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Part VIII

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ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY

Part VIII: Spiral Grooved Floating Ring Journal Bearing

J. Vohr
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Mechanical Technology Incorporated

TECHNICAL REPORT AFAPL-TR-65-45, PART VIII

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Air Force Aero Propulsion Laboratory
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184

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184

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FOREWORD

This report was prepared by Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110 under USAF Contract No. AF33(615)-3238. The contract was initiated under Project No. 3048, Task No. 304806. The work was administered under the direction of the Air Force Aero Propulsion Laboratory, with Mr. Michael R. Chasman (APFL) acting as project engineer.

This report covers work conducted from 1 May 1967 to 1 May 1968.

This report was submitted 31 July 1968. This report is Part VIII of final documentation issued in multiple parts.

This technical report has been reviewed and is approved.

Arthur V. Churchill

ARTHUR V. CHURCHILL, Chief
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ABSTRACT

In this volume is presented an analysis of the static and dynamic characteristics of the spiral-grooved journal bearing operating with incompressible lubricant in both laminar and turbulent regimes. Both single film and floating ring bearing configurations are considered. Extensive design data are presented giving load capacity, attitude angle, bearing torque, bearing flow rate, stiffness and damping coefficients and critical rotor mass for limit of stable operation. In addition, two computer programs accompany the volume, and instructions and listings of the programs are included. These programs may be used to obtain data for cases not covered by the presented design data.

This abstract is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Air Force Aero Propulsion Laboratory (APFL), Wright-Patterson Air Force Base, Ohio 45433.

TABLE OF CONTENTS

ILLUSTRATIONS

Illustrations (Continued)

SYMBOLS

$B_{xx}, B_{xy}, B_{yx}, B_{yy}$	Bearing damping coefficients, lb-sec/in.
$\bar{B}_{xx}, \bar{B}_{xy}, \bar{B}_{yx}, \bar{B}_{yy}$	Dimensionless damping for overall floating ring bearing, $\bar{B}_{xx} = \frac{B_{xx} C_1}{\mu L D_1} \left(\frac{C_1}{R_1}\right)^2$ etc.
$(\bar{B}_{xx})_1, (\bar{B}_{xy})_1, (\bar{B}_{yx})_1, (\bar{B}_{yy})_1$	Dimensionless damping for inner film, $(\bar{B}_{xx})_1 = 2\pi \frac{B_{xx1} C_1}{\mu L D_1} \left(\frac{C_1}{R_1}\right)^2$ etc.
$(\bar{B}_{xx})_2, (\bar{B}_{xy})_2, (\bar{B}_{yx})_2, (\bar{B}_{yy})_2$	Dimensionless damping for outer film, $(\bar{B}_{xx})_2 = \frac{B_{xx2} C_2}{\mu L D_2} \left(\frac{C_2}{R_2}\right)^2$ etc.
C	Bearing mean radial clearance, in.
D	Bearing diameter, in.
e	Eccentricity, in.
F_r	Radial component of bearing film force = $W\cos\phi$, lb.
F_t	Tangential component of bearing film force = $W\sin\phi$, lb.
F_x, F_y	Bearing film forces in x and y directions, lb.
$\bar{F}_r$	Dimensionless radial (cosine) component of bearing film force = $W\cos\phi\, C^2/\mu(N_i + N_o)R^4$
$\bar{F}_t$	Dimensionless tangential (sin) component of bearing film force = $W\sin\phi\, C^2/\mu(N_i + N_o)R^4$
G_x, G_z	Turbulent flow correction factors

h	Local film clearance, in.
h_g	Local film clearance in groove region, in.
h_r	Local film clearance in ridge region, in.
h'	Dimensionless clearance = h/C
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Bearing stiffness coefficients, lb/in.
$\bar{K}_{xx}, \bar{K}_{xy}, \bar{K}_{yx}, \bar{K}_{yy}$	Dimensionless stiffness for overall floating fing bearing, $\bar{K}_{xx} = \frac{K_{xx} C_1}{\mu N_1 L D_1} \left(\frac{C_1}{R_1}\right)^2$ etc.
$(\bar{K}_{xx})_1, (\bar{K}_{xy})_1, (\bar{K}_{yx})_1, (\bar{K}_{yy})_1$	Dimensionless stiffnesses for inner film, $(\bar{K}_{xx})_1 = \frac{K_{xx1} C_1}{\mu (N_1 + N_2) L D_1} \left(\frac{C_1}{R_1}\right)^2$ etc.
$(\bar{K}_{xx})_2, (\bar{K}_{xy})_2, (\bar{K}_{yx})_2, (\bar{K}_{yy})_2$	Dimensionless stiffness for outer film, $(\bar{K}_{xx})_2 = \frac{K_{xx2} C_2}{\mu N_2 L D_2} \left(\frac{C_2}{R_2}\right)^2$ etc.
L	Bearing length, in.
L_1	Length of grooved region, in.
M	Shaft mass, lb.
M_c	Critical mass for threshold of whirl instability in floating ring bearing, lb.
M_{c1}	Inner film critical mass.
M_{c2}	Outer film critical mass.

SYMBOLS (continued)

M_r	Radial component of restoring moment, in-lb.
M_t	Tangential component of restoring moment, in-lb.
$\bar{M}_c$	Dimensionless critical mass = $\dfrac{M_c N_1 C_1^3}{\mu R_1^2 L D_1}$
$\bar{M}_{c1}$	Dimensionless critical mass = $\dfrac{M_{c1}(N_1 + N_2)C_1^3}{\mu R_1^2 \, L D_1}$
$\bar{M}_{c2}$	Dimensionless critical mass = $\dfrac{M_{c2} N_2 C_2^3}{\mu R_2^2 \, L D_2}$
$\bar{M}_r$	ensionless radial (cosine) component of bearing film force moment = $M_r \, C^2/\mu(N_i + N_o)R^5$
$\bar{M}_t$	Dimensionless tangential (sin) component of bearing film force moment = $M_t \, C^2/\mu(N_i + N_o)R^5$
m	Ring mass, lb.
N	Speed = $N_1 + N_2$, rps
N_1	Speed of journal, rps
N_2	Ring speed, rps
N_i	Speed of inner member, rps
N_o	Speed of outer member, rps
n	Ring speed ratio = N_2/N_1
P	Pressure, lb/in^2
P_s	Supply pressure, lb/in^2

SYMBOLS (continued)

$\bar{P}$	Dimensionless pressure = $PR_1^2/\mu N_1 C_1^2$
$\bar{P}$	Smoothed, "overall" pressure distribution around spiral grooved journal, lb/in^2
$\bar{P}_s$	Dimensionless supply pressure = $P_s R_1^2/\mu N_1 C_1^2$
$\bar{P}_{s1}$	Dimensionless supply pressure, inner film = $P_s R_1^2/\mu(N_1 + N_2)C_1^2$
$\bar{P}_{s2}$	Dimensionless supply pressure, outer film = $P_s R_2^2/\mu N_2 C_2^2$
Q	Total lubricant flow rate, $in^3/sec.$
Q_p	Lubricant flow due to self pumping of spiral grooves, $in^3/sec.$
Q_s	Lubricant flow due to pressurization of supply, $in^3/sec.$
$\bar{Q}$	Dimensionless total lubricant flow rate = $Q/R^2C(N_i + N_o)$
R	Bearing radius, in.
Re	Overall Reynolds number = $2\pi\ N_1 R_1 C_1/\nu$
R_h	Local Reynolds number = $2\pi\ (N_i - N_o)Rh/\nu$
Re_1	Inner film Reynolds Number = $2\pi\rho(N_o - N_2)R_1 C_1/\mu$
Re_2	Outer film Reynolds Number = $2\pi\rho\ N_2 R_2 C_2/\mu$
S	Overall Sommerfeld Number = $\dfrac{\mu N_1 D_1 L}{W}\left(\dfrac{R_1}{C_1}\right)^2$
S_1	Inner film Sommerfeld Number = $\dfrac{\mu(N_1 + N_2)D_1 L}{W}\left(\dfrac{R_1}{C_1}\right)^2$
S_2	Outer film Sommerfeld Number = $\dfrac{\mu N_2 D_2 L}{W}\left(\dfrac{R_2}{C_2}\right)^2$
T_B	Bearing torque, in-lb.

SYMBOLS (continued)

T_j	Journal Torque, in-lb.
$\overline{T}_B$	Dimensionless bearing torque = T_B/WC
$\overline{T}_j$	Dimensionless journal torque = T_j/WC
t	Time, sec.
U	Surface velocity of bearing, in/sec.
$\overline{u}$	Mean flow velocity in direction of rotation, in/sec.
$\overline{u}_p$	Mean flow velocity due to pressure gradient = $\overline{u}$ - V/2, in/sec.
V	Surface speed of journal, in/sec.
W	Bearing load, lb.
W^{ξ}, W^{η}	Mass flow components, lb/(sec-in^2)
$\overline{w}$	Mean flow velocity in axial direction, in/sec.
x,y	Coordinates in direction of and normal to load vector, in.
$\overline{Y}$	Ratio of length of grooving to total length of bearing = L_1/L
$Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy}$	Complex dimensionless bearing impedences
Z'	Dimensionless Z coordinate = Z/L

Greek

α	Ratio of groove width to groove plus land width = $a_g/(a_g + a_r)$
β	Groove angle, deg. (radians in equations)
Γ	Ratio of groove clearance to ridge clearance = h_g/h_r
γ	Misalignment angle, deg. (radians in equations)

SYMBOLS (continued)

$\bar{\gamma}$	Whirl frequency ratio = ν/ω
ϵ	Eccentricity ratio = e/C
η, ξ	Skewed coordinates
θ, Z	Cylindrical coordinates
λ	Dimensionless parameter = $\frac{\mu RL}{\pi}(C/R)^2$
μ	Viscosity, lb-sec/in^2
ν	Kinematic viscosity, in^2/sec.
ν	Whirl frequency, radians/sec.
ρ	Density, lb/in^3
ϕ	Attitude angle, deg. (radians in equations)
ϕ'	Moment attitude angle, deg. (radians in equations)
ω	Total rotational speed = $(\omega_1 + \omega_2)$, radians/sec.
ω_1	Rotational speed journal, radians/sec.
ω_2	Rotational speed ring, radians/sec.

Subscripts

1	Refers to inner film
2	Refers to outer film

INTRODUCTION

In many applications of rotating machinery it is desirable to lubricate bearings with the process fluid in order to avoid the complication of a separate lube system with accompanying seal problems. In many instances, process fluids may have low kinematic viscosities which result in operation of the bearings in the turbulent flow regime.

Bearing power loss rises rapidly with Reynolds number in turbulent film bearings. In a number of prototype systems involving rotary machines operating in the turbulent regime, the bearing power loss has been an appreciable percentage of the net system output. In addition to the effect on system efficiency, high bearing power losses mean that large quantities of lubricant must be circulated through the bearing for cooling.

A very attractive bearing for use in applications where power loss is important is the floating ring bearing. This is a journal bearing in which a loose ring is fitted between the shaft and the bearing housing. This ring is free to rotate when the journal rotates, and by so doing, can reduce the rate of shear between adjacent bearing surfaces thereby reducing power loss.

In plain bearing form, the principal disadvantage of the floating ring bearing is the rather poor stability characteristics of plain journal bearings, particularly when lightly loaded. There are a variety of configurations of journal bearings which offer improved stability over plain journal bearings. These include the tilting pad journal bearing, the Rayleigh step journal bearing, multi-lobed journal bearings, and the spiral-grooved journal bearing. Of these, the most suitable for use in a floating ring configuration is the spiral-grooved bearing. In addition to improved stability characteristics over plain bearings, the spiral-grooved bearing also has several other attractive advantages. These include the ability to self-pump lubricant axially through the bearing film and the ability to operate without cavitation at high eccentricity ratios without using a pressurized lubricant supply.

Because of the potential advantages of spiral-grooved floating ring journal bearings for many applications, an analysis was performed of this bearing for both laminar and turbulent flow regimes. This analysis and the results obtained thereof form the subject of this report. Extensive performance data are presented for both floating ring and single film configuration of the spiral-grooved bearing. These data include stiffness and damping coefficients and evaluations of critical mass for fractional frequency whirl instability.

DESCRIPTION OF SINGLE FILM AND FLOATING-RING SPIRAL GROOVED JOURNAL BEARINGS

A typical spiral-grooved journal bearing is shown schematically in Fig. 1. The configuration shown is that in which the grooving on the ends of the journal tends to pump inward, thereby pressurizing the interior of the bearing. Because of the symmetry of this configuration, there is no net flow of lubricant through the bearing.

Other possible configurations for the single film spiral-grooved bearing are shown in Fig. 2. Configuration 1 is as described above. Configuration 2 is an arrangement wherein spiral grooving is inscribed on one end of the bearing in a manner so as to pump fluid axially through the bearing toward the smooth or seal end. The axial flow of lubricant can remove heat from the bearing. The flow through the bearing can be increased by pressurizing the grooved end of the bearing to a supply pressure P_s as shown.

Configuration 3 shows a symmetrical arrangement wherein lubricant is pumped by spiral grooves outward from the center of the bearing. Lubricant may be supplied to the center of the bearing at an elevated pressure P_s. Configuration 3 consists, essentially, of two bearings of configuration 2 placed "back to back".

In all the configurations illustrated in Figs. 1 and 2, the spiral-grooving is shown inscribed on the journal which is considered to be the rotating member. One could, instead, inscribe the grooving on the stationary member and, for an incompressible lubricant, the performance of the bearing would be essentially the same. In this report, we shall only consider the case where the grooves are inscribed on the rotating member, this being the most common situation in practice.

The performance of a spiral-grooved bearing depends on the values of the groove parameters β (groove angle), α (ratio of groove width to total width), Γ (ratio of groove clearance to ridge clearance), and $\overline{Y}$ (ratio of length of grooving to total length of bearing). These groove parameters are defined in Fig. 1. By a suitable choice of these parameters, one can optimize various performance characteristics of the bearing. In this report, we have chosen to present performance data for the case where the groove parameters are set at those values which yield maximum radial component of bearing stiffness at zero eccentricity.

The radial component of bearing stiffness at zero eccentricity is defined as the limiting value of $W \cos \phi/e$ as e approaches zero where W is bearing load, e is the journal displacement from the center of the bearing and ϕ is the attitude angle for the bearing (see Fig. 3). In determining the optimum values of groove parameters for maximum radial stiffness, it turned out that optimum values of α, β and $\bar{Y}$ did not vary appreciably with Reynolds number or with L/D ratio in the range $0.5 \leq L/D \leq 1.0$. For simplicity, therefore, a fixed set of optimum values for these parameters was settled upon as valid for all Reynolds numbers and all L/D ratios between 0.5 and 1.0. The optimum value of groove depth ratio Γ however, did vary significantly with Reynolds number. Optimum values of α, β, $\bar{Y}$ and Γ selected for configuration 1 and configuration 2 type bearings are given in Tables 1 and 2 below.

TABLE 1

OPTIMUM VALUES OF GROOVE PARAMETERS SELECTED FOR CONFIGURATION 1 BEARING

Reynolds No.	α	β	$\bar{Y}$	Γ
laminar, Re < 500	0.5	151.5°	0.75	2.1
1,000	"	"	"	2.7
5,000	"	"	"	3.0
9,000	"	"	"	3.0

TABLE 2

OPTIMUM VALUES OF GROOVE PARAMETERS SELECTED FOR CONFIGURATION 2 BEARING

Reynolds No.	α	β	$\bar{Y}$	Γ
laminar, Re < 500	0.55	149°	0.67	2.4
1,000	"	"	"	3.1
5,000	"	"	"	3.8
9,000	"	"	"	3.8

Fig. 4 shows a schematic drawing of a floating-ring bearing. A floating-ring bearing is a journal bearing in which there is a loose ring between the shaft and the bearing housing. In this way, the fluid film is separated into an inner film and an outer film. The quantities connected with the inner film are identified by subscript 1, whereas subscript 2 refers to the outer film. The floating-ring bearing shown in Fig. 4 is shown with spiral-grooving on the journal and on the outer surface of the floating ring. The configuration of the grooving is such as to pump lubricant outward from the axial midplane of the bearing. Lubricant is supplied to the two lubricant films at supply pressure P_s via supply holes in the bearing and in the floating ring. Circumferential grooves machined at the midplane of the journal and the floating ring distribute the lubricant at uniform pressure around the journal and around the outside of the floating ring. The ring is free to rotate, and under the influence of shear stress from the revolving journal, turns at some ring speed N_2 less than the speed N_1 of the journal. Since in journal bearings, load capacity is essentially proportional to $(N_1 + N_2)$ whereas power loss is roughly proportional to $(N_1 - N_2)$, it follows that rotation of the ring will improve the load capacity of the inner film while reducing the power loss. This is the principle of operation of the floating ring bearing.

Calculation of the performance characteristics of the floating ring bearing requires calculation and matching of the performance characteristics of the individual lubricant films. These individual performance characteristics depend on the values selected for the groove parameters, with the possibility of having different groove parameters for the inner and outer films. For the calculations presented in this report, the groove parameters were taken to be the same for the inner and outer film and were selected to be those which provide maximum radial component of bearing stiffness for each individual bearing film. (See Table 2).

A description of the analysis of the turbulent single film and floating ring spiral-grooved journal bearings is given in the next section.

ANALYSIS

Analysis of the performance characteristics of the turbulent, spiral-grooved, single film and floating-ring journal bearing is based on the concept of solving for the "overall", "smoothed" pressure distribution around the bearing, neglecting the local zig-zag details of the pressure profiles which arise due to the discontinuous groove-ridge geometry. The theoretical basis for this analytical approach is discussed in detail in Reference 1. Essentially, this analytical approach is valid in the limit as the number of grooves approaches an infinite number, but practically speaking, the analysis proves to be quite accurate when applied to bearings having a reasonable number of grooves. Experimental verification of this analytical approach has been provided by a number of investigations (References 2 and 3).

The differential equation that had been derived in Reference 1 for the smoothed, overall pressure distribution around a spiral-grooved journal bearing was re-derived in the present study to take account of the effects of turbulence in the bearing film. This derivation is presented in Appendix 1. The effects of turbulence in the bearing film are accounted for by means of the linearized turbulent lubrication theory developed by Ng and Pan (Reference 4). In this theory, which is based on the concept of a turbulent eddy viscosity, there are developed turbulent flow correction factors G_x and G_z which relate the mean pressure flow in the direction of rotation (x direction) and the axial direction (z direction) to the pressure gradients in these respective directions. The relationships developed are

$$\bar{u}_p = -\frac{h^2}{\mu} G_x \frac{\partial P}{\partial x} \qquad (1)$$

$$\bar{w} = -\frac{h^2}{\mu} G_z \frac{\partial P}{\partial z} \qquad (2)$$

where

$\bar{u}_p = \bar{u} - \frac{V}{2}$ = the mean flow velocity in the x direction minus 1/2 the surface velocity of rotation

$\bar{w}$ = the mean flow velocity in the axial direction

h = local film clearance

μ = viscosity

In the generalized theory for turbulent fluid films developed by Elrod, Ng and Pan (Reference 5), G_x and G_z are functions of the pressure gradient in the film, the angle between the pressure gradient and the direction of rotation, and the Reynolds number based on rotational velocity. In the linearized theory of Ng and Pan, G_x and G_z are functions only of the local Reynolds number $R_h = \rho V h/\mu$. Values of G_x and G_z, plotted as a function of R_h, are shown in Figure 5.

A discussion of the theoretical basis of the linearized theory of turbulence is beyond the scope of this present report. For such a discussion, the reader can consult Reference 4. In this report, we have simply applied the results of this theory to derive the differential equation for a spiral-grooved journal bearing with turbulent, incompressible lubricant. This differential equation, obtained in Appendix I, is given below in dimensionless form.

$$\frac{\partial W^{\xi}}{\partial \theta} + \frac{\cos\beta}{R}\frac{\partial W^{\eta}}{\partial \theta} + \sin\beta\frac{\partial W^{\eta}}{\partial z}$$

$$+ \left(\frac{\partial}{\partial t} + \frac{V}{R}\frac{\partial}{\partial \theta}\right)\left\{R\,\rho\,\sin\beta\left[\alpha\, h_g + (1-\alpha)h_r\right]\right\} = 0 \qquad (3)$$

where

$$W^{\xi} = R \sin\beta \left\{ - \frac{\rho}{12\mu} h_r^{\;3} \left[G_{1r} \left(\bar{A}_2 \frac{\partial \bar{P}}{\partial \xi} - \alpha \bar{B}_1 \frac{\partial \bar{P}}{\partial \eta} - \alpha \bar{B}_2 \right) + G_{2r} \frac{\partial \bar{P}}{\partial \eta} \right] + \rho\, h_r \frac{(U-V)}{2R} \right\}$$

$$W^{\eta} = R \sin\beta \frac{\rho}{12\mu} \left\{ \alpha\, h_g^{\;3} \left[G_{3g} \frac{\partial \bar{P}}{\partial \eta} + G_{2g} \left(\bar{A}_2 \frac{\partial \bar{P}}{\partial \xi} - \alpha \bar{B}_1 \frac{\partial \bar{P}}{\partial \eta} - \alpha \bar{B}_2 \right) \right] + (1-\alpha) h_r^{\;3} \left[G_{3r} \frac{\partial \bar{P}}{\partial \eta} + G_{2r} \left(\bar{A}_2 \frac{\partial \bar{P}}{\partial \xi} - \alpha \bar{B}_1 \frac{\partial \bar{P}}{\partial \eta} - \alpha \bar{B}_2 \right) \right] \right\}$$

$$\frac{\partial \bar{P}}{\partial \eta} = \frac{\partial \bar{P}}{\partial \theta} \frac{\cos\beta}{R} + \frac{\partial \bar{P}}{\partial z} \sin\beta$$

$$\frac{\partial \bar{P}}{\partial \xi} = \frac{\partial \bar{P}}{\partial \theta}$$

$$A_1 = G_{1r} h_r^{\;3}$$

$$A_2 = - G_{1g} h_g^{\;3}$$

$$A_3 = A_2 - \alpha(A_2 + A_1)$$

$$B_1 = G_{2g} h_g^{\;3} - G_{2r} h_r^{\;3}$$

$$B_2 = \frac{6\mu(h_r - h_g)}{R} \, (U-V)$$

$$\bar{A}_1 = A_1/A_3$$

$$\bar{A}_2 = A_2/A_3$$

$$\bar{B}_1 = B_1/A_3$$

$$\bar{B}_2 = B_2/A_3$$

and where G_{1r}, G_{1g}, G_{2r}, G_{2g}, etc. are lumped, turbulent flow correction factors defined by Eqs. (65),(66) and (67) in Appendix I.

Eq. (3) was solved numerically on a digital computer using the method of column-wise influence coefficients developed by Castelli and Shapiro (Ref. 6) and Castelli and Pirvics (Ref. 7). Two separate computer programs were developed, one to obtain results for the static and dynamic characteristics of a single film spiral-grooved bearing, and the second to calculate the overall static performance characteristics of a spiral-grooved floating-ring bearing.

<u>Calculation of Performance of Single Film, Spiral-Grooved Journal Bearing</u>

The computer program for calculating the performance characteristics of a single-film spiral-grooved journal bearing requires that the following quantities be specified. (Symbols are defined in the nomenclature).

Reynolds number based on mean radial clearance in the seal region of the bearing $= \dfrac{2\pi\rho(N_i - N_o)RC}{\mu}$

L/D ratio

C/R ratio

Dimensionless pressure at both ends of the bearing = $\frac{p}{\mu(N_i + N_o)} (\frac{C}{R})^2$

Speed ratio factor = $\frac{N_o - N_i}{N_i + N_o}$

Dimensionless rate of change of eccentricity ratio = $\frac{\partial\epsilon}{\partial t}/2\pi(N_i + N_o)$

Dimensionless whirl speed ratio = $\frac{\partial\phi}{\partial t}/2\pi(N_i + N_o)$

Dimensionless rate of change of misalignment angle = $\frac{\partial\gamma}{\partial t}/2\pi(N_i + N_o)$

Eccentricity ratio, ϵ

Angle of misalignment, γ

Groove geometry parameters α, β, Γ, $\bar{Y}$

In addition, one must specify whether the bearing is of configuration 1 or configuration 2 (see Fig. 2). Subject to the above input conditions, Eq. (3) is solved by the computer program to determine the dimensionless pressure distribution $\bar{P}$. From this pressure distribution, the computer program then determines the following performance characteristics of the bearing.

Dimensionless tangential (sin) component of bearing film force,

$$\bar{F}_t = \frac{W \sin \phi}{\mu(N_i + N_o)R^2} (\frac{C}{R})^2 \quad \text{(see Fig. 3)}$$

Dimensionless radial (cos) component of bearing film force,

$$\bar{F}_r = \frac{W \cos \phi}{\mu(N_i + N_o)R^2} (\frac{C}{R})^2 \quad \text{(see Fig. 3)}$$

Dimensionless tangential (sin) component of the moment exerted by the bearing film force about an axis through the initial end of the bearing

$$\bar{M}_t = \frac{M \sin \phi'}{\mu(N_i + N_o)R^3} (\frac{C}{R})^2$$

Dimensionless radial component of the moment exerted by the bearing film force about an axis through the initial end of the bearing,

$$\overline{M}_r = \frac{M \cos \phi'}{\mu(N_i + N_o)R^3} \left(\frac{C}{R}\right)^2$$

Dimensionless flow through the journal, $\overline{Q} = \frac{Q}{R^2 C(N_i + N_o)}$

Dimensionless bearing torque, $\overline{T}_B = \frac{T_B}{WC}$

Dimensionless journal torque, $\overline{T}_j = \frac{T_j}{WC}$

The problem of cavitation of the bearing film is handled by the approximate method of setting all sub-ambient fluid film pressure equal to zero before integrating for loads and flows. Experience with plain journal bearings indicates that this approach yields values for load which are on the order of 5% to 10% conservative when compared to a more exact treatment (Ref. 8). For spiral-grooved bearings, the extent of cavitation is much less than for plain bearings. Thus, one would expect that the error introduced by the approximate method of handling cavitation would not be significant in the case of spiral-grooved bearings.

In the calculation of bearing torque, it is assumed that regions of sub-ambient pressure are cavitated and therefore do not contribute to the shear stress on the journal or bearing.

The program for the single film bearing calculates values of the radial and tangential components of fluid film force F_r and F_t as functions of the steady state eccentricity of the journal, e, the instantaneous rate of change of eccentricity of the journal, $\partial e/\partial t$, and the instantaneous whirl velocity of the journal $\partial \phi/\partial t$. Let us now see how this program may be used to obtain stiffness and damping coefficients for the bearing. Consider the reference axes x and y shown in Fig. 3 where x is taken as the direction of the steady state load, W, and y is normal to this. The stiffness and damping coefficients are defined by

$$dF_x = - K_{xx}\, x - B_{xx} \frac{\partial x}{\partial t} - K_{xy}\, y - B_{xy} \frac{\partial y}{\partial t} \qquad (4)$$

$$dF_y = - K_{yx}\, x - B_{yx} \frac{\partial x}{\partial t} - k_{yy}\, y - B_{yy} \frac{\partial y}{\partial t} \qquad (5)$$

It is shown in Appendix II that for bearings possessing rotational symmetry, K_{xx}, K_{yx} etc. may be determined from derivatives of F_r and F_t with respect to e and $\dot{e}$ by means of the following expressions

$$K_{xx} = \frac{\partial F_r}{\partial e} \cos^2 \phi + \frac{\partial F_t}{\partial e} \cos \phi \sin \phi \qquad (6)$$

$$B_{xx} = \frac{\partial F_r}{\partial \dot{e}} \cos^2 \phi + \frac{\partial F_t}{\partial \dot{e}} \cos \phi \sin \phi - \frac{\sin \phi}{e}\left[\cos \phi \frac{\partial F_r}{\partial \dot{\phi}} + \sin \phi \frac{\partial F_t}{\partial \dot{\phi}}\right] \qquad (7)$$

$$K_{xy} = \frac{\partial F_t}{\partial e} \sin^2 \phi + \frac{\partial F_r}{\partial e} \cos \phi \sin \phi \qquad (8)$$

$$B_{xy} = \frac{\partial F_t}{\partial \dot{e}} \sin^2 \phi + \frac{\partial F_r}{\partial \dot{e}} \cos \phi \sin \phi + \frac{\cos \phi}{e}\left[\cos \phi \frac{\partial F_r}{\partial \dot{\phi}} + \sin \phi \frac{\partial F_t}{\partial \dot{\phi}}\right] \qquad (9)$$

$$K_{yx} = - \frac{\partial F_t}{\partial e} \cos^2 \phi + \frac{\partial F_r}{\partial e} \cos \phi \sin \phi - \frac{W}{e} \sin \phi \qquad (10)$$

$$B_{yx} = - \frac{\partial F_t}{\partial \dot{e}} \cos^2 \phi + \frac{\partial F_r}{\partial \dot{e}} \cos \phi \sin \phi - \frac{\sin \phi}{e}\left[\sin \phi \frac{\partial F_r}{\partial \dot{\phi}} - \cos \phi \frac{\partial F_t}{\partial \dot{\phi}}\right] \qquad (11)$$

$$K_{yy} = \frac{\partial F_r}{\partial e} \sin^2 \phi - \frac{\partial F_t}{\partial e} \cos \phi \sin \phi + \frac{W}{e} \cos \phi \qquad (12)$$

$$B_{yy} = \frac{\partial F_r}{\partial \dot{e}} \sin^2 \phi - \frac{\partial F_t}{\partial \dot{e}} \cos \phi \sin \phi + \frac{\cos \phi}{e}\left[\sin \phi \frac{\partial F_r}{\partial \dot{\phi}} - \cos \phi \frac{\partial F_t}{\partial \dot{\phi}}\right] \qquad (13)$$

where $\dot{e} = \frac{\partial e}{\partial t}$ is the instantaneous rate of change of eccentricity and $\dot{\phi} = \frac{\partial \phi}{\partial t}$ is the whirl velocity of the journal.

The determination of the derivatives $\partial F_r/\partial e$, $\partial F_r/\partial \dot{e}$ etc. from the single film program is tedious but essentially straight forward. Care must be taken, however, that the changes in e, $\dot{e}$ and $\dot{\phi}$ used to evaluate these derivatives be chosen sufficiently small such that the results accurately apply to infinitesimally small amplitude motions of the journal center about a steady state position*. It is recommended that one keep $\Delta e/C < .05$, $\Delta\dot{e}/2\pi(N_i + N_o)C < .05$ and $e\dot{\phi}/2\pi(N_i + N_o)C < .05$ for evaluation of the above mentioned derivatives.

A detailed description of how to prepare input for the single film spiral-grooved journal bearing program is given in Appendix III together with a listing of the program.

Calculation of Steady State Performance of Floating Ring Bearing

The program for calculating the performance of a floating ring, spiral-grooved bearing consists, essentially, of two parts. The first part contains the program for a single film, spiral grooved journal bearing described above. This is used to calculate the individual performance characteristics of the inner and outer films of the floating ring bearing. The second part of the program consists of the logic required to determine the correct ring speed and eccentricity ratio of the outer film such that the load capacity of the outer film is equal to the load capacity of the inner film and the torque exerted by fluid shear stresses on the

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* The linear formulation represented by Eqs. (4) and (5) implicitly carries the assumption that the motions x, y, $\partial x/\partial t$, $\partial y/\partial t$ are vanishingly small.

inner surface of the ring is balanced by the fluid shear stresses on the outer surface of the ring. In detail, the computational procedure works as described below

1. To determine the overall preformance of the floating ring bearing requires that the following quantities be specified as input to the program.

Radius ratio of ring and journal = R_2/R_1 where R_2 is the outside radius of the ring and R_1 is the radius of the journal.

Overall Reynolds number under which the bearing is to be operated $Re = 2\pi\rho N_1 R_1 C_1/\mu$

Eccentricity ratio of inner film to be examined = ϵ_1

Clearance to radius ratio for inner film = C_1/R_1

Clearance to radius ratio for outer film = C_2/R_2

Dimensionless supply pressure to center of bearing $=(P_s/\mu N_1)\ (C_1/R_1)^2$

Length to inner diameter ratio for bearing = L/D_1

Groove geometry parameters α, β, Γ and $\bar{Y}$

2. Given this input, the first thing the program does is to calculate and store in tabular form the following performance data for the inner and outer films of the floating ring bearing.

<u>Inner film</u>

Sommerfeld No. $S_1 = \dfrac{\mu(N_1 + N_2)D_1 L}{W} \left(\dfrac{R_1}{C_1}\right)^2$

Dimensionless torque on journal $(\bar{T}_J)_1 = \dfrac{(T_J)_1}{WC_1}$

Dimensionless torque on inside of ring $(\overline{T}_B)_1 = \frac{(T_B)_1}{WC_1}$

Attitude angle ϕ_1

Outer film

Sommerfeld No. $S_2 = \frac{\mu N_2 D_2 L}{W} \left(\frac{R_2}{C_2}\right)^2$

Dimensionless torque on outside of ring $(\overline{T}_J)_2 = \frac{(T_J)_2}{WC_2}$

Dimensionless torque on inside of bearing $(\overline{T}_B)_2 = \frac{(T_B)_2}{WC_2}$

Attitude angle ϕ_2

These performance data are stored in tables as functions of the eccentricity ratio and Reynolds number for the film concerned. Values are calculated for three predetermined values of eccentricity ratio and three predetermined values of Reynolds number. Thus, 9 separate calculations of performance characteristics must be made for each film (18 calculations altogether). The Reynolds number for the inner film is defined as

$$Re_1 = \frac{2\pi\rho(N_1 - N_2)R_1C_1}{\mu} \qquad (14)$$

while the Reynolds number for the outer film is defined as

$$Re_2 = \frac{2\pi\rho N_2R_2C_2}{\mu} \qquad (15)$$

Note that since the overall Reynolds number Re is specified, Re_1 and Re_2 are determined uniquely by the ring speed ratio N_2/N_1. Typically, the three values of Re_1 and Re_2 for which film characteristics are calculated are those corresponding to $N_2/N_1 = 0.25$, 0.35 and 0.45. Typical values of eccentricity ratio for which the inner film characteristics are determined are $\epsilon_1 = 0.2$, 0.3 and 0.5. The three values of ϵ_2 selected for calculation of outer film data are chosen in accordance with anticipated operating eccentricities of the outer film.

3. Once the tables of inner and outer film characteristics are prepared, the program next considers an initial guess for the ring speed ratio N_2/N_1. This initial guess is read in as input to the program. From this ring speed ratio, the program then calculates a value for Re_1. Corresponding to this value for Re_1, and the value of ϵ_1 read into the program, there will be specific values of S_1 and $(\overline{T}_B)_1$ which the program will determine from the tabular data for the inner film characteristics. The program will interpolate within the tables if necessary.

4. The program next determines S_2 from the condition that the load capacities of the inner and outer film must be equal. This condition is expressed by

$$S_2 = S_1 \left(\frac{N_2}{N_1 + N_2}\right) \frac{R_2}{R_1} \left(\frac{R_2}{C_2}\right)^2 \left(\frac{C_1}{R_1}\right)^2 \tag{16}$$

The program also determines Re_2 corresponding to the guessed value of ring speed ratio.

5. With S_2 and Re_2 calculated, the program next determines the corresponding values of, $(\overline{T}_J)_2$ and ϵ_2 from the tabular data for the outer film performance characteristics. The program then checks to see if

$$(\overline{T}_J)_2 = (\overline{T}_B)_1 \frac{C_1}{C_2} \tag{17}$$

i.e. the program checks to see if the torque on the outside of the floating ring matches the torque on the inside. If they match, the solution is complete. If the torque on the outside of the ring is lower (higher), then a slightly higher (lower) value for ring speed is guessed and the process is repeated until a convergent solution is obtained.

When a convergent solution is obtained for the floating ring bearing, the program prints out various performance data for the individual films and for the overall floating ring bearing. This output is described fully in Appendix IV. Detailed instructions for preparing the input for the floating ring program are also provided in this appendix along with a listing of the program.

Stiffness and Damping Coefficients for Floating Ring Bearing

In order to determine the overall stiffness and damping coefficients for the floating ring bearing, it is necessary to first determine the stiffness and damping coefficients for each individual bearing film. For each steady state solution for the floating ring bearing, the steady state operating conditions for each film are established. Stiffness and damping coefficients for each film can therefore be determined by the single-film, spiral-grooved journal bearing program as described earlier.

Let us denote the stiffness and damping coefficients for the inner film by the subscript 1 and those for the outer film by the subscript 2, i.e., K_{xx1}, K_{xx2}, etc. The overall stiffness and damping coefficients for the floating ring bearing are denoted simply as K_{xx}, K_{xy}, etc. Consider that the shaft moves in sychronous whirl with a frequency $\omega_1 = 2\pi N_1$ radians/sec. and with amplitude components $\bar{x}_1 e^{i\omega_1 t}$. The overall damping and stiffness coefficients are defined by the following relationships.

$$\frac{F_x e^{i\omega_1 t}}{W} = -\left(\frac{C_1 K_{xx}}{W} + i\,\frac{C_1 \omega_1 B_{xx}}{W}\right)\frac{\bar{x}_1}{C_1}\,e^{i\omega_1 t} - \left(\frac{C_1 K_{xy}}{W} + i\,\frac{C_1 \omega_1 B_{xy}}{W}\right)\frac{\bar{y}_1}{C_1}\,e^{i\omega_1 t} \tag{18}$$

$$\frac{F_y e^{i\omega_1 t}}{W} = -\left(\frac{C_1 K_{yx}}{W} + i\,\frac{C_1 \omega_1 B_{yx}}{W}\right)\frac{\bar{x}_1}{C_1}\,e^{i\omega_1 t} - \left(\frac{C_1 K_{yy}}{W} + i\,\frac{C_1 \omega_1 B_{yy}}{W}\right)\frac{\bar{y}_1}{C_1}\,e^{i\omega_1 t} \tag{19}$$

or in short:

$$\begin{Bmatrix} \frac{F_x}{W} \\ \frac{F_y}{W} \end{Bmatrix} = -\begin{Bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{Bmatrix} \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} \tag{20}$$

where:

$$Z_{xx} = \frac{C_1 K_{xx}}{W} + i\,\frac{C_1 \omega_1 B_{xx}}{W} \quad \text{(analogous for } Z_{xy}, Z_{yx}, Z_{yy}) \tag{21}$$

$$x_1 = \frac{\bar{x}_1}{C_1} \tag{22}$$

$$y_1 = \frac{\bar{y}_1}{C_1} \tag{23}$$

Let the center of the ring have whirl amplitudes $\bar{x}_2 e^{i\omega_1 t}$ and $\bar{y}_2 e^{i\omega_1 t}$ and let:

$$x_2 = \frac{\bar{x}_2}{C_1} \tag{24}$$

$$y_2 = \frac{\bar{y}_2}{C_1} \tag{25}$$

The dimensionless dynamic coefficients for the inner and outer film are obtained in the form:

Inner film:

$$\frac{C_1 K_{xx1}}{W}, \quad \frac{C_1(\omega_1 + \omega_2)B_{xx1}}{W}, \text{ etc} \tag{26}$$

Outer film:

$$\frac{C_2 K_{xx2}}{W}, \quad \frac{C_2 \omega_2 B_{xx2}}{W}, \text{ etc} \tag{27}$$

Define:

Inner film:

$$Z_{xx1} = \frac{C_1 K_{xx1}}{W} + i \frac{1}{1+n} \frac{C_1(\omega_1 + \omega_2)B_{xx1}}{W} \tag{28}$$

(similarly for Z_{xy1}, Z_{yx1}, Z_{yy1})

Outer film:

$$Z_{xx2} = \frac{C_1}{C_2} \frac{C_2 K_{xx2}}{W} + i \frac{C_1}{C_2} \frac{1}{n} \frac{C_2 \omega_2 B_{xx2}}{W} \tag{29}$$

(similarly for Z_{xy2}, Z_{yx2}, Z_{yy2})

where:

$$n = \frac{N_2}{N_1}$$

Hence, the dynamic forces acting on the shaft become:

$$\begin{Bmatrix} \frac{F_x}{W} \\ \\ \frac{F_y}{W} \end{Bmatrix} = - \begin{Bmatrix} Z_{xx1} & Z_{xy1} \\ \\ Z_{yx1} & Z_{yy1} \end{Bmatrix} \begin{Bmatrix} x_1 - x_2 \\ \\ y_1 - y_2 \end{Bmatrix} . \tag{30}$$

In order to bring Eq. (30) into the same form as Eq. (20) and thereby determine the overall dynamic coefficients it is necessary to eliminate x_2 and y_2 from the equations. This is done by setting up the equations of motion for the ring with mass m:

$$-\frac{C_1 m\omega_1^2}{W}\begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} = -\begin{Bmatrix} Z_{xx1} & Z_{xy1} \\ Z_{yx1} & Z_{yy1} \end{Bmatrix}\begin{Bmatrix} x_1 - x_2 \\ y_1 - y_2 \end{Bmatrix} - \begin{Bmatrix} Z_{xx2} & Z_{xy2} \\ Z_{yx2} & Z_{yy2} \end{Bmatrix}\begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} \tag{31}$$

or:

$$\begin{Bmatrix} (Z_{xx1} + Z_{xx2} - \frac{C_1 m\omega_1^2}{W}) & (Z_{xy1} + Z_{xy2}) \\ (Z_{yx1} + Z_{yx2}) & (Z_{yy1} + Z_{yy2} - \frac{C_1 m\omega_1^2}{W}) \end{Bmatrix}\begin{Bmatrix} x_1 - x_2 \\ y_1 - y_2 \end{Bmatrix} =$$

$$\begin{Bmatrix} (Z_{xx2} - \frac{C_1 m\omega_1^2}{W}) & Z_{xy2} \\ Z_{xy2} & (Z_{yy2} - \frac{C_1 m\omega_1^2}{W}) \end{Bmatrix}\begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} \tag{32}$$

Substitute Eq. (32) into Eq. (30) and compare with Eq. (20) to get:

$$\begin{Bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{Bmatrix} = \begin{Bmatrix} Z_{xx1} & Z_{xy1} \\ Z_{yx1} & Z_{yy1} \end{Bmatrix}\begin{Bmatrix} (Z_{xx1} + Z_{xx2} - \frac{C_1 m\omega_1^2}{W}) & (Z_{xy1} + Z_{xy2}) \\ (Z_{yx1} + Z_{yx2}) & (Z_{yy1} + Z_{yy2} - \frac{C_1 m\omega_1^2}{W}) \end{Bmatrix}^{-1}$$

$$\begin{Bmatrix} (Z_{xx2} - \frac{C_1 m\omega_1^2}{W}) & Z_{xy2} \\ Z_{yx2} & (Z_{yy2} - \frac{C_1 m\omega_1^2}{W}) \end{Bmatrix} \tag{33}$$

Solving for Z_{xx}, Z_{xy}, etc. and using Eq. (21) yields the overall dynamic coefficients for the floating-ring bearing.

In general the mass of the ring is relatively small such that:

$$\frac{C_1 m\omega_1^2}{W} << \frac{C_2 K_{xx}}{W}$$

in which case it can be ignored in the calculations. This condition applies to all the numerical results given in the present report.

<u>Stability Calculation for Floating Ring Bearing</u>

The overall stability of the floating ring journal bearing to self-excited whirl may be calculated from the overall dynamic coefficients described above. In this present study, the stability calculations were performed by means of the computer program developed under USAF contract No. AF 33(615)-3238 and described in part V of the final documentation issued under this contract (Ref. 9). A brief description of the analysis upon which the stability calculations are based is provided below.

At any given rotor speed and with a known static load on the bearing, the journal center occupies a certain unique equilibrium position relative to the bearing center. When the journal whirls around this equilibrium in a small orbit, the dynamic forces F_x and F_y generated in the bearing fluid film can be expressed in linearized form as:

$$F_x = - K_{xx} x - B_{xx} \frac{dx}{dt} - K_{xy} y - B_{xy} \frac{dy}{dt} \qquad (34)$$

$$F_y = - K_{yx} x - B_{yx} \frac{dx}{dt} - K_{yy} y - B_{yy} \frac{dy}{dt} \qquad (35)$$

where x and y are the whirl amplitudes measured from the static equilibrium position, t is time, and the four spring coefficients (the K - coefficients) and the four damping coefficients (the B - coefficients) would be determined for the floating ring bearing from the analysis described above. For a given bearing geometry and known lubricant properties, the 8 coefficients are functions of the bearing load and the rotor speed and, if the lubricant is compressible like a gas, they are also functions of the whirl frequency. In the latter case, Eqs. (34) and (35) are only valid for harmonic motions such that:

$$x = x_c \cos(\nu t) - x_s \sin(\nu t) \tag{36}$$

$$y = y_c \cos(\nu t) - y_s \sin(\nu t) \tag{37}$$

where ν is the angular whirl frequency. These equations can also be written:

$$x = \mathrm{Re}\left\{(x_c + ix_s)e^{i\nu t}\right\} \tag{38}$$

$$y = \mathrm{Re}\left\{(y_c + iy_s)e^{i\nu t}\right\} \tag{39}$$

where $i = \sqrt{-1}$ and "Re{ }" means that only the real part of the bracketed expression applies. For convenience the "Re{ }" and the $e^{i\nu t}$ are dropped whereby Eqs. (38) and (39) are written

$$x = x_c + ix_s \tag{40}$$

$$y = y_c + iy_s \tag{41}$$

When these equations are used in the analysis their complete meaning is defined through Eqs. (38) and (39).

With this notation Eqs. (34) and (35) can be written:

$$F_x = -\bar{Z}_{xx}\, x - \bar{Z}_{xy}\, y \tag{42}$$

$$F_y = -\bar{Z}_{yx}\, x - \bar{Z}_{yy}\, y \tag{43}$$

where:

$$\bar{Z}_{xx} = K_{xx} + i\left(\frac{\nu}{\omega}\right)\omega B_{xx} = K_{xx} + i\bar{\gamma}\omega B_{xx} \tag{44}$$

$$\bar{\gamma} = \frac{\nu}{\omega} \tag{45}$$

and similarly for $\bar{Z}_{xy}$, $\bar{Z}_{yx}$ and $\bar{Z}_{yy}$. Here, ω is the angular speed of rotation and $\bar{\gamma}$ gives the ratio between the whirl frequency and the rotational frequency. In this form, the equations are equally valid for an incompressible and a compressible lubricant.

To illustrate the procedure for calculating the threshold of instability, assume for simplicity that the rotor is rigid and symmetric such that the two bearings support an equal mass M which equals half the mass of the rotor. Then the equations of motion for a journal become:

$$M\frac{d^2x}{dt^2} = F_x$$

$$M\frac{d^2y}{dt^2} = F_y \tag{46}$$

By substitution from Eqs. (41) through (43), these equations can be written in matrix form:

$$\begin{Bmatrix} (\bar{Z}_{xx} - M\nu^2) & \bar{Z}_{xy} \\ \bar{Z}_{yx} & (\bar{Z}_{yy} - M\nu^2) \end{Bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \tag{47}$$

At the threshold of instability, a non-zero solution of x and y must exist which means that the determinant Δ of the matrix should be zero:

$$\Delta = \Delta_c + i\Delta_s = (\bar{Z}_{xx} - M\nu^2)(\bar{Z}_{yy} - M\nu^2) - \bar{Z}_{xy}\,\bar{Z}_{yx} = 0 \tag{48}$$

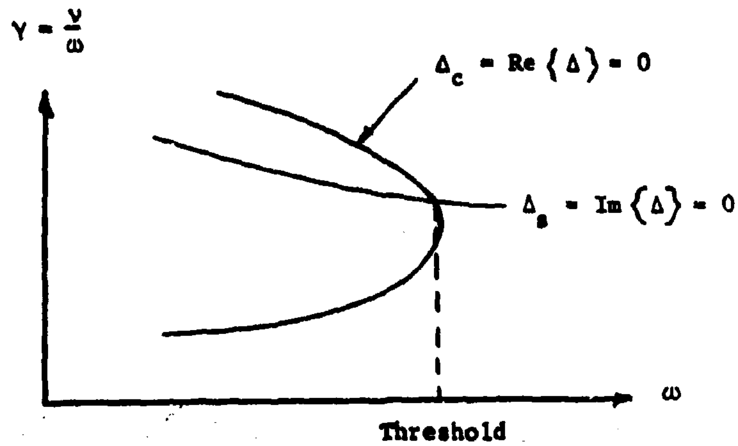
or

$$\Delta_c = R_e\left\{\Delta\right\} = (K_{xx} - \bar{\gamma}^2 M\omega^2)(K_{yy} - \bar{\gamma}^2 M\omega^2) - K_{xy}\,K_{yx}$$

$$- \bar{\gamma}^2 \left[\omega B_{xx}\,\omega B_{yy} - \omega B_{xy}\,\omega B_{yx}\right] = 0 \tag{49}$$

$$\Delta_s = \mathrm{Im}\left\{\Delta\right\} = \bar{\gamma} \left[(K_{xx} - \bar{\gamma}^2 M\omega^2)\omega B_{yy} + (K_{yy} - \bar{\gamma}^2 M\omega^2)\omega B_{xx} - K_{xy}\omega B_{yx} - K_{yx}\omega B_{xy}\right] = 0 \tag{50}$$

These two equations must be satisfied simultaneously at the threshold of instability. They contain two unknowns, namely the whirl frequency ratio, $\bar{\gamma}$, and the angular speed of rotation, ω. In the general case, the 8 dynamic fluid film coefficients are functions of both $\bar{\gamma}$ and ω, making a closed form solution impossible, and the solution is most conveniently obtained graphically. For any fixed value of ω, Δ_c and Δ_s can be plotted as functions of $\bar{\gamma}$ to find their zero points. With $\bar{\gamma} > 0$ it is seen that Δ_s has one zero point and Δ_c has up to two zero points (only true in this simple case). The calculation is repeated for several values of ω and the results may be plotted as shown:



The intersection of the two curves define the speed at which instability sets in.

