AN ANALYSIS OF LEARNING CURVE THEORY AND THE FLATTENING EFFECT AT THE END OF THE PRODUCTION CYCLE

THESIS

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Abstract

The premise of this research is to identify and model modifications to the prescribed learning curve model, provided by the Air Force Cost Analysis Handbook, such that the estimated learning rate is modeled as a decreasing learning rate function over time as opposed to the constant learning rate that is currently in use. The current learning curve model mathematically states that for every doubling of units there will be a constant gain in efficiency.

The purpose of this thesis was to determine if a new learning curve model could be implemented to reduce the error in the cost estimates for weapon systems across the DoD. To do this, a new model was created that mathematically allowed for a "flattening effect" later in the production process. This model was then compared to Wright's learning curve, which is the prescribed method to use throughout the Air Force.

The results showed a statistically significant reduction in error through the measurement the two error terms, Sum of Squared Errors and Mean Absolute Percent Error. This paper will explain in detail how the new learning curve was formulated as well as how the testing was conducted to compare the different learning curve methodologies.

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This thesis is dedicated to my loving wife without whom none of this research would be possible.

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Capt Evan R. Boone, USAF

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AN ANALYSIS OF LEARNING CURVE THEORY AND THE FLATTENING EFFECT AT THE END OF THE PRODUCTION CYCLE

I. Introduction

Background

The United States Department of Defense (DoD) operates in a fiscally constrained and financially conscious environment. The Budget Control Act of 2011 magnified these financial concerns while demand for new and updated weapon systems continues to rise. Managers throughout the DoD are expected to maximize the utility out of every dollar as the Department's budget continues to shrink. The increased scrutiny adds greater emphasis on the accuracy of program office estimates to ensure acquisition programs across the DoD are sufficiently funded.

To ensure the DoD produces reliable cost estimates, cost estimating models and tools used by the DoD must be evaluated for their relevance and accuracy. Specifically, the DoD's cost estimating procedures for learning curves were developed in the 1930s (Wright, 1936). As automation and robotics increasingly replace human touch-labor in the manufacturing process, the current 80 year old learning curve model may no longer provide the most accurate approach for estimates. New learning curve methods that incorporate automated production or other factors that could lead to reduced learning should be examined as a possible tool for cost estimators in the acquisition process. Since the original learning curve model was developed, researchers have found other functions to model learning within the manufacturing process. The purpose of this research is to investigate new learning curve estimating methodology, develop the learning curve theory within the DoD, and pursue a more accurate and complete suite of cost estimation models.

Research Objectives/Questions/Hypotheses

The premise of this research is to identify and model modifications to the prescribed learning curve model, provided by the Air Force Cost Analysis Handbook, such that the estimated learning rate is modeled as a decreasing learning rate function over time as opposed to the constant learning rate that is currently in use. The current learning curve model in use today mathematically states that for every doubling of units there will be a constant gain in efficiency. For example, if the manufacturer observed a 10% reduction in man hours in the time to produce unit 10 from the time to produce unit 5, then they should expect to see the same 10% reduction in man hours in the time to produce unit 10 to the time to produce unit 20.

The basis of this research is that more accurate cost estimates could be made if a decay factor was taken into consideration in developing the learning curve model. The proposed modification may take this form:

$$Cost(X) = \; aX^{f(x)} = \frac{a}{X^{f(x)}}$$

Where:

Cost(X) = the cumulative average time (or cost) per unit
X = the cumulative number of units produced
a = time (or cost) required to produce the first unit
f(x) = the learning curve slope represented as a function of units produced

The function to be used for the slope is what this research will attempt to discover. Figure 1 shows the phenomena this research will attempt to model where the black (flatter) line depicts the traditional curve used to model learning and the Red (steeper) line represents the hypothesized learning structure, and the blue line represents actual data.

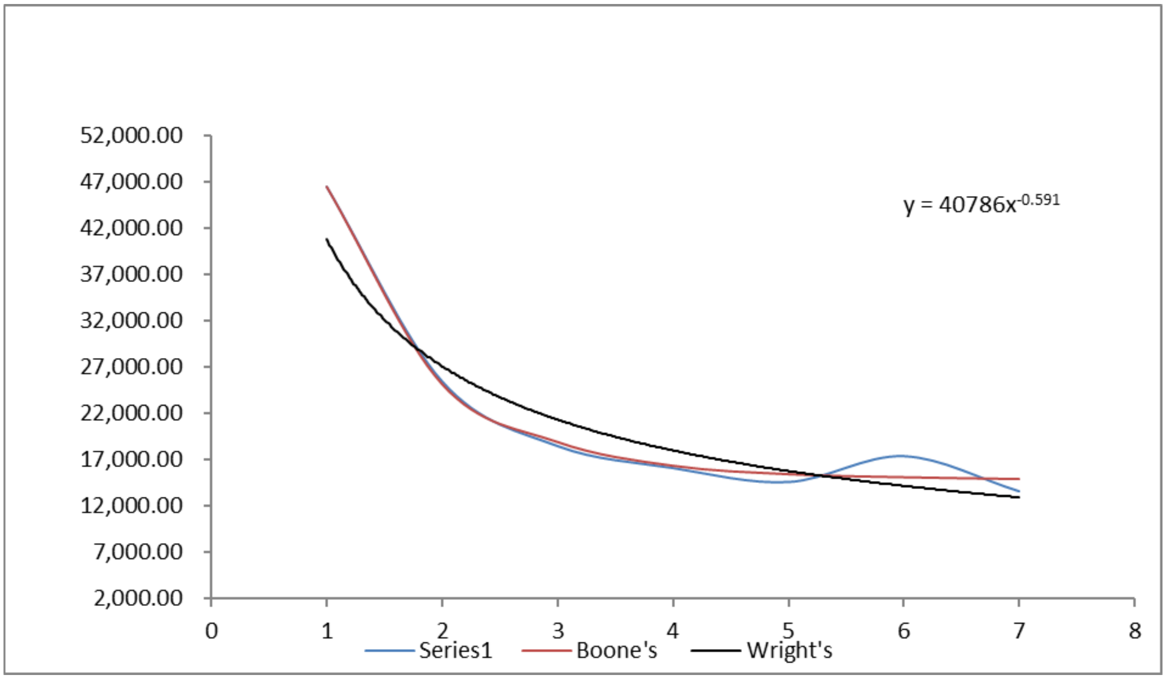

Figure 1: Learning Curve Depiction

This research aims to model a function that has the added precision of diminishing learning effects over time by answering the following questions:

1. How does the incorporation of a decay factor impact the accuracy of DoD cost estimates?
2. At what point in the production process does Wright's Learning Curve no longer reflect accurate costs compared to other learning models?
3. How will the development of a new model affect the amount of error compared to the current estimation models?

If the hypotheses are supported and diminishing learning effects are found to be significant, then this research may contribute by increasing the DoD cost estimating communities understanding by:

1. Developing a modeling tool that incorporates a decay factor into cost estimation techniques.
2. Providing the framework for a software tool that will allow cost estimators to easily calculate estimates based on the data they have available.
3. Refining the methodology of the estimation process so that it can be used in other areas of the finance and accounting world for the benefit of not only the DoD, but the public at large.

Methodology

Learning curves, specifically when estimating the expected cost per unit of complex manufactured items such as aircraft, are frequently modeled with a mathematical power function. The intent of these models is to capture the expected reduction in costs over time due to the learning effects, particularly in areas with a high percentage of human touch labor. Typically as production increases, manufacturers identify labor efficiencies and improve the process. If labor efficiencies are identified, it translates to unit cost savings over time. The general form of the learning curve model used today, as prescribed by the Air Force Cost Analysis Hand Book, is shown below:

$$Cost(x) = \; Ax^{-b} = \frac{A}{x^b}$$

Where:

Cost(x) = the cumulative average time (or cost) per unit

X = the cumulative number of units produced
A = time (or cost) required to produce the first unit
B = slope of the function when plotted on log-log paper
= log of the learning rate/log of 2

The cost of a particular production unit is modeled as a convex curvilinear function that decreases at a constant exponential rate. The problem is that the rate of decrease is not likely to be constant over time. It is proposed that the majority of cost improvements are to be found early on in a program, and fewer revelations are made later in the program as the manufacturer becomes more familiar with the process. As time progresses, the production process should normalize to a steady state and additional cost reductions prove less likely. For relatively short production runs, the basic form of the learning curve may be sufficient because the hypothesized efficiencies will not have yet been gained.

