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GRAVITATIONAL DETECTION OF SUBMARINES

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14. ABSTRACT

Because a submarine normally operates in a condition of nearly neutral buoyancy, its overall mass (including ballast) is nearly equal to the mass of water it displaces. Accordingly, a submarine produces no first order gravitational anomaly. However, in the interest of stability, a submarine is designed such that its center of mass is located below its center of buoyancy, thus giving rise to a net vertical gravitational dipole moment and a concomitant gravitational field anomaly. This report calculates gravitational "signals" and the possibility of detecting them by an airborne gravimeter or gravity gradiometer. It concludes that militarily useful detection ranges are not achievable.

15. SUBJECT TERMS

Submarine, Detection, Nonacoustic, Gravitation, Gravimeter, Gravity Gradient, Gravitational Dipole, Aircraft, Antisubmarine Warfare

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INTRODUCTION

In accordance with Newton's law of universal gravitation there exists a force of attraction between any two particles in the universe which is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. Because submarines have mass, it is possible, at least in principle, to detect them remotely by sensing the force they exert on a distant reference mass. The purpose of this study was to investigate and assess the feasibility of this potential means for detecting submarines. This report includes (a) calculations of the magnitudes of the gravitational and gravitational gradient "signals" from submarines under a variety of assumed conditions, (b) illustrative calculations of sensor stabilization/compensation requirements, (c) a discussion of gravitational noise and background levels, (d) a brief review of the sensitivities of current gravimeters and gravity gradiometers, (e) a commentary on the results of a previous study, (f) a discussion of limits to the present study, (g) conclusions and (h) acknowledgement.

SIGNAL CALCULATIONS

Because a submerged submarine normally operates in a condition of nearly neutral buoyancy, its overall mass (including ballast) is nearly equal to the mass of water it displaces. Accordingly, a submerged neutrally buoyant submarine produces no first order gravitational anomaly; however, because the distribution of mass throughout the submarine is not uniform, second and higher order effects can be produced. If the boat is properly trimmed, the forward half of its volume will have the same mass as the aft half and therefore it will not exhibit a gravitational dipole moment along its longitudinal axis; similar reasoning applied to port-starboard symmetry rules out the possibility of a dipole moment along the transverse principal axis, although higher order gravitational moments (perhaps quadrupole, octopole, etc) will exist along both those axes. For a submarine to be in stable equilibrium while submerged it is necessary that its center of mass be below its center of buoyancy (in submarine-fixed coordinates). That is, the mass of the lower half of a neutrally buoyant submarine's volume (including ballast) must be greater than the mass of the water it displaces whereas the mass of the upper half must be smaller than the mass of water it displaces. Therefore, a submerged submarine may be considered to possess a net vertical gravitational dipole moment having "negative mass" above and "positive mass" below.

The magnitudes of gravitational and gravitational gradient anomalies will be calculated for three cases: (a) a submarine assumed to be a point mass in empty space, (b) a submarine assumed to behave as a vertical gravitational dipole for which the entire (positive) mass of the submarine is considered to reside in its lower half and to be concentrated at the lower pole and the (negative) mass of the displaced water to reside in its upper half and to be concentrated at the upper pole, and (c) a submarine assumed to be similar to that of case (b) except that 55% of its mass resides in the lower half of its volume and 45% in its upper half.

The following terms, symbols, values and units will be used throughout this report:

Universal gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2/kg^2
Mass of the earth	$m_E = 5.98 \times 10^{24}$ kg
Radius of the earth	$\rho_E = 6.37 \times 10^6$ m
Acceleration of gravity at the earth's surface (standard)	$g_E = 9.81$ m/s^2
Effective mass of the submarine	m_s (kg)
Gravitational anomaly caused by the submarine	Δg (m/s^2)
Distance measured from the center of buoyancy of the submarine	r (m)
Distance from the center of the earth	ρ (m)

Assume that the submarine is of the fleet ballistic missile type (e.g., Ethan Allen class) and can be approximated by a cylinder of length 125 m (410 ft) and diameter 8.4 m (27.6 ft). If it is completely submerged in sea water of density 1028 kg/m^3 (64.2 lb/ft^3) it will displace a volume of 6927 m^3 (245 000 ft^3) having a mass of 7.12×10^6 kg (7850 tons). If the submarine is neutrally buoyant, it will have the same mass of 7.12×10^6 kg.