RESULTS

Single Film Bearing

Results of calculations of the performance of single film spiral-grooved journal bearings are shown in Figs. 7 through 18. In all cases, the results shown are for bearings having groove geometry optimized for maximum radial component of stiffness at $\epsilon = 0$. These optimum values of groove geometry parameters are given in Tables 1 and 2 presented earlier in the text.

Load capacity of the single film spiral-grooved journal bearing is shown in Figs. 7, 8, 9 and 10, in terms of dimensionless load $W/\mu N_1 LD\ (C/R)^2$ vs. eccentricity ratio ϵ. Fig. 7 shows results for a bearing of configuration 1 i.e. a bearing having no flow-through of lubricant (see Fig. 2). L/D ratio is taken to be 1.0. It is assumed that only the grooved journal is rotating. In this figure, it can be noted that the existence of turbulence does not significantly effect the linearity of the load vs. eccentricity curve, but only serves to increase the load capacity over that which would be obtained if flow remained laminar.

In Fig. 8 are shown load vs. eccentricity curves for a bearing of configuration 2 with L/D = 1.0, in which there is net flow of lubricant pumped through the bearing entirely by action of the spiral grooving ($P_s = 0$). The load capacity of this bearing is seen to be slightly less than that for the configuration 1 bearing. On the other hand, the through-flow of lubricant is useful in removing heat from tha bearing.

Fig. 9 shows load capacity of a configuration 2 type bearing with L/D = 0.5. As can be seen, unit load capacity, W/LD, decreases significantly with decrease in L/D ratio. As a rough guide, it is found that in the range $0.5 \leq L/D \leq 2.0$, unit load capacity is very nearly proportional to L/D ratio.

To provide a greater flow of lubricant through a configuration 2 bearing, one can supply the lubricant to the grooved end at an elevated pressure P_s. The effect of this on load capacity is relatively slight as shown by the curves in

Fig. 10. The results are shown for Re = 5000 but are typical of those obtained at all values of Reynolds number. The degree of pressurization considered is indicated by the dimensionless parameter $\bar{P}_s G_z D/L$ where $\bar{P}_s$ is the dimensionless supply pressure defined as $\bar{P}_s = \left[P_s/\mu(N_i + N_o)\right](C/R)^2$ and G_z is a turbulent viscosity correction factor corresponding to the mean Reynolds No. (G_z is obtained from Fig. 5). When the supply pressure parameter $\bar{P}_s G_z (D/L)$ is maintained at 0.35, the net flow of lubricant through the bearing due to pressurization is approximately equal to that due to self-pumping of the grooves, independent of Reynolds number. When this parameter doubles, flow due to pressurization doubles.

In general one may conclude that for values of $\bar{P}_s G_z (D/L)$ less than 0.7, the effect of pressurization on load capacity may be neglected. In any case, it is conservative to do so since pressurization tends to increase load capacity*.

Figs. 11, 12, and 13 show curves of attitude angle ϕ vs. eccentricity ratio for configuration 1 and configuration 2 bearings at different Reynolds numbers and L/D ratios. The effect of pressurization on attitude angle is also shown by the dashed curves in Figs. 12 and 13.

Referring to Fig. 11, which gives attitude angle for a configuration 1 bearing, we see that for laminar flow ϕ decreases slightly with eccentricity ratio. This decrease is mostly due to effects of cavitation in the bearing film. At higher values of Reynolds number, very little or no cavitation occurs in the bearing film out to $\epsilon = 0.7$ and, as a consequence, attitude angle shows very little dependence on ϵ. The extent of cavitation that does occur in the bearing will be discussed later in connection with predicted bearing torque.

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* Pressurization of the grooved end of a spiral-grooved journal bearing produces hydrostatic load capacity whether the journal is rotating or not. Such pressurization will help to promote rotation of the ring in a spiral-grooved floating ring bearing. Getting the ring to rotate can be somewhat of a problem in plain, cylindrical floating ring bearings. Whether this is true for spiral-grooved floating ring bearings remains to be seen.

For configuration 2 bearings, we find again that for laminar flow, ϕ decreases with ϵ due to cavitation whereas this effect is less pronounced at higher values of Reynolds number. We also find that pressurization tends to decrease attitude angle. In most instances, this effect is not great, although for laminar flow and L/D = 0.5, a pressurization of $\bar{P}_s G_z$ D/L = 0.7 produces approximately a 10 degree reduction in attitude angle.

Comparing Figs. 11 and 12, we see that a configuration 2 bearing of L/D = 1.0 has a slightly higher attitude angle than a configuration 1 bearing of the same L/D ratio. Comparing Figs. 12 and 13 we see that for configuration 2 spiral-grooved journal bearings, attitude angle decreases quite substantially as L/D ratio is decreased from 1.0 to 0.5. This latter effect does not occur in plain cylindrical bearings.

In general, we can observe that development of turbulence in spiral-grooved journal bearings reduces the attitude angle by a significant amount i.e. approximately 10 degrees at low eccentricity ratios for both configuration 1 and configuration 2 geometries.

The bearing through-flow, Q_p, that is generated by the self-pumping of spiral grooving is shown in Fig. 14. Results are plotted in terms of $Q_p/R^2C(N_i - N_o)$ vs. ϵ. The results shown were obtained neglecting effects of cavitation. This was done because the way in which cavitation is handled in the present analysis does not provide an accurate calculation of flow rate when cavitation appears. This is not a serious deficiency since spiral-grooved bearings usually operate with a full fluid film.

As Fig. 14 indicates, the "self-pumping" flow of an optimized bearing increases as turbulence develops in the bearing film. This is probably due to the fact that the optimum value of groove depth increases as turbulence develops. When turbulence is fully developed (Re = 5000) both flow and optimum pocket depth approach asymptotic values.

It should be kept in mind that the results shown in Fig. 14 pertain to a bearing with $\bar{Y} = 0.67$ i.e. a bearing with grooving on 67% of its length. Flow through the bearing can be increased by increasing $\bar{Y}$ slightly with very little penalty paid in terms of loss of load.

The flow of lubricant, Q_s, through a spiral-grooved bearing that results from pressurizing either end of the bearing is directly proportional to $P_s C^3 G_z/\mu$ (D/L) for any given eccentricity ratio and Reynolds number. Curves of dimensionless pressure flow, $Q_s \mu / G_z C^3 (L/D)$ are plotted vs. ϵ in Fig. 15. Use of the turbulent viscosity correction factor G_z in forming this dimensionless flow takes account of the influence of Reynolds number quite well although some slight dependence on Reynolds number still remains.

The dimensionless torque, $T_j C/\mu N_1 R^3 L$, on the journal of a spiral-grooved bearing is plotted in Fig. 16 for a configuration 1 bearing and in Fig. 17 for a configuration 2 bearing. The solid lines show the torque for a bearing with a complete fluid film while the dashed curve shows the torque taking account of cavitation that would be expected to occur. Cavitation is accounted for by assuming that all regions of subambient pressure are cavitated and that shear stresses in these regions are negligible. The discrepencies between the dashed and solid curves in Figs. 16 and 17 provide an indication of the extent of cavitation that develops as eccentricity increases.

Looking at Fig. 16, we see that for a configuration 1 bearing, cavitation does not set in until $\cong 0.3$ for laminar flow, until $\epsilon \cong 0.5$ for Re = 1000, and does not occur at all below $\epsilon = 0.7$ for Re = 5000 and 9000. For an unpressurized configuration 2 bearing (Fig. 17), cavitation occurs at lower values of ϵ than for a configuration 1 bearing and does occur at Re = 5000 and 9000. Cavitation can easily be eliminated, however, by modest pressurizing of the bearing.

The solid curves shown in Fig. 17 also apply with reasonable accuracy to bearings with L/D = 0.5. The dashed curves do not, however, because there is less tendency for a spiral-grooved bearing to cavitate as L/D ratio is decreased. For configuration 2 bearings with L/D = 0.5, no cavitation occurs for Re $>$ 1000 and $\epsilon <$ 0.7.

An important quantity to consider is how the dimensionless ratio of friction torque to load, T_j/WC, varies with Reynolds number. This is shown in Fig. 18 for $\epsilon = 0.2$, 0.5 and $\epsilon = 0.7$. As can be seen, torque increases more rapidly than does load when turbulence develops in the bearing film. One can note at this point that one of the advantages of the floating ring bearing is that the Reynolds number in each separate bearing film is less than the Reynolds number that would be obtained if the bearing had only a single film. Hence, due to this effect alone, the floating ring bearing can operate with a more favorable torque to load ratio than an equivalent bearing with only a single film.

Floating Ring Bearing

The floating ring configuration chosen for analysis was one having an overall length to diameter ratio of $L/D = 1.0$. (See Figure 4). This means that each half of the floating ring bearing had an effective length to diameter ratio of 0.5. Grooving parameters for each half of the floating ring bearing were those presented in Table 2. Grooving on the outside surface of the ring was the same as that on the shaft.

Two values of the ratio of inner clearance to outer clearance were considered i.e. $C_2/C_1 = 1.2$ and 0.8. Results were obtained for no pressurization of the bearing ($\bar{P}_s = 0$) and for a degree of pressurization corresponding to $\bar{P}_s\ G_z\ D/L = 0.35$.

The static performance data for the floating ring journal bearing are given in Table 3. Dynamic performance data are given in Tables 4 and 5. Much of this data is presented in graphical form in Figs. 19 through 36. These figures are discussed below.

Curves of dimensionless load, $\bar{W}$, vs the eccentricity ratio of the inner film ϵ, are presented in Fig. 19. The curves are for $C_2/C_1 = 1.2$ although they apply within a few percent accuracy to the case of $C_2/C_1 = 0.8$. As can be seen the degree of pressurization considered for the floating ring bearing (dashed curves) results in a substantial increase in load capacity over the unpressurized case (solid curves). For the single film bearing, the same apparent degree of pressurization resulted in only a slight increase in load capacity (See Figure 10). The reasons for the greater apparent effect of pressurization on the floating ring bearing are as follows.

First, the parameter $\overline{P}_s\ G_z D_1/L$ for the floating ring bearing is based on the overall L/D_1. The effective value of this parameter for each half of the floating ring bearing is actually twice the overall value. Second, the value for G_z used in establishing the parameters $\overline{P}_s\ G_z D_1/L$ is based on the overall Reynolds number $Re = 2\pi N_1 R_1 C_1/\nu$. Because of rotation of the ring, the individual Reynolds numbers for the inner and outer films are each less than the overall. Hence, the effect of pressurization of each film is greater than is indicated by the parameter $\overline{P}_s\ G_z\ D_1/L$ because, in turbulent flow, the lower the Reynolds number the greater the amount of flow for a given supply pressure. Third, the dimensionless supply pressure $\overline{P}_s$ is based on the shaft speed N_1. For spiral-grooved bearings, it turns out that rotation of the ring decreases rather than increases the load capacity of the inner film. Hence pressurization of the inner film becomes relatively more significant with respect to load capacity of the film.

Roughly speaking, the increase in load capacity resulting from pressurization of a floating ring bearing is linearly proportional to the guage supply pressure. One can therefore linearly interpolate between the curves shown in Fig. 19 to determine load capacity at supply pressures different from that considered.

Curves of overall attitude angle ϕ of the floating ring bearing are shown in Figs. 20 and 21. Overall attitude angle is defined in Fig. 4. Values for the attitude angles of the inner and outer films are given in Table 3. Due to rotation of the ring, which decreases the spiral-grooved pumping effect in the inner film, attitude angles for the inner film are considerably greater than for the outer film.

Values of dimensionless journal torque, $\overline{T}_j = T_j/WC$, are given in Fig. 22 for a floating ring bearing with $C_2/C_1 = 1.2$. Similar curves for a bearing with $C_2/C_1 = 0.8$ would run about 7% higher.

One of the primary advantages of the floating ring bearing is that, for a given eccentricity of the inner film, the ratio of torque to load is much lower than for a comparable single film bearing. This is particularly true for plain bearings for which load capacity of the inner film is proportional to the sum of the speeds of the shaft and ring. In spiral grooved bearings, however, the pumping effect of the grooves is proportional to the difference in speed between the shaft and ring. Load capacity of these bearings is due partly to this pumping effect and partly to the usual hydrodynamic effect which is proportional to the sum of the shaft and ring speed. The net effect is that as ring speed increases, load capacity of the inner film for a spiral-grooved bearing decreases slightly. Consequently, spiral-grooved floating ring bearings do not enjoy the same torque to load advantage possessed by plain floating ring bearings. Nonetheless, the spiral-grooved floating ring bearing does have a torque to load ratio better than that of a single film bearing operating at the same eccentricity ratio. This is evidenced by the curves shown in Fig. 23. The single film bearing used for comparison in this figure is a configuration 2 bearing with $L/D = 0.5$. This provides a fair comparison because each side of the centrally fed floating ring configuration we are considering consists, essentially, of an isolated bearing with $L/D = 0.5$.

Total flow pumped through the floating ring bearing by the self-pumping effect of the spiral grooves is plotted in Fig. 24. The increase in lubricant flow that would result from pressurization of the bearing can be calculated from the single film curves plotted in Fig. 15. These single film curves can be applied directly to each individual film of the floating ring bearing on either side of the central feeding groove. In applying these curves to calculate pressure flow, one must be careful to use the appropriate Reynolds number and L/D ratio corresponding to the individual film being considered. The pressure flow that is calculated can be added directly to the self-pumping flow calculated from Fig. 24.

Ring speed ratio, N_2/N_1, is plotted as a function of inner film eccentricity ϵ_1 in Figs. 25a and 25b. Results for laminar flow and Re = 9000 are shown. Results for Re = 5000 are nearly the same as for Re = 9000 while results for Re = 1000 lie between the Re = 9000 and laminar curves. Ring speed ratio for laminar flow tends to decrease with ϵ_1 while, for turbulent flow, ring speed ratio remains relatively constant with eccentricity. Note that ring speed ratio is not strongly affected by the clearance ratio C_2/C_1.

ϵ_2, the eccentricity ratio for the outer film, is plotted as a function of ϵ_1 in Fig. 25. In general, ϵ_2 increases approximately linearly with ϵ_1. As would be expected, ϵ_2 is considerably greater than ϵ_1 for $C_2/C_1 = 1.2$ and more nearly equal to ϵ_1 for $C_2/C_1 = 0.8$.

Dynamic data for the floating ring bearing are given in Tables 4 and 5. These data consist mostly of stiffness and damping coefficients for the individual bearing films and for the overall bearing. The data also include values of the dimensionless critical mass for the threshold of whirl instability.

Data for the individual bearing films are given in Table 4. Stiffness and damping for the inner film pertain to the fluid film forces that develop in the inner film due to <u>relative</u> motion between the shaft and the ring. Stiffness and damping for the outer film pertain to fluid film forces developed in that film due to <u>relative</u> motion between the ring and the outer bearing.

Stiffness and damping coefficients for the overall floating ring bearing are given in Table 5. These pertain to the forces developed on the shaft due to a relative motion between the shaft and the outer bearing including the effect of motion of the ring.

Overall stiffness and damping coefficients for the floating ring bearing are plotted vs. ϵ_1 for a number of representative situations in Figs. 27 through 32. Qualitatively, the behavior of the overall stiffness and damping coefficients as eccentricity ratio increases is similar to that for single film bearings.

The stiffness and damping coefficients presented in Tables 4 and 5 may be used to calculate a critical mass at the threshold of whirl instability. The critical masses determined from the inner or outer film damping and stiffness coefficients, while not of great physical significance, are still of interest to calculate. The critical mass M_{c1}, determined from the inner film coefficients, is the critical mass of the shaft within the rotating ring assuming that the ring were restricted from any translational motion i.e. assuming that the outer film were infinitely stiff. M_{c2}, the critical mass calculated from the outer film coefficients, is the critical mass for the ring rotating within the bearing neglecting any effect of the inner bearing film. Essentially, M_{c2} represents the critical mass for a single film bearing operating at Reynolds number Re_2 and eccentricity ratio ϵ_2.

M_c, the critical mass determined from the stiffness and damping coefficients for the overall floating ring bearing, represents the critical mass for the shaft rotating within the composite floating ring structure. It is the appropriate value of critical mass for the shaft of the floating ring bearing.

The critical masses M_{c1}, M_{c2} and M_c are presented in Tables 4 and 5 in the dimensionless form

$$\bar{M}_{c1} = \frac{M_{c1}\, C_1\, (N_1 + N_2)}{\mu (R_1/C_1)^2\, LD_1}$$

$$\bar{M}_{c2} = \frac{M_{c2}\, C_2\, N_2}{\mu (R_2/C_2)^2\, LD_2}$$

$$\bar{M}_{c} = \frac{M_{c}\, C_1\, N_1}{\mu (R_1/C_1)^2\, LD_1}$$

In attempting to compare the critical masses in these tables one should keep in mind that each is made non-dimensionless on a slightly different basis.

Values of overall critical mass vs. ϵ_1 are plotted in Figs. 33 and 34 for

$C_2/C_1 = 1.2$ and 0.8 respectively. In general, critical mass for hydrodynamic bearings increases with eccentricity ratio. However, when operating in the turbulent regime, ·al-grooved bearings exhibit the anomalous characteristic that critical mass decreases with eccentricity in certain ranges. This characteristic may be related to the fact that the attitude angle of spiral-grooved bearings sometimes increases with eccentricity ratio in the turbulent regime. This increase in attitude angle can be qualitatively explained on the basis that as eccentricity increases, the usual plain-bearing-type of hydrodynamic action becomes relatively more significant compared with the self-pressurizing hydrodynamic action of the spiral grooves, particularly in turbulent flow. Hence, there is a tendency for attitude angle to increase with eccentricity in spiral-grooved bearings.

It is interesting to note in Figs. 33 and 34 that pressurization of the floating ring bearing has a more significant effect on bearing stability than it does on bearing load capacity. It is obvious that pressure feeding of the bearings is an effective means of stabilization.

An important point to investigate is whether the spiral-grooved, floating ring bearing configuration is more or less stable than an equivalent spiral-grooved, single film bearing. Comparisons of the critical masses of the floating ring and single film configurations are shown in Figs. 35 and 36 for laminar and turbulent flow respectively. The single film values of critical mass were taken from the values for M_{c2} calculated for the outer film. The L/D ratio of the outer film is about 20% less than that for the inner film but this fact should not greatly effect the comparison.

For laminar flow, the single film bearing is more stable than the floating ring bearing with $C_2/C_1 = 0.8$ and, at low eccentricities, is also more stable than the floating ring bearing with $C_2/C_1 = 1.2$. However, at high eccentricities, the critical mass for the floating ring bearing with $C_2/C_1 = 1.2$ exceeds that of the single film bearing.

For turbulent flow, the stability of the single film configuration is clearly superior to that of the floating ring bearing.

The floating ring configuration of spiral-grooved bearings suffers in regard to stability for the following reason. The good stability of spiral-grooved bearings is due principally to the self-pressurizing effect of the spiral grooves. This self-pressurizing effect is proportional to the difference in speeds, $(N_1 - N_2)$. Consequently, in a floating ring configuration, rotation of the ring reduces this self-pressurizing effect in the inner film and, hence, reduces the stability of the inner film. The stability of the overall bearing suffers as a consequence.

Stability of the spiral-grooved configuration is compared with the stability of a plain floating ring bearing in Fig. 37. At low eccentricities, stability of the spiral-grooved configuration is better but, at high eccentricity, stability of the plain bearing increases rapidly due to the effect of cavitation and begins to surpass that of the spiral-grooved bearing.

An important point to keep in mind when comparing the stability of plain and spiral-grooved bearings is that the plain bearings achieves stability only as a result of cavitation in the bearing film. A plain bearing operating with a full fluid film is inherently unstable for any speed or mass. Often, particularly with liquid metals as lubricants, it is undesirable to operate with cavitated bearing films because of the problem of cavitation damage. If a pressurized supply is used with plain bearings to suppress cavitation, the stability of the bearings suffers drastically.

On the other hand, spiral-grooved bearings, even when unpressurized, operate without cavitation in the bearing film out to quite large eccentricity ratios due to the self-pressurizing action of the spiral grooves. Stability of these bearings is achieved through this self-pressurizing action. Moreover, use of a pressurized lubricant supply further enhances the stability of these bearings as is evidenced by the performance charts presented in this report.

SUMMARY AND CONCLUSIONS

An analysis has been performed of the turbulent, spiral-grooved, journal bearing with incompressible lubricant. Optimum values of groove parameters have been determined to provide maximum radial stiffness. Performance charts are presented for load capacity, attitude angle, lubricant flow rate and bearing frictional torque for both single film and floating ring configurations of the spiral-grooved journal bearing. In addition, stiffness and damping coefficients are presented for the floating ring configuration and values of the critical mass for threshold of whirl instability are determined. Performance data are presented for bearings operating with and without a pressurized supply of lubricant.

The following general conclusions can be drawn concerning the performance of spiral-grooved single film and floating ring journal bearings:

1) Compared with plain, cylindrical journal bearings, spiral-grooved bearings offer the following advantages: (a) They possess greater stability under lightly loaded conditions. (b) They tend to operate without cavitation due to the self-pressurizing effect of the spiral-grooving. (This is an important consideration when operating the bearing with liquid metal lubricants where the problem of cavitation damage is significant.) (c) The spiral grooving provides self-pumping of lubricant through the bearing eliminating the need for a pressurized supply. (d) All performance characteristics of spiral-grooved bearings can be easily enhanced by use of a pressurized supply of lubricant.

2) The floating ring configuration of spiral-grooved bearing operates with lower torque to load ratio than is achieved with a similarly loaded single film bearing. On the other hand, stability of the single film spiral-grooved bearing is generally better than that for the floating ring bearing.

3) In spiral-grooved bearings, development of turbulence results in an increase in frictional torque, an increase in load capacity, an increase in stability and a decrease in attitude angle. In general, the ratio of frictional torque to load capacity increases with development of turbulence.

TABLE 3 STATIC PERFORMANCE DATA FOR THE FLOATING RING JOURNAL BEARING

C_2/C_1	$\bar{P}_s$	Re	ϵ_1	ϵ_2	ϵ	N_2/N_1	Re_1	Re_2	S	S_1	S_2	T_j/WC_1	$\bar{P}_{s1}$	$\bar{P}_{s2}$	ϕ_1	ϕ_2	ϕ	$\bar{Q}$	$\bar{Q}_1$	$\bar{Q}_2$
0.8	0	LAM	.2	.254	.220	.396	LAM	LAM	1.87	2.45	1.58	21.7	0	0	52.7	29.8	41.2	4.22	2.12	4.00
			.3	.383	.331	.300	LAM	LAM	1.19	1.55	.965	14.0	0	0	51.3	29.5	40.3	4.20	2.18	3.94
			.5	.640	.560	.257			.628	.791	.437	7.72	0	0	45.8	29.5	37.6	4.13	2.39	3.78
0.8	0	1000	.2	.231	.210	.369	631	354	1.66	2.27	1.65	28.4	0	0	46.9	24.3	36.0	6.12	2.78	5.47
			.3	.349	.316	.366	634	351	1.07	1.46	1.06	18.8	0	0	46.7	24.7	36.1	6.07	2.79	5.47
			.5	.629	.551	.338	662	324	.575	.769	.524	10.7	0	0	45.0	27.7	36.3	5.90	2.93	5.09
0.8	0	5000	.2	.225	.207	.385	3071	1851	.912	1.26	.949	37.1	0	0	42.4	18.9	31.3	8.80	3.76	8.06
			.3	.344	.313	.385	3076	1846	.582	.806	.605	24.0	0	0	41.2	19.2	30.7	8.68	3.74	7.88
			.5	.576	.525	.381	3093	1830	.326	.451	.336	14.0	0	0	41.4	20.7	31.5	8.38	3.70	7.44
0.8	0	9000	.2	.209	.200	.392	5472	3386	.673	.938	.713	40.2	0	0	42.4	18.1	31.3	9.36	3.94	8.60
			.3	.314	.300	.391	5477	3381	.436	.606	.460	26.3	0	0	42.3	18.2	31.3	9.22	3.90	8.46
			.5	.547	.511	.386	5519	3341	.234	.324	.244	14.6	0	0	41.5	19.1	31.0	8.86	3.80	8.00
0.8	4.20	LAM	.2	.241	.216	.312	LAM	LAM	1.48	1.94	1.24	16.6	3.20	5.99	33.4	18.0	25.9	25.1	13.7	19.5
			.3	.356	.322	.309	LAM	LAM	.948	1.24	.791	11.0	3.21	6.04	33.6	18.2	26.1	26.2	14.4	20.7
			.5	.578	.530	.300	LAM	LAM	.503	.654	.407	6.68	3.23	6.23	34.5	19.5	27.2	29.7	16.3	24.6
0.8	5.13	1000	.2	.235	.214	.366	634	351	1.15	1.57	1.13	19.0	3.76	6.23	26.8	14.2	20.7	32.0	16.6	22.0
			.3	.355	.322	.362	638	348	.748	1.02	.731	12.7	3.77	6.30	27.2	14.8	21.2	33.0	17.1	23.1
			.5	.607	.545	.349	651	335	.420	.567	.396	7.83	3.80	6.52	28.9	17.2	23.1	36.4	18.8	27.4
0.8	10.0	5000	.2	.233	.213	.382	3085	1837	.618	.855	.639	24.1	7.23	11.6	24.5	11.2	18.1	40.6	20.0	29.4
			.3	.362	.326	.381	3093	1830	.388	.537	.400	15.3	7.24	11.6	23.6	11.6	17.7	42.0	20.6	30.4
			.5	.601	.542	.378	3106	1817	.223	.307	.228	9.23	7.25	11.7	24.6	13.3	19.1	45.6	22.4	33.4
0.8	14.2	9000	.2	.218	.206	.389	5498	3361	.460	.639	.483	26.2	10.2	16.2	24.8	11.0	18.4	43.8	21.6	30.6
			.3	.327	.309	.388	5502	3357	.299	.416	.314	17.3	10.2	16.2	25.0	11.2	18.6	44.6	21.8	31.8
			.5	.563	.525	.385	5531	3329	.163	.226	.170	9.87	10.2	16.3	25.2	12.3	19.1	47.2	22.8	35.0

TABLE 3 (continued)

C_2/C_1	$\bar{P}_s = \frac{PC_1^2}{\mu N_1 R_1^2}$	$Re = \frac{2\pi N_1 R_1 C_1}{\nu}$	ϵ_1	ϵ_2	ϵ	N_2/N_1	$Re_1 = \frac{2\pi(N_1 - N_2)R_1 C_1}{\nu}$	$Re_2 = \frac{2\pi N_2 R_2 C_2}{\nu}$	$S = \frac{DD_1 \mu N_1 R_1^2}{WC_1^2}$	$S_1 = LD_1 \frac{\mu(N_1 + N_2)R_1^2}{WC_1^2}$	$S_2 = \frac{LD_2 \mu N_2 R_2^2}{WC_2^2}$	T_j/WC_1	$\bar{P}_{s1} = \frac{PC_1^2}{\mu(N_1 + N_2)R_1^2}$	$\bar{P}_{s2} = \frac{PC_2^2}{\mu N_2 R_2^2}$	ϕ_1	ϕ_2	ϕ	$\bar{Q} = \frac{Q}{R_1^2 C_1^2 N_1}$	$\bar{Q}_1 = \frac{Q_1}{R_1^2 C_1^2 (N_1 + N_2)}$	$\bar{Q}_2 = \frac{Q_2}{R_2^2 C_2^2 N_2}$
1.2	0	LAM	.2	.409	.306	.391	LAM	LAM	1.88	2.61	.881	19.3	0	0	57.4	29.4	37.4	5.13	1.78	3.53
			.3	.587	.447	.354	LAM	LAM	1.21	1.64	.514	12.6	0	0	54.3	29.1	36.6	4.96	1.84	3.42
			.5	.870	.694	.289	LAM	LAM	.639	.824	.222	7.41	0	0	47.1	28.9	34.8	4.76	2.24	3.73
1.2	0	1000	.2	.370	.286	.406	594	584	1.71	2.41	.834	26.9	0	0	49.5	23.1	31.2	7.62	2.54	5.73
			.3	.540	.420	.397	603	572	1.10	1.54	.525	18.0	0	0	49.0	24.2	32.0	7.40	2.57	5.5[illegible]
			.5	.929	.729	.340	659	491	.577	.773	.236	10.6	0	0	45.0	30.0	34.7	6.98	2.91	5.20
1.2	0	5000	.2	.340	.270	.408	2960	2938	.946	1.33	.463	36.3	0	0	44.1	18.5	26.8	10.8	3.57	8.2[illegible]
			.3	.507	.405	.405	2977	2913	.605	.849	.294	23.6	0	0	42.9	19.1	26.9	10.6	3.57	7.9[illegible]
			.5	.831	.673	.393	3135	2829	.333	.464	.157	13.8	0	0	42.3	23.6	29.8	10.0	3.60	7.3[illegible]
1.2	0	9000	.2	.332	.266	.408	5327	5288	.690	.972	.338	39.4	0	0	43.4	17.8	26.3	11.5	3.80	[illegible].74
			.3	.483	.392	.406	5342	5267	.444	.625	.217	25.7	0	0	43.0	18.1	26.5	11.2	3.74	8.[illegible]
			.5	.811	.661	.395	5448	5115	.237	.331	.112	14.5	0	0	42.0	22.1	28.8	10.4	3.74	7.64
1.2	4.2	LAM	.2	.336	.270	.399	LAM	LAM	1.47	2.06	.705	14.3	3.0	10.5	35.2	13.1	20.4	42.4	14.2	32.8
			.3	.485	.395	.388	LAM	LAM	.947	1.32	.441	9.71	3.01	10.8	35.3	13.7	21.0	45.4	14.8	37.0
			.5	.795	.655	.329	LAM	LAM	.504	.669	.199	6.42	3.16	12.8	35.1	18.0	23.9	54.0	16.5	56.6
1.2	5.13	1000	.2	.320	.263	.406	594	585	1.16	1.63	.567	17.3	3.65	12.6	27.2	10.1	16.0	50.3	15.9	37.8
			.3	.480	.390	.401	599	578	.759	1.06	.396	11.7	3.66	12.8	27.7	10.9	16.7	53.2	17.5	41.4
			.5	.870	.700	.356	644	512	.422	.572	.180	7.74	3.78	14.4	29.0	18.0	21.5	64.4	18.9	63.0
1.2	10.0	5000	.2	.307	.256	.408	2959	2938	.631	.889	.309	22.9	7.10	24.5	24.8	8.7	14.4	60.6	20.5	45.0
			.3	.467	.388	.405	2974	2917	.400	.562	.195	14.7	7.11	24.7	24.1	9.1	14.3	63.9	20.9	49.3
			.5	.790	.655	.394	3032	2834	.226	.316	.107	8.97	7.18	25.4	24.9	12.7	16.9	72.8	22.9	60.1
1.2	14.2	9000	.2	.303	.254	.408	5332	5282	.465	.654	.227	25.0	10.1	34.8	24.8	8.6	14.3	64.8	22.1	47.9
			.3	.446	.376	.407	5336	5275	.300	.423	.147	16.4	10.1	34.9	24.7	8.9	14.6	66.1	22.3	49.4
			.5	.758	.637	.398	5417	5159	.165	.231	.079	9.60	10.2	35.7	25.3	11.7	16.5	71.8	23.2	57.3

TABLE 4 DYNAMIC PERFORMANCE DATA FOR THE FLOATING RING JOURNAL BEARING

C_2/C_1	$\bar{P}_s = \frac{PC_1^2}{\mu N_1 R_1^2}$	$Re = \frac{2\pi N_1 R_1 C_1}{\nu}$	ϵ_1	$(\bar{K}_{xx})_1$	$(\bar{K}_{xy})_1$	$(\bar{K}_{yx})_1$	$(\bar{K}_{yy})_1$	$(\bar{B}_{xx})_1$	$(\bar{B}_{xy})_1$	$(\bar{B}_{yx})_1$	$(\bar{B}_{yy})_1$	$(\bar{K}_{xx})_2$	$(\bar{K}_{xy})_2$	$(\bar{K}_{yx})_2$	$(\bar{K}_{yy})_2$	$(\bar{B}_{xx})_2$	$(\bar{B}_{xy})_2$	$(\bar{B}_{yx})_2$	$(\bar{B}_{yy})_2$	$(M_c)_1 = \frac{M_{c1}(N_1+N_2)}{2\mu L}(\frac{C_1}{R_1})^3$	$(M_c)_2 = \frac{M_{c2}N_1}{2\mu L}(\frac{C_2}{R_2})^3$
1.2	0	LAM	.2	1.10	1.72	-1.58	1.07	3.19	.165	.013	3.27	3.45	1.94	-1.32	2.43	3.08	.377	.225	2.65	.109	.258
			.3	1.30	1.81	-1.52	1.36	3.14	.453	.321	3.36	5.15	2.90	-1.70	2.85	4.43	.925	.670	3.28	.129	.363
			.5	2.45	2.64	-1.38	2.07	3.93	1.32	1.05	3.46	17.5	9.69	-2.55	4.53	21.0	8.29	8.37	7.34	.181	1.24
1.2	0	1000	.2	1.43	1.67	-1.58	1.34	3.13	.174	-.023	3.14	3.54	1.51	-1.37	2.94	2.68	.220	.063	2.38	.134	.266
			.3	1.50	1.84	-1.65	1.39	3.32	.292	.064	3.34	4.20	1.90	-1.83	3.00	3.49	.507	.282	2.70	.143	.270
			.5	2.36	2.37	-1.63	2.01	4.02	.95	.620	3.78	31.8	17.5	-2.81	4.39	81.6	40.3	46.8	28.2	.211	1.59
1.2	0	5000	.2	2.86	2.75	-2.60	2.72	5.18	.253	-.061	5.17	7.16	2.44	-1.93	5.95	4.16	.240	.014	3.81	.272	.605
			.3	3.23	3.07	-2.56	2.92	5.44	.404	.048	5.40	7.76	2.67	-2.65	6.21	5.00	.479	.173	4.09	.299	.557
			.5	3.85	3.55	-3.01	3.09	6.45	.991	.492	6.26	13.8	5.93	-4.89	6.26	14.9	3.47	3.06	5.68	.341	.869
1.2	0	9000	.2	4.15	3.97	-3.28	3.97	7.09	.315	-.055	7.06	9.30	2.97	-2.98	8.44	5.81	.342	.062	5.22	.403	.789
			.3	4.42	3.96	-3.76	3.97	7.58	.610	.167	7.48	11.3	3.72	-3.41	8.93	6.90	.624	.239	5.66	.415	.880
			.5	6.30	5.67	-4.16	4.34	9.07	1.47	.829	8.76	17.4	7.10	-6.11	9.34	16.5	3.55	3.18	7.84	.471	1.22
1.2	4.2	LAM	.2	2.14	1.51	-1.42	1.96	3.28	.172	.016	3.19	5.00	1.15	-1.09	4.08	2.85	.149	.025	2.41	.259	.748
			.3	2.42	1.71	-1.51	2.03	3.59	.318	.129	3.39	6.31	1.53	-1.52	4.44	3.72	.316	.126	2.67	.271	.895
			.5	3.51	2.47	-1.97	2.26	4.82	.916	.593	4.12	11.1	3.60	-5.54	4.85	11.2	2.4	1.90	4.50	.314	.750
1.2	5.13	1000	.2	2.83	1.46	-1.44	2.69	3.18	.157	-.040	3.09	5.91	1.04	-1.13	5.41	2.57	.115	-.027	2.28	.331	.756
			.3	3.00	1.57	-1.57	2.70	3.43	.254	.026	3.21	6.46	1.24	-1.58	5.56	3.18	.217	.023	2.46	.331	.675
			.5	3.66	2.02	-2.14	2.80	4.48	.668	.331	3.76	15.6	5.34	-4.74	5.27	20.9	4.80	4.96	4.66	.333	1.28
1.2	10.0	5000	.2	5.38	2.45	-2.38	5.12	5.24	.227	-.087	5.11	11.3	1.76	-1.75	10.2	4.07	.154	-.060	3.74	.618	1.47
			.3	6.25	2.87	-2.26	5.33	5.57	.344	-.011	5.27	12.2	1.94	-2.39	10.9	4.79	.262	-.019	3.93	.687	1.29
			.5	6.47	3.03	-3.34	5.44	6.85	.801	.302	5.85	13.0	2.91	-7.31	10.5	12.0	1.74	1.22	5.32	.619	.937
1.2	14.2	9000	.2	8.00	3.75	-2.82	6.99	7.17	.282	-.089	6.99	15.1	2.22	-2.49	14.6	5.69	.212	-.051	5.12	.932	1.02
			.3	7.73	3.41	-3.62	7.33	7.79	.506	.063	7.25	19.7	3.09	-3.12	14.8	6.64	.359	.012	5.39	.887	1.22
			.5	10.5	4.98	-4.44	7.42	9.65	1.19	.551	8.12	18.9	3.86	-8.00	15.5	13.2	1.58	.905	6.91	.902	1.40

TABLE 4 (continued)

C_2/C_1	$\bar{P}_s$	Re	ϵ_1	$(\bar{K}_{xx})_1$	$(\bar{K}_{xy})_1$	$(\bar{K}_{yx})_1$	$(\bar{K}_{yy})_1$	$(\bar{B}_{xx})_1$	$(\bar{B}_{xy})_1$	$(\bar{B}_{yx})_1$	$(\bar{B}_{yy})_1$	$(\bar{K}_{xx})_2$	$(\bar{K}_{xy})_2$	$(\bar{K}_{yx})_2$	$(\bar{K}_{yy})_2$	$(\bar{B}_{xx})_2$	$(\bar{B}_{xy})_2$	$(\bar{B}_{yx})_2$	$(\bar{B}_{yy})_2$	$(\bar{M}_c)_1$	$(\bar{M}_c)_2$
0.8	0	LAM	.2	1.36	1.78	-1.60	1.26	3.22	.179	.023	3.26	2.55	1.46	-1.19	2.17	2.51	.156	.053	2.36	.125	.208
			.3	1.53	1.91	-1.55	1.50	3.33	.395	.200	3.47	3.26	1.84	-1.28	2.38	2.95	.328	.188	2.59	.152	.248
			.5	2.62	2.70	-1.40	2.17	4.03	1.29	1.06	3.57	6.06	3.43	-1.56	3.18	5.11	1.26	.940	3.35	.197	.483
0.8	0	1000	.2	1.60	1.71	-1.61	1.50	3.19	.176	-.021	3.19	2.58	1.16	-1.11	2.37	2.07	.107	-.016	1.97	.149	.202
			.3	1.76	1.87	-1.68	1.54	3.38	.297	.068	3.38	2.85	1.31	-1.22	2.42	2.29	.186	.039	2.07	.157	.205
			.5	2.38	2.38	-1.65	2.02	4.03	.949	.618	3.79	3.86	2.03	-1.90	2.41	3.69	.763	.518	2.68	.211	.222
0.8	0	5000	.2	3.10	2.81	-2.66	2.94	5.30	.256	-.063	5.27	4.69	1.61	-1.56	4.41	2.94	.123	-.039	2.83	.293	.388
			.3	3.65	3.30	-2.35	3.37	5.56	.408	.047	5.50	5.09	1.77	-1.70	4.50	3.22	.202	.010	2.93	.357	.394
			.5	4.04	3.61	-3.11	3.17	6.54	.998	.495	6.31	5.89	2.22	-2.50	4.58	4.41	.572	.285	3.36	.345	.379
0.8	0	9000	.2	4.13	3.80	-3.59	3.94	7.22	.321	-.056	7.18	6.53	2.03	-2.28	6.56	4.06	.126	-.069	3.96	.396	.580
			.3	5.11	4.60	-3.38	4.39	7.63	.574	.138	7.53	7.13	2.31	-2.33	6.56	4.37	.212	-.016	4.08	.455	.586
			.5	5.77	5.09	-4.42	4.27	9.17	1.48	.831	8.82	8.79	3.00	-3.06	6.85	5.75	.619	.267	4.51	.471	.619
0.8	4.2	LAM	.2	2.34	1.57	-1.49	2.14	3.28	.169	.013	3.19	3.60	1.17	-1.06	3.15	2.51	.110	.010	2.31	.273	.415
			.3	2.63	1.75	-1.53	2.30	3.60	.312	.122	3.38	4.28	1.41	-1.20	3.33	2.91	.208	.077	2.46	.285	.464
			.5	3.64	2.50	-1.98	2.35	4.84	.907	.584	4.10	6.40	2.26	-1.99	3.80	4.57	.658	.401	3.01	.322	.586
0.8	5.13	1000	.2	2.98	1.50	-1.48	2.82	3.24	.158	-.039	3.14	3.84	.976	-1.00	3.62	2.08	.089	-.034	1.96	.341	.404
			.3	3.17	1.63	-1.61	2.85	3.50	.256	.027	3.27	4.12	1.08	-1.17	3.68	2.34	.144	-.003	2.04	.341	.388
			.5	3.70	2.04	-2.15	2.83	4.50	.668	.331	3.77	4.75	1.47	-2.12	3.68	3.67	.481	.245	2.44	.335	.330
0.8	10	5000	.2	5.60	2.53	-2.45	5.33	5.37	.231	-.089	5.23	6.94	1.39	-1.41	6.64	2.96	.109	-.054	2.81	.635	.742
			.3	6.80	3.04	-2.19	5.71	5.68	.333	-.026	5.38	7.41	1.53	-1.62	6.75	3.29	.171	-.025	2.91	.766	.723
			.5	6.65	3.09	-3.38	5.57	6.95	.810	.305	5.93	7.94	1.88	-2.76	6.86	4.74	.473	.174	3.32	.631	.599
0.8	14.2	9000	.2	7.38	3.44	-3.31	7.02	7.32	.287	-.091	7.13	11.1	2.16	-1.60	9.44	4.11	.125	-.075	3.96	.848	1.54
			.3	9.30	4.21	-3.06	7.47	7.84	.486	.049	7.35	10.1	1.98	-2.19	9.66	4.42	.181	-.050	4.05	1.00	1.05
			.5	9.17	4.30	-4.64	7.52	9.78	1.20	.555	8.24	11.9	2.56	-3.23	10.0	6.00	.494	.130	4.46	.886	1.00

TABLE 5 DYNAMIC PERFORMANCE DATA FOR THE FLOATING RING JOURNAL BEARING

C_2/C_1	$\bar{P}_s = \frac{P_s C_1^2}{\mu N_1 R_1^2}$	$Re = \frac{2\pi N_1 R_1 C_1}{\nu}$	ϵ_1	K_{xx}	K_{xy}	K_{yx}	K_{yy}	B_{xx}	B_{xy}	B_{yx}	B_{yy}	$\bar{M}_c = \frac{M_c C_1 N_1}{\mu L D_1}\left(\frac{C_1}{R_1}\right)^2$	$(\nu/\omega)_c$
1.2	0	LAM	.2	.785	.762	- .616	.637	1.32	.629	- 0	1.41	.084	.50
			.3	1.00	.928	- .653	.745	1.52	.990	.123	1.64	.102	.50
			.5	2.69	2.05	- .33	1.28	2.09	2.61	.833	2.23	.530	.29
1.2	0	1000	.2	.909	.652	- .592	.782	1.29	.541	- .041	1.31	.106	.48
			.3	1.06	.790	- .676	.799	1.51	.743	.020	1.46	.113	.49
			.5	3.18	2.42	.006	1.58	.199	2.82	.859	2.43	.174	.57
1.2	0	5000	.2	1.81	1.04	- .928	1.58	2.10	.842	- .087	2.16	.223	.46
			.3	1.97	1.15	- .985	1.62	2.40	1.03	- .021	2.28	.244	.45
			.5	3.44	2.07	-1.27	1.71	3.84	2.40	.555	2.92	.356	.44
1.2	0	9000	.2	2.44	1.38	-1.19	2.23	2.97	1.09	- .089	2.93	.337	.43
			.3	2.85	1.58	-1.45	2.34	3.33	1.40	- .016	3.20	.336	.46
			.5	4.88	2.91	-1.72	2.49	5.38	3.15	.697	4.10	.538	.42
1.2	4.2	LAM	.2	1.26	.528	- .492	1.06	1.44	.431	- .039	1.36	.243	.36
			.3	1.51	.655	- .588	1.11	1.73	.607	.004	1.48	.258	.37
			.5	2.72	1.34	- .965	1.20	3.30	1.60	.417	2.19	.453	.33
1.2	5.13	1000	.2	1.54	.473	- .482	1.42	1.38	.392	- .051	1.31	.315	.35
			.3	1.68	.542	- .582	1.44	1.62	.506	- .038	1.39	.308	.36
			.5	3.71	1.47	- .688	1.44	3.18	1.96	.514	1.98	-1.33	.21
1.2	10	5000	.2	2.97	.802	- .784	2.74	2.25	.637	- .097	2.18	.595	.35
			.3	3.17	.869	- .814	2.76	2.53	.744	- .063	2.20	.649	.34
			.5	4.11	1.32	-1.63	2.80	4.83	1.65	.312	2.88	.693	.36
1.2	14.2	9000	.2	4.00	1.08	- .938	3.69	3.10	.808	- .098	2.88	.907	.33
			.3	4.84	1.28	-1.34	3.99	3.51	1.14	- .067	3.18	.830	.37
			.5	5.77	1.80	-2.05	3.95	6.18	2.09	.261	3.77	.999	.35

TABLE 5 (continued)

C_2/C_1	$\bar{R}_e$	Re	ϵ_1	K_{xx}	K_{xy}	K_{yx}	K_{yy}	B_{xx}	B_{xy}	B_{yx}	B_{yy}	$\bar{M}_c$	$(\nu/\omega)_c$
0.8	0	LAM	.2	1.21	1.31	-1.15	1.09	2.04	.794	- .096	2.25	.105	.57
			.3	1.39	1.43	-1.12	1.24	2.18	1.08	.016	2.46	.127	.56
			.5	2.43	2.12	-1.07	1.77	2.88	2.26	.572	2.79	.179	.57
0.8	0	1000	.2	1.41	1.19	-1.13	1.31	1.94	.799	- .120	2.11	.122	.58
			.3	1.56	1.32	-1.18	1.33	2.08	.956	- .067	2.23	.127	.58
			.5	2.16	1.77	-1.21	1.62	2.76	1.82	.294	2.72	.178	.55
0.8	0	5000	.2	2.67	1.82	-1.73	2.52	3.11	1.19	- .179	3.33	.241	.55
			.3	3.03	2.06	-1.54	2.72	3.42	1.50	- .105	3.49	.301	.52
			.5	3.53	2.44	-2.15	2.65	4.14	2.09	.148	4.00	.278	.57
0.8	0	9000	.2	3.59	2.45	-2.38	3.48	4.24	1.44	- .201	4.49	.322	.55
			.3	4.31	2.91	-2.27	3.77	4.67	1.92	- .125	4.78	.382	.54
			.5	5.06	3.37	-3.01	3.71	5.58	2.73	.243	5.44	.376	.58
0.8	4.2	LAM	.2	1.90	1.03	- .95	1.71	2.28	.781	- .138	2.31	.266	.43
			.3	2.18	1.19	-1.03	1.77	2.52	.982	- .075	2.44	.278	.44
			.5	3.15	1.79	-1.40	1.90	3.46	1.74	.202	2.95	.318	.47
0.8	5.13	1000	.2	2.35	.946	- .939	2.21	2.19	.794	- .153	2.20	.337	.43
			.3	2.51	1.03	-1.03	2.23	2.40	.925	- .123	2.28	.335	.44
			.5	3.06	1.38	-1.47	2.22	3.20	1.45	.048	2.66	.328	.47
0.8	10.0	5000	.2	4.32	1.48	-1.45	4.11	3.43	1.16	- .210	3.44	.615	.43
			.3	4.94	1.68	-1.33	4.25	3.87	1.41	- .147	3.56	.781	.39
			.5	5.26	1.92	-2.19	4.27	4.88	1.95	- .025	3.97	.614	.46
0.8	14.2	9000	.2	6.14	2.16	-1.98	5.60	4.50	1.62	- .255	4.69	.781	.45
			.3	6.86	2.39	-1.90	5.87	5.26	1.82	- .180	4.85	.990	.41
			.5	7.34	2.64	-2.97	5.97	6.43	2.51	- .003	5.36	.819	.47

