AD-780 904

RANGE PERFORMANCE IN CRUISING FLIGHT

D. H. Peckham

Royal Aircraft Establishment
Farnborough, England

March 1974

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

UDC 533.6.013.61 : 533.6.015.74

ROYAL AIRCRAFT ESTABLISHMENT

Technical Report 73164

Received for printing 10 October 1973

RANGE PERFORMANCE IN CRUISING FLIGHT

by

D. H. Peckham

SUMMARY

The classical theory of range performance is reviewed, and some further relationships derived. The first part of the paper deals with 'specific range', which is the instantaneous range performance of an aircraft at a point on a cruise trajectory, in terms of distance covered per unit quantity of fuel consumed. The second part of the paper is concerned with the integration of specific range over a given flight trajectory, to give the 'integral range' on a given quantity of fuel; comparisons are made between the ranges obtained using various cruising techniques, and some numerical examples are included.

This Report is a combination of two Technical Memoranda issued in 1970, and replaces these in a form suitable for wider distribution.

Departmental Reference: Aero 3323

58

CONTENTS

1 INTRODUCTION

There are three main ingredients in the calculation of the range of an aircraft:

(i) it's performance during climb, cruise and descent, for a range of conditions of weight, speed and altitude,

(ii) estimation of fuel available after taking into account payload and reserve fuel requirements, and

(iii) choice of flight trajectory such as cruise speed and height, climb and descent paths, distance for diversion, and time for holding*.

The procedure for estimating range, given the above information, is described in Ref.1; since this Report was not given a wide circulation, much of it is repeated in Appendix A for convenience of the reader. If such information is not available, range performance has to be obtained from estimates of drag, thrust and fuel consumption characteristics. The most important part of this, except for very-short-range aircraft, is the performance during cruise, generally referred to as the 'specific range', which is the instantaneous value of distance covered per unit quantity of fuel consumed (at a given aircraft weight, speed and altitude). Typically, specific range is quoted in nautical miles per pound of fuel; miles per gallon is now rarely used because of possible confusion between US and Imperial gallons**, and because of differences in density between various fuels.

For civil aircraft, specific range is normally the subject of guarantees between the manufacturer and airlines, and checks on specific range performance at a number of speeds and altitudes form an important part of the flight-test programme of a new aircraft. The results of such tests are also of great interest to the airframe and engine designers, because they provide a means of checking original design estimates for drag and fuel consumption[2].

Estimation of specific range is described in section 2, and the theory for the particular case of a parabolic drag polar is discussed in section 3. Section 4 deals with maximum specific range in general, and section 5 with maximum specific range for the classical case when a parabolic drag polar can

---

* This Report is written in terms of a typical civil transport mission; however, much of the Report is also relevant to military aircraft missions.

** One Imperial gallon = 0.1605 ft^3 = 1.201 US gallons = 4.546 litres.

be assumed; relationships for cruise at constant speed, constant engine setting, and constant altitude are derived. Numerical examples are given in Appendix B.

Integration of the specific range over a given flight trajectory, for a change of aircraft weight equal to the fuel consumed, gives the range. This is discussed in sections 6 and 7 for cruising techniques such as the Breguet 'cruise-climb', and cruising at constant altitude with either lift-to-drag ratio, speed, or thrust kept constant. Comparisons are made of the ranges obtained using these various cruising techniques and numerical examples are given in Appendix C. All theory in these sections is based on the assumption that the specific fuel consumption remains essentially constant along the cruise trajectory considered. This is a reasonable assumption when cruise speed is constant, but may not be strictly correct if there is a large change of speed over the cruise trajectory. The use of a power-law approximation to the variation of specific fuel consumption with speed is discussed in Appendix D. There can also be a variation of specific fuel consumption with temperature, i.e. with variation of height in the troposphere. However, since most cruise-climb trajectories are used only by long-range aircraft flying in the stratosphere, this is rarely a problem.

2 ESTIMATION OF SPECIFIC RANGE

The rate of change of aircraft weight, being equal to the rate at which fuel is consumed, is given by

$$\frac{dW}{dt} = -cT \qquad (1)$$

where c = specific fuel consumption (sfc)

t = time

T = thrust

W = aircraft weight.

The instantaneous value of range in still air is then given by

$$dR = Vdt = -\frac{V}{cT}\,dW \qquad (2)$$

where R = range

V = cruise true airspeed.

Now in steady level cruising flight, because the incidence and attitude are small, it can be assumed that lift is equal to weight, and that thrust is equal to drag, so the expression for specific range becomes

$$-\frac{dR}{dW} = \frac{V}{cT} = \frac{1}{W}\frac{V}{c}\frac{L}{D} \qquad (3)$$

which, for speed in knots, sfc in lb/lb/h, and aircraft weight in pounds, has the units of nautical miles per pound of fuel.

In some theoretical studies (e.g. Ref.3), it is more convenient to work in terms of an overall efficiency of the powerplant, η_P, defined as

$$\eta_P = \frac{V}{cH} \qquad (4)$$

where H = the calorific value of the fuel.

The expression for specific range then becomes:

$$-\frac{dR}{dW} = \frac{\eta_P H}{T} = \eta_P \frac{H}{W}\frac{L}{D} \quad . \qquad (5)$$

The calorific value of kerosene is 18550 btu/lb. For use in a range equation, H needs to be expressed in appropriate length units Thus it should be noted that

$$18550 \text{ btu/lb} = 18550 \times 778 \text{ ft lb/lb} = 14.43 \times 10^6 \text{ ft}$$

$$14.43 \times 10^6 \text{ ft} = 4.398 \text{ Mm}$$

$$= 2733 \text{ mile (statute)}$$

$$= 2373 \text{ n mile (UK)}$$

$$= 2375 \text{ n mile (international)}.$$

3 SPECIFIC RANGE BASED ON A PARABOLIC DRAG POLAR

The simplest expression for the total drag coefficient of an aircraft, and one which accords very closely with actual drag characteristics for a wide range of aircraft types is

$$C_D = C_{D_0} + C_{D_i} = C_{D_0} + \frac{K}{\pi A} C_L^2 \qquad (6)$$

where C_{D_0} = drag coefficient at zero lift*

C_{D_i} = lift-dependent drag coefficent

K = lift-dependent drag factor

A = aspect ratio

C_L = lift coefficient $\equiv L/\frac{1}{2}\rho V^2 S = W/qS$

ρ = air density

S = wing area.

Thus the ratio of drag to lift is then

$$\frac{D}{L} = \frac{C_D}{C_L} = \frac{C_{D_0}}{C_L} + \frac{K}{\pi A} C_L \qquad (7A)$$

which on substitution for C_L gives

$$\frac{D}{L} = \frac{1}{2}\rho C_{D_0}\left(\frac{V^2}{W/S}\right) + \frac{2K}{\pi\rho A}\left(\frac{W/S}{V^2}\right) \quad . \qquad (7B)$$

Thus the lift-to-drag ratio, required for use in equation (3), can be obtained by taking the reciprocal of D/L in equations (7A) or (7B), depending on which is the most convenient form to use.

When a number of results are required, over a range of speeds for example, a convenient method is to base the calculation on conditions for minimum drag. Differentiation of equation (7A) with respect to C_L gives the condition $\partial(D/L)/\partial C_L = 0$ for maximum lift-to-drag ratio (and for minimum drag) as:

$$C_{D_0} = \frac{K}{\pi A} C_{L_{md}}^2 \quad \text{and} \quad C_{D_{md}} = 2C_{D_0} \quad . \qquad (8)$$

where the suffix md is for minimum-drag conditions.

It follows that

$$C_{L_{md}} = \left(\frac{\pi A C_{D_0}}{K}\right)^{\frac{1}{2}} \qquad (9)$$

* It must be admitted that, for a cambered wing, the term 'drag coefficient at zero lift' is not very meaningful. For the purpose of performance computation, however, equation (6) can be used as a 'best fit' over the range of lift coefficients which are of interest.

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2}\left(\frac{\pi A}{KC_{D_0}}\right)^{\frac{1}{2}} \tag{10}$$

and

$$V_{md} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{\pi AC_{D_0}}\right)^{\frac{1}{2}} . \tag{11}$$

Combining equation (10) with equations (7A) and (7B) gives

$$\frac{\left(\frac{L}{D}\right)_{max}}{\left(\frac{L}{D}\right)} = \frac{D}{D_{min}} = \frac{1}{2}\left(\frac{C_{L_{md}}}{C_L} + \frac{C_L}{C_{L_{md}}}\right) = \frac{1}{2}\left[\left(\frac{V}{V_{md}}\right)^2 + \left(\frac{V_{md}}{V}\right)^2\right] = \frac{1}{2}\left(m^2 + \frac{1}{m^2}\right) \quad \ldots (12)$$

where $m^2 = \dfrac{V^2}{V_{md}^2} = \dfrac{C_{L_{md}}}{C_L}$.

Equation (12) is a perfectly general function that applies to any aircraft whose drag characteristics can be represented by equation (6). The variation of lift-to-drag ratio with speed is tabulated below.

$m = V/V_{md}$	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$(L/D)/(L/D)_{max}$	0.9782	1.0	0.9821	0.9370	0.8765	0.8096	0.7423

Using such values, specific range can be obtained from the expression

$$-\frac{dR}{dW} = \frac{V}{cW}\frac{L}{D} = \frac{V}{cW}\frac{2\left(\frac{L}{D}\right)_{max}}{m^2 + \frac{1}{m^2}} . \tag{13}$$

Equations (7B), (12) and (13) above are in a form from which it is convenient to calculate D , L/D and specific range at a given speed. If one wishes to estimate the speed and specific range of an aircraft at a given thrust, then equation (7B) has to be solved as a quadratic in V^2 , i.e.

$$(V^2)^2 - \frac{2T}{\rho SC_{D_0}}(V^2) + \frac{4KW^2}{\pi A\rho^2 S^2 C_{D_0}} = 0$$

the solution of which is

$$V^2 = \frac{T}{\rho S C_{D_0}}\left[1 + \sqrt{1 - \frac{W^2}{\left(\frac{L}{D}\right)^2_{max} T^2}}\right] = \frac{T}{\rho S C_{D_0}}\left[1 + \sqrt{1 - \frac{D^2_{min}}{T^2}}\right] \qquad (14)$$

i.e.

$$m^2 = \frac{V^2}{V^2_{md}} = \frac{T}{D_{min}}\left[1 + \sqrt{1 - \frac{D^2_{min}}{T^2}}\right] \quad . \qquad (15)$$

A numerical example is given in Appendix B.

4 MAXIMUM SPECIFIC RANGE

Maximum specific range, at a given aircraft weight, will occur when (V/c)(L/D) is a maximum, and different values will be obtained depending on the cruise condition specified, i.e. whether constant speed, constant engine setting, or constant altitude. On the assumption that engine specific fuel consumption does not vary along the cruise trajectory for each of these cruise techniques, the relationships between lift and drag to obtain maximum specific range are derived below, and it is shown that these relationships are independent of the way in which drag varies with lift.

4.1 Constant speed

For cruise at a constant true airspeed, it follows directly from equation (3) that maximum specific range will occur when drag is a minimum (i.e. when the ratio of lift to drag is a maximum). The same is also true for cruise at a constant Mach number in the stratosphere, where the temperature and speed of sound are constant.

4.2 Constant engine setting

At a given engine rev/min, engine performance in the stratosphere (where temperature is constant) is such that thrust may be assumed to be directly proportional to air density, and specific fuel consumption remains constant. Usually, also, thrust is sensibly independent of speed (at subsonic speeds). For these conditions, a simple relationship between lift and drag to obtain best specific range can be derived, provided that there is sufficient thrust at the maximum cruise rating of the engines for flight in the stratosphere to be attained. The required relationship can be obtained by substituting in equation (3)

$$V = \left(\frac{2D}{\rho S C_D}\right)^{\frac{1}{2}} = \left(\frac{2T}{\rho S C_D}\right)^{\frac{1}{2}} \tag{16}$$

and it follows that

$$-\frac{dR}{dW} = \frac{1}{cW}\left(\frac{2T}{\rho S}\right)^{\frac{1}{2}}\left(\frac{C_L}{C_D^{3/2}}\right) . \tag{17}$$

Thus the condition for maximum specific range in the stratosphere, at a given engine setting, occurs when $C_L/C_D^{3/2}$ is a maximum (or when $C_L^{2/3}/C_D$ is a maximum).

4.3 Constant altitude

The obtain the relationship between lift and drag for maximum specific range at a constant cruise altitude, V in equation (3) can be eliminated by substituting

$$V = \left(\frac{2W}{\rho S C_L}\right)^{\frac{1}{2}} \tag{18}$$

giving

$$-\frac{dR}{dW} = \frac{1}{c}\left(\frac{2}{\rho S W}\right)^{\frac{1}{2}} \frac{C_L^{1/2}}{C_D} . \tag{19}$$

Thus the condition for maximum specific range at a given altitude (ρ constant) occurs when $C_L^{1/2}/C_D$ is a maximum.

4.4 Summary of conditions for maximum specific range

In the three previous sections it has been shown that maximum specific range in the following cruise conditions is obtained when

For constant speed - C_L/C_D is a maximum.

For constant engine setting - $C_L^{2/3}/C_D$ is a maximum.

For constant altitude - $C_L^{1/2}/C_D$ is a maximum.

It is emphasised, once again, that these conditions have been derived without making any assumptions in regard to the way in which lift and drag vary with speed. However, they apply only to the case of specific fuel consumption constant.

It should be noted that $\rho^{\frac{1}{2}}$ appears in the denominators of the expressions for specific range, and this is one fundamental reason why high cruising altitudes are chosen for jet aircraft when good range performance is required.

5 MAXIMUM SPECIFIC RANGE WITH A PARABOLIC DRAG POLAR

The relationships between lift and drag for maximum specific range obtained in section 4, are now used with the theory of section 3, to obtain relationships for maximum specific range, for the case where an aircraft's drag characteristics can be represented by equation (6). Numerical examples are given in Appendix B.

5.1 Constant speed

From section 4.1, we have that maximum specific range at a constant speed occurs when drag is a minimum, and equations (9), (10) and (11) then apply. In particular, we get from equation (11) that

$$V_{md}\sqrt{\sigma} = \left(\frac{2W}{\rho_0 S}\right)^{\frac{1}{2}}\left(\frac{K}{\pi A C_{D_0}}\right)^{\frac{1}{4}} = (V_e)_{md} \qquad (20)$$

is a constant for a given aircraft at a given weight, where $(V_e)_{md}$ is the minimum drag speed in eas $(V_e = V\sqrt{\sigma})$, and σ is the air density relative to sea level conditions.

Hence the maximum specific range condition for any required true airspeed, V_{reqd}, is obtained at an altitude defined by

$$\sqrt{\sigma} = \frac{(V_e)_{md}}{V_{reqd}} \quad . \qquad (21)$$

5.2 Constant engine setting

Maximum specific range in this case is obtained when $C_L^{2/3}/C_D$ is a maximum, and equation (7A) gives

$$\frac{C_D}{C_L^{2/3}} = \frac{C_{D_0}}{C_L^{2/3}} + \frac{K}{\pi A} C_L^{4/3}$$

which on differentiation with respect to $C_L^{2/3}$ gives

$$C_{D_0} = \frac{2K}{\pi A} C_{L_{es}}^2 \quad \text{i.e.} \quad C_{D_{es}} = \frac{3}{2} C_{D_0} \qquad (22)$$

where the suffix es refers to conditions at a constant engine setting. It follows that

$$C_{L_{es}} = \left(\frac{\pi A C_{D_0}}{2K}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} C_{L_{md}} = 0.707\ C_{L_{md}} \tag{23}$$

$$\left(\frac{C_L}{C_D}\right)_{es} = \frac{\sqrt{2}}{3}\left(\frac{\pi A}{K C_{D_0}}\right)^{\frac{1}{2}} = \frac{2\sqrt{2}}{3}\left(\frac{L}{D}\right)_{max} = 0.943\left(\frac{L}{D}\right)_{max} \tag{24}$$

$$V_{es} = \left(\frac{2W}{\rho S C_{L_{es}}}\right)^{\frac{1}{2}} = 2^{\frac{1}{4}} V_{md} = 1.189\ V_{md} \ . \tag{25}$$

5.3 Constant altitude

Maximum specific range in this case is obtained when $C_L^{\frac{1}{2}}/C_D$ is a maximum, and equation (7A) gives

$$\frac{C_D}{C_L^{\frac{1}{2}}} = \frac{C_{D_0}}{C_L^{\frac{1}{2}}} + \frac{K}{A} C_L^{3/2}$$

which on differentiation with respect to $C_L^{1/2}$ gives

$$C_{D_0} = \frac{3K}{\pi A} C_{L_h}^2 \qquad \text{i.e.} \quad C_{D_h} = \frac{4}{3} C_{D_0} \tag{26}$$

where the suffix h refers to conditions at a constant altitude. It follows that

$$C_{L_h} = \left(\frac{\pi A C_{D_0}}{3K}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{3}} C_{L_{md}} = 0.577\ C_{L_{md}} \tag{27}$$

$$\left(\frac{L}{D}\right)_h = \frac{\sqrt{3}}{4}\left(\frac{\pi A}{K C_{D_0}}\right)^{\frac{1}{2}} = \frac{\sqrt{3}}{2}\left(\frac{L}{D}\right)_{max} = 0.866\left(\frac{L}{D}\right)_{max} \tag{28}$$

$$V_h = \left(\frac{2W}{\rho S C_{L_h}}\right)^{\frac{1}{2}} = 3^{\frac{1}{4}} V_{md} = 1.316\ V_{md} \ . \tag{29}$$

The effects of variation of sfc with speed (at constant altitude) on specific range are discussed in Appendix D.

