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Hiroaki Nishikawa

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14. ABSTRACT
Unstructured-grid methods are essential for computations with complex geometries such as rotorcraft simulations, but its potential has been limited by a higher cost than structured-grid methods as well as inaccuracy in gradient predictions (e.g., diffused or oscillatory vorticity predictions). Grid irregularities are hard to avoid particularly once grid adaptation is performed, which is a critical technique especially for high-order unstructured-grids methods to be practical. Current state-of-the-art Navier-Stokes (NS) codes are known to produce highly erratic viscous stress

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Report Title

Intrinsically Efficient and Accurate Viscous Simulations via Hyperbolic Navier-Stokes Systems

ABSTRACT

Unstructured-grid methods are essential for computations with complex geometries such as rotorcraft simulations, but its potential has been limited by a higher cost than structured-grid methods as well as inaccuracy in gradient predictions (e.g., diffused or oscillatory vorticity predictions). Grid irregularities are hard to avoid particularly once grid adaptation is performed, which is a critical technique especially for high-order unstructured-grids methods to be practical. Current state-of-the-art Navier-Stokes (NS) codes are known to produce highly erratic viscous stress and heating distributions. Resolution of these problems is very important for justifying the use of high-fidelity models in aerodynamic design and optimization. In this project, we address these issues by developing a Navier-Stokes solver based on a novel first-order hyperbolic system method. The new solver is expected to yield O(1/h) acceleration in convergence over existing solvers, where h is the typical grid spacing, as well as achieve high-accuracy in auxiliary quantities,
viscous stresses, heating rates, and vorticity, on unstructured grids. These improvements will be achieved
by the new method in which the Navier-Stokes equations are discretized as a first-order hyperbolic system including the auxiliary quantities as additional variables. The new code will enable complex large-scale simulations with the current hardware and meet the challenge of highly efficient and accurate multi-scale unsteady aerodynamic computations of Army's interest: vortex-dominated flows, separated flows, wake
interaction, and dynamic stall of rotor-craft, helicopter blades, high-speed missiles, gun-launched projectiles, micro air vehicles, and micro adaptive flow control, which require especially accurate vorticity predictions. The new solver is implemented in the framework of a practical flow solver used by Army, NASA's fully unstructured and parallel 3D RANS code, FUN3D. It will be, therefore, applied immediately to Army applications such as unsteady simulations of a rotorcraft and helicopter blades.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
02/25/2015	10.00	Hiroaki Nishikawa. Accuracy-preserving boundary flux quadrature for finite-volume discretization on unstructured grids, Journal of Computational Physics, (01 2015): 0. doi: 10.1016/j.jcp.2014.10.033
08/11/2014	6.00	Alireza Mazaheri, Hiroaki Nishikawa. Very efficient high-order hyperbolic schemes for time-dependent advection–diffusion problems: Third-, fourth-, and sixth-order, Computers & Fluids, (10 2014): 0. doi: 10.1016/j.compfluid.2014.06.020
08/12/2014	9.00	Hiroaki Nishikawa. First, second, and third order finite-volume schemes for advection–diffusion, Journal of Computational Physics, (09 2014): 0. doi: 10.1016/j.jcp.2014.05.021
10/22/2013	4.00	Hiroaki Nishikawa. First-, second-, and third-order finite-volume schemes for diffusion, Journal of Computational Physics, (01 2014): 0. doi: 10.1016/j.jcp.2013.09.024

TOTAL: **4**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received	Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

-"Hyperbolize It" by Hiroaki Nishikawa, Conference on the Future Directions in CFD Research, sponsored by NASA, August 6-8, 2012. Hampton, VA.

- "Past or Future?: A Never-Ending Story of CFD Algorithm Development" by Hiroaki Nishikawa, Four Decades of CFD: Looking Back and Moving Forward, A Symposium Celebrating the Careers of Antony Jameson, Phil Roe and Bram van Leer, sponsored by AFOSR and University of Kansas, June 22-23, 2013. The Westin San Diego, 400 West Broadway, San Diego, California 92101.

