. Geometries are shown to scale.



Figure 2.4: x-y plane shock structure normalized by shock generator height h, demonstrating effect of sweep for various incoming Mach numbers. Shock generator with deflection $\theta_{sg} = 12.5^{\circ}$ is shown in grey. Swept shocks at $\psi = 22.5^{\circ}$ are shown with solid lines, unswept shocks are shown with dashed lines.

2.3.2 Model limitations

The model above relies on analytical inviscid flow relations. It is therefore not applicable when inviscid shock detachment occurs as flow is deflected beyond the maximum turning angle for a given local upstream Mach number. In such situations, the shock will translate upstream and curve towards the wall to form a local subsonic region (in a plane normal to the sweep angle) that enables flow to achieve the required deflection. The structure of this region is not analytically solvable [40] and is therefore beyond the scope of this model. Regardless, the onset of this regime is clearly defined such that at conditions below the detachment onset boundary the present model is valid. Prediction of the onset of shock detachment is particularly relevant in swept SBLI studies as this has been proposed to act as a mechanism that changes the interaction from cylindrical (*i.e.* parallel lines of separation and reattachment) to conical (*i.e.* divergent separation and reattachment lines) [41].

As with unswept oblique shocks, there are two potential inviscid shock solutions for a given sweep-Mach number combination: a steep *strong* shock with subsonic downstream flow (when observed in the sweep-aligned x_n -y plane), or a shallower *weak* shock which typically exhibits local supersonic downstream flow (except at conditions close to the maximum turning angle where it too may return subsonic flow). While it is *weak* shock solutions that would usually form in nature, *strong* oblique shocks may be induced in situations of high imposed back-pressure [65]. Full implementation of the above model requires that flow must be supersonic downstream of the incident shock (when viewed in the x_n -y plane), thus limiting the incident shock solution to the supersonic weak case only. The reflected shock may exhibit either solution through selection of the appropriate β_n . If the required deflection to redirect flow parallel to the floor is greater than the maximum turning angle then the reflected shock will detach to form a Mach Reflection [66] (as opposed to the Regular Reflection studied herein) and the model becomes invalid.

Figure 2.5 demonstrates the onset of the maximum turning angle limitations with respect to shock generator x-y deflection and sweep, which is of importance when considering the proposed onset of conical/cylindrical SBLI development [41]. Increasing the upstream Mach number expands the available envelope for both sweep and shock generator deflection angles. Sweep at low angles of shock generator deflection is limited by the local upstream Mach number component normal to the sweep is subsonic and does not induce a shock. However, below this upper limit in sweep excessive flow deflection beyond the maximum turning angle will result in shock detachment. At the zero-sweep limit this aligns with unswept 2D shock detachment. The behavior across the reflected shock is similar in nature to the incident shock, albeit shifted towards lower shock generator deflection angles. This is due to the reduction in Mach number across the incident shock, an effect which diminishes at low deflection angles, and is evident by maintaining the same zero-deflection sweep limit as for the incident shock case.



Figure 2.5: Onset of shock detachment in the oblique shock reflection configuration. Geometries that fall below or to the left of the contours represent flows that would induce an attached shock.

2.4 Discussion

2.4.1 Collapse of sweep influence

Initial observation of the results would suggest that flow configuration is subject to three input terms: i) upstream Mach number M_{∞} , ii) x-z plane sweep ψ , and iii) x-y plane deflection angle θ_{sg} . This results in a reasonably complex model imposing difficulties in extracting certain behaviors, as demonstrated by shock detachment predictions given in figure 2.5. However, certain observations can be made in construction of the model to simplify results.

Determination of the shock strength and associated parameters $(M_{2n}, p_2/p_1, T_2/T_1, etc.)$ are all either defined in the swept x_n -y plane or defined as a scalar. Therefore, the three-variable problem stated above can be rephrased in this coordinate system to constrain the influence of sweep only in its effect on the Mach number and deflection angle. The upstream Mach number and shock generator deflection angle should therefore be expressed as the respective component experienced in the plane normal to the sweep $(x_n-y \text{ plane})$ as given by equation (2.4) and equation (2.5), respectively. This manipulation results in the collapse of the three-parameter model into two parameters only, simplifying the parametric approach required.

This approach has been followed in unswept sharp fin studies which have typically resolved Mach number normal to the swept shock as the relevant scaling term [34]. Since the shock induced by an unswept sharp fin is not inclined in the x-y plane and occurs on a flat plate, it follows that the inviscid flow deflection normal to the sweep is zero and the shock acts as a normal shock with a cross flow component.

Addition of sweep to the sharp fin leading edge (or shock generator leading edge) therefore necessitates inclusion of this effect when considering flow parameters across the inviscid shock. The effectiveness of this approach is demonstrated below through a revised prediction of shock detachment, as has been discussed prior in the three-parameter form.

2.4.2 Detachment phenomena

Shock detachment is particularly relevant in swept SBLI research as it has been postulated as a mechanism that brings about a substantial change in interaction behavior [41]. Moderately swept SBLIs with low deflection angles have been observed to develop in the spanwise direction towards a quasi-infinite interaction with length scale L_{int} , presenting *cylindrical* geometric similarities when scaled [41] (figure 2.6a). Conversely, highly swept SBLIs with more significant deflection angles demonstrate a continual linear geometric growth of the interaction structure in the spanwise direction described by *conical* similarities [41, 36] (figure 2.6b). The mechanism responsible for this change in behavior is not well understood. However, it has been associated with the following concepts: i) a fundamental change in 3D separated flow mass entrainment/ventilation [41], ii) the onset of shock detachment [41], iii) sustained influence of the inception region in finite span domains [43], or iv) effect of non-planar shock distortion at the root [40, 36].

Determination of the detachment onset boundary has been presented in figure 2.5 in terms of typical parameters used in experiments and simulations $(M_{\infty}, \theta_{sg}, \text{ and } \psi)$. By instead considering the incoming Mach number and flow deflection components in the x_n -y plane, this can be plotted as shown in figure 2.7. The resultant curves now collapse to form unique detachment onset boundaries when solved in a two-parameter domain.

Inviscid model results for both one-shock solutions (*i.e.* compression ramp or sharp fin) and two-shock solutions (*i.e.* shock reflection) are shown in figure 2.7a. As observed prior, the qualitative nature of the two boundaries appear similar, albeit shifted to higher incoming Mach numbers for detachment of the reflected shock due to the reduction in local upstream Mach number caused by the incident shock.

Experimental data from swept compression ramps [41], and unswept/swept sharp fins [40, 34] in Mach 2.95 and 3.95 flows are included in figure 2.7b for reference. The Mach number in the ordinate axis is instead defined here using M_{slip} rather than freestream Mach number M_{∞} , as proposed by Settles and Teng [41]. They argue that onset of shock detachment is dictated by lower-speed fluid within the boundary layer, at an effective height where the flow can be considered inviscid when simplified into a two-layer model [67]. A value of $M_{slip} = 0.75 M_{\infty}$ is found to result in a good collapse of data between experimental observations and the inviscid model presented here. Similar behaviour is not witnessed in sharp fin interactions, suggesting another mechanism is responsible for promoting conical behavior in that instance (*e.g.* forced reattachment due to presence of sharp fin surface [68]).

The similarity between experimental compression ramp data using M_{slip} and results of the inviscid model is encouraging. While some level of uncertainty is to be expected when estimating the similarity state of an



Figure 2.6: Schematic diagram of simplified SBLI spanwise development behaviours (adapted from [41]). Wall topology shows: U - limit of upstream influence, S - separation line, I - inviscid shock location, R - reattachment line, L_i – SBLI inception length, and z_{VCO} – spanwise location of virtual conical origin. Shaded region indicates quasi-infinite development of separated flow.

interaction, only one data point for the experimental compression ramp results does not observe the apparent trend ($M_{\infty} = 2.95$, $\theta_{ramp} = 24^{\circ}$, $\psi = 20^{\circ}$) and reports cylindrical similarities when conical is predicted.



(a) Inviscid model results for single shock (black) and shock reflection solutions (grey), for freestream Mach number M_{∞} .



(b) Experimental comparison of single shock solution, using slip Mach number $M_{slip} = 0.75 M_{\infty}$ when shock located in a viscous environment ($M_{slip} = M_{\infty}$ for freestream shocks or inviscid model results).

Figure 2.7: Normalised shock detachment conditions. The solid black line indicates detachment onset for incident shock, the solid grey like indicates detachment onset for reflected shock. Square markers represent data for swept compression ramps in Mach 2.95 flow [41]. Triangular markers represent data for unswept and swept sharp fin interactions in Mach 2.95 and 3.95 flows [40, 34] (\triangle corresponds to shock generated at the sharp fin itself, whereas ∇ corresponds to shock at the flat plate, where $\cos \psi' = \sin \beta'_0$). Interactions reported with conical similarities are shown filled in black (shocks at sharp fin are determined whether detached using present model and are filled in grey).



Figure 2.8: Normalised pressure rise coefficient across various shock systems. The left column (a and c) represents conditions across a single shock, the right column (b and d) represents conditions across a shock reflection where the first shock is a *weak* shock. The top row (a and b) features systems with a *weak* final shock, the bottom row (c and d) features systems with a *strong* final shock. The shock detachment boundary is shown by a solid black line. Note, the color scale differs between each column.

2.4.3 Interaction strength characterization

A suitable strength parameter for scaling interactions has been demonstrated in two-dimensional flows as the pressure rise coefficient [61, 60]. In addition to affecting the scale of interactions, this parameter can also significantly influence the onset of unsteadiness within transitional SBLIs [69, 70]. The pressure rise coefficient equates to the interaction pressure rise normalized by the incoming flow dynamic pressure. Once extended to the x_n -y plane (as discussed prior) this returns equation (2.21).

$$C_{p_n} = \frac{p_{post} - p_{pre}}{q_{\infty} \cos^2 \psi} = \frac{2}{\gamma M_{\infty}^2 \cos^2 \psi} \left(\frac{p_{post}}{p_{pre}} - 1\right)$$
(2.21)

The pressure rise coefficients for a range of shock systems are shown in figure 2.8. Single shock systems representing *weak* and *strong* solutions are shown in figure 2.8a and figure 2.8c, respectively. Dual shock systems representing *weak-weak* and *weak-strong* are shown in figure 2.8b and figure 2.8d, respectively.

Typical swept compression ramp conditions are given directly by the weak solution (see figure 2.8a) where the interaction strength increases as: i) Mach number (M_{∞}) is decreased, ii) x-y plane ramp deflection angle (θ_{sg}) is increased, or iii) sweep angle (ψ) is increased. The freestream strength of swept sharp fin shocks can also be determined using this figure, providing the reference frame is considered such that the x-y plane shock generator angle θ is defined in a plane perpendicular to the swept plane.

Alternatively, one can also refer to the strong solution data in figure 2.8c by considering the equivalences between shock geometries [62]. For example, an unswept sharp fin shock could be defined with the *weak* solution data using a sweep angle of ψ and x-y plane shock generator angle θ . It could also be defined in a reference frame

shown in figure 2.8d for completeness.

perpendicular to this such that sweep is given by $\psi = 90^{\circ} - \beta$ and the new x-y plane shock generator angle is 0°. In this new reference frame the two solutions correspond to a Mach wave (*weak* solution) or an oblique shock (*strong* solution). The strong shock produces an inverse trend compared to the *weak* case. Namely, the strength

iii) sweep angle (ψ) is increased. Figure 2.8b describes the pressure rise across two *weak* reflected shocks, which forms the main focus of this study. As found previously, shock generator angles are more limited in the two-shock system resulting in a smaller envelope of permissible conditions that will result in attached shocks. At high Mach numbers and/or low sweep angles, the interaction becomes largely insensitive to both Mach number and sweep (owing to small angle approximations) and is therefore primarily governed by the *x-y* plane shock generator deflection angle. This implies that in these conditions there is minimal spanwise deflection of the flow and it therefore behaves in a quasi-2D manner. However, at low Mach numbers and/or moderate to high sweep angle, such a simplification is not viable. Qualitatively, the response agrees with that of a single *weak* shock (see figure 2.8a) where the interaction increases in strength for: i) decreased freestream Mach number, ii) increased sweep, or iii) increased *x-y* plane shock generator deflection angle. Disregarding any of these effects when scaling the strength of an SBLI may result in misinterpretation of fundamental mechanisms present. The *weak-strong* two-shock system is

now decreases as: i) Mach number (M_{∞}) is decreased, ii) x-y plane ramp deflection angle (θ_{sq}) is increased, or

The pressure rise coefficient has been used in unswept turbulent SBLIs to characterize the onset of flow separation. By establishing an empirical database of such experiments Souverein, Bakker and Dupont [60] demonstrated that the onset of fully separated flow existed when $C_{p_n} > 0.33$ (for low Reynolds number flows, where $\text{Re}_{\theta} < 10^4$), or $C_{p_n} > 0.40$ (for high Reynolds number flows, where $\text{Re}_{\theta} > 10^4$). A similar normalized approach may be taken to assess Mach number variation across the shock structure by proposing an equivalent temperature rise coefficient. Equation (2.22) normalizes the dimensional temperature rise across a shock structure by the difference between stagnation and the local static temperature measured normal to the sweep (*i.e.* local dynamic temperature). When used in conjunction with equation (2.14), this returns an assessment of Mach number throughout the shock reflection, reduced into the two-parameter domain as discussed prior (see equation (2.23)).

$$C_{T_n} = \frac{T_{post} - T_{pre}}{(T_0 - T_\infty)\cos^2\psi} = \frac{2}{(\gamma - 1)M_\infty^2\cos^2\psi} \left(\frac{T_{post}}{T_{pre}} - 1\right)$$
(2.22)

$$M^{2} = \frac{2M_{\infty}^{2} \left(1 - C_{T} \cos^{2} \psi\right)}{2 + (\gamma - 1) C_{T} M_{\infty}^{2} \cos^{2} \psi}$$
(2.23)

Demonstration of this temperature rise coefficient is shown in figure 2.9. Results are dominated by the strength of the final shock, such that a strong final shock produces high temperature coefficients (indicating slower flow) which increase with incoming Mach number and induced flow deflection (in x_n -y plane). Weak-only shock systems produces notably smaller temperature coefficients, although qualitatively retain a similar behaviorr.

Finally, one can use equation (2.14) again to return an alternative formulation for the downstream sideslip angle η . The temperature ratio T/T_{∞} is constant regardless of the reference frame in which Mach number is determined. One can use this feature to rewrite this expression as equation (2.24).

$$\frac{T}{T_{\infty}} = \frac{2 + (\gamma - 1)M_{\infty}^2 \cos^2 \psi}{2 + (\gamma - 1)M_n^2}$$
(2.24)

Equation (2.25) defines the relationship between the Mach number measured normal to the sweep and the absolute local Mach number for a given flow deflection. Using equation (2.24) to return M_n and equation (2.22) to substitute C_{Tn} , this may be written as equation (2.26) and then employed to calculate the downstream sideslip angle η . This presents as an alternate formulation of equation (2.17). Since the terms for temperature coefficient C_{Tn} and x_n -y plane deflection angle α_n both collapse in the two-variable domain, one can use this equation to determine the slideslip angle η for a given shock solution with a defined swept angle ψ .

$$\frac{M^2}{M_n^2} = \sin^2 \alpha_n + \frac{\cos^2 \alpha_n}{\cos^2 (\psi + \eta)}$$
(2.25)

$$\frac{\tan(\psi+\eta)}{\tan\psi} = \frac{1}{\cos\alpha_n\sqrt{1-C_T}}$$
(2.26)



Figure 2.9: Normalised temperature rise coefficient across various shock systems. The left column (a and c) represents conditions across a single shock, the right column (b and d) represents conditions across a shock reflection where the first shock is a *weak* shock (required to induce a second shock). The top row (a and b) features systems with a *weak* final shock, the bottom row (c and d) features systems with a *strong* final shock. The shock detachment boundary is shown by a solid black line.

The relation $\tan(\psi + \eta) / \tan \psi$ can therefore be considered as the appropriate term for the normalized spanwise flow deflection, as shown in figure 2.10. For a given shock generator sweep, the sideslip deflection is observed to be greater for the *weak-weak* shock reflection when compared to the single *weak* shock. Both cases experience increased sideslip for increased shock generator deflection angles and decreased freestream Mach numbers. *Strong* shocks significantly increase sideslip deflection over the *weak* shock counterparts. In this case, trends with shock generator deflection angle and Mach number invert. Also, sideslip is reduced for increased shock generator deflection angles and decreased freestream Mach numbers.

Attention is now returned to the three-parameter domain to demonstrate the effect of modifications to a given shock reflection environment. A nominal flow field is defined using a shock generator with $\psi = 22.5^{\circ}$ and $\theta_{sg} = 12.5^{\circ}$ in Mach 3.0 flow. Figure 2.11 demonstrates the influence of these parameters with respect to the pressure rise coefficient C_{p_n} (as defined by equation (2.21)). Inspection of the data suggests that the normalized pressure rise is most sensitive to changes in shock generator deflection angle. The effect of sweep and Mach number appear to be less significant, a feature clearly evident on the broad bands in figure 2.11c. By considering each parameter independently, one can establish the following partial derivatives at the nominal condition defined above ($\psi = 22.5^{\circ}$, $\theta_{sg} = 12.5^{\circ}$, $M_{\infty} = 3.0$). This determines the relative influence on the interaction pressure rise: $\partial C_p/\partial \psi = 0.0121 \text{ deg}^{-1}$, $\partial C_p/\partial \theta_{sg} = 0.102 \text{ deg}^{-1}$, and $\partial C_p/\partial M_{\infty} = -0.146$. This approach can be followed for any arbitrary configurations.



Figure 2.10: Normalised sideslip across various shock systems. The left column (a and c) represent conditions across single shock, the right column (b and d) represent conditions across dual-shock reflection systems where the first shock is a *weak* shock. The top row (a and b) features systems with a *weak* final shock, the bottom row (c and d) features systems with a *strong* final shock. The shock detachment boundary is shown by a solid black line. Note, the color scale differs between each row.



Figure 2.11: Influence of sweep angle, deflection angle, and Mach number on the pressure rise across a weak-weak shock reflection. Nominal case is for a $\psi = 22.5^{\circ} \theta_{sg} = 12.5^{\circ}$ shock generator in Mach 3.0 flow (indicated by the circle marker). Each figure demonstrates the effect of varying two parameters while keeping the third constant. Filled contours represent C_{p_n} increments of 0.1.



Figure 2.12: Schematic diagram of shock structure in inviscid flow past a swept sharp fin (adapted from [40]). L represents an inviscid inception length, characterizing the spanwise extent of the curved root shock.

2.4.4 Comparisons to other interaction types

Flow under swept shock generators can be considered identical to that over an inverted inviscid swept compression ramp. Both configurations experience flow deflection routinely defined in the x-y plane normal to the swept plane, and therefore can be contrasted directly. As such, results from the model measured across the incident shock agree with prior compression ramp-specific models that have been presented [41, 62]. The present model expands upon this knowledge and characterizes the shock reflection against a wall parallel to the shock generator.

Comparisons with other supersonic deflected flow configurations can also be made, but consideration of the reference frame is vital. To illustrate this one may consider the swept sharp fin configuration, as shown in figure 2.12, which is similar to the shock structure at the root plane of a swept impinging oblique SBLI. Prior research has characterized the shock structure and surface topology beneath the fin [40]. Away from the wall, the shock develops at a consistent angle; however, near to the wall, the shock is shallower. This near-wall behavior matches similar observations at the root of a swept impinging oblique SBLIs [51], and in analogous delta wing flows [71, 72]. A state of equivalence between these flows can be made since each configuration demonstrates highly swept oblique shocks interacting with non-porous plane. However, a crucial difference between these configurations is in the definition of the shock generator geometries which needs to be understood to enable comparison.

The swept sharp fin in figure 2.12 is essentially a flat plate with a swept sharp leading edge mounted normal to the wall at an incidence to the incoming flow [40]. By this definition, the leading edge is not in the x-y' plane, but at a plane rotated around the y' axis by an angle θ' . Figure 2.13 demonstrates the implication of this when compared to the swept shock generator and compression ramp definitions discussed previously. The leading edge is held constant between the two configurations, producing a difference in the wall location at the base of the shock due to rotation around the z-axis of ξ , characterised by equation (2.27).

Since inviscid conditions collapse well in the shock generator xyz reference frame, it is desirable to transform the swept sharp fin geometry into a swept impinging oblique shock generator such that a similar analysis can be followed. The effect of this reference frame transformation is that the sharp fin sweep and deflection angles are no longer equal to that defined for the effective 2D shock generator. Instead, equation (2.28) and equation (2.29) are used to redefine these variables, respectively.

$$\tan \xi = \tan \psi' \sin \theta' \tag{2.27}$$

$$\tan\psi = \tan\psi'\cos\theta'\cos\xi \tag{2.28}$$

$$\tan \theta_{sg} = \tan \xi / \tan \psi \tag{2.29}$$

Understanding the differences between these geometries is vital when comparing data from swept SBLIs induced by various geometries. A focus of this article is the collapse of data with appropriate consideration of


Figure 2.13: y-z plane comparison between swept sharp fin and shock reflections. The reference frame for the swept sharp fin flow is given by the x, y', and z' vectors.



Figure 2.14: Shock equivalency for a range of shock generator geometries showing data associated with the incident shock. Solutions along black contour lines indicate identical equivalent shocks, plotted with increments of $\Delta M_{\infty} \cos \psi = 0.2$ at $\tan \theta_{sg} / \cos \psi = 0$. Color contour levels reflect pressure ratio across shock, plotted with increments of $\Delta p/p_{\infty} = 1$. Solution data is symmetrical around $\tan \theta_{sg} / \cos \psi = 0$. For convenience, strong solution is plotted on left side of figure, and weak solution plotted on the right.

the sweep. A similar collapse of inviscid swept sharp fin data requires substitution of appropriate terms into the abscissa and ordinate definitions from figure 2.8. Equation (2.30) and equation (2.31) give the appropriate relations, written in terms of the sharp fin geometry definition.

$$\cos\psi = \cos\psi' \sqrt{\tan^2\psi' \sin^2\theta' + 1}$$
(2.30)

$$\tan \theta_{sg} = \tan \theta' \sqrt{\tan^2 \psi' \sin^2 \theta' + 1}$$
(2.31)

It is clear that terms cancel when defining the abscissa variable, such that $\tan \theta' / \cos \psi' = \tan \theta_{sg} / \cos \psi$. However, the ordinate variable must now included the effect of deflection angle in addition to sweep, such that it is defined as $M_{\infty} \cos \psi' \sqrt{\tan^2 \psi' \sin^2 \theta' + 1}$, which equates to $M_{\infty} \cos \psi' / \cos \xi$, if equation (2.27) is utilized.

2.4.5 Equivalent Rotation

Providing the leading edge of a shock generator is located parallel to a shock defined in an equivalent 2D plane with incidence β_e , and that the shock generator surface induces a deflection in the same plane of θ_e then the

resultant shock and downstream flow is identical, albeit transformed to an alternate reference frame. This means infinite combinations of shock generator geometries can be used to induce a matching shock, providing a certain relationship between sweep and deflection angle is followed. To maintain an identical shock, one can relate such shock generator geometries to those of the equivalent plane flow, with the knowledge that these must remain constant, as shown by equation (2.19) and equation (2.32). In the instance that $\tan \psi = 0^{\circ}$ and $\sin \phi = 0^{\circ}$ (true for any unswept configuration), the equivalent shock angle remains undefined using equation (2.32) alone, equation (2.20) should therefore be used instead.

$$\tan \beta_e = \frac{\sin \phi}{\tan \psi} \tag{2.32}$$

The fin inclusion angle ξ is defined previously using equation (2.27), meaning the relation for θ_e given by equation (2.29) can be rewritten as equation (2.33). For any given rotation ϕ , a suitable combination of ψ and θ_{sq} can therefore be defined which would result in an identical shock.

$$\frac{1}{\tan \theta_e} = \sin \phi \tan \psi + \frac{\cos \phi}{\tan \theta_{sq}}$$
(2.33)

This observation has significance because it separates the shock (typically the focus of most studies) from the body used to induce it. As a result, a wide range of bodies can thus be designed to promote the same shock, providing flexibility in the experimental design process. In addition, it highlights the continuum of shock configurations avoiding common over-classification between SBLI configurations.

A simple example of this occurrence is the equivalency between an unswept sharp-fin induced shock and a swept normal shock. Given a certain shock generator deflection angle θ_{sg} and freestream Mach number M_{∞} , an oblique shock will be induced with angle β (sweep is $\psi = 0^{\circ}$ by definition of the unswept fin). Since the lack of sweep precludes any sideslip downstream of the shock, the angle of the equivalent plane is coincident with the x-y plane, such that $\phi = 0^{\circ}$. If one now considers a reference frame taken perpendicular to this (where $\tilde{\phi} = 90^{\circ}$), then it follows that $\tilde{\theta}_{sg} = 0^{\circ}$ and $\tilde{\psi} = 90^{\circ} - \beta$. Using equation (2.20), the newly defined shock angle in the x-y plane is given as $\tilde{\beta} = 90^{\circ}$. The transformed configuration is thus equivalent to a normal shock, swept with angle $\tilde{\psi}$.

