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STOCHASTIC FINITE ELEMENT METHODS FOR STRUCTURAL RELIABILITY ANALYSIS

by

I.R. Orisamolu - Q. Liu

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CONTRACTOR REPORT

Prepared for

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

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TABLE OF CONTENTS

TABLE OF CONTENTS - Continued

LIST OF TABLES

LIST OF FIGURES

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ABSTRACT

A research and development study of stochastic finite element methods (SFEM) for structural reliability analysis is presented. A detailed state-of-the-art literature review of various SFEM methodologies and the diverse technical subjects relating to probabilistic finite element analysis is summarized. Case studies of two existing SFEM-based reliability analysis computer programs were conducted, with a discussion of the methodology, main features, structure, and capabilities of both systems. The design and implementation of a software package for random field discretization (RANFLD) and SFEM-based reliability analysis (STOVAST) is then presented. The package, developed on the basis of the finite element reliability method (FERM) approach, and designed for operation with the VAST commercial finite element analysis system, was verified using several problems. Also, in this work, the quantification and analysis of bias and modelling errors/uncertainties in engineering analysis are discussed. Sources of bias and modelling errors/uncertainties in SFEM-based reliability analysis are identified and suggestions are offered for the application of realistic formulations for accounting for this class of uncertainties. Finally, recommendations are given on requirements for further development work that will broaden the analysis scope of the STOVAST system and permit straightforward applications to ship structures.

RÉSUMÉ

On présente une étude de recherche et développement portant sur l'application à la fiabilité structurale des méthodes stochastiques à éléments finis (MSÉF). On résume une analyse bibliographique détaillée des connaissances de pointe sur diverses MSÉF et les divers aspects techniques reliés à l'analyse probabiliste par éléments finis. On a effectué des études de cas de deux programmes informatiques d'analyse de fiabilité basée sur les MSÉF et on a examiné la méthode, les caractéristiques principales, la structure et les capacités des deux systèmes. On traite ensuite de la conception et de la mise en application d'un progiciel de discrétisation à champ aléatoire (RANFLD) et d'analyse de fiabilité basée sur les MSÉF (STOVAST). Le progiciel basé sur la méthode d'analyse de fiabilité par éléments finis (MFÉF) et destiné à être utilisé avec le système commercial d'analyse par éléments finis VAST a été vérifié par la résolution de plusieurs problèmes. Ce travail traite également de la quantification et de l'analyse de l'erreur systématique et des erreurs/incertitudes de modélisation dans l'analyse technique. Les sources d'erreur systématique et d'erreurs/incertitudes de modélisation dans l'analyse de fiabilité basée sur les MSÉF sont déterminées et des suggestions sont faites en vue de l'application de formules réalistes permettant de tenir compte de cette classe d'incertitudes. Enfin, on fait des recommandations au sujet des exigences en matière de travaux futurs de développement qui élargiront la portée de l'analyse du système STOVAST et permettront des applications directes aux structures des navires.

1. BACKGROUND

1.1 General

Issues of structural safety and reliability are of concern to design engineers as well as those entrusted with the responsibility for developing design criteria and operating/maintenance schedules and practices. While an engineer's primary task in designing a component or system is to adequately assure its satisfactory performance during its useful life, the planning, design, and operation of most (if not all) engineering systems are carried out under conditions of uncertainty. This uncertainty arises due to a myriad of factors. These include loads, structural strength determined by such uncertain parameters as geometry, material properties and sectional properties, boundary conditions, conditions of manufacture, fabrication, construction, storage, and environmental conditions. There are also uncertainties associated with mathematical modelling which usually employ some simplifying assumptions, the limited accuracy of the numerical methods and computing machines employed, and unintended human errors. A comprehensive description of various types of uncertainty that may be associated with structural reliability can be found in the monograph by Melchers [1].

Even though the presence of these uncertainties is well recognized, traditional approaches to structural design and performance assessment have treated the uncertain parameters as deterministic constants and accounting for the possible variabilities in these quantities by means of empirical safety factors. These safety factors are, in many cases, wrongly assumed or wholly unknown and are not founded on any rational philosophy. Associated with a given nominal safety factor, no matter how conservative it may be, there is invariably some underlying probability of failure. This is why engineering structures designed with the safety factor approach are still known to fail. The message from this is that, because of unavoidable uncertainties, the assurance of structural safety or performance cannot be absolute and so there is always invariably some underlying probability of failure. Unfortunately, conventional deterministic methods of analysis, hereinafter referred to as the traditional approach, is not capable of quantifying this probability of failure.

The probabilistic reliability approach is proving to be a very effective and rational procedure for accounting for the uncertainties inherent in any engineering system. The approach has been so successful in the characterization of uncertainties and the prediction of structural behaviour in the aerospace and nuclear industries that it has formed an important part of design, analysis, maintenance, and inspection planning strategies. Currently, reliability-based approaches are being developed and applied by NASA in conjunction with Southwest Research Institute (SwRI) to the design of the next generation Space Shuttle main engine. There is no doubt that the reliability approach is gaining wider acceptance in the engineering community, especially as it is capable of assisting in optimization in design and economization in maintenance practices.

1.2 Reliability of Ship Structures

In the same manner as for aerospace, nuclear, and other structures, reliability concepts offer the potential for achieving greater understanding of the safety and performance assessment of ship structures. Although the most obvious source of uncertainties in the case of ship structures is the random loading induced by the random ocean wave environment, there is no doubt that the structural systems themselves contain inherent uncertainties in material properties, section properties, structural geometry, damping, and boundary conditions. Random vibration analysis theory is applicable to cases in which only the loading is considered to be random. However, when the variabilities in structural and material properties are significant enough, a full scale probabilistic analysis is called for.

Mansour [2] was the first to put forward the idea of probabilistic design concepts in marine structures in his analysis of a Mariner ship. Since then, the field has expanded enormously with the majority of applications directed to offshore structures. A review of reliability methods as applied to ship structures up to 1980 was given by Stiansen et al. [3]. In those early applications, probabilistic analysis and reliability methods were mainly utilized to check and compare the safety level of existing designs. In more recent times, however, some civil and offshore engineering regulatory bodies and classification societies have implemented reliability concepts for determining their code requirements. They are also increasingly relying on the use of probabilistic methods for assessing the uncertainties underlying the design or performance variables and for selecting the associated partial safety factors suitable for use in standards and codes. A comparison of various approaches to estimating reliability (ranging from first order methods to exact (Level III) formulations) was made by Mansour [4] and applied to 18 ocean going vessels.

Interest in ship structural reliability is increasing as evidenced by the level of research activity in the field. The American Bureau of Shipping (ABS) and the Ship Structure Committee (SSC) of the United States Coast Guard are good examples of agencies that are continuously working on a program of action to effect applications of probabilistic techniques for the assessment of ship structural reliability.

Recently, there has been some interest in developing tools for reliability assessment of ship structures within a framework that permits detailed three-dimensional analysis. Just like its deterministic counterpart, the stochastic finite element method is the most viable vehicle for achieving this objective. This was exemplified in a recent work by Moore [5].

The Defence Research Establishment Atlantic (DREA) has embarked on an impressive research and development program on the applications of probabilistic mechanics technology to ship structures in the last four years. These efforts include characterization of the ocean wave environment, the probabilistic characterization of wave-induced loads within the framework of the finite element method (FEM), three-dimensional finite element random response computations, stochastic fatigue damage estimation, and applications of structural reliability theory.

1.3 The Stochastic Finite Element Method

Over the years, the fields of probabilistic mechanics and computational mechanics have proceeded vigorously to what may be safely referred to as a mature stage of development. Progress in the two fields, however, has proceeded rather independently for two major reasons. First, until recently, and except for a few exceptions, research in each of the above fields have been conducted by separate groups that are essentially mutually exclusive. Secondly, the application of stochastic methods to engineering structures generally requires considerable computational resources and resulted in concerns regarding feasibility of the tool for complex models. The advent of powerful computer hardware and the development of novel numerical techniques are making it possible to address problems on a more realistic basis. Furthermore, there is a better exchange among these groups of researchers through evaluation and verification of ideas on the application of computational methods to problems in stochastic mechanics.

1.4

The field of computational stochastic mechanics has emerged to provide the marriage of advanced techniques in probabilistic mechanics and computational mechanics. The finite element method in conventional deterministic analysis has gained overwhelming acceptance in the mechanics community because of its robustness and suitability for the modelling of complicated structures. For the same reason, the stochastic finite element method (SFEM) is emerging as a very powerful tool in probabilistic structural analysis. The basic objective of SFEM is to compute the probabilistic structural response in terms of either the response statistics or the probability of failure corresponding to a particular performance criterion (or set of criteria) from the probabilistic description of the structure. The latter objective is especially significant from a practical point of view and constitutes the basis of stochastic finite element based structural reliability analysis. Several important technical issues that are of direct relevance to the application of stochastic finite elements for structural reliability analysis were discussed in a recent technical note by the first author [6].

1.4 Scope of the Present Work

The objectives of the current contract work consist of the following:

i. A state-of-the-art literature review of methodologies for stochastic finite element-based structural reliability;

ii. Formulation and implementation of a model computer program for demonstrating the best approaches for SFEM-based reliability analysis; and

iii. Study of bias and modelling errors associated with SFEM-based reliability analysis.

These objectives were met in the work reported in the present document. The report has seven chapters. Chapter 2 summarizes the literature review effort. As an extension of the literature review, case studies of two existing SFEM based computer programs are given in Chapter 3. The design and implementation of the model computer program (STOVAST) for performing SFEM-based reliability analysis is described in Chapter 4. This also includes the development of the random field discretization module (RANFLD) which acts as a pre-processor to program STOVAST.

Some demonstration/verification problems for the model code are presented in Chapter 5. In Chapter 6, a discussion on bias/modelling errors associated with SFEM-based reliability analysis is given. Chapter 7 concludes the report with discussions on the level of success achieved in the project and recommendations for future work that will ensure that the laudable objectives of the project will be vigorously pursued.

2. A STATE-OF-THE-ART REVIEW OF STOCHASTIC FINITE ELEMENT BASED RELIABILITY ANALYSIS METHODOLOGIES

2.1 Introduction

In this chapter, a comprehensive review of methodologies for stochastic finite element (SFEM) based reliability analysis is presented. This review covers the pertinent subject areas which include random field discretization, general approaches for SFEM-based reliability analysis, reliability analysis algorithms, and utilization of existing FEA computer programs.

2.2 Representation and Discretization of Random Fields

2.2.1 Definition

A random field is a family of random variables depending on more than one deterministic parameter and, as such, it may be viewed as a multi-dimensional random process. They are used to represent physical systems that have attributes which exhibit complex patterns of spatial and temporal variations. Examples of random fields include random pressure loading due to ocean waves, stresses induced by turbulent boundary-layer pressure, material property (such as Young's modulus) with random spatial variations, and temperature fields in the main engine of a space vehicle.

Random field models of complex stochastic systems serve multiple purposes. They provide a format for efficient description of the random variation. They provide the basis for predicting system response and performance. They also permit the assessment of the impact of alternative strategies in diverse situations (such as in system design or data requisition). In general, it is usually impractical and uneconomical to sample, for example, material properties of a random medium at all locations. Therefore, the tasks of prediction, analysis, and decision-making must usually proceed on the basis of incomplete information about the medium. An important purpose of random field description is to permit a meaningful representation that can be used for modelling. A detailed exposition of the theory of random fields and their engineering applications is given in the monograph by Vanmarcke [7].

2.2.2 Random Field Element Discretization Techniques

2.2.2.1 Objective of the Discretization Process

The terminology "stochastic finite element analysis" (SFEA) is loosely employed to describe the explicit stochastic treatment of uncertainty in any system quantity within the framework of the finite element method. In the strict sense, however, the distinguishing feature of stochastic finite element analysis is that it involves the discretization of the parameter space of a random field of material properties and/or loads.

For SFEM-based structural reliability analysis, in particular, discretization is required to transform the spatial distribution of the field to "point" values, i.e. random variables. The resulting reliability model then becomes of the (generally correlated) random variable type, for which the vast majority of existing computational algorithms for reliability analysis were designed for. In this connection, the primary statistics of the random variables required for the reliability analysis are the mean values and the covariance matrix which are obtained from the discretization process.

The available techniques may be broadly classified into three groups. In one group of methods, the domain of the field is discretized into a mesh of random field elements (not necessarily coinciding with the finite element mesh) and the value for each element is described by a single random variable. The spatial averaging methods, the midpoint method, the nodal point method, and the interpolation method fall into this category. In another approach, the discretization of the domain of the field is avoided, and instead series expansion methods are used to model the field as a series of shape functions with random coefficients. Finally, the third group of methods is based on simulation. The procedures pertaining to each of these groups of approaches are briefly highlighted in what follows.

2.2.3 The Spatial Averaging Method

The theory of local averages of homogeneous random scalar fields over rectangular domains is well established. A detailed discussion on the subject was provided by Vanmarcke in his textbook [7], and forms the basis of the spatial averaging method.

The spatial averaging method, suggested by Vanmarcke and Grigoriu [8], uses the local average of the field over a random field element to represent the random quantity for the element. For a random field W(X), the discretized value for an element is given by:

$$W_i = \frac{1}{\Omega_i} \int_{\Omega_i} W(X)\, d\Omega \qquad (2.1)$$

where Ω_i is the domain of the element. For homogeneous fields and rectangular elements whose edges are parallel to the coordinate axes, expressions for the covariances of the discretized variables W_i in terms of the autocovariance function of W(X) were derived by Vanmarcke [7].

According to Der Kiureghian and Ke [9], the spatial averaging method yields accurate results (even for coarse meshes) for Gaussian random fields. However, it has two major shortcomings in the context of application to finite element reliability analysis. First, for a two-dimensional or three-dimensional continuum of arbitrary shape it is not always possible to discretize the domain into rectangular elements. Several approximation schemes that have been suggested in the literature to deal with cases involving non-rectangular elements are known to introduce errors in the computed covariance matrix. These errors may lead to very inaccurate reliability results and are indeed capable of causing the FORM and SORM algorithms to breakdown in some cases [10].

The efforts of Zhu and coworkers are among the most notable attempts to relax the restrictions on the use of local averages. Zhu and Ren [11] extended the theory to deal with homogeneous and rectangular isotropic random vector fields. A generalization of the work by Zhu and Wu [12] then followed in which the condition of isotropy was relaxed to one of quadrant symmetry. In addition, the work provided the capability for modelling nonhomogeneous random vector fields and permitted application to nonrectangular domains. The allowance for irregular domains was made possible by the use of Gaussian quadrature for evaluating the means and covariances of local averages. The procedure was demonstrated in a recent work involving applications to SFEM analysis of static, eigenvalue, and stress-intensity factor problems [13].

Another drawback of the spatial averaging method is that the probability distribution of W_i is difficult or impossible to obtain, except when the underlying random field is Gaussian, in which case W_i is also Gaussian. Essentially, therefore, the use of this method is restricted to Gaussian fields.

2.2.4 The Midpoint Method and the Nodal-point Method

The midpoint and nodal-point methods are two point discretization methods which represent the uncertainties of a random field by the values at some specific points. In the midpoint method [9, 14], the element random variable is defined as the value of the random field at the centroid of the element. Hence, the randomness in a random field element i is represented by the random variable:

$$W_i = W(\tilde{X}_i) \tag{2.2}$$

in which,

$$\tilde{X}_i = \frac{1}{NNODE} \sum_{j=1}^{NNODE} X_j^d \quad , \tag{2.3}$$

are the coordinates of the centroid, where NNODE is the number of nodes of the random field element, and X_j^d are the nodal coordinates.

The nodal-point method represents the random field in terms of the values at the nodal points of the finite element mesh. In this method, the randomness of the field at node i is represented by:

$$W_i = W(\tilde{X}_i) \tag{2.4}$$

where:

$$\tilde{X}_i = X_i^d \tag{2.5}$$

are the coordinates of node i.

In both the midpoint method and the nodal-point method, the mean, variance, and marginal distribution of W_i are the same as those of the process at point $\tilde{X}_i$. The correlation coefficient matrix of W_i is directly computed in terms of the auto-correlation coefficient function of the random field,

$$\rho_{W_iW_j} = \rho_{WW}(\tilde{X}_i, \tilde{X}_j) \tag{2.6}$$

and the joint distribution for any set of W_i is given by the specified distribution of the random field.

As pointed out by Der Kiureghian and Ke [9], the midpoint method tends to overrepresent the variability of the field within each element, and it does not provide as accurate a result as the spatial averaging method for a coarse random field mesh. However, these point discretization methods have three advantages. First, no complicated computations are required for the covariance matrix and the method is easy to implement. Second, the correlation coefficient matrix obtained by Equation (2.6) is always positive-definite, provided a valid autocorrelation function is specified. Hence, the numerical stability problem arising in the spatial averaging method does not exist in this case. Most importantly, the distribution information on the discretized variables W_i is retained and the method is not restricted to Gaussian random fields.

2.2.5 The Interpolation Method

The approach employed in the interpolation method suggested by Liu et al. [15] is to represent the random field in terms of an interpolation rule involving a set of deterministic shape functions and the random nodal values of the field. Thus, the random field W(X) is discretized into q random variables, W_i, i = 1, ..., q. The value at an arbitrary point is obtained by the following interpolation rule:

$$\tilde{W}(X) = \sum_{i=1}^{q} N_i(X) W_i \tag{2.7}$$

where W_i is the value of W(X) at node X_i, and $N_i(X)$ are shape functions. The number q is not necessarily equal to the number of finite elements and the shape functions $N_i(X)$ need not be the same as the finite element interpolation functions for the displacement field. Since the choice of the q nodal points and shape functions is arbitrary, the interpolation method constitutes a class of random field discretization methods. In particular, it is easy to observe that if the nodes are chosen to be the centroids of the random field elements and the shape functions are assumed to be unity inside each element and zero elsewhere, the interpolation method becomes identical to the midpoint method described in Section 2.2.4.

Liu et al. [15] further suggested a method to reduce the number of random variables W_i. The random vector W is transformed into an uncorrelated random vector C by:

$$C = \psi^T W \tag{2.8}$$

such that the covariance matrix of C, i.e. Cov (C, C), is diagonal. The orthogonalization matrix ψ is obtained by solving the eigenproblem:

$$Cov(W,W)\psi = \psi\Lambda \tag{2.9}$$

where Λ is the eigenvalue matrix containing the variances of C. Liu et al. [15] noted that a good approximation (within the context of second-moment analysis) of the random field can be obtained by retaining only the C_i with large variances, thus reducing the number of random variables. It should be emphasized, however, that this reduction is only applicable to Gaussian random fields. This is because the distribution of C is generally unknown or difficult to obtain unless W(X) is Gaussian.

2.2.6 Series Expansion Methods

Two series expansion methods have been suggested for second-moment stochastic finite element analysis. One is the basis random variable method proposed by Lawrence [16], and the other is the kernel expansion method proposed by Spanos and Ghanem [17].

In the first method, the random field is expanded into a double series in the form:

$$W(X) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} W_{ij} e_i \phi_j(X) \tag{2.10}$$

in which $\phi_j(X)$ are a set of linearly independent shape functions, e_i are independent basis random variables having the properties:

$$E[e_i] = \begin{cases} 1 & i=0 \\ 0 & i=1,2,... \end{cases} \tag{2.11a}$$

$$E[e_i e_j] = \delta_{ij} \tag{2.11b}$$

in which δ_{ij} is the Kronecker delta, and the coefficients W_{ij} are determined by least-square fitting to the moment functions of the function field. In reference [16], Legendre polynomials were suggested for the shape functions, $\phi_j(X)$. In applications, only a few dominant terms are included in the expansion.

The kernel expansion method [17] employs the Karhunen-Loeve orthogonal expansion to decompose a one-dimensional random field. The random field is expanded into the sum of its mean function and a single series:

$$W(X) = \mu_W(X) + \sum_{i=0}^{\infty} W_i \sqrt{\lambda_i}\, \phi_i(X) \tag{2.12}$$

where W_i are random coefficients independent of X, and λ_i and $\phi_i(X)$ are the eigenvalues and eigenfunctions of the covariance kernel, respectively. The latter are obtained as the solutions of the eigenvalue problem:

$$\int COV(X,t)\phi_i(X)dt = \lambda_i \phi_i(X) \tag{2.13}$$

The series in Equation (2.12) are truncated after the first few dominant terms just as in the basis random variable method. Since this series has a zero mean and the eigenfunctions are orthogonal, the random coefficients W_i have properties similar to the basis random variables, i.e. Equations (2.11)a and (2.11)b. One major obstacle of this method is the difficulty in solving the eigenvalue problem in Equation (2.13) for arbitrary geometry and boundary conditions. As noted by Liu and Der Kiureghian [10], this would be a particularly difficult task if the method is to be extended to two-dimensional or three-dimensional random fields. Details of this method are extensively discussed and exemplified in a recent monograph by Ghanem and Spanos [18].

Both series expansion methods described above are strictly applicable only to Gaussian random fields by virtue of the central-limit theorem. Therefore, they are appropriate for second-moment analysis, or for reliability analysis when the random fields are truly Gaussian.

2.2.7 Simulation Methods

Digital simulation is a technique of last resort to solve complex engineering problems involving random variation. Assuming a deterministic solution method is available, the simulation technique consists of repeatedly generating sets of observations of all the random variables or functions involved in the calculation, solving the deterministic problem associated with each set of the observations, and evaluating appropriate statistics of response and performance measures.

Shinozuka has developed and advocated the use of Monte Carlo simulation techniques in the field of engineering mechanics over the last two decades. A synopsis of his works in this regard is well documented in a recent review article co-authored with Deodatis [19]. In particular, Yamazaki and Shinozuka [20] proposed an interactive procedure to simulate non-Gaussian stochastic fields and later introduced statistical preconditioning to reduce the sample size in another work [21].

Recently, Fenton and Vanmarcke [22] presented a method of generating realizations of a discrete "local average" Gaussian random process using a local average subdivision technique. One of the advantages cited by the authors is the suitability for finite element models that employ efficient low-order interpolation functions in which each local average becomes an element property. No structural applications of this technique has been reported in the open literature.

More recent contributions in connection with the application of simulation schemes for modelling and discretization of random fields include the works of Bielewicz et al. [23] on nonhomogeneous scalar fields, Grigoriu [24] on the application of the sampling theorem and Poirion [25] on the simulation of non-Gaussian fields.

A major disadvantage of simulation schemes is the enormous amounts of computation times associated with their application. For a large scale structure like a ship, therefore, these procedures are not recommended.

2.2.8 Other Methods for Random Field Discretization

A new technique was recently introduced for random field discretization by Li and Der Kiureghian [26]. The technique is referred to as the optimal linear estimation (OLE) method. For a random field defined in a domain Ω, a linear function of the nodal values $W(x_i)$ is proposed in the form:

$$\hat{W}(x) = a(x) + \sum_{i=1}^{N} b_i(x)^T v(x_i) \ , \ x \in \Omega \qquad (2.14)$$

where N denotes the number of nodal points in the domain. The functions a(x) and $b_i(x)$ are determined optimally by minimizing the variance of the error $W(x) - \hat{W}(x)$. The minimization is subject to the condition that $\hat{W}(x)$ is an unbiased estimator in the mean, that is,

$$E\left[W(x) - \hat{W}(x)\right] = 0 \ . \qquad (2.15)$$

This results in the representation:

$$\hat{W}(x) = \mu(x) + \Sigma^T_{W(x)W} \ \Sigma^{-1}_{WW} \ (W - \mu), x \in \Omega \qquad (2.16)$$

in which W denotes the vector of nodal values (i.e. the random variables) and $\Sigma_{W(x)W}$ denotes an Nx1 vector containing the covariances of W(x) with the elements of W.

The OLE method is believed to be always superior to the midpoint, spatial averaging, and shape function methods, and its efficiency can be further improved by using eigenvalue expansion. The OLE method can also be applied for non-Gaussian random fields by utilizing the Nataf model of Liu and Der Kiureghian [27]. The Nataf model assumes that a transformed process z(x) can always be found such that

$$W(x) = \Phi^{-1} [F(z(x))] \ , \qquad (2.17)$$

where Φ is the standard normal cumulative distribution function. The key to the superior accuracy and efficiency of this method lies in the utilization of shape functions that take the correlation structure of the random field into account. The technique, therefore, appears to be a very promising tool in stochastic finite element analysis where it is always desirable to represent a random field with as few random variables as possible.

2.3 Methodologies for SFEM-Based Structural Reliability Analysis

2.3.1 Merit of SFEM-Based Approach

In a stochastic finite element based approach to structural reliability analysis, the methods of structural reliability provide the basis for modelling and analysis of uncertainties and computation of probabilities, while the finite element method provides the necessary computational framework for analyzing complex structures. The application of these two concepts in a closely integrated fashion is the basis of SFEM-based reliability analysis, and results in a powerful tool for realistically dealing with practical engineering problems.

The merit of the SFEM-based approach was clearly demonstrated by Orisamolu et al. [28] in a recent Martec study. In that study, one of the most powerful and internationally recognized general purpose commercial probabilistic analysis programs (PROBAN®) was applied for the systems reliability analyses of the two-storey braced frame, and then the tubular steel jacket offshore platform whose finite element models are shown in Figures 2.1a and 2.2a, respectively. The external loads (as shown in Figures 2.1b and 2.2b) and some other strength and modelling parameters were considered to be stochastic. Details of the description and analysis of this problem are given in reference [28]. The offshore platform problem is discussed further below since it sufficiently illustrates the merit of the integrated approach.

For the offshore platform problem, three failure modes were considered for the two-dimensional tubular members. These include yielding, stability, and punching. Performance functions corresponding to these failure modes are described below.

The performance function, g_y, corresponding to yielding failure of a two-dimensional tubular element is given by:

$$g_y(X) = Z_1 - \left(\frac{N}{N_F}\right)^2 - \frac{|M|}{M_F} \tag{2.18}$$

where Z_1 is a modelling uncertainty variable. N is the axial force on the element defined as:

$$N = \sum_{j=1}^{NLOAD} a_j P_j \tag{2.19}$$

where NLOAD is the number of independent loads acting, a_j are force coefficients of influence and P_1, P_2,..., P_{NLOAD} are the external loads modelled as stochastic variables. Figure 2c shows the section forces used in modelling this type of failure element. The two end-moments M_1 and M_2 are determined from:

$$M_i = \sum_{j=1}^{NLOAD} b_{ij} P_j \quad , \; i=1,2 \tag{2.20}$$

where b_{ij} are moment coefficients of influence. The resultant moment in the actual end is, for end I,

$$M = M_1 \tag{2.21}$$

and for end J,

$$M = M_2 \tag{2.22}$$

N_F in Equation (2.18) is determined from the relation:

$$N_F = Y\pi \; (d^2 - (d-t)^2)/4 \tag{2.23}$$

while M_F is given by

$$M_F = Y \; (d^3 - (d-t)^3)/6 \tag{2.24}$$

where Y is the yielding stress, d is the tubular diameter and t is the tubular thickness. This failure is assumed to have a brittle behaviour, i.e. it is assumed that the element is removed by failure.

For stability failure, the performance function, g_s, for a two-dimensional tubular element is given by:

$$g_s(X) = Z_2 - \frac{-N}{N_U} - \frac{M}{\left(1 - \frac{-N}{N_E}\right) M_P} \tag{2.25}$$

where:

$$N_P = YA = Y\pi dt \tag{2.26)a}$$

$$M_P = Yd^2t \tag{2.26)b}$$

$$N_U = \begin{cases} YA\,(1-\lambda^2/4) & , \lambda \le \sqrt{2} \\ YA/\lambda^2 & , \lambda > \sqrt{2} \end{cases} \tag{2.26)c}$$

$$N_E = \frac{AY}{\lambda^2} \tag{2.26)d}$$

The buckling parameter (λ) is determined by the expression:

$$\lambda = \frac{Z_3 L}{Y} \frac{1}{\pi} \sqrt{\frac{Y}{E}} \tag{2.27}$$

In the above relations, Y is the yielding stress, Z_2 is a model uncertainty variable for this mode of failure, Z_3 is a model uncertainty variable associated with modelling effective column lengths, A is the section area, d is the tubular diameter, t is the tubular thickness, Y is the cross-section radius of gyration, L is the column length, and E is the Young's modulus. Again, the section forces used in modelling this failure element are as shown in Figure 2.2c. The axial force N and the two moments M_1 and M_2 are determined by:

$$N = \sum_{j=1}^{NLOAD} b_{1j} P_j \tag{2.28a}$$

$$M_1 = \sum_{j=1}^{NLOAD} b_{2j} P_j \tag{2.28b}$$

$$M_2 = \sum_{j=1}^{NLOAD} b_{3j} P_j \tag{2.28c}$$

where b_{1j}, b_{2j}, and b_{3j} are coefficients of influence and $P_1, P_2, \ldots, P_{NLOAD}$ are the external loads modelled as stochastic variables. The moment M is defined as:

$$M = \text{Max}\ (M_1, M_2) \tag{2.29}$$

The punching failure element has the performance function, g_p, given by:

$$g_p(X) = Z_4 - \frac{N_b}{Z_5 N_U} - \left(\frac{M}{Z_6 M_U}\right)^{1.2} \tag{2.30}$$

where:

$$N_u = \frac{YT^2}{\sin\theta}(3.4 + 19\beta)\mu \tag{2.31a}$$

$$M_u = \frac{YT^2}{\sin\theta}\, 0.8d\, (3.4 + 19\beta)\mu \tag{2.31b}$$

$$\mu = \min\left(1,\ 1.22 - \frac{|N_c|}{2Y\pi DT}\right) \tag{2.31c}$$

in which Y is the yielding stress, and Z_4 , Z_5, and Z_6 are modelling uncertainty variables. This failure element models punching failure of the plane K-joint shown in Figure 2.2d. It is assumed that the punching failure of each branch can be considered separately. The brace has a diameter d and thickness t, while the chord has a diameter D and thickness T. The angle between the brace and the chord is θ. These parameters defining the geometry of the joint are shown in Figure 2.2d. The axial force in the brace, N_b, and in the chord, N_c, are given, respectively, as:

$$N_B = \sum_{j=1}^{NLOAD} a_{jb} P_j \tag{2.32}$$

$$N_c = \sum_{j=1}^{NLOAD} a_{jc} P_j \tag{2.33}$$

The moment in the actual branch is given as:

$$M = \sum_{j=1}^{NLOAD} b_j P_j \tag{2.34}$$

In the application of PROBAN for the probabilistic analysis of this offshore platform, each of the above failure modes had to be coded and linked into the PROBAN limit state function library. It can be seen from the above limit state equations that the force and moment influence coefficients are involved. For this complicated structure, these coefficients are best determined by performing a linear-elastic finite element analysis. For this case the Martec FEA code (VAST) was used to determine these influence coefficients. They are to be supplied as part of the input data for each of the components that makeup the structural system in the description of the probabilistic model to PROBAN. Now, for this offshore platform, a total of about 62 failure elements consisting of 36 yielding elements, 18 stability elements, and eight punching elements are involved. If all these 62 failure elements are to be accounted for in the evaluation of systems reliability, there is a lot of tedium associated with the process of preparing the enormous amount of input data required. This is especially so since, for the punching failure function, the calculation of angles between the members (which is not an output of the FEA results) is required. All these efforts are required because PROBAN does not include finite element analysis. The computer program is aimed at the analysis of problems whose probabilistic models are well defined in terms of explicit limit state functions and statistical characteristics of the random variables.

A fully integrated and coupled finite element reliability analysis program is the most suitable for the analysis of the offshore platform problem described above, and even more so for continuum structures. For such an integrated program, the statistical properties of the random variables are merely supplied as additional information to the finite element input data and the probabilistic analysis proceeds automatically. The user is, therefore, alleviated of the tedium involved in the detailed description of the probabilistic model. In addition, the risk of errors in defining the input data is considerably reduced. For example, the calculation of the influence coefficients will be unnecessary as this operation will be internally taken care of.

Furthermore, in the use of SFEM-based reliability analysis programs for problems involving semi-ductile or ductile elements (such as the stability failure element described above) the redefinition of the loads is automatically accounted for. The use of semi-ductile or ductile elements, in strictly probabilistic analysis programs involves the tedious operation of analysis, input data definition, then re-analysis and redefinition of input data, and so on. Also, for continuum structures (or even for discrete structures subjected to distributed random loads) the calculation of the random loads acting at several locations is best achieved via deterministic or stochastic finite element discretization procedures. Also, this scheme is indispensable if the tubular members were to have stochastic spatial variability in the Young's modulus or yield stress. Thus, it is obvious that problem formulation and analysis can be more efficiently performed within the framework of an integrated finite element reliability analysis computer package.

The merit of SFEM-based reliability analysis is well recognized in the research community. As such, it represents a very active area of investigation. Currently available methodologies may be classified into four major groups. These include the perturbation-based SFEM approach, the reliability-based SFEM approach, response surface methods, and Monte Carlo simulation methods. The weighted integral method has also recently been introduced [29, 30] to avoid a direct discretization of the random fields during a stochastic finite element analysis. The basic features pertaining to each of these groups of techniques are briefly highlighted in what follows. With the exception of the Monte Carlo simulation and the weighted integral techniques, these methods are generally based on the combination of stochastic finite element discretization techniques and the well known first order reliability methods (FORM) and second order reliability methods (SORM).