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received		Paper
08/03/2016	16.00	. Hyperbolic Navier-Stokes Method for High-Reynolds-Number Boundary-Layer Flows, 55th AIAA Aerospace Sciences Meeting. 09-JAN-17, Gaylord, TX. : ,
08/03/2016	17.00	. Third-Order Inviscid and Second-Order Hyperbolic Navier-Stokes Solvers for Three-Dimensional Unsteady Inviscid and Viscous Flows, 55th AIAA Aerospace Sciences Meeting. 09-JAN-17, Gaylord, TX. : ,

TOTAL: **2**

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received		Paper
08/01/2016	7.00	Ali R. Mazaheri, Hiroaki Nishikawa. High-Order Residual-Distribution Hyperbolic Advection-Diffusion Schemes: 3rd-, 4th-, and 6th-Order, 7th AIAA Theoretical Fluid Mechanics Conference. 16-JUN-14, Atlanta, GA. : ,
08/01/2016	8.00	Hiroaki Nishikawa. First, Second, and Third Order Finite-Volume Schemes for Navier-Stokes Equations, 7th AIAA Theoretical Fluid Mechanics Conference. 16-JUN-14, Atlanta, GA. : ,
08/01/2016	1.00	Hiroaki Nishikawa. First, Second, and Third Order Finite-Volume Schemes for Diffusion, 51st AIAA Aerospace Sciences Meeting, 7 - 10 January, Grapevine, Texas, 2013. 07-JAN-13, Grapevine, TX. : ,
08/01/2016	11.00	Hiroaki Nishikawa. Alternative Formulations for First-, Second-, and Third-Order Hyperbolic Navier-Stokes Schemes, 22nd Computational Fluid Dynamics Conference. 22-JUN-15, Dallas, TX. : ,
08/01/2016	12.00	Alireza Mazaheri, Hiroaki Nishikawa. High-Order Hyperbolic Residual-Distribution Schemes on Arbitrary Triangular Grids, 22nd AIAA Computational Fluid Dynamics Conference. 22-JUN-15, Dallas, TX. : ,
08/01/2016	15.00	. Third-Order Inviscid and Second-Order Hyperbolic Navier-Stokes Schemes for Three-Dimensional Inviscid and Viscous Flows, 46th AIAA Fluid Dynamics Conference. 13-JUN-16, Washington, D.C.. : ,
08/23/2016	18.00	. First, Second, and Third Order Finite-Volume Schemes for Advection-Diffusion, 21st AIAA Computational Fluid Dynamics Conference. 24-JUN-13, San Diego, CA. : ,

TOTAL: **7**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received	Paper

TOTAL:

Number of Manuscripts:

Books

Received | Book

TOTAL:

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Hiroaki Nishikawa was awarded Associate Fellow in The American Institute of Aeronautics and Astronautics (AIAA) in 2015, Class of 2016.

Graduate Students

NAME	PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Post Doctorates

NAME	PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Hiroaki Nishikawa	0.50	
FTE Equivalent:	**0.50**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
Yi Liu	0.50
FTE Equivalent:	**0.50**
Total Number:	**1**

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

The developed hyperbolic Navier-Stokes scheme has been implemented in NASA's FUN3D code, which is used by Army as a near-body solver in Army's Helios code. The implementation has been performed by Dr. Yi Liu of National Institute of Aerospace in the framework of FUN3D development team. The development continues under NASA funding, towards generating an economical third-order scheme for turbulent flow applications.

A low-order version of the developed hyperbolic Navier-Stokes scheme has been implemented into a commercial CFD code at Software CRADLE, based on the publication [AIAA2015-2451]. It is still under development, and not officially released yet.

Alireza Mazaheri at NASA Langley Research Center showed an interest in the hyperbolic method and approached Hiroaki Nishikawa in 2013. He coded it with different discretization methods (residual-distribution and discontinuous Galerkin) for model problems, under the supervision of Hiroaki Nishikawa. Alireza Mazaheri obtained NASA's internal funding in 2014, and intends to extend a hyperbolic discontinuous-Galerkin method to the Navier-Stokes equations for hypersonic-flow applications. Hiroaki Nishikawa no longer works with Alireza Mazaheri.

Hiroaki Nishikawa interacted with Mario Ricchiuto at INRIA, France, in 2012. Ricchiuto came up with an idea of extending the hyperbolic method to dispersion equations (third derivatives). Later, Alireza Mazaheri (NASA) coded the idea, and a paper was published [JCP2016].

