

UNCLASSIFIED

AD NUMBER
AD809825
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; OCT 1966. Other requests shall be referred to Air Force Materials Lab., Wright-Patterson AFB, OH 45433.
AUTHORITY
USAFML ltr, 29 Mar 1972

THIS PAGE IS UNCLASSIFIED

809825

A NOTE ON TENSILE TESTING AT HIGH STRAIN RATES

T. NICHOLAS

TECHNICAL REPORT AFML-TR-66-341

OCTOBER 1966

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

**AIR FORCE MATERIALS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO**

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

Copies of this report should not be returned to the Research and Technology Division unless return is required by security considerations, contractual obligations, or notice on a specific document.

A NOTE ON TENSILE TESTING AT HIGH STRAIN RATES

T. NICHOLAS

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Metals and Ceramics Division (MAM), Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division, under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals." The research work was conducted in the AF Materials Laboratory, Research and Technology Division, Wright-Patterson Air Force Base, Ohio, by Dr. T. Nicholas of AFML.

This report covers work performed from July 1966 to August 1966.

The manuscript was released by the author August 1966, for publication as an RTD Technical Report.

This technical report has been reviewed and is approved.



W. J. TRAPP
Strength and Dynamics Branch
Metals and Ceramics Division
Air Force Materials Laboratory

ABSTRACT

The response of an elastic uniaxial tension specimen to a constant velocity applied at one end is presented. The application to the problem of testing specimens with high rate testing machines where the velocity of the ram approaches the dilatational wave velocity of the material is discussed. The concept of high strain rates is shown to break down under high loading rates.

INTRODUCTION

The last several years have seen an increased interest in the properties of materials at high rates of strain. These properties have been investigated by various techniques including wave propagation and impact studies as well as by the conventional uniaxial tension test. The advent of high speed testing machines has made it possible to conduct tests at relatively high rates of strain, in some cases approaching the dilatational wave velocity of the material for some soft materials.

It is the purpose of this note to show the theoretical response of an elastic material to a constant cross-head velocity applied at one end of a uniaxial tension specimen.

THEORY

Consider a prismatic bar of length l which is fixed at one end as shown in Fig. 1. The other end is given a uniform velocity v_0 at time $t=0$. If $u(x,t)$ denotes the displacement of any point in the x -direction, the one dimensional equation of motion neglecting rotational inertia is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (1)$$

$$\text{where } c = \sqrt{E/\rho} \quad (2)$$

is the dilatational wave velocity, E is Young's modulus and ρ the mass density of the material. The bar is initially at rest, i.e.

$$\begin{aligned} u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned} \quad (3)$$

and the boundary conditions are

$$u(0,t) = 0$$

$$u(l,t) = \begin{cases} 0 & t < 0 \\ v_0 t & t \geq 0 \end{cases} \quad (4)$$

Taking the Laplace transform of (1) and incorporating the initial conditions (3) we get²

$$s^2 \bar{u}(x,s) = c^2 \frac{d^2 \bar{u}(x,s)}{dx^2} \quad (5)$$

where $\bar{u}$ is the Laplace transform of u with respect to the parameter s . The solution to (5) is

$$\bar{u} = c_1 \sinh \frac{sx}{c} + c_2 \cosh \frac{sx}{a} \quad (6)$$

with the associated transformed boundary conditions

$$\bar{u}(0,s) = 0 \quad (7)$$

$$\bar{u}(l,s) = \frac{v_0}{s^2}$$

Substitution of (7) into (6) gives the solution

$$\bar{u}(x,s) = \frac{v_0}{s^2} \frac{\sinh(\frac{3x}{c})}{\sinh(\frac{sl}{c})} \quad (8)$$

which can be rewritten for solution purposes as

$$\bar{u}(x,s) = \frac{v_0}{2s^2} [e^{sx/c} - e^{-sx/c}] \operatorname{csch}(\frac{sl}{c}) \quad (9)$$

