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On a Theory of Broadband Absorption Suppression in Magnetic Composites

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FLORIDA INSTITUTE OF TECHNOLOGY, INC.

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14. ABSTRACT
The scientific focus and main objective of the PIs research project was on the development of an effective theory of broadband absorption suppression in magnetic composites. During the report period (April 1, 2015-March 31, 2018), the PI has made steady progress towards this objective and has had several accomplishments which are described in more detail in this final performance report.

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Abstract

The scientific focus and main objective of the PI's research project was on the development of an effective theory of broadband absorption suppression in magnetic composites. During the report period (April 1, 2015-March 31, 2018), the PI has made steady progress towards this objective and has had several accomplishments which are described in more detail in this report and briefly summarized as follows.

My paper (with co-author Alex Figotin) entitled, `On overdamping phenomena in gyroscopic systems composed of high-loss and lossless components," was published in the Journal of Mathematical Physics and the referee who reviewed the paper rated the quality of research and presentation as ``Excellent."

My two book chapters (with co-authors Graeme Milton and Maxence Cassier) entitled, ``Analyticity of the Dirichlet-to-Neumann map for the time-harmonic Maxwell's equations" and `` A rigorous approach to the field recursion method for two-component composites with isotropic phases" were published in the book: *Extending the Theory of Composites to Other Areas of Science*, edited by G. W. Milton. Reviews of this book and our chapters (in J. Appl. Mech. and SIAM Review) have been extremely positive, saying, for instance, ``The book under review…is impressive both in scope and depth…," or ``It is a thought-provoking work, containing many new ideas that could be fueling new research for years," or ``I personally found inspiration for several new research directions." And about my chapter on the field recursion method that it ``is mathematically the most elegant chapter in the book. It is a rigorous justification…using a beautiful functional-analytic argument."

Two workshops and four minisymposiums, co-organized by the PI, have been or will be held which were directly related to developments that occurred during the PI's grant research. Moreover, the interdisciplinary workshops are in prestigious international research institutes, namely, the Institute Mittag-Leffler in Sweden and the Banff International Research Station in Canada, with several world-class speakers and attendees.

Nineteen talks and two poster have been presented on the PI's grant research, plus three talks to be given by the PI in the near future, one of which the PI has the honor of being a key speaker. Of all the presentations, one talk was given by a collaborator of the PI's, namely, Robert Viator (IMA, UMN) based on joint research related to the PI's grant, started while the PI was a long-term visitor at the IMA. Three other talks were given by the PI's collaborators, namely, one by Alex Figotin (UCI) and two by Graeme Milton (Univ. of Utah), based on their joint work with the PI related to the YIP grant. One of the poster presentations, which won the award ``Best in Show for Mathematical Sciences," was given by an undergraduate student, Daniel McCormick, whose research was supervised by the PI and based on the PI's AFOSR sponsored research.

The PI held a long-term visiting research professor position, from Sept. 10-Dec. 15, 2016, at the Annual Thematic Program on Mathematics and Optics at the Institute for Mathematics and its Applications (IMA) at the Univ. of Minnesota (UMN).

Moreover, several contacts were made including a government lab (Dr. Joao Romano, General Manager and Technical Lead, Precision Armament Laboratory), a director of a prestigious institute (Fadil Santosa, Director, Institute for Mathematics and its Applications) which resulted in the long-term visit at the IMA, and an executive editor of a well-known and respected publisher (Rochelle Kronzek, World Scientific Publishing) about their interest in publishing a math-physics monograph based on the joint research of the PI and Alex Figotin.

Finally, a very fruitful and long-term collaboration was developed with one of the world's leading researcher (Graeme Milton, Distinguished Professor, University of Utah) on the theory of composites and a brilliant young applied mathematician (Maxence Cassier, CNRS researcher, Institut Fresnel, Marseille, France), whose interactions have significantly contributed to the PI's research program.

I. RESEARCH OBJECTIVES.

Develop an effective theory of broadband absorption suppression in magnetic composites which can provide guidance for the realization of novel magnetic composites with strong magnetic properties/functionality similar to or better than the individual constitute magnetic components and with similar dimensions, but with losses drastically reduced within a broad frequency range of operational bandwidth.

II. STATUS and ACCOMPLISHMENTS.

Publications of the PI with collaborators:

1. In April 2016, the PI and Alex Figotin published a paper (47 pages) in the Journal of Mathematical Physics (see item III.1 in this report) entitled, ``On overdamping phenomena in gyroscopic systems composed of high-loss and lossless components." In this paper, they continued their study of mechanisms of broadband absorption suppression in composites, using perturbation theory and the Lagrangian framework that they introduced in previous works. They studied overdamping phenomena in dissipative-gyroscopic systems, with finite-degrees of freedom (e.g. electric circuits with resistors and gyrators), using two-component model of a composite with a high-loss and a lossless component. Three points worth mentioning are the following:
    i. This paper was accepted without revision (i.e. publish as is) in this journal and the referee who reviewed the paper rated the quality of research and presentation as ``Excellent."
    ii. The paper is significant since it shows that the selective overdamping phenomena in dissipative-gyroscopic systems occurs generically and hence provides a mechanism for selective enhancement of the underdamped eigenmodes (they are low-loss with high quality factor) of the system and simultaneous suppression of the overdamped eigenmodes (they are non-oscillatory). It also provides upper bounds on the amount of loss required in the lossy component of the composite for the selective overdamping phenomena to occur as well as estimates on the quality factor of the underdamped modes.
    iii. The paper also demonstrated numerically, via a simple electric circuit example with resistors and gyrators, a novel phenomenon (which we are currently referring to as *finite-interval*

overdamping) that appears to have gone unnoticed before in studies of overdamping which may be exploitable for AFOSR applications, namely, complete or selective overdamping WITHOUT the need of highly lossy components. Let me brief elaborate on this point.