Case (a) Submarine Considered as a Point Mass in Empty Space

From the law of universal gravitation

$$\Delta g = \frac{Gm_s}{r^2} = \frac{6.67 \times 10^{-11} \times 7.12 \times 10^6}{r^2} = \frac{4.75 \times 10^{-4}}{r^2}\, \mathrm{m/s^2}.$$

The gradient of the gravitational field anomaly is

$$\frac{d(\Delta g)}{dr} = \frac{-2\, G\, m_s}{r^3} = \frac{-9.50 \times 10^{-4}}{r^3}\, \mathrm{s^{-2}}.$$

Values of the gravitational anomaly and its gradient calculated from these equations are given in table 1.

Range	Gravitational Anomaly	Gravitational Anomaly	Vertical Gradient Of Gravitational Anomaly	Vertical Gradient of Gravitational Anomaly
r	Δg	Δg	$\frac{d(\Delta g)}{dr}$	$\frac{d(\Delta g)}{dr}$
(m)	(m/s^2)	(μGal)	(s^{-2})	(Eötvös Units)
10	4.75×10^{-6}	475	9.50×10^{-7}	950
20	1.19×10^{-6}	119	1.19×10^{-7}	119
50	1.90×10^{-7}	19.0	7.60×10^{-9}	7.60
100	4.75×10^{-8}	4.75	9.50×10^{-10}	0.950
200	1.19×10^{-8}	1.19	1.19×10^{-10}	0.119
500	1.90×10^{-9}	0.190	7.60×10^{-12}	7.60×10^{-3}
1000	4.75×10^{-10}	4.75×10^{-2}	9.50×10^{-13}	9.50×10^{-4}

Table 1. Gravitational Field and Gradient of the Gravitational Field of a Point Mass of 7.12×10^6 kg in Empty Space as a Function of Range

Case (b) Submarine Considered as a Vertical Dipole with its Entire Mass Concentrated in the Lower Half of its Volume

In this case the (positive) mass of the submarine is assumed concentrated at the centroid of the semicircular cross-section of the lower half of the submarine and the (negative) mass of the submarine-displaced water is concentrated at the centroid of its upper half as shown in figure 1. The "net mass" of the entire submarine, relative to the surrounding sea water is zero for the neutrally buoyant submarine.

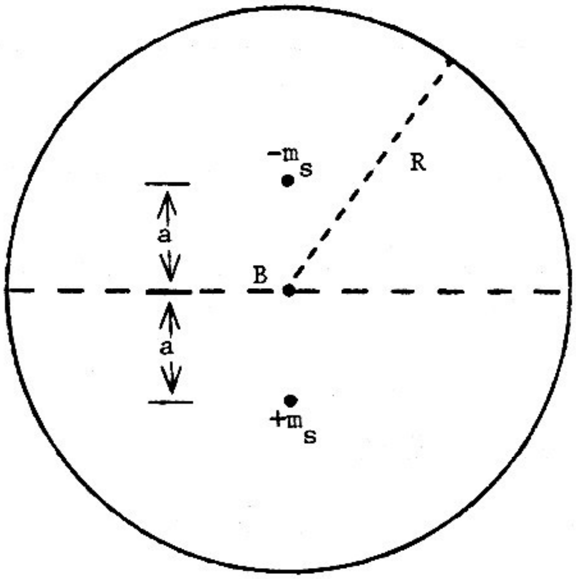

Figure 1. Location of Equivalent Mass Points of a Submarine at the Centroids of its Upper and Lower Halves.

