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MTR-2469

INTERFERENCE EFFECTS IN DIGITAL MATCHED FILTERS

P. P. Bratt

DECEMBER 1972

Prepared for

DEPUTY FOR PLANNING AND TECHNOLOGY

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 511A

Approved for public release; distribution unlimited.

Prepared by

THE MITRE CORPORATION
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FOREWORD

The work described in this report was carried out under the sponsorship of the Deputy for Planning and Technology, Project 511A, by The MITRE Corporation, Bedford, Massachusetts under Contract No. F19628-71-C-0002.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

NEIL P. ANDERSON, Acting Director
Communications System Plng
Deputy for Planning and Technology

ABSTRACT

This MTR contains a summary of some of the more important theoretical results related to the performance of a digital matched filter receiver designed to receive a biphase modulated signal in the presence of interference. Both the synchronization and data performance of the receiver are evaluated when the signal is received in the presence of a) Gaussian noise, b) Random phase sinusoidal interference, c) Fixed phase sinusoidal interference. Specific attention is paid to the effect of processing the received signal using one bit quantization.

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GLOSSARY

A = Amplitude of the received biphase signal

B = Amplitude of the interfering sinusoid

E_b = Energy per bit

$I_o(x)$ = Modified Bessel function of zero order and argument x

k = Number of correct digits fed into either the I or Q channel shift registers

n = Number of chips integrated before a decision is made (number of stages in shift register)

N = Noise power in receiver I.F.

N_o = Noise power per unit bandwidth

p_i = Probability of an incorrect digit being fed into the I channel shift register

p_q = Probability of an incorrect digit being fed into the Q channel shift register

p_e = Bit error probability

p_s = Probability of double signal suppression (both the inphase and quadrature components of the received signal being suppressed)

P_F = Probability of false alarm

P_D = Probability of detecting the synchronizing preamble given that double signal suppression does not occur

$P(sync)$ = Probability of synchronizing

$(\frac{S}{N})_i$ = Signal to noise power ratio in the receiver IF filter

$(\frac{S}{J})$ = Signal to interference power ratio measured in the receiver I.F. filter

V_I = Output voltage from I channel shift register

V_q = Output voltage from Q channel shift register

x = Voltage at the input to the 1 bit A/D converter in the I channel

y = Voltage at the input to the 1 bit A/D converter in the Q channel

Z = Output voltage from square root circuit

Z_T = Threshold voltage set at the output of the square root circuit

σ_I = rms noise at the input to the I channel A/D converter

σ_q = rms noise at the input to the Q channel A/D converter

θ = Phase angle of the received signal

χ = Phase angle of the interfering sinusoid

ω = Angular zone in which the phase angle of the interfering sinusoid must lay to cause double signal suppression

SECTION I

INTRODUCTION

This MTR is a summary of some of the more important results related to the performance of the PN modem being built by D-8 of the MITRE Corporation. MTR-2446 by R. Haggarty will contain the detailed circuit design of the modem together with details of the RF transmitter and receiver units and laboratory test data on the modem performance.

In order to increase the utility of the work in this MTR, where possible, the results obtained have been stated in terms of the important parameters of a more general model of the modem than that actually being built. Since the modem was designed for use in a tactical environment emphasis is placed on the anti-jam performance.

In this communication system information in binary form is transmitted by sending one of two uncorrelated signals. The signals are derived by choosing one of two pseudo-random sequences, which is used as a rectangular waveform to directly phase modulate a sine wave carrier. The optimum receiver consists of two matched filters, each matched to one of the two possible transmitted signals, followed by a level comparison circuit. This implementation assumes that the receiver has derived the received phase of the RF carrier. If it does not, the matched filter for each of the two possible signals is replaced by two filters each matched to the same signal but 90° out of carrier phase with one another. The outputs of each pair of quadrature matched filters are squared and added before entering the level comparison circuit.

A matched filter for the above mentioned signal is commonly implemented by a mixer to convert the RF signal to baseband followed by an analog tapped delay line, with plus-and-minus-one tap gains corresponding to the appropriate pseudo-random sequence, and connected to a single adder. The tapped delay line can be replaced by an analog-to-digital converter followed by a Q-level quantizer and a shift register with taps again matched to the appropriate pseudo-random sequence. If the number of levels Q used by the quantizer is large the performance of the receiver approaches that of an analog delay line implementation. Both the analog delay line and the required amount of storage for a digital implementation using a large number of voltage levels Q, tend to be expensive. The simplest and cheapest implementation is do to it digitally using only two quantization levels (i.e., Q = 2), however, one expects to pay a price in performance for this simplification.

The majority of the results given in this MTR are concerned with the performance both in the synchronization and data modes when the received signal is processed using one bit quantization.

SECTION II

DESCRIPTION OF THE MATCHED FILTER RECEIVER MODEL

2.1 Basic Matched Filter

Figure 1 shows a block diagram of the basic digital matched filter. The received bi-phase modulated sequence is separated into inphase and quadrature baseband components by the two mixers shown in the diagram. Both the inphase and quadrature components are low pass filtered before being hard limited by the 1 bit A/D converters. The output of the two one bit A/D converters are sampled at the PN rate and the "ones" and "zeros" (corresponding to plus and minus voltages) are shifted into the appropriate shift register. A copy of the transmitted PN sequence is stored in comparison shift registers and the contents of the I and Q channel shift registers are continually compared against the contents of the comparison registers. The sum of agreements minus disagreements in both the I and Q channels (i.e., V_I and V_q) are continuously calculated, squared and added and the square root of the sum Z calculated. When the received sequence is correctly aligned in the I and Q shift registers there is a large output from the square root circuit, i.e., the received sequence is highly correlated with the stored replica.

2.2 Binary Receiver Using Two Matched Filters

For the transmission of binary information two orthogonal PN sequences would be selected, one corresponding to a binary "one", the other to a "zero". The receiver would contain two digital matched filters as shown in Figure 2, one filter being matched to the "zero" sequence and the other matched to the "one" sequence. Assuming that the receiver is synchronized the outputs Z and Z_1 from the two filters would be compared at the end of a received information bit and a decision as to the reception of a "one" or "zero" would be based on the larger of the two outputs Z or Z_1.

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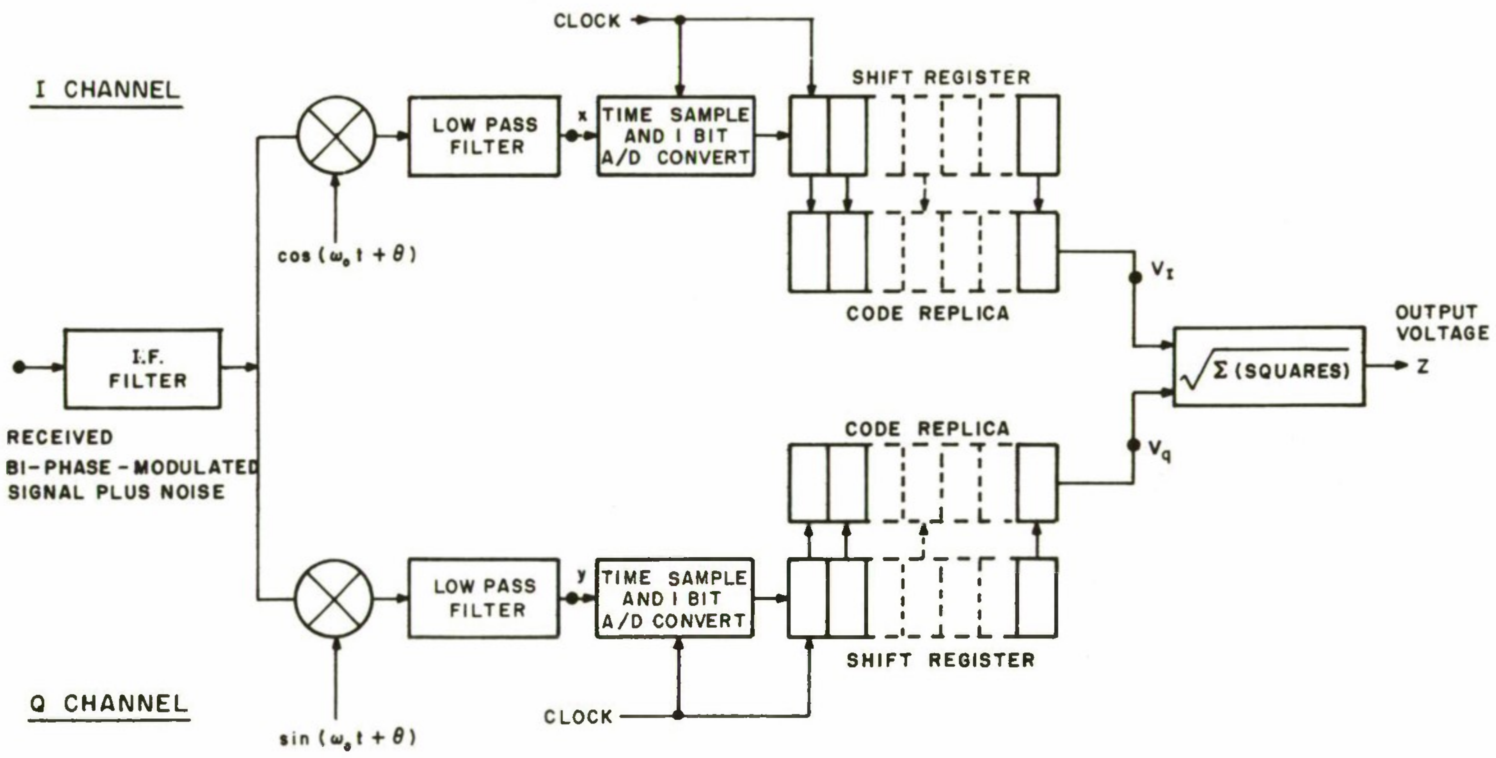