- Figs. 4 & 5, p. 20 of this paper show graphs of the all the frequencies vs the loss parameter β of the electric circuit shown in Fig. 1, p. 14. Numerically, there is an a nonempty open interval contained in the finite-interval $[\beta_0, \beta_1]$ of the loss parameter in which all the eigenmodes of the system are overdamped (i.e., the frequencies are all zero) yet, all our current theory could not predict this phenomenon which we are referring to as *finite-interval overdamping*.
    - The reason for the use of the term ``finite-interval" is that for all investigations of overdamping phenomena that we are aware of, including our own, the theory is associated only with the high-loss regime in which the loss parameter is in a semi-infinite interval with infinity as an endpoint or limit point of the interval.
- This phenomena and its potential applications is begin currently investigated by the PI for general circuits and mechanical systems with multiple degrees-of-freedom (DOF).
    - The PI along with an undergraduate research student, Daniel McCormick (FIT), begun numerical and theoretical studies of simple two DOF systems which have already yielded new, interesting, and informative results and were presented in a poster on these results in the undergraduate research showcase at FIT in April 2017 which won the award ``Best in Show for Mathematical Sciences."

iv. Recent developments by the PI, using tools from the Herglotz-Nevanlinna-Stieltjes function theory and methods from the theory of composites, but applied in a novel way to the study of overdamping, have led to a much better understand of both the selective overdamping and finite-interval overdamping, and give us better estimates and bounds on the frequencies, damping factors, and Q-factor of the eigenmodes. An overview of the PI's results on this will be presented in a talk in the upcoming July 16-20, 2018 ETOPIM11 Conference, in which the PI is a key speaker (see item IV.24 in this report).

2. The PI together with Graeme Milton and Maxence Cassier wrote a book chapter (see item III.2 in this report) entitled, ``Analyticity of the Dirichlet-to-Neumann map for the time-harmonic Maxwell's equations" which is chapter 4 in the book, published in Aug. 2016: *Extending the Theory of Composites to Other Areas of Science*, edited by Milton.
    i. This chapter studies the analyticity properties of the electromagnetic Dirchlet-to-Neumann map (DtN) for Maxwell's equations in passive linear multicomponent composite media focusing on two different types of geometries, namely, layered media and bounded media with Lipschitz domains. In the chapter they prove that the DtN map is an operator-valued function which is analytic in the dielectric permittivities and magnetic permeabilities of each phase and belongs to a special class of functions known as Herglotz functions.
    ii. This chapter is significant since Herglotz functions have been useful in the theory of composites, such as in developing bounds on effective tensors of composites containing multiphases, for instance, the work of Ken Golden and George Papanicolaou, David Bergman, and Graeme Milton.

iii. This DtN map and its analytic properties are expected to play a key role in the PI's research program and objectives for broadband absorption suppression in magnetic composite media for Maxwell's equations (such as in magnetophotonic crystals with high-loss magnetic components in the unit cell or bounded domains in which one of the components is a high-loss magnetic/gyrotropic material).

3. The PI together with Graeme Milton and Maxence Cassier wrote another chapter (see item III.3 in this report) entitled, ``A rigorous approach to the field recursion method for two-component composites with isotropic phases" which is chapter 10 in the book, published in Aug. 2016: *Extending the Theory of Composites to Other Areas of Science*, edited by Milton.
   i. This paper gives a rigorous derivation of the field equation recursion method in the abstract theory of composites for two-component composites with isotropic phases.
   ii. This paper is significant as the method is of great interest since it has proven to be a powerful tool in developing sharp bounds for the effective tensor of a composite material. It is a first step in the development of the method for the more general case when one of the components has an anisotropic phase (such a magnetic/gyrotropic-dielectric composite) and then applying it to the electromagnetic DtN map in Maxwell's equation with two-component magnetic-dielectric composites.
   iii. For the PI this is a step toward the research objective of discovering basic principles for achieving magnetic-dielectric composites that are optimized with respect to the trade-off between suppression of losses and enhancement of magnetic properties/functionality inherited from the magnetic components. And the idea is to use the methods and results from the abstract theory of composites to achieve this goal.
4. Two reviews of this book, including my chapters I co-authored, have been published now by two external reviewers, in J. Appl. Mech. and SIAM Review, and are very positive in regards for the content, quality, and scientific merit. For relevant highlights of these reviews, see item III in this report.

Invited talks given and posters presented based on the PI's research:

5. Nineteen talks and two poster were presented based on joint work of PI and his collaborators, plus three talks to be given by the PI in the near future. For one of these talks, the PI has been invited to be a key speaker based on his AFOSR YIP research achieved during this grant period. For more details on these presentations, see item IV in this report.

Workshops and Minisymposiums co-organized or being co-organized by PI on his research:

6. The PI co-organized a minisymposium, together with Robert P. Lipton and Stephen P. Shipman, entitled `` Mathematical and Computational Issues in Electromagnetic Materials" at the SIAM Conference on Mathematical Aspects of Materials Science in Philadelphia, PA on May 8-12, 2016. For more details on this minisymposium, see item V.1 in this report.
7. The PI co-organized a minisymposium, together with Graeme W. Milton, Maxence Cassier, and Mihai Putinar, entitled ``Complex Analysis, Optimization, and Herglotz Functions in Passive Electromagnetics and Composite Media" at the SIAM Conference on Mathematical Aspects of Materials Science in Philadelphia, PA on May 8-12, 2016. For more details on this minisymposium, see item V.2 in this report.

8. A workshop was co-organized by the PI, together with Mats Gustafsson, Annemarie Luger, and Mihai Putinar, at the Institute Mittag-Leffler in Djursholm, Sweden in May 8-12, 2017 and entitled ``Herglotz-Nevanlinna Functions and their Applications." For more details on this workshop, see item V.3 in this report.
9. A minisymposium is being co-organized by the PI and Ornella Mattei and entitled ``Applications of Herglotz-Nevanlinna Function Theory to Electromagnetics, Composites, and Dirichlet-to-Neumann Maps," will be held at SIAM Conference on Mathematical Aspects of Materials Science, Portland, OR, July 9-13, 2018. For more details on this minisymposium, see item V.4 in this report.
10. A minisymposium is being co-organized by the PI, together with Elena Cherkaev and Ornella Mattei, entitled ``Herglotz-Nevanlinna Function Theory and its Applications," will be held at the 11th ETOPIM Conference, Krakow, Poland, July 16-20, 2018. For more details on this minisymposium, see item V.5 in this report.
11. A workshop (currently accepted) is being co-organized by the PI, together with Andrea Alu, Mats Gustafsson, and Annemarie Luger, and entitled ``Herglotz-Nevanlinna Theory Applied to Passive, Causal and Active Systems," will be held at the Banff International Research Station, Banff, Alberta, Canada, Oct. 6-11, 2019. For more details on this workshop, see item V.6 in this report.

