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A METHOD FOR PLACING SOURCES

IN MULTISTATIC SONAR NETWORKS

by

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The allocation of underwater sensors for surveillance purposes is a fundamental problem in naval operations. Passive receivers have been used in recent decades; however, modern submarines are increasingly difficult to detect with receivers alone. The idea of deploying non-collocated sources and receivers is a promising alternative to passive sensor fields and to traditional monostatic sonar fields made up of collocated sources and receivers. Such a multistatic sonar network has a number of advantages, but it is much more difficult to deploy optimally. In this work, we consider the problem of optimally positioning active multistatic sonar sources for a point coverage application where all receivers and targets are fixed and stationary. We formulate exact and approximation algorithms for this problem using a definite range sensor model.

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ABSTRACT

The allocation of underwater sensors for surveillance purposes is a fundamental problem in naval operations. Passive receivers have been used extensively in recent decades; however, modern submarines are increasingly difficult to detect with receivers alone. The idea of deploying non-collocated sources and receivers is a promising alternative to passive sensor fields and to traditional monostatic sonar fields made up of collocated sources and receivers. Such a multistatic sonar network has a number of advantages, but it is much more difficult to deploy optimally. In this work, we consider the problem of optimally positioning active multistatic sonar sources for a point coverage application where all receivers and targets are fixed and stationary. We formulate exact and approximation algorithms for this problem using a definite range sensor model.

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I. INTRODUCTION

Situational awareness of the undersea environment is critical in undersea warfare. Although passive sensors have historically been used to detect enemy submarines with great success, recent declines in submarine acoustic-source levels have sparked a renewed interest in active undersea sensing (Lilley, 2014). An emerging tool for active undersea sensing is multistatic sonar. A multistatic sonar network (MSN) consists of two types of devices: sources and receivers. The basic operating concept of such a network is that the sources emit sound energy into the water; this energy is then reflected by objects in the environment, and the reflections are detected by receivers and used to make inferences about the surrounding objects. This is the same operating concept as that of a traditional monostatic sonar device consisting of a collocated source and receiver as shown in Figure 1(a); the distinction is that in a MSN, sources and receivers are not necessarily collocated (see Figure 1(b)). This distinction has important implications for the performance of the network as well as the analytical tractability of MSN design.

A MSN generally consists of a field of geographically dispersed sources and receivers (Fewell and Ozols, 2011). Such a network has a number of advantages. Key among these is covertness: although the locations of sources are readily apparent to any entity in the vicinity of the MSN, the locations of the receivers are more difficult to ascertain. This fact significantly complicates enemy countermeasures. A second advantage is cost. Amanipour and Olfat (2011) and Washburn and Karatas (2015) indicate that sources are significantly more expensive than receivers; thus, a multistatic network containing more receivers than sources could provide equivalent sensing capability to a monostatic network at a lower cost. Additionally, fewer sources result in reduced undersea noise pollution, an important environmental consideration (Jasny, 2005). MSNs can allow greater coverage, multi-angle observations, and improved tracking accuracy relative to equivalently sized monostatic networks (Cox, 1989). Furthermore, MSNs can facilitate multi-platform operations. For example, a source might originate from a ship or dipping helicopter, while receivers are deployed as sonobuoys (Washburn, 2010). For these reasons and others, MSNs are promoted as a way forward for anti-submarine warfare operations (Lilley ,2014).

However, these benefits come at a cost. Although the performance of a single monostatic sensor is relatively straightforward to model, the performance of a MSN is more complicated and depends strongly on the relative positions of the sources and receivers (Wang et al., 2008). In this study, our goal is to determine an optimal configuration for a MSN for a point coverage application. In particular, we are interested in optimizing the placement of static sources in a network of fixed receivers and targets. Although our motivation comes primarily from underwater detection systems, some aspects of our approach are generalizable to radar or geolocation systems.

This report is organized as follows. In Section 2, we review a selection of prior work relating to multistatic systems. Section 3 describes the aspects of MSN performance modeling relevant to our work. In Section 4 we describe new exact and approximations for optimizing the placement of static sources using definite range sensor model in a network of fixed receivers and targets. Section 5 contains the conclusions and our recommendations for future work.

