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Effects of Singular Dispersion Relation on Amplification in Electromagnetic Periodic Structures

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14. ABSTRACT

The goal of the project was to develop concepts and theory, and to indicate possible benefits in modern technology, associated to singular dispersion relations for electromagnetic (EM) fields that either evolve in time in resonators or propagate in waveguides, focusing on spectral singularities and exploring the effect of gain. The UC Irvine research team has linked this fundamental research to various important applications from microwave to optical frequencies, like oscillators, antennas, amplifiers, lasers, sensors, etc. The spectral singularities investigated under this contract are referred to as exceptional points of degeneracy (EPDs) since they are all about a degeneracy point in a parameter space describing the spectral dispersion of electromagnetic fields either evolving in time or propagating in space.

The UCI group has classified EM structures that develop EPDs of various orders into four categories, in relation to gain and losses and space/time invariance or periodicity. The UCI group provided different examples of systems that lie under each category.

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I. Introduction

This report presents the main accomplishments of the research group at UC Irvine (UCI) during the three years under the FA9550- 15-1-0280 grant managed by Dr. Arje Nachman. The work has been conducted by the Principal Investigator Prof. Filippo Capolino, from the Department of Electrical Engineering and Comp. Sci., at UCI, with his PhD students and some of the topics have been developed in collaboration with Prof. Alex Figotin, Department of Mathematics. The UCI group has worked on different aspects of the project ranging from advancing theory to proposing novel principles of operations and possible applications. The accomplishments made during the three years of the FA9550- 15-1-0280 project are here outlined. More details can be found in the various publications referenced under each topic. A list of publications from the UCI group (refs. [1-18]) as a result of this research topic as well as conference talks (refs. [C1-C30]) and seminars given during the project period are included in the report.

II. Summary of objectives and achievements

The goal of the project was to develop concepts, associated theory, and indicate possible benefits in modern technology, associated to singular dispersion relations for electromagnetic (EM) fields that evolve in time or propagate in space, focusing on spectral singularities and exploring the effect of gain. Importantly, the UCI group has linked this fundamental research to various important applications.

In the PI's point of view, this project has led to fine theoretical and conceptual advancements with clear indication of how they can be used in certain applications like oscillators, antennas, amplifiers, lasers, sensors, etc. In the following, the details will be given of how all the novel principles developed under this project could be used for boosting the performance of various devices and active electromagnetic systems, from microwave to optical frequencies. The spectral singularities investigated under this contract are referred to as exceptional point of degeneracy (EPDs) since they are all about a degeneracy point in a parameter space describing the spectral dispersion of electromagnetic fields either evolving in time or propagating in space.

The UCI group has provided a general picture of the different EM structures that can exhibit EPDs, of various orders, in relation to gain and losses and space/time periodicity in a guiding system. The UCI group has classified EM structures that develop EPDs into four categories and provided different examples of EM systems that lie under each category. The case of PT-symmetry (i.e., Parity-Time symmetry, a property under which non-Hermitian systems develop EPDs) in photonics, that has been a hot topic of research in the last years, is a particular case under the big picture (Fig. 1) provided by the UCI group.

The UCI group has discovered various new electromagnetic waveguide structures that support EPDs of various orders. Moreover, the team has carried out for the first time a direct experimental observation of the degenerate band edge (DBE), that is a 4th order EPD in circular metallic waveguides and transmission line microwaves circuits. Also, the team has explored the characteristics of EM systems when operating at or in proximity of an EPD, defining the concept of hyperdistance, i.e., how "close" a system is to an EPD. Recently, the group has investigated an alternative view of EPDs associated to branch points and bifurcation theory.

III. Summary of possible applications and impact

The new concepts described in the above section lead to possible advantages in lasers, following the paper "Degenerate band edge laser," where The UCI group show low-threshold single-mode laser regimes based on the DBE principle. Furthermore, the team has proposed a possible new approach for designing array antenna oscillators based on EPD. The team is currently working on demonstrating superior performance in EPD oscillators in terms of phase noise and power efficiency. The UCI group used some of the concepts described in the above section that lead to advancing new schemes for high power microwave generation using electron beam vacuum technology, though this was the main topic of another grant received from the AFOSR.

The UCI group has been observing that EPDs like the DBE can be very important for conceiving new oscillator regimes, operating at any frequency range, from microwave to optical frequencies.

The EPD regime based on the "gain and loss balance" condition is important to conceive devices that generate *very high power*. This regime is different from the DBE, since the DBE is an EPD that exists in absence of losses and gain. However, controlling EPDs with the simultaneous presence of loss and gain is extremely important for achieving high power devices, since one can benefit from having an EPD and, at the same time, gain and losses that indeed the latter may represent power extraction. Therefore, in this "gain and loss balance" regime, the UCI group suggests that losses are indeed "radiation losses", i.e., they represent power extraction from the system. In principle, there is no theoretical limit to the amount of gain involved in the EM system and in the amount of power extracted by a system, possibly leading to new highly power efficient and high radiation intensity devices. These last aspects have not been demonstrated yet, but the work done in these three years paved the way for these important developments.

For brevity, the report summarizes the various main achievements with the details contained in the journal papers listed at the end of this report.

IV. Accomplishments and Detailed topics

We first list the principal accomplishment of the UCI team. For the sake of brevity, each summarized accomplishment points to a specific publication and reports a representative result, skipping the theoretical details altogether.

1) **General picture of different electromagnetic systems that exhibit EPDs (uniform/periodic in space/time)**

2) **Hyperdistance in four-dimensional complex vector space for identifying "vicinity" to a fourth order EPD**

3) **Development of EM structures supporting EPDs**
   a) **EPDs induced in *spatially uniform* structures**
      i) Theory of EPDs in uniform coupled waveguides with gain and loss balance
      ii) Exceptional points of degeneracy and branch points for transmission-line problems – linear Algebra and bifurcation theory perspectives
      iii) Realization of the DBE (4^{th} order EPD) in *uniform* coupled lossless/gainless waveguides (the first time a 4^{th} order is shown in a uniform structure)
   b) **EPDs induced in temporally uniform resonators:** Third order EPD in coupled resonators
   c) **EPDs in periodic waveguides**
      i) Fourth order EPD in periodically-coupled waveguides and the interplay of gain and radiation loss
      ii) Fourth order EPD condition in multilayer waveguide structure with grating
      iii) Radio frequency double ladder oscillators with DBE
      iv) Experimental demonstration of DBE in metallic waveguides
      v) Various orders of EPDs at optical frequency in coupled resonator optical waveguides (CROW)
      vi) Correlation between EPDs and PT-symmetry in photonic coupled chains of scatterers
      vii) Sixth order modal degeneracy: triple ladder lumped circuit and microstrip implementation
      viii) Sixth order EPD at optical frequencies based on CROW
   d) **EPDs in EM systems, periodic in time:** EPDs induced in linear time periodic resonator

4) **EPDs properties and proposed applications**
   a) **High sensitivity**
      i) Lossless/gainless linear time periodic LC sensor
      ii) Highly-sensitive directive leaky wave antenna
      iii) DBE optical switch with very low switching threshold voltage
   b) **High quality factor**
      i) DBE laser: low-threshold single-mode laser
      ii) Low-threshold grid array oscillators with EPD
      iii) Pulse generation devices based on modal degeneracy
   c) **Other applications associated to EPDs**
      i) Gain enhancement near stationary inflection points 3^{rd} order EPD (SIP)
      ii) Space-time variance and induced EPDs for non-reciprocal devices

1) General picture of electromagnetic systems that exhibit EPDs

The UCI group has formulated a general picture of possible EM structures that can exhibit EPDs and they have classified such structures into mainly four categories as shown in Fig. 1. These categories are: a) coupled mode structures that are spatially uniform (not periodic) but have wave propagation, e.g., two uniform coupled waveguides; b) coupled mode structures that are uniform in time, e.g., two coupled resonators; c) coupled mode structures that are spatially periodic, e.g., photonic crystals or periodic stack of anisotropic layers; and finally d) structures with time-periodic elements and such structures are completely new and the UCI group is the first to show EPDs in such time periodic varying structures even in a very simple LC tank.

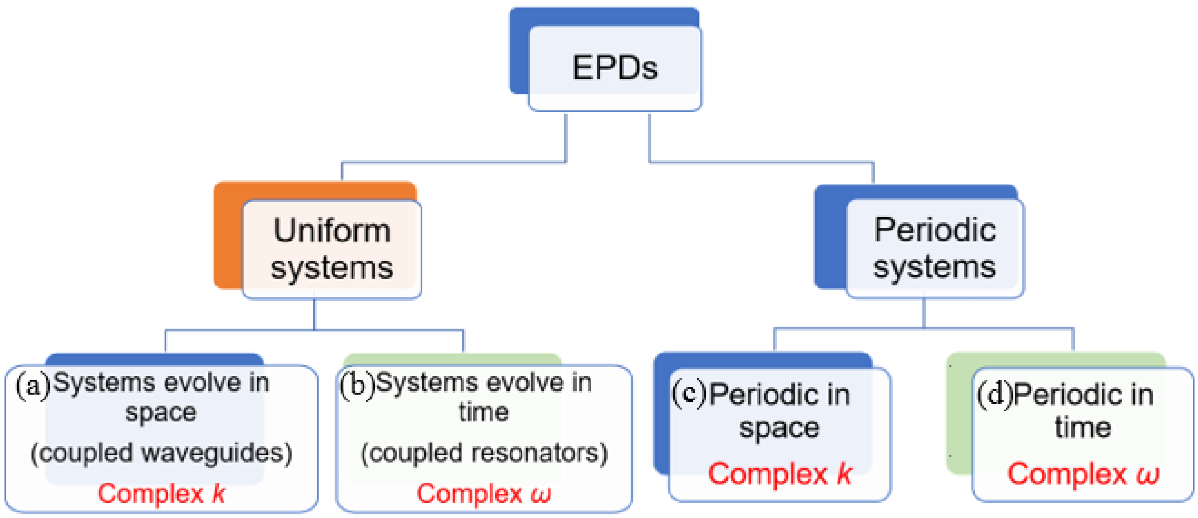


Fig. 1. Classification of the different structure-topologies that can exhibit exceptional points of degeneracy (EPDs). (a,b) Uniform electromagnetic systems means two kind of systems: those invariant in space (i.e., waveguides) and systems that are invariant in time (resonators). Box (c) is about periodic waveguides. Box (d) is about EM systems that are periodically time modulated. During this project new EM structures expressing EPDs under each of these cases have been found.

In all cases the EM system is described as the evolution of a multidimensional state vector $\mathbf{\Psi}(\zeta)$ as

$$\frac{\partial}{\partial \zeta}\mathbf{\Psi}(\zeta) = -j\underline{\mathbf{M}}(\zeta)\mathbf{\Psi}(\zeta) \qquad (1)$$

where ζ can be either a space coordinate (e.g., $\zeta \equiv z$), representing cases (a) and (c) in Fig. 1, or time (i.e., $\zeta \equiv t$), representing cases (b) and (d). Furthermore, the system matrix $\underline{\mathbf{M}}(\zeta)$ can be either constant in ζ, representing the evolution in "uniform" EM systems, or periodic in ζ, representing the evolution in periodic EM systems. The UCI group has done a lot of work in periodic EM systems under this grant, inspired by the pioneering work by Figotin and Vitebskiy in the early and mid 2000's. In analogy to the existence of EPDs in periodic waveguides, the UCI group has proposed and demonstrated the existence of EPDs that is directly induced in time periodic EM systems (Case d).

EPDs can be found in all the cases described in Fig. 1, and this happens when two or more eigenmodes of the EM system coalesce, i.e., two or more eigenmodes (in eigenvectors and eigenvalues) become identical.

In uniform EM systems (i.e., invariant in z or invariant in time) this happens when the matrix $\underline{\mathbf{M}}$ in Eq. (1) is similar to a matrix that contains a non-trivial Jordan block. This happens when $\underline{\mathbf{M}}$ is non-Hermitian.

Periodic systems are more complicated (either periodic in z, like a periodic waveguide, or periodic in time, like in periodically time varying systems) and one need to resort to the concept of transfer matrix, as shown in [1], [3-15], [18]. If ζ_p is the period, then a periodic system is described by the transfer matrix $\underline{\mathbf{T}}_{\mathrm{U}}$ relative to a "unit periodic cell" according to the equation

$$\mathbf{\Psi}\left(\zeta+\zeta_p\right)=\underline{\mathbf{T}}_{\mathrm{U}}\,\mathbf{\Psi}\left(\zeta\right) \tag{2}$$

In periodic EM systems that express EPDs, the transfer matrix $\underline{\mathbf{T}}_{\mathrm{U}}$ is similar to a matrix that contains a non-trivial Jordan block, and this happens also in the case of absence of losses and gain.

The above general description is considered the common initial mathematical setting for all EM systems that express EPDs and relate them to the existence of a Jordan block describing the evolution in time or propagation in space. The details of how to solve these systems, and which EM systems can express EPDs is given below and especially in the papers authored by the UCI group listed at the end of this report.

In the following, a detailed summary of the accomplishments done by the UCI group is listed and classified according to the previous categories. However before listing the accomplishments another general concept is introduced. This concept was developed by the UCI team that is useful to address the presence or not of an EPD in practical terms. Indeed, an EPD is a mathematical point, but when can practical systems express EM properties proper of EPDs? This is answered by resorting to the concept of hyperdistance, i.e., the distance between eigenvectors in a multidimensional space, indicating when these become almost parallel.

2) Hyperdistance in four-dimensional complex vector space for identifying "vicinity" to a fourth order EPD

The UCI group developed a figure of merit to assess the occurrence of fourth-order EPD in the presence of tolerances and losses [18]. They called this figure of merit the hyperdistance which is useful to assess the quality of such EPD subject to any kind of perturbation, like losses, frequency detuning, or tolerances of parameters. A perfect degeneracy condition like the DBE corresponding to a lossless structure does not exist in practice when losses are present but can be met in an approximate way and still retains the main features of the four coalescing eigenvectors. Since a 4th order EPD occurs when four eigenvectors coalesce into a degenerate eigenvector, the UCI group has developed the concept of hyperdistance $D_H(\omega)$ between the four eigenvectors of the transfer matrix of one-unit cell $\underline{\mathbf{T}}_{\mathrm{U}}$ to determine the closeness to an EPD. Indeed at an EPD the eigenvectors become parallel and the distance vanishes. Various choices could be made for its definition, and here the UCI group defined the hyperdistance that represents a figure of merit as

$$D_H = \frac{1}{6}\sum_{\substack{m=1,n=1\\ m\neq n}}^{4} \sin(\theta_{mn}), \quad \cos(\theta_{mn}) = \frac{\mathrm{Re}(\mathbf{\Psi}_m \cdot \mathbf{\Psi}_n)}{\|\mathbf{\Psi}_m\|\|\mathbf{\Psi}_n\|} \tag{3}$$

with θ_{mn} representing the angle between two vectors $\mathbf{\Psi}_m$ and $\mathbf{\Psi}_n$ in a four-dimensional complex vector space with norms $\|\mathbf{\Psi}_m\|$ and $\|\mathbf{\Psi}_n\|$. Angles are defined via the inner product $(\mathbf{\Psi}_m \cdot \mathbf{\Psi}_n) = \mathbf{\Psi}_m^\dagger \mathbf{\Psi}_n$, where the dagger symbol † denotes the complex-conjugate transpose operation, and D_H is defined to be always positive. This FOM yields a hyperdistance ideally equal to zero when four eigenvectors coalesce, i.e., when the system experiences a fourth order EPD. Mathematically this is described by the transfer matrix of the unit cell $\underline{\mathbf{T}}_{\mathrm{U}}$ becoming similar to a 4×4 Jordan Block (for more details see [18]). The concept of hyperdistance can be generalized to any EPD order.