If the radius of the submarine is R, the centroids of its upper and lower halves are displaced from the center of buoyancy B by an amount

$$a = \frac{1}{2}\frac{0.5\,\pi\,R^2}{2\,R} = \frac{\pi}{8}R = 1.65\ \text{m}.$$

The separation d of the two mass points is therefore $d = 2\,a = 3.30$ m and the gravitational dipole moment (in this assumed case)

$$m_s\,d = 7.12 \times 10^6\ \text{kg} \times 3.30\ \text{m} = 2.35 \times 10^7\ \text{kg m}.$$

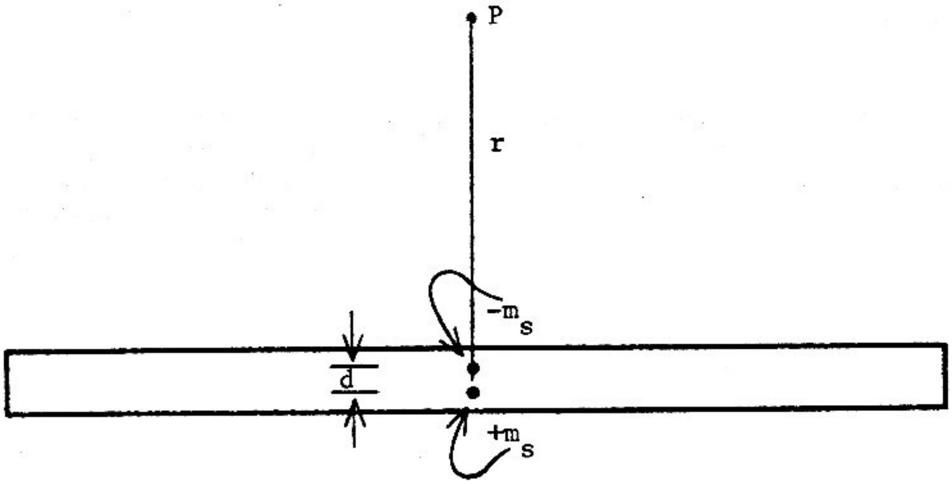

Figure 2. Geometry Assumed in Calculation of Dipole Gravitational Field of a Submarine

The magnitude of the gravitational anomaly at a point P on the axis of a mass dipole (See figure 2.) in terms of previously defined quantities is

$$\Delta g = \frac{G\ m_s}{(r + a)^2} - \frac{G\ m_s}{(r - a)^2} = -\frac{4\ a\ r\ G\ m_s}{(r^2 - a^2)^2}$$

$$\text{For } a << r,\ \Delta g = -\frac{4\ a\ G\ m_s}{r^3} = -\frac{2\ G\ m_s\ d}{r^3}$$

$$= \frac{-2 \times 6.67 \times 10^{-11} \times 7.12 \times 10^6 \times 3.30}{r^3}$$

$$= \frac{-3.134 \times 10^{-3}}{r^3}\ \text{m/s}^2$$

$$\text{and} \quad \frac{d(\Delta g)}{dr} = \frac{6\ G\ m_s\ d}{r^4} = -\frac{3}{r}\Delta g = \frac{9.403 \times 10^{-3}}{r^4}\ s^{-2}.$$

Note that since $\Delta g < 0$ the earth's gravitational field is reduced over the submarine by an amount that varies inversely as range raised to the third power and that the gradient varies inversely as range to the fourth power.

Values of the gravitational anomaly and its gradient calculated from these equations are given in table 2.

Range	Gravitational Anomaly	Gravitational Anomaly	Gradient of Gravitational Anomaly	Gradient of Gravitational Anomaly
r	Δg	Δg	$\frac{d(\Delta g)}{dr}$	$\frac{d(\Delta g)}{dr}$
(m)	(m/s^2)	(μGal)	(s^{-2})	(Eötvös Units)
10	-3.13×10^{-6}	-313	9.40×10^{-7}	940
20	-3.92×10^{-7}	-39.2	$5.88 - 10^{-8}$	58.8
50	-2.51×10^{-8}	-2.51	1.50×10^{-9}	1.50
100	-3.13×10^{-9}	-0.313	9.40×10^{-11}	9.40×10^{-2}
200	-3.92×10^{-10}	-3.92×10^{-2}	5.88×10^{-12}	5.88×10^{-3}
500	-2.51×10^{-11}	-2.51×10^{-3}	1.50×10^{-13}	1.50×10^{-4}
1000	-3.13×10^{-12}	-3.13×10^{-4}	9.40×10^{-15}	9.40×10^{-6}