Figure 1 BASIC DIGITAL MATCHED FILTER

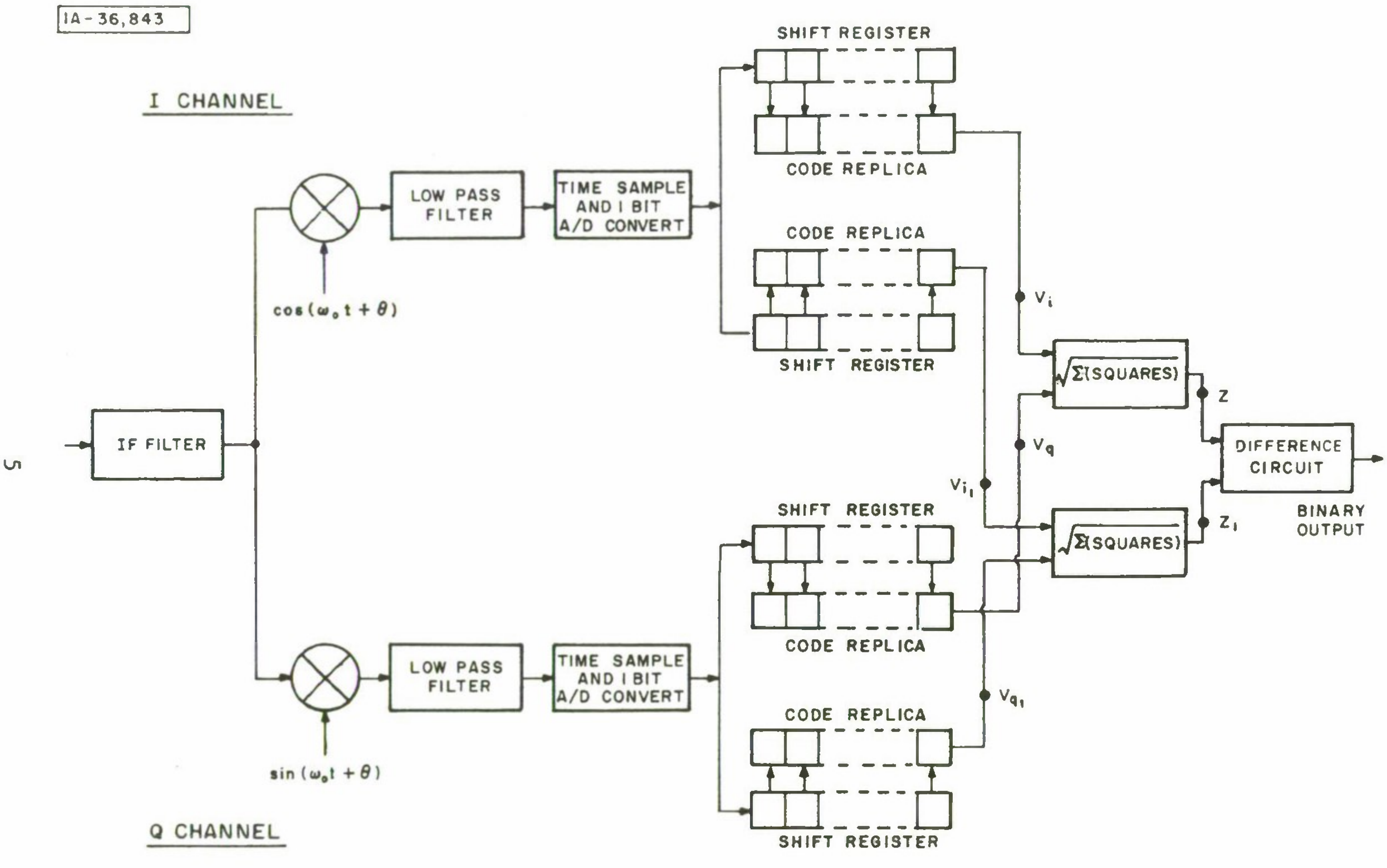

Figure 2 BINARY RECEIVER USING 2 DIGITAL MATCHED FILTERS

SECTION III

PROBABILITY DENSITY FUNCTION FOR THE OUTPUT VOLTAGE OF THE DMF

In order to compute such performance parameters as the bit error probability, the probability of falsely synchronizing and the probability of missing sync we first need the probability density function for the output voltage from the digital matched filter (DMF). In this section, we give this probability density function when the bi-phase modulated signal is received together with additive

(a) white Gaussian noise
(b) random phase sinusoidal interference
(c) fixed phase sinusoidal interference

The actual derivation of the probability density functions are given in Appendices A, B and C.

In order to simplify the analysis of the DMF performance the following simplifying assumptions were made in the appendices. They are,

(1) that the low pass filter in each channel is wide enough to allow the pulse amplitude to build up to full amplitude within a chip time.* In the actual design of the filter there will be the usual conflict of requiring the filter to be wide to allow fairly rapid build up to full pulse amplitude but at the same time keeping it as narrow as possible to reject as much noise as possible.

(2) that sampling of the I and Q channel A/D converters takes place towards the end of each chip, that is when the low pass filter outputs have reached maximum amplitude. To ensure that

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*A chip time is the duration of the basic signaling element and consists of the transmitted carrier in one of two possible phases (0° or 180°).

this is achieved it is usual to duplicate the matched filter assembly two or more times and have the sampling pulses for the matched filters staggered in time. For example, if there were three separate DMF assemblies the sampling pulses to each would be offset from each other by one third of a chip time. This would ensure that at least one DMF was being sampled with a timing error no greater than one-sixth of a chip period, i.e., within one-sixth of a chip period of the end of each chip.

3.1 Gaussian Noise

It is shown in Appendix A (equation 17) that when the bi-phase modulated signal is received together with white Gaussian noise and the signal is correctly aligned in the DMF the probability density function (PDF) for the output voltage Z of the DMF is given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n}\left\{Z^2 + \frac{4n^2}{\pi}\left(\frac{S}{N}\right)_i\right\}\right] I_o\left(\frac{Z}{n}\frac{2n}{\sqrt{\pi}}\sqrt{\left(\frac{S}{N}\right)_i}\right) \tag{1}$$

where n = the number of chips in the transmitted sequence

$\left(\frac{S}{N}\right)_i$ = signal to noise ratio in the receiver IF

$I_o(x)$ = modified Bessel function of zero order and of argument x.

The PDF for the output voltage of the DMF when a sequence orthogonal to that to which the DMF is matched is obtained by putting $(S/N)_i = 0$ in equation 1, i.e.,

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2}{2n}\right] \tag{2}$$

This is also the PDF for the output voltage from the DMF for a noise only input.

3.2 Random Phase Sinusoidal Interference

It is shown in Appendix B (equation 16) that when the bi-phase modulated signal is received together with an inband interfering sinusoid of random phase and the signal is correctly aligned in the DMF, the PDF for the output voltage Z of the DMF is given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \left| Z^2 + \frac{4n^2}{\pi^2} \left(\frac{S}{J}\right) \right| \right] I_o \left(\frac{Z}{n} \frac{2}{\pi} n \sqrt{\left(\frac{S}{J}\right)} \right) \qquad (3)$$

where n = number of chips in the transmitted sequence

$(\frac{S}{J})$ = ratio of signal power to interference power measured in the receiver IF

$I_o(x)$ = modified Bessel function of zero order and argument x.

Similarly for a random phase sinusoidal interference input only, or a random phase sinusoidal interference input plus a bi-phase modulated sequence orthogonal to the sequence to which the DMF is matched, the PDF of the output voltage Z of the DMF is given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2}{2n} \right] \qquad (4)$$

3.3 Fixed Phase Sinusoidal Interference

It is shown in Appendix C that for fixed phase sinusoidal interference the output of the matched filter can be represented by two distinct distributions. Again we assume that the received sequence is correctly aligned in the matched filter. One distribution occurs when the phase of the interfering signal is such that it causes both the in-phase and quadrature components of the demodulated signal to be completely suppressed. We call this effect double suppression and it occurs when the phase angle of the interfering sinusoid χ is such that

$$\chi \quad < \chi < \chi_i \tag{5}$$

where

$$\chi_i = \cos^{-1} \left(\frac{A}{B} \cos \theta\right) \tag{6}$$

$$\chi_q = \sin^{-1} \left(\frac{A}{B} \sin \theta\right) \tag{7}$$

and

A = amplitude of the received signal

B = amplitude of the interfering sinusoid

θ = phase of the received signal

When double signal suppression occurs the output Z of the DMF is noise only and has a Rayleigh PDF as given by equation 4.

The second distribution occurs when the phase of the interfering signal is such that it causes only the inphase or the quadrature component of the received signal to be completely suppressed. This occurs when

$\chi < \chi_i$ (In-phase channel suppressed)

$\chi > \chi_q$ (Quadrature channel suppressed)

where χ_i and χ_q are as previously defined.

Equation 19 of Appendix C gives the probability density function for the output Z of the DMF receiver when the phase of the interfering sinusoid causes only one of the quadrature components to be suppressed, i.e.,

$$p(Z)\, dZ = \frac{2}{\sqrt{2\pi n}} \exp - \left(\frac{Z^2 - n^2}{2n}\right) \frac{Z}{\sqrt{Z^2 - n^2}}\, dZ \qquad (Z \geq n) \qquad (8)$$

The output Z can never be less than n since the quadrature component which is not suppressed will always give an output of $\pm$ n which, when combined with the noise component from the other quadrature channel (by taking the square root of the sum of the squares) will always result in $Z \geq n$.