6 PRELIMINARY REMARKS ON INTEGRAL RANGE

The distance covered during cruising flight (in still air) is obtained by integration of the 'specific range' over a change of aircraft weight equal to the weight of fuel consumed, i.e.

$$\text{cruise range} \quad R = -\int_{W_i}^{W_i - W_F} \left(\frac{dR}{dW}\right) dW \tag{30A}$$

and

$$dW = dW_F \tag{30B}$$

where $-\frac{dR}{dW} = \frac{V}{cT}$ = specific range from equation (3)

W_i = aircraft weight at start of cruise

W_F = weight of fuel consumed.

It is often sufficiently accurate to obtain cruise range by multiplying a mean specific range by the weight of fuel consumed, since the variation of specific range with weight is usually close to linear.

Thus, we have, approximately, that either

$$R = \left(\frac{dR}{dW}\right)_{W_i - \frac{W_F}{2}} \times W_F \tag{31}$$

or

$$R = \frac{1}{2}\left[\left(\frac{dR}{dW}\right)_i + \left(\frac{dR}{dW}\right)_f\right] W_F \tag{32}$$

where the subscripts i and f refer to initial and final conditions, respectively.

For instance, in the example from an aircraft performance manual given in Fig.1, for an initial cruise weight of 260000 lb and 80000 lb of fuel consumed, we get

Method	Method $\left(\frac{dR}{dW}\right)$ n mile/lb	Range n mile	Error %
Integration	0.0380	3040	-
Equation (31)	0.0382	3056	+0.5
Equation (32)	0.0377	3016	-0.8

However, for theoretical work and early project studies, it is more convenient to obtain cruise range from direct integration of equation (30A) over a chosen cruise trajectory. In some cases, it is then necessary to assume a law for the variation of aircraft drag with speed, such as those given in equations (6) and (7).

The integration of equation (30A) is discussed in the next section, which collects together and extends the classical theory of range performance of jet aircraft in cruising flight, for conditions where engine specific fuel consumption can be assumed to remain essentially constant during the flight.

7 RANGE EQUATIONS

7.1 General remarks

It can be seen from inspection of equation (3), that one simple* class of cruise trajectories that can be considered, consists of those where speed and/or the ratio of lift to drag are kept constant throughout the flight. Keeping the ratio of lift to drag constant, also means that the aircraft incidence and lift coefficient will remain constant.

Examination of the basic equation

$$L = W = C_L q S = C_L \tfrac{1}{2}\rho V^2 S \qquad (33)$$

shows that (ignoring Mach number effects) flight at constant lift coefficient can be achieved in the following ways:

(i) speed (V) constant. This requires the cruise altitude of the aircraft to be steadily increased as fuel is consumed, in a way such that air density is proportional to the weight of the aircraft,

or (ii) altitude (ρ) constant. This requires the speed of the aircraft to be steadily reduced as fuel is consumed, in a way such that V^2 is proportional to the weight of the aircraft,

or (iii) dynamic pressure (q) proportional to the weight of the aircraft. However, an infinite variety of combinations of ρ and V is then possible, and this case is not amenable to a general theoretical approach.

Case (i) above is generally referred to as the Breguet 'cruise-climb' technique; it may not be acceptable in many situations because of the

---

* i.e. simple from the analytical point of view.

equirements of Air Traffic Control. Case (ii) which requires a steady *decrease* in speed during the cruise is unlikely to be acceptable to airlines as a normal operational procedure. These somewhat 'academic' cases are considered in sections 7.2.1 and 7.2.2, respectively.

The more practical procedures from the operational point of view, of cruising at constant altitude with either speed or engine thrust kept constant, are considered in sections 7.3.1 and 7.3.2, respectively.

Also in sections 7.2 and 7.3, each of the equations derived for various methods of cruising at constant altitude are compared with the Breguet equation for cruise-climb, for the same initial conditions of aircraft weight, speed, altitude and fuel-fraction. Values of the 'ratios of range' so obtained are plotted. This has been done in order to show the loss of range relative to the Breguet cruise-climb technique, and also to simplify computation. Thus, once a 'Breguet-range' has been obtained, for given initial cruise conditions, the range using other cruise techniques can quickly be found by application of the appropriate 'ratio of range', rather than having to substitute values into each range equation in turn.

A typical variation of specific range with aircraft weight and speed is shown in Fig.2, with cross-plots showing the various types of cruise trajectory, at constant altitude, that have been considered. The worked examples in Appendix C relate to this Figure.

7.2 Cruising with lift-to-drag ratio constant (C_L and α constant)

7.2.1 Speed constant, altitude increasing ($\rho \propto W$)

Integration of equation (3) with $(V/c)(L/D)$ constant throughout the cruise, gives the Breguet equation

$$R_{Br} = \frac{V}{c}\frac{L}{D}\log_e\left(\frac{W_i}{W_f}\right) \tag{34A}$$

i.e.

$$R_{Br} = \frac{V}{c}\frac{L}{D}\log_e\left(\frac{1}{1 - W_F/W_i}\right) \quad . \tag{34B}$$

7.2.2 Altitude constant, speed decreasing ($V^2 \propto W$)

An example of such a cruise trajectory is shown by the line AB in Fig.2.

From equations (3) and (33) we get

$$-dR = \frac{1}{c}\left(\frac{2}{\rho S}\right)^{\frac{1}{2}}\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)\frac{dW}{W^{\frac{1}{2}}}$$

which on integration gives

$$R = \frac{2}{c}\left(\frac{2W_i}{\rho S}\right)^{\frac{1}{2}}\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)\left[1 - \left(1 - \frac{W_F}{W_i}\right)^{\frac{1}{2}}\right] \quad (35A)$$

i.e.

$$R = 2\frac{V_i}{c}\frac{L}{D}\left[1 - \left(1 - \frac{W_F}{W_i}\right)^{\frac{1}{2}}\right] \quad (35B)$$

where $V_i = \left(\frac{2W_i}{\rho SC_L}\right)^{\frac{1}{2}}$ = initial cruise speed.

7.2.3 Comparison of range at constant altitude with Breguet range

For the same conditions at start of cruise (i.e. aircraft weight, speed and altitude), and L/D constant throughout the flight, the ratio of the cruise ranges from equations (35B) and (34B) is given by

$$\frac{\text{Range at constant altitude}}{\text{Breguet range}} = \frac{2[1 - (1 - W_F/W_i)^{\frac{1}{2}}]}{\log_e\left(\frac{1}{1 - W_F/W_i}\right)} \quad . \quad (36)$$

Equation 36 gives

W_F/W_i	0.1	0.2	0.3	0.4	0.5
Ratio of ranges	0.974	0.946	0.916	0.883	0.845

The above values are plotted in Fig 3.

Thus there is a significant loss in range, particularly at the larger values of W_F/W_i, if flight is constrained to the cruise technique at constant altitude of section 7.2.2. In addition, this technique gives a lower average speed.

7.3 Cruising at constant altitude

7.3.1 Speed constant ($C_L \propto W$)

For this method of cruising, the aircraft incidence has to be steadily decreased as fuel is consumed, in such a way that the lift coefficient is proportional to the aircraft weight. In addition, the engine thrust needs to be steadily decreased during the course of the flight. An example of such a cruise trajectory is shown by the line AC in Fig.2.

From equation (3), and noting from equation (7A) that

$$\frac{C_D}{C_L} = \frac{C_{D_0}}{C_L} + \frac{K}{\pi A} C_L = \frac{qSC_{D_0}}{W} + \frac{KW}{\pi A q S}$$

we obtain

$$-dR = \frac{V}{cqSC_{D_0}} \frac{dW}{\left(1 + KW^2/\pi A q^2 S^2 C_{D_0}\right)}$$

which on integration gives

$$R = \frac{V}{c}\left(\frac{\pi A}{KC_{D_0}}\right)^{\frac{1}{2}}\left[\tan^{-1}\frac{W_i}{qS}\left(\frac{K}{\pi AC_{D_0}}\right)^{\frac{1}{2}} - \tan^{-1}\frac{W_f}{qS}\left(\frac{K}{\pi AC_{D_0}}\right)^{\frac{1}{2}}\right] \tag{37A}$$

and using equations (9) and (10) we get

$$R = 2\frac{V}{c}\left(\frac{L}{D}\right)_{max}\left(\tan^{-1}\frac{C_{L_i}}{C_{L_{md}}} - \tan^{-1}\frac{C_{L_f}}{C_{L_{md}}}\right) \tag{37B}$$

which can be rearranged as

$$R = 2\frac{V}{c}\left(\frac{L}{D}\right)_{max}\tan^{-1}\left(\frac{C_{L_i} - C_{L_f}}{C_{L_{md}} + C_{L_i}C_{L_f}/C_{L_{md}}}\right) . \tag{37C}$$

Furthermore, since at constant speed and altitude

$$C_{L_f} = C_{L_i}\left(1 - \frac{W_F}{W_i}\right)$$

and using equation (12), equation (37C) becomes

$$R = \frac{V}{c}\left(\frac{L}{D}\right)_i\left(m_i^2 + \frac{1}{m_i^2}\right)\tan^{-1}\left[\frac{W_F/W_i}{m_i^2 + (1 - W_F/W_i)/m_i^2}\right] . \tag{37D}$$

For the same conditions at start of cruise, the ratio of the ranges from equations (37D) and (34B) is given by

$$\frac{\text{Range at constant speed and altitude}}{\text{Breguet range}} = \frac{\left(m_i^2 + \dfrac{1}{m_i^2}\right) \tan^{-1}\left[\dfrac{W_F/W_i}{m_i^2 + (1 - W_F/W_i)/m_i^2}\right]}{\log_e \left(\dfrac{1}{1 - W_F/W_i}\right)} \quad \text{... (38)}$$

The above ratio of ranges is plotted in Fig.4, and it can be seen that for the same conditions at start of cruise, cruising at constant speed and altitude always results in a loss of range relative to the Breguet cruise-climb technique. However, for low values of m_i (i.e. less than about 1.3 at short range and about 1.2 at long range), it gives a greater range than the method of cruising discussed in section 7.2.2; this can be seen by comparing Figs.3 and 4.

7.3.2 Thrust constant

The method of cruising described in section 7.3.1 above, requires that the engine thrust be steadily reduced during the course of the flight. A possibly more convenient method from the operational point of view is to leave thrust constant, and to allow the aircraft speed to increase steadily during the course of the flight*, so that

$$T(\text{constant}) = \frac{W}{(L/D)} = \frac{1}{2} \frac{W}{(L/D)_{max}} \left(m^2 + \frac{1}{m^2}\right) \quad (39A)$$

i.e.

$$m^2 + \frac{1}{m^2} = \frac{2(L/D)_{max}}{(L/D)} = \text{constant} \quad (39B)$$

giving

$$m^2 = \frac{V^2}{V_{md}^2} = \frac{(L/D)_{max}}{(L/D)} + \left[\frac{(L/D)_{max}^2}{(L/D)^2} - 1\right]^{\frac{1}{2}} . \quad (39C)$$

Further since

$$\frac{V_{md}^2 (L/D)_{max}}{(L/D)} = V_{md}^2 \frac{T}{D_{min}} = \frac{T}{\rho S C_{D_0}}$$

---

* Provided that limits on operating speed are not exceeded; in addition, the relationships derived are not valid if the increase of speed results in a drag increment due to the effects of compressibility.

$$V^2 = \frac{T}{\rho S C_{D_0}}\left\{1 + \left[1 - \frac{W^2}{(L/D)^2_{max} T^2}\right]^{\frac{1}{2}}\right\} \quad (39D)$$

and

$$\frac{V_f}{V_i} = \left\{\frac{1 + \left[1 - 4(1 - W_F/W_i)^2/\left(m_i^2 + 1/m_i^2\right)^2\right]^{\frac{1}{2}}}{1 + \left[1 - 4/\left(m_i^2 + 1/m_i^2\right)^2\right]^{\frac{1}{2}}}\right\}^{\frac{1}{2}} . \quad (39E)$$

It should also be noted that when cruising at constant thrust, specific range is directly proportional to speed (see equation (3)). Hence in Fig.2, lines of constant thrust are a family of rays from the origin, each one being tangential to a specific-range curve (at a particular aircraft weight) at the speed for minimum drag, V_{md} . One example is the line AD in Fig.2.

From equations (3) and (39D) we get that the expression for specific range at constant thrust and altitude is

$$-\frac{dR}{dW} = \frac{1}{c}\left\{\left(\frac{1}{\rho S C_{D_0} T}\right)\left[1 + \left(1 - \frac{W^2}{(L/D)^2_{max} T^2}\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{2}}$$

which an integration gives

$$R = \frac{2}{3}\left[\frac{V}{c}\frac{L}{D}\left(1 + \frac{T}{\rho S C_{D_0} V^2}\right)\right]_f^i \quad (40A)$$

i.e.

$$R = \frac{2}{3}\left[\frac{V}{c}\frac{L}{D}\left(1 + \frac{(L/D)_{max}}{m^2 (L/D)}\right)\right]_f^i . \quad (40B)$$

On substitution of the limits of integration, and after some manipulation, we obtain that

$$R = \frac{2}{3}\frac{V_i}{c}\left(\frac{L}{D}\right)_i\left[\left(1 + \frac{m_i^2 + 1/m_i^2}{2m_i^2}\right) - \left(1 - \frac{W_F}{W_i}\right)\left(\frac{V_f}{V_i} + \frac{m_i^2 + 1/m_i^2}{2m_i^2}\frac{V_i}{V_f}\right)\right] \quad (41)$$

where V_i/V_f and V_f/V_i are functions of m_i and W_F/W_i , and can be obtained from equation (39E).

Equation (41) is not in a very suitable form to compare with equation (34B) for Breguet range, and it is convenient to introduce the concept of a mean cruise speed. Now for flight at constant thrust, cT = constant , and hence the total cruise time is given by

$$t = \int dt = -\frac{1}{cT}\int dW_F = \frac{W_F}{cT} \tag{42}$$

and the mean speed is given by

$$V_{mean} = \frac{\text{Total distance}}{\text{Total time}} = \frac{R}{W_F/cT} \quad . \tag{43}$$

Thus range can be expressed in the form

$$R = \frac{W_F}{cT} V_{mean} = \frac{W_i}{cT}\frac{W_F}{W_i} V_{mean}$$

i.e.

$$R = \frac{V_i}{c}\left(\frac{L}{D}\right)_i \frac{W_F}{W_i}\frac{V_{mean}}{V_i} \tag{44}$$

where, from equations (41) and (43)

$$\frac{V_{mean}}{V_i} = \frac{2}{3}\frac{W_i}{W_F}\left[\left(1 + \frac{m_i^2 + 1/m_i^2}{2m_i^2}\right) - \left(1 - \frac{W_F}{W_i}\right)\left(\frac{V_f}{V_i} + \frac{m_i^2 + 1/m_i^2}{2m_i^2}\frac{V_i}{V_f}\right)\right] \tag{45}$$

and V_i/V_f and V_f/V_i are functions of m_i and W_F/W_i which can be obtained from equation (39E).