3) Development of EM structures supporting EPD

a) EPDs induced in *spatially uniform* structures

i. Theory of EPDs in uniform coupled waveguides with gain and loss balance

The UCI group has demonstrated a transmission line theory of guided waves in coupled *uniform* waveguides with EPDs [7] and how it is related to PT-symmetry. In optics, PT-Symmetry requires that the system's refractive index obeys

$$n(x) = n^*(-x) \tag{4}$$

where x is a coordinate in the system, and * denotes complex conjugation, that in the case of two parallel waveguides means that one waveguide is lossy and the other has symmetric gain. The team proposed two types of degeneracies that may occur in coupled waveguides, namely, a second-order EPD in balanced gain–loss uniform waveguides, first satisfying PT-symmetry (i.e., gain and loss is distributed in a symmetric fashion) as shown in Fig. 2(a) and Fig. 2(c) and also, importantly, a gain and loss balance condition without gain and loss symmetry shown in Fig. 2(b) and Fig. 2(d). The UCI group demonstrated that *PT-symmetry is not a necessary condition for realizing EPDs* by showing that *EPD is also obtained with asymmetric distributions of gain and loss* in uniform CTLs. In addition, The UCI group has shown an operational principle for *leaky-wave antennas (LWA) based on EPD*s, with gain and balance condition, with the capability of beam and directivity control as well as enhanced sensitivity as it will be shown later in the application part.

The EPD associated to gain and loss balance condition has many applications in different operating regimes such as: (i) high-power traveling wave tubes, promising high efficiency when electron beams act as source of gain while distributed loads represent mechanism of losses or radiation "losses", (ii) low-threshold microwave and terahertz sources, antennas, and RF circuits, including radiating and synchronized array oscillators.

ii. Exceptional points of degeneracy and branch points for transmission-line problems – linear algebra and bifurcation theory perspectives

The UCI group in collaboration with Prof. George Hanson, University of Wisconsin-Milwaukee and Prof. Alexander B. Yakovlev, University of Mississippi, has provided a new angle to investigate exceptional points of degeneracy (EPD) relating the current linear-algebra point of view to bifurcation theory [16], where the team concluded that the eigenvalues at EPD can be

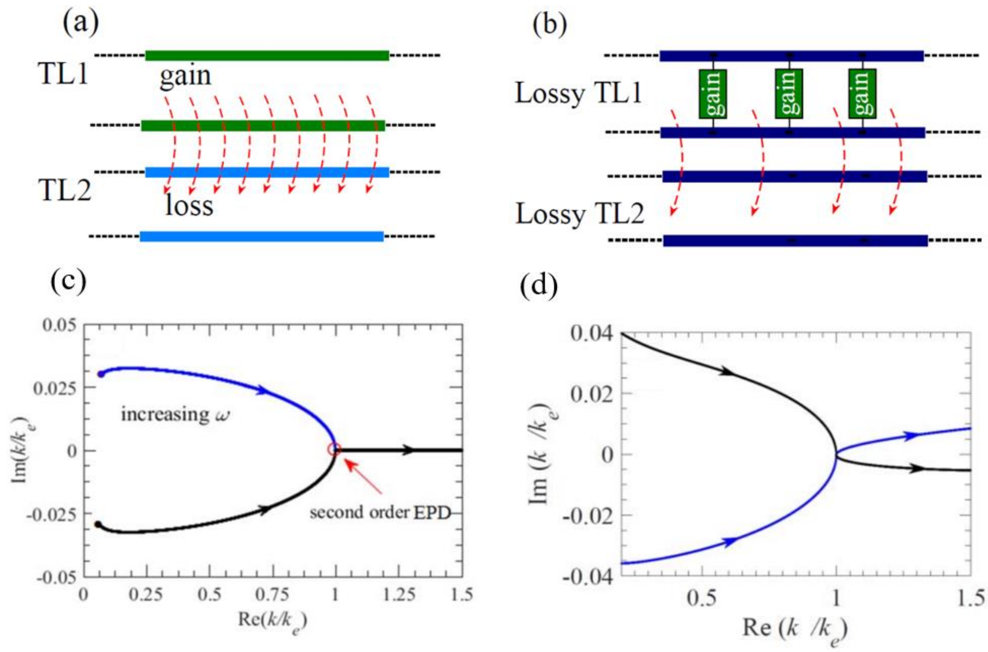


Fig.2. Configurations of uniform CTLs that may exhibit EPD. (a) Uniform CTL with distributed and symmetric gain and loss (PT-symmetric). (b) Uniform CTL with distributed loss in both TLs and gain in only one (non PT-symmetric). (c) and (d) Trajectory of complex k in the Re(k)-Im(k) plane varying as a function of ω near a second-order EPD for the two configurations in (a) and (b), respectively (see [7]).

expressed in term of the degenerate eigenvalue, for example the eigenvalues in the vicinity of the 2nd order EPD expressed as

$$k(\omega) = k_e \pm \alpha\sqrt{\omega - \omega_e} \ . \tag{5}$$

The above equation shows the occurrence of the branch-point singularity in the complex-frequency plane which is basically resulting from the square-root function.

The UCI group has shown that EPDs are singular points of the dispersion function associated with the fold bifurcation connecting multiple branches of dispersion spectra. This provides an important connection of various modal interaction phenomena known in guided-wave structures with recent interesting effects observed in quantum mechanics, photonics, metamaterials, and EM systems described in terms of the algebraic EPD formalism. Since bifurcation theory involves only eigenvalues, the UCI group has also established the connection to the linear-algebra point of view by casting the system eigenvectors in terms of eigenvalues, analytically showing that the coalescence of two eigenvalues results automatically in the coalescence of the two respective eigenvectors. Therefore, for the studied two-coupled transmission-line problem shown in Fig. 3, the eigenvalue degeneracy explicitly implies an EPD. The UCI group has explained the concept of EPDs from a point of view based on singular fold points from bifurcation theory. This is very important because it allows to derive many conclusions about the EPD just based on the properties of branch points (BPs) in the complex-frequency plane. Based on the concept of singular fold points from bifurcation theory several analytics formulas can be derived as explained in [16].

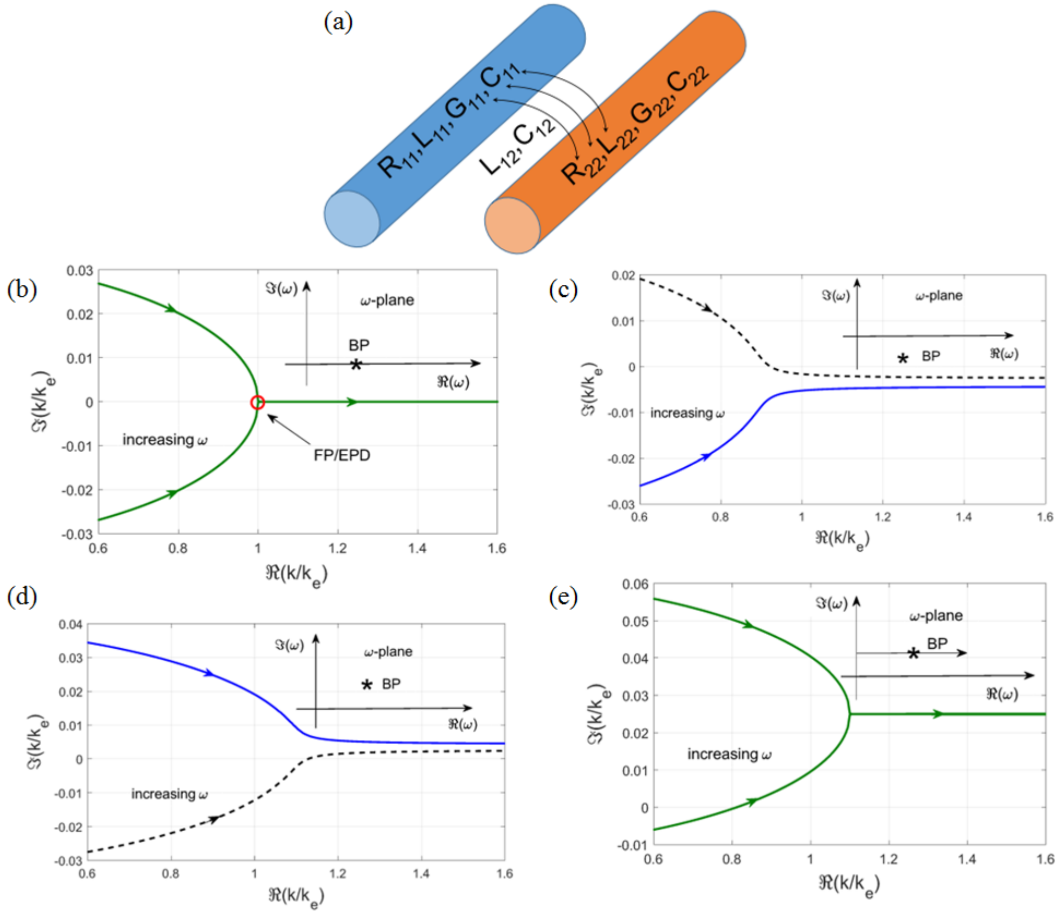


Fig. 3. (a) Two coupled transmission lines with mutual capacitive and inductive coupling, invariant along-z. Dispersion behavior near an EPD for coupled transmission lines in (a) with (b) $R_{11} = -R_{22} = -73.172$ ohms (PT-symmetric case), as ω varies from 0.5 ω_e to 1.5 ω_e along the real-ω axis. (c) Same as (b) but for $R_{11} = -1.2\ R_{22}$, wherethe EPD lies above the real-frequency axis. (d) Same as (b) but for $R_{11} = -0.8\ R_{22}$, such that the EPD is below the real-frequency axis. (e) Same as (a) but for Re(ω) varying from 0.5 ω_e to 1.5 ω_e at a constant value Im(ω) = Im(ω_s) = (0.022 ω_e). In all cases the pair (k_e; ω_e) are the values at PT-symmetry, (k_e; ω_e) = (28.649 m^{-1}; 2π10^9 s^{-1}), R_{22} = 73.172 ohms, and the star indicates the BP/EPD.

iii. Realization of the DBE (4th order EPD) in uniform coupled *lossless/gainless* waveguides

The team proposed for the first time the general conditions to obtain EPDs with any order in *uniform* coupled waveguides without the need of *loss and gain* [C26]. Considering two uniform waveguides, as shown in Fig. 4(a), where each waveguide (when uncoupled) supports either forward propagating mode, backward propagating mode or evanescent mode along both the positive and negative z-direction. This structure can be modeled by a generic a per-unit-length distributed parameters as shown in Fig. 4(b). They UCI group has showed the novel and important fact that EPDs may be supported in uniform lossless/gainless CTLs when there is coupling between either: (i) propagating mode and evanescent mode, (ii) forward and backward propagating modes, or (iii) two evanescent modes. The team have particularly concluded that uniform CTL can exhibit DBE when its impedance and admittance matrices satisfy

$$\mathrm{Trace}(\mathbf{ZY}) = \mathrm{Det}(\mathbf{ZY}) = \mathbf{0} \qquad (6)$$

A possible structure of *uniform gainless/lossless* CTLs that can support EPD, i.e., satisfies (6), is shown in Fig. 4(c) where the dispersion diagram that exhibits the DBE at $\omega = \omega_e$ is shown in Fig. 4(d). The DBE in such design can only be found at the center of the Brillouin zone, i.e., when the wavenumber $k = 0$, but the surprising thing is its order that is equal to 4. Also, the structure exhibits second order EPD at $\omega = 0.4\omega_e$, where two modes coalesce as $k = \pm k_e$. To further promote the proposed concept, in Fig.4(e) the team has also developed a microstrip implementation of the uniform CTLs which is realized with a subwavelength period.

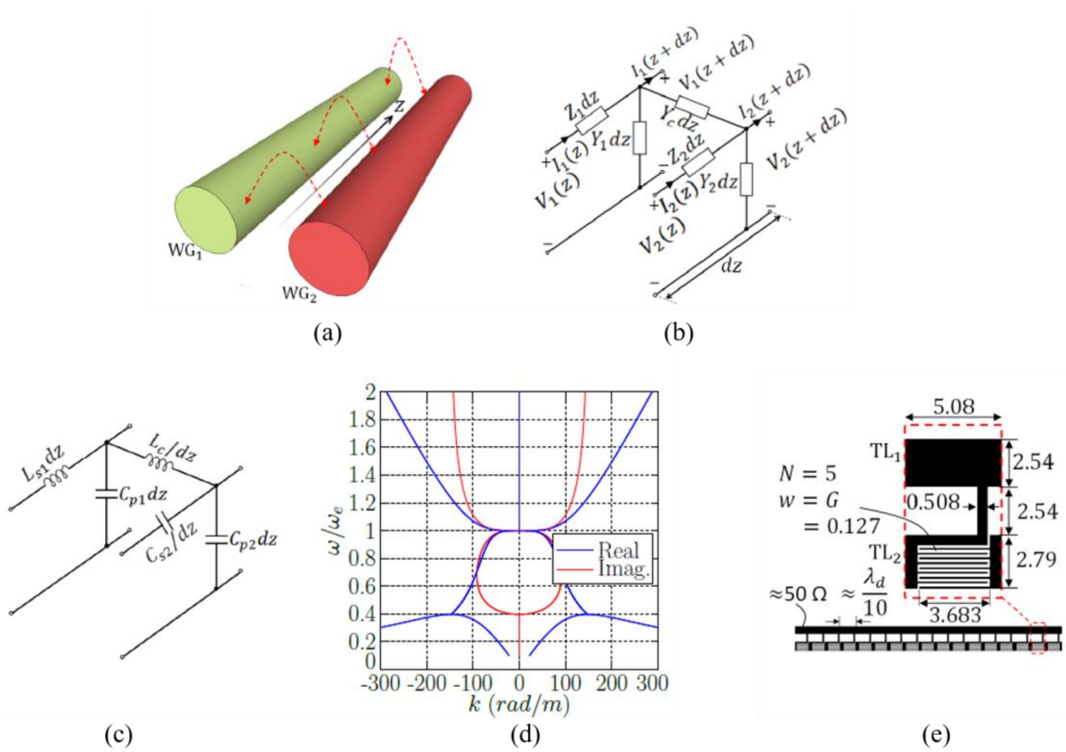


Fig. 4. (a) Two uniform coupled waveguides. (b) Generalized distributed circuit model for infinitesimal section of the two coupled transmission line (CTL) structure. (c) The per unit length parameters of two uniform coupled transmission lines (CTLs) where one supports forward propagating mode and the other supports evanescent mode. (d) The dispersion diagram of the CTLs showing both the real and imaginary parts of the wavenumbers of the four supported eigenmodes where the DBE can be seen at k = 0. (e) A microstrip sub-wavelength implementation of the CTLs in (c) with period d=5.08mm and EPD frequency $f_e = 5$GHz. All the given dimensions in (e) are in mm.