Figure 2.14 demonstrates these relations by comparing various shock generator geometries to the pressure ratio across the shock. The pressure ratio is dependent on only the 2D equivalent shock, therefore is constant for any combination of shock generator geometries that would result in an identical shock. Figure 2.14 indicates this behavior is true for both weak and strong shock solutions. Returning to the example above, if the shock induced by the sharp fin were weak, then the contours reflecting the weak shock solution in figure 2.14 would be followed. Likewise, if the shock were induced as a strong solution, the strong contours would be followed. Both solutions result in a normal shock when transformed, only with different values of sweep to reflect the differences in shock strength between the weak and strong solution. The resultant shock is effectively the swept shock experienced at the floor at the root of the sharp fin. Since it is a normal shock, its strength scales with incident Mach number $M_{\infty} \cos \tilde{\psi}$ (or $M_{\infty} \sin \tilde{\beta}$) alone, as widely reported in literature [36, 34].

To extend this example, consider that the shock experienced at the wall is no-longer normal but inclined such that it impinges upon the wall (*i.e.* to impose a shock reflection). Now both $\tilde{\beta} \neq 90^{\circ}$ and $\tilde{\theta}_{sg} \neq 90^{\circ}$. The *x-y* shock angle $\tilde{\beta}$ can now be set such that a *strong* or *weak* shock solution is possible. Following the contours in figure 2.14, it is clear a wide range of solutions are possible that satisfy the criteria of generating an identical shock. The shock generator may be located with leading edge parallel to the floor to maintain the present reference frame, or it may be rotated by varying ϕ to any prescribed angle. Indeed, the shock may be equally induced by a forward swept sharp fin mounted perpendicular to the floor. This approach may prove appealing as it forcefully prevents viscous contamination at the root of a swept impinging SBLI as has been reported in associated studies [51, 52]. The *x-z* deflection angle of such a fin η' is defined using equation (2.34), where $\tilde{\phi}$ reflects the equivalent 2D plane rotation associated with a horizontal shock generator with leading edge at constant *y*.

$$\tan \eta' = \frac{\tan \theta_{sg} \cos \xi}{\cos \left(\phi - \tilde{\phi} - \xi\right)}$$
(2.34)





(a) Horizontal shock generator; $\psi = 22.5^{\circ}$, $\theta_{sg} = 12.5^{\circ}$, $\Delta \phi = 0.0^{\circ}$, $\xi = 5.2^{\circ}$, $\theta_n = 13.5^{\circ}$.

(b) Delta wing shock generator: $\psi = 29.4^{\circ}$, $\theta_{sg} = 12.2^{\circ}$, $\Delta \phi = -7.0^{\circ}$, $\theta' = 12.1^{\circ}$, $\xi = 7.0^{\circ}$, $\theta_n = 14.0^{\circ}$.



(c) Swept vertical sharp fin: $\psi = -51.4^{\circ}$, $\theta_{sg} = 5.6^{\circ}$, (d) Skewed & swept sharp fin: $\psi = -46.8^{\circ}$, $\theta_{sg} = 9.4^{\circ}$, $\Delta \phi = 90.0^{\circ}$, $\eta' = 5.6^{\circ}$, $\xi = 7.0^{\circ}$, $\theta_n = 8.9^{\circ}$. $\Delta \phi = 72.1^{\circ}$, $\eta' = 10.5^{\circ}$, $\xi = 10.0^{\circ}$, $\theta_n = 13.6^{\circ}$.

Figure 2.15: Examples of equivalent shock generators with Mach 2.3 freestream flow, plotted with normalized domain width 1.5*h*. Impinging shock is shaded in red, and the reflected shock shaded in blue. The black dotted line indicates the equivalent 2D plane for the impinging shock. Black dashed lines indicate streamlines emanating from $[x, y, z]/h = [-\infty, 1, 0]$ and $[x, y, z]/h = [-\infty, 0, 0]$. Dashed-dotted lines in (a) and (b) indicate the impinging shock end-effect influence region as determined by the intersection of the impinging shock and the downstream Mach wave cone (assuming flow expansion at spanwise edges of shock generators). Similar features in (c) and (d) have been omitted as they span the entire shock.

Figure 2.15 demonstrates four shock generators that would produce an identical shock. The horizontal shock generator in figure 2.15a retains the reference frame of the reflection and should therefore be used when quantifying flow in this region. Using a simple transformation procedure the remaining shock generator geometries can be defined such that an identical shock is induced. Delta wing generators (see figure 2.15b) are attractive options for experimentation due to the relative ease of construction, and the reduction of inviscid relief effects at the root as opposed to the horizontal generator.

A forward swept sharp fin can similarly be defined to create the shock, see figure 2.15c. The inviscid relief effects at root across the impinging shock are now mitigated as the angle of the fin surface runs parallel to the induced flow deflection by definition. However, downstream of the reflected shock this no-longer remains true as the spanwise deflection acts to induce a similar relief effect and will result in a localized weaker shock $(\eta' \neq \eta_r)$. Finally, figure 2.15d demonstrates a skewed and swept shock generator that induces the desired impinging shock, but also runs parallel to the flow downstream of the reflected shock such that $(\eta' = \eta_r)$, thus mitigating localized weakening of the reflected shock.



Figure 2.16: Attached two-shock configuration domain for Mach 3.0 flow defined normal to sweep. The two shock angles are prescribed relative to the incoming flow, normal to the sweep (with positive local deflection downwards and upwards for the first and second shock, respectively, see figure 2.2 and inset subfigure for reference). Colored contours of pressure coefficient are plotted in increments of 0.1. Black contours show downstream deflection α_{2_n} in 5° increments, where thin dashed lines are negative, thin solid lines are positive, and the thick solid line shows zero deflection (*i.e.* a flat wall). The magenta line denotes when the second shock is locally normal. The red line identifies the weak/strong solution. The blue line outlines the region of supersonic downstream flow ($M_{3_n} > 1$).

A further point of concern is the influence region by which edge-effects of the generator can influence the impinging shock and thus the shock reflection. By identifying the intersection of the downstream Mach wave cone and the shock itself, one can establish the affected region [73, 74]. This approach assumes that flow expansion occurs in these regions such that the affected Mach wave cone decreases in size thereby also reducing its region of influence. Should compression occur, the influence zone would increase in size and may induce additional shocks.

The highly forward-swept fins of figure 2.15c and figure 2.15d are particularly susceptible to end effects since the influence of the upper edge of the fin spans the entire induced shock. In addition, this shock represents the *strong* local shock solution at the fin and therefore would likely not be induced in typical flow without augmentation of back pressure. This shows that, while the fins reduce the localized weakening of the shock system at the root, these generators cannot be used in isolation. They should therefore be combined with a rearward-sweep shock generator (such as those in figure 2.15a and figure 2.15b) to form a hook-like model to move the edge influence region away from the root. This induces a back pressure requirement that will promote the *strong* fin shock along with the matching *weak* generator shock. Providing the equivalent shock generator relations are followed when defining any transitions between geometries, the change may be gradual or discontinuous.

The equivalence between the interactions outline above suggests that the means by which shocks are induced has no relevance on the shock reflection itself (in an inviscid sense only). Instead, one can consider the reflection behavior as a function of the shock angles alone. It has been shown previously that resolving flow normal to the imposed sweep removes dependency of the shock system on sweep. Figure 2.16 demonstrates this approach for an incoming Mach number normal to the sweep of 3.0. The resulting two-parameter domain shows the full range of attached one- and two-shock systems. Commonly studied SBLI configurations can be overlaid to understand the variety across the domain, and to highlight the continuous nature that describes any differences between these configurations. Single shock flows associated with compression ramps and normal shocks are found assuming an infinitely weak impinging shock, such that $\beta_{1n} = \mu_{1n}$ (where $\mu_{1n} = 19.47^{\circ}$ for $M_{1n} = 3.0$). Similarly, shock reflections with a flat wall are found by following the zero deflection contour where $\alpha_{2n} = 0^{\circ}$.



Figure 2.17: Schematic showing curved shock region near the root plane (from Roe [75], adapted from Babaev [76]).

2.4.6 Shock Structure at the Root

Typical impinging shocks experienced outside of fundamental research environments are unlikely to be induced via the unique condition shown in figure 2.15d, where the domain spanwise boundary planes (in this case the shock generator) are aligned with downstream sideslip angles so as to prevent influencing the inviscid shock. It is therefore important to gain an understanding of the broader environment in which such an effect is experienced.

Impinging oblique SBLIs are commonly induced by a shock generating surface located away from the wall. When swept, a spanwise component of velocity is induced. If a slip plane is introduced to the fluid that restricts such flow (*i.e.* sidewalls of a engine inlet, or symmetry plane beneath delta wing) this could induce either a secondary compression (forming a shock reflection) or a local expansion region, depending on the inclination of the fluid to the plane [72]. In the former case, the reflected shock can be solved directly using the inviscid model presented above, providing the appropriate reference frame is considered and the shock is attached. However, determination of latter case that exhibits local expansion and acceleration of the fluid is significantly more challenging and is addressed below.

A commonly studied configuration which exhibits this behavior is flow near the centerline beneath a delta wing model. The local expansion acts to relieve the downstream flow deflection requirements, weakening the shock, and moving the shock-wall impingement location further downstream. This is shown schematically in figure 2.12 for an inviscid swept sharp fin configuration which is identical to the delta wing flow (albeit split at the centerline symmetry plane). Figure 2.17 shows how the outboard region of the flow is conically supersonic, equal to the uniform flow predicted by the present inviscid model. The central region is conically subsonic and bound by the Mach wave emanating from flow at the apex.

The solution for the shock shape and downstream flow conditions in this region is not directly solvable [40]. Various numerical solutions have been proposed [76, 77, 72]. In addition, several simplified models of this region have been created [78, 73, 75]. However, their suitability in modeling shock shape at moderate Mach numbers is generally poor as they rely on assumptions proposed to address hypersonic flows [79]. Messiter [79] scales the dimensional variables using ε , defined as the inverse density ratio across an equivalent unswept shock (*i.e.* $\varepsilon = \rho_1/\rho_2$, see equation (2.35)). As the shock attitude is not known a priori it is assumed to be parallel to the unswept shock generator surface ($\beta' = \theta'$) inclined to the flow running parallel to the shock generator surface, a reasonable assumption in hypersonic flows with significant flow deflection.

$$\varepsilon = \frac{\rho_{\infty}}{\rho} = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M_{\infty}^2 \sin^2 \theta'}$$
(2.35)

Dimensional terms are normalised using ε in equation (2.36), equation (2.37), equation (2.38), and equation (2.39). The terms noted with a bar accent (e.g. \bar{x}) are dimensional and defined in a reference frame rotated around the y axis, such that the \bar{x} axis runs parallel to the shock generator surface with the origin at the gener-

ator apex. The spanwise extent of the shock generator is normalized using equation (2.37) to define $\check{z} = \Omega$ when $z/\bar{x} = b$, such that \check{z} varies between 0 at the central plane to Ω at the tip.

$$\check{y} = \frac{\bar{y}/\bar{x}}{\varepsilon \tan \theta'} \tag{2.36}$$

$$\check{z} = \frac{z/x}{\varepsilon^{\frac{1}{2}} \tan \theta'} \tag{2.37}$$

$$\check{w} = \frac{w}{\varepsilon^{\frac{1}{2}} U_{\infty} \sin \theta'} \tag{2.38}$$

$$\check{p} = \frac{1}{\varepsilon} \left[\frac{p - p_{\infty}}{\rho_{\infty} U_{\infty}^2 \sin^2 \theta'} - 1 \right] = \frac{1}{\varepsilon} \left[\frac{C_p}{\sin^2 \theta'} - 1 \right]$$
(2.39)

At high-Mach numbers and significant flow deflections, ε tends towards $(\gamma - 1) / (\gamma + 1)$ and closely resembles conditions downstream of a real shock over a matching wedge. However, at low-Mach numbers and small flow deflections the disparity between this estimation and the real flow can be striking, resulting in significant errors when estimating shock shape and strength [72, 75].

Roe's delta wing induced shock estimation[75] is attractive as the solution is continuous, derived from the equations of motion, and relies on simple formulation. A summary of the approach is provided below, but the reader is directed to the source papers for full details of derivation [79, 75]. Employing Messiter's non-dimensionalization approach[79] with ε (returning \check{y} and \check{z} , etc.), Roe reduces the shock structure to three zones: i) uniform outer flow with gradient $d\check{y}/d\check{z} = w_0$, ii) intermediate parabolic blending region, and iii) parabolic central region with zero gradient at $\check{z} = 0$.

i)
$$\check{y}_s = -\check{w}_0 \left(\check{z} - \Omega\right)$$

ii) $\check{y}_s = 1 - \check{w}_0 + \frac{\check{w}_0^2}{2} - \frac{\left(\check{z} - 1\right)^2}{2}$
 $1 + \check{w}_1 \le \check{z} < 1 + \check{w}_0$

iii)
$$\check{y}_s = 1 - \check{w}_0 + \frac{\check{w}_0^2}{2} + \frac{\check{w}_1}{2} - \frac{\check{w}_1}{1 + \check{w}_1} \frac{\check{z}^2}{2} \qquad 0 \le \check{z} < 1 + \check{w}_1 \qquad (2.40)$$

The terms \check{w}_0 and \check{w}_1 represent non-dimensionalized spanwise velocities and are determined using equation (2.41) and equation (2.42). They also act to define the matching points between the three curves listed above defining the shock shape, such that the central curved shock region is bound by $\check{z} = 1 + \check{w}_1$ and the outer uniform shock region is bound by $\check{z} = 1 + \check{w}_0$. Solving equation (2.41) for \check{w}_0 also reveals further significance of the spanwise term Ω , where if it has a value less than 2 there is no solution and the shock is fully detached across the entire span.

$$\Omega = \frac{\check{w}_0^2 + 1}{w_0} \tag{2.41}$$

$$(1+w_1)^2 = 1 + 3\check{w}_0^2 - 2\check{w}_0^3 \tag{2.42}$$

Finally, the shock structure is transformed back to the upstream-aligned reference frame using equation (2.43) and equation (2.44).

$$\frac{y'_s}{x} = \frac{\tan\psi' + \bar{y}/\bar{x}}{1 + (\bar{y}/\bar{x})\tan\psi'}$$
(2.43)

$$\frac{z'}{x} = \frac{\bar{z}/\bar{x}}{\cos\theta' - (\bar{y}/\bar{x})\sin\theta'} \tag{2.44}$$

In a similar manner, the model also returns estimation of the non-dimensionalized pressure downstream of the shock using equation (2.45).



Figure 2.18: Shock angles for Mach 2.95 delta wing experiment [40]. Markers indicate experimental data for the root shock angle, from Settles and Lu (1985). Solid and dashed bold lines indicate the inviscid quasi-infinite span shock angle and root shock angle (using ε^*), respectively, defined in a matching reference frame away from the wall using the present inviscid model. Similarly, the corresponding faded solid and dashed lines indicate inviscid shock angles and root shock angles, respectively, with results extracted from Roe's uncorrected model for the shock structure (using ε).

i)
$$\check{p} = 1 + \frac{\check{w}_0^2}{2}$$

ii) $\check{p} = 1 + \frac{\check{w}_0^2}{2} + 2(\check{z} - 1 - \check{w}_0)$
 $1 + \check{w}_1 \le \check{z} < 1 + \check{w}_0$

iii)
$$\check{p} = A + B\check{y}^2$$
 $0 \le \check{z} < 1 + \check{w}_1$ (2.45)

where the terms A and B are given by equation (2.46) and equation (2.47), respectively.

$$A = \left(1 - w_0^2\right) + \frac{w_1}{2} \left(6w_1^2 + 9w_1 + 4\right) + 3w_1^2 \left(1 + w_1\right)^2 \log\left(\frac{w_1}{1 + w_1}\right)$$
(2.46)

$$B = -3w_1 \left[\frac{1}{\left(1+w_1\right)} + \frac{1}{2\left(1+w_1\right)^2} + \log\left(\frac{w_1}{1+w_1}\right) \right]$$
(2.47)

While the model offers useful estimation of the induced shock shape and strength, there are notable discrepancies at low Mach numbers and strong flow deflections [75]. Comparison to such experimental data is shown in figure 2.18 for various delta wing geometries in $M_{\infty} = 2.95$ flow [40]. The experimental centerline shock angle is derived from Schlieren imaging and compared to the theoretical value determined from Roe's model using $\tan \beta'_0 = y'_s/x$ (at z' = 0). Significant differences are observed between the data which appear to become worse as the deflection angle is reduced (consistent with strengthening invalidity of assumptions when defining ε as a scaling variable).

As the uniform outer flow is now directly determined using the inviscid infinite-span model presented in this paper, one can use the resultant shock geometries to define the non-dimensionalised shock gradient \check{w}_0 and, in turn, estimate a revised scaling parameter, termed ε^* for clarity. After some manipulation, this returns equation (2.48), which replaces the previous definition of ε given in equation (2.35).



Figure 2.19: Schematic diagram demonstrating quasi-infinite physical geometries used to define normalization variable ε^* (to scale for $M_{\infty} = 2.95$, $\psi' = 40^{\circ}$, $\theta' = 15^{\circ}$).

$$\varepsilon^* = \frac{\tan\left(\beta' - \theta'\right)}{\tan\theta'} - \tan^2\left(\beta' - \theta'\right)\tan^2\psi' \tag{2.48}$$

This new term is thus used to normalise dimensional variables following Messiter's approach [79]. Visualized in figure 2.19, this parameter physically represents $\varepsilon^* = (|\vec{B}|/|\vec{C}|) - (|\vec{B}|/|\vec{D}|)^2$ (where each vector is defined relative to vertex A).

The revised estimation of root shock shapes for the delta wing experiments presented in figure 2.18 shows significant improvement when using ε^* over the previous scaling term ε . The modeled root shock structures become shallower in all cases, closely matching experimental observations at moderate and low angles of sweep. At higher sweep angles (within approximately 5° of shock detachment), some divergence is seen between experimental and modeled root shocks. This divergence is associated with invalid assumptions that guide formation of Roe's model. As conditions for a detached shock are approached, the curved root shock structure extends over a greater extent of the shock generator span and errors due to imposed assumptions are exacerbated. The quasi-infinite shock model is an analytical solution to the shock relations, but predictions of the root structure remain a simplified model. Thus, errors grow since this model is required to predict a greater extent of the shock shape. Regardless, the error at high sweep is still less than that of the previous uncorrected model in most cases.

In addition, since the present motivation of modeling the root shock structure is to understand the impingement pattern for an attached reflected shock, it follows that the deflection at the shock generator will be significantly below conditions required for a detached impingement shock (see figure 2.7). Therefore, use of the model correction is justified providing the impinging shock is not close to detachment.

Sample shock structures and downstream pressure profiles are shown in figure 2.20 compared to various validation datasets. Implementation of the revised scaling approach demonstrates a significant improvement against all other models when predicting the shock structure. Experimental data for the Mach 2.95 shock shape in figure 2.20a was determined by observing Mach waves emanating from a delta wing model with a notched leading edge [40]. Excellent agreement is observed between the experimental data and the revised Roe model in the root region. However, the model predicts a shallower shock in the outer region of the flow. This discrepancy is an experimental artifact due to an oversight when interpreting the schlieren observations of the notch-induced shock ripples. It is assumed that the shock ripples propagate along the shock in a strictly 2D sense (*i.e.* z = constant); however, an infinitesimal notch will produce a Mach wave cone that intersects with the shock in two locations, which are dependent on the shock attitude and the direction and speed of the downstream flow. In this case, the upstream shock ripple extends from the notch towards the root as it is observed further downstream. By assuming this to be within the same spanwise plane as the notch, the shock angle is over-predicted resulting in



(a) Experimental validation: M = 2.95, $\psi' = 40.0^{\circ}$, and (b) Numerical validation: M = 4.0, $\psi' = 50.0^{\circ}$, and $\theta' = 15.0^{\circ}$. $\theta' = 15.0^{\circ}$.

Figure 2.20: Application of revised scaling approach for root-limited swept shocks using: (a) experimental [40], and (b) numerical [77, 80] reference data. Other shock models are shown for comparison [72, 75].

the over-estimate observed in figure 2.20a. The reduction of shock sweep near the root decreases this disparity to zero as the centerline of the delta wing is approached, thus giving more accurate shock estimation in this region.

The shock location is established directly in numerically derived results [77] shown in figure 2.20b, and return an excellent matches across the entire shock when compared to the revised model using ε^* . Surface pressure coefficients derived from the model agree well with the numerical data [77, 80], but fail to fully characterise the smoothing of the spanwise pressure rise as is the case for all proposed models in literature. Regardless, the revised model offers an excellent prediction of the centerline pressure and agrees with the infinite span results (by definition by using ε^*). Other proposed models [75, 72] tend to over-predict the steepness of the shock and the associated shock-induced pressure rise.

2.4.7 Swept shock impingement

The above estimation of the shock shape at a delta-wing root enables prediction of the shape of the impingement line exhibited by a swept impinging oblique shock induced by any given shock generator (providing the resultant shocks remain attached). The procedure is as follows:

- 1. The shock generator geometry is transformed into a reference frame such that the generator leading edge is perpendicular to the no-penetration plane delimiting the generator root $(\theta_{sg}, \psi, M_{\infty})$.
- 2. The quasi-infinite swept shock solution is determined using the prescribed model (to return ϕ , section 2.2).
- 3. An equivalent shock generator is found that satisfies $\xi = \Delta \phi$, such that the leading edge matches a corresponding delta-wing model, and that the root plane is oriented to be coincident with root plane (θ' and ψ' , section 2.3.2.4.4 and section 2.3.2.4.5).
- 4. The root shock structure is modeled using Roe's method, corrected using knowledge of the quasi-infinite shock solution (ε^* , section 2.3.2.4.6).
- 5. The resultant shock shape is transformed for a given wall position, to return the impingement line $(x_{imp.}, z_{imp.})$.

This approach is demonstrated in Mach 2.3 flow in figure 2.21 for $\theta_{sg} = 12.5^{\circ}$ shock generators installed parallel to the floor plane, with a range of sweep angles. Away from the root, the shock reaches its quasi-infinite span orientation. However, the floor impingement line at the root noticeably demonstrates a shallower sweep





(a) Inviscid swept oblique shock impingement line, calculated from Roe's model[75] using ε^* correction.

(b) Gross' interaction footprint [53] ($\psi = 40^{\circ}$) with contours of pressure. Box shows domain from (a).

Figure 2.21: Mach 2.3 oblique shock floor impingement footprint ($\theta_{sq} = 12.5^{\circ}$).

angle, reducing to zero sweep at the root plane thus satisfying the zero-crossflow condition mandated by the no-penetration root plane. The location of the transition zone between these regions is associated with the Mach cone downstream of the shock at the root apex. Thus, in the zero sweep limit, the corner location for figure 2.21a is at z/H = 0.731 (see equation (2.49)). As sweep is increased, the conditions for the shock to become detached are approached and a greater extent of the shock is curved. Thus, the corner location tends to $z = \infty$ for a semi-infinite span shock generator. In addition, the transition region becomes more smoothed as sweep is increased.

$$\frac{z_{c,\psi=0^{\circ}}}{H} = \frac{1}{\sin\beta} \sqrt{\frac{\cos^2\left(\beta - \theta\right)}{1 - \frac{1}{M_0^2}} - 1}$$
(2.49)

This development at the root is reminiscent of the inception length observed near the root of other swept SBLIs such as sharp fins and swept compression ramps [41, 35, 43]. However, in those cases, such an inception length is solely due to the viscous flow describing the development of the separation bubble as z = 0 at the wall. In the present configuration, this feature is partly due to inviscid effects, motivating the used of the term 'inviscid inception region'. This length is imposed by curvature of the shock near the root since the shock responds to the no-penetration condition here. Once viscous features are included, the actual inception length will be affected by viscous and inviscid factors, thus affecting the spanwise development of the interaction (see figure 2.21b).

These observations indicate that the swept impinging oblique shock interaction can be considered a dimensional interaction when root shock curvature is present. Its features are scaled by these length scales in addition to the boundary layer encountered by the interaction [17]. However, a non-dimensional form of the configuration can be induced providing effective design of the shock generator is undertaken thus, enabling a fundamental breakdown of features (see section 2.4.5).