2.3.2 Perturbation-Based SFEM Approach

The perturbation approach to probabilistic structural analysis was introduced about two decades ago. Initial applications were directed at the study of the eigenvalue problem related to the free vibration of structures with stochastic mass and stiffness matrices and the solutions of linear static problems involving loading and system stochasticity. The work of Hisada and Nakagiri [14] represents one of the modern applications of this approach to structural safety and reliability analysis. In that work, SFEM was applied for the evaluation of the reliability index and design point within the framework of the Advanced First-Order Second Moment (AFOSM) method.

Consider the linear finite element equation

$$KU = F \tag{2.35}$$

where K is the global stiffness matrix, U is the vector of nodal displacement and F is the global nodal load vector. These stochastic quantities can be expressed as

$$K = K_0 + \Delta K \quad \text{(2.36)a}$$

$$U = U_0 + \Delta U \quad \text{(2.36)b}$$

$$F = F_0 + \Delta F \quad \text{(2.36)c}$$

where K_0, U_0 and F_0 are considered to be deterministic parts and the increments ΔK, ΔU and ΔF are considered to be the stochastic parts of K, U, and F, respectively. Substituting Equations (2.36)a-c into (2.35) gives

$$[K_0 + \Delta K][U_0 + \Delta U] = [F_0 + \Delta F] \; . \tag{2.37}$$

on neglecting the product ($\Delta K \Delta U$) and separating the deterministic and stochastic parts of Equation (2.37) gives:

$$K_0 U_0 = F_0 \tag{2.38}$$

and

$$K_0 \Delta U = \Delta F - \Delta K U_0 \tag{2.39}$$

Equation (2.38) gives the finite element solution at the deterministic expansion point. From the solution of Equation (2.39), the second-order variation of the response may be computed.

A more rigorous and general formulation of the perturbation approach can be constructed using Taylor series expansion. This also paves the way for higher-order approximations. The stiffness matrix in Equation (2.35) may be expanded about a deterministic stake as:

$$K = K_0 + \sum_{i=1}^{n} K_i'(x-x_i) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1]}^{n} K_{ij}'(x-x_i)(x-x_j), \tag{2.40}$$

where K_i' and K_{ij}' are the first-order and second-order partial derivatives of the stiffness matrix with respect to the basic variables X. The displacement and load vectors in Equation (2.35) may be expanded in a similar fashion and the response computed as previously described.

When the mean state is chosen as the expansion point the approach is referred to as the mean-centred perturbation approach. Using mean-centred perturbation results in the computation of reliability indices according to the First-Order Second Moment (FOSM) approach of structural reliability analysis. In the AFOSM, however, the performance function is expanded not about the mean values of the basic variables, but about the most probable failure point. Hisada and Nakagiri utilized this approach and also presented a second-order perturbation formulation in reference [14]. A notable feature of the formulations is that the stiffness matrix is inverted only once in contrast to simulation or response surface methods in which many inversions of the stiffness matrix are required. The key to successful solution using the perturbation approach is the ability to compute and assemble partial derivative matrices for stiffness, displacements, and loads. Second-order approximations are obviously more accurate than the first-order approximations; however, these involve the computation and assembly of second-order partial derivative matrices.

Numerous applications of the perturbation approach have been reported in the open literature. Prominent in this connection are the works of Liu et al. [15, 31, 32], in which applications were investigated for linear and nonlinear structural dynamics and a variational formulation of probabilistic finite elements established.

Although the formulation of the perturbation approach is mathematically elegant, its application to reliability analysis has several disadvantages. The mean-centred perturbation method suffers from the well known invariance problem associated with FOSM. Furthermore, the perturbation methods do not use the distribution information about the basic random variables, even if it is available. This is a serious limitation, unless for the exceptional cases in which all the variables are normally distributed. The method is also not capable of producing accurate results when there are large variations in the random variables defining a problem.

2.3.3 The Reliability-Based SFEM Approach

Current methods for reliability analysis compute the reliability index β by solving the limit state equation $g(X)=0$ explicitly. However, search algorithms that do not rely on an explicit solution of the limit state equation are available. These algorithms only need the value and the gradient of the performance function at each iteration point.

The reliability approach has been formulated and applied for several structural problems by Der Kiureghian and his co-workers. In [9, 33] the first-order reliability method was used for static analysis of linear structures with random properties and in [16] for dynamic analysis. In a more recent work [10], a general framework for finite element reliability analysis based on FORM and SORM was presented. New expressions for the required gradients of the response of geometrically nonlinear structures were derived and implemented. This work represents the first application of the finite element reliability method (FERM) in conjunction with SORM with non-Gaussian random fields, and with system reliability analysis. Arnbjerg-Nielsen and Bjerager [35] and Mahadevan [36] has also developed and implemented a reliability-based SFEM approach and applied it to the modelling of frame structures.

The reliability approach has a significant advantage over the perturbation approach in that information about the distribution of the random variable is used. Furthermore, in the reliability approach, the probability density function of the response variable (not just second moment statistics) can be obtained. Since the computation of response gradients is a key operation in the implementation of this procedure, the use of the adjoint method has been recommended for this operation [10, 37]. This technique is estimated to be capable of reducing computation times by a factor equal to the number of random variables.

The reliability-based approach to SFEM structural reliability analysis is especially compatible with the algorithmic structure of existing FEA codes and is about the best strategy of all presently available methodologies.

2.3.4 Response Surface Methods

The response surface method is a classical statistical technique in which a complex (computer) model is approximated by a simple functional relationship between the output quantities and the input (basic) variables. The approximation is usually based on polynomial functions and, often, linear or quadratic response functions are applied. Adopting the simpler response functions allows an efficient repeated computation, for example, as may be needed in simulations or parameter studies in structural reliability analysis. This is because the approximation to the response surface rather than the original limit state function is used in the calculation of failure probabilities.

The concept of response surface methods has been used when approximating costly to compute and/or non-differentiable limit state functions. Within the framework of the stochastic finite element method, the steps required for the implementation of the response surface technique were described by Favavelli [38]. This involves the application of regression analysis to obtain the polynomial coefficients involved in the representation of the limit state function using the results of several numerical experiments.

The explicit representation of the limit state function g(x), for the quadratic approximation for example, takes the form:

$$\bar{g}(x) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_i x_j \qquad (2.41)$$

where n is the number of basic random variables (x_i) and the coefficients a, b, and c are to be determined from numerical experiments.

The works of Schuëller et al. [39], Bucher and Bourgund 40[] and Böhm and Brückner-Foit [41] are among recent efforts at promoting the application of response surface methods. The work of

Böhm and Brückner-Foit [41], in particular, introduced a special lack of fit measure and formulated criteria for accepting response surface models in structural reliability analysis. Ghanem and Spanos [42] proposed a Galerkin-based response surface approach in which the surface is approximated by its projection onto a complete set of polynomials that are orthogonal to the Gaussian measure. These polynomials are known as polynomial chaos functions and are believed to be capable of yielding accurate approximations of the response surface.

So far, no general scheme has been developed to efficiently establish linear and quadratic response surfaces for reliability computations. Further research is needed to establish general, efficient, and robust response surface methods for reliability analysis. Nevertheless, this methodology appears to be a promising tool for large scale structures as recently demonstrated in a study dealing with the fatigue reliability of a container ship structure [43].

2.3.5 Monte Carlo Simulation Methods

The direct Monte Carlo simulation method was used in many early works in stochastic finite element analysis. In this brute force method, deterministic analysis is carried out for a *series of* parameters generated in accordance with their probability distribution. The desired statistics of the response quantities, such as the mean, variance, and exceedance probabilities, are then evaluated based on the generated sample. Applications of this procedure can be found in Vanmarcke et al. [44] and Takada [45].

The Monte Carlo simulation method has the advantage that it is adaptable to all types of problems and the results can be obtained to desired accuracy. However, for practical problems with many random variables or small failure probabilities this procedure is usually too expensive, since a large number of solutions are needed to obtain reliable results. Shinozuka and his co-workers [46] have introduced the Neumann (Monte Carlo simulation) expansion technique. Computation time for this technique is reduced significantly since only the mean stiffness matrix needs to be decomposed with this formulation. Other schemes have also been proposed to improve the efficiency of the simulation method. However, for ship structures, this procedure is not recommended.

2.3.6 The Weighted Integral Method

Under certain conditions, discretization of the random fields in SFEM may be avoided through the application of the weighted integral method. This method was recently introduced by Deodatis [29]. It was applied by Deodatis and Shinozuka [30] for the calculation of the response variability and reliability of stochastic frame structures and the calculation of the response variability of two-dimensional stochastic systems by Deodatis et al. [47].

Basically, the method assumes that the elastic constitutive matrix (D) in the element stiffness formulation can be expressed as a product of a scalar random field E(x) and a deterministic matrix D_o, that is:

$$D = E(x)D_o \tag{2.42}$$

Then, due to the polynomial nature of the strain matrix, B, the element stiffness matrix can be written as:

$$K_e = \sum_{i=1}^{n} X_i K_{ei} \tag{2.43}$$

where n denotes the number of distinct polynomial terms in the coordinates x in the matrix product B^TB, K_{ei} are deterministic matrices, and X_i are random variables defined by:

$$X_i = \int_{\Omega_e} P_i(x)\ E(x)\ d\Omega_e \tag{2.44}$$

in which $P_i(x)$ denotes the i-th distinct polynomial term in the coordinates x. The random variables X_i are interpreted as "weighted integrals" for all elements and completely define the SFEM problem without the need to discretize the random fields.

The major advantage of this procedure has been cited as the circumvention of the restriction (in random field discretization) that the finite element size has to be a fraction of the correlation distance of the stochastic field involved in the problem. This restriction makes necessary the use of a fine mesh to accurately describe stochastic fields characterized by short correlation distances.

The method, however, has two main disadvantages. First, the assumption of a product form assumed in Equation (2.42) is generally difficult to realize in practice. Secondly, the method assumes a Gaussian random field which is a serious drawback in reliability analysis where probabilities are sensitive to tails of the underlying distributions.

2.3.7 Utilization of Commercial FEA Programs

A few applications involving the use of existing general purpose commercial finite element programs for reliability analysis have been reported. Gopalakrishna and Donaldson [48] utilized ANSYS within the framework of the advanced first-order second moment (AFOSM) method to estimate failure probabilities. The work took advantage of the response surface based design optimization routine already available in ANSYS. The solution of the reliability problem is constructed such that the basic random variables represent the design variables, the limit state function constitutes the constraint function and the reliability index is treated as the objective function.

Riha et al. [49] also presented a coupling of a fast probability integration scheme with the finite element code MSC/NASTRAN. Again, in this work, advantage was taken of the capability for design sensitivity analysis (DSA) available in the finite element system for the computation of structural response derivatives. The DSA capability however, is not applicable to some boundary condition and load variables and so cannot be used for the calculation of the partial derivatives of such random variables. Provision was made, however, for the use of the finite difference scheme.

The probabilistic analysis capabilities described in the preceding couple of paragraphs do not fall into the general framework of stochastic finite element methods. This is because there is no capability for characterizing or discretizing random fields.

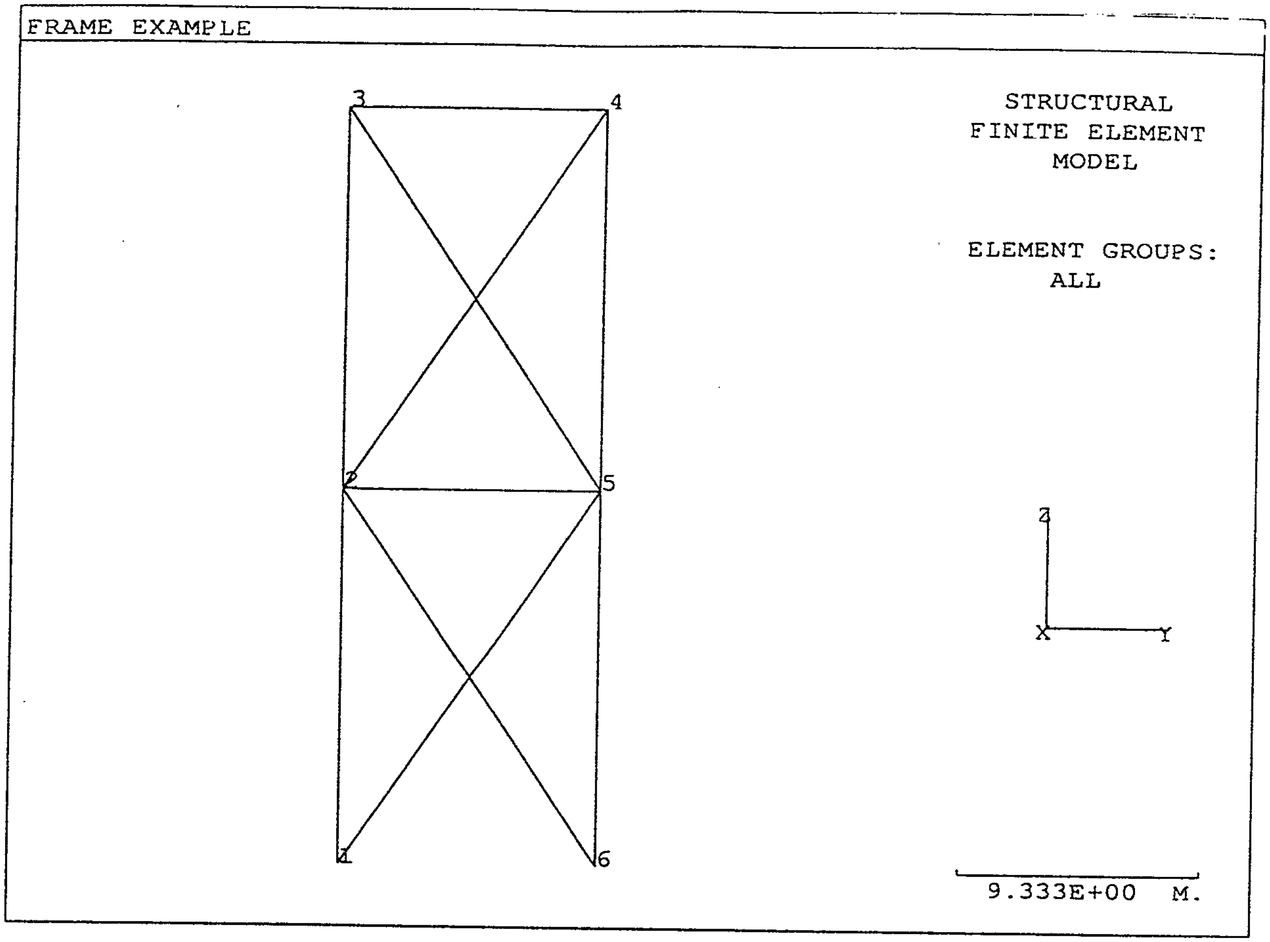

FIGURE 2.1a: Finite Element Model of a Two-Storey Braced Frame

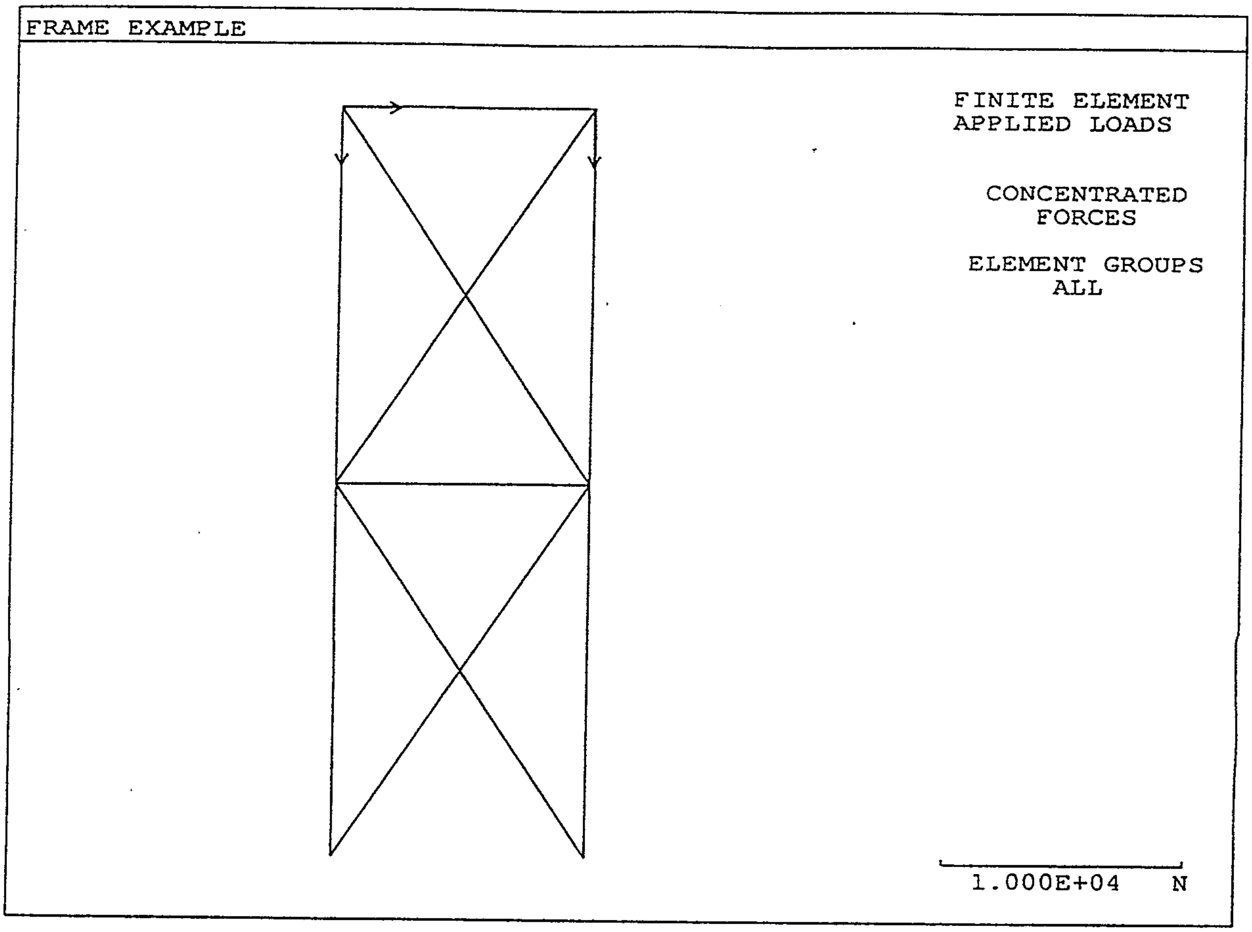

FIGURE 2.1b: External Concentrated Loads on the Two-Storey Braced Frame

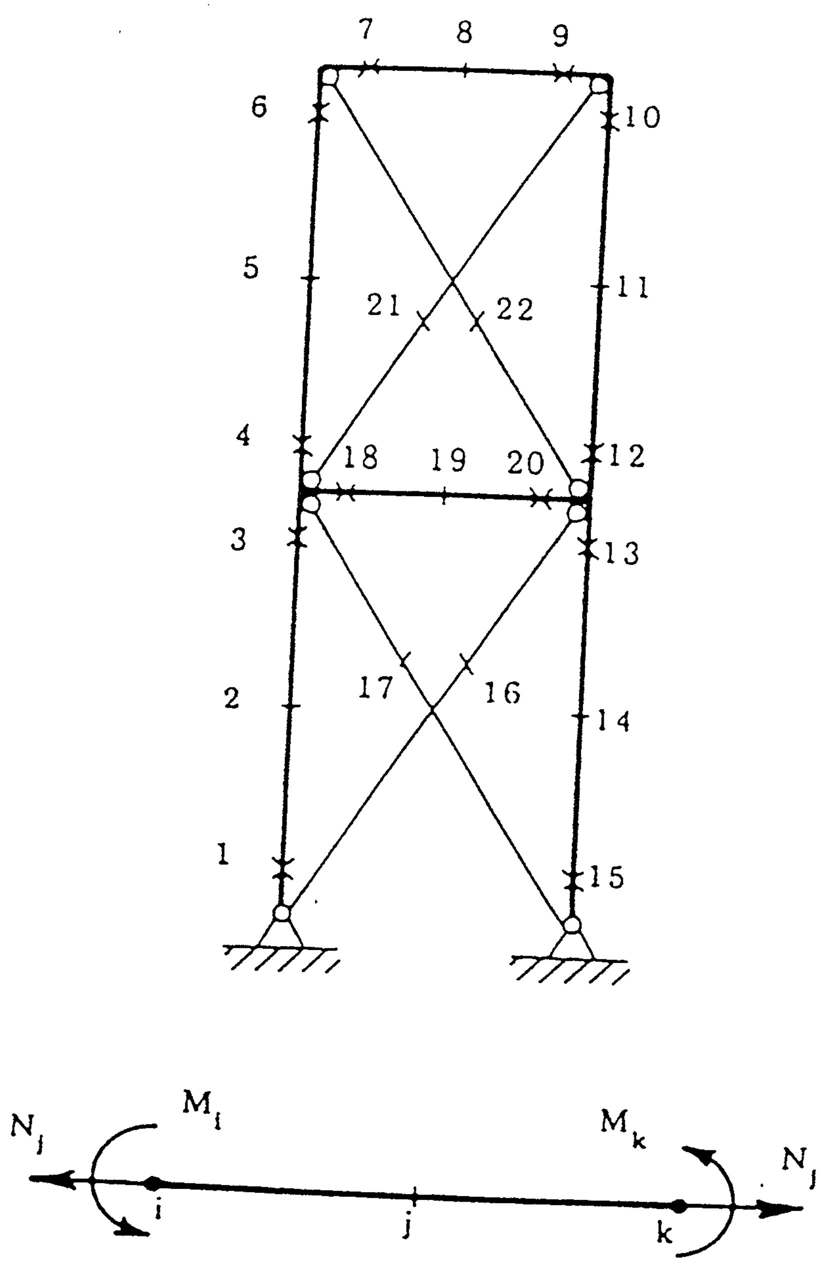

FIGURE 2.1c: Failure Elements and Section Forces for the Two-Storey Braced Frame

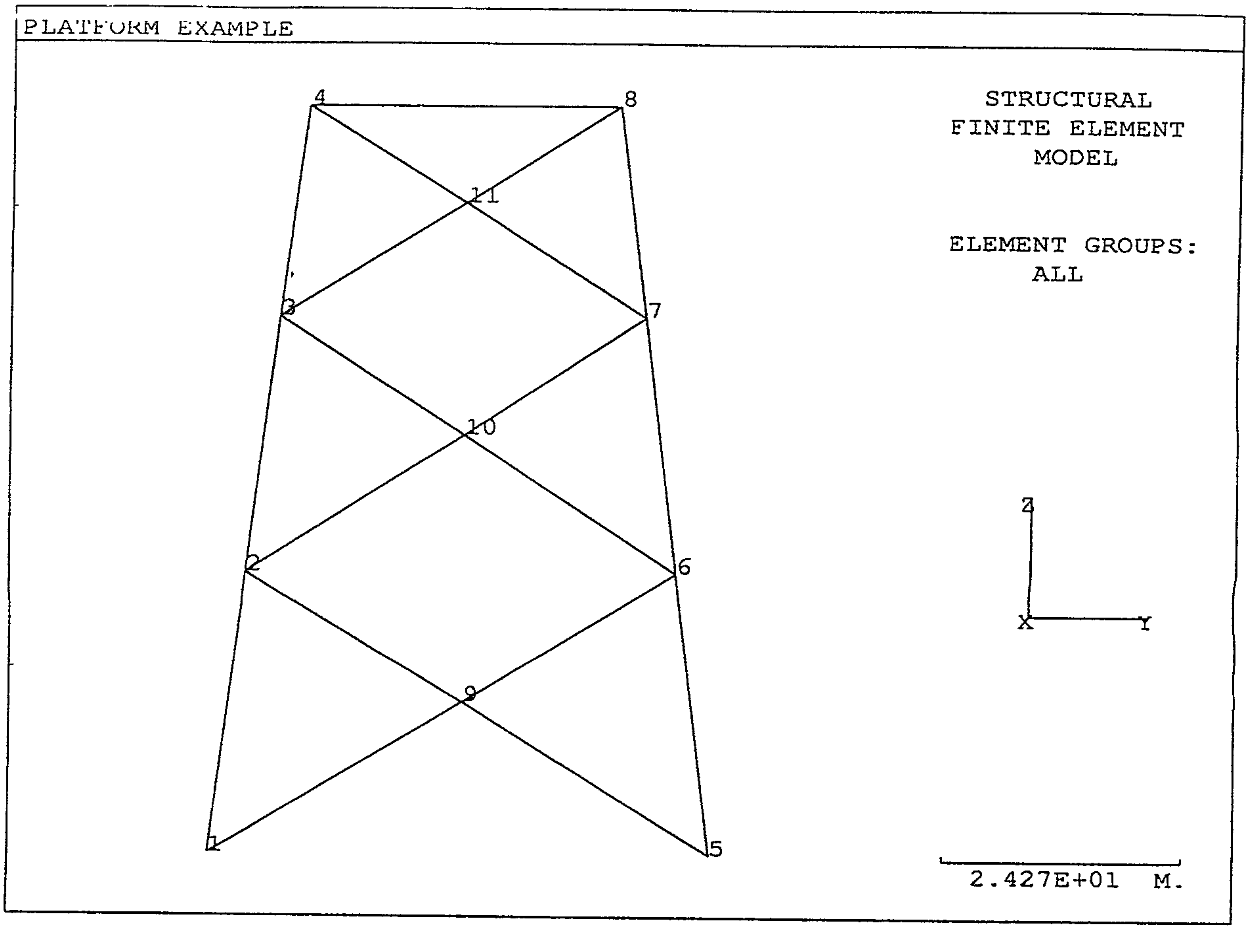

FIGURE 2.2a: Finite Element Model of an Offshore Platform

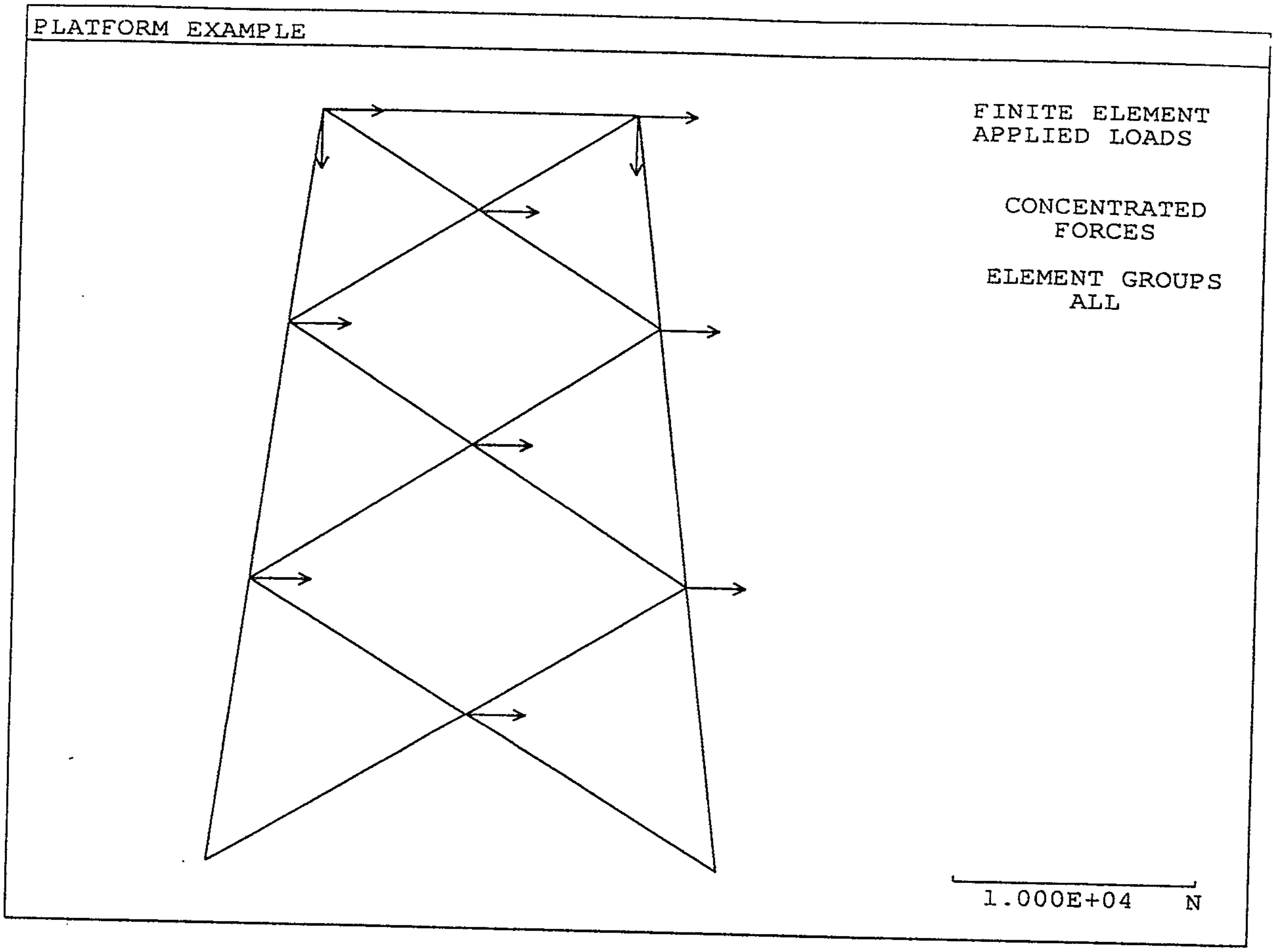

FIGURE 2.2b: External Concentrated Loads on the Offshore Platform

FIGURE 2.2c: Section Forces for a Tubular Member

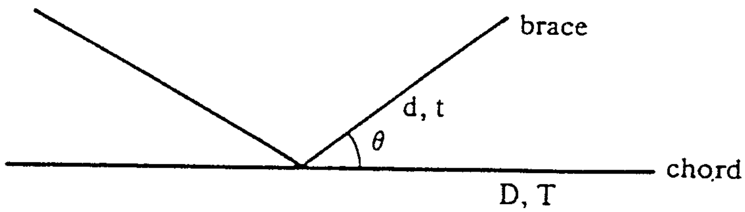

FIGURE 2.2d: Geometric Parameters of a Plane K-joint

3. CASE STUDIES OF SFEM-BASED COMPUTER PROGRAMS

3.1 Introduction

As of the time of writing this document, there are only a few computer programs available in the public domain for performing SFEM-based reliability analysis. The two most prominent are NESSUS developed by Southwest Research Institute (SwRI) in San Antonio, Texas on behalf of NASA (Lewis Research Center) and the CALREL-FEAP system developed at the University of California at Berkeley. In this chapter, a discussion of the main features, structure, and capabilities of both systems are discussed.

3.2 The NESSUS® System

3.2.1 General Description

The NESSUS® software system (acronym for Numerical Evaluation of Stochastic Structures Under Stress) is a general purpose probabilistic computer program for predicting stochastic structural response due to random loads, material properties, part geometry, and boundary conditions. This program is the outcome of an ongoing effort at Southwest Research Institute, San Antonio, Texas under contract to the National Aeronautics and Space Administration, Lewis Research Center (NASA/LeRC). The project, referred to as "Probabilistic Structural Analysis Methods" (PSAM) for Select Space Propulsion System Components is being carried out in two five-year phases. The first five-year PSAM program (1985-1989) developed tools for predicting the probabilistic structural response of large scale structural components. The second five-year program (1990-1994) is extending this capability to include more general failure definitions, and addressing more complicated issues such as reliability analysis for structural systems, certification, and health monitoring [50].

The NESSUS system has several software components. These include an EXPERT system, a probabilistic finite element code (PFEM), a probabilistic boundary element code (PBEM), a fast probability integrator (FPI) module, and a pre-processor module.

The NESSUS/EXPERT module is an interactive menu driven expert system that provides information to assist in the use of the probabilistic finite element code NESSUS/PFEM and the fast probability integrator NESSUS/FPI. This module has the objective of capturing and utilizing PSAM knowledge and experience for guiding and assisting the engineer in the effective and efficient use of probabilistic structural analysis technology.