The realization of high order EPDs (such as the DBE) in uniform coupled waveguides is very important for implementing on-chip slow wave structures (SWSs) which will lead to potential performance enhancement for various application as filters, oscillators, and pulse forming networks for high speed communications.

b) EPDs induced in *temporally uniform* resonators: Third order EPD in coupled resonators

The UCI group has proposed the existence of a *third order EPD* in coupled LC oscillators with balanced gain and loss. The proposed coupled oscillator system consists of an LC tank associated with C and L_a which is coupled with a pair of coupled RLC oscillators, one with gain and the other with loss, where the coupled system is shown in Fig. 5(a). In this coupled system the team has only considered the electric coupling between the oscillators, through the capacitors connecting the LC oscillators. The coupled RLC oscillators are an ideal PT-symmetric dimer, studied by other group where a breaking point between the exact and broken phase was shown and defined as the PT-symmetry breaking point. Here, the extra two coupling capacitors couple the additional LC tank to the ideal PT dimer which is necessary for the emergence of the 3rd order EPD. The dispersion diagram of such system versus the gain and loss parameter is shown Fig. 5(b). Moreover, the team has also shown the necessary conditions on the system parameters to exhibit a 3rd order EPD. The details are contained in a Master Thesis deposited at the UCI library in Spring 2018 [MT1].

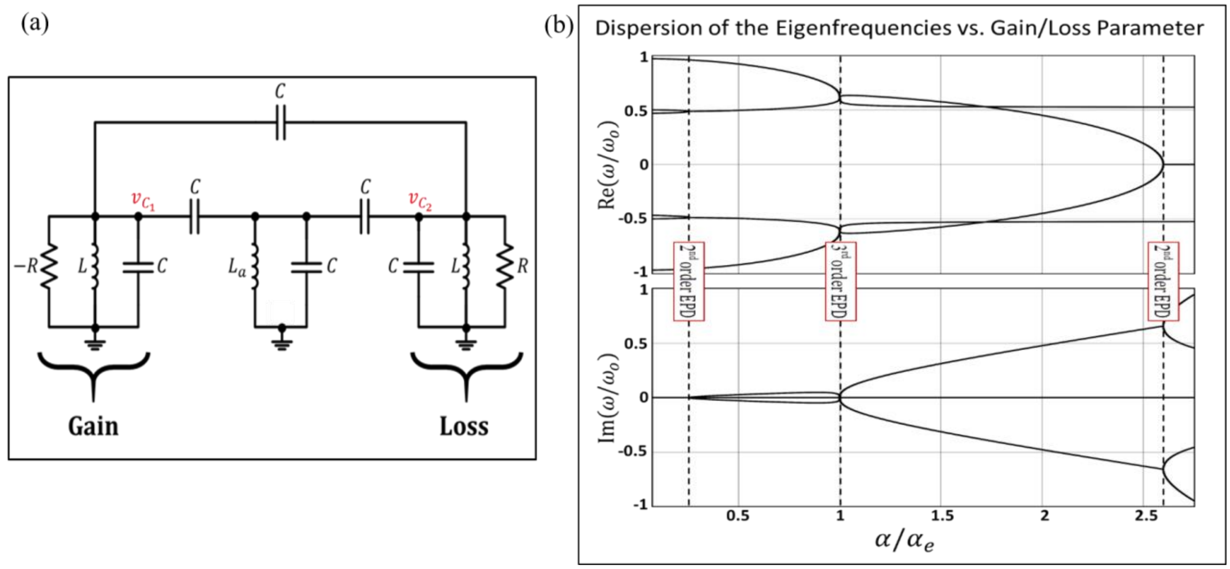


Fig. 5. (a) The proposed circuit to exhibit a *third order* EPD that consists of three coupled LC oscillators. (b) Dispersion diagram of the real and imaginary parts of the eigenfrequencies of the circuit (a) versus the normalized gain/loss parameter. The third order EPD is located at the point where three branches coalesce. Despite it is not shown here, the three associated eigenvectors coalesce as well.

c) EPDs induced in *spatially periodic* structures

i. Fourth order EPD in periodically-coupled waveguides and the interplay of gain and radiation loss

The team has experimentally demonstrated for the first the time the existence of fourth order EPD (the DBE) in microstrip CTLs at microwave frequencies [18] through four-port measurement of a unit cell leading to the dispersion relation shown in Fig. 6 and also through the transmission characteristics of a finite-length CTL as shown in Fig. 7. They have used the developed hyperdistance concept to assess the quality of such EPD subject to any kind of perturbations, imperfect coupling, losses and any other perturbation that may arise from fabrications or numerical simulation.

The hyperdistance concept is a way to measure the eigenvector "distance" in a multidimensional vector space. Based on the defined figure of merit, an experimental verification has confirmed for the first time that the main EPD features of almost parallel eigenvectors can still exist in radiating arrays, i.e., periodic structure with radiating losses.

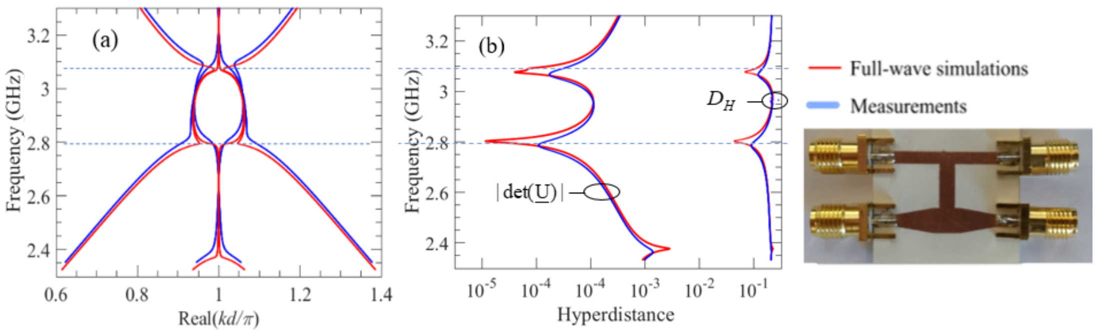


Fig. 6. (a) Dispersion relation and (b) hyperdistance measurement versus full-wave simulation of the unit cell of a periodic CTL exhibiting a 4th EPD. The results show that the 4th order EPD occurs at 2 different frequencies in the range shown in this plot. Full-wave simulations are performed with Keysight ADS using the method of moments (MoM).

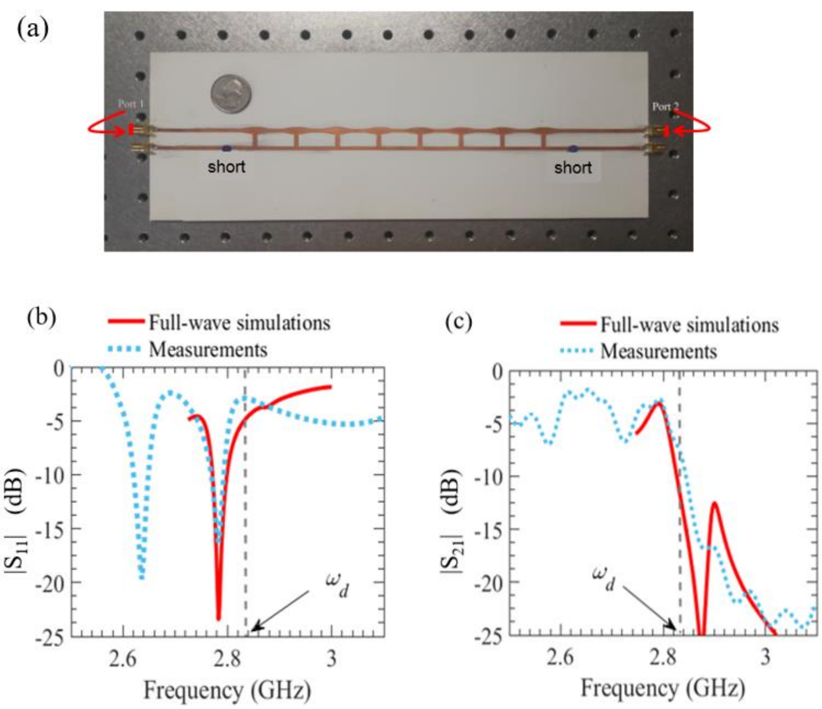


Fig. 7. (a) A fabricated microstrip 8 cell array with an EPD. The CTL is based on the unit cell shown in Fig. 6. (b-c) Simulations and measurements of the scattering parameters S_{11} (b) and S_{21} (c) for an 8-unit-cell array; where a good agreement between full-wave simulations and measurements is clear at the resonance frequency associated with the EPD.

Furthermore, the UCI group showed that in lossy waveguides, the "gain and loss balance", that is different from the PT-symmetry concept, in a CTL leads to recovering the EPD as shown in

Fig. 8 which can be verified via the hyperdistance concept, i.e., by the minimum of the hyperdistance and observing that the minimum is much smaller than unity. This allows for utilizing the multi-eigenmode degenerate scheme in designing array-oscillator paving the way to a new class of coherent EPD-based single-frequency radiating oscillators based on gain and loss balance. The potential benefits of such oscillators may include power efficiency and spectral purity.

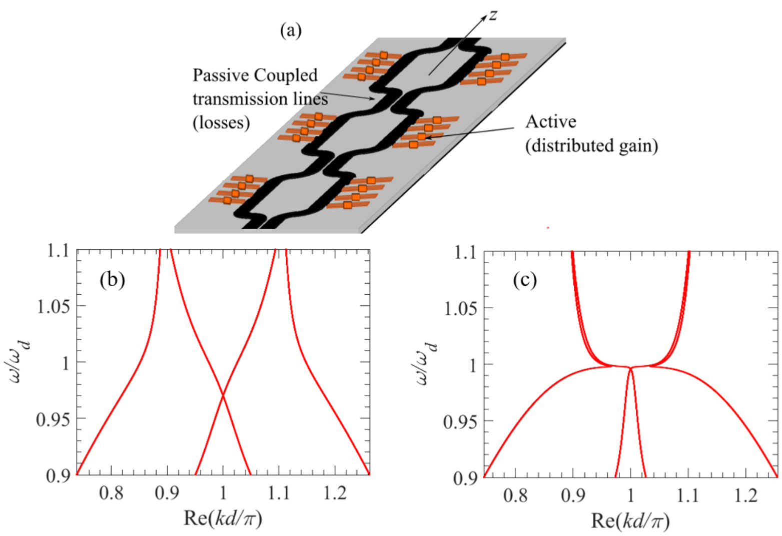


Fig. 8. (a) Example geometry of coupled microstrip lines with distributed gain and "loss" (loss is actually a distributed radiation, that is seen as a loss from the circuit point of view). (b)The dispersion diagram for the "lossy" structure (i.e. before introducing gain) where losses are modeled as series resistance R_n such that all TLs have $Q_{\mathrm{TL}} = 100$ (see [18]). (c) The corresponding dispersion diagram after introducing the appropriate gain associated to the minimum hyperdistance (i.e. gain is modeled by introducing negative shunt conductance in all TLs) to achieve balanced gain and loss condition, showing the flatness of 4^{th} order EPD. This microwave circuit may be used as an efficient high intensity radiator, generating high spectral purity with high power efficiency.

ii. Fourth order EPD condition in multilayer waveguide structure with grating

The UCI group has proposed the existence of degeneracy conditions of four modes in multilayer waveguide structures composed of two coupled dielectric slab with grating as shown in Fig. 9(a). The structure in Fig. 9 is usually related to distributed feedback lasers. The UCI group has showed that this structure can be designed to have an EPD of order 4, paving the way to new laser regimes. Indeed, a single slab waveguide with a grating on top exhibits a second order degeneracy, namely, it is a regular band edge due to coalescing of two identical modes propagating opposite to each other. Therefore, to obtain a fourth order degeneracy or a degenerate photonic band edge, i.e., coalescing of four modes, one should have at least two coupled slab waveguides with grating, as shown in Fig. 9(a). Such structure can support four modes, which are for instance even and odd mode propagating in positive and negative z-direction for symmetrical structure. Based on the coupled mode theory formulism, Fig. 9(a) illustrates a possible situation where a DBE can be obtained. From Fig. 9(b), it is clear that four modes merge to a degenerate mode when $\omega = \omega_0$, which indicates the existence of a 4^{th} order EPD. These finding are included in a Master Thesis, June 2018 [MT2].

Operating at photonic band edges leads to several promising characteristics of the photonic devices, such as frequency stability and selectivity, giant gain enhancement, and high-quality factors, furthermore, photonic devices operating near a DBE have been proved to provide much better performance compared to those operating near an RBE [1], [9], [11], [12].

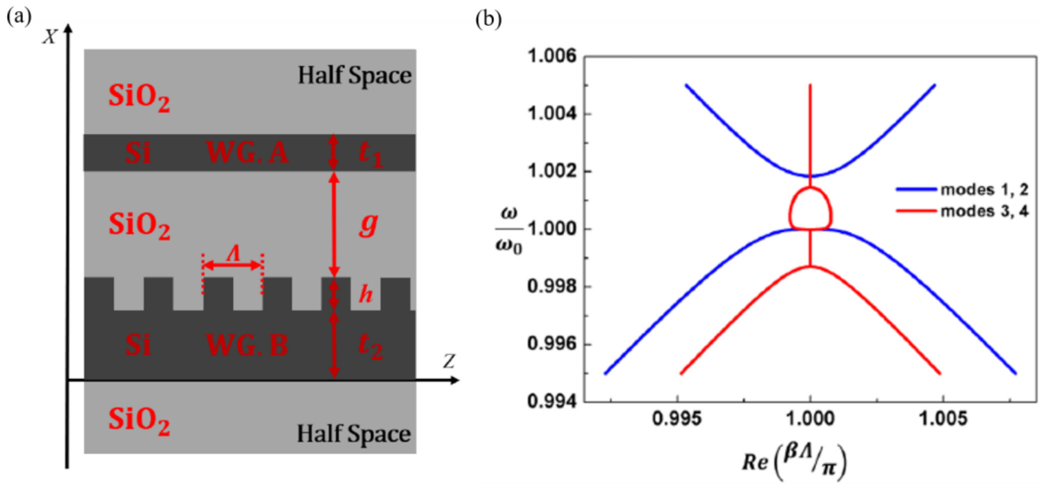


Fig. 9. (a) Proposed multi-layer waveguide structure with grating exhibiting a DBE. (b) The dispersion diagram of such structure based on coupled mode theory formulation showing a DBE, i.e., the coalescence of four eigenvalues. Though not shown here, at that frequency also the four eigenvectors (polarization states) are coalescing.

