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The Effects of Irregularity Scale on Usable Antenna Aperature

by

L. A. Manning

July 1968

TECHNICAL REPORT NO. 141

Prepared under
Office of Naval Research Contract
Nonr 225(64), NR 088 019, and
Advanced Research Projects Agency ARPA Order 196

RADIOSCIENCE LABORATORY
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ABSTRACT

It is assumed that a wave is incident on a broadside antenna array after passing through a propagation medium containing refractive-index fluctuations having a Gaussian correlation function. The influence of the irregularities on usable array directivity and effective power gain is explored as a function of the magnitude and scale of the refractive-index fluctuations, and of the extent and position of the scattering region. The domains of Fresnel diffraction (geometrical optics) and Fraunhofer diffraction (wave theory) are distinguished, and within these domains the nature of the array performance is categorized and explored for relative parameter classes.

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I. INTRODUCTION

The purpose of this study is to investigate the influence of irregularities in the ionosphere on the usable directivity, aperture and effective power gain of an antenna array. This influence is to be studied as a function of the magnitude and scale of fluctuations in the refractive index of the ionosphere, and also of the extent and position of the scattering region.

Irregularities in the ionosphere cause a propagated radio wave to be scattered, with a resultant random deviation or broadening of the beam transmitted by an isolated mode. If a signal is received from a point source, the source position (as estimated at the receiving location) will be uncertain, and in addition the source may appear diffuse. There is a limit to the amount of antenna directivity that is usable under these conditions. If the aperture of the receiving antenna is too wide, the signal may a) drift out of the beam as the irregularities move, b) be wider than the beam so that increased gain and/or resolution is not achieved, or c) be weakened by inclusion of uncorrelated signal components in different parts of the aperture. Which of these possibilities occurs, and so determines the size of the usable aperture, depends on a number of factors, including the relations between irregularity scales (sizes), the Fresnel zone length, the phase deviation caused by the irregularities, and the aperture dimensions. The range of magnitude within which each of these factors falls determines the signal behavior. Hence the total number of distinct forms of behavior is large. Kaydanovskiy and Smirnova (1965) have published a partial consideration of the problem that is valid in limiting cases, with primary emphasis being placed on radio-astronomical applications. In the present study the classes of cases investigated are more extensive, and the nature of the signal behavior is investigated in more detail. In order to avoid dealing with the exact theory, which is very complex, the signal behavior is explored within parameter regions where a clear-cut performance description may be sought, and no attention is given to behavior in transitional regions. Thus only order-of-magnitude calculations are made. It is assumed also, for clarity, that the irregularities have only one scale of Gaussianly distributed autocorrelation function. The behavior of the signal in the presence of two or more simultaneous irregularity scales is not considered in the present report, which is only intended to clarify the performance factors applicable for a given antenna aperture and for ionosphere irregularities of a given but simple description.

II. ANALYSIS

Preliminaries

Since a determination of the effect of ionospheric scatter on usable antenna aperture is one of the principal purposes of this study, it has been assumed, as shown in Fig. 1, that energy from a transmitter T is refracted in an ionospheric region of refractive index μ, and returned to a receiver R. Because of the existence of refractive-index irregularities having a mean-square deviation from the average index of $\overline{(\Delta\mu)^2}$, the energy will suffer phase and amplitude perturbations, as well as alterations in the direction of propagation. For the purposes of the

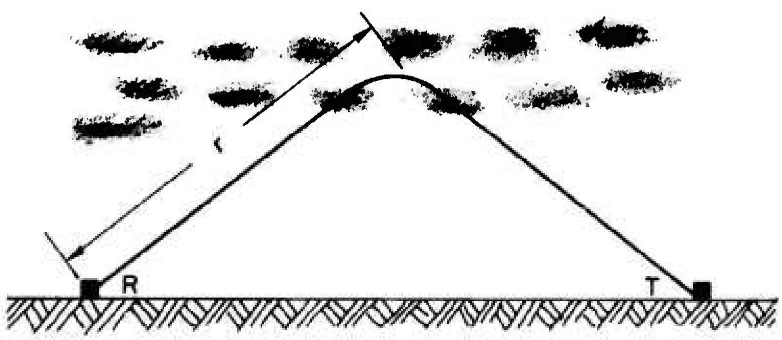


Fig. 1. GEOMETRY OF AN IONOSPHERIC PATH.

present classification of cases, the refractive effects caused by the smooth variation of refractive index μ will be disregarded, and only the perturbations caused by the irregularities will be considered. Figure 2 shows the geometry corresponding to this simplification. It will be assumed that the transmitter and receiver are both at the same

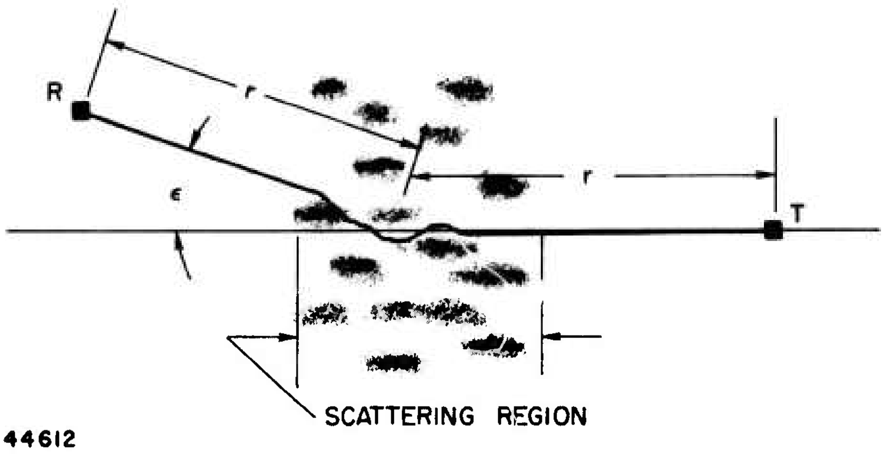


Fig. 2. RAY PATH WITH FIRST-ORDER REFRACTIVE EFFECTS REMOVED. Only deviations due to scattering irregularities are retained.

distance r from the scattering region. In those cases in which it is correct to speak of the deviation in direction of a ray by refraction in the ionosphere irregularities, the total angle of refractive perturbation will be denoted by the symbol ϵ. The refractive index fluctuations will produce a phase shift along a ray path which will be called φ.

The effects of ionosphere irregularities, in fact, are not likely to be describable by a single scale relationship. However, it will be assumed at this stage that the spatial autocorrelation function, ρ, of the refractive index caused by the irregularities is

$$\rho = e^{-\frac{x^2}{\Lambda^2}} , \qquad (1)$$

where x is a linear spatial coordinate, and Λ determines the mean dimensions of the irregularities. The mean-square magnitude of the fluctuations, described by $\overline{(\Delta\mu)^2}$, together with the scale Λ, will determine φ and ϵ.

It will be assumed that the receiving antenna has a rectangular aperture which is aligned broadside to the received ray direction, and which has the dimension w_x times w_y. The half-width of the antenna beam will be taken to be $\alpha = \lambda/w_x$ and $\beta = \lambda/w_y$ radians. The scattering region will be assumed to have length L, volume V, and thus a cross-sectional area $S = V/L$. The quantity $F = \sqrt{r\lambda}$, with λ the wavelength, will be referred to as the Fresnel length; it is within a factor of $\sqrt{2}$ of being the length of the first Fresnel zone at the scattering region.

Depending on the size of F relative to Λ, we will distinguish two cases, as follows:

Case I: Near zone, Fresnel scatter.

Case II: Far zone, Fraunhofer scatter.

Case I--Near Zone, Fresnel Scatter

The behavior of energy received after traveling through the scattering region depends markedly on whether the irregularity scale Λ is less than or greater than the Fresnel distance F. When $\Lambda \gg F$, the scatter is of Fresnel type, and the methods of geometrical optics are applicable. In contrast, when $\Lambda \ll F$, the scatter is of Fraunhofer type, and wave theory is needed for a solution. This second case will be referred to as Case II, to be discussed later.