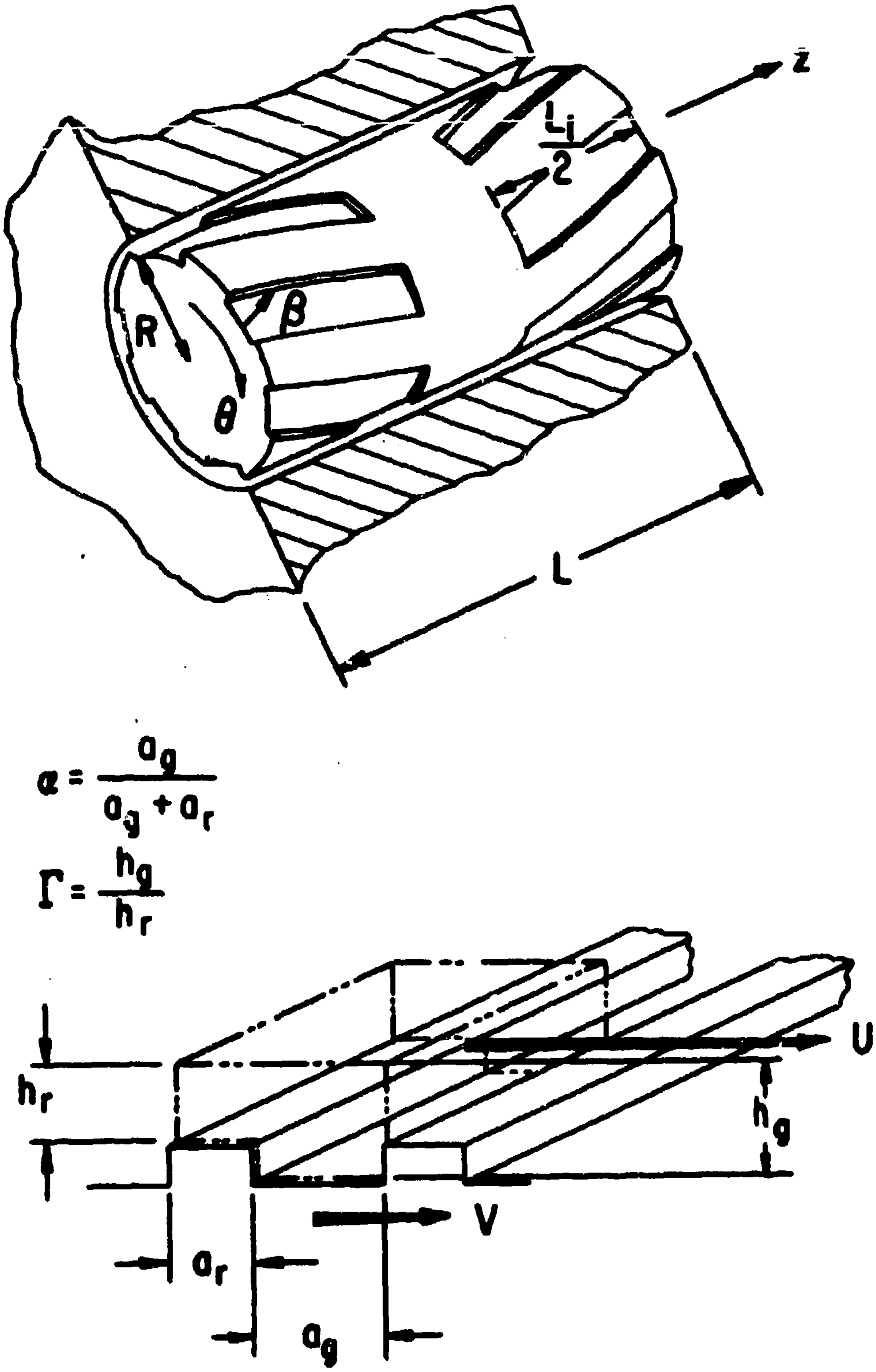

Figure 1. Schematic of Spiral-Grooved Journal Bearing

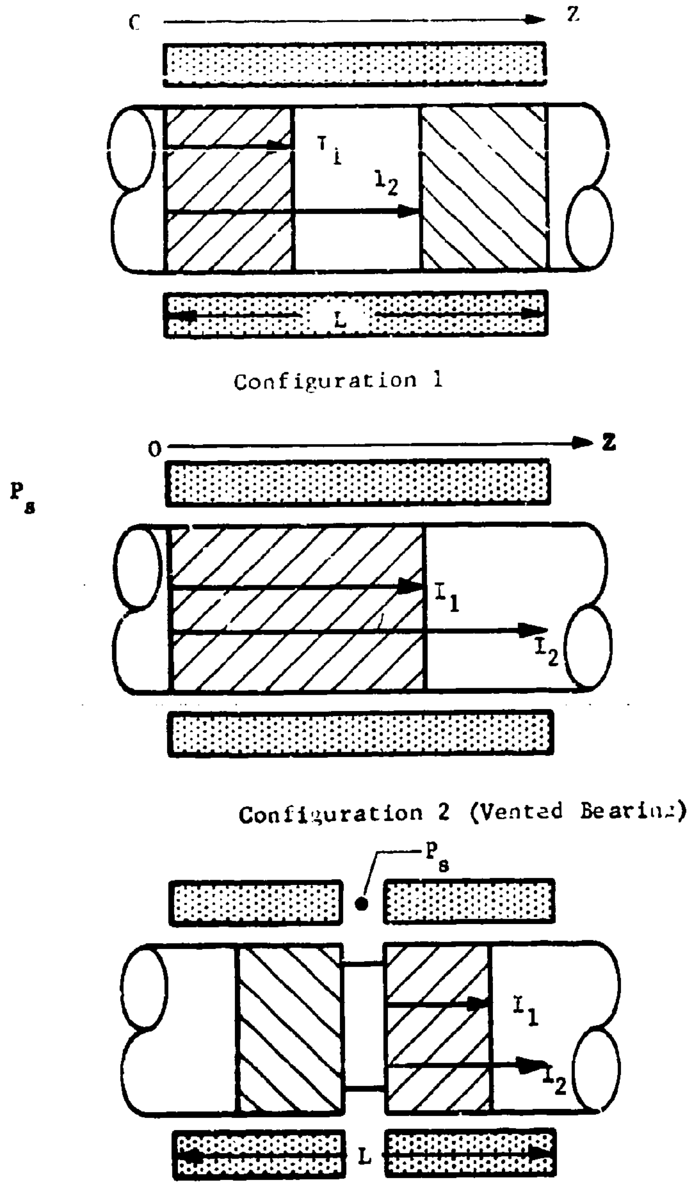

Figure 2. Various Configurations of Spiral-Grooved Journal Bearings

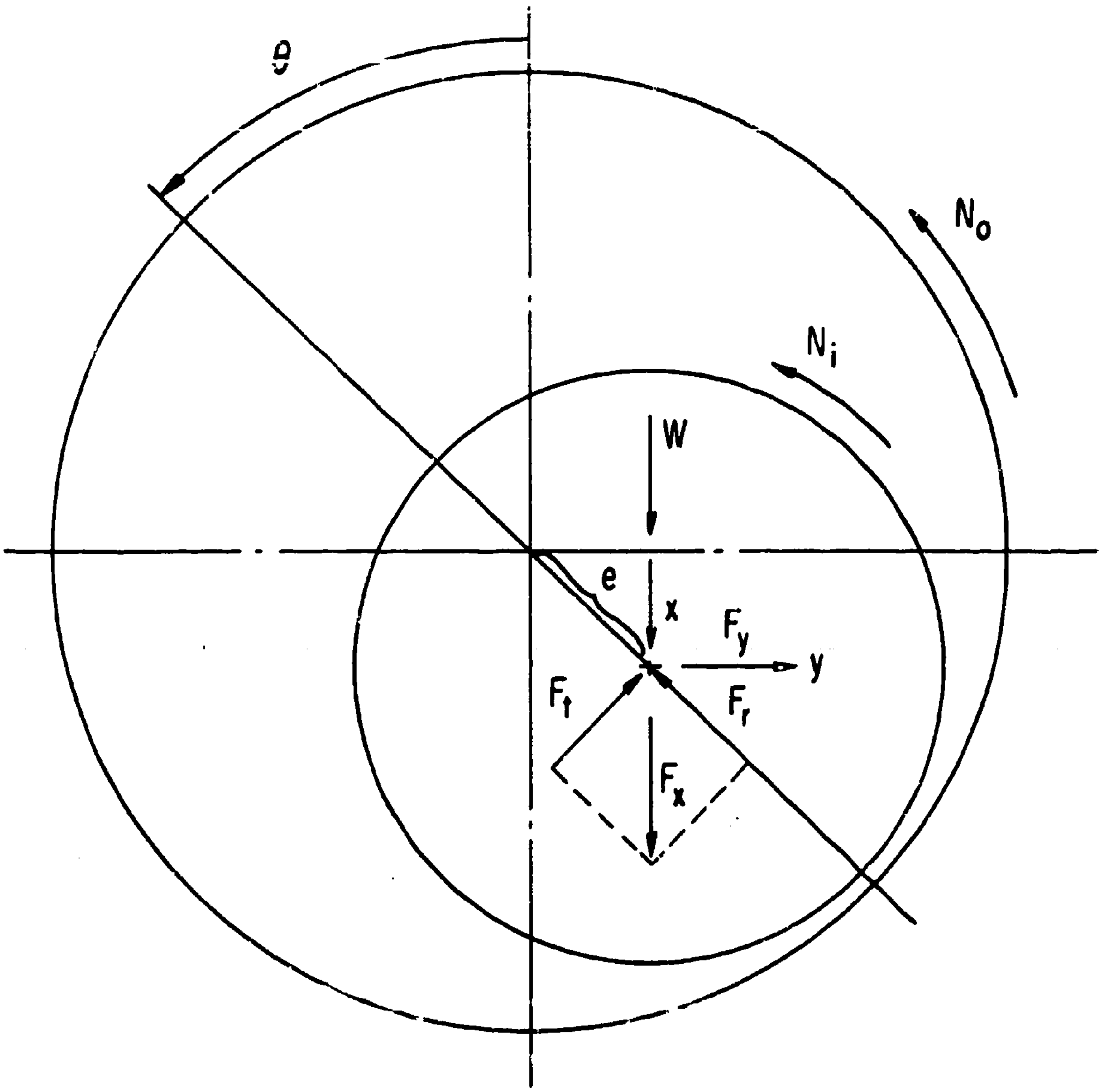

Figure 3. Coordinate System for Forces and Displacements

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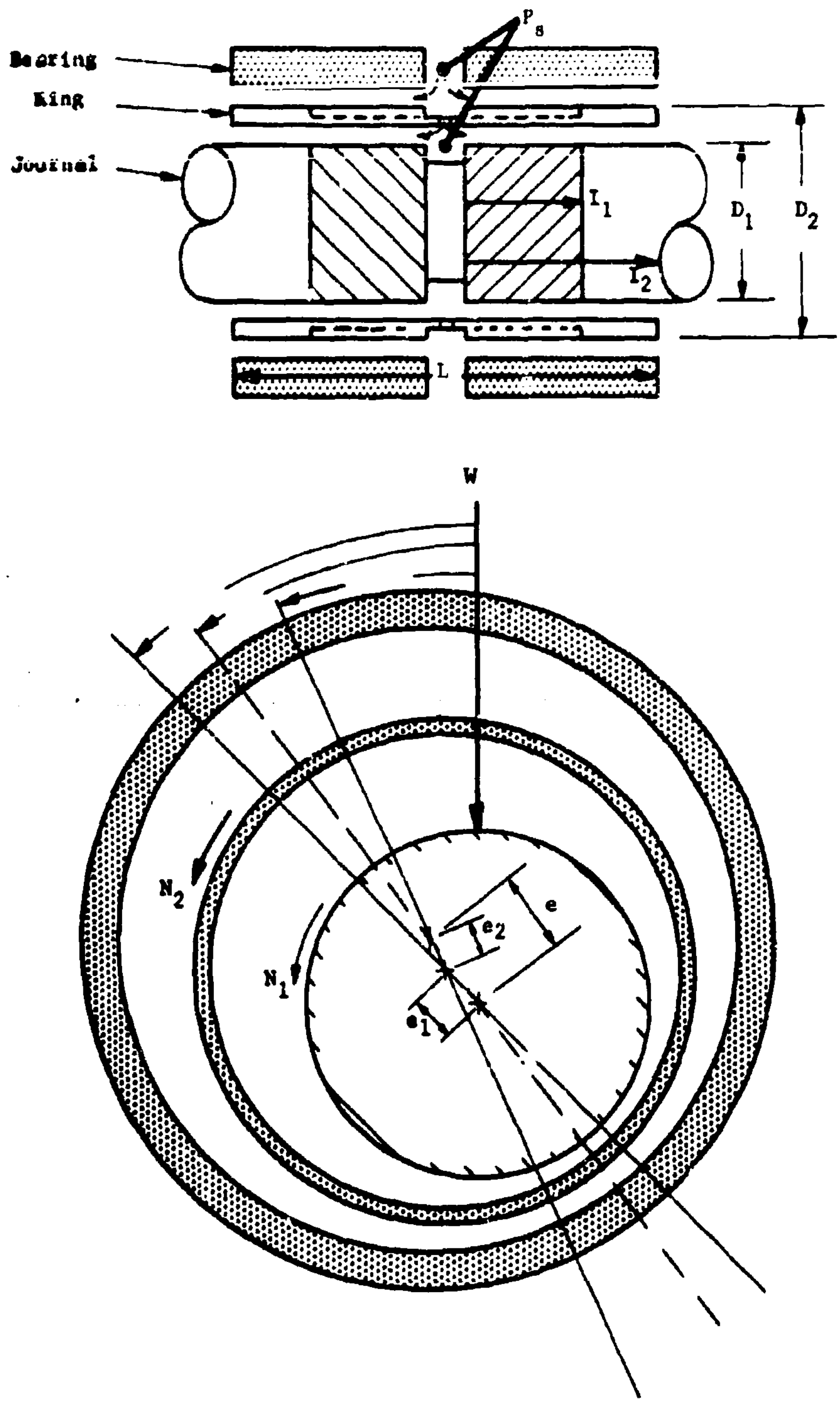

Figure 4. Geometry of Floating Ring Bearing

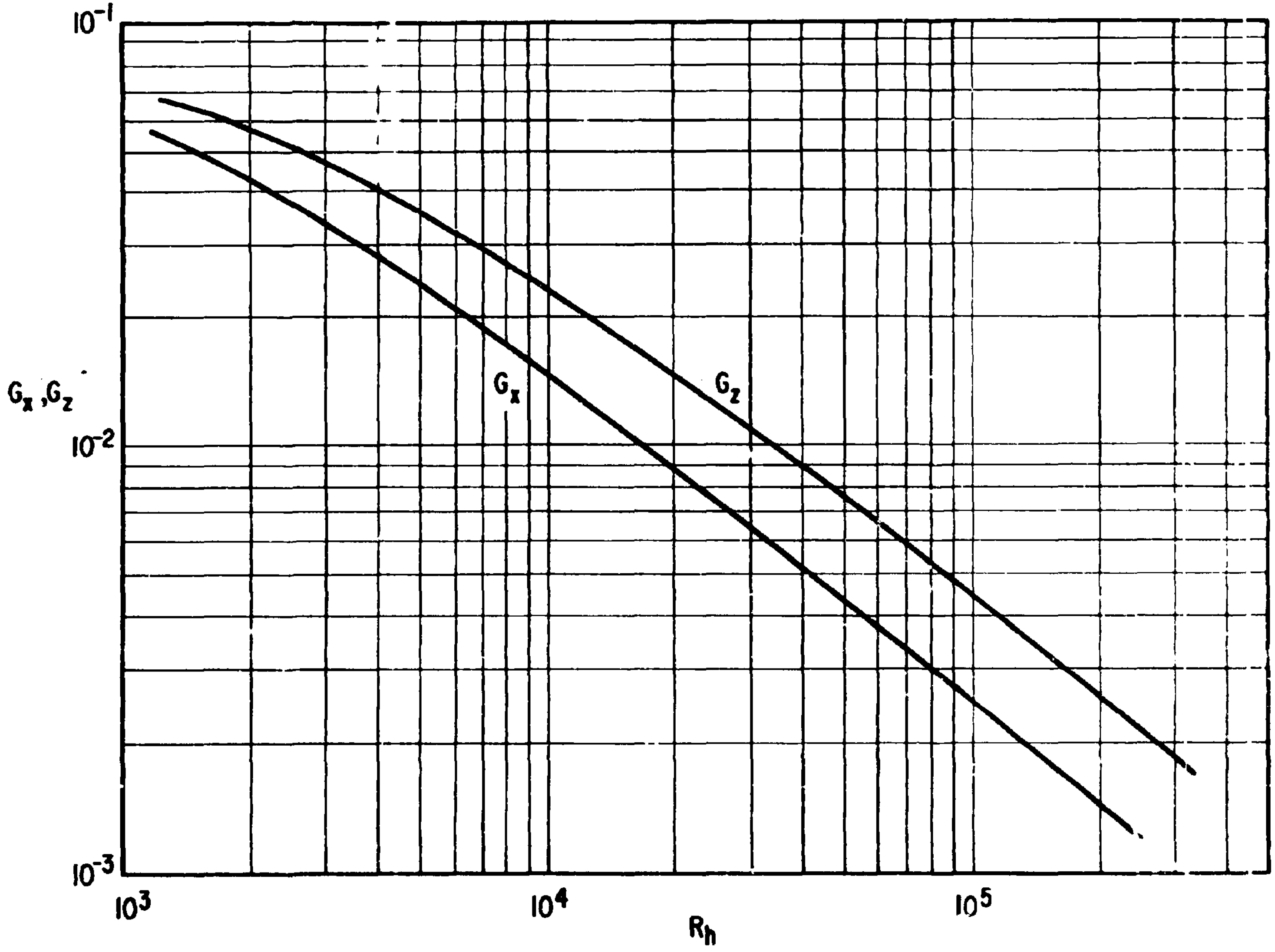

Figure 5. G_x and G_z vs. Local Reynolds Number R_h

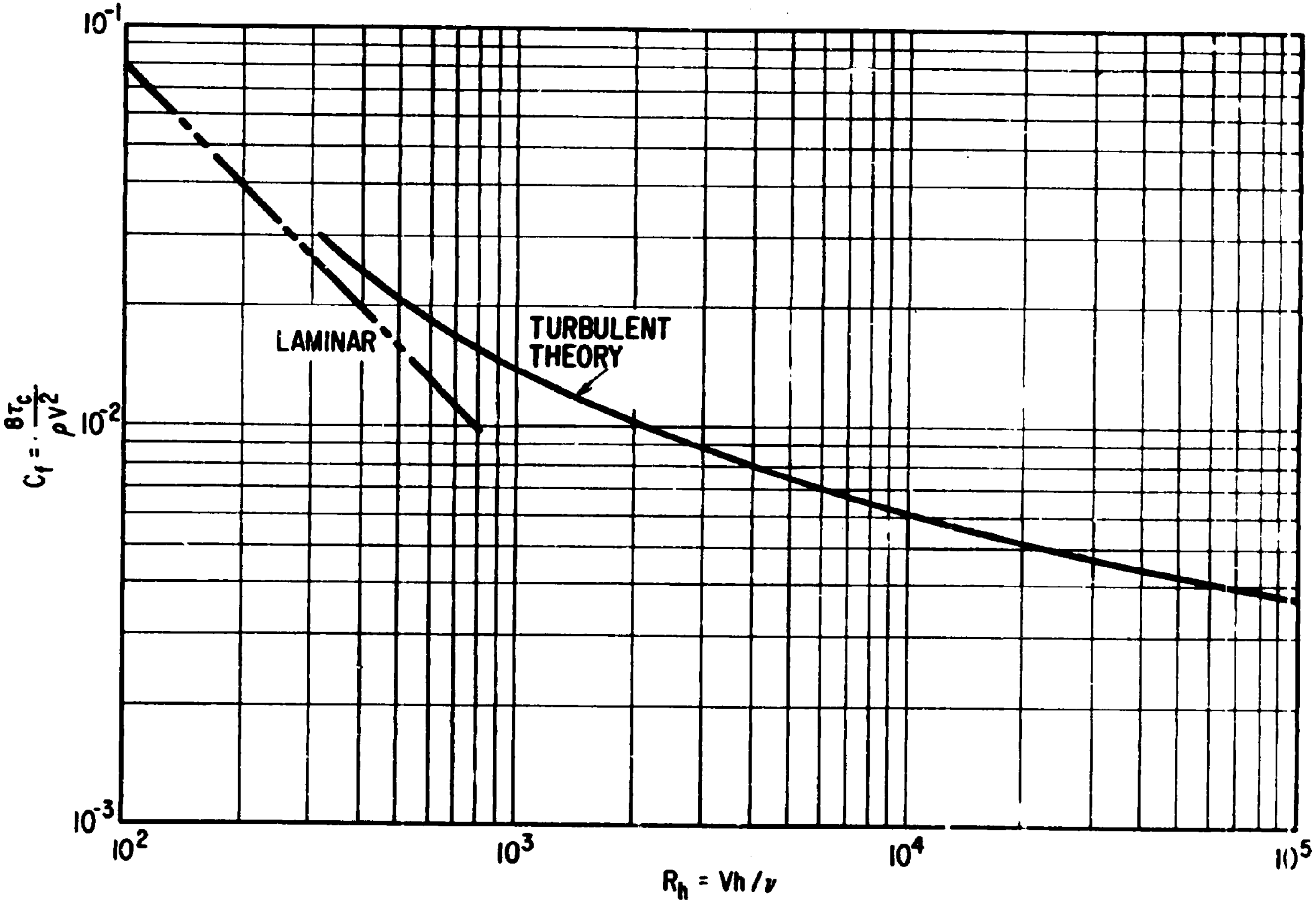

Figure 6. Coefficient of Friction vs. Local Reynolds Number R_h

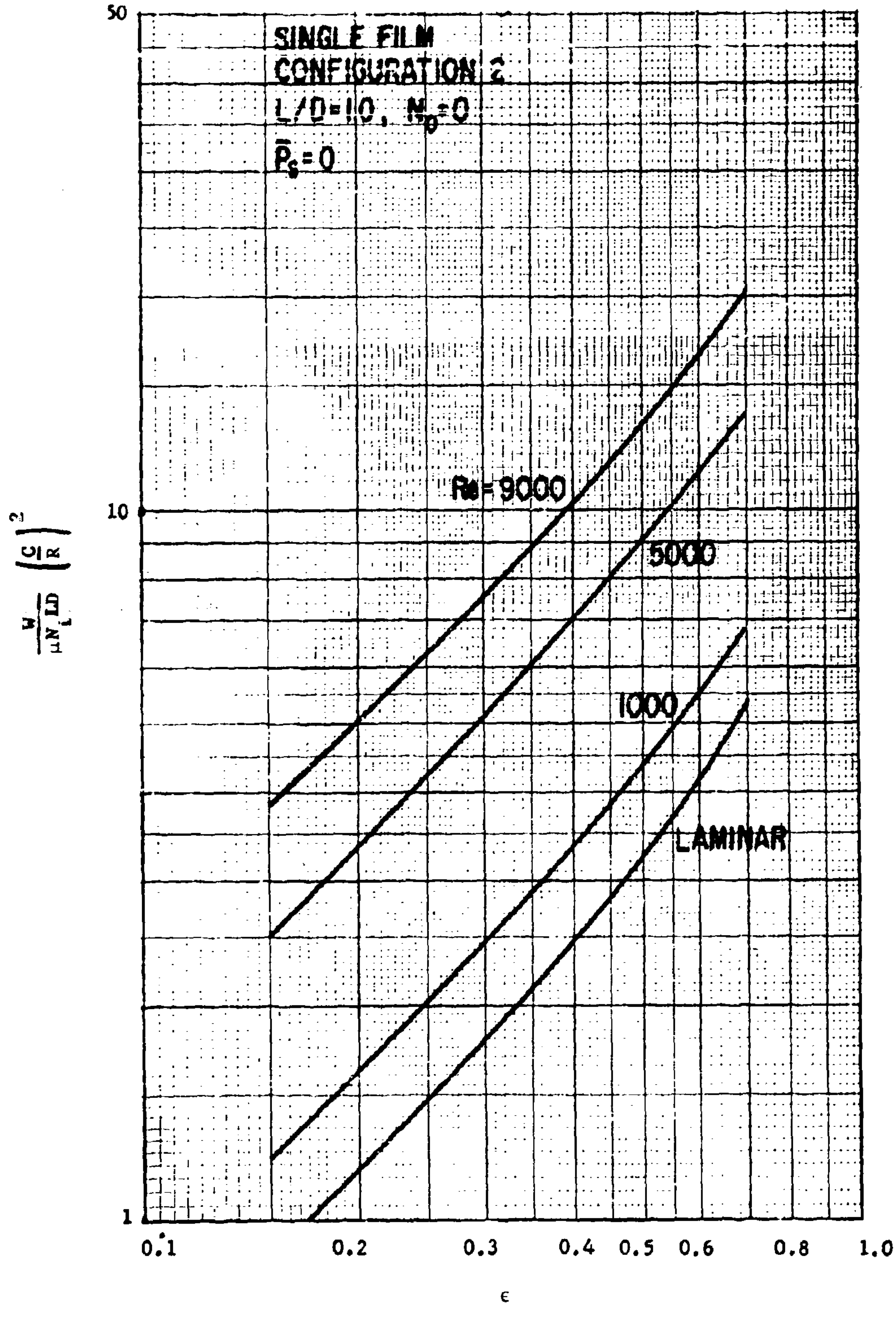

Figure 7. Load vs. Eccentricity, Single Film Bearing

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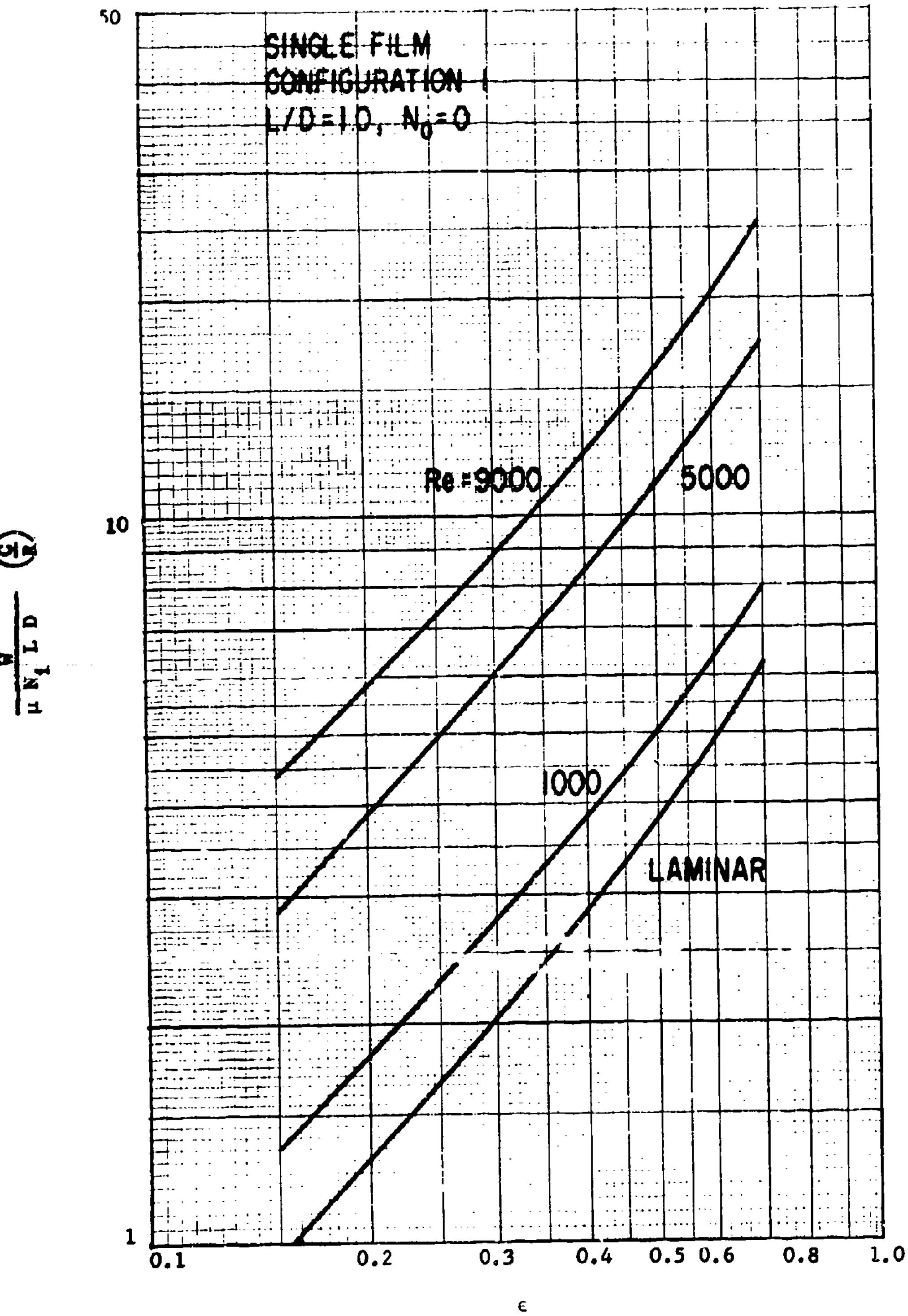