However, when estimating production runs over longer periods of time, the basic learning curve would likely underestimate the unit costs of those farthest out in the future. The underestimation would occur because the model would calculate a constant learning rate, while actual learning would diminish, causing the actuals to be higher than the estimate. The current learning curve could miss a significant cost when dealing with high unit cost items such as those in major acquisition programs, because a small error in the percentage an estimate is off can still be large in terms of dollars. The goal is to add to previous research to determine what modifications can be made to the above function to make the model more accurate. By using curve fitting techniques, a comparison can be made to determine which models best predict learning within the production process. These curve fitting techniques include minimizing the sum of squared error (SSE) terms and changing the parameters within the equations previously listed. Minimizing the sum of squared error is common practice in regression analysis. Many programs like Microsoft excel will do this for you by manipulating the variables in the equation to minimize this error term. As stated above, the intent of this research is to model the learning curve slope as a function of the number of units produced to allow for a slowing of learning over time.

Implications

The goal of this research is to address questions immediately relevant to understanding the links between learning curve models and program cost estimation. Collectively, the proposed work will improve the understanding of how production cycles and time influence the accuracy of current cost estimation techniques, as well as what parameters should be used when making estimates to deliver the most accurate predictions possible. Beyond the Department of Defense, the researchers believe other branches of the government, both state and federal, as well as the civilian sector might use the results of this research to improve their acquisition processes and cost estimating techniques, specifically as they relate to learning. By generating more accurate estimates the DoD will be able to more adequately manage its many projects by giving the programs the resources they need when they need them.

II. Literature Review

Chapter Overview

This chapter will highlight the current theory behind Learning Curve analysis by summarizing published works in the field and it will examine how it applies to the DoD today. It will also lay the groundwork for a discussion on how the production process has changed over the years and how this may or may not affect the way the DoD predicts the effects of learning on the production process. Learning curve research dates back to 1936 when Wright published the original learning curve equation that predicted the effects learning had on production.

Learning Curve Theory Review

First published in 1936, Theodore Paul Wright recognized the mathematical relationship that exists between the time it takes for a worker to complete a single task and the number of times the worker has previously performed that task (Wright, 1936). The mathematical relationship developed from this hypothesis is that as workers completed the same process, they get better at it. Specifically, he realized that the rate at which they get better at that task is constant. The relationship between these two variables is as follows: as the number of units produced doubles, the worker will do it faster by a constant rate (Wright, 1936). He proposed that this relationship takes the form of:

$$F = N^x \quad \text{Or} \quad \mathrm{X} = LogF/LogN$$

"Where F = a factor of cost variation proportional to the quantity N. The reciprocal of F then represents a direct percent variation of cost vs. quantity." (Wright, 1936) The relationship between these variables can be modified to find the expected cost of a given unit number in production by multiplying the factor of cost variation by the theoretical first cost of the aircraft. That equation takes the form of:

$$Y = aX^b$$

Where:

Y = the cumulative average time (or cost) per unit
X = the cumulative number of units produced
a = time (or cost) required to produce the first unit
b = slope of the function when plotted on log-log paper
= log of the learning rate/log of 2

Visually this function is be modeled by the figure below.

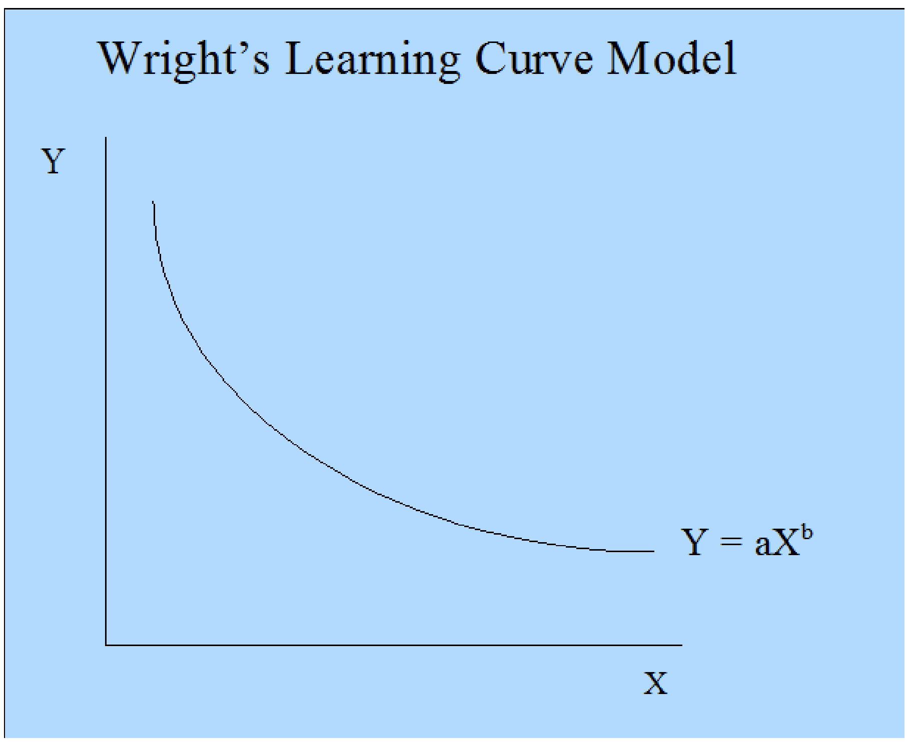

Figure 2: Wright's Learning Curve Model (Martin, N.d.)

As previously stated, this relationship is a log linear relationship through an algebraic manipulation. The logarithmic form of this equation (taking the natural log of both sides of the equation) allows practitioners to run linear regression analysis on the data to find what slope best fits their data using a straight line. (Martin, N.d.)

The goal of using learning curves within the DoD is to increase the accuracy of cost estimates. Having accurate cost estimates allows the government to efficiently budget while providing as much operational capability as possible. According to the GAO Cost Estimating and Assessment Guide, “The ability to generate reliable cost estimates is a critical function, necessary to support the Office of Management and Budget’s (OMB) capital programming process.” The guide also states “cost increases often mean that the government cannot fund as many programs as intended or deliver them when promised.” Even though the use of learning curves focuses on creating accurate cost estimates learning curves often use the number of labor hours it takes to perform a task. When the theory originated, Wright proposed the theory in terms of time to produce, not production cost. However, the DoD does learning curve analysis on both production cost and time to produce, depending on the data available. Even when using labor hour data, the cost estimator still uses that information to estimate a cost based on other factors such as labor rates, and other associated values. Using labor hours allows a common comparison over time without the effects of inflation convoluting the results, but the same goal can be achieved by using inflation adjusted cost values.

Wright’s model has been compared to some of the more contemporary models that have surfaced in the years since the original learning curve theory was established (Moore, 2015). Moore compared the Stanford-B model, Dejong’s model, and the S-

Curve Method to Wright's model to see if any of these functions could provide a more accurate estimate of the learning phenomenon (Moore, 2015). In his research, he used data from the F-15 C/D/E programs. His rationale was that using models that incorporate incompressibility factors would allow for more accurate estimation (Moore, 2015). Both the Dejong model and the S-curve take incompressibility factors into account.

The incompressibility factor is used to account for the percentage of automation in the production process. Therefore, values of the incompressibility factor can range from zero to one where zero is all touch labor and one is 100% automation. Moore found that when using an incompressibility factor between zero and .1, the Dejong and S-Curve models were more accurate (Moore, 2015). Therefore, when a production process has very little automation and high amounts of touch labor, the newer learning curve models are more accurate. For all other incompressibility factors, Wright's model was more accurate. Moore proposed that further research should be conducted on the incompressibility factor values within different DoD industries (Moore 2015). Knowing how much automation is in a process could allow the DoD to use different methods for different industries and ultimately end up with more accurate estimates in industries with low incompressibility factors.

Recently Johnson (2016) followed on Moore's research. Johnson hypothesized that there was a flattening effect at the end of the production process and that learning does not continue to happen at a constant rate toward the end of a production cycle. Johnson also states "it is human nature for people to lose focus or concentration at certain times when performing repetitive tasks." (Johnson, 2016) A loss in concentration could be one of the reasons for the flattening effect Johnson referred to in his research. Using the same models as Moore, Johnson explored the difference in accuracy between Wright's model and contemporary models early in the production process versus later in the production process. He had similar findings to Moore in that Wright's model was most accurate except in cases where the incompressibility factors were extremely low. When the incompressibility factor is low there is more touch labor in the process, and when there is more touch labor in the process more learning can occur. He also found that Wright's learning curve was more accurate early in the production process and Dejong's model and the S-Curve were more accurate later in the production process (Johnson, 2016). Johnson suggested that further research should look into finding a heuristic for when to use one of the more advanced learning curve methods. He believed this would give a more accurate estimate compared to just using Wright's learning curve throughout the production process.