Since the PSAM project is designed to perform probabilistic structural analysis on realistic engineering structures under complex loading environments, finite element analysis has a key role to play. The NESSUS/FEM module is a nonlinear probabilistic finite element code for determining structural response and sensitivities. This program module is based on a mixed finite element variational formulation in which the nodal degrees of freedom includes stresses, strains, as well as displacements. NESSUS/FEM includes special "perturbation" algorithms to obtain the response to a perturbed set of random variables. Perturbation algorithms provide efficient ways (compared to the finite difference method) of computing the sensitivity of the structural response to small fluctuations of the random variables about a deterministic value. Analysis types that can be handled by this program module include static, natural frequency, buckling, harmonic excitation, and random vibration. The element library includes beam, plate, plane stress, plane strain, axisymmetric, and 3-D solid elements. Random variables may include geometry parameters, loads, and material properties. The program also has the capability to deal with material and geometric nonlinearities.

NESSUS/FPI is a fast probability integration analysis program that evaluates point probability estimates and cumulative probability distributions (CDF) for selected structural response variables. The FPI procedure implemented in NESSUS is based on an extension of the Rackwitz-Fiessler algorithm [51]. A procedure called the Advanced Mean Value First Order Method (AMVFO) is used to couple the finite element and FPI codes to efficiently obtain CDF's of large scale complex engineering structures whose solution times are computationally expensive. Improved versions of this procedure (such as the AMV+ method) have also being developed and implemented into NESSUS by SwRI personnel. The AMV+ method is an iterative algorithm designed to improve the accuracy of the advanced mean value method. This is achieved by using the exact most probable point locus (MPPL) to define the higher order terms in the Taylor series expansion of the performance function about the mean value [50]. Two different AMV+ algorithms are available in NESSUS. The AMV+ p-level procedure is used to compute the response corresponding to a specified probability, while the AMV+ z-level procedure is for the computation of the probability corresponding to a prescribed overall response z(x). NESSUS/FPI also includes a Monte-Carlo simulation scheme. Further details of the AMV/AMV+ algorithms are given in Subsection 3.2.2.

A schematic representation of the NESSUS (Version 5.0) code structure is shown in Figure 3.1. In addition to a library of precoded functions, user programmable subroutines are provided to allow any performance function to be defined. The user has an option of selecting the method of structural response calculation (NESSUS/FEM, user subroutine, user input, etc.). One of the new features of NESSUS is the facility to store probabilistic results in a database for later post-processing.

New features of NESSUS include systems reliability analysis, adaptive importance sampling and probabilistic fault tree analysis. A facility for probabilistic fracture mechanics analysis is also under development [52].

The NESSUS system is available from SwRI for a yearly license fee of about $20,000 (Cdn.). An additional first time installation fee of about $3,600 applies to all licenses. The annual license includes usage, maintenance, and support of the software. Two variations of the annual license are:

i. Maintenance and telephone support license, which includes software usage, unlimited telephone support, and periodic code update and enhancements; and

ii. Maintenance and engineering support license, which includes software usage, 120 hours of SwRI engineering (including telephone) support and periodic code updates and enhancements. This second option is available in the first year only.

There was no access to this code during the course of this work. However, its theoretical foundation and some of its applications are documented in conference proceedings and journal articles, many of which were thoroughly studied by the principal investigator prior to and during the course of the present contract.

NESSUS is a probabilistic finite element analysis package that permits an integrated response analysis and reliability analysis of complicated structures. The AMV algorithm developed at SwRI is unique to NESSUS and has the major advantage that it efficiently performs probabilistic analysis by a procedure that minimizes the number of limit state function evaluations in comparison with conventional optimization schemes. The algorithm has been successfully applied to many practical problems in the aerospace industry. However, further rigorous applications are required to fully establish the robustness of the AMV algorithm.

3.2.2 NESSUS AMV/AMV+ Algorithms

As was pointed out in the preceding subsection the so-called AMV/AMV+ algorithms were developed at SwRI and are unique to the NESSUS system. In this subsection, these algorithms are elaborated upon.

Consider a response (or performance) function $g=g(x)$, where x is a vector of N random variables and g is either explicit or implicit. The function $g(x)$ is usually implicit when dealing with complex structures where structural response is evaluated by means of the finite element method. The AMV procedure has the fundamental objective of constructing reasonably accurate cumulative distribution functions (CDFs) of g, namely F_g, with a very minimum number of performance function evaluations. A description of the iterative procedure follows:

i. Approximate g as a linear function of X via a Taylor series expansion about a representative point, usually taken as the mean value;

ii. Based on the approximate explicit function obtained in step (i), use reliability methods to approximate probabilities in selected points in the sample space of g. This first approximation to F_g is called the mean-value first-order (MVFO) method, which is usually inaccurate;

iii. Evaluate the function g at each design point to improve the estimate of F_g obtained in step (ii). This is referred to as the "first move" in the AMV method;

iv.
   a. Obtain a linear approximation (Taylor's series) to g at each of the updated design points, and
   b. Use a fast probability integration (FPI) scheme to obtain point probability estimates in the "next move" to construct F_g;

v. Evaluate the g function at the design points computed in step (iv); and

vi. Repeat steps (iv) and (v) to improve the estimate of g until desired accuracy is achieved.

When the algorithm is terminated at step (iii), it is referred to as the AMV method (or AMVFO, i.e. AMV first-order, the first-order referring to the linear approximation in step (v)). For a problem in which M points are selected for the CDF construction (i.e. M probability levels) in the sample space of g, the AMVFO method requires a total of J_G^{AMV} function evaluations where

$$J_G^{AMV} = N + M + 1 \; . \tag{3.1}$$

This number is based on the assumption that the gradients of the performance function are computed using the perturbation method as is done in NESSUS. When the algorithm includes steps (iv-vi), the procedure has been referred to as the AMV+ algorithm based on the most probable point locus (MPPL). The number of g-function evaluations for the AMV+ procedure in a solution requiring L full iterations is given by

$$J_G^{AMV+} = (N+M+1) + LM(N+2) \; . \tag{3.2}$$

A full iteration here is defined as an iteration required to update the linear approximation to g at the most current design points, the calculation of point probability estimates corresponding to this approximation, and "moves" to correct the g-function values corresponding to the probability estimates. The process is referred to as the p-level version of the AMV+ algorithm because probability levels are selected and appropriate response values are determined for the probability levels.

The z-level AMV+ algorithm, on the other hand, is for estimating probabilities corresponding to given response levels, Z_0. The steps involved may be summarized as follows:

i. Use the mean-value first-order (MVFO) method to obtain the intercept and slope of the cumulative distribution function (CDF) curve at the 50% probability level;

ii. Let $z_1(x) = Z_0$ and compute the MPP and CDF using MVFO method;

iii. Recompute Z_0 at the most probable point (MPP);

iv. Use corrected CDF point and information from step (i) to fit a quadratic CDF curve, and then use this curve to predict the probability level P_0 for specified Z_0;

v. Set the value of the CDF to P_0 and use $z_1(x)$ to find the corresponding response value Z_0 and the MPP; and

vi. Use the MPP found as a starting point for repeated iterations about Z_0 until the probability converges; the solution is expected to converge quickly because of the quadratic curve fitting scheme.

The main attractive feature of the AMV/AMV+ algorithm is the relatively small number of FE calculations required to establish the CDF of a given performance function as compared to conventional FORM/SORM algorithms. The algorithm may, however, have difficulties in the analysis of problems involving non-monotonic or highly nonlinear performance functions. Some thoughts on how to exploit the nice features of the AMV algorithms in conjunction with schemes to improve the robustness of the algorithms were generated during the course of this work. These ideas form the basis of some of the recommendations for future work given in Chapter 7.

3.3 The CALREL-FEAP System

3.3.1 General Description

The CALREL-FEAP system is a combined package for performing integrated finite element/probabilistic modelling of the reliability analysis of stochastic structures. CALREL is a general purpose structural reliability analysis program that has been developed by Prof. A. Der Kiureghian and his research group at the University of California, Berkeley [53]. FEAP is a finite element analysis package developed by Prof. R.L. Taylor also of the University of California at Berkeley. The two programs are connected via user defined subroutines for limit state function evaluations, the gradients of the limit state functions, and the possibility of user provided probability distributions. The development of the integrated CALREL-FEAP system was directed by Prof. Der Kiureghian [10] based on the finite element reliability method (FERM). Figure 3.2 illustrates the structure of the CALREL-FEAP system.

3.3.2 CALREL-FEAP Algorithms

CALREL has four general techniques for computing the failure probability corresponding to a given performance function. These include FORM, SORM, directional simulation (with exact or approximate surfaces), and Monte-Carlo simulation. The second-order component reliability analysis is performed by means of either the point fitting method or the curvature fitting method, or both, in the standard normal space. Facilities are available for the computation of first-order reliability bounds and PNET approximation for series systems, first-order reliability sensitivity analysis with respect to probability distribution functions and limit-state function parameters, and Monte-Carlo simulation for general systems.

The program has a library of 11 standard probability distribution functions and provision is made for user-defined subroutines for the addition of other distribution functions. Correlations between the basic random variables are permitted.

The calculation of gradients in the CALREL-FEAP system is based on the adjoint variable method in conjunction with the use of analytically derived partial derivative element matrices and load vectors. There is provision for user-supplied gradients in the system.

Current features include the finite element reliability analysis of geometrically nonlinear stochastic structures using the wide variety of the finite elements that are available in the FEAP program which serves as a subroutine in this system. The SORM procedure for this system is based on the point-fitting paraboloid method developed by Der Kiureghian et al. [54]. Further details on CALREL-FEAP algorithms can be found in reference [10].

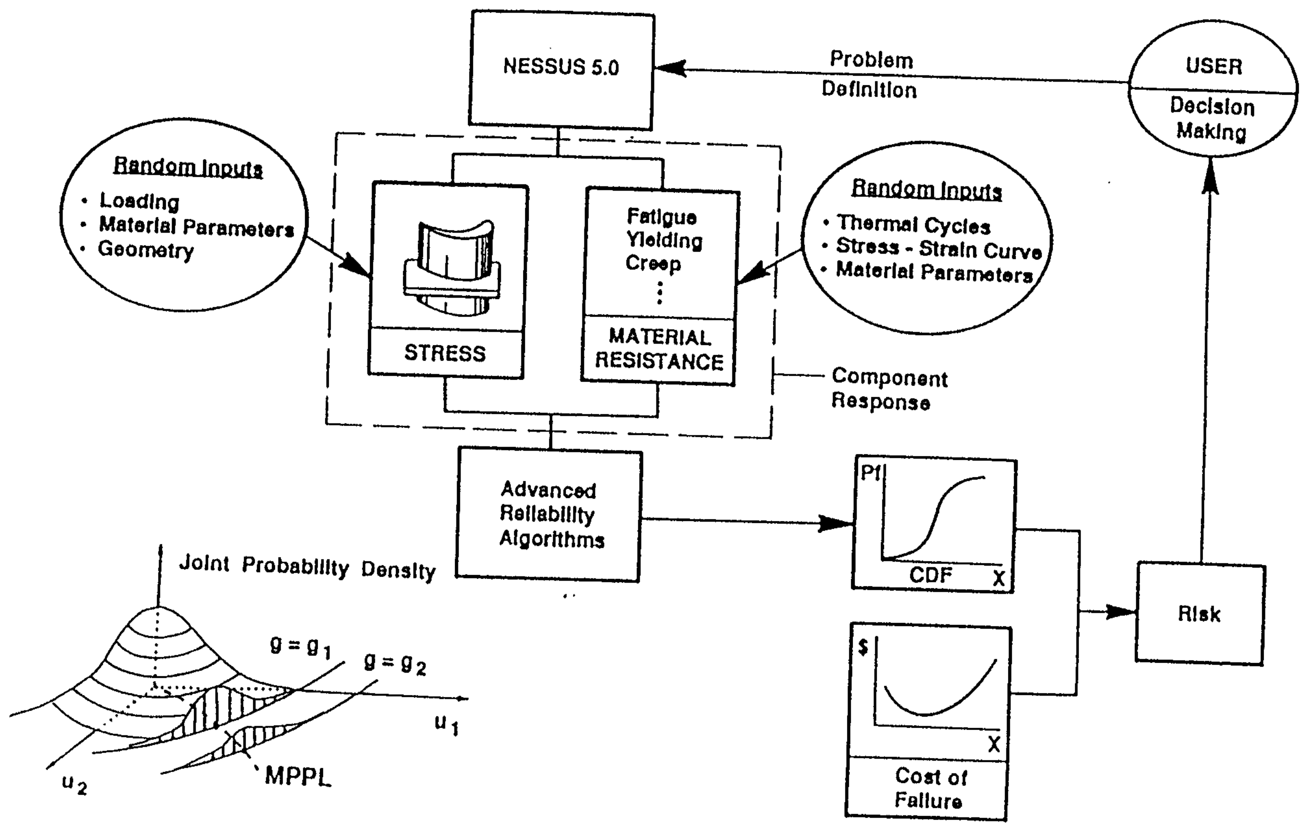

FIGURE 3.1: Schematic Illustration of the NESSUS System

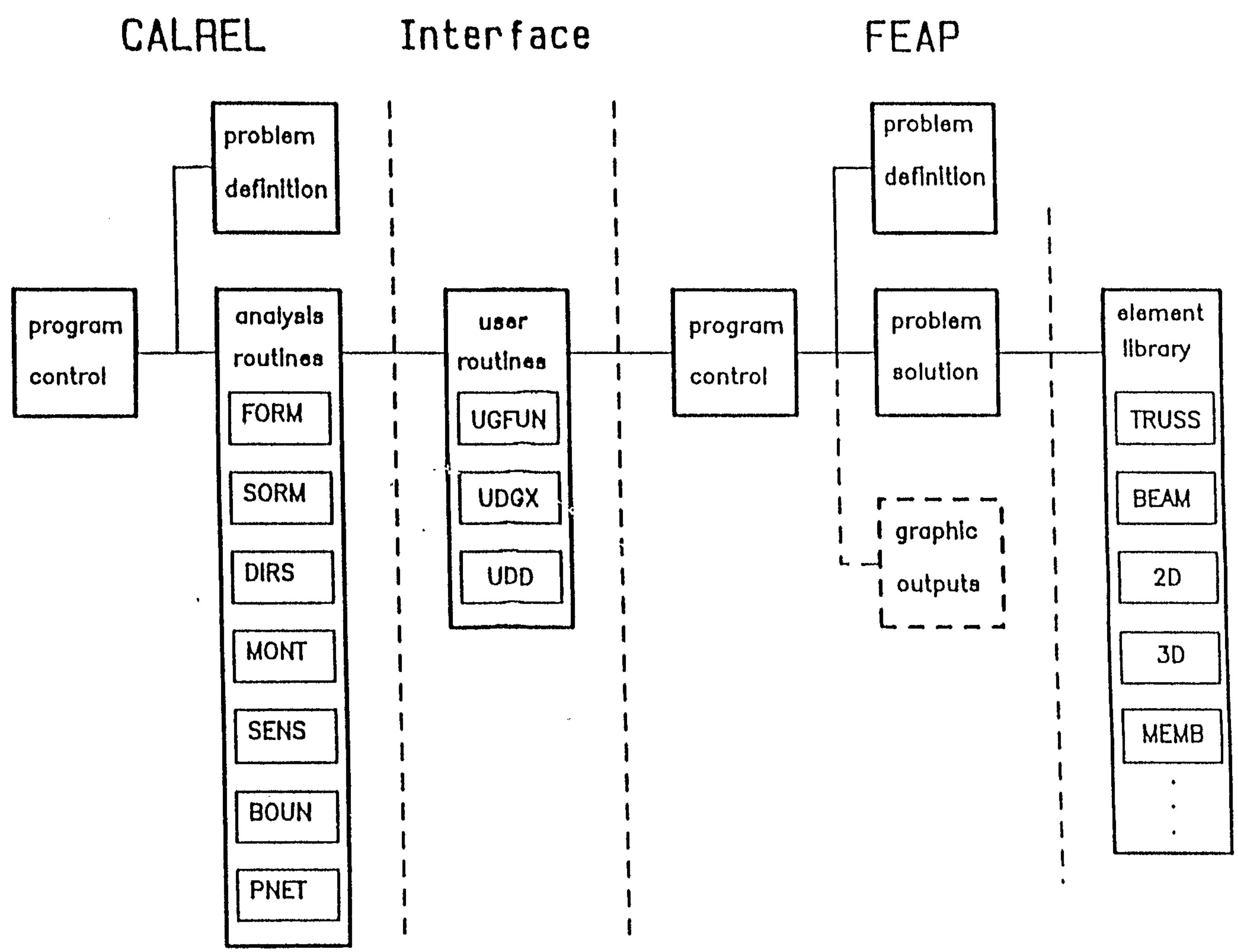

FIGURE 3.2: The Structure of the CALREL-FEAP System

4. DESIGN AND IMPLEMENTATION OF THE MODEL SFEM COMPUTER PROGRAMS

4.1 Basic Requirements of Computer Programs for SFEM-Based Structural Reliability Analysis

There are several components of analysis capabilities required of any SFEM-based reliability analysis software package. In the present work, the finite element reliability method (FERM) developed by Der Kiureghian and his coworkers [9, 10], was selected because of its generality and the other advantages alluded to in Chapter 2. The specific components that constitute a FERM analysis capability are highlighted below.

One of the first requirements is the capability for discretizing random fields into random variables. This is a necessary pre-processing step as the most advanced reliability analysis algorithms are formulated on the basis of random variable models. A main program is required to drive the entire probabilistic finite element analysis process. This module will accept input, control the flow of the various modules and algorithms, and control analysis output.

A finite element analysis (FEA) platform is required for the calculation of the structural responses. In addition to this, a module for the computation of response gradients is required. This is a requirement of virtually all strategies for SFEM-based reliability analysis, except the Monte Carlo simulation scheme.

The next major module is the reliability analysis module, which performs the calculation of failure probabilities and probabilistic sensitivity factors. This module requires the capability of transforming generally correlated non-Gaussian random variables to uncorrelated standard normal variables. Then a fast probability integration (FPI) scheme is required to evaluate the integral that defines the failure probability, P_f, namely:

$$P_f = \int_\Omega \cdots \int f_x(x)dx \ , \tag{4.1}$$

where $f_x(x)$ is the joint probability density function of the random variables x. Also, the scope of the present work includes the calculation of various probabilistic sensitivity measures.

Finally, a limit state function library is required for the definition of typical failure modes of interest. However, in addition to this, a provision could be made for a user to program limit state functions of interest.

In the following sections, the design and computer implementation of the above requirements are presented. The design of the overall computer program was done in a way that ensures easy enhancements and implementation of new strategies in the future. The selection of specific algorithms was also done with careful considerations of extension to more advanced capabilities that are likely to be of interest in the future.

4.2 The Random Field Discretization Module (RANFLD)

Program RANFLD is an important pre-processor for stochastic finite element analysis involving random fields. Two discretization procedures were implemented in RANFLD: these are the midpoint method and the nodal point method. These techniques were selected because they are the most versatile as they are applicable to all types of random fields. It is recommended that material property fields (namely: Young's modulus, Poisson's ratio, and density) be discretized using the midpoint method, while distributed random load fields could employ the nodal point method. This scheme is the most compatible with the VAST FEA system.

Stochastic finite element (SFE) meshes are similar to conventional finite element meshes in that they are used to discretize a given domain. SFE meshes are recommended to be generally coarser (or at least not finer) than the mechanical finite element mesh. RANFLD is designed on the basis that the finite element model generation program VASGEN [55] will be utilized for the generation of stochastic finite element meshes. Provision is made, however, for the user to manually define an SFE mesh in terms of a conventional finite element mesh without a physical domain discretization of the random field.

For the midpoint method, the centroids of the stochastic finite elements are automatically computed. The user has the options of specifying the centroidal point or the preferred location for computing the correlation matrix. This point may be specified in terms of a node number of a FE mesh or in terms of the global (Cartesian) coordinates. Provision is made in RANFLD for the use of the same mesh for the SFEM and the FEM. Multiple random fields are permitted but, in the present version, there is the restriction of using the same SFEM mesh for material property fields. The material properties are classified into four types as follows:

i. Type 1: Young's modulus;
ii. Type 2: Poisson's ratio;
iii. Type 3: mass density; and
iv. Type 4: any other type.

This classification is mainly for the information of the gradient calculation programs and the "type" of every random variable generated by RANFLD is tagged to that variable throughout the analysis.

The input to RANFLD consists of probabilistic descriptions of all the random fields and the descriptions of the stochastic finite element meshes to be used for discretization. Each random field is defined via the specification of the probability distribution function, mean function, variance function, and the correlation function. Acceptable probability distribution functions include any of the 16 distributions available in the COMPASS [56] library and also in the FORMREL module. Four commonly used analytical correlation function models were implemented as a start-up library in RANFLD. It should be noted that random fields can also be described in terms of the variance function ($\delta(t)$) and the scale of fluctuation (θ) instead of the correlation function. However, the former framework is usually employed in conjunction with spatial averaging techniques. The RANFLD requirement is based on practical information availability considerations and is by no means a restriction on its capabilities. This is especially so as there exists a well defined analytical relationship between the variance function and the correlation function [7].

The correlation function models implemented in RANFLD during the present work include the exponential, Gaussian, triangular and the second-order autoregessive process models. These schemes were implemented assuming the fields to be homogeneous. (A random field is called homogeneous if all the joint probability distribution functions remain the same when the set of locations, δ_1, δ_2, ..., δ_n is translated (but not rotated) in the parameter space.) A homogeneous random field is the spatial counterpart of a stationary random process.

The exponential correlation function model is associated with a first-order autoregessive (or Markov) process and has the form:

$$\rho(\tau) = \exp(-|\tau|/d) \ , \tag{4.2}$$

where $\tau = |\mathbf{r}_1 - \mathbf{r}_2|$ is the distance between points $\mathbf{r}_1$ and $\mathbf{r}_2$, and d is the correlation distance. The Gaussian (or squared exponential) model has the form

$$\rho(\tau) = \exp\left[-(|\tau|/d)^2\right] . \tag{4.3}$$

The triangular correlation function decreases linearly from 1 to 0 as $|\tau|$ goes from 0 to d,

$$\rho(\tau) = \begin{cases} 1 - |\tau|/d \ , & |\tau| \leq d \\ 0 & |\tau| > d \end{cases} \tag{4.4}$$

The second-order autoregessive process correlation model has the mathematical representation:

$$\rho(\tau) = \left[1 + \frac{|\tau|}{d}\right] \exp(-|\tau|/d) \tag{4.5}$$

Another important capability that was provided for the RANFLD program is the spectral decomposition of the covariance matrix. This procedure (which is used by NESSUS) is only applicable to Gaussian fields. However, it has the advantage in SFEA of being able to reduce the number of random variables by truncating the probabilistic correlation modes. Two eigenvalue orthogonalization subroutines (SPECD1 and SPECD2) were implemented to achieve this.

Also in RANFLD, names are automatically assigned to every random variable generated. Apart from the fact that the variable names are used by the reliability analysis module, they serve as useful identification for the element groups and property type (eg. E, ν or ρ).

Figure 4.1 illustrates the schematic flow of program RANFLD. The input file format is described in Appendix A1 and is named PREFX.RFD, where, as in other VAST analysis files, "PREFX" is a five-character prefix prompted for at the beginning of the program run. Necessary files are opened and the centroids of the stochastic finite element meshes are computed (if required) using the main subroutines CENTRD. This subroutine, in turn, has several subsidiary subroutines for dealing with each element.

The current capability is available for seven elements in the VAST finite element library. These include the 8-noded thick/thin shell element (IEC=1), the two-noded general beam element (IEC=3), the three-noded triangular plate element (IEC=4), the four-noded quadrilateral shell element (IEC=5), the two-noded bar element (IEC=8), the three-noded triangular membrane element (IEC=9), and the eight-noded isoparametric membrane element (IEC=20).

The output of program RANFLD consists of the following:

i. An echo of the random field description;

ii. Upper triangular portion of the correlation matrix printed row-wise; and

iii. Eigenvalues and eigenvectors of the covariance matrix, arranged in descending order of eigenvalues.

Two input files are produced. A formatted output file PREFX.LPD permits the user to view the results of the random field discretization. A binary file PREFX.RVB contains essentially the same information and is the file utilized by the stochastic finite element program (STOVAST) for the definition of random variables generated from random fields.

4.3 Algorithms for Gradient Computation

The most important feature of a stochastic finite element based reliability analysis program is the calculation of the gradients of finite element structural responses which appear in the performance functions. This is because algorithms for first-order reliability methods (FORM) and second-order reliability methods (SORM) which are efficiently utilized for the approximate evaluation of failure probabilities rely on the accurate (repeated) calculation of gradients of the limit state functions.

Several strategies are available for the computation of the gradients of finite element responses. These include the finite difference method, perturbation method, and the adjoint method.

For the present work, the adjoint method first suggested by Arora and Haug [57] and discussed extensively in the monograph by Haug et al. [58] was the selected strategy for gradient computation. This method is superior in terms of both computational efficiency and accuracy to the perturbation method used by Hisada and Nakagiri [14] and Liu et al. [15, 31], the iterative perturbation algorithm proposed by Dias and Nakazawa [59], and of course the finite difference method. The merit of the adjoint method was emphasized in the work of Liu and Der Kiureghian [10] and was implemented in the CALREL-FEAP system. The efficiency of the adjoint method was also demonstrated by Reh et al. [37], especially for problems involving a large number of random variables.

Consider the most general representation of the limit state function, g, defined in terms of response variables, namely: displacements (U), strains (ϵ) and stresses (σ), and additionally some (finite element or non-finite element) basic (primitive) variables (v). The mathematical representation of g is then given by:

$$g = g(v,U,\epsilon,\sigma) \tag{4.6}$$

or

$$g = g(v,U(v),\epsilon(v,U(v)),\ \sigma(v,\epsilon(v,U(v))) \tag{4.7}$$

The gradient of g with respect to a basic random variable (v), $\nabla_v g$, is then given by:

$$\nabla_v g = \frac{\partial g}{\partial v} + \frac{\partial g}{\partial U}\nabla_v U + \frac{\partial g}{\partial \epsilon}\nabla_v \epsilon + \frac{\partial g}{\partial \sigma}\nabla_v \sigma \tag{4.8}$$

where $\nabla_v U$, $\nabla_v \epsilon$, and $\nabla_v \sigma$ are the gradients (with respect to v) of the displacement, strain and stress fields, respectively. Using the chain rule of differentiation and the respective functional dependencies of U, ϵ and σ, it is easily shown that

$$\begin{aligned}\nabla_v g = {} & \frac{\partial g}{\partial v} + \frac{\partial g}{\partial \epsilon}\frac{\partial \epsilon}{\partial v}\Big|_U + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial v}\Big|_\epsilon + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial \epsilon}\Big|_v\frac{\partial \epsilon}{\partial v}\Big|_U \\ & + \left[\frac{\partial g}{\partial U} + \frac{\partial g}{\partial \epsilon}\frac{\partial \epsilon}{\partial U}\Big|_v + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial \epsilon}\Big|_v\frac{\partial \epsilon}{\partial U}\Big|_v\right]\nabla_v U\end{aligned} \tag{4.9}$$

Equation (4.8) is the most general representation of the gradient of typical performance functions of interest in structural mechanics. It may be more compactly represented as:

$$\nabla_v g = f_v + f_U \nabla_v U , \tag{4.10}$$

where

$$f_v = \frac{\partial g}{\partial v} + \frac{\partial g}{\partial \epsilon}\frac{\partial \epsilon}{\partial v}\Big|_U + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial v}\Big|_\epsilon + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial \epsilon}\Big|_v \frac{\partial \epsilon}{\partial v}\Big|_U , \tag{4.11}$$

and

$$f_U = \frac{\partial g}{\partial U} + \frac{\partial g}{\partial \epsilon}\frac{\partial \epsilon}{\partial U}\Big|_v + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial \sigma}\Big|_v \frac{\partial \epsilon}{\partial U}\Big|_v . \tag{4.12}$$

For the special case of displacement-type limit states the functional representation of g is of the form:

$$g = g(v,U(v)) \tag{4.13}$$

and f_v and f_U in Equations (4.11) and (4.12), respectively reduce to:

$$f_v = \frac{\partial g}{\partial v} , \tag{4.14}$$

and

$$f_U = \frac{\partial g}{\partial U} . \tag{4.15}$$

Similarly, stress-type limit state conditions in engineering practice are usually defined in terms of a stress threshold (typically yield stress of the material) and some other stress components. The functional representation then takes the form:

$$g = g(v, \sigma(v,U(v))) , \tag{4.16}$$

and f_v and f_U in Equations (4.11) and (4.12), respectively reduce to:

$$f_v = \frac{\partial g}{\partial v} + \frac{\partial g}{\partial \sigma}\frac{\partial \sigma}{\partial v}\Big|_U , \tag{4.17}$$

and

$$f_U = \frac{\partial g}{\partial \sigma} \frac{\partial \sigma}{\partial U}\Big|_v . \tag{4.18}$$

It can be appreciated from the above equations that the calculation of the gradients of limit state functions within the framework of a finite element analysis procedure is a very complicated process. A major requirement of the procedure laid out above is the calculation of the gradients of the displacement vector with respect to the basic random variables, that is $\nabla_v U$.

The direct method of evaluating $\nabla_v g$ is to compute f_v, f_U, $\nabla_v U$ and then the product $f_U \nabla_v U$ as required in Equations (4.10), (4.11) and (4.12). For a linear static formulation of the finite element method (which is the scope of this contract), the assembled equilibrium equations appear in the form:

$$KU = F , \tag{4.19}$$

where K is the assembled stiffness matrix and F is the assembled force vector, both random in general. From this equation, it follows that:

$$K\nabla_v U = \frac{\partial F}{\partial v}\Big|_U - \frac{\partial K}{\partial v} U . \tag{4.20}$$

The direct computation of $\nabla_v U$ from Equation (4.20) is very time consuming. For a structure with m degrees-of-freedom and n basic random variables, this approach requires n solutions of m simultaneous equations for each limit state function of interest.

The adjoint method avoids the above costly computations by solving the adjoint equation:

$$\lambda^T K = f_U , \tag{4.21}$$

only once for an adjoint displacement vector λ. The product $f_U \nabla_v U$ required in Equation (4.10) is then directly computed from the expression

$$\lambda^T K \nabla_v U = f_U \nabla_v U = \lambda^T \left(\frac{\partial F}{\partial v}\Big|_U - \frac{\partial K}{\partial v}\Big|_U U \right) , \tag{4.22}$$

in which advantage of Equations (4.20) and (4.21) have been taken.

The evaluation of $f_U \nabla_v U$ then requires the solution of one equilibrium-type equation in which the stiffness matrix involved is the same stiffness matrix for the structural response in the current reliability iteration, the computation of load gradients, and the gradients of the stiffness matrix with respect to the basic random variables. For this reason, two major modules for gradient computations were designed and developed in this work for the calculation and assembly of stiffness and load gradients. A summary of the entire adjoint method procedure for the calculation of limit state function gradients is schematically illustrated in Figure 4.2.

The element gradient module ELEMGD (similar to ELEMS1 in VAST) directs the computation of the gradients of element matrices, with element-specific subroutines implemented in a submodule ELGRAD (similar to ELSUB1/ELSUB2 in VAST). The assembly of the element partial derivative matrices is performed by a module ASSEMG (similar to ASSEM1 in VAST), and the solution of the adjoint problem is computed using a module called STODIS (similar to DISP1 in VAST).

The second gradient module is LOADGD for the calculation of load gradients, with element-specific subroutines for calculating element distributed loads assembled in a submodule LDGRAD.

The computation of gradients of concentrated nodal loads and the assembly of all load gradients is done within the load gradient module. Both of these modules (i.e. ELEMGD/ELGRAD and LOADGD/LDGRAD) are driven by a subprogram called GRADEN. Subprogram GRADEN includes a facility for computing the required adjoint force and displacement vectors for displacement limit states, and directs the calculation of gradients in the entire reliability analysis process.

Three elements were implemented in the gradient modules. These are the two-noded general beam element (IEC=3), the four-noded quadrilateral shell element (IEC=5) and the three-noded triangular membrane element (IEC=9). The formulation of the element and load gradients are element-specific. However, a typical finite element has a stiffness matrix representation of the form:

$$K_e = \int_{\Omega e} B^T D B \, dV , \qquad (4.23)$$

where B is the strain-displacement matrix, D is the constitutive matrix, V is the volume and Ω_e is the domain over which the element is defined. The gradient with respect to random material property v_i can be expressed as:

$$\frac{\partial K_e}{\partial v_i} = \int_{\Omega e} B^T \frac{\partial D}{\partial v_i} B \, dV \, , \qquad (4.24)$$

since B is generally independent of material properties.

For element IEC=5, for example, the (isotropic) constitutive matrix, D, has the form

$$D = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2k} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2k} \end{bmatrix} , \qquad (4.25)$$

where E is the Young's modulus, ν is the Poisson's ratio, and k is the shear displacement factor. This leads to the following gradients:

$$\frac{\partial D}{\partial E} = \frac{1}{E}\,[D]\ ,$$

$$\frac{\partial D}{\partial \nu} = E \begin{bmatrix} \frac{2\nu}{(1-\nu^2)^2} & \frac{1+\nu^2}{(1-\nu^2)^2} & 0 & 0 & 0 \\ \frac{1+\nu^2}{(1-\nu^2)^2} & \frac{2\nu}{(1-\nu^2)^2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2(1+\nu)^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2k(1+\nu)^2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2k(1+\nu)^2} \end{bmatrix} \qquad (4.26)$$

For the beam element, the gradients are computed with respect to E, ν, ρ, A, I_x, I_y, and I_z. The gradients of the element loads are similarly computed with respect to distributed load parameters. The use of the above direct differentiation technique for gradient calculation ensures the accuracy and efficiency especially when used in conjunction with the adjoint method.

