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14. ABSTRACT
In compressive sensing, one basic issue is the robustness of signal recovery solutions in the presence of uncertainties. The main objective of this project is to analysis the robustness of compressive sensing solutions, and derive, through minimax optimization, solutions that are robust to uncertainties (or perturbations) in modeling and in measurements. Exact solutions of compressive sensing solutions to perturbations were obtained. Algorithms for sensitivity reduction in sparse signal recovery solutions we designed. Algorithms for obtaining robust compressive

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Final Report: Minimax Compressed Sensing Reconstruction

ABSTRACT

In compressive sensing, one basic issue is the robustness of signal recovery solutions in the presence of uncertainties. The main objective of this project is to analysis the robustness of compressive sensing solutions, and derive, through minimax optimization, solutions that are robust to uncertainties (or perturbations) in modeling and in measurements. Exact solutions of compressive sensing solutions to perturbations were obtained. Algorithms for sensitivity reduction in sparse signal recovery solutions we designed. Algorithms for obtaining robust compressive sensing solutions under the worst-case perturbations were obtained through the Alternating Direction Method of Multipliers. Finally, the optimality of Wiener filter was established under non-Gaussian distributions of signals.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
09/12/2014	3.00	Jin Tan, Danielle Carmon, Dror Baron. Signal Estimation With Additive Error Metrics in Compressed Sensing, IEEE Transactions on Information Theory, (01 2014): 0. doi: 10.1109/TIT.2013.2285214
09/12/2014	4.00	Dror Baron, Liyi Dai, Jin Tan. Wiener Filters in Gaussian Mixture Signal Estimation With <inline-formula> <tex-math notation="TeX">\(\ell _\infty \) </tex-math></inline-formula>-Norm Error, IEEE Transactions on Information Theory, (10 2014): 0. doi: 10.1109/TIT.2014.2345260
10/20/2015	10.00	Liyi Dai . Sensitivity Analysis of Compressive Sensing Solutions, Frontiers in Robotics and AI: Sensor Fusion and Machine Perception , (09 2015): 2. doi:

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Number of Papers published in peer-reviewed journals:

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09/30/2016	24.00	. Performance regions in compressed sensing from noisy measurements, 2013 47th Annual Conference on Information Sciences and Systems (CISS 2013). 20-MAR-13, Baltimore, MD. : ,
09/30/2016	23.00	. Performance of parallel two-pass MDL context tree algorithm, 2014 IEEE Global Conference on Signal and Information Processing (GlobalSIP). 03-DEC-14, Atlanta, GA, USA. : ,
10/20/2015	11.00	Frederick D. Gregory , Liyi Dai. Multisensory information processing for enhanced human-machine symbiosis, Human Interface and the Management of Information: Information and Knowledge Design (HIMI 2015). 02-AUG-15, . : ,

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Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received		Paper
09/12/2014	1.00	Jin Tan, Danielle Carmon, Dror Baron. Optimal estimation with arbitrary error metrics in compressed sensing, 2012 IEEE Statistical Signal Processing Workshop (SSP). 05-AUG-12, Ann Arbor, MI, USA. : ,
09/12/2014	2.00	Dror Baron, Junan Zhu, Marco F. Duarte. Complexity-adaptive universal signal estimation for compressed sensing, 2014 IEEE Statistical Signal Processing Workshop (SSP). 29-JUN-14, Gold Coast, Australia. : ,
09/12/2014	6.00	J. Tan, D. Baron, and L. Dai. Signal Estimation with Low Infinity-Norm Error by Minimizing the Mean p-Norm Error, Conf. Inf. Sciences Systems, Princeton. 19-MAR-14, . : ,
09/12/2014	7.00	N. Krishnan, D. Baron, and M. K. Mihcak. A Parallel Two-Pass MDL Context Tree Algorithm for Universal Source Coding, IEEE Int. Symp. Inf. Theory. 29-JUN-14, . : ,
10/20/2015	13.00	Xiao Bian, Hamid Krim, Alex Bronstein, Liyi Dai. Sparse null space basis pursuit and analysis dictionary learning for high-dimensional data analysis, ICASSP 2015 - 2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). 19-APR-15, South Brisbane, Queensland, Australia. : ,
10/20/2015	12.00	Liyi Dai . Compressive sensing solutions through minimax optimization, SPIE Sensing Technology + Applications. 24-APR-15, Baltimore, Maryland, United States. : ,