II. LITERATURE REVIEW

Previous studies concerning MSNs mainly focus on areas such as tracking and data fusion, localization, imaging, ping scheduling, cost effectiveness, and sensing coverage problems. In many of these areas, the spatial configuration of the MSN is already determined and the focus is on optimizing its operation (e.g., ping scheduling) or interpreting the information collected by the MSN (e.g., data fusion). In contrast, our study takes place in the sensing coverage domain, in which the goal is to determine the spatial configuration of sensors that results in the best possible coverage according to some metric.

The sensing coverage domain can be further sub-categorized into three specialties: area coverage, barrier coverage, and point coverage (Cardei and Wu, 2004). Each of these specialties has a specific purpose: in the area coverage problem, the goal is to position sensors so as to cover as much of a two- or three-dimensional space as possible with adequate sensing capability. The barrier coverage problem is similar to the area coverage problem, but the space to be covered is a belt region or a one-dimensional line segment that separates one region from another. In the point coverage problem, the space to be covered is further restricted to a discrete set of target locations.

Among the sensing coverage studies indicated above, Krieger et al. (2003) analyze the interferometric performance of three different multistatic radar formations, including a set of independent microsatellites flying in close formation. Tharmarasa et al. (2009) also use a genetic algorithm; their goal is to determine the optimal locations and paths of mobile sources in order to maximize the tracking performance in a field of stationary receivers. Ozols and Fewell (2011) compare 27 different sonobuoy field designs to determine the most cost effective layout when a large area must be covered. Gong et al. (2013) study the barrier coverage problem for multistatic radar networks, and they determine the placement order and optimal placement spacing of the sensors on a line segment for minimizing barrier's vulnerability. Most of the aforementioned studies assume that the locations of the sources and receivers are to be determined by the user; in contrast, Washburn and Karatas (2015) consider a field of randomly-placed sources and receivers and develop a simple analytic theory for predicting the coverage of the MSN.

They then use this theory to study pattern optimization and cost/effectiveness maximization in area coverage scenarios. Using analytic results of Washburn and Karatas (2015) as a benchmark, Karatas and Craparo (2015) and Karatas et al. (2016) use Monte Carlo simulations to measure the direct blast effect in MSNs and to evaluate the area coverage performance of mobile multistatic search operations, respectively. In a more recent study Craparo et al. (2017) consider the point coverage problem for MSNs and develop the Divide Best Sector (DiBS) algorithm which seeks to determine an optimal source position assuming fixed receivers.

III. MULTISTATIC THEORY

Most prior work has focused on area coverage and barrier coverage problems. In contrast, we focus on the point coverage problem, in which the goal is to achieve good coverage of a number of discrete points in space, henceforth referred to as targets. An advantage of this modeling framework is that it allows the decision maker to explicitly model regions of the space that are unimportant (and thus contain no targets) and to distinguish the relative importance of other regions by associating a weight with each target. Targets may represent specific locations of strategic importance such as oceanic bastions, high value units and naval assets, oil platforms, offshore drilling stations, or ports that must be protected and kept under surveillance. Alternatively, an area coverage or barrier coverage problem can be transformed into a point coverage problem via discretization of the area of interest.

In an operational setting, receivers are sometimes deployed in advance throughout an area by an aircraft or ship, and the decision maker does not have any control over their positions (Tharmarasa et al., 2009). Given the locations of the receivers and targets, we consider the problem of optimally deploying a limited number of sources so as to monitor the discrete set of targets as well as possible. We assume that all sources, receivers, and targets remain stationary once placed, and for simplicity we consider a two-dimensional setting, though our approach generalizes to three dimensions.

A multistatic sonar system involves two important distances: the distance from the target to the source, denoted as $d_{t,s}$, and the distance from the target to the receiver, denoted $d_{t,r}$.

A general expression for transmission loss in multistatic systems can be written by introducing a parameter $\rho_{t,s,r}$, known as the *equivalent monostatic detection range* of target t from source s and receiver r (Fewell and Ozols, 2011; Washburn, 2010).

$$\rho_{t,s,r} = \sqrt{d_{t,s} d_{t,r}} \qquad (1)$$

If we first consider a binary detection model in which source *s* and receiver *r* detect target *t* if and only if $d_{t,s} d_{t,r} \le \rho^2$, where ρ is a threshold value for the equivalent monostatic detection range (see Figure 2), the detection probability $p_{t,s,r}$ of target t with source *s* and receiver *r* is written as:

$$p_{t,s,r} = \begin{cases} 1 & \text{if } d_{t,s}d_{t,r} \leq \rho^2 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Such a sensor model is sometimes called a *definite range* approximation or a cookie-cutter sensor. The inequality $d_{t,s}d_{t,r} \leq \rho^2$ defines the interior of a region called a *Cassini oval* (Cox, 1989).