Long-term visiting positions held by the PI:

12. The PI had the honor of begin a long-term visiting research professor in the Annual Thematic Program on Mathematics and Optics at the Institute for Mathematics and its Applications (IMA) at the Univ. of Minnesota (UMN) from Sept. 10-Dec. 15, 2016. The IMA paid the living expenses of the PI while at Minnesota (MN) and helped to buy out all of his teaching load at FIT with the IMA paying $18,000 in the PI's behalf toward this buyout and the remaining paid through the PI's grant. For more details on this long-term visit, see item VI in this report.

III. PUBLICATIONS.

Articles in refereed journals:

1. A. Figotin and A. Welters, ``On overdamping phenomena in gyroscopic systems composed of high-loss and lossless components," J. Math. Phys. 57, 042902 (2016). doi: http://dx.doi.org/10.1063/1.4944721.

Book chapters:

2. M. Cassier, A. Welters, and G. W. Milton, ``Analyticity of the Dirichlet-to-Neumann map for the time-harmonic Maxwell's equations," in *Extending the Theory of Composites to Other Areas of Science* (G. W. Milton, Editor), Chapter 4, Milton-Patton Publishing, 2016. ISBN-10: 1483569195.
3. M. Cassier, A. Welters, and G. W. Milton, ``A rigorous approach to the field recursion method for two-component composites with isotropic phases," in *Extending the Theory of Composites to Other Areas of Science* (G. W. Milton, Editor), Chapter 10, Milton-Patton Publishing, 2016. ISBN-10: 1483569195.

Note: *The book can be found on amazon at https://www.amazon.com/Extending-Theory-Composites-Other-Science/dp/1483569195/ and two good reviews of the book can found in the following articles:*

- *J. Appl. Mech. 84(3):036501 (Jan. 25, 2017); doi:10.1115/1.4035525, written by Prof. P. Sharma, Dept. of Mech. Eng. and Physics at the Univ. of Houston.*
- *SIAM Rev., 60(2), pp. 475-481 (June 2018); https://doi.org/10.1137/18N97456X, book review by Professor Yury Grabovsky, Dept. of Math. at Temple University.*

- Highlights from the 2017 review, by Prof. P. Sharma, of our book:
  - Prof. P. Sharma says in his review of this book that ``The goal of this book is simple: present a synthesis and distillation of the field of homogenization (i.e., theory of composites) and then discuss a suite of scientific topics across which the key ideas can be utilized for enhanced insights. The book succeeds wonderfully in this mission!"
  - ``The abstract construction is worth the effort since it teases out the unification across the various scientific disciplines…"
  - And in regards to an important new idea, he says that this book introduces ``the idea that finding the so-called Dirichlet-to-Neumann map is analogous to finding the effective property of a composite material. This notion allows then the straightforward use of several tools and tricks of composites to tackle problems (in equal footing) related to acoustics, electromagnetism, and elastodynamics of heterogeneous materials."
  - `` I personally found inspiration for several new research directions even though it will take me a few more readings to fully appreciate all the deeper nuances of the book."
- Highlights from the 2018 review, by Prof. Y.Grabovsky, of our book:
  - In the Book Reviews section in the June 2018 SIAM Review, the section editor, Volker H. Schultz, says about our book among the eight book reviews in this issue that ``Among them, one is on a particularly far-reaching book…" and Y. Grabovsky gives a review of our book that ``is a very detailed discussion of an advanced amalgam of modeling and PDE analysis with lots of novel perspectives. Our reviewer recommends the book as ``a thought-provoking work, containing many new ideas that could be fueling new research for years.""
  - Y. Grabovsky says in his reviews that `` This new book… composites, promotes the bold idea that a broad range of phenomena modeled by linear partial differential equations (from electrostatics to elastodynamics to quantum mechanics) can be recast in a common framework that originally appeared in the context of composite materials. This process of abstraction is very familiar to pure mathematicians. In applied mathematics, generalizations of such a magnitude are rare."
  - `` The book under review…is impressive both in scope and depth…"
  - `` A prominent theme that permeates the entire book is that the Herglotz property is ubiquitous in science and, once established, permits one to make predictions even in problems of such complexity that precludes direct analysis…" and as example of this mentions the fact that one can derive, using the ``analytic function method" or ``field equation recursion method" in the theory of composites, ``bounds on complex permittivity and other characteristics of composites from their Herglotz properties."
  - In connection to this and the Dirichlet-to-Neumann (DtN) map, ``Readers interested in the Maxwell system can then continue to **Chapter 4, written with Maxence Cassier and Aaron Welters**, where the authors give a rigorous proof of the analytic dependence of the Dirichlet to Neumann (DtN) map on frequency in the case of a heterogeneous medium (not necessarily a composite)."

- ``a second major theme of the book is the development of the new language of subspace collections, which are the actual carriers of the Herglotz property, since, as Milton and his coauthors prove, it is preserved under the operations on subspace collections described in the book…" with the importance of this Grabovsky says ``that, as in the theory of composites, virtually all physical models described by systems of linear PDEs admit a common formulation…," namely, ``subspace collections." In this regards he says, ``It is not often that a new mathematical object appears on the scene. In this book Milton introduces "subspace collections" that replace partial differential equations as the mathematical language in which models are formulated."
- In connection to this, ``**Chapter 10, written with Maxence Cassier and Aaron Welters**, is mathematically the most elegant chapter in the book. It is a rigorous justification of the field equation recursion method, using a beautiful functional-analytic argument. In such arguments topological methods are intertwined with linear algebraic constructions." He goes on to say that ``Those who have enjoyed Chapter 10 as much as I have can then go ahead and read the rest of Chapter 7…" on the theory of subspace collections.
- His review concludes with saying the book
  - ``is a bold attempt at an ambitious generalization reaching far beyond composites, driven by the recognition that a wealth of powerful tools…stemming from a few universal features of physical laws can be brought to bear on multiple areas of science."
  - ``It is a thought-provoking work, containing many new ideas that could be fueling new research for years."