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(d) Manuscripts

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09/30/2016	5.00	Y. Ma, D. Baron, and D. Needell . Two-Part Reconstruction with Noisy-Sudocodes, IEEE Transactions on Signal Processing (06 2014)
09/30/2016	8.00	J. Tan, Y. Ma, and D. Baron. Compressive Imaging via Approximate Message Passing with Image Denoising, IEEE TRANSACTIONS ON Information Theory (05 2014)
09/30/2016	9.00	N. Krishnan and D. Baron. A Universal Parallel Two-Pass MDL Context Tree Compression Algorithm, IEEE Trans. Signal Process., (06 2014)

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Number of Manuscripts:

Books

Received Book

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Patents Submitted

Provisional Patent: IMAGE MOSAICKING" for which we filed United States Provisional Patent Application No. 62/240748 October 13, 2015 (Atty. File No. 030871-9052-US00); (hereinafter the "U.S. provisional patent application"); S.H. Clouse, X. Bian, H. Krim, A. Gentimis.

Patents Awarded

Awards

Liyi Dai, Fellow of the IEEE, 2014
Liyi Dai, Adjunct Professor at NCSU, 2016

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Xiao Bian	0.30	
Amrutha Nadarjan	0.08	
Shahin Mahdizadehaghdam	0.25	
FTE Equivalent:	**0.63**	
Total Number:	**3**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
K Krim	0.16	
FTE Equivalent:	**0.16**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

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Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
Xiao Bian	
Total Number:	**1**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

Final Progress Report: Minimax Compressed Sensing Reconstruction

Dror Baron – North Carolina State University

1 Introduction

This report summarizes progress made during the project "Minimax Compressed Sensing Reconstruction." Below we state the problem in Section 2, and then summarize the important results in Section 3.

2 Statement of Problem

Compressed sensing (CS) [1, 2] has sparked a tremendous amount of research activity in recent years, because it performs signal acquisition and processing using far fewer samples than required by the Nyquist rate. Breakthroughs in CS have the potential to greatly reduce the sampling rates in numerous signal processing applications such as cameras [3], medical scanners, fast analog to digital converters [4, 5], and high speed radar [6].

The intellectual foundations underlying CS rely on the ubiquitous compressibility of signals: in an appropriate basis, most of the information contained in a signal often resides in just a few large coefficients. Traditional sensing and processing first acquires the entire data, only to later throw away most coefficients and retain the few significant ones [7]. Interestingly, the information contained in the few large coefficients can be captured by a small number of random linear projections [8]. The ground-breaking work in CS [1, 2, 6] has proved for a variety of settings that the signal can then be reconstructed in a computationally feasible manner from these random projections.

System model: To be precise and concrete, consider the linear system,

$$\mathbf{w} = \mathbf{\Phi}\mathbf{x}, \tag{1}$$

where the input $\mathbf{x} \in \mathbb{R}^N$ is independent and identically distributed (i.i.d.), and the random linear mixing matrix $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is known, and typically $M < N$. The measurements $\mathbf{w} \in \mathbb{R}^M$ are passed through a bank of separable channels characterized by conditional distributions,

$$f_{\mathbf{Y}|\mathbf{W}}(\mathbf{y}|\mathbf{w}) = \prod_{i=1}^{M} f_{Y|W}(y_i|w_i). \tag{2}$$

Note that the channels are general and are not restricted to additive noise such as Gaussian. We observe the channel output $\mathbf{y}$, and want to estimate the original input signal $\mathbf{x}$ from $\mathbf{y}$ and $\mathbf{\Phi}$.