Thus for the same conditions at start of cruise, the ratio of the cruise ranges from equations (44) and (34B) is given by

$$\frac{\text{Range at constant thrust and altitude}}{\text{Breguet range}} = \frac{V_{mean}}{V_i}\frac{W_F/W_i}{\log_e\left(\frac{1}{1 - W_F/W_i}\right)} \quad . \tag{46}$$

The ratio V_{mean}/V_i is plotted in Fig.5, and the ratio of ranges in Fig.6. In general, there is a loss of range relative to the Breguet technique, except at low values of m_i and W_F/W_i. Comparison of Figs.4 and 6, shows that at the higher values of m_i and W_F/W_i , there is very little difference in range between the constant-speed and constant-thrust techniques (for the same initial cruise conditions).

8 INITIAL CRUISE CONDITIONS FOR MAXIMUM RANGE

All the 'ratios of ranges' given so far have been based on the same initial cruise conditions of aircraft weight, speed and altitude. In many circumstances, this is a fair basis of comparison, but on occasions it may be more realistic to make comparisons on the basis of initial cruise speeds in each case which give maximum range, (but still retain the same initial weight and altitude).

For the cruise techniques of sections 7.2.1 and 7.2.2, in which the lift coefficient is held constant throughout the flight, it can be seen from equations (34B) and (35B) that maximum range will be obtained when VL/D is a maximum. From equations (28) and (29) we have that $(VL/D)_{max}$ is obtained when

$$m = \frac{V}{V_{md}} = 3^{\frac{1}{4}} \approx 1.316 \quad \text{and} \quad \frac{L}{D} \approx 0.866 \left(\frac{L}{D}\right)_{max}$$

and since

$$\frac{VL}{D} \propto \frac{m}{m^2 + 1/m^2}$$

we obtain that

m_i	1.0	1.1	1.2	1.3	1.316	1.4	1.5
$\frac{\text{Breguet range}}{(\text{Breguet range})_{max}}$	0.8774	0.9479	0.9865	0.9998	1.000	0.9996	0.9769

... (47)

Thus the ratios of range obtained by cruising at constant speed, or constant thrust, to maximum Breguet range, can be found by multiplying the 'ratios of range' from equations (38) and (46), respectively, by the ratio of Breguet range to maximum Breguet range given in (47) above.

For example, for cruising at constant thrust and $W_F/W_i = 0.2$, we get

m_i	R/R_{Br} (same V_i equation (46))	$R_{Br}/R_{Br_{max}}$ (equation (47))	$R/R_{Br_{max}}$
1.0	1.062	0.8774	0.932
1.1	0.989	0.9479	0.937
1.2	0.953	0.9865	0.940
1.3	0.934	0.9998	0.934
1.4	0.922	0.9996	0.922
1.5	0.915	0.9769	0.894

The above table also shows that the initial speed to give maximum range, using the constant-thrust technique, is about 1.2 V_{md} in this particular case.

9 NOTES ON UNITS

In the context of aircraft performance (civil aircraft performance in particular), agreement has not yet been reached on the most appropriate metric units to be used. In fact, current international agreements, such as ICAO standards, are largely in terms of British units. The main difficulties relate to:

Distance (range)

The SI unit is metres, and for typical ranges kilometres (km) or megametres (Mm) would be used. However the international nautical mile* (1852 m exactly) is internationally recognised by ICAO and is used in a 'navigational' context; it is also a recognised unit in the metric system.

Speed

The SI unit is metres per second, but if ranges are quoted in nautical miles, then to be consistent speeds should be quoted in knots. Moreover, the majority of airspeed indicators are calibrated in knots, current Air Traffic Control speed instructions are given in knots**, and meteorological information on wind speeds are quoted in knots.

Height

The SI unit is metres. However, the majority of altimeters are calibrated in feet, and ATC instructions and meteorological information relating to height are given in feet**.

Force, weight and mass

The SI unit of force is the newton (1 N = 0.224809 lbf) and the unit of mass is the kilogramme (1 kg = 2.20462 lb). If forces such as thrust, drag, lift are quoted (quite correctly) in newtons, then it is logical to quote aircraft weight, fuel weight etc. in newtons also. An alternative procedure is always to write weight as $W = mg$, and to quote aircraft mass, fuel mass

---

* The present UK nautical mile, 6080 ft (1853.18 m) is roughly 0.06% greater than the international nautical mile.

** ICAO standard.

etc. in kilogrammes. Unfortunately, in many countries weights are quoted in kilogrammes (kilogramme-force, or kiloponds, being meant, which is not a recognised SI unit).

The position becomes even more complicated with derived units such as specific fuel consumption. On a weight basis, the units of sfc are N/N/h, which has the same numerical value as the British unit. If fuel mass is used as the basis, the units of sfc would probably be kg/N/h.

Appendix A

A.1 INTRODUCTION

The procedure for estimating the range performance of a civil aircraft is shown in Fig.A1; there are three main ingredients in the calculation:

(i) Performance during climb, cruise and descent, for a range of conditions of weight, speed and altitude. Manufacturers' brochures normally present the information for climb and descent in terms of fuel used, time, and distance covered, and for cruise in terms of miles per pound of fuel. Examples are given in Figs.A2, A3 and A4 and described in section A.2.1.

(ii) Fuel load. This is obtained by subtracting from the all-up weight of the aircraft, its empty weight and payload. The fuel available for range is the fuel load less the fuel reserves and various allowances for taxying, take-off, approach and landing. Definitions of these weights and allowances are given in section A.2.2.

(iii) Flight trajectory. This involves choice of cruising speed and height, climb and descent paths, and the distance for diversion and time for holding. This is discussed further in section A.2.3.

The range is then obtained by calculating the distance covered, and fuel used, during the climb and descent phases, the remaining fuel available being used for cruise. When doing this calculation it is also convenient to calculate the time spent during each phase of the flight, since this information will be needed for estimating the direct operating cost. A sample calculation is given in section A.3, and the presentation of a payload-range diagram is discussed in section A.4.

A.2 BASIC INFORMATION REQUIRED

A.2.1 Climb, cruise and descent performance

Typical examples of manufacturers' presentations of such performance data are given in Figs.A2, A3 and A4. Use of these charts is straightforward, but the following points need to be watched:

(i) Climb data does not usually include an allowance for the fuel and time consumed, and distance covered, during take-off and acceleration to climb speed. The distance covered during this phase is not normally credited towards range.

(ii) Similarly, descent data does not usually include an allowance for the fuel and time consumed, and distance covered, during deceleration to circuit speed. The distance covered during this phase is not normally credited to range.

(iii) Occasionally, allowance is made for acceleration to cruise speed from climb speed, and acceleratior or deceleration from cruise speed to descent speed. The effects are small, however, and can normally be neglected.

(iv) The fuel consumption rates are usually based on the engine manufacturers' estimated values. In some instances, though, a guarantee tolerance may have been applied. Assumptions regarding engine off-takes and losses should be noted.

The above pitfalls can be avoided provided that introductory remarks and footnotes in brochures are studied before commencing work. This is very important if proposals from different manufacturers are being compared, since there are nearly always small differences in accounting and presentation between firms.

A.2.2 Estimation of fuel available for range

At first sight this is straightforward, the basic equation being:

Fuel for range = (Aircraft 'all-up' weight)
− (aircraft empty weight + payload + fuel reserves
+ fuel allowances) .

However, each of these items needs to be defined precisely if errors in calculating the fuel available are to be avoided.

A.2.2.1 'All-up' weight

This is a term sometimes used to denote the weight of an aircraft prior to a flight, but it does not have a precise definition. To avoid confusion, the following definitions are preferable:

Ramp weight

This is the weight of an aircraft prior to engine starting. Its maximum value is fixed by structural considerations.*

* In the sense that a given take-off weight, or ramp weight, implies an initial-cruise weight, at which various manoeuvre and gust loadings have to be met.

Maximum take-off weight

This is the maximum allowable weight of the aircraft prior to commencement of take-off, fixed by structural considerations.* Typically, for an aircraft weighing around 300 000 lb, the maximum take-off weight would be 1000 lb to 2000 lb less than the maximum ramp weight.

It should be noted that a take-off weight less than the maximum may be dictated by airfield limitations such as runway length, engine-cut climb gradient, obstacle clearance, and noise-abatement procedures.

The difference between the ramp weight and the take-off weight is the fuel used during engine starting, warm-up taxying and waiting for take-off. It does not affect the range performance of the aircraft, but must be included in any calculation of the total fuel used, e.g. when estimating the direct operating cost.

A.2.2.2 Empty weight

Care is needed in calculating the empty weight of an aircraft. The weight required is the 'operating weight empty' (OWE), made up as follows:

Manufacturers' empty weight

This is the weight of the standard model of the aircraft as it leaves the factory. It includes basic furnishing.

Empty equipped weight

This is the manufacturers' empty weight to which has been added the weight of requirements of a particular airline in regard to additional equipment and furnishing, e.g. higher standard of radio fit, heavier seats, extra galleys etc.

Operating weight empty

This is the empty equipped weight to which has been added the weight of 'operators' items' such as crew and their baggage, and catering supplies.

When comparing the range performance of a number of aircraft it is essential that a common basis of definition of empty weight is used, and especial care must be taken that items are not left out, or included twice.

---

* In the sense that a given take-off weight, or ramp weight, implies an initial-cruise weight, at which various manoeuvre and gust loadings have to be met.

Examples of such offending items are unusable fuel, water, and toilet fluid, which are sometimes included in empty equipped weight, and sometimes in operators' items. These can easily amount to a few per cent of payload. Another pitfall is that some aircraft proposals include a contingency for weight growth,* while others do not.

A.2.2.3 Payload

Range calculations are normally made with full passenger payload at a certain defined seating standard. Sometimes an additional freight payload is included. The conventional (tourist) allowance is 200 lb for a passenger and his baggage, but other values within the range 190 lb to 210 lb are sometimes used. A maximum payload is usually quoted by manufacturers, representing an adequate allowance for passengers plus a modest freight load. This maximum payload, added to the operating weight empty, must not exceed the maximum 'zero-fuel weight' (ZFW), which is a structural design condition for the aircraft.

A.2.2.4 Fuel reserves

A typical flight plan (Fig.A5) calls for the ability to overshoot** and divert from the destination airfield to an alternate airfield 200 miles away, and to hold in the vicinity of this alternate airfield for a period of 45 minutes at 5000 ft. In order to calculate the diversion fuel, the weight of the aircraft at beginning of diversion is needed, and similarly, to calculate the holding fuel the weight at the end of diversion (i.e. start of hold) is needed. Guesses at these weights, followed by one or two iterations are usually sufficient to arrive at values of diversion and hold fuel to sufficient accuracy.

The diversion is usually flown using a 'best-range' technique, for a 200 mile diversion a cruise height of around 25000 ft would probably be chosen, and the 'best range' cruise speed would typically be a Mach number of about 0.6. Holding is done at a speed close to the speed for a minimum drag, in order to minimise fuel consumption; a speed 10%-15% above the speed for minimum drag is normally chosen, in order to ease control in turbulent conditions.

---

* Engine contingency allowances must be checked, as well as airframe.

** Overshoot fuel is typically three-quarters of the allowance for a normal take-off (see section A.2.2.5).

In addition to the above fuel reserves, an 'en-route' allowance of 5% of the stage fuel is often included, to allow for deviations from the specified flight path, and non-optimum handling of the aircraft. For the purposes of estimating direct operating cost it is usual to assume that this en-route fuel allowance is consumed (whereas diversion and hold fuels are not).

A.2.2.5 Fuel allowances

In addition to the fuel used during climb, cruise and descent, allowance must be made for fuel consumed during:

(i) start, warm-up, taxying and waiting for take-off. The fuel used here does not affect design range (see section A.2.2.1), but must be accounted for in direct operating cost.

(ii) Take-off and acceleration to 1500 ft.

(iii) Deceleration to approach speed.

(iv) Circuit, approach and land.

(v) Taxi-in.

Warm-up fuel is usually assumed to be at idling thrust. Taxi fuel is based on the thrust required to overcome folling friction ($\sim$0.025 × taxi weight), plus an allowance to cover ground manoeuvring and braking. Take-off fuel is normally based on the use of take-off thrust for a period of $1\frac{1}{2}$ minutes. Fuel used during deceleration to approach speed, circuit and land will depend on the assumptions made regarding the distance covered during this manoeuvre (not normally credited to range), and the aircraft configuration.

A.2.3 Flight trajectory

A typical flight plan is shown in Fig.A5. The climb schedule chosen will depend on whether the flight is to be made using a technique appropriate to minimum flying time, maximum range or minimum direct operating cost; in some cases, angle of climb is the criterion, in order to reach an Air Traffic Control reporting point at a given altitude, for example. Similar remarks apply to the cruise and descent conditions chosen. The cruise is normally made at constant speed and altitude, but for long ranges a 'stepped-cruise' of several stages at various cruise altitudes is sometimes used; in areas where traffic is light, ATC sometimes allow a 'cruise-climb' technique in which speed is held constant but cruise altitude steadily increases as the aircraft weight decreases.

Descent rates are normally limited so that the cabin altitude rate of descent does not exceed 300 feet per minute.

Diversion is assumed to start from ground level, and a long-range climb technique would be used. In theory, for a short diversion, the minimum-fuel trajectory would be one consisting solely of a climb directly followed by a descent; in practice, a maximum height during diversion of around 25000 ft is normally imposed by ATC, so a short cruise phase is necessary. The diversion descent is assumed to be broken at 5000 ft by a holding manoeuvre; a typical chart giving holding fuel consumption is given in Fig.A6.

A.3 EXAMPLES OF RANGE CALCULATION

	Weight (lb)	Fuel (lb)	Distance (nm)	Time (min)
(1) Ramp weight	267 600			
(2) Start, warm-up, taxi, wait for take-off		600	-	10.0
(3) Take-off weight, (1)-(2)	267 000			
(4) Operating weight empty	170 000			
(5) Payload	50 000			
(6) Zero-fuel weight (4)+(5)	220 000			
(7) Flight fuel load (3)-(6)		47000		
Reserves				
Hold at 1.1 V_{md}/5000 ft/45 min	223 300 (mean)	6320	-	-
Diversion descent from 25000 ft	227 000	450	69 } 200	-
Diversion climb 1500 ft to 25000 ft	232 500 (start)	3860	62 } 200	-
Diversion cruise M = 0.6/25000 ft*	228 000 (mean)	1660	69 } 200	-
Diversion overshoot to 1500 ft		600	-	-
5% stage fuel (0.05 × 30500)		1525	-	-
(8) TOTAL RESERVES		14415		
Allowances				
Take-off and acceleration to 1500 ft		800	-	1.5
Deceleration, circuit, approach, land, taxi		1300	-	10.0
(9) TOTAL ALLOWANCES		2100		
(10) FUEL FOR RANGE (7)-(8)-(9)		30485		
Long-range descent 30000 ft to 1000 ft	236 000 (mean)	520	86	17.3
Long-range climb to 30000 ft	266 200 (start)	6320	121	19.2
Cruise at M = 0.8/30000 ft**	248 000 (mean)	23645	903	115.0
TOTAL RANGE AND TIME		(30485)	1110	173.0

TOTAL FUEL USED = 30485 + 2100 + 1525 + 600 = 34710 lb

* At 0.0416 nm/lb.

** At 0.0382 nm/lb.

A.3.1 Description of calculation method

(i) The flight fuel load is given by:
(Take-off weight) - (operating weight empty + payload)
These items are defined in section A.2.2.

(ii) The reserve fuel requirements are calculated by an iterative process, starting from the weight at the end of diversion, which is the zero-fuel weight (220 000 lb) plus an allowance for landing and taxying (say about 300 lb), i.e. about 220 300 lb. A first rough estimate of the holding fuel, from Fig.6, gives the holding fuel as approximately 6000 lb (¾ h at 8000 lb/h). Using a mean weight during holding of 223 300 lb, gives the fuel for ¾h hold as 6320 lb. A further iteration does not increase the accuracy with which Fig.6 can be read.

(iii) From (ii) above, it follows that the mean weight during the diversion descent would be about 227 000 lb, giving a descent fuel of 450 lb from Fig.A3(c), and a descent distance of 69 nm from Fig.A3(b).

(iv) One now needs to calculate the fuel used, and distance covered, during the diversion climb. A first estimate, from Fig.A2(c) shows the climb fuel to be about 4000 lb, and the distance covered (Fig.A2(b)) to be about 60 miles; thus the diversion cruise distance is about 70 nm (200-69 descent - 60 climb) and the diversion cruise fuel about 1500 lb (70 nm ÷ 0.0416 nm/lb). Thus the weight at the start of the diversion climb is approximately 5500 lb (4000 lb climb + 1500 lb cruise) greater than the weight during diversion descent, i.e. approximately 232 500 lb. Using this weight gives 3860 lb fuel and 62nm distance for the diversion climb. Hence the diversion cruise distance is 69 nm.