Hiroaki Nishikawa interacted with Professor Hubert Baty at University of Strasbourg, France, who contacted Nishikawa about applying the hyperbolic method to a model for magnetohydrodynamic (MHD) flows he proposed recently. Baty coded the method and demonstrated its superior properties for MHD problems of his interest [H. Baty and H. Nishikawa, Hyperbolic Method for Magnetic Reconnection Process in Steady State Magnetohydrodynamics, Monthly Notices of the Royal Astronomical Society, 459, pp.624-637, 2016.]

Intrinsically Efficient and Accurate Viscous Simulations via Hyperbolic Navier-Stokes System

Final report to ARO regarding W911NF-12-1-0154 (August 1, 2015 - July 31, 2017)

Hiroaki Nishikawa
National Institute of Aerospace,
100 Exploration Way, Hampton VA 23666, USA

1 Foreword

This is a final report for the research funded by ARO under W911NF-12-1-0154 in the period August 1, 2012 - July 31, 2016. The fourth year was added for the implementation of the hyperbolic Navier-Stokes scheme into NASAs FUN3D code, which was originally planned in the third year but could not be completed due to a newly added effort of developing an economical third-order edge-based scheme.

2 Statement of the Problem Studied

Unstructured-grid technologies are essential for flow simulations over complex geometries such as rotorcraft simulations, but their potential have been limited by a higher cost than structured-grid methods as well as inaccuracy in gradient predictions (e.g., diffused or oscillatory vorticity predictions). Grid irregularities are hard to avoid particularly once grid adaptation is performed, which is a critical mechanism for high-order unstructured-grid methods to be practical. Even current state-of-the-art Navier-Stokes (NS) codes are known to produce highly erratic viscous stress and heating distributions [1, 2]. Resolution of these problems is very important for justifying the use of high-fidelity models in aerodynamic design and optimization for complex applications.

In this project, we address these issues by developing a Navier-Stokes solver based on a novel first-order hyperbolic system method. The new solver is expected to yield $O(1/h)$ acceleration in convergence over existing solvers, where h is the typical grid spacing, as well as achieve high-accuracy in auxiliary quantities, viscous stresses, heating rates, and vorticity, on unstructured grids. These improvements will be achieved by the new method in which the Navier-Stokes equations are discretized as a first-order hyperbolic system including the auxiliary quantities as additional variables. The new code will enable complex large-scale simulations with the current hardware and meet the challenge of highly efficient and accurate multi-scale unsteady aerodynamic computations of Armys interest: vortex-dominated flows, separated flows, wake interaction, and dynamic stall of rotor-craft, helicopter blades, high-speed missiles, gun-launched projectiles, micro air vehicles, and micro adaptive flow control, which require especially accurate vorticity predictions. The new solver will be implemented in the framework of a practical flow solver used by Army, NASA's fully unstructured and parallel 3D RANS code, FUN3D. It will be, therefore, applied immediately to Army applications such as unsteady simulations of a rotor-craft and helicopter blades.

3 Summary of the Most Important Results

3.1 Economical Third-Order Edge-Based Discretization

In the early stage of the project, it was recognized that the hyperbolic Navier-Stokes method enables the construction of a highly economical third-order finite-volume scheme. The PI decided to pursue the development of third-order Navier-Stokes schemes with the concurrence of the then-ARO program manager, Dr. Frederick Ferguson. The economical third-order edge-based method was discovered by Katz and Sankaran [3], but its applicability was limited to a hyperbolic system of conservation laws, e.g., the Euler equations. Extension to diffusion terms was not clear. Also, it was not clear how to preserve third-order accuracy at boundary nodes. This project addresses these issues by formulating the viscous terms as a hyperbolic system and deriving a boundary quadrature formula that preserves the design accuracy at boundary nodes [4].