Figure 8. Load vs. Eccentricity, Single Film Bearing

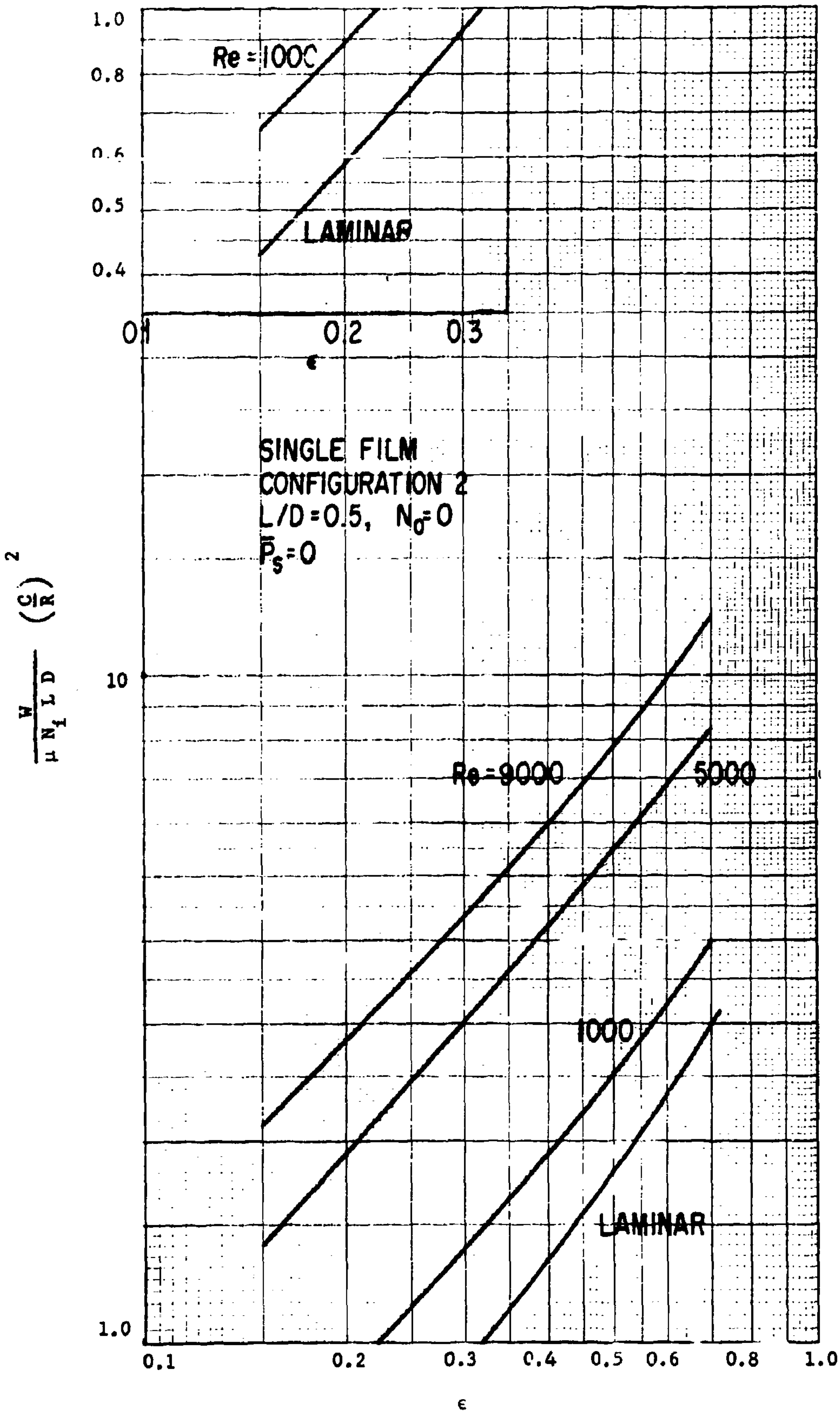


Figure 9. Load vs. Eccentricity, Single Film Bearing

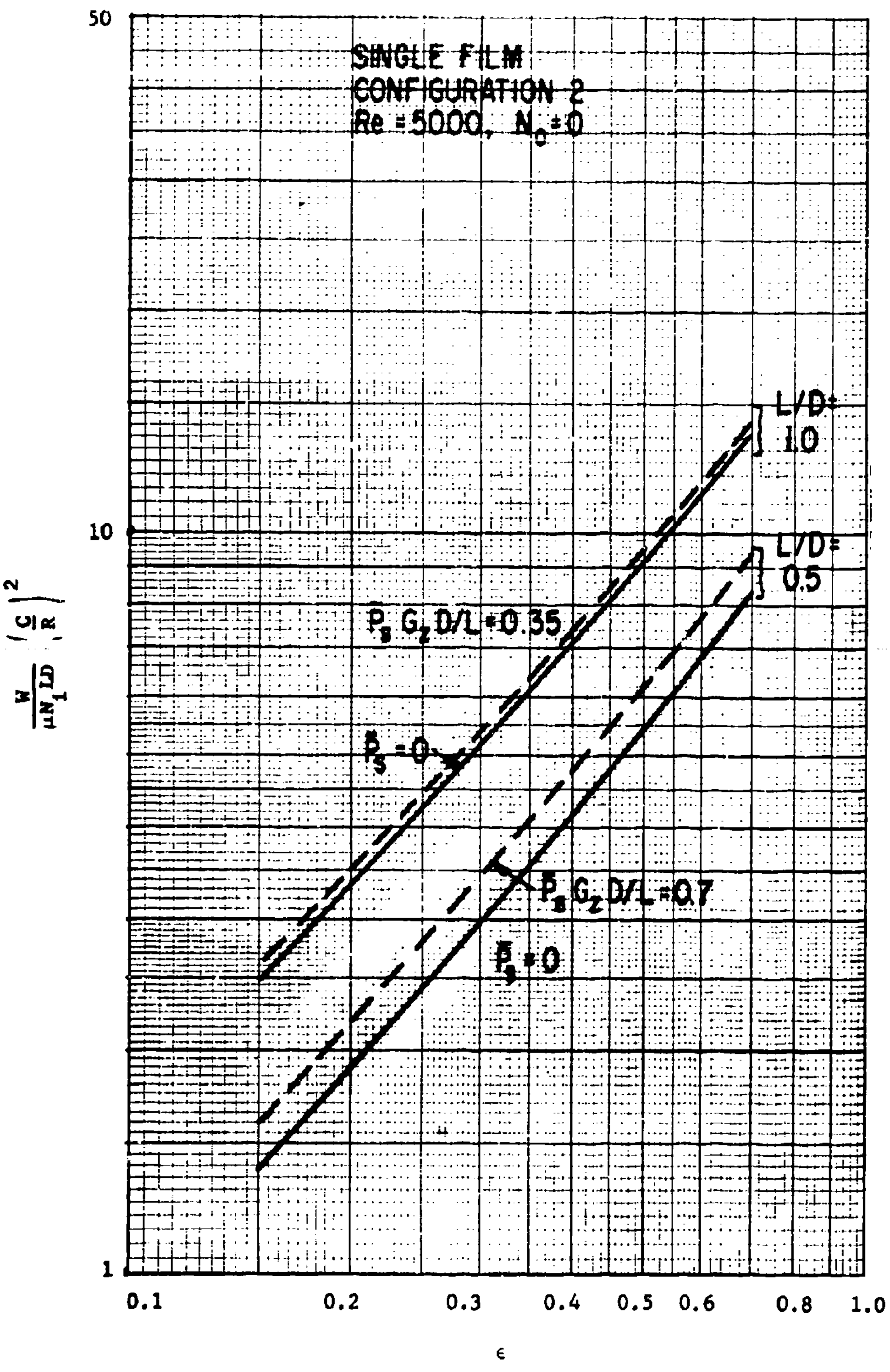


Figure 10. Load vs. Eccentricity, Effect of Pressurization

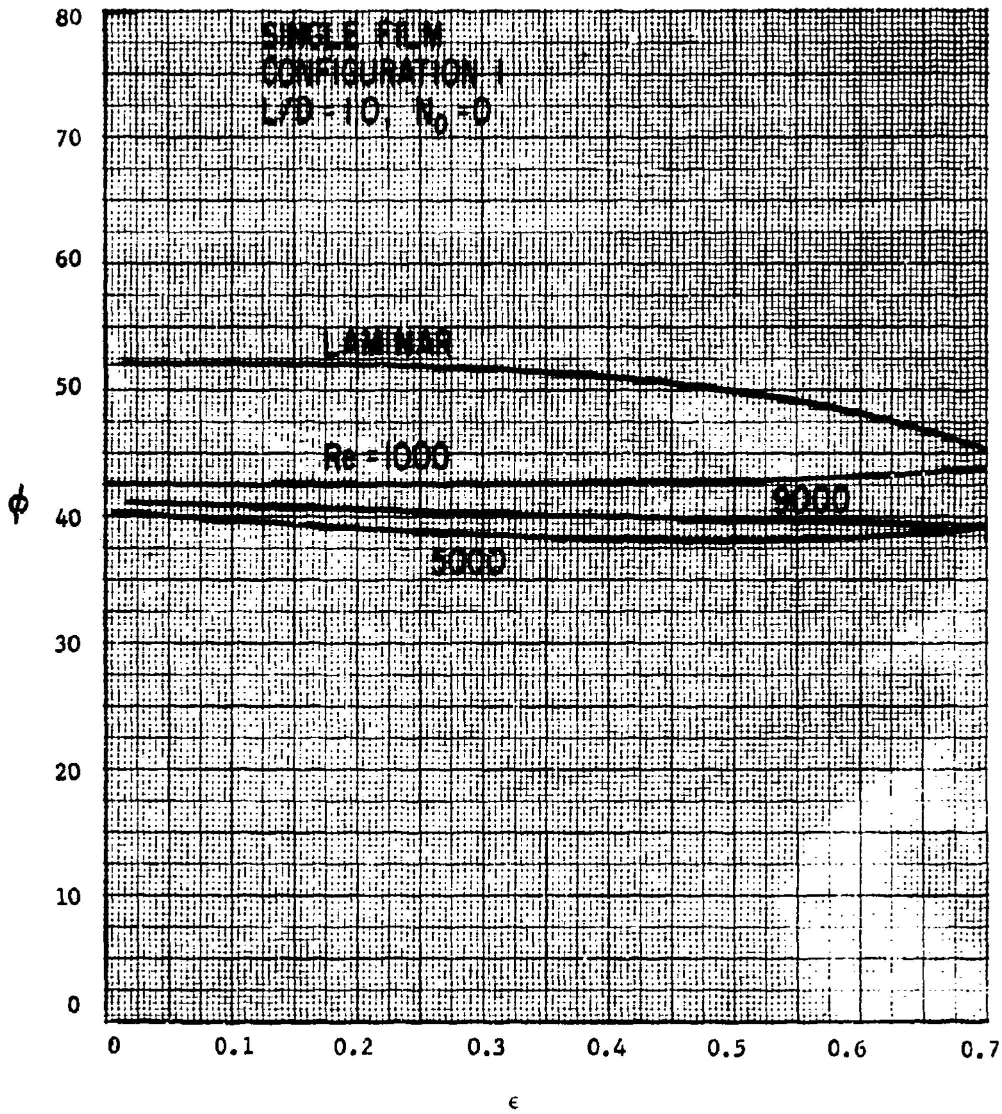


Figure 11. Attitude Angle vs. Eccentricity, Single Film Bearing

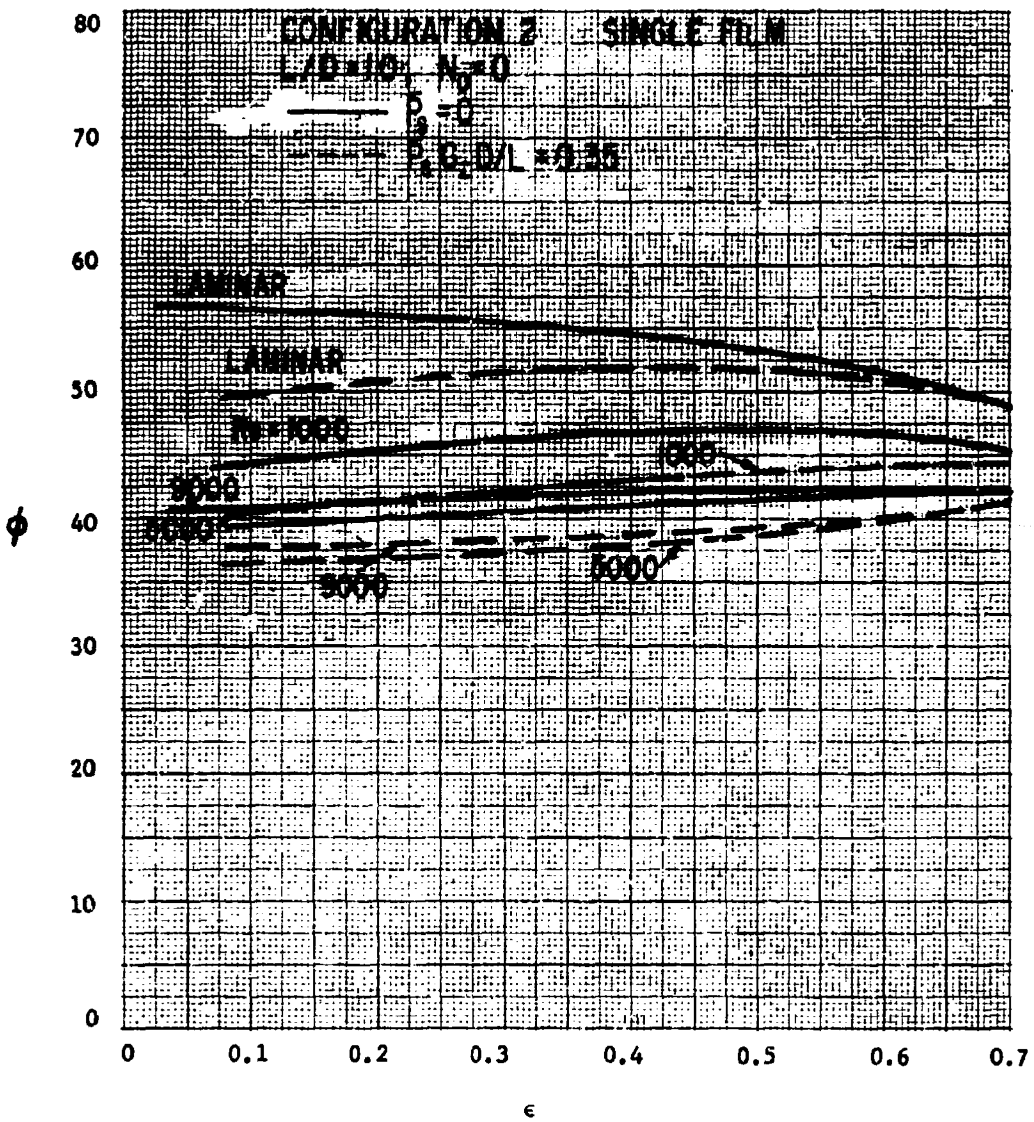


Figure 12. Attitude Angle vs. Eccentricity, Single Film Bearing

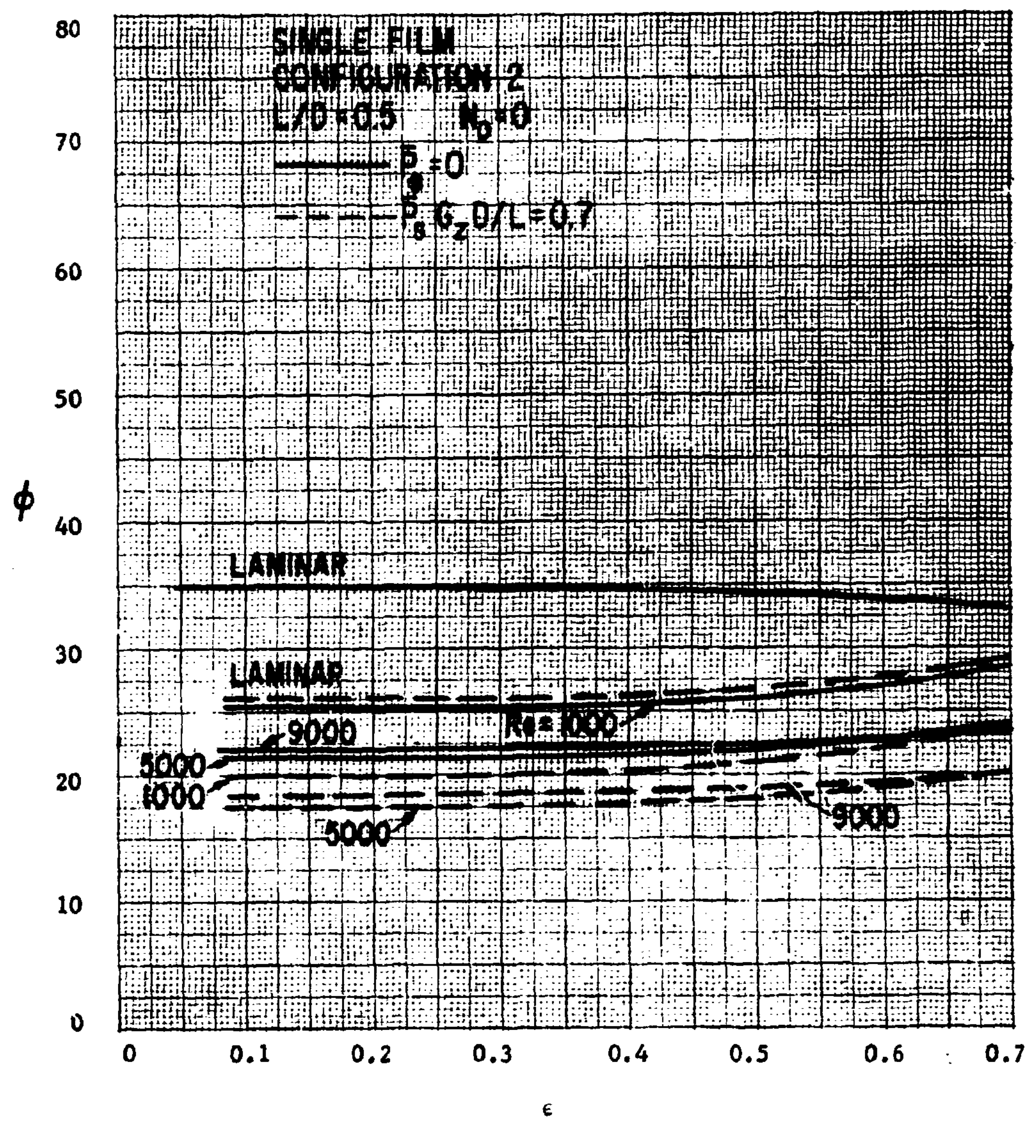


Figure 13. Attitude Angle vs. Eccentricity, Single Film Bearing

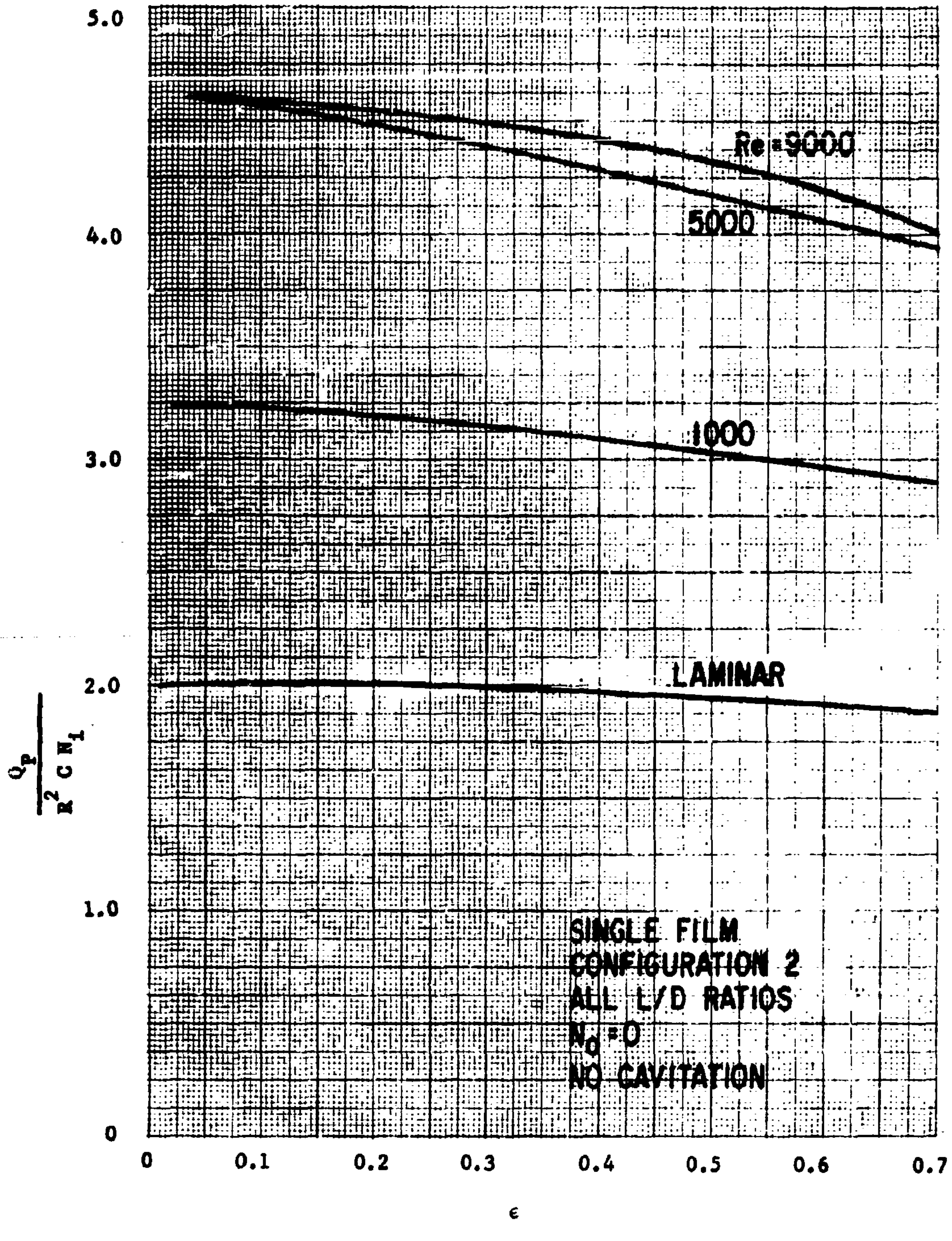


Figure 14. Single Film Bearing Flow Due to Self Pumping Effect of Spiral Grooves

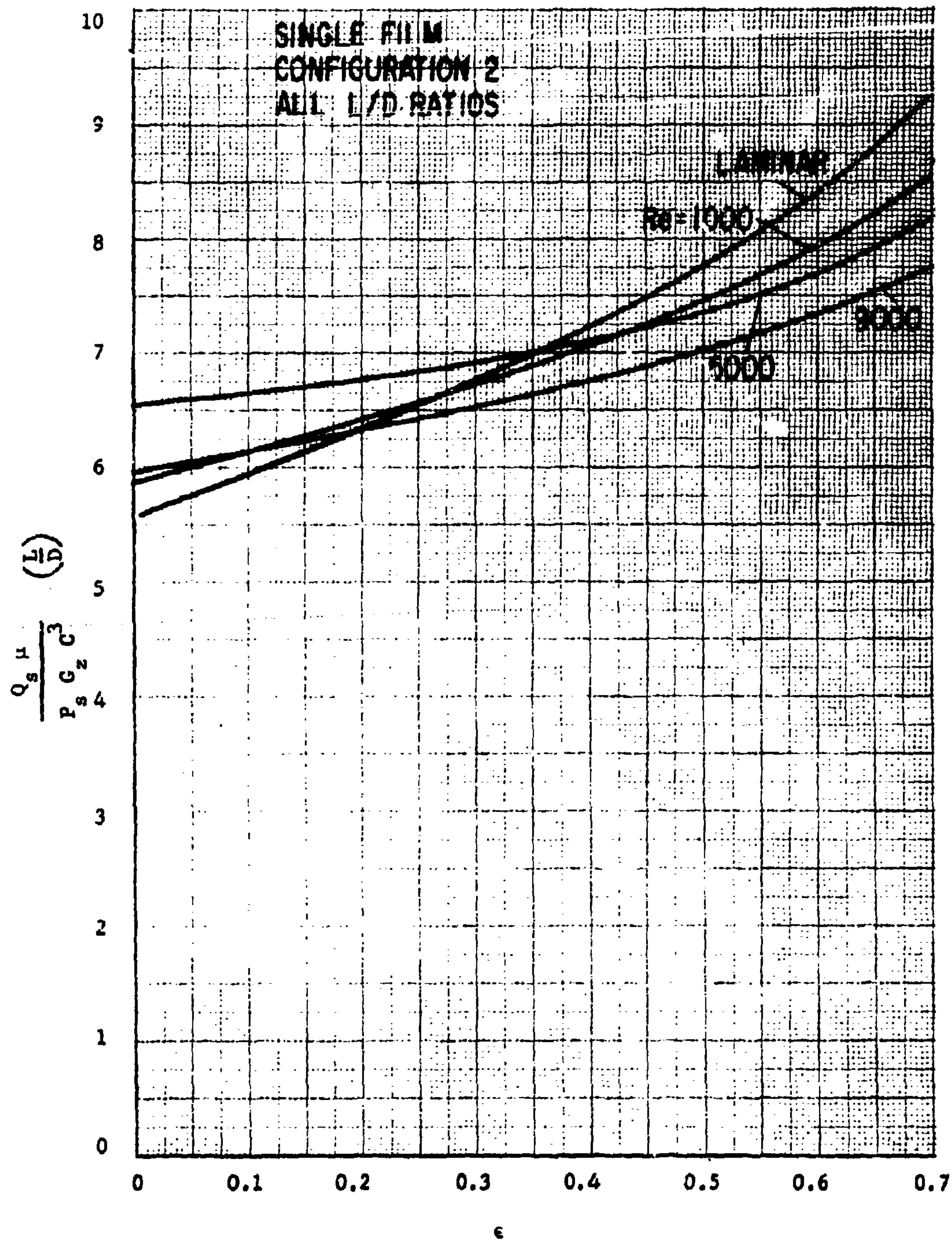


Figure 15. Single Film Bearing Flow Due to Pressurized Supply

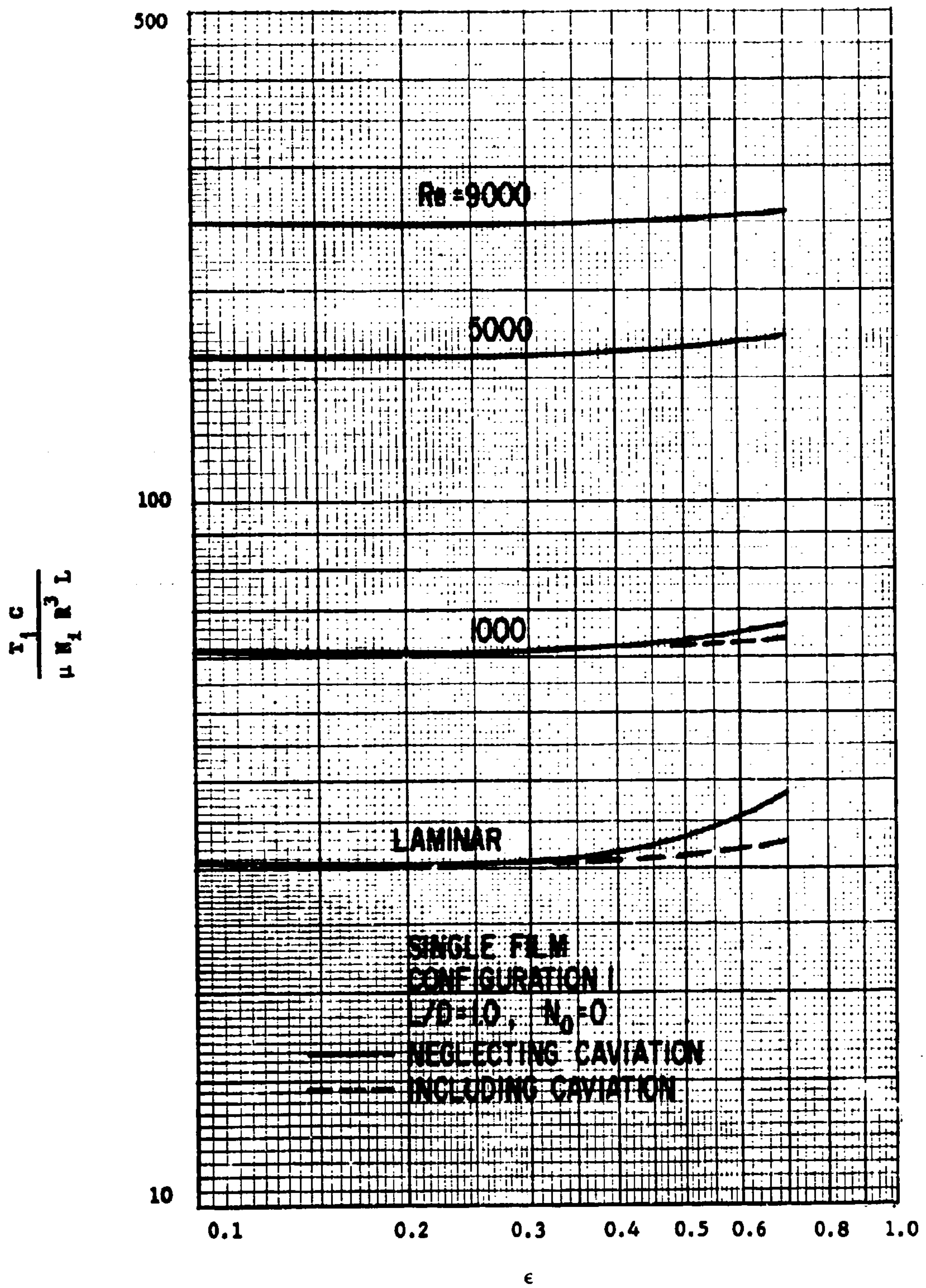


Figure 16. Torque vs. Eccentricity, Single Film Bearing

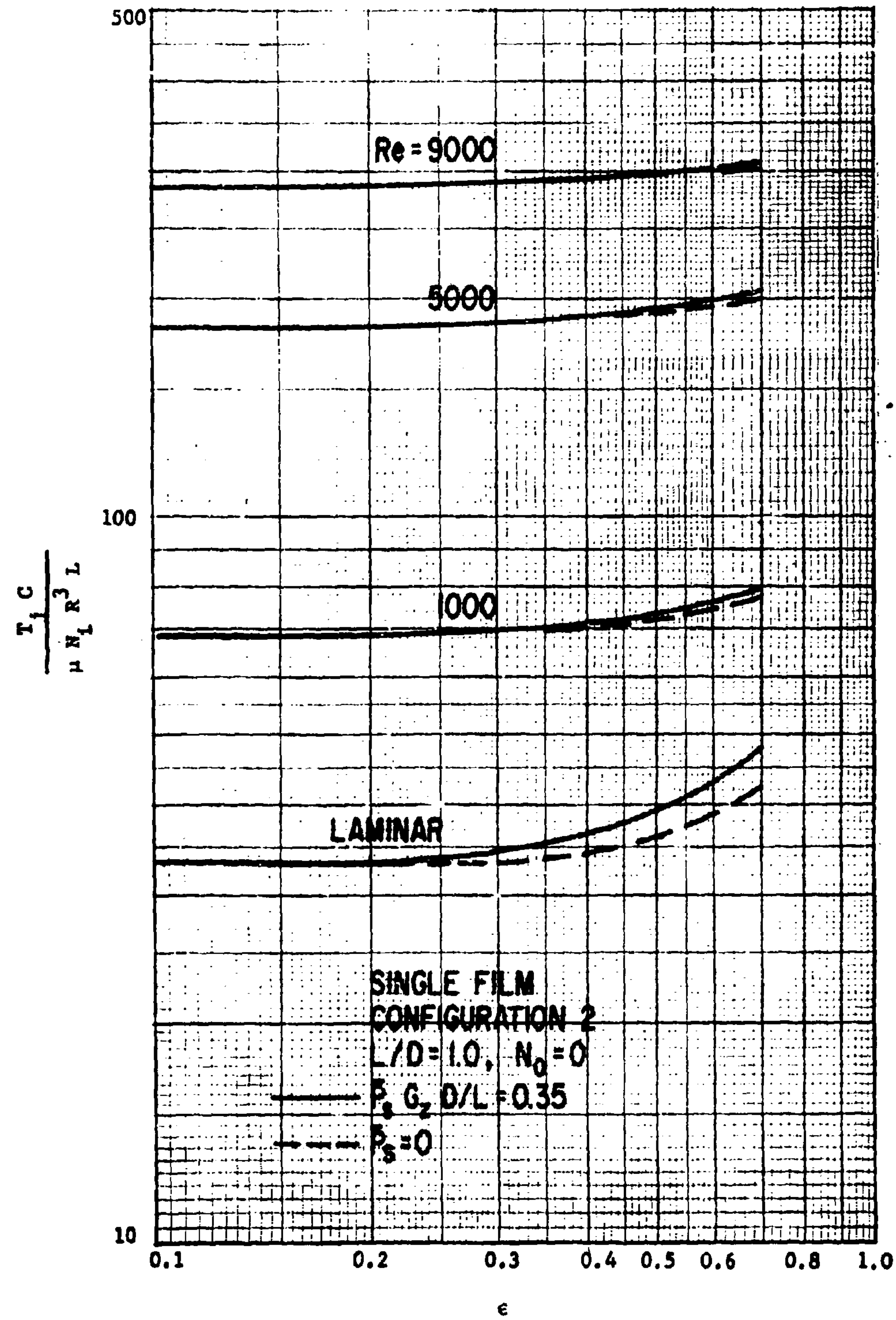


Figure 17. Torque vs. Eccentricity, Single Film Bearing

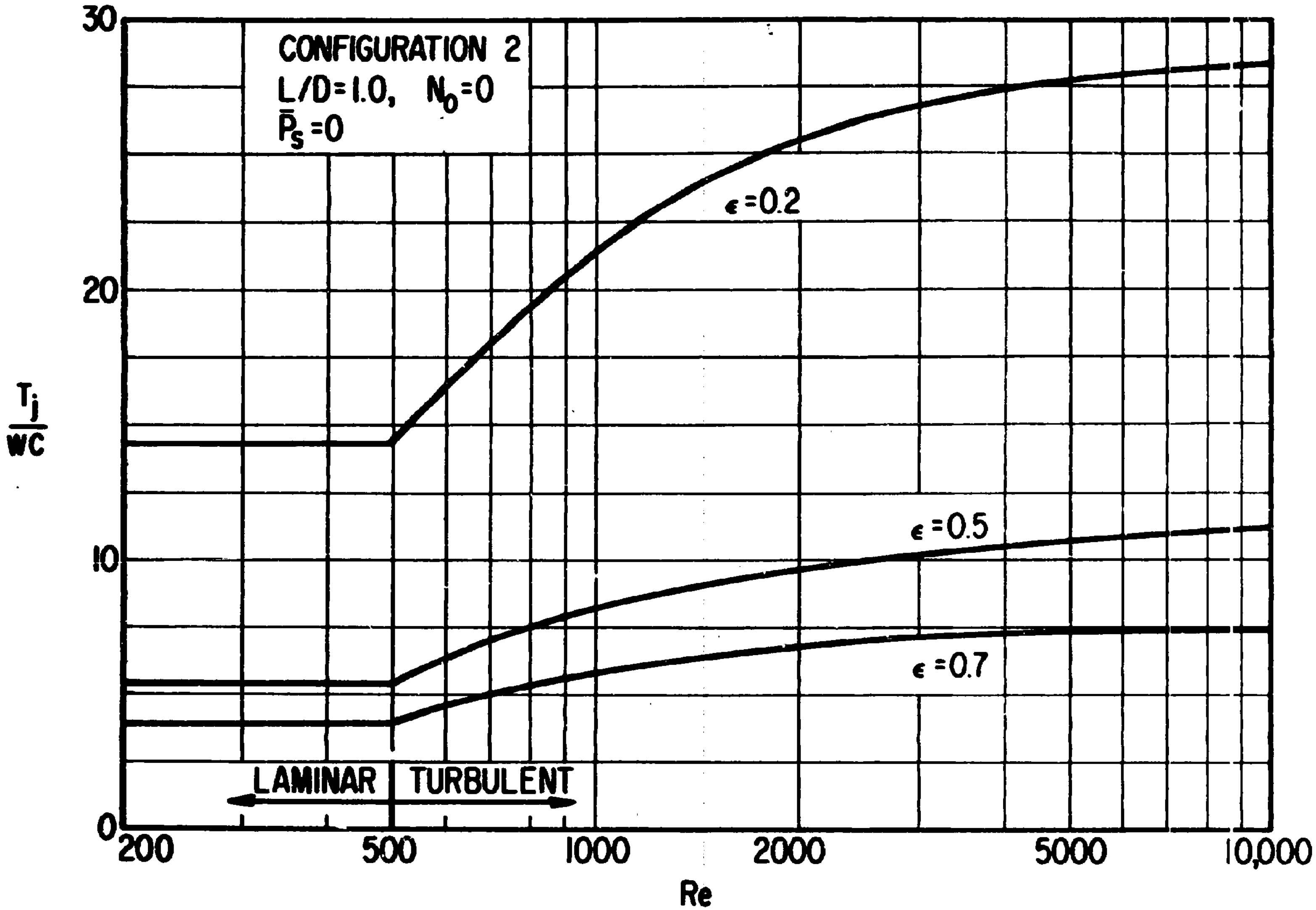


Figure 18. Torque to Load Ratio vs. Reynolds Number, Single Film Bearing

MTI-6623

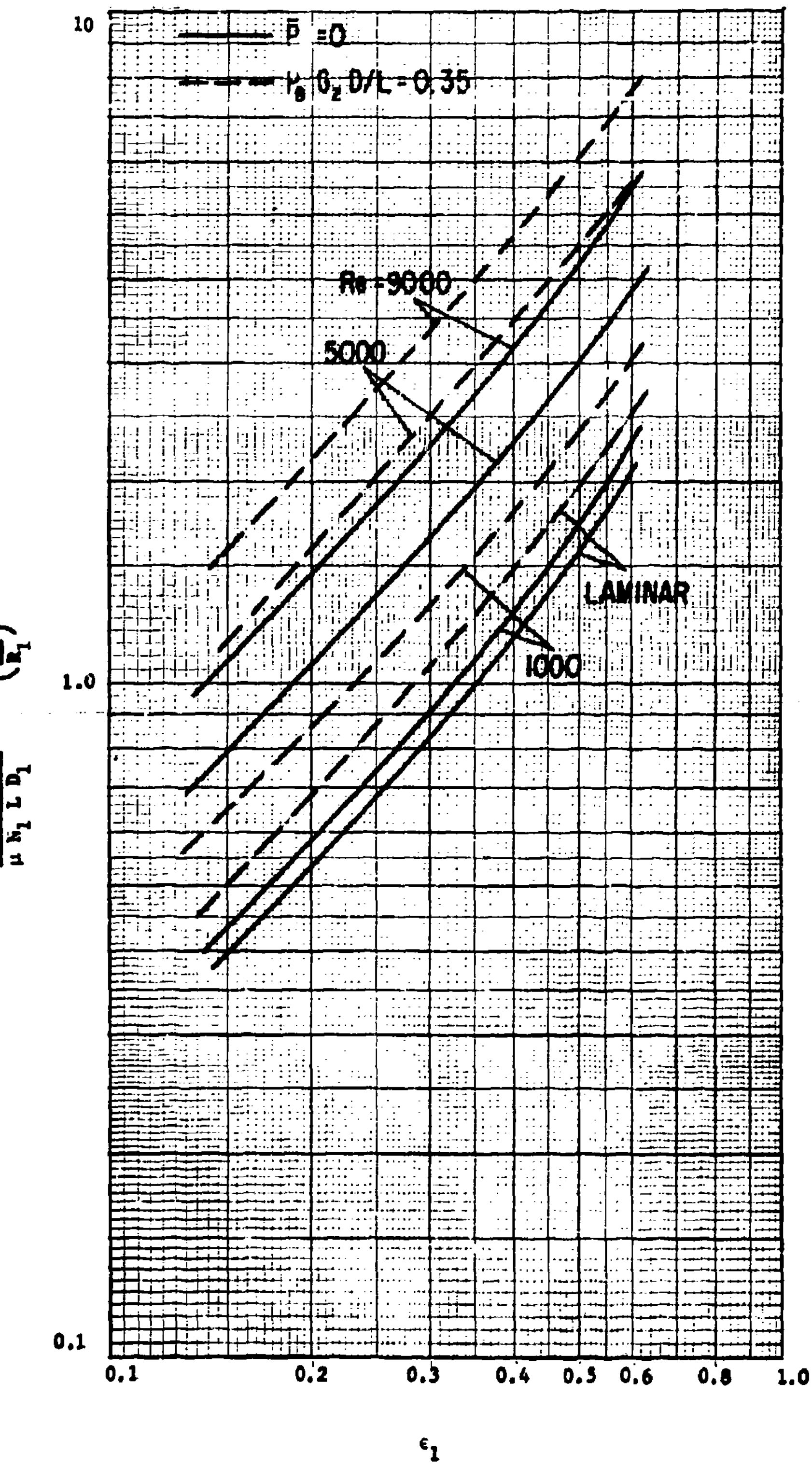


Figure 19. Load vs. Inner Film Eccentricity Ratio, Floating Ring Bearing

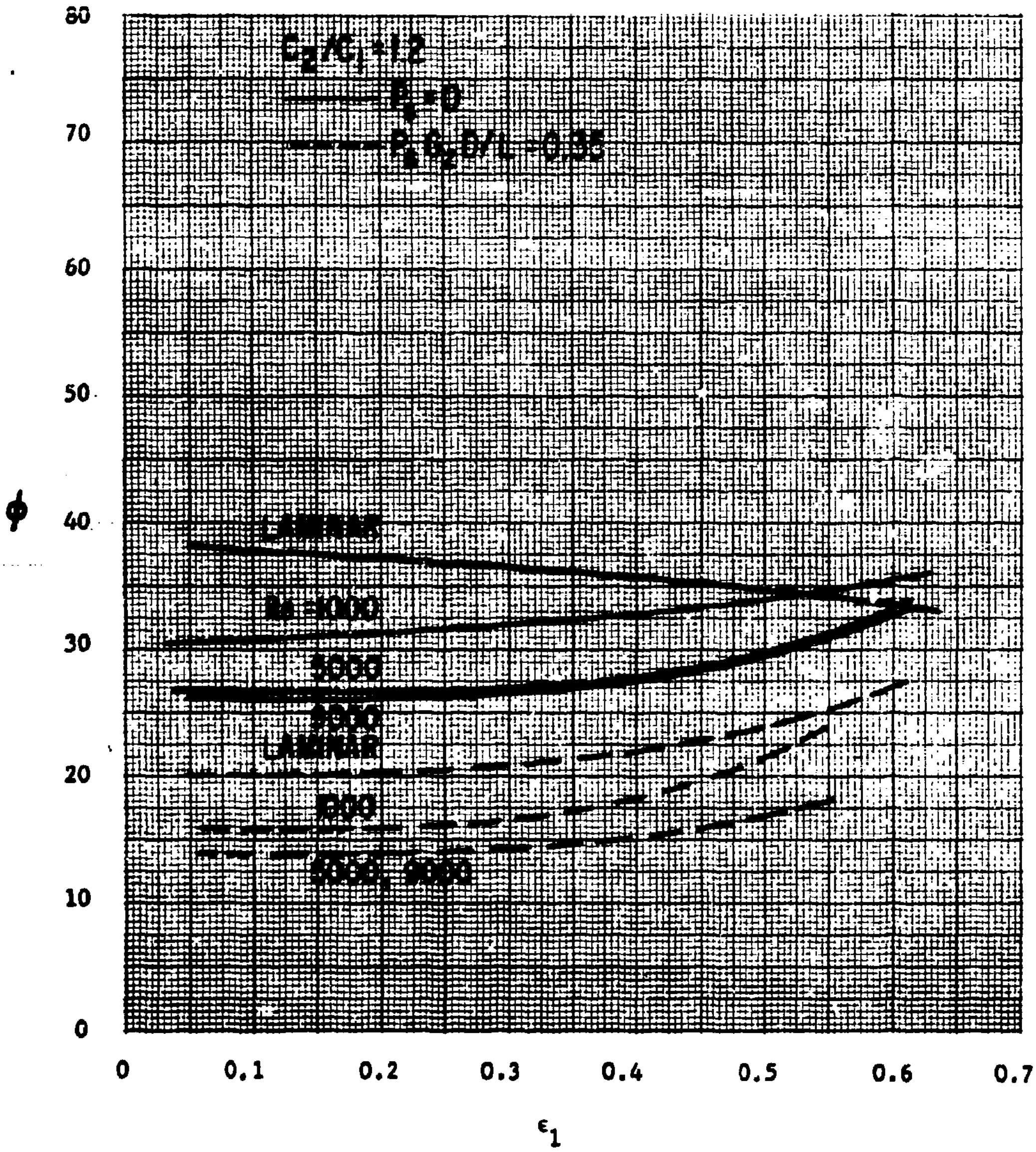


Figure 20. Attitude Angle vs. Inner Film Eccentricity Ratio, Floating Ring Bearing

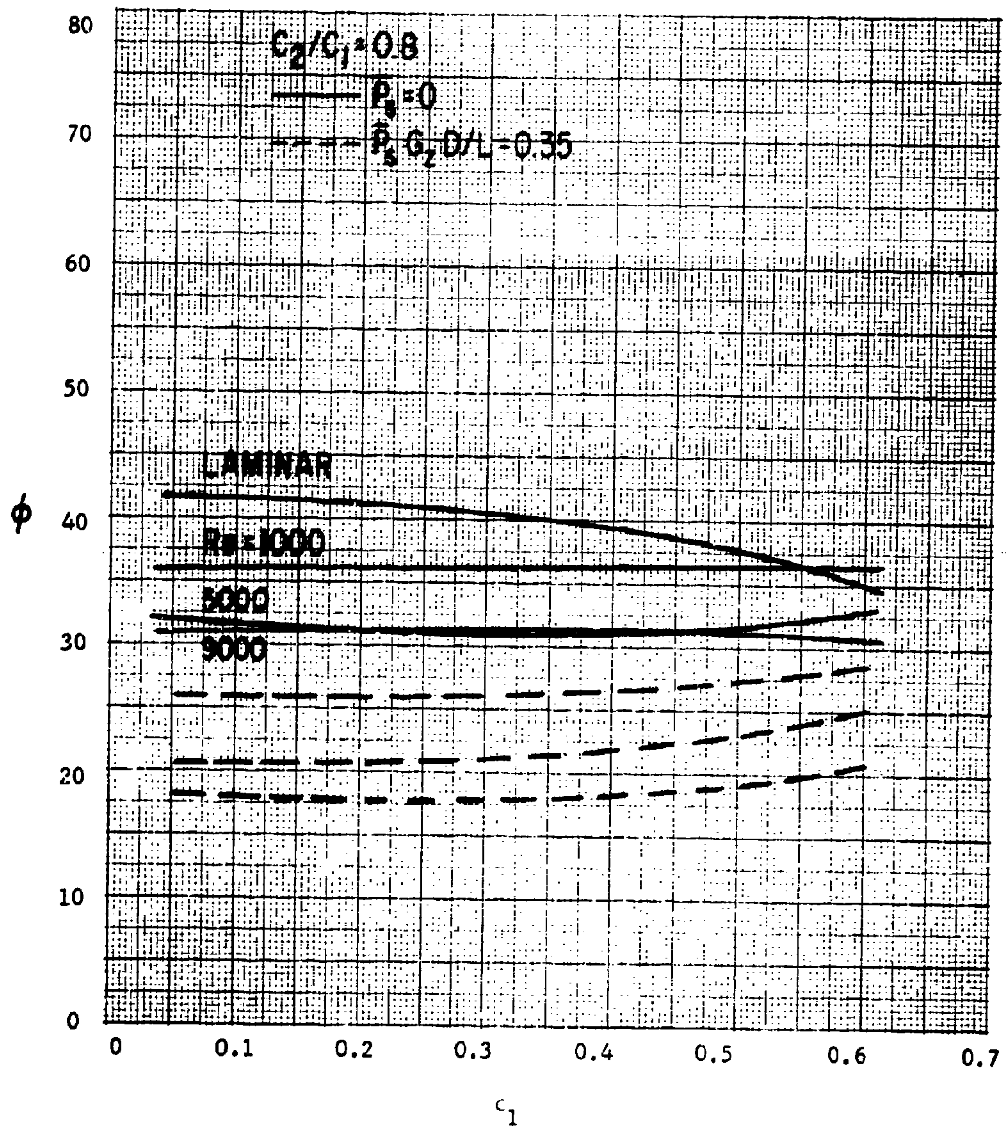


Figure 21. **Attitude Angle vs. Inner Film Eccentricity Ratio, Floating Ring Bearing**

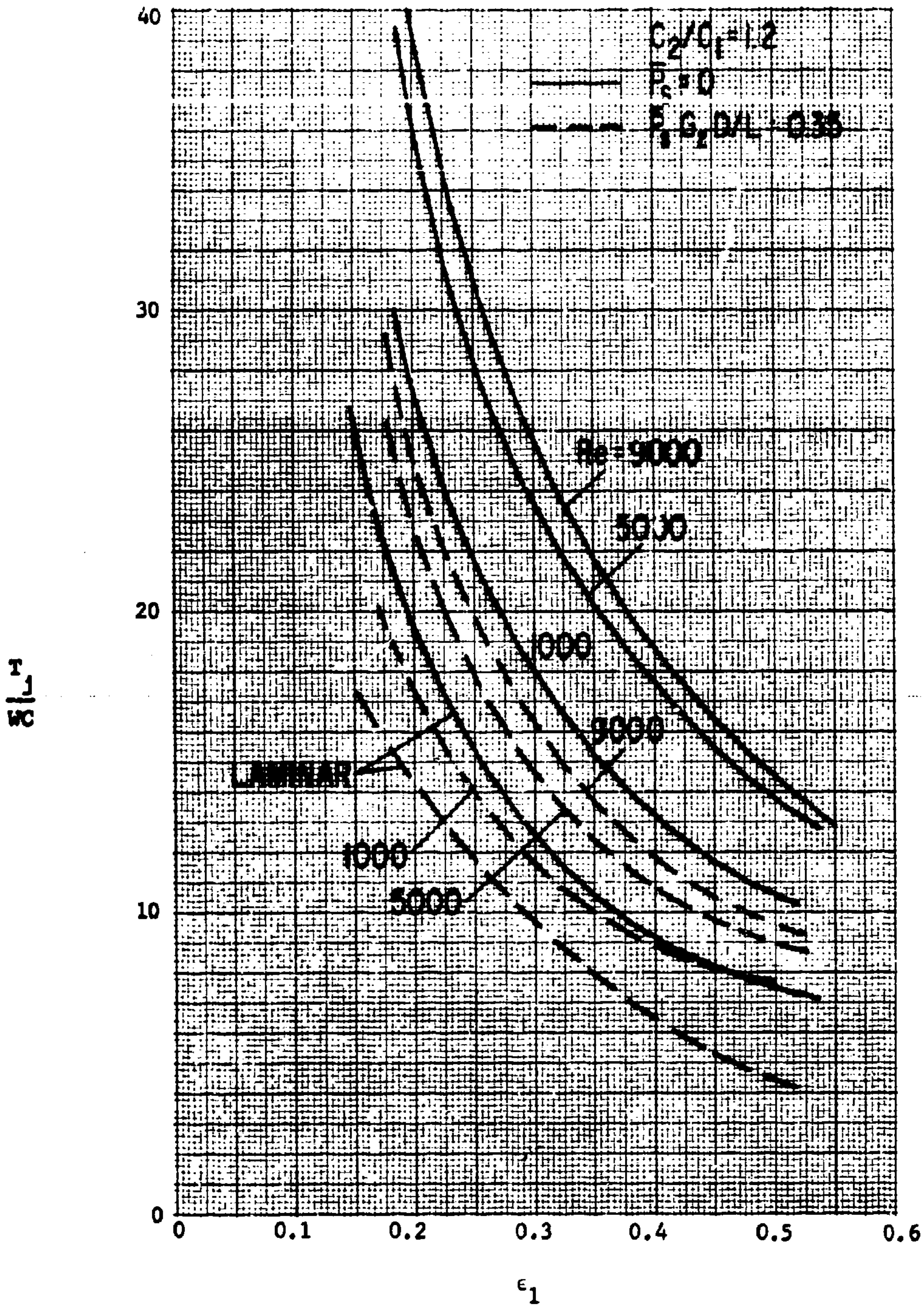


Figure 22. Torque to Load Ratio vs. Inner Film Eccentricity, Floating Ring Bearing

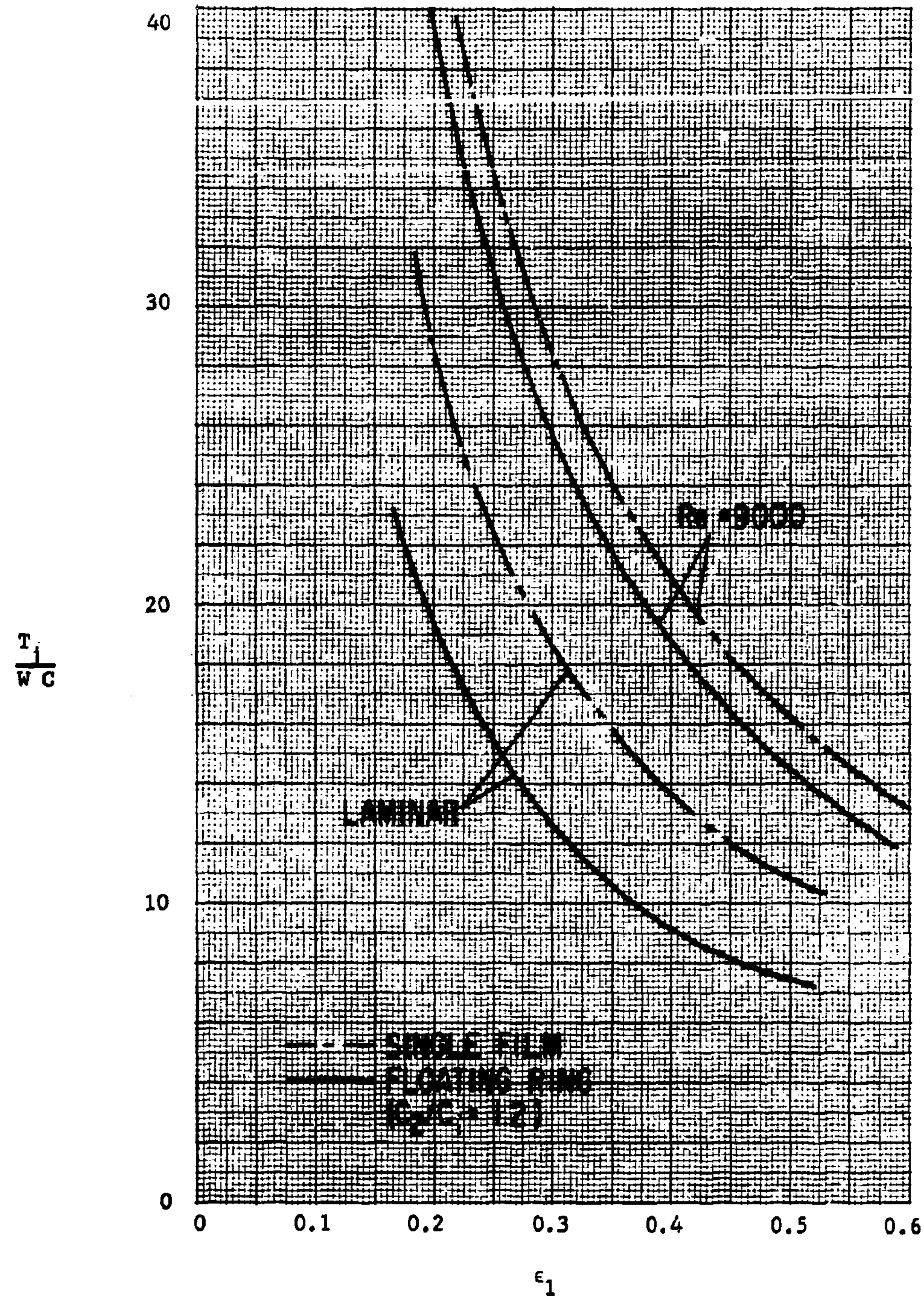


Figure 23. Torque to Load Ratio, Comparison between Single Film and Floating Ring Bearing

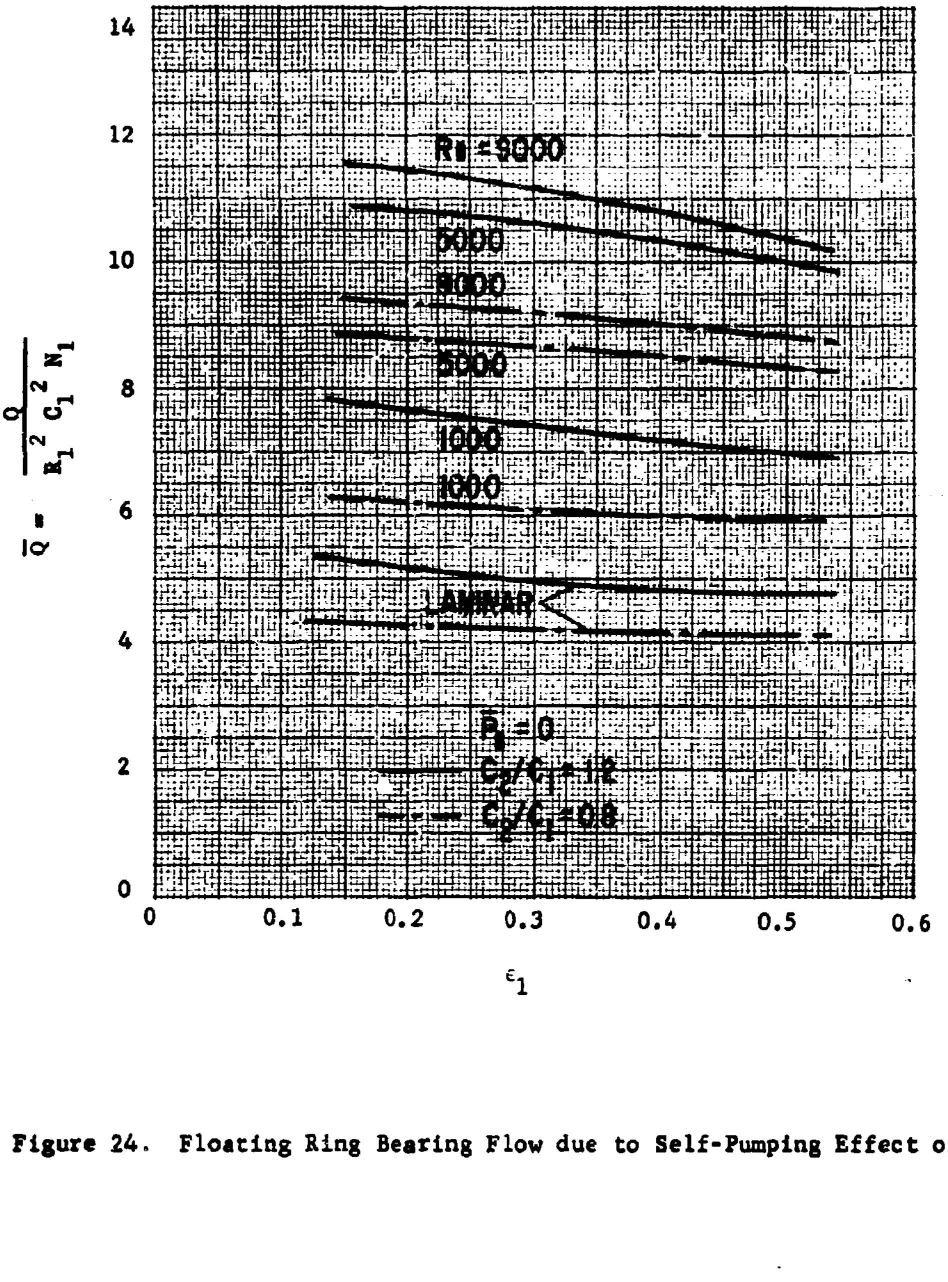
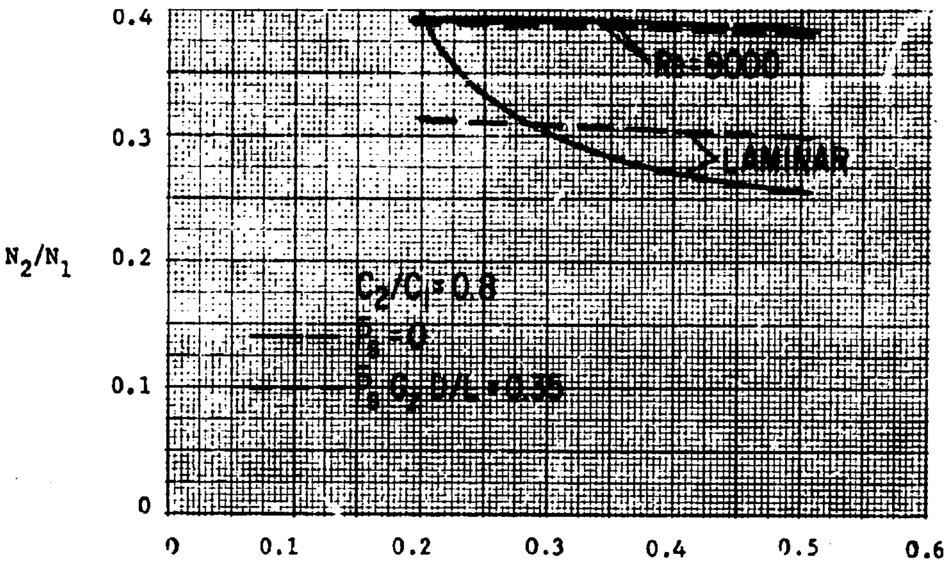


Figure 24. Floating Ring Bearing Flow due to Self-Pumping Effect of Spiral Grooves



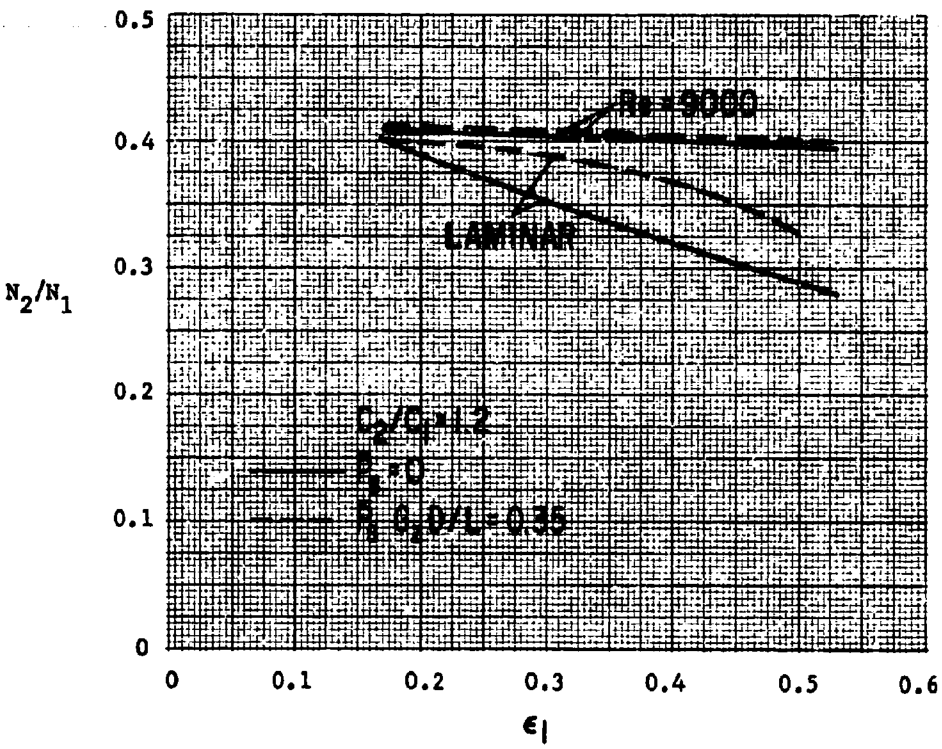


Figure 25. Ring Speed Ratio vs. Inner Film Eccentricity Ratio, Floating Ring Bearing

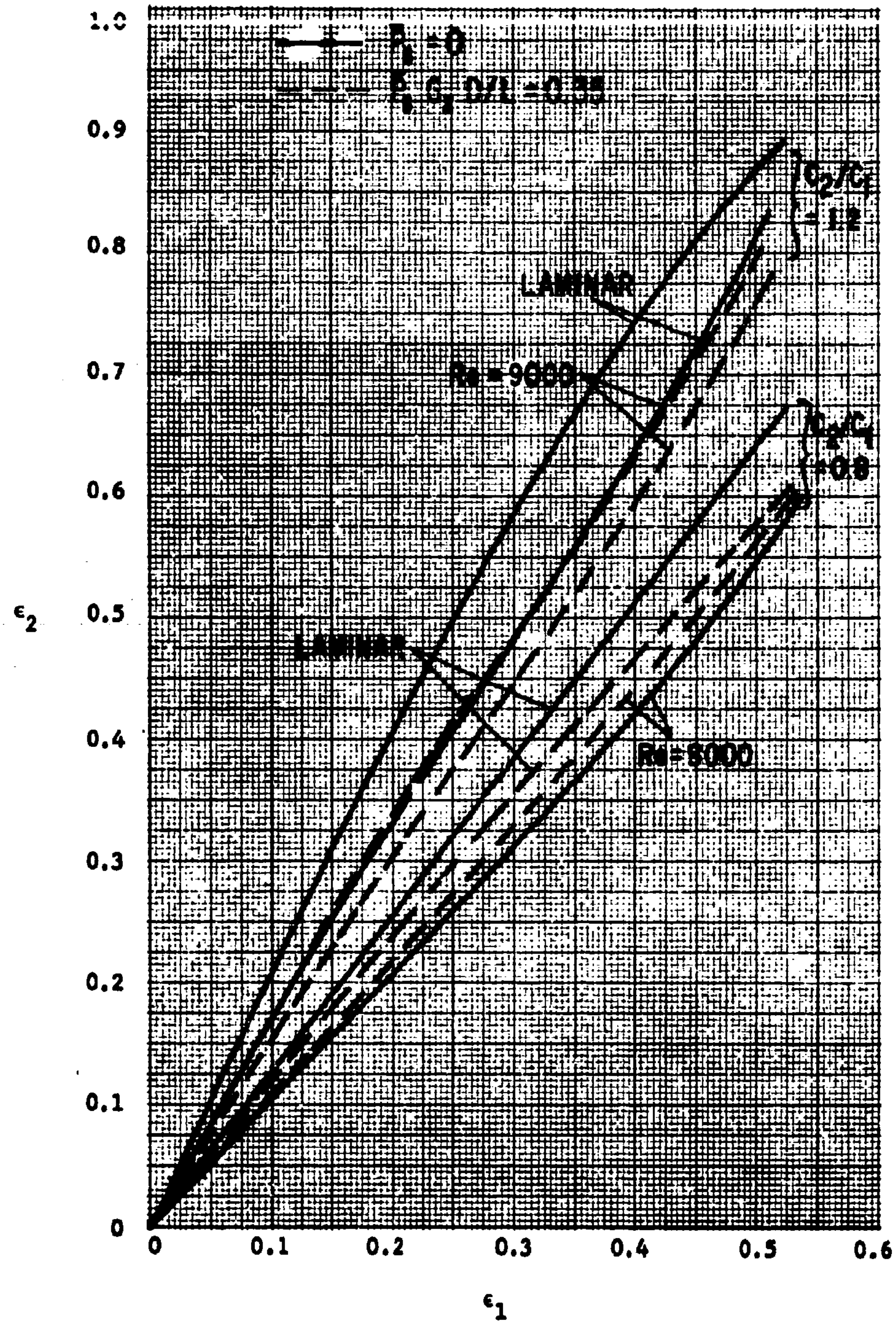


Figure 26. Outer Film Eccentricity Ratio vs. Inner Film Eccentricity Ratio, Floating Ring Bearing