Another key concept in learning curve estimation and modeling is the idea of a forgetting curve (Honious, 2016). A forgetting curve explains how configuration changes in the acquisition of a product can cause a break in production and cause the producer to lose some of the efficiency that they had previously gained. When a configuration change occurs the production process changes. This change could be using different material, different tooling, adding additional pieces to the process, or could even be attributed to workforce turnover. This new process affects how the workers do their work, which causes some of what they had learned to be lost and new potential efficiencies to be available. This research looked into the aspect of the learning curve with the goal of providing the DoD an idea of how configuration changes affect the learning process (Honious, 2016). If the DoD does not take into account a break in learning when a configuration change or any other production break occurs, then they will underestimate the total effort by the contractor to produce the product. She found that configuration changes do significantly change the learning curve and that the new learning curve slope is steeper than the previous steady slope before the configuration change (Honious, 2016).The distinction between pre and post configuration change is important when developing the heuristics mentioned above by Johnson (2016) in order to make sure both effects are taken into account. If practitioners use a flatter learning curve slope later in a project and begin using a different model that more accurately reflects this flatness, then they should also be sure to look at which model is used after a configuration change. It may not be appropriate to use a new model late in a production process after a configuration change.

Prior to Honious's research on forgetting due to configuration changes, Badiru (2012) looked into the impact of forgetting caused by natural effects. Badiru concluded that forgetting was important to factor into a learning curve evaluation and that half-life analysis is important to consider when estimating the effects of the learning curve. The concept of half-life is "the amount of time it takes for a quantity to diminish to half of its original size through natural processes." (Badiru, 2012) Forgetting in the production process can be caused by both internal and external factors (Badiru, 2012). Internal events can range anywhere from complacency of the workforce to policy changes. External impacts include anything from natural disasters to drastic market swings that cause a halt in production. Badiru focused on the actual phenomenon of learning that is apparent in production and less on how the government can use that analysis to generate accurate cost estimates. Nevertheless, his work on half-life learning interpretations and forgetting within the production process put a spotlight on how important it is to accurately estimate the amount of time it will take to produce a product and how this analysis should be viewed moving forward (Badiru, 2012). Ultimately Badiru (2012) recommends that "future efforts to develop learning curve models should also attempt to develop the corresponding half-life expressions to provide full operating characteristics of the models." (Badiru, 2012) This research will help form how we look for a heuristic to apply different learning curve models. If the learning curve behaves differently after reaching its half-life, then the DoD should take that into account and incorporate that modeling technique.

The International Cost Estimating and Analysis Association (ICEAA) published a learning curve guide/training in 2013. While presenting the basics of learning curves, they also presented some rules of thumb for learning. The first rule is that learning curves are steepest when the amount of touch labor is the highest in the production process. Conversely, learning curves are the flattest when the production process is "highly automated" (All, 2013). These rules of thumb make sense when you consider machines do not learn or necessarily improve at a process like humans do. Another key piece of information is that adding new work to the process can affect the cost. ICEAA states new work essentially adds a new curve for the added work and increases the time associated with the new work to the original curve (All 2013). The equation is as follows:

$$Y = a_1 X^{b_1} + a_2 (X - L)^{b_2}$$

Where:
Y = Unit Cost
a_1 = Original Theoretical first cost
a_2 = New Work Theoretical first cost
X = Current Unit Number

L = Last unit before addition of new work
b_1 = exponent for original Learning Curve Slope
b_2 = exponent for new work Learning Curve Slope (Typically the same as b_1)

This equation is important to take into consideration when generating an estimate after a major configuration change or engineering change proposal (ECP). An example of this would be while producing the 8th unit of an aircraft the government realizes they need to drastically change the radar on the aircraft. Learning has already taken place on the first 8 aircraft; the new radar has not yet been installed and therefore there has been no learning. To accurately take into account the new learning the radar would be treated as a second part to the equation ensuring we account for the learning on the 8 aircraft while also accounting for no learning on the new radar.

Lastly, the ICEAA guide speaks to production breaks and the effects they have on a learning curve. These production breaks can cause a direct loss of learning which can fully or partially reset the learning curve. For example, a 50% loss of learning would result in half of the cost reduction that has occurred to be lost (All, 2013). This information is important when analyzing past data to ensure that these breaks in production are accounted for. Without accounting for these breaks within the data, one could inadvertently end up with a model that does not represent the scenario that actually occurred.

In this portion of the chapter, we laid out the fundamental building blocks for learning curve theory and how they will apply to the research in this paper. Wright's learning curve formula established the method by which the DoD estimates the effects of learning in our procurement process. Since Wright's findings, many have proposed and developed new methods for estimating learning. Those methods include accounting for breaks in production, natural loss of learning over time, incompressibility factors, and half-life analysis. This research will add to the discussion by diving deeper into the flattening affect and how different models predict learning at different times in the production process.

Production Process Theory Review

When discussing learning curve theory and the effects learning has on production, one must look into the production process they are trying to estimate. Since Wright established learning curve theory in 1936, automation and technology in factories have grown tremendously and continue to grow. Contemporary learning curve methods try to account for this automation. To get the best understanding, we must look at how things are produced in general and how the aircraft industry, specifically, behaves in relation to the rest of the manufacturing industry.

The aircraft industry, when compared to other industries, has relatively low automation (Henneberer and Kronemer, 1993). Kronemer and Henneberer state "although the industry assembles a high-tech product, its assembly process is fairly labor intensive, with relatively little reliance on high-tech production techniques." (Henneberer and Kronemer, 1993) Specifically, they list three main reasons why manufacturing aircraft is so labor intensive. First, aircraft manufacturers typically build multiple models of the same aircraft just for the commercial sector alone. These different aircraft models mean different tooling and configurations are needed to meet the demand of the customer. Second, aircraft manufacturers deal with a very low unit volume when compared to other industries in manufacturing. The final reason for low levels of automation is the fact that aircraft are highly complex and have very tight tolerances. In order to attain these specifications, manufacturers must continue to use highly skilled touch laborers or spend extremely large amounts of money on machinery to replace them (Henneberer and Kronemer, 1993). For these reasons, we should typically see or use low incompressibility factors in the learning curve models when estimating within the aircraft industry. This conclusion, combined with the previous research that stated contemporary learning curve models are more accurate at low incompressibility factors, provides the basis for looking at models other than Wright's to estimate cost within the DoD (Moore, 2015).

Although the aircraft industry remains largely unaffected by the shift to machine production from human touch labor, most industries are seeing a rise in the percentage of the manufacturing process that is automated. In an article posted in 2012 by the *Wall Street Journal*, the author shows how companies have been increasing the amount of money spent on machines and software while spending less on manpower. They propose part of the reason behind this shift is a temporary tax break "that allowed companies in 2011 to write off 100% of investments in the first year." (Aeppel, 2012) Combining that tax incentive with extremely low interest rates has given industry incentive to invest toward future production (Aeppel, 2012). With the burden of the initial investment being softened by the government, the manufacturing industry is investing in technology. This investment is increasing the incompressibility factor that would be used when estimating the effects of learning in production.

In a separate article for the *Wall Street Journal*, Kathleen Madigan also pointed out the increase in spending on capital investments in relation to labor. She states that “businesses had increased their real spending on equipment and software by a strong 26%, while they have added almost nothing to their payrolls.” (Madigan, 2011) Figure 3 illustrates that statement from 2009 to 2011:

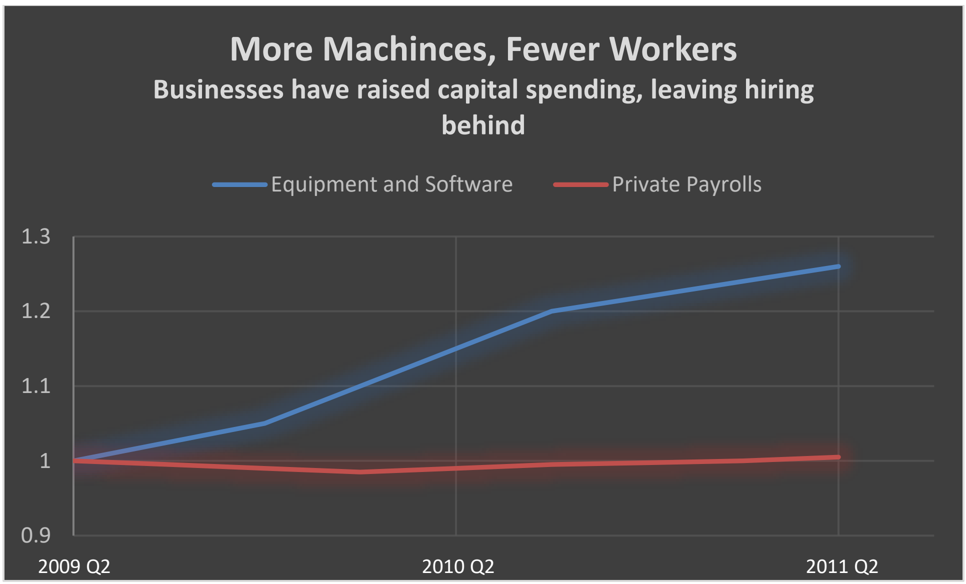

Figure 3: Capital vs Payroll Growth

Again, this figure illustrates an ever changing manufacturing process that continues to transition away from touch labor in favor of a more automated machine oriented process.