3.1.1 Very Accurate and Efficient Third-Order Diffusion Scheme

The hyperbolic method enables us to write a diffusion term as a hyperbolic system, and therefore the third-order scheme is directly applicable. This was demonstrated in Ref.[5], which was later published in Journal of Computational Physics [6]. *The resulting scheme was demonstrated to achieve third-order accuracy in the solution and gradients on irregular grids, and achieve $O(1/h)$ convergence acceleration.*

3.1.2 Simplified Construction of Hyperbolic Advection Diffusion Scheme and Artificial Hyperbolic Diffusion

The third-order diffusion scheme was extended to advection diffusion in Ref.[7], which led to another journal publication [8]. This paper introduced a simplified construction of hyperbolic schemes: add an advective scheme and a hyperbolic diffusion scheme, which enabled the extension of the hyperbolic method to more complicated systems such as the Navier-Stokes equations. Also, the study revealed that the hyperbolic diffusion scheme can produce accurate gradients even with a vanishingly small viscosity. It means that a hyperbolic diffusion scheme can be introduced just to produce accurate gradients with a sufficiently small coefficient. Later, this technique was used in the name of *artificial hyperbolic diffusion* to generate accurate density gradients in hyperbolic Navier-Stokes schemes.

3.1.3 Achieved Third-Order Accuracy without Curved Elements

A general boundary quadrature formula was derived, which preserves third-order accuracy at boundary nodes [4] on linear elements (straight/planer boundary elements). This is a significant contribution in that this third-order scheme does not require a challenging task of generating curved high-order grids for curved geometries, which has been identified as a significant bottleneck in many modern high-order unstructured-grid methods. The third-order edge-based scheme does not require high-order grid generation, and is immediately applicable to existing grids used in second-order computations.

3.2 Demonstrated Time-Accuracy of Hyperbolic Schemes

Time accuracy of hyperbolic diffusion schemes was demonstrated in Refs.[9, 10]. This was the first demonstration of hyperbolic diffusion schemes for unsteady problems. It had been obvious (just use a steady solver in combination with an implicit time integration scheme), but was never demonstrated numerically before.

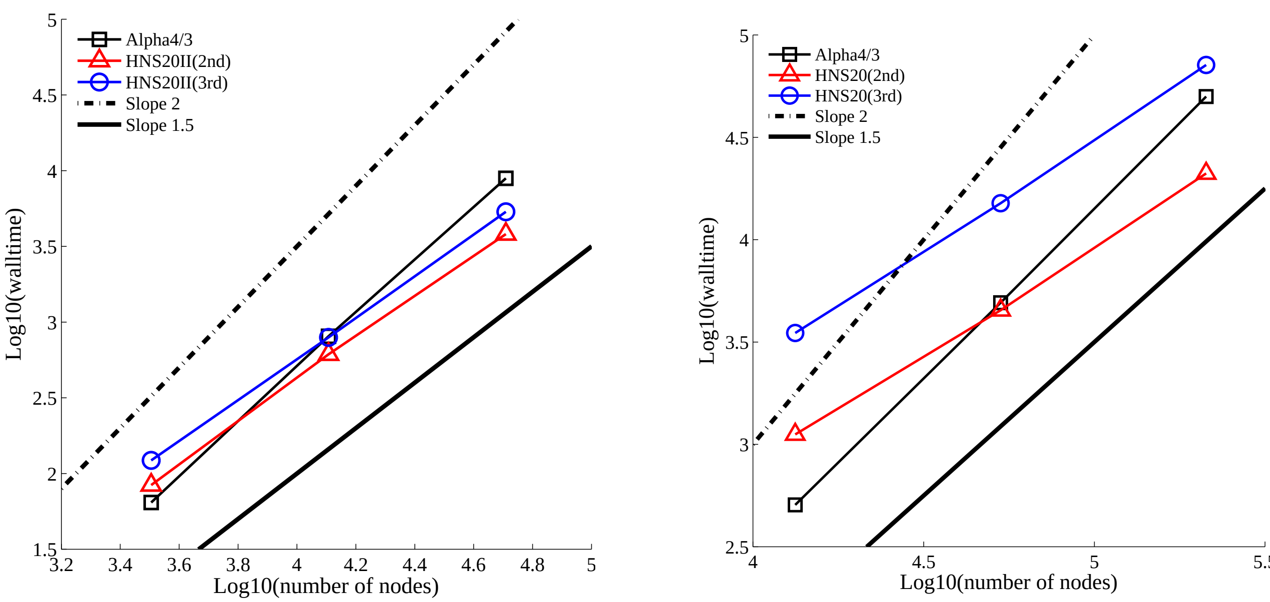
3.3 Extended to Dispersion Problem

The hyperbolic method was extended to the dispersion equation (third-order derivative) [11, 12]. While it has applications to dispersion phenomena (e.g., water waves), a significant finding was that the hyperbolic dispersion scheme can produce the solution, gradients, and Hessian (second derivatives) to the same order of accuracy on irregular grids. This is significant in comparison with conventional schemes, which lose one order of accuracy in the gradients, and two orders in Hessian. It shows a potential for improving anisotropic grid adaptation methods, where directions of refinement are determined by Hessian information.