Table 2. Gravitational Field Anomaly and the Vertical Gradient of the Gravitational Field Anomaly along the Axis of a Vertical Mass Dipole of Moment 2.35×10^7 kg m as a Function of Range

Case (c) Submarine Considered as a Vertical Dipole with 55% of its Mass Contained within the Lower Half of its Volume and 45% in its Upper Half

If the lower half of the submarine's volume contains 55% of its mass, then the lower half will have a mass of $0.55 \times 7.12 \times 10^6 = 3.92 \times 10^6$ kg but it will displace a mass of water of $0.50 \times 7.12 \times 10^6 = 3.56 \times 10^6$ kg, thereby possessing a "net positive mass" of $3.92 \times 10^6 - 3.56 \times 10^6 = 3.6 \times 10^5$ kg. Similarly, the upper half will have a mass of $0.45 \times 7.12 \times 10^6 = 3.20 \times 10^6$ kg and will displace $0.50 \times 7.12 \times 10^6 = 3.56 \times 10^6$ kg of water thereby yielding a "net negative mass" of $3.20 \times 10^6 - 3.56 \times 10^6 = -3.6 \times 10^5$ kg. Thus the gravitational dipole moment (in this assumed case) $m_s d = 3.6 \times 10^5 \text{ kg} \times 3.30 \text{ m} = 1.19 \times 10^6$ kg m.

From previous Case (b) for $a << r$,

$$\Delta g = -\frac{2\,G\,m_s\,d}{r^3} = -\frac{2 \times 6.67 \times 10^{-11} \times 3.6 \times 10^5 \times 3.30}{r^3} = -\frac{1.584 \times 10^{-4}}{r^3}\ m/s^2$$

$$\text{and } \frac{d(\Delta g)}{dr} = \frac{6\,G\,m_s\,d}{r^4} = -\frac{3}{r}\Delta g = \frac{4.754 \times 10^{-4}}{r^4}\ s^{-2}.$$

Values of the gravitational anomaly and its gradient calculated from these equations are given in table 3.

Range	Gravitational Anomaly	Gravitational Anomaly	Gradient of Gravitational Anomaly	Gradient of Gravitational Anomaly
r	Δg	Δg	$\frac{d(\Delta g)}{dr}$	$\frac{d(\Delta g)}{dr}$
(m)	(m/s^2)	(μGal)	(s^{-2})	(Eötvös Units)
10	-1.58×10^{-7}	-15.8	4.75×10^{-8}	47.5
20	-1.98×10^{-8}	-1.98	2.97×10^{-9}	2.97
50	-1.27×10^{-9}	-0.127	7.60×10^{-11}	7.60×10^{-2}
100	-1.58×10^{-10}	-1.58×10^{-2}	4.75×10^{-12}	4.75×10^{-3}
200	-1.98×10^{-11}	-1.98×10^{-3}	2.97×10^{-13}	2.97×10^{-4}
500	-1.27×10^{-12}	-1.27×10^{-4}	7.60×10^{-15}	7.60×10^{-6}
1000	-1.58×10^{-13}	-1.58×10^{-5}	4.75×10^{-16}	4.75×10^{-7}

Table 3. Gravitational Field Anomaly and the Vertical Gradient of the Gravitational Field Anomaly along the Axis of a Vertical Mass Dipole of Moment 1.19×10^6 kg m as a Function of Range.

The data of tables 1, 2 and 3 are plotted in figures 3 and 4.

STABILIZATION/COMPENSATION REQUIREMENTS

If a gravitational sensor is to be employed on a moving platform such as an aircraft, attention must be directed toward the "noise" generated by motion of the platform and toward steps to be taken to mitigate such effects. Two such factors will be considered: (a) vertical displacements of the aircraft and (b) vertical accelerations of the aircraft.