(v) The 5% stage fuel reserve needs to be calculated by an iterative process. One can see that the total of reserves plus allowances will be approximately 17000 lb, leaving 30000 lb stage fuel as the first estimate.

(vi) Once these reserves have been calculated for one case, only a slight modification to the 5% stage fuel item is needed for other values of flight fuel load.

(vii) A similar process to the above is used for estimating fuels, distances and times for the stage fuel.

A.3.2 Notes on above calculation

(i) A slight difficulty arises in regard to the en-route allowance of 5% stage fuel (which is normally assumed to be consumed) since it affects the calculation of aircraft weight during the flight. In the above example, it is accounted for at the end of the first descent, i.e. immediately prior to diversion. If the purpose of the calculation is to *compare* ranges of various aircraft designs, rather than to obtain an *absolute* value of range, it is convenient to omit this item.

(ii) A further simplification is to calculate the reserve fuels at a fixed weight; the landing weight at the destination airfield is usually chosen in this instance.

(iii) The above calculation represents a typical procedure used to present aircraft performance. In actual operations, further allowances would be made for headwinds and non-standard atmospheric conditions.

A.4 THE PAYLOAD-RANGE DIAGRAM

From the example in section A.3, it is obvious that the range of an aircraft, operated at its maximum take-off weight, will decrease as payload is increased, and *vice versa*, payload and fuel having to be interchanged. This is given by the line AB in Fig.A7 which shows for the aircraft example of section A.3 (operated at its maximum take-off weight of 267 000 lb) that a theoretical payload of 97000 lb is possible for zero range, and that with zero payload a theoretical range of about 2300 nm would be obtained (on a fuel load of 97000 lb). The line AB is usually nearly straight; its slope is determined by two factors:

(i) The performance of the aircraft in terms of miles per pound of fuel - the 'specific range'.

(ii) The variation of reserve fuel with range.

If the maximum take-off weight of the aircraft is increased (say by 10000 lb, with a consequent increase in OWE of 1000 lb) the point A will be raised by 9000 lb to A', and the range at a given payload will be increased, as shown by the line A'B'. Thus lines AB, A'B' etc. give the payload-range performance at a given take-off weight.

However, it will not normally be possible to operate an aircraft throughout the complete range AB for two reasons:

(i) A volumetric limit on fuel tankage may prevent operation over long ranges at small payloads. The line JK on Fig.A7 represents, for example, a maximum tankage limit of 60000 lb.

(ii) As well as a structural limitation on maximum take-off weight, there are also structural limitations related to particular stressing cases at the maximum zero-fuel weight, and the maximum landing weight. It is necessary that the maximum zero-fuel weight should be not less than the sum of the maximum payload and the operating weight empty (say 50000 lb passengers + baggage, 10000 lb freight and 170 000 operating weight empty, giving 230 000 lb for the present example), and that the maximum landing weight should not be less than the maximum zero-fuel weight plus the fuel reserves (giving 245 000 lb for the present example). The maximum zero-fuel weight limitation is shown as the line EF in Fig.A7. There may also be a volumetric limitation on the payload that can be carried, due to the size of the passenger cabin and the freight holds.

Thus the normal operating limits for the aircraft are given by the boundary EFJK. To operate in the area AEF would involve increasing the maximum zero-fuel weight (and possibly the maximum landing weight also) with consequent increases in structure weight. This would result in the lowering of the line AB, and loss of range at a given payload in the region FJ. To obtain more than 1600nm range, by operating in the area JKB, would require extra fuel tankage to be fitted, again at the expense of some weight penalty. Such a modification would need to be associated with an increase in take-off weight (to A'B' say) if a reasonable payload is to be carried at long range; however, this would not be necessary if the increased range is required only for ferry purposes.

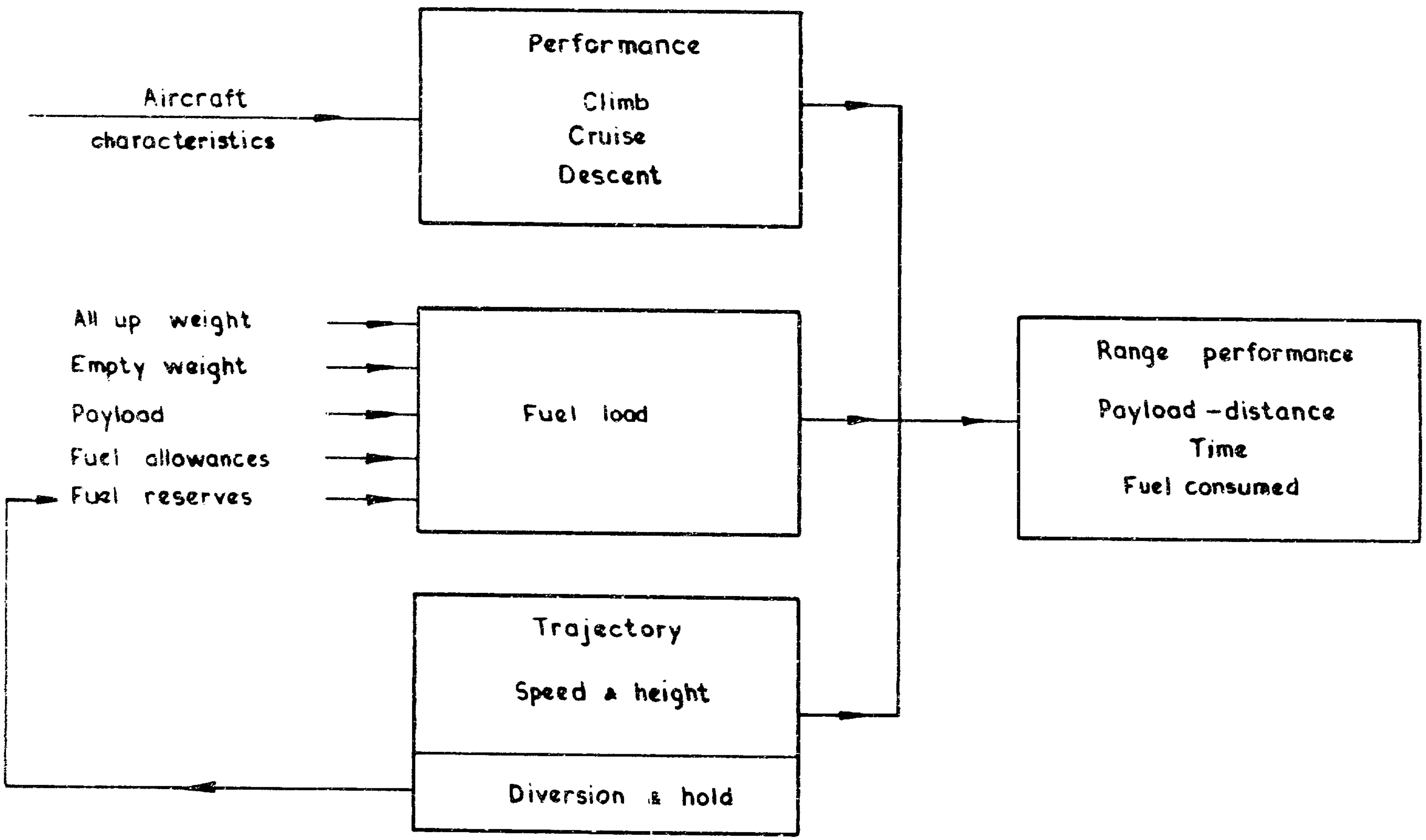
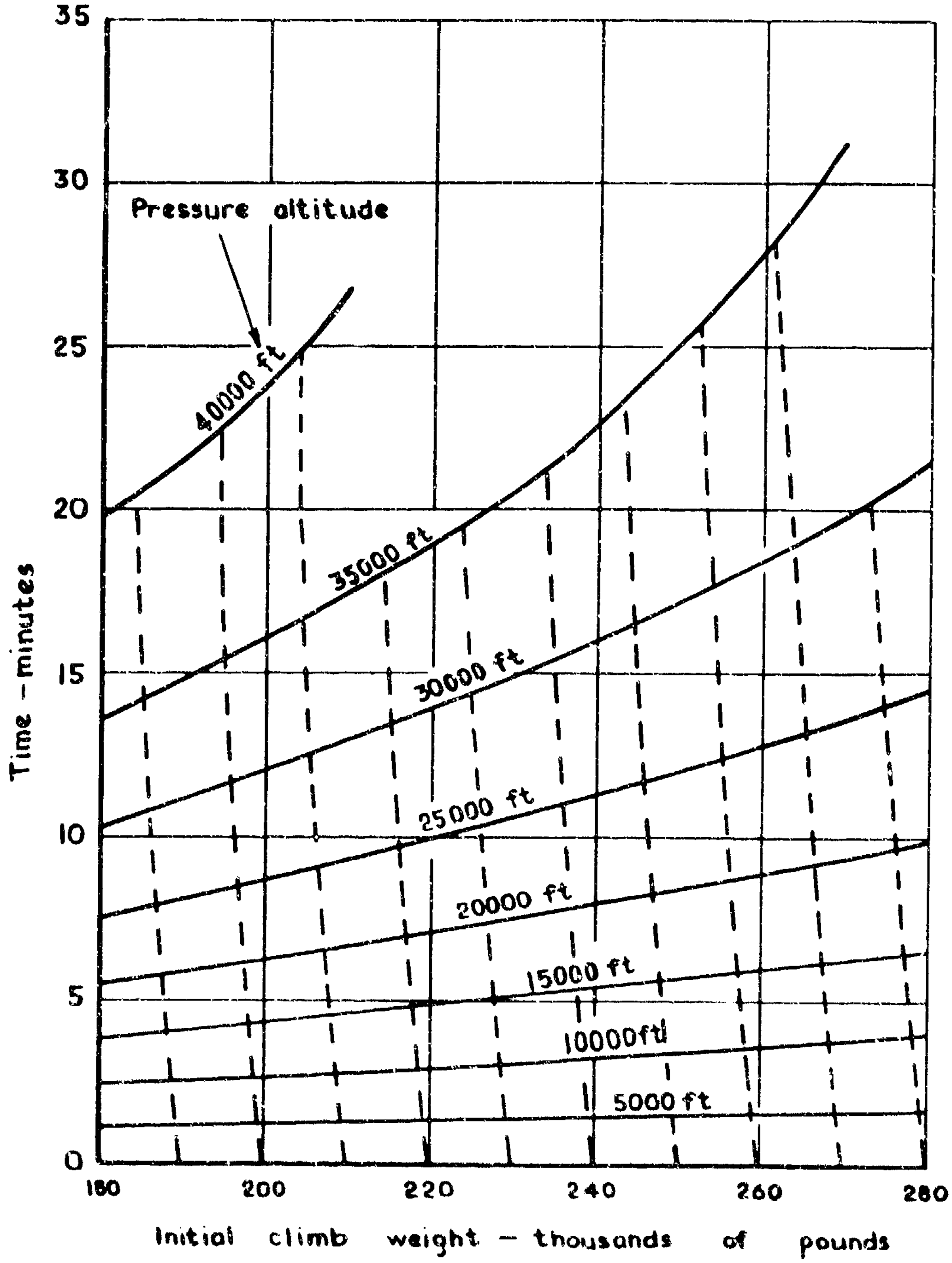