The finite difference scheme for gradient calculation was also implemented in the reliability analysis module (RELAMM) during this work.

4.4 Reliability Analysis Algorithms

The general reliability problem is usually formulated in terms of a finite set of basic random variables $X = (X_1, X_2, \ldots, X_n)$ and a limit state function $g = g(X)$ in which g is the failure or performance function. Failure is defined by the event $\{g(X) \leq 0\}$, while $\{g(X) > 0\}$ identifies a safe state. The probability of failure, P_f, is defined as:

$$P_f = \text{Prob}\{g(X) \leq 0\} = \int_{g(X) \leq 0} f_x(x)dx \quad , \tag{4.27}$$

where $f_x(x)$ is the multivariate joint probability density function of X.

The integral in Equation (4.27) is in general very difficult to evaluate and approximate procedures have evolved as practical tools for efficiently calculating the integral. Prominent examples in this connection include the first-order reliability methods (FORM) and the second-order reliability methods (SORM) which are well discussed in classical monographs on structural reliability; see, for example, Madsen et al. [60] and Melchers [1]. These procedures are based on the evaluation of a reliability index (β) from which the failure probability (P_f) can then be computed. The failure probability is related to the reliability index (which is the widely accepted probabilistic measure of safety) through the expression:

$$P_{f_1} = \Phi(-\beta) \quad , \tag{4.28}$$

where P_{f_1} is the FORM failure probability, and Φ is the standard normal cumulative distribution function (CDF). The relationship in Equation (4.28) is an important one and must be available in any reliability analysis program. This relationship is graphically illustrated in Figure 4.4.

The scope of the present work is limited to FORM computations. The FORM algorithm (FORMREL) implemented in the reliability analysis module (RELAMM) follows the one implemented in the Martec reliability analysis package COMPASS [56]. The main steps in the FORM-based computation of β are as follows:

i. Transformation of the vector of basic random variables from the original x-space to the standard normal u-space as shown in Figure 4.5;

ii. A search (usually in the u-space) for the point (u^*) on the limit state surface $g(u) = 0$ that has the highest joint probability density; this point is popularly referred to as the design point, failure point, or the most probable point (MPP);

iii. An approximation at the MPP of the failure surface in u-space; and

iv. A computation of the distance from the origin to the MPP called β and hence the failure probability.

The probability transformation (step (i) above) is presently accomplished in four substeps. First, there is a transformation from the x-space to an (generally non-Gaussian) intermediate y-space of equivalent normal variables y_i by equating their CDFs and PDFs, that is:

$$\Phi\left(\frac{y_i - \mu_{yi}}{\sigma_{y_i}}\right) = F(x_i) \ , \qquad i=1,2,...,n \tag{4.29a}$$

$$\phi\left(\frac{y_i - \mu_{yi}}{\sigma_{y_i}}\right) = f(x_i) \ , \qquad i=1,2,...,n \tag{4.29b}$$

where Φ is the standard normal PDF, $F(x_i)$ is the marginal CDF of the basic random variable x_i and $f(x_i)$ is the corresponding PDF. Equations (4.29)a and (4.29)b lead to the evaluation of the mean value and standard deviation of the resulting equivalent normal distribution; namely:

$$\mu_{y_i} = x_i - \sigma_{yi}\ \Phi^{-1}\left[F(x_i)\right] \ , \tag{4.30a}$$

$$\sigma_{y_i} = \frac{\phi\{\Phi^{-1}[F(x_i)]\}}{f(x_i)} \tag{4.30b}$$

The next substep then is to transform the correlation matrix from the x-space to the y-space. This correlation matrix is then, in turn, transformed to independent standard normal variables via an orthogonal transformation:

$$y' = T_y^T y \ , \tag{4.31}$$

in which T_y is the orthogonal matrix of eigenvectors (R) of the correlation matrix in the intermediate space.

(Please note that the superscript T in Equation (4.31) denotes the matrix transpose operator.) Finally, the normalized variables y' are transformed to the required standard normal u-space via a linear transformation:

$$U = [S]^{-1} [H]^T y' , \tag{4.32}$$

where H is the nxn matrix of scaled eigenvectors $\overline{R}$ and [S] is the diagonal matrix of the eigenvalues. The resultant transformation from the original x-space to the u-space is usually denoted by the transformation operator T such that:

$$U = T(X) . \tag{4.33}$$

The above probability transformation scheme (illustrated schematically in Figure 4.5) has been verified to yield very accurate results in COMPASS as is also demonstrated in Chapter 5. A more robust algorithm that has been recommended by Liu and Der Kiureghian [27] will be considered in future work.

The search referred to in step (ii) above is conducted by means of the solution of an optimization problem. The optimization problem pertaining to the calculation of the Hasofer-Lind reliability index in the u-space may be posed as follows:

$$\underline{\text{minimize}}\ D = \sqrt{u_i^T u_i} = \beta , \tag{4.34a}$$

$$\text{subject to } g(u_i) = 0 . \tag{4.34b}$$

The solution of this problem locates the MPP and the n-dimensional position vector of this point, U^*, is given by:

$$U^* = -\alpha^* \beta , \tag{4.35}$$

where α^* is the unit normal vector at the MPP, i.e.

$$\alpha^* = \frac{\nabla g(U^*)}{|\nabla g(U^*)|} . \tag{4.36}$$

The optimization algorithms implemented include the HL-RF algorithm originally proposed by Hasofer and Lind [61] and later extended by Rackwitz and Fiessler [51] to include distribution information. This is currently the most widely used method for solving the constrained optimization problem in structural reliability [62]. The method is based on the recursive formula:

$$U_{k+1} = \frac{1}{\nabla g^{T}(U_k)\nabla g(U_k)} \left(\nabla g^{T}(U_k)U_k - g(U_k)\right)\nabla g(U_k) \qquad (4.37)$$

Experience shows that for most situations the HL-RF algorithm converges rapidly.

Liu and Der Kiureghian [62] introduced a modified HL-RF procedure, referred to hereafter as the MHL-RF algorithm, which introduces a merit function m(U) to monitor the convergence of the scheme. The non-negative merit function proposed has the form:

$$m(U) = \frac{1}{2}\left|U - \frac{\nabla g(U)U}{|\nabla g(U)|^2}\nabla g(U)\right|^2 + \frac{1}{2} c g(U)^2 , \qquad (4.38)$$

where c is a positive constant. In the MHL-RF procedure, the new iteration point is selected by a line search along the direction vector

$$d_k = \frac{1}{|\nabla g(U_k)|^2}\left[\nabla g(U_k)U_k - g(U_k)\right] \nabla g(U_k)^T - U_k \qquad (4.39)$$

until a sufficient decrease in m(U) is achieved. The merit function defined in Equation (4.38) is believed to be a convenient guide for selecting the step size. Numerical examples were given in reference [62] to illustrate the improved robustness of the MHL-RF method over the original HL-RF algorithm. The MHL-RF algorithm was implemented in this work because of this robustness property especially for finite element limit state functions. The implementation of the algorithm is schematically illustrated in Figure 4.6.

4.5 Computation of Sensitivity Measures

Modern probabilistic reliability analysis programs must include capabilities for computing sensitivity measures. This is because the identification of the main sources of uncertainty which have significant influences on the reliability of a system is as important as the calculation of failure probabilities. The importance of probabilistic sensitivity estimation was demonstrated in a recent work by the authors [63].

Sensitivity measures are utilized for two major purposes in reliability analysis. First, they can be used to identify the variables or distribution parameters (such as means, standard deviations and correlations) which have major influences on failure probability. Second, they may be used to gain insight into the complex behaviour of structural systems. In the first connection, especially, these quantitative measures provide guidance in the assessment of the validity of reliability estimates and in the definition of the roles of the random variables in subsequent analysis. The first-order reliability method (FORM) provides an excellent tool for assessing sensitivities because they can be determined with relatively little extra computational effort, once the failure probability has been computed.

Parametric sensitivity factors express the sensitivity of the reliability to small changes in statistical parameters that define the distribution of the basic random variables X or deterministic parameters of the performance function. The FORM expressions for parametric sensitivities of the first-order reliability index (β) and failure probability (P_f) are as follows [60, 63]:

i. For a distribution parameter ω_i,

$$\frac{\partial \beta}{\partial \omega_i} = \frac{1}{\beta} u^{*T} \frac{\partial u^*}{\partial \omega_i} = \frac{1}{\beta} u^{*T} \frac{\partial T(x^*)}{\partial \omega_i} \qquad (4.40)$$

ii. For a performance function parameter p_i,

$$\frac{\partial \beta}{\partial p_i} = \frac{1}{\beta} u^{*T} \frac{\partial u^*}{\partial p_i} = \frac{\frac{\partial}{\partial p_i} g(u^*)}{|\nabla g(u^*)|} \qquad (4.41)$$

In the above, u* is the point in the standard normal space (u-space) on the limit state surface that has the highest joint probability density: the most probable failure point (MPP).

iii. For any parameter q_i (ω_i or p_i),

$$\frac{\partial P_f}{\partial q_i} = \frac{\partial}{\partial q_i} \Phi(-\beta) = -\phi(\beta)\frac{\partial \beta}{\partial q_i} , \qquad (4.42)$$

where ϕ is the standard normal probability density function and Φ is the normal cumulative distribution function.

Importance factors express the relative importance of the different sources of uncertainty associated with the basic random variables that define a problem. The scaled and normalized sensitivity of the reliability index with respect to the design point in the space of the original basic random variables is mathematically expressed as [9]:

$$\gamma(x) = \frac{D\nabla\beta_x}{|D\nabla\beta_x|} = \frac{D\alpha(u*)^T J_{u,x}}{|D\alpha(u*)^T J_{u,x}|} , \qquad (4.43)$$

where D is the diagonal matrix of standard deviations of X, $\nabla\beta$ is the gradient of β with respect to x at the linearization point, $\alpha(u^*)$ is the unit vector normal to the limit state surface at the MPP directed towards the failure set, and $J_{u,x}$ is the Jacobian of the probability transformation from the u-space to the x-space. The importance factor, γ_i^2 for each random variable X_i is defined as the square of the corresponding component of the unit vector defined in the preceding equation.

The above sensitivity measures were implemented in the COMPASS reliability analysis computer program developed at Martec Limited and were made available to the reliability analysis module of program STOVAST: RELAMM/FORMREL in a submodule called SENSCMP.

4.6 Other Development Efforts

In a SFEM-based structural reliability analysis program, the finite element analysis is driven by the reliability algorithm. For this reason, the VAST FEA program was converted to a subroutine (VASTM) so that VAST is the FEA module of the main SFEM program STOVAST. The programs developed during the course of this work should, therefore, not be looked upon as providing a stochastic finite element analysis capability for VAST. Rather, a stochastic finite element analysis program (STOVAST) has been developed in which VAST is used as the finite element solver. This point is raised here to emphasize that although developments in STOVAST will be geared towards compatibility with the VAST system, they will remain as two separate programs, each with its own user manual. This situation will most probably remain so until, hopefully, stochastic finite element analysis becomes the accepted standard for FEA.

In order to make VAST serve as the platform for FEA in this work, certain modifications were required in existing VAST modules. First the geometry input file had to be modified to permit the specification of random element properties and random loads. This triggered modifications to the element subroutines (ELEMS1/ELSUB1/ELSUB2) and the load module and its element library (LOAD1).

It was also necessary to transfer some computation parameters to the various modules and to suppress printout of intermediate (FE) reliability iteration results. Limit state functions for displacements and stresses were also made available in program STOVAST.

4.7 Input Requirements for Running Program STOVAST

Detailed descriptions of the input formats for running the random field pre-processing program RANFLD and SFEM-based reliability analysis program (STOVAST) are described in Appendix A1 and A2, respectively. For STOVAST, the input data files required consists of all VAST FEA input files (modified as described in Appendix A2) and the main input file PREFX.SFE. If random fields have been discretized using RANFLD, then the random variables generated in file PREFX.RVB must also be available.

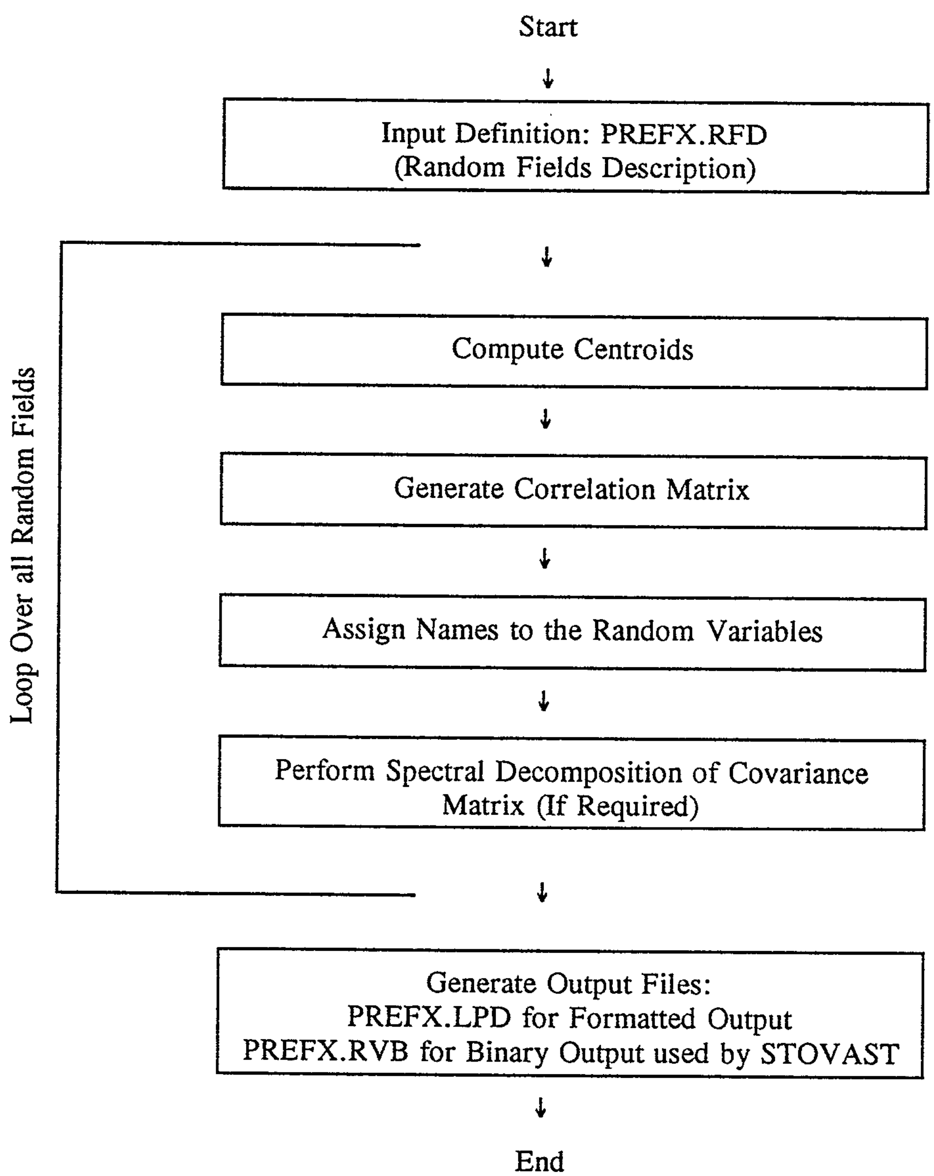

FIGURE 4.1: Illustration of the Flow of Program RANFLD

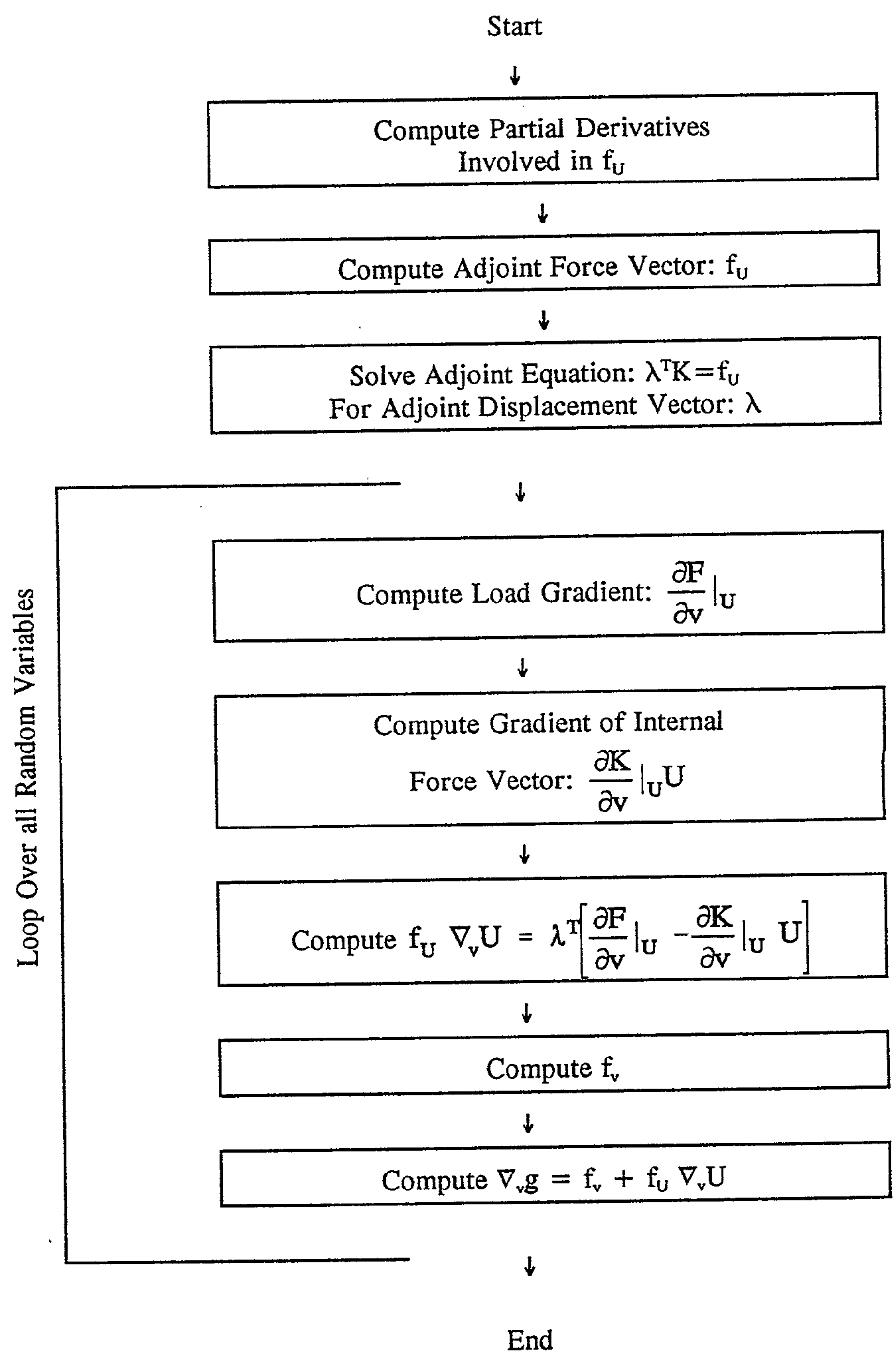

FIGURE 4.2: Summary of The Adjoint Method for Gradient Computation

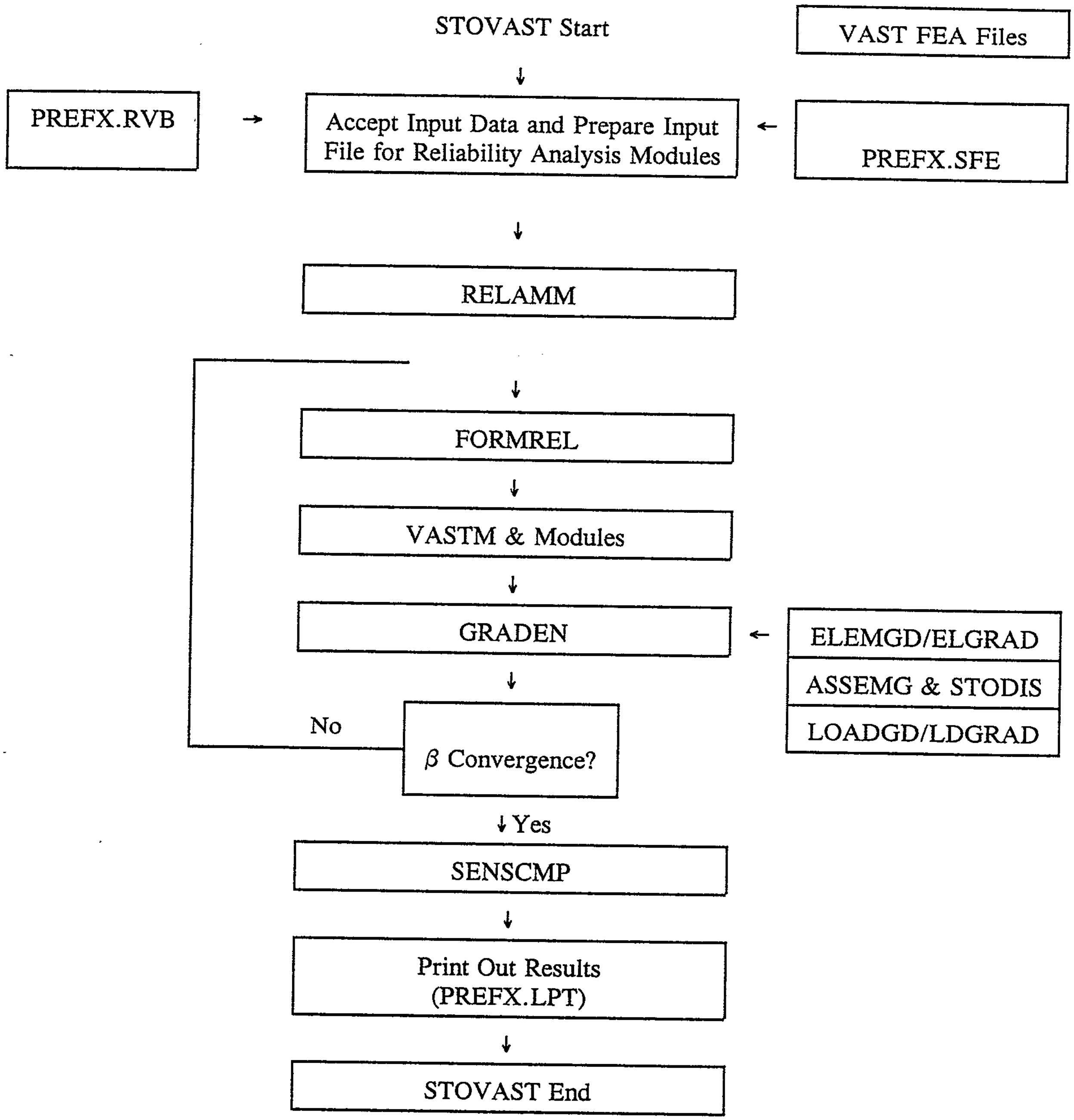

FIGURE 4.3: Illustration of the Flow of Program STOVAST

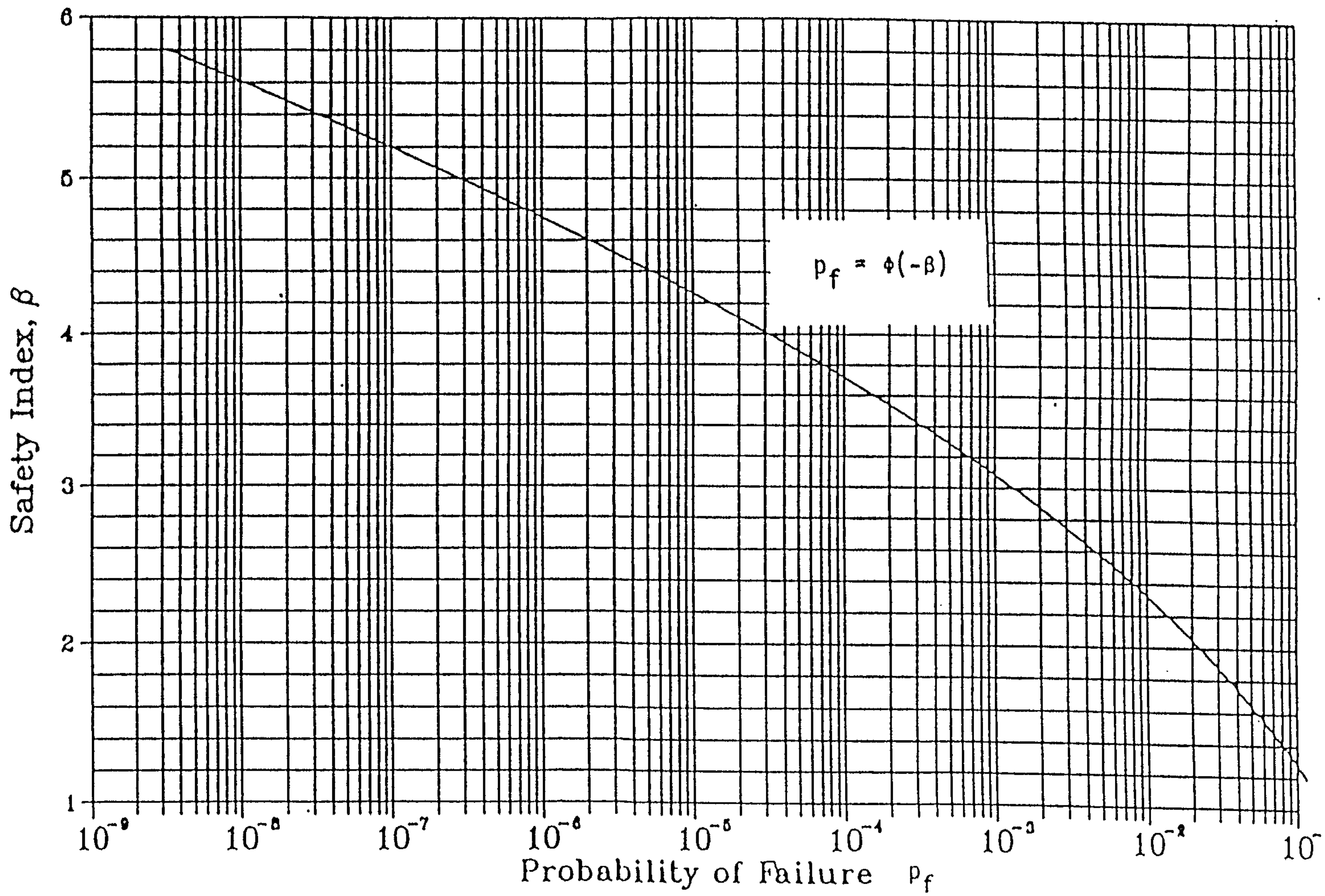

FIGURE 4.4: Relationship Between Failure Probability and Reliability Index

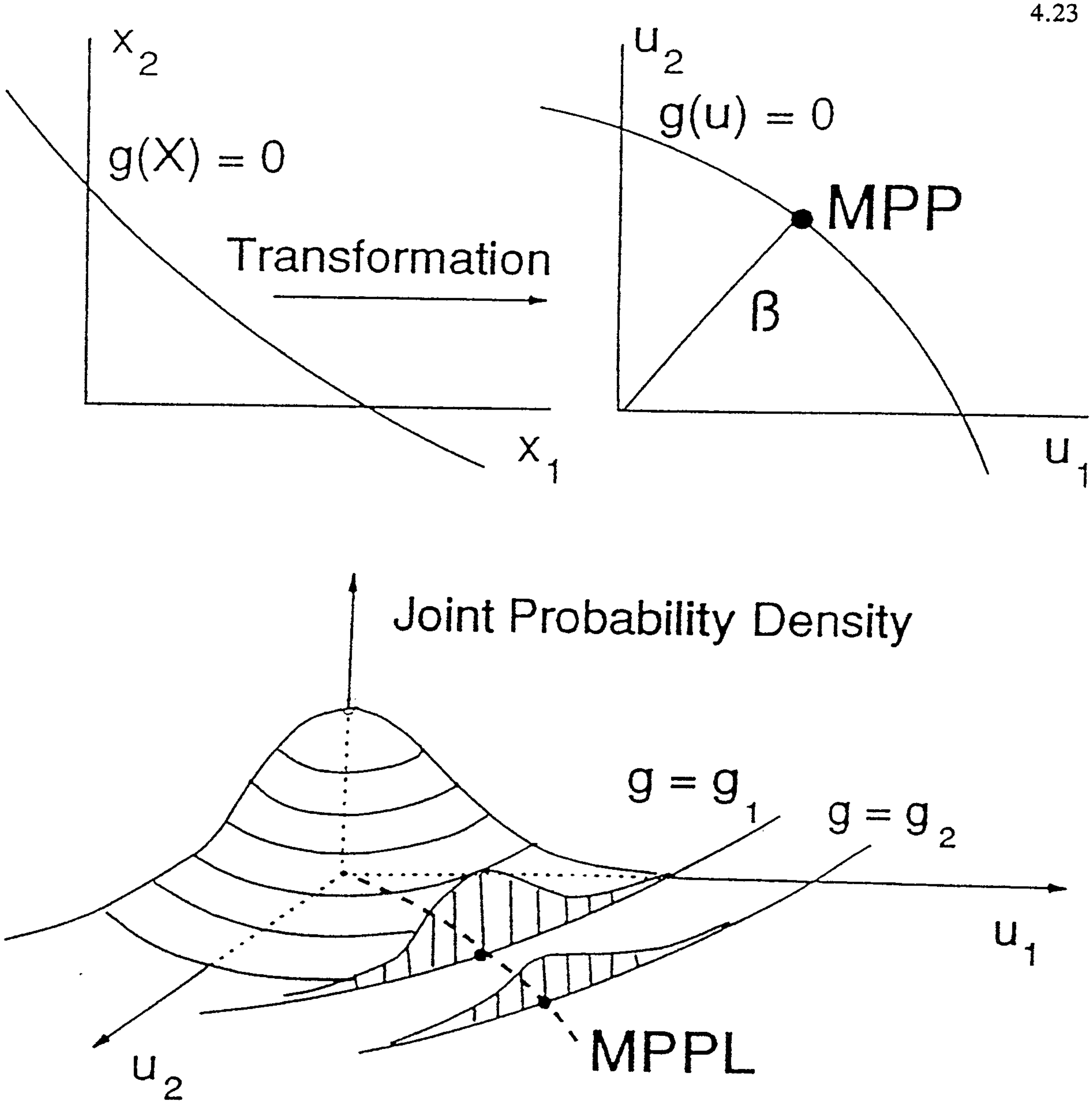

FIGURE 4.5: Illustration of the Transformation of Original Basic Variables (x-space) to Standard Normal Variables (u-space)

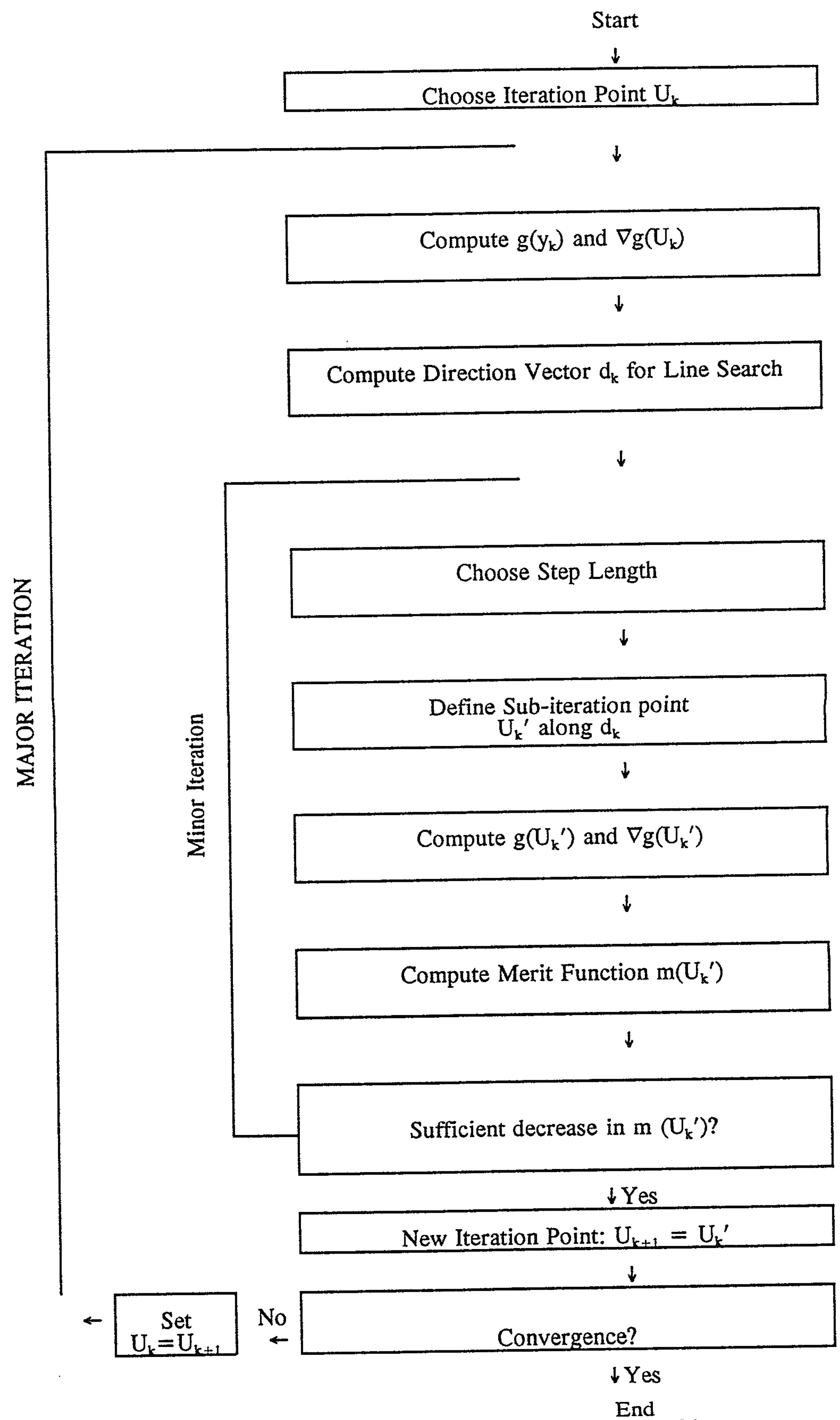

FIGURE 4.6: Illustration of the MHL-RF Optimization Algorithm

5. VERIFICATION PROBLEMS FOR PROGRAMS RANFLD AND STOVAST

5.1 Random Field Discretization for a Stochastic Beam

Consider a stochastic beam of length L whose Young's modulus is best modelled as a Gaussian random field, with a correlation function:

$$\rho(x_i, x_j) = \exp\left(-\frac{\Delta x}{\lambda L}\right), \tag{5.1}$$

where Δx is the distance between points x_i and x_j, and λ is a dimensionless correlation parameter. The random field has a mean value (RMV) of 2.90E+04 ksi and a standard deviation (STD) of 1.74E+03 ksi. It may be recalled (see Chapter 4) that Equation (5.1) corresponds to the exponential correlation function model with a correlation distance $d = \lambda L$. For a value of $\lambda = 0.25$ used by Mahadevan [36] and a beam length of 192", this gives a correlation distance d = CORD = 48" for the preparation of the preparation of the PREFX.RFD file. Program VASGEN [55] was used to prepare the stochastic finite element mesh used for the random field discretization.