2.5 Summary

This section presents an overview of the formulation and results obtained from an inviscid model used to predict the baseline shock structure and effects of sweep for an impinging SBLI. A fundamental consideration of features pertinent to an inviscid shock reflection system has been conducted. Using an analytical model, this flow has been parameterized and considered for a variety of Mach numbers, sweep angles, and flow deflection angles. A collapse of resultant flows into a two-parameter domain has been demonstrated, namely the shock generator deflection angle and incoming Mach number, both measured normal to the swept leading edge. Application of the model to other configurations has been demonstrated including posing the relevant transformation terms required to extend this two-parameter domain to swept sharp fin/delta wing flows. The onset of shock detachment, which has been associated with conical development of swept SBLIs, collapses into the simplified two-variable domain. Experimental observations of cylindrical/conical SBLI similarities in swept compression ramp configurations appear to agree with the hypothesis of Settles and Teng [41] in that a reduced Mach number M_{slip} within the boundary layer dictates the onset of shock detachment and ultimately the transition between cylindrical and conical SBLI development. However, explanation of this collapse still lacks an established physical mechanism. Shock detachment does not appear to be the dominant mechanism when sharp-fins are assessed.

The effect of Mach number and shock generator geometry on shock-induced pressure rise is demonstrated for a variety of one and two shock configurations featuring *weak* and/or *strong* shock solutions. At low Mach numbers (below approximately 3.5), the strength of *weak-weak* shock reflections is observed to rise with increased sweep, increased shock generator deflection angle, and decreased incoming Mach number. At higher-Mach numbers, the relation between incoming Mach number and interaction strength inverts. Such observations are especially critical when assessing transitional SBLIs as subtle variations in strength can correspond to significant changes in flow structure and unsteady behavior.

Using the prescribed model, the equivalence between various one- and two-shock systems is established. It is shown that a given shock can be induced through a continuous range of shock generator geometries that satisfy given relations. Using this observation, a wealth of information on delta-wing and swept sharp fins is utilized to further develop understanding of the inviscid swept impinging shock interaction. A model for shock curvature at the root of a delta wing flow is revised using information of the quasi-infinite span oblique shock, and applied to the root of a swept impinging shock. The resultant estimation of shock shape is demonstrated to accurately predict experimental and simulation validation cases.

Finally, a method to estimate the shape of the wall impingement line for an arbitrary shock generator geometry is defined. The presence of an inviscid inception length is demonstrated when the shock generator root plane restricts spanwise flow, resulting in curvature of the shock. Thus, a length scale associated with the height of the shock generator apex is imposed on the resultant interaction, acting in addition to the viscous inception length reported in literature for a range of other swept SBLIs. Information is also provided to develop a shock generator that eliminates the inviscid inception length, enabling fundamental investigations in which data can be directly compared to other SBLI configurations that have been presented in literature.

3. DNS Investigation of SBLIs in Supersonic Flows

The interaction between an impinging oblique shock-wave and a laminar boundary layer on a flat plate is investigated using direct numerical simulations. The two-dimensional separation bubble resulting from the SBLI at freestream Mach number of 2.3 for the approach flow is investigated in detail. The flow parameters used for the present investigation match the laboratory conditions in the experiments conducted at the UA. In addition to the steady flow field calculations, in order to study the linear stability behavior of the separation bubble, the response to low-amplitude disturbances is investigated using linearized Navier Stokes calculations. For comparison, both the development of two-dimensional and three-dimensional (oblique) disturbances are studied with and without the impinging oblique shock. Furthermore, the effects of the shock incidence angle and Reynolds number are also investigated. Finally, three-dimensional direct numerical simulations are performed in order to investigate the laminar-turbulent transition process in the presence of a laminar separation bubble generated by an impinging shock-wave.

3.1 Physical Problem and Computational Setup

The flow parameters in our present investigation of the interaction between an oblique shock-wave and a laminar boundary layer on a flat plate match the conditions in the experiments by Little and co-workers [81] at the UA. A schematic of the computational setup for the direct numerical simulations is presented in figure 3.1. The approach flow in the UA experiments has a Mach number of 2.3. A shock generator plate is used to create an oblique shock that impinges on the boundary layer, causing separation. A number of different shock inclinations and configurations were studied by Little and co-workers, but in this paper we focus on a 2D configuration. The variation of viscosity with the temperature is assumed to obey the Sutherland's law, for which the value of the Sutherland constant temperature is $T_2^* = 110.4$ K and the free-stream temperature is $T_{\infty}^* = 144.80$ K. The relevant simulation parameters used for the results presented in this paper are provided in table 3.1.

Note that in the UA experiments the approach boundary layer is completely turbulent. In the present work, however, we are considering the interaction of a shock–wave and a laminar boundary layer in order to provide possible insights into the instability mechanisms of the more complicated turbulent SBLI. In future research, we will also investigate SBLI with a completely turbulent approach boundary layer as in the experiments.

Flow Parameters :		
Re/m	[-]	$5.0\cdot 10^6$
M	[-]	2.3
T^*_{∞}	[K]	144.80
Pr	[-]	0.72
γ	[-]	1.40
θ	[deg.]	3.0
σ	$[\deg.]$	28.10

Table 3.1: Flow parameters used in the simulations presented in this paper (based on the conditions in the experiments conducted by Little and co-workers [81] at the University of Arizona). "CASE A".



(a) SBLI test section of the supersonic wind tunnel at the University of Arizona.



(b) Schematic of the computational setup used for the direct numerical simulations.

Figure 3.1: Comparison of experimental and numerical simulation schematic.

3.2 Governing Equations and Numerical Methods

3.2.1 Governing Equations

The physical problem under consideration is governed by the compressible Navier–Stokes equations, consisting of conservation of mass, momentum and total energy. The fluid is assumed to be an ideal gas with constant specific heat coefficients. For simplicity, all equations in this section are presented in tensor notation.

The non-dimensional continuity, momentum and the energy equations are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k} \left(\rho u_k \right) = 0, \tag{3.1}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_k}[\rho u_i u_k + p\delta_{ik} - \tau_{ik}] = 0, \qquad (3.2)$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_k} \left[\rho u_k \left(E + \frac{p}{\rho} \right) + q_k - u_i \tau_{ik} \right] = 0.$$
(3.3)

The stress tensor and the heat-flux vector are computed as

$$\tau_{ik} = \frac{\mu}{Re} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} \frac{\partial u_j}{\partial x_j} \delta_{ik} \right), \tag{3.4}$$

$$q_k = \frac{-\mu}{(\gamma - 1)M^2 P r R e} \frac{\partial T}{\partial x_k} , \qquad (3.5)$$

respectively, where $\gamma = 1.4$, and Pr = 0.72. To close the system of equations, the pressure is obtained from the non-dimensional equation of state for ideal gas

$$p = \frac{1}{\gamma M^2} \rho T, \tag{3.6}$$

and the viscosity is calculated using Sutherland's law as follows:

$$\mu = T^{\frac{3}{2}} \frac{1 + \frac{T_2^*}{T_{\infty}^*}}{T + \frac{T_2^*}{T_{\infty}^*}},\tag{3.7}$$

with

$$T_2^* = 110.4 \text{ K}$$
 (3.8)

Furthermore, the nondimensional parameters in the above equations are defined as

$$Re = \frac{\rho_{\infty}^{*} U_{\infty}^{*} L^{*}}{\mu_{\infty}^{*}}, \quad Pr = \frac{\mu_{\infty}^{*} c_{p\infty}^{*}}{k^{*}} \quad \text{and} \quad M = \frac{U_{\infty}^{*}}{c_{\infty}^{*}} = \frac{U_{\infty}^{*}}{\sqrt{\gamma p_{\infty}^{*} / \rho_{\infty}^{*}}}, \tag{3.9}$$

with c, k, c_p being the speed of sound, the thermal conductivity and the specific heat at constant pressure, respectively. The subscript ∞ denotes free-stream values and the superscript * denotes dimensional quantities.

3.2.2 Numerical Scheme

The Navier–Stokes equations are integrated in time with a standard 4^{th} -order accurate Runge–Kutta scheme. The spatial discretization is based on high-order accurate finite differences. In particular, the derivatives of the viscous terms and the source term are calculated by 6^{th} -order non-compact central finite differences in the streamwise direction and by 4^{th} -order central finite differences in the wall-normal direction. The inviscid fluxes are divided into an upwind flux and a downwind flux using van Leer's splitting.] [82] Then, grid centered upwind differences [83] with 9^{th} -order accuracy are applied to evaluate the derivatives for these fluxes. These grid centered upwind differences are derived using the factor α , which prescribes the degree of upwinding,

$$\frac{\partial \phi_i}{\partial x} = \sum_{k=i-N}^{i+N} c_k \phi_k - \alpha \overline{\Delta x} \frac{\partial^{2N-1} \phi_i}{\partial x^{2N-1}}.$$
(3.10)

Hereby ϕ_i denotes the flow variable at the grid point *i*, the c_k 's are the stencil coefficients and $\overline{\Delta x}$ is the averaged grid spacing over the stencil interval. The parameter *N* determines the number of grid points in the stencil. For example the 9th-order upwind scheme is derived by setting N = 5 and $\alpha = -1500$. Note that for $\alpha = 0$ the upwind scheme reduces to a central difference scheme. All stencil coefficients are derived on a stretched grid. To capture shock waves, high order WENO schemes are employed.

This high–order accurate finite difference code was developed in our CFD Laboratory to perform supersonic/hypersonic transition research for flat–plate and conical geometries. For a more detailed description of the code see Laible et al. [84, 85]

3.2.3 Boundary Conditions

The inflow for supersonic boundary layer simulations is split into two regions: a subsonic region (M < 1) close to the wall and a supersonic region (M > 1). In the supersonic region, Dirichlet conditions for u, v, w, T, p and ρ are specified (obtained from similarity solution). For the subsonic region in the boundary layer, the non-reflecting boundary condition suggested by Poinsot & Lele [86] is adopted.

Flow Parameters :		
Re	[-]	2.96×10^5
M	[-]	2.00
T^*_{∞}	[K]	288.00
Pr	[-]	0.72
γ	[-]	1.40
θ	$[\deg.]$	3.40
σ	$[\deg.]$	32.58

Table 3.2: Flow parameters in the experiments conducted by Hakkinen et al. [88].

On the flat-plate, the no-penetration (v = 0) and the no-slip (u = 0, w = 0) conditions are enforced. The wall is set to be isothermal with temperature equal to the laminar adiabatic wall temperature. The value of the pressure at the wall boundary is obtained from the y-momentum equation. Finally, density is computed using the equation of state (equation 3.6).

At the outflow, the second derivatives of the primitive variables are set to zero: $\partial^2 u/\partial x^2 = 0$, $\partial^2 v/\partial x^2 = 0$, $\partial^2 u/\partial x^2 = 0$, $\partial^2 u/\partial x^2 = 0$, $\partial^2 u/\partial x^2 = 0$. Density is then determined from temperature and pressure by using the equation of state (equation 3.6). For the pulse disturbance simulations and the three–dimensional (transitional SBLI) simulations, a buffer domain technique is applied, where finite amplitude disturbances are ramped down to zero at the outflow.

The boundary conditions at the freestream boundary are applied in a different manner for the simulations with no shock and the SBLI. To calculate the steady flat-plate solution with no shock a characteristic boundary condition [87] is used. At the beginning of the SBLI simulations, the steady solution with no shock is specified within the whole integration domain as initial condition. Also at the beginning of the simulation, the shock is introduced into the freestream boundary. For several grid points upstream and downstream of the shock location the relevant flow variables are held constant. Elsewhere, the characteristic boundary condition is used. The variables downstream of the shock location at the freestream boundary are calculated by the Rankine–Hugoniot relations.

3.3 Validation

3.3.1 Shock Induced Laminar Separation bubble in a Mach 2 Boundary Layer

For validating the computational setup used in the present investigation of laminar SBLI, the experiments of Hakkinen et al. [88] were considered first. The approach flow in the Hakkinen experiments has a Mach number of 2.0 and a Reynolds number of 2.96×10^5 (based on the impingement location in an inviscid case). Several shock inclinations were studied by Hakkinen et al., but for validation purposes we chose a 2D configuration with a wedge angle of $\theta = 3^{\circ}$, corresponding to a shock angle $\sigma = 32.58^{\circ}$ and an overall pressure ratio between pressure after the reflected shock and the upstream pressure equal to 1.4.

Skin friction and wall pressure along the plate are presented in figure 3.2 and are compared with results from Hakkinen et al. [88], Katzer [89] and Sandham et al. [29, 90]. As expected, the results show rapid changes near the separation and reattachment and a pressure plateau in the bubble region. The pressure ratio reaches the value of 1.4 downstream of the reflected shock in accordance with the Rankine-Hugoniot relations. Both upstream and downstream of the separated flow region, the skin friction curve follows the laminar solution (Eckert [29]).

The calculation results (both ours and others) are qualitatively similar to the experimental measurements. However, the calculated skin friction and wall pressure differ quantitatively from the benchmark experiment. The calculated separation bubble is longer in the present study and the computations by others. Possibly, three-dimensional effects present in Hakkinen's experiment, and/or the effects of freestream turbulence may reduce the bubble length when compared to our two-dimensional numerical calculations and those published in the literature [89, ?]. Katzer [89] also obtained a longer bubble than in the experiments, however it is shorter than ours and those from other recent two-dimensional numerical calculation. The reasons for these differences have been investigated recently by Sansica et al. [29]. They also studied the effect of using different models for calculating the viscosity as a function of temperature on the bubble length. They concluded that the different methods used to calculate the viscosity does not affect the bubble length. Our results closely match those by Sandham and co-workers [29].

Contours of streamwise velocity, wall–normal velocity, pressure, temperature, density and Mach number are shown in figures 3.3 and 3.4. In can be observed that, as the shock wave impinges on the laminar boundary layer, it causes separation of the flow. Oblique shocks, compression waves in the separation and reattachment regions and an expansion fan emanating from the top of the bubble are visible in the pressure plot. The domain size is chosen such that the reflected waves of the impinging shock leave the domain at the outflow boundary. Contours of density together with the sonic line and streamlines and the wall–normal density gradient ("Schlieren") are plotted in figure 3.5. The complex shock system is visible in the density contours and the impinging oblique shock wave and the reflected wave is clearly visible in the contours of wall–normal density gradient. The asymmetric structure of the bubble can be seen in figure 3.6, where streamlines are plotted (see also in figure 3.5b). This asymmetric behavior of the flow within the separation bubble is confirmed by the asymmetrical distribution of the skin friction in figure 3.2, with a lower minimum at the back of the bubble due to the center of recirculation being shifted downstream.

3.3.2 Linearized Navier–Stokes Calculations of Spatial Stability for a Mach 2 Boundary Layer

In the present work we are employing a LNS solver [91] developed in our CFD laboratory to study the stability of the flow in the presence of a laminar separation bubble that is generated by the SBLI. The linearized Navier– Stokes solver was first validated by performing stability calculations for a Mach 2 flat–plate boundary layer without SBLI and comparing the results with DNS and LST. We performed stability calculations by introducing a short duration (localized) pulse disturbance of sufficiently small amplitude with a broad frequency spectrum. The pulse disturbance develops into a wave packet that propagates downstream. The wave packets will have a broad spectrum in frequency and wavenumber (see for example Sivasubramanian & Fasel [92]). The flow parameters used are the same as in the experiments of Hakkinen et al. [88] (see table 3.2).

According to linear stability theory (see figure 3.7), in supersonic boundary layers, the dominant disturbances (most strongly amplified) are three-dimensional (oblique disturbance waves). Therefore, we calculated the development of a three-dimensional pulse disturbance for spanwise wave number 0.9/mm (the most amplified spanwise wave number at the impingement location in the experiment of Hakkinen (see Sivasubramanian & Fasel [93])). The streamwise development of the amplitude of the waves within the wave packet generated by the three-dimensional pulse disturbance are extracted and compared in figure 3.8. In particular, the streamwise development of the maximum u-velocity disturbance amplitude and wall pressure disturbance amplitude obtained from LNS and DNS are compared for several three-dimensional waves with spanwise wave number, $\beta = 0.9/mm$. There is an excellent agreement between the amplitudes obtained from LNS and DNS.



Figure 3.2: Comparison of various streamwise distributions. $M = 2.0, T^*_{\infty} = 288.0$ K, $\theta = 3^{\circ}$, Re = 296,000.



Figure 3.3: Flow visualization: contours of (a) streamwise velocity, (b) wall–normal velocity and (c) pressure. $M = 2.0, T_{\infty}^* = 288.0$ K, $\theta = 3^{\circ}$, Re = 296,000.



Figure 3.4: Flow visualization: contours of (a) temperature, (b) density and (c) Mach number. M = 2.0, $T^*_{\infty} = 288.0$ K, $\theta = 3^{\circ}$, Re = 296,000.



Figure 3.5: Flow visualization: contours of (a) density with sonic line, (b) density with streamlines and (c) wall–normal density gradient ("Schlieren image"). $M = 2.0, T_{\infty}^* = 288.0$ K, $\theta = 3^{\circ}$, Re = 296,000.



Figure 3.6: Flow visualization: (a) contours of the streamwise velocity, (b) contours of the wall-normal velocity and (b) streamlines. $M = 2.0, T^*_{\infty} = 288.0$ K, $\theta = 3^{\circ}$, Re = 296,000.

Note that a linear (low-amplitude) wave packet can be considered to be composed of a sum of independent normal modes, each of which behaving exactly according to linear stability theory. Therefore, for the lowamplitude wave packet considered here, all individual wave components of the packet can be tracked independently from each other all the way from the disturbance hole to the outflow boundary of the domain. Then the amplitudes (see figure 3.8) and growth rates of all individual wave components were extracted. In particular, wall-normal disturbance amplitude profiles ("eigenfunctions") and phases were obtained by performing Fourier transformations of the time signals for the various flow variables. In numerical simulations based on the complete Navier–Stokes equations, the non–parallel effects are included and therefore, the spatial growth rate, " $-\alpha_i$ " depends on the criterion used [96]. The specific criterion or quantity used for obtaining the disturbance amplitude affects not only the growth rates, but also the neutral curve [96]. For our Mach 2 validation case, a comparison of the streamwise wavelength, α_r and spatial growth rates, α_i for three oblique disturbance waves (with spanwise wave number, $\beta = 0.9/mm$) with different frequencies with linear stability theory results is shown in figure 3.9. The spatial growth rate and the streamwise wavenumbers were calculated based on the wall pressure disturbance as follows:

$$\alpha_{i} = -\frac{d}{dx} \left[\ln \left(A\left(x\right) |_{p'_{wall}} \right) \right], \quad \alpha_{r} = \frac{d}{dx} \left[\theta\left(x\right) |_{p'_{wall}} \right].$$
(3.11)

In general, the streamwise wave number α_r is less sensitive to the non-parallel effects and therefore less sensitive to the criterion used. Hence the agreement between streamwise wave number from LNS and LST is very good for the three frequencies shown in figures 3.9. Close to the forcing location the streamwise wave number α_r and the spatial growth rate α_i calculated from the LNS data are modulated by the superposition of damped waves. This modulation is more pronounced for the spatial amplification rate α_i than for the streamwise wave number α_r . The reason for the small differences in spatial growth rate α_i is most likely due to non-parallel effects.

The wall-normal amplitude and phase distributions for the streamwise velocity disturbance, density disturbance and pressure disturbance from LNS are compared to LST results in figure 3.10. The amplitude and phase distributions are taken at the downstream location $x^* = 0.061m$ ($R_x = 600$) for an oblique disturbance wave with frequency F = 8.5116E - 05. The amplitude distributions from LNS and LST are normalized by their respective maximum values. The excellent agreement between LNS and LST results in figure 3.10 is an indication that the numerical simulations and post-processing tools can accurately capture the linear disturbance development resulting from a localized pulse disturbance.



Figure 3.7: Contours of constant amplification rate α_i in the $R_x - F$ plane for (a) two-dimensional disturbances and (b) three-dimensional (oblique) disturbances with spanwise wave number 0.9/mm (most amplified spanwise wavenumber at the shock impingement location) and in the $\beta - F$ plane for (c) $R_x = 544$ (corresponds to the shock impingement location, see section 3.33.3.1) and (d) $R_x = 700$. Amplification (growth) rates were computed using Mack's solver [94, 95]. The vertical dashed lines in red indicate the beginning and the end of the computational domain used in the LNS and DNS. Furthermore, the dashed lines in black indicate the streamwise extend of the laminar separation bubble in section 3.33.3.1. $M = 2.0, T_{\infty}^* = 288.0$ K, Re = 296,000.



Figure 3.8: Detailed comparison of the streamwise development of the maximum u-velocity disturbance amplitude and wall pressure disturbance amplitude for 3D disturbances (spanwise wave number, $\beta = 0.9/mm$) between LNS and DNS. $M = 2.0, T_{\infty}^* = 288.0$ K, Re = 296,000.





Figure 3.9: Comparison of streamwise wavelength (α_r) and spatial growth rates (α_i) for three 3D disturbance waves (with spanwise wave number, $\beta = 0.9/mm$) with different frequencies. Lines are from LNS and symbols are from LST. LST results were computed using Mack's solver [94, 95]. $M = 2.0, T_{\infty}^* = 288.0$ K, Re = 296,000.



Figure 3.10: Comparison of wall-normal amplitude (a,b,c) and phase distribution (d,e,f) of the streamwise velocity, density and pressure disturbance to theoretical predictions from LST for frequency F = 8.5116E - 05 and spanwise wave number, $\beta = 0.9/mm$ at $R_x = 600$. LST results were computed using Mack's solver [94, 95]. $M = 2.0, T_{\infty}^* = 288.0$ K, Re = 296,000.

3.4 Results and Discussion

In the present investigation of the interaction of an oblique shock–wave and a laminar boundary layer, the flow parameters match those of the experiments conducted at the University of Arizona [81]. In the experiments a shock generator plate is used to create an oblique shock that impinges on the boundary layer and eventually causes separation (see section 3.1). In the present work we focus on 2D configurations with several different wedge angles and Reynolds numbers. We started with a lower Reynolds number of Re = 360,000 (based on the impingement location in an inviscid case) and a wedge angle of $\theta = 3^{\circ}$ (see table 3.1), which gave a shock angle equal to $\sigma = 28.10^{\circ}$ (CASE A).

Figures 3.11 and 3.12 show contours of streamwise velocity, wall-normal velocity, pressure, temperature, density and Mach number from CASE A. As observed before for the validation case (in section 3.33.3.1), as the oblique shock-wave impinges on the laminar boundary layer, it causes the flow to separate and form a laminar separation bubble. In the pressure plot in figure 3.11c the oblique shocks and compression waves in the separation and reattachment regions are clearly visible. Also visible are the expansion fan emanating from the top of the separation bubble. Note that the height of the computational domain is chosen such that the reflected waves of the impinging shock leave the domain at the outflow boundary (see figures 3.11 and 3.12). Streamlines plotted in figure 3.13 shows the asymmetric structure of the bubble. This asymmetric behavior of the flow within the separation bubble is similar to the validation case discussed in section 3.33.3.1.

Skin friction and wall pressure distributions along the plate from CASE A are presented in figure 3.14. The results look qualitatively similar to the results obtained for the conditions of Hakkinen et al. [88]. Furthermore, the asymmetrical distribution of the skin friction confirm the asymmetric behavior of the separation bubble. It has a lower minimum at the back of the bubble due to the center of the recirculation region being shifted downstream.

3.4.1 Linear Stability Investigations

We also computed the development of low-amplitude disturbances in order to investigate the stability of the flow in the presence of a laminar separation bubble that is generated by the SBLI. Towards this end, two-dimensional and three-dimensional pulse disturbances with a broad spectrum in frequency (see for example Sivasubramanian & Fasel [92]) were introduced into the boundary-layer for both cases, with and without SBLI. The pulse disturbance develops into a wave packet with a broad spectrum in frequency and wavenumber that propagates downstream. This allows to investigate the response of the shock-induced separation bubble to both low- and high-frequency disturbances. For these computations we employ a linearized compressible Navier–Stokes solver developed in our CFD laboratory [91] (see section 3.33.3.2).

In figure 3.16 the development of two-dimensional disturbances in a boundary-layer without SBLI is presented. Shown are contours of the streamwise velocity disturbance amplitude for four different time instances. These snapshots illustrate the development of the pulse disturbance into a wave packet that propagates downstream. It can be observed that the wave packet spreads in downstream direction and its amplitude levels increase as it propagates downstream. Figure 3.17 shows the corresponding results for the boundary-layer with SBLI. As before, in the case of the boundary layer without SBLI, the pulse disturbance develops into a wave packet that propagates downstream. However the downstream development is strongly affected by the presence of the impinging shock wave, the associated adverse pressure gradient and the resulting laminar separation bubble. The disturbance waves within the wave packet are now much more strongly amplified in the case of the boundary-layer with SBLI (as can be observed in figure 3.17).

According to the stability diagrams in figure 3.15, the dominant (most strongly amplified) disturbance waves in a Mach 2.3 boundary layer are three–dimensional (oblique disturbance waves). Therefore, we also investigated the development of three–dimensional disturbance using a pulse for spanwise wave number 0.67/mm (the most amplified spanwise wave number at the impingement location). The results are presented in figures 3.18 and 3.19 for the boundary–layer without and with SBLI, respectively. Similar to the two–dimensional disturbance behaviors, the three–dimensional (oblique) disturbances are also amplified in the downstream direction. However, as expected, it can also be observed that the three–dimensional disturbances are more strongly amplified compared to the two–dimensional ones. As observed before for two–dimensional disturbances, the three–dimensional (oblique) disturbances with spanwise wave number 0.67/mm are more strongly amplified for the case with SBLI present.