When ray theory is applicable--i.e., in Case I--the mean-square phase deviation $\overline{\varphi^2}$ caused by refractive-index perturbations is

$$\overline{\varphi^2} = \sqrt{\pi}\ \overline{(\Delta\mu)^2}\ k^2\ \Lambda L \quad , \tag{2}$$

where $k = 2\pi/\lambda$, $\Delta\mu$ is the refractive index fluctuation, Λ is the irregularity scale in the ray direction, and L is the length of the scattering region (Chernov, 1960). The mean-square angular ray-directional perturbation $\overline{\epsilon^2}$ is

$$\overline{\epsilon^2} = \frac{1}{\pi^2}\left(\frac{\lambda}{\Lambda}\right)^2 \overline{\varphi^2} \quad , \tag{3}$$

where Λ is now the transverse irregularity scale.

If the deflected rays are to fall within the receiving beam, it is necessary that the beam widths α and β in the x and y directions be greater than $\sqrt{\overline{\epsilon_x^2}}$ and $\sqrt{\overline{\epsilon_y^2}}$, where x and y are rectangular coordinate axes aligned with the rectangular aperture of the receiving antenna. That is, the rays will fall within the beam provided that, with $\alpha = \lambda/w_x$ and $\beta = \lambda/w_y$,

$$w_x \ll \frac{\pi\Lambda_x}{\sqrt{\overline{\varphi^2}}} \qquad \text{and} \qquad w_y \ll \frac{\pi\Lambda_y}{\sqrt{\overline{\varphi^2}}} \quad . \tag{4}$$

With these landmarks, we may now distinguish several subcases depending on the magnitude of the phase perturbations $\sqrt{\overline{\varphi^2}}$ which occur when the ray leaves the scattering volume V.

System of Notation: Subcases

A system of literal subscripts and superscripts will be used to distinguish the various cases, and will be explained throughout the text.

The format used will be I_{abc}^{df} or II_{ab}^{def} for the Fresnel and Fraunhofer cases, respectively. The meaning of each letter subscript or superscript will be given as the cases are developed. These meanings are recapitulated in Table 1, Section III. Each of the letters a through f may appear in either lower case or capital form, thus distinguishing the relative value of a particular parameter with respect to a transition value.

<u>Cases I_c and I_C</u>

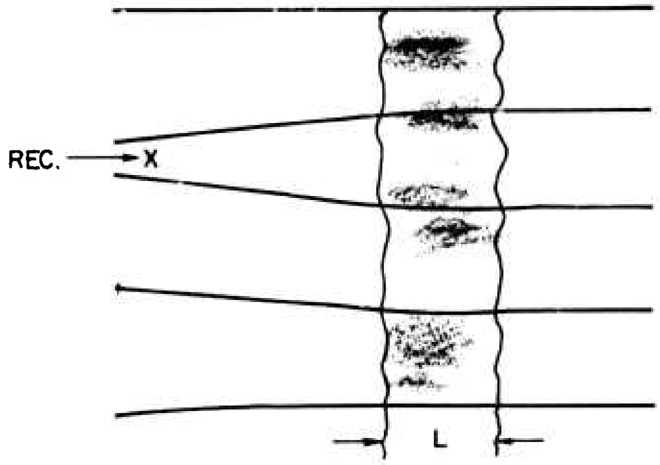
With respect to the magnitude of the phase perturbations $\sqrt{\overline{\varphi^2}}$, distinction will be made with respect to the subscript c as follows:

$$\text{Case } I_c: \quad \overline{\varphi^2} \ll 16\left(\frac{\Lambda}{F}\right)^2 \gg 16,$$

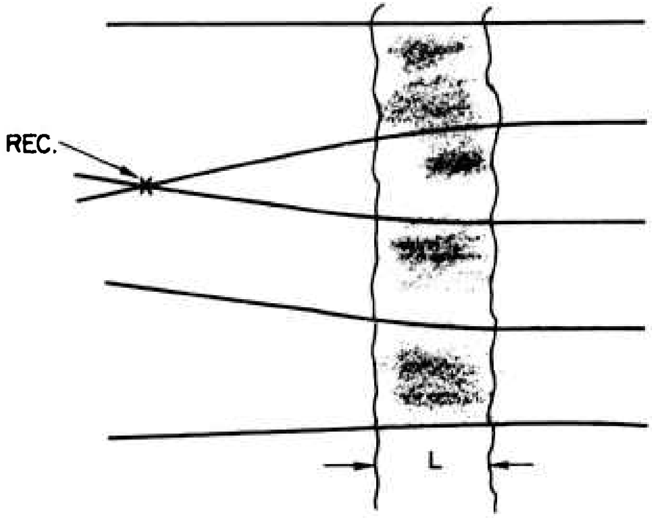
$$\text{Case } I_C: \quad \overline{\varphi^2} \gg 16\left(\frac{\Lambda}{F}\right)^2 \gg 16, \tag{5}$$

where the right-hand inequality is characteristic of Fresnel scatter, Case I rather than Case II. [When (later on) we treat Case II, the relation of Λ to F will not be of importance, since $(\Lambda/F) \ll 1$, and the condition $\overline{\varphi^2} \ll 1$ (subscript a) will make the use of the c subscript unnecessary.]

In Case I_c, only a single ray will reach a given receiving point. In Case I_C, N rays will reach a single receiving point. Figure 3 shows the difference in ray behavior in these two cases. In Case I_C the ray deviation is great enough so that the ray paths cross, while in Case I_c



CASE I_c



CASE I_C

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Fig. 3. IN CASE I_c THE RAYS DO NOT CROSS UNTIL AFTER PASSING THE LOCATION OF THE RECEIVER; IN CASE I_C CROSSING DOES OCCUR.

they do not cross at positions within the distance r from the scattering region (F is a function of r). To distinguish between these cases, as in inequalities (5), it is necessary to investigate the deviation of rays statistically, and so to obtain an estimate of the expected number of rays received at a single point. This calculation is made in the Appendix, with the following result: If the scattering region is one-dimensional, as for cylindrical or trough-like scatterers, the expected number of rays at a receiving point P will be

$$N_1 \simeq 1 + \frac{\sqrt{2}}{\pi^{3/2}} \frac{F}{\Lambda} \sqrt{\overline{\varphi^2}} \quad , \tag{6}$$

where Λ is the scale across the irregularities. If the scattering is two-dimensional, and the irregularities have the same scale in all directions perpendicular to the ray direction, the expected number of rays at each receiving point will be $N_2 = N_1^2$. Since $\pi^{3/2}/2^{1/2} \cong 4$, Eq. (6) leads to the distinction between cases contained in inequalities (5).

<u>Cases I_{ac} and I_{Ac}</u>

Now we shall examine Case I_c, and find a still further distinction, to be defined as that between Case I_{ac} for which $\overline{\varphi^2} \ll 1$, and Case I_{Ac} for which $\overline{\varphi^2} \gg 1$ though still less than $16(\Lambda/F)^2$. The characteristic feature of Case I_{ac}, the small phase perturbation $\overline{\varphi^2} \ll 1$, is denoted by the lower-case subscript a. This criterion insures that rays reaching all parts of the receiving aperture will have no significant phase differences other than those that may result from deviations in ray directions. An angular deviation of ϵ radians sustained over a portion of the aperture of width h will cause a phase deviation across the aperture of $2\pi \epsilon h/\lambda$ radians. Taking the width h across the aperture containing correlated deviations to be of order Λ, the transverse irregularity correlation scale, the phase deviations are either $2\pi \epsilon \Lambda/\lambda$, or $2\pi \epsilon (w_x \text{ or } w_y)/\lambda$, depending on whether the aperture is greater or less than the irregularity scale.

If

$$w_x \text{ and } w_y \ll \Lambda \ll \frac{\pi\Lambda}{\sqrt{\overline{\overline{\varphi^2}}}} , \tag{7}$$

the aperture is small in both dimensions. The inequality I_c in (5) insures that $N = 1$, so that only a single ray will appear at any point in the aperture. That is, a cone of rays traveling through different blobs (irregularities) will not arrive at a single receiving point. Since w_x and $w_y \ll \Lambda$, all rays in the aperture pass through the same blobs, and so exhibit the same lateral deviation. The width of the aperture is not enough to extend laterally to noncrossing rays that have passed through transversely uncorrelated irregularities. The inequality w_x and $w_y \ll \pi\Lambda/\sqrt{\overline{\overline{\varphi^2}}}$ implies that the beam widths λ/w_x and λ/w_y are large compared with the ray deviation given by Eq. (3); the phase difference $2\pi \epsilon (w_x \text{ or } w_y)/\lambda$ across the aperture due to ray deviation, equal to $2(w_x \text{ or } w_y)\sqrt{\overline{\overline{\varphi^2}}}/\Lambda$, is less than one. Hence only a single, slightly deviated ray is received in the array aperture by a wide beam, and thus the full power gain of the beam is achieved. This gain is proportional to the product $w_x w_y$, and so in this case, gain and directivity may advantageously be increased by use of larger aperture dimensions.