The output of program RANFLD (PREFX.LPD file) for this problem is shown in Table 5.1, where it may be noted that the midpoint method was used. The variable names for the four random variables corresponding to the four SFEMs are given in the output file as YM-G#001, YM-G#002, YM-G#003, YM-G#004, respectively. These names indicate that in the SFEA that may later be required, all deterministic finite elements within SFEM no. 1 are best grouped together as the first element group (hence the designations G#001, etc.). The YM is a reminder that this random variable represents a Young's modulus.

The correlation coefficient matrix is printed row-wise as noted and shown in Table 5.1. These results agree, of course, with those of Mahadevan [36]. Only the upper triangular portion is printed because a correlation matrix is symmetric. Further down the table, eigenvalues and eigenvectors of the covariance matrix (not correlation matrix) are also printed in descending order of the magnitude of the eigenvalues.

5.2 Random Field Discretization of a Stochastic Plate

The second random field discretization example is a stochastic rectangular plate. The Young's modulus of the plate is modelled as a homogeneous, lognormal field with a mean value of 1.0E+06 units and coefficient of variation of 10%. The autocorrelation coefficient function is assumed to have the isotropic form:

$$\rho(\Delta x) = \exp\left(\frac{\Delta x^T \Delta x}{(0.25L)^2}\right), \tag{5.2}$$

where L is the width of the plate (=16 units). Equation (5.2) represents the Gaussian correlation model with a correlation distance of 4.0. This random field is discretized using a mesh of 64 stochastic finite elements as shown in Figure 5.1. (This utilized the VAST four-noded quadrilateral shell element (IEC=5).) The result includes a correlation matrix (64 x 64) written into the binary output file used in the stochastic finite element analysis problem described under Section 5.4.

5.3 SFEM Reliability Analysis of a Stochastic Beam

The stochastic beam shown in Figure 5.2 was first analyzed for the verification of program STOVAST. The beam has a random Young's modulus, cross-sectional area and second moment of area and is subjected to a random field load. The random field has properties earlier described in Section 5.1 above. The beam was first analysed using a single stochastic finite element to discretize the load field. This is the same as considering the random field as a random variable (WLOAD). Table 5.2(a) gives a summary of the basic variables involved in this problem. The cross-sectional area is Gaussian with a mean value of 7.68 in^2 and a coefficient of variation (COV) of 5%, the second moment of area I_z is also normally distributed with a COV of 5%. The Gaussian load has a mean value of 0.08 k/in and a COV of 20%. The stochastic finite element reliability analysis is conducted with reference to the limit state that the vertical deflection at the free end of the beam exceeds 0.5% of the beam length. This translates to a limit state function

$$g = u_{thr} - u_5^y \tag{5.3}$$

where u_{thr} (= 0.5% of L) is the displacement threshold and u_5^y denotes the vertical displacement at node 5 of the structural finite element mesh. The results shown in Table 5.2b, indicate a reliability index, β, of 1.8627 and a failure probability of 0.31253E-01. Importance factors are also shown in Table 5.2b. These answers are exact (as demonstrated below) indicating the STOVAST solution is perfect. Four finite elements were used to model the beam.

The closed-form expression for the tip deflection of a cantilever beam of length L subjected to a uniform distributed load w unit length is given by:

$$u^y = \frac{WL^4}{8EI}, \tag{5.4}$$

where E is the Young's modulus and I is the second moment of area. A reliability analysis was earlier conducted for this beam using COMPASS based on the same limit state condition defined in Equation (5.3) but with the tip displacement computed using the explicit representation in Equation (5.4). The results are exactly the same as those summarized above and detailed in Table 5.2(b) for both β and the probabilistic sensitivity factors. A detailed STOVAST output for the 4-SFEM case is shown in Appendix B1.

This STOVAST result not only illustrated the successful solution of a stochastic finite element problem but demonstrates two important points in the development and application of SFEM procedures. The first point is that if accurate (deterministic) FEA solutions can be found (as was the case for this beam problem), then for well-behaved limit state functions such as the present one, very accurate reliability estimates can be expected. The other notable point is the power and utility of response surface methods. Equation (5.4) is a response surface representation of the finite element random variable u_5^y - i.e. the vertical displacement of node 5. Thus, for more complicated models, if reliable (i.e. reasonably accurate) response surfaces can be constructed in the neighbourhood of critical structural locations, then reliability analysis results obtained can be depended upon.

5.4 SFEM Reliability Analysis of a Stochastic Plate

The stochastic plate described in Section 5.2 was also analyzed for STOVAST verification. The plate with a lognormal homogeneous random field Young's modulus is subjected to two random point loads in the global x and y directions [64]. The loads have mean values: μ_x and μ_y of 4,000 and 1,000 units, respectively and COVs of 10% each. Both loads are assumed to be normally distributed but statistically independent. The Poisson ratio is assumed to be deterministic (i.e. fixed) with a value of 0.2. The failure criterion examined was the exceedance of the horizontal displacement at the point of load application above a threshold of 0.03 units.

Three analyses were conducted for this problem, all utilizing a mechanical FE mesh of 64 quadrilateral shell elements. The first analysis considered the Young's modulus random field as a single random variable and yielded a reliability index, β, of 1.7714 with a corresponding failure probability of 3.825%. A detailed printout of STOVAST output file, PREFX.LPT for this analysis is provided in Appendix B2. This output contains the description of the input random variables, finite element input data, intermediate FE reliability iteration results for both structural response, loads, and response gradients, and the reliability analysis results. (Please note that some of the output data were printed out to monitor the convergence performance of the FPI algorithms and a provision would be made in future (i.e. release) versions to suppress some of the output information.)

The second analysis was conducted to check the robustness of the probability transformation scheme for highly correlated random variables. To achieve this, the random field was discretized using 64 SFEMs but assuming perfect correlation structure (i.e. ρ_{ij} = 1.0) between the resulting random variables. This correlation option is the null correlation choice in the RANFLD correlation function library and may be selected by setting the parameter ICOREM (see Appendix A1) to zero. This analysis gave exactly the same reliability results (as it should) as obtained in the first analysis, demonstrating that RELAMM can adequately handle very closely correlated (in this case perfectly correlated) random variables.

The third analysis conducted for this problem used a 64 SFEM mesh to discretize the Young's modulus random field but this time utilizing the actual correlation function model described in Section 5.2, that is Equation (5.2). This analysis yielded a reliability index of β = 2.1139 or P_f = 1.726%. We note, as expected, that the value of the reliability index has gone up with reference to the other analyses. This is because the single random variable model of the random field over-represents the variability in the parameter and hence a higher failure probability. This last analysis also demonstrates that random variable models of parameters that are truly random fields give very conservative estimates of safety. The example also underlines the importance of random field discretization capabilities and the merit of the SFEM approach itself. All the above problems were solved using both the HL-RF and MHL-RF algorithms.

5.5 SFEM Reliability Analysis of a Perforated Stochastic Plate

Figure 5.3 shows a quarter finite element model of a stochastic plate with a circular hole. The square plate is of dimensions 24" x 24" with a central hole of radius 2". The plate is subjected to a random distributed load at the edges. This load is assumed to be normally distributed with a mean value of (2 ksi and a cov of 20%). The Young's modulus of the plate is considered to be a homogeneous random field with a mean value of 0.290E+05 ksi and a cov of 10%. The correlation of the field is assumed to be Gaussian, with a correlation distance of 6".

The discretization (mechanical and stochastic) was performed with the four-noded quadrilateral shell element as shown in Figure 5.3. The four bold elements denote the stochastic finite elements used for random field discretization, while the 24 lighter elements denote the mechanical finite elements used for discretizing the continuum structure.

Stochastic finite element reliability analysis of the plate is conducted with reference to the displacement limit state function:

$$g(x) = u_{thr} - u_5^x \qquad (5.5)$$

where u_{thr} is the displacement threshold of 0.600E-02" and u_5^x denotes the horizontal displacement at finite element node number 20 (see Figure 5.3) of the plate. The reliability analysis results are:

5.6

β	=	2.5746
P_f	=	0.50183E-02

The importance factors are 27.71% for the combined random variables describing the Young's modulus and 72.29% for the random load.

TABLE 5.1: RANFLD Output for Stochastic Beam Problem

```
***   R A N D O M    F I E L D    D I S C R E T I Z A T I O N   ***

PREFIX DEFINING SFEM-REL. ANALYSIS FILES :  WALS8
 STOCHASTIC BEAM WITH RANDOM YOUNGS MODULUS
TOTAL NO. OF RANDOM FIELDS -        1
NO. OF SFEM MESHES USED    -        1

*** DISCRETIZATION OF RANDOM FIELD NO.     1
--------------------------------------------
ICATFL -   1      ITYPE -   1
RANDOM VARIABLE CLASSIFICATION : KRVT -   2
ICDF   -   2
RMV -     0.290000E+05      STD -     0.174000E+04
ICOREM -      1     IRHOSP -      1     CORD -  0.480E-02
PREFIX DEFINING SFEM MESH FOR THIS FIELD :  WALS8
CENTROIDAL COORDS. OF SFEMs COMPUTED AUTOMATICALLY
 CORRELATION COEFFICIENT MATRIX PRINTED ROWWISE
 NOTE : ONLY UPPER TRIANGULAR PORTION IS PRINTED

 VARIABLE NUMBER :      1
 VARIABLE NAME   : YM-G#001
ROW #      1       HAS        4      MEMBERS
  0.100000E+01   0.367879E+00   0.135335E+00   0.497871E-01
 VARIABLE NUMBER :      2
 VARIABLE NAME   : YM-G#002
ROW #      2       HAS        3      MEMBERS
  0.100000E+01   0.367879E+00   0.135335E+00
 VARIABLE NUMBER :      3
 VARIABLE NAME   : YM-G#003
ROW #      3       HAS        2      MEMBERS
  0.100000E+01   0.367879E+00
 VARIABLE NUMBER :      4
 VARIABLE NAME   : YM-G#004
ROW #      4       HAS        1      MEMBERS

  0.100000E+01

EIGENVALUES AND EIGENVECTORS OF COVARIANCE MATRIX :
 ***  PROBABILISTIC MODE NO. ***   :       1
      EIGENVALUE -     0.525768E+07
      CORRESPONDING EIGENVECTOR :

            0.417922E+00
            0.570387E+00
            0.570387E+00
            0.417922E+00
 ***  PROBABILISTIC MODE NO. ***   :       2
      EIGENVALUE -     0.324831E+07
      CORRESPONDING EIGENVECTOR :

           -0.625406E+00
           -0.329950E+00
            0.329950E+00
            0.625406E+00
 ***  PROBABILISTIC MODE NO. ***   :       3
      EIGENVALUE -     0.206205E+07
      CORRESPONDING EIGENVECTOR :

           -0.570387E+00
            0.417922E+00
            0.417922E+00
           -0.570387E+00
 ***  PROBABILISTIC MODE NO. ***   :       4
      EIGENVALUE -     0.154237E+07
      CORRESPONDING EIGENVECTOR :

            0.329950E+00
           -0.625406E+00
            0.625406E+00
           -0.329950E+00
```

TABLE 5.2a: Random Variables Describing the Stochastic Beam Problem (RELAMM is the Reliability Analysis Module of STOVAST)

RELAMM VARIABLE LISTING

NO.	NAME	DISTR.TYPE	MEAN	STAND.DEV.	DISTR.PAR.
1	YMOD	NORMAL	0.2900E+05	0.1740E+04	0.2900E+05 0.1740E+04
2	AREA	NORMAL	0.7680E+01	0.3840E+00	0.7680E+01 0.3840E+00
3	AIZ	NORMAL	0.3010E+03	0.1505E+02	0.3010E+03 0.1505E+02
4	PR	FIXED	0.3000E+00		0.3000E+00
5	DEN	FIXED	0.1000E+01		0.1000E+01
6	ZERO	FIXED	0.0000E+00		0.0000E+00
7	WLOAD	NORMAL	0.8000E-01	0.1600E-01	0.8000E-01 0.1600E-01
8	UTHR	FIXED	0.9600E+00		0.9600E+00

TABLE 5.2b: Finite Element Reliability Analysis Results for Stochastic Beam Problem

	COMPASS Solution (Analytical Limit State Function)	STOVAST (FERM) Solution With [1 SFEM for Random Field]	STOVAST (FERM) Solution With [4 SFEMs for Random Field]
β	1.8627	1.8627	2.3887
P_f	0.31253 E-01	0.31253 E-01	0.84537 E-02
E^{PIF}	3.32%	3.32%	5.56%
I_Z^{PIF}	2.33%	2.33%	3.93%
W^{PIF}	94.34%	94.35%	90.52% (i.e. sum of all the W_{SFEM} PIFS)
W_{SFEM-1}^{PIF}	N/A	N/A	0.26%
W_{SFEM-2}^{PIF}	N/A	N/A	63.18%
W_{SFEM-3}^{PIF}	N/A	N/A	2.01%
W_{SFEM-4}^{PIF}	N/A	N/A	25.07%

Note: PIF denotes probabilistic importance factors of random variables.

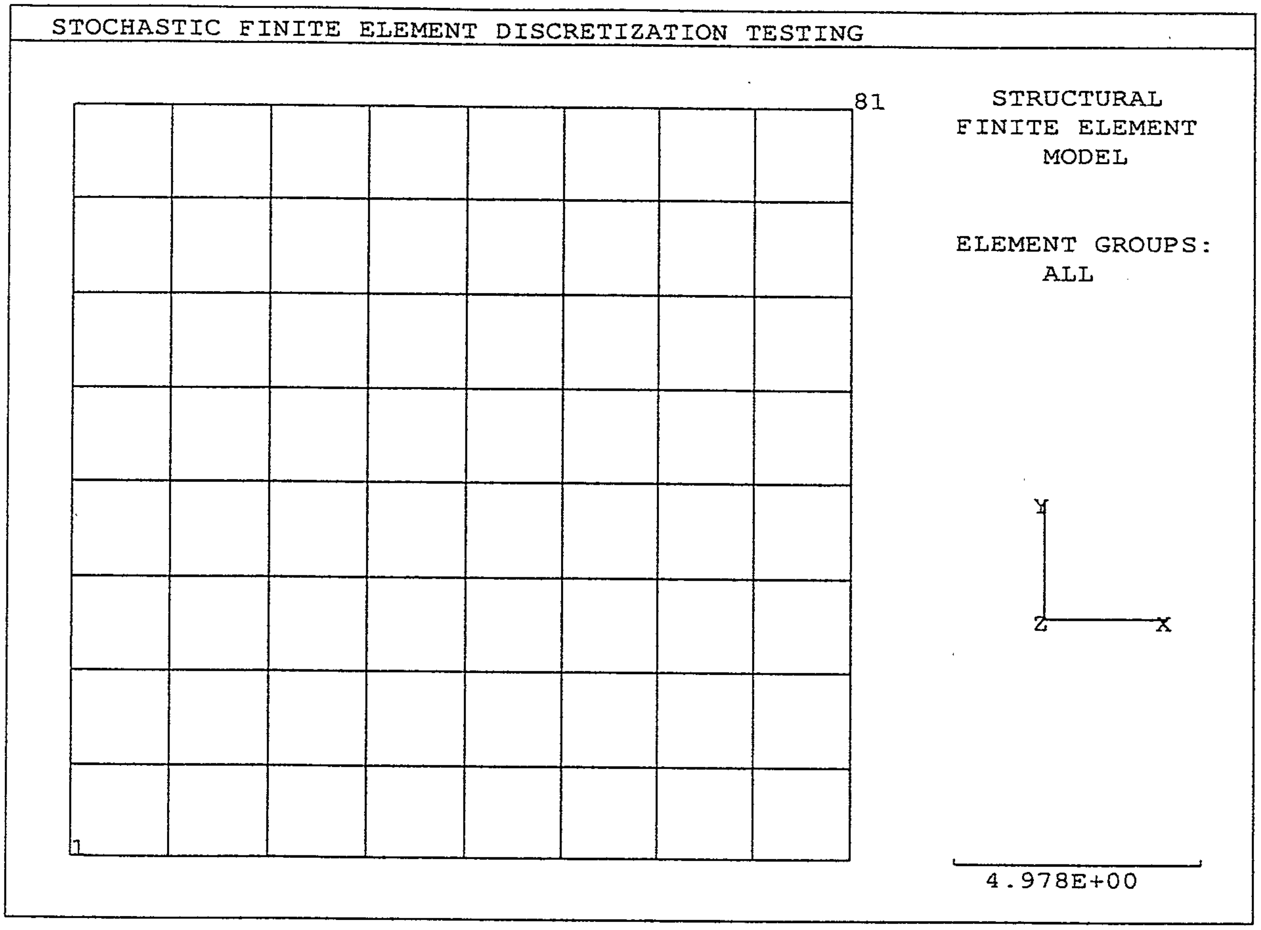

FIGURE 5.1: Random Field Discretization of a Stochastic Rectangular Plate

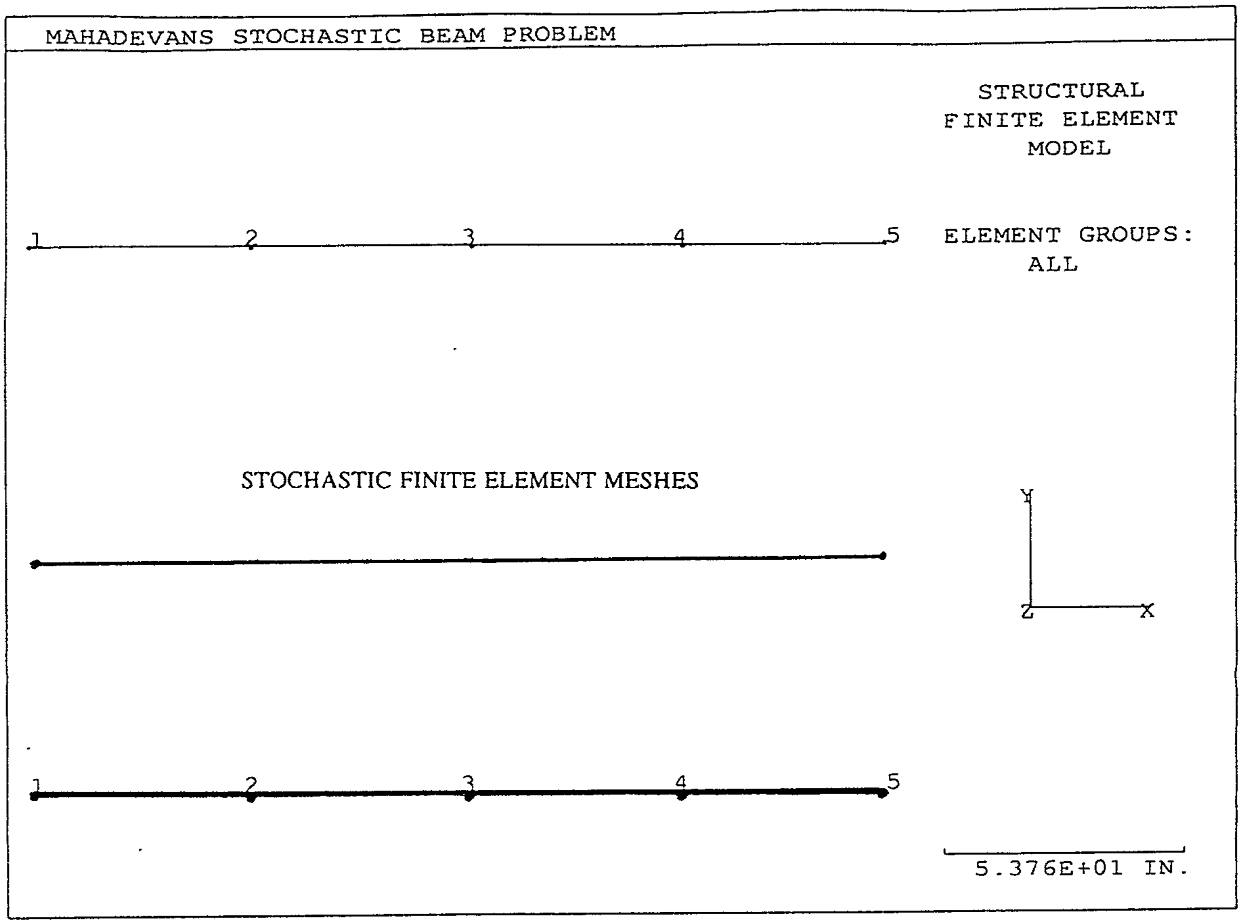

FIGURE 5.2: Structural and Stochastic Finite Element Meshes for a Stochastic Beam

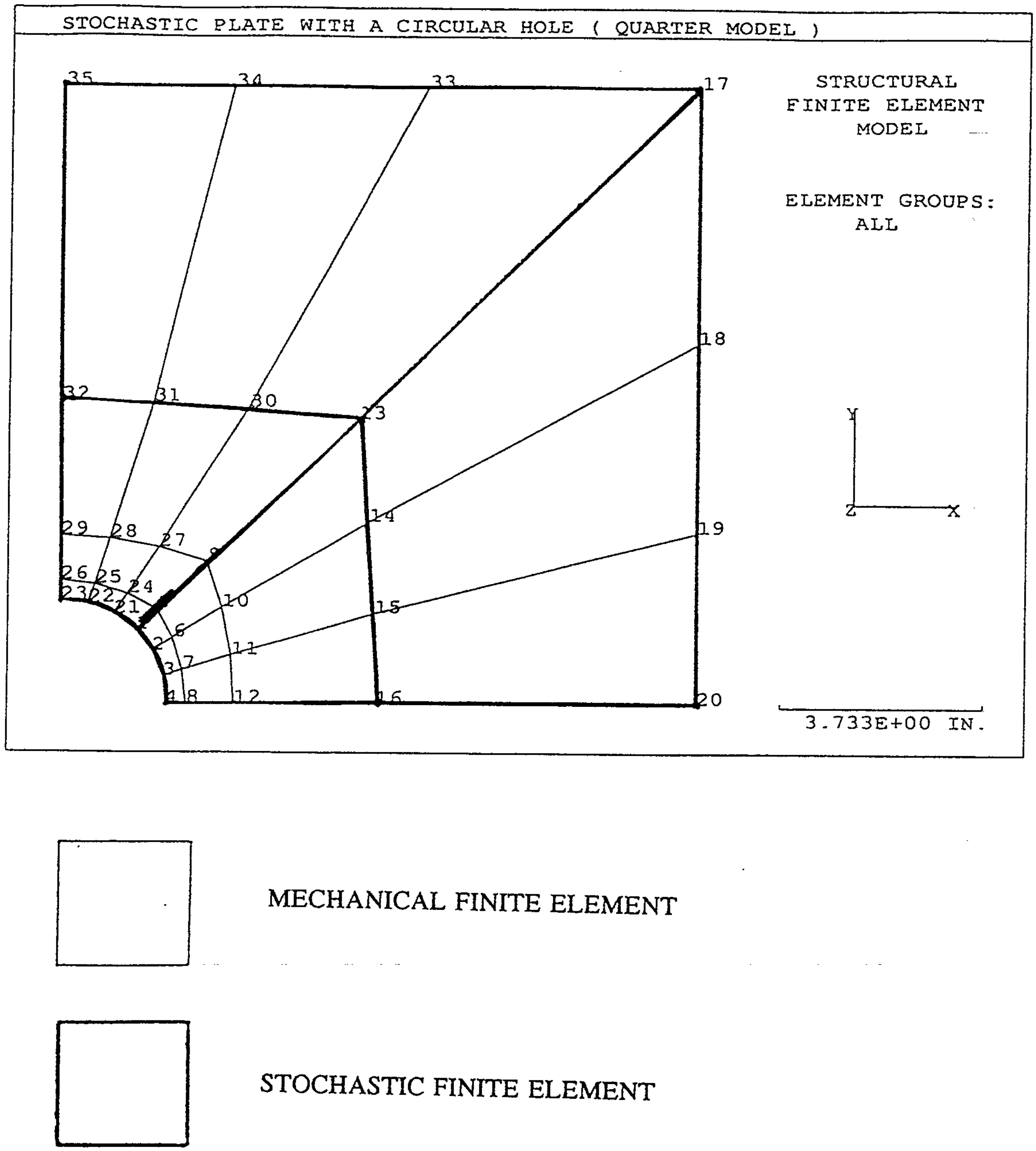

FIGURE 5.3: Mechanical and Stochastic Finite Element Meshes of a Stochastic Plate

6. BIAS AND MODELLING ERRORS IN SFEM-BASED RELIABILITY ANALYSIS

6.1 Classification of Uncertainties

There are several possible sources of uncertainty when dealing with stochastic structures for which we have highly recommended the application of stochastic finite-element methodologies. These include inherent randomness, estimation errors, model imperfections and human error. Inherent randomness is usually due to natural variabilities in physical phenomena or processes. Examples of these include environmental loads imparted on marine structures by the inherently random ocean environment, material properties, member sizes and geometry. This class of uncertainties is usually referred to as Category I or aleatory uncertainties. The other class of uncertainties called Category II or epistemic uncertainties encompasses estimation errors, model imperfections, and human errors. They are generally due to lack of sufficient data, knowledge or understanding of a complex phenomenon (i.e. ignorance), or use of simplified models. Examples of this latter category include errors in analysis results due to inadequate analytical procedures for load/response computation and human errors induced during finite element model generation or other input descriptions.

There exists a fundamental distinction between aleatory and epistemic uncertainties. Whereas inherent randomness is intrinsic to nature and usually beyond the engineer's control, epistemic uncertainties are extrinsic and to some extent reducible. Reducibility in the latter class stems from the fact that an analyst may choose to obtain additional information to improve the accuracy of the estimation procedures. This may be in the form of gathering more data for more accurate probabilistic characterization of governing engineering parameters, the execution of appropriate experiments to gain a better understanding of the relevant phenomena, or the development and application of more sophisticated models for analysis. Uncertainty due to human error may also be reduced by implementing rigorous quality control measures in the analysis process [65]. It is pertinent to point out that the avenues for reducing epistemic uncertainties are associated with costs in terms of money and time. However, epistemic uncertainties are information sensitive, unlike aleatory uncertainties.

6.2 Quantification of Uncertainties

Two important measures of uncertainties are the bias factor and the coefficient of variation (COV). The bias factor for a parameter x with a true value x_t is defined as:

$$B = \frac{x_t}{x_p}, \tag{6.1}$$

where x_p is the predicted value of the parameter. This parameter measures the systematic error in the prediction of the average value of the desired quantity. The bias factor is often applied constantly to predicted values and may be considered to be a random variable. The COV, δ, is the ratio of the standard deviation, σ_x, to the mean value, $\bar{x}$, and relates to the standard error pertaining to the statistical imperfection in the estimated control value (i.e. mean or median):

$$\delta = \frac{\sigma_x}{\bar{x}} \tag{6.2}$$

The credibility of uncertainty measures is important for the obvious reason that the validity of calculated structural responses or failure probabilities depend on these measures.

6.3 Analysis of Uncertainties

The bias factor corrects predicted results to give:

$$x_t = Bx_p \tag{6.3}$$

This model is popularly referred to as the Ang-Cornell model [66]. Ditlevsen's model [67] has a more general representation of the form:

$$x_t = Bx_p + b \tag{6.4}$$

where b is a Gaussian random variable which is statistically independent of x. The Ang-Cornell model is, nevertheless, often preferred in the propagation of uncertainties because it is simpler and requires less information in order to determine the statistics of its parameters. The Ang-Cornell model is particularly suitable for lognormal random variables, especially when expressed as a product of several other variables. Detailed illustrations and applications of this application can be found in the works of Nikolaidis and Kaplan [68] and Bea [69]. However, the basic principle for propagating uncertainties using the first-order second moment method is summarized below following the work of Bea [69].

For the addition or subtraction of two normal random variables, $(X \pm Y) = Z$, the mean of the resultant distribution can be calculated as:

$$\overline{Z} = \overline{X} \pm \overline{Y} \tag{6.5}$$

The standard deviation of the resultant distribution can be calculated as:

$$\sigma_Z = \sqrt{\sigma_X^2 + \sigma_Y^2 \pm 2\rho\,\sigma_x\sigma_Y} \tag{6.6}$$

For the multiplication of two random variables, $(X\ Y) = Z$, the mean of the resultant distribution can be calculated as:

$$\overline{Z} = \overline{X}\ \overline{Y} \tag{6.7}$$

The standard deviation of the resultant distribution can be calculated as:

$$\sigma_Z = \overline{X}\ \overline{Y}\sqrt{(1+\rho^2)\left[V_X^2 + V_Y^2 + \left(V_X^2 V_Y^2\right)\right]} \tag{6.8}$$

for $\rho = 0$.

$$V_Z = \sqrt{V_X^2 + V_Y^2 + \left(V_X^2 V_Y^2\right)} \tag{6.9}$$

For the division of two random variables, $(X / Y) = Z$, the mean of the resultant distribution can be calculated as:

$$\overline{Z} = \frac{\overline{X}}{\overline{Y}} \tag{6.10}$$

The standard deviation of the resultant distribution can be calculated as:

$$\sigma_Z = (\overline{X}^2/\overline{Y}^2)\sqrt{V_X^2 + V_Y^2 - 2\rho(V_X V_Y)} \tag{6.11}$$

or for $\rho = 0$

$$V_Z = \sqrt{\frac{V_X^2 + C_Y^2}{1 + V_Y^2}} \tag{6.12}$$

When the random variables X and Y can be considered independent ($\rho=0$), and V_X and V_Y are small ($V << 1$), then for either the multiplication or quotient of two random variables:

$$V_Z \approx \sqrt{V_X^2 + V_Y^2} \tag{6.13}$$

For the square of X ($Z = X^2$):

$$\overline{Z} = \overline{X}^2 + \sigma_X^2$$
$$\sigma_Z = \sqrt{4\ \overline{X}^2\ \sigma_Z^2 + 2\ \sigma_Z^4} \tag{6.14}$$

More generally, if $Z = X\ Y^\alpha$

$$\overline{Z} = \overline{X}\ \overline{Y}^\alpha$$
$$\sigma_Z = \sqrt{\sigma_X^2 + \alpha\sigma Y^2} \tag{6.15}$$

For lognormal distributed variables (logarithms of the variables are normally distributed):

$$\sigma_X = \sigma_{lnX} \text{ and } \overline{X} = X_{50} \exp\ (0.5\ \sigma_{lnX}^2) \tag{6.16}$$

and relations similar to the above can be utilized for propagating uncertainties.