The amplitude of the waves within the wave packet are extracted and compared in figures 3.20 and 3.21 for the two-dimensional and three-dimensional disturbances respectively. In figures 3.22 and 3.23, the amplitudes of four

different two-dimensional and three-dimensional disturbance waves from the boundary layer with an impinging shock wave is compared to the boundary layer with no impinging shock. Once again these results confirm the observations made above. That is, both, two-dimensional and three-dimensional (oblique) disturbances are more strongly amplified with the SBLI.

3.4.2 Effect of Shock Angle and Reynolds Number

In order to investigate the effect of the incident shock angle the shock generator angle (θ) was increased to 6 degrees (CASE B). This produced a stronger pressure jump and consequently a larger separated flow region (see figure 3.24). The larger separation results in a longer pressure plateau and leads to secondary separation with in the separated flow region (see figure 3.24a). In addition, the separated flow region becomes unsteady and starts to shed vortices (see figure 3.24b). The time history of the wall pressure and the corresponding frequency spectra obtained using Fourier transformation are presented in figure 3.25. The frequency spectra show distinct peaks at higher frequencies that correspond to the vortex shedding. The high frequency peaks are due to a Kelvin-Helmholtz type instability mechanism arising from the inflectional velocity profile in the shear layer that forms between the flow inside the separation bubble and the flow outside the bubble.

Subsequently, to investigate the effect of increased Reynolds number, the Reynolds number was increased to Re = 720,000 (CASE C) and Re = 1,080,000 (CASE D). The results from CASE C are presented in figures 3.26 and 3.27 and the results from CASE D are presented in figures 3.28 and 3.29. Compared to the lower Reynolds number case (CASE B), the bubble is now shedding vortices even more strongly in CASE C and D (see figures 3.26b and 3.28b). With the increased Reynolds number, the boundary layer thickness near the separation is now reduced. Therefore, the length of the separation bubble is also reduced. The stronger shedding may also be responsible for the reduction in length.

The time history of the wall pressure and the corresponding frequency spectra for the high Reynolds number cases are plotted in figures 3.27 and 3.29. The time history of the wall pressure for the high Reynolds number cases (CASE C and D) look very different from the time history for the low Reynolds number case (CASE B). The stronger shedding of vortices could be clearly seen in the time history in figures 3.27a and 3.29a. The shedding manifests itself as distinct peaks in the frequency spectra in figures 3.27b and 3.29b. It can also be observed that as the Reynolds number increases the spectrum broadens over higher frequencies and the amplitude of the peaks in the frequency spectra increases.

3.4.3 Three-dimensional Simulations

We also performed three–dimensional simulations for the high Reynolds number case to investigate the laminar– turbulent transition process in the presence of SBLI (CASE E). Results from the three–dimensional direct numerical simulation are presented in figures 3.30 and 3.31. Contours of instantaneous streamwise velocity, wall normal velocity, temperature and density are shown in figure 3.30 in the x - y plane for a fixed spanwise location ($z^* = 0.0$). The vortex shedding that results from the shear layer instability (Kelvin-Helmholtz) is clearly visible and downstream of the re-attachment location the flow seems to transition to turbulence.

Figure 3.31 shows contours of streamwise velocity in the x - z plane (parallel to the wall) for several wall normal positions starting from close to the wall to away from the wall. This figure illustrates various flow features in the early turbulent region close to the wall and farther away from the wall. Remarkable streamwise structures seem to appear in figure 3.31 for all wall normal positions before the flow breaks down into small–scale structures. These streamwise structures may be a consequence of a dominant physical mechanism playing a role in the natural transition process in laminar SBLI. We plan to investigate these mechanisms in detail in future research. Note that such streamwise structures have also been observed in the numerical simulations of transitional shock/boundary–layer interactions in hypersonic flows by Sandham et al. [97].

3.5 Summary

The interaction between an impinging oblique shock–wave and a laminar boundary layer on a flat plate was investigated using direct numerical simulations. First, for validating the computational setup used in the present investigation, the experiments of Hakkinen et al. [88] were considered. The skin friction and pressure distributions from the direct numerical simulation were compared to the experimental measurements and other numerical results published in the literature. Our results confirmed the asymmetric nature of the separation bubble previously reported in the literature.

In the present investigation of the interaction of an oblique shock-wave and a laminar boundary layer, the flow parameters closely match the experiments conducted at the UA [81]. In this section we focused on 2D configurations with several different wedge angles and Reynolds numbers. We started with a lower Reynolds number of Re = 360,000 (based on the impingement location in an inviscid case) and a wedge angle of $\theta = 3^{\circ}$ (CASE A). The impinging shock wave causes the boundary layer to separate and as a result a laminar separation bubble is formed. The separation bubble looked qualitatively similar to the bubble in the validation case.

We investigated the development of low-amplitude disturbances generated by a short duration pulse in order to shed light on the linear stability behavior of the flow in the presence of SBLI. For these computations we employed a linearized compressible Navier-Stokes solver developed in our Computational Fluid Mechanics (CFD) laboratory [91]. For comparison, both the development of two-dimensional and three-dimensional (oblique) disturbances was investigated for two cases, with and without the SBLI. Not surprisingly, it was found that both the two-dimensional and three-dimensional disturbances were more strongly amplified for the cases with SBLI present, due to the increased instability effects of the adverse pressure gradient and boundary layer separation caused by the impinging shock wave.

The effect of the shock incidence angle was investigated by increasing the shock angle to 6° (CASE B). The larger shock angle produces a stronger pressure jump and thus a larger and stronger separated flow region. As a consequence, the flow became unsteady and the bubble started to shed vortices likely due to a Kelvin-Helmholtz instability. The time history and frequency spectra of the wall pressure indicated the presence of distinct high-frequency disturbances. These higher frequencies corresponded to the vortex shedding. We also investigated the effect of Reynolds number by increasing the Reynolds number to Re = 720,000 (CASE C) and Re = 1,080000 (CASE D). With the increase in the Reynolds number the thickness of the boundary layer decreased and the streamwise extent of the separated flow region was reduced. Also, the bubble started shedding more strongly with the increase in the Reynolds number. The time history of the wall pressure for the higher Reynolds number cases clearly showed the stronger vortex shedding activities. And from the corresponding frequency spectra it was observed that, as the Reynolds number increased the frequency spectrum broadened and the amplitude of the peaks with in the spectrum increased.

In addition, three–dimensional direct numerical simulations were performed for the higher Reynolds number case to investigate the laminar–turbulent transition process in the presence of SBLI (CASE E). Flow visualization from the three–dimensional simulation revealed a Kelvin-Helmholtz instability mechanism that lead to the development of spanwise rollers in the early stage of transition. Further downstream, the contours of the streamwise velocity component revealed streamwise structures ("streaks") before the flow breaks down to small scales as it transitions to turbulence. These streamwise structures could be due to a secondary instability mechanism that may be relevant in a "natural" transition process.



Figure 3.11: Flow visualization: contours of (a) streamwise velocity, (b) wall–normal velocity and (c) pressure. $M = 2.3, T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.12: Flow visualization: contours of (a) temperature, (b) density and (c) Mach number. M = 2.3, $T^*_{\infty} = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.13: Flow visualization: (a) contours of the streamwise velocity, (b) contours of the wall-normal velocity and (b) streamlines. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.14: Streamwise distribution of (a) skin friction and (b) wall pressure. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.15: Contours of constant amplification rate α_i in the $R_x - F$ plane for (a) two-dimensional disturbances and (b) three-dimensional (oblique) disturbances with spanwise wave number 0.67/mm (most amplified spanwise wavenumber at the shock impingement location) and in the $\beta - F$ plane for (c) $R_x = 600$ (corresponds to the shock impingement location) and (d) $R_x = 1000$. Amplification (growth) rates were computed using Mack's solver [94, 95]. The vertical dashed lines in red indicate the beginning and the end of the computational domain used in the LNS and DNS. Furthermore, the dashed line in black indicate the shock impingement location in an inviscid case. M = 2.3, $T_{\infty}^* = 144.8$ K, Re = 360,000.



Figure 3.16: Development of two–dimensional disturbances in a boundary layer without SBLI. Shown are contours of streamwise velocity disturbance amplitude. M = 2.3, $T_{\infty}^* = 144.8$ K, Re = 360,000.



Figure 3.17: Development of two–dimensional disturbances in a boundary layer with SBLI. Shown are condours of streamwise velocity disturbance amplitude. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.


Figure 3.18: Development of three–dimensional (spanwise wave number, $\beta = 0.67/mm$) disturbances in a boundary layer without SBLI. Shown are condours of streamwise velocity disturbance amplitude. M = 2.3, $T_{\infty}^* = 144.8$ K, Re = 360,000.



Figure 3.19: Development of three–dimensional (spanwise wave number, $\beta = 0.67/mm$) disturbances in a boundary layer with SBLI. Shown are condours of streamwise velocity disturbance amplitude. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.20: Detailed presentation of the streamwise development of the wall pressure disturbance amplitude for 2D disturbances. (a) Without SBLI (top) and (b) with SBLI (bottom). M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.21: Detailed presentation of the streamwise development of the wall pressure disturbance amplitude for 3D disturbances (spanwise wave number, $\beta = 0.67/mm$). (a) Without SBLI (top) and (b) with SBLI (bottom). $M = 2.3, T_{\infty}^* = 144.8$ K, $\theta = 3^\circ$, Re = 360,000.



Figure 3.22: Comparison of development of wall pressure disturbance amplitude for 2D disturbances in a boundary layer without SBLI and with SBLI. (a) F = 5.3197E - 5, (b) F = 6.3837E - 5, (c) F = 7.4476E - 5 and (d) F = 8.5116E - 5. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.23: Comparison of development of wall pressure disturbance amplitude for 3D disturbances (spanwise wave number, $\beta = 0.67/mm$) in a boundary layer without SBLI and with SBLI. (a) F = 4.2558E - 5, (b) F = 5.3197E - 5, (c) F = 6.3837E - 5 and (d) F = 7.4476E - 5. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 3^{\circ}$, Re = 360,000.



Figure 3.24: Flow visualization: contours of (a) streamwise velocity, (b) wall–normal velocity, (c) temperature and (d) density in a x - y plane. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 6^{\circ}$, Re = 360,000.

(a)



(b)



Figure 3.25: (a) Time history of wall pressure and (b) frequency spectra obtained from wall pressure. M = 2.3, $T^*_{\infty} = 144.8$ K, $\theta = 6^{\circ}$, Re = 360,000.



Figure 3.26: Flow visualization: contours of (a) streamwise velocity, (b) wall–normal velocity, (c) temperature and (d) density in a x - y plane. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 6^{\circ}$, Re = 720,000.

(a)



(b)



Figure 3.27: (a) Time history of wall pressure and (b) frequency spectra obtained from wall pressure. M = 2.3, $T^*_{\infty} = 144.8$ K, $\theta = 6^{\circ}$, Re = 720,000.



Figure 3.28: Flow visualization: contours of (a) streamwise velocity, (b) wall–normal velocity, (c) temperature and (d) density in a x - y plane. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 6^{\circ}$, Re = 1,080,000.

(a)



(b)



Figure 3.29: (a) Time history of wall pressure and (b) frequency spectra obtained from wall pressure. M = 2.3, $T^*_{\infty} = 144.8$ K, $\theta = 6^{\circ}$, Re = 1,080,000.



Figure 3.30: Flow visualization: contours of (a) streamwise velocity, (b) wall–normal velocity, (c) temperature and (d) density in a x - y plane obtained from the three dimensional simulation. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 6^{\circ}$, Re = 1,080,000.



Figure 3.31: Contours of the instantaneous streamwise velocity in the x-z plane for four wall-normal positions (a - b: from close to the wall to away from the wall) from the three dimensional simulation. M = 2.3, $T_{\infty}^* = 144.8$ K, $\theta = 6^{\circ}$, Re = 1,080,000.

4. Numerical Analysis of Turbulent Shock-Wave Boundary Layer Interactions

Hybrid turbulence model and large-eddy simulations of turbulent interactions at a freestream Mach number of 2.3 were carried out for sweep angles of 0° and 40° . The simulations for the unswept interaction reveal the shedding of spanwise coherent structures as well as intermittent spanwise deformations of the separation line that are referred to as ripples in the literature. The frequency associated with the ripples is similar to the reported low-frequency content of turbulent interactions. By adding an approach flow cross-flow component, a comparable turbulent interaction with 40° sweep is obtained. For the swept interaction, the overall level of the pressure fluctuations is higher than for the unswept case but the ripples are absent. In addition to spanwise coherent structures, traveling oblique structures were observed downstream of the interaction.

4.1 Methodology

4.1.1 Governing Equations and Discretization

A research computational fluid dynamics code by Gross and Fasel [98, 99] was employed for the present investigations. The code solves the compressible Navier-Stokes equations in curvilinear coordinates. Sutherland's law was invoked for computing the laminar dynamic viscosity, μ . The Subgrid Stress (SGS) tensor was modeled as

$$\tau_{ij} = -2\mu_T \left(S_{ij} - \frac{1}{3} S_{kk} \delta_{ij} \right) + \frac{1}{3} \tau_{kk} \delta_{ij} , \qquad (4.1)$$

where

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$
(4.2)

is the strain rate tensor of the resolved scales. The deviatoric part is proportional to a SGS (unresolved) eddyviscosity, μ_T , which is obtained either from the Renormalization Group (RG) hybrid model by De Langhe et al. [100, 101] or from the Wall-Adapting Local Eddy-Viscosity (WALE) LES model by Nicoud and Ducros [102]. According to Erlebacher et al. [103], the isotropic part, τ_{kk} , is small compared to the thermodynamic pressure and hence neglected. The heat flux is computed as

$$q_i = -c_p \left(\frac{\mu}{Pr} + \frac{\mu_T}{Pr_T}\right) \frac{\partial T}{\partial x_i}, \qquad (4.3)$$

with specific heat, c_p , and laminar and turbulent Prandtl numbers, Pr = 0.72 and $Pr_T = 0.9$. For the one-equation RG model [100, 101], a transport equation for the turbulent dissipation rate,

$$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial \rho u_i \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\left(\mu + \alpha \mu_T \right) \frac{\partial \varepsilon}{\partial x_i} \right] + \frac{\mu + \mu_T}{\rho} \left[\min\left(\Lambda_c, \Lambda_0 \right) \right]^2 \left(C_{\varepsilon 1} P_k - C_{\varepsilon 2} \rho \varepsilon \right) , \tag{4.4}$$

with $\alpha = 1.39$, $C_{\varepsilon 1} = 4/3$, and $C_{\varepsilon 2} = 2$ is solved. The filter-width wave-number is $\Lambda_c = \max \left[\pi/\Delta, \pi/(2.2y) \right]$, with cell diagonal, Δ , and wall distance, y. The dissipation wave-number is taken as, $\Lambda_0 = 0.215 \left(\rho^3 \varepsilon / \mu^3 \right)^{0.25}$, and the unresolved eddy-viscosity is obtained from

$$\mu_T = \mu \left\{ \left[1 + \frac{c_\mu \rho^3 \varepsilon}{\mu^3} \max\left(0, \Lambda_c^{-4} - \Lambda_0^{-4}\right) \right]^{\frac{1}{3}} - 1 \right\}.$$
(4.5)

with $c_{\mu}=0.1$. For the WALE model [102], the subgrid (unresolved) eddy-viscosity is modeled as

$$\mu_T = \bar{\rho} \Delta^2 C_w^2 \frac{(S_{ij}^* S_{ij}^*)^{3/2}}{(S_{ij} S_{ij})^{5/2} + (S_{ij}^* S_{ij}^*)^{5/4}}$$
(4.6)

with $S_{ij}^* = \left(g_{ij}^2 + g_{ji}^2\right)/2 - g_{kk}^2 \delta_{ij}/3$, $g_{ij}^2 = g_{ik}g_{kj}$, $g_{ij} = \partial u_i/\partial x_j$, and model constant $C_w = 0.5$. The grid length-scale, Δ , was computed as the geometric average of the cell dimensions.

The convective terms of the Navier-Stokes equations were discretized with a ninth-order-accurate weighted essentially non-oscillatory method based on the van Leer [82] flux vector splitting scheme. The discretization of the viscous terms was fourth-order-accurate. A second-order-accurate discretization was employed for the convective and diffusive terms of the RG model. The implicit trapezoid rule was employed for integrating the governing equations in time.

4.1.2 Non-Dimensionalization

Velocity and density were made dimensionless with the respective approach flow values, u_1^* and ρ_1^* . The reference length scale, L_{ref}^* , and Reynolds number, Re, were 0.1 m and 1 million. Temperature was non-dimensionalized with the approach flow temperature, $T_1^* = 145.77$ K. The approach flow pressure was $p_1^* = 7,397.5$ Pa. Pressure was made dimensionless with $\rho_1^* u_1^{*2}$, time was normalized by L_{ref}^*/u_1^* , and the eddy-viscosity was made dimensionless with the freestream dynamic viscosity, $\mu_1^* = 1.0017 \times 10^{-5}$ kg/(ms). The reference Mach number, M_1 , was 2.3.

4.1.3 Computational Grids

Table 4.1 provides the block dimensions and number of cells for the hybrid (RG) simulation and the LES. The streamwise and spanwise grid resolution for the LES is considerably better than for the hybrid simulation. The computational domains for both approaches were split up into two blocks: Block 1 was employed for temporal boundary layer simulations that provided turbulent inflow data. Block 2 was used for the spatial simulations of the turbulent interactions. Figure 4.1 provides the near-wall grid resolution in wall units. Georgiadis et al. [104]

Case	Block 1		Block 2	
	Dimensions	Cells	Dimensions	Cells
RG, unswept	$1.2 \times 0.8 \times 0.6$	$156{\times}128{\times}192$	$3.381 \times 0.8 \times 0.6$	$416 \times 128 \times 192$
LES, unswept and swept	$1.2 \times 0.8 \times 0.6$	$468{\times}138{\times}576$	$3.381 \times 0.8 \times 0.6$	$1248{\times}138{\times}576$

Table 4.1: Grid block dimensions and number of cells (streamwise \times wall-normal \times spanwise direction).

recommend grid resolutions of $10 \le \Delta x^+ \le 20$, $\Delta y^+ \le 1$, $5 \le \Delta z^+ \le 10$ for direct numerical simulations and $50 \le \Delta x^+ \le 150$, $\Delta y^+ \le 1$, $15 \le \Delta z^+ \le 40$ for LES. Accordingly, the grid resolution of the present simulations is sufficient for LES. Since the present LES are wall-resolved, the near-wall grid resolution had to be increased considerably compared to the hybrid simulation to capture the log-layer.

4.1.4 Temporal Boundary Layer Simulations

Temporal boundary layer simulations were run in parallel with the spatial simulations to provide turbulent inflow data. For the temporal simulations, periodic boundary conditions were employed in the streamwise and spanwise directions. The inflow and outflow data were shifted with respect to each other in the spanwise direction by a predefined distance, d_s , to prevent the spanwise locking of large-scale structures. According to Munters at al. [105] for $d_s = 0.5\delta$, which is roughly equal to the spanwise length-scale of the largest turbulent structures, the approach is most effective. For the present simulations, the boundary layer thickness was $\delta_{99} \approx 0.05$ and the shift was $d_s = 0.025$.

The following approach was taken to prevent the boundary layer from growing: After each timestep, the streamwise and spanwise average of the primitive variables, $\bar{\mathbf{q}}(y)$, was computed. The average was then sub-tracted from the instantaneous data to obtain the fluctuations, $\mathbf{q}(x, y, z)' = \mathbf{q}(x, y, z) - \bar{\mathbf{q}}(y)$. The flow field was then overwritten with the sum of a turbulent profile from a precursor RANS simulation and the fluctuations,



Figure 4.1: Near-wall grid resolution in wall units.

 $\mathbf{q}(x, y, z) \leftarrow \bar{\mathbf{q}}_{RANS}(y) + \mathbf{q}(x, y, z)'$. With this approach, a temporal turbulent boundary layer could be maintained. To prevent an accumulation of disturbances in the freestream, the conservative variables, \mathbf{Q} , were forced to the RANS profile in the freestream via, $\partial \mathbf{Q}/\partial t = (\mathbf{Q} - \mathbf{Q}_{RANS}) \times f$ where

$$f = \begin{cases} 0 & y < 0.2\\ \frac{y - 0.2}{0.4 - 0.2} & 0.2 \le y \le 0.4\\ 1 & 0.4 < y \end{cases}$$
(4.7)

4.1.5 Freestream and Boundary Conditions

$\lambda \; [deg]$	0	40		
M_1	2.3	3.0024		
u_1		1		
v_1		0		
w_1	0 0.83910			
p_1	0.13503			
T_1	1			
ϑ [deg]	12.5			
σ [deg]	36.880			
M_2	1.8124 2.5068			
u_2/u_1	0.85739			
v_2/u_1	-0.19008			
w_2/u_1	0 0.83910			
p_2/p_1	2.0562			
T_{2}/T_{1}	1.2420			

Table 4.2: Inviscid flow states.

The shock generator angle for the simulations was $\vartheta = 12.5^{\circ}$ and the sweep angle was $\lambda = 0^{\circ}$ and 40° . Using the oblique shock relationships, the shock angle in a z=const. plane, σ , and the pressure, temperature, and velocity ratios across the shock were computed (Tab. 4.2). For the swept interaction, instead of sweeping the impinging shock-wave, a spanwise velocity component, $w_1 = \tan \lambda$, was added to the approach flow which raises the approach flow Mach number to $M_1 = 2.3/\cos \lambda = 3$. For the unswept case, the reflected shock-wave angle is $\sigma = 47.003^{\circ}$ and the conditions downstream of the reflected shock-wave are $M_3 = 1.3651$, $u_3/u_1 = 0.72673$, $v_3/u_1 = w_3/u_1 = 0$, $p_3/p_1 = 3.8726$, and $T_3/T_1 = 1.4992$.

The flow states upstream and downstream of the impinging oblique shock-wave were prescribed at the freestream boundary. Dirichlet boundary conditions were employed at the inflow boundary of the spatial simulations for feeding in the unsteady data from the separate temporal turbulent boundary layer simulations. All

flow quantities were extrapolated at the outflow boundary. Periodic boundary conditions were employed in the spanwise direction. No-slip and no-penetration boundary conditions were employed on the flat plate. The flat plate was assumed to be adiabatic. For the RG model, the turbulent dissipation rate at the wall was computed from $\varepsilon = 0.22\nu \left(\frac{\partial u}{\partial y}\right)^2$.

4.1.6 Startup of Simulations

Both simulations of the unswept interaction were continued from earlier simulations. For the LES, the orderof-accuracy of the discretization of the convective terms was increased from seven (earlier simulation) to nine and the simulation was advanced over a dimensionless time interval of six (to let the flow adjust to this change) before flow data was analyzed. The LES of the infinite swept interaction was started from an instantaneous flow field of the unswept interaction. The *w*-velocity component was initialized according to

$$w \leftarrow w + \min(\max(u, 0), 1) \tan \lambda, \qquad (4.8)$$

with $\lambda = 40^{\circ}$ at t = 0 and flow data was analyzed for t > 5.5.

4.1.7 Proper Orthogonal Decomposition

The proper orthogonal decomposition (POD) by Lumley [106] was developed for extracting coherent structures from turbulent flows. When temporal data are available in discrete form, the computationally more efficient "snapshot" method by Sirovich [107] can be employed. Typically, the POD kernel is computed from the square of the velocity vector. The magnitude of the POD eigenvalues, λ_i , does then correspond to two times the kinetic energy contents of the respective modes. The POD captures traveling waves with mode pairs where the respective time-coefficients, a_i , and modes are nearly identical except for a phase shift of 90°. It was decided to employ the POD for analyzing the wall pressure, c_p , and streamwise skin-friction coefficient, c_f . The wall pressure is more representative of flow structures away from the wall while the skin-friction is more representative of boundary layer structures. For the analysis of the wall pressure coefficient, the POD kernel was computed from the square root of the wall pressure, $p = c_p/2 + 1/(\gamma M^2)$. The POD eigenvalues are then proportional to the magnitude of the static pressure fluctuations associated with the modes. For the analysis of the skin-friction coefficient, the POD kernel was computed from c_f and the eigenvalues are proportional to c_f^2 . The modes were sorted according to their eigenvalue magnitude.