Cases I^d_{abc}, I^D_{abc} and I^D_{aBc}

The condition $w \ll \Lambda / \sqrt{\overline{\varphi^2}}$ will be denoted by the subscript b, and its failure by the subscript B. The condition $w \ll \Lambda$ will be described by the superscript d, and when $w \gg \Lambda$, the superscript will be D. Thus Eq. (7) refers to Case I^d_{abc}.

If

$$\Lambda \ll (w_x \text{ and/or } w_y) \ll \frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} , \tag{8}$$

which will be referred to as Case I^D_{abc}, the situation differs from that for Eq. (7) in that in one or both dimensions of the beam, the aperture exceeds the irregularity transverse correlation distance Λ. Since inequality I_c of (5) is still satisfied, only one ray reaches each point of the aperture. Because the phase deviation $\sqrt{\overline{\varphi^2}}$ due to the irregularities is less than one, Case I_a, the only significant phase deviations in the aperture are those due to angular ray deviation. Since $(w_x$ and/or $w_y)$ $\gg \Lambda$, Case I^D, the phase deviations in the aperture due to ray deviation are of order $2\pi\, \epsilon\, \Lambda/\lambda$, which reduces to $2\sqrt{\overline{\varphi^2}} \ll 1$. Hence all elements of the aperture are excited in phase, and the full aperture power gain, proportional to $w_x w_y$, is achieved. The rays in different parts of the aperture, from different blobs, do not have the same angular deviation, so that the received energy is in effect in the form of a diffuse cone. However, the beam directivity remains insufficient to resolve the ray deviations. The inequality $(w_x$ and/or $w_y) \ll \pi\Lambda / \sqrt{\overline{\varphi^2}}$, Case I_b, implies that the angular beam width is greater than the ray dimension ϵ, and so full directivity is achieved. An increase in aperture dimension results

If

$$\Lambda \ll \frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll w_x \text{ and/or } w_y \quad , \tag{9}$$

to be referred to as Case I^D_{aBc}, one or both dimensions of the aperture are now great enough to resolve the angular ray deviation, provided that the full aperture is able to function coherently. There are again rays in different parts of the aperture which have been deflected to differing extents by different blobs; but by inequalities (5) there is still only one ray at each point in the aperture. As in the previous case, the phase deviation $\sqrt{\overline{\varphi^2}}$ in the aperture caused by the differences in phase paths through different blobs is negligible, since $\sqrt{\overline{\varphi^2}} \ll 1$, Case I_a. Also, as in the case of inequalities (8), the phase deviation in the aperture due to angular deviation is restricted to $2\pi\ \epsilon\ \Lambda/\lambda$ because of the limited aperture width over which the same deviations ϵ are observed. Hence the phase deviations in the aperture elements due to angular deviation are restricted to the order $2\pi\ \epsilon\ \Lambda/\lambda = 2\varphi \ll 1$, and are negligible. Hence all rays contribute to the aperture in phase with one another, and the full power gain, proportional to $w_x w_y$, is achieved.

In the last case considered, the rays in the aperture in effect collectively constitute a bundle of width $\sqrt{\overline{\epsilon^2}}$. The beam directivity is sharper than the width of the bundle, and is achieved despite the bundle width. If the beam is turned away from the mean ray direction by an angle greater than $(w_x \text{ or } w_y)/\lambda$, but less than $\sqrt{\overline{\epsilon^2}}$, the geometrically induced phase deviations reduce the array output. Because of the limited correlation distance of phase changes associated with individual ray angular deviations, these phase deviations are restricted to small values. Hence, to the aperture, the incoming rays are indistinguishable from a plane wave, even though the aperture is wide enough to resolve an angular deviation of a plane wave smaller than that achieved by the individual rays that enter parts of the aperture. For a single incoming deflected ray the effective aperture is Λ, and the ray is accepted. However, if the beam is slewed, the aperture is w_x or w_y, and the directivity is realized. Note that the source direction as inferred by the full aperture is determined as the normal to the approximately plane wavefront, rather than as the mean ray deviation. Thus the precision of the mean increases with aperture dimension, not with its square root.

It may be noted that notational Case I^d_{aB} is impossible.

Case I_{Ac}

If the refractive index fluctuations are greater than in Case I_{ac}, destructive interference occurs between array element outputs excited by rays passing through uncorrelated blobs. However, the behavior will depend on whether more than one ray reaches a single receiving point. In Case I_{Ac} it is still assumed that $\overline{\varphi^2}$ meets the requirement of Case I_c, (5), and rays do not cross at the receiver; N = 1. Case I_{Ac} is distinguished by the condition

$$1 \ll \overline{\varphi^2} \ll 16 \frac{\Lambda^2}{F^2} \gg 16 \quad . \tag{10}$$

Note that the subscript A implies $\overline{\varphi^2} \ll 1$; subscript c implies the second inequality in (10); and Case I is implied by the third inequality.

Cases I^d_{Abc} and I^d_{ABc}

Depending on the aperture dimensions, several subcases may now be distinguished. If

$$w_x \text{ and } w_y \ll \frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll \Lambda \quad , \tag{11}$$

subscript b and superscript d ($w \ll \Lambda$) apply; the aperture is small compared with the correlation distance Λ in both aperture dimensions. The received rays all have substantially the same angular deviation, and have received substantially the same phase shift in the irregularities. The wavefront in the aperture will be turned through the angle ϵ, but the directivity of the array is not adequate to resolve the incoming deviations. Thus the full directivity of the beam is maintained, as is the power gain, which is proportional to the aperture area ab.

If one or both aperture dimensions are large enough to resolve the ray deviation angle ϵ, but are within the correlation distance Λ, the condition is

$$\frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll \left(w_x \text{ and/or } w_y\right) \ll \Lambda \quad , \tag{12}$$

and the second subscript becomes B, with superscript d as before. Since the aperture dimensions are less than the correlation distance Λ, the rays in the aperture still have substantially the same angular deviation. However, in one or both directions the beam directivity is excessive, and if the beam remains directed along the unperturbed ray path, the deflected rays will drop in and out of the beam acceptance angle as the irregularities move and change with time. As a result, deep fading will occur. The signal duty cycle will be, roughly, $\pi^2 \Lambda_x \Lambda_y \Big/ \left(\overline{\varphi^2} w_x w_y\right)$ for a two-dimensional aperture, or $\pi\Lambda_x \Big/ \sqrt{\overline{\varphi^2}}\, w_x$ for a one-dimensional aperture when Λ_x and Λ_y are the transverse scales in the directions of aperture dimensions w_x and w_y. Thus at any given moment the maximum usable aperture to avoid signal dropout with a fixed beam is

$\pi\Lambda/\sqrt{\overline{\varphi^2}}$, where Λ is the scale factor parallel to the aperture dimension. Within this size [the condition b shown by (11)], full directivity is effective, and the beam power gain proportional to $w_x w_y$ is achieved. Wider apertures maintain the power gain only at the peak of the fading cycle.