iii. Radio frequency double ladder oscillators with DBE

The team has proposed a new theory [10] and a new scheme of distributed oscillator [15] based on a periodic, double ladder resonant circuit (see Fig. 10(a)) exhibiting a degenerate band edge (DBE) in the dispersion diagram of its phase-frequency eigenstates. The double ladder features unique characteristics related to its giant resonance, compared to single ladder or a conventional LC tank circuit. Besides the new theory [10], [15] the proposed oscillator leads to an important conclusion: a starting oscillation threshold that is almost half that of a single LC ladder circuit having the same total quality factor, and thus is more robust than an LC oscillator in the presence of losses as shown in Figs. 10(b) and (c). Furthermore, the output amplitude and frequency of the double-ladder oscillator depends much less on the output loading compared to single-ladder oscillators. This oscillator is shown to have an *oscillation threshold that is half that of a single LC ladder circuit having the same total quality factor*, and thus is more robust than an LC oscillator in the presence of losses. Moreover, the double ladder oscillators have a unique mode selection scheme that leads to stable single-frequency oscillations even when the load is varied. It is also shown that the output amplitude of the double-ladder oscillator is much *less sensitive to the output loading* compared to single-ladder oscillators. The team show the analysis and design of such oscillators that potentially lead to enhancing the efficiency of RF components and sources. Importantly, preliminary calculations have shown that the *double ladder oscillator consumes seven time less power than a simple LC tank oscillator*, since the LC tank oscillator requires buffers while the double ladder oscillator does not.

iv. Experimental demonstration of DBE in metallic waveguides

The UCI group has successfully demonstrated experimentally for the first time ever the existence of the degenerate band edge (DBE) in metallic circular waveguides with periodic metallic loading (Fig. 11), in collaboration with the University of New Mexico, with Christos Christodoulou group. Fig. 11(a) shows the fabricated structure, while in Fig. 11(b) the full-wave simulated dispersion diagram and the measured one are proven to be in a good agreement. This

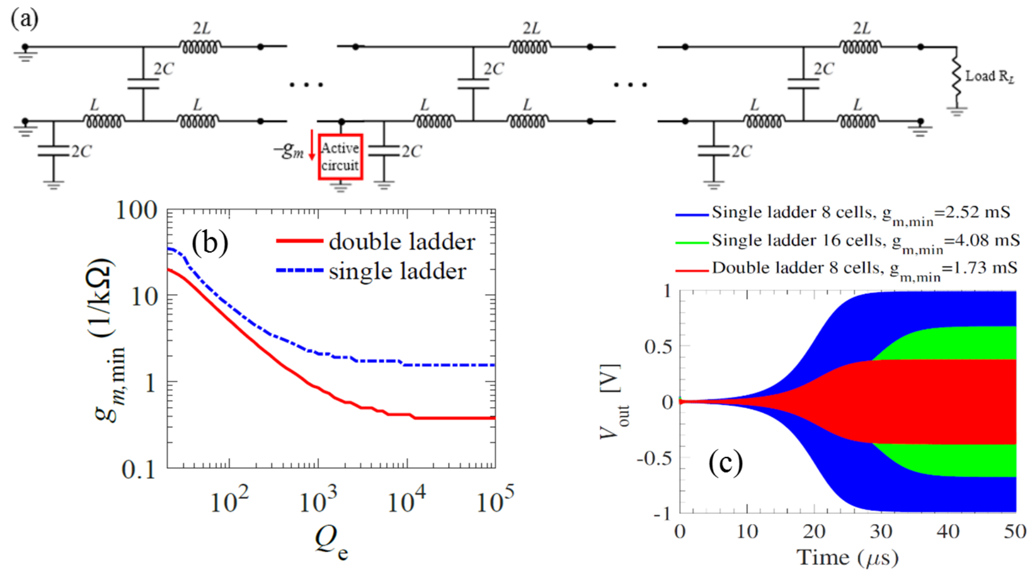


Fig. 10. (a) Double ladder periodic oscillator structure that supports a DBE. It has 8 unit cells with terminations and negative resistance $(-g_m)$ acting as the active device (b) Minimum g_m needed start oscillations versus different values for quality factor of elements (Qe) for double and single ladder structures. The double ladder needs lower g_m to start oscillation. (c) Oscillation behavior for three different cases of single-ladder and double-ladder circuits in comparison and the minimum g_m needed to start oscillation. All data is reported in [15].

figure clearly confirms the existence of the DBE in the metallic waveguide. Moreover, a giant scaling of the quality factor as L^5, where L is the length of the SWS, is reported in [6].

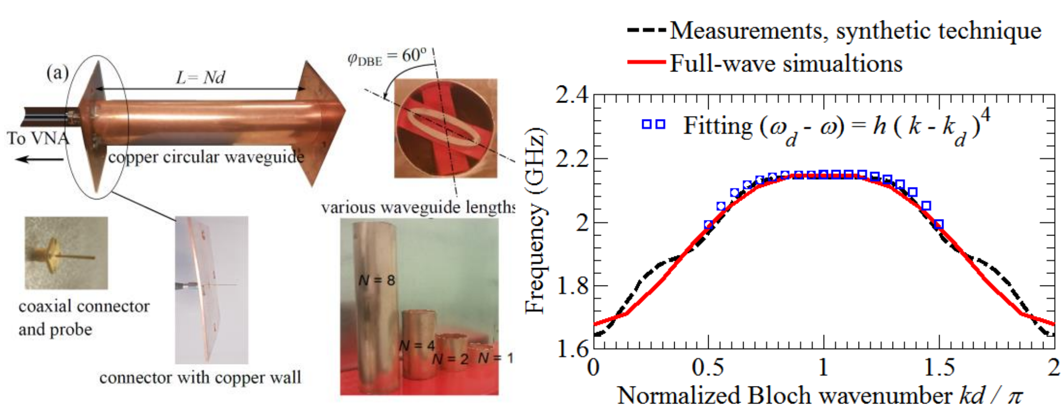


Fig. 11. (Left) Fabricated waveguide that develops a DBE at S-band. (Right) Dispersion relation showing good agreement between both measurement and full-wave simulations using CST microwave studio.

v. Various orders of EPDs at optical frequency in coupled resonator optical waveguides (CROW)

The UCI group has presented a novel approach and a theoretical framework for generating high order exceptional points of degeneracy (EPD) in photonic structures based on periodic coupled resonators optical waveguides (CROWs) [9], [12]. The UCI group has modified a conventional CROW that is consisting of a chain of ring resonators coupled to each other with coupling coefficients alternating form ring to another as κ_1' and κ_2'. The EPDs arise here by introducing symmetry breaking in the conventional CROW through periodic coupling to an adjacent uniform optical waveguide with alternating coupling coefficients κ_1 and κ_2 as shown in Fig. 12(a). The team has shown for the first time the *capability of such proposed CROWs to exhibit EPDs of various order*; including the degenerate band edge (DBE) which is a 4th order EPD and the stationary inflection point (SIP) that is a 3rd order EPD besides the regular band edge (RBE) as can be seen within a narrow frequency band in Fig. 12(b).

The existence of EPDs in such structure has led to unique modal characteristics that cannot be realized in conventional CROWs. Such remarkable characteristics include *high quality factors (Q-factor) and strong field enhancement*, even without any mirrors at the two ends of a cavity. The proposed CROW of finite length shows enhanced quality factor when operating near the DBE as shown in Fig. 12(c), and the *Q*-factor exhibits an *unconventional scaling with the CROW's length* which is proportional to N^5 with N being the number of rings in the CROW. The team has developed the theory of EPDs providing the *necessary conditions of each EPD order to exist* in such unconventional CROW. This was done using the coupled-wave formulation approach, and also the team has derived an *analytical expression for the dispersion relation* [9]. The team has also investigated the robustness to structural perturbations of the quality factor and resonance frequency in [12]. In addition, in [12] the team has also provided an *analytical expression to estimate the flatness of the dispersion diagram at the EPD*, which is a useful parameter to control the density of states and the scaling of the quality factor.

The proposed unconventional CROW concepts have various potential applications including *Q*-switching, nonlinear devices, lasers, and extremely sensitive sensors.

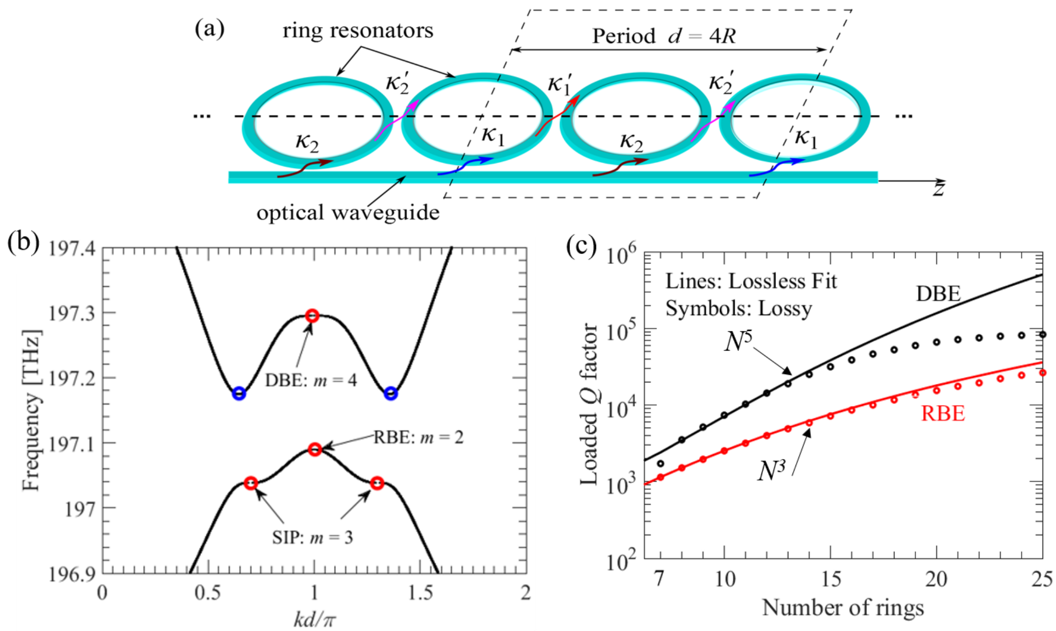


Fig. 12. (a) The proposed CROW is consisting of a chain of coupled ring resonators optical waveguides of radius R side coupled to a rectangular straight waveguide. (b) The Floquet-Bloch wavenumber dispersion diagram of the CROW unit cell shown in (a), viewing three different kinds of EPDs within a narrow frequency band: an RBE, a DBE; and an SIP. (c) The calculated loaded quality factor (Q) of a finite CROW plotted versus the number of rings for the lossless and lossy CROW.

vi. Correlation between EPDs and PT-symmetry in photonic coupled chains of scatterers

The UCI group has shown how EPDs are manifested in EM structures with non-Hermitian degeneracies (when loss or gain is present) and in EM structures that have no loss or gain, with a single unified formulation under which all EPDs can be described [7]. PT-symmetry implies balanced, and spatially symmetric gain and loss and the UCI group has shown how these properties leads to EPDs in chains of scatterers [8]. Moreover, the team has investigated several cases where EPDs occurs within a more general framework that *does not require PT-symmetry*. Indeed, EPDs can be achieved in a variety of EM structures as shown in Fig. 1. As an important example relevant to optical interactions and applications the team has developed the theory of EPDs in coupled chains of dipolar scatterers [8] using the Green's function method as shown in Fig. 13. Two different regimes based on PT-symmetry have been identified for large optical wavelengths compared to the period (with $\alpha_1 = \alpha_2^*$ where $\alpha_{1,2}$ are the polarizabilities of the spheres in each chain, meaning that one exhibits gain while the other exhibits loss, as shown in Fig. 13), yielding both 2nd order and 4th order EPDs. Fig. 13(right) shows the fourth order EPD at which four curves merge (red line). It has been shown that some regimes of operation do not necessitate PT-symmetry in order for such EPDs to occur. The theory provides new insights into how EPDs of order 2 and 4 can manifest in general discrete coupled mode structures. These properties can also be harnessed for sensing applications, enhancing non-linear effects and super-resolution at optical wavelengths. Though working with PT-symmetry require working with systems that are marginally unstable (i.e., very close to instability) which should be further investigated.

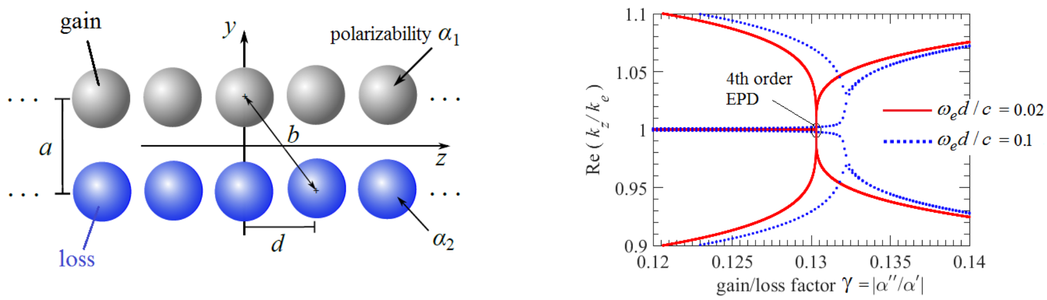


Fig. 13. (Left) Two coupled chains of polarizable particles with gain and loss. (Right) Evolution of the complex wavenumber varying the gain/loss parameter in the scatterers polarizabilities. The system is PT-symmetric for low frequencies $\omega_e d/c$=0.01 (with the condition for $\alpha_1 = \alpha_2^*$ where $\alpha_{1,2}$ are the scatterers' polarizabilities) while such property is not necessarily achieved for high frequencies. All data is reported in [8].

vii. Sixth order modal degeneracy: triple ladder lumped circuit and microstrip implementation

The team has introduced a novel design based on a simple *triple* ladder circuit (unit cell is shown in Fig. 14(a)) realized with lumped reactive components that provides a *sixth order degenerate band edge* (6DBE) in the phase shift-frequency dispersion diagram whose dispersion relation varies as $\Delta\omega \propto \Delta\varphi^6$ as shown in Fig. 14(b). The proposed design exhibits unique resonance features of triple ladders of finite length, forming a cavity associated with a high loaded *Q*-factor. An example of a unit cell of a periodic circuit that can develop a six-order degeneracy is depicted in Fig. 14(a). The unit cell is composed of eight reactive components (five inductors and three capacitors) where the simulated dispersion diagram of such unit cell is shown in Fig. 14(b). An implementation of these lumped circuit models using coupled transmission lines (CTLs) is shown in Fig. 14(c), which basically consists of three CTLs. The dispersion diagram in Fig. 14(d) is carried out using full-wave simulations of the S-parameters performed using Keysight Technologies ADS based on the Method of Moments showing the 6DBE at frequency 2.85 GHz. The team also investigated the filtering characteristics of a finite-length structure that is terminated with loads to highlight the important qualities of the 6DBE compared to the RBE and the 4DBE where the 6DBE show some leverages over previously introduced double ladder [10] (exhibiting DBE) and single ladder (RBE) circuits of the equal length (*N*), specifically in terms of the *Q*-factor. The circuit framework introduced here with a 6DBE can be exploited in designing novel high *Q*-factor oscillators, filters, and pulse shaping networks.

