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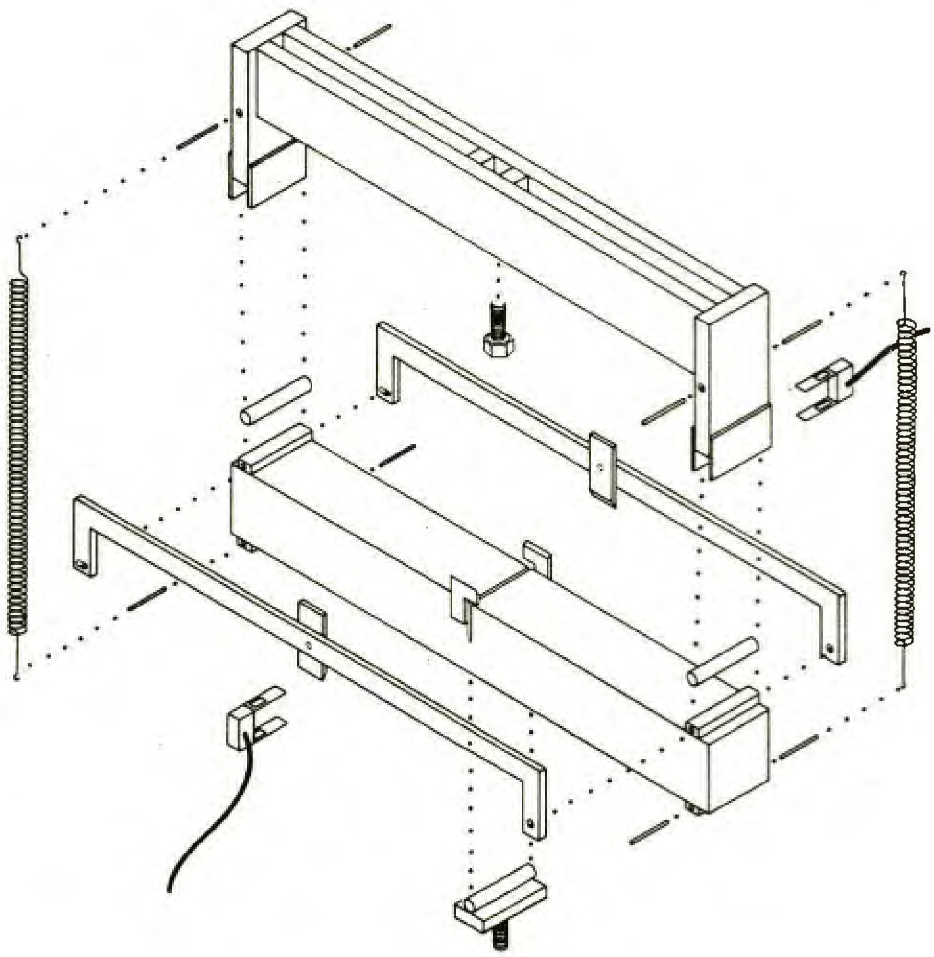
Fracture Energy for Three Point Bend Tests on Single Edge Notched Beams: Proposed Evaluation



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ABSTRACT

The research in fracture mechanics of concrete is reviewed in an attempt to reconcile the different and sometimes contradictory approaches for evaluation of the fracture energy, G_f. An improved method to measure G_f for beams in three-point bending is presented. This method is expected to provide a more reliable characterization of the fracture toughness of concrete.

SUMMARY

The research in fracture mechanics of concrete is reviewed in an attempt to reconcile the different and sometimes contradictory approaches for evaluation of the fracture energy, G_f. An improved method is presented to measure G_f for beams in three-point bending using the load versus load point deflection plot. In this method, the specimen is loaded with the notch on the upper face. By unloading at the point of instability, the energy spent on process zone formation, slow crack growth, and outside the crack zone can be evaluated. Dye application allows for determination of the true crack length, a_p, at that moment. Upon reloading, the energy spent on crack propagation, U, is found as the difference between the total area under the curve and the energy previously consumed. The fracture energy is then defined as:

$$G_f = U/B(W-a_p).$$

The test method compensates for the specimen weight and accounts for the inelastic indentation at the loading points. This method is expected to provide a more reliable characterization of the fracture toughness of concrete.

This report was generated within work unit YR-23.03.01.009, Fracture and Fatigue of Concrete, in the Structural Modeling Project, which is part of NAVFAC's 6.1 Basic Research Program Subelement 23, Mechanics, of Program Element 61153N. Application of fracture mechanics of concrete will impact design of gas turbine engine test cells and waterfront structures of reinforced and prestressed concrete. This work unit continues with measurement of concrete fracture energy using methodology described in this report. This work unit should be transitioned to 6.2 in FY89 for development of design criteria.

CONTENTS

INTRODUCTION

The existing approaches for fracture energy evaluation of concrete are reviewed. Drawing from this previous work, a more accurate and size independent method is developed for measuring the fracture energy of concrete, G_f, using a three-point bend test specimen.

Fracture mechanics and Griffith's theory (Ref 1) were first applied to concrete in the early sixties. Kaplan (Ref 2) first determined the crack extension force (or critical strain energy release rate), G_c, using beams in three-and four-point bending. He first observed its dependency on specimen size and attributed it to slow crack growth. Romualdi and Batson (Ref 3) later performed a series of tension tests and observed an increase of G_c with crack length. Glucklitch (Ref 4) noted the dissipative effects at the crack tip, charged them to microcracking, and concluded that G_c was greater than twice the surface energy of concrete. Hughes and Chapman (Ref 5) first published in 1966 a complete stress-deformation curve for concrete in tension.

Since 1966 there have been several attempts at characterizing the toughness of concrete by measure of the critical strain energy release rate. Most of them used single-edge notched beams in three- and four-point bending (3PB and 4PB). An extensive review was presented by Mindess (Ref 6), Carpinteri (Ref 7), ACI Committee 224 (Ref 8), and Gustafsson (Ref 9). RILEM Technical Committee 50-FMC on fracture mechanics of concrete has attempted setting a standard for determination of the fracture energy of concrete by 3PB tests (Refs 10, 11). Toward this goal, 14 laboratories in 9 countries have been involved and 700 beams tested (Refs 12, 13, 14). However, the method has not yet been fully accepted since its results are size dependent and approximate. The fracture energy is overestimated (Refs 15 through 19) and so is the corresponding ligament area (Refs 20, 21, 22).

EVALUATION OF THE FRACTURE ENERGY, G_f

Fracture Energy and Critical Strain Energy Release Rate

For a beam in three-point bending (Figure 1a) a typical plot of load versus load point deflection (LLPD) is shown in Figure 1b. The area, U, under the LLPD curve represents the energy required to break the specimen and the fracture energy, G_f, was defined as:

$$G_f = U/A$$

where A is the uncracked area at the notch (ligament area) (Refs 15, 23, 24).