IV. INVITED TALKS and PRESENTATIONS (given by the PI and/or collaborators on joint work)

1. A. Welters (speaker) and A. Figotin, ``On Selective Overdamping Phenomena in Gyroscopic-Dissipative Composites," Electrical, Transport and Optical Properties of Inhomogeneous Media, Tenth International Symposium (ETOPIM 10), Neveh Ilan, Israel, June, 2015.
2. A. Welters (poster presenter) and A. Figotin, ``On Selective Overdamping Phenomena in Gyroscopic-Dissipative Composites." ETOPIM 10, Neveh Ilan, Israel, June, 2015.
3. A. Welters, ``On linear translation-invariant systems, causality, passivity, and Herglotz functions: Fundamental properties and relations." KMRS Seminar, Kaist University, Daejeon, South Korea, July, 2015 (Invited Talk).
4. A. Welters, ``Generalizing speed-of-light limitations to arbitrary passive linear media." 2nd International Workshop on the Neumann-Poincare Operator and Related Fields, KAIST University, Daejeon, South Korea, July, 2015 (Invited Talk).
5. A. Welters (speaker) and A. Figotin, ``Toward a Theory of Broadband Absorption Suppression in Magnetic-Dielectric Composites." Mathematics and Optics minisymposium, 8th International Congress on Industrial and Applied Mathematics (ICIAM 2015), Beijing, China, Aug., 2015.
6. A. Welters, ``Toward a Theory of Broadband Absorption Suppression in Magnetic Composites." Applied Mathematics Seminar, Department of Mathematics, University of Utah, Salt Lake City, UT, Nov., 2015 (Invited Talk).
7. A. Welters, ``Toward a Theory of Broadband Absorption Suppression in Magnetic Composites." AFOSR EM Contractors Review Meeting, Arlington, VA, Jan., 2016 (Invited Talk).

8. M. Cassier, G. W. Milton, and A. Welters (speaker), ``Analyticity of the Dirichlet-to-Neumann map for Maxwell's equations in passive composite media." Spectral Theory of Novel Materials Conference, Centre International de Recontres Mathematiques (CIRM), Marseille, France, Apr. 2016 (Invited Talk).
9. A. Welters and A. Figotin (speaker), ``Overdamping in gyroscopic systems composed of high-loss and lossless components." Spectral Theory of Novel Materials Conference, Centre International de Recontres Mathematiques (CIRM), Marseille, France, Apr. 2016 (Invited Talk).
10. M. Cassier, G. W. Milton, and A. Welters (speaker), ``Analyticity of the Dirichlet-to-Neumann map for Maxwell's equations in passive composite media." Minisymposium on Complex Analysis, Optimization, and Herglotz Functions in Passive Electromagnetism and Composite Media, SIAM Conference on Mathematical Aspects of Materials Science, Philadelphia, PA, May 2016 (Invited Talk).
11. A. Welters (speaker) and A. Figotin, ``Mechanism of absorption suppression in gyroscopic systems composed of high-loss and lossless components." Minisymposium on Mathematical and Computational Issues in Electromagnetic Materials, SIAM Conference on Mathematical Aspects of Materials Science, Philadelphia, PA, May 2016 (Invited Talk).
12. M. Cassier, G. W. Milton, and A. Welters (speaker), ``Analyticity of the Dirichlet-to-Neumann map for Maxwell's equations in passive composite media." Operators, Operator Families and Asymptotics Conference, Department of Mathematical Sciences, University of Bath, Bath, UK, May. 2016 (Invited Talk).
13. A. Welters (poster presenter) and A. Figotin, ``On Selective Overdamping Phenomena in Gyroscopic-Dissipative Composites." LMS Durham Research Symposium on the Mathematical and Computational Aspects of Maxwell's Equations, Department of Mathematical Sciences, Durham University, Durham, UK, July 2016.
14. M. Cassier, G. W. Milton (speaker), and A. Welters, ``Brief review of parts of `Extending the Theory of Composites to Other Areas of Science'." LMS Durham Research Symposium on the Mathematical and Computational Aspects of Maxwell's Equations, Department of Mathematical Sciences, Durham University, Durham, UK, July 2016 (Invited Talk).
15. A. Welters (speaker), ``Toward a Theory of Broadband Absorption Suppression in Magnetic Composites." Annual Program Seminar Series, Institute for Mathematics and Its Applications (IMA), University of Minnesota, Minneapolis, MN, Dec. 2016 (Invited Talk).
16. M. Cassier, G. W. Milton (speaker), and A. Welters, ``New Approaches to Imaging." IAS Workshop on Inverse Problems, Imaging and Partial Differential Equations, The Hong Kong University of Science and Technology (HKUST), Hong Kong, Dec. 2016 (Invited Talk).
17. A. Welters (speaker), ``On the dissipative properties of electromagnetic fields in stratified magneto-dielectric media." AFOSR EM Contractors Review Meeting, Arlington, VA, Jan., 2017 (Invited Talk). Note: This talk is based on joint work with several collaborators including M. Cassier (Univ. of Utah), G. W. Milton (Univ. of Utah), and R. Viator (IMA).
18. R. Viator (speaker) and A. Welters, ``Analysis of Maxwell's equations in passive layered media." Postdoc Seminar Series, Institute for Mathematics and Its Applications (IMA), University of Minnesota, Minneapolis, MN, Feb. 2017 (Invited Talk). Abstract: https://www.ima.umn.edu/2016-2017/PS9.27.16-5.23.17/26086.
19. D. McCormick (poster presenter) and A. Welters, `` Finite-Interval Overdamping: Phenomena, Analysis, & Applications." FIT Senior Design Showcase, Florida Institute of Technology, Melbourne,

FL, April 7-8, 2017. Note: This poster won the award ``Best in Show for Mathematical Sciences" at this showcase.