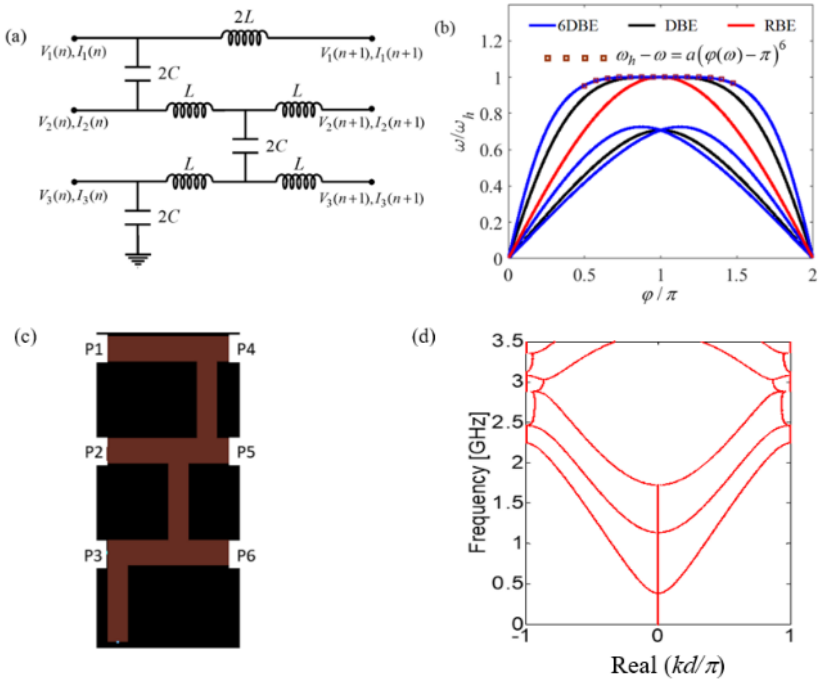
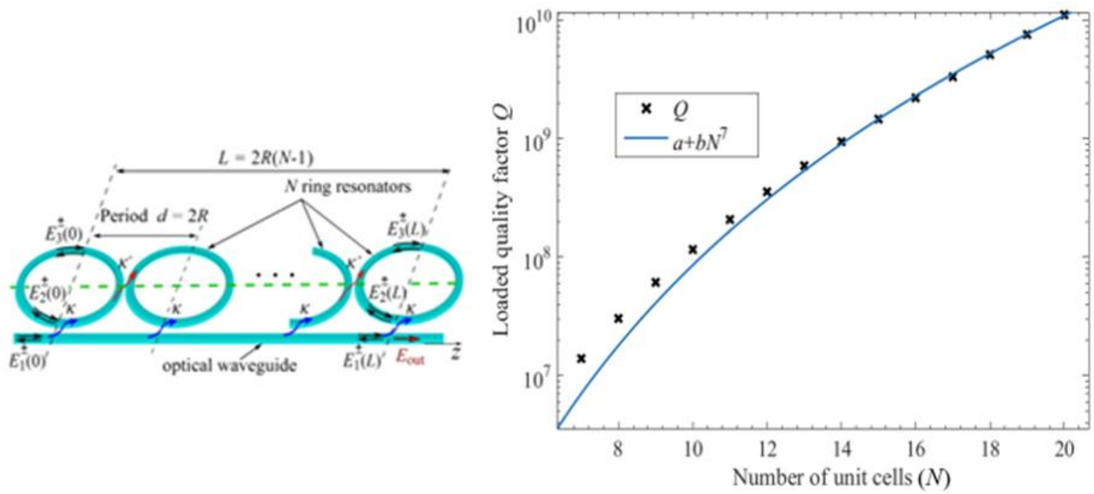


Fig. 14. (a) Unit cell of a periodic triple-ladder lumped circuit that develops a sixth order degeneracy at an angular frequency $\omega_h = 1/\sqrt{LC}$ (b) Dispersion diagram of the infinitely long periodic triple-ladder circuit exhibiting a 6DBE at angular frequency ω_h . (c) Microstrip implementation unit cell of a periodic CTL in (a) that develops a sixth order degeneracy (d) Dispersion diagram showing the real part of the dispersion of the infinitely long periodic structure of unit cell in (c) exhibiting 6^{th} order EPD at frequency = 2.85 GHz.

viii. Sixth order EPD at optical frequencies based on CROW

In addition to obtaining the 6DBE at radio frequencies using microstrip lines, the team has succeeded to obtain *the 6DBE at optical frequencies* using the proposed CROW shown in Fig. 15(a). The quality factor of a periodic CROW operating near a 6DBE scales with the number of unit cells N as $Q \propto N^7$ and such unique scaling is shown in Fig. 15(b). The team has also provided the design equations governing the various parameters of the CROW so that one can easily obtain the 6^{th} order DBE at any desired frequency. This is the first time an actual, realistic, structure has been proposed with a 6^{th} order EPD.



(a) (b)

Fig. 15. (a) Loaded CROW consists of a chain of rings coupled to each other. (b) Loaded quality factor (Q) of the lossless EPD-CROW calculated at different number of unit cells N. The values of Q denoted by cross symbols are calculated using the group delay method while the solid line represents the fitting curve with the equation $a+bN^7$.

d) EPDs induced in *linear time periodic* resonator

The UCI group has proposed a new and interesting type of EM systems which exhibit exceptional points of degeneracy (EPD) due to the existence of a time periodic element in the system. This has been inspired from the observation that lossless and gainless periodic waveguides exhibit EPDs, hence the UCI team proposed for the first time how to realize EPDs in periodic time varying circuits. The UCI group has presented a general theory of EPDs in periodically time-variant systems that do not necessarily require the presence of loss or gain, [17] analogous to the EPDs found in spatially periodic structures without the need of loss and gain [9-11]. They also demonstrate the conditions for EPDs to exist in time-periodic systems that may be lossless/gainless or with loss and/or gain. The team has shown the existence of EPDs in a single lossless/gainless LC resonator with a linear time-periodic capacitor as shown in Fig. 16(a), where the time domain simulations of this system are shown in Fig. 16(b). The team has also clarified that a system with zero time-average loss/gain exhibits EPDs with purely real resonance frequencies as shown in Fig. 16(c), yet the resonator energy grows algebraically in time, see Fig. 16(d). Energy is inserted in the system by the time modulation. Moreover, the UCI group has provided the analytic sufficient conditions for such EPDs to exist and they have illustrated how such temporally induced EPDs may have potential application in conceiving highly-accurate sensing devices.

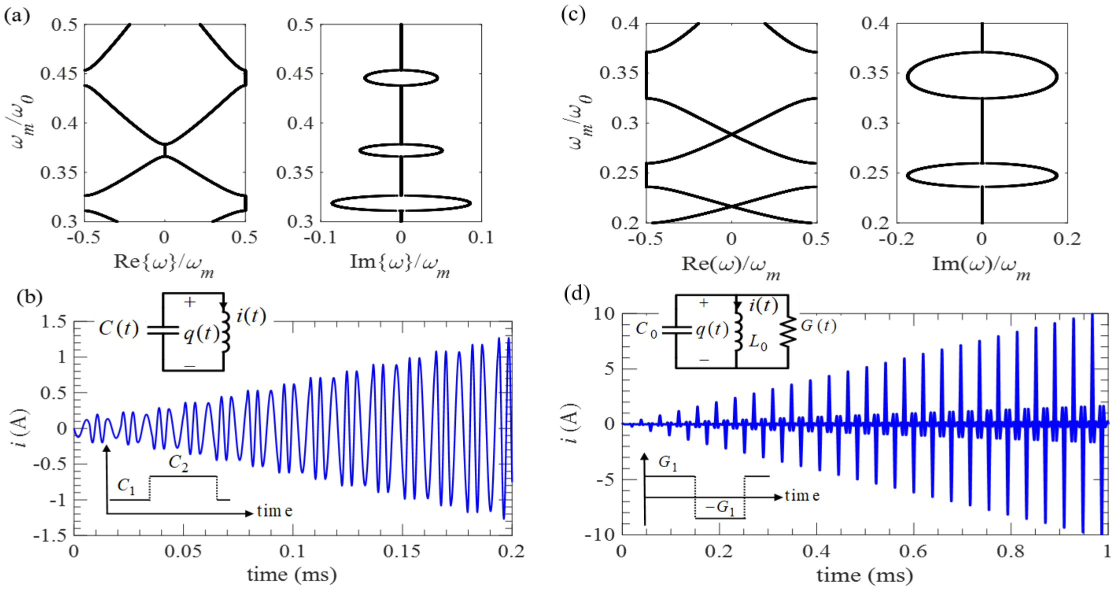


Fig. 16. (a) and (c) Real and imaginary parts of dispersion diagram of the eigenfrequencies versus the normalized angular modulation frequency of the resonators depicted in (b) and (d), respectively. (b) and (d) The linear algebraic growth of the inductor current at an EPD for an LC resonator with time-varying capacitor and for and RLC resonator with time varying conductance, respectively.

4) EPDs properties and proposed applications

The UCI group has been investigating and understanding the properties of EM systems with EPDs. Moreover, the UCI group has employed these properties for conceiving better performance and features in devices that are otherwise hard to obtain in conventional EM systems. In this section, important properties of EM systems operating close to EPDs are addressed and their corresponding applications are proposed. Then the enhancement of some EM systems operating at EPDs is shown in terms of better performance compared to conventional EM systems.

a) High sensitivity

One of the principal advantages of EPDs lie in their large (exceptional) sensitivity to external perturbations. Indeed, as the order of degeneracy increases, the system becomes more and more sensitive to perturbations. Here sensitivity is defined as ratio of the change in a characteristic quantity of the system (wavenumber, group delay, quality factor, etc) with respect to a change in a certain parameter (frequency, coupling parameter, etc). In guiding EM systems with EPDs the basic principle, according to Puiseux series, is that a small change $\Delta\varepsilon$ of a physical parameter ε will result in changing the system degenerate eigenvalue λ (may represent a propagation wavenumber, an eigenfrequency, etc.) according to

$$\Delta\lambda \propto (\Delta\varepsilon)^{1/m} \tag{7}$$

where m is the order of degeneracy. Therefore, a tiny change $\Delta\varepsilon$ will result in a huge change in the degenerate eigenvalue λ due to the existence of the m^{th} root, especially for large m.

Such unprecedented sensitivity is very beneficial towards various applications, such as highly sensitive sensors, low threshold switches and highly sensitive leaky wave antennas, as shown next with some details.

i. Lossless/gainless linear time periodic LC sensor

Consider an LC resonator with time-variant capacitance [17] as described in Fig. 17(a), but with a lossy inductor with quality factor of 100 and assume C_2 is now perturbed from its nominal value as $(1 + \varepsilon)\, C_2$. When C_2 is not perturbed the system exhibits an EPD with an eigenvalue for a modulation frequency. Fig. 17(b) shows the two perturbed eigenvalues versus the perturbation ε calculated from the exact eigenvalue problem and by using the Puiseux series approximation. The most important thing to be noticed from Fig. 17(b) is that an extremely small perturbation in the capacitor C_2 will lead to a much larger change in the eigenvalues of the system. This property is actually one of the most exceptional physical properties associated to EPDs and it can be exploited in designing extremely sensitive sensors. The large perturbation of the eigenvalues in turn implies a sharp change in the complex resonance frequency of the LTP LC resonator as shown in Fig. 17(c). For positive but very small ε-perturbation the imaginary part of the complex resonance frequency shows a sharp change while its real part is kept constant. Furthermore, a very small negative ε-perturbation causes a rapid change of the real part of the resonance frequency. For example, a +0.1% perturbation in the dielectric permittivity of the capacitor C_2 (i.e., $\varepsilon = 0.001$) will change the imaginary part of the resonance frequency by $0.3\omega_o$; which implies 76% change in the quality factor of the resonator. Hence, this highly sensitive system can be employed to measure the dielectric permittivity (that can be changed by an environment parameter, like acidity or humidity, or a gas presence) with very high accuracy.

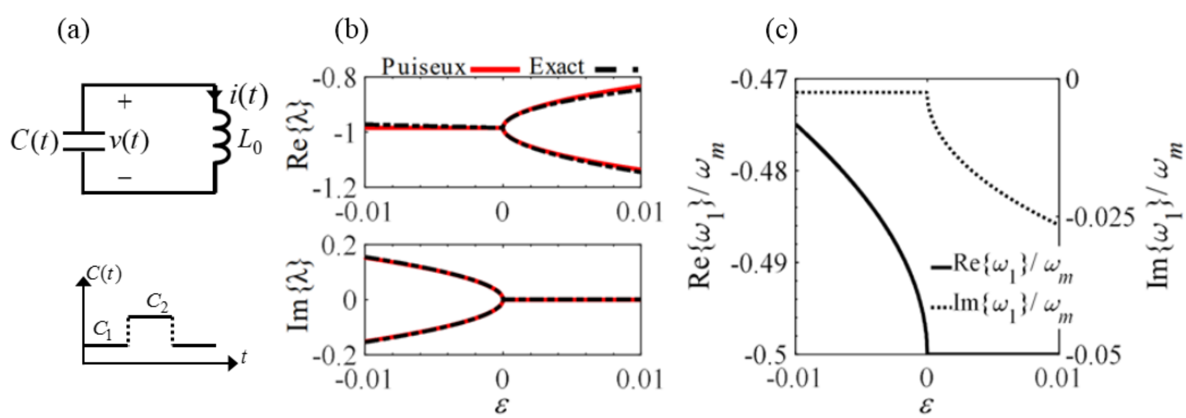


Fig. 17. (a) LC resonator with a periodic time varying capacitor exhibits a 2nd order EPD; (b) change of the eigenvalues due to perturbation in the capacitor value C_2 and (c) change in the complex resonance frequency ω_1 corresponding to the change in the eigenvalue in (b). A +0.1% perturbation in the dielectric permittivity of the capacitor C_2 implies a 76% change in the quality factor of the resonator, demonstrating the exceptional sensitivity.

ii. Highly-sensitive directive leaky wave antenna

Gain and loss balance condition is considered a remarkable framework upon which highly sensitive sensors can be brought to light, though the use of gain may induce unwanted oscillations. However, EPD-based high sensitivity feature may also not rely on gain and loss balance and could be further harnessed from all-passive systems.

A concept for RF and microwave sensors proposed by the UCI group for a highly sensitive radiative system is based on leaky-wave antennas and was proposed and published in [7]. The technique is based on enhancing the sensitivity of leaky-wave antenna' *directivity* utilizing an EPD with gain and loss balance as shown in Fig. 18(a). The 1D leaky-wave antenna is characterized by a TL with shunt distributed conductance loading moldering radiation losses G_1. The active TL is also characterized by a distributed shunt negative conductance representing small signal gain G_2. Coupling between the two systems is capacitive though proximity in this case. The balance condition in such example is shown when $G_1 = -G_2$ which is the PT-symmetry conditions. Other scenarios will also be investigated. This is a topic that may generate a new class of ultra-sensitive sensors in general. This scheme may also lead to active antennas that radiate and oscillate at the same time, with high power efficiency and high spectral purity, if gain is brought above the threshold.