The fracture energy, G_f, has typically been used as an approximation to the critical strain energy release rate, G_c, which is defined as the elastic energy per unit crack surface area that is available for infinitesimal crack extension (Ref 1). Unfortunately, both notations have been used indistinctively (Refs 15 and 25 through 28), creating some confusion in the literature. For linear elastic fracture mechanics (LEFM), an exact relation between G_c and G_f for a beam in 3PB is presented by Turner (Ref 29) and Plati and Williams (Ref 30). They showed that:

$$G_c = rG_f$$

with r varying between 0.6 and 2 for crack length to specimen depth ratios a/W from 0.1 to 0.6, depending on the configuration (r = 1.67 for S/W = 8 and a/W = 0.5). This agrees with the simplification of the J integral approach formulated by Rice et al (Ref 31) for a/W > = 0.6:

$$J_c = 2U/A$$

(Refs 30 and 32 through 37), which is the definition used in ASTM E813 (Ref 38). In effect, Rice showed for an elastic-plastic material and small scale yielding (Ref 39):

$$J_c = G_c$$

Turner (Ref 29) and Plati and Williams (Ref 30) based their relation on the formulas derived by Srawley in 1966 (Ref 40), which represent boundary collocation K-calibrations for single-edge notched plate specimens in 3PB. The values they obtained for r, together with Rice's (r = 2 for a/W > = 0.6), are plotted in Figure 2. In 1976, Srawley derived a more general formula (Ref 41) for a/W from 0 to 1 which is now used in ASTM E399 (Ref 42). Using Srawley's formula and Turner's derivation, the corresponding values of r are obtained as follows:

$$K = \frac{PS}{BW^{3/2}} \left\{ \frac{3(a/W)^{1/2}\ [1.99-(a/W)(1-a/W)(2.15-3.93a/W+2.7a^2/W^2)]}{2(1+2a/W)(1-a/W)^{3/2}} \right\}$$

where S is the span and B the width of the specimen. If the nominal bending stress is defined as (Refs 40, 29):

$$\sigma = \frac{6M}{BW^2} = \frac{3PS}{2BW^2}$$

Where M is the maximum moment for conventional bending theory,

then: $K = Y \sigma a^{1/2}$

$$\text{where} \quad Y = \frac{1.99 - a/W(1-aW)(2.15-3.93a/W+2.7a^2/W^2)}{(1+2a/W)(1-a/W)^{3/2}}$$

and applying Turner's derivation (Ref 29):

$$r = \frac{Y^2(a/W)(1-a/W)}{\int Y^2(a/W)\ d(a/W) + S/18W}$$

These values of r have also been plotted in Figure 2. It is apparent that all approaches give an upper limit for r of approximately 2 ($a/W > 0.5$). Go, et al, (Ref 43) proposed other relations for the boundary collocation K-calibrations which agree closely for $S/W = 4$ but are somewhat different for $S/W = 8$.

Load Versus Load Point Deflection Plot

As shown in Figure 1b, the LLPD plot has three stages of behavior (Refs 44, 45, 46). In the first stage, the deflection increases linearly with the load. The crack is opened but does not extend (Ref 45). During the second stage, a fracture process zone develops, microcracks form, and slow crack growth is observed (Refs 47 through 51). In the third stage (called the strain softening zone) rapid crack growth is observed that may lead to instability (Refs 52, 53). This instability can be avoided by choosing an adequate specimen configuration (Refs 15, 17, 54, 55), but depends upon the stiffness of the testing apparatus. Stability will be achieved if the rate of energy dissipation by the fractured zone and the testing apparatus is greater than the rate of release of the energy stored in the whole specimen and the loading frame (Refs 15, 23, and 56 through 59). Gurney and Hunt simply stated "the stability is decreased by the flexibility of the testing machine" (Ref 60). RILEM advises a minimum machine stiffness of 10 kN/mm (57 kips/in) for tests carried out on its smallest standard specimen (Ref 11).

During strain softening most of the damage to the specimen is concentrated in a narrow zone (Ref 61). This concentration is higher as the load carrying capacity decreases (Refs 50, 62). It has been observed that energy dissipation occurs through a single major crack (Refs 46, 63). Strain softening is considered a material characteristic (Refs 64, 65, 66). Using a power relation to represent the strain softening, the exponent of the relation itself has been considered a measure of the true toughness (Ref 66).

Current Approaches

Several methods of measuring the fracture energy, G_f, using the LLPD plot have been proposed. Nakayama (Refs 23, 67), Tattersall and Tappin (Ref 59), Davidge and Tappin (Ref 24), and Petersson (Ref 15) used the total area under the curve U_o (Figure 3b). This method was the basis for the RILEM recommendation (Refs 10, 11), which includes a correction for the beam weight (Ref 62):

$$G_f = (U_o + mgd)/A$$

where A = ligament area = $B(W-a_o)$
B = width
a_o = notch depth
mg = weight of the specimen
d = load point deflection at fracture

For elastic-brittle materials like glass, Turner (Ref 29) obtained satisfactory results using only the area under the curve, U_1, (Fig 3a) up to peak load or up to the point of instability which is the point where the load starts to drop off. Other researchers applied this approach to hardened cement paste (HCP) and concrete (Refs 22, 68, 69, 70). Go (Ref 70) introduced a roughness coefficient, C_a = 1.15, and used:

$$G_f = U_1/C_aA$$

Swartz, et al, (Ref 22) measured the extended crack length, a_p (this is the crack length at the point of instability), and used $A=B(W-a_p)$. They also proposed a modified RILEM method using U_1 (Refs 19, 71) and a correction for the beam weight:

$$G_f = (U_1 + mgd_p)/A$$

where d_p is the midspan deflection at the point of instability. Swartz also proposed testing a beam loaded upward (Ref 45) to simplify dye application (Ref 72) and the corresponding beam weight correction (Ref 73).

These test approaches have the following shortcomings:

1. The energy dissipation outside of the fracture zone cannot be neglected (Refs 53, 65, 74, 75) and depends on the specimen size and notch depth (Refs 15, 17, 75). This dissipation will be more important if the notch size decreases or if the beam size increases. This explains the choice of specimen dimensions by RILEM (Refs 10, 11) in an attempt to minimize this loss. Hillerborg (Refs 26, 64) tried to evaluate this effect for the case of a tensile specimen by loading a similar but uncracked specimen. This is suspected to be one of the causes of the size dependence of G_f (Refs 75, 76).

2. Slow crack growth precedes fracture (Refs 7, 47, 48, 75, 77) and is also suspected to be a cause of size effect on G_f (Refs 2, 51, 78). In the first stages of loading, energy is dissipated in creating and extending a process zone (Refs 64, 79) by debonding aggregates and opening

microcracks. This is a direct result of the heterogeneity of concrete (Refs 4, 56). Consequently, the real (extended) crack length differs from the notch depth used by most researchers. Only a few authors (Refs 20, 21, 22, 56, 80) report evaluating the extended crack length, generally using compliance techniques.