To invert this transform it is noted from the table of transforms of Roberts and Kaufman³ that

$$L^{-1} \left[\frac{1}{s^2} \operatorname{csch} \left(\frac{sl}{c} \right) \right] = \begin{cases} 0 & 0 < t < \frac{l}{c} \\ 2n(t - \frac{ln}{c}) & (2n-1)\frac{l}{c} < t < (2n+1)\frac{l}{c} \end{cases} \quad (10)$$

If the dimensionless variable $\tau = t \frac{c}{l}$ is introduced, a function $f(\tau)$ can be defined from (10) as

$$f(\tau) = \begin{cases} 0 & 0 < \tau < 1 \\ 2n(\tau - n) & (2n-1) < \tau < (2n+1) \end{cases} \quad (11)$$

A graph of this function is shown in Fig. 2. The term $e^{-sx/c}$ in the transformed solution (9) has the property of shifting the inverse transform (10) to the right by an amount (x/c) along the time axis, or by (x/l) along the τ axis. The function $e^{sx/c}$ has a similar property of shifting the solution to the left by the same amount, provided the solution is zero in the region $0 < t < x/c$. The displacement can then be written for any point as

$$u(x, \tau) = \frac{v_0 l}{2c} \left[f\left(\tau + \frac{x}{l}\right) - f\left(\tau - \frac{x}{l}\right) \right] \quad (12)$$

Introducing the following dimensionless variables,

$$\begin{aligned} \xi &= x/l \\ u^* &= u/l \\ v^* &= v_0/c \end{aligned} \quad (13)$$

the dimensionless displacement becomes

$$u^*(\xi, \tau) = \frac{v^*}{2} [f(\tau + \xi) - f(\tau - \xi)] \quad (14)$$

where $f(\tau)$ is defined by equation (11). The displacement at any interior point ξ of the bar is shown in Fig. 3, plotted as u^*/v^* against τ . The dashed curve is the displacement of the end in motion, $\xi=1$, which is $u^* = v^*$, or $u = v_0 t$.

The strain at any point is easily derived from $\epsilon_x = \partial u / \partial x$ or in terms of dimensionless variables,

$$\epsilon_x = \frac{\partial u^*}{\partial \xi} \quad (15)$$

From the definition of $f(\tau)$ in (11) and (14), we find

$$\epsilon_x = v^*(n_1 + n_2) \quad (16)$$

where n_1 and n_2 are defined in the relations

$$\begin{aligned} (2n_1 - 1) < \tau + \xi < (2n_1 + 1) \\ (2n_2 - 1) < \tau - \xi < (2n_2 + 1) \end{aligned} \quad (17)$$

The strain function is shown in Fig. 4. It is to be noted that the average value of the strain is the same at any point in the bar, i.e. $\epsilon_{av} = vt/\ell$.

This average strain is what is observed in ordinary slow speed tests of uniform cross-head or ram velocity as on an Instron tester.

The stress is calculated from $\sigma = \epsilon E$ and is plotted for both ends of the bar in Fig. 5. The stress at $\xi = 1$ is what the machine applies; the stress at $\xi = 0$ is what would be observed with a load cell at the fixed end.

DISCUSSION

From the diagrams of stress and strain along the bar several conclusions may be drawn concerning the use of constant cross-head or ram velocity machines for determining stress strain properties at very high loading rates. First, the machine must be capable of applying a step type loading as shown in Fig. 5 if the velocity of the ram is to remain constant during the test. Secondly, the observed stress at the fixed end (see Fig. 5) does not give a stress-strain curve for the material when plotted against average strain because of the discrete jumps in the response. This phenomenon has been observed in the high speed testing of

rubbers by Dannis⁴. Finally, the concept of constant strain rate is no longer valid at very high loading rates because the problem now becomes one of a wave propagating through the material.

The constant cross-head velocity experiment thus becomes an impact or wave propagation test when the loading rates approach the dilatational wave velocity of the material. This, along with the mechanical difficulties of constructing such a machine, limits the range of usefulness of these high rate universal testing machines. Above certain loading rates, which depend on the stiffness of the material, wave propagation techniques are necessary for the determination of material properties. However, the data obtained from constant loading rate tests may be used for determining material properties if interpreted properly in conjunction with the equations and diagrams presented here.