SECTION IV

SYNCHRONIZATION PERFORMANCE

Each transmission in the TDMA system will start with a synchronization preamble consisting of an n digit segment of a pseudo random sequence. The n ones and zeros in the synchronization preamble bi-phase modulate the transmitted carrier and are demodulated at the receiver by a digital matched filter as shown in Figure 1. The particular n digits of the PR sequence used for synchronization is assumed to be stored in the shift registers denoted by "code replica" in Figure 1. Arrival of the synchronization preamble is assumed when the output voltage Z of the matched filter exceeds a preset threshold. The system parameters which determine just where the threshold is set are the probability of detecting synchronization P_D and the false alarm probability P_F. These two quantities are in turn related to the probability density function (PDF) of the output voltage of the digital matched filter (DMF). This will be shown in the next three sections.

4.1 Gaussian Noise Performance

Equation 1 gives the PDF of the output voltage Z of the DMF when the synchronization sequence is correctly aligned in the DMF. Equation 1 may be written as

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \left| Z^2 + \frac{4n^2}{\pi} \left(\frac{S}{N}\right)_i \right| \right] I_o \left[\frac{Z}{n} \sqrt{\frac{4n^2}{\pi} \left(\frac{S}{N}\right)_i} \right] \tag{9}$$

The probability of detecting synchronization P_D is the probability that Z exceeds some preset threshold Z_T, that is

$$P_D = \int_{Z_T}^{\infty} p(Z)\, dZ \tag{10}$$

Using equations 9 and 10 we have

$$P_D = \int_{Z_T}^{\infty} \frac{Z}{n} \exp - \left[\frac{1}{2n} \left\{ Z^2 + \frac{4n^2}{\pi} \left(\frac{S}{N}\right)_i \right\} \right] I_o \left[\frac{Z}{n} \sqrt{\frac{4n^2}{\pi} \left(\frac{S}{N}\right)_i} \right] dZ \quad (11)$$

Making substitutions,

$$\frac{Z}{\sqrt{n}} = v \quad (12)$$

and

$$\frac{Z_T}{\sqrt{n}} = \beta, \quad (13)$$

equation 11 may be rewritten as

$$P_D = \int_{\beta}^{\infty} v \exp - \left[\frac{v^2 + \frac{4n}{\pi} \left(\frac{S}{N}\right)_i}{2} \right] I_o \left[v \sqrt{\frac{4n}{\pi} \left(\frac{S}{N}\right)_i} \right] dv \quad (14)$$

If we let

$$2n \left(\frac{S}{N}\right)_i = a_1^{\,2} \quad (15)$$

and

$$K_1^{\,2} = \frac{2}{\pi} \quad (16)$$

Equation 14 may be written as

$$P_D = \int_{\beta}^{\infty} v \exp - \left[\frac{v^2 + (K_1 a_1)^2}{2} \right] I_o \left[v K_1 a_1 \right] dv \quad (17)$$

This equation is the Marcum Q function and is tabulated in [5]. Equation 17 gives the same probability of detection one would obtain for the envelope of a sinusoid plus Gaussian noise where the input signal-to-noise ratio was given by

$$\left(\frac{S}{N}\right)_e = \frac{2}{\pi} n \left(\frac{S}{N}\right)_i \tag{18}$$

With the threshold set at β, the probability of false alarm in the noise only case is given by putting $a_1^2 = 0$ in equation 17, i.e.,

$$P_F = \int_\beta^\infty v \exp - \left[\frac{v^2}{2}\right] dv$$

$$= \exp - \frac{\beta^2}{2} \tag{19}$$

Figure 3 is a set of receiver operating characteristics (ROC's) for the envelope of a sinusoid in Gaussian noise for various values of $(S/N)_e$. These ROC's are a plot of the probability of detection P_D versus probability of false alarm P_F at a fixed signal to noise ratio $(S/N)_e$. For convenience a threshold scale is provided along the abscissa and indicates where the threshold at the output of the receiver should be set to obtain specific values of P_D and P_F. The units used for the theshold are in terms of multiples of the rms noise, σ, into the envelope detector. In the next section we illustrate how they can be used to assess synchronization performance.

4.1.1 Use of the Receiver Operating Characteristics (ROC) to Assess Synchronization Performance in Gaussian Noise

In designing a synchronization system based on using a digital matched filter the quantities to be specified are:

a. the synchronization sequence length, n
b. the synchronization input signal to noise ratio $(S/N)_i$
c. where the threshold, Z_T, should be set at the output of the digital matched filter.

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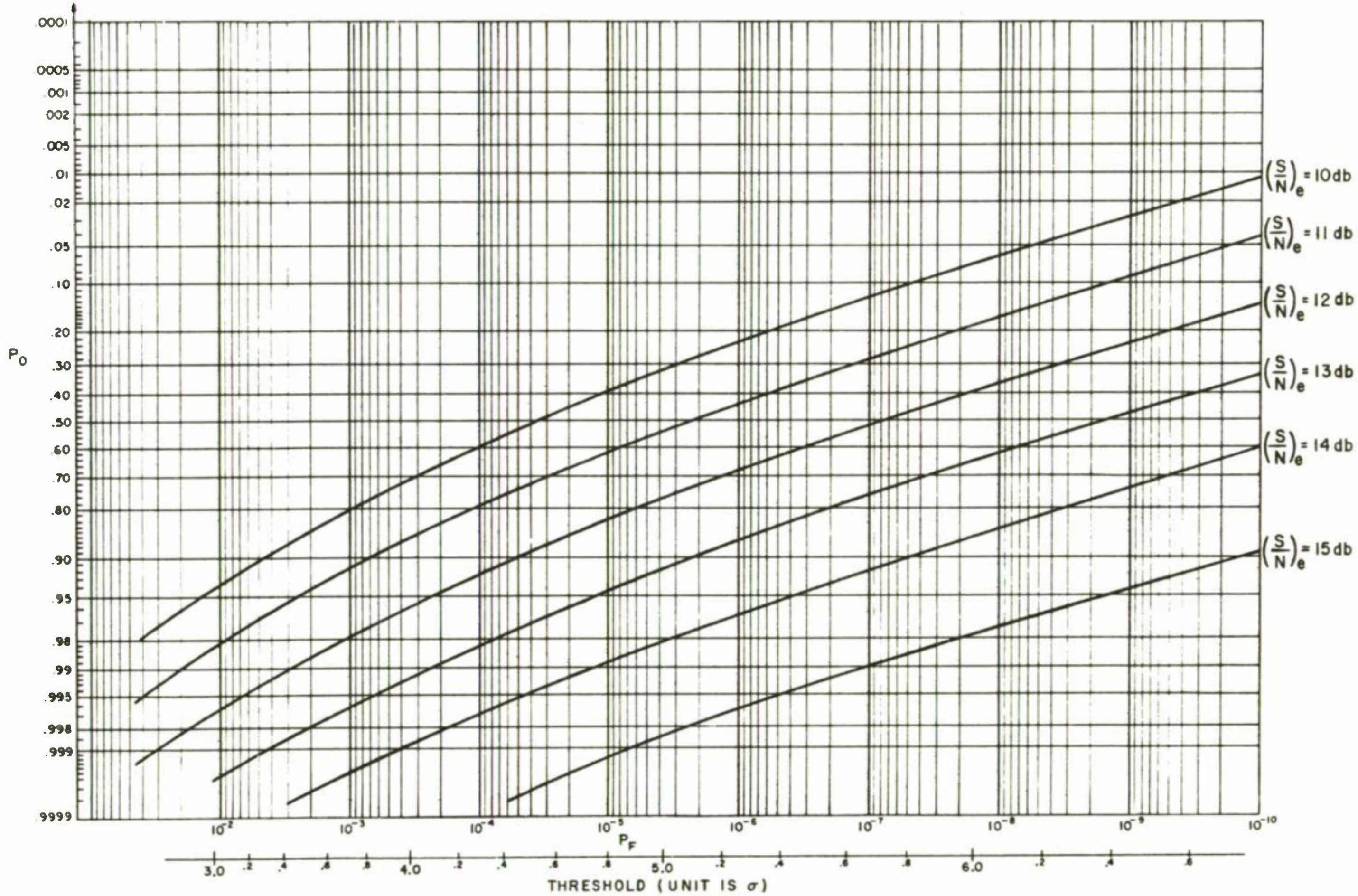

Figure 3 RECEIVER OPERATING CHARACTERISTICS FOR THE DIGITAL MATCHED FILTER DURING SYNCHRONIZATION

These three quantities are in turn dependent on the required probabilities of detection and false alarm, P_D and P_F which are determined from such operational requirements as message error rate, probability of two consecutive messages being in error, etc.

We therefore assume that P_D and P_F are specified and that we require the three quantities given in a, b, and c in order to design the DMF to be used for synchronization. As a specific example in use of the ROC's assume we require

$$P_D = .99$$

and

$$P_F = 10^{-7}$$

Looking at Figure 3 we see that the lower curve denoted by $(S/N)_e = 15$ dB passes through the intersection of the lines $P_D = .99$ and $P_F = 10^{-7}$ so that we require at least $(S/N)_e = 15$ dB to obtain the required values of P_D and P_F.

Using equation 18 we have

$$\left(\frac{S}{N}\right)_e = \frac{2}{\pi} n \left(\frac{S}{N}\right)_i \qquad (20)$$

Recalling that the factor $2/\pi$ represents a loss of 2 dB we have for $(S/N)_e = 15$ dB, that

$$n \left(\frac{S}{N}\right)_i = 17 \text{ dB} \qquad (21)$$

If we now choose a reasonable length sequence n so that the DMF does not require too many shift register stages, both $(S/N)_i$ and the threshold value are fixed.

