Multi-Channel Digital Equalization to Enable Wideband Digital Arrays

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***Abstract*—In a digital beamforming system, equalization in the digital domain is utilized to help in calibration and reducing errors induced by the system processing chain. This is done by precisely matching amplitude and phase between elemental channels. Without this critical processing step, the desired beamforming (i.e., main beam and null) of next generation radar systems cannot be achieved. This paper addresses the problem of implementing equalization in real time to enable wideband digital arrays. Several equalization algorithms were evaluated and implemented to produce FIR coefficients including our polyphase based equalizer. To our knowledge this type of equalization implementation has not been utilized before in this way. These concepts were demonstrated on data collected from a benchtop four-channel RF system prototype. Additionally, a beamforming environment was created to show the importance of equalization in forming nulls and main lobes. A true-time delay (TTD) FIR filter was combined with the chosen minimum-mean-square-error (MMSE) equalization FIR filter followed by a wideband adaptive digital beamforming (ADBF) FIR filter to display results.**

***Keywords*—equalization; adaptive; digital beamforming; phased array; channelizer; wideband**

I. Introduction

In a digital beamforming system, equalization in the digital domain is utilized to help in calibration and reducing errors induced by the system processing chain. This is done by precisely matching amplitude and phase between elemental channels. Without this critical processing step, the desired beamforming (i.e., main beam and null) of next generation radar systems cannot be achieved. This importance is discussed in further detail in [1] with several techniques for calibrating and aligning different aspects of a digital phased array demonstrated in [2].

Many papers discuss ways of implementing equalization in digital systems as well as incorporating them into the digital beamforming processing chains [3]–[7]. Equalization in the digital domain is typically implemented using complex FIR filters with methods such as zero-forcing (ZF) equalization, minimum-mean-square-error (MMSE) equalization, blind equalization, adaptive equalization, subbanding and so forth. True-time delay (TTD) and adaptive digital beamforming (ADBF) can also be implemented with complex FIR filters. Additional ways of improving FIR filter coefficient calculation is shown in [8].

This paper addresses the problem of implementing equalization in real time to enable wideband digital arrays through algorithm evaluation and simulation of a beamforming environment using multi-channel laboratory data to show the importance of equalization on beamforming performance.

II. Technical Approach

To begin, several equalizer algorithms and implementations for next generation radar systems were evaluated using a data set captured from a benchtop four-channel RF system prototype with a metric known as the channel pair cancellation ratio (CPCR) to evaluate performance [9]. These equalizers were implemented to process the incoming sampled data and correct for mismatches using a multi-channel FIR filter. The equalization of channels was based on a reference signal that was either known prior, such as a loop-back test signal with a flat response, or based on a chosen incoming auxiliary channel depending on application. The following evaluates the ZF and MMSE equalization across the entire band as well as subbanding techniques of STFT and polyphase decomposition that rely on MMSE in each subband. These can be implemented in FIR filters for the full band and STFT approaches and FIR filters in each channel for the polyphase approach. These equalizer coefficients are combined with TTD coefficients for wideband digital beamforming followed by an adaptive digital beamforming filter that nulls out interferers. These will be explained further in the Beamforming Environment Section.

A. Zero Forcing Equalization

First, a ZF equalization algorithm was analyzed and implemented. Computing FIR coefficients for a ZF equalizer relies on the inverse of the relative mismatch responses across the frequency band between the reference signal and auxiliary signal. Based on the desired filter length, the group delay response is combined with the inverse of the relative mismatch response resulting in the desired response of the filter. Using a $\mathbf{T}$ frequency matrix and the desired desired filter response $\mathbf{d_m}$, the complex coefficients $\mathbf{h_m}$ can be solved for. This is done by solving the linear system of equations using the inverse

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of the overdetermined system and multiplying by the desired response like shown in the following equation.

$$\mathbf{h}_m = \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{d}_m$$

Some drawbacks of a ZF equalization is that it can perform poorly in the presence of noise or gaps in the frequency response.