It must also be noted here that the "strain rates" which a material undergoes during the propagation of a wave are average values of step type functions (cf. Fig. 4).

The strain rates that actually occur are infinite for a zero time interval and are then zero for a finite time interval. The concept of material response as a function of strain rate should be carefully reviewed in conjunction with this observation. An analysis of what is really happening can only lead one to conclude that claims by experimentalists of achieving "strain rates" in the laboratory of 10^4 and 10^5 sec^{-1} are really meaningless statements.

REFERENCES

1. Timoshenko, S., Vibration Problems in Engineering, 3rd ed., D. Van Nostrand, New York, 1955.
2. Churchill, R. V., Operational Mathematics, 2nd ed., McGraw Hill, New York, 1958.
3. Roberts, G. E. and Kaufman, H., Table of Laplace Transforms, W. B. Saunders Co., Philadelphia, 1966, p. 279.
4. Dannis, M. L., "Stress-Strain Testing of Rubbers at High Rates of Elongation", in High Speed Testing, Vol. 3, Interscience, New York, 1962.

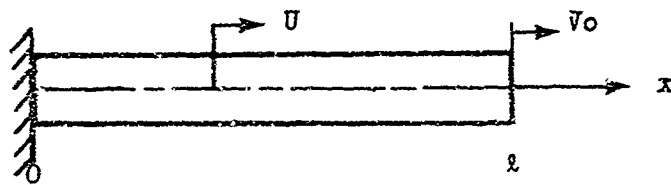


