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14. ABSTRACT The main objective of this proposal is to develop efficient and accurate reduced-order models comprised of multiscale and multiphysics characteristics amenable for fast simulation of large-scale problems of flow and assessment of uncertainty in highly heterogeneous porous media. This effort will incorporate multiscale methods and system theory (reduced-order modeling) for nonlinear systems for a broad spectrum of applications, ranging from single-phase, to multiphase flow and transport phenomena. In our approach, we develop a framework which balances the error from global reduced-order models and local multiscale					
15. SUBJECT TERMS model reduction, local-global model reduction, multiscale methods, porous media flow simulation					
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## Report Title

### Local-Global Model Reduction for Large-Scale Models Integrating Systems-Theoretical Properties

#### ABSTRACT

The main objective of this proposal is to develop efficient and accurate reduced-order models comprised of multiscale and multiphysics characteristics amenable for fast simulation of large-scale problems of flow and assessment of uncertainty in highly heterogeneous porous media. This effort will incorporate multiscale methods and system theory (reduced-order modeling) for nonlinear systems for a broad spectrum of applications, ranging from single-phase, to multiphase flow and transport phenomena. In our approach, we develop a framework which balances the error from global reduced-order models and local multiscale approximations. Another unique feature of the proposed work involves the development of extensions to nonlinear uncertain parameter-dependent problems in subsurface flow simulation. The project will attempt to achieve the following results: (1) development of a new local-global multiscale model reduction framework} based system theory and multiscale techniques for processes in highly heterogeneous porous media; (2) development of multiscale methods for complex nonlinear systems of two-phase flows; (3) derivation of error estimators for reduced large-scale discretized models} for characterizing model solution accuracy based on system-theoretical properties; (4) extensions of the proposed techniques to nonlinear and stochastic (parameter-dependent) systems

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**Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

<u>Received</u>	<u>Paper</u>
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**TOTAL:**

**Number of Papers published in peer-reviewed journals:**

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**(b) Papers published in non-peer-reviewed journals (N/A for none)**

<u>Received</u>	<u>Paper</u>
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**TOTAL:**

**Number of Papers published in non peer-reviewed journals:**

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**(c) Presentations**

see attached final report

Number of Presentations: 25.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

01/30/2013	1.00	Eduardo Gildin, Yalchin Efendiev. Local-Global Model Reduction for Large-Scale Models Integrating Systems-Theoretical Properties - Report 1, (01 2012)
08/20/2013	2.00	Eduardo Gildin, Yalchin Efendiev. Local-Global Model Reduction for Large-Scale Models Integrating Systems-Theoretical Properties, Working Paper (08 2013)

TOTAL: 2

Number of Manuscripts:

Books

Received      Book

TOTAL:

Received      Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

see attached final report

Graduate Students

NAME	PERCENT SUPPORTED	Discipline
Sardar Afra	1.00	
Yanfang Yang	1.00	
Mohammadreza Ghasemi	1.00	
FTE Equivalent:	3.00	
Total Number:	3	

Names of Post Doctorates

NAME	PERCENT SUPPORTED
Alexander Lozoviskiy	1.00
FTE Equivalent:	1.00
Total Number:	1

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### Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Eduardo Gildin	1.00	
Yalchin Efendiev	1.00	
<b>FTE Equivalent:</b>	<b>2.00</b>	
<b>Total Number:</b>	<b>2</b>	

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### Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

### Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ..... 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: ..... 0.00

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### Names of Personnel receiving masters degrees

<u>NAME</u>
<b>Total Number:</b>

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### Names of personnel receiving PHDs

<u>NAME</u>
Sardar Afra
Mohammadreza Ghasemi
Yanfang Yang
<b>Total Number:</b>

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### Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

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## **Sub Contractors (DD882)**

## **Inventions (DD882)**

## **Scientific Progress**

See attachment

## **Technology Transfer**

We have closely interacted with Drs. Chris Kees and Matthew Farthing from the Coastal and Hydraulics Laboratory, US Army Engineer Research and Development Center (ERDC). Other forms of technology transfer were the standard channels as main conferences in the subject areas and the Foundation CMG Summits in which Dr. Gildin participates every year.