3.4 Versatile Hyperbolic Navier-Stokes System (HNS20)

The hyperbolic method was extended and demonstrated to the compressible and incompressible Navier-Stokes equations first in Ref.[13]. Convergence acceleration and high-order/quality gradients (i.e., viscous stresses) were demonstrated for viscous flow problems in 2D.

A more versatile hyperbolic formulation of the Navier-Stokes equations was developed and presented in Ref.[14]. The new system called HNS20 enables the same order of accuracy in the primitive variables and their gradients. Consequently, a second-order HNS scheme achieves third-order accuracy in the inviscid terms. Significant results in Ref.[14] include the superior iterative convergence by the HNS solver. As shown in Figure 1, the HNS solver gets faster for finer grids; this is because of the elimination of the numerical stiffness associated with the second derivatives in the viscous terms. Slopes of 1.5 and 2.0 are theoretical predictions for the HNS solver and a conventional solver, respectively. Note that all solvers are implicit solvers. The most significant fact is that the third-order accurate solver can converge faster than a second-order conventional solver, which is typically impossible (because a higher-order scheme is always more expensive on a given grid than a second-order scheme).



(a) Viscous flow over a cylinder at $Re_\infty = 40$. (b) Viscous flow over a flat plate at $Re_\infty = 10^4$.

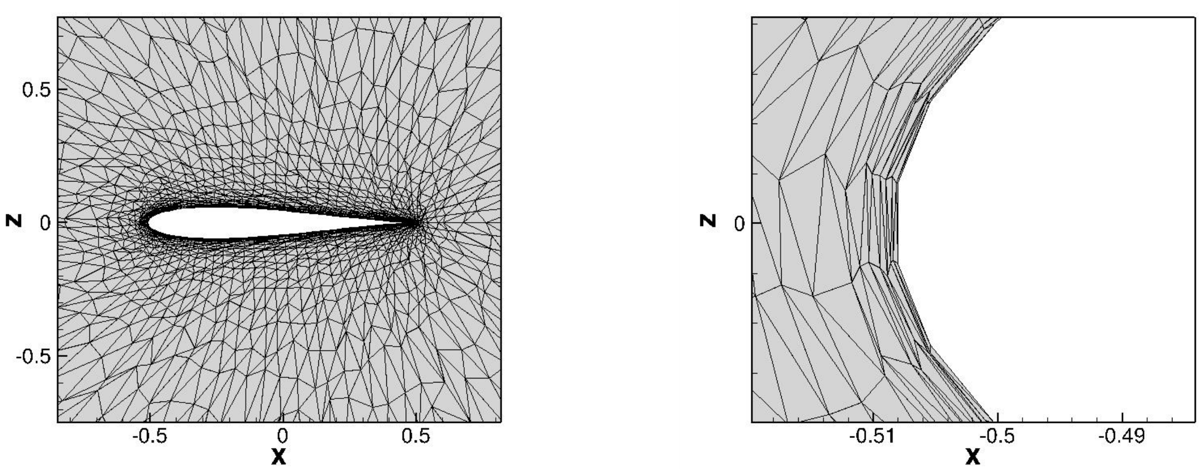
Figure 1: CPU time versus nodes. Alpha4/3 indicates a conventional second-order finite-volume Navier-Stokes solver. All computations were performed on triangular grids.

