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Unified Theory and Algorithm for Solving Challenging Problems in Mathematical Physics and Complex Systems with Applications

David Gao FEDERATION UNIVERSITY AUSTRALIA

04/20/2018 Final Report

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# Final Report for AFOSR Grant (AOARD) FA2386-16-1-4082 July 1, 2016 – December 31, 2017

**PI:** David Y Gao, Alex Rubinov Professor of Mathematics, *Federation University Australia* **Title**: A Breakthrough Theory and Algorithms for Solving Chaotic Dynamics and NP-Hard Problems in Mathematical Physics and Complex Systems

**Program Manager**: DR. KRISTOPHER H AHLERS, Lt Col USAF AFMC AFOSR/IOA **Email**: <u>kristopher.ahlers@us.af.mil</u>

## Personnel Supported:

Senior Research Fellow: Dr. Ning Ruan Visiting Scientists:

- 1. Professor Peter Olver, University of Minnesota
- 2. Professor Z.C. Cai, Purdue University

## **Other Research Collaborators:**

- 1. Dr. Eldar Hajilarov, Federation University
- 2. Dr. Vittorio Latorre, University of Rome.

## Accomplishments/New Findings:

Supported by this AOARD grant, the PI and his post-doctor and co-workers have successfully developed/improved a breakthrough canonical duality theory and its associated algorithms for solving a large class of challenging problems in mathematical physics and complex systems. Within one year, he has published 1 book by Springer, 1 journal special issue (Springer), and about 29 papers (11 are journal papers).

The most significant achievement is the solution to the well-known **knapsack problem**, which is listed as one of 21 NP-complete problems in computer science. By using the canonical duality theory, this 0-1 integer programming problem can be equivalently converted to a non-smooth concave maximization problem with only one dual variable, which can be solved very easily, therefore, this so-called NP-complete problem can be obtained analytically via this canonical dual solution. Application to computational physics leads to **a powerful deterministic algorithm** for solving the most challenging bi-level mixed integer programming problem in structural topology optimization. Research results can be directly applied for **light-weight design** of aircrafts, fight jets, and much more (see Figures 1 and 2).

Another important achievement of this project is a revolutionarily new global optimal method for solving chaotic dynamical systems. Instead of traditional linear iteration methods, a nonlinear dynamical system is first formed as a global optimization problem via the least squares method. Then using the canonical duality theory, this nonconvex minimization problem can converted as a concave maximization problem in dual space, which can be solved easily via the well-developed convex optimization techniques. If this canonical dual has a unique solution, the nonlinear system is not chaotic. Otherwise, the primal global optimization problem is NP-hard, and in this case, the nonlinear system has chaotic solutions. Therefore, the connection between chaos in complex systems and the NP-hardness in computer science is revealed for the first time.

Remaining significant achievements include a unified model for multi-scale complex systems, a unified methodology for solving a class of fully nonlinear partial differential equations, and a unified algorithm for solving a class of so-called NP-Hard problems in decision science and global optimization.

The projects proposed in the proposal are fully completed.



Fig. 1. Design domain for a long cantilever beam with external load

TOP88: volfrac= 0.5, nex= 180, ney= 60, It.= 60, C=184.3573, Time= 0.16725

(a)

BESO: Vc=0.5, mu=0.975, nex=180, ney=60, lt=32, C=173.2986, T=4.0202



(b)





(c)

Fig. 2. Topology optimization results obtained by SIMP (a), BESO (b), and the canonical duality theory CDT (c).

Co-sponsored by Federation University, this AFOSR grant has been used to support one senior research fellow, one PhD student, several international visitors, and international research activities.

## Keynote, Plenary, and Invited Lectures at International Conferences.

- 1. Invited Lecture, <u>Symposia on Intelligent Technologies for Advancing and</u> Safeguarding Australia. 15 August 2017, Deakin University, Geelong, Australia.
- 2. Invited Lecture, 12<sup>th</sup> International Symposium on Health Informatics and Management, 2-3 June, 2017, National Chiao Tung University, Taiwan.
- 3. Plenary Lecture, <u>International Conference in Mathematics Trends and</u> <u>Developments</u> 2017 (ICMTD-17), 28 – 30 Dec. 2017, Cairo, Egypt.
- 4. Plenary Lecture, Frontier Forum on Intelligent Control and Decision Optimization, 23-25 August, 2017, Central South University, Changsha, China
- Plenary Lecturer, The 2<sup>nd</sup> International Conference and Summer School on <u>Numerical Computations: Theory and Algorithms</u>, 19 – 25 June 2016 Club Med Resort "Napitia" Pizzo Calabro, Calabria, Italy.
- 6. Keynote Lecturer, International Symposium of Heterogeneous Materials Mechanics, June 4-6, 2016, Chongqing, China.
- 7. Invited Lecturer, <u>The 10<sup>th</sup> International Conference on Scientific Computing and</u> <u>Applications</u>, June 7-10, 2016, Toronto, Ontario