Fig. A1 The range estimation process

Fig. A2a



TR 73164

Fig. A2a Time to climb

TR 73164

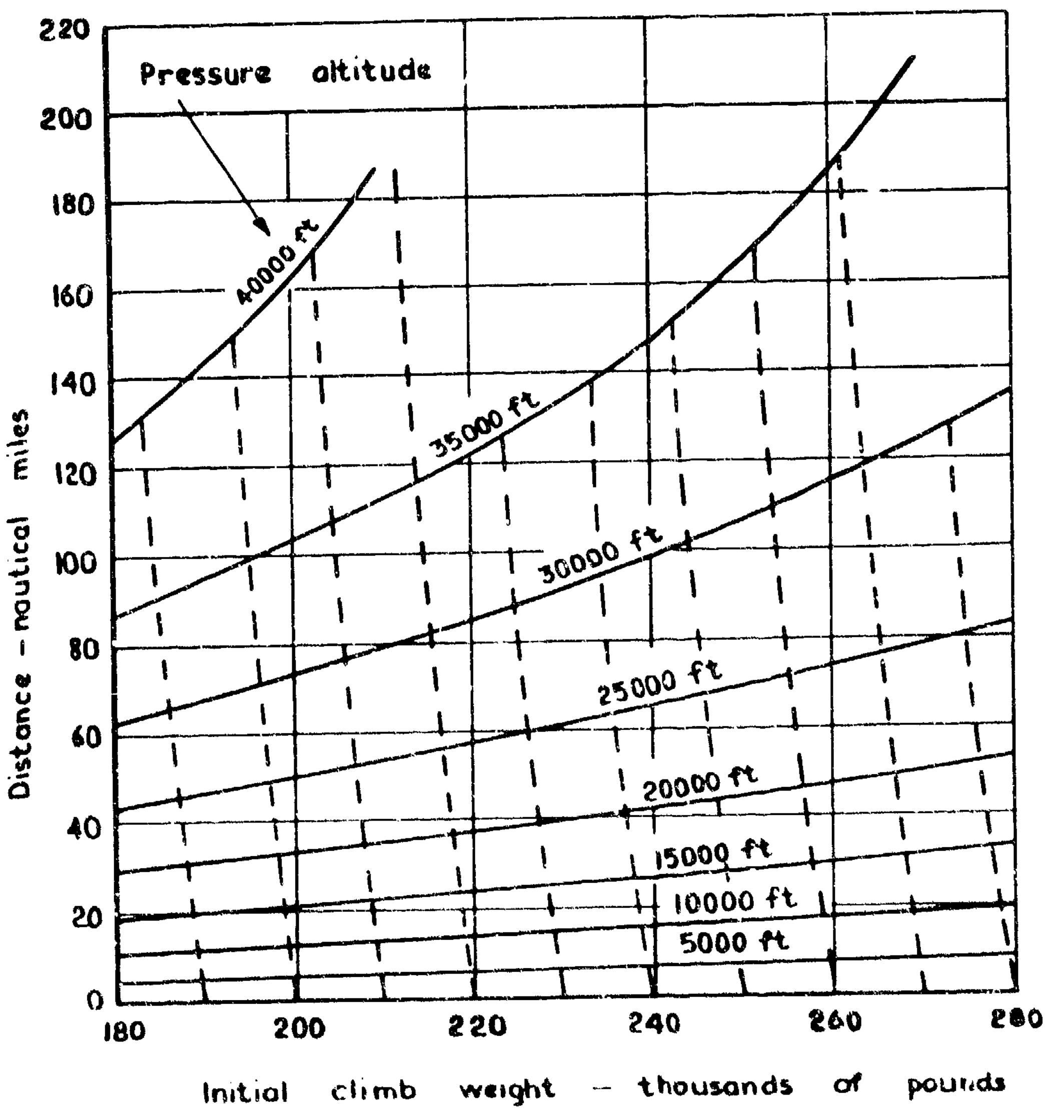


Fig. A2b Distance to climb

Fig. A2c

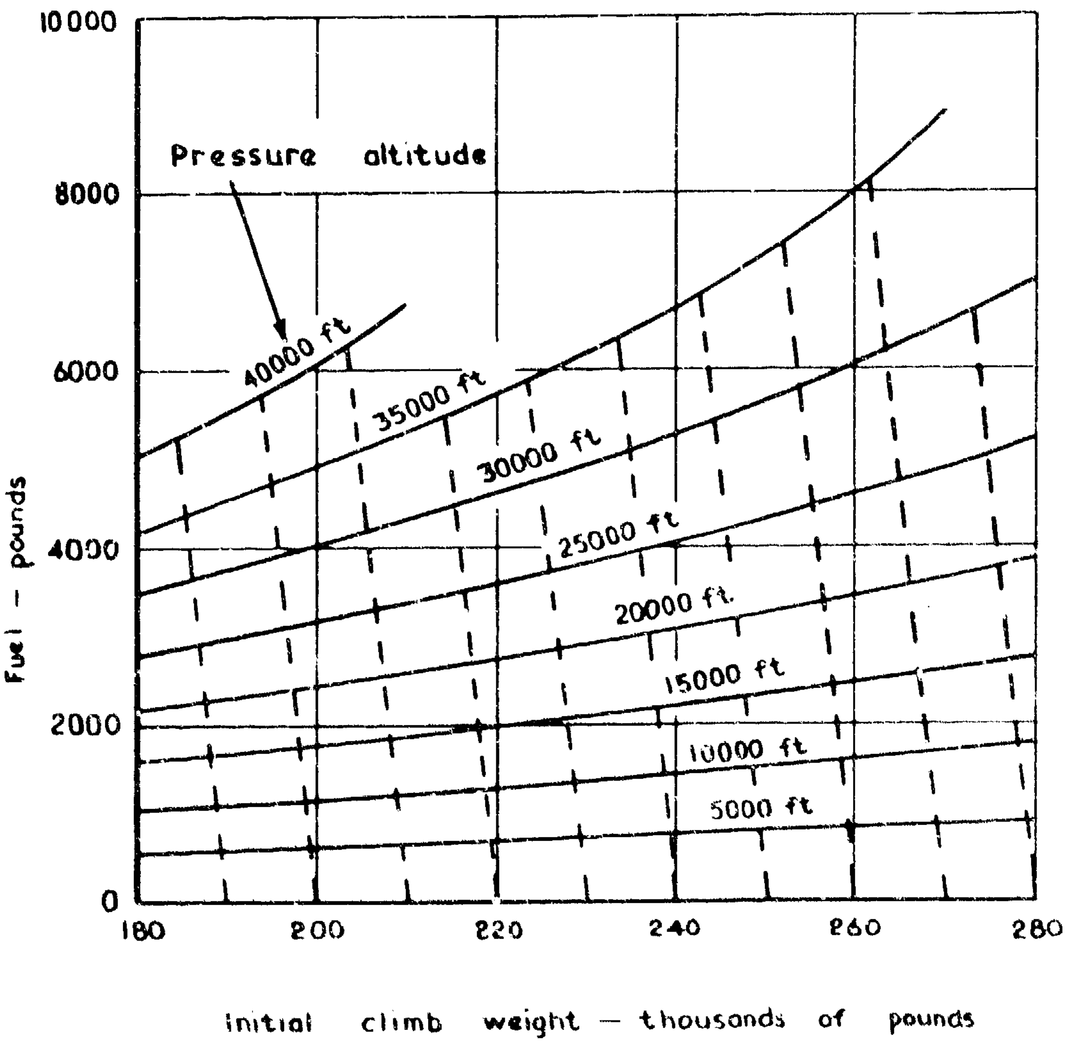
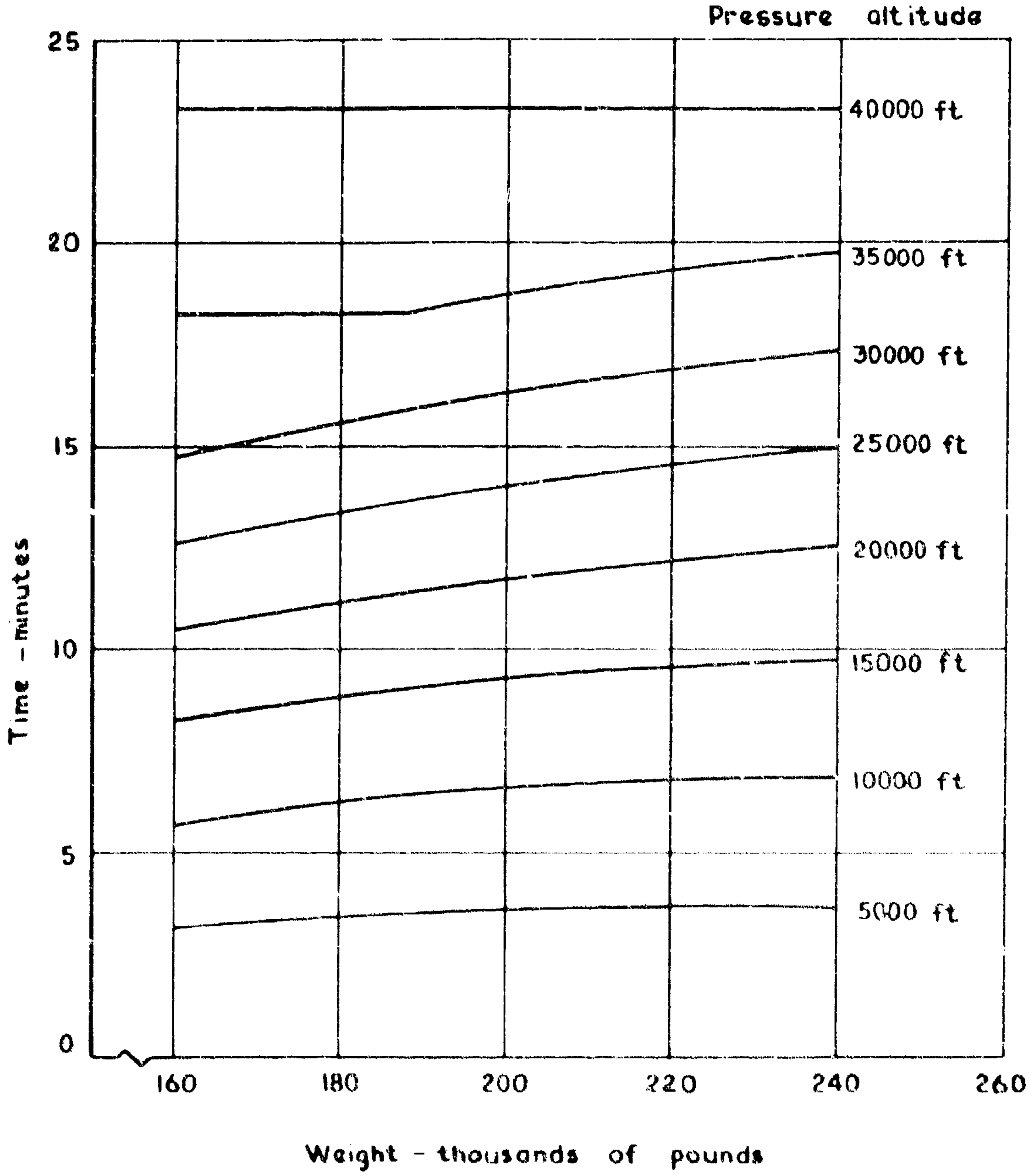


Fig. A2c Fuel to climb



TR 73164

Fig. A3a Time to descend

32 - 9

Fig. A3b

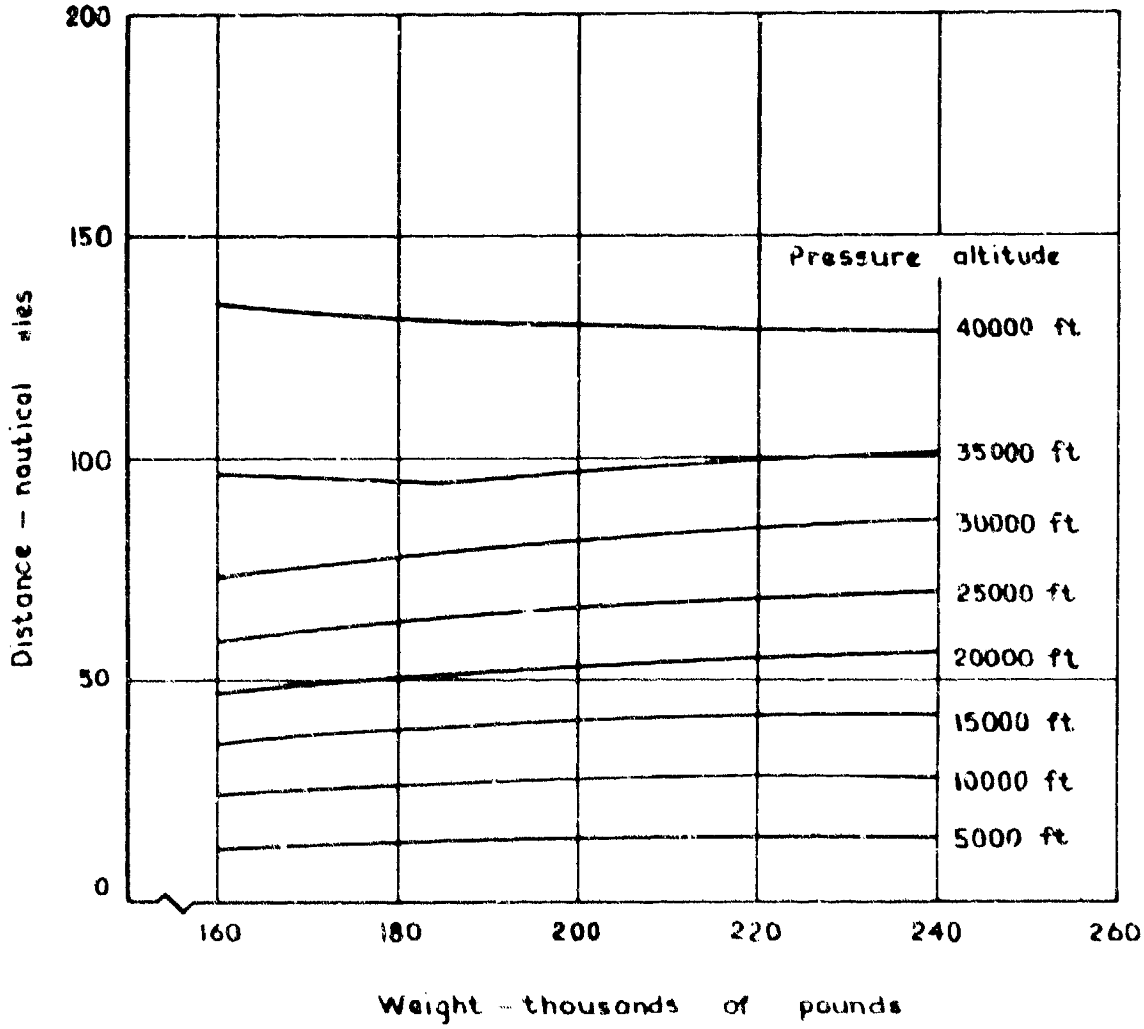
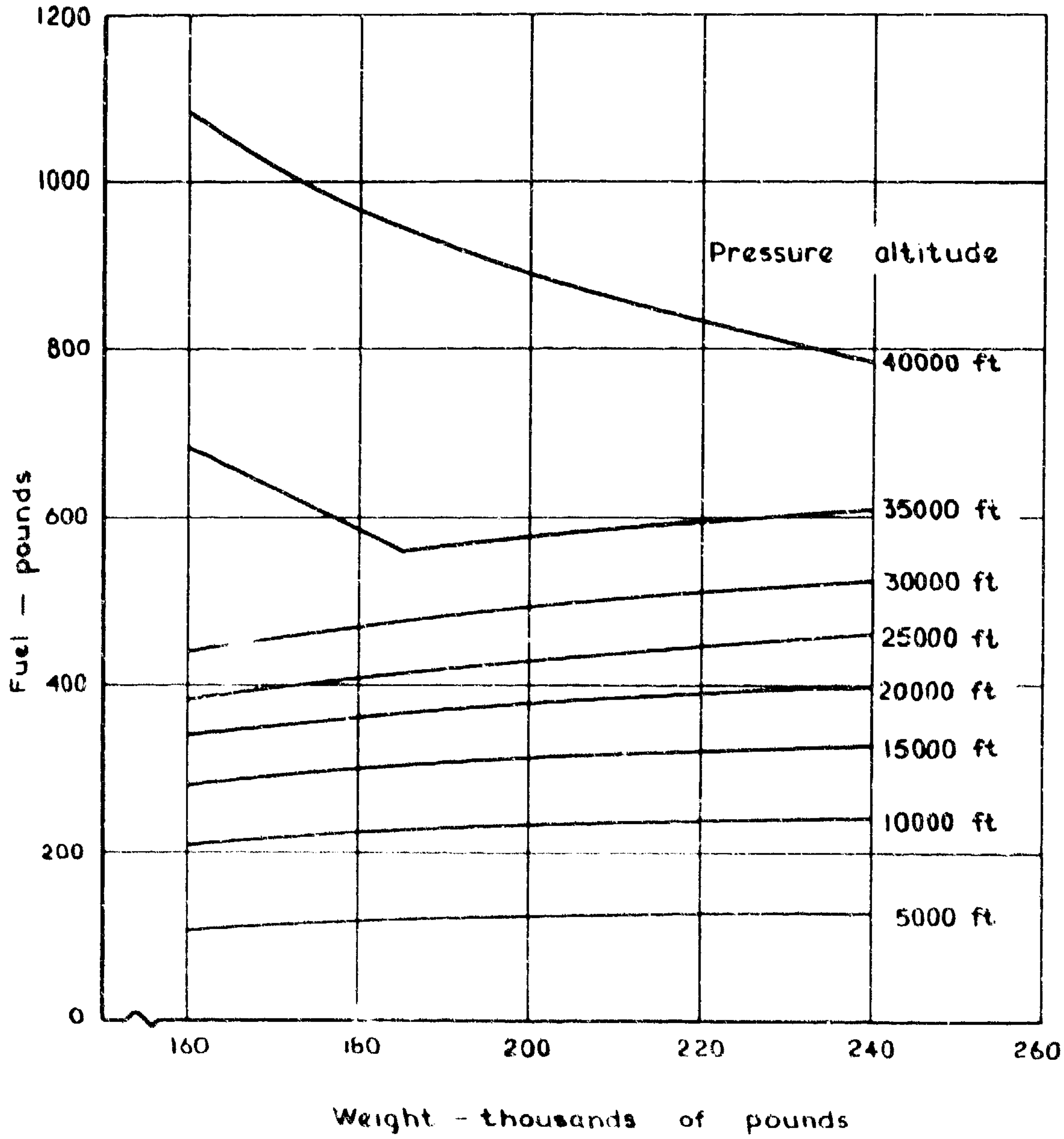