20. M. Cassier, G. W. Milton, and A. Welters (speaker), ``Analyticity of the Dirichlet-to-Neumann map for Maxwell's equations in passive composite media." Institut Mittag-Leffler Workshop on Herglotz-Nevanlinna Functions and Their Applications, Djursholm, Sweden, May 2017 (Invited Talk).
21. M. Cassier, G. W. Milton, and A. Welters (speaker), ``On the Field Recursion Method in the Abstract Theory of Composites for Two-Component Composites and Applications." Department of Mathematical Sciences, KAIST University, Daejeon, South Korea, July 2017 (Invited Seminar Talk).
22. M. Cassier, G. W. Milton, and A. Welters (speaker), ``On the Field Recursion Method for Two-Component Composites." Minisymposium on Applications of Herglotz-Nevanlinna Function Theory to Electromagnetics, Composites, and Dirichlet-to-Neumann Maps, SIAM Conference on Mathematical Aspects of Materials Science, Portland, OR, July 2018. (Upcoming Invited Talk – July 13, 2018).
23. M. Cassier, G. W. Milton, and A. Welters (speaker), ``On the Field Recursion Method for Two-Component Composites." Minisymposium on Herglotz-Nevanlinna Function Theory and its Applications, 11th ETOPIM Conference, Krakow, Poland, July 2018 (Upcoming Invited Talk – July 20, 2018).
24. A. Welters, `` Overdamping Phenomena in Gyroscopic-Dissipative Composites: an Application of Herglotz-Stieltjes Function Theory." 11th ETOPIM Conference, Krakow, Poland, July 2018 (Upcoming Invited Talk as a Key Speaker).

V. SYMPOSIUM and WORKSHOPS (co-organized or being co-organized by the PI)

1. Minisymposium: *Mathematical and Computational Issues in Electromagnetic Materials*, SIAM Conference on Mathematical Aspects of Materials Science, Philadelphia, PA, May 8-12, 2016.
    - Co-organizers: Aaron Welters (FIT), Robert P. Lipton (LSU), and Stephen P. Shipman (LSU).
    - Description: The aim of this symposium was to bring together researchers working on fundamental questions related to electromagnetic waves and materials and to highlight recent progress in this area. Topics of focus in the symposium included slow light, band gap solitons, photonics, metamaterials, and resonance.
    - Website and partial list of speakers: http://www.siam.org/meetings/ms16/l.php
2. Minisymposium: *Complex Analysis, Optimization, and Herglotz Functions in Passive Electromagnetics and Composite Media*, SIAM Conference on Mathematical Aspects of Materials Science, Philadelphia, PA, May 8-12, 2016.
    - Co-organizers: Aaron Welters (FIT), Graeme W. Milton (Univ. of Utah), Maxence Cassier (Univ. of Utah), and Mihai Putinar (Univ. of California at Santa Barbara and Newcastle Univ., UK).
    - Description: The aim of this symposium was to bring together a diverse group of researchers from mathematics, engineering, and physics that, although working on seemingly distinct topics, use similar tools from complex analysis or optimization theory in which Herglotz or Stieltjes functions are important. This symposium helped to bring new insights and advancements in the theory of Herglotz functions and its applications by highlighting progress and challenges in the areas of multivariate complex analysis, electromagnetics, and the theory of composites. Topics included Herglotz functions, passive systems, complex

analysis, optimization theory, dispersion relations, sum rules, bounds, and applications in electromagnetics and composites.

- Website and partial list of speakers: http://www.siam.org/meetings/ms16/c.php

3. Workshop: *Herglotz-Nevanlinna Functions and their Applications*, Institut Mittlag-Leffler, Djursholm, Sweden, May 8-12, 2017.
    - Co-organizers: Aaron Welters (FIT), Mats Gustafsson (Lund Univ., Lund, Sweden), Annemarie Luger (Stockholm Univ., Stockholm, Sweden), and Mihai Putinar (Univ. of California at Santa Barbara and Newcastle Univ., UK).
    - Description: The purpose of the workshop is to bring together mathematicians, physicists, and engineers who are interested in bounded analytic interpolation. The focus lies on Herglotz-Nevanlinna functions in connection with passive, casual, and active systems (in particular, electromagnetic design, composite materials, and Dirichlet-to-Neumann maps) and hence the main tools are within complex analysis and functional analysis as well as convex optimization.
    - Website: http://www.mittag-leffler.se/workshop/herglotz-nevanlinna-functions-and-their-applications
    - Several noteworthy scientists and mathematicians with be speaking, for instance, Barry Simon (https://en.wikipedia.org/wiki/Barry_Simon) the IBM Professor of Mathematics and Physics at Caltech. For a more complete list of program participants, see http://www.mittag-leffler.se/workshop/herglotz-nevanlinna-functions-and-their-applications/participants.
4. Minisymposium (upcoming, 2018): *Applications of Herglotz-Nevanlinna Function Theory to Electromagnetics, Composites, and Dirichlet-to-Neumann Maps*, SIAM Conference on Mathematical Aspects of Materials Science, Portland, OR, July 9-13, 2018.
    - Co-organizers: Aaron Welters (FIT) and Ornella Mattei (Univ. of Utah).
    - Description: Herglotz-Nevanlinna functions play a crucial role in materials science, especially in the study of passive, causal, and active linear systems. The aim of this minisymposium is to bring together experts from Mathematics, Physics, and Electrical Engineering to discuss the newest ideas and developments involving the applications of Herglotz-Nevanlinna functions and their multivariate analogs. In particular, the main focus of this minisymposium is on those applications concerning electromagnetism, composites, and Dirichlet-to-Neumann maps for heterogeneous bodies, with special attention given to the development of new mathematical tools. Consequently, topics will also include complex analysis, functional analysis, and convex optimization.
    - Website and partial list of speakers: http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=64303; http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=64304
5. Minisymposium (upcoming, 2018): *Herglotz-Nevanlinna Function Theory and its Applications*, 11th ETOPIM Conference, Krakow, Poland, July 16-20, 2018.
    - Co-organizers: Aaron Welters (FIT), Elena Cherkaev (Univ. of Utah), and Ornella Mattei (Univ. of Utah).
    - Description: This minisymposium focuses on novel applications of spectral analysis and Herglotz-Nevanlinna-Stieltjes functions to the theory of composites, inverse problems, optimal design, metamaterials, and synthesis problems. Our aim is to bring together

mathematicians, physicists, and engineers to discuss a broad range of topics in which Herglotz-Nevanlinna or Stieltjes functions play a crucial role. Such applications include performance bounds, sum rules, and optimization of electromagnetic, acoustic, and mechanical passive, causal, and active systems.