In practice, there exists a small ellipsoidal *dead zone* between the source and receiver in which targets cannot be detected (Fewell and Ozols, 2011). The dead zone exists because within this region, the reflected sound from the target arrives at the receiver at nearly the same time as the original ping. This causes the reflected signal to be obscured, drastically reducing detection probability. However, as shown by Fewell and Ozols (2011), the effects of the dead zone phenomena can be greatly reduced by pulse compression techniques. Thus, for simplicity, we do not model the dead zone in this paper.

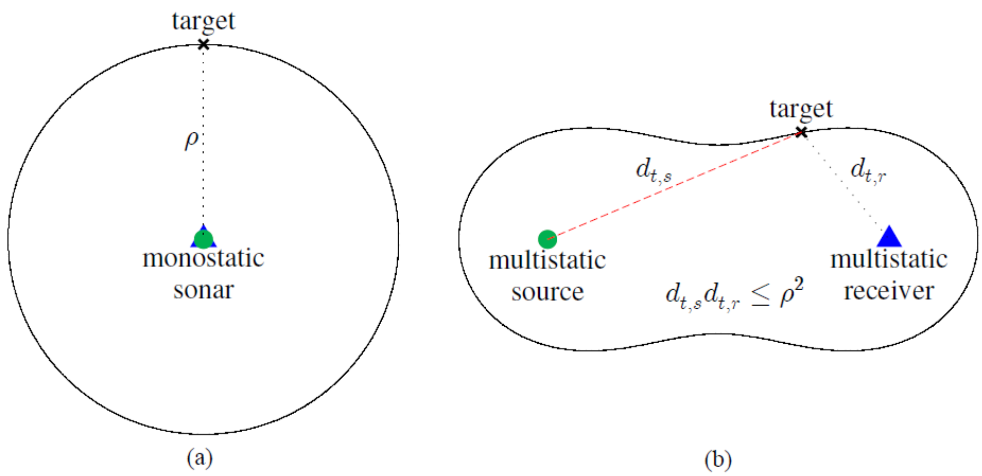

Figure 1. (a): Monostatic sensor with a disk shaped detection region of radius ρ. (b): Multistatic source and receiver detecting targets inside the region satisfying $d_{t,s}d_{t,r} \leq \rho^2$. Targets, receivers, and sources are denoted by $\times$, #, $\bullet$ symbols respectively.

The boundary of a Cassini oval is symmetric with respect to the line segment passing through the source and receiver, as well as an axis perpendicular to this line at its midpoint. The particular shape of a Cassini oval depends on the dimensionless separation parameter a/ρ, where a is the semi-distance between source and receiver. Figure 2 shows a set of Cassini ovals for different values of a/ρ, where without loss of generality ρ is fixed to 1.

As Figure 2 demonstrates, separation of a source and receiver can allow coverage of spatially disjoint regions not possible with a monostatic sensor. Depending on the spatial distribution of targets, this property can enable much more efficient coverage. However, the particular geometry of the Cassini oval coverage profile presents challenges not present in analysis of monostatic sonar systems.

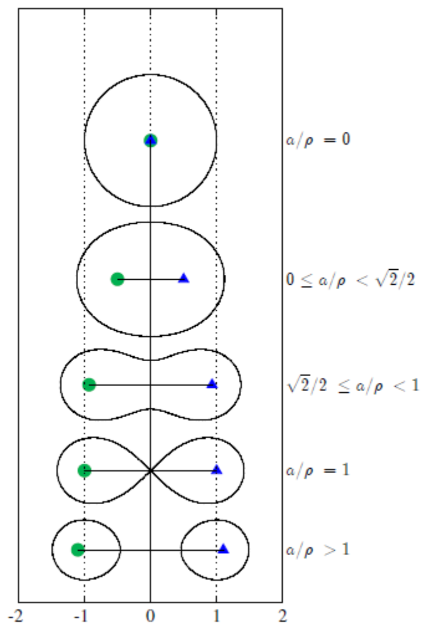

Figure 2. Cassini ovals corresponding to various values of a/ρ. Receivers and sources are denoted by # and • symbols respectively. For $a/\rho = 0$ the oval is a circle. For $a/\rho < \sqrt{2}/2$ the curve is a single loop. For $\sqrt{2}/2 \le a/\rho < 1$ the oval attains a dent on top and bottom. When $a/\rho = 1$ the curve is a lemniscate. For $a/\rho > 1$ the curve splits into two ovals (Karataş, 2013; Karataş and Akman, 2015).