4.2 Results

4.2.1 Approach Flow Boundary Layer

Temporal and spanwise averages of the approach flow were computed over time-intervals of 100 (0 < t < 100, hybrid simulation of unswept interaction), 26 (6 < t < 32, LES of unswept interaction), and 29 (5.5 < t < 34.5, LES of swept interaction). For the computation of the boundary layer displacement and momentum thickness, the boundary layer edge was taken as the wall-normal distance where the dimensionless spanwise vorticity, ω_z , dropped below 0.2. This is illustrated in Figure 4.2. In Figure 4.3a, the displacement, δ^* , and momentum thickness, ϑ , as well as the shape factor, H, are plotted versus the x-coordinate. For the LES of the swept interaction, the boundary layer is growing slightly faster in x which can be attributed to the swept freestream. The incompressible shape factor upstream of the interaction is roughly 1.4 for the hybrid simulation and 1.3 for the LES (Figure 4.3b) and thus in the range that is typical for turbulent boundary layers. Upstream of the interaction, the momentum thickness Reynolds number reaches roughly 4,000 (Figure 4.4a) and the skin-friction coefficient is slightly lower than the van Driest transformed [108] incompressible skin-friction coefficient (Figure 4.4b). In Figure 4.5 velocity profiles in wall units [109]

$$y^+ = \frac{yu_\tau}{\nu_w} \tag{4.9}$$

$$u^* = \frac{1}{\sqrt{\tau_w}} \int \sqrt{\rho} du \tag{4.10}$$

are compared with the law of the wall, $u^* = 5 + \ln y^+ / 0.41$, and the relationship for the laminar sublayer $u^* = y^+$. The friction velocity is obtained from the wall shear stress, $u_\tau = \sqrt{\tau_w / \rho_w}$. The profile for the hybrid simulation



Figure 4.2: Iso-contours of spanwise vorticity, ω_z , (left) and *u*-velocity (right) with estimated boundary layer edge (solid lines) for a) hybrid simulation and b) LES of unswept interaction.



Figure 4.3: a) Displacement thickness, momentum thickness, and shape factor. b) Incompressible shape factor.



Figure 4.4: a) Momentum thickness Reynolds number. b) Skin-friction coefficient.

follows the reference profile near the wall (where the model is in RANS mode) and then deviates from it for $y^+ > 100$ (LES mode). This "log-layer mismatch" is known to reduce the skin-friction (also see Figure 4.4b). The LES profiles in Figure 4.5 lie slightly above the reference profile which is an indication of insufficient grid resolution. Simulations by Morgan et al. [30] using a coarse mesh with $\Delta x^+ = 42$ and $\Delta z^+ = 21$ exhibited a similar behavior. When the near-wall grid resolution was increased to $\Delta x^+ = 30$ and $\Delta z^+ = 16$, a closer match



Figure 4.5: Velocity profiles in wall units at x=-0.8.

with the law of the wall was obtained.

4.2.2 Analysis of Separated Region for Unswept Interactions

According to free interaction theory (Chapman et al. [110]), the pressure rise across the separation shock can be linked to the skin-friction coefficient,

$$\frac{p - p_i}{q_i} = \sqrt{\frac{2c_{f,i}}{\sqrt{M_i^2 - 1}}} F, \qquad (4.11)$$

where the subscript "i" refers to incipient separation. The function F is 4.22 at the onset of separation and 6 when a significant change of the flow field from the attached boundary layer state can be observed. Assuming that the flow state at "i" is equivalent to the freestream state, Eq. 4.11 becomes

$$c_p = F_{\sqrt{\frac{2c_{f,i}}{\sqrt{M_1^2 - 1}}}}.$$
(4.12)

The pressure rise across the separation shock is

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} \left[(M_1 \sin \sigma)^2 - 1 \right] \,. \tag{4.13}$$

With dynamic pressure $q_1 = 1/2\gamma M_1^2 p_1$, from this a second expression for the wall pressure coefficient can be obtained,

$$c_p = \frac{4\left[(M_1 \sin \sigma)^2 - 1\right]}{(\gamma + 1)M_1^2} \,. \tag{4.14}$$

For the RG model simulation, the mean separation and reattachment are at x = -0.61040 and x = 0.10640(Figure 4.6a). A curve fit provides an estimate of the skin-friction coefficient at separation, $c_{f,fit} = 0.0018206$, which according to Eq. 4.12 is equivalent to $c_p = 0.17694$ for F = 4.22 and $c_p = 0.25157$ for F = 6. According to Figure 4.6a, the Chapman criterion approximately captures the pressure plateau associated with the boundary layer separation. From Eq. 4.14 separation shock angles of $\sigma = 32.910^{\circ}$ (F = 4.22) and $\sigma = 35.667^{\circ}$ (F = 6) can be computed. The exact reflected shock angle is $\sigma - \vartheta = 47.003 - 12.5 = 34.503^{\circ}$. In Figure 4.6b iso-contours of the magnitude of the density gradient ("numerical Schlieren") are superimposed with streamlines and lines that indicate the inviscid impinging and reflected shock-waves as well as the angles corresponding with the Chapman pressure rise. Although the separation in the present hybrid simulation is substantial, the best agreement with the fit for the reflected shock-wave is obtained for F = 4.22. According to Chapman et al. [110] this value is to be used for barely separated flows (onset of separation).

The displacement and momentum thickness are $\delta_1^* = 0.020886$ and $\vartheta_1 = 0.010490$ at separation and $\delta_3^* = 0.072916$ and $\vartheta_3 = 0.018365$ at reattachment (Figure 4.7). The length of the separated region is $L_s = 0.7168$ and the interaction length, which is the difference between the inviscid shock impingement location and the



Figure 4.6: a) Skin-friction and wall pressure coefficient (vertical dashed lines indicate mean separation and reattachment locations) and b) iso-contours of density gradient from hybrid simulation.



Figure 4.7: Displacement and momentum thickness from hybrid simulation.

approximate foot of the reflected shock-wave (when extended to the wall; fit in Figure 4.6b), is $L_{int} = 0.73$ (corresponding to $L_{int}/\delta_1^* = 35.0$). Based on a control volume analysis for the continuity and momentum equations, Souverein et al. [60] proposed two estimates for the interaction length,

$$L_{int,1} = \frac{\frac{\rho_3}{\rho_1} \frac{u_3}{u_1} \delta_3^* - \delta_1^*}{-\frac{\rho_2}{\rho_1} \frac{v_2}{u_1}} = 0.3686, \qquad (4.15)$$

$$L_{int,2} = \frac{\frac{\rho_3}{\rho_1} \left(\frac{u_3}{u_1}\right)^2 \left(\vartheta_3 + \delta_3^*\right) - \left(\vartheta_1 + \delta_1^*\right)}{-\frac{\rho_2}{\rho_1} \frac{u_2}{u_1} \frac{v_2}{u_1}} = 0.3453.$$
(4.16)

For the estimates, δ_3^* and ϑ_3 were taken directly at reattachment which implies larger estimated length scales than when δ_3^* and ϑ_3 were taken downstream of the interaction where the reattached boundary layer recovers its equilibrium state. Both numbers fall short of the observed interaction length, $L_{int} = 0.73$, which suggests that the extent of the separated region is overpredicted in the hybrid simulation.

For the LES, the flow separates at x = -0.35878 and reattaches at x = 0.069844 (Figure 4.8a). The fitted skinfriction coefficient at separation is $c_{f,fit} = 0.0018845$. According to Chapman, this corresponds to $c_p = 0.18002$ for F = 4.22 and $c_p = 0.25595$ for F = 6. The associated separation shock angles are $\sigma = 33.026^{\circ}$ and $\sigma = 35.826^{\circ}$ (Eq. 4.14) and close to both the fitted (at the shock foot) and theoretical (inviscid) reflected shockwave angles (Figure 4.8a). According to Figure 4.9a the displacement and momentum thickness are $\delta_1^* = 0.015692$ and $\vartheta_1 = 0.0053461$ at separation and $\delta_3^* = 0.057020$ and $\vartheta_3 = 0.012807$ at reattachment. The length of the separated region is $L_s = 0.4286$ and the interaction length is $L_{int} = 0.4$ ($L_{int}/\delta_1^* = 25.5$). The estimates by



Figure 4.8: a) Skin-friction and wall pressure coefficient (vertical dashed lines indicate mean separation and reattachment locations) and b) iso-contours of density gradient from LES.



Figure 4.9: a) Displacement and momentum thickness and b) skin-friction and wall pressure coefficient from LES.

Souverein et al. [60] are $L_{int,1} = 0.2903$ and $L_{int,2} = 0.2751$. The proposed scaling for the height of the separated flow region [60] is $\delta_1^* L_{int} = 0.00628$ (compare to Figure 4.8b). Souverein et al. [60] also proposed an inviscid separation criterion,

$$S_e^* = k \frac{\Delta p}{q_e} \,, \tag{4.17}$$

where Δp is the pressure increase across the separation shock and q_e is the boundary layer edge dynamic pressure. A constant k of 3 is recommended for $Re_{\vartheta} \leq 10^4$ (this is the case here) and 2.5 otherwise. Separation occurs for $S_e^* \geq 1$ which implies $c_p \geq 1/3$. Based on Figure 4.9b this suggests that the flow would separate at x = -0.24393 which constitutes a less accurate estimate than the prediction based on the Chapman criterion.

Based on the comparison with the existing correlations from the literature it was decided that the LES did likely provide a more truthful representation of the flow. In the following, data from both simulations are analyzed with respect to the unsteady fluid dynamics.

4.2.3 Analysis of Unsteady Flow

Unswept Interaction

Instantaneous visualizations of the Q vortex identification criterion [111] obtained from the simulations for the unswept interaction in Figure 4.10 illustrate the turbulent approach flow boundary layer, the interaction region, and the thick reattached turbulent boundary layer. Compared to the hybrid simulation, for the LES considerably finer unsteady flow structures are resolved and the separated region is smaller. Also shown in Figure 4.10 are instantaneous visualizations of the streamwise skin-friction coefficient, $c_f = 2\omega_z/Re$, that reveal streaky structures upstream of the interaction. Such structures are typical for turbulent boundary layers. For the particular time instances shown in Figure 4.10, neither the separation nor the reattachment lines are straight. Instantaneous iso-



Figure 4.10: Unswept interaction. Iso-surfaces of Q=10 flooded by streamwise velocity and iso-contours of skin-friction coefficient, $-0.005 < c_{f,x} < 0.005$, for a) hybrid simulation and b) LES.



Figure 4.11: Unswept interaction. Iso-contours of density gradient (left) and temperature (right) in z=const. plane obtained from a) hybrid simulation and b) LES.

contours of the density gradient magnitude (numerical Schlieren) and temperature in a constant spanwise plane in Figure 4.11 reveal large-scale spanwise-coherent structures in the interaction region that are transporting hot near-wall fluid into the freestream.

The wall pressure and streamwise skin-friction coefficient were recorded at constant time intervals of $\Delta t = 0.02$ for the hybrid simulation and 0.05 for the LES (unswept and swept). The wall pressure data were then Fouriertransformed in the spanwise direction. Figure 4.12 provides the Fourier mode amplitudes and phases for mode k = 0 (spanwise average) and for modes k = 1 through k = 3 with wavelengths $\lambda_z = 0.6/k$. For the hybrid



Figure 4.12: Unswept interaction. Amplitude and phase of spanwise Fourier modes of wall pressure coefficient obtained from a) hybrid simulation and b) LES.

simulation, the mean pressure increase (mode k = 0) associated with the separation line is noticeably moving back and forth in time. Time-periodic traces downstream of reattachment for mode k=0 (spanwise average) are a consequence of the shedding of spanwise coherent flow structures. The k = 1 mode phase at separation varies very slowly and intermittently the k = 1 amplitude attains large values. This behavior can be associated with the random appearance of "bulges" or ripples of the separation shock as shown in Figure 4.13a. The ripples resemble the "teepee" structures in separated rocket motor flows [112]. Intermittently, modes 2 & 3 attain larger amplitudes at separation which can be associated with the appearance of multiple ripples (e.g. t = 30.78). Downstream of reattachment the flow is highly unsteady and the mode k = 2, 3 amplitudes are noticeably increased. A similar but less dramatic behavior can be observed for the LES (Figs. 4.12b & 4.13b).

The spanwise wavelength of the ripples in Figure 4.13 is many times larger than the wavelength of the nearwall streaks. Ganapathisubramani et al. [10] and Humble et al. [25] found that spanwise deformations of the separation line can be correlated to superstructures in the approach boundary layer. According to Elsinga et al. [113] the spanwise spacing of the dominant flow structures in supersonic boundary layers is approximately identical to the boundary layer thickness. For the present simulations, the boundary layer thickness upstream of separation is $\delta_{99} \approx 0.1$ and thus still six times smaller than the wavelength of the observed ripples which suggests that the structures may not be related to the upstream boundary layer but that they rather arise as a consequence of an instability of the separated region.

To check if the spanwise wavelength of the ripples was artificially fixed by the spanwise domain extent, the



Figure 4.13: Unswept interaction. Wall pressure coefficient, $0 < c_p < 0.6$, and skin-friction coefficient, $-0.005 < c_{f,x} < 0.005$, iso-contours obtained from a) hybrid simulation and b) LES.



Figure 4.14: Wall pressure coefficient iso-contours ($0 < c_p < 0.6$) obtained from hybrid simulation of unswept interaction with wider domain.

spanwise domain extent for the hybrid simulation was doubled. Visualizations of the wall-pressure coefficient again reveal structures of varying intensity and wavelength at random spanwise locations (Figure 4.14). The wedge angle of the ripples in Figs. 4.13a and 4.14 is identical which suggests an underlying inviscid compressible mechanism.

Instantaneous visualizations for one ripple at t=49 are shown in Figure 4.15. The wall pressure coefficient associated with the structure has a diamond shape (Figure 4.15b). Off the wall, the leading edge of the structure becomes rounded (Figure 4.15c). For y=0.1761 a second diamond-shaped shock structure can be observed downstream of the primary structure. The wall pressure coefficient ahead of the separation shock, c_{p1} , inside the first diamond structure, c_{p2} , and immediately downstream of the structure, c_{p3} , were extracted from the simulation (Tab. 4.3). A triple point (TP) is assumed at the interface of the diamond-shape structure and the unswept separation shock (Figure 4.16a). In analogy to shock-polars for two-dimensional flow, using the oblique



Figure 4.15: Instantaneous flow visualizations for t = 49 obtained from hybrid simulation of unswept interaction. a) $|\nabla \rho| = 6$ iso-surfaces flooded by pressure coefficient. Pressure coefficient contours at b) wall and c) y=0.1761.

shock relationships [53], for a given Mach number, M, all possible flow deflection angles in the wall-normal (ϑ) and spanwise (η) direction can be computed (Figure 4.16b). The associated surfaces are axissymmetric and shaded by the sweep angle of the oblique shock with respect to the flow direction.

-	1	2	3
c_p	0	0.028	0.066
λ [°]		43.35	38.37
M	2.3	2.075	1.857
ϑ [°]		4.809	9.891
η [°]		3.206	0

Table 4.3: Flow states for diamond structure.



Figure 4.16: a) Sketch of separation shock system in y=const. plane (x points to the right). b) Possible flow states (shock pressure ratio, p_2/p_1 , and deflection angles in wall-normal, ϑ , and spanwise, η , direction) flooded by sweep angle in degrees.

The pressure coefficient after the first swept shock, $c_{p2}=0.028$, corresponds to a pressure ratio of $p_2/p_1 = 2c_{p2}\gamma M_1^2 + 1 = 1.415$, or $\ln(p_2/p_1) = 0.3469$. By cutting the $M_1 = 2.3$ surface in Figure 4.16b at that pressure ratio, the curves in Figure 4.17a were generated. For a sweep angle of $\lambda_2 = 43.35^{\circ}$, an after-shock Mach number of $M_2=2.075$ and deflection angles of $\vartheta_2 = 4.809^{\circ}$ and $\eta_2 = 3.206^{\circ}$ are obtained (all values are in approximate agreement with the simulation). A second surface was added in Figure 4.16b for the after-shock Mach number M_2 . Similar to the shock-polar based 2-D TP analysis, the surface was displaced by $\ln(p_2/p_1)$, ϑ_2 , as well as η_2 , and flipped in the η -direction. The pressure coefficient after the second diamond shock, $c_{p2}=0.066$, corresponds to a pressure ratio of $p_3/p_1 = 2c_{p3}\gamma M_1^2 + 1 = 1.978$ or $\ln(p_3/p_1) = 0.6819$ and an after-shock Mach number of $M_3 = 1.857$. Distributions of the deflection angles and Mach number for this pressure ratio are provided in Figure 4.17b. Assuming a zero spanwise deflection after the second oblique shock, $\eta_3 = 0$, a wall-normal deflection angle of $\vartheta_3 = 9.891$ is obtained for $\lambda_3 = 38.37^{\circ}$. Assuming that the pressure across the slip line (Figure 4.16a) is



Figure 4.17: Curves of constant a) $\ln(p_2/p_1)$ and b) $\ln(p_3/p_1)$.

constant, $c_{p2'} = c_{p3}$, an after-shock Mach number of $M_{2'} = 1.8416$ (very close to M_2) and a deflection angle of $\vartheta_{2'} = 11.8^{\circ}$ (slightly more than ϑ_3) are obtained for the unswept interaction. The similarity of the after-shock conditions (3 and 2') may explain why the ripples can form.



Figure 4.18: Unswept interaction. POD eigenvalues for a) hybrid simulation for -0.962 < x < 0.982 and 40 < t < 100, and b) LES for -0.500 < x < 0.994 and 6 < t < 32.

The wall data for the hybrid simulation (time-interval of 40 < t < 100, frame rate $\Delta t = 0.02$) and the LES $(0 < t < 26 \text{ and } \Delta t = 0.05)$ were analyzed with the POD. Overall, the eigenvalue magnitudes for the hybrid simulation are higher than for the LES which implies larger wall fluctuation amplitudes (Figure 4.18). The eigenfunctions and time-coefficients for the hybrid simulation of the unswept interaction are compared in Figure 4.19. Concerning the wall pressure data, modes 2-4 capture the shedding of spanwise coherent structures (similar eigenvalue magnitudes as well as eigenfunctions and time-coefficients that are similar except for a phase shift). Mode 6 and 7 capture 3D structures at separation and have a low-frequency content and may thus be related to the ripples. The POD based on the skin-friction coefficient is more difficult to interpret. Modes 2-4 appear to capture streamwise structures downstream of reattachment.

The POD modes and time-coefficients for the LES are quantitatively similar to the respective results for the hybrid simulation (Figure 4.20). With respect to the wall pressure data, modes 1-3 capture low-frequency 3D structures at separation and modes 4 & 5 must be related to the shedding of spanwise coherent structures. The skin-friction POD modes 1-3 also capture low-frequency 3D structures at separation that are related to streamwise structures downstream of reattachment.

The time-coefficients were Fourier-transformed in time (Figure 4.21) and the frequency was normalized by the streamwise extent of the separated region (Tab. 4.4). Concerning the wall pressure data for the hybrid simulation, modes 2-4 (shedding of spanwise coherent structures) have a frequency of roughly 0.5 and modes 5 &



Figure 4.19: Hybrid simulation of unswept interaction. POD modes and time-coefficients computed from a), b) \sqrt{p} (-0.01 < A < 0.01) and c), d) c_f (-0.0005 < A < 0.005) for -0.962 < x < 0.982 and 40 < t < 100.

Case	L_s
hybrid, unswept	0.7168
LES, unswept	0.4286
LES, swept	0.9235

Table 4.4: Streamwise extent of separated region.

6 (3D structures at separation) have a frequency of roughly 0.05. For the LES, the mode 4 & 5 (spanwise coherent structures) frequency is roughly 0.55 and the modes 1-3 frequency is roughly 0.05. The low-frequency contents for both the hybrid simulation and the LES is of the same order as the reported low-frequency unsteadiness of $f \times L_{int} \approx 0.01$ for unswept ramps and $f \times L_{int} \approx 0.03$ for swept ramps [3]. The present results thus suggest that the low-frequency unsteadiness may be coupled to the spanwise deformations (ripples) of the separation line.

Swept Interaction

Finally, the results for the swept interaction are discussed. Instantaneous flow visualizations in Figs. 4.22 & 4.23 for the swept interaction resemble those for the unswept interaction (Figs. 4.10 & 4.11), although the interaction region appears longer for the swept interaction. For the hybrid turbulence model simulation of the swept interaction, the separated flow region opened up with time (Figure 4.24). This was not the case for the LES. Therefore, in the following only the LES data for the swept interaction are analyzed. In Figure 4.25 the skin-friction and wall pressure coefficient for the LES of the swept and unswept interaction are compared. The



Figure 4.20: LES of unswept interaction. POD modes and time-coefficients computed from a), b) \sqrt{p} (-0.01 < A < 0.01) and c), d) c_f (-0.0005 < A < 0.005) for -0.500 < x < 0.994 and 6 < t < 32.



Figure 4.21: Unswept interaction. Fourier transforms of POD time-coefficients for a) hybrid simulation for -0.962 < x < 0.982 and 40 < t < 100 as well as b) LES for -0.500 < x < 0.994 and 6 < t < 32.

time-average for the swept interaction was computed from t = 5.5 to t = 34.5. The swept flow separates earlier at x = -0.51824 and reattaches later at x = 0.18922 compared to the unswept flow. The separation length in the streamwise direction is $L_s = 0.7075/\cos \lambda = 0.9235$. Similar to the hybrid simulation for the unswept interaction,



Figure 4.22: Swept interaction. Iso-surfaces of Q=10 flooded by x-velocity component and iso-contours of skinfriction coefficient, $-0.005 < c_{f,x} < 0.005$, for a) hybrid simulation and b) LES.



Figure 4.23: LES of swept interaction. Iso-contours of density gradient (left) and temperature (right) in z=const. plane.



Figure 4.24: Hybrid simulation of swept interaction. Spanwise average of wall pressure coefficient.

the wall pressure distribution features a plateau.

Amplitudes and phases for the first four spanwise Fourier modes of the wall pressure coefficient are shown in Figure 4.26. Starting from the initial condition, the size of the interaction region is seen to gradually increase up to $t \approx 5$ and then fluctuate slowly in the *x*-direction. As for the unswept interaction (Figure 4.12b), the mode k = 0 mode amplitude suggests the shedding of spanwise coherent structures (Figure 4.26). However, different from the unswept interaction, the mode k = 1 phase at separation is changing rapidly which suggests the absence of ripples. In fact, instantaneous visualizations of the wall pressure and skin-friction coefficient



Figure 4.25: Comparison of LES skin-friction and wall pressure coefficient.



Figure 4.26: LES of swept interaction. Amplitude and phase of spanwise Fourier modes of wall pressure coefficient.



Figure 4.27: LES of swept interaction. Wall pressure, $0 < c_p < 0.6$, and skin-friction coefficient, $-0.005 < c_f < 0.005$, iso-contours.

display no pronounced rippling of the separation line (Figure 4.27).

The wall data for 5.5 < t < 34.5 (frame rate: $\Delta t = 0.05$) were analyzed with the POD. The eigenvalues (Figure 4.28) are slightly larger than for the unswept case (Figure 4.18b) which suggests overall larger fluctuations. Eigenvalues 2 & 3 and 4 & 5 have similar magnitudes. Figure 4.29 provides the POD modes and time-coefficients. Mode 1 of the wall pressure data captures an adjustment of the mean-flow for 5.5 < t < 20 that is followed by a low-frequency expansion and contraction ("breathing") of the separation bubble (only 1.5 periods were captured).



Figure 4.28: LES of swept interaction. POD eigenvalues for -2.126 < x < -0.588 and 5.5 < t < 34.5.



Figure 4.29: LES of swept interaction. POD modes and time-coefficients computed from a), b) \sqrt{p} (-0.01 < A < 0.01) and c), d) c_f (-0.0005 < A < 0.005) for -2.126 < x < -0.588 and 5.5 < t < 34.5.

Modes 2 & 3 can be associated with spanwise coherent structures. Interestingly, modes 4 & 5 capture traveling oblique structures which are often an indication of cross-flow instability. Because periodic boundary conditions were employed in the spanwise direction, only certain spanwise wavenumbers are possible, $\beta = 2\pi n/0.6$, where $n = 0, 1, \ldots$ is a positive integer number. The width of the computational would have to be increased to get around this limitation. Different from the POD for the unswept interaction (Figure 4.20a), no 3D structures are

captured at separation. With respect to the skin-friction coefficient, the POD captures streamwise structures downstream of reattachment.



Figure 4.30: LES of swept interaction. Fourier transforms of POD time-coefficients for -2.126 < x < -0.588and 5.5 < t < 34.5.

Spectra of the POD time-coefficients are provided in Figure 4.30. Compared to the results for the unswept interaction (Figure 4.21b), the low-frequency contents is noticeably reduced. Considering the spectra for the wall pressure coefficient, the frequency associated with the 2-D shedding (modes 2 & 3) is roughly 1.3 and the frequency associated with the oblique structures is around 2.6. The skin-friction coefficient unsteadiness is centered around a frequency of roughly one which is close to the frequency of the spanwise coherent structures. Interestingly, the frequency contents for the oblique structures is missing which suggests that the oblique structures are situated too far from the wall to affect the skin-friction coefficient.