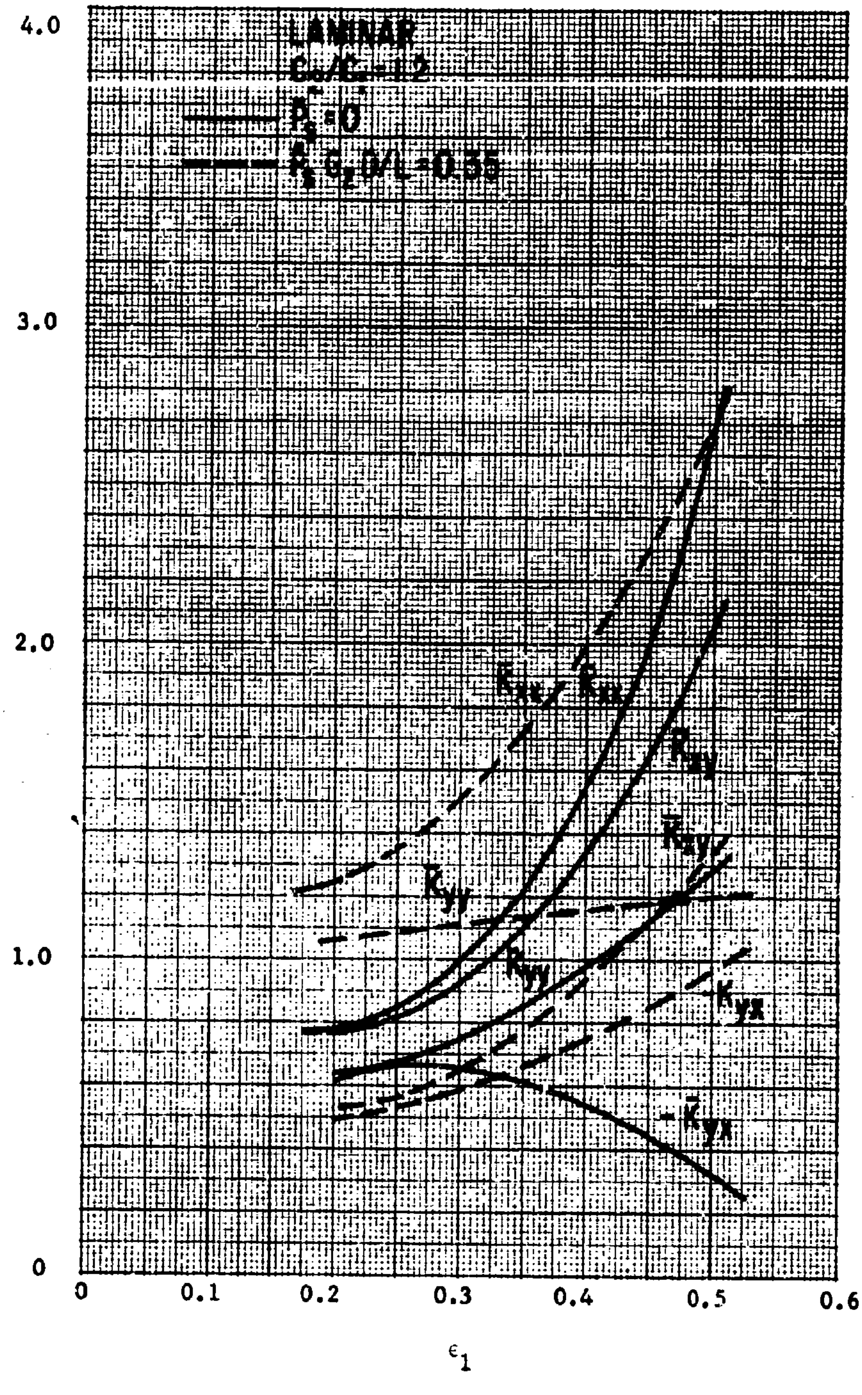


Figure 27. Stiffness Coefficients, Floating Ring Bearing

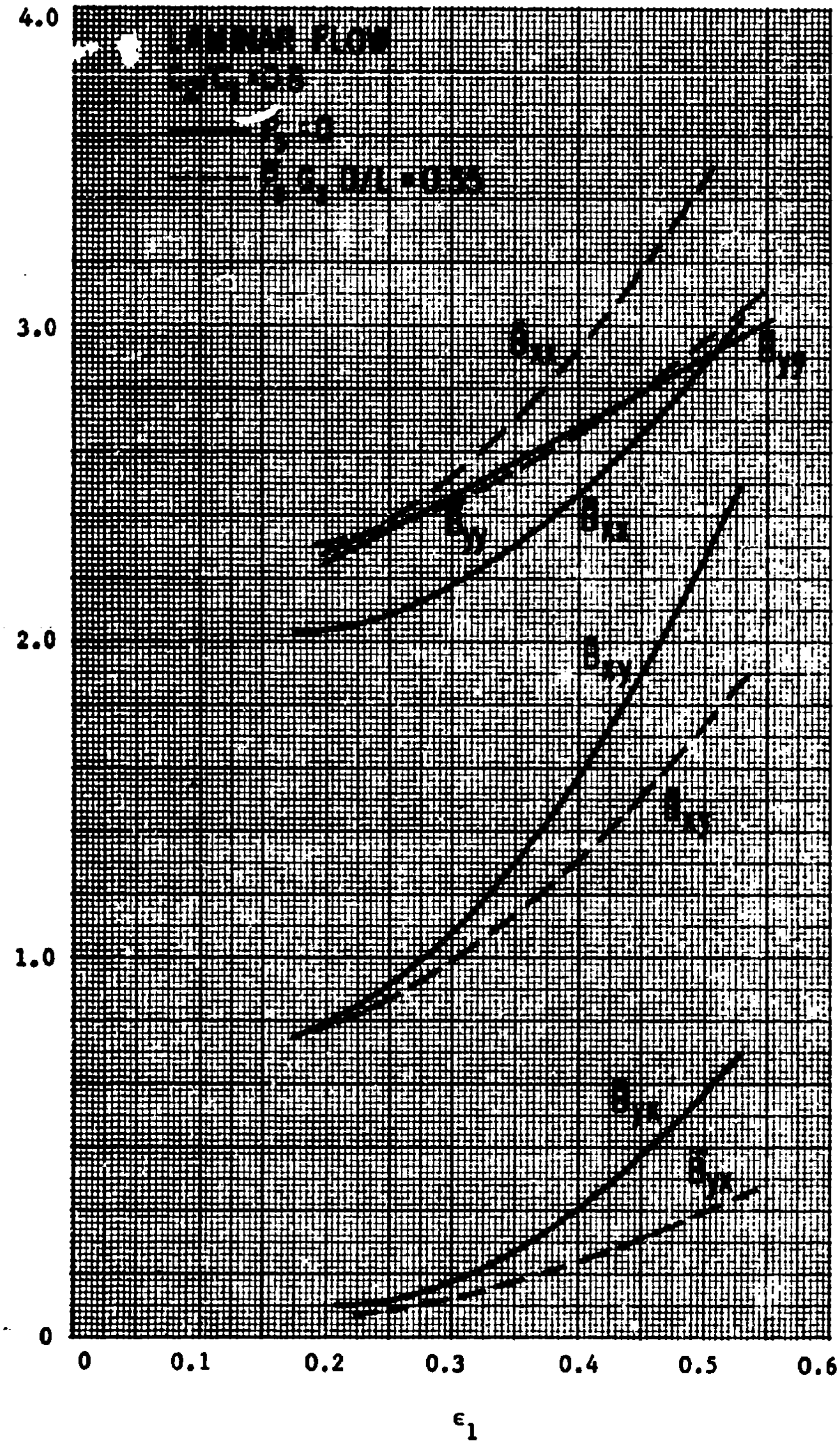


Figure 28. Damping Coefficients, Floating Ring Bearing

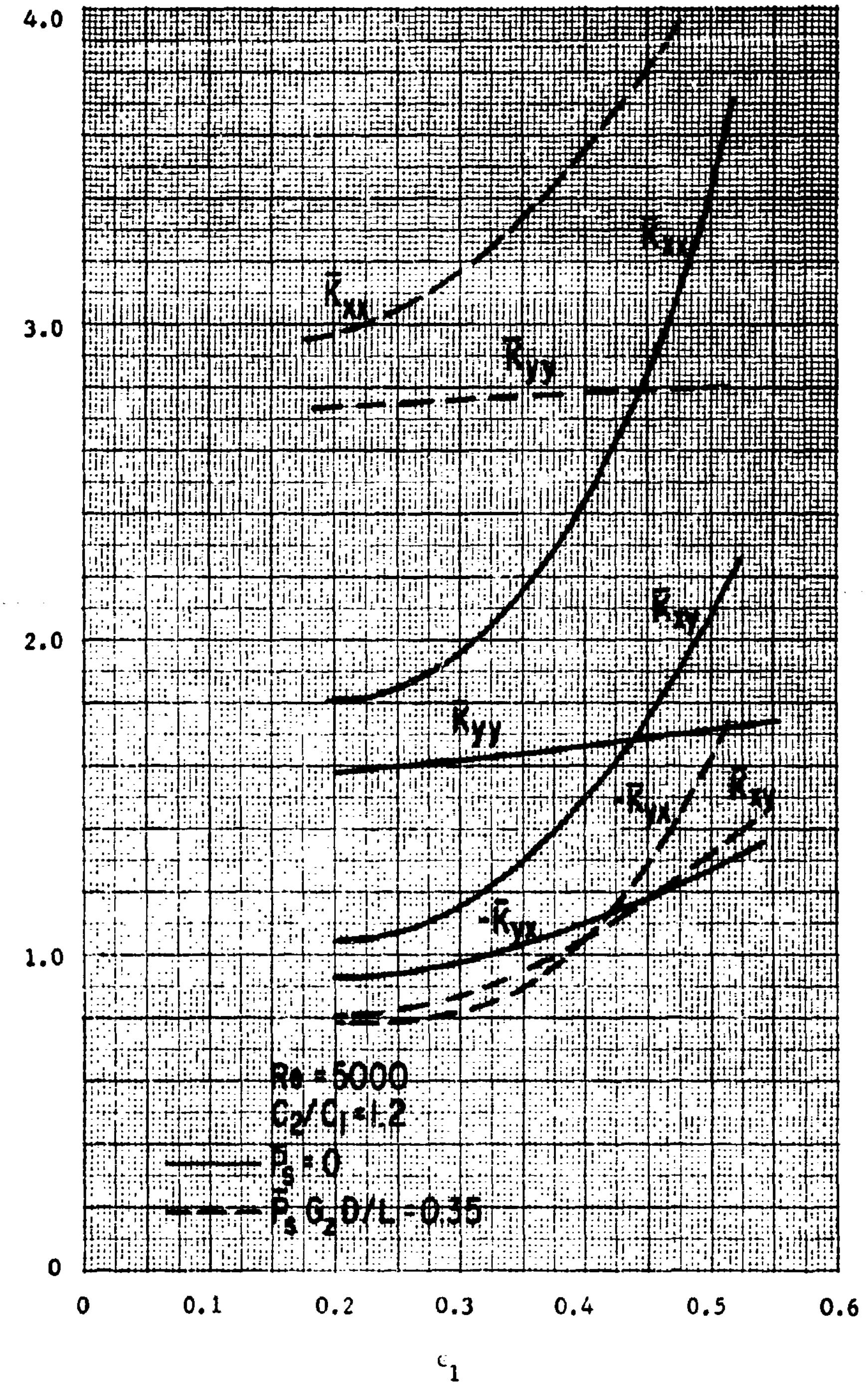


Figure 29. Stiffness Coefficients, Floating Ring Bearing

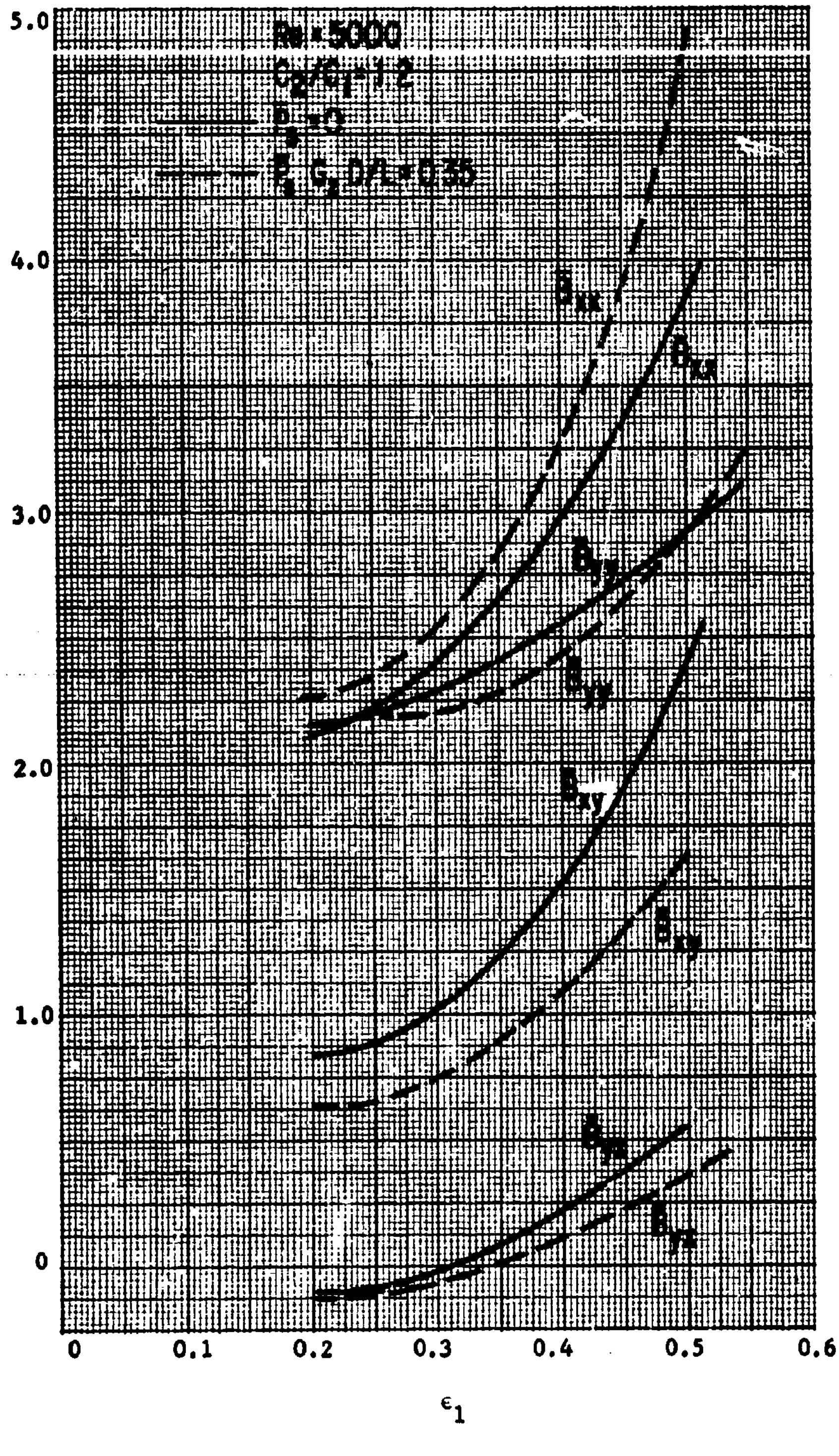


Figure 30. Damping Coefficients, Floating Ring Bearing

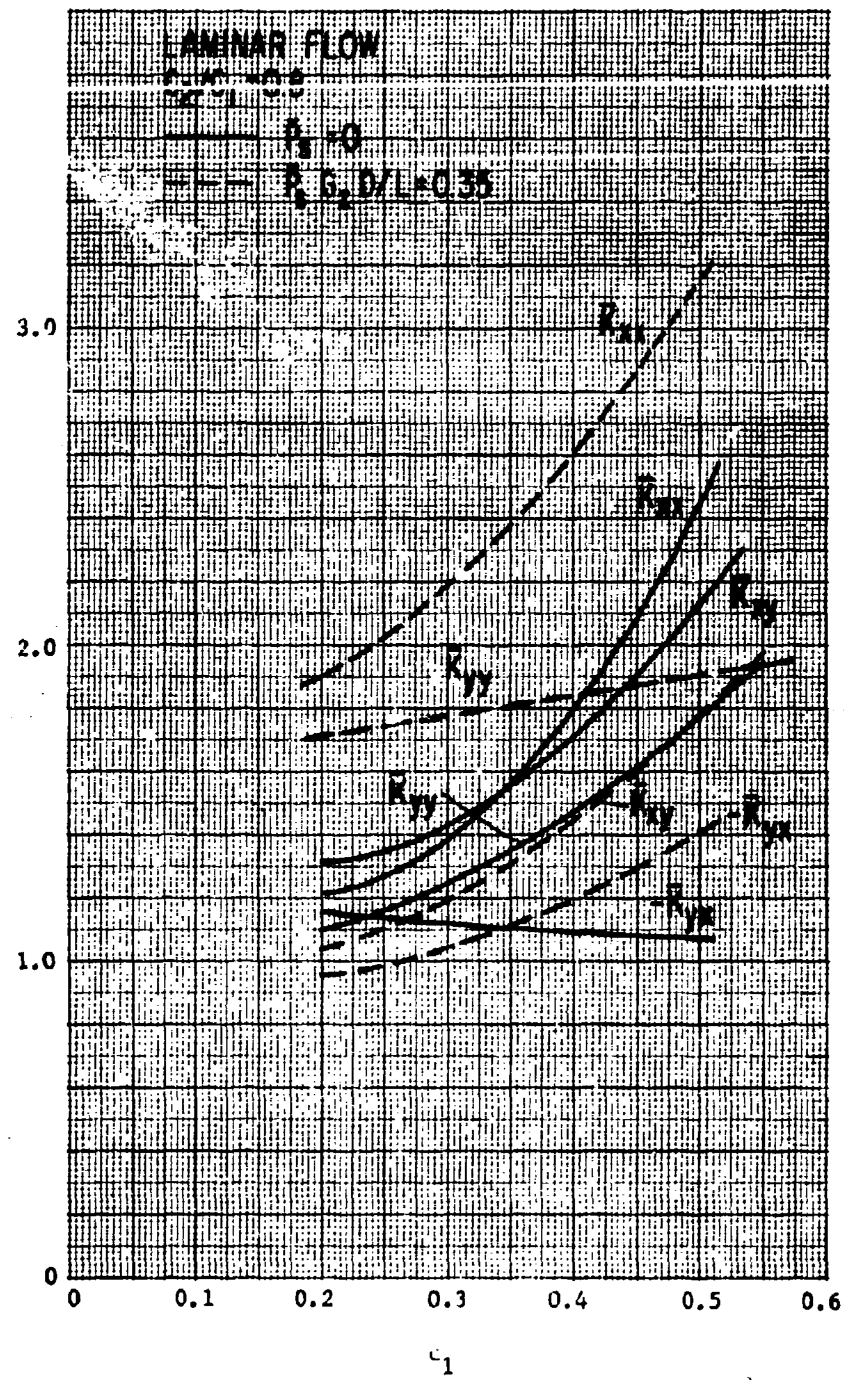


Figure 31. Stiffness Coefficients, Floating Ring Bearing

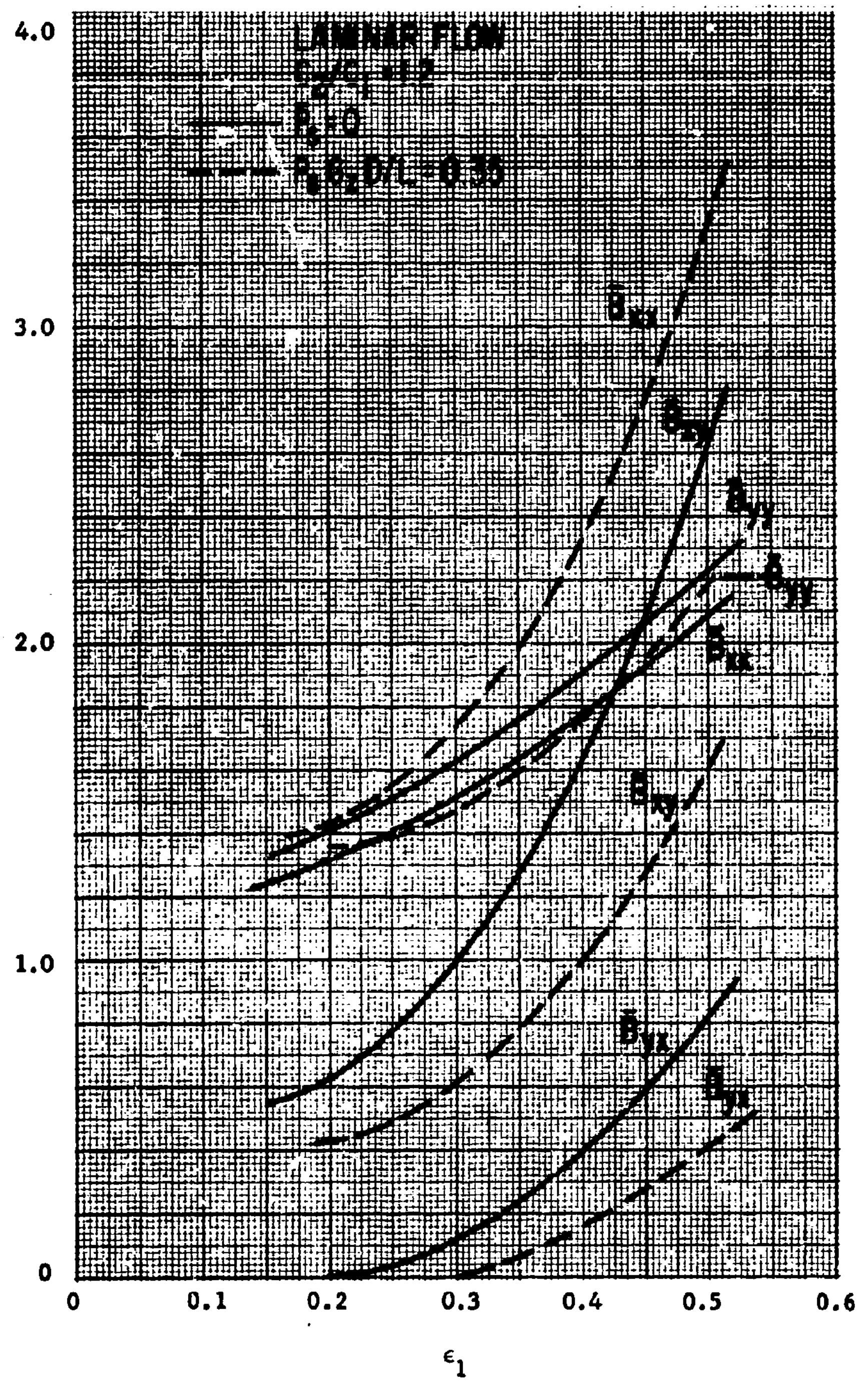


Figure 32. Damping Coefficients, Floating Ring Bearing

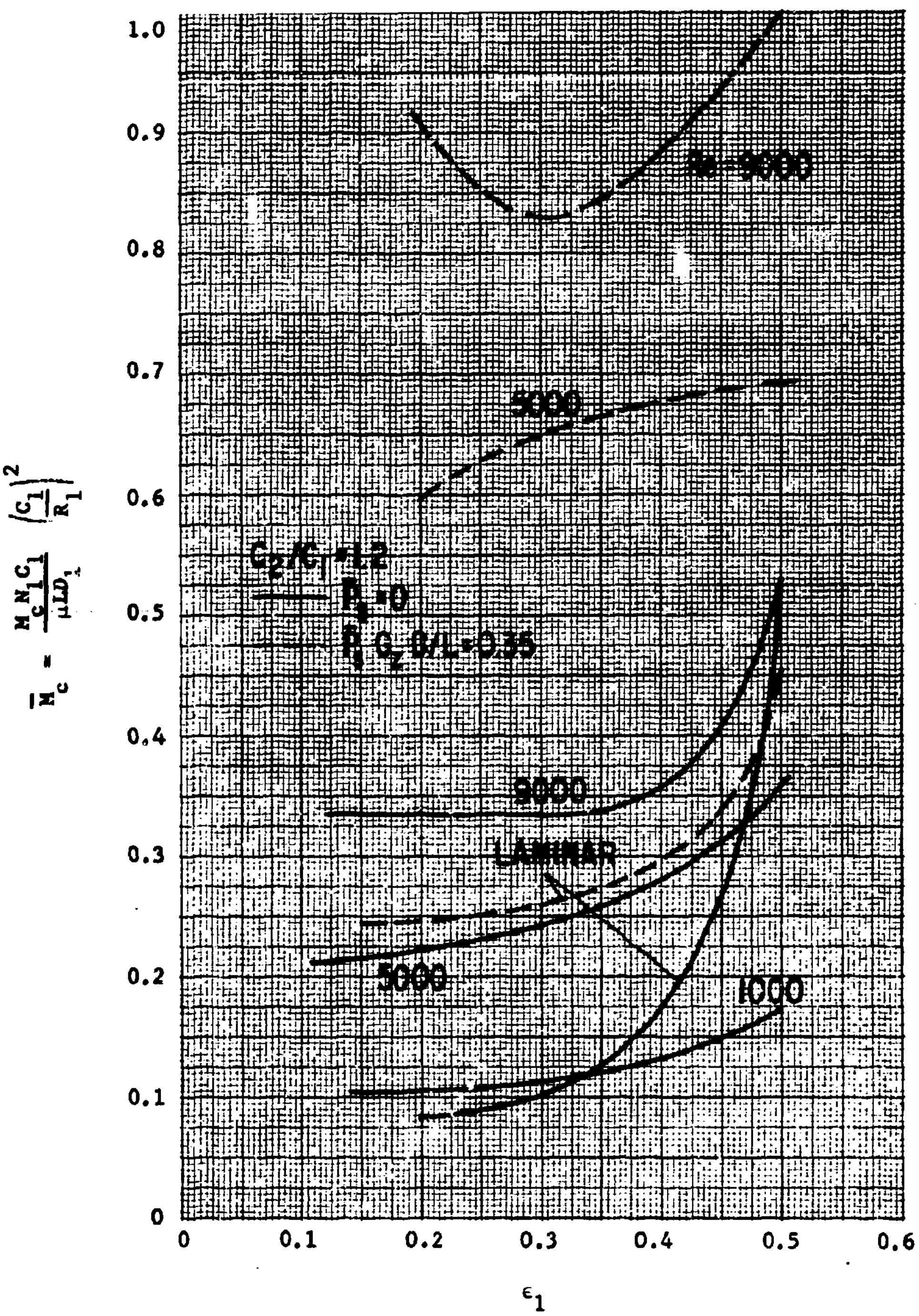


Figure 33. Critical Journal Mass, Floating Ring Bearing

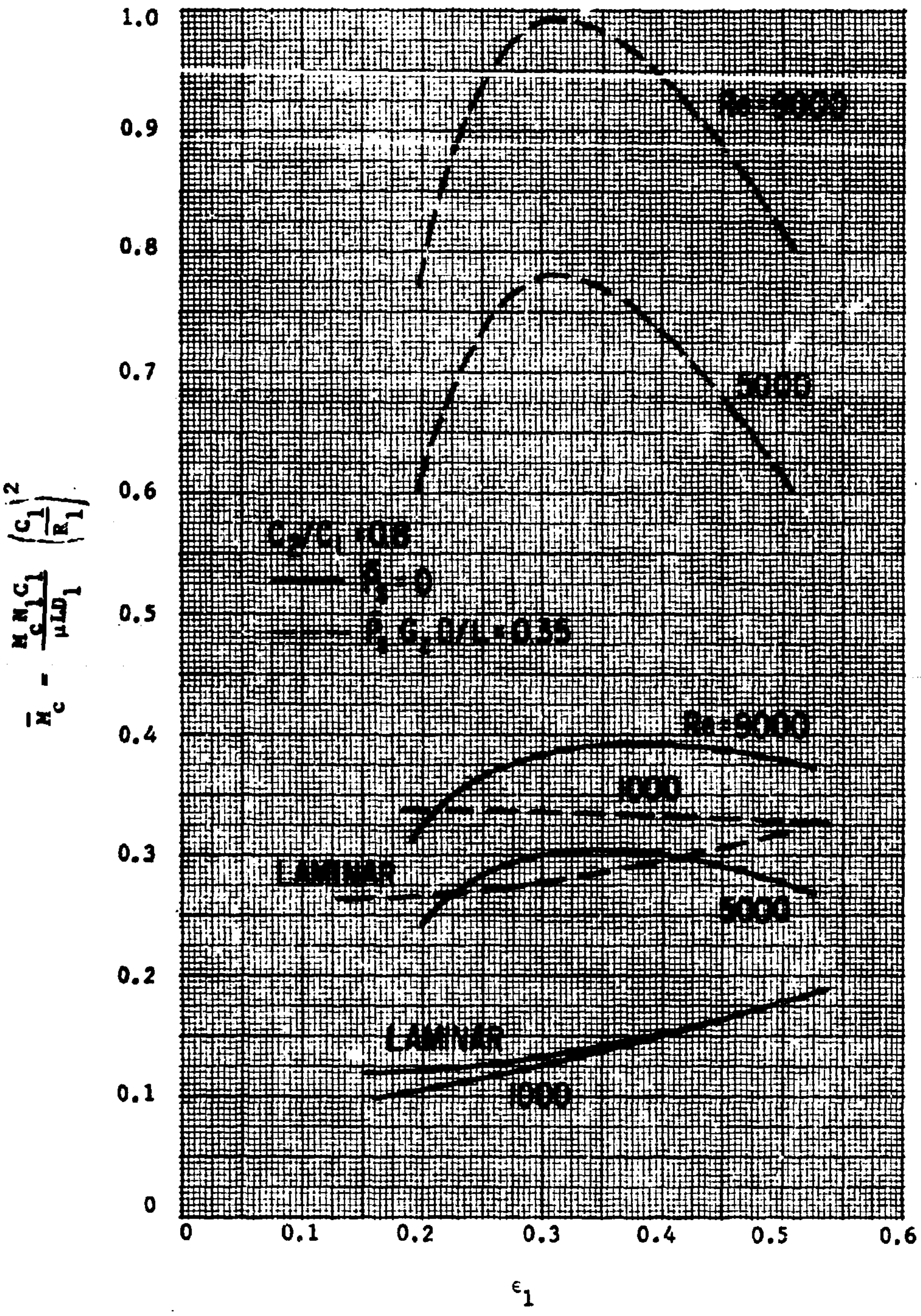


Figure 34. Critical Journal Mass, Floating Ring Bearing

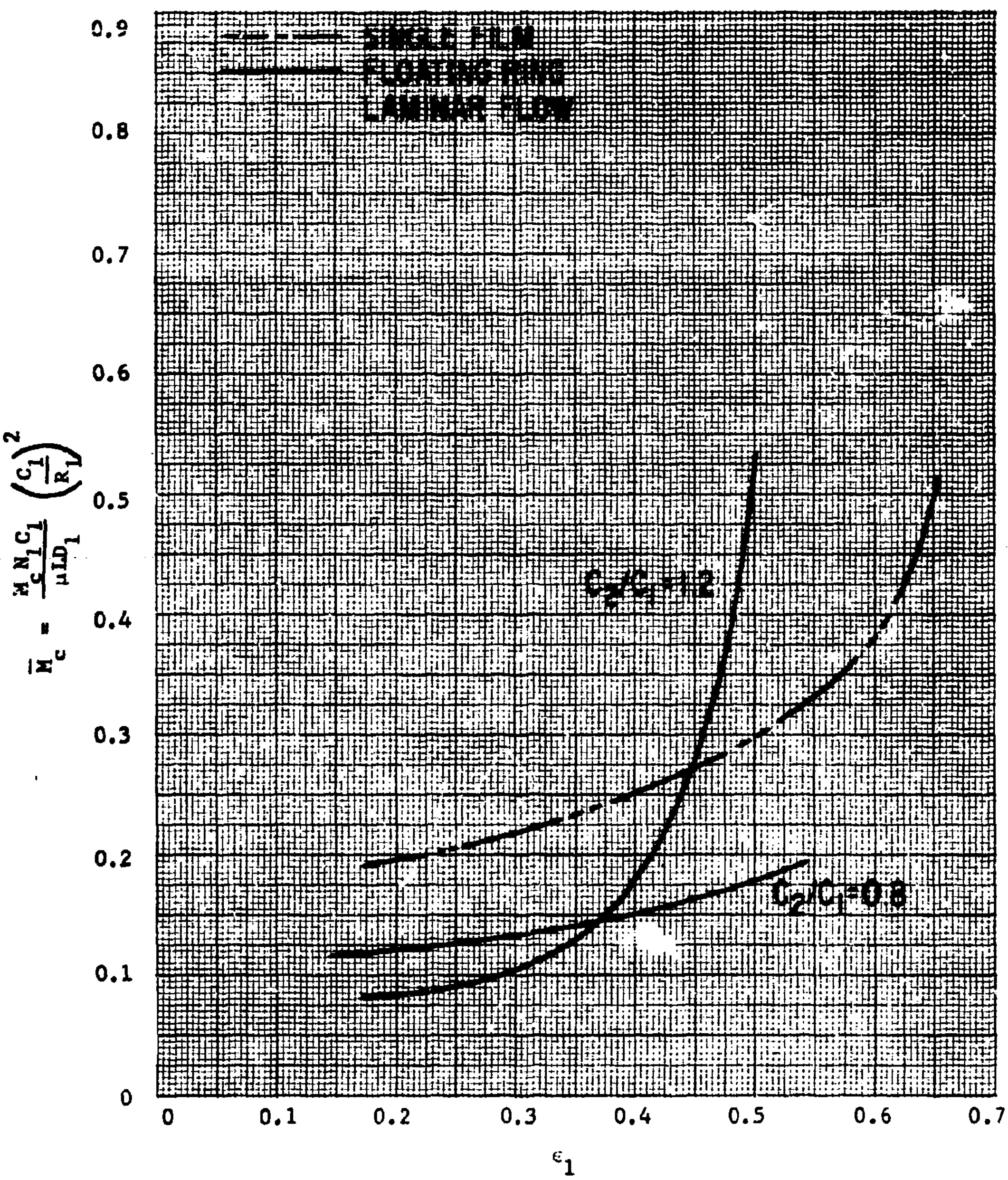


Figure 35. Critical Journal Mass, Comparison between Plain and Spiral-Grooved Floating Ring Bearings

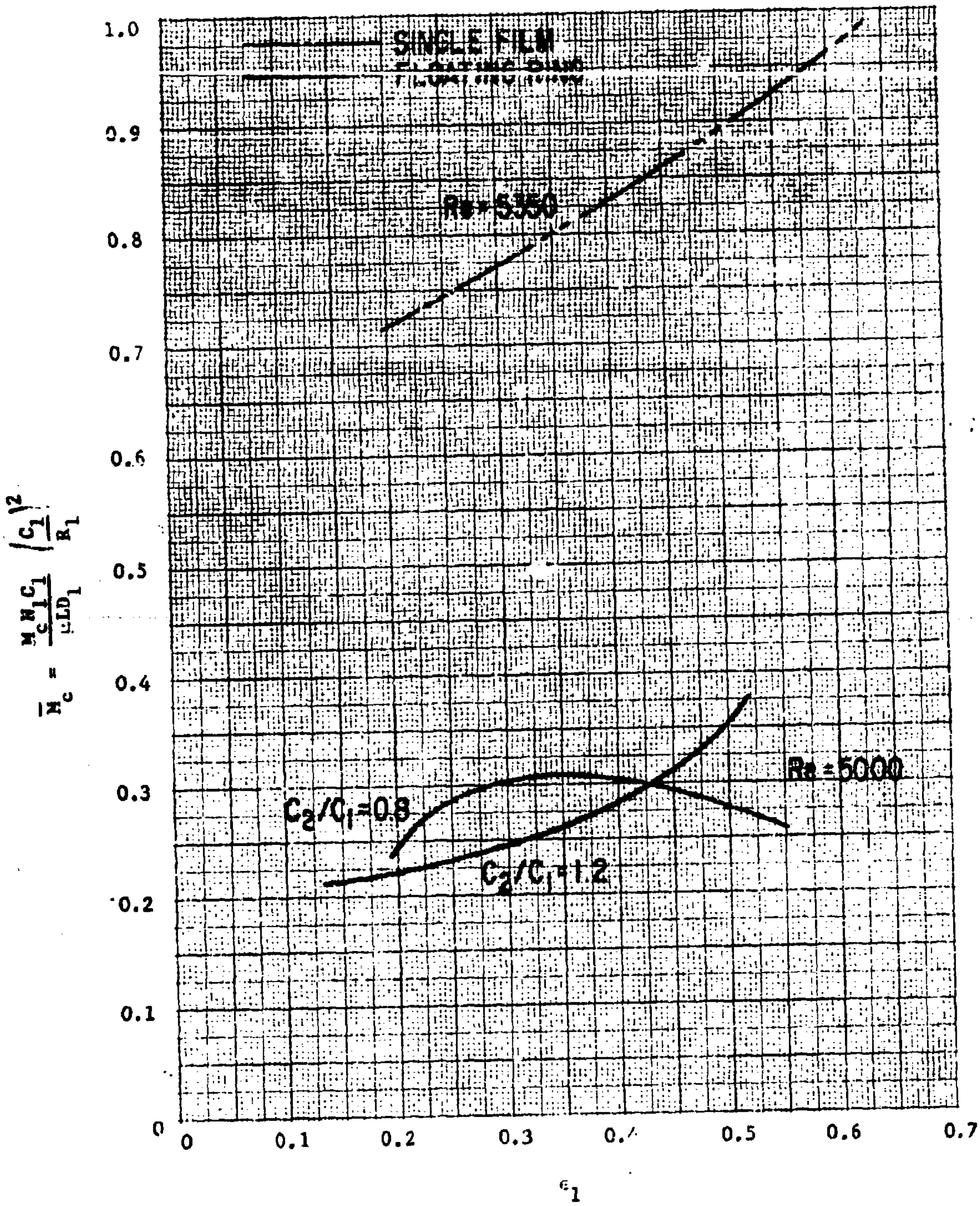


Figure 36. Critical Journal Mass, Comparison between Plain and Spiral-Grooved Floating Ring Bearings

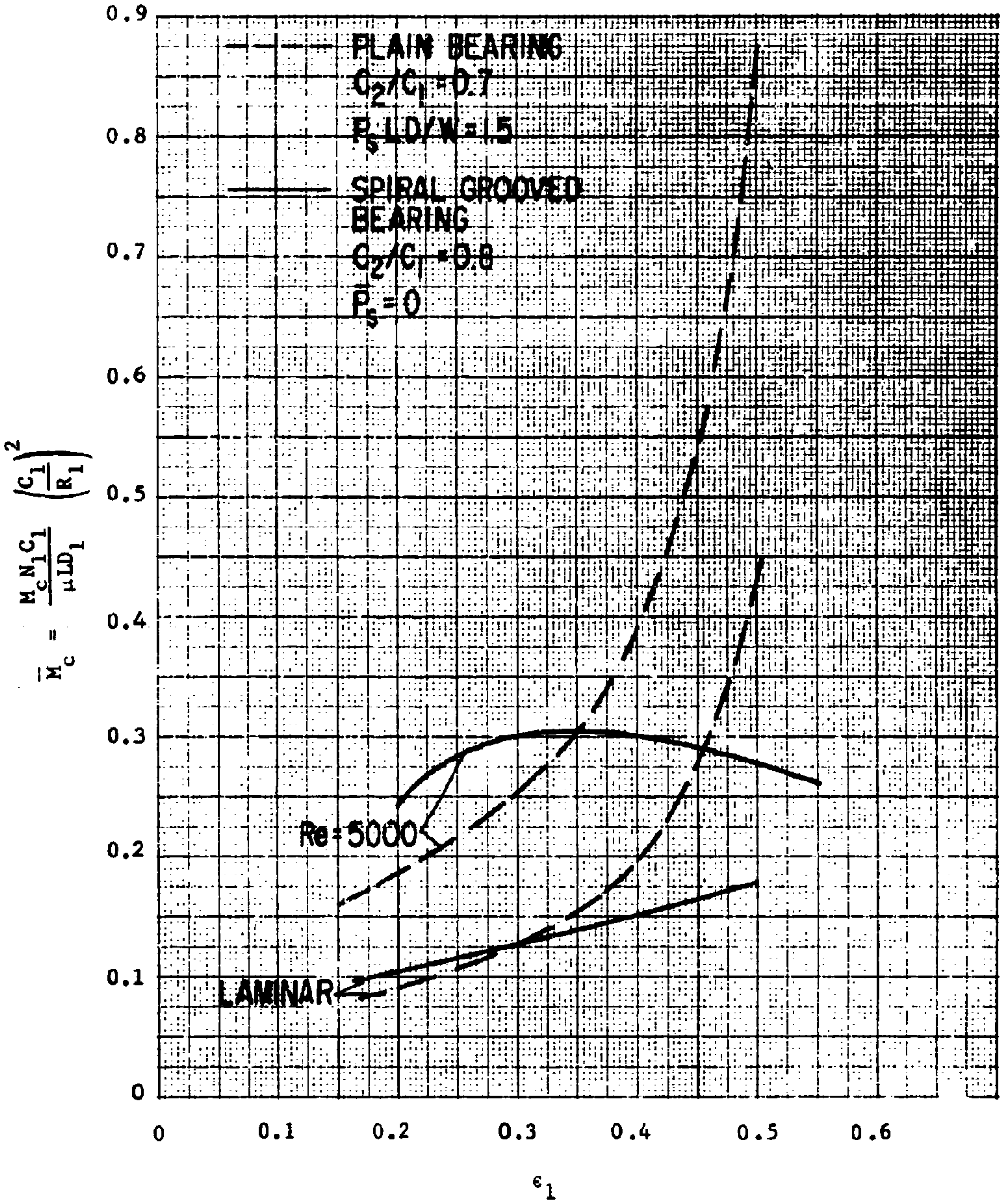


Figure 37. Critical Journal Mass, Comparison between Single Film and Floating Ring Spiral-Grooved Bearings

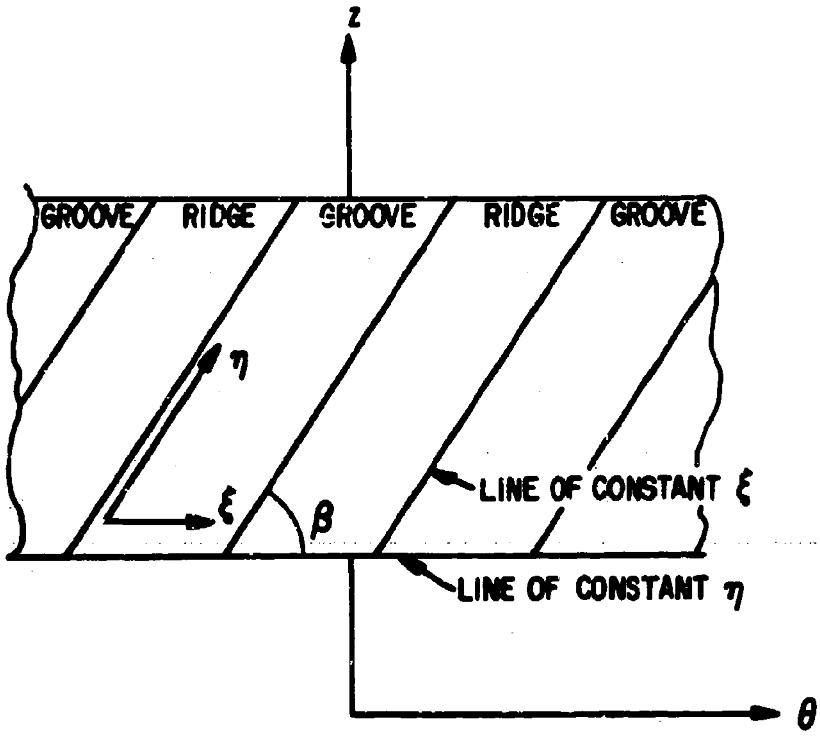


Figure 38. η and ξ Coordinates for Grooved Surface

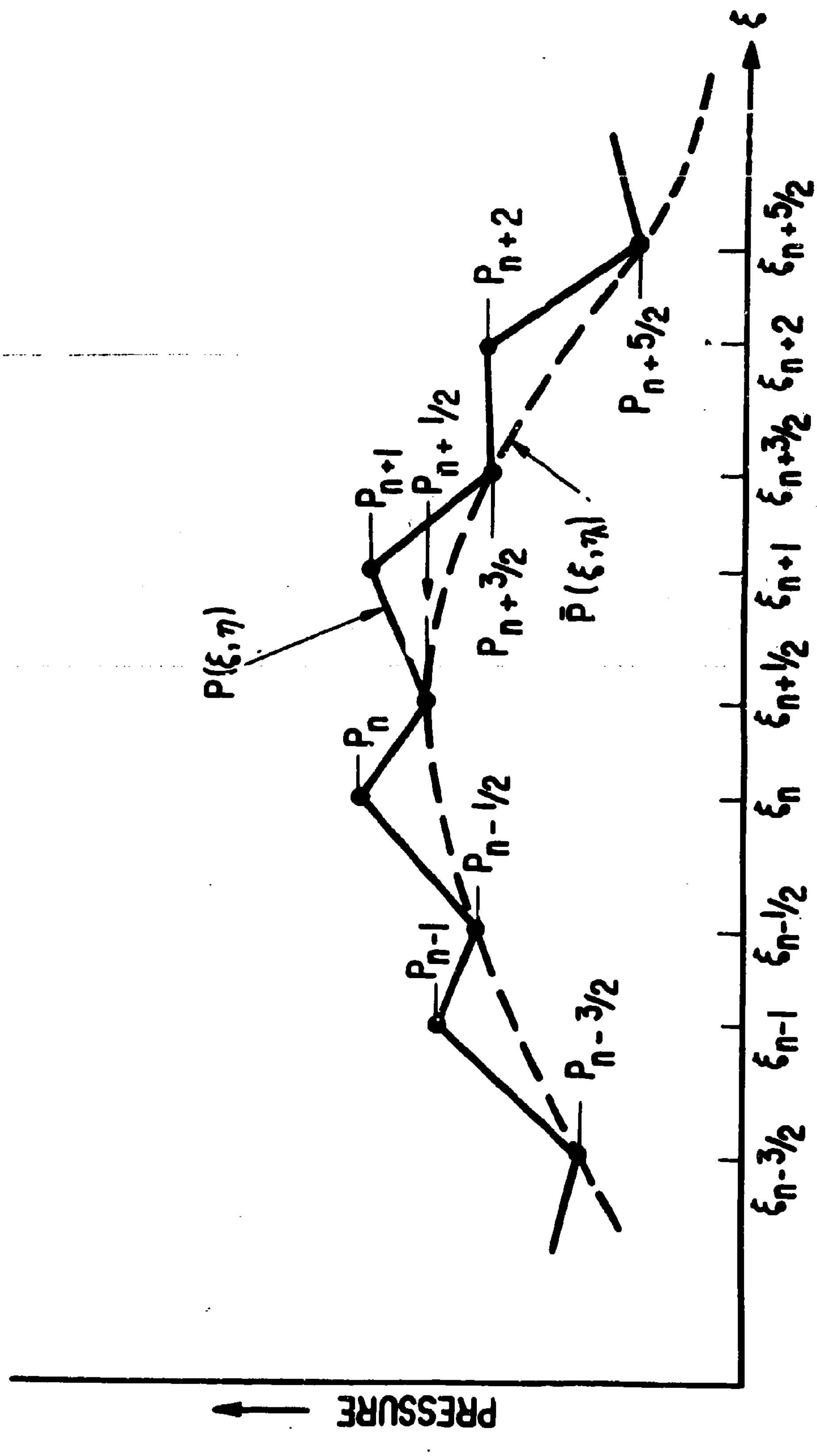


Figure 39. Pressure Distribution around Spiral-Grooved Journal Bearing

MTI-6612

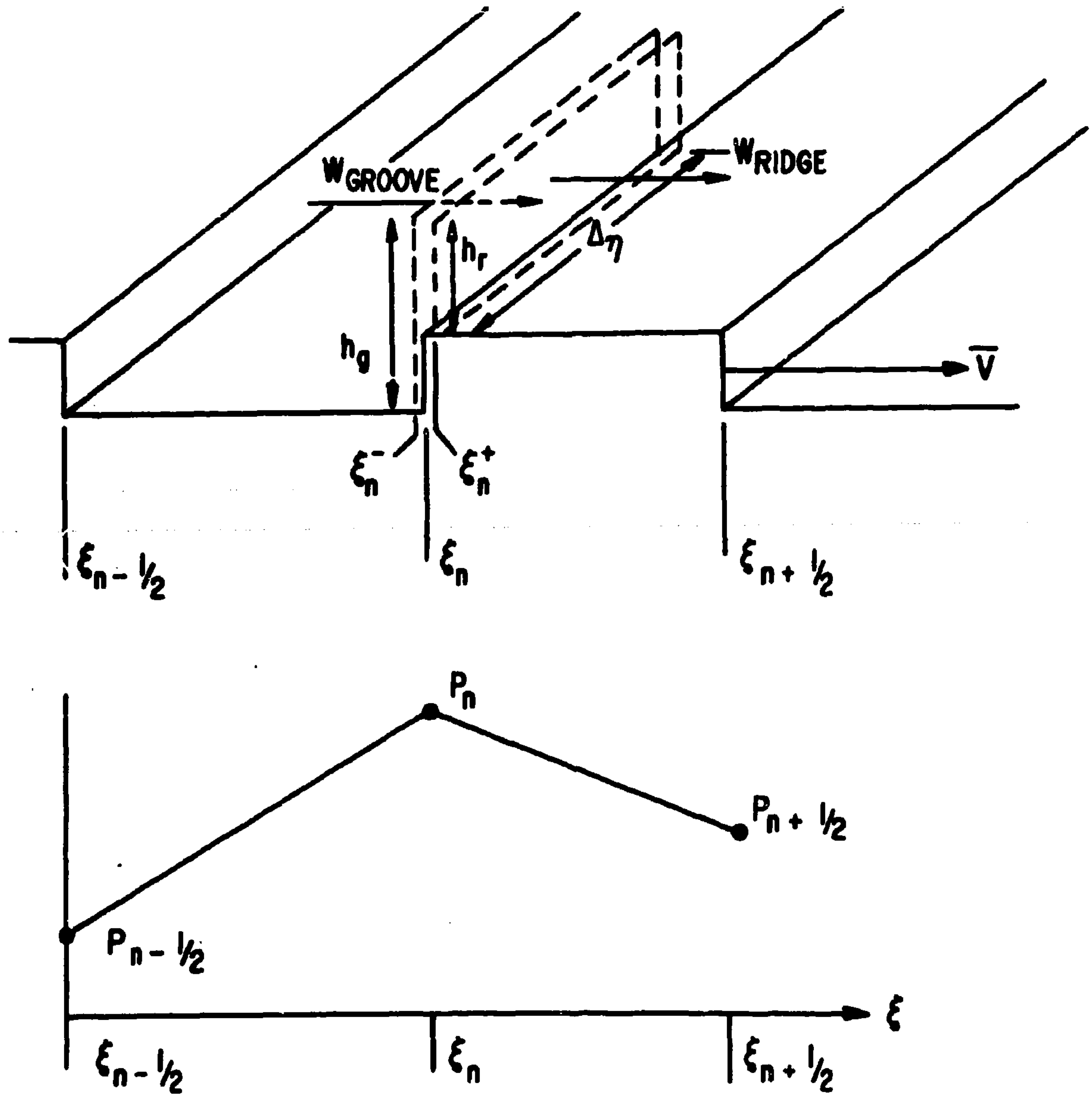


Figure 40. Continuity of Mass Flow across Groove-Ridge Interface and Pressure Variation across Groove-Ridge Pair

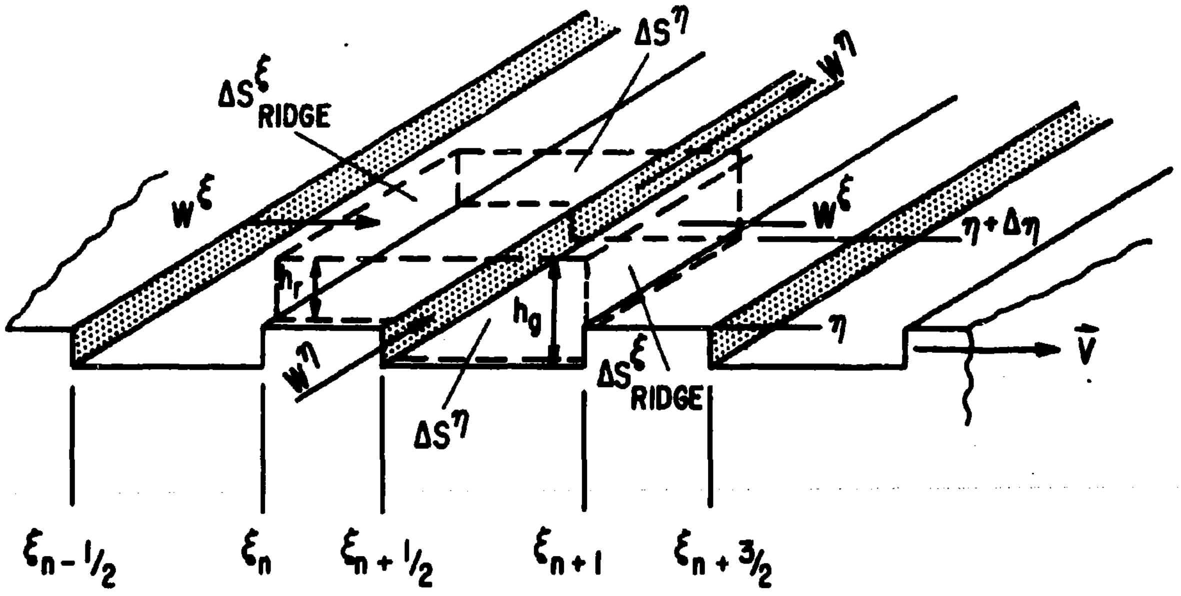


Figure 41. Control Volume for Mass Flow Continuity Analysis

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APPENDIX I

DERIVATION AND SOLUTION OF EQUATION FOR TURBULENT SPIRAL-GROOVED JOURNAL BEARING

The differential equation for the pressure distribution around a spiral-grooved journal bearing has been derived previously by the present authors for the case of laminar flow (Ref. 1). In this appendix will be presented a re-derivation of this equation in which the effects of turbulence in the bearing film will be accounted for by means of the linearized turbulent lubrication theory developed by Ng and Pan (Ref. 4). The derivation will involve four major steps:

1. Express the local mass flows in the ridge and groove regions in terms of the local pressure gradients in those two regions.