Fig. A3b Distance to descend

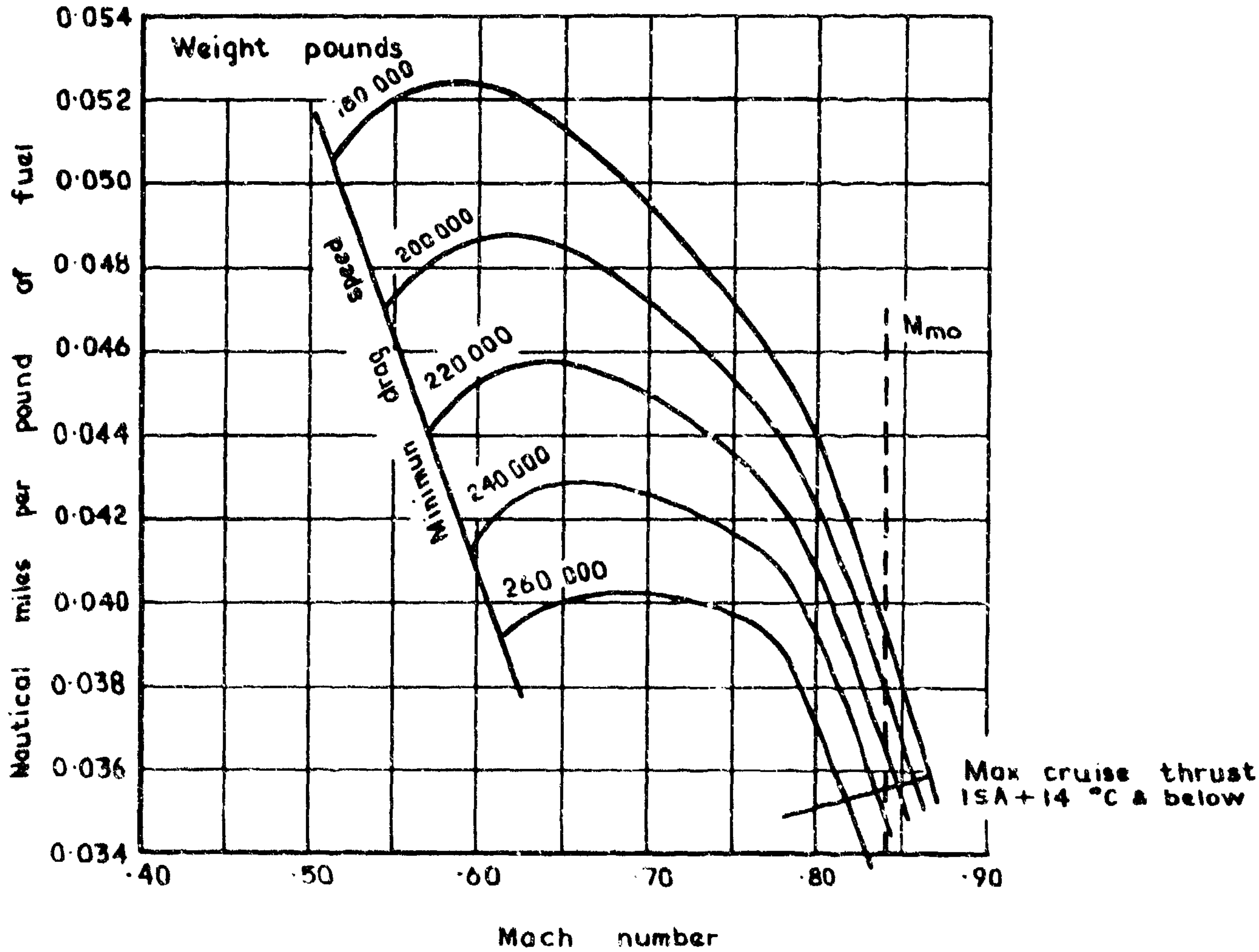


TR 73164

Fig. A3c Fuel to descend

32·8

Fig. A4



TR 73164

Fig. A4 Specific range at 30000 ft

Fig.A5

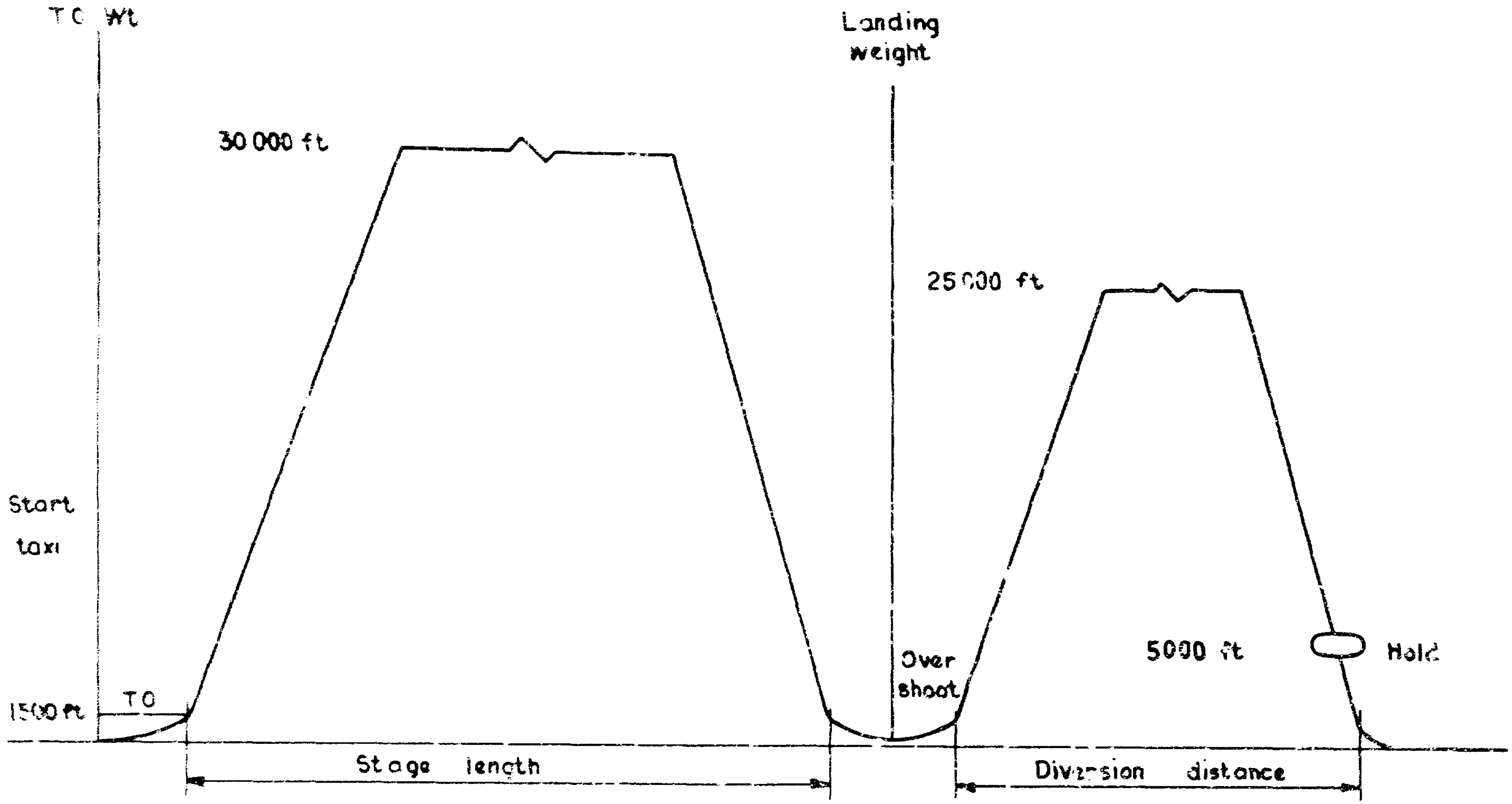


Fig. A5 Typical flight plan

Fig. A6

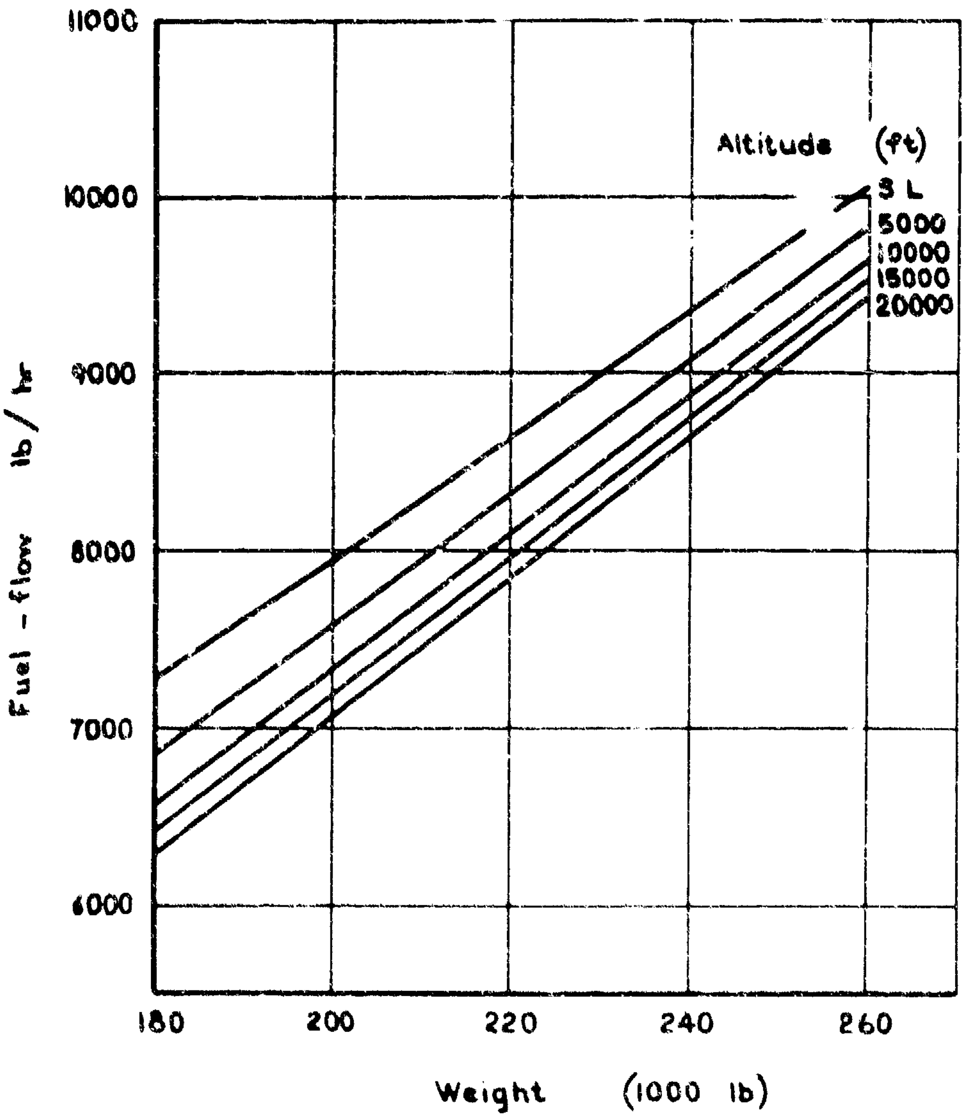


Fig. A6 Holding fuel flow at 1·1 Vmd

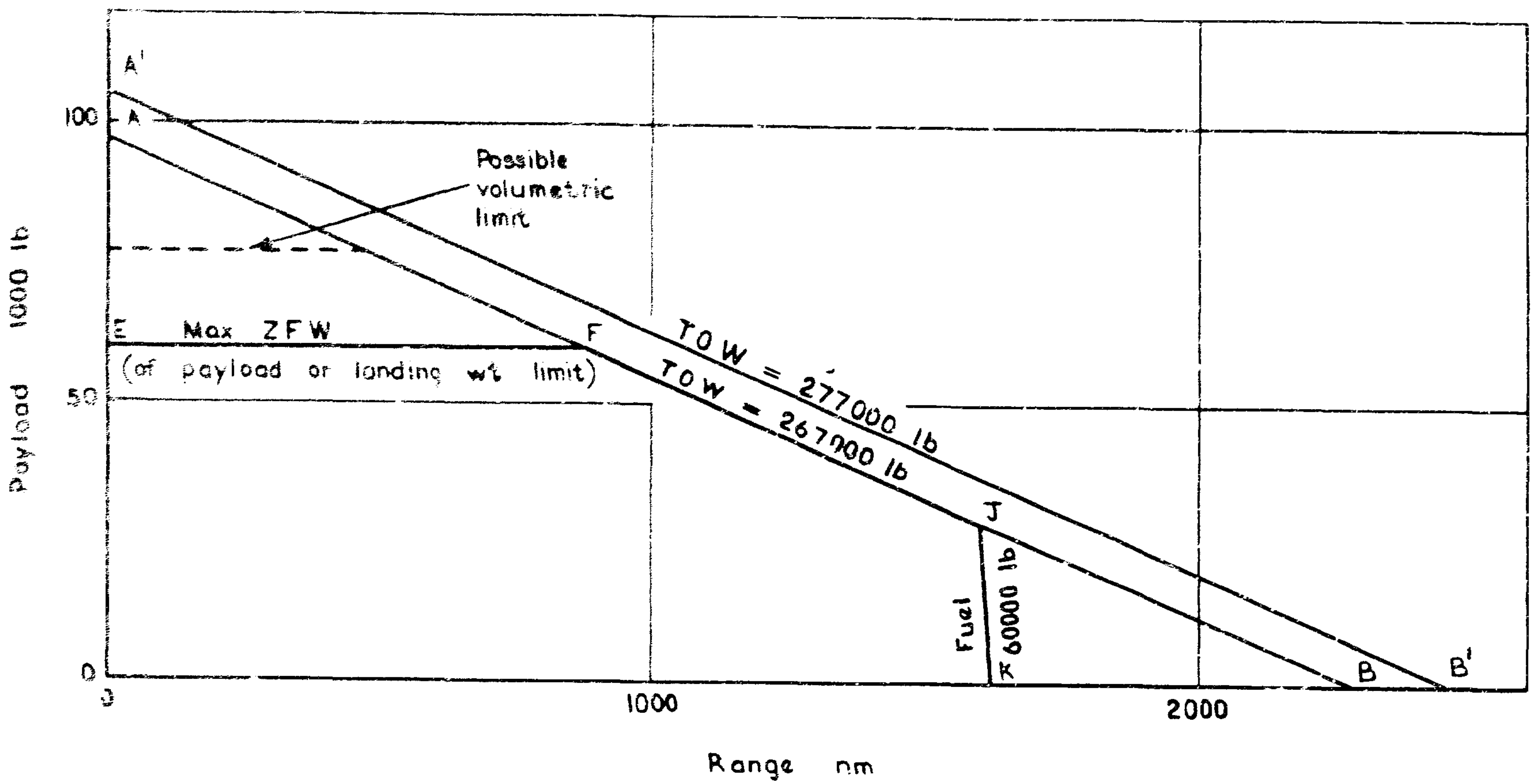


Fig. A7 Typical payload - range diagram

Appendix B

EXAMPLES OF SPECIFIC RANGE CALCULATIONS

For the purposes of these examples, and those in Appendix C, an aircraft with the following characteristics is assumed:

Initial cruise weight	=	300 000 lb
Final cruise weight	=	200 000 lb
Initial (or constant) cruise height	=	30 000 ft (σ = 0.3747)
Wing reference area	=	3 000 ft^2

$$C_{D_0} = 0.02 \quad ; \quad \frac{\pi A}{K} = 20 \quad ; \quad c = 0.7 \text{ lb/lb/h} .$$

Thus we have from equations (9), (10) and (11)

$$C_{L_{md}} = (20 \times 0.02)^{\frac{1}{2}} = \underline{0.6325}$$

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2}\left(\frac{20}{0.02}\right)^{\frac{1}{2}} = \underline{15.811} \quad ; \quad 0.866\left(\frac{L}{D}\right)_{max} = \underline{13.693}$$

$$(V_{md})_i = \left(\frac{200}{0.000891 \times 0.6325}\right)^{\frac{1}{2}} = 595.7 \text{ ft/s} = \underline{352.7 \text{ kn}} .$$

The specific-range performance is plotted in Fig.2. The variation of specific range with speed is calculated as follows:

For W = 300 000 lb ,

$$\frac{1}{cW}\left(\frac{L}{D}\right)_{max} = \frac{15.811}{0.7 \times 300\ 000} = \underline{75.29 \times 10^{-6} \text{ h/lb}}$$

$$D_{min} = \frac{W}{(L/D)_{max}} = \frac{300\ 000}{15.811} = \underline{18974 \text{ lb}}$$

m		1.0	1.1	1.2	1.3	1.4
$V = mV_{md}$	kn	352.7	387.9	423.1	458.4	493.6
$(L/D)/(L/D)_{max}$	equation (12)	1.0	0.9821	0.9370	0.8765	0.8096

therefore

$\frac{V}{cW}\left(\frac{L}{D}\right)$	n mile/lb	0.0265	0.0287	0.0298	0.0303	0.0301
T	lb	18974	19320	20250	21650	23440

The speed at a given thrust is obtained from equation (15); for example, for a thrust of 20000 lb

$$m^2 = \frac{20000}{18974}\left[1 + \sqrt{1 - \left(\frac{18974}{20000}\right)^2}\right] = 1.3874 \quad ; \qquad m = 1.1779$$

therefore

$$V = 1.1779 \times 352.7 = \underline{415.3 \text{ kn}} \quad .$$

The specific range for a thrust of 20000 lb is then given by equation (13) as

$$\frac{V}{cW}\frac{L}{D} = \frac{415.3}{0.7 \times 300\,000}\left(\frac{2 \times 15.811}{1.1779^2 \times 1.1779^{-2}}\right) = \underline{0.0296 \text{ n mile/lb}} \quad .$$

Examples are now given of estimation of maximum specific range for the following cases

(i) A Mach number of 0.8 (in the stratosphere).

(ii) An engine setting as in (i) above, speed and altitude free.

(iii) A fixed altitude of 30000 ft.