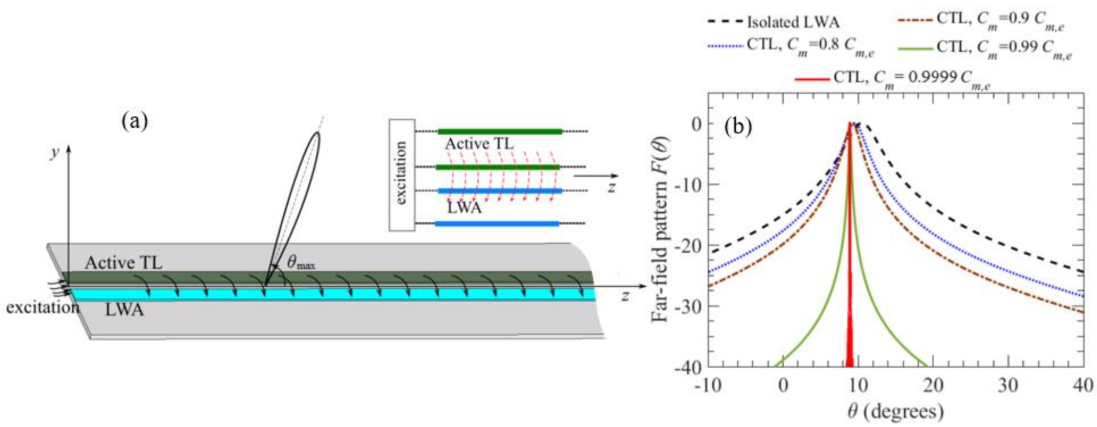


Fig. 18. Coupled transmission lines (CTLs) are composed of one leaky-wave antenna (LWA) coupled to an active TL (distributed amplifier). (d) Radiation pattern for the CTL with an LWA for different values of C_m. The beam is narrowest near the EPD and the radiation beam width is very sensitive to variation of the distributed coupling capacitance C_m. Gain is introduced in the active TL as a negative shunt distributed conductance, while the radiation loss in the LWA is modeled using a positive shunt distributed conductance, see details in [7].

iii. DBE optical switch with very small switching threshold voltage

Based on the high sensitivity of the eigenvalues near the EPD, the UCI group has conceived a new regime for low-threshold and fast switches. Consider a finite CROW structure made of 9 unit-cells that are designed to exhibit a 4th order EPD as shown in Fig. 19(a) and Fig. 19(b). The coupled chain of ring resonators is made of an electro-optic material so that the effective refractive index of the rings (n_r) changes according to the applied voltage. By applying voltage to the electrodes across the rings, the rings permittivity is very slightly perturbed however resulting in a big change in the transfer function of the structure as shown in Fig. 19 (c). It is obvious that the switching happens with very small change in n_r which means very small switching threshold voltage and possible high speed.

b) High quality factor

One of the unique features associated with EPDs is the giant unconventional scaling of the quality factor with the structure length of any system operating near an EPD where the group velocity is vanishing. This unconventional giant scaling of Q with the structure length L is given as

$$Q \propto L^{m+1} \tag{8}$$

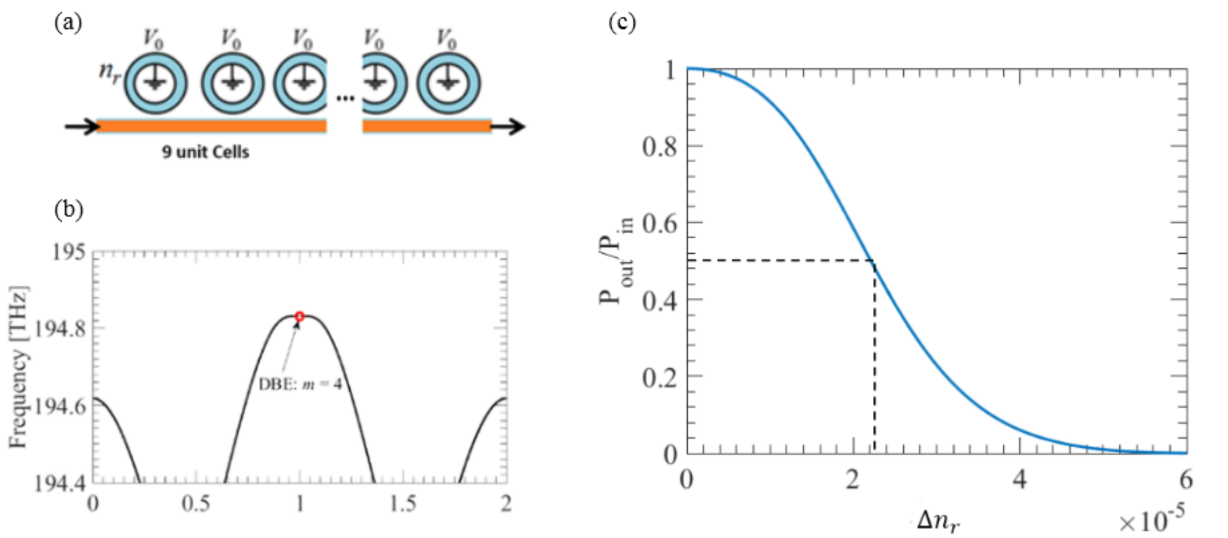


Fig. 19. (a) The proposed CROW is consisting of a chain of coupled ring resonators that are side-coupled to a uniform optical waveguide. (b) The dispersion diagram of the CROW unit cell showing a 4th order EPD $f_e = 194.8\,\text{THz}$ and (c) power transfer function of the proposed structure in (a) versus change in the refractive index of the rings.

where m is the order of the EPD. The word "giant" has been used to describe this unconventional scaling law encountered also in other geometries supporting the DBE (4th order EPD) [9], [11], [12]. This can be inherently understood from the fact that the quality factor is inversely proportional to the group velocity of the Floquet-Bloch wave v_g (i.e. $Qv_g = \text{constant}$). This statement implies that if the EPD resonance $\omega_{r,d}$ (usually different than the EPD frequency due to being the structure finite) coincides with the EPD frequency ω_d at which $v_g = 0$, then Q-factor will be infinite (ideally, in a lossless waveguide). However, the Q-factor is finite because $\omega_{r,d} \neq \omega_d$, though they are very close following the asymptotic trend $\left(\omega_d - \omega_{r,d}\right) \propto 1/N^m$. Hence, the Q-factor rapidly increases with the number of cells N, because this latter formula describes the rate at which the resonance angular frequency $\omega_{r,d}$ gets closer to ω_d with growing N, which in turn leads to special scaling with N of the increase of group delay and Q-factor.

i. DBE laser: low-threshold single-mode Laser

Demonstration of *single-mode coherent* laser is a classical contest in the optics realm. The team proposed for the first time a novel class of low threshold single-mode lasers based on a fourth order eigenmode degeneracy in a multimode waveguide [11]. Such a class of lasing action is based on the degenerate band edge (DBE). The "DBE laser", that is how this regime of operation has been named, involves four degenerate eigenmodes in a cavity made of a finite number of coupled optical waveguide sections doped with an optically-pumped active medium. The team demonstrated that the DBE laser has the *lowest lasing threshold* in comparison to a regular band edge laser or to a conventional laser in cavities with the same loaded quality (Q) factor and length as shown in Fig. 20(b). Importantly, the DBE laser, without mirror reflectors, is shown to exhibit a lasing threshold which is an order of magnitude lower than that of a uniform FPC laser of the same length shown in Fig. 20(a, right). The DBE laser *does not need mirrors* at the two ends of the cavity and this may simplify certain realizations. In Fig. 20(a, left) the straight waveguide is continued without mirrors, since reflectivity is given by a mismatch of the Floquet-Bloch mode in the DBE cavity.

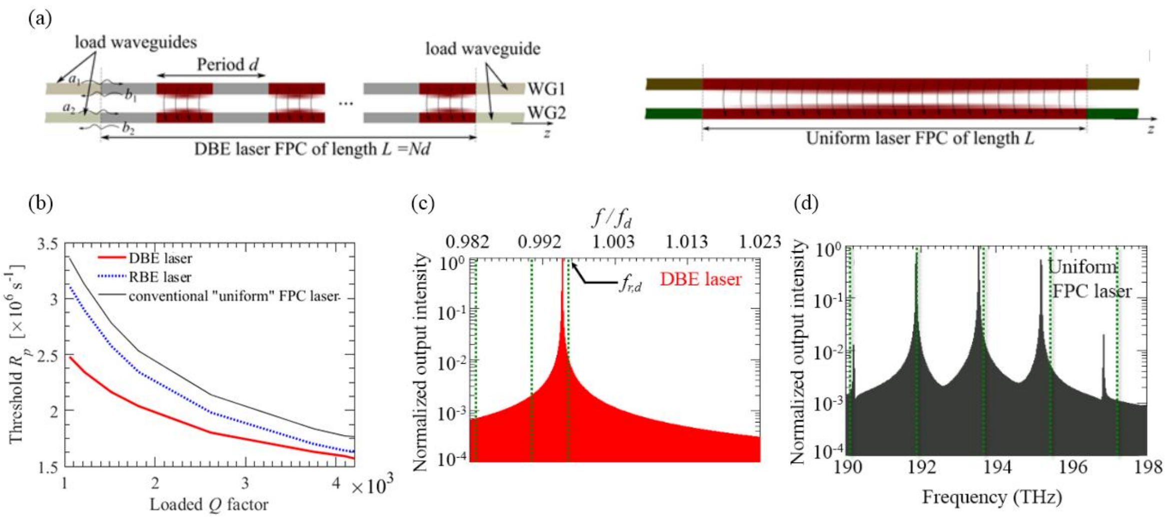


Fig. 20. (a) Schematic representation of an FPC with DBE(Left) composed of a finite number of periodic unit cells, each made of a coupled and an uncoupled section and the conventional uniform FPC(Right). (b) Comparison between the lasing threshold pumping rate of the proposed DBE laser with the conventional uniform FPC and the RBE lasers of equal length. (c) and (d) output spectrum of the DBE laser and the long uniform FPC laser at the same pumping rate, respectively.

Moreover, the team has shown that the mode selection scheme associated with the DBE ensures coherent *single-lasing* mode (i.e., single frequency) operation of the DBE laser in contrast to the multimode nature of conventional uniform FPC lasers as shown in Fig. 20 (c) and Fig. 20(d). The lasing platform has also been developed by the team for realistic implementation including coupled resonator optical waveguides (CROWs) as in [9], [12] shown in Fig. 20(a, left). This laser exhibits unique scaling of the lasing threshold versus cavity length (in unit cells). The DBE threshold is the lowest and it is the one that decreases the fastest with cavity length. The comparative plot in Fig. 20(b) clearly shows that the DBE laser has a significantly lower lasing threshold as compared to the RBE and the conventional uniform FPC lasers having the same Q-factor.

This laser regime show great potentials for low or moderate laser power levels, in terms of power efficiency and spectral accuracy. This research also suggest an analogous mechanisms involving distributed power extraction and distributed gain may lead to very high power lasers with high power efficiency and spectral purity.

ii. Low-threshold grid array oscillators with EPD

Generation of microwave and mm-wave radiating with high efficiency is challenging, especially at mm-waves. The UCI team is exploring for the first time a new paradigm for highly efficient array-antenna oscillators utilizing EPDs ([C1], [C25], and another manuscript in preparation). This condition is realized in CTLs loaded with multiple active devices and radiating elements when a “gain and loss balance” is achieved. Gain and loss balance means the particular condition with gain and loss distribution that generates the EPD. (The PT-symmetry condition is a very special case based on pure structural and gain symmetry.) In the proposed case the “loss” is actually distributed power extraction (i.e., radiation “loss”, from a circuit point of view), so this device can be used for high intensity radiation. The team has demonstrated the EPD condition in microstrip CTLs and conceived a novel scheme to design either (i) low-threshold radiating grid oscillators, where the radiation threshold of the active device is very low as shown in Fig. 21, or (ii) very high power radiating grid oscillators. Indeed, this new operational principle would enhance the efficiency of spatial power combiners and active antennas, robustness of oscillations, stability of oscillation frequency as well as the tunability and reconfigurability of antenna arrays for mm-waves and 5G applications.

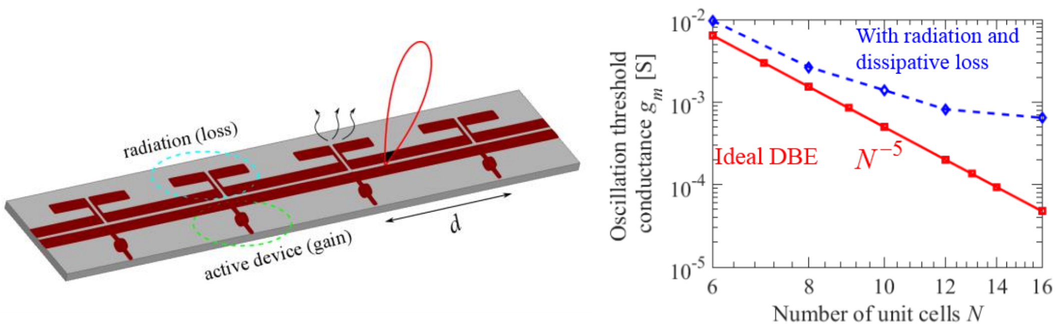


Fig. 21. (Left) Configurations of CTLs that exhibit exceptional points of degeneracy (EPD) based on coupled transmission lines (CTL) with gain and loss, [C1], [C25]. The CTL is loaded with gain and loss in the form of active devices and radiating antennas, respectively. (Right) The minimum amount of the gain conductance per unit cell required to start oscillations is shown to exhibit a unique trend typical of the DBE, by increasing the number of the unit cells.

iii. Pulse generation devices based on modal degeneracy

The UCI group is investigating and proposing a *new paradigm for microwave pulse compression devices* based on EPDs in resonators [C12]. The technique relies on Q-switching of a high-Q cavity once it is full of RF energy that leads to generation of a high-power sharp pulse [see Fig. 22]. The proposed approach benefits from the structured energy distribution inside a DBE cavity to form a pulse with high gain and very short temporal width. The cavity is filled with energy while it possesses and EPD, and hence it freezes light inside the cavity. Once the switches are operated the cavity does not possess the EPD anymore, so the group velocity is large, and the electromagnetic energy can escape quickly.

Compared to a conventional way to produce pulses from *Q*-switching of a uniform cavity with the same *Q*-factor, the proposed technique seems to produce at least 5 times higher output pulse power and very large compression ratio (defined as the ratio of the feeding pulse temporal width to that of the output compressed pulse). The technique proposed by the team is based on microstrip lines loaded with switches; however, it will be also extended to waveguides for high power microwave pulse generation, and also for short laser pulses.