- Conference Website: http://etopim11.up.krakow.pl/

6. Workshop (accepted, 2019): *Herglotz-Nevanlinna Theory Applied to Passive, Causal and Active Systems*, Banff International Research Station, Banff, Alberta, Canada, Oct. 6-11, 2019.
    - Co-organizers: Aaron Welters (FIT), Andrea Alu (Univ. of Texas, at Austin), Mats Gustafsson (Lund Univ., Lund, Sweden), and Annemarie Luger (Stockholm Univ., Stockholm, Sweden).
    - Objectives: There are not so many instances where mathematicians, who are interested in theory and applications, and electrical engineers working towards developing applications, who recognized the power of mathematics, are actually meeting. It is the purpose of the proposed workshop to bring together representatives from these groups. Toward this goal the workshop organizers include two electrical engineers and two mathematicians. With joint forces it is possible to treat new applications of old and well understood (mathematical) objects as well as advancing the classical study of them. Recent advances in electrical engineering have shown that mathematical tools from functional theory can be brought to bear on finding fundamental limits on the performance of metamaterials and composites, generally, as well as defining limitations to broadband cloaking. It is also hoped that the results of this workshop would ultimately improve imaging methods. This is of obvious importance to medical imaging, geophysical prospecting, and homeland security.
    - Website: http://www.birs.ca/events/2019/5-day-workshops/19w5209.

VI. LONG-TERM VISITS

1. Visiting Research Professor (Sept. 10-Dec. 15, 2016), Annual Thematic Program on Mathematics and Optics, Institute for Mathematics and its Applications (IMA), Univ. of Minnesota (UMN), Minneapolis, MN. See https://www.ima.umn.edu/2016-2017 for information about this program.

    Points of significance in regards to this long-term visit relating to the objectives of the PI's grant:

    i. The IMA paid the living expense of the PI while at Minnesota (MN) and helped to buy out all of his teaching load (two courses) at FIT with the IMA paying $18,000 in the PI's behalf toward this buyout and the PI's YIP grant buying out the rest of his courses.

    ii. With no teaching duties, the very productive stay at the IMA allowed the PI to continue working closely with his new collaborators Graeme Milton (one of the world's leading researchers on the theory of composite) and Maxence Cassier who were also visiting the IMA at the same time as the PI. The collaboration and interactions with these two have and are continuing to significantly contribute to the PI's research objectives for this YIP grant.
        - For instance, besides the book chapters and continued studies on composites and the DtN map, the research direction of the PI with these collaborators led to co-organizing one of the minisymposiums with the PI and helped lead to two workshops, see items V.3 and V.6 of this report.

    iii. The PI developed a new collaboration with Robert Viator, an IMA postdoc and recent LSU PhD graduate student whose PhD advisor was Robert Lipton (Lipton is the S.B. Barton Prof. of Math. at LSU and a leading expert in multiscale analysis, photonic crystals, and

metamaterials who was a recent PI of an AFOSR MURI on Metamaterials with Alex Figotin of UCI).

- Results pertaining to this research collaboration have already started to be discussed in two talks by the PI and R. Viator, see item IV.17 and IV.18 of this report.
- The PI continued this collaboration with R. Viator, who visited the PI at FIT to further push their research project which was started at the IMA.

VII. COLLABORATORS

The following is a list of the PI's collaborators during this report period.

Collaborator's Name, Title, Institution, Email, and Homepage:

1. Alexander Figotin, Professor of Mathematics, Department of Mathematics, University of California—Irvine, Irvine, CA 92697. Email: afigotin@math.uci.edu. Homepage: https://www.math.uci.edu/~afigotin/
2. Graeme W. Milton, Distinguished Professor of Mathematics, Department of Mathematics, The University of Utah, Salt Lake City, UT. Email: milton@math.utah.edu. Homepage: https://www.math.utah.edu/~milton/
3. Maxence Cassier, CNRS researcher, Institut Fresnel, Marseille, France. Email: cassier@fresnel.fr. Homepage: http://fresnel.fr/perso/cassier/index.html
4. Robert Viator, Jr., Visiting Professor, Department of Mathematics, Southern Methodist University, Dallas, TX. Email: rviator@smu.edu. Homepage: not available.
5. Stephen P. Shipman, Professor, Department of Mathematics, Louisiana State University, Baton Rouge, LA. Email: shipman@lsu.edu. Homepage: https://www.math.lsu.edu/~shipman/

The following is a list of meetings with these collaborators, including dates and locations, to progress the PI's research agenda for this grant:

- Alex Figotin and PI: July 25-Aug. 1, 2015 at FIT in Melbourne, FL; Dec. 13-19, 2015 at UCI in Irvine; Apr. 18-22, 2016 at CIRM in Marseille, France; June 19-26, 2016 at FIT in Melbourne, FL; July 11-21, 2016 at Durham Univ. in Durham, UK; Oct. 9-16, 2016 at UCI in Irvine, CA; Dec. 12-16, 2016 at the IMA, UMN in MN; May 8-12, 2017 at Institut Mittag-Leffler in Djursholm, Sweden.
- Graeme W. Milton, Maxence Cassier, and PI: June 19-27, 2015 at ETOPIM10 conference in Neveh Ilan, Israel; July 18-25, 2015 at KAIST University in Daejeon, South Korea; Sept. 1-6, 2015 at Univ. of Utah in Salt Lake City, Utah; Oct. 12-19, 2015 at FIT in Melbourne, FL; Nov. 15-12, 2015 at Univ. of Utah in Salt Lake City, Utah; Feb. 10-15, 2016 at Univ. of Utah in Salt Lake City, Utah; May 8-12, 2016 at SIAM Conference on Mathematical Aspects of Materials Science in Philadelphia, PA; May 16-19, 2016 at Operators, Operator Families and Asymptotics Conference held at the University of Bath in Bath, UK; July 11-21, 2016 at the LMS Symposium held at Durham Univ. in Durham, UK; Sept. 10-Dec. 15, 2016 at the IMA, UMN in MN while the PI, Milton, and Cassier were long-term visitors; Apr. 21-28, 2017 at FIT in Melbourne, FL; May 8-12, 2017 at Institut Mittag-Leffler in Djursholm, Sweden; June 17-24, 2017 at Univ. of Utah in Salt Lake City, Utah; July 9-22, 2017 at KAIST University in Daejeon, South Korea.
- Robert Viator and PI: Sept. 10-Dec. 15, 2016 at the IMA, UMN in MN while the PI was a visitor and Viator was a postdoc; Dec. 12-16, 2016 at the IMA, UMN in MN; Feb. 19-25, 2017 at FIT in Melbourne, FL.