The earth's gravitational field is given by the expression

$$g = \frac{G\, m_E}{\rho^2} .$$

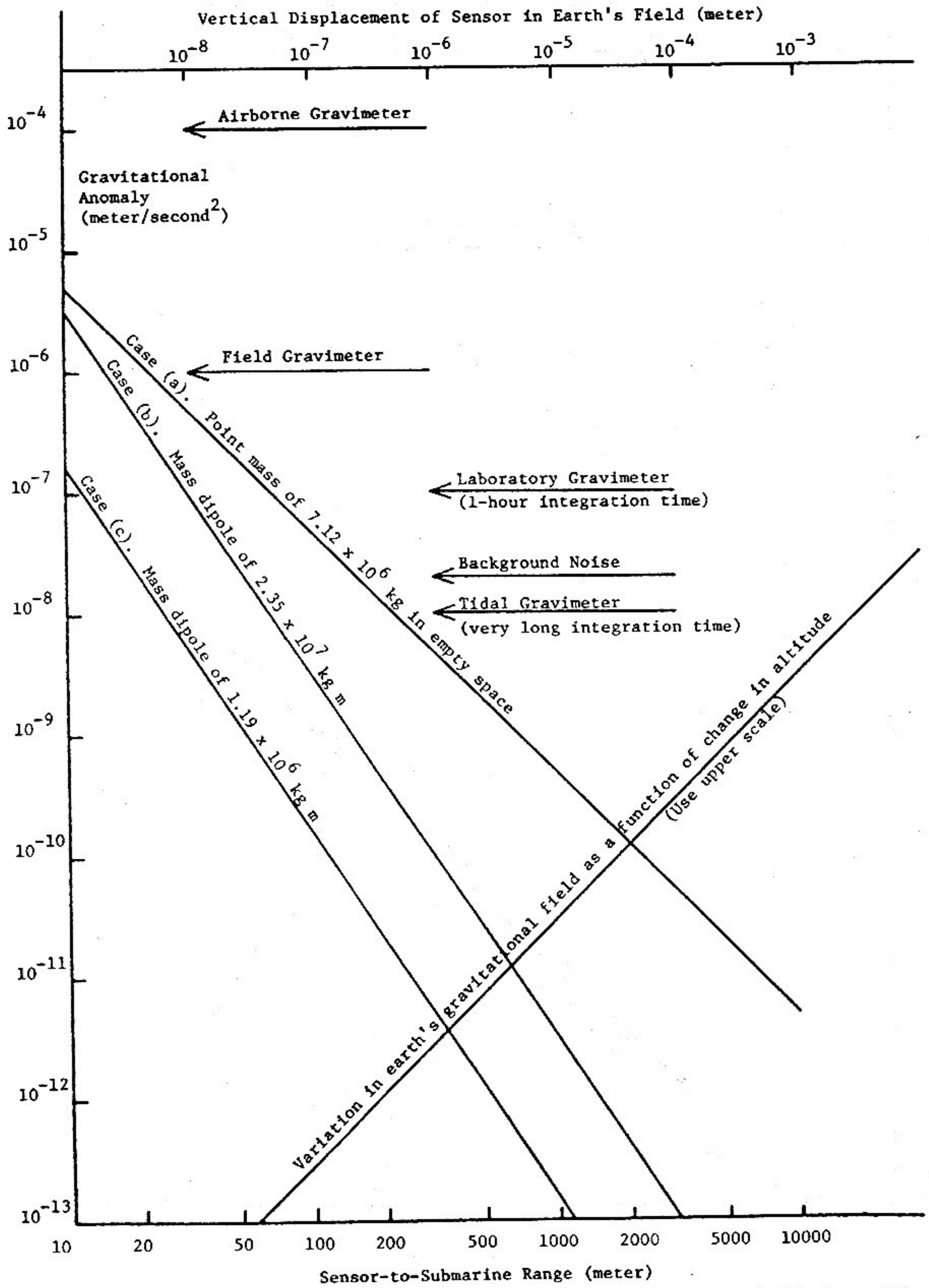

Figure 3. Calculated Gravitational Anomaly Directly Above a Submarine as a Function of Range