2. Define an "overall" pressure profile around the bearing neglecting the zig-zag ripples in the profile which arise due to the discountinuous groove-ridge geometry.

3. Use the requirement that the flow normal to a groove-ridge interface be continuous across the interface in order to solve for local groove-ridge pressure gradients in terms of the "overall" pressure gradient.

4. Apply the principle of conservation of mass to obtain a differential equation for the overall pressure profile.

According to the linearized turbulent lubrication theory of Ng and Pan, the local turbulent mass flows in a bearing film can be expressed as

$$\rho\bar{u}h = -\frac{\rho h^3 G_x}{\mu}\frac{\partial P}{\partial x} + \rho\frac{(U+V)}{2}h \qquad (51)$$

$$\rho\bar{w}h = -\frac{\rho h^3 G_z}{\mu}\frac{\partial P}{\partial z} \qquad (52)$$

where x and z are the coordinates in the direction of rotation and the axial direction respectively and where U and V are the surface velocities of the bearing and journal respectively. The factors G_x and G_z can be considered, essentially, to be turbulent viscosity correction factors. In the linearized turbulent lubrication theory G_x and G_z are considered to be functions of the local Reynolds number, R_h, in the bearing film where $R_h = \rho(V-U)h/\mu$. A plot of G_x and G_z vs. R_h is shown in Fig. 5.

Next, we introduce the "skewed" ξ, η coordinate system shown in Fig. 38 in which lines parallel to groove-ridge interfaces are lines of constant ξ while planes perpendicular to the axis of the bearing are planes of constant η. The relationships between ξ and η and the cylindrical coordinates θ and z are given below

$$\xi = \theta - \frac{z \cot \beta}{R} \tag{53}$$

$$\eta = z/\sin\beta \tag{54}$$

$$\theta = \xi + \frac{\eta \cos \beta}{R} \tag{55}$$

$$z = \eta \sin \beta \tag{56}$$

note also that

$$\frac{\partial P}{\partial \theta} = \frac{\partial P}{\partial \xi} \tag{57}$$

$$\frac{\partial P}{\partial z} = \frac{\partial P}{\partial \eta} \frac{1}{\sin \beta} - \frac{\partial P}{\partial \xi} \frac{\cot \beta}{1} \tag{58}$$

$$\frac{\partial P}{\partial \xi} = \frac{\partial P}{\partial \theta} \tag{59}$$

$$\frac{\partial P}{\partial \eta} = \frac{\partial P}{\partial \theta} \frac{\cos \beta}{R} + \frac{\partial P}{\partial z} \sin \beta \tag{60}$$

In the ξ, η coordinate system, the pressure gradient $\partial P/\partial \eta$ is continuous everywhere in the bearing film because the geometry has no discontinuities in the η direction. On the other hand, the gradient $\partial P/\partial \xi$ is discontinuous at groove-ridge interfaces due to the discontinuity in film height. Consequently the pressure profile in the ξ (circumferential)direction will have the "zig-zag" appearance as shown schematically by the solid line in Fig. 39 . In this figure the symbols ξ_n, ξ_{n+1}, etc., denote the interfaces at the beginning of ridge region while $\xi_{n+1/2}$, $\xi_{n+3/2}$, etc. denote the interfaces at the beginning of groove regions. Now,by neglecting the saw-toothed ripples in the actual pressure distribution, one can conceive of an approximate, smoothed "overall" pressure distribution around the journal shown in Fig. 39 as the dashed line through the pressures $P_{n-3/2}$, $P_{n-1/2}$, etc., at $\xi_{n-3/2}$, $\xi_{n-1/2}$, etc. Since the local pressure gradients within each groove and ridge region are bounded in magnitude,the saw-toothed fluctuations in pressure due to alternating groove and ridge regions will reduce to a negligible magnitude as the width of the groove-ridge pair becomes very small, i.e., as the number of grooves becomes very large. In the limit, as the width of each groove-ridge pair becomes very small, the smooth "overall" pressure distribution through the discrete points $P_{n-3/2}$, $P_{n-1/2}$, etc., will approach a continuous distribution $\overline{P}(\xi,\eta)$ which should provide a very good approximation to the actual "saw-toothed" pressure distribution around the journal. Formally, one can define the slope of $\overline{P}(\xi,\eta)$ in the ξ direction at the point ξ_n, as

$$\left.\frac{\partial \overline{P}(\xi,\eta)}{\partial \xi}\right|_{\xi_n} = \lim_{\Delta\xi \to 0} \frac{P_{n+1/2} - P_{n-1/2}}{\Delta\xi} = \lim_{\Delta\xi \to 0} \left[\alpha \frac{\partial P_g}{\partial \xi} + (1-\alpha) \frac{\partial P_r}{\partial \xi} \right] \qquad (61)$$

where $\Delta\xi = \xi_{n+1/2} - \xi_{n-1/2}$ and where $\partial P_g/\partial\xi$ and $\partial P_r/\partial\xi$ refer to the local pressure gradients within the groove and ridge region respectively.

Having defined the overall gradient $\partial\overline{P}/\partial\xi$, we next consider writing an expression for the mass flux W^{ξ} normal to a moving groove-ridge interface (see Fig. 39). In terms of the mass fluxes ρuh and ρwh, W^{ξ} is expressed as

$$W^{\xi} = \rho \left[(u-V) \sin\beta - w \cos\beta \right] h \tag{62}$$

Substituting for u and w in Eq. (62) by means of Eqs. (51) and (52) and noting that $\partial P/\partial x \equiv \partial P/R\partial\theta$ we obtain

$$W^{\xi}_{\text{groove region}} = -\rho \left\{ \left[\frac{h_g^{\,3} G_{xg}}{\mu} \frac{\partial P_g}{R\partial\theta} - \frac{(U-V)}{2} h_g \right] \sin\beta - \frac{h_g^{\,3} G_{zg}}{\mu} \frac{\partial P_g}{\partial z} \cos\beta \right\} \tag{63}$$

$$W^{\xi}_{\text{ridge region}} = -\rho \left\{ \left[\frac{h_r^{\,3} G_{xr}}{\mu} \frac{\partial P_r}{R\partial\theta} - \frac{(U-V)}{2} h_r \right] \sin\beta - \frac{h_r^{\,3} G_{zr}}{\mu} \frac{\partial P_r}{\partial z} \cos\beta \right\} \tag{64}$$

Note that the factors G_x and G_z are subscripted r and g because they have different values in the ridge and groove regions.

Next we make the following useful definitions

$$G_1 = 12(G_x + G_z \cot^2\beta)/R^2 \tag{65}$$

$$G_2 = -\frac{12G_z \cos\beta}{R \sin^2\beta} \tag{66}$$

$$G_3 = -\frac{12\, G_z}{\sin^2\beta} \tag{67}$$

If, in Eqs. (63) and (64), we transform the derivatives of P with respect to θ and z into derivatives with respect to ξ and η (see Eqs. (57) and (58)) and collect terms, we obtain

$$W^{\xi}_{\text{groove region}} = R \sin\beta \left\{ - \frac{\rho}{12\mu} h_g^{\;3} \left[G_{1g} \frac{\partial P_g}{\partial \xi} + G_{2g} \frac{\partial P}{\partial \eta} \right] + \rho h_g \frac{(U-V)}{2R} \right\} \tag{68}$$

$$W^{\xi}_{\text{ridge region}} = R \sin\beta \left\{ - \frac{\rho}{12\mu} h_r^{\;3} \left[G_{1r} \frac{\partial P_r}{\partial \xi} + G_{2r} \frac{\partial P}{\partial \eta} \right] + \rho h_r \frac{(U-V)}{2R} \right\} \tag{69}$$

Note that the derivative $\partial P/\partial \eta$ does not have to be identified by a subscript g or r since it is continuous everywhere within the bearing film.

Next we note that by continuity of mass flow

$$W^{\xi}_{\text{groove region}} = W^{\xi}_{\text{ridge region}} \tag{70}$$

Eqs. (61) and (70) constitute two linear equations in the "unknowns" $\partial P_g/\partial \xi$ and $\partial P_r/\partial \xi$. Eqs. (61) and (70) may therefore be solved to yield

$$\frac{\partial P_g}{\partial \xi} = (1-\alpha)\, \overline{B}_1 \frac{\partial P}{\partial \eta} + (1-\alpha)\, \overline{B}_2 - \overline{A}_1 \frac{\partial \overline{P}}{\partial \xi} \tag{71}$$

$$\frac{\partial P_r}{\partial \xi} = -\alpha \bar{B}_1 \frac{\partial P}{\partial \eta} - \alpha \bar{B}_2 + \bar{A}_2 \frac{\partial \bar{P}}{\partial \xi} \tag{72}$$

where

$$\left.\begin{aligned}
A_1 &= G_{1r} h_r^3 \\
A_2 &= - G_{1g} h_g^3 \\
A_3 &= A_2 - \alpha(A_2 + A_1) \\
B_1 &= G_{2g} h_g^3 - G_{2r} h_r^3 \\
B_2 &= \frac{6\mu(h_r - h_g)}{R} (U-V) \\
\bar{A}_1 &= A_1/A_3 \\
\bar{A}_2 &= A_2/A_3 \\
\bar{B}_1 &= B_1/A_3 \\
\bar{B}_2 &= B_2/A_3
\end{aligned}\right\} \tag{73}$$

Eqs. (71) and (72) may be substituted in either Eq. (68) or (67) to yield an expression for the mass flux W^{ξ} in terms of the overall pressure gradients $\partial\bar{P}/\partial\xi$ and $\partial\bar{P}/\partial\eta$ (note that $\partial P/\partial\eta$ may be written as $\partial\bar{P}/\partial\eta$ because of the continuous nature of this derivative). The expression obtained is

$$W^{\xi} = R \sin\beta \left\{ - \frac{\rho}{12\mu} h_r^{3} \left[G_{1r} \left(\bar{A}_2 \frac{\partial \bar{P}}{\partial \xi} - \alpha \bar{B}_1 \frac{\partial \bar{P}}{\partial \eta} - \alpha \bar{B}_2 \right) + G_{2r} \frac{\partial \bar{P}}{\partial \eta} \right] + \rho h_r \frac{(U-V)}{2R} \right\} \tag{74}$$

Next, consider the control volume ΔV, for one ridge-groove pair shown in Fig. 41. We take the control volume to move with the grooved surface at velocity $\vec{V}$ and we consider that, at the instant shown in Fig. 41, the various ridge-groove interfaces are located at ξ_n, $\xi_{n+1/2}$, ξ_{n+1} etc., as depicted. The mass flows entering and leaving the moving control volume through the surfaces ΔS^{ξ}_{ridge} we denote as $W^{\xi}_{ridge}\,\Delta\eta$ whereas the mass flows entering and leaving the control volume through the surfaces ΔS^{η} we denote as $W^{\eta}(\xi_{n+1} - \xi_n)$. The total time rate of ch nge of the mass, ΔM contained in ΔV is given by

$$\frac{\partial}{\partial t} (\Delta M) + V \cdot \nabla (\Delta M) \tag{75}$$

By the conservation of mass we obtain that

$$\left[W^{\xi}_{ridge}\Big|_{\xi_{n+1}} - W^{\xi}_{ridge}\Big|_{\xi_n} \right] \Delta\eta + \left[W^{\eta}\Big|_{\eta+\Delta\eta} - W^{\eta}\Big|_{\eta} \right] \left(\xi_{n+1} - \xi_n \right) + \frac{\partial}{\partial t} (\Delta M) + \vec{V} \cdot \vec{\nabla} (\Delta M) = 0 \tag{76}$$

Dividing Eq. (76) through by $\Delta\xi\Delta\eta$ where $\Delta\xi = \xi_{n+1} - \xi_n$, and taking the limit as $\Delta\xi$ and $\Delta\eta$ go to zero we have

$$\frac{\partial W^{\xi}}{\partial \xi} + \frac{\partial W^{\eta}}{\partial \eta} + \frac{\partial}{\partial t} \frac{(\Delta M)}{\Delta\xi\,\Delta\eta} + \vec{V} \cdot \vec{\nabla} \frac{(\Delta M)}{\Delta\xi\,\Delta\eta} = 0 \tag{77}$$

where $\partial W^{\xi}/\partial\xi$ is defined to be

$$\frac{\partial W^{\xi}}{\partial\xi} = \lim_{\Delta\xi\to 0} \frac{W^{\xi}\Big|_{\xi_{n+1}} - W^{\xi}\Big|_{\xi_n}}{\Delta\xi} \tag{78}$$

The subscript ridge has been dropped from W^{ξ} because of the equality condition expressed by Eq. (70). The expression for W^{ξ} is given by equation (74).

The expression for the mass flux W^{η} is

$$W^{\eta} = -\rho\left[\frac{(1-\alpha)h_r^{\ 3}}{\mu}\ \frac{\partial P_r}{\partial z}\ G_{zr} + \frac{\alpha h_g^{\ 3}}{\mu}\ \frac{\partial P_g}{\partial z}\ G_{zg}\right] \tag{79}$$

If we transform $\partial P_r/\partial z$ and $\partial P_g/\partial z$ into ξ - η coordinates by means of Eq. (58) we will obtain an expression for W^{η} in terms of $\partial P/\partial\eta$, $\partial P_r/\partial\xi$ and $\partial P_g/\partial\xi$. We then can substitute for $\partial P_r/\partial\xi$ and $\partial P_g/\partial\xi$ by means of Eqs. (71) and (72) to obtain, finally

$$\begin{aligned} W^{\eta} = R \sin\beta\ \frac{\rho}{12\mu}\Bigg\{&\alpha\, h_g^{\ 3}\left[G_{3g}\ \frac{\partial\bar{P}}{\partial\eta} - G_{2g}\left(-\bar{A}_1\frac{\partial\bar{P}}{\partial\xi} + (1-\alpha)\ \bar{B}_1\ \frac{\partial\bar{P}}{\partial\eta} + (1-\alpha)\ \bar{B}_2\right)\right] \\ &+ (1-\alpha)h_r^{\ 3}\left[G_{3r}\ \frac{\partial\bar{P}}{\partial\eta} - G_{2r}\left(\bar{A}_2\ \frac{\partial\bar{P}}{\partial\xi} - \alpha\ \bar{B}_1\ \frac{\partial\bar{P}}{\partial\eta} - \alpha\ \bar{B}_2\right)\right]\Bigg\} \end{aligned} \tag{80}$$

where $\bar{A}_2$, $\bar{B}_1$ and $\bar{B}_2$ were defined previously by Eqs. (73)

The quantity $\Delta M/\Delta\xi\Delta\eta$ in Eq. (77) is

$$\frac{\Delta M}{\Delta\xi\Delta\eta} = \rho\ R\ \sin\beta\ \left[(1-\alpha)h_r + \alpha h_g\right] \tag{81}$$

while

$$\vec{V} \cdot \vec{\nabla} = \frac{V}{R} \frac{\partial}{\partial \xi} \tag{82}$$

Substitution of Eqs. (74), (80), (81) and (82) into Eq. (77) yields a second-order differential equation in $\overline{P}(\xi,\eta)$ the overall pressure distribution around the spiral-grooved journal. To solve this equation, we must first transform it back into the orthogonal θ-z coordinate system. This is done by means of Eqs. (59) and (60). The result is

$$\frac{\partial W^{\xi}}{\partial \theta} + \frac{\cos\beta}{R} \frac{\partial W^{\eta}}{\partial \theta} + \sin\beta \frac{\partial W^{\eta}}{\partial z}$$

$$+\left(\frac{\partial}{\partial t} + \frac{V}{R} \frac{\partial}{\partial \theta}\right) \left\{R \rho \sin\beta \left[\alpha h_g + (1-\alpha)h_r\right]\right\} = 0 \tag{83}$$

Expressions for W^{ξ} and W^{η} in the θ-z coordinate system may be obtained by substituting Eqs. (59) and (60) into Eqs. (74) and (80).

The Numerical Solution

Eq. (83) is solved for any combination of groove and seal arrangement with or without feed or vent in the middle of the journal (Fig. 2). Thus, it can provide the solution for a seal-groove, groove-seal, fully grooved or herringbone journal containing up to three distinct sections. The basic technique is first to divide the bearing into a numerical grid with dimensions mxn. Then write the differential equation into a finite difference form of three columns. Thus, each point in concern is related to the five neighboring points. At the boundaries, the pressures are given. Therefore mxn equations are established for mxn unknowns. By means of the columnwise matrix inversion solution routine developed by Castelli, Pirvics and Shapiro, the pressure field is obtained.

This technique is fully discussed in References 6 and 7.

Once the pressure field is obtained, loads, moments, torques, etc. are obtained by numerically evaluating the following expressions.

Radial component of force (cosine component)

$$F_r = -\int_0^L \int_0^{2\pi} (\bar{P} - P_a)\, R \cos\theta \, d\theta \, dz \tag{84}$$

Tangential component of force (sine component)

$$F_t = \int_0^L \int_0^{2\pi} (\bar{P} - P_a)\, R \sin\theta \, d\theta \, dz \tag{85}$$

Radial component of moment (cosine component)

$$M_r = -\int_0^L \int_0^{2\pi} (\bar{P} - P_a)\, Rz \cos\theta \, d\theta \, dz \tag{86}$$

Tangential component of moment (sine component)

$$M_t = \int_0^L \int_0^{2\pi} (\bar{P} - P_a) \, Rz \sin\theta \, d\theta \, dz \tag{87}$$

Attitude angles

$$\phi = \tan^{-1} \frac{F_r}{F_t}, \quad \phi' = \tan^{-1} \frac{M_r}{M_t} \tag{88}$$

Bearing torque

$$T_B = \frac{R\rho(U-V)^2}{8} \int_0^z \int_0^{2\pi} C_f \, R d\theta \, dz + \frac{R}{2} \int_0^z \int_0^{2\pi} h \frac{\partial P}{\partial \theta} \, d\theta \, dz \tag{89}$$

where C_f is the Couette friction factor which is plotted in Fig. 6 against R_{h_e}. $\partial P/\partial\theta$ is the local pressure gradient. By Eqs. (71), (72), (59) and (60) the second term of Equation (89), the Poiseuille torque, can be written in terms of the overall pressure gradients.

$$\frac{R}{2} \int_0^z \int_0^{2\pi} h \frac{\partial P}{\partial \theta} \, d\theta \, dz = \frac{R}{2} \int_0^z \left\{ \alpha h_g \left[(1-\alpha)\bar{B}_1 \left(\frac{\partial \bar{P}}{\partial z} + \frac{\cot\beta}{R} \frac{\partial \bar{P}}{\partial \theta}\right) \sin\beta \right. \right.$$

$$\left. + (1-\alpha)\bar{B}_2 - \bar{A}_1 \frac{\partial \bar{P}}{\partial \theta} \right] + (1-\alpha) h_r \left[\bar{A}_2 \frac{\partial \bar{P}}{\partial \theta} - \alpha \bar{B}_1 \left(\frac{\partial \bar{P}}{\partial z} + \frac{\cot\beta}{R} \frac{\partial \bar{P}}{\partial \theta}\right) \sin\beta \right.$$

$$\left. \left. - \alpha \bar{B}_2 \right] \right\} dz \tag{90}$$

Journal torque

$$T_j = T_B + \epsilon C W \sin \phi \tag{91}$$

Where $W = \text{load} = \sqrt{F_r^{\,2} + F_t^{\,2}}$

Flow

$$Q = R \int_0^{2\pi} W^{\eta} \, d\theta \tag{92}$$

where W^{η} is defined by Eq. (80).

APPENDIX II

DERIVATION OF RELATIONSHIPS FOR CALCULATING STIFFNESS AND DAMPING COEFFICIENTS

Consider the reference axes shown in Fig. 3. The relationship between the forces F_x and F_y and the forces F_r and F_t can be written in matrix form as

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} -W \\ 0 \end{Bmatrix} = -\begin{Bmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{Bmatrix} \cdot \begin{Bmatrix} F_r \\ F_t \end{Bmatrix} \tag{93}$$

For an infinitesimally small motion around the steady state position the dynamic forces become

$$\begin{Bmatrix} dF_x \\ dF_y \end{Bmatrix} = -\begin{Bmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{Bmatrix} \cdot \begin{Bmatrix} dF_r + F_t\, d\phi \\ dF_t - F_r\, d\phi \end{Bmatrix} \tag{94}$$

The infinitesimal dynamic motion of the journal center is described by the coordinates (x,y):

$$x = d(e\cos\phi) \qquad y = d(e\sin\phi)$$

or

$$\begin{Bmatrix} de \\ e d\phi \end{Bmatrix} = \begin{Bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{Bmatrix} \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} \tag{95}$$

The velocities transform similarly:

$$\begin{Bmatrix} d\dot{e} \\ e d\dot{\phi} \end{Bmatrix} = \begin{Bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{Bmatrix} \cdot \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} \tag{96}$$

If, in the differential equation for pressure P obtained in Appendix I, we introduce the following dimensionless variables

$$\left.\begin{aligned} \omega &= (U+V)/R \\ z' &= z/L \\ h' &= h/C \\ \bar{P} &= \frac{2\pi P}{\mu\omega}\left(\frac{C}{R}\right)^2 \\ \epsilon &= e/C \\ \frac{\dot{\epsilon}}{\omega} &= \frac{1}{\omega}\frac{\partial\epsilon}{\partial t} \\ \dot{\phi} &= \frac{1}{\omega}\frac{\partial\phi}{\partial t} \end{aligned}\right\} \tag{97}$$

we obtain the result that the dimensionless pressure $\bar{P}$ in a spiral-grooved bearing of fixed geometry is a function only of the dimensionless variables ϵ, $\dot{\epsilon}/\omega$ and $\dot{\phi}/\omega$ i.e.

$$\bar{P} = \bar{P}\left(\epsilon, \frac{\dot{\epsilon}}{\omega}, \frac{\dot{\phi}}{\omega}\right). \tag{98}$$

The resulting fluid film forces in radial and tangential directions are:

$$F_r = -\lambda\omega \iint \bar{P} \cos\theta \, dx' \, dz'$$

$$= \lambda\omega \, \bar{F}_r \left(\epsilon, \frac{\dot{\epsilon}}{\omega}, \frac{\dot{\phi}}{\omega} \right) \tag{99}$$

$$F_t = \lambda\omega \iint \bar{P} \sin\theta \, dx' \, dz'$$

$$= \lambda\omega \, \bar{F}_t \left(\epsilon, \frac{\dot{\epsilon}}{\omega}, \frac{\dot{\phi}}{\omega} \right) \tag{100}$$

where:

$$\lambda = \frac{\mu RL}{\pi} \left(\frac{R}{C}\right)^2$$

$$\bar{F}_r = \frac{F_r}{\mu NDL} \left(\frac{C}{R}\right)^2 \tag{101}$$

$$\bar{F}_t = \frac{F_t}{\mu NDL} \left(\frac{C}{R}\right)^2$$

Differentiating Eqs. (99) and (100) we obtain

$$dF_r = \frac{\lambda\omega}{C} \left[\frac{\partial \bar{F}_r}{\partial \epsilon} de + \frac{1}{\omega} \frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\epsilon}}{\omega}\right)} d\dot{e} + \frac{1}{\epsilon\omega} \frac{\partial \bar{F}_r}{\partial \left(\frac{\dot{\phi}}{\omega}\right)} e \, d\dot{\phi} \right] \tag{102}$$

$$dF_t = \frac{\lambda\omega}{C}\left[\frac{\partial\bar{F}_t}{\partial\epsilon}\,de + \frac{1}{\omega}\,\frac{\partial\bar{F}_t}{\partial(\frac{\dot{\epsilon}}{\omega})}\,d\dot{e} + \frac{1}{\epsilon\omega}\,\frac{\partial\bar{F}_t}{\partial(\frac{\dot{\phi}}{\omega})}\,e\,d\dot{\phi}\right] \tag{103}$$

By substitution of Eqs. (102 and (103) into Eq. (94)

$$\begin{Bmatrix} dF_x \\ \\ dF_y \end{Bmatrix} = -\frac{1}{C}\lambda\omega\begin{Bmatrix} \cos\phi & \sin\phi \\ \\ \sin\phi & -\cos\phi \end{Bmatrix}\cdot\left[\begin{Bmatrix} \frac{\partial\bar{F}_r}{\partial\epsilon} & \frac{\bar{F}_t}{\epsilon} \\ \\ \frac{\partial\bar{F}_t}{\partial\epsilon} & \frac{-\bar{F}_r}{\epsilon} \end{Bmatrix}\begin{Bmatrix} de \\ \\ ed\phi \end{Bmatrix} + \frac{1}{\omega}\begin{Bmatrix} \frac{\partial\bar{F}_r}{\partial(\frac{\dot{\epsilon}}{\omega})} & \frac{\partial\bar{F}_r}{\epsilon\partial(\frac{\dot{\phi}}{\omega})} \\ \\ \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\epsilon}}{\omega})} & \frac{\partial\bar{F}_t}{\epsilon\partial(\frac{\dot{\phi}}{\omega})} \end{Bmatrix}\begin{Bmatrix} d\dot{e} \\ \\ ed\dot{\phi} \end{Bmatrix}\right]$$

The stiffness and damping coefficients are defined by:

$$\begin{aligned} dF_x &= -K_{xx}x - B_{xx}\dot{x} - K_{xy}y - B_{xy}\dot{y} \\ dF_y &= -K_{yx}x - B_{yx}\dot{x} - K_{yy}y - B_{yy}\dot{y} \end{aligned} \tag{105}$$

To determine the 8 coefficients, substitute Eq. (95) and (96) into Eq. (104) and collect the terms in accordance with Eq. (105) to get:

$$K_{xx} = \frac{1}{C}\lambda\omega\left[\frac{\partial\bar{F}_r}{\partial\epsilon}\cos^2\phi + \frac{\partial\bar{F}_t}{\partial\epsilon}\cos\phi\sin\phi - \frac{\bar{F}_y}{\epsilon}\sin\phi\right] \tag{106}$$

$$\omega B_{xx} = \frac{1}{C}\lambda\omega\left[\frac{\partial\bar{F}_r}{\partial(\frac{\dot{\epsilon}}{\omega})}\cos^2\phi + \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\epsilon}}{\omega})}\cos\phi\sin\phi\right.$$

$$\left. - \frac{\sin\phi}{\epsilon}\left(\frac{\partial\bar{F}_r}{\partial(\frac{\dot{\phi}}{\omega})}\cos\phi + \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\phi}}{\omega})}\sin\phi\right)\right] \tag{107}$$

$$K_{xy} = \frac{1}{C}\lambda\omega\left[\frac{\partial\bar{F}_t}{\partial\epsilon}\sin^2\phi + \frac{\partial\bar{F}_r}{\partial\epsilon}\cos\phi\sin\phi + \frac{\bar{F}_y}{\epsilon}\cos\phi\right] \tag{108}$$

$$\omega B_{xy} = \frac{1}{C}\lambda\omega\left[\frac{\partial\bar{F}_t}{\partial(\frac{\dot{\epsilon}}{\omega})}\sin^2\phi + \frac{\partial\bar{F}_r}{\partial(\frac{\dot{\epsilon}}{\omega})}\cos\phi\sin\phi\right.$$

$$\left. + \frac{\cos\phi}{\epsilon}\left(\frac{\partial\bar{F}_r}{\partial(\frac{\dot{\phi}}{\omega})}\cos\phi + \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\phi}}{\omega})}\sin\phi\right)\right] \tag{109}$$

$$K_{yx} = \frac{1}{C}\lambda\omega\left[-\frac{\partial\bar{F}_t}{\partial\epsilon}\cos^2\phi + \frac{\partial\bar{F}_r}{\partial\epsilon}\cos\phi\sin\phi + \frac{\bar{F}_x}{\epsilon}\sin\phi\right] \tag{110}$$

$$\omega B_{yx} = \frac{1}{C}\lambda\omega\left[-\frac{\partial\bar{F}_t}{\partial(\frac{\dot{\epsilon}}{\omega})}\cos^2\phi + \frac{\partial\bar{F}_r}{\partial(\frac{\dot{\epsilon}}{\omega})}\cos\phi\sin\phi\right.$$

$$\left. - \frac{\sin\phi}{\epsilon}\left(\frac{\partial\bar{F}_r}{\partial(\frac{\dot{\phi}}{\omega})}\sin\phi - \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\phi}}{\omega})}\cos\phi\right)\right] \tag{111}$$

$$K_{yy} = \frac{1}{C}\lambda\omega\left[\frac{\partial\bar{F}_r}{\partial\epsilon}\sin^2\phi - \frac{\partial\bar{F}_t}{\partial\epsilon}\cos\phi\sin\phi - \frac{\bar{F}_x}{\epsilon}\cos\phi\right] \tag{112}$$

$$\omega B_{yy} = \frac{1}{C}\lambda\omega\left[\frac{\partial\bar{F}_r}{\partial(\frac{\dot{\epsilon}}{\omega})}\sin^2\phi - \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\epsilon}}{\omega})}\cos\phi\sin\phi\right.$$

$$\left. + \frac{\cos\phi}{\epsilon}\left(\frac{\partial\bar{F}_r}{\partial(\frac{\dot{\phi}}{\omega})}\sin\phi - \frac{\partial\bar{F}_t}{\partial(\frac{\dot{\phi}}{\omega})}\cos\phi\right)\right] \tag{113}$$

where, in the coordinate system selected,

$$\bar{F}_x = \frac{-W}{\mu NLD} \left(\frac{R}{C}\right)^2 \tag{114}$$

$$\bar{F}_y = 0 \tag{115}$$

and all forces and derivatives are calculated for the given steady state position, defined by (ϵ_o, ϕ_o).

APPENDIX III - COMPUTER PROGRAM PN 412

PERFORMANCE OF A HERRINGBONE JOURNAL BEARING

OPERATED IN THE TURBULENT REGIME

Input

All input data appears in the form of name list. A full description of name list can be found in Ref. 10. The input are contained in the namelist "INPUT". The input data are:

1. REN = Reynolds number

   $$= \frac{2\pi \left| N_i - N_o \right| RC}{\nu}$$

   where N_i and N_o are the rotational velocities of the journal and bearing in cycles/sec., respectively.

   R = Radius of journal, in.

   C = Nominal radial clearance, in.

   ν = Kinematic viscosity, in^2/sec.

2. ALOVD = Length-diameter ratio

   = L/D

   where L = Length of journal, in.

   D = Diameter of journal, in.

3. COR = Film clearance ratio

   = C/R

4. PFIX = Dimensionless gage pressures at the ends and the middle of the bearing.

   $$= \frac{P \text{ (Pressure, PSIG)}}{\mu \left(\frac{R}{C}\right)^2 (N_i + N_o)}$$

   where μ = dynamic viscosity, lb-sec/in^2.

   The pressure at the initial end appears first, then at the middle and finally at the final end.

5. I1 = Number of axial grid points in the first region of the bearing counted from the initial end of journal (see Fig. 2). An odd number is required and also it must be at least 2 less than I2. The minimum permissible value for I1 is 3.

6. I2 = Number of axial grid points counted from the initial end of the journal to the end of the second region. For a vented bearing, I2 represents the grid points of one half of a bearing. For a three region, non-vented bearing, I2 represents the grid points for the initial end up to the interface of the second and third region (see Fig. 2). The minimum permissible value for I2 is 5 and the maximum is 17. Again, I2 must be an odd number.

7. MORE = Indicator to specify whether or not there is another set of input cards to follow the present set.
MORE = F, last set of input
MORE = T, another set of input to follow

8. SIGN = $\dfrac{N_o - N_i}{N_o + N_i}$

9. N = Number of grid points in the circumferential direction. The maximum permissible value for N is 19.

10. EDOT = $\dot{\epsilon}/2\pi\ (N_i + N_o)$
where $\dot{\epsilon}$ = time rate change of eccentricity ratio of the journal.

11. PHDOT = Dimensionless whirl velocity ratio
= $\dot{\phi}/2\pi\ (N_i + N_o)$
where $\dot{\phi}$ = journal whirl velocity, rad/sec.

12. GAMDOT = $\dot{\gamma}/2\pi\ (N_i + N_o)$
where $\dot{\gamma}$ = time rate change of angular misalignment.

13. EPS = Eccentricity ratio
= e/c
where e = eccentricity of journal, in.

14. GAM = Misalignment of journal, degrees.

15. VENT = Indicator to specify whether or not the middle of bearing is vented. Set VENT = T only for a bearing vented at the middle. For non-vented bearings, set VENT = F.

16. BETA = Groove angle in degrees. Beta must be specified for the first region of the bearing (first value) and the second region of the bearing (second value). If VENT = F, the value of BETA in the third region must be specified. For a pump-in design, with grooving in the first region, and a smooth seal in the second region, BETA should be read in as an obtuse angle, the same value for BETA being read in for the second region as the first region. For a pump-out design, with grooving in the second region of the bearing and a seal in the first region, BETA should be read in as acute angle with the same value of BETA being read in for the first region as for the second region. If VENT = F, the value of BETA in the third region should be given. The third value of BETA should be a conjugate of the first value of BETA, i.e. BETA(3)=180°-BETA(1). Never set BETA = 0.

17. DEP = Groove recess ratios in two or three regions of the journal with the first value referring to the first region, etc.

$= \delta/c$

where δ = groove recess, in.

To impose the condition of a smooth bearing (no grooving) in either region, set DEP = 0 for that region.

18. ALPHA = Fractional groove width in two or three regions of the journal with the first value referring to the first region, etc.

$$= \frac{a_g}{a_g + a_r}$$

where a_g and a_r are the widths in inches in the groove and ridge portion respectively. To impose the conditions of a smooth portion in either region, set ALPHA = 0 for that region.

19. PPOUT = Indicator to specify whether the pressure distribution in the bearing is required as a part of output.

PPOUT = T, pressure distribution is printed out; no pressure is printed out when PPOUT = F.

Output

1. Under the heading of INPUT, the complete set of input in the namelist "INPUT" is printed out. Each quantity is identified by the same symbol as used in the namelist "INPUT".

2. In the case of PPOUT = T, a heading of "Final Pressure Distribution" is printed out. Below the heading, if VENT = T, there are a total of I2 number of lines of pressures with the first line referring to the initial end of the bearing. If N is less or equal to 10, each line contains N number of pressures starting at $\phi = 0$ (see Fig. 1). For the case of $N > 10$, the number of lines are double. If VENT = F, there are a total of (I2 + I1-1) lines when $N \leq 10$. The number of lines will be double for the case of $N > 10$.

3. Regular Output:
There are a total of nine quantities in one line under the heading of REN NO., ECC., TORQUE J., TORQUE B., RADIAL LOAD, TANG. LOAD, FLOW, COS. MOMENT, SIN MOMENT, which are defined below.

   a. REN NO. = Same as the input.

   b. EPS = Same as the input.

   c. TORQUE J. = Dimensionless torque on bearing

   $$= \frac{T_j}{WC}$$

   where T_j = torque on journal, in-lb.
   W = total load, lb.

   d. TORQUE B. = Dimensionless torque on bearing

   $$= \frac{T_b}{WC}$$

   where T_b = torque on bearing, in-lb.

   e. RADIAL LOAD = Dimensionless radial component of load

   $$= \frac{F_r}{\mu\ (N_i + N_o)\ (\frac{R}{C})^2\ R^2}$$

   where F_r = radial component of load, lb.

f. TANG. LOAD = Dimensionless tangential component of load

$$= \frac{F_t}{\mu\ (N_i + N_o)\ (\frac{R}{C})^2\ R^2}$$

where F_t = tangential component of load, lb.

g. FLOW = Dimensionless flow

$$= \frac{Q}{R^2 C\ (N_i + N_o)}$$

where Q = flow, cu. in/sec.

h. COS. MOMENT = Dimensionless radial (cosine) component of moment, about the initial end of journal.

$$= \frac{M_r}{\mu\ (N_i + N_o)\ (\frac{R}{C})^2\ R^3}$$

where M_r = radial component of moment, lb-in.

i. SIN MOMENT = Dimensionless tangential (sin) component of moment, about the initial end of journal.

$$= \frac{M_t}{\mu\ (N_i + N_o)\ (\frac{R}{C})^2\ R^3}$$

where M_t = tangential component of moment, lb-in.