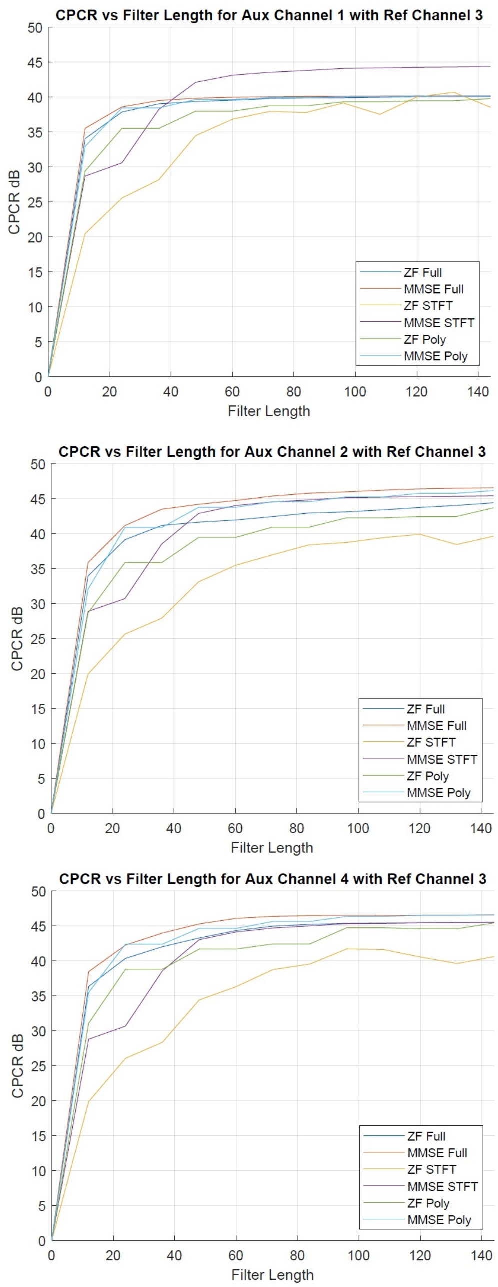

Fig. 1. The CPCR vs filter length to evaluate performance of ZF and MMSE equalization across the full band, STFT subbanding, and polyphase subbanding. Using the four-channel benchtop data equalizers were implemented and evaluated. The third channel was used as a reference to compute the equalizer coefficients for the channels. The CPCR for each method is shown below vs filter length.

B. Minimum Mean Square Error Equalization

Next, a MMSE Equalization algorithm was analyzed and implemented. One benefit of this method is that the frequency responses need not to be computed like in ZF equalization because it relies on second order statistics of the incoming sampled data, making it possible to compute in the time domain. In the following it is assumed the second order statistics are known i.e. the covariance matrix can be computed from the captured data. In MMSE equalization the mean square error between channels is minimized by solving for the optimal filter coefficients using the Wiener solution. This is based on the computing the covariance matrix using the data matrix $\mathbf{A}_m$ for the mth channel needing equalization. The data matrix is composed of temporal samples across the desired filter taps. The cross-correlation between the data matrix $\mathbf{A}_m$ needing equalization and the reference signal $\mathbf{b}$ delayed to account for the group delay are also computed. The complex filter coefficients $\mathbf{h}_m$ are computed using a linear system of equations where the inverse of the covariance matrix is multiplied by the cross-correlation matrix like shown in the following equation.

$$\mathbf{h}_m = \left(\mathbf{A}_m^H\mathbf{A}_m\right)^{-1}\mathbf{A}_m^H\mathbf{b}$$

Iterative algorithms can be used to further calculate filter coefficients like shown in [8], but were not fully implemented here.

C. STFT Based Equalization

Next, a STFT Based Equalization algorithm was analyzed. The short time Fourier transform (STFT) [10] with 50% overlapping and Tukey windowing was utilized to view the time varying data of the reference and auxilary channels as stationary segments of data rather than the entire window like previously. Implementing and evaluating both the ZF and MMSE equalization methods described prior, sets of equalizer coefficients were computed for each subband of data using the segmented reference and auxilary channels. These coefficients are then cycled or updated as the signal is received. This method performs quite well when there exists higher levels of mismatch that the other methods fall short on. One drawback is the incoming signal needs to be known in real time so the coefficients can be cycled through or updated appropriately for the frequency band of interest.

D. Polyphase Channelizer Based Equalization

Lastly, our polyphase channelizer based equalization algorithm was implemented and evaluated. This involves breaking the full band up into subbands which can help with computational complexity of computing coefficients because each subband will have less taps than an equalizer would across the full band. The polyphase subbanding utilizes a near perfect reconstruction algorithm modeled after the algorithm found in [11]. The novelty comes from utilizing the channelization mentioned prior, then applying either the MMSE or ZF equalizers in each subband to form a polyphased based