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IV. PROBLEM FORMULATION

A. BASIC ASSUMPTIONS

We consider a set of receivers, *R*, and a set of targets, *T*, each of which is associated with a point in the two-dimensional Euclidean plane. We assume that the locations of both the receivers and targets are fixed and known; this assumption can reflect a scenario in which inexpensive receivers are dropped from an aircraft in order to provide coverage of a given set of target locations, for instance. We have a set of sources *S* available and would like to deploy them in such a way as to achieve good coverage of the targets. Each target $t \in T$ is associated with a weight w_t that reflects its relative importance. We assume that the environmental conditions in the plane are homogeneous, that all sources and receivers are omnidirectional, and that the aspect dependence of the targets' return signal strength is negligible.

B. MSN POINT COVERAGE MODEL FORMULATION

Because we consider fixed receivers and only wish to decide the locations of the sources, it is useful to rewrite $p_{t,s,r}$ as

$$p_{t,s,r} = \begin{cases} 1 & \text{if } d_{t,s} \le \rho^2 / d_{t,r} \\ 0 & \text{otherwise} \end{cases} \quad \forall t \in T, r \in R, s \in S \tag{3}$$

Given this detection model, we wish to position our sources in such a way as to maximize the total weight of the targets detected.

Note that although many source-receiver pairs could potentially detect target t, since $p_{t,s,r}$ is binary and nonincreasing in both $d_{t,s}$ and $d_{t,r}$, we need only consider the nearest source and nearest receiver to target t. We can therefore rewrite the overall detection probability of target t, $\overline{p}_t$, as a function of the distance to target t's nearest source $s^*(t)$ and nearest receiver $r^*(t)$:

$$\overline{p}_t = \begin{cases} 1 & \text{if } d_{t,s^*(t)} \le \rho^2 / d_{t,r^*(t)} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Equation (4) implies that for every target t, there exists a disk-shaped region δ_t that constitutes the set of possible locations at which a source can be placed that will detect target *t*. In other words, if target t's closest source $s^*(t)$ is contained within δ_t, then target t is detected with probability 1. If $s^*(t)$ is not contained within δ_t, then target t is not detected. Detection disk δ_t has its center at the location of target t, and its radius is $\rho^2 / d_{t,r^*(t)}$. In order to construct an algorithm for maximizing the number of targets detected, we denote the set of detection disks for all targets by $D = \{\delta_1, \delta_2, ..., \delta_T\}$. Note that if two or more detection disks overlap, then multiple targets can be detected by placing a single source in the region of overlap. We now construct a finite set of candidate locations for our sources.

C. GENERATING CANDIDATE LOCATIONS FOR SOURCES

We say that disk set $\bar{D} \subseteq D$ is a *mutually overlapping detection disk set* if there is at least one point x in the plane such that x is covered by δ_t for all $\delta_t \in \bar{D}$. Let O denote the set of all mutually overlapping detection disk sets. For a mutually overlapping disk set $\bar{D}_o \in O$, we define $\Pi\left(\bar{D}_o\right) \equiv \{x \mid x$ is covered by δ_t for all $\delta_t \in \bar{D}_o\}$. In other words, $\Pi\left(\bar{D}_o\right)$ is the set of points in the plane simultaneously covered by all disks in $\bar{D}_o$

Furthermore, we define a *maximal set of mutually overlapping detection disks* to be a mutually overlapping detection disk set that is not a subset of any other mutually overlapping detection disk set. In other words, a mutually overlapping detection disk set $\bar{D}_o$ is maximal if, for every point $x \in \Pi\left(\bar{D}_o\right)$, x is *not* covered by any $\delta_t \notin \bar{D}_o$. Let $M \subseteq O$ denote the set of all maximal sets of mutually overlapping detection disks. Note that the set of targets detected by a source placed at a location $x \in \Pi\left(\bar{D}_o\right)$ for some $\bar{D}_o \in M$ is a maximal set of targets that can be detected by a single source. Denote the set of targets detected by a source placed at a location $x \in \Pi\left(\bar{D}_o\right)$ as τ_o.