An article published by *The Economist* in 2012 lays out how machines and robots are taking over major sections of the manufacturing sector, however we are still a long way from a world where manufacturing floors are 100% automated. There are some areas, of the manufacturing process that need little to no human intervention for days or even weeks. Laser cutting machines and additive manufacturing machines, also known as 3D printers, can run without humans. Therefore, shops that specialize in these tasks have been able to eliminate the need for human touch labor (“Making the Future The Economist” 2012). Some issues with assembly robots include that they are still “too costly and too inflexible” (“Making the Future The Economist” 2012). These issues show that for a highly unique sector, like aircraft manufacturing, the percentage of automation will be low and should remain low in the near future. This fact may change as technology continues to advance but for now humans are still a necessary part of the manufacturing and assembly process, especially in highly unique and technical sectors.

Summary

In this chapter we discussed the fundamental theory behind Learning Curves as well as the basis for the argument that the production process is becoming more automated as we move toward a more technologically advanced society. Learning curves are a significant part of the estimation process for the DoD and as such should be modeled in the most effective manner possible. This thesis will focus on the models that account for incompressibility factors, learning decay, and shifts in the production process and testing whether those models are in fact more accurate than Wright’s model developed approximately 80 years ago. Specifically, this research will explain how a new model, utilizing the ideas of existing models, can affect the accuracy of cost estimates. The next chapter focuses on the methodology used in this research and will include the data used, the methods performed, and the general assumptions made while doing the research.

III. Methodology

Chapter Overview

This chapter will focus on the methods used to select, collect, and analyze the data. The goal is to show how the methods used are appropriate to answer the research questions and hypotheses presented in this paper. Learning curve analysis is crucial in accurately predicting the cost of a weapon system over time. Regression analysis is often performed when measuring the accuracy of a learning curve and that methodology holds true in this analysis as well. This chapter will also lay out how the researchers measure the significance of these tests using statistical tools. By using a wide range of historical weapon systems cost and labor hour data, we hope to gather a broad picture of how the DoD can more accurately predict the effects of learning within the defense community.

Research Questions and Hypotheses

As stated in the introduction of this thesis, this research hopes to answer the following questions:

- How does the incorporation of a decay factor impact on the accuracy of DoD cost estimates?
- At what point in the production process does Wright's Learning Curve no longer reflect accurate costs compared to other learning models?
- How will the development of a new model reduce error compared to the current estimation models?

The following sections of this chapter lay out how the research addressed each of these questions and the methodology used to address them.

Model Formulation

In order to come up with a new learning curve equation the first step was to figure out the characteristics of the curve we expected to best fit the data. The hypothesis is that a curve whose slope decreases over time would fit the data better than Wright's curve. In order to change the rate at which the curve flattens, the "b" value from Wright's learning curve, or the exponent in the power function, needs to be adjusted. Specifically, to make the curve get flatter the exponent in the power curve must decrease as a function of some value. As stated in the introduction chapter, our goal was to have the exponent value vary as a function of "x" the unit number, or time. Initially we modified Wright's existing formula by dividing by a modifier of X, with an equation of:

$$Y = aX^{b/X}$$

Where:

Y = the cumulative average time (or cost) per unit
X = the cumulative number of units produced
a = time (or cost) required to produce the first unit
b = slope of the function when plotted on log-log paper
    = log of the learning rate/log of 2

The change in the exponent value was too dramatic and the resulting prediction line did not meet the intent of the research in this form. In order to lessen the effect of the modifying value we added a qualifier by changing the denominator from "X" to "(1+X/c)". The addition of the c term allows the exponent to change by 1/c for each additional unit produced rather than by 1 for each unit produced. The new equation took the form of:

$$Y = \; aX^{b/(1+\frac{X}{c})}$$

Where:

Y = the cumulative average time (or cost) per unit
X = the cumulative number of units produced
a = time (or cost) required to produce the first unit
b = slope of the function when plotted on log-log paper
= log of the learning rate/log of 2
c = Boone's Decay Value (Range from 0-5000 in this research)

This equation allows for a decrease in the slope as the number of units increases and ultimately provides a curve that can be steeper in the early stages of production and flatter in the later stages of production when compared to Wright's learning curve. The decay variable was added to ensure the rate at which the slope decreases best fit the data for a specific aircraft.

Population and Sample

In order to answer these questions, the researchers looked at quantitative data from several airframes in order to give a comprehensive understanding of how learning affects the cost of lot production. The costs used in this analysis are the direct cost for a single unit and excludes things such as Research, Development, Test, & Evaluation (RDT&E), support items, and spares. This data specifically includes Prime Mission Equipment (PME) only as the researchers believed these cost are the cost that experience an effect due to learning. To ensure we were only comparing things that should be compared, we used inflation and rate adjusted PME cost data for each production lot of the selected weapon systems. The PME cost data was adjusted using escalation rates for materials using the Office of Secretary of Defense (OSD) rate tables when applicable. In order to remain unbiased, we looked at all of the data we could find on major weapon systems. Initially the only requirement for the data to be included in the analysis was that there were at least three production lots. The requirement for three lots was put in place to ensure that either curve wasn't over-fitting the data. After some analysis however we decided to also re run the analysis using only production runs with at least 5 lots. The change to 5 lots was to again ensure we were not overfitting a curve with so few data points. We used data from fighters, bombers, and cargo aircraft, as well as missiles and munitions. Using a diverse dataset ensures that we provide a full picture when discussing whether it is appropriate to use different learning curve estimation techniques. All of the data used for this analysis came from Air Force weapon systems and was pulled from government form 1921s as described in the next section.

Data Collection

Most of the data used was pulled from the Cost Assessment Data Enterprise (CADE) through form 1921s by a member of the Air Force Life Cycle Management Center cost staff. CADE is a resource available to DoD cost analysts that stores historical data on weapon systems. Some of the older data also came from AFLCMC/FZC Research Library in the form of cost summary reports. The data used can be broken out by Work Breakdown Structure (WBS) or Contract Line Item Number (CLIN). For this research the PME cost data was broken out by work breakdown structure element, then rolled up into top line, finished product elements and used for the regression analysis.

This system, as previously stated, provides access to a variety of aircraft models as well as missiles and munitions. It was important to include a wide variety of data to give the Air Force and the DoD a broad sample in order to draw conclusions on the appropriate learning curve methodology. In total we analyzed 46 weapon system platforms. This data can be seen in Table 1.

Data Analysis

In order to test which model is most accurate in estimating the data, we performed regression analysis. When testing how well a model estimates a given set of data using regression analysis, the goal is to minimize the sum of squared errors (SSE). The sum of squared errors is calculated by taking the vertical distance between the actual data point, in this case lot midpoint PME cost, and the prediction line, or estimate. That error term is then squared and the sum of those squared error terms is the value for comparing which model is a more accurate predictor. However, when creating a new model that is similar to the old model with an extra variable the new model should be able to maintain or decrease the SSE in every case. In the case of this research as the decay factor in Boone's learning curve equation approaches infinity the equation approaches Wright's learning curve formula. With this in mind the researchers also looked at the Mean Absolute Percent Error (MAPE). MAPE takes the same error term that is found in the SSE equation and divides it by the actual value of the unit, then takes the mean (arithmetic average) of all of the data points. By putting the error in terms of a percentage it allows a comparison of how the model works between aircraft as well as the accuracy of different learning curve models. Using MAPE could allow the researchers to determine if the equation works better for different groups of aircraft such as Cargo/Fighter/Munitions or if it works better based on another factor such as total cost. If the new model reduces both SSE and MAPE when compared to the SSE and MAPE of Wright's prediction, then the researchers would conclude that the new model would be a more accurate / better model to use when conducting learning analyses.

As stated in previous chapters, Wright's learning curve follows a log-linear curve. A log linear curve is a curve that when one looks at the log of the x-axis and the log of the y-axis, the curve is a linear function. As Wright proposed, this linear transformation occurs because learning happens at a constant rate throughout the production cycle. If learning happens at unequal rates, then the curve in log-log space would no longer be linear. This conclusion means the regression used to compare models will not be linear regression in log space, and we will instead use the tools in Microsoft excel to fit the curves on a standard x and y axis.