3.5 Implementation and Demonstration in NASA's FUN3D Code

The HNS20 scheme was extended to three dimensions, and implemented in NASA's FUN3D code. Dr. Yi Liu (National Institute of Aerospace) was hired for this particular task in 2015, and the HNS20 solver was demonstrated for three-dimensional steady viscous flows. Important results were presented at the 46th AIAA Fluid Dynamics Conference [15]. The HNS20 scheme denoted by HNS20-I(Q), which is second-order but achieves third-order accuracy in the inviscid terms, has been implemented and demonstrated for a series of 3D unstructured tetrahedral grids over a Joukowsky wing. Highly-skewed nature of the grids can be seen in Figure 2. Comparison of the default second-order FUN3D scheme and the HNS20-I(Q) is shown in Figures 3-5 on the second coarsest grid. These results demonstrate the ability of the HNS20-I(Q) scheme to produce highly smooth and accurate gradients and Hessian on irregular grids. These results are significant and quite encouraging in that arbitrary grid adaptation can be applied to viscous problems without degrading accuracy of derivative quantities.

Furthermore, the superior drag convergence by the HNS20-I(Q) scheme has been demonstrated as shown in Figure 6. In the figure, FUN3D is the default FUN3D scheme, FUN3D-i3rd is the FUN3D scheme with a third-order inviscid scheme, HNS20-I is a standard second-order HNS scheme, and HNS20-I(Q) is a version that can achieve third-order accuracy in the inviscid terms. It is shown that the HNS20-I(Q) scheme gives much more accurate drag coefficient than FUN3D does for the same number of discrete unknowns. Compared for the same number of discrete unknowns, the HNS solver has been found to require less CPU time than the default second-order FUN3D solver.

The HNS scheme in FUN3D has been extended to unsteady flows, and results will be presented at 55th AIAA Aerosapce Sciences Meeting in January 2017 [16]. Also, a new formula for improving the performance of the HNS scheme for very high-Reynolds-number flows has been proposed, and a detailed study will be presented at 55th AIAA Aerosapce Sciences Meeting in January 2017 [17].

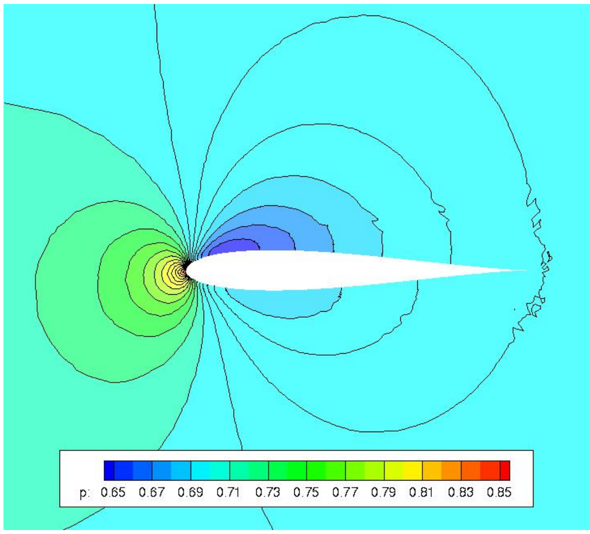


(a) Near-field view. (b) Near-field view close to the viscous wall.

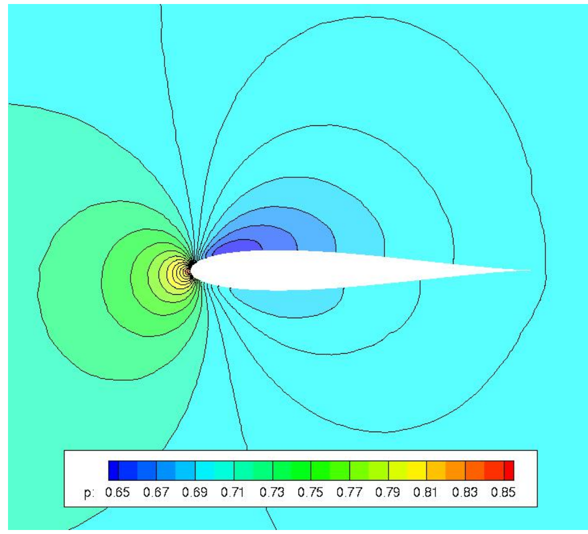
Figure 2: Near-field views in (x, z)-plane of a 3D tetrahedral grid over a Joukowsky wing (Grid1).

References

[1] P. A. Gnoffo. Multi-dimensional, inviscid flux reconstruction for simulation of hypersonic heating on tetrahedral grids. AIAA Paper 2009-599, Orlando, Florida, January 2009.