3. Using the whole area under the curve leads to an overestimate of G_f, as recognized by Petersson (Ref 15) and others (Refs 16, 44, 65).

4. Using only the area up to peak load (or instability) assumes concrete to be a perfectly elastic-brittle material. Strain softening for a purely homogeneous material theoretically leads to an instantaneous vanishing of the stress (Refs 52, 81, 82) but this is not the case for concrete (Ref 83). For typical laboratory size specimens, LLPD plots from References 11, 15, 19, 22, and 56, show that this assumption disregards more than half the total energy spent in breaking the specimen. However it does seem to apply to hardened cement paste (Refs 6, 9, 21, 56, 84, 85, 86) which appears as a quasi-homogeneous material (although References 8, 87, and 88 still specify a minimum specimen size). It should be recalled that Griffith's fracture criterion was first postulated for glass (Refs 1, 86). Only for very large sizes does concrete behave in a brittle fashion. Unfortunately, this would require a beam depth of at least 230 mm according to Walsh (Ref 76), or 650 mm according to Carpinteri (Ref 89), or 2000 mm according to Modeer (Ref 90) and others (Ref 15, 18, and 26).

PROPOSED TEST METHOD

From the preceding observations it was concluded that a true measure of the toughness of concrete would be obtained with a three-point bend beam specimen and the following procedure:

1. Install the beam with the notch on the top surface. This will help in dying the cracked surface (Refs 53, 72).

2. Load the specimen up to the point of instability (Refs 19, 56, 71) defined as the point where the load drops to 95 percent of its maximum value, then remove the load completely. The area enclosed in this load-unload loop includes the energy spent on formation of a process zone, slow crack growth, and the inelastic energy spent outside of the crack zone.

3. Insert a dye through the notch and allow it to flow into the crack. This will highlight the true crack length, a_p (Refs 22, 28, 72).

4. Reapply the load and obtain the strain softening zone. Unloading and reloading once will not significantly affect the LLPD curve (Refs 53, 60, 91). Cyclic loading has been routinely performed for compliance measurements or damage evaluation (Refs 51, 53, 75).

5. Define U as the total area under the LLPD curve minus the area in the load-unload loop indicated in the second step (see Figure 3c).

6. Define G_f as the energy spent on developing one major crack divided by the ligament area existing at that moment:

$$G_f = U/B(W-a_p)$$

Brown and Srawley proposed the same formula using the energy U_{bs} as defined in Figure 3d, discarding an area similar to the load-unload loop of Step 2, which "represents the energy contribution associated with stable crack extension during the increase of the crack extension resistance from G_{1c} (at first crack) to G_c (at peak load)" (Ref 40).

A similar measure was also attempted for a specimen in tension. Petersson recognized that part of the area under the curve to the left of the peak load should not be taken into account (Ref 15). Unloading at the instability point would eliminate a similar area. Carpinteri (Ref 50) and Bazant (Ref 61) similarly measured G_f for a tensile specimen on the strain softening side of the curve. Hillerborg, while presenting the theoretical basis of the RILEM recommendation, discarded the energy spent outside the fracture zone for a specimen in tension, but did not extend it to the RILEM specimen (Ref 17).

These corrections also follow the generalized fracture mechanics approach developed by Andrews, et al (Refs 92, 93, 94) where the critical energy release rate, G_c (or J_c), is only a part of the total energy, G_o (or J_o), actually needed to form a unit area of crack:

$$G_c = zG_o$$

where z is a loss function. Part of this loss is attributed to inelastic behavior. The importance of inelastic behavior for metallic materials was recognized as early as 1948 by Irwin and Orowan (Refs 29, and 95 through 97) in what is known as the Irwin-Orowan concept. Although it is not as important, inelastic behavior is also present in concrete.

DETAILS OF EXPERIMENTAL PROCEDURE AND METHODOLOGY

Fracture Area

During strain softening, a crack will actually follow a surface that is not flat but governed by the aggregate size and relative hardness (Refs 6, 99). A roughness coefficient is not required since the roughness will depend on the type of aggregate and will be related to toughness and included in its measure (Refs 24, 59). The actual area of fracture is actually much larger than the area of a single crack even when including a roughness coefficient of 1.15 (Refs 56, 85).

Precracking

Precracking of the specimen (or fatigue cracking) is not necessary in the proposed method. Measurement of the energy takes place only after a sharp crack has been formed, and does not depend on the initial condition, whether it is precracked, form notched, or saw notched.

Rate of Loading

Mindess reported that both the work of fracture and the strain energy release rate only show a small increase (about 15 percent) for cross-head deflection rates from 5.10^{-7} to 5.10^{-5} m/sec (2.10^{-5} to 2.10^{-3} in/sec) (Ref 100). RILEM recommends reaching peak load after 30 to 60 seconds which corresponds to a rate in the order of 5.10^{-6} m/sec (2.10^{-4} in/sec).

Beam Weight

Techniques to compensate for specimen weight have been derived following the works of Petersson (Ref 102) and Hilsdorf and Brameshuber (Ref 53). Petersson's correction is approximate (Ref 73). On the other hand, by supporting half of the beam weight at the ends, Hilsdorf and Brameshuber obtained a complete LLPD curve, which actually starts after the applied load equals the other half of the weight. The discarded area (1/2 weight by midspan deflection at total fracture) corresponds to work being transformed into potential energy as the center of gravity of the beam is forced up. It should be noted that RILEM guidelines invalidate tests carried out on large beams where the effect of specimen weight is excessive (Refs 101, 102).

Displacements

According to RILEM recommendations, the midspan deflection can be measured with reference to the loading apparatus as long as the inelastic deformations at the loading points do not exceed 0.01 mm (0.0004 inch) (Refs 10, 11). For nonstandard specimens with small span/depth ratios (e.g., S/W = 4) the inelastic indentations at the loading points are not negligible. They have to be considered (Refs 21, 36, 44, 53) or else the deflection has to be measured on the beam itself (Refs 21, 34, 44). This problem has not always been addressed (Refs 28, 73, 103), thereby producing uncertain results.

The error caused by inelastic indentation may be quantified as follows for a 27.65 MN/m^2 (4,000 psi) concrete specimen with dimensions 102 by 76 by 406 mm (4 by 3 by 16 inch) (depth by width by span), with an initial notch depth a_o = 25 mm (1 inch), and bearing directly on 51 mm (2 inches) diameter rollers. A maximum load of approximately 3.12 kN (700 pounds) should be expected. The minimum bearing area at the center roller is 3.12/0.02765 = 113 mm^2 (0.175 in^2) and the minimum bearing width is 113/76 = 1.5 mm (0.058 inch). This implies an indentation of the flat surface at the center roller only of 0.75 by 0.75/25.5 = 0.022 mm (about 0.001 inch). In this case the indentation represents about 25 percent of the midspan deflection at peak load.