Case (i)

True airspeed at $M = 0.8$ in the stratosphere = 774.5 ft/s
Speed for minimum drag in eas $(V_e)_{md}$ = 364.5 ft/s

therefore

Relative air density for cruise at $M = 0.8 = \left(\frac{364.5}{774.5}\right)^2 = 0.2215$

Hence cruise height (from atmosphere tables) = 42200 ft

From section 5.1, maximum specific range at a constant speed is obtained when (L/D) is a maximum, and from equation (3)

$$\left[-\left(\frac{dR}{dW}\right)_{max}\right]_{M=0.8} = \frac{774.5}{1.688} \times \frac{15.811}{0.7 \times 300\ 000} = \underline{0.0345\ \text{nm/lb}}$$

therefore

$$\frac{T}{\rho} = \frac{D_{min}}{\rho} = \frac{18974}{0.2215 \times 0.00238} = \underline{36 \times 10^6\ \text{lbft}^3/\text{slug}} \ .$$

Case (ii)

From section 5.2, maximum specific range at a fixed engine setting occurs when $(L/D) = 0.943\ (L/D)_{max}$, i.e.

$$\frac{L}{D} = 0.943 \times 15.811 = \underline{14.91}$$

therefore

$$\text{thrust required} = \frac{300\ 000}{14.91} = 20120\ \text{lb}$$

and for the value of T/ρ given above, cruise height is given by

$$\rho = \frac{20120}{36 \times 10^6} = 0.000559\ , \quad \text{i.e.}\ h = \underline{41100\ \text{ft}} \quad \text{and} \quad \sigma = \underline{0.2348} \ .$$

From equation (25), the speed for maximum specific range is

$$(V_e)_{es} = 1.189\ (V_e)_{md} = 1.189 \times 364.5 = 433.4\ \text{ft/s eas}$$

therefore

$$V_{es} = \frac{433.4}{(0.2348)^{\frac{1}{2}}} = 894.4\ \text{ft/s} = \underline{429.6\ \text{kn}} \ .$$

Therefore the maximum specific range at an engine setting giving $T/\rho = 36 \times 10^6\ \text{lbft}^3/\text{slug}$ is

$$\left(-\frac{dR}{dW}\right)_{max} = \frac{529.6 \times 14.91}{0.7 \times 300\ 000} = \underline{0.0376\ \text{n mile/lb}} \ .$$

The above value of specific range is 9% better than the value obtained in case (i).

Case (iii)

From section 5.3, maximum specific range at a given altitude is obtained when $(L/D) = 0.866\ (L/D)_{max}$, i.e.

$$\frac{L}{D} = 0.866 \times 15.811 = \underline{13.69} \quad .$$

From equation (29), the speed for maximum specific range is

$$(V_e)_h = 1.316\ (V_e)_{md} = 1.316 \times 364.5 = 479.7 \text{ ft/s eas} = 284.0 \text{ kn eas}$$

therefore speed for maximum specific range at 30000 ft is

$$V_h = \frac{284.0}{(0.3747)^{\frac{1}{2}}} = \underline{464.2 \text{ kn}} \quad .$$

Hence the maximum specific range at a cruise height of 30000 ft is given by

$$\left(-\frac{dR}{dW}\right)_{max} = \frac{464.2 \times 13.69}{0.7 \times 300\ 000} \simeq \underline{0.0302 \text{ n mile/lb}} \quad .$$

Appendix C

EXAMPLES OF RANGE CALCULATIONS

From equation (47) we have that the speed for maximum Breguet range, for a given initial cruise altitude, is $3^{\frac{1}{4}}(V_{md})_i$, which in the present example is

$$3^{\frac{1}{4}} \times 352.7 = \underline{464.2 \text{ kn}} \ .$$

This speed is marked as point A in Fig.2. The Breguet cruise-climb will be at a constant ratio of lift to drag, given by equation (29) as

$$0.866 \left(\frac{L}{D}\right)_{max} = \underline{13.693} \ .$$

For the various methods of cruising at constant altitude, with the initial cruise speed of 464.2 kn calculated above, the constant C_L and L/D trajectory is shown by the line AB in Fig.2, the constant-speed trajectory by the line AC, and the constant-thrust trajectory by the line AD.

The Breguet range is given by equation (34B) as

$$R_{Br} = \frac{464.2}{0.7} \times 13.693 \log_e \left(\frac{1}{1 - 0.33\dot{3}}\right) = \underline{3682 \text{ n mile}} \ .$$

Since W/σ = constant

$$\sigma_F = \frac{W_F}{W_i}\sigma_i = \frac{2}{3} \times 0.3747 = 0.2498 \quad \text{giving} \quad h_f = \underline{39800 \text{ ft}} \ .$$

For cruising at constant C_L and altitude, equation (35B) gives

$$R = 2 \times \frac{464.2}{0.7} \times 13.693 \left[1 - (1 - 0.33\dot{3})^{\frac{1}{2}}\right] = \underline{3333 \text{ n mile}} \ .$$

Alternatively, from Fig.3

$$R \approx 0.905 R_{Br} = \underline{3330 \text{ n mile}} \ .$$

Since W/V^2 = constant

$$V_f = V_i \left(\frac{W_f}{W_i}\right)^{\frac{1}{2}} = 464.2 \times 0.66\dot{6}^{\frac{1}{2}} = \underline{379.0 \text{ kn}} \ .$$

For cruising at constant V and altitude, equation (37D) gives

$$R = \frac{464.2}{0.7} \times 13.693(3^{\frac{1}{2}} + 3^{-\frac{1}{2}}) \tan^{-1}\left(\frac{0.33\dot{3}}{3^{\frac{1}{2}} + 0.66\dot{6} \times 3^{-\frac{1}{2}}}\right) = \underline{3274 \text{ n mile}} \ .$$

Alternatively, from Fig.4

$$R \simeq 0.890 \ R_{Br} = \underline{3280 \text{ n mile}} \ .$$

For cruising at constant T and altitude, equation (39E) gives

$$\frac{V_f}{V_i} = \left\{\frac{1 + [1 - 4(1 - 0.33\dot{3})^2/(3^{\frac{1}{2}} + 3^{-\frac{1}{2}})^2]^{\frac{1}{2}}}{1 + [1 - 4/(3^{\frac{1}{2}} + 3^{-\frac{1}{2}})^2]^{\frac{1}{2}}}\right\}^{\frac{1}{2}} = 1.10045$$

therefore

$$V_f = 1.10045 \times 464.2 = \underline{510.8 \text{ kn}} \ .$$

From equation (45)

$$\frac{V_{mean}}{V_i} = \frac{2}{3} \times 3\left[\left(1 + \frac{3^{\frac{1}{2}} + 3^{-\frac{1}{2}}}{2 \times 3^{\frac{1}{2}}}\right) - \frac{2}{3}\left(1.10045 + \frac{3^{\frac{1}{2}} + 3^{-\frac{1}{2}}}{2 \times 3^{\frac{1}{2}}} \times \frac{1}{1.10045}\right)\right] = 1.05832 \ .$$

From equation (44)

$$R = \frac{464.2}{0.7} \times 13.693 \times \frac{1}{3} \times 1.05832 = \underline{3203 \text{ n mile}} \ .$$

Alternatively, from Fig.6

$$R \simeq 0.869 \ R_{Br} = \underline{3200 \text{ n mile}} \ .$$

Appendix D

EFFECT OF VARIATION OF SFC WITH SPEED

A good approximation to the variation of specific fuel consumption with speed (at constant altitude) can be obtained by using a power-law equation of the form:

$$c = c_0 V^x \ .$$

On this assumption, the value of m (= V/V_{md} , see equations (12) and (29)) for maximum specific range, instead of being $3^{\frac{1}{4}}$ becomes

$$m = \left(\frac{3 - x}{1 + x}\right)^{\frac{1}{4}} \ .$$

For pure-jet engines, $x \simeq 0$, but it is found that for engines of low bypass ratio, such as the Avon, Spey and Conway, $x \simeq 0.2$, and for modern engines of high bypass ratio such as the RB211, $x \simeq 0.4$. The values of m for optimum specific range at a given altitude then become:

x	0	0.2	0.4
m_{opt}	1.316	1.236	1.167

Thus, other things being equal, one would expect the optimum cruise speed of an aircraft with an engine of high bypass ratio to be slightly less than that of an aircraft with engines of low bypass ratio.

With the expression for m given above, the relationships for C_D , C_L and (L/D) for optimum cruise at a given altitude become:

$$C_{D_h} = \frac{4}{3 - x} C_{D_0}$$

$$C_{L_h} = \left[\left(\frac{\pi A C_{D_0}}{K}\right)\left(\frac{1 + x}{3 - x}\right)\right]^{\frac{1}{2}}$$

$$\left(\frac{L}{D}\right)_h = \frac{1}{2}\left(\frac{L}{D}\right)_{max} [(1 + x)(3 - x)]^{\frac{1}{2}}$$

These relationships should be compared with equations (26), (27) and (28) given earlier.

SYMBOLS

A	aspect ratio
c	specific fuel consumption
C_D	overall drag coefficient
C_{D_0}	zero-lift drag coefficient
C_{D_i}	lift-dependent drag coefficient
C_L	overall lift coefficient
D	drag
H	calorific value of fuel
K	lift-dependent drag factor
L	lift
m	V/V_{md} , defined in equation (12)
q	dynamic pressure, $\frac{1}{2}\rho V^2$
R	range
S	wing reference area
t	time
V	true airspeed
V_e	equivalent airspeed
W	all-up weight
W_F	fuel weight
x	exponent in power-law relationship for specific fuel consumption
η_P	overall propulsive efficiency
ρ	air density
σ	ρ/ρ_0 , relative air density

Suffix

Br	Breguet range
es	engine setting
f	final conditions
h	constant altitude
i	initial condition
md	minimum-drag condition

REFERENCES

No.	Author	Title, etc.
1	D.H. Peckham	Range estimation for civil aircraft using manufacturers brochures. RAE unpublished work (1969)
2	A.A. Woodfield	Thrust measurement in flight: the requirements, current situation and future possibilities. RAE unpublished work (1969)
3	D. Küchemann J. Weber	An analysis of some performance aspects of various types of aircraft designed to fly over different ranges at different speeds. Progress in Aeronautical Sciences, 9 Pergamon (1968)
4	A.D. Edwards	Performance estimation of civil jet aircraft. Aircraft Engineering, p.95 (1950)
5	D.O. Dommasch S.S. Sherby T.F. Connolly	Airplane aerodynamics. Pitman, London
6	C.D. Perkins R.E. Hage	Airplane performance, stability and control. Wiley, London
7	-	Estimation of range. Royal Aero. Soc. Data Sheet EG 4/1
8	A. Miele	Flight mechanics. Pergamon, London, Vol.1

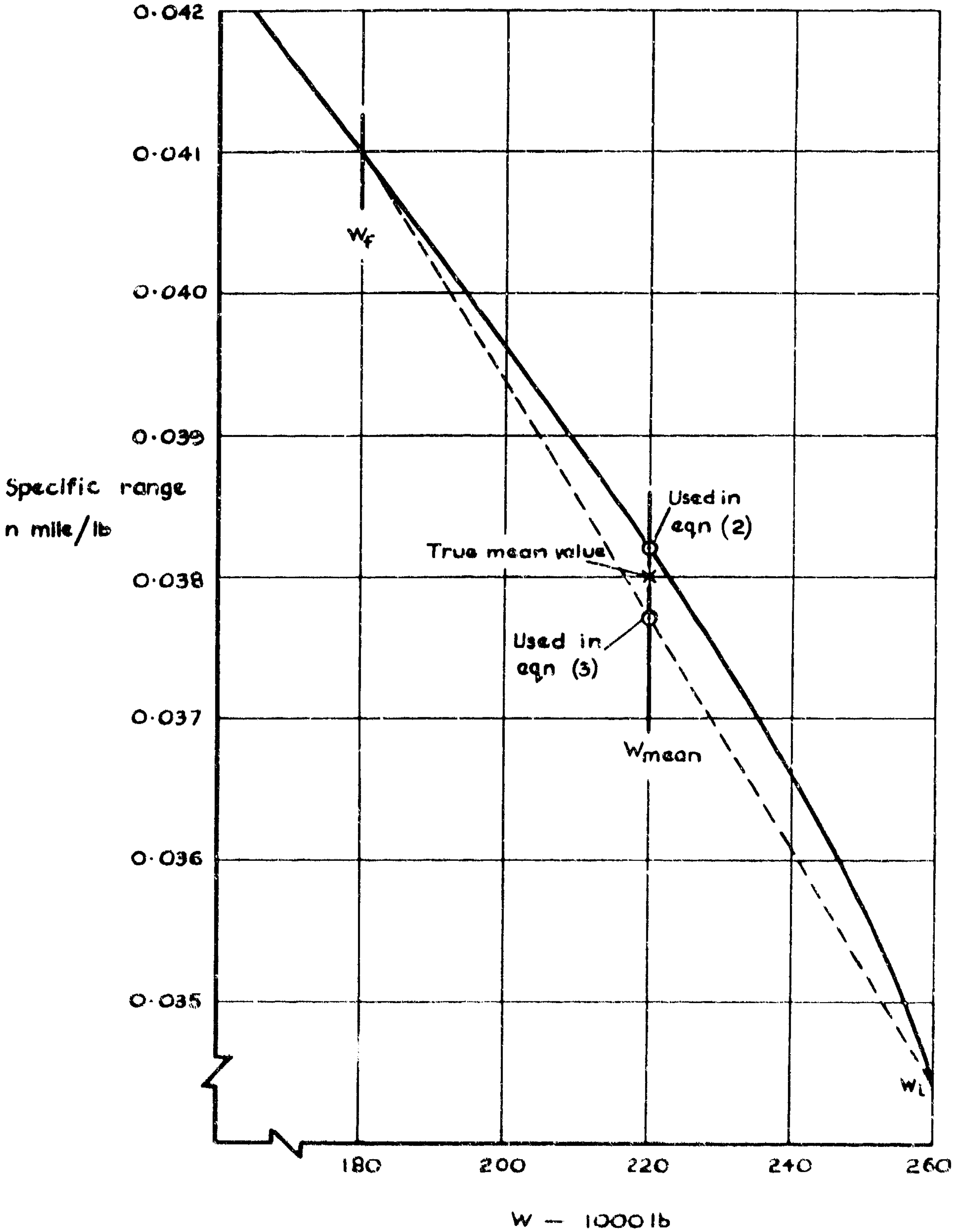


Fig. 1 Typical variation of specific range with weight, at constant speed and altitude