Specifically, we used Microsoft Excel's Solver package to minimize the SSE by adjusting the factors for T1 (the theoretical first unit), b (the learning curve slope), and C (Boone's decay value). Using the GRG Non-linear Solver model, Excel cycles values for those 3 variables with the goal of minimizing the sum of squared errors. In order to solve in this format each of the variables needs bounds. The bounds set for each of the variables were based off of the values a cost estimator would use for Wright's curve. These are values that are easy to obtain for any dataset, as they are provided by Microsoft Excel when fitting a power function or by using the "linest()" function in Excel. The researchers used this as a starting point because Wright's curve is currently used throughout the DoD. For the T1 variable, the lower bound was one half of Wright's T1 and the upper bound was 2 times Wrights T1. These values were used to give the solver model a wide enough range to not limit the value but small enough to ease the search for the optimal values. Neither of these limits were found to be binding limits. For the exponent variable values we used between 3 and -3 times Wright's exponent value. In theory the value of the exponent should never go above 0 due to positive learning leading to a decrease in cost, but in practice there are some data sets that go up over time and we wanted to be able to account for those scenarios if necessary. Again, these values between 3 and -3 times Wright's exponent value were never found to be binding limits for the model. Lastly, for the decay variable, values were used between 0 and 5000. These numbers were estimated through initial observation. As Boone's decay value approaches infinity, Boone's learning equation approaches Wright's learning curve equation. Only positive values were used because positive values decrease the slope over time while negative values would have negative learning over time, which is not the focus of this research. The 5000 value was found to be a binding constraint in the solver on several of the data observations. In practice analysts should bound the value as high as possible to reduce error, but in the case of this research we used 5000 as we did not see a significant change from 5000 to infinity.

Statistical Significance Testing

Once the SSE and MAPE values are collected for each learning curve equation the researchers will test for significance to decide whether the difference between the error values for the two equations are statistically different, or if the difference is due to random chance. Specifically, the researchers conducted a T-test on the distribution of the differences in error terms between Wright's and Boone's learning curve equations. The T-test used an alpha value of .05. If the p-value for the test is less than .05 then the results will be considered significant. This T-Test was conducted for both SSE and MAPE values separately.

Summary

This chapter explained how the researchers collected the data, what data was used in this research, and how the data was analyzed. Using regression analysis, the researchers will be able to conclude if Boone's learning curve model provides more accuracy in estimation than the currently prescribed Wright's learning curve model. This comparison will test which model has better SSE and MAPE values and test to see if there is a statistically significant difference between models across different portions of the data. Specifically, we will test which model is better overall and if the models predict better with different weapon system types due to various factors. The next chapter will provide the results and analysis of this research.

IV. Analysis and Results

Chapter Overview

The following section contains the results from the methodology described in the previous section. Chapter IV attempts to answer the three primary research questions proposed earlier in this research: first, how does the incorporation of a decay factor impact the accuracy of DoD cost estimates; second, at what point in the production process does Wright's Learning Curve no longer reflect accurate costs compared to other learning models, and third, how will the development of a new model affect the amount of error compared to the current estimation models? The following graphs and charts will attempt to explain how we used the methodology laid out previously to answer our research questions. This analysis, as previously explained, was conducted on data from 46 weapons systems across the Air Force. This Chapter will focus only on displaying the results and analysis while drawing conclusions from these results and the implications this research may have will be discussed in Chapter V.

Direct Comparison Wright's vs Boone's

Table 1 shows a list of each of the programs used as well as the SSE and MAPE values for Wright's learning curve and Boone's learning curve for each program. This Table can also be seen in the Appendix sorted by Boone's MAPE, SSE Difference, and MAPE difference. The last two columns are the difference in SSE and Difference in MAPE in terms of percentage. This percentage was calculated by taking the difference of Boone's error term minus Wright's error term divided by Wright's error term. Negative values represent programs where Boone's learning curve had less error than Wrights learning curve, and positive values represent programs where Wright's curve had less error than Boone's curve.

Table 1: Results

PROGRAM	Wright's SSE	Wright's MAPE	Boone SSE	Boone MAPE	SSE Difference	MAPE Difference
F-35 - Joint Strike Fighter (JSF) Program	2.78E+08	5.3%	2.17E+08	4.8%	-22%	-10%
F-22A Production	4.88E+08	5.4%	4.90E+08	5.6%	0%	5%
A-10 Aircraft Production	1.58E+07	10.8%	4.51E+05	2.1%	-97%	-80%
C-17 Production	6.56E+10	22.1%	6.02E+10	24.5%	-8%	11%
B-1B System	1.14E+09	6.2%	1.10E+09	5.6%	-4%	-9%
AEA - Prime - E/A-18G - Electronic Variant of the F/A-18 Aircraft	1.94E+06	4.6%	1.95E+06	4.6%	0%	1%
F-18A Production	7.14E+08	13.6%	6.28E+08	12.9%	-12%	-5%
F/A-18E/F Production	5.49E+06	4.6%	5.00E+06	4.0%	-9%	-13%
T-45TS Production	1.30E+09	18.6%	1.21E+09	23.8%	-7%	28%
F-15A/B Production	7.90E+06	3.9%	6.12E+06	3.6%	-23%	-8%
S-3A Production	2.18E+07	6.0%	7.48E+06	3.2%	-66%	-47%
EA-6B Production	1.06E+08	9.6%	1.05E+08	9.7%	-1%	0%
F-4B Production	1.49E+07	10.7%	1.48E+07	13.4%	0%	26%
A-6A Aircraft Production	9.92E+08	16.3%	7.67E+07	10.0%	-92%	-39%
A-6E Aircraft Production	1.81E+08	13.0%	1.78E+08	14.0%	-1%	7%
KC-135A Production	1.71E+07	6.3%	7.96E+06	4.7%	-53%	-26%
A-7D Production	8.00E+06	10.1%	4.11E+06	7.6%	-49%	-25%
B-58A Production	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
F-14A Production	5.00E+07	6.2%	4.89E+07	6.1%	-2%	-2%
F-16A/B Production	4.01E+07	11.1%	5.45E+06	6.5%	-86%	-41%
T-38A Production	1.19E+06	8.8%	1.34E+06	7.8%	13%	-11%
C-5A Production	1.60E+09	10.6%	1.74E+02	0.0%	-100%	-100%
C-5B Production	1.39E+09	6.4%	1.38E+09	6.4%	-1%	0%
C-141A Production	7.61E+08	18.1%	3.18E-01	0.0%	-100%	-100%
F-16C/D Production	6.81E+05	3.3%	1.10E+06	4.1%	62%	26%
F-16A/B Production Blk 25	2.12E+06	7.5%	1.57E+06	6.8%	-26%	-9%
P-3C Production	2.66E+07	5.0%	2.73E+07	5.5%	2%	10%
B-58A Production	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
F-14D Production	3.81E+07	5.9%	2.45E+07	4.5%	-36%	-24%
B-2A Production	3.03E+11	21.9%	1.34E+11	16.7%	-56%	-24%
E-6A Production	1.04E+09	10.0%	1.03E+09	10.3%	-1%	3%
AEA - E/A-18G - Electronic Variant of the F/A-18 Aircraft	9.01E+05	5.1%	6.94E+05	4.0%	-23%	-23%
C-5 Wing Modification	8.20E+06	5.9%	1.77E+06	3.7%	-78%	-37%
AWACS Blk40/45 Upgrade	6.40E+06	10.8%	6.11E+06	9.8%	-4%	-9%
MH-60S VERTICAL REPLENISHMENT HELICOPTER	1.47E+07	8.2%	5.22E+06	5.4%	-65%	-35%
MH-60R Naval Hawk Helicopter	4.95E+07	10.0%	4.98E+07	10.7%	1%	6%
MH-60R Avionics	5.99E+07	19.8%	5.69E+07	20.4%	-5%	3%
JSTARS Radar Subsystem (E-8C)	1.50E+10	12.9%	1.43E+10	14.8%	-5%	15%
H1 UPGRADE Production Program - AH-1Z	1.29E+07	5.5%	1.28E+07	5.4%	-1%	-3%
H1 UPGRADE Production Program - UH-1Y	4.99E+06	3.7%	3.02E+06	3.4%	-39%	-9%
F-35 Joint Strike Fighter (JSF) Program - F-135 Engine	9.63E+07	21.9%	9.45E+07	21.5%	-2%	-2%
F-22 Propulsion (F119 Engine)	1.18E+06	3.1%	1.22E+06	3.4%	3%	7%
B-1B COMPUTER UPGRADE PROGRAM	2.77E+03	3.4%	1.19E-05	0.0%	-100%	-100%
C-5 Avionics Moderization Pgm	1.84E+06	17.3%	1.82E+06	18.0%	-1%	4%
C-5 Reliability Enhancement & Reengineering (RERP)	3.27E+06	1.3%	1.09E+00	0.0%	-100%	-100%
Advanced Anti-Radiation Guided Missile (AARGM) Program	1.98E+03	2.8%	1.19E+03	1.7%	-40%	-40%

Based on these results, we observed that Boone's learning curve equation reduced the sum of squared error in approximately 84% of programs and reduced MAPE in 67% of programs. The mean amount off SSE reduced was 27% and the mean amount of MAPE was reduced by 17%. As previously mentioned these values were based on using both learning curve equations to minimize the SSE for each aircraft data set. This is standard practice in the DoD as prescribed by Air Force Cost Analysis Handbook when predicting the cost of subsequent units or subsequent lots.