4.3 Summary

Simulations of an unswept and swept interaction at Mach 2.3 were carried out with a hybrid turbulence model and a subgrid stress model. The setup of the simulations was guided by related experiments at the University of Arizona [50, 51, 52]. Based on a comparison with available correlations from the literature, it was determined that the hybrid simulation of the unswept interaction likely over-predicted the extent of the separation. For both simulations, 3D structures developed intermittently at the foot of the separation shock. Such structures are referred to as ripples in the literature [25, 63, 52]. In accordance with Doehrmann et al. [52], the ripples were found to be correlated with the low-frequency contents of the interaction. A high-frequency unsteadiness downstream of the interaction was attributed to the shedding of spanwise coherent structures.

A spanwise freestream velocity was added to obtain an approach flow sweep angle of 40° relative to the incoming shock-wave. Because periodic boundary conditions were employed in the spanwise direction, the setup of the simulation forces the separated region to be cylindrically symmetric, and disturbances that "leave" the domain at one spanwise boundary are entering the domain again at the other spanwise boundary. No closed separation could be obtained with the hybrid turbulence model. For the LES, the ripples were noticeably absent and existing low-frequency contents could be related to a breathing motion of the bubble. In addition to spanwise coherent structures, traveling oblique structures could be identified downstream of the swept interaction.

5. Volumetric Study of Swept Impinging Oblique SBLIs

A swept impinging oblique SBLI is investigated in Mach 2.3 flow induced by a shock generator with sweep $\psi = 30.0^{\circ}$ and x-y plane deflection of $\theta = 12.5^{\circ}$. The incoming flow is a naturally turbulent boundary layer developing over the flat wind tunnel wall with $\text{Re}_{\theta} = 5.5 \times 10^3$. A combination of Stereo PIV and Tomographic PIV is used to characterize both the undisturbed incoming boundary layer and the resultant complex geometries of the swept SBLI. Linear Stochastic Estimation is used to identify statistically significant boundary layer vortical structures and document changes to their topology at various heights in the boundary layer. Three-dimensional velocity snapshots throughout the swept SBLI show both large-scale growth/collapse of the interaction and prominent streamwise streaks with a notable spanwise periodicity. Finally the mean structure for this configuration is documented for the first time. This is an initial summary of a large PIV data set (over 3 TB) that already provides a valuable contribution to the understanding of this complex flow.

5.1 Experimental Methodology

5.1.1 Experimental Facility

Experiments have been conducted in the new Supersonic Wind Tunnel facility at the University of Arizona. The working section and inlet of this tunnel have recently been redeveloped, replacing the flexible nozzle of the prior facility which had been in use since the 1960s. Solid aluminum nozzle blocks designed using the Method of Characteristics with boundary layer corrections now define the throat of the vacuum driven in-draft tunnel. The present tests have been conducted at a nominal Mach number of 2.3 (see Table 5.1), but Mach 4.0 nozzles are also available (Mach 3.0 nozzles are also in development). The parallel working section measures 121.9 mm \times 81.3 mm \times 609.6 mm (4.8" \times 3.2" \times 24", W \times H \times L) and features a series of interchangeable panels and liner pieces to facilitate a range of experimental setups. The naturally dry Arizona air provides excellent stagnation conditions enabling tests at constant T_0 and p_0 , which can be challenging in blow-down facilities. An adjustable second throat is employed downstream to optimize run times, before air is exhaust to a 34.0 m^3 (1200 ft^3) vacuum chamber. Typical run times are of the order 15 s. The present tests have been conducted using a shock generator mounted to the tunnel side-wall. Optical access for Particle Image Velocimetry (PIV) is achieved through small windows in the shock generator side-panel (for the laser access) and then large opposing side-panel windows which span the entire working section height (see Figure 5.1). Unless stated otherwise the streamwise, wall normal, and spanwise dimensions are defined relative to the throat, tunnel floor and the spanwise centerline, respectively. When presenting SBLI configurations, the spanwise dimension is defined relative to the root plane (tunnel side wall).

5.1.2 Shock Generator

The impinging shock is induced using a shock generator with the leading edge in a plane running parallel to the floor and geometry defined according to Figure 1.3a. The aluminum generator is 88.9 mm (3.5") wide and has maximum thickness below the leading edge of 8.9 mm (0.35"). The leading edge is swept at an angle of $\psi = 30.0^{\circ}$

M_{∞} [-]	T_0 [K]	p_0 [Pa]	$U_{\infty} [\mathrm{m/s}]$	${\rm Re}_{\infty}/L \ [1/m]$
2.28	300.5	$0.933 imes 10^5$	554.8	9.99×10^6
± 0.02	± 0.2	$\pm 0.002 \times 10^5$	± 2.6	$\pm \ 0.13 \times 10^6$

Table 5.1: Working section test environment.



Figure 5.1: Experimental facility (schematic diagram drawn to scale).

(defined in the x-z plane) with the downstream active surface inclined at $\theta = 12.5^{\circ}$ (defined in the x-y plane). A photo of the shock generator underside is shown in Figure 5.2.



Figure 5.2: Photo of shock generator ($\psi = 30.0^\circ$, $\theta = 12.5^\circ$, W = 88.9 mm, t = 8.9 mm).

The shock generator is mounted to the side wall of the tunnel using a plug installed in the side panel. The height of the locating dowel pins (and thus the aspect ratio of the SBLI) can be varied by interchanging different plugs. The current investigation is conducted using a plug to mount the shock generator such that the leading edge is at a height of 48.26 mm (1.90") from the wall. It is located in the streamwise direction such that the root apex is located at x = 462.0 mm (18.23") from the throat. Projection of the quasi-infinite shock to the tunnel floor (see Table 5.2) shows that the quasi-infinite impingement line root apex is located at x = 518.5 mm (20.41").

Flow parameters across the swept impinging shock have been determined for the quasi-infinite span case[55], as shown in Table 5.2 where errors in shock generator angles are assumed to be $\pm 0.1^{\circ}$. While a similar array of flow properties can often be calculated across the reflected shock also, such is not possible for the present configuration. This is because the required deflection to realign flow with the wall would be beyond the maximum turning angle for the flow downstream of the impinging shock. In unswept flows the reflected shock would detach, resulting in a Mach reflection. In swept shock reflections the behaviour is less characterized. Some researchers have proposed that it is the detachment of the swept reflected shock that is responsible for the conical spanwise growth of the SBLI[41]. However, a full understanding of the influence of the detached shock remains unexplored.

It is prudent to note that the impinging shock may be induced through a range of shock generating model geometries[55]. For brevity, only the standard shock generator is investigated here. Table 5.3 shows the equivalent geometries that would be required from a delta wing generator to produce an identical swept impinging oblique shock.

β_1	p_2/p_1	T_{2}/T_{1}	$ ho_2/ ho_1$	U_{2}/U_{1}	M_2	$ heta_2$	η_2	ϕ_2
41.00°	2.116	1.254	1.687	0.869	1.770	11.77°	5.97°	26.66°
$\pm 0.61^{\circ}$	± 0.019	± 0.004	± 0.010	± 0.003	± 0.023	$\pm 0.10^{\circ}$	$\pm 0.18^{\circ}$	$\pm 0.58^{\circ}$

Table 5.2: Flow properties across quasi-infinite span swept oblique shock[55].

Geometry	Sweep	Deflection angle
Shock generator	$\psi = 30.00^{\circ}$	$\theta = 12.50^{\circ}$
Delta-wing generator	$\psi' = 37.93^{\circ}$	$\theta' = 11.78^{\circ}$

Table 5.3: Equivalent swept shock-inducing model geometries in Mach 2.3 freestream flow (see Figure 1.3).

5.1.3 Diagnostic Tools

Two types of PIV have been used to assess velocity fields within the tunnel. SPIV returns three-component velocities in a 2D plane (aligned with *y*-*z* plane in current investigation), while TPIV returns a full 3D domain with three-component velocities. While TPIV has lower spatial resolution, it is able to capture the full velocity gradient tensor so 3D vortices and complex instantaneous 3D flow structures can be visualized. The tunnel was seeded similarly for both SPIV and TPIV tests using a Pea Soup Smoke Generator to release atomized oil particles into the lab, which were then ingested by the tunnel as it draws upon air from the room. The desired intensity of seed varied drastically, with TPIV needing far less. The seed particle size is estimated to be 300 nm, with an associated Stokes number of Stk = $\rho_p d_p^2 U_{\infty}/18\delta_0 \mu_{\infty} \approx 0.04$.

Stereo PIV

Seed particles are illuminated through a optical window using a Quantel Evergreen 200 dual cavity Nd:YAG laser with pulse energy 200 mJ operating at 15 Hz. The time delay between laser pulses was measured using a ThorLabs DET10A photodiode (1 ns response time) to be $dt = 365 \pm 2$ ns. The laser beam was focused using a bespoke optical rail system then clipped by a rectangular aperture to form the final beam measuring 15 mm in height and approximately 1.2 mm thick. This resulted in freestream particles passing through approximately 20% of the light sheet thickness between laser pulses. Two Lavision 2560×2160 pix sCMOS cameras are positioned on the opposite side of the tunnel from the laser such that both utilize forward scatter. They are angled slightly towards the tunnel floor by 15° , then split in the streamwise direction such that the cameras view the measurement plane with $\pm 30^{\circ}$. A scheimpflug lens adapter was used to focus on the angled measurement plane. In order to achieve a high-magnification, each camera was fitted with a $2\times$ teleconverter, a Nikkor 85 mm lens, and a +2 diopter close-up lens filter. The latter component was required due to the positioning of the cameras being too close for standard focusing of the lens. As a result, the final image magnification is 80.4 pix/mm. Data acquisition and processing is performed using Lavision Davis 8.3. Calibration is performed using the Lavision 058-5 3D calibration plate placed parallel to the desired y-z measurement plane. Final velocity vectors are calculated using 4:1 ellipse weighted 24×24 pix interrogation windows with 75% overlap (stretched in z axis to assist near-wall measurement). Erroneous vectors were identified and removed, then iteratively replaced with other correlation peaks with suited the neighboring vectors. The SPIV dataset contains 448 velocity fields. Assuming a ± 0.1 pix accuracy on seed displacement, this returns an accuracy for determining the freestream velocity of $\pm 0.6\% U_{\infty}$.

Tomographic PIV

Seed particles are illuminated using a Quantel Evergreen HP 15-340-S dual cavity Nd:YAG laser with pulse energy 340 mJ operating at 10 Hz. Laser pulse delay was set as dt = 600 ns, with negligible observed timing jitter. Four Lavision 2560×2160 pix sCMOS cameras were used to capture images of the illuminated seed particles. Mounted to a frame, they were spread linearly to cover a range of observation angles. They were positioned on the opposite side of the tunnel to the laser such that they all operated in forward-scatter to optimize light intensity. Cameras were fitted with a similar lens assembly as installed for the SPIV setup, but fitted with lower strength +1 diopter close-up lenses. The main limit on camera positioning for this configuration was the ability to focus on the x-z plane as the scheimpflug adapter was at the maximum angle. As a result, the cameras needed to be moved back from the tunnel slightly, reducing image magnification to 39.9 pix/mm. After alignment, the cameras are calibrated by combining six views of the Lavision 058-5 3D calibration plate to obtain a 3D pinhole calibration
Configuration	dt [ns]	Freq. [Hz]	Magnification [pix/mm]	Int. Window [pix]	Overlap
SPIV	365	15 Hz	80.4	24×24	75%
TPIV	600	10 Hz	39.9	$32 \times 32 \times 32$	75%

Configuration	Domain size: $(x \times y \times z) / \delta_0$	Vector Density δ_0	Resolution δ_0
SPIV	0.2 imes 2.3 imes 4.6	87.5	0.04
TPIV	$6.9\times1.2\times5.1$	32.6	0.12

Table 5.4: Summary of PIV setup.

Table 5.5: Summary of PIV geometries.

accurate to 0.5 pix. Using the self-calibration disparity correction, the calibration accuracy was increased to 0.05 pix. The 3D volume was then reconstructed using Lavision routines. With sufficient illumination and correct seed density, the presence of ghost particles was limited to a Signal to Noise Ratio (SNR) of > 3. Velocity vectors are calculated with a final interrogation window size of $32 \times 32 \times 32$ pix with an overlap of 75%. The TPIV dataset contains 149 velocity fields. Calculation of a similar uncertainty limit as done for the SPIV system returns $\pm 0.8U_{\infty}$. A comparison of pertinent PIV parameters for each of the setups is summarize in Table 5.4 and Table 5.5.



Figure 5.3: TPIV four-camera setup.

5.1.4 3D Signal Analysis

The presence of a vortex is visualized using the Q-criterion[111], which represents the second invariant Q of the local velocity gradient tensor ∇V (see Equation 5.1). When greater than zero, this indicates that the rotation rate dominates over the strain rate forming a swirling flow topology[113].

$$Q = \frac{1}{2} \left(\left| \Omega \right|^2 - \left| S \right|^2 \right) = \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] - \frac{\partial v}{\partial x} \frac{\partial u}{\partial x} - \frac{\partial w}{\partial x} \frac{\partial u}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial v}{\partial z}$$
(5.1)

where $S = \frac{1}{2} \left[\nabla V + (\nabla V)^T \right]$ is the strain tensor and $\Omega = \frac{1}{2} \left[\nabla V - (\nabla V)^T \right]$ is the vorticity tensor

However, as the Q-criterion returns a scalar field, it does not offer further insight into the direction in which a vortex is acting. To achieve this a swirling strength criterion is used to return a directional estimate of swirling topology[114, 113]. This criterion is calculated as the imaginary part of the complex conjugate eigenvalues of ∇V_{2D} (see Equation 5.2 for swirl strength criterion in the y-z plane). Since $\lambda_{ci} \geq 0$, a sign is artificially applied to the scalar term using the sign of the local vorticity, sign (ω_i). The final swirl strength criterion can thus be expressed as a vector field by combining swirl components in each orthogonal plane.

$$\lambda_{ci,x} \cdot \operatorname{sign}(\omega_x) = \operatorname{Im}\left\{\sqrt{\frac{1}{4}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 - \frac{\partial v}{\partial y}\frac{\partial w}{\partial z} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y}}\right\} \cdot \operatorname{sign}(\omega_x)$$
(5.2)

LSE is used to identify statistically significant structures present within the velocity fields associated with a point event variable E(x, y, z) (see Equation 5.3). This technique[115, 116, 113] provides a linear estimation of the conditional average between observatory parameters \hat{V}' (defined relative to local average) and the event variable E. r_i terms indicate local distances from the event position x'_i . A final conditionally averaged field based on E would thus be constructed as $V = \bar{V} + \check{V} \times E$.

$$\frac{1}{N}\sum \left[\hat{V}'\left(x'+r_{x},y'+r_{y},z'+r_{z}\right)|E\left(x',y',z'\right)\right] \approx E\left(x',y',z'\right) \times \frac{\sum \left[V'\left(x'+r_{x},y'+r_{y},z'+r_{z}\right)\cdot E\left(x',y',z'\right)\right]}{\sum E\left(x',y',z'\right)^{2}} \approx E\left(x',y',z'\right) \times \check{V}\left(x'+r_{x},y'+r_{y},z'+r_{z}\right)$$
(5.3)

where summations are added through various fields and/or event locations; N is the number of summations; x', y', and z' represent the location of the event; r_x , r_y , and r_z represent distances from the event which is being observed in field V'.

5.2 Boundary Layer Characterization

5.2.1 Instantaneous Turbulent Structures

Characterization of the naturally turbulent tunnel wall boundary layer was conducted using SPIV located at $\text{Re}_x = 5.45 \times 10^6$. Example instantaneous fields are shown in Figure 5.4 for three independent samples. The fields are noticeably variable in spatial structure and unsteady in time, as would be expected for a turbulent boundary layer. Low velocity ejection events are observed near the wall (for example at $y/\delta_0 = 0.10$, $z/\delta_0 = -0.05$, top field) where the wall-normal velocity component is positive showing that the low-velocity region is being convected away from the wall. Likewise, there is evidence of high-speed fluid being entrained within the boundary layer (for example at $y/\delta_0 = 0.35$, $z/\delta_0 = -1.90$, middle field) with downward wall-normal velocity component bringing fluid with higher than average velocity towards the wall. Large scale structures appear to have size of the order $1\delta_0$.

The low-level of erroneous vectors is clearly evident, with minimal void regions (white). In addition, vectors are determined very low within the boundary layer (within the lower 2% of δ_0), enabling high-quality statistical characterization of this region (see Section 5.2.5.2.3).

Visualization of the turbulent boundary layer is also performed in a three dimensional domain using TPIV. As the full velocity tensor can now be defined, one may also visualize vortical structures within the domain in addition to velocity components. Figure 5.5 demonstrates this visualization for a domain within the boundary layer. Dimensions are shown normalized as a Reynolds numbers (*i.e.* $\operatorname{Re}_y = \rho_\infty U_\infty y/\mu_\infty$) rather than by normalizing with boundary layer height as this is variable through the streamwise length of the domain. However, as the turbulent growth is relatively slight within such a small region of the working section, one can similarly scale with an approximate boundary layer taken from $\operatorname{Re}_x = 5.45 \times 10^6$, returning $\operatorname{Re}_{\delta_0} \approx 6.52 \times 10^4$. The location of the SPIV plane is shown to be central within the TPIV domain.

Two types of isosurface are plotted within the domain: i) low-speed flow where $u/U_{\infty} = 0.8$, and ii) vortical structures identified using normalized Q-criterion $Q\delta_0^2/U_{\infty}^2 = 5 \times 10^8$. The vortical structure surfaces are colored by the local proportion of swirl components. This helps to convey the nature of the vortex beyond simple Q-criterion which just states that a vortex is present[113]. Through inclusion of the swirl components, one can establish the local behavior of the vortex. As a result, vortices are colored with the following RGB color components: i) streamwise swirl $\lambda_{ci,x}/|\lambda_{ci}|$ (red), ii) wall-normal swirl $\lambda_{ci,y}/|\lambda_{ci}|$ (green), and iii) spanwise swirl $\lambda_{ci,z}/|\lambda_{ci}|$ (blue).

Figure 5.6 presents four example views zoomed-in near low-speed streamwise streak features within the lower boundary layer. Wall-normal swirling structures (green) are often observed along the spanwise edges of the streaks as high-velocity surrounding fluid passes the low-velocity region (giving rise to cane-type vortices[113]). Spanwise swirl structures (blue) are typically present above the low-speed streak, consistent with a rolling vortex (sometimes part of a hairpin-type vortex when combined with wall-normal swirling legs). Streamwise swirl



Figure 5.4: Example instantaneous y-z plane velocity fields (SPIV) within inner boundary layer at $\operatorname{Re}_x = 5.45 \times 10^6$. Colors represent the through-plane streamwise velocity component u/U_{∞} (with black contours at $0.1u/U_{\infty}$ intervals). Regions with removed erroneous vectors shown in white. Black vectors represent in-plane v/U_{∞} and w/U_{∞} velocity components (plotted with $1/8 \times$ magnification factor with respect to the y/δ_0 and z/δ_0 axes). The corresponding 24×24 pixel PIV interrogation window size is shown in the top left (white box) for reference. The mean streamwise boundary layer profile model fit $\bar{u}_{model}/U_{\infty}$ (see Figure 5.12) is shown against the left side of the axes (with magnification $1/2 \times$ with respect to the z/δ_0 axes, thick solid line colored according to u/U_{∞} colormap), with associated freestream velocity (white dashed lines). Axes are shown normalized by local boundary layer height δ_0 and also in Reynolds Number form (*i.e.* $\operatorname{Re}_y = \rho_{\infty} U_{\infty} y/\mu_{\infty}$).

structures (red) appear in more variable circumstances, sometimes as a cane-type vortex being ejected into the outer boundary layer, or as a transition region within a hairpin-vortex as the wall-normal swirling legs are skewed along the streamwise direction.



 $u/U_{\infty} = 0.8$ shown in white. Isosurface of normalized Q-criterion at $Q\delta_0^2/U_{\infty}^2 = 5 \times 10^8$. Color represents addition of $\lambda_{ci,x}/|\lambda_{ci}|$, $\lambda_{ci,y}/|\lambda_{ci}|$, and $\lambda_{ci,z}/|\lambda_{ci}|$ in RGB colorspace, respectively (*i.e.* a green surface indicates vortex with swirl entirely in wall-normal direction, and a maroon surface indicates vortex with swirl of equal proportions in streamwise and spanwise direction). Colored planes at the back of the domain show the streamwise velocity component u/U_{∞} (with black contours at $0.1u/U_{\infty}$ intervals). Co-located black vectors represent appropriate in-plane velocity components with mean velocity subtracted (plotted with 2× magnification factor with respect to the axes). Dashed black rectangle indicates region of SPIV data. The base of the domain is shown Figure 5.5: Example instantaneous volumetric boundary layer velocity field (TPIV data, frame: 001). Isosurface of normalized streamwise velocity at in black, with lightened regions indicating vertical projection of isosurfaces. Re $_z$ is defined relative to the spanwise center of the domain.



Figure 5.6: Zoomed views of instantaneous volumetric boundary layer velocity fields (TPIV) focused on vortical structures surrounding low-speed streak regions. See caption of Figure 5.5 for full description of plot format. Q-criterion isosurface volumes disconnected from main streamwise velocity isosurface volumes have removed for clarity.

Three example structures from Figure 5.6 have been extracted and shown in isolation in Figure 5.7. A classical hairpin-type vortex is shown in Figure 5.7a, with wall-normal swirling legs (green) extending from the lower boundary layer either side of a low-speed region (direction of swirl in each leg is opposite). The legs join above the low-speed streak to form a small spanwise swirling region (with negative sign). A region of streamwise swirl is also observed towards the top of the structure, consistent with a streamwise skew since the leg at $\text{Re}_z = -5 \times 10^4$ is rooted further upstream. The vortical structure in Figure 5.7b is similarly a hairpin-type structure, however one leg blends into a streamwise/spanwise vortical structure that is present within the low-speed region upstream of the main vortical structure and continues to extend further downstream than the blending point. Finally, Figure 5.7c shows another complex 3D vortical structure. A hairpin-vortex is observed upstream where one leg blends with a sizable wall-normal swirling structure. This seems to branch into two cane-type vorticies that curve back over the low-velocity streak.



Figure 5.7: Example single Q-criterion isosurface structures extracted from Figure 5.6. See caption of Figure 5.5 for description of surface coloring format.

5.2.2 Visualization of Boundary Layer Structures Using LSE

Characterization of vortical structures within the boundary layer is performed using LSE on the TPIV velocity fields. This approach is used to find statistically significant structures that result from certain events defined at a point. For the purposes of identifying key vortical structures, three events are investigated: i) streamwise swirl $\lambda_{ci,x} \cdot \text{sign}(\omega_x)$, ii) wall-normal swirl $\lambda_{ci,y} \cdot \text{sign}(\omega_y)$, and iii) negative spanwise swirl $\lambda_{ci,z} \cdot \text{sign}(\omega_z) < 0$. The selection of negative-only swirl events follows similar analysis that identified this as an effective means to observe hairpin vortices[113]. To increase statistical convergence, the effective sample size was increased by employing a similarity assumption that the growth of the boundary layer in the streamwise (or spanwise) direction is sufficiently small, such that multiple x-z positions within the domain can be used as event points, drastically increasing the sample size[113]. For the present analysis, 45 streamwise locations and 39 spanwise location are used to increase the LSE sample size, with sampling intervals of $\Delta \text{Re}_x = \Delta \text{Re}_z \approx 0.9$. However, as large structures are somewhat coherent within a boundary layer length scale δ_0 , this reduces the number of independent sample positions to approximately 6 and 5 in the streamwise and spanwise directions, respectively (giving approximately 30 times more effective samples than if using a single event point). The excessive sampling frequency helps to reduce experimental noise which is independent of defined event position, thus the spacial super-sampling results in an increase to effective sample size by approximately three orders of magnitude.

After the LSE fields have been calculated, vortical structures are identified using a Q-criterion isosurface. Since the magnitude of vectors within this field is arbitrary, there is no easily definable velocity scale with which to define the Q-criterion isosurface level. Therefore, the isosurface was set to be at the 90th percentile of positiveonly Q-criterion values within the LSE domain. While this approach works well when statistically significant vortices are present, it breaks down somewhat when they are not. As the Q-criterion is set according to a percentile, structures will always be identified. However, if not statistically significant they will be fragmented and incoherent.

Figure 5.8, Figure 5.9, and Figure 5.10 show the resultant vortical structure observed in the LSE field when using streamwise, wall-normal, and negative spanwise swirl events, respectively, as the event variables. While structures are plotted on an inclined line for reference, it is not the actual development path of the structure, but rather a helpful way to visualize[117].