A framework for incorporating model uncertainties in advanced reliability methods employing full distributional information was developed by Der Kiureghian [70]. This work, formulated on the Bayesian approach, derived simple formulas that show the effect of model uncertainty on the reliability index. The Der Kiureghian model is more general than both the Ang-Cornell and Ditlevsen models and permits the calculation of the variance of the reliability index to reflect the influence of modelling errors or uncertainties.

In our opinion, the Der Kiureghian model is the most realistic vehicle for accounting for bias and modelling uncertainties in SFEM-based structural reliability analysis. This is for the obvious reason that stochastic finite element analysis is a sophisticated process that is difficult to express in simple analytical forms such as are required by the Ang-Cornell and Ditlevsen models.

6.4 Sources of Bias and Modelling Errors

The sources of bias and modelling errors in SFEM-based structural reliability analysis include the following:

i. Data collection/characterization for material properties and structural geometry;

ii. Probabilistic description of random fields, i.e. whether the most suitable probability distribution function or correlation function has been used to define the field. Errors in these can be reduced by collecting more data and by validation of the stochastic character of the field. Validation can be accomplished by the reverse process of generating Monte Carlo simulations of the random field and comparing these to measured values of this field at selected locations;

iii. Discretization of random fields. Some probabilistic information is lost when the field over a domain is represented by pointwise values (i.e. random variables);

iv. Probability transformation scheme. The transformation of generally correlated variables to standard normal (statistically independent variables) involve approximations, and hence some information is also lost in this process;

v. Finite element model generation. Discretization errors are incurred in the process of converting a continuum structure into several discrete elements;

vi. Finite element formulation. These are errors due to inappropriate use of finite elements or boundary conditions. This may also be due to inadequate modelling of the physical phenomena, for example, using linear instead of nonlinear finite element analysis, failing to utilize inappropriate constitutive models, or failing to account for interaction of certain phenomena (eg. thermomechanical coupling in processes involving large inelastic deformations);

vii. Calculation of FE structural response such as displacements and stresses. These are the solution errors in FEA which may sometimes be significant (eg. in nonlinear FE analysis);

viii. Calculation of gradients of the structural response;

ix. Calculation of the failure probability (and hence the reliability). These calculations usually employ approximate procedures which introduce errors in the computed reliability indices;

x. Mathematical definition of the failure conditions (i.e. limit states) of interest;

xi. Definition of loads; and

xii. Human error.

For marine structures, Nikolaidis and Kaplan conducted [68] a study dealing with uncertainties in stress analysis on behalf of the U.S. Ship Structure Committee. The study dealt mainly with load characterization and response computation, but did not deal with reliability estimation. The above list, although generic in form, is therefore, more comprehensive as the scope of stochastic finite element analysis is much broader than the focus of the study in reference [68].

As we pointed out in the preceding subsection, the work of Der Kiureghian [70] is recommended as a framework for realistically accounting for model uncertainties in SFEA. Another strategy that is worthy of consideration is the use of confidence bounds on the computed values of failure probabilities (or reliability indices). This is clearly an area that would benefit from further research investigations.

7. CONCLUSIONS AND RECOMMENDATIONS

A research and development program for the application of stochastic finite element-based reliability analysis has been successfully executed in this work. The detailed state-of-the-art literature review of various SFEM methodologies permitted a solid background for the evaluation of the different strategies and substrategies pertaining to the diverse technical requirements of a probabilistic finite element analysis. Case studies of the CALREL-FEAP system and the NESSUS system permitted practical evaluation of some of the available strategies.

The random field modelling capabilities implemented in RANFLD are the most versatile because they are applicable to non-Gaussian fields and have no restrictions on the geometrical shape of the stochastic finite element meshes. A respectable library of correlation models were made available to permit easy applications. Furthermore, the RANFLD strategy does not suffer from numerical stability problems that may result in the reliability analysis module because the correlation coefficient matrix is always positive definite.

The finite element reliability method (FERM) implemented in program STOVAST under this contract was demonstrated to efficiently give very accurate results. The verification problems solved in Chapter 5 demonstrate not only the overall proper operation of program STOVAST but its various algorithms such as those for gradient computations, fast probability integration, and sensitivity computations. Also very important, program STOVAST was designed to permit easy extension of its capabilities either in terms of additional reliability analysis procedures or the implementation of new SFEM strategies.

The sources of bias/modelling errors and uncertainties in the application of SFEM-based structural reliability analysis were also examined during this work. Available procedures for propagating uncertainties in engineering analysis were briefly reviewed and some suggestions were made on a suitable approach for the SFEM process.

The verification of program STOVAST and the research studies conducted in this work have produced very encouraging results. Based on this observation, and noting the unique promise of the stochastic finite element method as the most viable strategy for realistically accounting for the various uncertainties present in ship structural analysis, it is recommended that further work be pursued.

First, it is desirable to consolidate, expand, and further verify the existing capabilities of program STOVAST. This will include the addition of more elements to the stochastic finite element library and the upgrading of the routines in both RANFLD and STOVAST for compatibility with the nonlinear version of VAST (Version 7). The capability for modelling random nodal coordinates was not implemented because of the additional complications in the calculation of the partial derivative of load vectors and element matrices. This capability should be provided in future work so that a full stochastic structural geometry representation of the mechanical finite elements will be permitted. A more fully automated facility for the creation of modified (STOVAST) PREFX.GOM files from results of random field discretization and the conventional VAST geometry files will also be useful.

Further enhancements to the reliability analysis algorithms would also be beneficial. We believe the development of the Nataf model for the probability transformation from x-space to u-space will provide a more robust platform. An additional optimization algorithm suitable for locating the MPP for large scale problems should be implemented. This will further complement the existing HL-RF and MHL-RF algorithms. Also, information on probabilistic sensitivity factors could be used to speed up the reliability iterations.

The second area that would require further work is the development of a new (alternative) SFEM strategy. Some advanced algorithmic concepts which have been developed, tested, and (in some cases) commercialized by other leading researchers should be seriously considered for implementation in STOVAST.

Recent and current DREA research efforts in the area of probabilistic mechanics in general and in the subject of SFEM-based reliability analysis in particular represent novel attempts to apply this new technology to ships. However, several technical issues that are unique to the size scale, operational environment, and specialized architecture of ship structures need to be researched. Included in this last are topics such as various procedures for the optimization of computational efficiency, efficient implementation of ship-specific engineering failure modes (both serviceability and ultimate) and development of time-dependent reliability models.

Overall, we believe the execution of this project is a major step towards the practical application of SFEM-based reliability analysis to ship structural integrity assessment. It is hoped that the scientific authority continues to maintain interest and give the dedicated commitment and support for the vigorous pursuit of this laudable objective.

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APPENDIX A1

INPUT FORMAT FOR THE RANDOM FIELD DISCRETIZATION PROGRAM: RANFLD

FORMAT OF RANFLD INPUT DATA: PREFX.RFD

Card 1 (A72)

TITLE = Title describing the random field problem

Card 2 (I5)

NRFLDS = Number of random fields involved in the problem

Card 3 (I5)

NMESH = Number of stochastic finite element meshes used

Cards 4-11 are to be provided NRFLDS times.

Card 4 (A8)

RANFIELD = Eight character literal constant as header for the beginning of a random field description

Card 5 (3I5)

ICATFL = Category of field
= 1 for material property random field
= 2 for distributed random load field
= 3 for random structural geometry*
= 4 for random boundary conditions*

Note: * not presently available

ITYPE = 1 for Young's modulus random field
= 2 for Poisson's ratio random field
= 3 for mass density random field
= 4 for any other material property type

ISORT = 0 not to sort
= 1 to sort eigenvalues

(parameter for classifying material property random fields)

Card 6 (I5)

ICDF = Marginal cumulative distribution function (CDF) defining the probabilistic character of the random field

ICDF Values Corresponding to Various Probability Distributions	
ICDF Value	Probability Distribution
1	Fixed-valued variable
2	Normal
3	One-sided normal
4	Lognormal
5	Uniform
6	Triangular
7	Exponential
8	Beta
9	Chi-square
10	Gamma
11	Gumbel (EVD-Type I)
12	Maxwell (not presently available)
13	Rayleigh
14	Student-t
15	Weibull
16	Evd Type II maximum (Frêchet)
17	EVD Type III maximum
Note: EVD = Extreme Value Distribution	

Card 7 (2E10.3)

RMV	=	Mean value of the field
STD	=	Standard deviation of the field

Card 8 (2I5, E10.3)

ICOREM	=	Correlation function type for this random field
	= 0	for fully correlated field
	= 1	for exponential correlation
	= 2	for Gaussian correlation

= 3 for triangular correlation
= 4 for second-order autoregessive process correlation

IRHOPS = Parameter that defines the space in which the correlation coefficient function is defined
= 1 for original space of basic random variables (x-space)
= 2 for transformed standard normal space (u-space)

CORD = Correlation distance characterizing the random field

Card 8 (AS, I5)

PREFX = Five character prefix describing the SFEM mesh (i.e. geometry file) to be used for discretizing this field

IPRF = 0 no physical geometry file available for describing this SFEM mesh
= 1 PREFX.GOM file to be used for the SFEM mesh (i.e. identical to mechanical FE mesh)
= 2 PREFX.GOM file to be used for the SFEM mesh

Card 9 (2I5)

ICENT = 0 for automatic computation of the centroidal coordinates the SFEMs
= 1 for user-supplied centroidal node number
= 2 for user-supplied global coordinates of the centroid or some preferred location for discretization

NSFEM = Number of stochastic finite elements

Card 10 to be provided NSFEM times.

Card 10a (I6I5) (provide only if IPRF=2) (omit if ICENT=0 and IPRF=1)

JSFE = SFEM number

NFE = Number of mechanical finite elements in this stochastic finite element

JGRP = Element group number of the mechanical finite elements involved

JFEM(k) (k=1,2, ...,NFE) = Element numbers for the (NFE) mechanical finite elements

Card 10b (16I5) (provide only if IPRF=0)

JSFE = SFEM number

NNODES	=	Number of nodes defining the boundary of the stochastic finite element
NBOUND(k) (k=1,2, ..., NNODES)	=	Node numbers defining the NNODES boundary nodes

Card 11a (I5) (provide only if ICENT=1)

INODE	=	Node number to be used as discretization reference point (representing a SFEM centroid or some other preferred location within the SFEM)

Card 11b (3E10.3) (provide only if ICENT=2)

XC(I)	=	Global x-coordinate of discretization point
YC(I)	=	Global y-coordinate of discretization point
ZC(I)	=	global z-coordinate of discretization point

Note: as in Card 11a, the discretization point is a point representing a SFEM centroid or some other preferred location within the SFEM.

APPENDIX A2

INPUT FORMAT FOR THE SFEM-BASED RELIABILITY ANALYSIS PROGRAM: STOVAST

INPUT FORMAT OF PROGRAM STOVAST: PREFX.SFE

Card 1 (A72)

TITLE	=	Master title of the SFEM-based reliability analysis

Card 2 (7I5)

Seven parameters for defining the master control codes:

IRANFLD	= 0	No random field definition or discretization involved in this analysis
	= 1	Random field discretization involved in this analysis
IVBLDFN	= 0	No definition of random variables required other than those generated from random fields
	= 1	Random variables are defined in addition to those generated from random fields
IRESPNS	= 1	Limit state functions (LSF) or response functions defined in terms of VAST finite element analysis
	= 2	LSF defined via RESLIB library
ISYSDEF*	= 0	No systems defined in this analysis
	= 1	Some systems are defined for analysis
ICOMREL	= 1	Component reliability analysis via FORM
ISENSTY	= 0	Probabilistic sensitivities not of interest
	= 1	Computation of probabilistic sensitivities of the reliability index or failure probability are of interest
ISYSREL*	= 0	No systems reliability analysis required
	= 1	Systems reliability analysis required

*Please note that these systems are not yet available in STOVAST. Provisions were made in the code to permit these definitions for easy extensions of STOVAST capabilities in future work.

Omit Card 3 if IRANFLD = 0

Card 3a (A7)

HEADR = Seven character literal constant 'IRANFLD'

Card 3b (I5)

IOPT = 1 Random variables generated from random fields are available in PREFX.RVB file

Omit Cards 4 and 5 if IVBLDFN=0

Card 4a (A7)

HEADR = Six character literal constant 'IVBLDFN'

Card 4b (I5)

NBRV = No. of basic random variables defined for this analysis

Provide Card 5 NBRV times)

Card 5a (A8)

VNAME = Variable name (maximum of eight characters)

Card 5b (2I5)

ICDF = Probability distribution function for this random variable

(Admissible values of ICDF are shown below)

ICDF Values Corresponding to Various Probability Distributions	
ICDF Value	Probability Distribution
1	Fixed-valued variable
2	Normal
3	One-sided normal
4	Lognormal
5	Uniform
6	Triangular
7	Exponential
8	Beta
9	Chi-square
10	Gamma
11	Gumbel (EVD-Type I)
12	Maxwell (not presently available)
13	Rayleigh
14	Student-t
15	Weibull
16	Evd Type II maximum (Frêchet)
17	EVD Type III maximum
Note: EVD = Extreme Value Distribution	

KRVT
= 0 Fixed variable (i.e. deterministic constant)
= 1 FE geometrical nodal coordinates random variable
= 2 FE material property random variable (eg. E, ν, ρ)
= 3 FE size property random variable (eg. A, I, t)
= 4 FE distributed load parameter random variable
= 5 Fe concentrated load parameter random variable
= 6 Non-finite element threshold random variable
= 7 Any other type of random variable

Card 5c (I5)

IOPX = 1 Random variables defined in terms of mean values (μ), standard deviations (σ), and bounds

= 2 Random variables defined in terms of μ, coefficient of variation (COV), and bounds

= 3 Random variables defined in terms of distribution parameters

Card 5d (3E10.3) (required only if KRVT$\neq$0)

RMVL = Mean value (or other distribution parameter)

STDV = Standard deviation

BOUND = Bounds

Card 5e (E10.3) (required only if KRVT=0 or ICDF=1)

FIXVAL = Fixed value of the variable

Card 5f (2I5)

ICOREL = 0 No correlation between the basic random variables

= 1 Correlations between random variables defined in the original x-space

= 2 Correlations between random variables defined in the transformed standard normal u-space

ICOPT = 1 Correlation described in terms of the upper triangular portion of the correlation matrix

= 2 Correlation defined in terms of correlation pairs

Card 5g (omit if OCOREL=0 or ICOPT=2) (provide J times: J=1,2,...,NBRV)

Card 5g.1

MM = No. of terms in row number J of the correlation matrix starting from the diagonal term and going only up to the last non-zero term)

Card 5g.2 (free format)

AC(k) (k=1,...,mm)	=	Values for the current row of the correlation matrix

Card 5h (omit if OCOREL=0 or ICOPT=1)

Card 5h.1 (I5)

NCORPS	=	Number of correlation pairs

Card 5h.2 (free format) (provide NCORPS times)

KI	=	First variable number in the current correlated pair
KJ	=	Second variable number in this pair
RHO	=	Correlation coefficient value

Card 6a (A7)

HEADR	=	Seven character literal constant 'IRESPNS'

Card 6b (I5)

IOPT	= 1	For limit state (i.e. component) definition

Card 6c (I5)

NCOMPS	=	Total number of components to be defined in this problem

Card 6d to be provided NCOMPS times.

Card 6d (A20)

COMPNAME	=	Component name (maximum of 20 characters)

Card 6e (4I5)

NUMC	=	Identification number of the current component
LSFT	= 1	For displacement-type limit state function
	= 2	For stress-type limit state function
	= 3	For buckling limit states (not presently available)

NAVS	=	Number of additional random variables (i.e. in addition to those generated from random fields) to be used for defining this limit state
LTHR	=	Number of the random variables defining the limit state threshold

Card 6f (16I5)

JVN(k) (k=1,2, ..., NAVS)	=	Variable numbers of the NAVS random variables for the limit state

Card 6g (2I5)

LSDNN	=	Limit state node number
LSDOF	=	Limit state degree-of-freedom number for displacements or stress component number for stresses

Note: The convention used is as follows:

Displacements	Stresses
u = 1	$\sigma_{xx} = 1$
v = 2	$\sigma_{yy} = 2$
w = 3	$\sigma_{zz} = 3$
$\sigma_x = 4$	$\sigma_{xy} = 4$
$\sigma_y = 5$	$\sigma_{yz} = 5$
$\sigma_z = 6$	$\sigma_{zx} = 6$

Omit Card 7 for ISYSREL=0.

Card 7a (A7)

HEADR	=	Seven character literal constant ‘ISYSDEF’

Card 7b (I5)

NSYS	=	Number of systems defined in this analysis

Provide Card 7c NSYS times.

Card 7c.1 (A20)

SYSNAME	=	System name (maximum of 20 characters)
NCOMS	=	Number of components defining this system

Card 7c.2 (2I5)

ISYST	= 1	For series system
	= 2	For parallel system
JSYS(k) (k=1, NCOMS)	=	Component numbers of the components that define this system

Card 8a (A7)

HEADR	=	Seven character literal constant 'ICOMREL'

Card 8b (I5)

IOPT	= 1	Component reliability analysis based on the Rackwitz-Ang-Tang FORM algorithm (not presently active in STOVAST)
	= 2	Component reliability analysis based on the HL-RF FORM algorithm
	= 3	Component reliability analysis based on the MHL-RF FORM algorithm

Card 8b (free format)

TOL	=	Tolerance of the reliability index to be used for consequence
ITERMAX	=	Maximum number of iterations permitted

Card 8c (free format) (omit if IOPT=1 or 2)

CMERIT	=	Constant c in the definition of the merit function (MHL-RF algorithm).

Omit if ISENSXTY=0.

Card 9a (A7)

HEADR	=	Seven character literal constant 'ISENSTY'

Card 9b (free format)

IPP	= 1	Compute parametric sensitivities of β and P_f for distribution parameters only
	= 2	Compute sensitivities for mean values and standard deviations
	= 3	Compute 1 and 2 above

Omit if ISYSREL=0.

Card 10a

HEADR = Seven character literal constant 'ISYSREL'

Card 10b (I5)

NSAN = Number of systems to be analyzed

Cards 10 to 10e are to be provided NSAN times.

Card 10c (16I5)

NSYS(I) (I=1,2, ...,NSAN) = System numbers of selected systems to be analyzed

Card 10d (3I7)

IB1	= 1	Perform systems reliability analysis using the uni-modal bound technique
IB2	= 1	Compute relaxed bi-modal bounds
	= 2	Compute regular bi-modal bounds
	= 3	Compute improved bi-modal bounds
	= 4	Compute all of the above
IPNT	= 1	Compute PNET estimation of the failure probability for this system

Card 10e (E10.3) (omit if IPNET$\neq$1)

RHODEM = Demarcation correlation coefficient to be used in the PNET procedure

(Note that systems reliability analysis capabilities are currently not available in STOVAST. So the parameters ISYSDEF and ISYSREL should be set equal to zero for all STOVAST analyses.)

APPENDIX B1

STOVAST OUTPUT FILE FOR STOCHASTIC BEAM PROBLEM

*** P R O G R A M S T O V A S T ***

STOCHASTIC FINITE ELEMENT BASED RELIABILITY ANALYSIS

HEADER DATA

PREFX = WALBB IOUTPT = 0 IDFLT = 0 IKOPTC = 0

TITLE : PFEA OF MAHADEVAN'S STOCHASTIC BEAM

MASTER CONTROL CODES FOR PROBABILISTIC ANALYSIS :

```
IRANFLD = 1
IVBLDFN = 1
IRESPNS = 1
ISYSDEF = 0
ICOMREL = 1
ISENSTY = 1
ISYSREL = 0
```

PROBLEM TITLE:	PFEA OF MAHADEVAN'S STOCHASTIC BEAM
INPUT FILE: OUTPUT FILE: COMMAND LOG FILE:	WALBB.INP WALBB.LPT N/A
TASK:	VDFN CDFN FORM PSNS

* RELAMM VARIABLE LISTING

NO.	NAME	DISTR.TYPE	MEAN	STAND.DEV.	DISTR.PAR.
1	PL-G#001	NORMAL	0.8000E-01	0.1600E-01	0.8000E-01 0.1600E-01
2	PL-G#002	NORMAL	0.8000E-01	0.1600E-01	0.8000E-01 0.1600E-01
3	PL-G#003	NORMAL	0.8000E-01	0.1600E-01	0.8000E-01 0.1600E-01
4	PL-G#004	NORMAL	0.8000E-01	0.1600E-01	0.8000E-01 0.1600E-01
5	YMOD	NORMAL	0.2900E+05	0.1740E+04	0.2900E+05 0.1740E+04
6	AREA	NORMAL	0.7680E+01	0.3840E+00	0.7680E+01 0.3840E+00
7	AIZ	NORMAL	0.3010E+03	0.1505E+02	0.3010E+03 0.1505E+02
8	PR	FIXED	0.3000E+00		0.3000E+00
9	DEN	FIXED	0.1000E+01		0.1000E+01
10	ZERO	FIXED	0.0000E+00		0.0000E+00
11	UTHR	FIXED	0.9600E+00		0.9600E+00

RELAMM VARIABLE CORRELATION LISTING

VARIABLE NO.	VARIABLE NO.	CORR. COEF. OF Z	CORR. COEF. OF U
1	2	0.36788E+00	0.36788E+00
1	3	0.13534E+00	0.13534E+00
1	4	0.49787E-01	0.49787E-01
2	3	0.36788E+00	0.36788E+00
2	4	0.13534E+00	0.13534E+00
3	4	0.36788E+00	0.36788E+00

RELAMM COMPONENT LISTING

COMPONENT NUMBER: 1
COMPONENT NAME : BEAMDEFLECT
LIMIT STATE FUNCTION: FEA DISPLACEMENT

VAR.NO. IN LSF	VARIABLE NAME	DISTRIBUTION TYPE
1	PL-G#001	NORMAL
2	PL-G#002	NORMAL
3	PL-G#003	NORMAL
4	PL-G#004	NORMAL
5	YMOD	NORMAL
6	AREA	NORMAL
7	AIZ	NORMAL
8	PR	FIXED
9	DEN	FIXED
10	ZERO	FIXED
11	UTHR	FIXED

TITLE : MAHADEVANS STOCHASTIC BEAM PROBLEM

GEOMETRY AND ELEMENTS (V61)

IOPT = 1 ISTIF = 0 IMASS = 0 NTGE = 1 IPTC = 1

EXECUTION SPECIFICATIONS:

- -ELEMENT MATRIX COMPUTATION MODE
- -STRUCTURAL STIFFNESS MATRICES GENERATED
- -NO ELEMENT MASS MATRICES GENERATED
- -GEOMETRY AND ELEMENT DATA READ FROM "PREFX.GOM"

SUBTITLE : MAHADEVANS STOCHASTIC BEAM PROBLEM

NUMBER OF SUBSTRUCTURES - 0

UNITS FOR EXTERNAL PRE/POST-PROCESSING:
LENGTH (NIL)
FORCE (NIL)

```
IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

TITLE :                         MAHADEVANS STOCHASTIC BEAM PROBLEM
-------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.000000E+00      IY = 0.181194E+08      IZ = 0.181194E+08
   X = 0.960000E+02       Y = 0.000000E+00       Z = 0.000000E+00
   W = 0.147456E+04 X G

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =   0.180618D+01 . THIS OCCURED IN ROW =    26
      WHICH CORRESPONDS WITH FREEDOM 2 OF NODE    5 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.170584D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.428299D+00

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

TITLE :                         MAHADEVANS STOCHASTIC BEAM PROBLEM
-------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.000000E+00      IY = 0.181194E+08      IZ = 0.181194E+08
   X = 0.960000E+02       Y = 0.000000E+00       Z = 0.000000E+00
   W = 0.147456E+04 X G

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =   0.180618D+01 . THIS OCCURED IN ROW =    26
      WHICH CORRESPONDS WITH FREEDOM 2 OF NODE    5 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.170584D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.428299D+00

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

TITLE :                         MAHADEVANS STOCHASTIC BEAM PROBLEM
-------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.000000E+00      IY = 0.181194E+08      IZ = 0.181194E+08
   X = 0.960000E+02       Y = 0.000000E+00       Z = 0.000000E+00
   W = 0.147456E+04 X G

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =   0.180618D+01 . THIS OCCURED IN ROW =    26
```

```
 IELC = 1      ICLC = 0

ELEMENT GROUP #    1

BEAM ELEMENT LOADS

IDISTL  = 1    IPOINT  = 0    ISTRN   = 0

                         DISTRIBUTED LOADS                    INITIAL                POINT LOADS
ELEM         FY1       FY2            FZ1        FZ2          STRAIN          XL          FY          FZ

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   1      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   2      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   3      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   4      0.100E+01 0.100E+01      0.000E+00 0.000E+00      0.000E+00

ASSEMBLED LOAD VECTOR

NODE       1         2         3         4         5         6

    1   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    2   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    3   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    4   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00 0.192E+03
    5   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00-0.192E+03

TOTALS:
       0.000E+00 0.480E+02 0.000E+00 0.000E+00 0.000E+00 0.000E+00

TITLE :                           MAHADEVANS STOCHASTIC BEAM PROBLEM
-------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.000000E+00      IY = 0.181194E+08      IZ = 0.181194E+08
   X = 0.960000E+02       Y = 0.000000E+00       Z = 0.000000E+00
   W = 0.147456E+04 X G

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =    0.180618D+01 . THIS OCCURED IN ROW =    26
      WHICH CORRESPONDS WITH FREEDOM 2 OF NODE     5 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.170584D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.428299D+00

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1
```

```
TOTALS:
       0.000E+00 0.480E+02 0.000E+00 0.000E+00 0.000E+00 0.000E+00

GRADIENTS OF DISTRIBUTED & POINT LOADS COMPUTED


IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1

SUBTITLE :   STOCHASTIC BEAM WITH RANDOM FIELD LOAD

IDYN = 0     IACC = 0     IROT = 0     ILSF = 0     IHSP = 0

 IELC = 1       ICLC = 0

ELEMENT GROUP #    1

BEAM ELEMENT LOADS

IDISTL  = 1    IPOINT  = 0    ISTRN   = 0

                         DISTRIBUTED LOADS                     INITIAL               POINT LOADS
ELEM         FY1       FY2            FZ1        FZ2           STRAIN          XL         FY          FZ

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   1      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   2      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   3      0.100E+01 0.100E+01      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   4      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

ASSEMBLED LOAD VECTOR

NODE        1          2          3          4          5          6

    1   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    2   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    3   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00 0.192E+03
    4   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00-0.192E+03
    5   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00

TOTALS:
       0.000E+00 0.480E+02 0.000E+00 0.000E+00 0.000E+00 0.000E+00

GRADIENTS OF DISTRIBUTED & POINT LOADS COMPUTED


IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1

SUBTITLE :   STOCHASTIC BEAM WITH RANDOM FIELD LOAD

IDYN = 0     IACC = 0     IROT = 0     ILSF = 0     IHSP = 0
```

```
   4      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

ASSEMBLED LOAD VECTOR

NODE       1          2          3          4          5          6

    1   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00 0.192E+03
    2   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00-0.192E+03
    3   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    4   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    5   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00

TOTALS:
       0.000E+00 0.480E+02 0.000E+00 0.000E+00 0.000E+00 0.000E+00

GRADIENTS OF DISTRIBUTED & POINT LOADS COMPUTED


IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

SUBTITLE :   STOCHASTIC BEAM WITH RANDOM FIELD LOAD

IDYN = 0    IACC = 0    IROT = 0    ILSF = 0    IHSP = 0

 IELC = 1      ICLC = 0

ELEMENT GROUP #    1

BEAM ELEMENT LOADS

IDISTL  = 1    IPOINT  = 0    ISTRN   = 0

                     DISTRIBUTED LOADS                  INITIAL               POINT LOADS
ELEM         FY1       FY2            FZ1       FZ2         STRAIN          XL         FY         FZ

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   1      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   2      0.100E+01 0.100E+01      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   3      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   4      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

ASSEMBLED LOAD VECTOR

NODE       1          2          3          4          5          6

    1   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    2   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00 0.192E+03
    3   0.000E+00 0.240E+02 0.000E+00 0.000E+00 0.000E+00-0.192E+03
    4   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    5   0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00
```

```
 = 1 FREE FORMATTED INPUT

ELEMENT GROUP #    1

NUMBER       AX        AIX        AIY        AIZ        AIYZ       SCY        SCZ        SFY        SFZ

    1    0.768E+01 0.000E+00 0.000E+00 0.301E+03 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00


NUMBER     STRESS        Y1         Y2         Y3         Y4         Z1         Z2         Z3         Z4         C
           POINTS
    1      1 -  4     0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.100E+01

ELEM
NUMB      N1     N2     N3   ICODE  NSECT         P          XN3  YN3        ZN3          EY          EZ,6X,VOLUME

   1       1      2      0     1       1       0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   2       2      3      0     1       1       0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   3       3      4      0     1       1       0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   4       4      5      0     1       1       0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03

THE TOTAL VOLUME FOR THIS GROUP OF ELEMENTS IS 0.147E+04
THE TOTAL WEIGHT FOR THIS GROUP OF ELEMENTS IS 0.147E+04 X G

 ASSEMBLY OF ELEMENT GRADIENT MATRICES :


GRADIENTS OF DISTRIBUTED & POINT LOADS COMPUTED


IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1

SUBTITLE :    STOCHASTIC BEAM WITH RANDOM FIELD LOAD

IDYN = 0     IACC = 0     IROT = 0     ILSF = 0     IHSP = 0

 IELC = 1       ICLC = 0

ELEMENT GROUP #    1

BEAM ELEMENT LOADS

IDISTL  = 1    IPOINT  = 0    ISTRN   = 0

                      DISTRIBUTED LOADS                  INITIAL                POINT LOADS
ELEM         FY1        FY2         FZ1        FZ2          STRAIN          XL         FY         FZ

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   1      0.100E+01 0.100E+01      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   2      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :


   3      0.000E+00 0.000E+00      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :
```

```
   1      1    2     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   2      2    3     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   3      3    4     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   4      4    5     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03

THE TOTAL VOLUME FOR THIS GROUP OF ELEMENTS IS 0.147E+04
THE TOTAL WEIGHT FOR THIS GROUP OF ELEMENTS IS 0.147E+04 X G

 ASSEMBLY OF ELEMENT GRADIENT MATRICES :


 COMPUTATION OF GRADIENTS OF ELEMENT MATRICES :


NUMBER OF GEOMETRIC NODES          -    5
NUMBER OF DISPLACEMENT NODES       -    5
NUMBER OF ELEMENT GROUPS           -    1
NUMBER OF SKEW COORDINATE SYSTEMS -    0
CODE FOR GLOBAL ROTATIONS          -    0
 = 0 ROTATIONS ARE IN LOCAL SYSTEM
 = 1 ROTATIONS ARE IN GLOBAL SYSTEM
CODE FOR COORDINATE FORMAT         -    0
 = 0 FORMATTED INPUT
 = 1 FREE FORMATTED INPUT


ELEMENT GROUP #    1


NUMBER      AX        AIX        AIY        AIZ        AIYZ       SCY        SCZ        SFY        SFZ

    1    0.768E+01 0.000E+00 0.000E+00 0.301E+03 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00


NUMBER     STRESS        Y1         Y2         Y3         Y4         Z1         Z2         Z3         Z4         C
           POINTS
    1      1 -  4     0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.100E+01

ELEM
NUMB     N1    N2    N3   ICODE  NSECT        P         XN3  YN3       ZN3         EY          EZ,6X,VOLUME

   1      1    2     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   2      2    3     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   3      3    4     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03
   4      4    5     0    1       1      0.000E+00 0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.369E+03

THE TOTAL VOLUME FOR THIS GROUP OF ELEMENTS IS 0.147E+04
THE TOTAL WEIGHT FOR THIS GROUP OF ELEMENTS IS 0.147E+04 X G

 ASSEMBLY OF ELEMENT GRADIENT MATRICES :


 COMPUTATION OF GRADIENTS OF ELEMENT MATRICES :


NUMBER OF GEOMETRIC NODES          -    5
NUMBER OF DISPLACEMENT NODES       -    5
NUMBER OF ELEMENT GROUPS           -    1
NUMBER OF SKEW COORDINATE SYSTEMS -    0
CODE FOR GLOBAL ROTATIONS          -    0
 = 0 ROTATIONS ARE IN LOCAL SYSTEM
 = 1 ROTATIONS ARE IN GLOBAL SYSTEM
CODE FOR COORDINATE FORMAT         -    0
 = 0 FORMATTED INPUT
```

```
    4     0.00000000E+00  0.10399119E+01  0.00000000E+00  0.00000000E+00  0.00000000E+00  0.10642374E-01
    5     0.00000000E+00  0.15568272E+01  0.00000000E+00  0.00000000E+00  0.00000000E+00  0.10811300E-01

NODAL POINT REACTIONS

   ( * INDICATES REACTION AT SUPPORT D.O.F.)
   ( $ INDICATES REACTION AT PRESCRIBED DISPLACEMENT D.O.F.)

NODE            1            2            3            4            5            6

    1     0.0000E+00*-0.1536E+02* 0.0000E+00* 0.0000E+00* 0.0000E+00*-0.1475E+04*

REACTION SUMMATION
         0.0000E+00 -0.1536E+02  0.0000E+00  0.0000E+00  0.0000E+00 -0.1475E+04

STRAIN ENERGY =    0.478205433254665324E+01

FORCE AND MOMENT RESIDUALS

NODE            1            2            3            4            5            6

    1     0.0000E+00 -0.5995E-14  0.0000E+00  0.0000E+00  0.0000E+00 -0.2842E-13
    2     0.0000E+00 -0.3719E-14  0.0000E+00  0.0000E+00  0.0000E+00 -0.1137E-12
    3     0.0000E+00 -0.3719E-14  0.0000E+00  0.0000E+00  0.0000E+00 -0.1137E-12
    4     0.0000E+00  0.1760E-13  0.0000E+00  0.0000E+00  0.0000E+00  0.2274E-12
    5     0.0000E+00 -0.2673E-13  0.0000E+00  0.0000E+00  0.0000E+00 -0.1272E-12

CONDITIONING NUMBER EPS =  -0.279332E-14
ESTIMATE OF DIGITS LOST =   0.344612E+01

 G R A D I E N T     C O M P U T A T I O N   :

 COMPUTATION OF GRADIENTS OF ELEMENT MATRICES :

NUMBER OF GEOMETRIC NODES         -    5
NUMBER OF DISPLACEMENT NODES      -    5
NUMBER OF ELEMENT GROUPS          -    1
NUMBER OF SKEW COORDINATE SYSTEMS -    0
CODE FOR GLOBAL ROTATIONS         -    0
 = 0 ROTATIONS ARE IN LOCAL SYSTEM
 = 1 ROTATIONS ARE IN GLOBAL SYSTEM
CODE FOR COORDINATE FORMAT        -    0
 = 0 FORMATTED INPUT
 = 1 FREE FORMATTED INPUT

ELEMENT GROUP #    1

NUMBER       AX        AIX        AIY        AIZ        AIYZ       SCY        SCZ        SFY        SFZ

    1     0.768E+01 0.000E+00 0.000E+00 0.301E+03 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00

NUMBER      STRESS         Y1         Y2         Y3         Y4         Z1         Z2         Z3         Z4         C
            POINTS
    1       1 -  4      0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.100E+01

ELEM
NUMB      N1     N2     N3   ICODE  NSECT         P          XN3  YN3        ZN3          EY          EZ,6X,VOLUME
```

```
LOADS: DISTRIBUTED AND CONCENTRATED (V61)
=========================================

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

SUBTITLE :   STOCHASTIC BEAM WITH RANDOM FIELD LOAD

IDYN = 0    IACC = 0    IROT = 0    ILSF = 0    IHSP = 0

 IELC = 1       ICLC = 0

ELEMENT GROUP #    1

BEAM ELEMENT LOADS

IDISTL  = 1    IPOINT  = 0    ISTRN   = 0

                       DISTRIBUTED LOADS                    INITIAL              POINT LOADS
ELEM          FY1        FY2           FZ1        FZ2           STRAIN          XL         FY         FZ

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   1       0.800E-01 0.800E-01      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   2       0.800E-01 0.800E-01      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   3       0.800E-01 0.800E-01      0.000E+00 0.000E+00      0.000E+00

MEAN VALUES OF STOCHASTIC BEAM ELEMENT LOADS :

   4       0.800E-01 0.800E-01      0.000E+00 0.000E+00      0.000E+00

ASSEMBLED LOAD VECTOR

NODE       1          2          3          4          5          6

    1   0.000E+00 0.192E+01 0.000E+00 0.000E+00 0.000E+00 0.154E+02
    2   0.000E+00 0.384E+01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    3   0.000E+00 0.384E+01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    4   0.000E+00 0.384E+01 0.000E+00 0.000E+00 0.000E+00 0.000E+00
    5   0.000E+00 0.192E+01 0.000E+00 0.000E+00 0.000E+00-0.154E+02

TOTALS:
       0.000E+00 0.154E+02 0.000E+00 0.000E+00 0.000E+00 0.000E+00

NODAL DISPLACEMENTS   ( TRANSLATIONS AND ROTATIONS )

   ( * INDICATES SUPPORT D.O.F.)
   ( # INDICATES INACTIVE D.O.F.)
   ( $ INDICATES PRESCRIBED DISPLACEMENT D.O.F.)

NODE            1               2               3               4               5               6

    1      0.00000000E+00* 0.15360000E-28* 0.00000000E+00* 0.00000000E+00* 0.00000000E+00* 0.14745600E-26*
    2      0.00000000E+00  0.16419662E+00  0.00000000E+00  0.00000000E+00  0.00000000E+00  0.62502830E-02
    3      0.00000000E+00  0.55137631E+00  0.00000000E+00  0.00000000E+00  0.00000000E+00  0.94598877E-02
```

```
ASSEMBLY OF HIGHEST-LEVEL ELEMENT MATRICES (V61)
================================================

IBRC = 2      ICOL = 0      IPTC = 0

MATRICES ASSEMBLED USING NO RENUMBERING


      NUMBER OF DISPLACEMENT NODES =            5
      NUMBER OF DEGREES OF FREEDOM PER NODE =   6
      STIFFNESS MATRIX SIZE =                  30
      STIFFNESS MATRIX SEMI-BANDWIDTH =        12


STIFFNESS MATRIX ADDITIONS (V61)
================================

NTSM =    1      IPTC =    1

NSK =    1  DEFAULT SPRING STIFFNESS = 0.100E+31

SPRING STIFFNESSES:

NODE        DIRECTIONS
            1 2 3 4 5 6         SK1        SK2        SK3        SK4        SK5        SK6

   1        1 1 1 1 1 1     0.100E+31 0.100E+31 0.100E+31 0.100E+31 0.100E+31 0.100E+31

NEGST =    0

NDDOF =    0

  MATRIX MODIFICATION METHOD IS EMPLOYED TO TAKE INTO ACCOUNT THE MPCs !



MATRIX DECOMPOSITION (V61)
==========================

IOPT = 1      IPTC = 0

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =   0.180618D+01 . THIS OCCURED IN ROW =    26
      WHICH CORRESPONDS WITH FREEDOM 2 OF NODE    5 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.170584D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.428299D+00

 DECOMPOSED MATRIX HAS BEEN STORED ON    1 BLOCKS OF      154 TERMS EACH.

MEAN VALUES OF STOCHASTIC LOADS USED IN 1ST ITER. :
```

```
NUMBER OF GEOMETRIC NODES          -     5
NUMBER OF DISPLACEMENT NODES       -     5
NUMBER OF ELEMENT GROUPS           -     1
NUMBER OF SKEW COORDINATE SYSTEMS  -     0
CODE FOR GLOBAL ROTATIONS          -     0
 = 0 ROTATIONS ARE IN LOCAL SYSTEM
 = 1 ROTATIONS ARE IN GLOBAL SYSTEM
CODE FOR COORDINATE FORMAT         -     0
 = 0 FORMATTED INPUT
 = 1 FREE FORMATTED INPUT
```