Maximal sets of mutually overlapping detection disks are of special interest in the multistatic point coverage problem, as evidenced by the following lemma.

Lemma 1. *Let* $X = \bigcup_{\bar{D}_o \in M} \Pi\left(\bar{D}_o\right)$. *Then, it is possible to restrict the sources to occupy only locations in X without sacrificing optimality.*

Proof of Lemma 1: Consider an optimal solution to the multistatic detection problem in which a source s is placed at location $x_s \notin X$, and denote the set of targets detected by s as $\tau\left(s\right)$. Since all targets $t \in \tau\left(s\right)$ are detected by a single source, clearly $\tau\left(s\right) \subseteq \tau_o$ for some $\bar{D}_o \in M$. Thus, it is possible to move source s to a location in $\Pi\left(\bar{D}_o\right)$ while still detecting all targets in $\tau\left(s\right)$. In this way, one can always construct an optimal solution in which each source occupies a location in *X*. □

Lemma 1 implies that for each $\bar{D}_o \in M$, it is sufficient to consider only one location $x \in \Pi\left(\bar{D}_o\right)$ as a possible source location, since no additional benefit would be realized from placing a second source at location $x' \in \Pi\left(\bar{D}_o\right)$ where $x' \neq x$. Thus, in order to optimally place all sources, one need only consider a finite number of candidate source locations; $\left|M\right|$ such locations, to be exact. Theorem 1 describes a procedure for generating such locations. In this theorem, we let x_t denote the location of target t and let $i_{t,t'}$ denote an intersection point of the boundaries of detection disks δ_t and $\delta_{t'}$, i.e., the points x for which $d_{t,x} = \rho^2 / d_{t,r^*(t)}$ and $d_{t',x} = \rho^2 / d_{t',r^*(t')}$. Note that there may be zero, one, or two such points, assuming $x_t \neq x_{t'}$; we consider all such points.

Theorem 1. *For each* $\bar{D}_o \in M$, *either* $x_t \in \Pi\left(\bar{D}_o\right)$ *for some* t, *or* $i_{t,t'} \in \Pi\left(\bar{D}_o\right)$ *for some* t *and* t'.

Proof of Theorem 1: For an arbitrary $\bar{D}_o \in M$, consider the boundary of region $\Pi\left(\bar{D}_o\right)$. This boundary consists of portions of the boundaries of the disks $\delta_t \in \bar{D}_o$, as shown in Figure 3. Suppose the boundary of $\Pi\left(\bar{D}_o\right)$ is exactly the boundary of a single disk δ_t, as shown in Figure 3(a). Then $x_t \in \Pi\left(\bar{D}_o\right)$. Now consider a region $\Pi\left(\bar{D}_o\right)$ whose boundary consists of portions of the boundaries of multiple detection disks, as shown in Figure 3(b). In this case, there must exist some t and t' such that $i_{t,t'} \in \Pi\left(\bar{D}_o\right)$. □