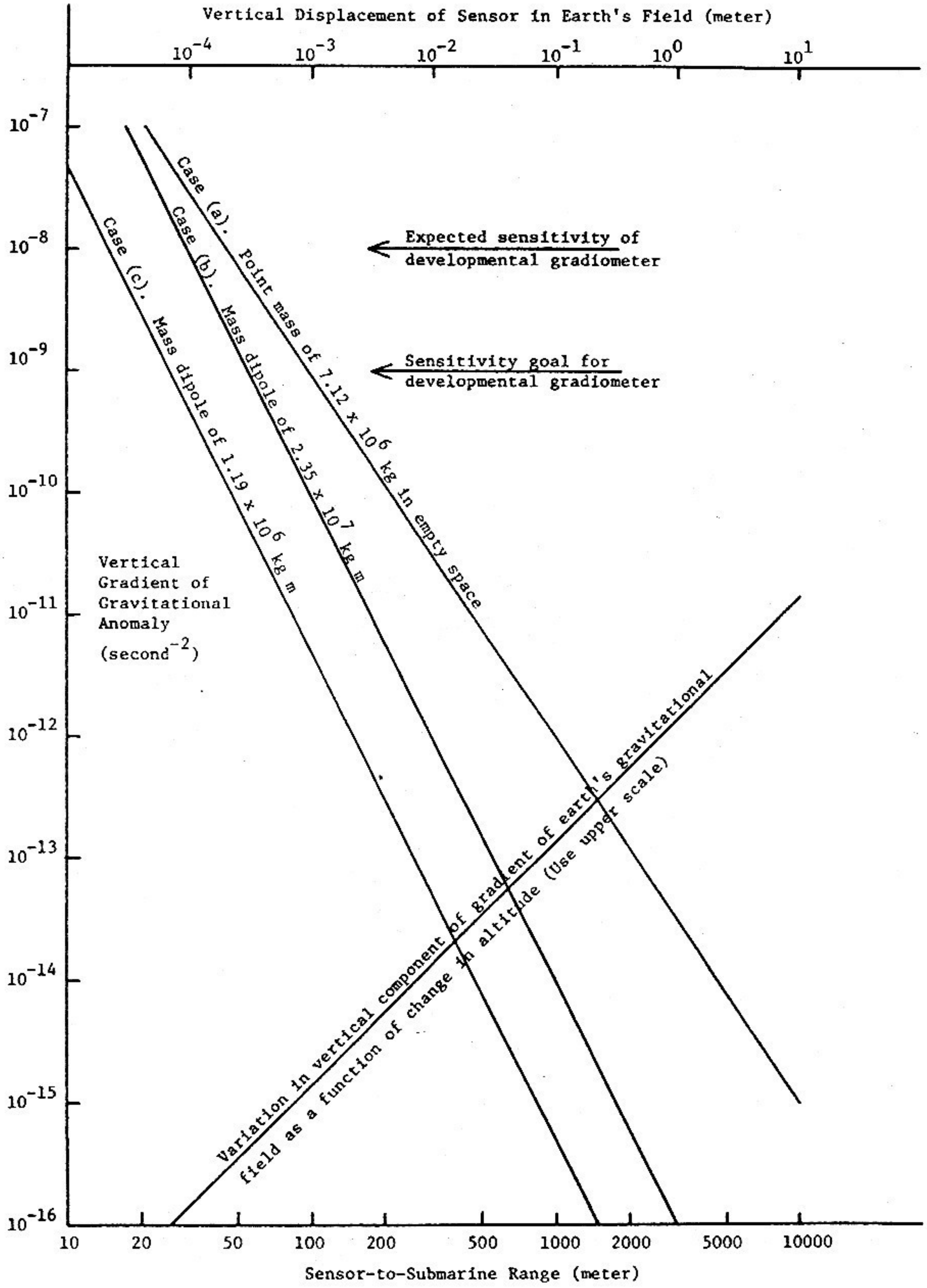

Figure 4. Calculated Vertical Component of Gradient of Gravitational Anomaly Directly Above a Submarine

The rate of change of the acceleration of gravity with radial distance from the earth is therefore

$$\frac{dg}{d\rho} = -\frac{2\,G\,m_E}{\rho^3} = -\frac{2g}{\rho}.$$

For $\rho = \rho_E$ and $g = g_E$,

$$\frac{dg}{d\rho} = -\frac{2 \times 9.81}{6.37 \times 10^6} = -3.08 = 10^{-6}\ s^{-2}$$

or $dg = -3.08 \times 10^{-6}\, d\rho$.