Assume n = 128 which using equation 21, gives the required input signal to noise ratio

$$(\frac{S}{N})_i = -4 \text{ dB}$$

Looking at Figure 3 we see that the threshold Z_T corresponding to $P_f = 10^{-7}$, is

$$Z_T = 5.63\ \sigma \qquad (22)$$

Since $\sigma = \sqrt{n}$, the required threshold Z_T is given by

$$\begin{aligned} Z_T &= 5.63 \sqrt{128} \\ &= 63.69 \\ &= 64 \text{ (nearest integer)} \end{aligned}$$

In summary, if we require $P_D = .99$ and $P_F = 10^{-7}$ and choose a DMF of length n = 128 we require an input signal to noise ratio of at least -4 dB and the threshold Z_T should be set at 64.

4.2 <u>Random Phase Sinusoidal Interference</u>

Equation 3 gives the PDF of the output voltage Z of the DMF when the synchronization sequence is correctly aligned in the DMF and the bi-phase signal is received together with a random phase sinusoidal interfering signal.

Equation 3 may be written as

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \left\{ Z^2 + \frac{4n^2}{\pi^2} (\frac{S}{J}) \right\} \right] I_o \left[\frac{Z}{n} \sqrt{\frac{4n^2}{\pi^2} (\frac{S}{J})} \right] \qquad (23)$$

The probability of detecting synchronization P_D is the probability that Z exceeds some preset threshold Z_T, that is

$$P_D = \int_{Z_T}^{\infty} p(Z)\ dZ \tag{24}$$

Using equations 23 and 24 we have

$$P_D = \int_{Z_T}^{\infty} \frac{Z}{n} \exp - \left[\frac{1}{2n} \left| Z^2 + \frac{4n^2}{\pi^2} (\frac{S}{J})\right|\right] I_o \left[\frac{Z}{n}\sqrt{\frac{4n^2}{\pi^2} (\frac{S}{J})}\right] dZ \tag{25}$$

We now make the same substitutions that were made in Section 4.1, i.e.,

$$\frac{Z}{\sqrt{n}} = v \tag{26}$$

$$\frac{Z_T}{\sqrt{n}} = \beta \tag{27}$$

Equation 25 may now be written as

$$P_D = \int_{\beta}^{\infty} v \exp - \left[\frac{v^2 + \frac{4n}{\pi^2} (\frac{S}{J})}{2}\right] I_o \left[v\sqrt{\frac{4n}{\pi^2} (\frac{S}{J})}\right] \tag{28}$$

Again if we let

$$2n\ (\frac{S}{J}) = {a_2}^2 \tag{29}$$

and

$${K_2}^2 = \frac{2}{\pi^2} \tag{30}$$

equation 28 may be written as

$$P_D = \int_{\beta}^{\infty} v \exp \left[\frac{v^2 + (K_2 a_2)^2}{2}\right] I_o \left[v K_2 a_2\right] \tag{31}$$

The function on the right hand side of equation 31 is the Marcum Q function and is tabulated in [1]. This equation gives the same probability of detection which one would obtain for the envelope of a sinusoid plus Gaussian noise where the input signal-to-noise ratio is given by

$$\left(\frac{S}{N}\right)_e = K_2^{\,2} \frac{a_2^{\,2}}{2} \tag{32}$$

which using equations 29 and 30 gives

$$\left(\frac{S}{N}\right)_e = \frac{2}{\pi^2}\, n \left(\frac{S}{J}\right) \tag{33}$$

Again with the threshold set at β the probability of false alarm in the case where the synchronizing sequence is misaligned in the DMF and the signal is received together with the interfering signal is obtained by putting $a_2^{\,2} = 0$ in equation 31, i.e.,

$$P_F = \int_{\beta}^{\infty} v \exp - \left[\frac{v^2}{2}\right] dv$$
$$= \exp - \left[\frac{\beta^2}{2}\right] \tag{34}$$

Figure 3 can be used to assess synchronization performance of the DMF in exactly the same way as that used for Gaussian noise in Section 4.1.1 provided the different loss factor

$$K_2^{\,2} = \frac{2}{\pi^2} \equiv 6.94 \text{ dB}$$

is used.

Using the previous example all numbers would be the same except the required input signal to noise ratio which is obtained from

$$\left(\frac{S}{N}\right)_e = \frac{2}{\pi^2} \; n \; \left(\frac{S}{J}\right)$$

or

$$\left(\frac{S}{J}\right) = \left(\frac{S}{N}\right)_e \frac{\pi^2}{2} \cdot \frac{1}{n}$$

$$= 15 \text{ dB} + 6.94 \text{ dB} - 21 \text{ dB}$$

$$= + .94 \text{ dB}$$

In summary if we require a $P_D = .99$ and $P_F = 10^{-7}$ and choose a DMF of length 128 we require an input signal to interference power ratio (S/J) of at least +.94 dB and the threshold Z_T should be set at 64.

4.3 Fixed Phase Sinusoidal Interference

The only case in which synchronization is possible is when the phase of the interfering sinusoid is in the single signal suppression zone (i.e., only the in-phase or the quadrature component of the signal is suppressed, not both). When this occurs and the synchronization sequence is correctly aligned in the DMF, the PDF for the output voltage Z is given by equation 8, i.e.,

$$p(Z) = \frac{2}{\sqrt{2\pi n}} \exp - \left(\frac{Z^2 - n^2}{2n}\right) \frac{Z}{\sqrt{Z^2 - n^2}} \qquad (35)$$

$$Z \geq n$$

The probability of detecting synchronization is given by the probability that Z exceeds some threshold voltage Z_T. That is

$$P_D = \int_{Z_T}^{\infty} p(Z) \, dZ \qquad (36)$$

using equations 35 and 36 we have

$$P_D = \int_{Z_T}^{\infty} \frac{2}{\sqrt{2\pi n}} \exp - \left(\frac{Z^2 - n^2}{2n}\right) \frac{Z}{\sqrt{Z^2 - n^2}} \, dZ \qquad (37)$$

Making the substitution

$$\frac{Z^2 - n^2}{2n} = x^2$$

equation 37 becomes

$$P_D = \int_{\sqrt{\frac{Z_T - n^2}{2n}}}^{\infty} \frac{2}{\sqrt{\pi}} \exp - x^2 \, dx$$

$$= 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{Z_T^2 - n^2}{2n}}} \exp - x^2 \, dx$$

$$\therefore \quad P_D = 1 - \operatorname{erf} \sqrt{\frac{Z_T^2 - n^2}{2n}} \qquad (38)$$

Equation 38 shows that if the threshold Z_T is set less than n the probability of detecting synchronization is unity. This point has already been covered in the paragraph immediately after equation 8.

The PDF for the output of the DMF when the interfering sinusoid or the interfering sinusoid plus a misaligned synchronization sequence is received is given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2}{2n}\right] \qquad (39)$$

If we have the same threshold voltage Z_T at the output of the DMF the probability of false alarm is given by P_F where

$$P_F = \int_{Z_T}^{\infty} \frac{Z}{n} \exp - \left[\frac{Z^2}{2n}\right]$$

$$= \exp - \left[\frac{{Z_T}^2}{2n}\right] \tag{40}$$

Equation 38 gives the probability of detecting the synchronization sequence given that the phase angle of the interfering sinusoid is in one of the single signal suppression zones. To obtain the overall probability of synchronizing we must weight the probability of detecting the synchronization sequence by the probability that the phase angle of the interfering sinusoid is in one of the single signal suppression zones. To do this requires that we make some assumptions regarding the distribution of the phase angle of the interfering sinusoid.

A reasonable assumption is that the phase angle remains constant for the duration of a synchronization sequence (less than 100 μsec) and that the phase angle for consecutive synchronization transmissions are independent and are uniformly distributed ($0 < \chi < 2\pi$). Based on this assumption the probability that the phase angle lies in the single signal suppression zone is given by P_1, where

$$P_1 = 1 - \text{(probability of double signal suppression)}$$

It is shown in Appendix C (Section C.3) that the probability of double signal suppression is given by

$$P_s = \frac{2}{\pi} \cos^{-1} \omega \tag{41}$$

where

$$\cos \omega = \frac{A}{B} \sqrt{2 - \frac{A^2}{B^2}} \cos (\theta - \alpha) \tag{42}$$

In equation 42

A = amplitude of the received signal

B = amplitude of the interfering signal

θ = phase of the received signal

ω = size of the angular zone in which the phase of the interfering signal must lie to cause double suppression

and

$$\alpha = \tan^{-1} \sqrt{\frac{1 - \frac{A^2}{B^2} \cos^2 \theta}{1 - \frac{A^2}{B^2} \sin^2 \theta}} \tag{43}$$

It is also shown in Section C.3 that the angular zone ω has its maximem size ω_{max} when $\theta = 0$ or $\pi/2$ and its minimum size ω_{min} when $\theta = \pi/4$, therefore using equations 42 and 43, we have

$$\cos \omega_{max} = \frac{A}{B} \tag{44}$$

and

$$\cos \omega_{min} = \frac{A}{B} \sqrt{2 - \frac{A^2}{B^2}} \tag{45}$$

Since

$$\frac{A^2}{B^2} = \frac{S}{J}$$

equations 44 and 45 may be written as

$$\cos \omega_{max} = \sqrt{\frac{S}{J}} \tag{46}$$

and

$$\cos \omega_{min} = \sqrt{\frac{S}{J}} \sqrt{2 - \frac{S}{J}} \tag{47}$$

Since the probability of double suppression is a maximum when $\omega = \omega_{max}$ and a minimum when $\omega = \omega_{min}$ we may obtain upper and lower bounds on the probability of double suppression by using equations 41 with 46 and 41 with 47, that is

$$P_{s,max} = \frac{2}{\pi} \cos^{-1} \omega_{max}$$

$$= \frac{2}{\pi} \cos^{-1} \sqrt{(\frac{S}{J})} \tag{48}$$

and

$$P_{s,min} = \frac{2}{\pi} \cos^{-1} \omega_{min}$$

$$= \frac{2}{\pi} \cos^{-1} \left[\sqrt{(\frac{S}{J})} \sqrt{2 - (\frac{S}{J})} \right] \tag{49}$$