Displacements are typically measured with linear variable differential transformers (LVDT) (Refs 34, 44) and clip gages (Refs 36, 103). The clip gage described by ASTM E399 (Ref 42) seems most appropriate, and may be manufactured out of high-strength aluminum (7075-T6 or 7178-T6) (Ref 104) which is more readily available and easier to machine than a titanium alloy. These aluminums present a ratio of yield strength to modulus of elasticity as high as 0.0069 (for 7178-T6) compared to a typical 0.0076 for titanium 13V-11Cr-3Al as proposed by ASTM E399 (Ref 42) (higher yield strengths may be obtained for titanium alloys). High strength aluminum ensures a large range of measurement without permanent deformation of the gage. Two clip gages are recommended, one on each side of the specimen, to mitigate errors due to asymmetry (Ref 36).

Point of Instability

The point of instability has been chosen as the point where the load decreases to 95 percent of its peak value, as recommended by Swartz and Yap (Ref 73). At this point, a small variation of the load close to peak value is accompanied by a small displacement on the LLPD curve. However, such a negligible amount of external work causes a significant crack advance (Ref 19). This is apparent on typical load-crack mouth opening displacement (LCMOD) plots (Refs 45, 73), where the CMOD increases significantly for almost constant maximum load. This instability is attributed to a redistribution of the energy inside the specimen, from elastic to surface energy. Measurement of the crack length at peak load would then yield unreliable values.

CONCLUSION

Different approaches for measuring the fracture energy, G_f, for beams in three-point bending have been evaluated and a new method proposed. In this method, the specimen is loaded with the notch on the upper face. By unloading at the point of instability, the energy spent on process zone formation, slow crack growth, and outside of the crack zone can be evaluated. Dye application allows for determination of the true crack length, a_p, at that moment. Upon reloading, the energy spent on crack propagation, U, is found as the difference between the total area under the curve and the energy previously consumed. The fracture energy is then defined as:

$$G_f = U/B(W-a_p)$$

The test method compensates for the specimen weight and accounts for the inelastic indentation at the loading points.

REFERENCES

1. A.A. Griffith. "The phenomena of rupture and flow in solids," in Philosophical Transactions, London, England, 1920. Royal Society of London, Series A221, pp 163-198, 1920.

2. M.F. Kaplan. "Crack propagation and the fracture of concrete," American Concrete Institute Journal, vol 58, no. 11, Nov 1961, pp 591-610.

3. J.P. Romualdi and G.B. Batson. "Mechanics crack arrest in concrete," American Society of Civil Engineers, Journal of the Engineering Mechanics Division, Jun 1963, pp 147-167.

4. J. Glucklich. "Fracture of plain concrete," American Society of Civil Engineers, Journal of the Engineering Mechanics Division, vol 89, no. EM6, Dec 1963, pp 127-138.

5. B.P. Hughes and G.P. Chapman. "The complete stress strain curve for concrete in direct tension," Bulletin RILEM, no. 30, Mar 1966, pp 95-97.

6. S. Mindess. "Fracture toughness testing of cement and concrete," Fracture Mechanics of Concrete (A. Carpinteri and A.R. Ingraffea, editors), Martinus Nijohff Publishers, Dordrecht, The Netherlands, 1984, pp 67-110.

7. A. Carpinteri. "Application of fracture mechanics to concrete structures," American Society of Civil Engineers, Journal of the Structural Division, vol 108, no. ST4, Apr 1982, pp 833-848.

8. American Concrete Institute Committee 224, "Control of cracking in concrete structures," Report No. ACI 224R-80, Concrete International, vol 2, no. 10, Oct 1980.

9. Lund Institute of Technology. Report TVBM-1007: Fracture mechanics studies of nonyielding materials like concrete: modeling of tensile fracture and applied strength analyses by P.J. Gustafson. Division of Building Materials, Lund, Sweden, 1985.

10. RILEM Technical Committee. 50-FMC, Proposed recommendation: Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams, Jan 1982, Revised Jun 1982.

11. RILEM Technical Committee. 50-FMC, Draft recommendation: Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams, Materials and Structures, no. 106, Jul-Aug 1985, pp 285-290.

12. A. Hillerborg. "Results of three comparative test series for determining the fracture energy of the concrete," Materials and Structures, no. 107, Sep-Oct 1985.

13. Lund Institute of Technology. Report TVBM-3015: Concrete fracture energy tests performed by 9 laboratories according to a draft RILEM recommendation, Report to RILEM TC50-FMC by A. Hillerborg, Division of Building Materials, S-221 00. Lund, Sweden, 1983.

14. Lund Institute of Technology. Report TVBM-3017: Additional concrete fracture energy tests performed by 6 laboratories according to a draft RILEM recommendation by A. Hillerborg. Division of Building Materials, S-221 00. Lund, Sweden, 1984.

15. P.E. Petersson. "Fracture energy of concrete: method of determination, practical performance and experimental results," Cement and Concrete Research, vol 10, 1980, pp 79-101.

16. S. Somayaji. A discussion of the paper: "Fracture energy of concrete: method of determination, practical performance and experimental results," by P.E. Petersson. Cement and Concrete Research, vol 10, 1980, pp 471-474.

17. A. Hillerborg. "The theoretical basis of a method to determine the fracture energy of concrete," Materials and Structures, no. 106, Jul-Aug 1985, pp 291-296.

18. P.J. Gustafsson and A. Hillerborg. "Improvements in concrete design achieved through application of fracture mechanics," Application of Fracture Mechanics to Cementitious Composites, (S.P. Shah editor) NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 487-500.

19. S.E. Swartz and H.C. Siew. "Load versus strain control in fracture energy measurements for concrete," in Proceedings of the 1986 System Engineering Management Spring Conference on Experimental Mechanics, Society for Experimental Mechanics, New Orleans, LA, Jun 1986, pp 16-22.

20. D.J. Cook and G. Crookham. A discussion of the paper: "Notch sensitivity and fracture toughness of concrete," Cement and Concrete Research, vol 8, 1978, pp 387-388.

21. J.H. Brown. "Measuring the fracture toughness of cement paste and mortar," Magazine of Concrete Research, vol 24, no. 81, Dec 1972, pp 185-196.

22. S.E. Swartz and S.M. Rood. "Fracture toughness testing of concrete beams in three-point bending: Phase I, small beams," in proceedings of the 1985 System Engineering Management Spring Conference on Experimental Mechanics, Society for Experimental Mechanics, Las Vegas, NV, pp 119-126.

23. J. Nakayama. "Direct measurement of fracture energies of brittle heterogeneous materials," Journal of the American Ceramic Society, vol 48, no. 9, Sep 1965, pp 583-587.

24. R.W. Davidge and G. Tappin. "The effective surface energy of brittle materials," Journal of Materials Science, vol 3, 1968, pp 165-173.

25. Society of Experimental Mechanics. Review of current notations on fracture toughness testing of concrete and rock, Report to the committee on fracture mechanics of concrete and rock, by M. Wecharatana and S.E. Swartz. Las Vegas, NV, Jun 1985.