Wall-normal swirl events appear most coherent lower within the boundary layer at a height of $y/\delta_0 \approx 0.5$ (see Figure 5.8). The principle structure is a vertical wall-normal vortex extending outwards from low within the boundary layer. Correlation coefficients with streamwise velocity of ± 0.4 are observed in elongated regions situated on either side (in span) of the wall-normal vortex. Thus, the structure is consistent with cane-vortex legs that are present along the edge of high/low velocity streaks. Secondary vortices are located at $\pm 0.5\delta_0$, these appear skewed and extend downstream. Streamwise swirl events appear most coherent towards the upper half of the boundary layer at a height of $y/\delta_0 \approx 0.7$ (see Figure 5.9). They share much of the same structure as the wall-normal swirl events, albeit with greater streamwise skew. Secondary structures are similarly located at $\pm 0.5\delta_0$ with similar skew to the principal structure. Negative spanwise swirl events appear most coherent near the outer limit of the boundary layer at a height of $y/\delta_0 \approx 0.9$ (see Figure 5.10). The principal structure is a spanwise vortex with two streamwise skewed legs extending downwards into the upstream boundary layer. This structure is consistent with hairpin vortices present in the outer boundary layer. Figure 5.11 shows the structures in an isometric view.

These findings support those made by Elsing et al.[113] who used the same technique to identify spanwise vortex structure within a compressible high-speed boundary layer. As the number of processed velocity fields increases the quality of the LSE structure will continue to improve. In addition, by acquiring data lower in the boundary layer further small-scale structures will be observed.

5.2.3 Statistical Characterization

The state of the boundary layer is assessed from SPIV data statistics, averaged across the span. If one accepts a estimated spanwise independence length scale of approximately $1\delta_0$ then this increases the effective sample size from 448 to approximately 2000, halving the corresponding confidence interval. In addition, experimental noise associated with high wavenumber features will be significantly attenuated as the effective sample size for these structures will be much higher.

The resultant mean boundary layer is shown in Figure 5.12a. The boundary layer is smooth and continuous, with the lowest measurement at $y/\delta_0 = 0.037$. In order to obtain a full boundary layer profile to determine integral statistics the experimental data was compared to two models for a turbulent boundary layer profiles with the associated Van Driest transformation [118] to allow for compressible effects. A semi-empirical model for the outer region of the boundary layer[119] was first fit to the experimental data (using the MATLAB function *fminsearch*) to obtain values for skin friction coefficient C_f , wall height offset error Δy_0 , and boundary layer height δ_0 . These values were then used to inform the lower boundary layer model[120] and ensure a continuous profile. While limited in tuning parameter, the models offered an excellent fit to the experimental data with $R^2 = 0.984$. Once corrected for compressibility (using Van Driest II[118]), the boundary layer could be expressed in wall-units (see Figure 5.12b). SPIV data is thus observed to extend from the freestream, through the wake and log-law region, into the start of the buffer layer with the lowest measurement point at $y^+ = 32$. A 95% confidence interval on this mean is also plotted (using dashed line), however is barely observable outside of very low in the boundary layer, further confirming the quality of the measurement. Data from TPIV at $\text{Re}_x = 5.45 \times 10^6$ is also included in Figure 5.12 for verification. TPIV appears to match SPIV closely (which is treated as quasi-truth data), with slight disparity consistent with the lower sample size and higher experimental uncertainty associated with the technique. Finally, boundary layer profiles for Reynolds stresses were calculated from the high-resolution SPIV data and shown in Figure 5.12c. The onset of the R_{uu} rise within the buffer region is observed and consistent with other reported compressible turbulent boundary layers[121, 122].

Parameters associated with the characterized boundary layer at $\text{Re}_x = 5.45 \times 10^6$ are shown in Table 5.6 for reference. Results are compared to initial boundary layer estimates made using the semi-empirical model by Tucker[123]. As expected[52], these differences were mostly within 20% and the model generally over-predicted the size of the boundary layer. Density within the boundary layer was estimated using the Crocco-Busemann relation[124], assuming an isothermal wall-temperature equal to stagnation conditions $T_w = T_0 = 300.5$ K (a reasonable assumption for short duration tunnel operations with large metal structures) and turbulent recovery factor $r = \Pr^{1/3} = 0.896$. Thus, minor heat transfer is expected to occur at the wall since the adiabatic wall temperature for $M_{\infty} = 2.28$ flow is $T_{aw} = 284.6$ K.

Source	$\delta_0 \; [\mathrm{mm}]$	δ^* [mm]	θ	[mm]	H	[-]	C_f [-]	u_{τ}/U_{∞} [-]	$\operatorname{Re}_{\theta}[-]$
SPIV	6.53	1.94	(1.02)	0.55	(0.74)	3.50	(1.39)	0.0020	0.044	5.5×10^3
Model	7.78	2.37	(1.21)	0.60	(0.88)	3.96	(1.40)	_	_	6.0×10^3
Difference	+19%	+22%	+19%	+9%	+19%	+13%	+1%	_	_	+9%

Table 5.6: Boundary layer statistics at $\text{Re}_x = 5.45 \times 10^6$ (incompressible values in parenthesis). Values compared to semi-empirical model by Tucker [123].

5.3 3D SBLI Structure Characterization

5.3.1 Instantaneous Visualization

TPIV offers a unique opportunity to visualize the instantaneous structure of this complex highly three dimensional SBLI. Prior measurements within this tunnel [51, 52] have sought to develop a three dimensional understanding of the interaction through statistical means, oil flow visualization (mean flow near the wall), multiple plane PIV

(three dimensional statistics only), shadowgraph/schlieren imaging (integrates structure across span), and simultaneous high-frequency pressure measurements (spatial/temporal relationships between point measurements). None of these other techniques are able to simultaneously visualize the entire interaction to gain a more complete understanding of the physical structure. In the wider community, recent development of a plenoptic PIV system shows promise in assessing complex SBLI[125], but is currently a technique less developed than TPIV which has attracted much attention from TU Delft[126, 26, 113].

Four examples of such instantaneous fields are shown in Figure 5.13 and Figure 5.14. Each subfigure features two isosurfaces which act as surrogate estimators of separation bubble shape and separation shock shape. The separation bubble surrogate is selected as a low streamwise velocity region where $u/U_{\infty} = 0.5$, this was selected over other alternatives (*i.e.* $u/U_{\infty} = 0$ or $\int_{0}^{y} \rho u \, dy = 0$) due to the lack of near-wall observation in this dataset. The separation shock surrogate was selected using the wall-normal velocity component as $v/U_{\infty} = 0.1$. This was selected as the most appropriate surrogate, as many other options (*i.e.* Ducros shock detector[127] or various velocity gradients) are highly susceptible to contamination with experimental noise. Sinceboth features are dominated by the respective surrogate parameter, they are appropriate to visualize flow structures. Spanwise locations are defined relative to the shock generator root plane (*i.e.* the side wall of the tunnel).

The surface of each surrogate is colored according to the local difference between the instantaneous and mean values. For the separation surrogate $(u - \bar{u})/U_{\infty} > 0$ (colored red), this corresponds to a region where the ensemble mean is lower than the measured instantaneous value, thus the separation bubble has shrunk from its mean structure. Inversely, if $(u - \bar{u})/U_{\infty} < 0$ (colored blue), then the mean value is higher, indicating the bubble has grown beyond the mean size. A similar approach was taken for the separation shock surrogate. If $(v - \bar{v})/U_{\infty} > 0$ (colored green), then the shock has moved forward resulting in reduction to the local mean value $\bar{v}/U_{\infty} \to 0$. Likewise, if the instantaneous shock moves downstream the new local mean would be higher, resulting in $(v - \bar{v})/U_{\infty} < 0$ (colored brown). Since the region of elevated v behind the shock is limited by the shear layer and the incident shock, these features are also visualized. The shear layer surface region is generally below the separation surrogate, but the incident shock is clearly visible. As v decreases across the incident shock, its effect on the difference to the mean is reversed compared to the separation shock. Therefore if the incident shock moves downstream it will be colored green, while if it moves upstream it will be brown. In addition, the fields are surrounded by instantaneous contours of u/U_{∞} to aid visualization of a large/small SBLI structure.

Figure 5.13 shows two examples of instantaneous fields which exhibit a series of high/low streamwise ridges along the bubble surrogate surface. These ridges extend from the separation foot, pass through the visualized shock system and along the top of the separation surrogate, then clearly influence the reattaching flow. Such examples can be seen at $\text{Re}_z = 36 \times 10^4$, $\text{Re}_z = 46 \times 10^4$, and $\text{Re}_z = 52 \times 10^4$ in Figure 5.13a, and at $\text{Re}_z = 40 \times 10^4$, $\text{Re}_z = 50 \times 10^4$, and $\text{Re}_z = 56 \times 10^4$ in Figure 5.13b. Inverted ridges are also seen to extend throughout the interaction, typically positioned between positive ridges. This ridge pattern appears to be correlated with the separation shock foot such that it locally moves upstream when the separation bubble region is enlarged. There also appears to be a spanwise periodicity to the ridge structures with wavelength close to the reference boundary layer height ($\Delta \text{Re}_z \approx \text{Re}_{\delta_0}$), this is highly reminiscent of the shock ripples that have recently been reported in other swept SBLI studies[52, 43, 53].

Figure 5.14 shows two other fields that exhibit large scale growth/collapse of the SBLI structure. In many ways, this is very similar to unswept SBLIs, that exhibit large-scale 'breathing' of the separation bubble[14], with localized shock jitter [128], but the presence of coherent ridges along the bubble has not been identified (possibly because such interactions are typically viewed in x-y plane). The large-scale growth of the separation region has a strong correlation with motion of the shock structure, moving the separation shock further upstream. The position of the incident shock does not appear to vary significantly.





Figure 5.9: Orthogonal views of morphing vortex structures visualized using Q-criterion isosurface, event: $\lambda_{ci,x} \cdot \operatorname{sign}(\omega_x)$. LSE fields are calculated at height increments of $\Delta y/\delta_0 = 0.092$, and artificially spread along x axis with separation of $\Delta x/\delta_0 = 3.2$. Q-criterion isosurface plotted with value equal to the 90th percentile of all values in the domain that are greater than 0. Solid black line between calculated event positions is for reference only. Dashed lines indicate the streamwise domain edges for the corresponding event calculation.



Figure 5.10: Orthogonal views of morphing vortex structures visualized using Q-criterion isosurface, event: $\lambda_{ci,z} \cdot \text{sign}(\omega_z) < 0$. Symmetry has been applied to increase effective sample size. See Figure 5.9 for full description of plot features.







(c) Reynolds stresses (SPIV only).

Figure 5.12: Boundary layer mean streamwise velocity profiles at $\text{Re}_x = 5.45 \times 10^6$. Semi-empirical models fitted to SPIV data only (upper model for $y^+ > 130$: Sun and Childs[119], lower model: Musker[120], coefficient of determination in (b) $R^2 = 0.984$).

5.3.2 Visualization of Ensemble Statistics

Statistics from the TPIV SBLI dataset have been assessed across the 149 velocity fields. Figure 5.15 depicts the various velocity components of the mean interaction structure. The upstream flow is consistent with a turbulent boundary layer having minimal content in wall-normal or spanwise directions. Upon approaching the SBLI, streamwise velocities decrease in a largely unswept manner, before the outer region of flow is deflected away from the wall due to the separation shock. The separation bubble continues to grow with minimal spanwise contributions until the incident shock is experienced. At this point, the flow is directed towards the wall, causing the bubble to reduce in height, and also imparts significant cross flow velocities close to the wall ($\bar{w}/U_{\infty} \approx 0.3$). After flow reattaches the downward velocity component abates as flow is directed more parallel to the wall. The spanwise velocity component also appears to reduce as the thickened boundary layer recovers downstream of the interaction. A quasi-conical structure is observed, with growth of the SBLI region as spanwise position is increased.

Unsteady velocity fluctuation magnitudes are similarly shown in Figure 5.16. The streamwise velocity component (Figure 5.16a) shows significant unsteadiness around separation and along the separated shear layer, much in a similar behavior to unswept SBLIs [18]. This is followed by a swept spanwise band of strong fluctuations after reattachment. Close observation of the wall-normal field in Figure 5.16b shows some unsteadiness in the separation shock (near $\text{Re}_x \approx 530 \times 104^4$, however the magnitude is relatively low in comparison to fluctuations after reattachment. Similarly in the spanwise component (Figure 5.16c), the fluctuating field is dominated by downstream unsteadiness. This post-reattachment unsteadiness is observed in all components, but appears to decrease in size as spanwise position is increased. This contradicts the quasi-conical structure that was apparent in the mean statistics, and suggests that significant unsteadiness is associated with the interaction root and is reduced somewhat as span is increased.

Characterizing separation in 3D is more complex[129] than merely identifying where $u/U_{\infty} < 0$ as would be done for 2D flows. Highly swept 'open' separated flows can exhibit limited flow reversal if any[43, 130]. Figure 5.17 and Figure 5.18 show y-plane streamlines (where $v/U_{\infty} = 0$) to visualize the separated flow structure at different heights. A clear separation region is observed at both heights. The line of separation appears upstream of any reversed streamwise flow in the mean at a swept angle below that of the shock generator $(\psi_S \ll \psi_{sg})$. Reattachment is significantly further downstream than any separation at a given span with an angle of sweep slightly greater than the shock generator $(\psi_R > \psi_{sg})$. Specific values are given in Table 5.7. The size of separated flow reduces as height from the wall is increased. An effective separation origin is defined at each height as the spanwise location where the extrapolated linear separation line meets the linear reattachment line $(\text{Re}_{z_0}(y))$. When projected the in-plane origin is projected to the floor, this returns the spanwise origin location as $\text{Re}_{z_0}(0) = 11.2 \times 10^4 (z_0/\delta_0 \approx 1.72)$. Such an origin is typically quoted negative (being beyond the root plane of the shock generating model)[131, 43]. This suggests that the root behavior is unique to this swept SBLI structure and is attributed to an effect of either the root boundary layer[52], or the inviscid inception length[55]. The unsteadiness in Figure 5.17b and Figure 5.18b show that the dominant unsteadiness occurs outside of the separated flow and translates in the spanwise with the in-plane origin.

Re_y	y/δ_0	ψ_S	ψ_R	$\operatorname{Re}_{z_0}(y)$
1.85×10^4	0.28	4.9°	31.6°	28.7×10^4
$2.61 imes 10^4$	0.40	9.4°	35.3°	$36.2 imes 10^4$

Table 5.7: Sweep angles of local flow separation and reattachment induced beneath shock generator that is swept at $\psi_{sg} = 30.0^{\circ}$.



Figure 5.13: Example instantaneous SBLI fields (TPIV) demonstrating streamwise streaks. Contours of u/U_{∞} plotted at spanwise limits of domain (black contour lines at $\Delta u/u_{\infty} = 0.1$, colored contours at $\Delta u/u_{\infty} = 0.05$, see ticks and colors on colorbar). SBLI bubble region visualized using isosurface of streamwise velocity at $u/u_{\infty} = 0.5$, surface colored by difference to mean field $(u - \bar{u})/u_{\infty}$, where blue indicates enlarged bubble and red indicates diminished bubble. The separation shock and incident shock are visualized using isosurface of wall-normal velocity at $v/u_{\infty} = 0.1$, surface colored by difference to mean field $(v - \bar{v})/u_{\infty}$, where brown indicates downstream shock green indicates upstream shock. The boundary layer detailed in Table 5.6 is shown at the nearside corner of the domain for reference, boundary layer height δ_0 is indicated by the dashed black line. Re_z is defined relative to the root plane (*i.e.* the tunnel sidewall). Background filled contour planes represent the instantaneous velocity u/U_{∞} . Dashed white line indicates the location of the mean bubble isosurface position $\bar{v}/U_{\infty} = 0.1$.



Figure 5.14: Example instantaneous SBLI fields (TPIV) demonstrating large scale bubble growth/collapse. See Figure 5.13 for full description of remaining plot format.



Figure 5.15: Mean velocity fields within SBLI (TPIV data). The central and nearside spanwise contour planes are displayed semi-transparent. Reversed streamwise velocity is indicated by the thick black contour. Re_z is defined relative to the root plane (*i.e.* the tunnel sidewall). See Figure 5.13 for full description of remaining plot format.



Figure 5.16: Unsteady velocity fields within SBLI (TPIV data). See Figure 5.13 and Figure 5.15 for full description of remaining plot format.



Figure 5.17: Streamlines in y plane at height of $\text{Re}_y = 1.85 \times 10^4 \ (y/\delta_0 = 0.28)$. Streamlines within separation region are shown in white. Streamlines upstream of separation and downstream of reattachment are shown in black. See Figure 5.15 for full description of remaining plot format.



Figure 5.18: Streamlines in y plane at height of $\text{Re}_y = 2.61 \times 10^4 (y/\delta_0 = 0.40)$. See Figure 5.17 for full description of remaining plot format.

By extracting features observed in various ensemble velocity fields, a simple model of the interaction can be constructed, as shown in Figure 5.19. An isosurface of $\bar{v}/U_{\infty} = 0.060$ (red) visualizes the separation shock, growing shear layer and the incident shock. Beneath which, a $\bar{u}/U_{\infty} = 0.160$ isosurface (blue) shows the conically developing bubble region with its maximum height at the impingement location of the incident shock. The region of high spanwise flow is visualized using a $\bar{w}/U_{\infty} = 0.190$ isosurface (green), which clearly shows this region is associated with reattachment, but is not limited to being either upstream or downstream of the actual reattachment line. Finally, unsteadiness within the interaction is represented by the cyan isosurface at $u'/U_{\infty} =$ 0.135 which highlights two key regions: i) developing shear layer over the first half of the separation bubble, and ii) significant non-conically developing unsteadiness downstream of reattachment.

5.4 Summary

An extensive collection of volumetric data has been presented to characterize a Mach 2.3 turbulent boundary layer and a swept impinging oblique SBLI. Stereo PIV was used for high-resolution boundary layer measurements



Figure 5.19: Notable features within SBLI. Blue isosurface at $\bar{u}/U_{\infty} = 0.160$ shows low-speed near-wall flow within SBLI (fluid surrounding separation bubble). Red isosurface at $\bar{v}/U_{\infty} = 0.060$ shows the separation shock, growing separated shear layer, and the incident shock. Green isosurface at $\bar{v}/U_{\infty} = 0.190$ shows flow around reattachment with strong spanwise component. Cyan isosurface at $u'/U_{\infty} = 0.135$ shows unsteadiness in separated shear layer and after reattachment. See Figure 5.13 for full description of remaining plot format.

in the streamwise plane, and then extended to Tomographic PIV to facilitate sampling instantaneous snapshots of the entire interaction, rather than building a statistical model through multiple independent measurements. In addition, the 3D velocity fields were used characterize vortical structures present in the boundary layer.

Boundary layer statistics show good agreement between the two PIV techniques. Careful setup of the Stereo PIV system enabled sampling within 0.25 mm of the wall $(y^+ = 32)$, penetrating the buffer layer. Reynolds stresses were characterized and agree with expected behaviors. Three-dimensional vortical structures have been identified within the boundary layer consistent with cane and hairpin vortices surrounding regions of low-velocity flow.

3D velocity measurements using Tomographic PIV have been used to characterize the complex structure of the swept impinging oblique SBLI. A spanwise growth of the interaction length scale is observed consistent with conical similarities found in other swept SBLI configurations. Within the interaction is an open-separation bubble with significant spanwise velocity components near reattachment. The bubble exhibits large-scale growth/decay in addition to localized streamwise ridges that extend throughout the interaction and occur with periodic regularity along the spanwise direction. Significant unsteadiness is observed in the separated shear layer, at the separation shock foot, and near reattachment.

This is an initial summary of the data set and the analysis will be expanded significantly in future publications. For example, Tomographic PIV data is currently being processed for the near-wall region and will eventually have a sample size similar to that of the Stereo PIV dataset. This will enable characterization of vortical structures close to the wall and with greater statistical confidence. In addition, the LSE approach will be applied within the SBLI to identify coupled flow features that could identify mechanisms for the interaction structure and unsteadiness. Other possibilities for data analysis abound including a myriad of modal decomposition techniques (POD, OMD, DMD, *etc.*).

6. Surface Measurements of Swept Impinging Oblique SBLI

An experimental investigation has been conducted on swept impinging oblique SBLIs at Mach 2.3. The incoming boundary layer is turbulent with ($\text{Re}_{\theta} = 5.5 \times 10^3$). The swept impinging oblique shock is induced by a shock generator mounted in the freestream with x-y plane deflection angle $\theta = 12.5^{\circ}$ and variable x-z plane sweep angles of 15.0°, 22.5°, 30.0°, and 40.0°. Oil flow visualization, PIV, mean pressure measurements and fast-response pressure transducers are used to provide detailed characterization of the mean and unsteady features of the SBLIs. Large scale separation is observed in all cases with spanwise growth evident at high shock generator sweep angles. At the onset of separation in the quasi-infinite region, mean pressures are independent of span and scale cylindrically. However, mean pressures at reattachment display a mild dependency on the span, suggesting the global structure of the SBLIs is conical in nature. This agrees with supporting tomographic PIV measurements. Unsteady pressure measurements beneath the separation shock foot for the $\psi = 30.0^{\circ}$ SBLI shows clear low frequency unsteadiness across the span at nearly constant frequency. Spanwise traveling ripples are present at the shock foot with considerable coherence in the low frequency range. The spanwise convection speed of these ripples increases with span suggesting that the wavelength also increases. Minimal upstream influence is associated with the low frequency unsteadiness, suggesting a source mechanism within the SBLI.

6.1 Experimental Methodology

Experiments were conducted in a supersonic wind tunnel at the University of Arizona. The inlet and the test section of the original in-draft tunnel from the 1960s have been recently modernized, with the adjustable nozzle being replaced by solid aluminum blocks designed using the Method of Characteristics with boundary layer corrections. This paper focuses on experiments performed at a nominal Mach number of 2.3. The test section has dimensions 121.9 mm \times 81.3 mm \times 609.6 mm ($4.8 \times 3.2 \times 24$, W \times H \times L) and features various interchangeable side panels to mount a variety of shock generators, windows and pressure taps. A second throat is employed to increase the run time (to approximately 15 s) before the air is drawn into the 34.0 m³ (1200 ft³) vacuum tank. The dry Arizona air provides excellent stagnation conditions, enabling tests to be performed at constant stagnation temperature and pressure. Table 6.1 shows relevant test parameters.

Tests have been conducted with a shock generator mounted on the tunnel sidewall positioned over an instrumentation plug, enabling the acquisition of pressure measurements within the SBLI [52]. The instrumentation plug has a diameter of 111.8 mm (4.40) and is located on the tunnel centerline, 149.2 mm (5.88) downstream of the nozzle exit plane. Rotation of the plug is accurate up to $\pm 0.1^{\circ}$ using a vernier scale on the outside of the tunnel. The available measurement zone is confined to the central 86.4 mm (3.40) diameter of the plug.

The boundary layer at Mach 2.3 has been characterized using Stereo PIV [132]. Relevant parameters for the boundary layer at $\text{Re}_x = 5.45 \times 10^6$ (which represents a location at the middle of the plug) are shown in Table 6.2. Near wall velocity profiles from PIV agree with compressible turbulent boundary layer models until $y^+ = 32$.

M_{∞}	T_0 [K]	p_0 [pa]	$U_{\infty} [{\rm m/s}]$	Re/L [1/m]
2.28	300.5	0.933×10^5	554.8	9.99×10^6
± 0.02	± 0.2	$\pm 0.002 \times 10^5$	± 2.6	$\pm 0.13 \times 10^{6}$

Table 6.1: Working section test environment.

$\delta_0 \; [\mathrm{mm}]$	$\delta^* \text{ [mm]}$	$\theta_0 \; [\mathrm{mm}]$	H[-]	C_f [-]	Re_{θ}
6.53	1.94	0.55	3.50	0.0020	5.5×10^3

Table 6.2: Compressible boundary layer statistics upstream of the interaction at $Re_x = 5.45 \times 10^6$ [132].

Table 6.3: Comparison of scaling variables for different sweep angles.

ψ [deg]	$\psi_s \; [\text{deg}]$	$\psi_R \; [\text{deg}]$	W/z_0 [-]
15.0	14.4	15.0	0.02
22.5	19.6	22.5	0.07
30.0	14.7	30.0	0.08
40.0	23.0	40.0	0.71

This investigation focuses on SBLIs produced by shock generators with an x-y plane angles of $\theta = 12.5^{\circ}$ and variable x-z plane sweep angles of 15.0° , 22.5° , 30.0° and 40.0° . The shock generators are mounted on to the tunnel sidewall 83.5 mm (3.31) downstream of the nozzle exit plane, and 38.1 mm (1.5) above the instrumentation plug. A schematic of the inviscid shock reflection (neglecting end effects) is shown in Figure 1.3. The shock generator is 88.9 mm (3.50) in width and has a maximum thickness below the leading edge of 8.9 mm (0.35). The aspect ratio is defined as the width of the shock generator relative to the height from the plug to the shock generator apex. The chosen height of 38.1 mm (1.5) yields an aspect ratio of about 2.3. Shock generators are located in the streamwise direction such that the SBLIs occur over the instrumentation plug, enabling detailed observations.