Statistical Difference Testing

The results of the significance testing can be seen in Figures 4 & 5. The researchers were testing against an alpha value of .05. P-values, as seen in the Prob > [t] line, that are less than .05 would be considered significant. When testing the significance of the difference in SSE we see that the mean is -.27 which translates to Boone’s learning curve having 27% less SSE on average. Figure 4 also shows a 95% confidence interval that ranges from -.16 to -.39. The p-value, highlighted in orange, shows “<.0001”. Since this value is less than .05 we can conclude that these results are statistically significant.

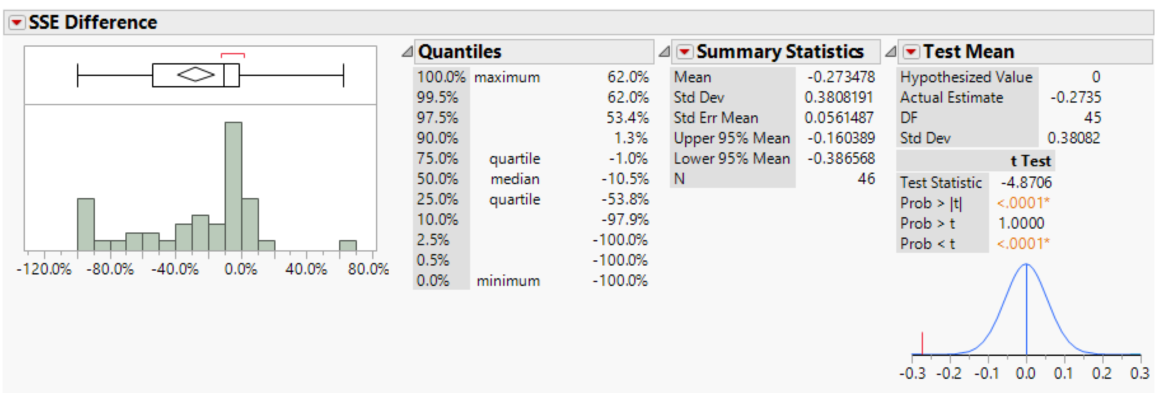

Figure 4: Difference in SSE

We applied the same test to the difference in the MAPE values from Boone’s learning curve and Wright’s learning curve where negative values represent Boone’s curve having a lower MAPE value. Figure 5 shows a mean of -.17 which translates to Boone’s curve reducing MAPE by 17% on average. The 95% confidence interval listed ranges from -.07 to -.27. The p-value, highlighted in orange, shows ".0011". Since this value is less than .05 we can conclude that these results are statistically significant as well.

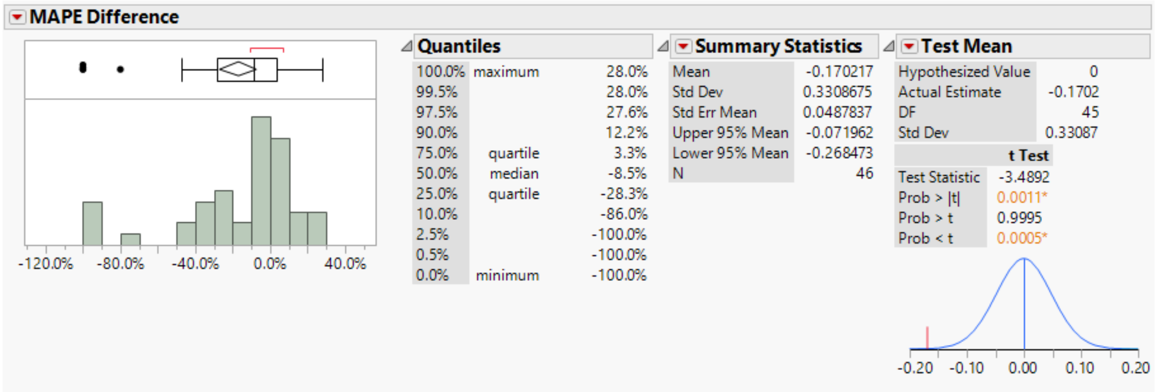

Figure 5: Difference in MAPE

Summary

The purpose of this chapter was to present the results from the analysis for both determining if the new learning curve equation could reduce error over Wright's learning curve and if the reduction in error is statistically different across the data set. The tables and figures in this section show how using Boone's learning curve equation affected the error in the cost estimate for each production run as well as if these results were statistically significant. The results showed that in both SSE and MAPE, Boone's learning curve reduced the error and that each of those values were statistically significant when using an alpha value of .05. Chapter V will show the practical significance of this research as well as recommendations for the DoD cost analysis community, and potential follow-on research topics to further enhance our understanding of the effects of learning in the manufacturing process.

V. Conclusions and Recommendations

Chapter Overview

This chapter will contain the context for the results provided in the previous chapter and the conclusions of the research. The significance of this research will be explained along with recommendations for the DoD cost analysis community and recommendations for further research in this area. The purpose of this thesis was to determine if a new learning curve model could be implemented to reduce the error in the cost estimates for weapon systems across the DoD. To do this, a new model was created that mathematically allowed for a “flattening effect” later in the production process. This model was then compared to Wright’s learning curve, which is the prescribed method to use throughout the Air Force. Along with conclusions and recommendations drawn from this research, the limitations of this study will also be addressed in this chapter.

Conclusions of Research

As stated in chapter 4 there was, on average, a 27% reduction in the SSE among the 46 programs analyzed. It was also concluded that these results were statistically significant. There was also a 17% reduction in the MAPE among the programs analyzed that was also found to be statistically significant. With this information we can conclude that Boone’s learning curve equation is able to reduce the overall error in our cost estimates, and allow the DoD to better allocate its resources. These conclusions answer the first and third question that this research set out to investigate. Also by finding that there is a model that is statistically more accurate throughout the production process we found an answer to question two. Specifically, we were looking for the point where

Wright's model became less accurate than other models but found a curve that was more accurate throughout the entire process. By doing the comparison to Wright's curve we also made the assumption that Boone's curve was more accurate than other learning curves through the conclusions of previous research that stated those curves were not statistically better than Wright's. We also could not directly compare Boone's curve to Dejong's curve because we did not have the incompressibility data needed for the analysis. By reducing the error in the estimates and properly allocating their resources, the DoD could potentially save large amounts of money over time and have better leverage in negotiations with contractors when awarding subsequent lots.

Limitations

Understanding the limitations of the research conducted is important when drawing conclusions and making recommendations. One limitation of this study is that all of the 46 weapon systems used were Air Force systems. While the list included many platforms spanning decades, it would be hard to draw conclusions outside of the Air Force without further research and analysis, however there is no reason to expect Boone's learning curve wouldn't generalize to other production settings. Another limitation in this research is the use of PME cost as opposed to man hours. Man-hour data is not readily available across many platforms, which lead to the use of PME cost. Additionally, the programs used in this study were limited to production lot data. There are inherently less lots than units so this may affect how the equation behaves when used on units. For this research we used the lot midpoint formula/method but further research should be conducted to ensure that the formula operates the same way with unitary data.

Recommendations for Action

By creating an add-in or macro in Microsoft Excel, the entire cost analysis community could have access to this learning curve method. By creating an add-in of some kind, it would make it easy to roll out and for cost estimators to be able to use. Currently, Excel makes it very simple to solve for the parameters in Wright's curve, since it uses the basic form of a power function. A cost estimator can either graph the points and fit a trend line, or use the "linest()" function to provide the intercept and slope parameters. We suggest an add-in similar to the "linest()" function be created to provide the intercept, slope, and decay parameters for Boone's equation as well. Logically, the use of a decay variable is easy to explain to the community as it accounts for the amount that learning decreases over time.

Recommendations for Future Research

As mentioned in the limitations section there is still more research that could be done to ensure the analysis is robust. Data outside of the Air Force should be examined in order to confirm that this equation applies broadly to programs, and not just to Air Force programs. Also, conducting the analysis with unitary data could confirm that this works for predicting subsequent units as well as subsequent lots, while reducing error over Wright's method. Additional research could also include modifications to Boone's formula to try and further reduce the error types listed in this research. Lastly, further research could examine whether the incorporation of multiple learning curve equations at different points in the production process would be beneficial to reducing additional error in the estimates.