Case I_{AB}^{D}

If one or both aperture dimensions are greater than the correlation distance Λ, the condition is D:

$$\frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll \Lambda \ll w_x \text{ and/or } w_y \quad . \tag{13}$$

This is Case I_{AB}^{D}. Now a two-dimensional aperture may be looked on as being filled by cells of width and height Λ, so that there will be of the order of $n = w_x w_y/\Lambda^2$ cells, each receiving rays of internally correlated angular deviation ϵ. Each of these cells may be looked on as a randomly phased element making up the whole array. Since the dimension Λ of each cell is greater than the aperture $\pi\Lambda/\sqrt{\overline{\varphi^2}}$ above which signal dropout occurs, the signals in the cells will individually fade with duty cycles of $\left(\pi^2/\overline{\varphi^2}\right) < 1$. The signals in each cell pass through uncorrelated irregularities, and so are randomly phased. Hence the average power gain of the array will be proportional to the product $w_x w_y$ (which is proportional to the gain of an inphase array), divided by the number of random cells, $n = w_x w_y/\Lambda^2$, times the average cell duty factor $\pi^2/\overline{\varphi^2}$. Hence the average power gain of the array is proportional to $\left(\pi^2\Lambda^2/\overline{\varphi^2}\right) < \Lambda^2$, and is independent of the array area when w_x and $w_y > \Lambda$, but is in inverse proportion to the square of the mean strength of the irregularities. The directivity is limited to that of a single cell, since the cells form a randomly phased array. The directivity is not increased by increasing the size of the array. This directivity is sharper than the mean ray deviation. In the presence of many cells, the fading will be Rayleigh, and sharp dropouts will not occur. As the beam is slewed through the signal, a rough cone of width ϵ will be perceived. Because of the finite number of cells, the variation of apparent signal strength with azimuth will be ragged, and the directivity of the cells cannot be fully realized. If the aperture $w_x w_y$ is greatly increased, the directional scan becomes smoother, so that the mean direction can be determined to a precision corresponding more nearly to the cell directivity $\lambda/(w_x \text{ or } w_y)$ than to $\sqrt{\overline{\epsilon^2}}$. Thus an increase of the aperture dimension beyond the correlation distance does not, in this case, increase either beam power gain or usable directivity, but does result in a less ragged fading pattern, and a smoother scan.

If the aperture is greater than Λ in only one dimension, no increase of gain or directivity is achieved in this direction. Moreover, if the aperture dimension is between $\pi\Lambda/\sqrt{\overline{\varphi^2}}$ and Λ in the other direction, sharp signal dropouts will still occur. If the smaller aperture dimension w is less than $\pi\Lambda/\sqrt{\overline{\varphi^2}}$, the average beam power gain will be $\left(w\Lambda\pi/\sqrt{\overline{\varphi^2}}\right) < w\Lambda$.

<u>Case I_C</u>

The difference between Case I_c and Case I_C is that in the latter case the phase deviation is great enough so that rays can arrive at a single receiving point simultaneously after having been deviated by uncorrelated irregularities. The distinguishing criterion is

$$16 \ll 16 \frac{\Lambda^2}{F^2} \ll \overline{\varphi^2} \quad , \tag{14}$$

which implies subscripts I_{AC}, since the second (C) inequality implies $\overline{\varphi^2} \gg 1$, which is condition A. The expected number of rays received at each point of the aperture will now be $N = N_1$ as given by Eq. (6) for the case of one-dimensional perturbations, and $N = N_2 = N_1 N_1'$ for two-dimensional perturbations, where N_1 and N_1' are the scales Λ measured in orthogonal transverse directions. Both N_1 and N_2 are greater than one.

Cases I^d_{AbC} and I^d_{ABC}

Consider now the situation in which the aperture is small compared with the aperture width that is capable of resolving the ray deviations. That is,

$$w_x \text{ and } w_y \ll \frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll \Lambda \quad , \tag{15}$$

Case I^d_{AbC}.

Since the aperture is small compared to the correlation distance, Case I^d, the phase is uniform across the aperture, and the aperture maintains its power gain and directivity. However, the directivity is insufficient to resolve the angular deviation ϵ, (Case I_b), and so the aperture may profitably be enlarged. At each point of the aperture there is an average of N randomly phased rays which have passed through separate blobs with large random phase deviations. In each of these rays, however, the average power density is proportional to $1/N$, so that the received power gain is independent of ray crossings, and remains proportional to the aperture area $w_x w_y$. The crossed rays will interfere, however, so that there will be Rayleigh fading of the signal as the irregularity positions and deviations change.

If

$$\frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll w_x w_y \ll \Lambda \quad , \tag{16}$$

(Case I^d_{ABC}), the aperture is still small compared to the correlation distance Λ. Hence the same phase-front perturbations exist across the whole aperture because of the effect of the irregularities. As in the case b where the aperture is less than $\pi\Lambda/\sqrt{\overline{\varphi^2}}$, the N rays incident on each point of the aperture do not affect the average received power, but do cause Rayleigh fading. Now, however, the aperture is large enough in one or both dimensions to resolve the ray deviations. Hence when a ray deviation is greater than the beam width, the ray will drop out. However, in the presence of N rays, the fading will be Rayleigh. The signal duty cycle for $N = 1$ would be $\pi^2\Lambda_x\Lambda_y/\overline{\varphi^2}\ w_x w_y$ if both dimensions of the beam were more directive than the incoming ray perturbations, and $\pi\Lambda_y/\sqrt{\overline{\varphi^2}}\ w_y$ if only the dimension w_y is larger. Since N rays with different random deviations arrive at each point, the probability of each ray being present is the same as the $N = 1$ duty cycle, and so is the average power. The average beam power gain is proportional to $w_x w_y$ reduced by the above duty factor, and so is $\left(\pi^2\Lambda_x\Lambda_y/\overline{\varphi^2}\right) \ll \Lambda_x\Lambda_y$

for the two-dimensional beam, and $\left(w\pi\Lambda_y/\sqrt{\overline{\varphi^2}}\right) \ll w\Lambda \ll \Lambda$ for the one-dimensional beam, where w is the smaller aperture dimension, $w < \left(\pi\Lambda/\sqrt{\overline{\varphi^2}}\right)$.

<u>Case I^D_{ABC}</u>

If

$$\frac{\pi\Lambda}{\sqrt{\overline{\varphi^2}}} \ll \Lambda \ll w_x w_y \quad , \tag{17}$$

(Case I^D_{ABC}), the aperture is larger than the correlation distance. There are now $n = w_x w_y/\Lambda^2$ cells within the aperture such that within each cell the ray arrival angles and phases are well correlated. Within each aperture cell there are N rays reaching the same receiving points after traveling through different blobs with different angular deviations. The average aperture power is not affected by the ray crossings, since each ray has an independent phase, but they cause the signals within the cells to fade. Since each blob cell receives a randomly phased signal, the calculation of beam gain is the same as for Case I_{Ac}. Because of out-of-phase contributions from the n apertures, the plane wave power gain proportional to area $w_x w_y$ is reduced to $w_x w_y/n = w_x w_y/(w_x w_y/\Lambda^2) = \Lambda^2$ independent of aperture size. (Note that the captured power increases with aperture, but the definition of power gain is affected also by radiation resistance change.) Since Λ is greater than $\pi\Lambda/\sqrt{\overline{\varphi^2}}$ (Case A), the directivity of each aperture cell is so high that only the least deviated incoming rays fall within the beam. For the two-dimensional array the fraction of the incoming rays that is received is $\alpha\beta/\epsilon^2 = \lambda^2/(\Lambda^2\epsilon^2) = (\pi^2/\varphi^2) \ll 1$. Since the rays in each cell are randomly phased, the received power is proportional to the number admitted. Hence the effective antenna power gain is proportional to $\left(\pi^2\Lambda^2/\overline{\varphi^2}\right) \ll \Lambda^2$ in the case of the two-dimensional array with both w_x and w_y greater than the correlation distance Λ. The apparent beam width on sweeping the beam will be controlled by $\sqrt{\overline{\epsilon^2}}$, and will be a ragged pattern unless the number of cells is large, and/or the number of crossing rays in each cell is high.

For an array with dimension w_x greater than Λ, and w_y between $\pi\Lambda/\sqrt{\overline{\varphi^2}}$ and Λ, the aperture will be partitioned into cells only in the x dimension. The effect of cells will be to reduce the plane-wave gain from a proportionality to $w_x w_y$, to a proportionality to $w_x w_y/n$, where $n = w_x/\Lambda$. Thus because of cells alone the gain would correspond to $w_y\Lambda$. The fraction of the rays received is approximately

$$\alpha\beta/\epsilon^2 = \frac{\lambda}{\Lambda\epsilon} \cdot \frac{\lambda}{w_y\epsilon} = \frac{\pi^2\Lambda}{w_y\overline{\varphi^2}} ,$$

so the effective aperture gain is $\pi^2\Lambda^2/\overline{\varphi^2}$ just as when both w_x and w_y are greater than Λ. If w_y is less than $\pi\Lambda/\sqrt{\overline{\varphi^2}}$, the fraction of rays in the beam is $\alpha/\epsilon = \lambda/\Lambda\epsilon = \pi/\sqrt{\overline{\varphi^2}}$, and the effective power gain is $\pi w_y\Lambda/\sqrt{\overline{\varphi^2}}$, and can be increased by increasing the aperture size in its smaller dimension.