Oil flow visualization is achieved with a TiO_2 , kerosene, and oleic acid mixture of 48% - 51% - 1% by weight. The oil is thoroughly mixed and applied to near the interaction region immediately before start-up. The mixture is distributed by the starting normal shock passing through the test section, then follows the local skin-friction gradients to reveal surface features. Images are calibrated for perspective distortion using a calibration plate and a bespoke MATLAB script.

Mean pressures are acquired using 35 pressure taps with a diameter of 0.5 mm (0.018). Tubes with diameter of 1.6 mm (0.063) transfer the slow-response pressure through a distance of 1000 mm (39.4) to a Scanivalve DSA 3217 pressure scanner with a sensor range of 15 psid referenced to ambient (stagnation) pressure. Eight channels were measured during each run with a sampling frequency of 62.5 Hz. The plug is rotated for approximately 30 tests to populate the mean pressure distribution.

Unsteady wall pressure data under the SBLI is captured using 10 holes with a diameter of 1.6 mm (0.063) in which fast response pressure transducers (Kulite MIC-062-10A) are flush mounted. Two transducers capture pressure data simultaneously at 262.144 kHz for 8 seconds using a National Instruments NI-9222 data acquisition module. The signals are amplified using a Kulite KSC-2 signal conditioner with a gain of 256 and filtered using a built-in analogue LP6F low-pass filter with a cutoff frequency of 95 kHz. Spectral analysis of the unsteady pressures is conducted with Welchs method with a Hamming window of length 8192 and 50% overlap. Spectral estimates are averaged across 511 windows, with a frequency resolution of 32 Hz. Tomographic PIV data is included in this paper to provide additional context and the experiment details can be found in Threadgill and Little [132].

6.2 Mean Results

Oil flow visualization and static pressure measurements were acquired to investigate the mean behavior of swept impinging oblique SBLIs. Figure 6.1 shows a representative example for $\psi = 30^{\circ}$. The inviscid shock impingement location is determined using the model developed by Threadgill and Little [133]. Oil flow visualization shows the upstream flow has uniform surface streaklines indicating it is void of any significant mean disturbances. The separation line is clear where the streaklines coalesce. Corner features are visible in the oil flow visualization at both the root and tip. The corner feature at the root is induced when the shock interacts with the side wall boundary layer. Curved streaklines in the root indicate flow separation, but no obvious reversed flow in the streamwise



Figure 6.1: Surface flow visualization and mean pressure distribution beneath a swept impinging oblique shock. The white dotted line representing upstream influence, The red line represents the separation region. The blue line represents reattachment line. Also, the magenta line represents the inviscid shock impingement and the green line is the shock generators projection along y-axis. White circles in both figures represent the diameter of the plug and the dotted circle indicates the area where the data can be taken. In the mean pressure contour figure, the white circle and white triangle represent the location of the Scanivalve pressure ports and Kulite pressure transducer respectively.

direction. The separation line near the middle of the tunnel appears quasi-infinite, but is eventually distorted by the corner separation with reversed flow at the tip. This paper focuses on the central quasi-infinite span region which, in all cases, is straight and displays a sweep angle lower than the shock generator. At reattachment, the surface streaklines diverge, in contrast to the behavior observed in the separation region. Reattachment features are somewhat more challenging to identify. However, it is clear that this occurs downstream of the inviscid shock impingement location and has an x - z plane angle similar to the shock generator sweep angle itself [51]. Like separation, quasi-infinite span features for reattachment appear near the middle of the tunnel. Downstream of reattachment, the flow is deflected in the direction of the freestream.

The 2D mean pressure distribution in Figure 6.1 is extracted from linear interpolation of 240 data points and shows good agreement with the oil flow visualization. There is a steep increase in pressure immediately upstream of separation, indicative of a separation-induced shock footprint. This is followed by a pressure plateau region with a reduced streamwise pressure gradient. In the reattachment region, a secondary rise in pressure is observed due to redirection of the separated fluid when it reattaches onto the wall. Downstream of the reattachment, expansion waves from the shock generator face trailing edge impart a drop in pressure. The corner features at the root and tip limit the quasi-infinite region of the interaction. To better isolate the quasi-infinite section, Regions of Interest (ROI) are created for each SBLI to mask the effects from the corners. First, an initial location for ROI is created based on the oil flow estimates of separation and reattachment lines. The collapse of pressure on the separation line and reattachment line is assessed, and the separation and reattachment angles are varied accordingly. This process is repeated until converged. This iterative process is outlined in detail in Doehrmann et al. [52], and it is applied here to isolate the quasi-infinite region where separate and reattachments are linear. The resulting separation (ψ_s) and reattachment (ψ_R) angles for the different cases are given in Table 6.3.

The mean pressure rise at separation is summarized for all cases in Figure 6.2 using data from Doehrmann et al. [52]. ROIs were created for different sweep angles, using the procedure outlined above. The ROI for the 30° shock generator is shown in Figure 6.2a. When mean pressures are projected normal to separation (z_S) , they collapses for all sweep angles (Figure 6.2b). This implies that the local features of separation are similar, in agreement with the free interaction concept [134]. Mean pressures at reattachment require further scaling. Doehrmann et al. [52] observed that conical scaling, using a virtual origin from the projection of separation and reattachment lines, did not collapse the data. This is because the mean pressure scaling along the separation line is locally cylindrical when aligned normal to separation. This quasi-cylindrical behavior persists downstream



(a) Coordinate system used for analyzing data along the separation line.

Figure 6.2: Mean pressure analysis for swept shock generator along the separation coordinate systems.

until the external influence of the impinging shock is encountered. Local scaling for pressures at reattachment (analogous to separation) also does not result in data collapse for higher sweep angles (30° and 40°). This implies that pressures at reattachment in highly-swept impinging obliques SBLIs are dependent on span. As the local interaction length increases along the span, the secondary pressure gradient will become milder. Thus, spanwise dependency must be accounted for in the scaling. To this aim, the location along reattachment (z_R) is offset by the spanwise virtual origin point (z_0), measured from the centerline of the tunnel parallel to reattachment. The reference frame and the reattachment axis are shown in Figure 6.3. For each sweep angle, the value of z_0 is found iteratively, resulting collapse of reattachment pressure for 22.5°, 30° and 40° sweep. However, the lowest shock generator sweep angle (15°) did not collapse using this method. This is attributed to the denominator of the pressure scaling term ($z_R - z_0$) tending to infinity (i.e. cylindrical similarity) and disrupting the scaling approach. z_0 values for different shock generators are shown in Table 6.3. Note that, the origins for all shock generators are outside the tunnel, which is similar to observations made for other sweet SBLIs [63].

Recently, Threadgill and Little [132] performed TPIV on the 30° sweep configuration. The mean streamwise velocity field shown in Figure 5.15 agrees with features present in the wall-pressure distribution and oil flow visualizations (Figure 6.1). As the boundary layer approaches the separation shock, the near wall velocity reduces and forms an open separation bubble. A quasi-conical global shock structure can be observed in the streamwise velocity contour. This agrees with recent work on swept compression ramps that suggests streamwise velocity components scale conically outside the inception region [44].

6.3 Unsteady Results

Unsteady wall pressure measurements were carried out using Kulite (MIC-062-10A) pressure transducers. The locations for Kulite pressure transducers for $\psi = 30^{\circ}$ are shown in Figure 6.1. The coarse nature of measurement locations when compared to mean pressure is apparent, but the overall behavior is clear. The spectral energy content is shown as pre-multiplied power spectral density (PSD), normalized by the variance. A similar normalization approach has been used to generate all PSD figures in this work. Normalizing by the variance shows the distribution of energies at the corresponding frequencies as a proportion of the total unsteadiness. Frequency is non-dimensionalized by defining a Strouhal number with incoming boundary layer thickness (δ_0) as a length scale. Unlike unswept interactions, which can be characterized by a single interaction length, swept interactions often have a variable interaction length that increases with span away from the root. Therefore, the boundary layer thickness is the only constant dimensional length scale to act upon the quasi-infinite span region.



(a) Coordinate system used for analyzing data along the reattachment line.

(b) Collapsed mean pressure at reattachment line.

Figure 6.3: Mean pressure analysis for swept shock generator along the reattachment coordinate systems.

Low frequency unsteadiness is observed near the separation shock foot in Figure 6.6a. This can be explained, in part, by interpreting the SBLI as a first-order low pass filter. This is well-established for unswept SBLIs [135, 136]. Doehrmann et al. [52] assessed these unsteady features for various sweep angles and found that as sweep increased, the low frequency values near the separation shock also increased (from $St_{\delta_0} \approx 0.015 - 0.045$, moving from $\psi = 15.0^{\circ}$ to 40.0° respectively) while the amplitude decreased (from $C_p' \approx 0.042 - 0.018$, moving from $\psi = 15.0^{\circ}$ to 40.0° respectively). This is qualitatively consistent with the findings of Erengil and Dolling [49] for swept compression ramps. Streamwise velocity fluctuations in Figure 5.16 show features that are consistent with unsteady pressure measurements. Significant unsteadiness is observed in the separated shear layer and near reattachment. Threadgill and Little [132] showed that, like unswept interactions, streamwise velocity fluctuations (u') are dominant near the separation shock foot. The streamwise velocity fluctuations decrease in the separated shear layer, followed by another increase near the reattachment region. The velocity fluctuations near the separation shock foot (no spanwise gradient in u') are consistent with the quasi-cylindrical structure observed in the mean pressure statistics. Along the separation shock foot, Doehrmann et al. [52] observed spanwise travelling ripples propagating away from the interaction root for 30° sweep angle. The spanwise travelling ripples moved at approximately 20% of the freestream velocity and increased to 25% along the span. It was also shown that the low frequency of the ripple was coherent with the frequency of the separation shock foot and remained constant along the span. However, Doehrmann et al. [52] studied this phenomenon at only a few discrete locations and additional data is required to fully cement the findings. To this aim, additional locations along the span of the separation shock foot are now surveyed.

Figure 6.5 shows the 8 locations where high bandwidth pressure measurements have been acquired. The resulting PSD values are plotted in pre-multiplied form, normalized by variance in Figure 6.6b. It is now clear that the low frequency content (peak at $St_{\delta_0} \approx 0.023$) dominates the unsteadiness along the span. As expected, there is excellent agreement between this data and the low frequency content in Figure 6.6a near separation shock foot. There is also evidence of high frequency content from the incoming boundary layer near $St_{\delta_0} = 0.6$, although the true upper limit of the high-frequency content is likely attenuated above 20kHz by the frequency response of the pressure sensors. It is interesting to note that the frequencies in Figure 6.6b are relatively constant along the span, especially in the quasi-infinite region, despite the increase in separation length. There are variable peak levels in the PSD near the separation shock foot. Near the root $(z/\delta_0 = 5)$, the frequency peak is quite broad and becomes sharper moving along the span in the direction of sweep. This behavior peaks near the tip of the interaction before encountering the corner region $(z/\delta_0 = -6)$.

To focus on the low frequency content of the shock foot alone, a bandpass filter with a range of $(0.0064 < St_{\delta_0} < 0.090)$ is applied when computing the variance (see Figure 6.7a). This range of St_{δ_0} is chosen because it



Figure 6.4: Mean and fluctuating streamwise velocity contours from TPIV. Reversed streamwise velocity is indicated by the thick black contour.

is symmetrical (in log) around the peak frequency $\operatorname{St}_{\delta_0} \approx 0.023$. This further isolates the low frequency energy content from the turbulent boundary/shear layer and emphasizes the appearance of first order low-pass filter behavior. To further assess the amplitude of the unsteadiness in quasi-infinite region, the band-passed pressure coefficient standard deviation (C'_p) is also estimated. The value of (C_p') in Figure 6.7b at each spanwise location clearly shows that the amplitude of the unsteadiness increases with the span. The increase in low frequency unsteadiness amplitude in the quasi-infinite region appears linear, which suggests that it is minimally affected by the corner features.

Correlations between sensor pairs are investigated to assess spanwise traveling waves. Cross-covariance between sensors pairs is shown in Figure 6.8a to present the lag between sensors and estimate the shock rippling speed (U_{τ_s}) . The distance between the sensors (Δx_s) is divided by the signal lag (τ) that corresponds to the peak in Figure 6.8a to yield shock rippling speed (U_{τ_s}) . Since the sensor is positioned parallel to the separation and located in the shock intermittency region, structures other than those exhibited in separation shock foot will yield a spurious signal. For example, the cross correlation between pairs 1-2 is estimated by lagging 1 with respect to 2. The cross correlation computed between 1 and 2 shows a positive lag which indicates the structure passes from 1 towards 2. The correlation pairs are always situated such that the root-most sensor is lagged relative to tip sensor.



Figure 6.5: Mean pressure contour showing the location of high bandwidth pressure sensors beneath the shock foot.



Figure 6.6: Power spectral density measurements done normal to separation and beneath the shock foot for $\psi = 30.0^{\circ}$.

The strongest correlation coefficient (0.57) is observed between sensor pairs 5-6 resulting in a shock rippling speed of 0.23 U_{∞} . Correlation coefficients are high when the sensors are near each other and decrease when farther apart due to natural dispersion effects. For example, sensor pairs 3-4 are separated by twice the distance as sensor pairs 5-6 and the correlation drops to 0.32. Sensor pairs 1-2, located in the root section, have weaker correlation and the distance between the sensors is the same as sensor pairs 3-4. Also, the structures observed along the span are much slower than the freestream ($U_{\tau_s} \approx 20\%$ of freestream velocity) and may be related to those observed in the separated region by Vanstone et al. [63]. Doehrmann et al. [52] reported that the shock rippling speed increases along span and this is observed in more detail in Figure 6.8b. The shock rippling speed in Figure 6.8b is slower at the root ($\approx 15\%$ of U_{∞}) and accelerates towards the tip ($\approx 25\%$ of U_{∞}).

Figure 6.9 assesses the possible influence of upstream turbulent boundary layer superstructures as reported by



Figure 6.7: Power spectral density beneath the shock foot with bandpass filter for the range of $(0.0064 < St_{\delta_0} < 0.090)$ to study the low frequency region.



Figure 6.8: a) Correlation of the sensor pairs located at the separation shock foot plotted against ratio of freestream velocity and shock rippling speed. b) Variation of shock rippling speed along the span and a linear fit drawn for shock rippling speeds at each end of sensor pair.

Ganapathisubramani et al. [10]. Two sensors are located in the upstream boundary layer and the others are near the shock foot (see Figure 6.9a). Sensor pair C-D is along the same z coordinate and there is some correlation (just above 0.1 see Fig 6.9b). This is because sensor D has a subtle peak in high frequency region due to its location just downstream of the separation shock foot, near the shear layer region (see PSD in Figure 6.9c). Sensor pair A - B shows no apparent correlation between the upstream boundary layer and the shock foot unsteadiness. Note that PSD for the individual sensor (B) is shown in Figure 6.9c and dominated by low frequency energy content. Conversely, sensor A is in the turbulent boundary layer with significant high frequency content (Figure 6.9c). The propagation of disturbances along the span (seen in e.g. Figure 6.8) coupled with the lack of correlation observed here suggest that super structures from incoming turbulent boundary layer do not influence the spanwise



Figure 6.9: Influence of the upstream boundary layer on the unsteady behavior behind the shock foot.



Figure 6.10: Coherence between the pressure sensor pairs located near the separation shock foot.

separation behavior observed here.

The relation between spanwise traveling waves and low frequency unsteadiness is evaluated using the coherence between sensors shown in Figure 6.10. The sensor pairs in the quasi-infinite region show a high level of coherence at lower frequencies below $St_{\delta_0} < 0.05$, which corresponds to the range of low frequency peaks exhibited beneath the shock foot shown in streamwise and spanwise PSD (see Figure 6.6). PSD and coherence show similar distributions suggesting that the spanwise ripple occurs at low frequencies consistent with motion of the separation shock foot. This is in agreement with the preliminary observations from Doehrmann et al [52].

Instantaneous 3D structures from TPIV show some indication of the dynamics associated with this behavior. Two instantaneous snapshots of the entire SBLI region are shown in Figure 6.11. The wall normal velocity ratio $\left(\frac{v-\bar{v}}{U_{\infty}}=0.1\right)$ is considered as the separation shock surrogate and $\left(\frac{u-\bar{u}}{U_{\infty}}=0.5\right)$ is considered as the separation bubble surrogate. All the velocity isosurfaces are subtracted from the mean to highlight deviations. The instantaneous velocity fields display some dynamics that are similar to unswept interactions. There is large scale collapse and growth of the separated region which is likely associated with low frequency unsteadiness. Threadgill and Little [132] also identified a ripple pattern (i.e. spanwise undulations) in the separation shock foot. The unsteady



Figure 6.11: 3D Instantaneous structure using TPIV a) Bubble collapse b) Bubble growth. See Threadgill and Little [132] for detailed description of plot format.

pressure measurements (see Figures 6.8-6.10) suggest that the ripple pattern is related to unsteadiness in the separation shock region. The growing unsteadiness amplitude in span (extending from the root) is obviously not seen in unswept SBLIs. This suggests that it could be related to inception length behavior near the root which stems from both viscous and inviscid effects [133].

6.4 Summary

Swept impinging oblique SBLIs have been investigated at Mach 2.3 with $\theta = 12.5^{\circ}$ deflection angle and sweep angles of $\psi = 15.0^{\circ}$, 22.5° , 30.0° and 40.0° . A detailed characterization of both mean and unsteady flow features has been presented with special attention to the unsteady pressure behavior along the span.

Significant separation is observed in oil flow visualizations and pressure measurements for all shock generator configurations. Strong corner features are present at the root and tip of the SBLI, but a quasi-infinite region is identified for each case in the center of the tunnel. Pressure measurements at separation show that the pressure rise is independent of span, with data collapsing when observed normal to the local separation line. This suggests that the local behavior of pressure at separation scales cylindrically and agrees with the free interaction concept. Pressures at reattachment show spanwise dependency, but collapse when the spanwise coordinate along reattachment is scaled using a virtual origin (z_0) that accounts for the increase in interaction length (and subsequent milder pressure rise) with increasing span. The global behavior for $\psi = 22.5^{\circ}$, 30.0° and 40.0° suggests that the interactions demonstrate conical scaling while 15° appears cylindrical. The idea of conical scaling in the global separation behavior is supported by TPIV results at $\psi = 30.0^{\circ}$.

Unsteady pressure measurements obtained using fast-response transducers show the separation shock exhibits significant low-frequency unsteadiness, in qualitative agreement with unswept SBLIs. Corner features located towards the root and tip influence the amplitude of unsteadiness locally, but within the quasi-infinite region the amplitude increases linearly with span. PSDs extracted at various spanwise locations show that the frequency distribution underneath the shock foot is constant across the span. A spanwise travelling ripple is observed in the pressure along the separation shock foot at speeds much lower than the freestream velocity ($\approx 15\%$ of U_{∞}). This ripple steadily accelerates along the shock foot reaching ($\approx 25\%$ of U_{∞}) near the tip. Significant coherence levels are observed along the span at frequencies that correspond with dominant low-frequency spectral peaks measured at the shock foot. The increasing wave speed at constant frequency suggests the spanwise ripples increase in wavelength along span. There appears to be minimal upstream influence associated with the low frequency unsteadiness supporting the idea of a source mechanism within the SBLI.

7. Conclusions

Our combined approach of experiments, simulations and stability analysis for investigating swept impinging oblique SBLI has resulted in the following contributions. Note that this is the only detailed investigation of this canonical and highly relevant SBLI to date. As such, there are ample opportunities for impactful future work which are the subject of current white papers and proposals.

- An analytical model for inviscid swept oblique shock reflections has been developed allowing the precise definition and unification of the shock-induced pressure rise across various configurations. The model outperforms existing treatments for swept shocks and delta wings providing more accurate predictions of past experimental and numerical observations. The inviscid model also allows for the design of swept shock generators that eliminate or at least minimize the inception region located near the root which we believe is a highly influential feature. Finally, it gives insight into the possible boundary between cylindrical and conical similarities which has been linked to the maximum turning angle based on experimental data.
- The mean flow topology for a Mach 2.3 swept impinging SBLI has been mapped out for various sweep angles with constant deflection angle of 12.5 degrees. The angle of the separation line asymptotically approaches a constant value that is independent of, but shallower than, the impinging shock angle. The angle of the reattachment line is nearly identical to the impinging shock in all cases but is offset downstream. Wall pressure distributions show local separation lengths that are constant with span for 15deg and increase with span for sweep angles of 22.5, 30 and 40 degrees. Mean pressures at separation collapse along the separation line, suggesting local agreement with the free interaction concept. Reattachment pressures can be similarly collapsed by adjusting for the growth in separation length along the span which results in milder pressure rise. For large sweep angles (22.5, 30 and 40 deg), the global flow structure appears quasiconical. Tomographic PIV has revealed the full volumetric character of the SBLI. The spanwise growth of the interaction length scale for a sweep angle of 30 degrees is consistent with surface data and other swept SBLI configurations. The SBLI displays an open separation bubble with significant spanwise velocity components near reattachment that increase with span. This sizeable data set is still be analyzed.
- Low-frequency unsteadiness is present even for swept interactions with conical similarity (open separation). Compared to the unswept interaction, the frequencies are higher with reduced amplitudes in agreement with compression ramp studies. It is noteworthy that some computational studies argue that low-frequency unsteadiness is not present for open separation due to the absence of an absolute instability that may be present in closed separations with cylindrical similarity. This stands in stark contrast to our experiments. Our experiments have also shown that upstream influence, which has been proposed as a source mechanism for low frequency unsteadiness, is negligible and another mechanism is at play in this case.
- The frequencies associated with the separation shock foot motion are surprisingly constant along the span and coherent with spanwise traveling ripples that travel at approximately 20% of the freestream velocity. The low frequency unsteadiness starts as broadband near the SBLI root and becomes more focused and high amplitude as it propagates along the span. The unsteadiness in wall pressure has been connected to instantaneous features of the volumetric flow from tomographic PIV including a large-scale growth and collapse of the separated region. We believe closed separation features at the inception region (root) of swept SBLIs play a role in the low-frequency unsteadiness and its propagation along the span.
- Simulations using both LES and a hybrid turbulence model have complemented the experiments throughout. Because periodic boundary conditions were employed in the spanwise direction, the setup of the simulation forced the separated region to be cylindrically symmetric, and disturbances that leave the domain at one spanwise boundary enter the domain again at the other spanwise boundary. No closed separation could be

obtained with the hybrid turbulence model. For the LES, the ripples were noticeably absent and existing low-frequency contents could be related to a breathing motion of the bubble. In addition to spanwise coherent structures, traveling oblique structures could be identified downstream of the swept interaction. This highlights the importance of various boundary conditions in establishing the SBLI character.

• The cause of low-frequency unsteadiness has been investigated using full and linearized DNS. To deliberately exclude the effect of turbulent boundary layer structures, only laminar and transitional SBLIs have been considered. The linearized simulations reveal a strong disturbance amplification in the separated region for unswept interactions. As for supersonic boundary layer transition, 3D oblique disturbances were most amplified at Mach 2.3. The full DNS revealed a low-frequency unsteadiness in the frequency range expected based on literature. 3D transitional simulations with forced breakdown also showed a low-frequency unsteadiness. The use of a laminar approach boundary layer means that an instability of the bubble itself rather than inflow turbulence is the major driver of the unsteadiness in agreement with our turbulent SBLI experiments mentioned above.

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9. Publications

9.1 Journal Publications

- Threadgill, J. A. S, and Little, J. C., "An Inviscid Analysis of Swept Oblique Shock Reflections", submitted to *Journal of Fluid Mechanics*.
- Threadgill, J. A. S, and Little, J. C., "Volumetric Study of a Turbulent Boundary Layer and Swept Impinging Oblique SBLI at Mach 2.3", under preparation for *Journal of Fluid Mechanics*.
- Padmanabahn, S., Doehrmann, A. C., Threadgill, J. A. S, and Little, J. C., "Unsteady Separation Shock Motion in Swept Impinging Oblique SBLI", under preparation for *AIAA Journal*.
- Gross, A., Little, J. C., and Fasel, H. F., "Numerical Analysis of Turbulent Shock-Wave Boundary Layer Interactions", under preparation for *AIAA Journal*.

9.2 Conference Publications

- Sivasubramanian, J., and Fasel, H. F., "Numerical Investigation of Shock-Induced Laminar Separation Bubble in a Mach 2 Boundary Layer", 45th AIAA Fluid Dynamics Conference, AIAA-2015-2641.
- Sivasubramanian, J., and Fasel, H. F., "Numerical Investigation of Shockwave Boundary Layer Interactions in Supersonic Flows", 54th AIAA Aerospace Sciences Meeting, AIAA-2016-0613.
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