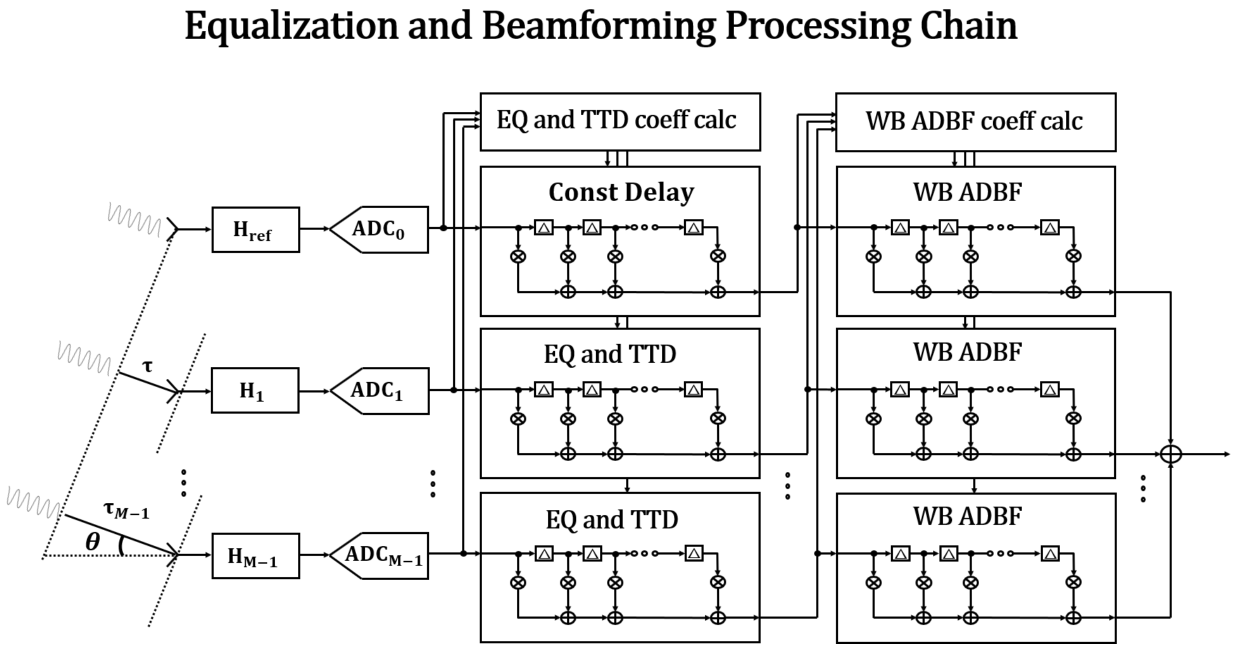

Fig. 2. Multi-channel beamforming processing chain used in the simulation environment. An interferer and desired signal arrive at specific angles. Each channel has a transfer function H_m that represent the mismatches due to system characteristics. The first channel is employed as the reference channel, so a constant group delay FIR is utilized. Equalization and TTD coefficients are computed and applied to the other channels using FIR filters. Additionally ADBF coefficients are computed using the LCMV algorithm and applied to each channel using FIR filters to null out undesired signals.

equalizer. The coefficients are computed and applied in the same way as the ZF and MMSE implementation but in each subband rather than across the full band. This can be useful for implementations where wideband incoming data needs to be processed in parallel. This has shown to perform well and results will be shown in the next section.

III. Laboratory Data and Equalization Results

Next, the multi-channel laboratory data are discussed. The data contained approximately 62 ksamp by four channels of 125 MSPS (mega samples per second) data received simultaneously. The excitation waveform was a chirp starting at -62 MHz going to +62 MHz over a duration of approximately 500 us. This transmit waveform was converted to RF around 2.8 GHz and coupled to the four channels through an RF splitter, analog down converted and digitized. The RF components caused several dB of variance across this bandwidth. Tones caused by artifacts of direct conversion transceivers impacted the received signal as well. Other artifacts included a DC offset caused by LO leakage, finite image rejection, and third-order nonlinearities. Applying equalization to the data based on the algorithms mentioned above results in STFT based equalization generally performing the best, followed by MMSE equalization across the full band, MMSE equalization in polyphase channels, ZF equalization across the full band, and ZF equalization in polyphase channels. STFT based equalization requires cycling through coefficients which requires knowing the frequency response of the incoming signal. Because MMSE equalization across the full band performs well and has static coefficients, it will be implemented in the following section for showing the necessity of equalization to result in deep nulls when beamforming. Figure 1 depicts the CPCR vs filter length results of each type of equalizer above including our MMSE based polyphase subbanding equalization implementation. The third channel was used as a reference with the rest of the channels serving as auxiliary channels. Using CPCR as a metric, our method performs just as well or better than other methods implemented and evaluated depending on channel.