- Stephen Shipman and PI: May, 8-12, 2016 at SIAM Conference on Mathematical Aspects of Material in Philadelphia, PA; Mar. 20-26, 2016 at FIT in Melbourne, FL.

VII. CONTACTS and INTERACTIONS

1. Dr. Joao Romano (CIV USARMY RDECOM ARDEC, General Manager and Technical Lead, Precision Armament Laboratory)
    a. Jan. 6, 2016, after my talk at the AFOSR EM Contractors Review Meeting, I had about an hour discussion with Dr. Romano on my research as he had interest in my YIP grant research that I presented at this meeting for which Dr. Romano was in attendance.
    b. Jan. 11, 2016 I was notified by my Program Manager, Dr. Arje Nachman, that Dr. Romano was interested in contacting me about my work so that he could get my presentation and ``share it with the TPOCs at ARDEC with the intent to hopefully start discussions," about my research area, ``and possible transitions of such work."
    c. Jan. 13, 2016 I sent Dr. Romano an email and included my talk slides and several of my papers related my YIP grant research.
2. Rochelle Kronzek (Executive editor, World Scientific Publishing Co.)
    a. The PI and Alex Figotin had several discussions (which began in 1/12/2016-1/16/2016 and continued again on 12/20/2016-12/28/2016) with Rochelle Kronzek, an Executive editor from the well-known and respected publisher, namely, World Scientific (http://www.worldscientific.com/), about their interest in publishing a math-physics monograph based on our joint work. We informed the editor that we would be interested in publishing a manuscript in the future, but not at this time.
3. Fadil Santosa (Professor of Mathematics, University of Minnesota and Director, Institute for Mathematics and its Applications)
    a. Aug. 10-14, 2015 at the ICIAM 2015 in Beijing, China, the PI had a very positive interaction with Prof. Santosa after I gave a talk on my YIP grant research (see item V.5 in this report) at the Mathematics and Optics minisymposium which he was a co-organizer of. This led to an informal invitation to attend a thematic annual program at the IMA being co-organized by Prof. Santosa during the Fall 2016 Semester.
    b. Jan. 11, 2016, a formal letter from Prof. Santosa inviting me to visit, from Sept. 10-Dec. 15 in 2016, the Institute for Mathematics and its Applications at the University of Minnesota to participate in the thematic annual program ``Mathematics and Optics" in which the IMA offered $18,000 to partially buy me out of my teaching duties at Florida Institute of Technology for this period of time. The PI's institution agreed to allow me to go with the remaining portion of my teaching duties being bought out using funds from my YIP grant.

VIII. STUDENTS.

The following students were trained, educated, and completed research related to the PI's AFOSR sponsored research during the term of this grant:

1. Daniel McCormick, Bachelor of Science (Magna Cum Laude, GPA: 3.82). Major: Mathematical Sciences; Minor: Physics. Graduated: May, 2018. Academic Advisor (Spring 2015-Spring 2018): Aaron Welters.

- Daniel (the first student I advised since becoming a faculty member at FIT in 2014) will be starting as a graduate student in the Department of Mathematics at the University of Utah and will pursue his goal of a PhD in Applied Mathematics.
- As an undergraduate, he published a paper (doi: 10.1016/j.laa.2018.06.024) on an inverse eigenvalue problem based on a REU project in Summer 2017.
- In Spring 2017 and Fall 2017, we worked together on an undergraduate research project, associated directly with my grant research, entitled ``Finite-Interval Overdamping: Phenomena, Analysis, & Applications."
    - The poster presentation he gave on this project won the award ``Best in Show for Mathematical Sciences" at the FIT Senior Design Showcase held on April 7-8, 2017.
    - The long-term goal of the initial research I did with Daniel is to develop a mechanism, based on the overdamping phenomena, for absorption/vibration suppression in circuits, mechanical, or electromagnetic systems, without the need for the lossy components to be ``high-loss."
    - The research was motivated by the objectives of my grant and based on A. Figotin and A. Welters' publication III.1. We observed numerically in composite dissipative-gyroscopic Lagrangian systems the existence of a new type of overdamping phenomena, which we coined ``finite-interval," in contrast to ``semi-infinite interval" overdamping that occurs in the selective overdamping phenomena studied in ref. III.1.
    - Daniel was able to prove as Theorems, using the Routh-Hurwitz stability criterion in linear control theory and Descartes' rule of signs on the roots of polynomials, that for such two-component composites systems with a lossy and a losses component, that this finite-interval overdamping does occur at least for systems with two degrees-of-freedom under certain assumptions on the associated Lagrangian. Important examples of such systems include RLC circuits with gyrators and Coriolis gyroscopes with constant angular velocity.
- With me, he also attended a conference, minisymposium, and workshop. First, the SIAM Conference on Mathematical Aspects of Materials Science in Philadelphia, PA, May 8-12, 2016 and was in attendance at the minisymposium I co-organized there entitled, ``Complex Analysis, Optimization, and Herglotz Functions in Passive Electromagnetics and Composite Media." Then, he was an invited participant (the only undergraduate in attendance) at the workshop I co-organized entitled, ``Herglotz-Nevanlinna Functions and their Applications" at the Institute Mittag-Leffler in Djursholm, Sweden.
- He will also be invited to the upcoming workshop I am co-organizing entitled, ``Herglotz-Nevanlinna Theory Applied to Passive, Causal and Active Systems," at the Banff International Research station in Oct. 5-11, 2019.