As stated previously, the probability that the phase angle of the interfering signal lies in the single signal suppression zone is given by

$$P_1 = 1 - \text{(probability of double signal suppression)}$$

$$= 1 - P_s \tag{50}$$

We may therefore obtain upper and lower bounds on P_1 by using equation 50 with 48 and 49, that is

$$P_{1,max} = 1 - P_{s,min}$$

$$= 1 - \frac{2}{\pi} \cos^{-1} \left[\sqrt{(\frac{S}{J})} \sqrt{2 - (\frac{S}{J})} \right] \tag{51}$$

and

$$P_{1,min} = 1 - P_{s,max}$$

$$= 1 - \frac{2}{\pi} \cos^{-1} \sqrt{(\frac{S}{J})} \tag{52}$$

Using the previous work we may now obtain upper and lower bounds on the probability of synchronizing since the probability of synchronizing P(sync) is given by the probability of detecting sync given that the phase angle of the interfering sinusoid is in one of the single signal suppression zones, that is

$$P(\text{sync}) = P_D \cdot P_1 \tag{53}$$

so that

$$P(\text{sync})_{max} = P_D \cdot P_{1,max} \tag{54}$$

and

$$P(\text{sync})_{min} = P_D \cdot P_{1,min} \tag{55}$$

Using equations 54, 51 and 38, we have as an upper bound on the probability of synchronizing

$$P(\text{sync})_{max} = \left[1 - \text{erf}\sqrt{\frac{Z_T^2 - n^2}{2n}}\right]\left[1 - \frac{2}{\pi}\cos^{-1}\left(\sqrt{(\frac{S}{J})}\sqrt{2 - (\frac{S}{J})}\right)\right] \quad (56)$$

Also using equations 55, 52 and 38 we have as a lower bound on the probability of synchronizing

$$P(\text{sync})_{min} = \left[1 - \text{erf}\sqrt{\frac{Z_T^2 - n^2}{2n}}\right]\left[1 - \frac{2}{\pi}\cos^{-1}\sqrt{(\frac{S}{J})}\right] \quad (57)$$

The probability of false alarm with the threshold set at Z_T is still given by equation 40, i.e.,

$$P_F = \exp - (\frac{Z_T^2}{2n}) \quad (58)$$

4.3.1 Comments on Synchronization When the Interfering Signal is a Fixed Phase Sinusoid

Looking at the equations for the probability of synchronization (equations 56 and 57) we see that there is no increase in the probability of sync if we set the threshold Z_T less than n since the first factor is unity for $Z_T \leq n$. For $Z_T \leq n$ the probability of synchronization is controlled solely by the second factor which is the probability that the phase angle of the interfering sinusoid is in one of the single signal suppression zones and depends on (S/J) only.

If we set the threshold $Z_T = n$ we see that using equation 58 the probability of false alarm is given by

$$P_F = \exp - \left[\frac{n}{2}\right] \quad (59)$$

which equals

$$1.2 \times 10^{-14} \text{ for } n = 64$$
$$1.6 \times 10^{-28} \text{ for } n = 128$$
and $$2.5 \times 10^{-56} \text{ for } n = 256$$

We see therefore that for the threshold Z_T set at n the probability of false alarm is extremely small for the values of n which would be considered and that the probability of synchronization is determined solely by the second term in equations 56 and 57. Figure 4 is a plot of upper and lower bounds on P_1 versus (S/J) which for $Z_T \leq n$ are identical to the upper and lower bounds on P(sync).

In summary, decreasing the threshold below $Z_T = n$ will increase P_F without increasing P(sync). Increasing the threshold above n will decrease P(sync) and decrease P_F which is extremely small anyway. We therefore conclude that the threshold Z_T should be set equal to n since this maximizes the probability of synchronization and results in a probability of false alarm P_F which is extremely small.

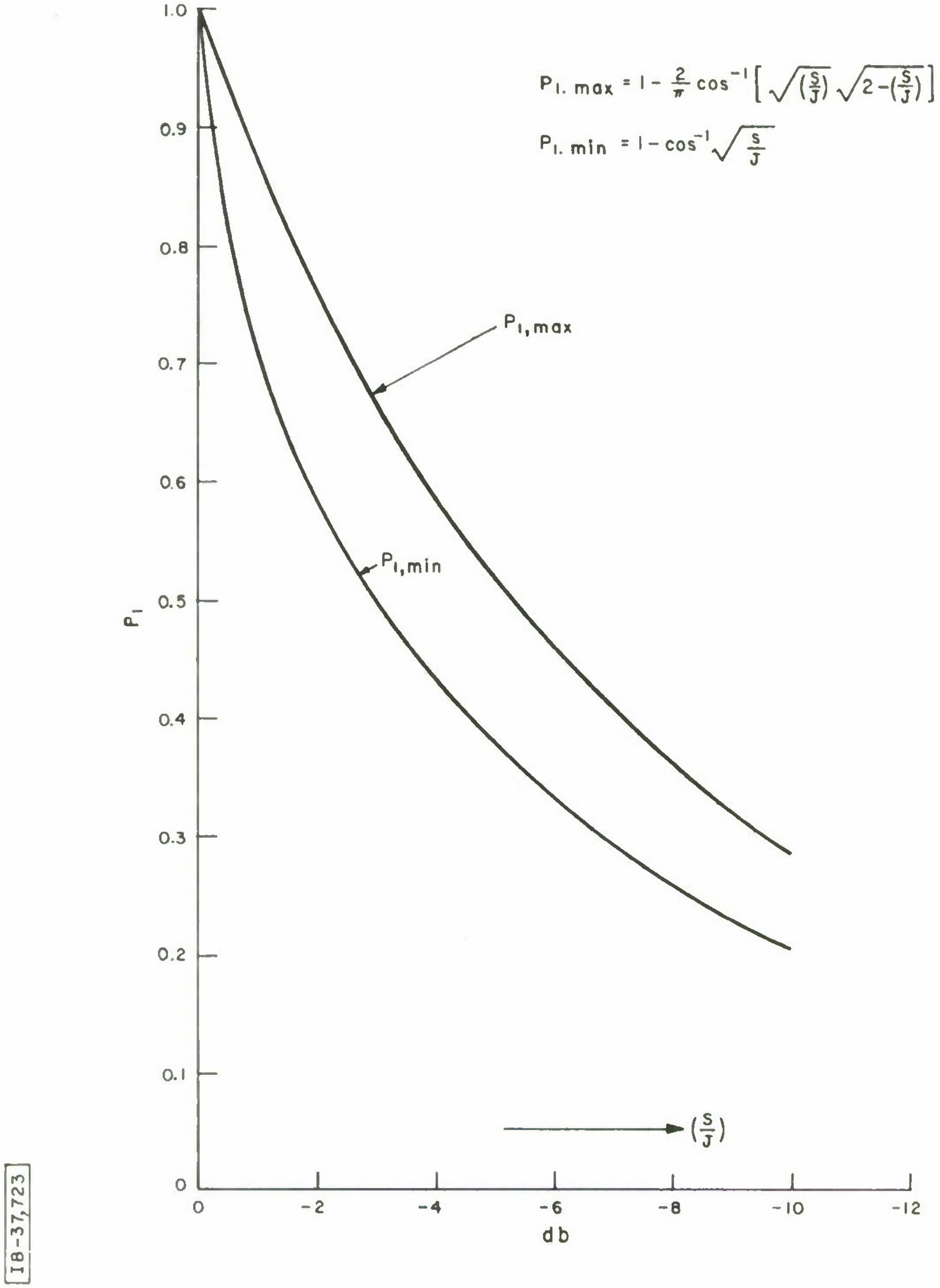

Figure 4 UPPER AND LOWER BOUNDS ON THE PROBABILITY P_1 THAT THE PHASE ANGLE OF THE INTERFERING SINUSOID LIES IN A SINGLE SIGNAL SUPPRESSION ZONE VERSUS $\left(\frac{S}{J}\right)$

SECTION V

DATA PERFORMANCE

In this section we compute the binary error rate for the receiver described in Section 2.2 when the received signal is received together with

a. white Gaussian noise

b. random phase sinusoidal interference

c. fixed phase sinusoidal interference

As described in Section 2.2 the receiver contains two digital matched filters as shown in Figure 2, one filter being matched to the "zero" sequence and the other matched to the "one" sequence. If the receiver is synchronized the outputs from the two matched filters would be compared at the end of a received information bit and a decision as to the reception of a "one" or zero would be based on the larger of the two DMF outputs.