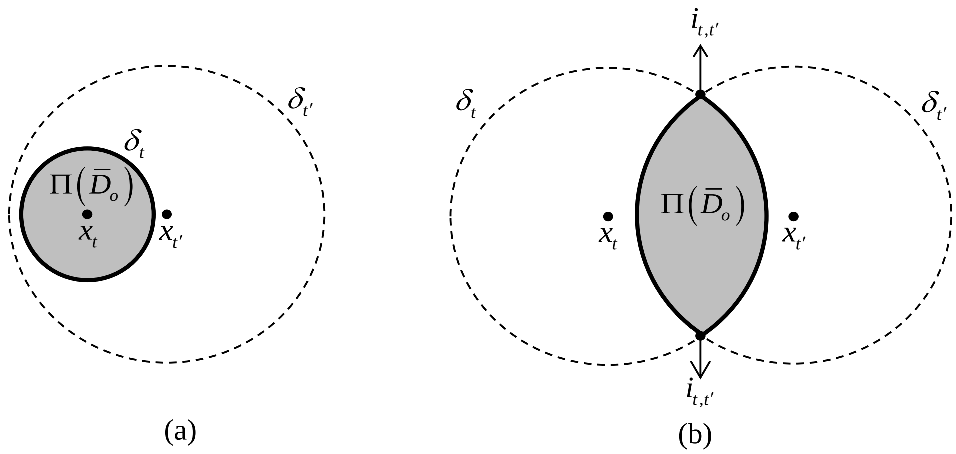

Figure 3. (a) The boundary of $\Pi\left(\bar{D}_o\right)$ denoted by solid line is exactly the boundary of a single disk δ_t (b): This boundary consists of portions of the boundaries of the disks $\delta_t, \delta_{t'}$.

Building on Theorem 1, we now construct algorithm LOC-GEN (see Figure 4), which generates a finite set of candidate locations for sources. We denote the set of all such candidate locations as *C*, indexed by $c \in C$, and we denote the position of candidate location c as x_c.

Algorithm LOC-GEN
Input parameters:

- Receiver set R
- Target set T
- Receiver locations $x_r \quad \forall r \in R$
- Target locations $x_t \quad \forall t \in T$
- Equivalent monostatic detection range ρ

1. Compute distances:

- Distances between targets and receivers: $d_{t,r} = \|x_t - x_r\| \quad \forall t \in T, r \in R$
- Set $d_{t,r^*(t)} = \min_r d_{t,r} \quad \forall t \in T$
- Distance between targets: $d_{t,t'} = \|x_t - x_{t'}\| \quad \forall t, t' \in T$

2. Create detection disks: disk $\delta_t \in D$ has center at x_t and radius $\rho^2 / d_{t,r^*(t)}$ $\forall t \in T$

3. Determine detection disk intersection points for all pairs of disks $\delta_t, \delta_{t'} \in D$ using equations (10) and (11) in Appendix A. Denote the set of intersection points of disks δ_t and $\delta_{t'}$ as $I_{t,t'}$, and note that $I_{t,t'}$ may contain zero, one, or two points. Denote the set of all disk intersection points as $\bar{I} = \bigcup_{t,t' \in T} I_{t,t'}$, indexed by $i \in \bar{I}$. Note that $|\bar{I}| \leq |T|^2 - |T|$.

4. Create an initial set of candidate locations as the union of the target locations and the set of intersection points: $C = \{x_1, x_2, ..., x_T\} \cup \bar{I}$.

5. Reduce C by removing candidate locations that do not represent maximal sets of mutually overlapping detection disks: Let $\tau(c)$ denote the set of all targets detected by a source placed at candidate location $c \in C$; that is, the set of all t such that $d_{t,c} \leq \rho^2 / d_{t,r^*(t)}$. Iterate over all $c, c' \in C$ such that $c \neq c'$. If $\tau(c) \subseteq \tau(c')$, remove c from C.

Output: Set C

Figure 4. Pseudo-code for generating candidate locations for sources. Algorithm LOC-GEN runs in time $O\left(|T|^5\right)$ and outputs a set C of size $O\left(|T|^2\right)$.

D. OPTIMAL AND NEAR-OPTIMAL SOURCE PLACEMENT

Given a finite set of candidate source locations as generated by LOC-GEN, we now formulate the integer linear program OPT-LOC, which selects the best set of $|S|$ source locations so as to maximize weighted coverage of the targets.

Indices and Sets

$t \in T$ targets

$c \in C$ candidate locations for sources

$\tau(c)$ set of targets detected by a source placed at candidate location c

Parameters

w_t value of target t

$|S|$ number of sources available

<u>Decision variables</u>

$$\varphi_c = \begin{cases} 1 & \text{if a source is placed at candidate location } c \\ 0 & \text{otherwise} \end{cases}$$

$$y_t = \begin{cases} 1 & \text{if target } t \text{ is detected} \\ 0 & \text{otherwise} \end{cases}$$

<u>Formulation OPT-LOC</u>

Objective Function:

$$\max_{\boldsymbol{\varphi},\mathbf{y}} \quad z = \sum_{t \in T} w_t y_t \tag{5}$$

Constraints:

$$y_t \le \sum_{c | t \in \tau(c)} \varphi_c, \ \forall t \in T \tag{6}$$

$$\sum_c \varphi_c \le |S| \tag{7}$$

$$0 \le y_t \le 1, \ \forall t \in T \tag{8}$$

$$\varphi_c \in \{0,1\}, \ \forall c \in C \tag{9}$$

The objective function (5) maximizes the total value of the targets detected. Constraint set (6) ensures that target t is detected only if a source is placed at a candidate location c that is close enough to enable detection. Constraint (7) states that the number of sources deployed cannot exceed the number available, and constraint sets (8) and (9) declare decision variable domains.