A Fortran listing of Program PN 406 is provided in the next few pages. Typical listings of input and output are also given.

```
C            HERRINGBONE JOURNAL
C            WITH LARGE ECCENTRICITY AND MISALIGNMENT
C      IN TURBULENCE REGIME
       COMMON MDIAG,DELX,DELZ,DELZS,M,MS,MG1,MG2,N,NP(17,19),A1(2,5,1
      19),B1(2,19),CC(2,19),AF1(17,19),AF2(17,19),AF3(17,19),AF4(17,19),A
      2F6(17,19),AF7(17,19),AF5(17,19),PHI(17,19)
       DIMENSION BETA(3),DEP(3),ALPHA(3),PFIX(3),          QQQ(19),H9(17
      1,19),QQQQ(19),PP(17),PPP(17),PPX(17),PPPX(17),AX1(17,19),AX2(17,19
      2),AX3(17,19),AX6(17,19),AX7(17,19),AX8(17,19),AX9(17,19),WS11(17,1
      39),WS12(17,19),WS13(17,19),WS14(17,19),XX(17),AXL(2,19),CZ(2,19)
       DIMENSION KUPT(17,19)
       LOGICAL PPOUT,VENT,TORQ,MORE
       NAMELIST/INPUT/REN,ALOVD,COR,PFIX,I1,I2,
      1MORE,SIGN,      N, EDOT,PHDOT,GAMDOT,EPS,GAM,VENT,BETA,DEP,ALPHA,
      1PPOUT
     2 FORMAT(7E14.7)
     4 FORMAT(/10(1X,F11.7))
     9 FORMAT(29HOFINAL PRESSURE DISTRIBUTION. //)
    11 FORMAT(6H1INPUT)
    10 READ(5,INPUT)
       WRITE(6,11)
       WRITE(6,INPUT)
         MDIAG=0
        MD=0
       RATLD=2.*ALOVD
       MG1=I1-1
       AMG=MG1
       MS=I2-I1
       IF(VENT) GO TO 102
       M=I2+I1-1
       BMM1=M-1
       MG2=MG1
       GO TO 101
   102 M=I2
         RATLD=2.*RATLD
       BMM1= M-1
       BMM1=BMM1*2.
       MG2=0
   101 DO 30 I=1,M
       DO 30 J=1,N
    30 NP(I,J)=0
         DO 20 J=1,N
        NP(1,J)=1
    20  NP(M,J)=1
    35 KK=1
   100 AN=N
       PI=3.14159265358979
       DTHETA=2.*PI/AN
       DTHE2=0.5/DTHETA
       TPS=2.*PI*SIGN
       CON1=6.*TPS
       RADIAN=.017453292519943
       GA1=GAM*RADIAN
       EDT=0.0
       CON2=CON1/REN
       TORQ=.FALSE.
       TPS1=TPS/8.0
       TRQ=TPS1*REN
       DELX=DTHETA
       DELZ=RATLD/BMM1
       TPS1=(2.*PI)**2*8.
```

```
      AMS=MS
      DELZS=DELZ
      ANG=0.0
      PHIMC=PHDOT-(1.-SIGN)*0.5
      IND=1
      ERO2=1.
      L1=1
      L2=I1
   38 DO 40 J=1,N
      CC(L1,J)=0.0
       AXL(L1,J)=0.
        CZ(L1,J)=0.
      A1(L1,2,J)=0.0
      A1(L1,3,J)=0.0
   40 AF7(L2,J)=0.0
      IF(IND.EQ.2)GO TO 48
      IF(VENT) GO TO 48
      L1=2
      L2=I2
      IND=2
      GO TO 38
   48 NR=1
      MN=MG1
      MM=I1
      ISS=1
      DEL=DELZ
      DZ=0.5/DELZ
      IS=1
      Z=0.0
  200 BET=BETA(NR)*RADIAN
      DEPH=DEP(NR)
      ALPH=ALPHA(NR)
      ALM1=1.-ALPH
      ALTAL1=ALPH*ALM1
      SINB=SIN(BET)
      COSB=COS(BET)
      SINB2=SINB*SINB
      COSB2=COSB*COSB
      COT2=COSB2/SINB2
      DLZ=DEL*DTHETA
      IF(TORQ) GO TO 1001
      DO 2000 I=IS,MM
      XX(I)=Z
      ZCOR=Z/COR
      EPZGH=EPS+GA1*ZCOR
      EDZR=EDOT+GAMDOT*ZCOR
      EPZG=EPZGH*PHIMC
      DO 2001 J=1,N
      SI=SIN(ANG)
      CO=COS(ANG)
      H=1.+CO*EPZGH
      H9(I,J)=H
      H3=H*H*H
      HG=H+DEPH
      HGHR=HG/H
      AX9(I,J)=(EDZR*CO+EPZG*SI)*SINB*PI*24.0
      HG3=HG**3
      HGHR3=HG3/H3
      S1=1./SINB2
      S2=-COSB/SINB2
      RENR=REN*H
```

```
      RENG=REN*HG
      GXR=GTCF(RENR)*12.
      GXG=GTCF(RENG)*12.
      GZR=GZCF(RENR)*12.
      GZG=GZCF(RENG)*12.
      S5=(GXR+GZR*COT2)
      S6=(GXG+GZG*COT2)
      S7=GZG*S2
      S8=GZR*S2
      S9=GZG*S1
      S10=GZR*S1
      S11=-S5/S6/HGHR3
      S12=1.-S11*ALPH-ALPH
      S13=(S7-S8/HGHR3)/S6/S12
      S14=RENR/HG3/S12*(1.-HGHR)/S6
      ALH3=ALPH*HG3
      A1S=SINB*(ALH3*S6*S11-ALM1*S5*H3)/S12
      A2=-ALM1*(ALPH*S5*S13+S8)*H3
      A2=(A2+ALH3*(ALM1*S6*S13-S7))*SINB
      A3=ALTAL1*S14*(S6*HG3-S5*H3)/REN
      A3=CON1*(A3+H*ALM1+HG*ALPH)*SINB
      DMY=H3*ALM1
      S7H=HGHR3*S7
      B1S=-H3/S12*(S8*ALM1-S7H*ALPH*S11)*SINB
      B2=ALH3*(S9-S7*S13*ALM1)+DMY*(S10+ALPH*S8*S13)
      B3=CON1*ALTAL1*S14*(S8-S7H)
      B2=-B2*SINB
      AX1(I,J)=A1S+A2*COSB
      B3=-B3*H3*SINB/REN
      AX2(I,J)=A2*SINB
      WS11(I,J)=S11/S12
      WS12(I,J)=1./S12
      WS13(I,J)=S13
      WS14(I,J)=S14
      AX3(I,J)=A3
      AX6(I,J)=B1S+B2*COSB
      AX7(I,J)=B2*SINB
      AX8(I,J)=B3
         IF(MD.NE.2)  GO TO 2001
            IF ( I.EQ.IS. AND.J.EQ.1)
     1               WRITE(6,2) AX1(I,J),AX2(I,J),AX3(I,J),AX6(I,J),AX7(
     1I,J),AX8(I,J),AX9(I,J)
 2001 ANG=ANG+DTHETA
      Z=Z+DEL
 2000 CONTINUE
      IF(NR.EQ.1) GO TO 2010
      IF(NR.EQ.3) GO TO 2014
      IF(VENT) GO TO 2014
      GO TO 2020
 2010 IK=1
      GO TO 3800
 2014 IK=MM
      GO TO 3800
      COMPUTE THE DIFFERENCES IN XS AND COEFFICIENT
 3800 I=IK
      DO 4010 J=1,N
      AF1(I,J)=0.0
      AF2(I,J)=0.0
      AF3(I,J)=0.0
      AF4(I,J)=0.0
      AF5(I,J)=0.0
```

```
      AF6(I,J)=1.0
      AF7(I,J)=-PFIX(NR)
 4010 CONTINUE
 2020 IST=IS+1
      MM1=MM-1
      DO 4000 I=IST,MM1
      I0=I-1
      I7=I+1
      DO 4000 J=1,N
      DTH=DTHE2
      IF(J.EQ.1.OR.J.EQ.N) GO TO 4004
      J0=J-1
      J1=J+1
      GO TO 4008
 4004 IF(J.EQ.1) GO TO 4006
      J0=N-1
      J1=1
      GO TO 4008
 4006 J0=N
      J1=2
 4008 AF1(I,J)=SINB*AX7(I,J)
      AF2(I,J)=(AX2(I,J1)-AX2(I,J0)+COSB*(AX7(I,J1)-AX7(I,J0)))*DTH
     1     +SINB*(AX7(I7,J)-AX7(I0,J))*DZ
      AF3(I,J)=AX2(I,J)+COSB*AX7(I,J)+SINB*AX6(I,J)
      AF4(I,J)=(AX1(I,J1)-AX1(I,J0)+COSB*(AX6(I,J1)-AX6(I,J0)))*DTH
     1     +SINB*(AX6(I7,J)-AX6(I0,J))*DZ
      AF5(I,J)=AX1(I,J)+COSB*AX6(I,J)
      AF6(I,J)=0.0
      AF7(I,J)=(AX3(I,J1)-AX3(I,J0)+COSB*(AX8(I,J1)-AX8(I,J0)))*DTH
     1   +SINB*(AX8(I7,J)-AX8(I0,J))*DZ+AX9(I,J)
       IF (MD.EQ.2 .AND. J. EQ. 1)
     1                    WRITE(6,2) AF1(I,J),AF2(I,J),AF3(I,J),AF4(I,J),AF5(
     1I,J),AF7(I,J)
 4000 CONTINUE
      IF(NR.EQ.1) GO TO 4020
      IF(NR.EQ.2) GO TO 4040
      II=2
      IE=I2
      GO TO 4090
 4020 II=1
      IE=MM
      GO TO 4090
 4040 II=1
      IE=IS
 4090 DO 4100 J=1,N
      A1(II,1,J)=0.0
      A1(II,5,J)=0.0
      TRS=AX7(IE,J)
      TRSZ=TRS*DZ*2.0
      A1(II,4,J)=-TRSZ
      A1(II,2,J)=A1(II,2,J)-TRSZ*ERO2
      DDT=AX6(IE,J)*DTHE2
      CC(II,J)=CZ(II,J)-DDT
       CZ(II,J)=DDT
      B1(II,J)=-CC(II,J)
      A1(II,3,J)=A1(II,3,J)+TRSZ
      AF7(IE,J)=AXL(II,J)-AX8(IE,J)
       AXL(II,J)=AX8(IE,J)
      AF1(IE,J)=0.0
      AF2(IE,J)=0.0
      AF3(IE,J)=0.0
```

```
      AF4(IE,J)=0.0
      AF5(IE,J)=0.0
      AF6(IE,J)=0.0
          IF(MD.EQ.2.AND.J.EQ.N)
     1                 WRITE(6,2) A1(II,2,J),A1(II,4,J),A1(II,3,J),B1(II,J
     1),CC(II,J),AF7(IE,J)
 4100 CONTINUE
      IF(NR.EQ.1) GO TO 4102
      GO TO 4150
 4102 IS=I1
      NR=2
      MM=I2
      ERO2=0.
      DEL=DELZS
      DZ=0.5/DELZS
      GO TO 200
 4160 IF(VENT) GO TO 4190
      IF(KK.EQ.1) GO TO 4180
      IS=I2
      MM=M
      NR=3
      ERO2=0.
      DEL=DELZ
      DZ=0.5/DELZ
      GO TO 200
 4180 IE=I2
      ERO2=1.
      KK=2
      II=2
      GO TO 4090
 4150 IF(NR.EQ.2) GO TO 4160
 4190 MTSR=MD
      MD =MTSR
      CALL CIC1 (MD)
          MD=0
      IF(PPOUT) WRITE(6,9)
      DO 575 I=1,M
      IF(PPOUT) WRITE(6,4)(PHI(I,J),J=1,N)
      DO 575 J=1,N
      KUPT(I,J)=0
      IF(PHI(I,J).GE.0.0) GO TO 575
      PHI(I,J)=0.0
      KUPT(I,J)=1
  575 CONTINUE
C     DIMENSIONLESS FLOW  Q/C(N1+N2)R*R
      IP=I1-1
 4204 AFLOW=0.0
      DO 4200 J=1,N
      IF(J.EQ.1.OR.J.EQ.N) GO TO 4220
      YF1=(PHI(IP,J+1)-PHI(IP,J-1))*DTHE2
 4210 AFLOW=(YF1*AX6(IP,J))+AX8(IP,J)+AX7(IP,J)*(PHI(IP+1,J)-PHI(IP-1,J)
     1)*DZ+AFLOW
      IF(MDIAG.EQ.2) WRITE(6,2)YF1,AFLOW
      GO TO 4200
 4220 IF(J.EQ.N) GO TO 4230
      YF1=(PHI(IP,2)-PHI(IP,N))*DTHE2
      GO TO 4210
 4230 YF1=(PHI(IP,1)-PHI(IP,N-1))*DTHE2
      GO TO 4210
 4200 CONTINUE
      AFLOW=AFLOW*DTHETA/12.
```

```
C      TORQUE DIVIDED BY MUXRXRXRXRXR/C
       NR=1
       TORQ=.TRUE.
       TQ=0.0
       IT1=1
       IT2=I1
       TC1=0.0
       GO TO 200
 1001  DO 1000 I=IT1,IT2
       DO 1000 J=1,N
       IF(KUPT(I,J).EQ.1) GO TO 1000
       IF(I.EQ.IT1.OR.I.EQ.IT2) GO TO 1800
       DPDZ=(PHI(I+1,J)-PHI(I-1,J))/DZ
       AFTR=1.0
 1200  IF(J.EQ.1.OR.J.EQ.N) GO TO 1600
       DPDT=(PHI(I,J+1)-PHI(I,J-1))*DTHE2
 1240  H=H9(I,J)
       BQ=H*ALTAL1*DEPH
       AQ=(-BQ*COSB*WS13(I,J)-H*ALM1*WS12(I,J)-ALPH*WS11(I,J)*(H+DEPH))*
      1DPDT
       AQ=AQ+BQ*(-CON2*WS14(I,J)-WS13(I,J)*DPDZ*SINB)
       TQ=TQ+AQ*DLZ*0.5*AFTR
       GO TO 1010
 1600  IF(J.EQ.N) GO TO 1610
       DPDT=(PHI(I,2)-PHI(I,N))*DTHE2
       GO TO 1240
 1610  DPDT=(PHI(I,1)-PHI(I,N-1))*DTHE2
       GO TO 1240
 1800  AFTR=.5
       IF(I.EQ.IT2) GO TO 1810
       DPDZ=(PHI(IT1+1,J)-PHI(IT1,J))/DELZ
       GO TO 1200
 1810  DPDZ=(PHI(IT2,J)-PHI(IT2-1,J))/DELZ
       GO TO 1200
 1010  RE=REN*H
       TC2=TCC(RE)*ALM1
       RE=REN*(H+DEPH)
       TC1=TC1+(TC2+TCC(RE)*ALPH)*TRQ*DLZ*AFTR
         IF (J.NE.1) GO TO 1000
       IF(MD.EQ.2.AND.I.EQ.IT1)
      1                  WRITE(6,2)AQ,TQ,RE,TC1,TC2
 1000  CONTINUE
       IF(MD.EQ.2)WRITE(6,2)WS11(I,J),WS12(I,J ),WS13(I,J),WS14(I,J)
       IF(NR.EQ.1) GO TO 1400
       IF(NR.EQ.2) GO TO 1410
       GO TO 578
 1400  NR=2
       IT1=I1
       IT2=I2
       GO TO 200
 1410  IF(VENT) GO TO 578
       NR=3
       IT1=I2
       IT2=M
       GO TO 200
  578  TQO= TQ+TC1
        IF (SIGN.LT.0.)  TQO=-TQO
       THE=0.0
       DO 580 J=1,N
       QQQ(J)=SIN(THE)
       QQQQ(J)=COS(THE)
```

```
580 THE=DTHETA+THE
    DO 590 I=1,M
    PP(I)=0.0
    PPP(I)=0.0
    DO 600 J=1,N
    DUM=PHI(I,J)
    PP(I)=PP(I)+QQQ(J)*DUM
600 PPP(I)=PPP(I)+QQQQ(J)*DUM
    PP(I)=PP(I)*DTHETA
    PPP(I)=PPP(I)*DTHETA
    PPX(I)=PP(I)*XX(I)
590 PPPX(I)=PPP(I)*XX(I)
    FSIN=SUM(PP,M,DELZ)
    FCOS=SUM(PPP,M,DELZ)
    FMSIN=SUM(PPX,M,DELZ)
    FMCOS=SUM(PPPX,M,DELZ)
       FMCOS=-FMCOS
     FCOS=-FCOS
     WLOAD=FCOS**2+FSIN**2
     WLOAD=SQRT(WLOAD)
      TQO=TQO/WLOAD
      TQI=TQO+EPS*FSIN/WLOAD
      WRITE (6,6)
      WRITE (6,7) REN,EPS,TQI,TQO,FCOS,FSIN,AFLOW,FMCOS,FMSIN
  6 FORMAT (112H REN NO.   EPS.  TORQUE J.    TORQUE B.    RADIAL LOA
   1D   TANG. LOAD       FLOW     COS.  MOMENT   SIN. MOMENT   )
  7 FORMAT (F9.2,1X,F6.3,7(1XE13.6))
599 IF(MORE) GO TO 10
    STOP
    END
```

```
      SUBROUTINE CIC1(MD)
      COMMON MDIAG,DELX,DELZ,DELZS,M,MS,MG1,MG2,N,NP(17,19),A1(2,5,1
     19),B1(2,19),CC(2,19),AF1(17,19),AF2(17,19),AF3(17,19),AF4(17,19),A
     2F6(17,19),AF7(17,19),AF5(17,19),PHI(17,19)
      DIMENSION A(17,17),B(17,17),C(17,17),D(17,17),F(20,17),AK(17,17),G
     1N(17,17),S(20,17),AFJI(17),HD(17,17),BF(17),DD(17,17),DS(17),G(17,
     217),GI(17,17),E(17,17)
      AN=1.0/DELX
        NC=0
      M=MS+MG1+MG2+1
        MN1=MG1+1
        MN2=MN1+MS
      DX1=AN*0.5
      DX2= AN**2
  203 DO 205 I=1,M
      DO 204 II=1,M
      E(I,II)=0.
  204 D(I,II)=0.
      F(1,I)=0.
  205 D(I,I)=1.
      DO 300 J=1,N
      WRITE(7)((E(I,II),D(I,II),I=1,M),II=1,M)
       IF (MD.NE.2) GO TO 240
      WRITE (6,103) AF1(1,J),AF2(1,J),AF3(1,J),AF4(1,J),AF5(1,J),AF6(1,J
     1),AF7(1,J)
      WRITE (6,103)
     1              A1(1,1,J),A1(1,2,J),A1(1,3,J),A1(1,4,J),A1(1,5,J),B1(MN1,
     1J),CC(MN1,J)
          WRITE (6,103)
     1                   A1(2,1,J),A1(2,2,4),A1(2,3,J),A1(2,4,J),A1(2,5,J),
     1B1(MN2,J),CC(MN2,J)
  240 DZ1=0.5/(DELZ)
      DZ2=1./(DELZ)**2
  241 DO242 I=1,M
      DO 242 II=1,M
      B(I,II)=0.
      A(I,II)=0.
  242 C(I,II)=0.
      DO 250 I=1,M
  206 IF(I-MG1-1) 212,210,212
  209 AFJI(I)=-AF7(I,J)
      A(I,I)=1.
      GO TO 250
  210 IF(MS) 211,212,211
  211 DZ1=0.5/DELZS
      DZ2=1./(DELZS)**2
      IF(NP(I,J))209,232,209
  232 IF(MG1) 233,212,233
  233 IF(NC.EQ.2) GO TO 600
      A(I,I-2)=A1(1,1,J)
      A(I,I-1)=A1(1,2,J)
      A(I,I)=A1(1,3,J)
      A(I,I+1)=A1(1,4,J)
      A(I,I+2)=A1(1,5,J)
      B(I,I)=B1(1,J)
      C(I,I)=CC(1,J)
  600 AFJI(I)=-AF7(I,J)
      GO TO 250
  212 IF(NP(I,J))209,234,209
  234 IF(I-MG1-MS-1) 215,213,215
  213 IF(MS) 214,215,214
```

```
214 DZ1=0.5/DELZ
    DZ2=(1./DELZ)**2
    IF(NC.EQ.2) GO TO 601
    A(I,I-2)=A1(2,1,J)
    A(I,I-1)=A1(2,2,J)
    A(I,I)=A1(2,3,J)
    A(I,I+1)=A1(2,4,J)
    A(I,I+2)=A1(2,5,J)
    B(I,I)=B1(2,J)
    C(I,I)=CC(2,J)
601 AFJI(I)=-AF7(I,J)
    GO TO 250
215 IF(NC.EQ.2) GO TO 602
    B(I,I-1)=AF3(I,J)*DX1*DZ1
    B(I,I)=AF5(I,J)*DX2-AF4(I,J)*DX1
    B(I,I+1)=-AF3(I,J)*DX1*DZ1
    A(I,I-1)=AF1(I,J)*DZ2-AF2(I,J)*DZ1
    A(I,I)=AF6(I,J)-2.0*(AF1(I,J)*DZ2 +AF5(I,J)*DX2)
    A(I,I+1)=AF1(I,J)*DZ2+AF2(I,J)*DZ1
    C(I,I-1)=-AF3(I,J)*DX1*DZ1
    C(I,I)=AF4(I,J)*DX1+AF5(I,J)*DX2
    C(I,I+1)=AF3(I,J)*DX1*DZ1
602 AFJI(I)=-AF7(I,J)
250 CONTINUE
    IF(NC.EQ.2) GO TO 603
    DO 260 I=1,M
    DO 260 II=1,M
    AK(I,II)=A(I,II)
    DO 260 III=1,M
260 AK(I,II)=AK(I,II)+B(I,III)*E(III,II)
    IF(MDIAG.LT.2) GO TO 262
    WRITE(6,101)J
    DO 261  I=1,M
261 WRITE(6,100) (AK(I,II),II=1,M)
262 CALL MATINV(AK,M,DUM,0,DUM1)
603 CONTINUE
    DO 404 I=1,M
    BF(I)=AFJI(I)
    F(J+1,I)=0.
    DO 404 II=1,M
    E(I,II)=0.
    BD(I,II)=0.
    DO 403 III=1,M
    E(I,II)=E(I,II)-AK(I,III)*C(III,II)
403 BD(I,II)=BD(I,II)+B(I,III)*D(III,II)
404 BF(I)=BF(I)-B(I,II)*F(J,II)
    DO 406 I=1,M
    DO 406 II=1,M
    D(I,II)=0.
    DO 405 III=1,M
405 D(I,II)=D(I,II)-AK(I,III)*BD(III,II)
406 F(J+1,I)=F(J+1,I)+AK(I,III)*BF(III)
300 CONTINUE
    DO 505 I=1,M
    DO 504 II=1,M
504 DD(I,II)=-D(I,II)
505 DD(I,I)=1.+DD(I,I)
    IF(MDIAG.LT.2) GO TO 264
    WRITE (6,101)
    DO 263 I=1,M
263 WRITE (6,100) (DD(I,II),II=1,M)
```

```
264 CALL MATINV(DD,M,DUM,0,DUM1)
    DO 507 I=1,M
    S(N,I)=0.
    DO 507 II=1,M
    GN(I,II)=0.
    DO 506 III=1,M
    GN(I,II)=GN(I,II)+DD(I,III)*E(III,II)
506 G(I,II)=GN(I,II)
507 S(N,I)=S(N,I)+DD(I,II)*F(N+1,II)
    J=N+1
    DO 512 K=2,N
    WRITE(8) ((G(I,II),I=1,M),II=1,M)
    BACKSPACE 7
    READ(7)  ((E(I,II),D(I,II),I=1,M),II=1,M)
    BACKSPACE 7
    IF(MDIAG.EQ.2) WRITE(6,100) ((E(I,II),D(I,II),I=1,M),II=1,M)
    J=J-1
    DO 509 I=1,M
    S(J-1,I)=F(J,I)
    DO 509 II=1,M
    GI(I,II)=0.
    DO 508 III=1,M
    GI(I,II)=GI(I,II)+D(I,III)*GN(III,II)+E(I,III)*G(III,II)
    IF(MDIAG.NE.2) GO TO 508
508 CONTINUE
509 S(J-1,I)=S(J-1,I)+E(I,II)*S(J,II)+D(I,II)*S(N,II)
    DO 512 I=1,M
    DO 512 II=1,M
512 G(I,II)=GI(I,II)
280 DO 511 I=1,M
    DO 510 II=1,M
510 DD(I,II)=-GI(I,II)
511 DD(I,I)=1.+DD(I,I)
    IF(MDIAG.LT.2) GO TO 266
    WRITE (6,101)
    DO 265 I=1,M
265 WRITE (6,100) (DD(I,II),II=1,M)
266 CALL MATINV(DD,M,DUM,0,DUM1)
    IF(MDIAG.LT.2) GO TO 270
    I=0
    WRITE (6,101)I
    DO 273 I=1,M
273 WRITE (6,100) (DD(I,II),II=1,M)
270 DO 515 I=1,M
    PHI(I,1)=0
    DO 515 II=1,M
515 PHI(I,1)=DD(I,II)*S(1,II)+PHI(I,1)
    DO 516 J=2,N
    BACK SPACE 8
    READ(8) ((G(I,II),I=1,M),II=1,M)
    BACKSPACE 8
    DO 516 I=1,M
    PHI(I,J)=S(J,I)
    DO 516 II=1,M
516 PHI(I,J)=PHI(I,J)+G(I,II)*PHI(II,1)
    REWIND 7
    REWIND 8
       IF (MD.EQ.2)  GO TO 110
    IF(MDIAG.LT.2) GO TO 268
110 WRITE (6,102)
    DO 267 I=1,M
```

```
267 WRITE (6,100) (PHI(I,J),J=1,N)
268 RETURN
100 FORMAT(1X,1P10E11.4)
101 FORMAT(5HOCIC1,I5)
102 FORMAT(1H0,30X10HFINAL PHI /1H0)
103 FORMAT (1X,1P7E11.4)
    END
```

```
      SUBROUTINE MATINV(A,N,B,M,DETER)
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
      DIMENSION IPIVO(17),A(17,17),B(17,1 ),INDEX(17,2 ),PIVOT(17)
      EQUIVALENCE (IROW,JROW), (ICOLU ,JCOLU ), (AMAX, T, SWAP)
C     INITIALIZATION
C
   10 DETER =1.0
   15 DO 20 J=1,N
   20 IPIVO (J)=0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVO (J)-1) 60, 105, 60
   60 DO 100 K=1,N
   70 IF (IPIVO (K)-1) 80, 100, 740
   80 IF (ABS (AMAX)-ABS (A(J,K))) 85, 100, 100
   85 IROW=J
   90 ICOLU =K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVO (ICOLU )=IPIVO (ICOLU )+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLU ) 140, 260, 140
  140 DETER =-DETER
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLU ,L)
  200 A(ICOLU ,L)=SWAP
  205 IF(M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLU ,L)
  250 B(ICOLU ,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLU
  310 PIVOT(I)=A(ICOLU ,ICOLU )
  320 DETER =DETER *PIVOT(I)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLU ,ICOLU )=1.0
  340 DO 350 L=1,N
  350 A(ICOLU ,L)=A(ICOLU ,L)/PIVOT(I)
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLU ,L)=B(ICOLU ,L)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLU ) 400, 550, 400
  400 T=A(L1,ICOLU )
  420 A(L1,ICOLU )=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLU ,L)*T
```

```
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLU ,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I  \
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLU =INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLU )
  700 A(K,JCOLU )=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

```
      FUNCTION TCC(RE)
      IF(RE.GT.100.0) GO TO 10
      TCC=8./RE
      GO TO 100
   10 IF(RE.GT.400.0) GO TO 20
      TCC=4.175/(RE)**.86
      GO TO 100
   20 IF(RE.GT.1000.0) GO TO 30
      TCC=.547/(RE)**.521
      GO TO 100
   30 IF(RE.GT.4000.0) GO TO 40
      TCC=.342/(RE)**.453
      GO TO 100
   40 TCC=.064/(RE)**.25
  100 RETURN
      END
```

```
      FUNCTION SUM(P,M,DX)
      DIMENSION P(17)
      K=2
      KK=M-1
      KKK=2
      SUM=0.0
   10 DO 20 I=K,KK,KKK
   20 SUM=SUM+P(I)
      GO TO (30,40,50),K
   30 SUM=SUM*DX/3.0
      RETURN
   40 K=3
   45 SUM=SUM*2.0
      GO TO 10
   50 K=1
      KK=M
      KKK=M-1
      GO TO 45
      END
```

```
      FUNCTION GZCF(REYN)
      RE=REYN
      IF(RE.LE.70.0)GO TO 20
      IF((RE.GT.70.0).AND.(RE.LE.4000.0))GO TO 30
   10 IF((RE.GT.4000.0).AND.(RE.LE.7.0E+03)) GO TO 40
      IF((RE.GT.7000.0).AND.(RE.LE.2.0E+04)) GO TO 50
      GZCF=25.6/(RE)**.756
      RETURN
   20 GZCF=1.0/12.0
      RETURN
   30 GZCF=1.858 E-09*(RE)**2-1.878E-05*RE+.0846
      RETURN
   40 GZCF=9.62/(RE)**.652
      RETURN
   50 GZCF=11.3/(RE)**.674
      RETURN
      END
```

```
      FUNCTION GTCF(REYN)
      RE=REYN
      IF(RE.LE.70.0)GO TO 20
      IF((RE.GT.70.0).AND.(RE.LE.2000.0)) GO TO 30
   10 IF((RE.GT.2000.0).AND.(RE.LE.5.5E+03))GO TO 40
      IF((RE.GT.5500.0).AND.(RE.LE.2.0E+04)) GO TO 50
      GTCF=20.5/(RE)**0.784
      RETURN
   20 GTCF=1.0/12.0
      RETURN
   30 GTCF=.619E-08*(RE)**2-3.465E-05*RE+.08569
      RETURN
   40 GTCF=4.90/(RE)**.628
      RETURN
   50 GTCF=10.35/(RE)**.716
      RETURN
      END
```

```
 INPUT

NAMELIST    INPUT
RRN      0.10000000F 02        ALOUD    0.80000000E 00      ERR     [illegible]
PFIX  (I)
     1   0.                     0.                 0.
I1                    3    12                      7
MORE     T
SIGN     -0.10000000E 01
N                    12
EDOT     0.                     PHDOT   0.50000000E 00      GAMDOT   0.                   EPS     0.50000000E-01
GAM      0.
VENT     T
BETA  (I)
     1   0.14900000E 03    0.31000000E 02    0.31000000E 02
DEP   (I)
     1   0.                0.24000000E 01    0.
ALPHA (I)
     1   0.                0.55000000E 00    0.55000000E 00
PPOUT    T
YOHR     0.
END    NAMELIST    INPUT

 FINAL PRESSURE DISTRIBUTION.

   0.          0.          0.          0.          0.          0.          0.          0.          0.          0.
   0.          0.

   0.6789010   0.6927785   0.7180269   0.7485889   0.7761385   0.7924057   0.7923074   0.7760784   0.7489701   0.7188989
   0.6937008   0.6702956

   1.3563577   1.3834361   1.4337954   1.4953438   1.5513639   1.5851544   1.5862490   1.5546936   1.5008398   1.4398732
   1.3883323   1.3872372

   1.0180076   1.0321377   1.0647648   1.1083063   1.1513494   1.1814016   1.1892039   1.1724800   1.1367066   1.0926132
   1.0521661   1.0252619

   0.6799845   0.6854405   0.7037073   0.7308586   0.7594643   0.7819976   0.7914106   0.7848128   0.7644812   0.7364897
   0.7088211   0.6884140

   0.3410142   0.3418344   0.3492073   0.3615078   0.3757176   0.3879441   0.3945387   0.3934595   0.3851826   0.3720764
   0.3581250   0.3468832

   0.          0.          0.          0.          0.          0.          0.          0.          0.          0.
   0.          0.
 REN NO.   EPS.   TORQUE J.     TORQUE B.    RADIAL LOAD   TANG. LOAD      FLOW        COS. MOMENT   SIN. MOMENT
  10.00   0.050   0.177719E 03  0.177713E 03  0.177580E 00  0.208569E-01 -0.230333E 01  0.101375E 00  0.5701988-02
```

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APPENDIX IV

COMPUTER PROGRAM PN 406

STATIC PERFORMANCE OF A SPIRAL-GROOVED, FLOATING-RING JOURNAL BEARING OPERATED IN THE TURBULENT REGIME

Input

All input data appear in the form of name list except when the characteristics of inner and outer film are known and need not be computed within the program. In that case, two sets of data, each containing 9 cards in a specified form as explained in detail below, are required to provide the information on the film characteristics.

For the readers who would like to be familiar with the format of namelist, it is recommended that he read pages 14 and 19 of Ref. (10).

The choice of whether to provide or to compute the film characteristics is indicated by the first word of the namelist "NGPUT". The preparation of input for each case is shown below:

Case I: The film data generated within the program - the namelist contains two listings; these are "NGPUT" and "INPUT".

A. "NGPUT" includes the following input:

1. INPRD, INPRD $\neq$ 1: The film characteristics will be generated within the program and the information in the namelist "INPUT" must be provided.
2. NEPS = Number of (inner film) eccentricities ratios to be examined (maximum 10).
3. NCASE, NCASE = 0: Last set of input.
   NCASE $\neq$ 0: More input follows, starting from the namelisting "NGPUT".
4. NN = Number of iterations to be allowed to achieve an equilibrium condition in the floating-ring system (recommend 10).
5. R2R1 = Radius ratio of ring and journal.
   = R_2/R_1, R_1 = radius of journal, in.; R_2 = radius of ring, in.
6. ANTT = Initial guess, on the speed ratio; ω_2/ω_1,
   ω_2 = speed of ring, rad/sec.; ω_1 = speed of journal, rad/sec.

7. DN = Incremental value of the ring speed ratio during the iteration (recommend .15).

8. REY = Overall Reynolds number under which the bearing is to be operated.

$= \frac{\omega_1 R_1 C_1}{\nu}$; C_1 = inner film thickness, in.,

ν = kinematic viscosity in^2/sec.

9. EPIS = Eccentricity ratios of inner film to be examined. There are NEPS number of eccentricity ratios to be provided (maximum 10).

10. COR1 = Inner film clearance ratio

$= C_1/R_1$

11. COR2 = Outer film clearance ratio

$= C_2/R_2$

12. PRES = Overall dimensionless gauge pressure measures at the end and the middle of the bearing.

$= 2\pi P$ (pressure, psig) / $\mu\omega_1(\frac{R_1}{C_1})^2$

where μ is the dynamic viscosity, $\frac{lb\text{-}sec}{in^2}$

The pressure at the end appears first.

B. "INPUT" includes the following input:

1. RINSP = The starting value of ring-speed ratio under which the film data will be generated. It is noted that $0 \leq RINSP \leq 1$. Since the film data of each film covers three different speeds, a small number for RINSP, say .25, is recommended.

2. DELSP = The incremental ring-speed ratio, DELSP, should be sufficiently small such that RINSP + 2 x DELSP ≤ 1. The recommended value for DELSP is 0.15.

3. BLOVD = Length diameter ratio, $L/2R_1$, where L is the length of journal.

4. I1 = Number of axial grid points in the first region of the bearing counted from the initial end of journal (see Fig. 2). An odd number is required and also it must be at least 2 less than I2. The minimum permissible value for I1 is 3.

B. 5. I2 = Number of axial grid points counted from the initial end of the journal to the middle of the journal. The minimum permissible value for I2 is 5 and the maximum is 17. Again, I2 must be an odd number. The number of grid points in the second region is I2-I1+1 which gives (I2-I1) intervals.

6. N9 = Number of grid points in the circumferential direction, maximum permissible value for N9 is 19.

7. EPS1 = An array of three values of ϵ_1 for which inner bearing film data are to be generated. The range of EPS1 should be wide enough to cover anticipated operating eccentricities.

8. BETA = Groove angle in degrees. BETA must be specified for the first region of the bearing (first value) and the second region of the bearing (second value). For a pump-in design, with grooving in the first region, and a smooth seal in the second region, BETA should be read in as an obtuse angle, the same value for BETA being read in for the second region as for the first region. For a pump out design, with grooving in the second region of the bearing and a seal in the first region, BETA should be read in as acute angle with the same value of BETA being read in for the first region as for the second region. Never set BETA = 0.

9. DEP = Groove recess ratios in the two regions of journal with the first value referring to the first region, etc.

= δ/c, where δ = groove recess, in.

and c = nominal, radial clearance, in.

To impose the condition of a smooth bearing (no grooving) in either region, set DEP = 0 for that region.

10. ALPHA=Fractional groove width in the two regions of a journal with the first value referring to the first region = $a_g / (a_g + a_r)$, where a_g and a_r are the widths in inches in the groove and ridge portion respectively. To impose the condition of a smooth bearing in either region, set ALPHA = 0 for that region.

11. EPS2= An array of three values of ϵ_2 for which outer bearing film data are to be generated. The range of EPS2 should be wide enough to cover anticipated operating eccentricity.

Case II: In the case of known film characteristics, only the namelisting "NGPUT" is required. In addition to that, 18 cards must be followed which contain the film data.

The content of namelisting "NGPUT" is essentially the same as those of Case I. provided INPRD must be equal to 1.

The inner film data appear first corresponding to three eccentricity ratios at three different inner film Reynolds numbers.

The data include (these symbols are defined below)

$$Re,\ \epsilon,\ \bar{T}_j,\ \bar{T}_B,\ S,\ \phi$$

which are punched on one card with the format of (1X, F8.1, F5.2, 4E13.6). The first three cards are for the same Reynolds number, the smallest of the three values. Each card refers to a different eccentricity ratio. Again, the order of the eccentricity ratio is ascending. The next three cards correspond to a higher Reynolds number but with the same set of eccentricity ratios. In total, there are 9 cards for the inner film.

Following the inner film data, there are 9 cards for outer film data arranged in an order similar to that for the inner film.

The six quantities used as input for the inner and outer film are defined below.

1. Re = Reynolds number

$$= \frac{(\omega_1 - \omega_2) R_1 C_1}{\nu} \quad \text{(for inner film)}$$

$$= \frac{\omega_2 R_2 C_2}{\nu} \quad \text{(for outer film)}$$

2. ϵ = Eccentricity (see Fig. 3)

$$= e_1/c_1 \quad \text{(for inner film)}$$

$$= e_2/c_2 \quad \text{(for outer film)}$$

3. $\bar{T}_j$ = Dimensionless Torque of Journal

$$= \frac{T_j}{WC_1} \quad \text{(for inner film), where } W = \text{load, pounds}$$

$$= \frac{T_j}{WC_2} \quad \text{(for outer film)}$$

Case II:

4. $\bar{T}_B$ = Dimensionless Torque of Bearing

$$= \frac{T_B}{WC_1} \quad \text{(for inner film)}$$

$$= \frac{T_B}{WC_2} \quad \text{(for outer film)}$$

5. S = Sommerfeld Number

$$= \frac{LD_1\mu(N_1+N_2)\left(\frac{R_1}{C_1}\right)^2}{W} \quad \text{(for inner film)}$$

$$= \frac{LD_2\mu N_2\left(\frac{R_2}{C_2}\right)^2}{W} \quad \text{(for outer film)}$$

where N_1 and N_2 are speed of journal and ring in rev./sec., respectively.

6. ϕ = Attitude angle, degree (see Fig. 3)

$= \phi_1$ (for inner film)

$= \phi_2$ (for outer film)

Output

The output appears under the title of "Floating-Ring with herringbone journal".

1. Main program input: it prints out the title of "Main program input". Immediately, there follows the title of "Namelist NGPUT" and the entire contents in that namelist. At the end, it prints out "end namelist NGPUT".

2. Input for subroutine "HERNB": it prints out the title of "Herringbone Bearing Input" and a title of "Namelist INPUT". Then, the entire contents of that namelist are printed out. At the end, it prints out "end namelist INPUT".

3. Single film data

   A. For Inner Film:

   There are three eccentricity ratios at three different Reynolds numbers corresponding to three different speed ratios. The output starts with a title of "inner film data" and then the headings REYNOLDS NO., ECCENTRICITY, INNER TORQUE, OUTER TORQUE, SOMMERFELD

NO., ATT. ANGLE, FLOW, on one line. Immediately, there follows 9 lines of data. Each line contains seven quantities under the appropriate heading. The first six of these seven quantities are defined above in the input list for NGPUT, Case II. The flow is defined as

$\bar{Q}$ -- dimensionless flow

$$= \frac{Q}{R_1^{\,2} C_1 (N_1+N_2)} \qquad \text{(for inner film)}$$

$$= \frac{Q}{R_2^{\,2} C_2 N_2} \qquad \text{(for outer film)}$$

Q = Flow, cu. in/sec.

B. For Outer Film:

After the output of the inner film, there are a set of output referring to the outer film just like those for the inner film. The output comprises the title of "Outer Film Data", the heading and 9 lines of data.

4. Final performance characteristics of the floating ring bearing at the steady state equilibrium condition. Under the title of output, there are four values in a line. These are:

ECCENTR/C_1 = e/C_1 = the overall eccentricity of the journal at equilibrium position divided by the nominal clearance of the inner film (see Fig. 4).

N_2/N_1, C_2/C_1 and R_2/R_1 as previously explained.

Next, there is a table of output referring to the inner film, the outer film, and the overall bearing. There are seven values in a line and a total of 3 lines. Each line contains

$$\text{REYNOLDS NO.} = \frac{(\omega_1-\omega_2)R_1C_1}{\nu} \qquad \text{(for inner film)}$$

$$= \frac{\omega_2 R_2 C_2}{\nu} \qquad \text{(for outer film)}$$

$$= \frac{\omega_1 R_1 C_1}{\nu} \qquad \text{(for overall)}$$

ECCENTRICITY $= e_1/C_1$ (for inner film) $= \varepsilon_1$

(see Figs. 3 and 4)

$$\epsilon_2 = e_2/C_2 \qquad \text{(for outer film)}$$

$$\epsilon = e/C_1+C_2 \qquad \text{(for overall)}$$

TORQUE = Dimensionless form defined as

$$= \left[\frac{T_1}{WC_1}\right] - \frac{1}{2} S_1 \epsilon_1 \bar{F}_{t1} \qquad \text{(for inner film)}$$

$$= \left[\frac{T_B}{WC_2}\right] + \frac{1}{2} S_2 \epsilon_2 \bar{F}_{t2} \qquad \text{(for outer film)}$$

$$= \left[\frac{T_1}{WC_1}\right] \qquad \text{(for overall)}$$

where $\bar{F}_{t1}$ and $\bar{F}_{t2}$ are defined on page 139.

SUPPLY PRES = Dimensionless supply pressure

$$= \frac{P}{\mu \left(\frac{R_1}{C_1}\right)^2 (N_1+N_2)} \qquad \text{(for inner film)}$$

$$= \frac{P}{\mu N_2 \left(\frac{R_2}{C_2}\right)^2} \qquad \text{(for outer film)}$$

$$= \frac{P}{\mu N_1 \left(\frac{R_1}{C_1}\right)^2} \qquad \text{(for overall)}$$

SOMMFD NO. = Sommerfeld Number

$$S_1 = \frac{LD_1\mu\ (N_1+N_2)\ \left(\frac{R_1}{C_1}\right)^2}{W} \text{(for inner film)}$$

$$S_2 = \frac{LD_2\mu N_2\ \left(\frac{R_2}{C_2}\right)^2}{W} \qquad \text{(for outer film)}$$

$$S = \frac{LD_1\mu N_1\ \left(\frac{R_1}{C_1}\right)^2}{W} \qquad \text{(for overall)}$$

ATT. ANGLE = Attitude angle in deg. (see Fig. 3)

= ϕ_1 (for inner film)

= ϕ_2 (for outer film)

= ϕ (for overall)

TANG. FORCE = Dimensionless form of tangential force

$$\bar{F}_{t1} = \frac{F_{t1}}{WS_1} \quad \text{(for inner film)}$$

$$\bar{F}_{t2} = \frac{F_{t2}}{WS_2} \quad \text{(for outer film)}$$

where F_{t1} = Tangential force of the inner film, pound

F_{t2} = Tangential force of the outer film, pound

A Fortran listing of program PN 406 is provided in the next few pages. Typical listings of input and output are also given.

```
C       FLOATING-RING IN CONJUNCTION WITH HERRINGBONE JOURNAL IN TURBULENT
C        REGIME
        DIMENSION P(6,3,3),Q(6,3,3),A(4,3,3),B(4,3,3),AP(4,3,3),BP(4,3,3)
        DIMENSION REY(1),C2C1(1),EPIS(10),BA(6),BC(6),AA(4,3),BB(4,3)
        DIMENSION AP1(4,3),BP1(4,3),A1(4,3),B1(4,3),SA(4),SB(4)
        DIMENSION EPS1(3),BETA(3),DEP(3),ALPHA(3),PRES(3),EPS2(3)
    500 FORMAT(1X,F8.1,F5.2,4E12.6)
    525 FORMAT(6(1PE12.5))
C       INPRO=1,DATA FOR HERRINGBONE BEARING READ IN
        NAMELIST/NGPUT/INPRC,NEPS,NCASE,MN,R2R1,     ANTT,DN,REY,
      1EPIS,COR1,COR2,PRES/INPUT/RINSP,DELSP,BLOVD,I1,I2,N9,
      1EPS1,BETA,DEP,ALPHA,EPS2
     52 NRE=1
        GAM=0.0
        EDOT=0.0
        MUST=0
        PHDOT=0.0
        KDIAG=0
        NSES1=0
        NSES2=0
        READ(5,NGPUT)
        IF(MUST.NE.1) GO TO 53
        KDIAG=1
        NSES1=1
        NSES2=1
     53 NCC=1
        WRITE(6,505)
    505 FORMAT(41H1 FLOATING-RING WITH HERRINGBONE JOURNAL )
        C2C1(1)=COR2/COR1*R2R1
        WRITE(6,210)
           PCS=PRES(2)
    210 FORMAT(20H MAIN PROGRAM INPUT )
        WRITE(6,NGPUT)
        IF(INPRC.EQ.1) GO TO 202
        READ(5,INPUT)
        WRITE(6,212)
    212 FORMAT(27H HERRINGBONE BEARING INPUT )
        WRITE(6,INPUT)
        MPAS=1
        REO=REY(1)
        CALL HERRB(BLOVD,COR1,COR2,PRES,I1,I2,N9,EDOT,PHDOT,EPS1,GAM,BETA,
      1DEP,ALPHA,RINSP,REO,R2R1,DELSP,MPAS,KDIAG)
        REWIND 9
        DO 1213 K=1,3
        DO 1213 J=1,3
        READ(9) (P(1,J,K),P(2,J,K),P(3,J,K),P(4,J,K),P(5,J,K),P(6,J,K))

ERROR MESSAGE NUMBER    1
   1213 CONTINUE
          REWIND 9
          GO TO 304
    202 READ(5,500) ((P(1,J,K),P(2,J,K),P(3,J,K),P(4,J,K),P(5,J,K),P(6,J,K
      1),J=1,3),K=1,3)
    304 IF(MUST.EQ.1)WRITE(6,500)((P(1,J,K),P(2,J,K),P(3,J,K),P(4,J,K),P(5
      1,J,K),P(6,J,K),J=1,3),K=1,3)
```

```
                                                                   03/04/69
       MAIN      -  EFN   SOURCE STATEMENT  -  IFN(S)  -

  204 IK=1                                                          105
      DO 1 I=1,6                                                    106
      DO 1 J=1,3                                                    107
      DO 1 K=1,3                                                    108
    1 C(I,J,K)=P(I,J,K)                                             109
    2 DO 5 J=1,3                                                    110
      DO 5 I=3,6                                                    111
      A(I-2,J,1)=0.C                                                112
      A(I-2,J,3)=0.C                                                113
      B(I-2,J,1)=0.C                                                114
      B(I-2,J,3)=0.C                                                115
      AP(I-2,J,1)=0.0                                               116
      AP(I-2,J,3)=0.0                                               117
      BP(I-2,J,1)=0.0                                               118
      BP(I-2,J,3)=0.0                                               119
      Y2=Q(I,J,1)                                                   120
      Y3=Q(I,J,2)                                                   121
      X2=Q(1,J,1)                                                   122
      X3=Q(1,J,2)                                                   123
      DO 5 K=2,2                                                    124
      Y1=Y2                                                         125
      Y2=Y3                                                         126
      X1=X2                                                         127
      X2=X3                                                         128
      Y3=Q(I,J,K+1)                                                 129
      X3=Q(1,J,K+1)                                                 130
      DT1=X2-X1                                                     131
      CT2=X3-X2                                                     132
      C1=Y2-Y1                                                      133
      C2=Y3-Y2                                                      134
      C3=X3-X1                                                      135
      C4=DT1/CT2                                                    136
      C24=C2*C4                                                     137
      A(I-2,J,K)=(C24-C1)/(CT1*C3)                                  138
      B(I-2,J,K)=(C24+C1/C4)/C3                                     139
      GO TC (3,5),IK                                                140
    3 AP(I-2,J,K)=A(I-2,J,K)                                        141
      BP(I-2,J,K)=B(I-2,J,K)                                        142
    5 CONTINUE                                                      143
      GO TC (7,9),IK                                                144
    7 IF(INPRC.EQ.1) GO TO 208                                      145
      MPAS=2                                                        146
      CALL HERNB(BLCVD,CGR1,CGR2,PRES,I1,I2,N9,EDOT,PHCOT,EPS2,GAM,BETA,  147
     1CEP,ALPHA,RINSP,REU,R2R1,DELSP,MPAS,KCIAG)                    148    131
      REWIND 10                                                     149    132
        DO 1228 K=1,3                                               150
        DO 1228 J=1,3                                               151
      READ(10) (Q(1,J,K),Q(2,J,K),Q(3,J,K),Q(4,J,K),Q(5,J,K),Q(6,J,K))  152  137

ERRCR MESSAGE NUMBER     2
 1228 CONTINUE                                                      153
        REWINC 10                                                          147
           GO TO 309                                                154
  208 READ(5,500)((C(1,J,K),Q(2,J,K),Q(3,J,K),Q(4,J,K),Q(5,J,K),C(6,J,K)  155
     1,J=1,3),K=1,3)                                                156    149
  309 IF(MUST.EQ.1)WRITE(6,50C)                                     157
```

```
    1              (((C(1,J,K),Q(2,J,K),Q(3,J,K),Q(4,J,K),Q(5,J,K),C(6,J,K)
     1,J=1,3),K=1,3)                                                    165
209 IK=2
    GO TC 2
  9 DO 201 IA=1,NRE
    RE=REY(IA)
    DO 201 IB=1,NCC
    CC=C2C1(IB)
    DO 201 IC=1,NEPS
    EP=EPIS(IC)
    ANRT=ANTT
    ND=0
    NA=0
 50 RE1=RE*(1.-ANRT)
    RE2=R2R1*CC*RE1/(1.0/ANRT-1.C)
    C1=P(1,1,3)
    IF(RE1-C1) 11,10,10
 10 KC=3
    GO TC 14
 11 DO 13 K=2,3
    C1=P(1,1,K)
    IF(RE1-C1) 12,13,13
 12 KC=K
    GO TC 14
 13 CONTINUE
 14 DO 18 I=1,4
    DO 18 J=1,3
    AR=AP(I,J,KC)
    BR=BP(I,J,KC)
    CR=P(I+2,J,KC)
    XR=P(1,J,KC)
    KL=KC-1
    AL=AP(I,J,KL)
    BL=BP(I,J,KL)
    CL=P(I+2,J,KL)
    XL=P(1,J,KL)
    C1=RE1-XR
    C1=CR+C1*(BR+C1*AR)
    C2=RE1-XL
    C2=CL+C2*(BL+C2*AL)
    IF(KC-2)15,15,16
 15 C2=C1
    GO TC 18
 16 IF(3-KC) 17,17,18
 17 C1=C2
 18 AA(I,J) =(C1+C2)/2.
    IF(NSES2.NE.1) GO TO 88
 87 WRITE(6,525)((AA(I,J),J=1,3),I=1,4)                                 235
 88 DO 19 I=1,4
    AP1(I,1)=0.0
    AP1(I,3)=0.0
    BP1(I,3)=0.0
    BP1(I,1)=0.0
    X2=P(2,1,1)
    X3=P(2,2,1)
    IF(I-4)101,102,101
```

```
 101 Y2=AA(I,1)*X2
     Y3=AA(I,2)*X3
     GO TO 103
 102 Y2=AA(I,1)
     Y3=AA(I,2)
 103 DO 19 J=2,2
     Y1=Y2
     Y2=Y3
     X1=X2
     X2=X3
     X3=P(2,J+1,1)
     IF(I-4) 104,105,104
 104 Y3=AA(I,J+1)*X3
     GO TO 106
 105 Y3=AA(I,J+1)
 106 DT1=X2-X1
     DT2=X3-X2
     C1=Y2-Y1
     C2=Y3-Y2
     C3=X3-X1
     C4=DT1/DT2
     C24=C2*C4
     AP1(I,J)=(C24-C1)/(DT1*C3)
     BP1(I,J)=(C24+C1/C4)/C3
  19 CONTINUE
     C1=P(2,3,1)
     IF(EP-C1) 21,20,20
  20 KA=3
     GO TO 24
  21 DO 23 J=2,3
     C1=P(2,J,1)
     IF(EP-C1) 22,23,23
  22 KA=J
     GO TO 24
  23 CONTINUE
  24 DO 109 I=1,4
     AR=AP1(I,KA)
     BR=BP1(I,KA)
     XR=P(2,KA,1)
     CR=AA(I,KA)
     IF(I-4) 131,132,132
 131 CR=CR*XR
 132 KB=KA-1
     AL=AP1(I,KB)
     BL=BP1(I,KB)
     XL=P(2,KB,1)
     CL=AA(I,KB)
     IF(I-4) 133,134,134
 133 CL=CL*XL
 134 C1=EP-XR
     C1=CR+C1*(BR+C1*AR)
     C2=EP-XL
     C2=CL+C2*(BL+C2*AL)
     IF(KA-2) 25,25,26
  25 C2=C1
     GO TO 28
```

```
 26 IF(3-KA) 27,27,28
 27 C1=C2
 28 IF(I-4) 107,108,107
107 SA(I)=(C1+C2)/(2.*EP)
    GO TO 109
108 SA(I)=(C1+C2)/2.
109 CONTINUE
    RF=SA(1)
    S1=SA(3)
    RCFM=(SA(1)+SA(2))/2.
    ATT1=SA(4)
    FT=(SA(1)-SA(2))/(EP*S1)
    C1=R2R1**3
    C2=CC*CC
    S2=S1*C1/(C2*(1.+1./ANRT))
    C1=Q(1,1,3)
    IF(RE2-C1) 31,30,33
 30 JC=3
    GO TO 34
 31 DO 33 K=2,3
    C1=Q(1,1,K)
    IF(RE2-C1) 32,33,33
 32 JC=K
    GO TO 34
 33 CONTINUE
 34 DO 38 I=1,4
    DO 38 J=1,3
    AR=A(I,J,JC)
    BR=B(I,J,JC)
    CR=Q(I+2,J,JC)
    XR=Q(1,J,JC)
    JL=JC-1
    AL=A(I,J,JL)
    BL=B(I,J,JL)
    XL=Q(1,J,JL)
    CL=Q(I+2,J,JL)
    C1=RE2-XR
    C1=CR+C1*(BR+C1*AR)
    C2=RE2-XL
    C2=CL+C2*(BL+C2*AL)
    IF(JC-2) 35,35,36
 35 C2=C1
    GO TO 38
 36 IF(3-JC) 37,37,38
 37 C1=C2
 38 BB(I,J)=(C1+C2)/2.
    IF(NSES2.NE.1) GO TO 90
 89 WRITE(6,525)((BB(I,J),J=1,3),I=1,4)          338
 90 DO 59 J=1,3
    NB=4-J
    C3=Q(2,J,1)
    C1=C3*C3
    BC(NB)=.C1/C1
    BA(NB)=BB(3,J)
 59 CONTINUE
    A1(3,1)=0.0
```

```
      A1(3,3)=C.0
      B1(3,1)=C.0
      B1(3,3)=0.0
      X2=BA(1)
      X3=BA(2)
      Y2=BC(1)
      Y3=BC(2)
      DO 39 J=2,3
      Y1=Y2
      Y2=Y3
      X1=X2
      X2=X3
      X3=BA(J+1)
      Y3=BC(J+1)
      DT1=X2-X1
      DT2=X3-X2
      C1=Y2-Y1
      C2=Y3-Y2
      C3=X3-X1
      C4=DT1/CT2
      C24=C2*C4
      A1(3,J)=(C24-C1)/(DT1*C3)
      B1(3,J)=(C24+C1/C4)/C3
   39 CONTINUE
      C1=BA(3)
      IF(S2-C1) 41,40,40
   40 NC=3
      GO TC 44
   41 DO 43 J=2,3
      C1=BA(J)
      IF(S2-C1) 42,43,43
   42 NC=J
      GO TC 44
   43 CONTINUE
   44 AR=A1(3,NC)
      BR=B1(3,NC)
      CR=BC(NC)
      XR=BA(NC)
      NL=NC-1
      AL=A1(3,NL)
      BL=B1(3,NL)
      CL=BC(NL)
      XL=BA(NL)
      C1=S2-XR
      C1=CR+C1*(BR+C1*AR)
      C2=S2-XL
      C2=CL+C2*(BL+C2*AL)
      IF(NC-2) 45,45,46
   45 C2=C1
      GO TC 48
   46 IF(3-NC) 47,47,48
   47 C1=C2
   48 C3=(C1+C2)/2.
      C2=1./C3
      EP2=.1*SCRT(C2)                                              413
      DO 49 I=1,4
```

```
      A1(I,1)=0.0
      A1(I,3)=0.0
      B1(I,1)=0.0
      B1(I,3)=0.0
      X2=Q(2,1,1)
      X3=Q(2,2,1)
      IF(I-4) 111,112,111
  111 Y2=BB(I,1)*X2
      Y3=BB(I,2)*X3
      GO TC 113
  112 Y2=BB(I,1)
      Y3=BB(I,2)
  113 DO 49 J=2,2
      Y1=Y2
      Y2=Y3
      X1=X2
      X2=X3
      X3=Q(2,J+1,1)
      IF(I-4) 114,115,114
  114 Y3=BB(I,J+1)*X3
      GO TC 116
  115 Y3=BB(I,J+1)
  116 DT1=X2-X1
      DT2=X3-X2
      C1=Y2-Y1
      C2=Y3-Y2
      C3=X3-X1
      C4=DT1/DT2
      C24=C2*C4
      A1(I,J)=(C24-C1)/(DT1*C3)
      B1(I,J)=(C24+C1/C4)/C3
   49 CONTINUE
      C1=Q(2,3,1)
      IF(EP2-C1)61,60,60
   60 LC=3
      GO TC 64
   61 DO 63 J=2,3
      C1=Q(2,J,1)
      IF(EP2-C1) 62,63,63
   62 LC=J
      GO TC 64
   63 CONTINUE
   64 DO 120 I=1,4
      AR=A1(I,LC)
      BR=B1(I,LC)
      XR=Q(2,LC,1)
      CR=BB(I,LC)
      IF(I-4) 135,136,136
  135 CR=CR*XR
  136 LL=LC-1
      AL=A1(I,LL)
      BL=B1(I,LL)
      XL=Q(2,LL,1)
      CL=BB(I,LL)
      IF(I-4) 137,138,138
  137 CL=CL*XL
```