# Local-Global Model Reduction for Large-Scale Models Integrating Systems-Theoretical Properties

Army Research Office - ARO

by

PI: Dr. Eduardo Gildin  
(Texas A&M University - Petroleum Engineering Department)

Co-PI: Dr. Yalchin Efendiev  
(Texas A&M University - Mathematics Department)

Final Report  
Contract No. W911NF-12-1-0206

August 14, 2016

# Final Report

September 26, 2016

**PI:** Eduardo Gildin<sup>1</sup>

**Co-PI:** Y. Efendiev<sup>2</sup>

## 1 Summary

This report summarizes the contribution of this project towards the development of efficient multiscale model reduction methods applicable to several problems in heterogeneous porous media. This project is a collaborative effort between the Petroleum Engineering and the Mathematics Departments at Texas A&M, and The DoD US Army Research Office (ARO) and the US Army Engineer Research and Development Center (ERDC).

**The main objective** of this project is to develop efficient and accurate reduced-order models comprised of multiscale and multiphysics characteristics amenable for fast simulation of large-scale problems of flow and assessment of uncertainty in highly heterogeneous porous media. The efforts realized thus far incorporated multiscale methods and system theory (reduced-order modeling) for nonlinear systems for a broad spectrum of applications, ranging from single-phase, to multiphase flow and transport phenomena. Multiscale solution methods are currently the method of choice for capturing the effects of fine-scale permeability variations through the calculation of specialized coarse-scale basis functions. Most of the multiscale techniques presented to date employ localization approximations in the calculation of these basis functions. However, in many applications global effects are important (e.g., channelized flows), requiring a global model reduction perspective. To this end, we developed methods that can couple multiscale algorithms with global model reduction techniques, giving rise to the idea of Global-Local Multiscale Model Reduction methods.

In our project, we explored efficient global model reduction by means of the projection framework. We employed the Galerkin projection using the proper-orthogonal decomposition (POD) for reducing the states of the system. As it is shown in many of our publications, the nonlinearities need to be treated carefully in order to resolve the dependencies on the fine scale dimensions. Efficient model reduction was achieved by means of the discrete empirical interpolation (DEIM) method and the POD-DEIM algorithm. Furthermore, we developed adaptive strategies for dealing with input test spaces (online step) that are far from the ones used in the training (offline step). In this project we also partnered with the US Army Engineer Research and Development Center (ERDC) for the systematic development of reduced order models for shallow water flows. This was achieved by a close interaction between our post-doc and ERDC personnel. In summary, we have considered (1) Systematic adaptive local and global-local model reduction techniques for highly heterogeneous flows; (2) System Theoretical Approaches in Model Reduction (e.g., DMD, Balanced Truncation and Bilinear Approximations); (3) Multilevel multiscale methods; and (4) Shallow Water Simulation (SWE) and Model Reduction. Below, we present a brief overview of each of these contributions.

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<sup>1</sup>Petroleum Engineering Department, Texas A&M University

<sup>2</sup>Department of Mathematics, Texas A&M University, College Station, TX 77843-3368



## 2 Systematic adaptive local and global-local model reduction techniques for highly heterogeneous flows [16, 17, 14, 15, 5, 4, 23, 40, 34, 37, 36, 35, 33, 32, 1, 9, 12, 11, 10, 13, 3, 21, 31, 14, 15]

The main idea of the proposed approach is to construct a small dimensional local solution space that can be used to generate efficient and accurate approximation to the multiscale solution with a potentially high dimensional input parameter space. In the proposed approach, we present a general procedure to construct the offline space that is used for a systematic enrichment of the coarse solution space in the online stage. The enrichment in the online stage is performed based on a spectral decomposition of the snapshot space consisting of all plausible functions in the space. In the online stage, for any input parameter, a multiscale space is constructed to solve the global problem on a coarse grid. The online space is constructed via a spectral decomposition of the offline space and by choosing the eigenvectors corresponding to the largest eigenvalues. The computational saving is due to the fact that the construction of the online multiscale space for any input parameter is fast and this space can be re-used for solving the forward problem with any forcing and boundary condition. This approach is used in conjunction with global mode decomposition techniques to come up with an efficient model reduction method. In previous years, POD-DEIM was introduced for porous media flow simulations. We tested the methodologies for cases of flow in highly heterogeneous media including gravity and capillary effects. The main issues overlooked in previous years were the efficient implementation of gravity and capillary [11, 12], as from the underlying equations, the reduced order model was still dependent upon fine scale computations for fractional flow. Also the assessment of good basis as the simulation progresses was not investigated in previous years. We also have studied the use of multiscale methods for solving multi-phase flow and some other applications in porous media. These results are reported in [16, 17, 14, 15, 5, 4, 23, 40, 41, 43, 41].