26. A. Hillerborg. "Analysis of one single crack," Fracture Mechanics of Concrete, Elsevier Science Publishers, Amsterdam, 1983 (F. H. Wittmann, editor), pp 223-249.

27. A. Hillerborg, M. Modeer and P.E. Petersson. "Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements," Cement and Concrete Research, 1976, vol 6, pp 773-782.

28. C.G. Go and S.E. Swartz "Energy methods for fracture toughness determination in concrete," in Proceedings of the Fifth International Congress on Experimental Mechanics, Montreal, Canada, 1984, pp 453-459. (also in Experimental Mechanics, vol 26, no. 3, Sep 1986, pp 292-298.)

29. C.E. Turner. "Fracture toughness and specific fracture energy: A re-analysis of results," Materials Science and Engineering, vol 11, 1973, pp 275-282.

30. E. Plati and J.G. Williams. "The determination of the fracture parameters for polymers in impact," Polymer Engineering and Science, vol 15, 1975, pp 470-477.

31. J. R. Rice, P.C. Paris, and J.G. Merkle. "Some further aspects of J-integral analysis and estimates," Progress in Flaw Growth and Fracture Toughness Testing, American Society for Testing and Materials, STP 536, 1973, pp 731-745.

32. University of Illinois at Urbana. Department of Civil Engineering NSF/ECS-81003: Evaluation of the fracture behavior of concrete with the J-integral, by R.C. Andren, T.J. Powell, F.V. Lawrence, and C.E. Kesler. Champaign, IL, Apr 1981.

33. G.T. Halvorsen. "J-integral study of steel fibre reinforced concrete," International Journal of Cement Composites, vol 2, no. 1, Feb 1980, pp 13-22.

34. G. T. Halvorsen. "Jm toughness comparison for some plain concretes," International Journal of Cement Composites, vol 2, 1980, pp 143-148.

35. J.E. Srawley. "On the relation of J to work done per unit uncracked area: "total" or component "due to crack," International Journal of Fractũre, vol 12, 1976, pp 470-474.

36. R.J. Buzzard and D.M. Fisher. "Load-displacement measurements and work determination in three-point bend tests of notched or precracked specimens," Journal of Testing and Evaluation, JTEVA, vol 6, no. 1, Jan 1978, pp 35-39.

37. H.P. Keller and D. Munz. "Comparison of different equations for calculation of J from one load-displacement curve for three-point bend specimens," International Journal of Fracture, vol 12, 1976, pp 780-784.

38. American Society for Testing and Materials. ASTM E 813-81: Standard test method for jic, A measure of fracture toughness, Annual Book of Standards, Philadelphia, PA, 1986.

39. J.R. Rice. "A path independent integral and the approximate analysis of strain concentration by notches and cracks," Transactions of the American Society of Mechanical Engineers, Journal of Applied Mechanics, vol 35, Series E, no. 2, Jun 1968, pp 379-386.

40. American Society of Mechanical Engineers. Special Technical Publication 410: Plane strain crack toughness testing of high strength metallic materials, by W.F. Brown and J.E. Srawley. New York, NY, 1966, pp 84-86.

41. J.E. Srawley. "Wide range stress intensity factor expressions for ASTM E-399 standard fracture toughness specimens," International Journal of Fracture, vol 12, 1976, pp 475-476.

42. American Society for Testing and Materials. ASTM E 399-83: Standard test method for plane strain fracture toughness of metallic materials," Annual Book of Standards, Philadelphia, PA, 1986.

43. C.G. Go, S.E. Swartz and K.K. Hu. "Stress intensity factors for single edge notch beam," Journal of Engineering Mechanics, American Society of Civil Engineers, vol 110, no. 6, Apr 1984, pp 629-633.

44. L. Nallathambi, B.L. Karihaloo and B.S. Heaton. "Effect of specimen and crack sizes, water/cement ratio and coarse aggregate texture upon fracture toughness of concrete," Magazine of Concrete Research, vol 36, no. 129, Dec 1984, pp 227-236.

45. Naval Civil Engineering Laboratory. Contract Report CR 86.006: Fracture mechanics: Applicability to cracking and fracture of concrete, by S.E. Swartz. Kansas State University, Feb 1986.

46. M. Wecharatana and S.P. Shah. "Predictions of nonlinear fracture process zone in concrete," American Society of Civil Engineers, Journal of Engineering Mechanics, vol 109, no. 5, Oct 1983, pp 1231-1246.

47. G.C. Sih. "Nonlinear response of concrete: Interaction of size, loading step and material property," Application of Fracture Mechanics to Cementitious Composites, (S.P. Shah, editor) NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 3-23.

48. G.C. Sih. "Mechanics of material damage in concrete," Fracture Mechanics of Concrete (A. Carpinteri and A.R. Ingraffea editors), Martinus Nijohff Publishers, Dordrecht, The Netherlands, 1984, pp 1-29.

49. S.P. Shah. "Dependence of concrete fracture toughness on specimen geometry and on composition," Fracture Mechanics of Concrete, (A. Carpinteri and A.R. Ingraffea, editors), Martinus Nijohff Publishers, Dordrecht, The Netherlands, 1984, pp 111-135.

50. A. Carpinteri. "Scale effects in fracture of plain and reinforced concrete structures," Fracture Mechanics of Concrete (G.C. Sih and A. DiTommaso, editors), Martinus Nijhoff Publishers, Dordrecht, The Netherlands, 1985, pp 95-140.

51. M. Wecharatana and S.P. Shah. "Slow crack growth in cement composites," American Society of Civil Engineers, Journal of the Structural Division, vol 108, no. ST6, Jun 1982, pp 1400-1413.

52. Z.P. Bazant. "Instability, ductility and size effect in strain-softening concrete," Journal of the Engineering Mechanics Divisions, American Society of Civil Engineers, vol 102, 1976, pp 331-344.

53. H.K. Hilsdorf and W. Brameshuber. "Size effects in the experimental determination of fracture mechanics parameters," Application of Fracture Mechanics to Cementitious Composites (S.P. Shah editor), NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 361-397.

54. K.L. Watson. "The estimation of fracture surface energy as a measure of the toughness of hardened cement paste," Cement and Concrete Research, vol 8, 1978, pp 651-656.

55. G.A. Cooper. "Optimization of the three-point bend test for fracture energy measurement," Journal of Materials Science, vol 12, 1977, pp 277-289.

56. F. Moavenzadeh and R. Kuguel. "Fracture of concrete," Journal of Materials, JMLSA, vol 4, no. 3, Sep 1969, pp 497-519.

57. J. Nakayama, H. Abe, and R.C. Bradt. "Crack stability in the work of fracture test: Refractory applications," Journal of the American Ceramic Society, vol 64, no. 11, Nov 1981, pp 671-675.