Summary

In conclusion, all three research questions were answered in this research. A new learning curve equation was created utilizing the concept of learning decay. This equation was tested against Wright's learning equation to see which equation provided the least amount of error when looking at both the sum of squared errors and the mean absolute percent error. It was found that the Boone's learning curve reduced error in both cases and that this reduction in error was shown to be statistically significant. With the understanding that Boone's learning curve is more accurate than Wright's, the Air Force could build an add-in in Microsoft Excel to be used by the Air Force cost community at large. Follow-on research in this field could lead to further discoveries and allow for broader use of this equation in the DoD cost community.

Appendix A: Results Boone's MAPE

PROGRAM	Wright's SSE	Wright's MAPE	Boone SSE	Boone MAPE	SSE Difference	MAPE Difference
B-1B COMPUTER UPGRADE PROGRAM	2.77E+03	3.4%	1.19E-05	0.0%	-100%	-100%
C-141A Production	7.61E+08	18.1%	3.18E-01	0.0%	-100%	-100%
C-5 Reliability Enhancement & Reengineering (RERP)	3.27E+06	1.3%	1.09E+00	0.0%	-100%	-100%
C-5A Production	1.60E+09	10.6%	1.74E+02	0.0%	-100%	-100%
Advanced Anti-Radiation Guided Missile (AARGM) Program	1.98E+03	2.8%	1.19E+03	1.7%	-40%	-40%
A-10 Aircraft Production	1.58E+07	10.8%	4.51E+05	2.1%	-97%	-80%
S-3A Production	2.18E+07	6.0%	7.48E+06	3.2%	-66%	-47%
F-22 Propulsion (F119 Engine)	1.18E+06	3.1%	1.22E+06	3.4%	3%	7%
H1 UPGRADE Production Program - UH-1Y	4.99E+06	3.7%	3.02E+06	3.4%	-39%	-9%
F-15A/B Production	7.90E+06	3.9%	6.12E+06	3.6%	-23%	-8%
C-5 Wing Modification	8.20E+06	5.9%	1.77E+06	3.7%	-78%	-37%
F/A-18E/F Production	5.49E+06	4.6%	5.00E+06	4.0%	-9%	-13%
AEA - E/A-18G - Electronic Variant of the F/A-18 Aircraft	9.01E+05	5.1%	6.94E+05	4.0%	-23%	-23%
F-16C/D Production	6.81E+05	3.3%	1.10E+06	4.1%	62%	26%
F-14D Production	3.81E+07	5.9%	2.45E+07	4.5%	-36%	-24%
AEA - Prime - E/A-18G - Electronic Variant of the F/A-18 Aircraft	1.94E+06	4.6%	1.95E+06	4.6%	0%	1%
KC-135A Production	1.71E+07	6.3%	7.96E+06	4.7%	-53%	-26%
F-35 - Joint Strike Fighter (JSF) Program	2.78E+08	5.3%	2.17E+08	4.8%	-22%	-10%
MH-60S VERTICAL REPLENISHMENT HELICOPTER	1.47E+07	8.2%	5.22E+06	5.4%	-65%	-35%
H1 UPGRADE Production Program - AH-1Z	1.29E+07	5.5%	1.28E+07	5.4%	-1%	-3%
P-3C Production	2.66E+07	5.0%	2.73E+07	5.5%	2%	10%
F-22A Production	4.88E+08	5.4%	4.90E+08	5.6%	0%	5%
B-1B System	1.14E+09	6.2%	1.10E+09	5.6%	-4%	-9%
F-14A Production	5.00E+07	6.2%	4.89E+07	6.1%	-2%	-2%
C-5B Production	1.39E+09	6.4%	1.38E+09	6.4%	-1%	0%
F-16A/B Production	4.01E+07	11.1%	5.45E+06	6.5%	-86%	-41%
F-16A/B Production Blk 25	2.12E+06	7.5%	1.57E+06	6.8%	-26%	-9%
A-7D Production	8.00E+06	10.1%	4.11E+06	7.6%	-49%	-25%
T-38A Production	1.19E+06	8.8%	1.34E+06	7.8%	13%	-11%
EA-6B Production	1.06E+08	9.6%	1.05E+08	9.7%	-1%	0%
AWACS Blk40/45 Upgrade	6.40E+06	10.8%	6.11E+06	9.8%	-4%	-9%
A-6A Aircraft Production	9.92E+08	16.3%	7.67E+07	10.0%	-92%	-39%
E-6A Production	1.04E+09	10.0%	1.03E+09	10.3%	-1%	3%
MH-60R Naval Hawk Helicopter	4.95E+07	10.0%	4.98E+07	10.7%	1%	6%
F-18A Production	7.14E+08	13.6%	6.28E+08	12.9%	-12%	-5%
F-4B Production	1.49E+07	10.7%	1.48E+07	13.4%	0%	26%
A-6E Aircraft Production	1.81E+08	13.0%	1.78E+08	14.0%	-1%	7%
JSTARS Radar Subsystem (E-8C)	1.50E+10	12.9%	1.43E+10	14.8%	-5%	15%
B-2A Production	3.03E+11	21.9%	1.34E+11	16.7%	-56%	-24%
C-5 Avionics Moderization Pgm	1.84E+06	17.3%	1.82E+06	18.0%	-1%	4%
B-58A Production	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
B-58A Production	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
MH-60R Avionics	5.99E+07	19.8%	5.69E+07	20.4%	-5%	3%
F-35 Joint Strike Fighter (JSF) Program - F-135 Engine	9.63E+07	21.9%	9.45E+07	21.5%	-2%	-2%
T-45TS Production	1.30E+09	18.6%	1.21E+09	23.8%	-7%	28%
C-17 Production	6.56E+10	22.1%	6.02E+10	24.5%	-8%	11%

Appendix B: Results SSE Difference

PROGRAM	Wright's SSE	Wright's MAPE	Boone SSE	Boone MAPE	SSE Difference	MAPE Difference
C-141A Production	7.61E+08	18.1%	3.18E-01	0.0%	-100%	-100%
B-1B COMPUTER UPGRADE PROGRAM	2.77E+03	3.4%	1.19E-05	0.0%	-100%	-100%
C-5A Production	1.60E+09	10.6%	1.74E+02	0.0%	-100%	-100%
C-5 Reliability Enhancement & Reengineering (RERP)	3.27E+06	1.3%	1.09E+00	0.0%	-100%	-100%
A-10 Aircraft Production	1.58E+07	10.8%	4.51E+05	2.1%	-97%	-80%
A-6A Aircraft Production	9.92E+08	16.3%	7.67E+07	10.0%	-92%	-39%
F-16A/B Production	4.01E+07	11.1%	5.45E+06	6.5%	-86%	-41%
C-5 Wing Modification	8.20E+06	5.9%	1.77E+06	3.7%	-78%	-37%
S-3A Production	2.18E+07	6.0%	7.48E+06	3.2%	-66%	-47%
MH-60S VERTICAL REPLENISHMENT HELICOPTER	1.47E+07	8.2%	5.22E+06	5.4%	-65%	-35%
B-2A Production	3.03E+11	21.9%	1.34E+11	16.7%	-56%	-24%
KC-135A Production	1.71E+07	6.3%	7.96E+06	4.7%	-53%	-26%
A-7D Production	8.00E+06	10.1%	4.11E+06	7.6%	-49%	-25%
Advanced Anti-Radiation Guided Missile (AARGM) Program	1.98E+03	2.8%	1.19E+03	1.7%	-40%	-40%
H1 UPGRADE Production Program - UH-1Y	4.99E+06	3.7%	3.02E+06	3.4%	-39%	-9%
F-14D Production	3.81E+07	5.9%	2.45E+07	4.5%	-36%	-24%
F-16A/B Production Blk 25	2.12E+06	7.5%	1.57E+06	6.8%	-26%	-9%
AEA - E/A-18G - Electronic Variant of the F/A-18 Aircraft	9.01E+05	5.1%	6.94E+05	4.0%	-23%	-23%
F-15A/B Production	7.90E+06	3.9%	6.12E+06	3.6%	-23%	-8%
F-35 - Joint Strike Fighter (JSF) Program	2.78E+08	5.3%	2.17E+08	4.8%	-22%	-10%
F-18A Production	7.14E+08	13.6%	6.28E+08	12.9%	-12%	-5%
B-58A Production	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
B-58A Production	1.48E+09	18.8%	1.31E+09	18.3%	-12%	-2%
F/A-18E/F Production	5.49E+06	4.6%	5.00E+06	4.0%	-9%	-13%
C-17 Production	6.56E+10	22.1%	6.02E+10	24.5%	-8%	11%
T-45TS Production	1.30E+09	18.6%	1.21E+09	23.8%	-7%	28%
MH-60R Avionics	5.99E+07	19.8%	5.69E+07	20.4%	-5%	3%
JSTARS Radar Subsystem (E-8C)	1.50E+10	12.9%	1.43E+10	14.8%	-5%	15%
AWACS Blk40/45 Upgrade	6.40E+06	10.8%	6.11E+06	9.8%	-4%	-9%
B-1B System	1.14E+09	6.2%	1.10E+09	5.6%	-4%	-9%
F-14A Production	5.00E+07	6.2%	4.89E+07	6.1%	-2%	-2%
F-35 Joint Strike Fighter (JSF) Program - F-135 Engine	9.63E+07	21.9%	9.45E+07	21.5%	-2%	-2%
A-6E Aircraft Production	1.81E+08	13.0%	1.78E+08	14.0%	-1%	7%
C-5 Avionics Moderization Pgm	1.84E+06	17.3%	1.82E+06	18.0%	-1%	4%
E-6A Production	1.04E+09	10.0%	1.03E+09	10.3%	-1%	3%
C-5B Production	1.39E+09	6.4%	1.38E+09	6.4%	-1%	0%
H1 UPGRADE Production Program - AH-1Z	1.29E+07	5.5%	1.28E+07	5.4%	-1%	-3%
EA-6B Production	1.06E+08	9.6%	1.05E+08	9.7%	-1%	0%
F-4B Production	1.49E+07	10.7%	1.48E+07	13.4%	0%	26%
AEA - Prime - E/A-18G - Electronic Variant of the F/A-18 Aircraft	1.94E+06	4.6%	1.95E+06	4.6%	0%	1%
F-22A Production	4.88E+08	5.4%	4.90E+08	5.6%	0%	5%
MH-60R Naval Hawk Helicopter	4.95E+07	10.0%	4.98E+07	10.7%	1%	6%
P-3C Production	2.66E+07	5.0%	2.73E+07	5.5%	2%	10%
F-22 Propulsion (F119 Engine)	1.18E+06	3.1%	1.22E+06	3.4%	3%	7%
T-38A Production	1.19E+06	8.8%	1.34E+06	7.8%	13%	-11%
F-16C/D Production	6.81E+05	3.3%	1.10E+06	4.1%	62%	26%