IV. Beamforming Environment

Based on the data, a four-channel beamforming simulation environment was created with an interferer and desired signal coming in at specific angles (-15 degrees and +30 degrees, respectively). This was used to show the importance of equalization in forming main lobes and nulls. MMSE equalization and TTD were combined into one FIR filter followed by linear constrained minimum variance (LCMV) beamforming [12] in an additional FIR filter. TTD beamforming requires fixing the linear phase shift response across frequency as well as the phase offset due to the angle of arrival causing a time delay τ across subsequent elements of the phased array. TTD filter coefficients $\mathbf{g_m}$ can be computed using a $\mathbf{T}$ frequency matrix and the desired phase response $\mathbf{p_m}$ of the filter across frequency. This is shown in the following equation.

$$\mathbf{g}_m = (\mathbf{T}^H\mathbf{T})^{-1}\mathbf{T}^H\mathbf{p}_m$$

These can be combined with the EQ filter coefficients in the frequency domain to result in the final coefficients. These coefficients are generally computed offline since the environment

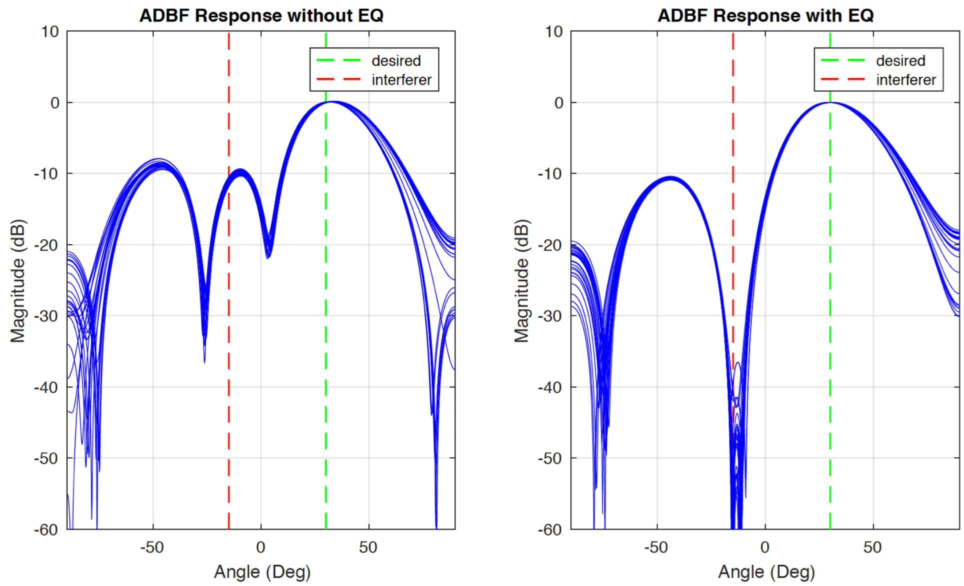

Fig. 3. (LEFT) without equalization a four-channel beamforming environment was created with an interferer (RED) and desired signal (GREEN) coming in at specific angles (-15 degrees and +30 degrees, respectively), and (RIGHT) beamforming environment with equalization. The processing steps taken were: equalization and TTD combined in one FIR filter followed by LCMV beamforming in an additional FIR filter. The overlaid blue lines are in 4.1 MHz steps over the entire 125 MHz.

and system characteristics are assumed static. LCMV adaptive beamforming can be implemented taking in a set of training data of length $5N$ for sufficient performance where N is the degrees of freedom. The coefficients are updated or cycled in real time based on new sets of training data. Forming a data matrix across spatial and temporal samples from the equalized and TTD signals, the covariance matrix $\mathbf{S}$ can be formed. Setting up constraints $\mathbf{C}$ for a broadside response, since the TTD beamforming corrected for the induced time delay due to the angle of arrival, the coefficients $\mathbf{w}$ can be computed in the following equations.

$$\mathbf{S} = \mathbf{x}^H \mathbf{x}$$

$$\mathbf{w} = \mathbf{S}^{-1}\mathbf{C}\left[\mathbf{C^H S^{-1} C}\right]^{-1}\mathbf{f}$$

Figure 2 illustrates the equalization and beamforming processing chain. Figure 3 shows the pattern results of adaptive beamforming without and with equalization applied to the four channels.

Conclusion

In summary: several equalization algorithms were studied, and our selected MMSE polyphase based equalizer performed well against the other methods and frequency banding techniques. Although STFT based equalization performs quite well, our method doesn't require cycling through coefficients based on a known input for equalization. Additionally this opens up ways of performing adaptive beamforming in each subband rather than across the full band leading to less computational complexity. It was also shown the necessity of equalization when generating deep nulls when beamforming.

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