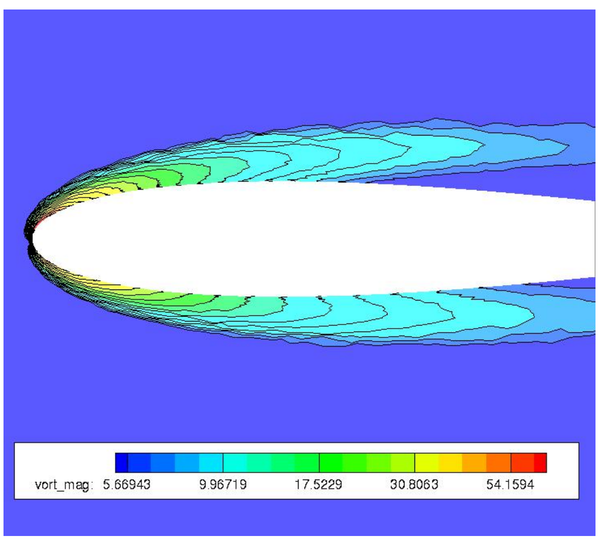


(a) FUN3D.

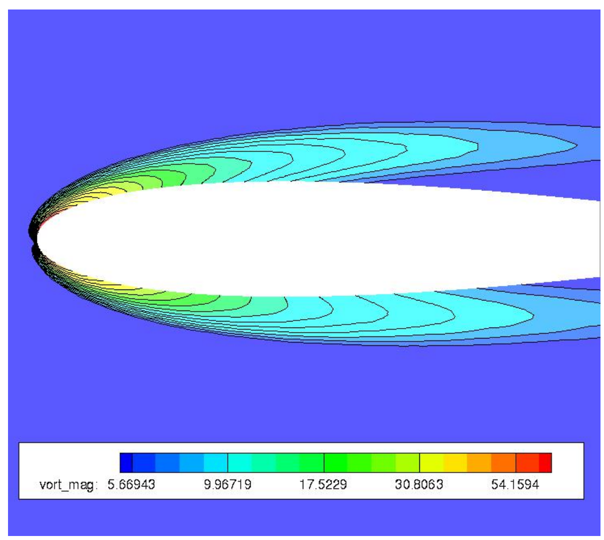


(b) HNS20-I(Q).

Figure 3: Pressure contours for Joukowsky wing (Grid2).

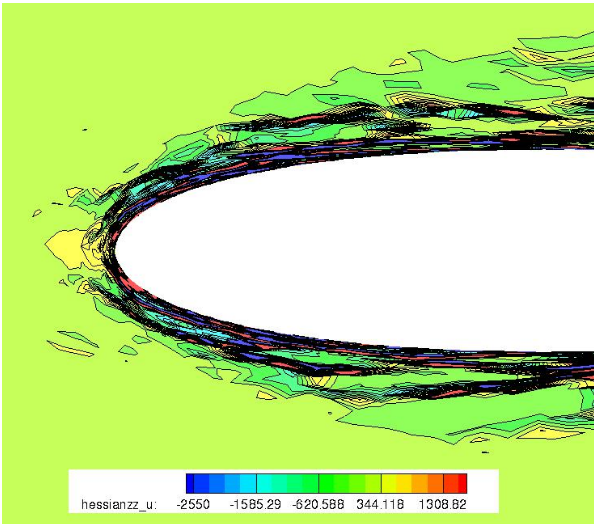


(a) FUN3D: Linear LSQ $|\text{curl}\,\mathbf{u}|$.

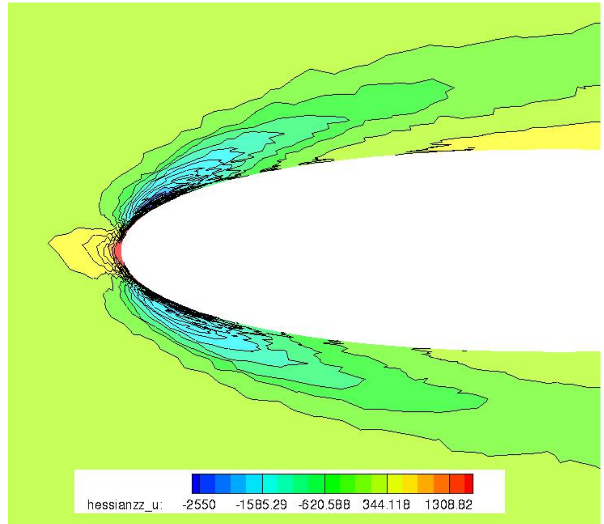


(b) HNS20-I(Q): Gradient variables that correspond to $|\text{curl}\,\mathbf{u}|$.