Fig. 1 Uniaxial tension test

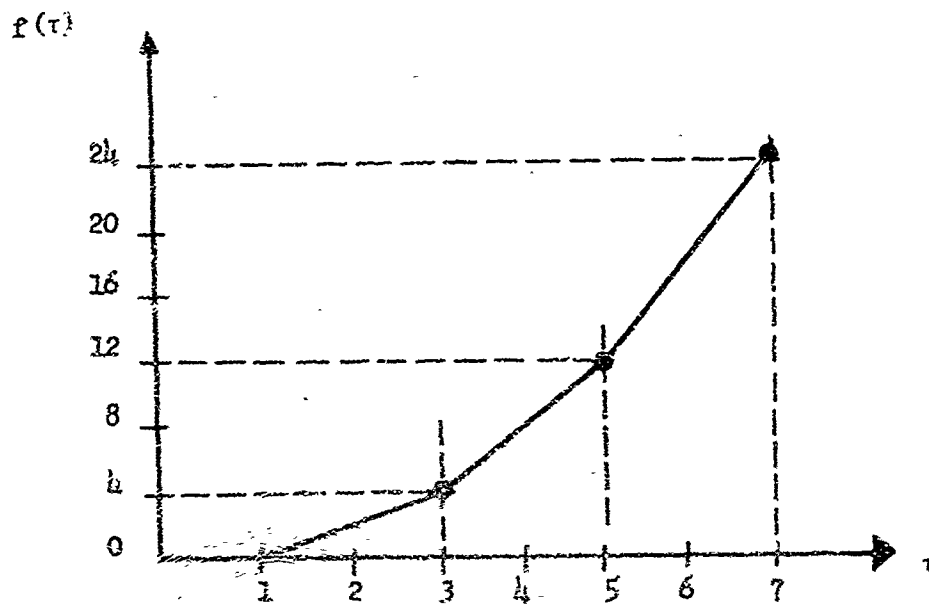


Fig 2 Graph of the Function $f(\tau)$

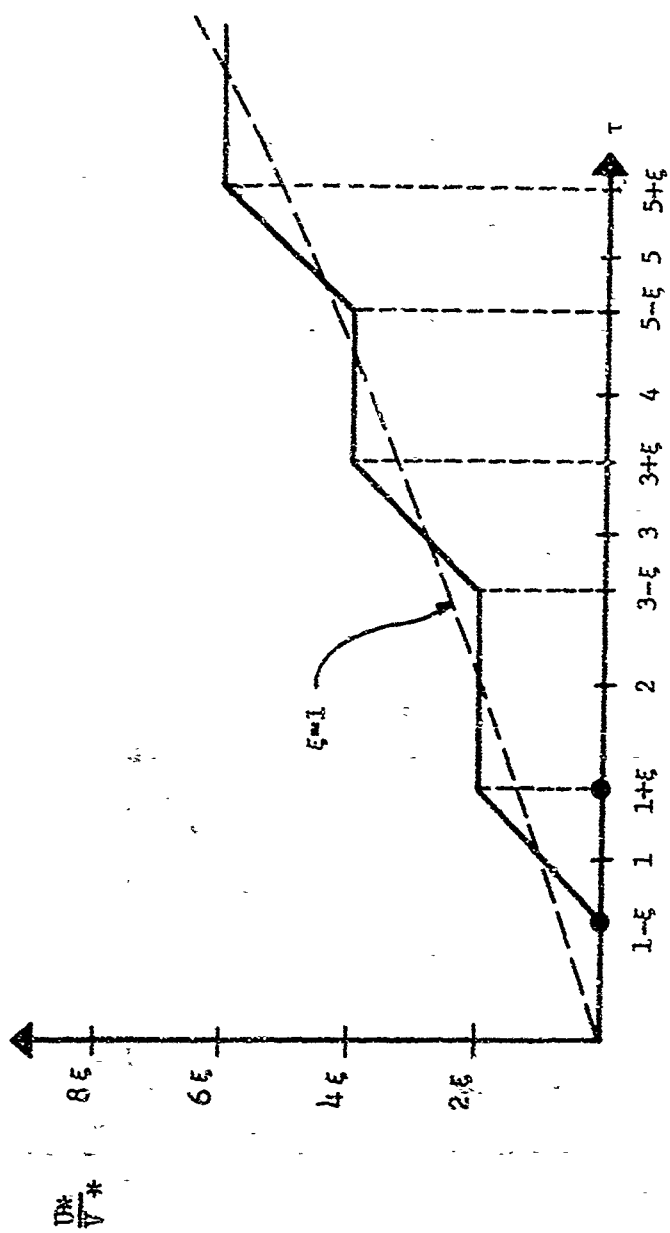


Fig 3 Displacement - time curve

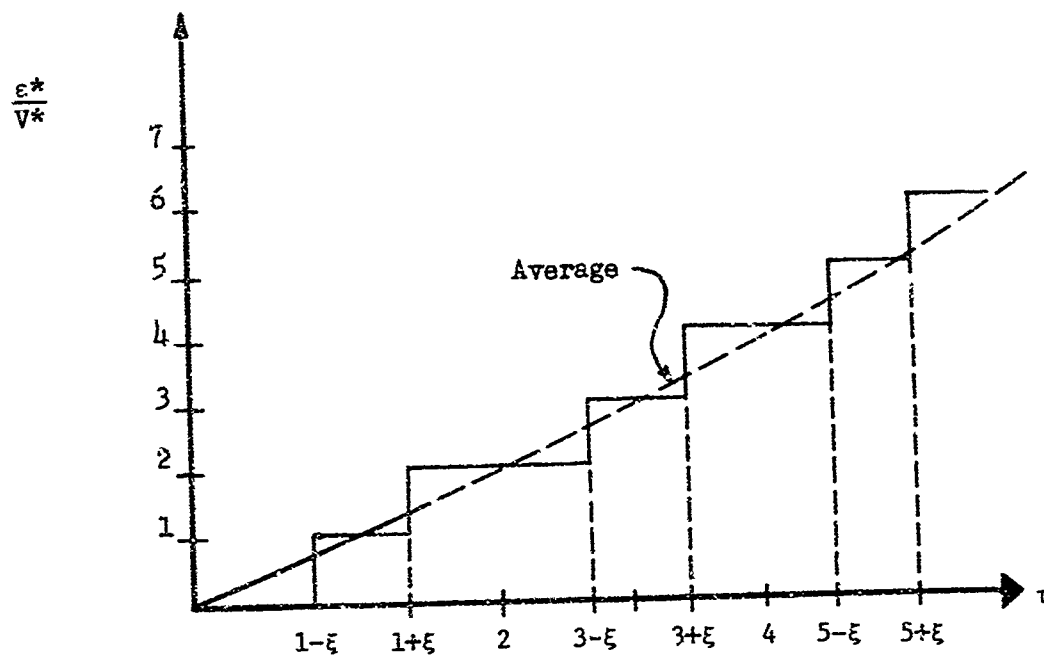


Fig 4 Strain - time curve

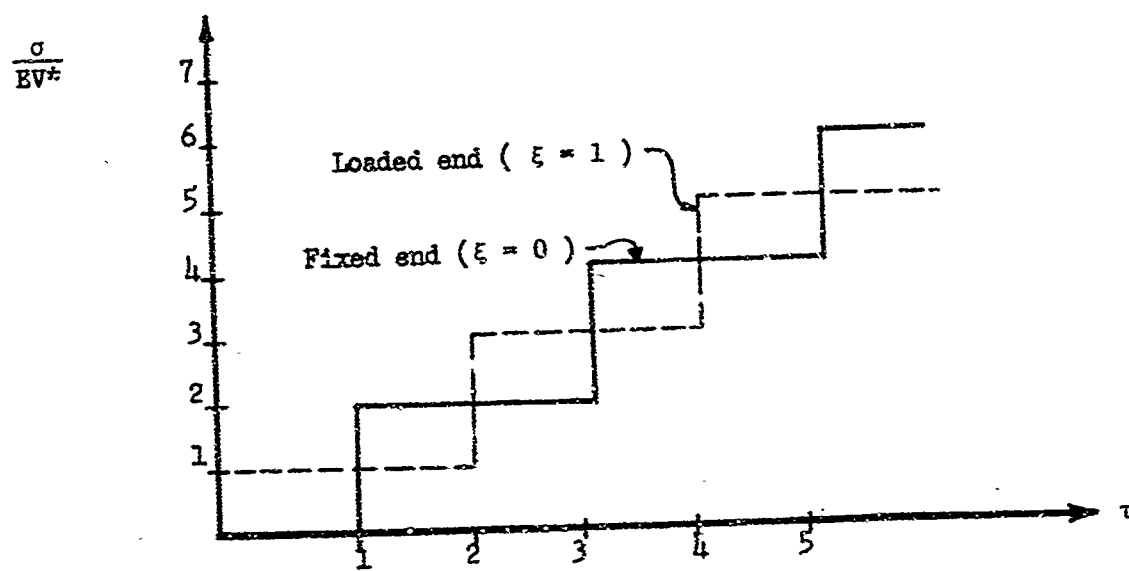


Fig 5 Stress at both ends of bar

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - RED		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1. ORIGINATING ACTIVITY (Company author)		2a. REPORT SECURITY CLASSIFICATION
Metals and Ceramics Division Air Force Materials Laboratory Wright-Patterson AFB, Ohio 45433		UNCLASSIFIED
3. REPORT TITLE		2b. GROUP
A NOTE ON TENSILE TESTING AT HIGH STRAIN RATES		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)		
Summary Report - July 1966 to August 1966		
5. AUTHOR(S) (Last name, first name, initial)		
T. Nicholas		
6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 1966	11	4
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO. 7351	AFML TR 66 -341	
c. TASK NO. 735106	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		
10. AVAILABILITY/LIMITATION NOTICE This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of AFML (MAMD), W-PAFB, Ohio 45433.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY	
	Metals and Ceramics Division Air Force Materials Laboratory Wright-Patterson AFB, Ohio 45433	
13. ABSTRACT		
The response of an elastic uniaxial tension specimen to a constant velocity applied at one end is presented. The application to the problem of testing specimens with high rate testing machines where the velocity of the ram approaches the dilatational wave velocity of the material is discussed. The concept of high strain rates is shown to break down under high loading rates.		

DD FORM 1473

1 JAN 64

UNCLASSIFIED

Security Classification

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
High Strain Rate Wave Propagation Tensile Testing Speed						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system number, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indicator of technical context. The assignment of links, rules, and weights is optional.

UNCLASSIFIED

Security Classification