1	0.0000	0.0000	0.0000	0
2	48.0000	0.0000	0.0000	0
3	96.0000	0.0000	0.0000	0
4	144.0000	0.0000	0.0000	0
5	192.0000	0.0000	0.0000	0

ELEMENTS

ELEMENT GROUP # 1

MEAN VALUES OF STOCHASTIC MATERIAL PROPERTIES :

BEAM ELEMENTS

YOUNGS MODULUS 0.290E+05
POISSONS RATIO 0.300E+00
DENSITY 0.100E+01

SECTION PROPERTIES:

NUMBER	AX	AIX	AIY	AIZ	AIYZ	SCY	SCZ	SFY	SFZ
1	0.768E+01	0.000E+00	0.000E+00	0.301E+03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

NUMBER	STRESS POINTS	Y1	Y2	Y3	Y4	Z1	Z2	Z3	Z4	C
1	1 - 4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.100E+01

ELEM NUMB	N1	N2	N3	ICODE	NSECT	P	XN3	YN3	ZN3	EY	EZ, 6X, VOLUME	
1	1	2	0	1	1	0.000E+00	0.000E+00	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.369E+03
2	2	3	0	1	1	0.000E+00	0.000E+00	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.369E+03
3	3	4	0	1	1	0.000E+00	0.000E+00	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.369E+03
4	4	5	0	1	1	0.000E+00	0.000E+00	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.369E+03

THE TOTAL VOLUME FOR THIS GROUP OF ELEMENTS IS 0.147E+04
THE TOTAL WEIGHT FOR THIS GROUP OF ELEMENTS IS 0.147E+04 X G

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

```
  IX = 0.000000E+00     IY = 0.181194E+08     IZ = 0.181194E+08
   X = 0.960000E+02      Y = 0.000000E+00      Z = 0.000000E+00
   W = 0.147456E+04 X G
```

```
WHICH CORRESPONDS WITH FREEDOM 2 OF NODE      5 .
AVERAGE SIGNIFICANT FIGURE LOSS WAS =         0.170584D+00
MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =     0.428299D+00

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1
```

RELAMM RESULT LISTING

COMPONENT NUMBER: 1
COMPONENT NAME : BEAMDEFLECT

METHOD: FORM (2) CPU TIME (Sec):	0.19800E+02
RELIABILITY INDEX:	0.23887E+01
PROBABILITY OF FAILURE:	0.84537E-02
TOLERANCE OF REL. INDEX:	0.10000E-03
NUMBER OF ITERATIONS:	4
NUMBER OF EVALUATIONS OF LIMIT STATE FUNCTION:	5
VALUE OF LIMIT STATE FUNCTION:	-0.55783E-06

VAR. NUMBER	VARIABLE NAME	DESIGN POINT	IMPORTANCE FACTOR
1	PL-G#001	0.73571E-01	0.26
2	PL-G#002	0.64709E-01	63.18
3	PL-G#003	0.53359E-01	2.01
4	PL-G#004	0.47867E-01	25.07
5	YMOD	0.29979E+05	5.56
6	AREA	0.76800E+01	0.00
7	AIZ	0.30813E+03	3.93
8	PR	0.30000E+00	0.00
9	DEN	0.10000E+01	0.00
10	ZERO	0.00000E+00	0.00
11	UTHR	0.96000E+00	0.00

RELAMM RESULT LISTING

COMPONENT NUMBER: 1
COMPONENT NAME : BEAMDEFLECT

METHOD: FORM BASED PARAMETER SENS. CPU TIME (SEC):	0.20000E-01
NUMBER OF VARIABLES:	11
NUMBER OF DISTRIBUTION PARAMETERS:	18

NAME	DISTR.TYPE	PAR.	PAR. VALUE	BETA SENSIT.	Pf SENSIT.
PL-G#0	NORMAL	PAR1	0.8000E-01	0.1519E+01	-0.3496E-01
		PAR2	0.1600E-01	-0.6105E+00	0.1405E-01
		MEAN	0.8000E-01	0.1519E+01	-0.3496E-01
		STDV	0.1600E-01	-0.6105E+00	0.1405E-01
PL-G#0	NORMAL	PAR1	0.8000E-01	0.9825E+01	-0.2260E+00
		PAR2	0.1600E-01	-0.9390E+01	0.2160E+00
		MEAN	0.8000E-01	0.9825E+01	-0.2260E+00

		STDV	0.1600E-01	-0.9390E+01	0.2160E+00
PL-G#0	NORMAL	PAR1	0.8000E-01	0.2421E+02	-0.5570E+00
		PAR2	0.1600E-01	-0.4031E+02	0.9274E+00
		MEAN	0.8000E-01	0.2421E+02	-0.5570E+00
		STDV	0.1600E-01	-0.4031E+02	0.9274E+00
PL-G#0	NORMAL	PAR1	0.8000E-01	0.4224E+02	-0.9718E+00
		PAR2	0.1600E-01	-0.8483E+02	0.1952E+01
		MEAN	0.8000E-01	0.4224E+02	-0.9718E+00
		STDV	0.1600E-01	-0.8483E+02	0.1952E+01
YMOD	NORMAL	PAR1	0.2900E+05	-0.1355E-03	0.3117E-05
		PAR2	0.1740E+04	-0.7625E-04	0.1754E-05
		MEAN	0.2900E+05	-0.1355E-03	0.3117E-05
		STDV	0.1740E+04	-0.7625E-04	0.1754E-05
AREA	NORMAL	PAR1	0.7680E+01	0.0000E+00	0.0000E+00
		PAR2	0.3840E+00	0.0000E+00	0.0000E+00
		MEAN	0.7680E+01	0.0000E+00	0.0000E+00
		STDV	0.3840E+00	0.0000E+00	0.0000E+00
AIZ	NORMAL	PAR1	0.3010E+03	-0.1318E-01	0.3032E-03
		PAR2	0.1505E+02	-0.6243E-02	0.1436E-03
		MEAN	0.3010E+03	-0.1318E-01	0.3032E-03
		STDV	0.1505E+02	-0.6243E-02	0.1436E-03
PR	FIXED	PAR1	0.3000E+00	0.0000E+00	0.0000E+00
		MEAN	0.3000E+00	0.0000E+00	0.0000E+00
DEN	FIXED	PAR1	0.1000E+01	0.0000E+00	0.0000E+00
		MEAN	0.1000E+01	0.0000E+00	0.0000E+00
ZERO	FIXED	PAR1	0.0000E+00	0.0000E+00	0.0000E+00
		MEAN	0.0000E+00	0.0000E+00	0.0000E+00
UTHR	FIXED	PAR1	0.9600E+00	-0.4230E+01	0.9733E-01
		MEAN	0.9600E+00	-0.4230E+01	0.9733E-01

EXECUTION OF PROGRAM STOVAST SUCCESSFULLY COMPLETED

APPENDIX B2

STOVAST OUTPUT FILE FOR STOCHASTIC PLATE PROBLEM

*** P R O G R A M S T O V A S T ***

STOCHASTIC FINITE ELEMENT BASED RELIABILITY ANALYSIS

HEADER DATA

PREFX = WALKK IOUTPT = 0 IDFLT = 0 IKOPTC = 0

TITLE : PFEA OF STOCHASTIC PLATE

MASTER CONTROL CODES FOR PROBABILISTIC ANALYSIS :

IRANFLD = 0
IVBLDFN = 1
IRESPNS = 1
ISYSDEF = 0
ICOMREL = 1
ISENSTY = 1
ISYSREL = 0

PROBLEM TITLE:	PFEA OF STOCHASTIC PLATE
INPUT FILE: OUTPUT FILE: COMMAND LOG FILE:	WALKK.INP WALKK.LPT N/A
TASK:	VDFN CDFN FORM PSNS

RELAMM VARIABLE LISTING

NO.	NAME	DISTR.TYPE	MEAN	STAND.DEV.	DISTR.PAR.
1	YMOD	LOGNORMAL	0.1000E+07	0.1000E+06	0.0000E+00
					0.9975E-01
					0.1381E+02
2	PR	FIXED	0.2000E+00		0.2000E+00
3	DEN	FIXED	0.1000E+01		0.1000E+01
4	P1	NORMAL	0.4000E+04	0.4000E+03	0.4000E+04
					0.4000E+03
5	P2	NORMAL	0.1000E+04	0.1000E+03	0.1000E+04
					0.1000E+03
6	UTHR	FIXED	0.3000E-01		0.3000E-01

RELAMM COMPONENT LISTING

```
| COMPONENT NUMBER:     1                                              |
| COMPONENT NAME  :    PLATEDEFLECT                                    |
| LIMIT STATE FUNCTION: FEA DISPLACEMENT                               |
|----------------------------------------------------------------------|
|  VAR.NO. IN LSF  |    VARIABLE NAME    |    DISTRIBUTION TYPE        |
|----------------------------------------------------------------------|
|        1         |        YMOD         |       LOGNORMAL             |
|        2         |        PR           |       FIXED                 |
|        3         |        DEN          |       FIXED                 |
|        4         |        P1           |       NORMAL                |
|        5         |        P2           |       NORMAL                |
|        6         |        UTHR         |       FIXED                 |
|----------------------------------------------------------------------|

TITLE :       STOCHASTIC FEA OF PLATE WITH RANDOM FIELD YOUNGS MODULUS
------

GEOMETRY AND ELEMENTS (V61)
===========================

IOPT = 1      ISTIF = 0      IMASS = 0      NTGE = 1      IPTC = 1

     EXECUTION SPECIFICATIONS:

       -ELEMENT MATRIX COMPUTATION MODE
       -STRUCTURAL STIFFNESS MATRICES GENERATED
       -NO ELEMENT MASS MATRICES GENERATED
       -GEOMETRY AND ELEMENT DATA READ FROM  "PREFX.GOM"

SUBTITLE :  STOCHASTIC FINITE ELEMENT DISCRETIZATION TESTING

NUMBER OF SUBSTRUCTURES      -    0

UNITS FOR EXTERNAL PRE/POST-PROCESSING:
     LENGTH (NIL)
     FORCE  (NIL)

NUMBER OF GEOMETRIC NODES         -   81
NUMBER OF DISPLACEMENT NODES      -   81
NUMBER OF ELEMENT GROUPS          -    1
NUMBER OF SKEW COORDINATE SYSTEMS -    0
CODE FOR GLOBAL ROTATIONS         -    0
 = 0 ROTATIONS ARE IN LOCAL SYSTEM
 = 1 ROTATIONS ARE IN GLOBAL SYSTEM
CODE FOR COORDINATE FORMAT        -    0
 = 0 FORMATTED INPUT
 = 1 FREE FORMATTED INPUT

      1          0.0000        0.0000        0.0000     0
      2          2.0000        0.0000        0.0000     0
      3          4.0000        0.0000        0.0000     0
      4          6.0000        0.0000        0.0000     0
      5          8.0000        0.0000        0.0000     0
      6         10.0000        0.0000        0.0000     0
      7         12.0000        0.0000        0.0000     0
      8         14.0000        0.0000        0.0000     0
      9         16.0000        0.0000        0.0000     0
     10          0.0000        2.0000        0.0000     0
     11          2.0000        2.0000        0.0000     0
     12          4.0000        2.0000        0.0000     0
     13          6.0000        2.0000        0.0000     0
     14          8.0000        2.0000        0.0000     0
     15         10.0000        2.0000        0.0000     0
     16         12.0000        2.0000        0.0000     0
     17         14.0000        2.0000        0.0000     0
```

18	16.0000	2.0000	0.0000	0
19	0.0000	4.0000	0.0000	0
20	2.0000	4.0000	0.0000	0
21	4.0000	4.0000	0.0000	0
22	6.0000	4.0000	0.0000	0
23	8.0000	4.0000	0.0000	0
24	10.0000	4.0000	0.0000	0
25	12.0000	4.0000	0.0000	0
26	14.0000	4.0000	0.0000	0
27	16.0000	4.0000	0.0000	0
28	0.0000	6.0000	0.0000	0
29	2.0000	6.0000	0.0000	0
30	4.0000	6.0000	0.0000	0
31	6.0000	6.0000	0.0000	0
32	8.0000	6.0000	0.0000	0
33	10.0000	6.0000	0.0000	0
34	12.0000	6.0000	0.0000	0
35	14.0000	6.0000	0.0000	0
36	16.0000	6.0000	0.0000	0
37	0.0000	8.0000	0.0000	0
38	2.0000	8.0000	0.0000	0
39	4.0000	8.0000	0.0000	0
40	6.0000	8.0000	0.0000	0
41	8.0000	8.0000	0.0000	0
42	10.0000	8.0000	0.0000	0
43	12.0000	8.0000	0.0000	0
44	14.0000	8.0000	0.0000	0
45	16.0000	8.0000	0.0000	0
46	0.0000	10.0000	0.0000	0
47	2.0000	10.0000	0.0000	0
48	4.0000	10.0000	0.0000	0
49	6.0000	10.0000	0.0000	0
50	8.0000	10.0000	0.0000	0
51	10.0000	10.0000	0.0000	0
52	12.0000	10.0000	0.0000	0
53	14.0000	10.0000	0.0000	0
54	16.0000	10.0000	0.0000	0
55	0.0000	12.0000	0.0000	0
56	2.0000	12.0000	0.0000	0
57	4.0000	12.0000	0.0000	0
58	6.0000	12.0000	0.0000	0
59	8.0000	12.0000	0.0000	0
60	10.0000	12.0000	0.0000	0
61	12.0000	12.0000	0.0000	0
62	14.0000	12.0000	0.0000	0
63	16.0000	12.0000	0.0000	0
64	0.0000	14.0000	0.0000	0
65	2.0000	14.0000	0.0000	0
66	4.0000	14.0000	0.0000	0
67	6.0000	14.0000	0.0000	0
68	8.0000	14.0000	0.0000	0
69	10.0000	14.0000	0.0000	0
70	12.0000	14.0000	0.0000	0
71	14.0000	14.0000	0.0000	0
72	16.0000	14.0000	0.0000	0
73	0.0000	16.0000	0.0000	0
74	2.0000	16.0000	0.0000	0
75	4.0000	16.0000	0.0000	0
76	6.0000	16.0000	0.0000	0
77	8.0000	16.0000	0.0000	0
78	10.0000	16.0000	0.0000	0
79	12.0000	16.0000	0.0000	0
80	14.0000	16.0000	0.0000	0
81	16.0000	16.0000	0.0000	0

ELEMENTS

ELEMENT GROUP # 1

QUADRILATERAL SHELL ELEMENTS

MEAN VALUES OF STOCHASTIC MATERIAL PROPERTIES :

YOUNGS MODULUS 0.100E+07
POISSONS RATIO 0.200E+00

DENSITY 0.100E+01

INTEGRATION POINTS ALONG MIDDLE SURFACE CURVILINEAR COORDINATE X = 3
INTEGRATION POINTS ALONG MIDDLE SURFACE CURVILINEAR COORDINATE Y = 3
INTEGRATION POINTS FOR COORDINATE Z NORMAL TO THE MIDDLE SURFACE = 3

ELEM NUMB	N1	N2	N3	N4	TK1	TK2	TK3	TK4	VOLUME
1	1	2	11	10	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
2	2	3	12	11	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
3	3	4	13	12	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
4	4	5	14	13	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
5	5	6	15	14	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
6	6	7	16	15	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
7	7	8	17	16	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
8	8	9	18	17	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
9	10	11	20	19	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
10	11	12	21	20	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
11	12	13	22	21	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
12	13	14	23	22	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
13	14	15	24	23	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
14	15	16	25	24	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
15	16	17	26	25	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
16	17	18	27	26	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
17	19	20	29	28	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
18	20	21	30	29	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
19	21	22	31	30	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
20	22	23	32	31	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
21	23	24	33	32	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
22	24	25	34	33	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
23	25	26	35	34	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
24	26	27	36	35	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
25	28	29	38	37	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
26	29	30	39	38	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
27	30	31	40	39	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
28	31	32	41	40	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
29	32	33	42	41	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
30	33	34	43	42	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
31	34	35	44	43	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
32	35	36	45	44	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
33	37	38	47	46	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
34	38	39	48	47	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
35	39	40	49	48	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
36	40	41	50	49	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
37	41	42	51	50	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
38	42	43	52	51	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
39	43	44	53	52	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
40	44	45	54	53	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
41	46	47	56	55	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
42	47	48	57	56	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01

43	48	49	58	57	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
44	49	50	59	58	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
45	50	51	60	59	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
46	51	52	61	60	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
47	52	53	62	61	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
48	53	54	63	62	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
49	55	56	65	64	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
50	56	57	66	65	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
51	57	58	67	66	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
52	58	59	68	67	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
53	59	60	69	68	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
54	60	61	70	69	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
55	61	62	71	70	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
56	62	63	72	71	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
57	64	65	74	73	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
58	65	66	75	74	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
59	66	67	76	75	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
60	67	68	77	76	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
61	68	69	78	77	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
62	69	70	79	78	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
63	70	71	80	79	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
64	71	72	81	80	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01

THE TOTAL VOLUME FOR THIS GROUP OF ELEMENTS IS 0.256E+03
THE TOTAL WEIGHT FOR THIS GROUP OF ELEMENTS IS 0.256E+03 X G

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

IX = 0.218667E+05 IY = 0.218667E+05 IZ = 0.436907E+05
X = 0.800000E+01 Y = 0.800000E+01 Z = 0.148400E-17
W = 0.256000E+03 X G

ASSEMBLY OF HIGHEST-LEVEL ELEMENT MATRICES (V61)

IBRC = 2 ICOL = 0 IPTC = 0

MATRICES ASSEMBLED USING NO RENUMBERING

NUMBER OF DISPLACEMENT NODES = 81
NUMBER OF DEGREES OF FREEDOM PER NODE = 6
STIFFNESS MATRIX SIZE = 486
STIFFNESS MATRIX SEMI-BANDWIDTH = 66

STIFFNESS MATRIX ADDITIONS (V61)

NTSM = 1 IPTC = 1

NSK = 17 DEFAULT SPRING STIFFNESS = 0.100E+20

SPRING STIFFNESSES:

NODE	DIRECTIONS 1 2 3 4 5 6	SK1	SK2	SK3	SK4	SK5	SK6

```
    1       1 1 1 1 1 1      0.100E+20 0.100E+20 0.100E+20 0.100E+20 0.100E+20 0.100E+20
    2       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    3       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    4       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    5       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    6       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    7       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    8       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
    9       0 1 1 0 0 0      0.000E+00 0.100E+20 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   10       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   19       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   28       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   37       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   46       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   55       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   64       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00
   73       1 0 1 0 0 0      0.100E+20 0.000E+00 0.100E+20 0.000E+00 0.000E+00 0.000E+00

NEGST =    0

NDDOF =    0

  MATRIX MODIFICATION METHOD IS EMPLOYED TO TAKE INTO ACCOUNT THE MPCs !



MATRIX DECOMPOSITION  (V61)
==========================

IOPT = 1      IPTC = 0

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =    0.230663D+01 . THIS OCCURED IN ROW =    483
      WHICH CORRESPONDS WITH FREEDOM 3 OF NODE    81 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.145043D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.226819D+00

 DECOMPOSED MATRIX HAS BEEN STORED ON     1 BLOCKS OF     23367 TERMS EACH.

MEAN VALUES OF STOCHASTIC LOADS USED IN 1ST ITER. :


LOADS: DISTRIBUTED AND CONCENTRATED (V61)
====================================

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

SUBTITLE :   PLATE CONCENTRATED LOAD FOR SFEM TESTING

IDYN = 0    IACC = 0    IROT = 0    ILSF = 0    IHSP = 0

 IELC = 0      ICLC = 1

NLF =    1      LTYPE = 0

CONCENTRATED LOADS  ( * INDICATES MASSES )
```

NODE	1	2	3	4	5	6
81	0.400E+04	0.100E+04	0.000E+00	0.000E+00	0.000E+00	0.000E+00

ASSEMBLED LOAD VECTOR

NODE	1	2	3	4	5	6
1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
5	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
9	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
11	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
15	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
16	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
17	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
18	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
19	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
20	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
21	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
22	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
24	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
25	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
26	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
27	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
28	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
29	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
30	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
32	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
33	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
34	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
35	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
39	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
40	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
41	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
42	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
43	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
44	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
45	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
46	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
47	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
48	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
50	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
51	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
52	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
53	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
54	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
55	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
56	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
57	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
58	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

59	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
60	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
61	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
62	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
63	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
64	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
65	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
66	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
68	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
69	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
70	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
71	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
73	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
74	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
75	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
76	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
77	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
78	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
79	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
80	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
81	0.400E+04	0.100E+04	0.000E+00	0.000E+00	0.000E+00	0.000E+00

TOTALS:

0.400E+04 0.100E+04 0.000E+00 0.000E+00 0.000E+00 0.000E+00

NODAL DISPLACEMENTS (TRANSLATIONS AND ROTATIONS)

(* INDICATES SUPPORT D.O.F.)
(# INDICATES INACTIVE D.O.F.)
($ INDICATES PRESCRIBED DISPLACEMENT D.O.F.)