2. Caleb Webb, Bachelor of Science, University of Utah. Dual Major: Physics and Mathematics. Graduated: May, 2018. REU Co-mentor: Sept. 2016-June 2017. Research project title: ``Spectral analysis of periodic, nonreciprocal systems consisting of high-loss and lossless components."
    - I was an REU (Research Experience for Undergraduates) co-mentor with Maxence Cassier (while he was a postdoc at the Univ. of Utah) of Caleb Webb from Sept. 2016-June 2017. The project was financed by the University of Utah.
    - My role in the REU project was to provide the model to study in the project based on objectives of the AFOSR YIP research project of the PI. Through teleconferencing on a weekly basis and the

occasion visit to the Univ. of Utah, I would advise and give directions on the overall research project and its trajectory as it developed.

- A manuscript (unpublished) entitled ``A classical model for infinite, periodic, nonreciprocal media with dissipative elements" was produced by Caleb based on his work in this project. Here is a copy of the abstract to give an idea of the project and its relevance to the PI's research: `` In previous work, the spectral properties of two component composite systems of finitely many degrees of freedom have been analyzed. Specifically, gyroscopic media consisting of high-loss and lossless components have been discussed. As a next step in modeling real composite materials with gyroscopic elements, we construct a general framework for analyzing periodic systems. Using a Lagrangian formalism, we derive a convenient model for studying infinite, periodic, nonreciprocal structures with arbitrarily many degrees of freedom in one (physical) dimension. We apply this model to derive the band structure of both a lossless, single component gyroscopic system and a two component, gyroscopic, system with dissipative elements."
- The main motivation for this project was to extend the results of the PI and Alex Figotin in their 2016 paper (see item III.1 of this report) and the work (see reference below) of Alex Figotin and Ilya Vitebskiy (AFRL) to lattice models based on the objectives of the PI's research.
    - A. Figotin and I. Vitebskiy, ``Spectra of Periodic Nonreciprocal Electric Circuits," SIAM J. Appl. Math., Vol. 61, No. 6, pp. 2008-2035 (2001). https://doi.org/10.1137/S0036139900370583
- The main model used in this project was first developed by the PI and his collaborators Stephen P. Shipman and Alex Figotin during their meetings and interactions support by the PI's grant.

3. Denzel Williams, Bachelor of Science. Major: Mathematical Sciences; Graduated: May, 2018. Undergraduate Research Advisor: Jan.-, 2018. Research project title: ``On interconnection formulas for the Dirichlet-to-Neumann map in electrical network theory."
    - Denzel will be starting in Fall 2018 as a graduate student in the Department of Mathematical Sciences at Florida Institute of Technology and will pursue his goal of a MS in Operations Research.
    - Since Jan., 2018, we have worked together on an undergraduate research project, associated directly with my grant research. It is expected that he will continue working with me until Aug. 2018 when he enters as a graduate student at FIT.
    - Here is the abstract of the research project associated with the PI's research program: ``Consider the Dirichlet problem for an oriented finite linear connected graph G with boundary nodes B which can be described as follows: If a boundary voltage f is given, the solution to the Dirichlet problem is a potential u defined throughout G, which agrees with f on the boundary B of G, and which satisfies the conductivity equation inside G. Interconnection of different graphs produce a new graph, thus a new Dirichlet problem to solve. In this paper we look at the relationship between solutions of the Dirichlet problem of the individual networks and that of the composite network."

I am currently supervising the research of the following Ph.D. student at FIT with the expectation that he will pursue a research topic in area related to the PI's research program developed during this report period:

4. Bader Alshammari (Ph.D. expected in Summer 2020); Currently he is preparing for Oral Qualifying Exams in Nov. 2018.

AFOSR Deliverables Submission Survey

1.

Report Type

Final Report

Primary Contact Email
Contact email if there is a problem with the report.

awelters@fit.edu

Primary Contact Phone Number
Contact phone number if there is a problem with the report

3216747202

Organization / Institution name

Florida Institute of Technology

Grant/Contract Title
The full title of the funded effort.

On a Theory of Broadband Absorption Suppression in Magnetic Composites

Grant/Contract Number
AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-15-1-0086

Principal Investigator Name
The full name of the principal investigator on the grant or contract.

Aaron Welters

Program Officer
The AFOSR Program Officer currently assigned to the award

Arje Nachman

Reporting Period Start Date

04/01/2015

Reporting Period End Date

03/31/2018

Abstract

The scientific focus and main objective of the PI's research project was on the development of an effective theory of broadband absorption suppression in magnetic composites. During the report period (April 1, 2015-March 31, 2018), the PI has made steady progress towards this objective and has had several accomplishments which are described in more detail in this final performance report.

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Archival Publications (published) during reporting period:

Articles in refereed journals:

1. A. Figotin and A. Welters, ``On overdamping phenomena in gyroscopic systems composed of high-loss and lossless components," J. Math. Phys. 57, 042902 (2016). doi: http://dx.doi.org/10.1063/1.4944721.

Book chapters:

2. M. Cassier, A. Welters, and G. W. Milton, ``Analyticity of the Dirichlet-to-Neumann map for the time-harmonic Maxwell's equations," in Extending the Theory of Composites to Other Areas of Science (G. W. Milton, Editor), Chapter 4, Milton-Patton Publishing, 2016. ISBN-10: 1483569195.

3. M. Cassier, A. Welters, and G. W. Milton, ``A rigorous approach to the field recursion method for two-component composites with isotropic phases," in Extending the Theory of Composites to Other Areas of Science (G. W. Milton, Editor), Chapter 10, Milton-Patton Publishing, 2016. ISBN-10: 1483569195.

New discoveries, inventions, or patent disclosures:

Do you have any discoveries, inventions, or patent disclosures to report for this period?

No

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