5.1 Gaussian Noise

When the bi-phase modulated signal is received together with Gaussian noise the PDF for the filter output Z which is matched to the received signal is given by equation 1, i.e.,

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \left| Z^2 + \frac{4n^2}{\pi} \left(\frac{S}{N}\right)_i \right| \right] \quad I_o \left(\frac{Z}{n} \frac{2n}{\sqrt{\pi}} \sqrt{\left(\frac{S}{N}\right)_i} \right) \qquad (60)$$

The PDF for the output voltage Z_1 of the filter matched to a sequence orthogonal to the first sequence is given by equation 2, i.e,

$$p(Z_1) = \frac{Z_1}{n} \exp - \left(\frac{Z_1^2}{2n}\right) \qquad (61)$$

The bit error probability is then given by the probability that $Z_1 > Z$ or

$$P_e = \text{Prob}\ (Z_1 > Z) \tag{62}$$

If we let the term

$$\frac{4n^2}{\pi} \left(\frac{S}{N}\right)_i = c^2 \tag{63}$$

in equation 60 we may use the result given by equation D10 of Appendix D, that is

$$P_e = \frac{1}{2} \exp - \left(\frac{c^2}{4n}\right) \tag{64}$$

Substituting in equation 64 for c^2 from equation 63 we obtain

$$P_e = \frac{1}{2} \exp - \left[\frac{n}{\pi} \left(\frac{S}{N}\right)_i\right] \tag{65}$$

So far it has been assumed that both the IF and low pass filter bandwidths of the receiver are wide enough to allow the pulse envelope in both the I and Q channels to come up to full amplitude by the end of a chip time. If we now wish to write equation 65 in terms of E_b/N_o we must make some assumptions regarding system bandwidths. If we assume that the IF bandwidth W_i is given by

$$W_i = \frac{1}{t} \tag{66}$$

where t is the chip duration we have that the energy per bit E_b is given by

$$E_b = \frac{A^2}{2}\ n\ t$$

so that

$$\frac{E_b}{N_o} = \frac{A^2}{2N_o} \frac{n}{W_i} = n \left(\frac{S}{N}\right)_i \tag{67}$$

Using equations 65 and 67 we have

$$p_e = \frac{1}{2} \exp - \left(\frac{1}{\pi} \frac{E_b}{2N_o} \right) \tag{68}$$

For ideal quadrature detection the corresponding expression for probability of error would be [2]

$$p_e = \frac{1}{2} \exp - \left(\frac{E_b}{2N_o} \right) \tag{69}$$

Figure 5 shows a plot of the probability of bit error versus E_b/N_o for the DMF using one bit quantization. The curve for ideal quadrature detection is shown for comparison.

5.2 Random Phase Sinusoidal Interference

The PDF for the output voltage Z of the filter matched to the incoming signal in the presence of random phase sinusoidal interference is given by equation 3, that is

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \left| Z^2 + \frac{4n^2}{\pi^2} \left(\frac{S}{J}\right) \right| \right] I_o \left(\frac{Z}{n} \frac{2}{\pi} n \sqrt{\left(\frac{S}{J}\right)} \right) \tag{70}$$

The PDF for the output voltage Z_1 of the filter matched to a sequence orthogonal to the first sequence and receiving this sequence together with random phase sinusoidal interference is given by equation 4, that is

$$p(Z_1) = \frac{Z_1}{n} \exp - \left(\frac{Z_1^2}{2n}\right) \tag{71}$$

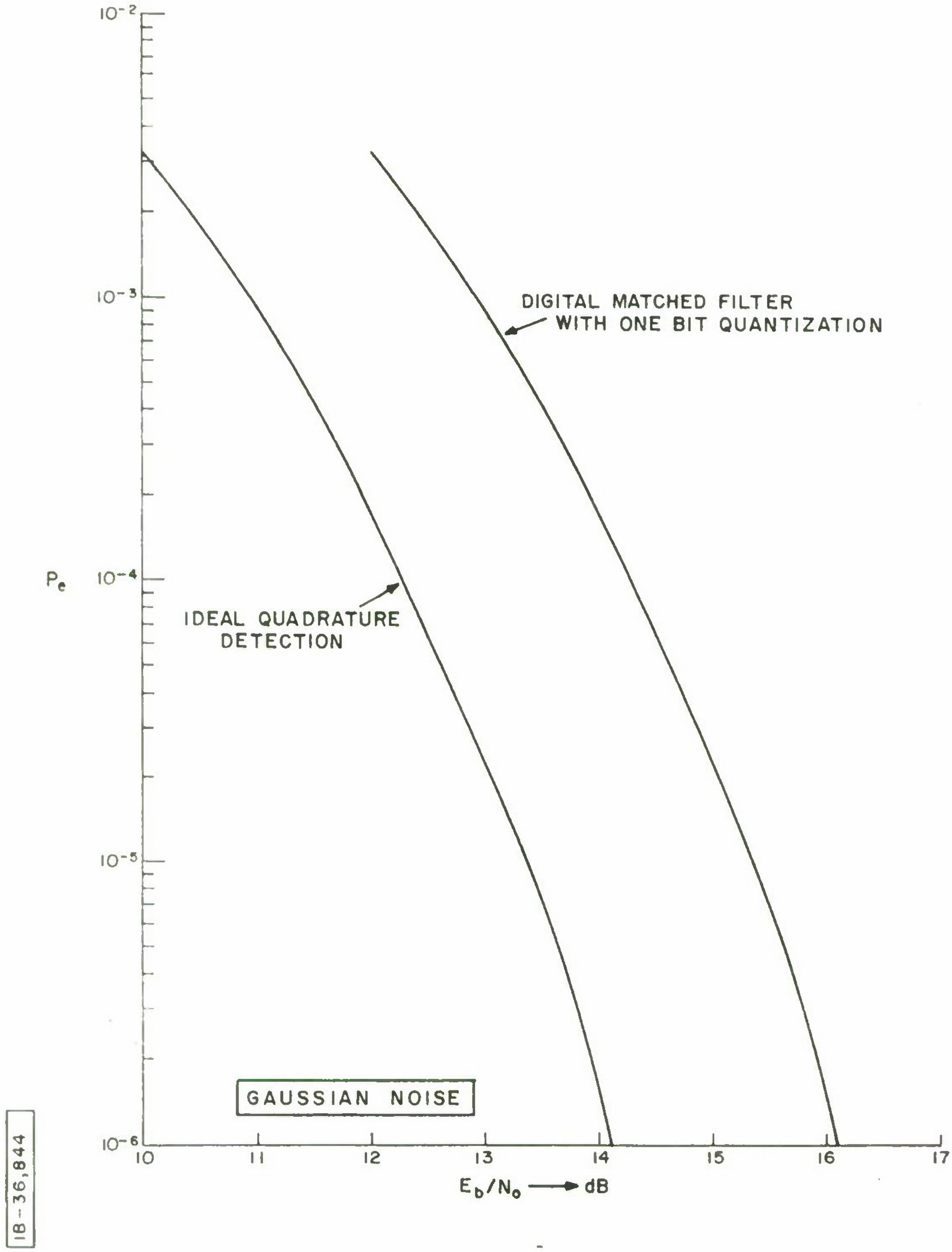

Figure 5 BIT ERROR PROBABILITY VERSUS E_b/N_o FOR A DIGITAL MATCHED FILTER USING ONE BIT QUANTIZATION

The bit error probability is then given by the probability that $Z_1 > Z$ or

$$P_e = \text{Prob}\ (Z_1 > Z)$$

As in the previous section if we now let the term

$$\frac{4n^2}{\pi^2} \left(\frac{S}{J}\right) = c^2 \tag{72}$$

in equation 70 we may again use the result given by equation D10 of Appendix D, that is

$$P_e = \frac{1}{2} \exp - \left(\frac{c^2}{4n}\right) \tag{73}$$

Substituting in equation 73 for c^2 from equation 72 we have

$$P_e = \frac{1}{2} \exp - \left[\frac{n}{\pi^2} \left(\frac{S}{J}\right)\right] \tag{74}$$

Figure 6 is a plot of the bit error rate P_e versus the product of the processing gain n and the signal to interference power ratio (S/J).

5.3 Fixed Phase Sinusoidal Interference

It has already been shown in Appendix C and summarized in Section 3.3 there are only two possible states for the output of the digital matched filter receiver. The first state occurs when the phase angle of the interfering sinusoid lies in the double signal suppression zone causing suppression of both the inphase and quadrature components of the signal. For this state, the output of the receiver is noise only resulting in a purely random output bit stream. The probability of a bit error for this condition is then given by