In terms of improving the quality of the reduced model, two new ideas were thought out in our formulation: (1) Adaptive basis, and (2) Efficient implementations. In the case of localized 3-dimensional models with capillarity and gravity, we made progress in implementing an algorithm that is almost not dependent in the fine scales computations. This was done in [12]. In the current period, we have worked on revising the works from previous periods, which are now accepted. We have also worked on several directions, which include the development of online multiscale methods, the global-local approaches, and re-iterated approaches. Below, we briefly describe them.

In [37], we propose an online adaptive Local-Global POD-DEIM model reduction method for flows in heterogeneous porous media. The main idea of the proposed method is to use local online indicators to decide on the global update, which is performed via a reduced cost local multiscale basis functions. This unique local-global online combination allows (1) developing local indicators that are used for both local and global updates (2) computing global online modes via local multiscale basis functions. The multiscale basis functions consist of offline and some online local basis functions. The approach used for constructing global reduced system is based on Proper Orthogonal Decomposition (POD) Galerkin projection. The nonlinearities are approximated by the Discrete Empirical Interpolation Method (DEIM). The online adaption is performed by incorporating new data, which become available at the online stage. Once the criterion for updates is satisfied, we adapt the reduced system online by changing the POD subspace and the DEIM approximation of the nonlinear functions. The main contribution is that the criterion for adaption and the construction of the global online modes are based on local error indicators and local multiscale basis function which can be cheaply computed. Since the adaption is performed infrequently, the new methodology does not add significant overhead associated with when and how to adapt the reduced basis. Moreover, the process of adaption employs inexpensive local order reduction techniques. Our approach is particularly useful for situations where it is desired to solve the reduced system for inputs or controls that results in a solution outside the span of the snapshots generated in the offline stage. Our method also offers an alternative of constructing a robust reduced system even if a potential initial poor choice of snapshots is used. Applications to single-phase and two-phase flow problems demonstrate the efficiency of our method.

### 3 System Theoretical Approaches: Model Reduction by DMD, Balanced Truncation and Bilinear Approximations [22, 20, 21, 6, 24, 25, 29, 30, 25, 26, 28]

We investigated the use of the more system's oriented model reduction, such as the Dynamic Model Decomposition (DMD) and the Balanced Truncation (BT) methods in order to accelerate computations by means of extracting the the dominant coherent structures of the snapshot matrix and and deriving the reduced-order models via Galerkin projection. In [20], POD was used in conjunction with DMD simulating the long-time dynamics. It has shown promising results when applied to a relatively large system as the SPE10 benchmark. We also investigated in [22], the combination of the generalized multiscale finite element method [17, 16] with mode decomposition methods to construct a robust local-global approach for model reduction of flows in high-contrast porous media. In [21], an effort was made to extend the Dynamic Mode Decomposition Concepts to multi-phase flow and transport. Looking at a more systems perspective, in [25], we developed a network of simple models that can be arranged in such a way to cover the entire spectrum of the full order system.

Regarding the Balanced truncation technique, we combined in a local-global framework the multiscale methods with balanced truncation. In [6], global model reduction based on balanced truncation methods is used to identify important global coarse-scale modes. This provides a substantial CPU savings as Lyapunov equations are solved for the coarse system. We also extended this framework to parameter depended problems where the permeability field is represented via a parameter. For parameter-dependent problems, we solved the two Lyapunov equations for each parameter, based on a coarser system written up by means of the Generalized Multiscale Finite Element Methods (GMsFEM). To this end, we introduced an offline and online stages for solving Lyapunov equation and showed that one can achieve a substantial CPU savings in this way.

We devoted some effort in developing an input-output invariant model reduction scheme [29, 30]. One of the main drawbacks of the POD framework is the fact that the optimal basis depend upon the training sets of input and outputs used to obtain the snapshot matrix. Once the operating conditions of the porous media flow system changes (e.g., uncertain parameters, different injection and production scenarios), the whole POD procedure needs to be recalibrate in order to incorporate these new modes. This is to say that the POD framework is not input-output invariant. In [29, 30], we used a departure from POD-DEIM in the sense that we work with the nonlinear system directly and recast it into the bilinear form. This allows ones to write the full nonlinear system of equations into an enhanced linearized model. The main tool used is the Carleman decomposition. It also enables one to use standard linear model reduction approaches, such as the celebrated Krylov subspaces methods. We also worked on designing of more accurate POD method using high-order SVD techniques (HOSVD) [3]. In the POD framework, it is necessary to vectorize the original set of snapshots, i.e., pressures and saturations are totally disconnected from their spatial correlation, leading to the loss of the higher order statistical information in the data. We developed an alternative to SVD-based methods, which preserves the spatial correlation by means of high-order singular value decompositions [26, 28]

### 4 Multilevel multiscale methods [18, 19, 39, 38, 42, 34, 33, 32, 36]

We have studied the use of multilevel approaches in upscaling. This allows simplifying the computations and achieving an optimal complexity. We have consider such approaches for upscaling problems and the use of generalized multiscale finite element concepts in designing optimal multi-level preconditioners. Below, we summarize our results.