Although formulation OPT-LOC provides an optimal solution to the source placement problem, its computation time may be prohibitively high for large problem instances. However, it is possible to obtain a near-optimal source placement in polynomial time. To see this, note that OPT-LOC is a special case of the *maximum coverage* problem. Hochbaum (1997) describes the maximum coverage problem as follows: "Given a set system S and a parameter k, the *maximum coverage* problem is to find k sets such that the total weight of elements covered is maximized." In OPT-LOC, the set system in question is the set of targets covered by a source at each candidate location $(\tau(c))$, and we would like to select $|S|$ candidate locations. Hochbaum (1997) further points out that the maximum coverage problem is NP-hard by reduction from the set cover problem, and she proves that a greedy approach to set selection has a performance guarantee of $1-\left(1-\frac{1}{|S|}\right)^{|S|} > 1-\frac{1}{e}$. (This performance guarantee is also easily obtained by showing that the weight of the targets detected is a submodular function of the set of source locations selected.)

Thus, to obtain a near-optimal solution to the source location problem, we construct algorithm GREEDY-LOC-DEF in Figure 5.

Algorithm GREEDY-LOC-DEF

Input parameters:

- Target set T
- Set of candidate source locations C
- Set of targets detected by a source at each location c $\tau(c)$
- Value of target t w_t
- Number of sources available $|S|$

1. Initialization:

- Initialize the set of source locations selected as $\bar{C} = \varnothing$
- Initialize the set of targets detected as $\bar{T} = \varnothing$

2. Greedy selection: While $|\bar{C}| < |S|$, do

$$\bar{C} \leftarrow \bar{C} \cup \arg\max_{c} \sum_{t \in \bar{T} \cup \tau(c)} w_t$$

$$\bar{T} \leftarrow \bigcup_{c \in \bar{C}} \tau(c)$$

Return $\bar{C}$

Figure 5. Pseudo-code for obtaining a near-optimal solution to the source location problem for definite range sensor model.

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V. CONCLUSIONS AND FUTURE WORK

We have considered optimal placement of sources in a MSN containing fixed receivers, with the goal of achieving good coverage of a number of discrete targets. Using a definite range sensor model, we have described a procedure for generating a finite set of candidate locations guaranteed to contain optimal source locations (LOC-GEN), as well as two algorithms for selecting among these candidate locations (OPT-LOC and GREEDY-LOC). Future work will extend the approaches described in this work to other sensor models.

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APPENDIX A

Consider two disks δ_i and δ_j with centers at (x_i, y_i) and (x_j, y_j) and radii r_i and r_j, respectively. Let d_{ij} denote their separation distance. Note that the disks intersect if $d_{ij} \le r_i + r_j$ and overlap if $d_{ij} < r_i + r_j$.

The intersection points of δ_i and δ_j, (x_{ij}, y_{ij}) and (x'_{ij}, y'_{ij}), can be computed with the following formulae:

$$x_{ij}, x'_{ij} = \frac{x_j + x_i}{2} + \frac{(x_j - x_i)(r_i^2 - r_j^2)}{2d_{ij}^2} \pm \frac{y_j - y_i}{2d_{ij}^2}\sqrt{\left((r_i + r_j)^2 - d_{ij}^2\right)\left(d_{ij}^2 - (r_i - r_j)^2\right)} \quad (10)$$

$$y_{ij}, y'_{ij} = \frac{y_j + y_i}{2} + \frac{(y_j - y_i)(r_i^2 - r_j^2)}{2d_{ij}^2} \pm \frac{x_j - x_i}{2d_{ij}^2}\sqrt{\left((r_i + r_j)^2 - d_{ij}^2\right)\left(d_{ij}^2 - (r_i - r_j)^2\right)} \quad (11)$$

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