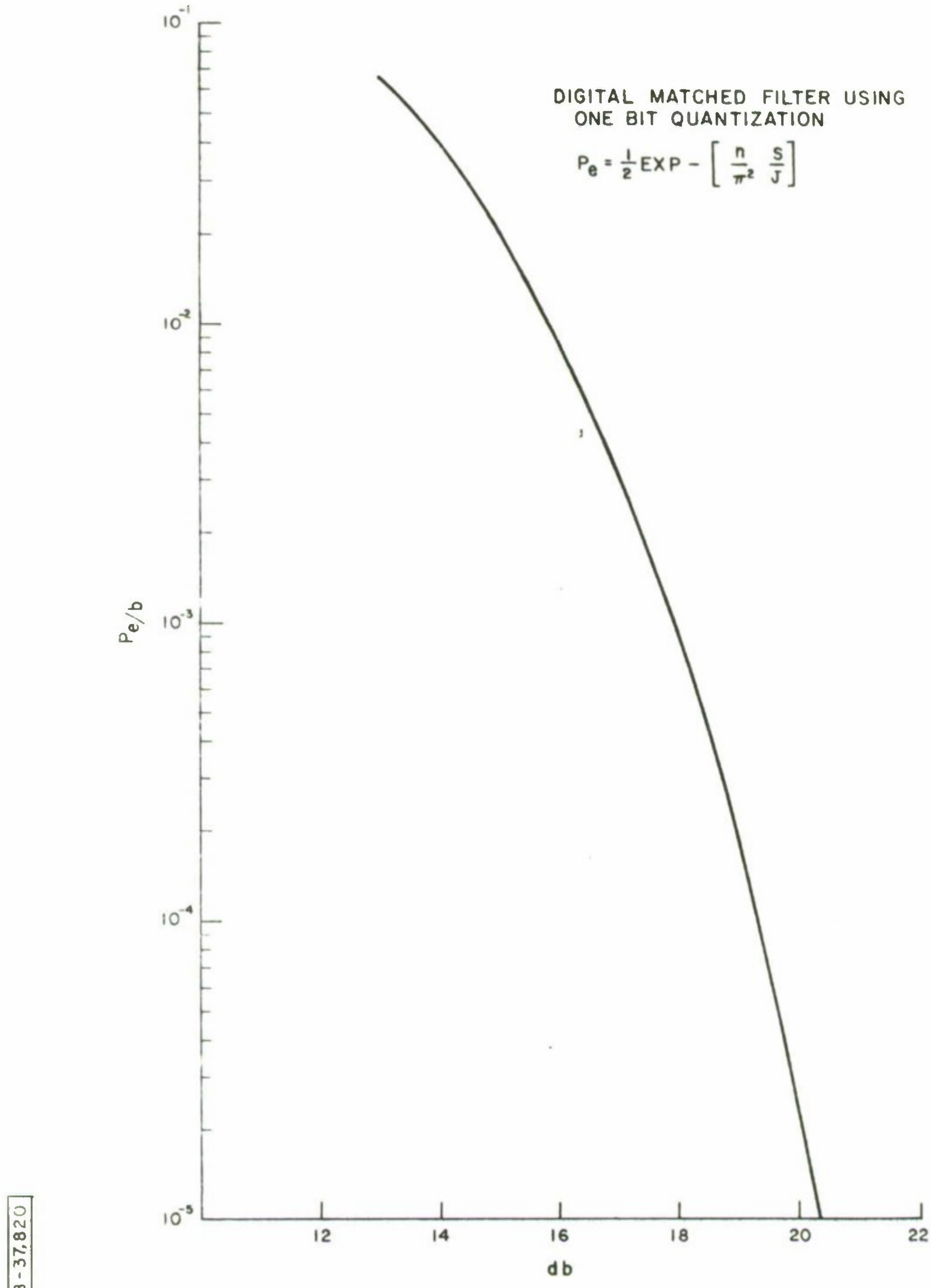

Figure 6 BIT ERROR PROBABILITY VERSUS THE PRODUCT OF PROCESSING GAIN AND SIGNAL TO JAMMING RATIO P_e VERSUS $n\left(\frac{S}{J}\right)$

$$P_e = \frac{1}{2} \tag{75}$$

The second state occurs when the phase of the interfering sinusoid lies in one of the single signal suppression zones causing suppression of either the inphase or quadrature component of the signal but not both. For this second state, the probability of a bit error is given by equation (26) of Appendix C, i.e.,

$$P_e = \frac{1}{\sqrt{2}} \exp - (\frac{n}{2}) \tag{76}$$

5.3.1 Comments on the Bit Error Rate Against Fixed Phase Sinusoidal Interference

(a) For values of n likely to be used in the system (n = 64, 128, 256) the value of P_e as given by equation (76) is extremely small ranging from 10^{-14} to 10^{-56}. It is therefore apparent that the error rate performance of the receiver would start to be controlled by the "front end" receiver noise well before an error rate like 10^{-14} could ever be reached.

(b) A necessary condition for a usable bit error rate is of course that the receiver by synchronized. If the receiver does not detect the synchronizing preamble the message will be missed since the timing circuits which control the sampling of the data portion of the transmission will not be enabled. We saw in Section 4.3 particularly Figure 13 that the probability of synchronizing against fixed phase sinusoidal interference was low, having upper and lower bounds of .52 and .38 for a signal to jamming power ratio of (S/J) = -5 dB. The upper and lower bounds at (S/J) = -10 dB are .28 and .20 respectively. We see therefore that for this type of interference at a signal to jamming ratio of -10 dB, on the average we will miss between 7 and 8 messages out of every 10 transmitted.

SECTION VI

CONCLUSIONS

The analysis of the performance of a digital matched filter (DMF) receiver using 1 bit quantization in both the inphase and quadrature channels of the DMF when the input signal is a biphase modulated signal in the presence of:

1) Gaussian noise
2) random phase sinusoidal interference
3) fixed phase sinusoidal interference

has given the following results:

6.1 <u>Gaussian Noise</u>

a. There is a loss of 1.96 dB ($2/\pi$) in detection efficiency compared with ideal quadrature detection (analog integration). This loss is solely attributable to the use of 1 bit quantization as opposed to analog integration.

b. The bit error rate for a receiver using this type of digital matched filter is given by

$$P_e = \frac{1}{2} \exp - \left[\frac{1}{\pi} \frac{E_b}{N_o} \right]$$

c. The output voltage Z of the DMF has a Rician probability density function (PDF) given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2 + \frac{4n^2}{\pi} (\frac{S}{N})_i}{2n} \right] I_o \left(\frac{Z}{n} \sqrt{\frac{4n^2}{\pi} (\frac{S}{N})_i} \right)$$

when samples of the received biphase signal are correctly aligned in the filter (the "in sync" condition).

The output voltage Z of the DMF has a Rayleigh PDF given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2}{2n} \right]$$

when the samples are either Gaussian noise alone, or Gaussian noise plus a biphase signal incorrectly aligned in the filter (the "out of sync" condition).

These two facts allow the synchronization performance of the DMF to be determined using standard [3,4] receiver operating characteristics.

6.2 Random Phase Sinusoidal Interference

a. There is a loss in detection efficiency of 6.94 dB ($2/\pi^2$) compared with ideal quadrature detection (analog integration). This loss is solely attributable to the use of one bit quantization as opposed to analog integration.

b. The bit error rate for a receiver using this type of digital matched filter is given by

$$p_e = \frac{1}{2} \exp - \left[\frac{n}{\pi^2} \left(\frac{S}{J}\right) \right]$$

where

n = receiver processing gain (i.e., number of chips/bit)

$\left(\frac{S}{J}\right)$ = signal to interference power ratio at the input to the receiver.

c. The output voltage Z of the digital matched filter has a Rician probability density function given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2 + \frac{4n^2}{\pi^2} \left(\frac{S}{J}\right)}{2n} \right] I_o \left(\frac{Z}{n} \sqrt{\frac{4n^2}{\pi^2} \left(\frac{S}{J}\right)} \right)$$

when samples of the received biphase signal are correctly aligned in the DMF (the "in sync" condition).

The output voltage Z of the DMF has a Rayleigh PDF given by

$$p(Z) = \frac{Z}{n} \exp - (\frac{Z^2}{2n})$$

when the samples are random phase sinusoid alone or a random phase sinusoid plus a biphase signal incorrectly aligned in the filter (the "out of sync" condition).

Again these two facts allow the synchronization performance of the DMF to be determined using the same set of receiver operating characteristics [3,4].

6.3 Fixed Phase Sinusoidal Interference

a. When the interference is a fixed phase sinusoid it is shown that there are only two possible states for the output of the digital matched filter for the case where the received sequence is correctly aligned in the matched filter. One state is that in which the phase of the interfering sinusoid causes both the inphase and quadrature components of the demodulated signal to be completely suppressed. We call this effect double suppression and it occurs when the phase angle of the interfering sinusoid χ is such that

$$\chi_q < \chi < \chi_i$$

where

$$\chi_i = \cos^{-1} (\frac{A}{B} \cos \theta)$$

$$\chi_q = \sin^{-1} (\frac{A}{B} \cos \theta)$$

and

A = amplitude of the received signal

B = amplitude of the interfering sinusoid

θ = phase of the received signal

When double signal suppression occurs the output of the receiver is noise only. The second state of the receiver output is that which occurs when only the inphase or quadrature component of the received signal is suppressed. This occurs when

$$\chi < \chi_i \quad \text{(inphase channel suppressed)}$$

$$\chi > \chi_q \quad \text{(quadrature channel suppressed)}$$

where χ_i and χ_q are as previously defined.

b. The bit error rate for the digital matched filter receiver is given by

$$P_e = \frac{1}{2}$$

when double signal suppression occurs, and by

$$P_e = \frac{1}{\sqrt{2}} \exp - (\frac{n}{2})$$

when single signal suppression occurs.

c. When double signal suppression occurs it is not possible for the receiver to synchronize correctly since the receiver output is noise only.

It is also shown that the probability of synchronizing when the samples of the signal are correctly aligned in the DMF has upper and lower bounds given by

$$P(\text{sync})_{max} = \left[1 - \frac{2}{\pi} \cos^{-1}\left(\sqrt{(\frac{S}{J})}\sqrt{2 - (\frac{S}{J})}\right)\right]$$

and by

$$P(\text{sync})_{min} = \left[1 - \frac{2}{\pi} \cos^{-1} \sqrt{(\frac{S}{J})}\right]$$

These results are based on the assumptions that

1. the phase angle of the interfering sinusoid remains constant for the duration of a synchronizing sequence,
2. the phase angle of the interfering sinusoid for consecutive synchronization transmissions are independent and are uniformly distributed ($0 < \chi < 2\pi$),
3. the voltage threshold at the output of DMF is set at a value $Z_T = n$. This value for Z_T is optimum as discussed in Section 4.3.1.