```
 138 C1=EP2-XR                                                    438
     C1=CR+C1*(BR+C1*AR)                                          439
     C2=EP2-XL                                                    440
     C2=CL+C2*(BL+C2*AL)                                          441
     IF(LC-2) 55,55,56                                            442
  55 C2=C1                                                        443
     GO TC 58                                                     444
  56 IF(3-LC) 57,57,58                                            445
  57 C1=C2                                                        446
  58 IF(I-4) 118,119,118                                          447
 118 SB(I)=(C1+C2)/(2.*EP2)                                       448
     GO TC 120                                                    449
 119 SB(I)=(C1+C2)/2.                                             450
 120 CONTINUE                                                     451
     ATT2=SB(4)                                                   452
     PI=3.1415926536                                              453
     RAD=PI/180.                                                  454
     C1=EP2*CC                                                    455
     C2=180.-ATT1+ATT2                                            456
     IF(C2-90.) 150,150,151                                       457
 151 C2= 180.-C2                                                  458
     C2=C2*RAD                                                    459
     CA=-COS(C2)                                                  460     491
     GO TC 152                                                    461
 150 C2=C2*RAD                                                    462
     CA=COS(C2)                                                   463     494
 152 CSA=SIN(C2)                                                  464     496
     EPP=SQRT(EP*EP+C1*C1-2.*C1*EP*CA)                            465     497
     CSB=C1*CSA/EPP                                               466
     CCB=SQRT(1.-CSB*CSB)                                         467     498
     CTB=CSB/CCB                                                  468
     TB=ATAN(CTB)                                                 469     499
     TR=ATT1*RAD-TB                                               470
     TD=TR/RAD                                                    471
     EPS=EPP/(1.+CC)                                              472
     RCF2=(SB(1)+SB(2))/2.                                        473
     FT2=(SB(1)-SB(2))/(S2*EP2)                                   474
     C1=(1.-ANRT)/(1.+ANRT)                                       475
      IF  (INPRD.NE.1)  C1=1.                                     476
     C2=RCFM*C1                                                   477
     C1=S1*FT*EP/2.                                               478
     RCFI=C2-C1                                                   479
     C1=EP2*S2*FT2/2.                                             480
     RCFO=RCF2+C1                                                 481
     RIO=RCFI/RCFO                                                482
     ER=RIO-CC                                                    483
     IF(NSES1.NE.1) GO TO 86                                      484
  85 WRITE(6,525)RE1,RE2,ANRT,CC,EP                               485     505
     WRITE(6,525)RCFI,RCFO,ER,S1,S2                               486     506
     WRITE(6,525)EP2,FT,FT2,RCFM,RCF2                             487     507
     WRITE(6,525) SB(1),SB(2),SB(3),SB(4),ATT1,ATT2               488     508
  86 IF(NA-1) 70,71,72                                            489
  70 D1=ER                                                        490
     Q1=ANRT                                                      491
     ANRT=Q1*CN                                                   492
     NA=1                                                         493
```

```
      GO TO 50
   71 IF(D1*ER) 73,74,74
   73 NA=2
      Q3=ANRT
      D3=ER
      ANRT=(Q1+Q3)/2.
      GO TC 50
   74 C1=ANRT
      D1=ER
      ANRT=Q1+CN
      ND=NC+1
      IF (ND-NN)  5C,700,700
  700 WRITE (6,702)  NN
  702 FORMAT (15H DIVERGED AFTER ,I5,6H TIMES)
       GO TO 201
   72 IF(NA-3) 76,75,79
   76 IF(D1*ER) 78,77,77
   77 C1=ANRT
      D1=ER
      NA=3
      ANRT=(Q1+Q3)/2.
      GO TO 50
   78 Q3=ANRT
      D3=ER
      ANRT=(Q1+Q3)/2.
      NA=3
      GO TO 50
   75 Q2=ANRT
      D2=ER
      NA=4
      DTA=Q3-C1
      C1=DTA*DTA
      C2=(D1+D3-2.*D2)*2./C1
      C3=(D3-D1)/DTA
      IF(C2) 80,81,80
   81 C4=-D2/C3
      GO TO 82
   80 DTA=.5*C3/C2
      C4=SQRT(DTA*DTA-D2/C2)
      IF(DTA) 83,84,84
   83 C4=-C4
   84 C4=C4-DTA
   82 ANRT=Q2+C4
      GO TO 50
   79 S3=S1/(1.+ANRT)
      IF( PDS) 91,92,91
   92 PD1=0.0
      PD2=0.0
      GO TC 93
   91 PD1=PDS/(1.+ANRT)
       PD2=PDS*(CCR2/COR1)**2/ANRT
   93 WRITE(6,501)
      WRITE(6,508)
      WRITE(6,600)EPP,ANRT,CC,R2R1
      WRITE(6,502)
      WRITE(6,522)RE1,EP,RCFM,PD1,S1,ATT1,FT
```

```
      WRITE(6,524)RE2,EP2,RCF2,PD2,S2,ATT2,FT2
      WRITE(6,526)RE,EPS,RF,PDS,S3,TD
  502 FORMAT(12X83H REYNOLDS NO ECCENTRIC      TORQUE   SUPPLY PRES  SOMM
     1FD  NO  ATT. ANGLE TANG. FORCE )
  501 FORMAT(7H OUTPUT)
  503 FORMAT(47H ECCENTR/C1       N2/N1        C2/C1          R2/R1   )
  600 FORMAT(4(1XE11.4))
  522 FORMAT(12H INNER RING ,7(1XE11.4))
  524 FORMAT(12H OUTER RING ,7(1XE11.4))
  526 FORMAT(12H    CVERALL  ,6(1XE11.4))
  201 CONTINUE
      IF(NCASE) 52,51,52
   51 STOP
      END
```

```
      SUBRCUTINE HERNB(BLOVD,COR1,COR2,PRES,I1,I2,N4,EDOT,PHDOT,EPS1,GAM
     1,BETA,DEP,ALPHA,RINSP,REO,R2R1,DELSP,MPAS,KDIAG)
C          HERRINGBONE JOURNAL ,CORRECT FOR TORQUE IN CAVITATED REGION,
C     1-16-68
C          WITH LARGE ECCENTRICITY AND MISALIGNMENT
C     IN TURBULENCE REGIME
      COMMON MDIAG,DELX,DELZ,DELZS,M,MS,MG1,MG2,N,NP(17,19),A1(2,5,1
     19),B1(2,19),CC(2,19),AF1(17,19),AF2(17,19),AF3(17,19),AF4(17,19),A
     2F6(17,19),AF7(17,19),AF5(17,19),PHI(17,19)
      DIMENSION BETA(3),DEP(3),ALPHA(3),PFIX(3),          QQQ(19),H9(17
     1,19),QQCC(19),PP(17),PPP(17),PPX(17),PPPX(17),AX1(17,19),AX2(17,19
     2),AX3(17,19),AX6(17,19),AX7(17,19),AX8(17,19),AX9(17,19),WS11(17,1
     39),WS12(17,19),WS13(17,19),WS14(17,19),XX(17),AXL(2,19),CZ(2,19)
      DIMENSION EPS1(3),PRES(3)
      DIMENSION KUPT(17,19)
      LOGICAL FPOUT,VENT,TORQ
    2 FORMAT(7E14.7)
    4 FORMAT(/10(1X,F11.7))
    3 FORMAT (89H REYNOLDS  NO.  ECCENTRICITY INNER TORQUE OUTER TORQUE
     1  SOMMERFELD NO ATT. ANGLE   FLOW )
    6 FORMAT(33H SIN AND COS FORCE COMPONENTS =   E14.7,2H, E14.7)
    7 FORMAT(31H SIN AND COS MOMENTS ABOUT Z=0 E14.7,2H, E14.7)
    9 FORMAT(29HOFINAL PRESSURE DISTRIBUTION. //)
      IF (KDIAG.EQ.1) WRITE (6,2) BLOVD,COR1,COR2,PRES(1),PRES(2),PRES(3
     1),        EDOT,PHDOT,EPS1(1),EPS1(2),EPS1(3),EPS1(4),EPS1(5),EPS1(6)
     1,GAM,BETA(1),BETA(2),BETA(3),DEP(1),DEP(2),DEP(3),ALPHA(1),ALPHA(2
     1),ALPHA(3),RINSP,REO,R2R1,DELSP                                          2
      IND3=KDIAG
      MDIAG=0
      N=N4
       MPASS=MPAS
       IF (MPAS.EQ.1) WRITE(6,5)                                               8
    5 FORMAT (1H1)
       IF (MPAS.EQ.1)  WRITE (6,1)                                             10
       IF (MPAS.EQ.2)  WRITE (6,10)                                            12
    1 FORMAT (15X17H INNER FILM DATA )
   10 FORMAT (15X17H OUTER FILM DATA )
       WRITE (6,3)                                                             13
        GAMDOT=0.0
      MD=0
      MG1=I1-1
      AMG=MG1
      MS=I2-I1
      VENT=.TRUE.
      IF(VENT) GO TO 102
      M=I2+I1-1
      BMM1=M-1
      MG2=MG1
      GO TO 101
  102 M=I2
      BMM1= M-1
      BMM1=BMM1*2.
      MG2=0
  101 IF(MPAS.EQ.2) GO TO 35
      DO 20 I=1,M
```

```
      DO 20 J=1,N                                           54
      NP(I,J)=0                                             55
   20 CONTINUE                                              56
      DO 30 J=1,N                                           57
      NP(1,J)=1                                             58
      NP(M,J)=1                                             59
   30 CONTINUE                                              60
   35 KK=1                                                  61
  100 AN=N                                                  62
      PI=3.14159265358979                                   63
      DTHETA=2.*PI/AN                                       64
      DTHE2=0.5/DTHETA                                      65
      PFIX(1)=PRES(1)                                       66
      SPRITO=RINSP                                          67
      PFIX(3)=PRES(3)                                       68
         IF (MPASS.EQ.1)  GO TO 105                         69
           GO TO 108                                        70
  105  SPRITO=2.*DELSP+RINSP                                71
       DELSI=-DELSP                                         72
           GO TO 107                                        73
  108  DELSI= DELSP                                         74
  107 DO 599 L8=1,3                                         75
      IF(MPASS.EQ.2) GO TO 120                              76
      ANM1=SPRITO-1.0                                       77
      ANP1=SPRITO+1.0                                       78
       COR=COR1                                             79
      SIGN=ANM1/ANP1                                        80
      PFIX(2)=PRES(2)*ANP1                                  81
      ALOVD=BLCVD                                           82
      REN=REO*ANM1*(-1.)                                    83
      GO TC 130                                             84
  120 SIGN=-1.                                              85
       COR=COR2                                             86
      PFIX(2)=PRES(2)*(COR2/COR1      )**2/SPRITO           87
      ALOVD=BLCVD/R2R1                                      89
      REN=REO*(R2R1)**2*COR2/COR1*SPRITO                    90
  130 ELVD=ALCVD                                            91
      RATLD=2.0*ALOVC                                       93
      DO 592 L7=1,3                                         94
      EPS=EPS1(L7)                                          95
      TPS=2.*PI*SIGN                                        96
      CON1=6.*TPS                                           97
      RADIAN=.0174532925199943                              98
      GA1=GAM*RADIAN                                        99
      EDT=0.0                                              100
      CON2=CON1/REN                                        101
      TORQ=.FALSE.                                         102
      TPS1=TPS/8.0                                         103
      TRQ=TPS1*REN                                         104
      DELX=DTHETA                                          105
      DELZ=RATLD/BNM1                                      106
      TPS1=(2.*PI)**2*2.*ELVC                              107
       AMS=MS                                              108
      DELZS=DELZ                                           109
      ANG=0.0                                              110
      PHIMC=PHCOT-(1.-SIGN)*0.5                            111
```

```
      IND=1
      EPO2=1.
      L1=1
      L2=I1
   38 DO 40 J=1,N
      CC(L1,J)=0.0
       AXL(L1,J)=0.
        CZ(L1,J)=0.
      A1(L1,2,J)=0.0
      A1(L1,3,J)=0.C
   40 AF7(L2,J)=0.0
      IF(IND.EC.2)GC TO 48
      IF(VENT) GO TC 48
      L1=2
      L2=I2
      IND=2
      GO TC 38
   48 NR=1
      MN=MG1
      MM=I1
      ISS=1
      DEL=DELZ
      DZ=C.5/DELZ
      IS=1
      Z=0.0
   00 BET=BETA(NR)*RACIAN
      DEPH=DEP(NR)
      ALPH=ALPHA(NR)
      ALM1=1.-ALPH
      ALTAL1=ALPH*ALM1
      SINB=SIN(BET)
      COSB=COS(BET)
      SINB2=SINB*SINB
      COSB2=CCSB*CCSB
      COT2=COSB2/SINB2
      DLZ=DEL*CTHETA
      IF(TCRQ) GO TC 1001
      DO 2000 I=IS,MM
      XX(I)=Z
      ZCOR=Z/CCR
      EPZGH=EPS+GA1*ZCOR
      EDZR=EDCT+GAMCCT*ZCOR
      EPZG=EPZGH*PHIMC
      DO 2001 J=1,N
      SI=SIN(ANG)
      CO=CCS(ANG)
      H=1.+CO*EPZGH
      H9(I,J)=H
      H3=H*H*H
      HG=H+DEPH
      HGHR=HG/H
      AX9(I,J)=(EDZR*CO+EPZG*SI)*SINB*PI*24.0
      HG3=HG**3
      HGHR3=HG3/H3
      S1=1./SINB2
      S2=-COSB/SINB2
```

```
      RENR=REN*H                                                   168
      RENG=REN*HG                                                  169
      GXR=GTCF(RENR)*12.                                           170   113
      GXG=GTCF(RENG)*12.                                           171   114
      GZR=GZCF(RENR)*12.                                           172   115
      GZG=GZCF(RENG)*12.                                           173   116
      S5=(GXR+GZR*CCT2)                                            174
      S6=(GXG+GZG*CCT2)                                            175
      S7=GZG*S2                                                    176
      S8=GZR*S2                                                    177
      S9=GZG*S1                                                    178
      S10=GZR*S1                                                   179
      S11=-S5/S6/HGFR3                                             180
      S12=1.-S11*ALPH-ALPH                                         181
      S13=(S7-S8/HGFR3)/S6/S12                                     182
      S14=RENR/HG3/S12*(1.-HGFR)/S6                                183
      ALH3=ALPH*HG3                                                184
      A1S=SINB*(ALH3*S6*S11-ALM1*S5*H3)/S12                        185
      A2=-ALM1*(ALPH*S5*S13+S8)*H3                                 186
      A2=(A2+ALH3*(ALM1*S6*S13-S7))*SINB                           187
      A3=ALTAL1*S14*(S6*HG3-S5*H3)/REN                             188
      A3=CCN1*(A3+H*ALM1+HG*ALPH)*SINB                             189
      DMY=H3*ALM1                                                  190
      S7H=HGHR3*S7                                                 191
      B1S=-H3/S12*(S8*ALM1-S7H*ALPH*S11)*SINB                      192
      B2=ALH3*(S9-S7*S13*ALM1)+DMY*(S10+ALPH*S8*S13)               193
      B3=CCN1*ALTAL1*S14*(S8-S7H)                                  194
      B2=-B2*SINB                                                  195
      AX1(I,J)=A1S+A2*COSB                                         196
      B3=-B3*H3*SINB/REN                                           197
      AX2(I,J)=A2*SINB                                             198
      WS11(I,J)=S11/S12                                            199
      WS12(I,J)=1./S12                                             200
      WS13(I,J)=S13                                                201
      WS14(I,J)=S14                                                202
      AX3(I,J)=A3                                                  203
      AX6(I,J)=B1S+B2*COSB                                         204
      AX7(I,J)=B2*SINB                                             205
      AX8(I,J)=B3                                                  206
         IF(MC.NE.2)  GO TO 2001                                   207
           IF ( I.EQ.IS. AND.J.EQ.1)                               208
     1              WRITE(6,2) AX1(I,J),AX2(I,J),AX3(I,J),AX6(I,J),AX7(   209
     1I,J),AX8(I,J),AX9(I,J)                                       210   130
 2001 ANG=ANG+CTHETA                                               211
      ANG=0.0                                                      212
      IF(I.EQ.MM) GO TO 2000                                       213
      Z=Z+DEL                                                      214
 2000 CONTINUE                                                     215
      IF(NR.EQ.1) GO TO 2010                                       216
      IF(NR.EQ.3) GO TO 2014                                       217
      IF(VENT) GO TO 2014                                          218
      GO TO 2020                                                   219
 2010 IK=1                                                         220
      GO TO 3800                                                   221
 2014 IK=MM                                                        222
      GO TO 3800                                                   223
```

```
C     COMPUTE THE DIFFERENCES IN XS AND COEFFICIENT
 3800 I=IK
      DO 4010 J=1,N
      AF1(I,J)=0.0
      AF2(I,J)=0.0
      AF3(I,J)=0.0
      AF4(I,J)=0.0
      AF5(I,J)=0.0
      AF6(I,J)=1.0
      AF7(I,J)=-PFIX(NR)
 4010 CONTINUE
 2020 IST=IS+1
      MM1=MM-1
      DO 4000 I=IST,MM1
      IO=I-1
      I7=I+1
      DO 4000 J=1,N
      DTH=DTHE2
      IF(J.EQ.1.OR.J.EQ.N) GO TO 4C04
      JO=J-1
      J1=J+1
      GO TO 4008
 4004 IF(J.EQ.1) GO TO 4006
      JO=N-1
      J1=1
      GO TC 4008
 4006 JO=N
      J1=2
 4008 AF1(I,J)=SINB*AX7(I,J)
      AF2(I,J)=(AX2(I,J1)-AX2(I,JO)+COSB*(AX7(I,J1)-AX7(I,JO)))*DTH
     1     +SINB*(AX7(I7,J)-AX7(IO,J))*DZ
      AF3(I,J)=AX2(I,J)+COSB*AX7(I,J)+SINB*AX6(I,J)
      AF4(I,J)=(AX1(I,J1)-AX1(I,JO)+COSB*(AX6(I,J1)-AX6(I,JO)))*DTH
     1     +SINB*(AX6(I7,J)-AX6(IO,J))*DZ
      AF5(I,J)=AX1(I,J)+COSB*AX6(I,J)
      AF6(I,J)=0.0
      AF7(I,J)=(AX3(I,J1)-AX3(I,JO)+COSB*(AX8(I,J1)-AX8(I,JO)))*DTH
     1     +SINB*(AX8(I7,J)-AX8(IO,J))*DZ+AX9(I,J)
       IF (MD.EQ.2 .AND. J. EC. 1)
     1                  WRITE(6,2) AF1(I,J),AF2(I,J),AF3(I,J),AF4(I,J),AF5(
     1I,J),AF7(I,J)                                                          228
 4000 CONTINUE
      IF(NR.EC.1) GC TO 4020
      IF(NR.EC.2) GC TO 4040
      I1=2
      IE=I2
      GO TC 4090
 4020 I1=1
      IE=MM
      GO TC 4090
 4040 I1=1
      IE=IS
 4090 DO 4100 J=1,N
      A1(I1,1,J)=0.0
      A1(I1,5,J)=0.0
      TRS=AX7(IE,J)
```

```
      TRSZ=TRS+DZ*2.0
      A1(II,4,J)=-TRSZ
      A1(II,2,J)=A1(II,2,J)-TRSZ*ERO2
      DDT=AX6(IE,J)*CTHE2
      CC(II,J)=CZ(II,J)-CDT
       CZ(II,J)=CCT
      B1(II,J)=-CC(II,J)
      A1(II,3,J)=A1(II,3,J)+TRSZ
      AF7(IE,J)=AXL(II,J)-AX8(IE,J)
       AXL(II,J)=AX8(IE,J)
      AF1(IE,J)=0.0
      AF2(IE,J)=0.0
      AF3(IE,J)=0.0
      AF4(IE,J)=0.0
      AF5(IE,J)=0.0
      AF6(IE,J)=0.0
         IF(MC.EQ.2.AND.J.EQ.N)
     1                    WRITE(6,2) A1(II,2,J),A1(II,4,J),A1(II,3,J),B1(II,J
     1),CC(II,J),AF7(IE,J)                                                    280
 4100 CONTINUE
      IF(NR.EQ.1) GO TO 4102
      GO TO 4150
 4102 IS=I1
      NR=2
      MM=I2
      ERO2=0.
      DEL=DELZS
      DZ=0.5/DELZS
      GO TO 200
 4160 IF(VENT) GO TO 4190
      IF(KK.EQ.1) GO TO 4180
      IS=I2
      MM=M
      NR=3
      ERO2=0.
      DEL=DELZ
      DZ=0.5/DELZ
      GO TO 200
 4180 IE=I2
      ERO2=1.
      KK=2
      II=2
      GO TO 4090
 4150 IF(NR.EQ.2) GO TO 4160
 4190 MTSR=MD
      MD =MTSR
      CALL CIC1 (MC)                                                          319
         MC=0
         PPOUT=.FALSE.
       IF (KDIAG.NE.1)  GO TO 571
      IF(IND3.EQ.1.OR.IND3.EQ.2) PPOUT=.TRUE.
      IF(PPOUT) WRITE(6,9)                                                    327
  571 DO 575 I=1,M
      IF(PPOUT) WRITE(6,4)(PHI(I,J),J=1,N)                                    332
      DO 575 J=1,N
      KUPT(I,J)=0
```

```
      IF(PHI(I,J).GE.0.0) GO TO 575
      PHI(I,J)=0.0
      KUPT(I,J)=1
  575 CONTINUE
C     DIMENSIONLESS FLOW
      IP=I1-1
 4204 AFLOW=0.0
      DO 4200 J=1,N
      IF(J.EQ.1.OR.J.EQ.N) GO TO 4220
      YF1=(PHI(IP,J+1)-PHI(IP,J-1))*DTHE2
 4210 AFLOW=(YF1*AX6(IP,J))+AX8(IP,J)+AX7(IP,J)*(PHI(IP+1,J)-PHI(IP-1,J)
     1)*DZ+AFLOW
      IF(NDIAG.EQ.2) WRITE(6,2)YF1,AFLOW                                  364
      GO TO 4200
 4220 IF(J.EQ.N) GO TO 4230
      YF1=(PHI(IP,2)-PHI(IP,N))*DTHE2
      GO TO 4210
 4230 YF1=(PHI(IP,1)-PHI(IP,N-1))*DTHE2
      GO TO 4210
 4200 CONTINUE
      AFLOW=AFLOW*DTHETA/12.
C     TORQUE DIVIDED BY MUXNXRXRXRXR/C
      NR=1
      TORQ=.TRUE.
      TQ=0.0
      IT1=1
      IT2=I1
      TC1=0.0
      GO TO 200
 1001 DO 1000 I=IT1,IT2
      DO 1000 J=1,N
      IF(KUPT(I,J).EQ.1) GO TO 1000
      IF(I.EQ.IT1.OR.I.EQ.IT2) GO TO 1800
      DPDZ=(PHI(I+1,J)-PHI(I-1,J))/DZ
      AFTR=1.0
 1200 IF(J.EQ.1.OR.J.EQ.N) GO TO 1600
      DPDT=(PHI(I,J+1)-PHI(I,J-1))*DTHE2
 1240 H=H9(I,J)
      BQ=H*ALTAL1*DEPH
      AQ=(-BQ*COSB*WS13(I,J)-H*ALPH*WS12(I,J)-ALPH*WS11(I,J)*(H+DEPH))*
     1DPDT
      AQ=AQ+BQ*(-CONZ*WS14(I,J)-WS13(I,J)*DPDZ*SINB)
      TQ=TQ+AQ*DLZ*0.5*AFTR
      GO TO 1010
 1600 IF(J.EQ.N) GO TO 1610
      DPDT=(PHI(I,2)-PHI(I,N))*DTHE2
      GO TO 1240
 1610 DPDT=(PHI(I,1)-PHI(I,N-1))*DTHE2
      GO TO 1240
 1800 AFTR=.5
      IF(I.EQ.IT2) GO TO 1810
      DPDZ=(PHI(IT1+1,J)-PHI(IT1,J))/DELZ
      GO TO 1200
 1810 DPDZ=(PHI(IT2,J)-PHI(IT2-1,J))/DELZ
      GO TO 1200
 1010 RE=REN*H
```

```
      TC2=TCC(RE)+ALPH                                          392    432
      RE=REN*(H+DEPH)                                           393
      TC1=TC1+(TC2+TCC(RE)*ALPH)*TRQ*DLZ*AFTR                   394    433
        IF (J.NE.1) GO TO 1000                                  395
      IF(MC.EQ.2.AND.I.EQ.IT1)                                  396
    1                     WRITE(6,2)AQ,TQ,RE,TC1,TC2            397    437
 1000 CONTINUE                                                  398
      IF(MC.EQ.2)WRITE(6,2)WS11(I,J),WS12(I,J ),WS13(I,J),WS14(I,J)  399    442
      IF(NR.EQ.1) GO TO 1400                                    400
      IF(NR.EQ.2) GO TO 1410                                    401
      GO TO 578                                                 402
 1400 NR=2                                                      403
      IT1=I1                                                    404
      IT2=I2                                                    405
      GO TO 200                                                 406
 1410 IF(VENT) GO TO 578                                        407
      NR=3                                                      408
      IT1=I2                                                    409
      IT2=M                                                     410
      GO TO 200                                                 411
  578 TQO=(TQ+TC1)                                              412
      THE=0.0                                                   413
      DO 580 J=1,N                                              414
      QQQ(J)=SIN(THE)                                           415    468
      CQQQ(J)=COS(THE)                                          416    470
  580 THE=DTHETA+THE                                            417
      DO 590 I=1,M                                              418
      PP(I)=0.0                                                 419
      PPP(I)=0.0                                                420
      DO 600 J=1,N                                              421
      DUM=PHI(I,J)                                              422
      PP(I)=PP(I)+CCC(J)*DUM                                    423
  600 PPP(I)=PPP(I)+CQQQ(J)*DUM                                 424
      PP(I)=PP(I)*CTHETA                                        425
      PPP(I)=PPP(I)*CTHETA                                      426
      PPX(I)=PP(I)*XX(I)                                        427
  590 PPPX(I)=PPP(I)*XX(I)                                      428
      FSIN=SUM(PP,M,DELZ)                                       429    501
      FCOS=SUM(PPP,M,DELZ)                                      430    502
      FMSIN=SUM(PPX,M,DELZ)                                     431    503
      FMCOS=SUM(PPPX,M,DELZ)                                    432    504
      FCOS=-FCOS
      FMCOS=-FMCOS
      IF (KDIAG.NE.1)  GO TO 594                                433
      WRITE(6,6)FSIN,FCOS                                       434    507
      WRITE(6,7)FMSIN,FMCOS                                     435    508
  594 WLOAD=FCOS**2+FSIN**2                                     436
      WLOAD=ABS(WLOAD)                                          437
      WLOAD=SQRT(WLOAD)                                         438    510
      SOMER=2.00*ELVC/WLOAD
        PHEE=ATAN2(FSIN,FCOS)                                   440    511
          PHEE=PHEE/RADIAN                                      441
      TQO=TQO/WLOAD                                             445
      TQI=TQO+EPS*FSIN/WLOAD                                    446
      WRITE (6,2) REN,EPS,TQI,TQO,SOMER,PHEE,AFLOW              447    512
      IF(MPAS.EQ.1) GO TO 593                                   448
```

```
      WRITE(10) REN,EPS,TQI,TQO,SOMER,PHEE          449   515
      GO TC 591                                     450
  593 WRITE(9) REN,EPS,TQI,TQO,SOMER,PHEE           451   517
  591 IND3=IND3+KDIAG                               452
  592 CONTINUE                                      453
  599 SPRITO=SPRITO+CELSI                           454
      RETURN                                        455
      END                                           456
```

```
      SUBRCUTINE CIC1(MD)
      COMMCN MCIAC,CELX,CELZ,DELZS,M,MS,MG1,MG2,N,NP(17,19),A1(2,5,1
     19),B1(2,19),CC(2,19),AF1(17,19),AF2(17,19),AF3(17,19),AF4(17,19),A
     2F6(17,19),AF7(17,19),AF5(17,19),PHI(17,19)
      DIMENSICN A(17,17),B(17,17),C(17,17),D(17,17),F(20,17),AK(17,17),G
     1N(17,17),S(20,17),AFJI(17),BC(17,17),BF(17),DD(17,17),DS(17),G(17,
     217),GI(17,17),E(17,17)
      AN=1.0/CELX
        NC=0
      M=MS+MG1+MG2+1
        MN1=MG1+1
        MN2=MN1+MS
      DX1=AN*0.5
      DX2= AN**2
  203 DO 205 I=1,M
      DO 204 II=1,M
      E(I,II)=0.
  204 D(I,II)=0.
      F(1,1)=C.
  205 D(I,I)=1.
      DO 300 J=1,N
      WRITE(7)((E(I,II),D(I,II),I=1,M),II=1,M)                         23
       IF (MD.NE.2) GO TO 240
      WRITE (6,103) AF1(1,J),AF2(1,J),AF3(1,J),AF4(1,J),AF5(1,J),AF6(1,J
     1),AF7(1,J)                                                        34
      WRITE (6,103)
     1             A1(1,1,J),A1(1,2,J),A1(1,3,J),A1(1,4,J),A1(1,5,J),B1(MN1,
     1J),CC(MN1,J)                                                      42
         WRITE (6,103)
     1                  A1(2,1,J),A1(2,2,4),A1(2,3,J),A1(2,4,J),A1(2,5,J),
     1B1(MN2,J),CC(MN2,J)                                               50
  240 DZ1=0.5/(CELZ)
      DZ2=1./(CELZ)**2
  241 DO242 I=1,M
      DO 242 II=1,M
      B(I,II)=0.
      A(I,II)=0.
  242 C(I,II)=C.
      DO 250 I=1,M
  206 IF(I-MG1-1) 212,210,212
  209 AFJI(I)=-AF7(I,J)
      A(I,I)=1.
      GO TC 250
  210 IF(MS) 211,212,211
  211 DZ1=0.5/CELZS
      DZ2=1./(CELZS)**2
      IF(NP(I,J))209,232,209
  232 IF(MG1) 233,212,233
  233 IF(NC.EC.2) GC TO 600
      A(I,I-2)=A1(1,1,J)
      A(I,I-1)=A1(1,2,J)
      A(I,I)=A1(1,3,J)
      A(I,I+1)=A1(1,4,J)
      A(I,I+2)=A1(1,5,J)
      B(I,I)=B1(1,J)
```

```
      C(I,I)=CC(1,J)
  600 AFJI(I)=-AF7(I,J)
      GO TO 250
  212 IF(NP(I,J))209,234,209
  234 IF(I-MG1-MS-1) 215,213,215
  213 IF(MS) 214,215,214
  214 DZ1=0.5/DELZ
      DZ2=(1./DELZ)**2
      IF(NC.EQ.2) GO TO 601
      A(I,I-2)=A1(2,1,J)
      A(I,I-1)=A1(2,2,J)
      A(I,I)=A1(2,3,J)
      A(I,I+1)=A1(2,4,J)
      A(I,I+2)=A1(2,5,J)
      B(I,I)=B1(2,J)
      C(I,I)=CC(2,J)
  601 AFJI(I)=-AF7(I,J)
      GO TO 250
  215 IF(NC.EQ.2) GO TO 602
      B(I,I-1)=AF3(I,J)*DX1*DZ1
      B(I,I)=AF5(I,J)*DX2-AF4(I,J)*DX1
      B(I,I+1)=-AF3(I,J)*DX1*DZ1
      A(I,I-1)=AF1(I,J)*DZ2-AF2(I,J)*DZ1
      A(I,I)=AF6(I,J)-2.0*(AF1(I,J)*DZ2 +AF5(I,J)*DX2)
      A(I,I+1)=AF1(I,J)*DZ2+AF2(I,J)*DZ1
      C(I,I-1)=-AF3(I,J)*DX1*DZ1
      C(I,I)=AF4(I,J)*DX1+AF5(I,J)*DX2
      C(I,I+1)=AF3(I,J)*DX1*DZ1
  602 AFJI(I)=-AF7(I,J)
  250 CONTINUE
      IF(NC.EQ.2) GO TO 603
      DO 260 I=1,M
      DO 260 II=1,M
      AK(I,II)=A(I,II)
      DO 260 III=1,M
  260 AK(I,II)=AK(I,II)+B(I,III)*E(III,II)
      IF(MDIAG.LT.2) GO TO 262
      WRITE(6,101)J                                                188
      DO 261  I=1,M
  261 WRITE(6,100) (AK(I,II),II=1,M)                               191
  262 CALL MATINV(AK,M,DUM,0,DUM1)                                 198
  603 CONTINUE
      DO 404 I=1,M
      BF(I)=AFJI(I)
      F(J+1,I)=0.
      DO 404 II=1,M
      E(I,II)=0.
      BD(I,II)=0.
      DO 403 III=1,M
      E(I,II)=E(I,II)-AK(I,III)*C(III,II)
  403 BD(I,II)=BD(I,II)+B(I,III)*C(III,II)
  404 BF(I)=BF(I)-B(I,II)*F(J,II)
      DO 406 I=1,M
      DO 406 II=1,M
      D(I,II)=0.
      DO 405 III=1,M
```

```
ISN    - EFN    SOURCE STATEMENT    -  IFN(S)  -

405 D(I,II)=C(I,II)-AX(I,III)*BD(III,II)
405 F(J+1,I)=F(J+1,I)+AM(I,II)*BF(II)
300 CONTINUE
    DO 505 I=1,M
    DO 504 II=1,M
504 DD(I,II)=-D(I,II)
505 DD(I,I)=1.+DD(I,I)
    IF(MDIAG.LT.2) GO TO 264
    WRITE (6,101)                                            267
    DO 263 I=1,M
263 WRITE (6,100) (DD(I,II),II=1,M)                          271
264 CALL MATINV(DD,M,DUM,0,CUM1)                             279
    DO 507 I=1,M
    S(N,I)=0.
    DO 507 II=1,M
    GN(I,II)=0.
    DO 506 III=1,M
    GN(I,II)=GN(I,II)+DD(I,III)*E(III,II)
506 G(I,II)=GN(I,II)
507 S(N,I)=S(N,I)+DD(I,II)*F(N+1,II)
    J=N+1
    DO 512 K=2,N
    WRITE(8) ((G(I,II),I=1,M),II=1,M)                        309
    BACKSPACE 7                                              317
    READ(7)  ((E(I,II),D(I,II),I=1,M),II=1,M)                318
    BACKSPACE 7                                              327
    IF(MDIAG.EQ.2) WRITE(6,100) ((E(I,II),D(I,II),I=1,M),II=1,M)  328
    J=J-1
    DO 509 I=1,M
    S(J-1,I)=F(J,I)
    DO 509 II=1,M
    GI(I,II)=0.
    DO 508 III=1,M
    GI(I,II)=GI(I,II)+D(I,III)*GN(III,II)+E(I,III)*G(III,II)
    IF(MDIAG.NE.2) GO TO 508
508 CONTINUE
509 S(J-1,I)=S(J-1,I)+E(I,II)*S(J,II)+D(I,II)*S(N,II)
    DO 512 I=1,M
    DO 512 II=1,M
512 G(I,II)=GI(I,II)
280 DO 511 I=1,M
    DO 510 II=1,M
510 DD(I,II)=-GI(I,II)
511 DD(I,I)=1.+DD(I,I)
    IF(MDIAG.LT.2) GO TO 266
    WRITE (6,101)                                            396
    DO 265 I=1,M
265 WRITE (6,100) (DD(I,II),II=1,M)                          400
266 CALL MATINV(DD,M,DUM,0,CUM1)                             408
    IF(MDIAG.LT.2) GO TO 270
    I=0
    WRITE (6,101)I                                           413
    DO 273 I=1,M
273 WRITE (6,100) (DD(I,II),II=1,M)                          417
270 DO 515 I=1,M
    HI(I,1)=0
```

```
                                                                  03/04/69
       CICI.     - EFN   SOURCE STATEMENT  -  IFN(S)  -

           DO 515 II=1,M                                          169
       515 PHI(I,1)=DD(I,II)*S(1,II)+PHI(I,1)                      170
           DO 516 J=2,N                                           171
           BACK SPACE 8                                           172     441
           READ(8) ((G(I,II),I=1,M),II=1,M)                       173     442
           BACKSPACE 8                                            174     450
           DO 516 I=1,M                                           175
           PHI(I,J)=S(J,I)                                        176
           DO 516 II=1,M                                          177
       516 PHI(I,J)=PHI(I,J)+G(I,II)*PHI(II,1)                    178
           REWIND 7                                               179     465
           REWIND 8                                               180     466
                IF (PC.EQ.2)  GC TC 110                           181
           IF(MDIAG.LT.2) GO TO 268                               182
       110 WRITE (6,102)                                          183     473
           DO 267 I=1,M                                           184
       267 WRITE (6,100) (PHI(I,J),J=1,N)                         185     477
       268 RETURN                                                 186
       100 FORMAT(1X,1P10E11.4)                                   187
       101 FORMAT(5F0CIC1,I5)                                     188
       102 FORMAT(1H0,30X10HFINAL PHI /1H0)                       189
       1C3 FORMAT (1X,1P7E11.4)                                   190
           END                                                    191
```

```
      SUBROUTINE MATINV(A,N,B,M,DETER)
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
      DIMENSION IPIVO(17),A(17,17),B(17,1 ),INDEX(17,2 ),PIVOT(17)
      EQUIVALENCE (IROW,JROW), (ICOLU ,JCOLU ), (AMAX, T, SWAP)
C     INITIALIZATION
C
   10 DETER =1.0
   15 DO 20 J=1,N
   20 IPIVO (J)=0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVO (J)-1) 60, 105, 60
   60 DO 100 K=1,N
   70 IF (IPIVO (K)-1) 80, 100, 740
   80 IF (ABS (AMAX)-ABS (A(J,K))) 85, 100, 100
   85 IROW=J
   90 ICOLU =K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVO (ICOLU )=IPIVO (ICOLU )+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLU ) 140, 260, 140
  140 DETER =-DETER
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLU ,L)
  200 A(ICOLU ,L)=SWAP
  205 IF(M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLU ,L)
  250 B(ICOLU ,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLU
  310 PIVOT(I)=A(ICOLU ,ICOLU )
  320 DETER =DETER *PIVOT(I)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLU ,ICOLU )=1.0
  340 DO 350 L=1,N
  350 A(ICOLU ,L)=A(ICOLU ,L)/PIVOT(I)
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLU ,L)=B(ICOLU ,L)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
C
```

```
  380 DO 550 L1=1,N
  390 IF(L1-ICOLU ) 400, 550, 400
  400 T=A(L1,ICOLU )
  420 A(L1,ICOLU )=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLU ,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLU ,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLU =INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLU )
  700 A(K,JCOLU )=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
```

```
      FUNCTICN GTCF(REYN)
      RE=REYN
      IF(RE.LE.70.0)GO TO 20
      IF((RE.GT.70.C).ANC.(RE.LE.2C00.0)) GO TO 30
   10 IF((RE.GT.200C.0).AND.(RE.LE.5.5E+03))GO TO 40
      IF((RE.GT.550C.0).ANC.(RE.LE.2.0E404)) GO TO 5C
      GTCF=20.5/(RE)**0.784                                    14
      RETURN
   20 GTCF=1.0/12.0
      RETURN
   30 GTCF=.619E-08*(RE)**2-3.465E-05*RE+.08565
      RETURN
   40 GTCF=4.90/(RE)**.628                                     18
      RETURN
   50 GTCF=10.35/(RE)**.716                                    20
      RETURN
      END
```

03/04/69

```
GZCF.        -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      FUNCTION GZCF(REYN)                                          2
      RE=REYN                                                      3
      IF(RE.LE.70.0)GO TO 20                                       4
      IF((RE.GT.70.C).AND.(RE.LE.4000.0))GO TO 30                  5
   10 IF((RE.GT.4000.0).AND.(RE.LE.7.0E+03)) GO TO 4C              6
      IF((RE.GT.7000.0).AND.(RE.LE.2.0E+04)) GO TO 90              7
      GZCF=25.6/(RE)**.756                                         8    14
      RETURN                                                       9
   20 GZCF=1.0/12.0                                               10
      RETURN                                                      11
   30 GZCF=1.858 E-09*(WRE)**2-1.878E-05*RE+.0846                 12
      RETURN                                                      13
   40 GZCF=9.62/(RE)**.652                                        14    18
      RETURN                                                      15
   50 GZCF=11.3/(RE)**.674                                        16    20
      RETURN                                                      17
      END                                                         18
```

```
      FUNCTION SUM(F,M,DX)
      DIMENSION F(17)
      K=2
      KK=M-1
      KKK=2
      SUM=0.0
   10 DO 20 I=K,KK,KKK
   20 SUM=SUM+F(I)
      GO TO (30,40,50),K
   30 SUM=SUM*DX/3.0
      RETURN
   40 K=3
   45 SUM=SUM*2.0
      GO TO 10
   50 K=1
      KK=M
      KKK=M-1
      GO TO 45
      END
```

03/06/69

```
      TCC.          -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      FUNCTION TCC(RE)
      IF(RE.GT.100.0) GO TO 10
      TCC=8./RE
      GO TC 100
   10 IF(RE.GT.400.0) GO TO 20
      TCC=4.175/(RE)**.86                                          9
      GO TC 100
   20 IF(RE.GT.1000.0) GO TO 30
      TCC=.547/(RE)**.521                                          14
      GO TC 100
   30 IF(RE.GT.4J00.J) GO TC 40
      TCC=.342/(RE)**.453                                          19
      GO TC 100
   40 TCC=.064/(RE)**.25                                           22
  100 RETURN
      END
```

```
FLOATING-RING WITH HERRINGBONE JOURNAL
MAIN PROGRAM INPUT

$INPUT

INPRC  =           0,

NEPS   =           3,

NCASE  =           1,

NN     =          10,

R2R1   =      0.12000000E 01,

ANTI   =      0.25000000E-00,

ON     =      0.09999999E-00,

REY    =      0.09999999E 02,

EPIS   =      0.20000000E-00,    0.30000000E-00,    0.50000000E 00,    -0.00000000E-19,    -0.00000000E-19,
             -0.00000000E-19,   -0.00000000E-19,   -0.00000000E-19,    -0.00000000E-19,    -0.00000000E-19,

COR1   =      0.99999999E-03,

COR2   =      0.99999999E-03,

PRES   =      0.            ,    0.41999999E 01,   -0.

$ END
HERRINGBONE BEARING INPUT

$INPUT

RINSP  =      0.25000000E-00,

DELSP  =      0.09999999E-00,

BLOVD  =      0.09999999E 01,

I1     =           3,

I2     =           7,

N9     =          12,

EPS1   =      0.20000000E-00,    0.30000000E-00,    0.50000000E 00,

BETA   =      0.14900000E 03,    0.30599999E 02,    0.09999999E 01,

DEP    =      0.            ,    0.13999999E 01,    0.            ,

ALPHA  =     -0.            ,    0.35000000E 00,    0.            ,

EPS2   =      0.50000000E 00,    0.59999999E 00,    0.80000000E 00,

$ END
```

```
                  INNER FILM CATA
REYNCLDS  NO.  ECCENTRICITY INNER TORQUE OUTER TORQUE   SOMMERFELD NO ATT. ANGLE   FLOW
0.5500000E 01 0.2000000E-00-0.1273566E 02-0.1285375E 02 0.2129888E 01 0.3618759E 02-0.7218548E 01
0.5500000E 01 0.3000000E-00-0.8305490E 01-0.8483987E 01 0.1369939E 01 0.3651181E 02-0.7565410E 01
0.5500000E 01 0.5000000E 00-0.4678258E 01-0.4985645E 01 0.7334086E 00 0.3793541E 02-0.8679913E 01
0.6500000E 01 0.2000000E-00-0.1537007E 02-0.1548261E 02 0.1993225E 01 0.3424303E 02-0.6979188E 01
0.6500000E 01 0.3000000E-00-0.1001766E 02-0.1018751E 02 0.1279345E 01 0.3448471E 02-0.7299742E 01
0.6500000E 01 0.5000000E 00-0.5639432E 01-0.5930835E 01 0.6808451E 00 0.3564804E 02-0.8330470E 01
0.7500000E 01 0.2000000E-00-0.1795031E 02-0.1805662E 02 0.1847429E 01 0.3211124E 02-0.6772657E 01
0.7500000E 01 0.3000000E-00-0.1168741E 02-0.1184764E 02 0.1183250E 01 0.3228307E 02-0.7066473E 01
0.7500000E 01 0.5000000E 00-0.6565635E 01-0.6839612E 01 0.6259595E 00 0.3322680E 02-0.8012116E 01
                  CUTER FILM CATA
REYNCLDS  NO.  ECCENTRICITY INNER TORQUE OUTER TORQUE   SOMMERFELD NO ATT. ANGLE   FLOW
0.3600000E 01 0.5000000E 00-0.6034926E 01-0.6116465E 01 0.3480383E-00 0.9385604E 01-0.2802687E 02
0.3600000E 01 0.6000000E 00-0.5024480E 01-0.5134692E 01 0.2722008E-00 0.1058461E 02-0.3048566E 02
0.3600000E 01 0.8000000E 00-0.4138659E 01-0.4349845E 01 0.1781508E-00 0.1530646E 02-0.3678925E 02
0.5040000E 01 0.5000000E 00-0.7139547E 01-0.7250334E 01 0.4060655E-00 0.1280149E 02-0.2057270E 02
0.5040000E 01 0.6000000E 00-0.5880299E 01-0.6025070E 01 0.3147292E-00 0.1396234E 02-0.2231984E 02
0.5040000E 01 0.8000000E 00-0.4703731E 01-0.4960753E 01 0.2008538E-00 0.1874024E 02-0.2681073E 02
0.6480000E 01 0.5000000E 00-0.7911438E 01-0.8042833E 01 0.4465474E-00 0.1523563E 02-0.1643149E 02
0.6480000E 01 0.6000000E 00-0.6468842E 01-0.6637570E 01 0.3439290E-00 0.1633265E 02-0.1778327E 02
0.6480000E 01 0.8000000E 00-0.5074363E 01-0.5361702E 01 0.2157208E-00 0.2104944E 02-0.2126711E 02
CUTPUT
ECCENTR/C1      N2/N1       C2/C1        R2/R1
 0.6001E 00   0.3941E-00   0.1200E 01   0.1200E 01
             REYNCLDS NO ECCENTRIC      TORQUE    SUPPLY PRES  SOMMFD  NO  ATT. ANGLE TANG. FORCE
INNER RING     0.6059E 01  0.2000E-00 -0.1427E 02   0.3013E 01   0.2055E 01   0.3512E 02  0.2800E-00
OUTER RING     0.5676E 01  0.3411E-00 -0.1201E 02   0.1066E 02   0.6970E 00   0.1390E 02  0.5694E 00
  CVERALL      0.1000E 02  0.2728E-00 -0.1421E 02   0.4200E 01   0.1474E 01   0.2083E 02
CUTPUT
ECCENTR/C1      N2/N1       C2/C1        R2/R1
 0.8641E 00   0.3947E-00   0.1200E 01   0.1200E 01
             REYNCLDS NO ECCENTRIC      TORQUE    SUPPLY PRES  SOMMFD  NO  ATT. ANGLE TANG. FORCE
INNER RING     0.6053E 01  0.3000E-00 -0.9344E 01   0.3011E 01   0.1321E 01   0.3541E 02  0.4388E-00
CUTER RING     0.5684E 01  0.4817E-00 -0.7917E 01   0.1064E 02   0.4485E-00   0.1389E 02  0.5425E 00
  CVERALL      0.1000E 02  0.3928E-00 -0.9257E 01   0.4200E 01   0.9468E 00   0.2121E 02
CUTPUT
ECCENTR/C1      N2/N1       C2/C1        R2/R1
 0.1404E 01   0.3597E-00   0.1200E 01   0.1200E 01
             REYNCLDS NO ECCENTRIC      TORQUE    SUPPLY PRES  SOMMFD  NO  ATT. ANGLE TANG. FORCE
INNER RING     0.6403E 01  0.5000E 00 -0.5694E 01   0.3069E 01   0.6861E 00   0.3588E 02  0.8542E 00
OUTER RING     0.5180E 01  0.7664E 00 -0.4987E 01   0.1148E 02   0.2178E-00   0.1798E 02  0.1432E 01
  CVERALL      0.1000E 02  0.6382E 00 -0.5548E 01   0.4200E 01   0.5046E 00   0.2426E 02
```

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13. ABSTRACT

In this volume is presented an analysis of the static and dynamic characteristics of the spiral-grooved journal bearing operating with incompressible lubricant in both laminar and turbulent regimes. Both single film and floating ring bearing configurations are considered. Extensived design data are presented giving load capacity, attitude angle, bearing torque, bearing flow rate, stiffness and damping coefficients and critical rotor mass for limit of stable operation. In addition, two computer programs accompany the volume, and instructions and listings of the programs are included. These programs may be used to obtain data for cases not covered by the presented design data.

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