<u>Case II--Far Zone, Fraunhofer Scatter</u>

When the Fresnel distance $F = \sqrt{\lambda r}$ is large compared with the correlation distance Λ, so that

$$\lambda \ll \Lambda \ll F, \tag{18}$$

the receiver is in the far zone of the scatterers, and the scatter is of Fraunhofer rather than Fresnel type. Geometrical optics no longer applies, and wave theory is needed. In treating Case II, we shall continue to assume a Gaussian correlation function of the phase irregularities.

<u>Case II_a</u>

If the phase perturbations are small we will distinguish Case II_a as:

$$\overline{\varphi^2} \ll 1 \quad . \tag{19}$$

Scattering in this case is described by the Fejer scatter formula (Fejer, 1953); it is similar to the Booker scatter formula, except that Booker assumed an exponential correlation function. In this case, the signal consists of a specular component plus scattered components. Assuming the wave incident on the scattering region to be plane, the specular component consists of a continuation of this plane wave through and beyond the scattering region. When $\overline{\varphi^2} \ll 1$, the specular component suffers a negligible decrease in amplitude on passing through the scattering region. The scattered component is distributed in a lobe centered on the specular direction. The width of the scattered beam depends only on the ratio of the irregularity scale to wavelength, Λ/λ, and in terms of angular deviation ϵ from the specular direction, the scattered power is proportional to

$$e^{-\left(\frac{2\pi\Lambda}{\lambda} \sin \frac{\epsilon}{2}\right)^2} \quad . \tag{20}$$

Thus $\sqrt{\overline{\epsilon^2}} = \lambda/\pi\sqrt{2}\,\Lambda$. The intensity of the scattered power may be described by the ratio of the on-axis scattered power to the specular power, p_s/p_o:

$$\frac{p_s}{p_o} = \frac{Vk^4 \Lambda^3 \overline{(\Delta\mu)^2}}{4\sqrt{\pi}\, r^2} \quad , \tag{21}$$

where V is the scattering volume, $k = 2\pi/\lambda$, Λ is the correlation distance of the irregularities, $\overline{\Delta\mu^2}$ is the mean-square refractive index deviation in the irregularities, and r is the distance from the irregularity volume to the receiving antenna. The phase perturbation may be related to the refractive-index perturbations by the equation

$$\overline{\varphi^2} = \frac{\sqrt{\pi}}{2} \overline{(\Delta\mu)^2}\, k^2 \Lambda L \quad , \tag{22}$$

where L is the length of the scattering region in the direction of wave propagation. [Note that Eq. (22) differs from Eq. (2) by a factor of 2; Eq. (2) applies to ray propagation, Eq. (22) to wave propagation.] Hence, in terms of the phase perturbations, the cross section of the scattering region S, the Fresnel length F, and the irregularity scale Λ, the on-axis ratio of scattered to specular power is

$$\frac{p_s}{p_o} = 2\pi \frac{S}{F^2} \frac{\Lambda^2}{F^2} \overline{\varphi^2} \quad . \tag{23}$$

Note that for Case II, $\Lambda \ll F$, and for Case II_a, $\overline{\varphi^2} \ll 1$. The effective scattering cross section S, unless reduced by sharp antenna beam width, is determined by the scattering angle. Then $S \cong r^2 \overline{\epsilon^2} = r^2\lambda^2/2\pi^2\Lambda^2$, or $S = F^4/2\pi^2\Lambda^2$. Putting in this value for S gives an approximate upper limit for p_s/p_o (valid when $\overline{\varphi^2} \ll 1$, and the beams admit all of the scattered rays) of

$$\frac{P_s}{P_o} \cong \frac{\overline{\varphi^2}}{\pi} \quad . \tag{24}$$

The scattered power is thus much less than the specular power, which is necessary if the specular power is to be unreduced by the scattered energy.

Cases II_{ab}^{de} and II_{ab}^{De}

Now consider the effect of the dimensions of the receiving aperture. Suppose first that

$$(w_x \text{ and } w_y) \ll \pi\sqrt{2}\,\Lambda \ll F \quad , \tag{25}$$

Case II_{ab}^{de}, where d implies $w \ll \Lambda$, and e implies $w \ll F$. In this case, $\alpha = \lambda/w_x$ and $\beta = \lambda/w_y$ are greater than $\sqrt{\overline{\epsilon^2}} = \lambda/\pi\sqrt{2}\,\Lambda$, and all of the scatter components are in the beam, as is the specular component. The scattered power, given by Eq. (24), is far less than the specular power. The beam gain is determined by the specular component, and is fully maintained, proportional to the product $w_x w_y$. The full directivity of the aperture is achieved.

If one or both aperture dimensions are greater than $\pi\sqrt{2}\,\Lambda$, case D, the beam width is less than the width of the scattered cone $\sqrt{\overline{\epsilon^2}}$, and the condition is

$$\pi\sqrt{2}\,\Lambda \ll (w_x \text{ and/or } w_y) \ll F \quad , \tag{26}$$

which is Case II_{ab}^{De}.

Both in the Fresnel and the Fraunhofer cases (Cases I and II), the transverse field correlation at the aperture is of the same order of magnitude as the correlation distance Λ of the refractive index fluctuations. Thus condition (26) implies that the aperture width exceeds the correlation distance Λ for the scattered component (case D).

On the specular component the full beam gain is realized. The scattered field is superposed on the aperture distribution of the specular component, and is correlated with cells of approximate width Λ. The phase of the scattered component is random from cell to cell, and the total signal due to the random component is randomly phased with respect to that due to the specular component. (Since $\overline{\varphi^2} \ll 1$, these phase differences are purely of geometrical origin. Points in the aperture a distance Λ apart, illuminated by a single point in the scattering region, fall within the scattering angle $\epsilon = \lambda/\sqrt{2}\pi\Lambda$, and are illuminated with unit phase difference.) Since the specular and scatter signal voltages are uncorrelated, their power gains and directivities may be computed separately. Since the aperture for scatter consists of coherent cells of dimension Λ, the maximum directivity for scatter is of order λ/Λ, comparable to the rms scatter angle $\lambda/\pi\sqrt{2}\,\Lambda$. (The factor $\pi\sqrt{2}$ is of order one, and arises from differences in definition of antenna and scatter beam widths.) Thus as the aperture is increased beyond roughly the correlation distance (case D), the effective scattering cross section S is not reduced, but the full beam gain is not realized for scatter because of a nonuniform aperture distribution. Thus the scattered intensity is reduced below the level of Eq. (24) by the factor $\Lambda^2/w_x w_y$, equal to the reciprocal of the number of aperture cells. (An equivalent result could have been obtained by reciprocity, considering the full aperture directivity to be used in transmitting, and only the central scatterers of the cross section S illuminated.) Since the undeviated specular component is dominant, the full beam gain, proportional to $w_x w_y$, is obtained. The directivity corresponding to the specular component might not necessarily be realized, since the smaller scattered signal of less precisely known azimuth adds to the specular signal vector randomly, and causes a broadening of the scanned specular lobe. The phase fluctuation across the aperture is roughly the square root of the relative scatter power, and so is $\theta = \Lambda\varphi/w\sqrt{\pi}$ (assuming a square aperture). The resultant tilt of the effective wave front is $(\theta/k)/w$, where θ/k is the phase irregularity expressed as a distance, reduced by $1/n$ where n is the number of phase-correlated cells. With $n = w^2/\Lambda^2$, the slope becomes

$$\frac{\theta}{kwn} = \frac{\theta\lambda}{2\pi\ wn} = \frac{\Lambda\ \varphi\ \lambda\ \Lambda^2}{w\sqrt{\pi}\ 2\pi w \cdot w^2} = \frac{\varphi\ \lambda\ \Lambda^3}{2\pi^{3/2}\ w^4} .$$

The angular beam width will be realized if the above quantity is less than λ/w, which requires

$$\varphi < 2\pi^{3/2}\frac{w^3}{\Lambda^3} ;$$

and since it is assumed that $w > \Lambda$, and $\varphi < 1$ (Case II_a^d), the full beam directivity will be realized on the specular component when the aperture is wider than the correlation distance Λ. (In the case $w < \Lambda$ treated previously, the directivity is also maintained despite perturbation of the specular component by the scatter component, since in that case, the beam width is greater than the width of the scatter cone.)