### **Contributions within Discipline:**

The canonical duality methodology can be used for modeling multi-scale complex systems. The canonical duality theory produces analytical solutions to a class of fully nonlinear partial differential equations in large deformation solid mechanics, post-buckling of thin-walled structures and phase transitions problems. The algorithm can be used for solving a large class of challenging problems in mathematical physics and complex systems.

## **Contributions to Other Disciplines:**

Canonical duality theory has been used successfully in neural networks optimization, sensor communication systems, filter design, signal processing, machine learning, chaotic dynamical systems, decision making, supply chain, scheduling problems, and computational mechanics, light-weight structural design, etc.

### **Publications:**

### **Books/Special Issue Published:**

- 1. Gao, DY, Latorre, V. and Ruan, N. (2017) <u>Canonical Duality Theory: Unified</u> <u>Methodology for Multidisciplinary Study</u>, Springer, 377pp.
- Floudas, C. and Gao, DY, (2016). <u>Special issue of 3<sup>rd</sup> World Congress of Global</u> <u>Optimization</u>, Journal of Global Optimization, Springer, Volume 64, Issue 3, March 2016

### Papers published in international journals:

 Gao, DY, Neff, P., Roventa, I., Thiel, C. (2017) <u>On the convexity of nonlinear elastic energies in the right Cauchy-Green tensor</u>, *Journal of Elasticity*, April 2017, Volume 127, Issue 2, pp 303–308 10.1007/s10659-016-9601-6. http://arxiv.org/abs/1508.05721

- 2. Jin, Z. and Gao, D.Y. (2017) On modeling and global solutions for d.c. optimization problems by canonical duality theory, *Applied Mathematics and Computation* 296 168–181
- Fang, S-C., Gao, D.Y., Lin, G.-X., Sheu, R.-L., Xing, W.X. (2017). Double Well Potential Function and Its Optimization in The n-dimensional Real Space -- Part I, *J. Industrial and Management Optimization*. 13(3): 1291-1305. doi: 10.3934/jimo.2016073 <u>http://arxiv.org/abs/1410.5925</u>
- 4. Gao, DY and Lu, X.J. (2016). On the extrema of a nonconvex functional with double-well potential in 1D, Z. Angew. Math. Phys. 67:62
- 5. Gao, DY (2016). Analytic solutions to general anti-plane shear problems in finite elasticity, *Continuum Mech. Thermodyn*, 28:175–194 Published online at <a href="http://link.springer.com/article/10.1007%2Fs00161-015-0412-y">http://link.springer.com/article/10.1007%2Fs00161-015-0412-y</a>
- Latorre, V. and Gao, DY (2016). Global Optimal Trajectory in Chaos and NP-Hardness, Int J. Bifurcation and Chaos, 26, 1650142 (2016) [14 pages] DOI: http://dx.doi.org/10.1142/S021812741650142X
- Chen Y., Gao D.Y. (2016). Global solutions to nonconvex optimization of 4thorder polynomial and log-sum-exp functions. *Journal of Global Optimization*, vol. 64, pp. 417-431, 10.1007/s10898-014-0244-5.
- Latorre, V. and Gao, DY (2016). Canonical duality for solving general nonconvex constrained problems, *Optimization Letters*, vol. 10, pp. 1763-1779, 10.1007/s11590-015-0860-0. <u>http://arxiv.org/abs/1310.2014</u>
- Zhou X., Gao D.Y., Simpson A.R. (2016). <u>Optimal design of water distribution</u> <u>networks by a discrete state transition algorithm</u>. *Engineering Optimization*, vol. 48, pp. 603-628, 10.1080/0305215X.2015.1025775.
- Zhou X., Gao D.Y., Yang C. (2016). Global solutions to a class of CEC benchmark constrained optimization problems. *Optimization Letters*, vol. 10, pp. 457-472, 10.1007/s11590-014-0784-0.
- 11. Zhou X., Gao D.Y., Yang C., Gui W. (2016). Discrete state transition algorithm for unconstrained integer optimization problems. *Neurocomputing*, vol. 173, pp. 864-874, 10.1016/j.neucom.2015.08.041.