This equation is plotted in figure 3. If it is desired to sense gravitational anomalies as small as 1.27×10^{-12} m/s^2, corresponding to a submarine at a range of 500 m (table 3), the sensor altitude must be stable/compensated to

$$d\rho = \frac{dg}{-3.08 \times 10^{-6}} = \frac{-1.27 \times 10^{-12}}{-3.08 \times 10^{-6}} = 4.12 = 10^{-7}\ m = 0.412\ \mu m.$$

That is, a vertical displacement of the sensor of only 0.412 μm will produce a spurious "signal" equal to the signal from a submarine at a range of 500 m.

Next, consider vertical accelerations of the sensor platform. Since the acceleration of gravity and the accelerations associated with non-uniform motion are indistinguishable by an instrument designed to measure acceleration, means must be undertaken to stabilize the sensor and/or to correct its output through data processing utilizing information from some independent source. Reference (a) provides acceleration data measured in an airborne towed vehicle of a sort that could conceivably be used to house a gravitational sensor. A representative value for the standard deviation of the vertical component of the linear acceleration of the center of gravity of the towed vehicle averaged over ten-second intervals is given as 0.05 g (0.49 m/s^2). If this figure is compared with the value of the gravitational anomaly of 1.27×10^{-12} m/s^2 (taken from table 3) calculated for a submarine at a range of 500 m, it is apparent that through some combination of stabilization, compensation and/or processing an improvement by a factor about 10^{11} must be achieved, if ranges comparable to conventional magnetic anomaly detection are to be realized. If it is assumed that the vertical motion of the airborne sensor is simple harmonic and the period of interest T is ten seconds and if it is desired to calculate the vertical acceleration d^2z/dt^2 of the sensor as a function of time by taking the second derivative of the sensor elevation, the precision Δz to which its altitude must be measured is given by

$$\Delta z = \frac{T^2}{4\pi^2}\frac{d^2z}{dt^2} = \frac{10^2}{4\pi^2} \times 1.27 \times 10^{-12} = 3.2 \times 10^{-12}\ m.$$

This value corresponds to about one-hundredth of the diameter of a medium size molecule! It is not obvious how altitude could be measured to this precision.

If a gravity gradiometer is to be used as the sensor, one must relate the gradient of the gravitational field anomaly of the submarine to the rate at which the earth's gradient varies with sensor altitude or with radial distance from the center of the earth. Thus

$$\frac{d}{d\rho}\left(\frac{dg}{d\rho}\right) = \frac{d}{d\rho}\left(-\frac{2\,G\,m_E}{\rho^3}\right) = \frac{6\,G\,m_E}{\rho^4} = 1.45 \times 10^{-12}\ m^{-1}\ s^{-2}$$

for the case of $\rho = \rho_E$. This equation is plotted in figure 4. From table 3, the vertical gradient of the gravitational anomaly from a submarine at a range of 500 m is 7.60×10^{-15} s^{-2}. Thus, a displacement

$$d\rho = \frac{7.60 \times 10^{-15}\ s^{-2}}{1.45 \times 10^{-12}\ m^{-1}\ s^{-2}} = 5.24 \times 10^{-3}\ m = 5.24\ mm$$

will produce a spurious gradient "signal" equal to the signal from a submarine at a range of 500 m. Accordingly, the stability problems for the vertical gradient sensor are considerably less severe than for the case of direct detection of the submarine's gravitational field anomaly but still formidable nevertheless.

GRAVITATIONAL BACKGROUND AND NOISE LEVELS

The acceleration of gravity at the earth's surface g_E is 9.81 m/s^2. The gravitational anomaly from a submarine at a range of 500 m is 1.27×10^{-12} m/s^2 (table 3). Thus a sensor designed to perform such a function must be capable of responding to changes of about one part in 10^{13}, (For comparison, current submarine magnetic anomaly detectors respond to variations of about one part in 10^7 relative to the earth's magnetic field.)