Case II^E

If the aperture dimensions exceed the Fresnel distance F, problems arise in defining the dimensions of the beam. For a sufficiently wide aperture, the beam divergence as determined from $\alpha = \lambda/w_x$ and $\beta = \lambda/w_y$ can be very small. However, the beam does not originate in a point source, but in an aperture of dimensions w_x and w_y greater than F. So unless the beam is convergently focused, it will not illuminate a scattering cross section smaller than $S = F^2$. If it is convergently focused to the scattering region, it will be out of focus for the specular component. If a very large aperture is focused on the source of the specular component, at twice the distance r to the scattering region, the illuminated cross section of the scattering region cannot be less than $S = w_x w_y/4$. That is, $S > (w_x w_y/4) > (F^2/4)$. Thus it is difficult to reduce the dimensions of the scattering region much below F, and the practical minimum value of scattered power for a large aperture, using Eq. (23), is about

$$\frac{P_s}{P_o} \cong \frac{\Lambda^2}{F^2} \overline{\varphi^2} \ll 1 \quad . \qquad (27)$$

It is still true that the full aperture can be utilized on the specular component both with respect to directivity and effective power gain, proportional to $w_x w_y$.

Case II$_A$

When $\overline{\varphi^2} \gg 1$ (and recall that $\lambda \ll \Lambda \ll F$ in all Case II), we get the condition for Case II$_A$. In this case the scattered power is comparable to or greater than the specular power. When $\overline{\varphi^2} \ll 1$, the scatter component is weak, and the scattered lobe width depends on scale Λ, but not on irregularity density. However, as the perturbations deepen, the increased power in the scatter component is obtained at the expense of the specular component, and the specular component begins to broaden and lose amplitude. Full beam gain is no longer achieved on the specular component, nor can unlimited directivity be used. When the phase perturbations are of order unity, the specular component no longer exists as such, and only a scatter component exists, of unit intensity relative to an unscattered plane wave, since the scatter is essentially in the forward direction, and of width of order $\sqrt{\overline{\epsilon^2}} = \lambda/\pi\sqrt{2}\,\Lambda$. With further increase in the refractive index perturbations, the scattered beam becomes increasingly broad, and eventually the scatter becomes isotropic. In this limit the scattered intensity to the receiver approaches zero, since half the incident wave is turned around and scattered out of the irregularity region on penetrating a distance sufficient to cause isotropic scattering; half of the remaining wave in the second section of the scattering region is returned to the first scattering region, and half of it travels through and out. The net effect is to make the scattering region (in the absence of loss) become attenuating and so a reflecting region in the limit. Because of multiple scattering, any effect of loss would be considerably augmented.

When $\overline{\varphi^2}$ is only slightly greater than one, the scattered wave has an angle $\sqrt{\overline{\epsilon^2}}$ of order $\lambda/\pi\sqrt{2}\,\Lambda$, and advantage with respect to directivity and power gain are achieved by using apertures w_x and w_y roughly as large as the correlation scale Λ. If larger apertures are used (case D), the increased resolution only defines the shape of the scattered cone. The effective beam gain does not go up with large apertures, since a smaller fraction of the scattering energy is received when the beam width is less than the angular size of the scattering cone. Thus when $\overline{\varphi^2}$ is near one, there is no advantage to apertures wider than the correlation distance of the irregularities, of order Λ.

When $\overline{\varphi^2} \gg 1$, the scattering lobe first widens, and then when approaching the isotropic, starts to lose energy in the total scattered beam. The energy scattered in the forward direction is a decreasing fraction of the total scattered energy as the beam widens prior to reaching the isotropic condition, so that there is a continual decrease of forward scattered axial signal strength as the strength of the phase fluctuations rises above unity. Under these circumstances the smallest aperture giving maximum angular discrimination and effective power gain is less than the correlation scale Λ. If a wider aperture is used, the beam power gain will be the same as that for a smaller aperture matching the irregularity scattering angle, while the beam will be sharper than the received energy cone.

III. REVIEW OF PARAMETER CLASSIFICATION

In the preceding sections, the extent to which antenna gain and directivity can be realized has been considered for a variety of parameter combinations. The rectangular aperture dimensions w_x and w_y have been related to φ, the phase deviation caused by the irregularities in the scattering medium; to Λ, the scale of the irregularities; and to F, the Fresnel distance. To distinguish the antenna behavior in the various parameter domains, a classification of cases was used according to the following notation: If the scale Λ is much greater than the Fresnel distance $F = \sqrt{\lambda r}$, with r the distance from the receiver to the scattering region and λ the wavelength, the scattering is describable by geometrical optics, and is called Case I. If $\Lambda \ll F$, wave theory must be used, and we speak of Case II. Further classification is made using subscripts and superscripts, in the format I_{abc}^{df} or II_{ab}^{def}. Each subscript or superscript is a binary digit represented by either a lower case or capital letter, and distinguishes the relative value of a parameter with respect to a critical value.

The subscripts a, b, and c, and the superscripts d, e, and f are defined by the inequalities in Table 1. The meaning of the inequalities can also be presented graphically. In Fig. 4 the coordinates are aperture width divided by irregularity scale versus the rms phase deviation. Each inequality divides the coordinate plane of Fig. 4 in a unique way. When all of the relevant inequalities are used to distinguish categories of propagation behavior, the diagrams of Figs. 5 and 6 result, corresponding to Cases I and II. The notation suggested by these figures has been used throughout the text.

TABLE 1
NOTATION

NOTATION	INEQUALITY	SIGNIFICANCE	NOTATION	INEQUALITY	SIGNIFICANCE
Case I	$\frac{\Lambda}{F} \gg 1$	Geometrical optics	Case II	$\frac{\Lambda}{F} \ll 1$	Wave theory
a	$\varphi_{rms} \ll 1$	Small phase deviation	A	$\varphi_{rms} \gg 1$	Large phase deviation
b	$w \ll \frac{\Lambda}{\varphi_{rms}}$	For Case I, beam broader than scatter	B	$w \gg \frac{\Lambda}{\varphi_{rms}}$	For Case I, beam sharper than scatter
c	$\varphi_{rms} \ll \frac{\Lambda}{F}$	For Case I, no crossing rays	C	$\varphi_{rms} \gg \frac{\Lambda}{F}$	For Case I, crossing rays
d	$w \ll \Lambda$	Field correlated in aperture	D	$w \gg \Lambda$	Field uncorrelated across aperture
e	$w \ll F$	Scatter region in focus	E	$w \gg F$	Scatter region out of focus
f	No intrusion of region DA into region Da.		F	Intrusions of region DA into region Da.	