### **Book Chapters and Papers in Refereed Proceedings**

- 12. Gao, D.Y., Ruan, N., and Latorre, V. (2017). <u>Canonical duality-triality: Bridge between nonconvex analysis/mechanics and global optimization</u>, in D.Y. Gao et al. (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 1-48.
- 13. Gao, D.Y. (2017). <u>Canonical Duality Theory for Topology Optimization</u>, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 263-276.
- 14. Gao, D.Y. (2017). <u>Remarks on Analytic Solutions and Ellipticity in Anti-plane</u> <u>Shear Problems of Nonlinear Elasticity</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 89-104.

- 15. Gao, D.Y. (2017). <u>Analytic Solutions to Large Deformation Problems Governed</u> <u>by Generalized Neo-Hookean Model</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 49-68.
- 16. Gao, DY and Hajilarov, E (2017). <u>Analytic Solutions to 3-D Finite Deformation</u> <u>Problems Governed by St Venant–Kirchhoff Material</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 69-88.
- 17. Gao, D.Y. and Wu, C. (2017). <u>Triality Theory for General Unconstrained Global</u> <u>Optimization Problems</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 127-154.
- 18. Latorre, V., Sagratella, S., Gao, DY (2017) Canonical Dual Approach for Contact Mechanics Problems with Friction, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 173-186. <u>http://arxiv.org/abs/1402.6909</u>
- 19. Wu, C. and Gao, DY (2017). Canonical Primal-Dual Method for Solving Nonconvex Minimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp.223-248, http://arxiv.org/abs/1212.6492
- 20. Liu, G.S., Gao, DY and Wang, SY (2017). Canonical Duality Theory for Solving Non-Monotone Variational Inequality Problems, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp.155-172.
- 21. Morales-Silva, D. and Gao, DY (2017). On Minimal Distance Between Two Surfaces, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp.359-372.
- 22. Ruan, N. and Gao, D.Y. (2017). Global Optimal Solution Computation of a Quadratic Integer Programming Problem with Linear Inequality Constraints, in D.Y. Gao et al. (eds.), Canonical Duality-Triality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 315-338. <u>http://arxiv.org/abs/1205.0856</u>
- 23. Jin, Z. and Gao, DY (2017). On D.C. Optimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 203-222.
- 24. Chen, Y. and Gao, DY (2017). <u>Global Solutions to Spherically Constrained</u> <u>Quadratic Minimization via Canonical Duality Theory</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 291-314.
- 25. Ruan, N. and Gao, DY (2017). <u>Canonical Duality Theory for Solving Nonconvex/Discrete Constrained Global Optimization Problems</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality-Triality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 187-202.
- 26. Lu, X.J. and Gao, D.Y. (2017). <u>Canonical Duality Method for Solving Kantorovich Mass Transfer Problem</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 105-126.
- 27. Wu, C. and Gao, DY (2017). <u>Canonical Primal–Dual Method for Solving Nonconvex Minimization Problems</u>, in D.Y. Gao *et al.* (eds.), *Canonical Duality Theory: Unified Methodology for Multidisciplinary Study*, Springer, pp. 223-248.

- Gao, D.Y. (2016). On Unified Modeling, Theory, and Method for Solving Multi-Scale Global Optimization Problems, *Numerical Computations: Theory and Algorithms*, AIP Conference Proceedings, vol. 1776, 10.1063/1.4965409.
- Ali E.J., Gao D.Y.(2016). Canonical finite element method for solving nonconvex variational problems to post buckling beam problem. *Numerical Computations: Theory and Algorithms*, AIP Conference Proceedings, vol. 1776, 10.1063/1.4965409.

#### Papers to appear:

- Gao, DY. On Unified Modeling in Multi-Scale Optimization and Canonical Duality-Triality Theory, <u>http://arxiv.org/abs/1605.05534</u>
- 2. Gao, DY. Duality in G. Saccomandi's Challenge on Analytical Solutions to Antiplane Shear Problem in Finite Elasticity, <u>http://arxiv.org/abs/1511.03374</u>
- 3. Latorre, V. and Gao, DY. Half-Quadratic regularization and canonical duality theory in image/signal process.
- 4. Ali, E. and Gao, DY. On SDP Method for Solving Canonical Dual Problem in Post Buckling of Large Deformed Elastic Beam.
- 5. Ruan, N. and Gao, DY. On Modelling and Complete Solutions to General Fixpoint Problems in Multi-Scale Systems with Applications