Fig. 2

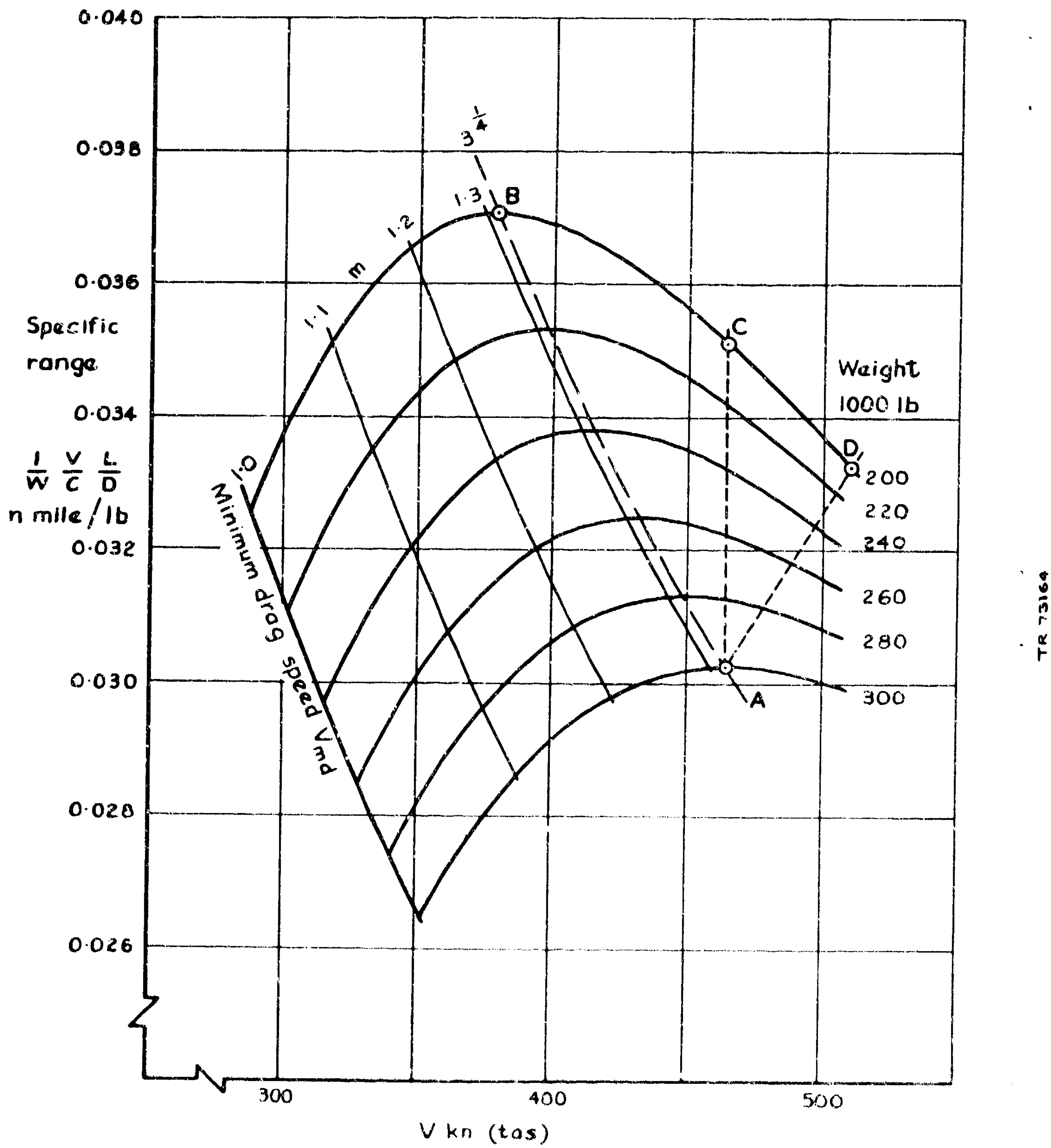
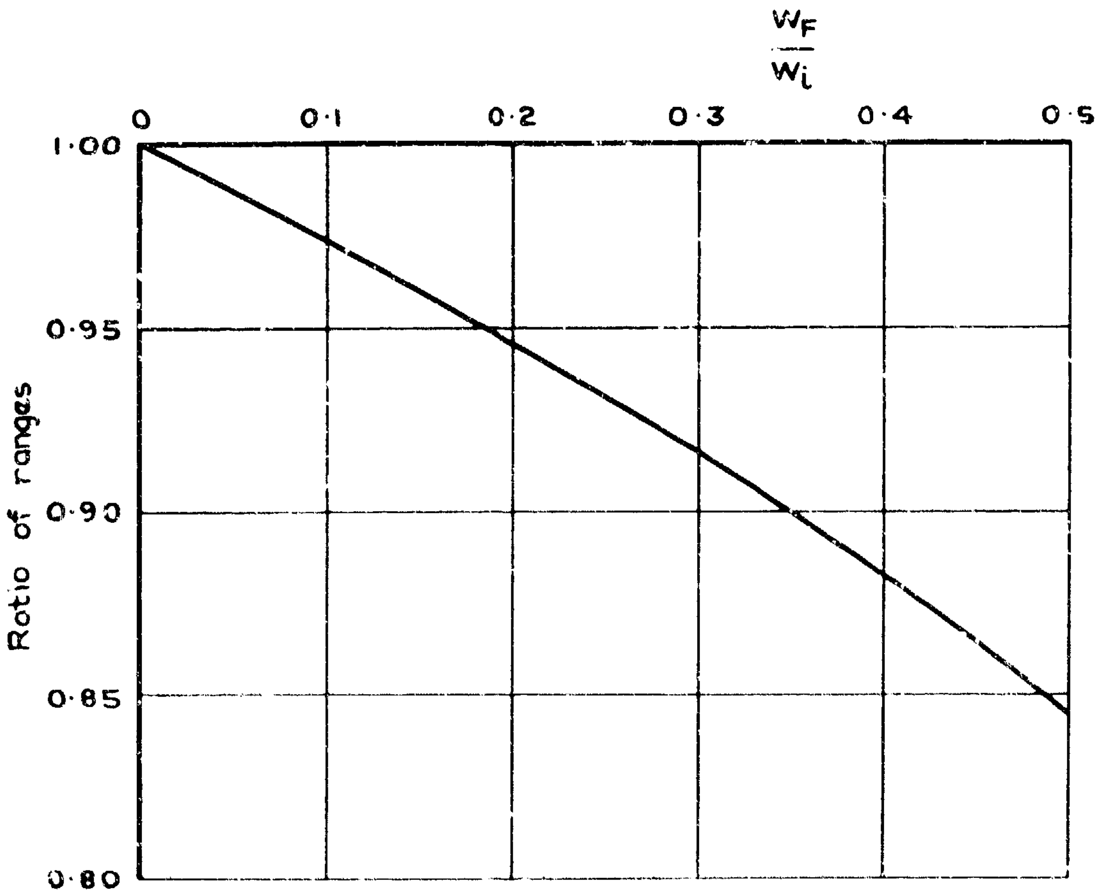


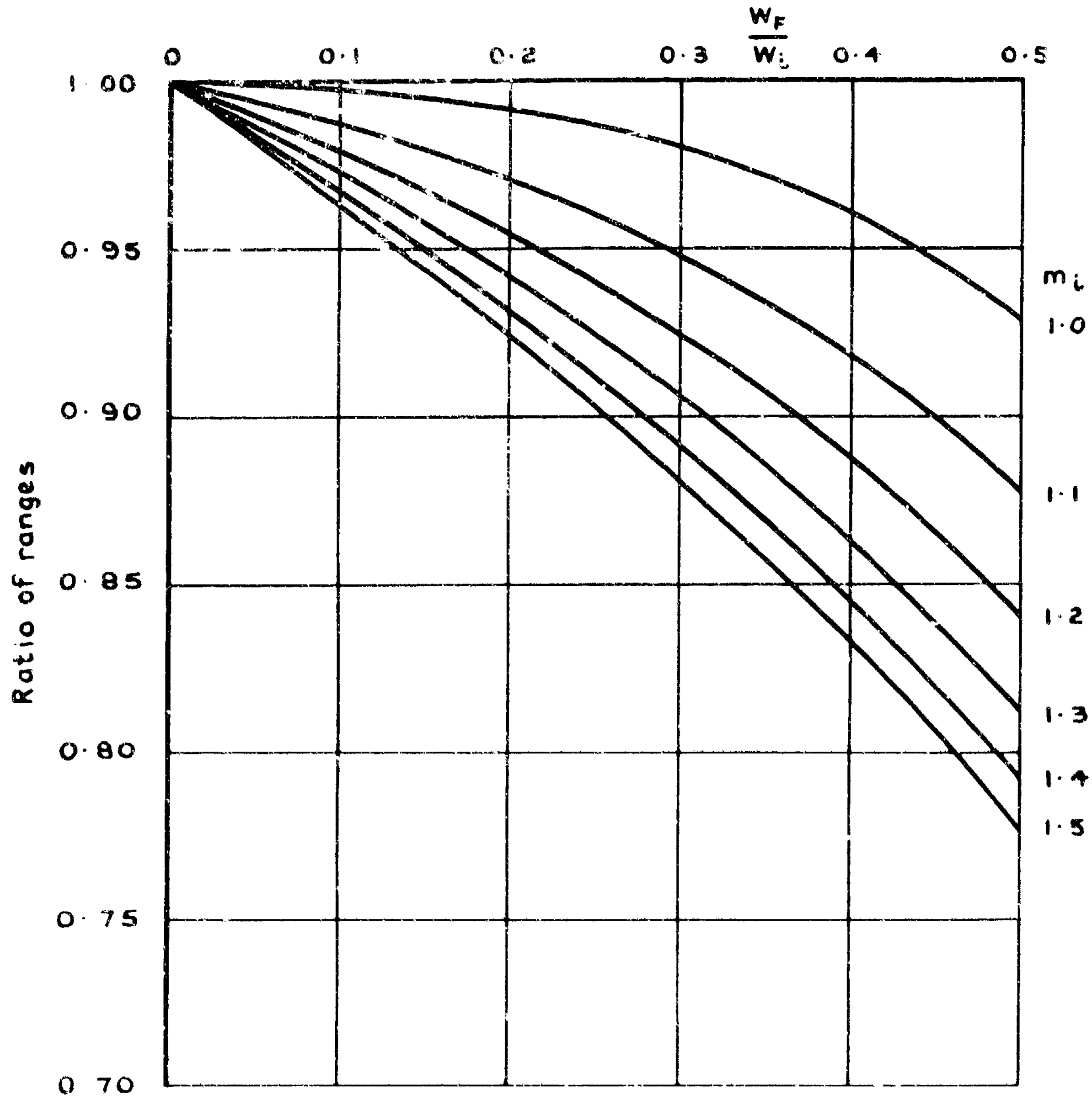
Fig. 2 Typical specific range performance (30000 ft)
$C_D = 0\cdot02 + 0\cdot05C_L^2$; $S = 3000\ ft^2$; $C = 0\cdot7$ lb/lb/hr



TR 73164

Fig.3 Ratio of range at constant lift-to-drag ratio and constant altitude to Breguet range, for the same conditions at start of cruise

Fig. 4



TR 73164

Fig. 4 Ratio of range at constant speed and altitude to Breguet range, for the same conditions at start of cruise

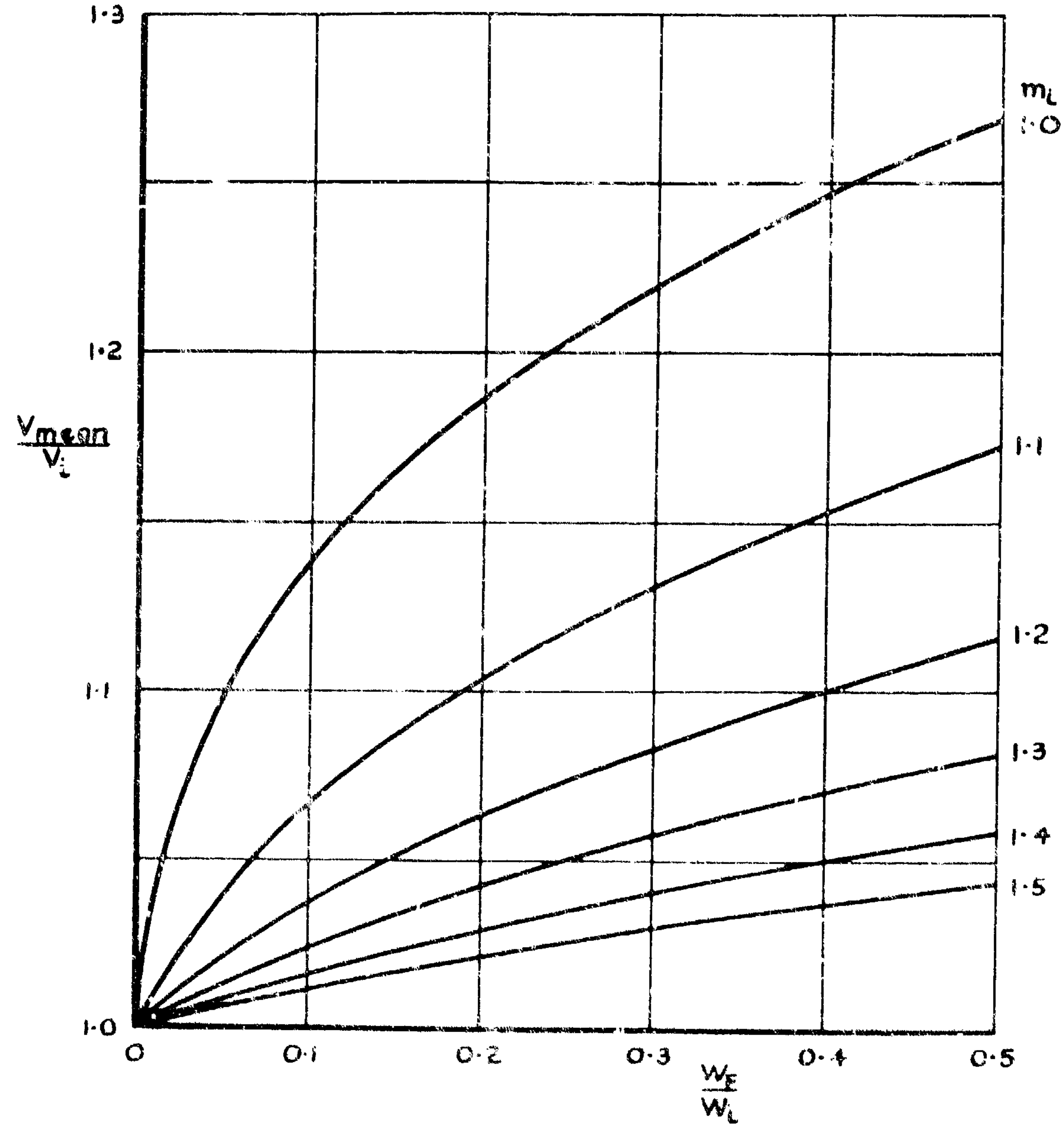
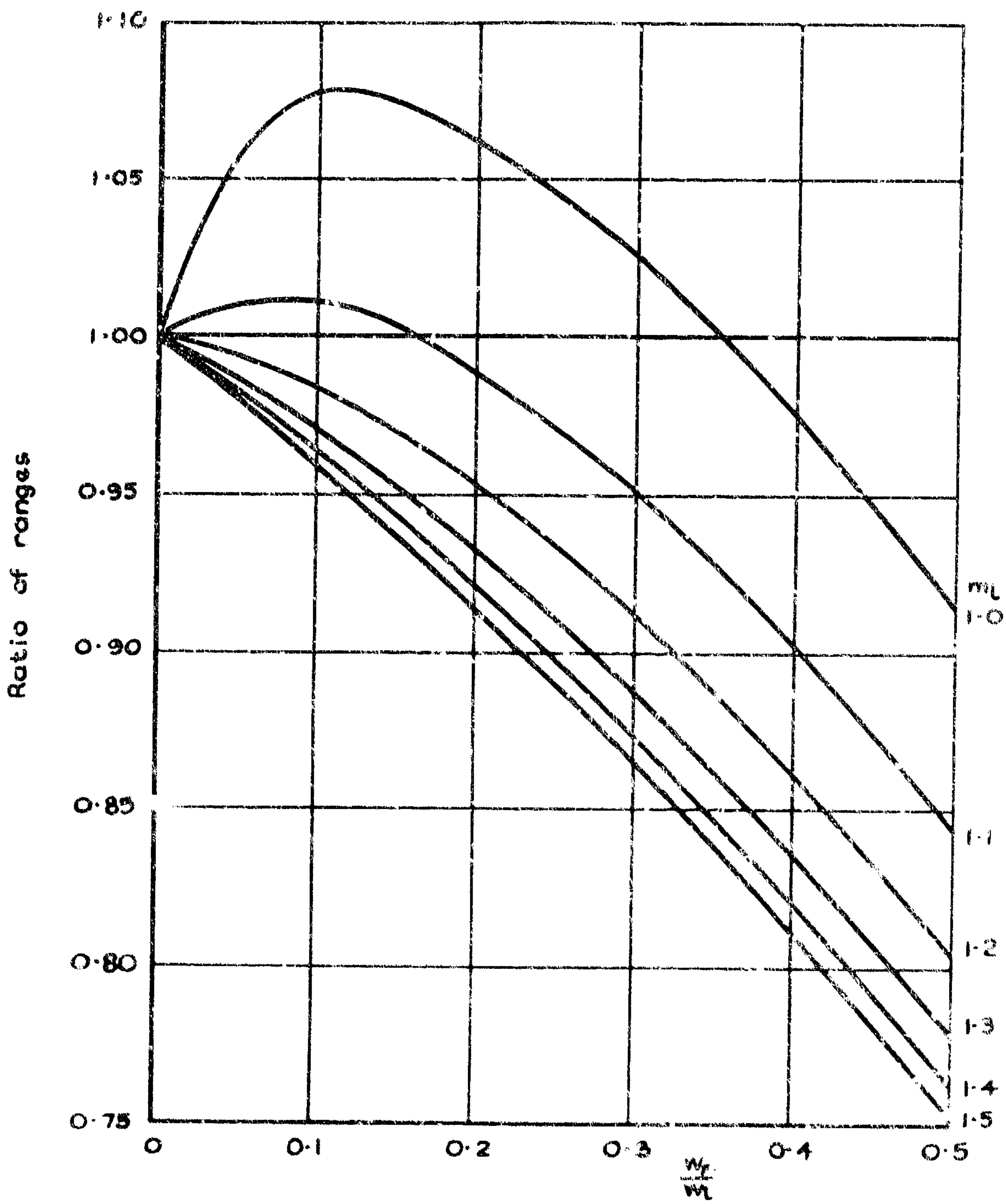


Fig. 5 Ratio of mean speed to initial speed for cruise at constant thrust and altitude

Fig. 6



TR 73164

Fig. 6 Ratio of range at constant thrust and altitude to Breguet range, for the same conditions at start of cruise