Appendix C: Results MAPE Difference

PROGRAM	Wright's SSE	Wright's MAPE	Boone SSE	Boone MAPE	SSE Difference	MAPE Difference
C-141A Production	7.61E+08	18.1%	3.18E-01	0.0%	-100%	-100%
B-1B COMPUTER UPGRADE PROGRAM	2.77E+03	3.4%	1.19E-05	0.0%	-100%	-100%
C-5A Production	1.60E+09	10.6%	1.74E+02	0.0%	-100%	-100%
C-5 Reliability Enhancement & Reengineering (RERP)	3.27E+06	1.3%	1.09E+00	0.0%	-100%	-100%
A-10 Aircraft Production	1.58E+07	10.8%	4.51E+05	2.1%	-97%	-80%
S-3A Production	2.18E+07	6.0%	7.48E+06	3.2%	-66%	-47%
F-16A/B Production	4.01E+07	11.1%	5.45E+06	6.5%	-86%	-41%
Advanced Anti-Radiation Guided Missile (AARGM) Program	1.98E+03	2.8%	1.19E+03	1.7%	-40%	-40%
A-6A Aircraft Production	9.92E+08	16.3%	7.67E+07	10.0%	-92%	-39%
C-5 Wing Modification	8.20E+06	5.9%	1.77E+06	3.7%	-78%	-37%
MH-60S VERTICAL REPLENISHMENT HELICOPTER	1.47E+07	8.2%	5.22E+06	5.4%	-65%	-35%
KC-135A Production	1.71E+07	6.3%	7.96E+06	4.7%	-53%	-26%
A-7D Production	8.00E+06	10.1%	4.11E+06	7.6%	-49%	-25%
B-2A Production	3.03E+11	21.9%	1.34E+11	16.7%	-56%	-24%
F-14D Production	3.81E+07	5.9%	2.45E+07	4.5%	-36%	-24%
AEA - E/A-18G - Electronic Variant of the F/A-18 Aircraft	9.01E+05	5.1%	6.94E+05	4.0%	-23%	-23%
F/A-18E/F Production	5.49E+06	4.6%	5.00E+06	4.0%	-9%	-13%
T-38A Production	1.19E+06	8.8%	1.34E+06	7.8%	13%	-11%
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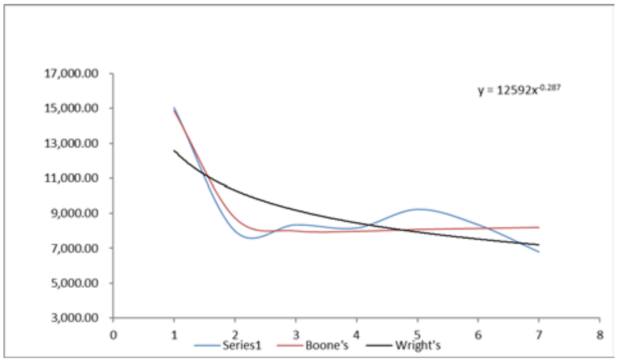
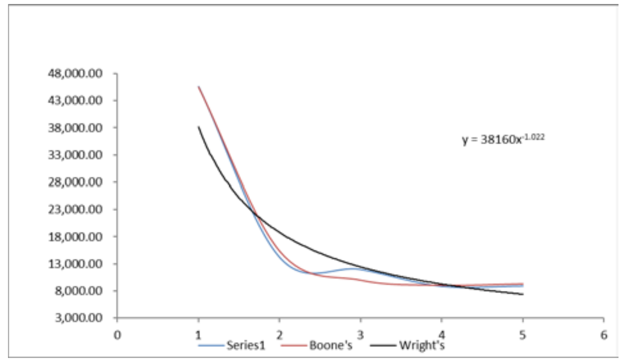
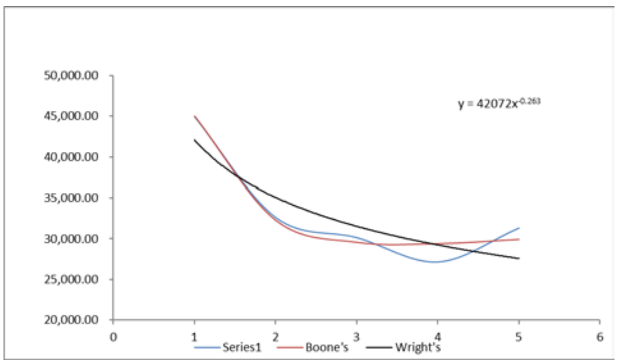
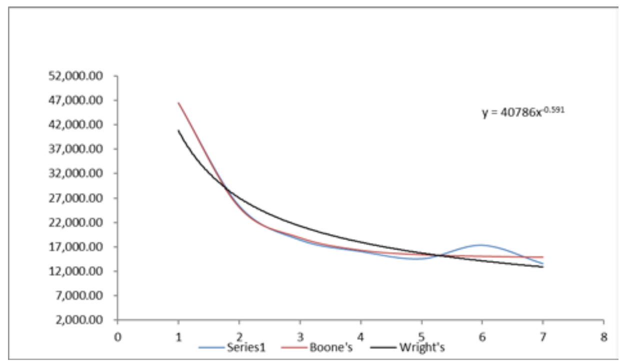
F-16C/D Production	6.81E+05	3.3%	1.10E+06	4.1%	62%	26%
F-4B Production	1.49E+07	10.7%	1.48E+07	13.4%	0%	26%
T-45TS Production	1.30E+09	18.6%	1.21E+09	23.8%	-7%	28%

Appendix D: Wright vs Boone Comparison Examples









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14. ABSTRACT

The premise of this research is to identify and model modifications to the prescribed learning curve model, provided by the Air Force Cost Analysis Handbook, such that the estimated learning rate is modeled as a decreasing learning rate function over time as opposed to the constant learning rate that is currently in use. The current learning curve model mathematically states that for every doubling of units there will be a constant gain in efficiency.

The purpose of this thesis was to determine if a new learning curve model could be implemented to reduce the error in the cost estimates for weapon systems across the DoD. To do this, a new model was created that mathematically allowed for a “flattening effect” later in the production process. This model was then compared to Wright’s learning curve, which is the prescribed method to use throughout the Air Force.

The results showed a statistically significant reduction in error through the measurement the two error terms, Sum of Squared Errors and Mean Absolute Percent Error. This paper will explain in detail how the new learning curve was formulated as well as how the testing was conducted to compare the different learning curve methodologies.

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