In [18], we consider an efficient approach for computing effective properties. Computation of effective properties for non-periodic microstructures can be prohibitively expensive as many local cell problems must be solved for different macroscopic points. The local problems may also exhibit complex geometries and scale disparity making them computationally expensive. When the microstructure varies slowly, we develop

an efficient numerical method for two-scales that achieves essentially the same accuracy as that for the full resolution solve of every local cell problem. In this method, we build a dense hierarchy of macroscopic grid-points and a corresponding nested sequence of approximation spaces. Essentially, solutions computed in high accuracy approximation spaces at select points in the hierarchy are used as corrections for the error of the lower accuracy approximation spaces at nearby macroscopic points. Such techniques have been used for high dimensional elliptic problems. We give a brief overview of slowly varying media and formal Stokes homogenization in such domains. We present a general outline the algorithm and list reasonable and easily verifiable assumptions on the partial differential equations, geometry, and approximation spaces. With these assumptions, we achieve the same accuracy as the full solve. We apply this algorithm to Stokes equations in a slowly porous medium where the microstructure is obtained from a reference periodic domain by a known smooth map. Using the Arbitrary Lagrange-Eulerian (ALE) formulation of the Stokes equations, we obtain modified Stokes equations with varying coefficients in the periodic domain. We show that the algorithm can be utilized in this setting. Finally, we implement the algorithm on the modified Stokes equations, using a simple stretch deformation mapping, and compute the effective permeability. We show that our efficient computation is of the same order as the full solve.

In [19], we construct and analyze multigrid methods with nested coarse spaces for second-order elliptic problems with high-contrast multiscale coefficients. The design of the methods utilizes stable multilevel decompositions with a bound that generally grows with the number of levels. The robustness, with respect to the contrast, is guaranteed due to the combined effect of the Schwarz smoothers used and the spectral construction of the coarse bases. More specifically, in order to obtain an optimal multilevel decomposition, we combine multigrid ideas in two-level approaches in the element-based algebraic multigrid methods (or AMGe), that use local spectral problems to enrich the coarse space. In general, the intermediate coarse spaces need to be enriched in order to get contrast-independent convergence. The general techniques presented here allow us to pose the problem of an optimal enrichment in the sense of enriching with a minimal number of extra coarse degrees of freedom. Thus, the methods we develop are optimal with respect to the contrast. Moreover, we have the potential to achieve this goal with a minimal number of coarse degrees of freedom. We present numerical results that illustrate our theoretical findings. To stabilize the growth with respect to the number of levels, we propose to use AMLI-cycle multigrid which, according to the analysis of standard multilevel methods, leads to an overall optimal cost algorithm that have the potential to be robust with respect to the contrast and the number of levels.

We have developed novel concepts within multiscale model reductions, which include online basis functions [39, 38] and sparsity [42]. Offline computation is an essential component in most multiscale model reduction techniques. This offline process can give a sufficient accuracy. However, there are cases in which the offline procedure is insufficient to give accurate representations of the solution, due to the fact that offline computations are typically performed locally and global information is missing in these offline information. These phenomena occur locally and in some of these regions that are identified using the proposed error indicators, we will need to develop online basis functions [39]. The resulting basis functions are able to capture the solution efficiently and accurately, and are added to the approximation iteratively. In [39], we design online multiscale basis functions and investigate their accuracy. In particular, we discuss that one needs a sufficient number of multiscale basis functions in order to achieve a fast convergence using online procedures.

In [34], we consider the Generalized Multiscale Finite Element Method (GMsFEM) and apply it iteratively to construct its multiscale basis functions. The main idea of the GMsFEM is to construct snapshot functions and then extract multiscale basis functions (called offline space) using local spectral decompositions in the snapshot spaces. The extension of this construction to several levels uses snapshots and offline spaces interchangeably to achieve this goal. At each coarse-grid scale, we assume that the offline space is a good approximation of the solution and use all possible offline functions or randomization as boundary conditions and solve the local problems in the offline space at the previous (finer) level, to construct snapshot space. We present an adaptivity strategy and show numerical results for flows in heterogeneous media and in perforated domains.