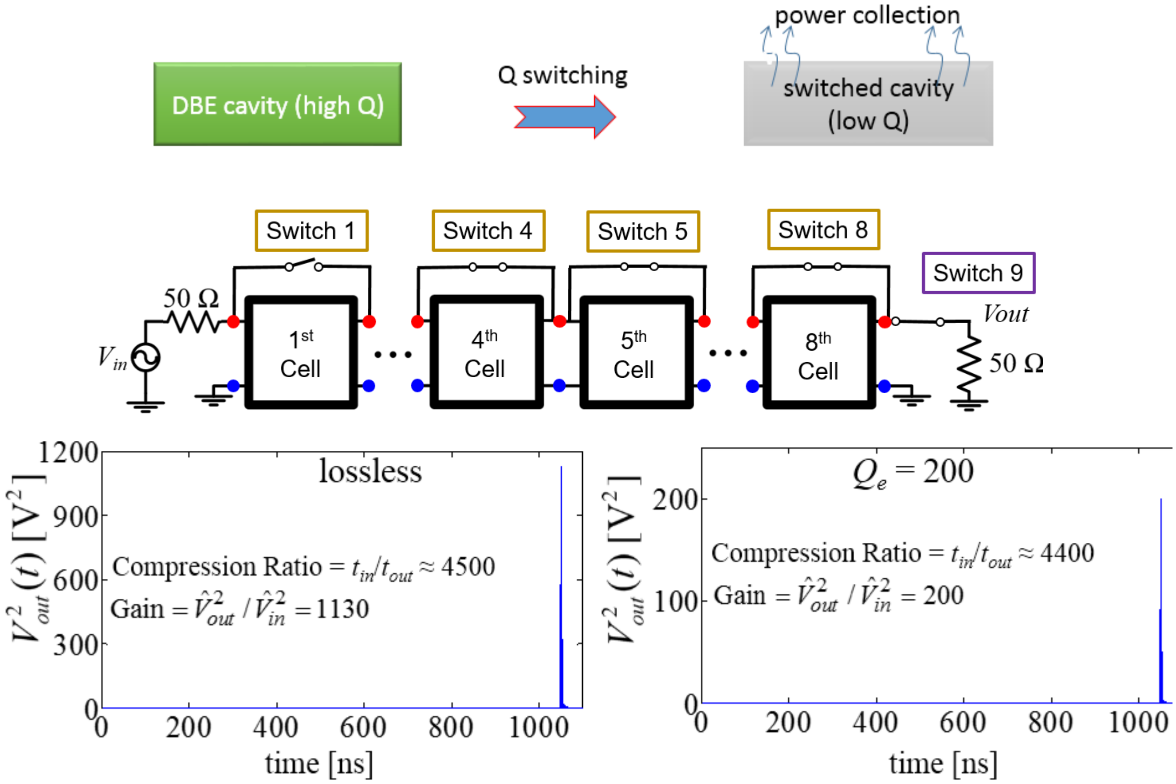


Fig. 22. (Top) Conceptual framework for a pulse compression device based on EPD and *Q*-switching, proposed in [C12]. (Middle) Schematic of a DBE resonator made of 8 unit cells with controlled switches that switch the high *Q* DBE cavity into a low-*Q* cavity for which energy can be extracted by shorting the switches on the upper line. (Bottom) Output load squared voltage for two cases of lossless and lossy DBE structures in which the compression ratio is ~4500 and a gain of at least 200 when losses are considered (Q_e is the element *Q* factor equal to 200).

c) Other applications associated to EPDs

i. Gain enhancement near stationary inflection points 3rd order EPD (SIP)

The team have investigated a new synchronization regime in traveling wave tubes (TWTs) based on slow wave structures (SWSs) operating near a third order degeneracy condition in their dispersion diagram [13]. This special three-eigenmode synchronization is associated with a stationary inflection point (SIP) that is manifested by the coalescence of three Floquet-Bloch eigenmodes in the SWS. The team has designed a periodic coupled transmission line that is consisting of three lines and two segments A and B in each unit cell, as shown in Fig. 23(a), to obtain SIP. The team after that has explored how electron beam can interact to this structure when it operates at SIP. One interesting feature of the proposed idea is that the slope of the dispersion relation can be manipulated to be positive or negative having small group velocities as shown in Fig. 23(b). It has been shown that positive slope SIP will result in higher bandwidth at the cost of gain/efficiency and negative slope SIP schemes can be used in low power oscillator designs due to lower thresholds needed to go to the oscillation regime compared to prior structures. When the SIP structure is synchronized with the electron beam potential benefits for amplification include (i) gain enhancement, (ii) gain-bandwidth product improvement, and (iii) higher power efficiency, when compared to conventional Pierce-like TWTs as shown in Fig. 23(c) and Fig. 23(d). The potential application of this concept will be in novel wideband power amplifiers due to the lenient dispersion relation and higher tolerance to electron beam current compared to DBE schemes while preserving high gains. This concept and its applications will be further developed by the UCI group as well as investigating a realistic implementation of the SIP in metallic SWS.

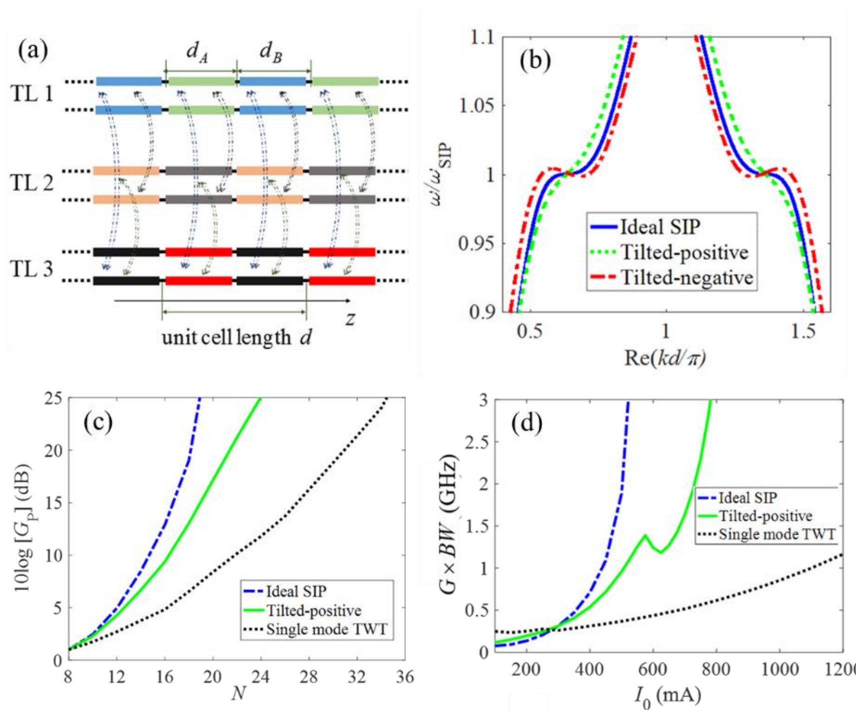


Fig. 23. (a) Schematic of a periodic coupled transmission line (CTL) consisting of three lines and two segments A and B in each unit cell having internal coupling through distributed inductance and/or capacitances (b) Dispersion relation of an infinitely long periodic coupled transmission line having SIP at angular frequency ω_{SIP} for three different cases of: ideal (zero slope), positive slope, and negative slope at SIP region. (c) Power gain (in dB) of the proposed ideal SIP and tilted-positive regimes of TWT operation, compared with a standard regime of amplification based on a finite, periodic TL, modeling a single mode TWT, having the same characteristic impedance and loading plotted versus the number of SWS unit cells, *N*. (d) Comparison of the gain-bandwidth product (G×BW) of TWT amplifiers based on SWS regimes based on ideal SIP and tilted-positive dispersion, versus electron beam current I_0. Results are also compared with a TWT made of a SWS with only one periodic TL (single mode TWT) having the same length and average characteristic impedance.

ii. Space-time variance and induced EPDs for non-reciprocal devices

Non-reciprocal devices are the core components of many microwave and optical communication systems. The most established and ubiquitous approach to induce nonreciprocity is to use ferromagnetic materials or ferrites biased by a static magnetic field. The resulting Faraday effects require large static magnetic fields and therefore Faraday rotators and isolators are usually very bulky. The team is investigating a new way to manipulate and engineer the dispersion diagram of modes in guiding systems utilizing linear and time-variant electrodynamical systems with space-time modulation. This enables an induced non-reciprocity and one-way propagation, unidirectionality, etc. Importantly, different EPDs, including the degenerate band edge (DBE) and the stationary inflection point (SIP), are perceived by tuning the space-time modulation parameters such as modulation depth and frequency in [C13]. In addition, the team is showing that Faraday effects and isolation can be greatly enhanced by employing the four degenerate eigenmodes associated with the DBE [see Fig. 24]. The UCI group is working on various aspects of designing such non-reciprocal devices such as (i) realizing high efficiency and low-phase noise space-time modulated oscillators and (ii) high power isolators and non-reciprocal traveling wave tubes.

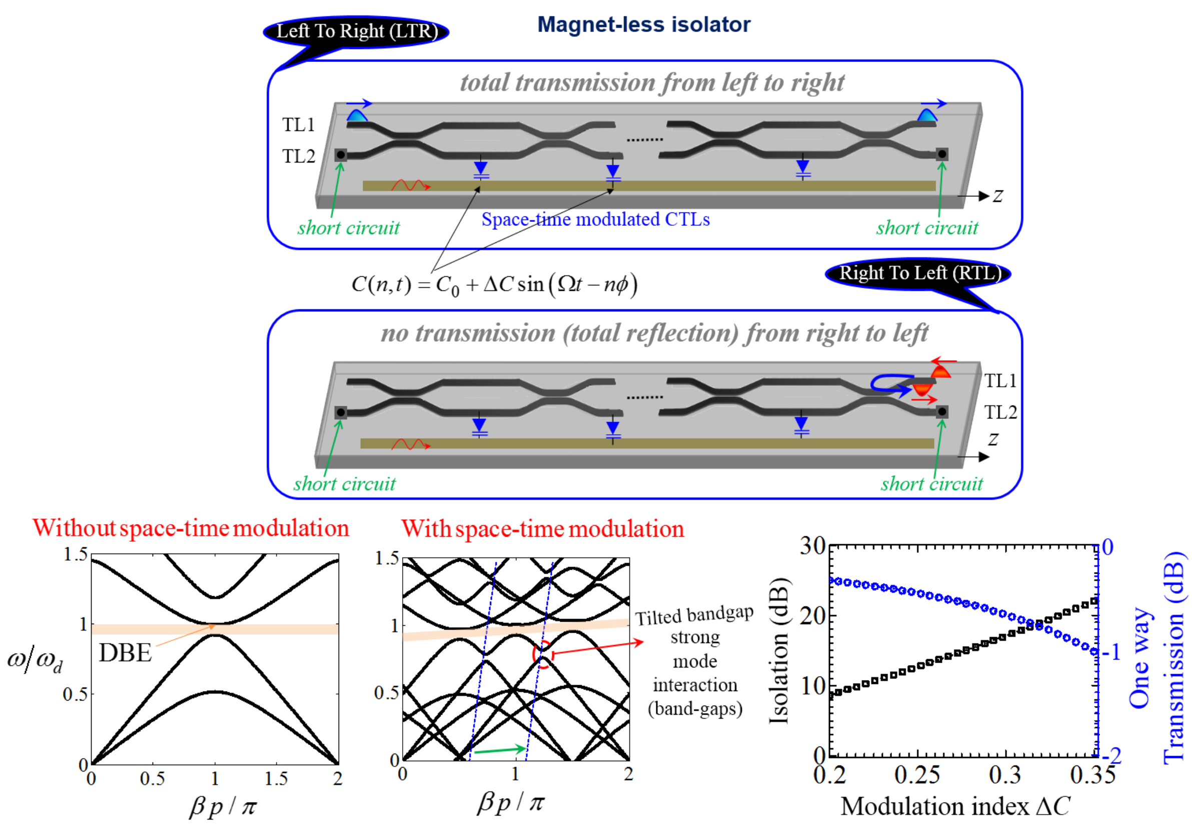


Fig. 24. (Top) Non-reciprocal coupled transmission lines (CTLs) with space-time modulations and EPDs, see [C13]. The two concepts intertwin to conceive a new class of a magnet-free isolators. (Bottom Left) Dispersion relation of periodically-coupled CTLs in the absence and presence of space-time modulations. (Bottom right) Transmission and isolation of the proposed EPD-based space-time modulated CTL showing non-reciprocity.

V. Faculty and Graduate Students Involved

PI: Prof. Filippo Capolino

PhD Students that graduated in Fall 2017: Mohamed A. K. Othman, Mehdi Veysi

PhD Students: Farshad Yazdi, Mohamed Y. Nada, Dmitry Oshmarin, Ahmed F. Abdelshafy, Hamidreza Kazemi, Tarek Mealy

MS Students: Muhannad R. S. Alshetaiwi (gradated Spring 2018), Puxi Zhou (graduated Summer 2018). Both Theses have been on subjects related to EPDs.

Collaborators:

- Prof. Alex Figotin, UC Irvine
- Prof. Christos Christodoulou, Prof. Edl Schamiloglu, University of New Mexico (MURI)
- Prof. Michael Green, UC Irvine (circuit oscillators, CMOS, and grid oscillating arrays)
- Prof. Ozdal Boyraz, UC Irvine (silicon photonics)
- Prof. Vincenzo Galdi, University of Sannio, Italy (dispersion engineering using spatial dispersion)
- Prof. Alexander B. Yakovlev, University of Mississippi
- Prof. George Hanson, University of Wisconsin-Milwaukee.

VI. Publications

Archival Publications (chronological order):

2016

[1] M. A. K. Othman, F. Yazdi, A. Figotin, and F. Capolino, "Giant gain enhancement in photonic crystals with a degenerate band edge," *Phys. Rev. B*, vol. 93, no. 2, p. 024301, Jan. 2016.

[2] V. A. Tamma, A. Figotin, and F. Capolino, "Concept for Pulse Compression Device Using Structured Spatial Energy Distribution," *IEEE Trans. Microw. Theory Tech.*, vol. 64, no. 3, pp. 742-755, Mar. 2016.

[3] M. A. K. Othman, M. Veysi, A. Figotin, and F. Capolino, "Giant Amplification in Degenerate Band Edge Slow-Wave Structures Interacting with an Electron Beam," *Phys. Plasmas 1994-Present*, vol. 23, no. 3, p. 033112, Mar. 2016.

[4] M. A. K. Othman, V. A. Tamma, and F. Capolino, "Theory and New Amplification Regime in Periodic Multi Modal Slow Wave Structures with Degeneracy Interacting with an Electron Beam," *IEEE Trans Plasma Sci*, vol. 44, no. 4, pp. 594 – 611, April 2016.

[5] M. A. K. Othman, M. Veysi, A. Figotin, and F. Capolino, "Low Starting Electron Beam Current in Degenerate Band Edge Oscillators," *IEEE Trans Plasma Sci*, vol. 44, no. 6, pp. 918-929 June 2016.

2017

[6] M. A. K. Othman, X. Pan, Y. Atmatzakis, and C. G. Christodoulou, "Experimental Demonstration of Degenerate Band Edge in Metallic Periodically-Loaded Circular Waveguide," *IEEE Microw. Theory Techn.*, vol. 1611, no. 8, p. 9, 2017.

[7] M. A. K. Othman and F. Capolino, “Theory of Exceptional Points of Degeneracy in Uniform Coupled-Waveguides and Balance of Loss and Gain,” *IEEE Trans. Antennas Propag.*, vol. 65, no. 10, pp. 1–15, 2017.

[8] M. A. K. Othman, V. Galdi, and F. Capolino, “Exceptional points of degeneracy and PT - symmetry in photonic coupled chains of scatterers,” *Phys. Rev. B 95*, 104305, 2017.

[9] M. Y. Nada, M. A. K. Othman, and F. Capolino, “Theory of coupled resonator optical waveguides exhibiting high-order exceptional points of degeneracy,” *Phys. Rev. B*, vol. 96, no. 18, p. 184304, Nov. 2017.

2018

[10] J. Sloan, M. Othman, and F. Capolino, “Theory of Double Ladder Lumped Circuits with Degenerate Band Edge,” *IEEE Trans. Circuits Syst. Regul. Pap.*, vol. 65, no. 1, pp. 3–13, 2018.