The quality of the signal reconstruction is often characterized by some error metric that quantifies the distance between the estimated and the original signals. For a signal $\mathbf{x}$ and its estimate $\widehat{\mathbf{x}}$, the error between them is the summation over the component-wise errors,

$$D(\widehat{\mathbf{x}}, \mathbf{x}) = \sum_{i=1}^{N} d(\widehat{x}_i, x_i). \tag{3}$$

For example, if the metric is absolute error, then $d(\widehat{x}_i, x_i) = |\widehat{x}_i - x_i|$; if the metric is squared error, then $d(\widehat{x}_i, x_i) = (\widehat{x}_i - x_i)^2$.

Commonly used error metrics: Squared error is one of the most popular error metrics in various problems, owing to many of its mathematical advantages; for example, minimum mean squared error (MMSE) estimation provides both variance and bias information about an estimator [9], and in the Gaussian case it is linear and thus often easy to implement [10]. However, there are applications where MMSE estimation is inappropriate, for example because it is sensitive to outliers [11, 12]. Therefore, alternative error metrics, such as mean absolute error (median) or Hamming distance can be used instead.

Mean-square optimal analysis and algorithms were introduced in [13, 14, 15, 16, 17] to estimate the original signal from Gaussian-noise corrupted measurements; in [18, 19, 20], further discussions were made given the circumstances where the output channel was arbitrary, while, again, the MMSE estimator was put forth.

Support recovery error is another metric of great importance, for example it relates to properties of the measurement matrices [21]. The authors of [22, 23, 21] discussed the support error rate when recovering a sparse signal from its noisy measurements; support-related performance metrics were applied in the derivations of theoretical limits on the sampling rate for signal recovery [24]. The readers may notice that previous work only paid attention to limited types of error metrics. What if absolute error, cubic error, or other non-standard metrics are required in a certain application?

The main problem addressed in this program was to see how to reduce the worst-case error in compressed sensing reconstruction problems. This minimax-style approach can be useful when one cares little about small errors but is very concerned by outlier errors.

3 Summary of Important Results

Tan and several coauthors [25, 26, 27, 28] provide several contributions related to minimizing for unconventional error metrics in CS reconstruction. First, an algorithm that minimizes the expected error in CS reconstruction was used for a general purpose additive error formulation. The main idea is that the output of relaxed belief propagation (relaxed BP) [29, 20] can be shown to correspond to the original input signal w corrupted by additive white Gaussian noise (AWGN), and we apply an appropriate denoising algorithm that minimizes the expected additive error. Second, we showed that applying a Wiener filter to the output of relaxed BP provides asymptotically optimal minimax performance. However, for finite length problems this approach may be sub-optimal, and a heuristic approach featuring an optimization of the mean ℓ_p error (with p gradually increasing as a function of the problem size) yields encouraging numerical results.

Secondary results: The project also partly funded the PI's work on several other indirectly related research projects. The first involved an image reconstruction work [30, 31]. The second features a two-part CS reconstruction algorithm that offers a trade-off between speed and precision and reconstruction [32]. The third involved fast parallel algorithms for data compression [33, 34, 35]. The fourth involves universal algorithms for signal recovery [36, 37], which estimates the input statistics on the fly from the actual noisy measurements while simultaneously recovering the input. The fifth involves an analysis of regions where the best-possible minimum mean square error (MMSE) of CS systems behaves differently [38]. Finally, the sixth related work involves an empirical Bayes denoising algorithm that automatically tunes for unknown parameters within the approximate message passing framework for solving CS reconstruction problems [39].

Yet another benefit of the project was the training of doctoral students. Ms. Jin Tan was completed supported by the project for three semesters, and graduated in September 2015. The project also funded part of the PI's summer salary, which indirectly contributed to the doctoral training of Mr. Nikhil Krishnan, Ms. Yanting Ma, and Mr. Junan Zhu.

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