In [33], we present a general overview of the GMsFEM and present several applications. In [32], we study multiscale problems with time and spatial scales. In [36], we develop space-time multiscale basis functions.

## 5 Shallow Water Simulation (SWE) and Model Reduction - ERDC Joint Work [12, 44, 7, 45]

In this project, we developed model reduction strategies applied to the free-surface flows, described via the shallow water equations (SWE). These equations are widely used to model depth-averaged free-surface hydrodynamics, whose phenomenological character is a typical of engineering applications involving risk assesment, optimal design, or parameter estimation. Among many, a branch of R&D that is significantly relevant to ERDC that relies on the utilization of free-surface flows in the evaluation of safety of coastal structures and the risk assessment under various wave and current conditions of interest. Another field includes bathymetry inversion for river basins.

Despite the popularity and relative simplicity of SWE compared to the fully coupled three-dimensional multi-phase flows based on Navier-Stokes equations, accurate resolution of shallow-water flows can still be very computationally demanding in practical simulations [12, 44]. Many parameter/risk estimation studies require fitting the sensitivities to design parameters by computationally modeling a given structure under a wide range of conditions (designs and wave/storm conditions). Such a study can require tens of thousands of simulations to fully explore the parameter space. Even with improvements of algorithms in numerical modeling and constantly growing performance of processors (in terms of GFlops/s) and memory bandwidth (in terms of Mbytes/s) in transistor-based technology, uncertainty quantification studies can takes hours to days for a single simulation. This constitutes a significant hurdle for inclusion of well-resolved shallow water models in engineering applications and, clearly, calls for efficient computational reduced-order models (ROM). Such models are expected to catch the principal character (physics) behind the process that is, ideally, supposed to be sufficient for efficient design analysis with reasonable accuracy. Here, we consider global model reduction for the SWE via projection. Specifically, we consider Galerkin-based projection via Proper Orthogonal Decomposition (POD) as well as Petrov-Galerkin approximations based on the Gauss-Newton with Approximate Tensors (GNAT). Our current project consisted of the theoretical construction of the reduced-order models for the shallow-water equations, computational (experimental) testing, and software development of the schemes represented by those models. The latter was performed on the basis of open-source simulator “Proteus” originating on ERDC site [7, 12, 44]. In our publications, the error statistics has been recorded for all the examined model reduction methods over a low-dimensional nonlinearity modal range.

We next studied the Petrov-Galerkin projection reduced order model, originating from the Gauss-Newton Approximate Tensor method. The idea of the method is to choose such reduced-order state that the discrete residual of the system is minimized. In this sense, this is a discrete optimal method, as opposed to Galerkin projection method described in [7]. The theoretical development of the method was performed for the stabilized SWE systems discretized semi-implicitly in time and compared with the Galerkin projection model. We showed that the snapshot strategy leading to consistency assumes collecting entire Jacobian matrices at every time step and thus is not feasible in practice. Alternative strategy, for which consistency in general does not hold, was incorporated. For computations, we examined both the Galerkin and Petrov-Galerkin methods from the point of view of accuracy, efficiency and consistency. A simple one-dimensional viscous Burgers equation discretized with the Finite Element method confirmed our theoretical predictions regarding consistency of hyper-reduction models and showed optimistic results for the cases when the snapshot collection strategy was not consistent. The paper [45] is in the process of being submitted.

## 6 Presentations

Below, we summarize the presentations streamed out of this project.

### YEAR 1

- Y. Efendiev, “Generealized multiscale finite element methods”, keynote speaker in East Asia SIAM Meeting, Taipei, 2012.

- E. Gildin, “Local-Global Multiscale model reduction using system theoretical properties for Flow in Heterogeneous Porous Media”, Presented at INTERPORE, Purdue University, May 14-16, 2012.
- Y. Efendiev, “Multiscale model reduction for flows in highly heterogeneous media”, SIAM UQ, 2012
- E. Gildin, “Local-Global Multiscale model reduction using system theoretical properties for Flow in Heterogeneous Porous Media”, Presented at 10th World Congress on Computational Mechanics (WCCM 2012); Sao Paulo, Brazil, July 09-13, 2012.