58. E.T. Brown, J.A. Hudson, and M.S. Marathe. Discussions of the paper: "Griffith fracture criterion and concrete," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, vol 98, no. EM5, Oct 1972, pp 1310-1312.

59. H.G. Tattersall and G. Tappin. "The work of fracture and its measurement in metals, ceramics and other materials," Journal of Materials Science, vol 1, 1966, pp 296-301.

60. C. Gurney and J. Hunt. "Quasi-static crack propagation," in Proceedings of the Royal Society of London, London, England, A 299, 1967, pp 508-524.

61. Z.P. Bazant. "Mechanics of fracture and progressive cracking in concrete structures," Fracture Mechanics of Concrete (G.C. Sih and A. DiTommaso, editors), Martinus Nijhoff Publishers, Dordrecht, The Netherlands, 1985, pp 1-94.

62. Lund Institute of Technology. Report TVBM-1006: Crack growth and development of fracture zones in plain concrete and similar materials, by P.E. Petersson. Division of Building Materials, S-221 00. Lund, Sweden, 1981.

63. N.M. Hawkins, A.N. Wyss and A.H. Mattock. "Fracture analysis of cracking in concrete beams," American Society of Civil Engineers, Journal of the Structural Division, vol 103, no. ST5, May 1977, pp 1015-1030.

64. A. Hillerborg. "Numerical methods to simulate softening and fracture of concrete," Fracture Mechanics of Concrete (G.C. Sih and A. DiTommaso, editors), Martinus Nijhoff Publishers, Dordrecht, The Netherlands, 1985, pp 141-170.

65. V.C. Li. "Fracture resistance parameters for cementitious materials and their experimental determinations," Application of Fracture Mechanics to Cementitious Composites (S.P. Shah, editor), NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 431-449.

66. H.W. Reinhardt. "Crack softening zone in plain concrete under static loading," Cement and Concrete Research, vol 15, 1985, pp 42-52.

67. J. Nakayama. "Bending methods for direct measurement of fracture energy of brittle materials," Japanese Journal of Applied Physics, vol 3, no. 7, 1964, pp 422-423.

68. Y.W. Mai. A discussion of the paper: "The estimation of fracture surface energy as a measure of the toughness of hardened cement paste," Cement and Concrete Research, vol 9, 1979, pp 537-539.

69. K.L. Watson. Reply to a discussion of: "The estimation of fracture surface energy as a measure of the toughness of hardened cement paste," Cement and Concrete Research, vol 9, 1979, pp 591-599.

70. C.G. Go. Fracture toughness techniques to predict crack growth and tensile failure in concrete, PhD dissertation, Kansas State University, Manhattan, Kansas, 1983, 109 p.

71. S.E. Swartz and S.T. Yap. "Evaluation of recently proposed recommendations for the determination of fracture parameters for concrete in bending," in Proceedings of the Eighth International Conference on Experimental Strain Analysis, Amsterdam, May 1986.

72. S.E. Swartz and C.G. Go. "Validity of compliance calibration to cracked concrete beams in bending," Experimental Mechanics, vol 24, no. 2, June 1984, pp 129-134.

73. S.E. Swartz and S.T. Yap. "Evaluation of proposed methods to determine fracture parameters for concrete in bending," Research Report 181, Kansas State University, Dordrecht, The Netherlands, Jun 1986.

74. Y.W. Mai. "Fracture measurements of cementitious composites," Application of Fracture Mechanics to Cementitious Composites (S.P. Shah, editor), NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 399-429.

75. Y.S. Jenq and S.P. Shah. "Nonlinear fracture parameters for cement based composites: Theory and experiments," Application of Fracture Mechanics to Cementitious Composite, (S.P. Shah, editor), NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 319-359.

76. P.F. Walsh. "Crack initiation in plain concrete," Magazine of Concrete Research, vol 28, no. 94, March 1976, pp 37-41.

77. B.I. Barr, E.B.D. Hasso and B.B. Sabir. "The effect of test specimen size on the fracture toughness of concrete," Cement and Concrete Research, vol 15, 1985, pp 833-841.

78. Y. Jenq and S.P. Shah. "Two parameter fracture model for concrete," Journal of Engineering Mechanics, American Society of Civil Engineers, vol 111, no. 10, Oct 1985, pp 1227-1261.

79. K. Visalvanitch and A.E. Naaman. "Fracture methods in cement composites," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, vol 107, no. EM5, Oct 1981, pp 1155-1171.

80. O.E. Gjorv, S.I. Sorensen and A. Arnesen. A reply to the discussion of the paper: "Notch sensitivity and fracture toughness of concrete," Cement and Concrete Research, vol 8, 1978, pp 389.

81. T. Belytschko, Z.P. Bazant, Y.W. Hyun and T.P Chang. "Strain-softening materials and finite-element solutions," Computers and Structures, vol 23, no. 2, 1986, pp 163-180.

82. Z.P. Bazant and L. Cedolin. "Blunt crack band propagation in finite element analysis," Journal of the Engineering Mechanics Division, American Society of Civil Engineering, vol 105, no. EM2, Apr 1979, pp 297-315.

83. P.C. Strange and A.H. Bryant. "Experimental tests on concrete fracture," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, vol 105, no. EM2, Apr 1979, pp 337-342.

84. S. Mindess. "The J-integral as a fracture criterion for fiber reinforced concrete," Cement and Concrete Research, vol 7, 1977, pp 731-742.

85. O.E. Gjorv, S.I. Sorensen and A. Arnesen. "Notch sensitivity and fracture toughness of concrete," Cement and Concrete Research, vol 7, 1977, pp 333-344.

86. S.P. Shah and F.J. McGarry. "Griffith fracture criterion and concrete," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, vol 97, no. EM6, Dec 1971, pp 1663-1676.

87. D.D. Higgins and J.E. Bailey. "Fracture measurements on cement paste," Journal of Materials Science, vol 11, 1976, pp 1995-2003.

88. P.C. Strange and A.H. Bryant. "The role of aggregate in the fracture of concrete," Journal of materials science, vol 14, 1979, pp 1863-1868.

89. A. Carpinteri. "Static and energetic fracture parameters of rocks and concrete," Materials and Structures, vol 14, no. 81, May-June 1981, pp 151-162.

90. Lund Institute of Technology. Report TVBM-1001: A fracture mechanics approach to failure analysis of concrete materials, by M.A. Modeer. Division of Building Materials, S-221 00. Lund, Sweden, 1979.

91. Lund Institute of Technology. Report TVBM-3021: Influence of beam size on concrete fracture energy determined according to a draft RILEM recommendation, by A. Hillerborg. Division of Building Materials, S-221 00. Lund, Sweden, 1985.

92. E.H. Andrews. "A generalized theory of fracture mechanics," Journal of Materials Science, vol 9, 1974, pp 887-894.

93. E.H. Andrews and E.W. Billington. "Generalized fracture mechanics, Part 2: Materials subject to general yielding," Journal of Materials Science, vol 11, 1976, pp 1354-1361.