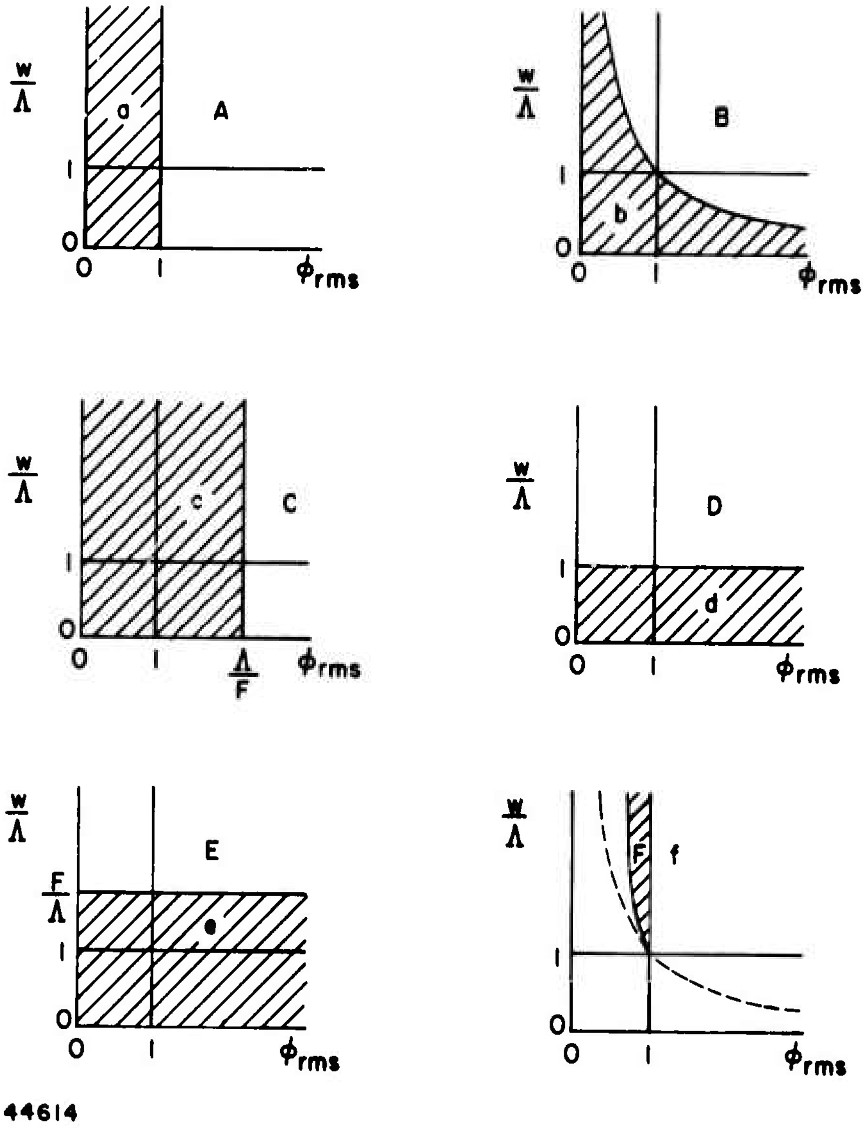

Fig. 4. GEOMETRICAL INTERPRETATION OF THE SUBSCRIPT NOTATION. The ordinates are aperture scale normalized by irregularity scale, and the abscissae are rms phase deviation.

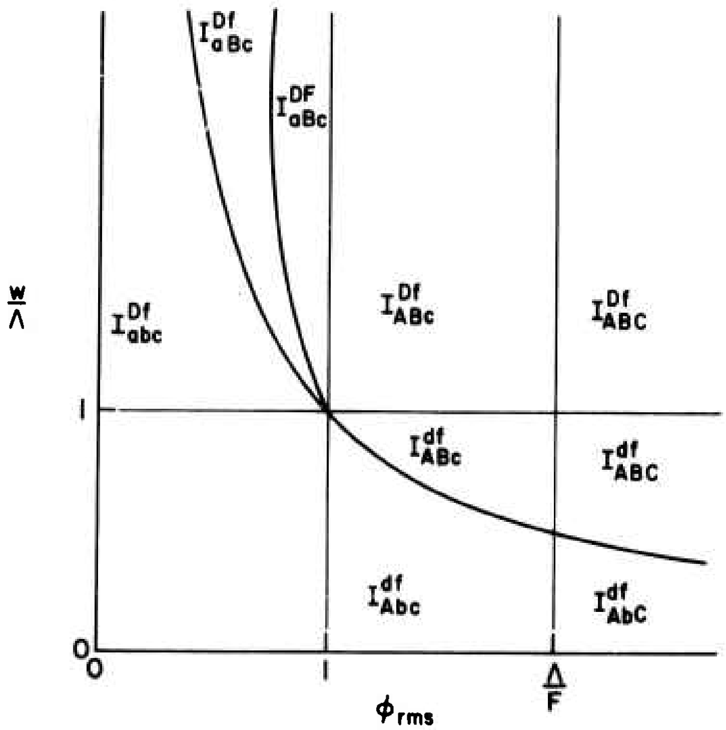

Fig. 5. THE NOTATION FOR CASE I.

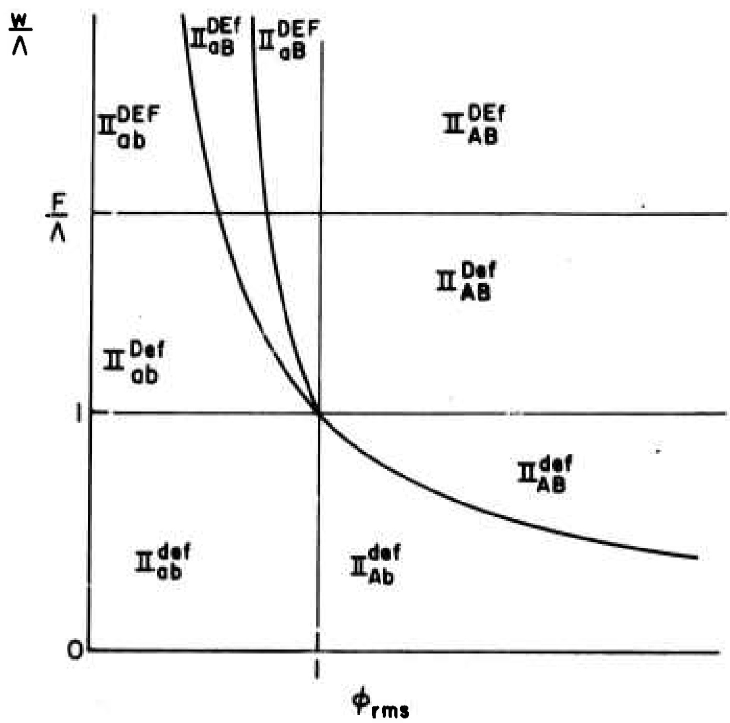

Fig. 6. THE NOTATION FOR CASE II.

IV. SUMMARY AND CONCLUSIONS

It has been shown that, by considering the results of each possible case, the optimum aperture to provide maximum beam power gain and directional accuracy can be found. First, if the irregularity scale is greater than the Fresnel distance $F = \sqrt{\lambda r}$, with r the distance from the receiver to the scattering region, and λ the wavelength, the scattering is describable by geometrical optics (Case I). If the rms phase perturbations in the medium are of order unity or less (Case I_a), full gain and directivity are achieved for any aperture size. If the phase perturbations are large, however $\left(\overline{\varphi^2} \gg 1, \text{ Case } I_A\right)$, full gain and directivity are maintained for apertures of dimension less than the order $\pi\Lambda/\sqrt{\overline{\varphi^2}}$ (Case I_{Ab}), where Λ is the irregularity scale, and φ is the phase perturbation. For larger apertures (Case I_{AB}) there will be either deep signal dropouts if $\overline{\varphi^2} \ll 16\Lambda^2/F^2$ (Case I_{ABc}) or Rayleigh fading and reduced gain and unrealized directivity if $\overline{\varphi^2} \gg 16\Lambda^2/F^2$ (Case I_{ABC}). If the aperture exceeds the correlation scale (Case I^D), gain and directivity are still unrealized, but the fading is Rayleigh even for $\overline{\varphi^2} \ll 16\Lambda^2/F^2$ (Case I^D_{ABc}).

When the irregularity scale Λ is smaller than the Fresnel distance F, wave theory must be used in place of geometrical optics (Case II). If the phase irregularities are small, $\overline{\varphi^2} \ll 1$ (Case II_a), the scatter is of Fejer type (for Gaussian irregularities), and consists of a scatter component of angular width determined by the irregularity scale, and of a specular component unreduced by scatter. As long as $\overline{\varphi^2} \ll 1$, full aperture gain and directivity are realized on the specular component.

However, as $\overline{\varphi^2}$ approaches one, the specular component is absorbed into the scatter component, and the maximum usable aperture shrinks to a value of the order of the irregularity scale. As the irregularities deepen (Case II_A), the scattered energy widens, and then weakens and vanishes for very deep irregularities. As the irregularities deepen beyond $\overline{\varphi^2} \cong 1$, the usable aperture decreases too, and in the limit is that aperture which fully illuminates the complete scattering region. (If the scattering region is a plane slab, the limiting aperture is zero.)

If the irregularities of more than one Gaussianly distributed scale are present in the scattering region simultaneously, the largest usable aperture will be no greater than that for the most limiting scale.

APPENDIX

THE EXPECTED NUMBER N OF RAYS AT A SINGLE RECEIVING POINT P

In Fig. 3 of the body of the report it was shown that separate rays may meet at a single receiving point if their angular deviation is sufficiently great. In Fig. 7 the geometry is shown in greater detail.