NODE	1	2	3	4	5	6
1	0.25440980E-17*	0.22259335E-16*	-0.50505563E-33*	0.32094103E-33*	-0.34119615E-33*	0.00000000E+00*
2	-0.47890830E-04	0.43519220E-16*	-0.30607073E-33*	0.14422547E-18	-0.59096696E-19	0.00000000E+00
3	-0.10617647E-03	0.40209580E-16*	-0.50213590E-35*	0.33085353E-18	-0.57464781E-19	0.00000000E+00
4	-0.17914148E-03	0.33753906E-16*	0.66714431E-34*	0.48610177E-18	-0.51696306E-19	0.00000000E+00
5	-0.26021240E-03	0.23122500E-16*	-0.47661871E-34*	0.63670719E-18	-0.50164541E-19	0.00000000E+00
6	-0.33080810E-03	0.76880910E-17*	0.11661172E-33*	0.76886111E-18	-0.43383541E-19	0.00000000E+00
7	-0.36520910E-03	-0.12026995E-16*	-0.15327402E-33*	0.87582192E-18	-0.37994612E-19	0.00000000E+00
8	-0.34214748E-03	-0.33820117E-16*	0.16947256E-33*	0.98077580E-18	-0.33110743E-19	0.00000000E+00
9	-0.25928453E-03	-0.24705519E-16*	0.30295923E-33*	0.10475599E-17	-0.37702639E-19	0.00000000E+00
10	0.76462803E-17*	0.43236980E-03	-0.34614607E-33*	0.53119537E-19	-0.14590709E-18	0.00000000E+00
11	-0.21177425E-04	0.42502745E-03	0.31584604E-18	0.14687972E-18	-0.14064597E-18	0.00000000E+00
12	-0.54961685E-04	0.39922434E-03	0.66461086E-18	0.34627384E-18	-0.14046610E-18	0.00000000E+00
13	-0.10798251E-03	0.34447491E-03	0.98483726E-18	0.49406142E-18	-0.12446830E-18	0.00000000E+00
14	-0.17570800E-03	0.24675150E-03	0.12858616E-17	0.65057907E-18	-0.11628580E-18	0.00000000E+00
15	-0.24005420E-03	0.95250589E-04	0.15467929E-17	0.78076163E-18	-0.97173588E-19	0.00000000E+00
16	-0.27337099E-03	-0.10787305E-03	0.17723669E-17	0.88950699E-18	-0.79618440E-19	0.00000000E+00
17	-0.25048545E-03	-0.33923263E-03	0.19499569E-17	0.99171219E-18	-0.65834666E-19	0.00000000E+00
18	-0.16591263E-03	-0.55457661E-03	0.21210011E-17	0.10354357E-17	-0.53822682E-19	0.00000000E+00
19	0.15337625E-16*	0.83703433E-03	-0.10685443E-33*	0.53416058E-19	-0.34559674E-18	0.00000000E+00
20	0.59877139E-04	0.82502553E-03	0.68135654E-18	0.15938394E-18	-0.35621499E-18	0.00000000E+00
21	0.10151034E-03	0.78149804E-03	0.13520033E-17	0.37121386E-18	-0.35215881E-18	0.00000000E+00
22	0.11151082E-03	0.68477727E-03	0.20130832E-17	0.51772826E-18	-0.34045766E-18	0.00000000E+00
23	0.87603494E-04	0.50357763E-03	0.26132823E-17	0.68835760E-18	-0.30442900E-18	0.00000000E+00
24	0.44603634E-04	0.20934780E-03	0.31553415E-17	0.82323903E-18	-0.26401098E-18	0.00000000E+00
25	0.15283825E-04	-0.20140928E-03	0.35984701E-17	0.93717273E-18	-0.21733814E-18	0.00000000E+00
26	0.36973680E-04	-0.68406188E-03	0.39708422E-17	0.10421158E-17	-0.17197460E-18	0.00000000E+00
27	0.12665306E-03	-0.11416532E-02	0.42485519E-17	0.10765711E-17	-0.15056862E-18	0.00000000E+00
28	0.28064206E-16*	0.11839276E-02	-0.12666284E-35*	0.44728487E-19	-0.52025343E-18	0.00000000E+00
29	0.19631957E-03	0.11718149E-02	0.10497318E-17	0.16801414E-18	-0.52029139E-18	0.00000000E+00
30	0.36865276E-03	0.11248713E-02	0.20975077E-17	0.39128569E-18	-0.50706548E-18	0.00000000E+00

31	0.49444175E-03	0.10109388E-02	0.30957343E-17	0.55636821E-18	-0.47079160E-18	0.00000000E+00
32	0.55767370E-03	0.77731094E-03	0.40435319E-17	0.74591156E-18	-0.44010628E-18	0.00000000E+00
33	0.56226668E-03	0.36420194E-03	0.48685050E-17	0.89471315E-18	-0.36953686E-18	0.00000000E+00
34	0.54316070E-03	-0.25870394E-03	0.55694494E-17	0.10202371E-17	-0.29667700E-18	0.00000000E+00
35	0.56172010E-03	-0.10355909E-02	0.61126198E-17	0.11335074E-17	-0.23572988E-18	0.00000000E+00
36	0.65865086E-03	-0.17977831E-02	0.65670784E-17	0.11976204E-17	-0.17256032E-18	0.00000000E+00
37	0.45199148E-16*	0.14409904E-02	-0.25353093E-33*	0.60430206E-19	-0.72644629E-18	0.00000000E+00
38	0.38431211E-03	0.14334672E-02	0.14585460E-17	0.18525196E-18	-0.73772114E-18	0.00000000E+00
39	0.74465822E-03	0.13989320E-02	0.28981725E-17	0.44119160E-18	-0.72002939E-18	0.00000000E+00
40	0.10526527E-02	0.12990502E-02	0.42997246E-17	0.62506028E-18	-0.70468149E-18	0.00000000E+00
41	0.12742893E-02	0.10631032E-02	0.56059350E-17	0.83531015E-18	-0.64299499E-18	0.00000000E+00
42	0.13810010E-02	0.58313207E-03	0.67773856E-17	0.10024384E-17	-0.54601584E-18	0.00000000E+00
43	0.13965539E-02	-0.23995505E-03	0.77159023E-17	0.11299030E-17	-0.43800231E-18	0.00000000E+00
44	0.14059197E-02	-0.13819663E-02	0.84824199E-17	0.12482439E-17	-0.35126112E-18	0.00000000E+00
45	0.15148639E-02	-0.25713892E-02	0.90636750E-17	0.12879342E-17	-0.28889566E-18	0.00000000E+00
46	0.65049804E-16*	0.15794651E-02	-0.59872337E-34*	0.56278996E-19	-0.96106278E-18	0.00000000E+00
47	0.60789651E-03	0.15789603E-02	0.19048753E-17	0.20512736E-18	-0.95981468E-18	0.00000000E+00
48	0.12039801E-02	0.15663975E-02	0.37995685E-17	0.48488480E-18	-0.93349303E-18	0.00000000E+00
49	0.17674154E-02	0.15052344E-02	0.56211463E-17	0.68369625E-18	-0.88134945E-18	0.00000000E+00
50	0.22570290E-02	0.13205609E-02	0.73604145E-17	0.92861300E-18	-0.82781296E-18	0.00000000E+00
51	0.26014596E-02	0.87451169E-03	0.88832421E-17	0.11128444E-17	-0.67884880E-18	0.00000000E+00
52	0.27231514E-02	-0.79251503E-04	0.10139642E-16	0.12631513E-17	-0.52634151E-18	0.00000000E+00
53	0.27479802E-02	-0.16730750E-02	0.11073966E-16	0.14037530E-17	-0.37740815E-18	0.00000000E+00
54	0.28470373E-02	-0.35373941E-02	0.11819121E-16	0.14029849E-17	-0.29112553E-18	0.00000000E+00
55	0.84504010E-16*	0.15844783E-02	-0.43876180E-33*	0.62432476E-19	-0.11763821E-17	0.00000000E+00
56	0.83308063E-03	0.15893521E-02	0.23928339E-17	0.22568435E-18	-0.11849179E-17	0.00000000E+00
57	0.16804403E-02	0.15952242E-02	0.47654990E-17	0.51358978E-18	-0.11710143E-17	0.00000000E+00
58	0.25495406E-02	0.15729321E-02	0.70850636E-17	0.75491465E-18	-0.11355926E-17	0.00000000E+00
59	0.34280434E-02	0.14619484E-02	0.92646381E-17	0.10027452E-17	-0.10591855E-17	0.00000000E+00
60	0.42524152E-02	0.11341298E-02	0.11244544E-16	0.12409720E-17	-0.92612139E-18	0.00000000E+00
61	0.48417497E-02	0.31027444E-03	0.12856067E-16	0.14349852E-17	-0.73668411E-18	0.00000000E+00
62	0.48952564E-02	-0.17554344E-02	0.14076393E-16	0.16376257E-17	-0.53002378E-18	0.00000000E+00
63	0.51143922E-02	-0.48448699E-02	0.14931339E-16	0.16747441E-17	-0.42780192E-18	0.00000000E+00
64	0.99238841E-16*	0.14676600E-02	0.36055706E-33*	0.62401402E-19	-0.14950514E-17	0.00000000E+00
65	0.10089333E-02	0.14728155E-02	0.29128235E-17	0.24908752E-18	-0.14973313E-17	0.00000000E+00
66	0.20646162E-02	0.14810270E-02	0.58236865E-17	0.55538315E-18	-0.14726547E-17	0.00000000E+00
67	0.32165164E-02	0.14689963E-02	0.86509677E-17	0.81874059E-18	-0.14100792E-17	0.00000000E+00
68	0.45185273E-02	0.13901532E-02	0.11371371E-16	0.11013681E-17	-0.13182230E-17	0.00000000E+00
69	0.60230118E-02	0.11563685E-02	0.13811596E-16	0.13515425E-17	-0.11290699E-17	0.00000000E+00
70	0.77271710E-02	0.54286375E-03	0.15895180E-16	0.15791546E-17	-0.87540594E-18	0.00000000E+00
71	0.92444577E-02	-0.10895480E-02	0.17507688E-16	0.18166503E-17	-0.64355622E-18	0.00000000E+00
72	0.89324789E-02	-0.66459251E-02	0.18697685E-16	0.20687311E-17	-0.41295184E-18	0.00000000E+00
73	0.52415987E-16*	0.12752940E-02	0.12071974E-32*	0.86342155E-19	-0.16985411E-17	0.00000000E+00
74	0.10792413E-02	0.12755350E-02	0.35167400E-17	0.27064728E-18	-0.16641323E-17	0.00000000E+00
75	0.22293455E-02	0.12688287E-02	0.69317601E-17	0.59772001E-18	-0.16426672E-17	0.00000000E+00
76	0.35318992E-02	0.12312374E-02	0.10377822E-16	0.89423668E-18	-0.16908445E-17	0.00000000E+00
77	0.50965964E-02	0.11170022E-02	0.13650104E-16	0.11816209E-17	-0.15379715E-17	0.00000000E+00
78	0.71016734E-02	0.83813655E-03	0.16645563E-16	0.15070933E-17	-0.14341556E-17	0.00000000E+00
79	0.99058277E-02	0.22313833E-03	0.19207447E-16	0.17113300E-17	-0.12223838E-17	0.00000000E+00
80	0.14460143E-01	-0.13912491E-02	0.21347210E-16	0.20506858E-17	-0.10353913E-17	0.00000000E+00
81	0.23002957E-01	-0.72005522E-02	0.22905561E-16	0.20960819E-17	-0.67169826E-18	0.00000000E+00

NODAL POINT REACTIONS

(* INDICATES REACTION AT SUPPORT D.O.F.)
($ INDICATES REACTION AT PRESCRIBED DISPLACEMENT D.O.F.)

NODE	1	2	3	4	5	6
1	-0.2544E+02*	-0.2226E+03*	0.5051E-14*	-0.3209E-14*	0.3412E-14*	0.0000E+00*
2	0.2665E-14	-0.4352E+03*	0.3061E-14*	-0.7396E-30	0.0000E+00	0.0000E+00
3	-0.6217E-14	-0.4021E+03*	0.5021E-16*	0.7716E-29	-0.3944E-30	0.0000E+00
4	-0.3553E-14	-0.3375E+03*	-0.6671E-15*	0.1388E-28	0.2958E-29	0.0000E+00
5	0.7550E-14	-0.2312E+03*	0.4766E-15*	0.5004E-29	-0.5522E-29	0.0000E+00
6	0.4441E-15	-0.7688E+02*	-0.1166E-14*	-0.4709E-29	-0.7889E-30	0.0000E+00

7	-0.1776E-14	0.1203E+03*	0.1533E-14*	0.5103E-29	0.1085E-29	0.0000E+00
8	-0.8882E-14	0.3382E+03*	-0.1695E-14*	-0.1162E-28	0.4930E-29	0.0000E+00
9	0.8882E-15	0.2471E+03*	-0.3030E-14*	-0.7161E-29	0.5719E-29	0.0000E+00
10	-0.7646E+02*	0.7105E-14	0.3461E-14*	0.0000E+00	-0.2367E-29	0.0000E+00
19	-0.1534E+03*	-0.7105E-14	0.1068E-14*	-0.1282E-29	-0.3944E-29	0.0000E+00
28	-0.2806E+03*	0.1421E-13	0.1267E-16*	-0.3747E-29	0.0000E+00	0.0000E+00
37	-0.4520E+03*	0.2842E-13	0.2535E-14*	0.3550E-29	-0.1578E-28	0.0000E+00
46	-0.6505E+03*	-0.2132E-13	0.5987E-15*	0.1893E-28	0.4733E-29	0.0000E+00
55	-0.8450E+03*	0.1630E-30	0.4388E-14*	0.3944E-29	-0.1104E-28	0.0000E+00
64	-0.9924E+03*	0.5576E-30	-0.3606E-14*	-0.8283E-29	-0.6311E-29	0.0000E+00
73	-0.5242E+03*	0.3109E-13	-0.1207E-13*	-0.5719E-29	0.9466E-29	0.0000E+00

REACTION SUMMATION
-0.4000E+04 -0.1000E+04 -0.2029E-26 -0.3209E-14 0.3412E-14 0.0000E+00

STRAIN ENERGY = 0.424056381949721608E+02

FORCE AND MOMENT RESIDUALS

NODE	1	2	3	4	5	6
1	-0.1332E-14	0.7105E-14	0.5729E-28	0.4684E-29	-0.2712E-29	0.0000E+00
2	0.2665E-14	0.7105E-14	0.1852E-27	-0.7396E-30	0.0000E+00	0.0000E+00
3	-0.6217E-14	0.7105E-14	-0.4330E-27	0.7716E-29	-0.3944E-30	0.0000E+00
4	-0.3553E-14	0.0000E+00	0.1442E-28	0.1388E-28	0.2958E-29	0.0000E+00
5	0.7550E-14	0.0000E+00	0.5374E-29	0.5004E-29	-0.5522E-29	0.0000E+00
6	0.4441E-15	0.3553E-14	-0.1363E-28	-0.4709E-29	-0.7889E-30	0.0000E+00
7	-0.1776E-14	0.1776E-14	0.1014E-19	0.5103E-29	0.1085E-29	0.0000E+00
8	-0.8882E-14	0.7105E-14	-0.4117E-29	-0.1162E-28	0.4930E-29	0.0000E+00
9	0.8882E-15	-0.3553E-14	0.5608E-20	-0.7161E-29	0.5719E-29	0.0000E+00
10	0.1776E-14	0.7105E-14	0.8875E-30	0.0000E+00	-0.2367E-29	0.0000E+00
11	0.3553E-14	0.1776E-13	-0.4733E-29	0.4437E-29	0.1972E-29	0.0000E+00
12	-0.1776E-14	0.2842E-13	0.6311E-29	0.1420E-28	-0.1972E-29	0.0000E+00
13	-0.3553E-14	0.1954E-13	0.6705E-29	0.1568E-28	-0.5128E-29	0.0000E+00
14	-0.3553E-14	0.6217E-14	0.4654E-28	0.4240E-29	0.5522E-29	0.0000E+00
15	-0.6661E-14	-0.7994E-14	0.1302E-28	-0.9861E-29	0.1026E-28	0.0000E+00
16	0.1421E-13	-0.3553E-14	0.3708E-28	0.1637E-28	-0.2406E-28	0.0000E+00
17	-0.2132E-13	0.1421E-13	0.1499E-28	-0.1962E-28	0.2268E-28	0.0000E+00
18	0.5329E-14	-0.1421E-13	0.3550E-28	-0.5017E-29	0.2761E-29	0.0000E+00
19	0.3553E-14	-0.7105E-14	-0.4958E-19	-0.1282E-29	-0.3944E-29	0.0000E+00
20	-0.7105E-14	-0.2132E-13	-0.1578E-29	-0.1262E-28	0.0000E+00	0.0000E+00
21	0.1066E-13	-0.1421E-13	0.3629E-28	0.9565E-29	-0.1578E-28	0.0000E+00
22	0.7105E-14	-0.1066E-13	-0.2051E-28	-0.2771E-28	-0.1104E-28	0.0000E+00
23	0.2665E-14	-0.1776E-14	0.7100E-29	-0.4970E-28	0.7100E-29	0.0000E+00
24	0.5240E-30	-0.4441E-14	0.1499E-28	-0.5719E-29	0.6311E-29	0.0000E+00
25	0.1716E-30	0.3553E-14	0.6469E-28	0.8382E-30	-0.9072E-29	0.0000E+00
26	-0.7105E-14	-0.3553E-13	0.1065E-28	0.5221E-28	0.3944E-29	0.0000E+00
27	0.5329E-14	0.3440E-30	0.3510E-28	0.3787E-28	-0.1183E-29	0.0000E+00
28	-0.7105E-14	0.1421E-13	-0.8046E-29	-0.3747E-29	0.0000E+00	0.0000E+00
29	0.2132E-13	0.7105E-14	-0.1735E-28	0.9861E-29	-0.2288E-28	0.0000E+00
30	0.2842E-13	-0.1421E-13	0.2840E-28	-0.4181E-28	-0.1420E-28	0.0000E+00
31	0.1066E-13	0.3553E-13	-0.1657E-28	-0.4082E-28	-0.3944E-28	0.0000E+00
32	0.3553E-13	-0.3197E-13	0.3155E-28	0.3944E-29	0.8677E-29	0.0000E+00
33	-0.4441E-14	0.4707E-13	0.8362E-28	-0.1223E-28	0.3392E-28	0.0000E+00
34	0.2132E-13	-0.7105E-14	0.3787E-28	0.8234E-28	0.5522E-29	0.0000E+00
35	0.2132E-13	-0.4263E-13	0.4457E-28	0.4733E-29	-0.2327E-28	0.0000E+00
36	-0.3553E-14	-0.4263E-13	0.5009E-28	-0.1417E-28	0.4654E-28	0.0000E+00
37	-0.7105E-14	0.2842E-13	-0.5719E-29	0.3550E-29	-0.1578E-28	0.0000E+00
38	-0.7105E-14	-0.7105E-14	0.3155E-29	0.1538E-28	-0.1262E-28	0.0000E+00
39	0.5701E-30	0.8527E-13	0.9466E-29	-0.1952E-28	-0.4575E-28	0.0000E+00
40	-0.4619E-13	0.8527E-13	0.4733E-28	-0.2820E-28	-0.3944E-29	0.0000E+00
41	-0.4974E-13	0.1030E-12	0.5995E-28	0.1345E-27	-0.6311E-29	0.0000E+00
42	-0.6484E-13	0.3650E-30	0.6074E-28	0.6981E-28	-0.1341E-28	0.0000E+00
43	0.1137E-12	-0.1421E-13	0.1657E-28	0.2100E-28	-0.4694E-28	0.0000E+00

44	0.1421E-13	0.7105E-13	-0.1988E-27	0.1410E-28	0.4812E-28	0.0000E+00
45	-0.3197E-13	-0.4263E-13	0.3629E-28	0.2633E-28	0.5483E-28	0.0000E+00
46	0.2842E-13	-0.2132E-13	-0.7396E-31	0.1893E-28	0.4733E-29	0.0000E+00
47	-0.2132E-13	0.4974E-13	0.5364E-28	0.5206E-28	-0.1578E-28	0.0000E+00
48	0.6750E-13	0.8527E-13	-0.3944E-28	0.6666E-28	-0.9466E-29	0.0000E+00
49	0.5684E-13	-0.7105E-14	-0.7889E-29	-0.4694E-28	0.2524E-28	0.0000E+00
50	-0.1421E-13	-0.1421E-13	-0.1941E-27	-0.6114E-29	0.4102E-28	0.0000E+00
51	0.1599E-13	-0.2665E-13	0.8677E-28	0.2820E-28	0.1657E-28	0.0000E+00
52	0.1990E-12	0.2842E-13	0.6390E-28	0.9210E-28	-0.1885E-27	0.0000E+00
53	-0.1421E-13	-0.2842E-13	0.7415E-28	-0.2869E-28	0.2367E-29	0.0000E+00
54	-0.4974E-13	0.5073E-30	0.6311E-28	-0.3875E-28	-0.1026E-28	0.0000E+00
55	-0.2842E-13	0.1630E-30	0.5522E-29	0.3944E-29	-0.1104E-28	0.0000E+00
56	-0.3553E-14	0.6395E-13	-0.3155E-28	0.3432E-28	-0.1735E-28	0.0000E+00
57	0.9948E-13	0.1421E-13	-0.8520E-28	-0.4220E-28	0.9466E-29	0.0000E+00
58	-0.1918E-12	0.1279E-12	-0.3155E-28	0.6784E-28	0.1735E-28	0.0000E+00
59	-0.3908E-13	-0.3553E-13	0.2367E-27	-0.7770E-28	0.3313E-28	0.0000E+00
60	-0.2167E-12	0.4974E-13	0.2524E-28	-0.3432E-28	-0.1751E-27	0.0000E+00
61	-0.3553E-13	0.8527E-13	-0.1735E-28	0.1637E-27	-0.4102E-28	0.0000E+00
62	0.5684E-13	0.5684E-13	-0.4733E-29	-0.2120E-28	-0.1124E-27	0.0000E+00
63	-0.2132E-13	-0.5684E-13	0.1215E-27	0.7238E-28	0.5049E-28	0.0000E+00
64	0.0000E+00	0.5576E-30	0.6607E-29	-0.8283E-29	-0.6311E-29	0.0000E+00
65	0.5684E-13	-0.4263E-13	0.8204E-28	-0.1499E-28	-0.4733E-29	0.0000E+00
66	0.2132E-13	0.2132E-13	0.7573E-28	0.1349E-27	-0.9624E-28	0.0000E+00
67	0.8572E-30	0.8065E-30	0.3155E-27	0.1455E-27	0.4733E-29	0.0000E+00
68	0.3197E-13	0.4974E-13	0.1388E-27	-0.9466E-29	-0.5049E-28	0.0000E+00
69	0.2238E-12	-0.2665E-13	-0.2903E-27	-0.5956E-28	-0.1310E-27	0.0000E+00
70	0.3020E-12	0.2132E-13	0.4418E-28	-0.5206E-28	0.2998E-28	0.0000E+00
71	0.1421E-12	-0.5684E-13	0.1956E-27	0.1672E-27	-0.1775E-27	0.0000E+00
72	-0.1137E-12	-0.5684E-13	0.3155E-28	0.5325E-29	0.1089E-27	0.0000E+00
73	-0.2842E-13	0.3109E-13	0.7889E-30	-0.5719E-29	0.9466E-29	0.0000E+00
74	0.1243E-13	0.1954E-13	0.1672E-27	-0.6942E-28	0.1262E-28	0.0000E+00
75	-0.2132E-13	0.5063E-13	0.2524E-28	-0.4319E-28	-0.7573E-28	0.0000E+00
76	-0.8527E-13	-0.6040E-13	0.1483E-27	0.3974E-28	-0.3155E-28	0.0000E+00
77	0.4352E-13	-0.4974E-13	-0.2398E-27	0.1501E-27	0.2840E-28	0.0000E+00
78	-0.5618E-13	-0.9481E-13	0.8204E-28	0.6932E-28	0.8835E-28	0.0000E+00
79	0.1599E-13	-0.7017E-13	-0.1515E-27	0.7711E-28	-0.9940E-28	0.0000E+00
80	0.1847E-12	-0.1066E-13	-0.2524E-28	0.1176E-27	0.6705E-28	0.0000E+00
81	0.5684E-13	0.5684E-13	0.1073E-27	-0.8184E-28	0.2840E-28	0.0000E+00

CONDITIONING NUMBER EPS = 0.970107E-16
ESTIMATE OF DIGITS LOST = 0.198682E+01

G R A D I E N T   C O M P U T A T I O N :

COMPUTATION OF GRADIENTS OF ELEMENT MATRICES :

NUMBER OF GEOMETRIC NODES - 81
NUMBER OF DISPLACEMENT NODES - 81
NUMBER OF ELEMENT GROUPS - 1
NUMBER OF SKEW COORDINATE SYSTEMS - 0
CODE FOR GLOBAL ROTATIONS - 0
= 0 ROTATIONS ARE IN LOCAL SYSTEM
= 1 ROTATIONS ARE IN GLOBAL SYSTEM
CODE FOR COORDINATE FORMAT - 0
= 0 FORMATTED INPUT
= 1 FREE FORMATTED INPUT

ELEMENT GROUP # 1

QUADRILATERAL SHELL ELEMENTS

MEAN VALUES OF STOCHASTIC MATERIAL PROPERTIES :

1	1	2	11	10	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
2	2	3	12	11	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
3	3	4	13	12	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
4	4	5	14	13	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
5	5	6	15	14	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
6	6	7	16	15	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
7	7	8	17	16	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
8	8	9	18	17	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
9	10	11	20	19	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
10	11	12	21	20	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
11	12	13	22	21	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
12	13	14	23	22	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
13	14	15	24	23	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
14	15	16	25	24	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
15	16	17	26	25	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
16	17	18	27	26	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
17	19	20	29	28	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
18	20	21	30	29	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
19	21	22	31	30	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
20	22	23	32	31	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
21	23	24	33	32	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
22	24	25	34	33	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
23	25	26	35	34	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
24	26	27	36	35	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
25	28	29	38	37	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
26	29	30	39	38	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
27	30	31	40	39	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
28	31	32	41	40	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
29	32	33	42	41	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
30	33	34	43	42	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
31	34	35	44	43	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
32	35	36	45	44	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
33	37	38	47	46	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
34	38	39	48	47	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
35	39	40	49	48	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
36	40	41	50	49	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
37	41	42	51	50	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
38	42	43	52	51	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
39	43	44	53	52	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
40	44	45	54	53	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
41	46	47	56	55	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
42	47	48	57	56	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
43	48	49	58	57	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
44	49	50	59	58	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
45	50	51	60	59	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
46	51	52	61	60	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
47	52	53	62	61	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
48	53	54	63	62	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
49	55	56	65	64	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
50	56	57	66	65	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
51	57	58	67	66	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
52	58	59	68	67	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
53	59	60	69	68	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
54	60	61	70	69	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
55	61	62	71	70	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
56	62	63	72	71	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
57	64	65	74	73	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
58	65	66	75	74	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
59	66	67	76	75	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
60	67	68	77	76	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
61	68	69	78	77	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
62	69	70	79	78	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
63	70	71	80	79	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01
64	71	72	81	80	0.100E+01	0.100E+01	0.100E+01	0.100E+01	0.400E+01

THE TOTAL VOLUME FOR THIS GROUP OF ELEMENTS IS 0.256E+03

THE TOTAL WEIGHT FOR THIS GROUP OF ELEMENTS IS 0.256E+03 X G

ASSEMBLY OF ELEMENT GRADIENT MATRICES :

GRADIENTS OF DISTRIBUTED & POINT LOADS COMPUTED

IOPT = 0   NTLD = 1   IPTC1 = 1   IPTC2 = 1

SUBTITLE :   PLATE CONCENTRATED LOAD FOR SFEM TESTING

IDYN = 0   IACC = 0   IROT = 0   ILSF = 0   IHSP = 0

IELC = 0   ICLC = 1

NLF = 1   LTYPE = 0

CONCENTRATED LOADS (* INDICATES MASSES)

NODE	1	2	3	4	5	6
81	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

ASSEMBLED LOAD VECTOR

NODE	1	2	3	4	5	6
1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
5	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
9	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
11	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
15	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
16	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
17	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
18	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
19	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
20	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
21	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
22	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
24	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
25	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
26	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
27	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
28	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
29	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
30	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
32	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
33	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
34	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
35	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

39	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
40	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
41	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
42	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
43	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
44	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
45	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
46	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
47	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
48	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
50	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
51	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
52	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
53	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
54	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
55	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
56	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
57	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
58	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
59	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
60	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
61	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
62	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
63	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
64	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
65	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
66	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
68	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
69	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
70	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
71	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
73	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
74	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
75	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
76	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
77	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
78	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
79	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
80	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
81	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

TOTALS:

0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00

GRADIENTS OF DISTRIBUTED & POINT LOADS COMPUTED

IOPT = 0   NTLD = 1   IPTC1 = 1   IPTC2 = 1

SUBTITLE :   PLATE CONCENTRATED LOAD FOR SFEM TESTING

IDYN = 0   IACC = 0   IROT = 0   ILSF = 0   IHSP = 0

IELC = 0   ICLC = 1

NLF = 1   LTYPE = 0

CONCENTRATED LOADS   (* INDICATES MASSES)

NODE	1	2	3	4	5	6
81	0.000E+00	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

ASSEMBLED LOAD VECTOR

NODE	1	2	3	4	5	6
1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
3	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
5	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
6	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
9	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
11	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
15	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
16	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
17	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
18	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
19	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
20	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
21	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
22	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
24	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
25	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
26	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
27	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
28	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
29	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
30	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
32	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
33	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
34	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
35	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
39	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
40	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
41	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
42	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
43	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
44	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
45	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
46	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
47	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
48	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
50	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
51	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
52	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
53	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
54	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
55	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
56	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
57	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
58	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
59	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
60	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
61	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
62	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

63	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
64	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
65	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
66	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
68	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
69	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
70	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
71	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
73	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
74	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
75	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
76	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
77	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
78	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
79	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
80	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
81	0.000E+00	0.100E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00

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TOTALS:
      0.000E+00 0.100E+01 0.000E+00 0.000E+00 0.000E+00 0.000E+00

TITLE :       STOCHASTIC FEA OF PLATE WITH RANDOM FIELD YOUNGS MODULUS
-------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.218667E+05      IY = 0.218667E+05      IZ = 0.436907E+05
   X = 0.800000E+01       Y = 0.800000E+01       Z = 0.148400E-17
   W = 0.256000E+03 X G

     MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
     OF DIAGONAL PIVOTS =    0.230663D+01 . THIS OCCURED IN ROW =   483
     WHICH CORRESPONDS WITH FREEDOM 3 OF NODE   81 .
     AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.145043D+00
     MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.226819D+00

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

TITLE :       STOCHASTIC FEA OF PLATE WITH RANDOM FIELD YOUNGS MODULUS
-------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.218667E+05      IY = 0.218667E+05      IZ = 0.436907E+05
   X = 0.800000E+01       Y = 0.800000E+01       Z = 0.148400E-17
   W = 0.256000E+03 X G

     MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
     OF DIAGONAL PIVOTS =    0.230663D+01 . THIS OCCURED IN ROW =   483
     WHICH CORRESPONDS WITH FREEDOM 3 OF NODE   81 .
     AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.145043D+00
     MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.226819D+00

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

IOPT = 0    NTLD = 1    IPTC1 = 1    IPTC2 = 1

TITLE :       STOCHASTIC FEA OF PLATE WITH RANDOM FIELD YOUNGS MODULUS
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MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.218667E+05      IY = 0.218667E+05      IZ = 0.436907E+05
   X = 0.800000E+01       Y = 0.800000E+01       Z = 0.148400E-17
   W = 0.256000E+03 X G

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =    0.230663D+01 . THIS OCCURED IN ROW =    483
      WHICH CORRESPONDS WITH FREEDOM 3 OF NODE   81 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.145043D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.226819D+00

IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1

IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1

TITLE :       STOCHASTIC FEA OF PLATE WITH RANDOM FIELD YOUNGS MODULUS
------

MOMENT OF INERTIAS, CENTRE OF GRAVITY AND TOTAL WEIGHT FOR STRUCTURE:

  IX = 0.218667E+05      IY = 0.218667E+05      IZ = 0.436907E+05
   X = 0.800000E+01       Y = 0.800000E+01       Z = 0.148400E-17
   W = 0.256000E+03 X G

      MAXIMUM NUMBER OF SIGNIFICANT FIGURES LOST IN CALCULATION
      OF DIAGONAL PIVOTS =    0.230663D+01 . THIS OCCURED IN ROW =    483
      WHICH CORRESPONDS WITH FREEDOM 3 OF NODE   81 .
      AVERAGE SIGNIFICANT FIGURE LOSS WAS =       0.145043D+00
      MEAN SQUARE SIGNIFICANT FIGURE LOSS WAS =   0.226819D+00

IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1

IOPT = 0     NTLD = 1     IPTC1 = 1     IPTC2 = 1
```

RELAMM RESULT LISTING

COMPONENT NUMBER: 1
COMPONENT NAME : PLATEDEFLECT

METHOD: FORM (2) — CPU TIME (Sec):	0.18921E+03
RELIABILITY INDEX:	0.17714E+01
PROBABILITY OF FAILURE:	0.38251E-01
TOLERANCE OF REL. INDEX:	0.10000E-04
NUMBER OF ITERATIONS:	4
NUMBER OF EVALUATIONS OF LIMIT STATE FUNCTION:	5
VALUE OF LIMIT STATE FUNCTION:	0.34411E-07

VAR. NUMBER	VARIABLE NAME	DESIGN POINT	IMPORTANCE FACTOR
1	YMOD	0.87894E+06	49.29
2	PR	0.20000E+00	0.00
3	DEN	0.10000E+01	0.00
4	P1	0.45003E+04	49.86
5	P2	0.98367E+03	0.85
6	UTHR	0.30000E-01	0.00

RELAMM RESULT LISTING

COMPONENT NUMBER: 1
COMPONENT NAME : PLATEDEFLECT

METHOD: FORM BASED PARAMETER SENS. CPU TIME (SEC): 0.20000E-01
NUMBER OF VARIABLES: 6
NUMBER OF DISTRIBUTION PARAMETERS: 10

NAME	DISTR.TYPE	PAR.	PAR. VALUE	BETA SENSIT.	Pf SENSIT.
YMOD	LOGNORMAL	PAR1	0.0000E+00	0.8007E-05	-0.6654E-06
		PAR2	0.9975E-01	-0.8753E+01	0.7274E+00
		PAR3	0.1381E+02	0.7038E+01	-0.5848E+00
		MEAN	0.1000E+07	0.7976E-05	-0.6628E-06
		STDV	0.1000E+06	-0.9385E-05	0.7799E-06
PR	FIXED	PAR1	0.2000E+00	0.0000E+00	0.0000E+00
		MEAN	0.2000E+00	0.0000E+00	0.0000E+00
DEN	FIXED	PAR1	0.1000E+01	0.0000E+00	0.0000E+00
		MEAN	0.1000E+01	0.0000E+00	0.0000E+00
P1	NORMAL	PAR1	0.4000E+04	-0.1765E-02	0.1467E-03
		PAR2	0.4000E+03	-0.2208E-02	0.1835E-03
		MEAN	0.4000E+04	-0.1765E-02	0.1467E-03
		STDV	0.4000E+03	-0.2208E-02	0.1835E-03
P2	NORMAL	PAR1	0.1000E+04	0.9218E-03	-0.7660E-04
		PAR2	0.1000E+03	-0.1505E-03	0.1251E-04
		MEAN	0.1000E+04	0.9218E-03	-0.7660E-04
		STDV	0.1000E+03	-0.1505E-03	0.1251E-04
UTHR	FIXED	PAR1	0.3000E-01	0.2346E+03	-0.1949E+02
		MEAN	0.3000E-01	0.2346E+03	-0.1949E+02

EXECUTION OF PROGRAM STOVAST SUCCESSFULLY COMPLETED

UNCLASSIFIED
SECURITY CLASSIFICATION OF FORM
(highest classification of Title, Abstract, Keywords)

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Halifax, N.S. B3J 3J8

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Stochastic Finite Element Methods for Structural Reliability Analysis

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Orisamolu, I.R., Liu, Q.

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A research and development study of stochastic finite element methods (SFEM) for structural reliability analysis is presented. A detailed state-of-the-art literature review of various SFEM methodologies and the diverse technical subjects relating to probabilistic finite element analysis is summarized. Case studies of two existing SFEM-based reliability analysis computer programs were conducted, with a discussion of the methodology, main features, structure, and capabilities of both systems. The design and implementation of a software package for random field discretization (RANFLD) and SFEM-based reliability analysis (STOVAST) is then presented. The package developed on the basis of the finite element reliability method (FERM) approach, and designed for operation with the VAST commercial finite element analysis system, was verified using several problems. Also, in this work, the quantification and analysis of bias and modeling errors/uncertainties in engineering analysis are discussed. Sources of bias and modeling errors/uncertainties in SFEM-based reliability analysis are identified and suggestions are offered for the application of realistic formulations for accounting for this class of uncertainties. Finally, recommendations are given on requirements for further development work that will broaden the analysis scope of the STOVAST system and permit straightforward applications to ship structures.

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