[11] M. Veysi, M. A. K. Othman, A. Figotin, and F. Capolino, “Degenerate band edge laser,” *Phys. Rev. B*, vol. 97, no. 19, p. 195107, May 2018.

[12] M. Y. Nada, M. A. K. Othman, O. Boyraz, and F. Capolino, “Giant Resonance and Anomalous Quality Factor Scaling in Degenerate Band Edge Coupled Resonator Optical Waveguides,” *J. Light. Technol.*, vol. 36, no. 14, pp. 3030 - 3039, 2018.

[13] F. Yazdi, M. A. K. Othman, M. Veysi, A. Figotin, and F. Capolino, “A New Amplification Regime for Traveling Wave Tubes with Third Order Modal Degeneracy,” *IEEE Trans. Plasma Sci.*, vol. 46, no.1, pp. 43 – 56, Jan. 2018.

[14] A. F. Abdelshafy, M. A. K. Othman, F. Yazdi, M. Veysi, A. Figotin, and F. Capolino, “Electron-Beam-Driven Devices with Synchronous Multiple Degenerate Eigenmodes,” *IEEE Trans. Plasma Sci.*, vol. 46, no.8, pp. 3126-3138, 2018.

[15] D. Oshmarin, F. Yazdi, M. A. K. Othman, J. Sloan, M. Radfar, M. M. Green, and F. Capolino, “Oscillator Based on Lumped Double Ladder Circuit with Band Edge Degeneracy,” arXiv preprint, *arXiv:1610.00415* (2016). Accepted in *IET Circuits, Devices & Systems*, 2018, subject to revision.

[16] G. W. Hanson, A. B. Yakovlev, M. Othman, and F. Capolino, "Exceptional Points of Degeneracy and Branch Points for Transmission-Line Problems - Linear Algebra and Bifurcation Theory Perspectives," arXiv preprint *arXiv:1804.03214* (2018). Accepted in *IEEE Trans. Antennas Propag.*, vol. 66, 2018, subject to revision.

[17] H. Kazemi, M. Y. Nada, T. Mealy, A. F. Abdelshafy, and F. Capolino, "Exceptional Points of Degeneracy Induced in Linear Time Periodic Systems," arXiv preprint *arXiv:1804.01075* (2018).

[18] A. F. Abdelshafy, M. Othman, D. Oshmarin, A. Al-Mutawa, and F. Capolino, " Exceptional Points of Degeneracy in Periodically-Coupled Waveguides and the Interplay of Gain and Radiation Loss: Theoretical and Experimental Demonstration," arXiv preprint *arXiv* (submitted in 13 Sept. 2018).

Conference Publications:

2016

[C1] M. A. K. Othman, M. Veysi, and F. Capolino, "Theory of gain enhancement in periodic structures with a degenerate band edge," *National Radio Science Meeting (USNC-URSI NRSM), 2016 United States National Committee of URSI*, Boulder, CO, Jan. 5-8 2016.

[C2] M. A. K. Othman, M. Veysi, A. Figotin, and F. Capolino, "Degenerate Band Edge Electron Beam Oscillators: Low Starting Current," *IEEE International Vacuum Electronics Conference (IVEC)*, Monterey, CA, April 19-21, 2016.

[C3] M. A. K. Othman, and F. Capolino, "Parity-Time Symmetry in Chain of Scatterers," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Fajardo, Puerto Rico, June 26 - July 1, 2016. [Invited to a special session, Novel Paradigms, Challenges, Perspectives in Wave Scattering & Propagation]

[C4] M. A. K. Othman, A. Figotin and F. Capolino, "Super Synchronous Operation of Traveling Wave Tubes Based on Band Edge Degeneracy," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Fajardo, Puerto Rico, June 26 - July 1, 2016.

[C5] M. Veysi, M. A. K. Othman, F. Capolino, "Time Domain Analysis of Coupled-Waveguides with Modal Degeneracies and Gain," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Fajardo, Puerto Rico, June 26 - July 1, 2016.

[C6] M. A. K. Othman, F. Yazdi, F. Capolino, "Exceptional Points of Degeneracy in Coupled-Mode Periodic Structures," *International Symposium on Electromagnetic Theory (EMTS)*, Espoo, Finland, August 14–18, 2016.

[C7] M. A. K. Othman and F. Capolino, "Coupled Waveguides with Exceptional Points of Degeneracies," *SPIE NanoScience + Engineering (OP16N)*, San Diego, California August 28 – September 1, 2016. [Invited to a special session, Non-Hermitian Photonics]

[C8] M. A. K. Othman, and F. Capolino, "Theory of Coupled Waveguides with Modal Degeneracies and Gain," *Metamaterials'2016. The 10th International Congress on Advanced Electromagnetic Materials in Microwaves and Optics*, Crete, September 17-22 2016.

2017

[C9] M. A. K. Othman and F. Capolino, "Theory of Exceptional Points of Degeneracy in Coupled Waveguides with Balanced Gain and Loss," *URSI National Radio Science Meeting (NRSM)*, Boulder, CO, Jan. 4-7, 2017.

[C10] M. A. K. Othman, M. Veysi, F. Yazdi, M. Nada, D. Oshmarin, A. Figotin, and F. Capolino, "Multimodal Waveguides with Exceptional Points of Degeneracy of Various Orders," *URSI National Radio Science Meeting (NRSM)*, Boulder, CO, Jan. 4-7, 2017. [Invited Talk to a special session, Advanced Analysis, Design, and Applications of Waveguiding Structures]

[C11] M. A. K. Othman, M. Veysi, A. Figotin, and F. Capolino, "Efficient Generation of High Power Microwaves Using Degenerate Band Edge Oscillators," *IEEE International Conference on Plasma Science (ICOPS)*, Atlantic City, NJ, May 21-25, 2017. [Plenary Talk]

[C12] D. Oshmarin, M. A. K. Othman, F. Capolino, "Microwave Pulse Compression Devices with Modal Degeneracy," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, San Diego, CA, July 9-14, 2017.

[C13] M. Veysi, M. A. K. Othman, H. Kazemi, F. Capolino, "Time Variant induced non-reciprocity enhanced by exceptional points of degeneracy," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, San Diego, CA, July 9–14, 2017.

[C14] M. A. K. Othman, X. Pan, G. Atmatzakis, C. Christodoulou, and F. Capolino., "Experimental Verification of Degenerate Band Edge Dispersion in Metallic Waveguides," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, San Diego, CA, July 9–14, 2017.

[C15] F. Yazdi, M. A. K. Othman, M. Veysi, F. Capolino, "Third Order Modal Degeneracy in Waveguides: Features and Application in Amplifiers," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, San Diego, CA, July 9–14, 2017.

[C16] M. A. K. Othman, and F. Capolino, "Coupled Transmission Line Array Antennas with Exceptional Points of Degeneracy," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, San Diego, CA, July 9-14, 2017.

[C17] M. A. K. Othman, M. Y. Nada, M. Veysi, A. Figotin, F. Capolino, "Low-threshold lasing in coupled resonator optical waveguides with exceptional points of degeneracy," *SPIE Optics+Photonics*, San Diego, CA, Aug. 8-10, 2017. [Invited Talk]

[C18] M. A. K. Othman, M. Veysi, F. Yazdi, M. Y. Nada, A. F. Abdelshafy, A. Figotin, and F. Capolino, "Exceptional Points of Degeneracy in Coupled Modes: Theory and Applications," *Metamaterials 2017, The 11th International Congress on Artificial Materials for Novel Wave Phenomena*, Marseille, France, Aug. 28-31, 2017. [Invited Talk]

[C19] M. Othman, F. Capolino, "Theory and Applications of Exceptional Points of Degeneracies in Gain and Loss Balanced Devices," *XXXIInd International Union of Radio Science URSI General Assembly and Scientific Symposium*, Montreal, Canada, Aug. 19-26, 2017.

2018

[C20] M. A. K. Othman, and F. Capolino, "New Paradigm in Coherent Radiating Oscillators Based on Waveguides with Exceptional Points of Degeneracy," *URSI National Radio Science Meeting (NRSM)*, Boulder, CO, Jan. 4-7, 2018. [Invited Talk to a special session, Advanced Analysis, Design & Applications of Waveguiding Structures]

[C21] M. Y. Nada, M. A. K. Othman, and F. Capolino, "High order exceptional points of degeneracy in coupled resonators optical waveguides," *SPIE Photonics West 2018*, San Francisco, CA, Jan. 27- Feb. 1, 2018

[C22] M. A. K. Othman, A. F. Abdelshafy, A. Figotin, and F. Capolino, "Exceptional points of degeneracy for enhanced interaction in multimode electron beam devices," *IEEE International Vacuum Electronics Conference*, Monterey, California, April 24-26, 2018.

[C23] M. Y. Nada, M. A. K. Othman, and F. Capolino, "Exceptional Points of Degeneracy in lossless Periodic Coupled Waveguides," *in Conference on Lasers and Electro-Optics, OSA Technical Digest (online) (Optical Society of America)*, paper FM3Q.5, San Jose, CA May 8-13, 2018.

[C24] F. Capolino et al., "Exceptional Points of Degeneracy in Coupled Mode Systems: Theory and Applications," *Int. Conf. Nanoplasm, New Frontiers in Plasmonics and Nano-photonics*, Cetraro, Italy, June 11-14, 2018. [Invited Talk]

[C25] A. F. Abdelshafy, M. A. K. Othman, D. Oshmarin, M. Y. Nada, and F. Capolino, "Exceptional Points of Degeneracy in a Linear Array Oscillator with Gain and Loss Balance," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Boston, MA, July 8–13, 2018.

[C26] T. Mealy, M. Y. Nada, and F. Capolino, "Realization of Fourth Order Exceptional Points of Degeneracy in Uniform Coupled-Waveguides," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Boston, MA, July 8–13, 2018.

[C27] M. Y. Nada, M. A. K. Othman, F. Yazdi, D. Oshmarin, A. F. Abdelshafy, and F. Capolino, "Unique Charactersitics and Applications of Systems with Exceptional Points of Degeneracy," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Boston, MA, July 8–13, 2018 [Invited Talk to a special session, Functional Material Platforms Enabling Exotic Scattering Phenomena]

[C28] M. Y. Nada, A. F. Abdelshafy, T. Mealy, F. Capolino, "Exceptional points of degeneracy in coupled mode structures: theory and applications," *SPIE Optics + Photonics (International Society for Optics and Photonics)*, San Diego, CA, Aug. 19-23, 2018 [Invited talk].

[C29] M. Y. Nada, A. F. Abdelshafy, T. Mealy, F. Capolino, "General conditions to realize exceptional points of degeneracy and applications," *International Congress on Advanced Electromagnetic Materials in Microwaves and Optics (Metamaterials'2018)*, Espoo, Finland, August 27–September 1, 2018 [Invited talk].

[C30] M. Y. Nada, A. F. Abdelshafy, T. Mealy, F. Yazdi, H. Kazemi, A. Figotin, F. Capolino, "Various Topologies of Coupled-mode Structures Exhibiting Exceptional Points of Degeneracy," *IEEE International Conference on Electromagnetics in Advanced Applications (ICEAA)*, Cartagena, Colombia, Sept. 10–14, 2018.

Master Theses

[MT1] Muhannad Alshetaiwi, "Third Order Exceptional Point of Degeneracy in Coupled Resonators at Radio Frequencies," M.S. thesis, Univ. of California, Irvine, CA, USA, 2018.

[MT2] Puxi Zhou, "Mode Coupling and Degeneracy Condition in Multilayer Waveguide Structure with Grating," M.S. thesis, Univ. of California, Irvine, CA, USA, 2018.

Ph.D Thesis

[PhD1] M. A. K. Othman, "DISPERSION ENGINEERING OF PHOTONIC AND HIGH-POWER DEVICES WITH EXCEPTIONAL POINTS OF DEGENERACY," Ph.D. thesis, Univ. of California, Irvine, CA, USA, 2017.

Invited Talks/Seminars at Workshops and at Universities:

2016

- “Parity-Time Symmetry in Chain of Scatterers,” *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Fajardo, Puerto Rico, June 26 - July 1, 2016. [Invited to a special session, Novel Paradigms, Challenges, Perspectives in Wave Scattering & Propagation]
- “Coupled Waveguides with Exceptional Points of Degeneracies,” *SPIE NanoScience + Engineering (OP16N)*, San Diego, California August 28 – September 1, 2016. [Invited to a special session, Non-Hermitian Photonics]
- “Novel concept of exceptional point of degeneracy for oscillators and other devices," *in the IEEE coastal Los Angeles section technical symposium (CLASTECH)*, Los Angeles, CA, October 28th, 2016. [Invited Talk]

2017

- “Multimodal Waveguides with Exceptional Points of Degeneracy of Various Orders,” *URSI National Radio Science Meeting (NRSM)*, Boulder, CO, Jan. 4-7, 2017. [Invited Talk to a special session, Advanced Analysis, Design, and Applications of Waveguiding Structures]
- “Low Threshold RF and Optical Oscillators Using Exceptional Points of Degeneracy," *in the Jacobs School of Engineering*, Invited Talk at University of California, San Diego, CA, Feb. 28th, 2017.
- “Exceptional points of degeneracy,” Invited Talk *at CREOL*, University of Central Florida, Orlando, FL, May 1st, 2017.
- “Efficient Generation of High Power Microwaves Using Degenerate Band Edge Oscillators,” *IEEE International Conference on Plasma Science (ICOPS)*, Atlantic City, NJ, May 20–23, 2017. [Invited as a **Plenary talk** for the ICOPS 2017].
- “Exceptional Points of Degeneracy in Coupled Modes: Theory and Applications,” *Metamaterials 2017, The 11th International Congress on Artificial Materials for Novel Wave Phenomena*, Marseille, France, Aug. 28-31, 2017. [Invited Talk]
- “Electromagnetics in Metamaterials and Applications of Exceptional Points of Degeneracy in Multimode Electromagnetic Systems,” Invited Talk *at AFRL*, Wright-Patterson Air Force Base, June 13th, 2017.

2018

- “New Paradigm in Coherent Radiating Oscillators Based on Waveguides with Exceptional Points of Degeneracy,” *URSI National Radio Science Meeting (NRSM)*, Boulder, CO, Jan. 4-7, 2018. [Invited Talk, Special session, Advanced Analysis, Design & Applications of Waveguiding Structures]

- "Exceptional Points of Degeneracy in Coupled Mode Systems: Theory and Applications," *Int. Conf. Nanoplasm, New Frontiers in Plasmonics and Nano-photonics*, Cetraro, Italy, June 11-14, 2018. [Invited Talk]
- "Unique Characteristics and Applications of Systems with Exceptional Points of Degeneracy," *IEEE International Symposium on Antennas and Propagation/USNC-URSI National Radio Science meeting*, Boston, MA, July 8–13, 2018 [Invited Talk, special session, Functional Material Platforms Enabling Exotic Scattering Phenomena]
- "Exceptional points of degeneracy in coupled mode structures: theory and applications," *SPIE Optics + Photonics* (*International Society for Optics and Photonics),* San Diego, CA, Aug. 19-23, 2018 [Invited talk].
- "General conditions to realize exceptional points of degeneracy and applications," *International Congress on Advanced Electromagnetic Materials in Microwaves and Optics (Metamaterials'2018)*, Espoo, Finland, Aug. 27–Sept. 1, 2018 [Invited talk].