Figure 4: Contours of the magnitude of the vorticity for Joukowsky wing (Grid2).

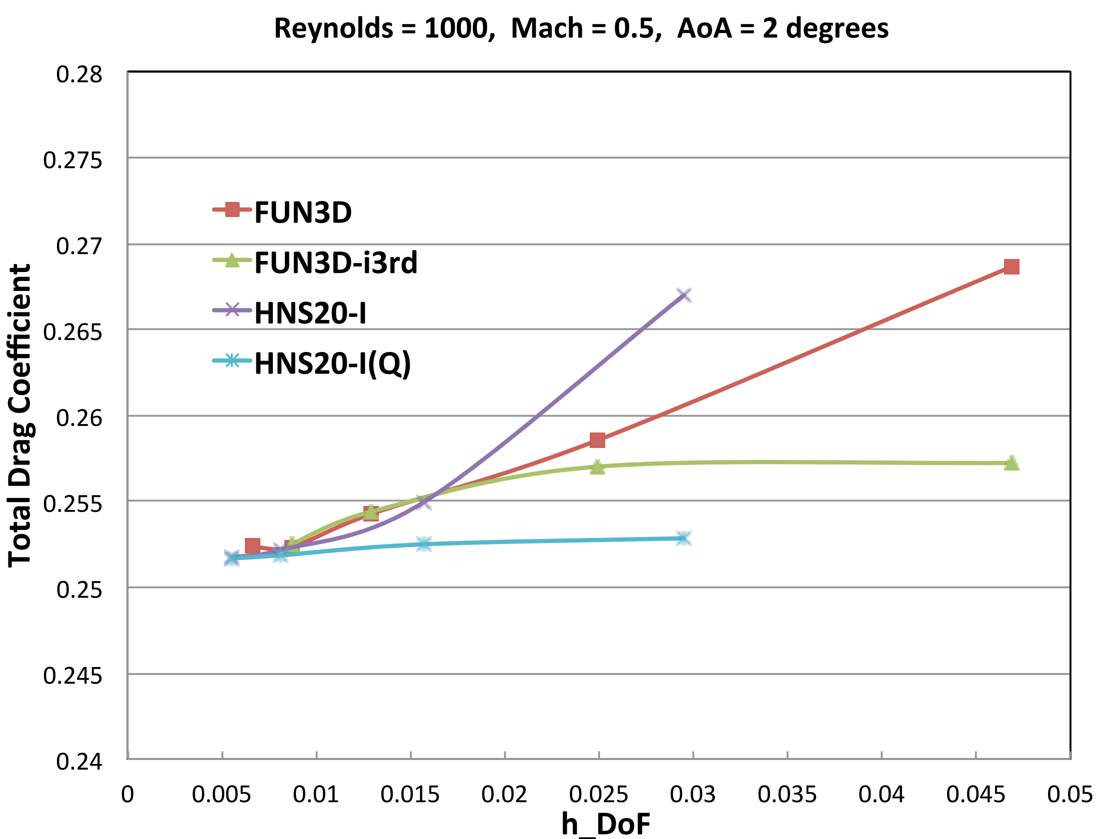


(a) FUN3D: Linear LSQ applied twice to the x-velocity component, u.



(b) HNS20-I(Q): Linear LSQ of the gradient variable that corresponds to $\partial_z u$.

Figure 5: Contours of the second-derivaitve $\partial_{zz} u$ for Joukowsky wing (Grid2).



(a) C_D versus h_{DoF}

Figure 6: Grid convergence of drag coefficient for Joukowsky wing. C_D is based on the chord length. The mesh spacing scale h_V is the L_1 norm of the cubic root of the control volume; $h_{DoF} = (4N)^{-1/3}$ for the HNS schemes and $h_{DoF} = N^{-1/3}$ for others, where N is the total number of nodes in the grid. NOTE: the same h_{DoF} means the same size of discrete problems.

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