Appendix H of reference (b) gives a value of 2×10^{-8} m/s^2 for the ambient gravitational noise observed under water east of San Clemente Island at a depth of 4000 ft. If it is assumed that this noise level was measured over a spectral band appropriate to a submarine anomaly detecting device, it appears from table 3 that ambient noise would limit submarine detection to ranges of about 20 m.

INSTRUMENT SENSITIVITIES

Sensitivities of gravimeters range over many orders of magnitude depending primarily upon the time available for making observations. For example, according to reference (c), gravimeters designed for recording tidal variations of gravity have been developed with sensitivities of about 10^{-8} m/s^2 (1 μGal). For measurements made over a period of one hour, an accuracy of about 10^{-7} m/s^2 (0.01 mGal) can be obtained. Gravimeters used in the field can easily achieve an accuracy of 10^{-6} m/s^2 (0.1 mGal). An accuracy to within about 10^{-4} m/s^2 (10 mGal) has been reported for a gravimeter in an airplane. For the task of detecting submarines from aircraft, accuracy of measurement is not important — only the ability to resolve small variations in gravitational field over time periods roughly equivalent to the time required for the sensor aircraft to pass through the anomaly (about ten seconds at a useful range). If it is assumed that variations can be detected that are one-tenth as great as the errors of absolute measurement for the above-mentioned instruments, reference to table 3 indicates that not even fixed laboratory type devices with very long integration times could provide useful detection ranges.

Gravity gradiometer developments are under way with sensitivity goals of about 10^{-9} s^{-2} (1 Eötvös Unit) but with expectations of actually achieving about 10^{-8} s^{-2}. Even if the goal were to be met, it could still yield a range of only about 26 m.

RESULTS OF A PREVIOUS STUDY

Unclassified appendix H of reference (b) provides calculated gravitational dipole detection ranges for gravimeter and gradiometer systems of 1300 m and 280 m, respectively. These values are one to two orders of magnitude more optimistic than those calculated in this report. This discrepancy is traceable primarily to an unfortunate six-order-of-magnitude error in the value of the universal gravitational constant used in reference (b). (A value of 6.6×10^{-5} with no units given was used instead of the correct value of 6.67×10^{-11} N m^2/kg^2.) Because the dipole gravitational field varies as r^{-3} and the gradient as r^{-4}, this introduces errors in the computations of range of factors of $(10^6)^{1/3}$ and $(10^6)^{1/4}$ or 100 and 32, respectively. Thus, for the conditions assumed in appendix H of reference (b), the detection range values calculated should be reduced to 13 m and 9 m for the direct and gradient modes, respectively.

Reference (b) points out correctly that the threshold detection ranges do not depend strongly on the dipole strength. However, one is perplexed by the peculiar set of submarine characteristics assumed; namely, a 200-ton vessel whose upper and lower halves have an effective mass separation of 10 m.

DISCUSSION

In this report, gravitational dipole field calculations are limited to points along the dipole axis (i.e., directly over the submarine) where the earth's field vector, the submarine's gravitational field vector and the submarine's gravitational gradient vector are all vertical. This was done in the interest of simplicity of computation. A more thorough treatment might have included computation of the vector gravitational field and the horizontal and vertical gradients for the entire space above the submarine. One might argue correctly, for example, that a technique involving sensing a horizontal component of the gradient would greatly reduce the sensor vertical motion stabilization/compensation problem. On the other hand, other noise sources such as ocean waves and the inability to compensate exactly for centrifugal effects due to the rotation of the earth (which requires precise knowledge of platform course, speed and latitude) have not been considered either. However, the results of this study are so pessimistic from so many points of view that it appears extremely unlikely that a more complete and/or more rigorous treatment is warranted.

Although the primary emphasis in this paper has been on airborne sensors, the results can be applied to fixed sensors, in which case some of the limitations are removed. Even in this case, however, the detection ranges that could be achieved under the best of conditions (about 20 m) are small in comparison with what can be achieved by other sensors (magnetic, acoustic, electric and mechanical pressure).

CONCLUSION

The concept of detecting submarines by means of detecting gravitational anomalies they produce, should be abandoned.

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(a) NAVAIRDEVCEN Report No. NADC-77238-20 "Instrumented Towed Vehicle Flight Test Report" of 31 May 1977

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