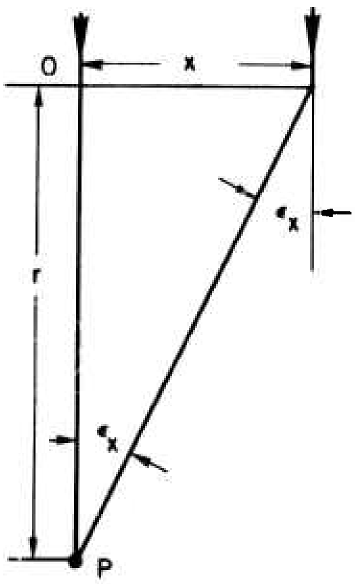

Fig. 7. THE PATH OF A DEVIATED RAY AFTER IT LEAVES THE REFRACTING REGION.

Let a ray passing through a plane scattering region be displaced by a distance x from the undeviated ray that is perpendicular to the plane and passes through the point P. This ray must be deviated by an angle $\epsilon_x = x/r$ for it to pass through the point P. Suppose that the x-component of angular deviation imparted to rays by the irregularities is plotted versus x as the irregular line in Fig. 8. Then if in Fig. 8 we also draw the straight line $\epsilon_x = x/r$, those rays for which the exit point x in the scattering plane is determined by the intersection between the straight and irregular curves in the figure will have just the

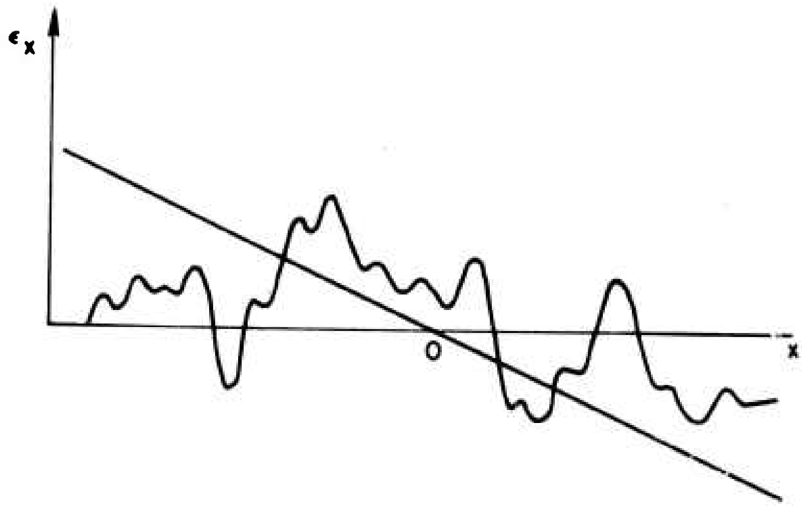

Fig. 8. LOCATION OF RAYS DEVIATED SUFFICIENTLY TO PASS THROUGH A GIVEN POINT.

correct x-component of angular deviation to pass through P. The calculations necessary to estimate the expected number of such intersections are similar to those made by Manning (1965) in treating the probable number of specular reflecting glints for a radar ray reflected from a distorted meteor trail.

Consider first a one-dimensional scattering region. Within a length dx the probable number of intersections in the geometry of Fig. 8 is

$$dN_1 = \frac{1}{\pi}\sqrt{\frac{m_2}{m_o}}\, e^{-\frac{x^2}{2r^2 m_o}}\, dx + \frac{1}{\sqrt{2\pi m_o}}\, e^{-\frac{x^2}{2r^2 m_o}}\, \frac{dx}{r} ,$$

where $m_o = \overline{\epsilon_x^2}$ is the mean-square angular deviation in the x direction, and it is assumed that the distribution function of the deviations is normal; $m_2 = \overline{\left(\frac{d\epsilon_x}{dx}\right)^2}$. The expected total number of intersections will be

$$N_1 = \int_{-\infty}^{\infty} \frac{dN_1}{dx}\, dx \quad .$$

The second term of the expression for dN_1 gives the probability of intersection of $\epsilon_x = x/r$ with the single-valued deviation curve, and so integrates to one. The first term gives the probability of intersection due to the horizontal component of x/r. By comparison with the second term, it can be seen to integrate to $r\sqrt{2m_2/\pi}$. The expected number of rays passing through the receiving point P in the one-dimensional case is then

$$N_1 = 1 + r\sqrt{\frac{2}{\pi}}\sqrt{m_2} \quad .$$

Hence

$$m_o = \overline{\epsilon_x^2} = \int_{-\infty}^{\infty} E(K)dK$$

and

$$m_2 = \overline{\left(\frac{d\epsilon_x}{dx}\right)^2} = \int_{-\infty}^{\infty} K^2 E(K)dK$$

may be identified as the zero and second-order moments of the spectral power density E(K) of the deviations $\epsilon_x(x)$, where $K = 2\pi/\Lambda$ is the spectral wave number. The power spectrum E(K) of $\epsilon_x(x)$ equals the Fourier transform of the autocorrelation function $\epsilon_x(x)$. The correlation of $\epsilon_x(x)$ is the same as that of the irregularity distribution, and has been taken to be Gaussian. Thus

$$\text{Correlation of } \epsilon_x = e^{-\frac{x^2}{\Lambda^2}}$$

and

$$m_2 = m_o \frac{m_2}{m_o} = \overline{\epsilon_x^2} \frac{\int_{-\infty}^{\infty} K^2 E(K)\,dK}{\int_{-\infty}^{\infty} E(K)\,dK} = \overline{\epsilon_x^2} \frac{2}{\Lambda^2} .$$

Hence in the one-dimensional case,

$$N_1 = 1 + \sqrt{\frac{2}{\pi}} \frac{\sqrt{2}\, r \sqrt{\overline{\epsilon_x^2}}}{\Lambda} .$$

From Eq. (3), $\overline{\epsilon^2} = \frac{1}{\pi^2} \left(\frac{\lambda}{\Lambda}\right)^2 \overline{\varphi^2}$, so that for one-dimensional scatter in the x direction,

$$\overline{\epsilon_x^2} = \frac{\epsilon^2}{2} = \frac{1}{\pi^2} \left(\frac{\lambda}{\Lambda_x}\right)^2 \overline{\varphi^2} ,$$

and with $F^2 = r\lambda$,

$$N_1 = 1 + \frac{\sqrt{2}}{\pi^{3/2}} \frac{F}{\Lambda} \sqrt{\overline{\varphi^2}}$$

and the constant $\sqrt{2}/\pi^{3/2} = 0.254$. To an order of magni[illegible], then, in the one-dimensional scatter case,

$$N_1 = 1 \text{ if } \overline{\varphi^2} \ll 16 \frac{\Lambda^2}{F^2} \quad : \quad \text{Case } I_c ,$$

and

$$N_1 \gg 1 \text{ if } \overline{\varphi^2} \gg 16 \frac{\Lambda^2}{F^2} \quad : \quad \text{Case } I_C .$$

If the scatterers are Gaussian in two dimensions, the number of rays refracted to a point P will be called N_2. If the cross section of the scatter volume has coordinates x and y, the condition of the one-dimensional case that $\epsilon_x = x/r$ must also apply simultaneously in the y dimension; $\epsilon_y = y/r$. If the x condition is satisfied at N_1 values of x, at each such x there will be N_1 values of y which will satisfy the condition. Therefore $N_2 = N_1^2$. Hence

$$N_2 = \left[1 + \frac{\sqrt{2}}{\pi^{3/2}} \frac{F}{\Lambda} \sqrt{\overline{\varphi^2}}\right]^2 ,$$

and the same conditions separate Cases I_c and I_C as in the one-dimensional case.

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13. ABSTRACT

It is assumed that a wave is incident on a broadside antenna array after passing through a propagation medium containing refractive-index fluctuations having a Gaussian correlation function. The influence of the irregularities on usable array directivity and effective power gain is explored as a function of the magnitude and scale of the refractive-index fluctuations, and of the extent and position of the scattering region. The domains of Fresnel diffraction (geometrical optics) and Fraunhofer diffraction (wave theory) are distinguished, and within these domains the nature of the array performance is categorized and explored for relative parameter classes.

14. KEY WORDS	LINK A		LINK B		LINK C	
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