## YEAR 2

- E. Gildin, “Nonlinear Complexity Reduction for Fast Simulation of Flow in Heterogeneous Porous Media”, Presented in the Reservoir Simulation Symposium, The Woodlands, February, 2013.
- E. Gildin, “Local-global model reduction techniques for porous media flow simulations”, SIAM CSE 2013, Boston, MA, February 2013.
- Y. Efendiev, “Multiscale Model Reduction for Porous Media Flows”, Interpore Annual Meeting, Prague, May 2013
- Y. Efendiev, “Generalized Multiscale Finite Element Method”, SIAM Geosciences, Padova, Italy, June 2013.
- Y. Efendiev, “Generalized Multiscale Finite Element Method”, ICES/DOE/USACM Thematic Workshop on Multiscale Modeling, to be held in Austin, Texas, April 29-May 1, 2013.
- Y. Efendiev, “Generalized Multiscale Finite Element Method and Its Applications”, South Central Conference on Advanced Numerical Methods and Applications, April 2013, Little Rock.
- Efendiev, 2 talks in Oberwolfach Conference on multiscale model reduction. February and April 2013

## YEAR 3

- E. Gildin, “Nonlinear Complexity Reduction for Fast Simulation of Flow in Heterogeneous Porous Media”, Presented in the Reservoir Simulation Symposium, The Woodlands, February, 2013.
- Gildin E., “Robust Reduced Complexity Modeling in Reservoir Simulation and Optimization”. Invited Speaker at 2014 Numerical Porous Media Center (NumPor) Annual Meeting. King Abdullah University of Science and Technology (KAUST), Saudi Arabia. March 1-3, 2014.
- Gildin E., “Research Initiatives in Reservoir Modeling and Optimization at Texas A&M University: The Search for the Optimal Closed-Loop Reservoir Management Practices”, SPE SPWLA Seo Brasil, Society of Petrophysicists and Well Log Analysts (SPE SPWLA) and BG Group, Brazil, Rio de Janeiro, July 16, 2013
- E. Gildin, “Robust Reduced Complexity Modeling in Reservoir Simulation and Optimization”. Invited Speaker at 2014 SPE Gulf Coast Section Reservoir Study Group Reservoir Technology Forum. To be held May 8, 2014, The Woodlands, TX
- E. Gildin, “Robust Reduced Complexity Modeling in Reservoir Simulation and Optimization”. Invited Speaker. Graduate Seminar at the Craft & Hawkins Department of Petroleum Engineering at Louisiana State University (LSU). February 21, 2014.
- E. Gildin, “Leveraging Real-time Data Can Maximize Asset Performance”, Invited Speaker at the Digital Oil Fields USA Summit. Oil & Gas iQPC. December 10-11, 2013. Houston, TX.
- Y. Efendiev, “Multiscale Model Reduction with Generalized Multiscale Finite Element Methods”, Invited 45 minute talk in International Congress of Mathematicians, Seoul, 2014

- Y. Efendiev, “Generalized Multiscale Finite Element Method”, Invited talk, Multiscale Methods and High Performance Computing, Edinburgh, UK, May 2014
- Y. Efendiev, ““Multiscale Model Reduction for porous media flows”, Invited talk. Porous Media Processes and Mathematics Annual Meeting, Edinburgh, UK, January 2014
- Y. Efendiev, “Generalized Multiscale Finite Element Method, one hour invited talk, Computational Multiscale Methods, Oberwolfach, June 2014

#### YEAR 4

- E. Gildin, Optimization of Placement of Hydraulic Fracture Stages in Horizontal Wells Drilled in Shale Gas Reservoirs. Invited Speaker at 2015 Unconventionals Completion Optimization (Hansonwade). March 23-24, 2015, Houston, TX
- E. Gildin, Multiscale Model Reduction in Heterogeneous Porous Media. Invited Presentation at TEES-COPPETEC Brasil Workshop. Held in Rio de Janeiro. August 06, 2014.
- Y. Efendiev, “Generalized Multiscale Finite Element Method”, Keynote Speaker, The Annual Meeting of International Porous Media, May, 2015.

#### YEAR 5

- E. Gildin, Robust Reduced Complexity Modeling in Reservoir Simulation and Optimization. Invited Speaker at University of Texas at Austin, Petroleum and Geosystem Engineering. Claude R. Hocott Graduate Seminar Series. February 29, 2016.
- E. Gildin, Complexity Reduction in Reservoir Simulation and Optimization: A Control System Perspective. Invited Speaker at ExxonMobil URC Seminar. February 25, 2016
- Y. Efendiev, “Multiscale Model Reduction for Heterogeneous Problems”, Invited Speaker, 7th Pacific Rim Conference, Korea.

## References

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