94. E.H. Andrews and Y. Fukahori. "Generalized fracture mechanics, Part 3: Prediction of fracture energies in highly extensible solids," Journal of Materials Sciences, vol 12, 1977, pp 1307-1319.

95. American Society of Metals. Fracture dynamics, fracturing of metals, by G.R. Irwin. Cleveland, OH, 1948, p 152.

96. E. Orowan. Fundamentals of Brittle Behaviour in Metals, MIT Symposium on Fatigue and Fracture in Metals, Wiley, 1950, p 139.

97. D. Maugis. "Subcritical crack growth, surface energy, fracture toughness, stick-slip and embrittlement," Journal of Materials Science, vol 20, 1985, pp 3041-3093.

98. Physical Society of London. Report on Progress in physics, vol 12: Fracture strength of solids, by E. Orowan. London, England, 1969.

99. S. Mindess and S. Diamond. "The cracking and fracture of mortar," Materials and Structures, vol 15, no. 86, 1982, pp 107-112.

100. S. Mindess. "Rate of loading effects on the fracture of cementitious materials," Application of Fracture Mechanics to Cementitious Composites (S.P Shah, editor), NATO-ARW, Sep 4-7, 1984, Northwestern University, Dordrecht, The Netherlands, pp 617-636.

101. S. Mindess. "The effect of specimen size on the fracture energy of concrete," Cement and Concrete Research, vol 14, 1984, pp 431-436.

102. Lund Institute of Technology. Report TVBM-3011: Comments on the method of determining the fracture energy of concrete by means of three-point bend tests on notched beams, by P.E. Petersson. Division of Building Materials, S-221 00. Lund, Sweden, 1982.

103. R.L. Gillespie, C.G. Go and S.E. Swartz. "Vertical-displacement gage fixture for Jic testing", Experimental Techniques, vol 8, no. 5, 1984, pp 32-33.

104. Aluminum Company of America. Alcoa Aluminum Handbook, Alcoa Building, Pittsburgh, PA, 1962.

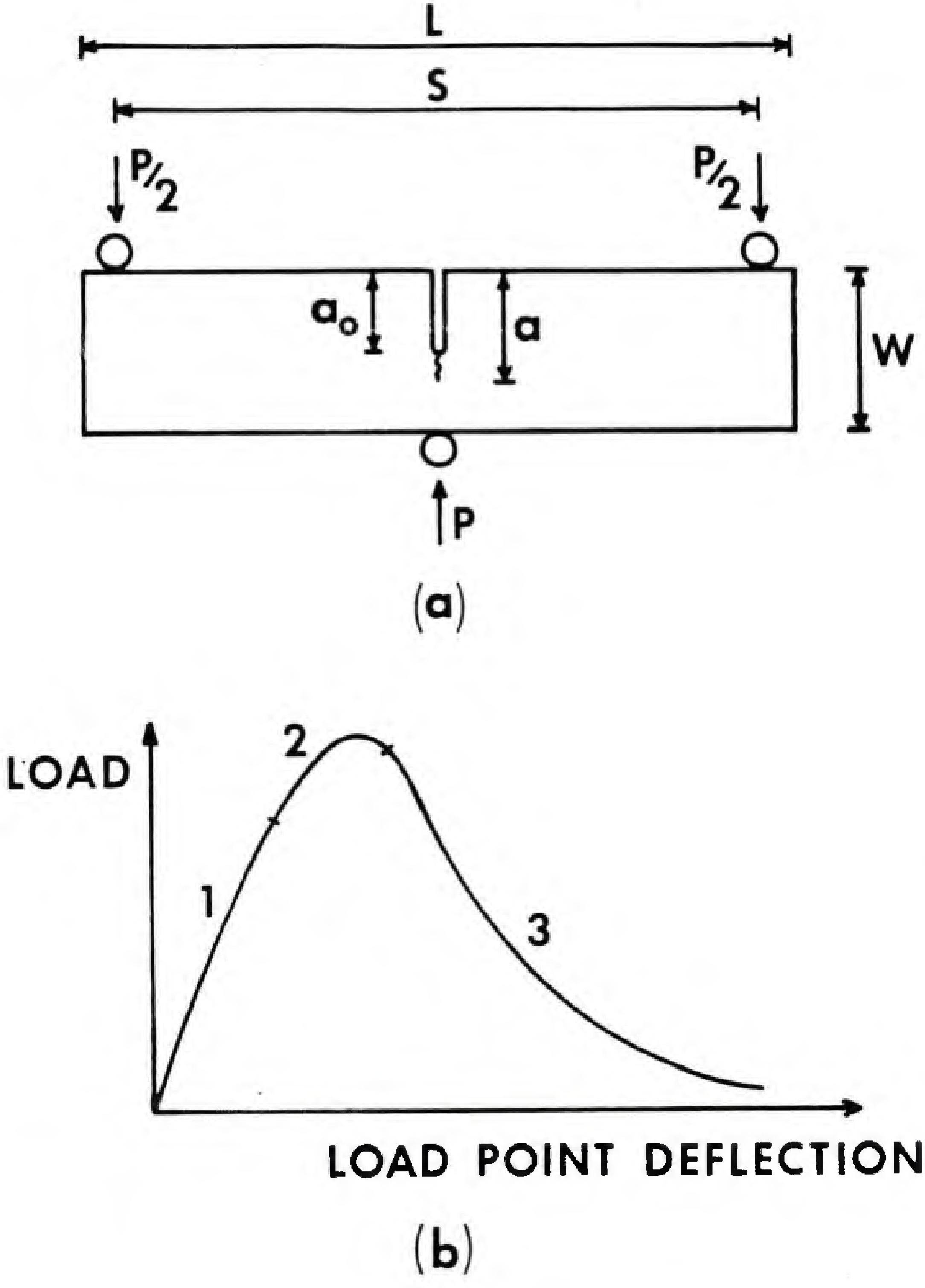

Figure 1. Three point bend test set up and load - load point deflection plot.

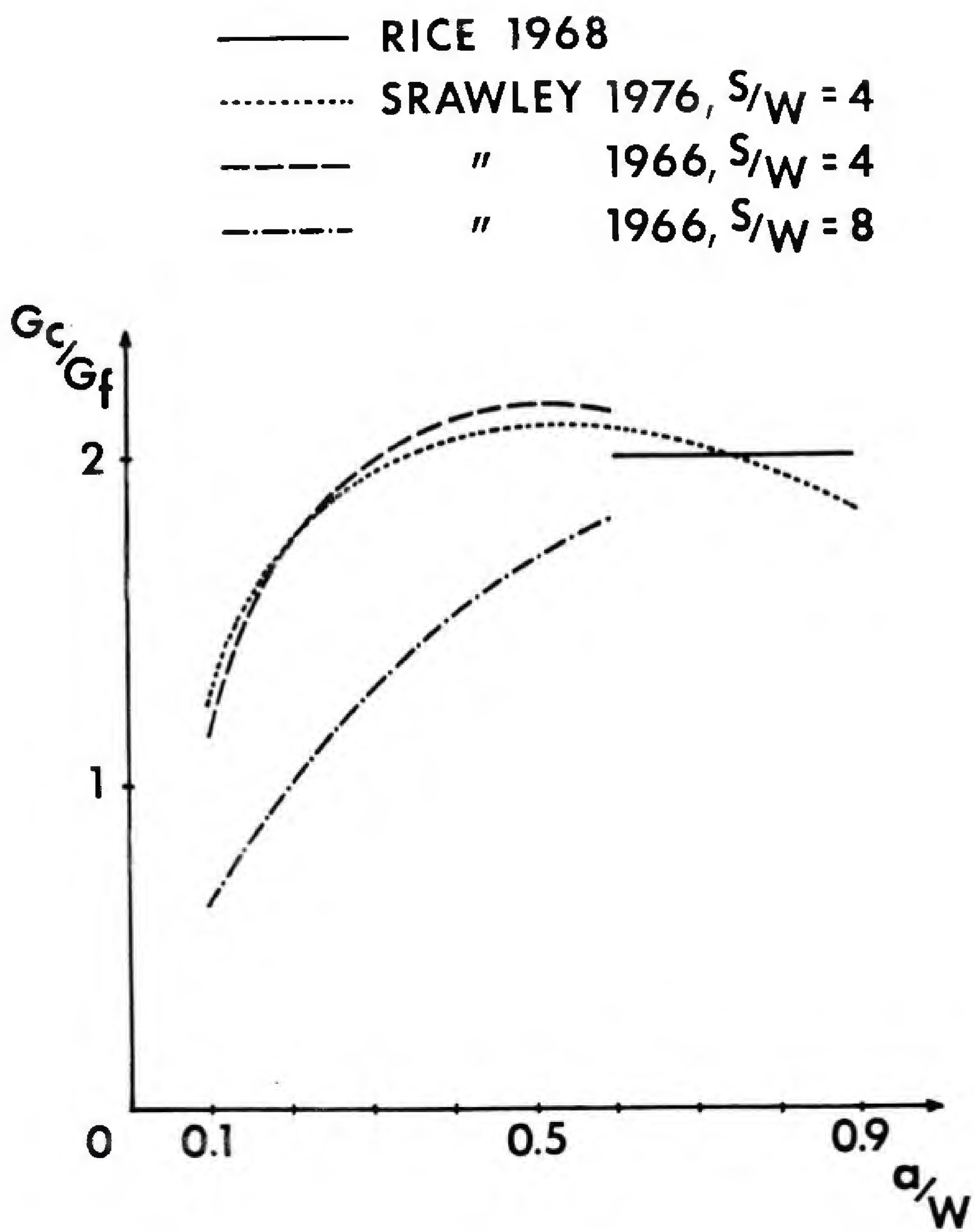

Figure 2. Ratio G_c/G_f for various crack depths.

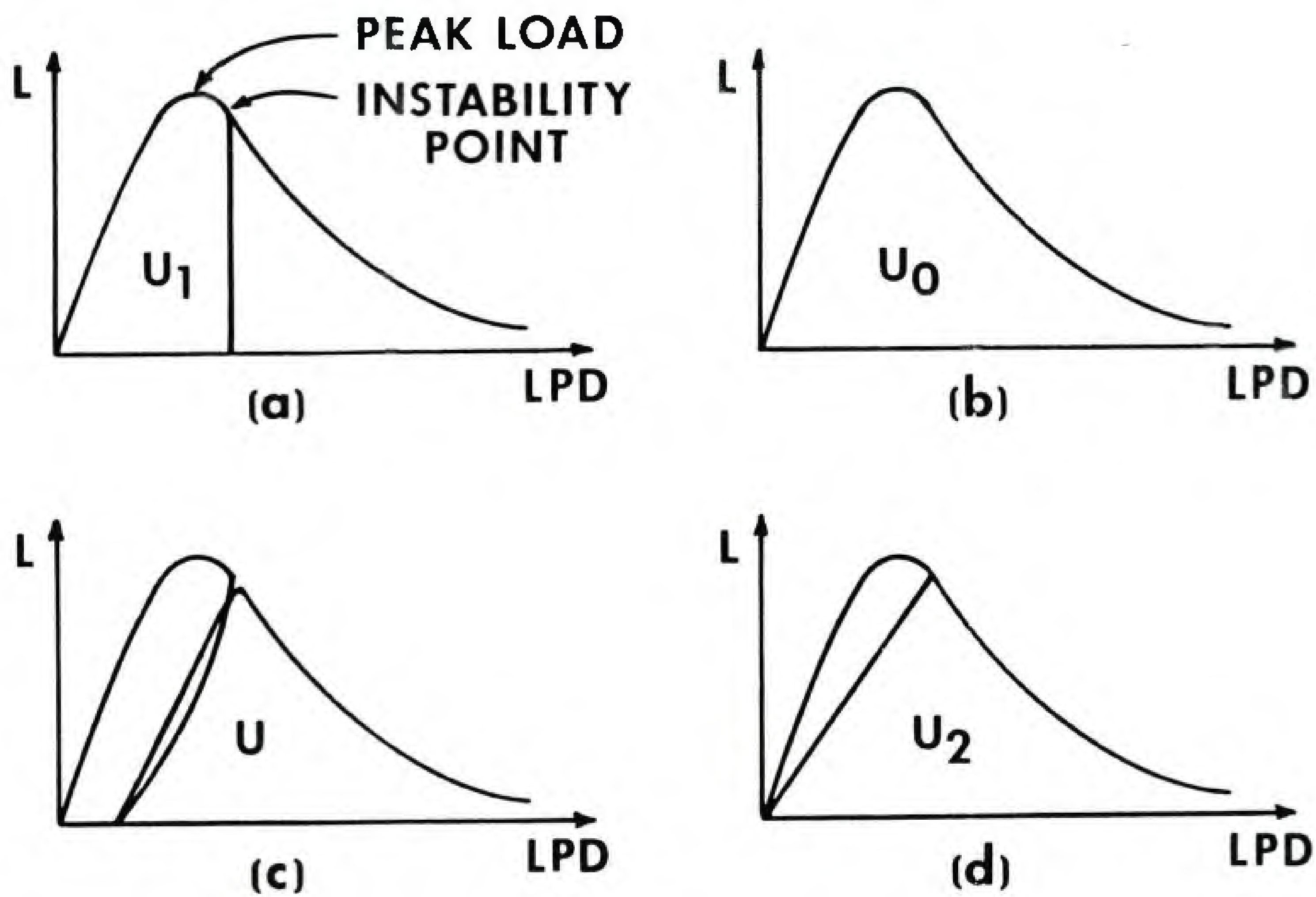

Figure 3. Energies from the load - load point deflection point.