APPENDIX A

PROBABILITY DENSITY FUNCTION FOR THE DMF OUTPUT WHEN THE SIGNAL IS RECEIVED TOGETHER WITH WHITE GAUSSIAN NOISE

For a bi-phase modulated signal plus Gaussian noise the probability density functions of the inputs to the I & Q channel are both Gaussian. The I channel input has a mean value A cos θ and a variance σ_I^2, the Q channel has a mean value A sin θ and variance σ_q^2. The PDF's are shown in Figure 7. Consider the I channel only; the density function for the input is

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma_I} \exp - \left(\frac{x - A \cos \theta}{\sqrt{2}\,\sigma_I}\right)^2 dx \qquad (1)$$

The probability p_i that a digit is shifted into the register with an incorrect sign (i.e., different to that stored in its matching position in the register) is simply the probability that it has the opposite sign to +A cos θ or to -A cos θ, depending on the phase of the received signal). Therefore

$$p_i = \int_{\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma_I} \exp - \left(\frac{x - A \cos \theta}{\sqrt{2}\,\sigma_i}\right)^2 dx \qquad (2)$$

This can be shown to be equal to

$$p_i = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{A \cos \theta}{\sqrt{2}\,\sigma_I}\right)\right]$$

Assuming that the input signal to noise ratio is small, i.e.,

$$\frac{A^2}{2\,\sigma_I^2} << 1$$

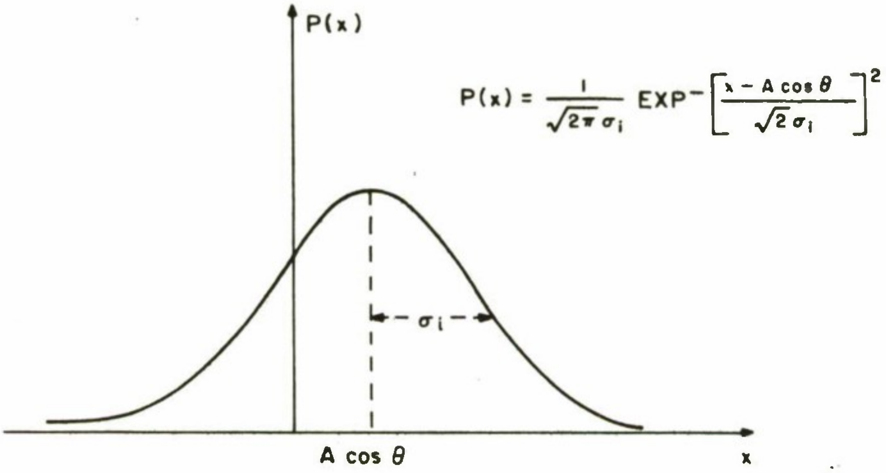

Figure 7a PROBABILITY DENSITY FUNCTION FOR THE INPUT TO THE I CHANNEL A/D CONVERTER

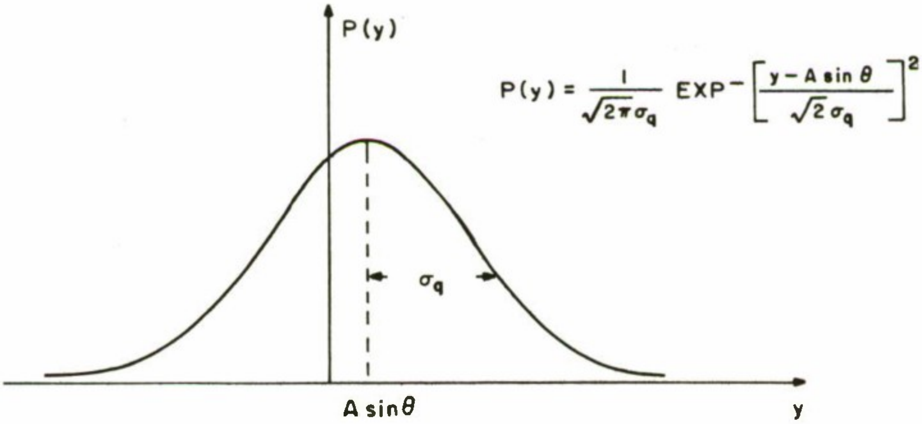

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Figure 7b PROBABILITY DENSITY FUNCTION FOR THE INPUT TO THE Q CHANNEL A/D CONVERTER the first term of the series expansion for erf may be used as a good approximation, i.e.,

$$\text{erf } \left(\frac{A \cos \theta}{2\ \sigma_I}\right) \approx \frac{2}{\sqrt{\pi}} \left(\frac{A \cos \theta}{2\ \sigma_I}\right). \tag{4}$$

Using equations 3 and 4 we have

$$p_i = \frac{1}{2} \left[1 - \frac{2}{\sqrt{\pi}} \frac{A \cos \theta}{2\ \sigma_I} \right]. \tag{5}$$

Similarly, it may be shown that the probability of an incorrect digit being fed into the Q channel shift register is

$$p_q = \frac{1}{2} \left[1 - \frac{2}{\sqrt{\pi}} \frac{A \sin \theta}{\sqrt{2}\ \sigma_q} \right]. \tag{6}$$

Equations 5 and 6 give the probabilities that a particular digit (1 or 0) in the I or Q shift register do not match that of the stored sequence.

Again restricting attention to the I channel, the probability that exactly k of the n digits in the shift register (for the in sync condition) are correct is given by the binomial distribution.

$$p(k) = \binom{n}{k} (1 - p_i)^k\ p_i^{\,n-k} \tag{7}$$

The digital number or voltage V_I at the output of the I channel shift register is given by

$$\begin{aligned} V_I &= \text{Number of correct digits} - \text{Number of incorrect digits} \\ &= k - (n - k) \qquad (8) \\ V_I &= 2k - n \end{aligned}$$

The number k is binomially distributed with a mean value $\overline{k}$ given by

$$\overline{k} = n\ (1 - p_i) \tag{9}$$

and variance

$$\sigma_k^{\ 2} = n\ (1 - p_i)\ p_i \tag{10}$$

Using equations 8 and 9 the mean value $\overline{V}_I$ is given by

$$\begin{aligned} \overline{V}_I &= 2\overline{k} - n \\ &= 2n\ (1 - p_i) - n \\ \overline{V}_I &= n\ (1 - 2p_i) \end{aligned} \tag{11}$$

Substituting for p_i from equation 5 in equation 11 we have

$$\overline{V}_I = \frac{2n}{\sqrt{\pi}}\ \frac{A \cos \theta}{\sqrt{2}\ \sigma_I} = \frac{2n}{\sqrt{\pi}}\ \sqrt{(\frac{S}{N})_i}\ \cos \theta \tag{12}$$

Also the variance $\sigma^2_{V_I}$ is given by

$$\sigma^2_{V_I} = (2\ \sigma_k)^2$$

so that using equations 5 and 10 we have

$$\sigma^2_{V_I} = 4n \left[\frac{1}{2}\ (1 + \frac{2}{\sqrt{\pi}}\ \frac{A \cos \theta}{\sqrt{2}\ \sigma_I})\ \frac{1}{2}\ (1 - \frac{2}{\sqrt{\pi}}\ \frac{A \cos \theta}{\sqrt{2}\ \sigma_I})\right]$$

$$\sigma^2_{V_I} \approx n \left[1 - (\frac{2}{\sqrt{\pi}}\ \frac{A \cos \theta}{\sqrt{2}\ \sigma_I})^2\right]$$

$$\sigma^2_{V_I} \approx n \qquad \text{for } (\frac{A}{\sqrt{2}\ \sigma_I})^2 = (\frac{S}{N})_i << 1 \tag{13}$$

Using a similar procedure it may be shown that the output V_q from the q channel shift register has a mean and variance given by

$$\overline{V}_q = \frac{2n}{\pi} \frac{A \sin \theta}{\sqrt{2}\ \sigma_q} = \frac{2n}{\sqrt{\pi}} \sqrt{\left(\frac{S}{N}\right)_i} \sin \theta \qquad (14)$$

and

$$\sigma^2_{V_q} \approx n \qquad (15)$$

For n large and p_i approaching .5 (i.e., small input signal to noise ratio) the binomial distribution for V_I (and V_q) may be closely approximated by a normal distribution with the same mean and variance.*

The final output Z from the digital matched filter is obtained by taking the square root of the sum of the squares of V_I and V_q.

To assess the performance of the DMF, we finally require the probability density function for Z.

It may be shown [5] that the square root of the sum of the squares of two normally distributed random variables with identical variances σ^2 and means m_1 and m_2 is given by

$$p(Z) = \frac{Z}{\sigma^2} \exp - \left[\frac{1}{2\sigma^2} \left\{ Z^2 + (m_1{}^2 + m_2{}^2) \right\} \right] I_o \left(\frac{Z \sqrt{m_1{}^2 + m_2{}^2}}{\sigma^2} \right) \qquad (16)$$

Substituting in equation 16 for the mean and variance of the two normal distributions from equations 12, 13, 14 and 15 we obtain for the PDF of the output voltage Z

---

* The accuracy of this approximation is good only in the region of the mean value of the density function, the accuracy increasing as p approached .5. Towards the tail of the density function the approximation is never good. See for example, "Probability and its Engineering Uses", T. C. Fry, Pages 227-234.

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \left\{ Z^2 + \frac{4n^2}{\pi} \left(\frac{S}{N}\right) \right\} \right] I_o \left(\frac{Z}{n} \frac{2n}{\sqrt{\pi}} \sqrt{\left(\frac{S}{N}\right)_i} \right) \qquad (17)$$

This is the required probability density function for the output of the DMF for the in sync condition.

APPENDIX B

PROBABILITY DENSITY FUNCTION FOR THE DMF OUTPUT WHEN THE SIGNAL IS RECEIVED TOGETHER WITH A RANDOM PHASE INTERFERING SINUSOID

Figure 8 shows a vector diagram of the received signal of amplitude A and the interfering sinusoid amplitude B. The phase angle of the interfering sinusoid is assumed to have a uniform distribution $-\pi < \chi < +\pi$, that is during the integration period of n chips all angles of ϕ are assumed equally probable. Figure 9 shows the probability density function of the input x to the I channel A/D converter while Figure 10 shows that of y the input to the Q channel A/D converter.

Looking at Figure 9 we see that the probability p_i that the output x from the I channel A/D converter and hence the probability that the digit (one or zero) fed into the shift register has a sign different to that stored in the matching position in the comparison register is given by

$$p_i = \frac{1}{\pi}\int_{-(B - A\cos\theta)}^{0} \frac{1}{\sqrt{B^2 - (x - A\cos\theta)^2}}\, dx$$

which, due to symmetry of the probability density function simplifies to

$$p_i = \frac{1}{\pi}\int_{A\cos\theta}^{B} \frac{1}{\sqrt{B^2 - x^2}}\, dx$$

or

$$p_i = \frac{1}{2} - \frac{1}{\pi}\sin^{-1}\left(\frac{A\cos\theta}{B}\right)$$

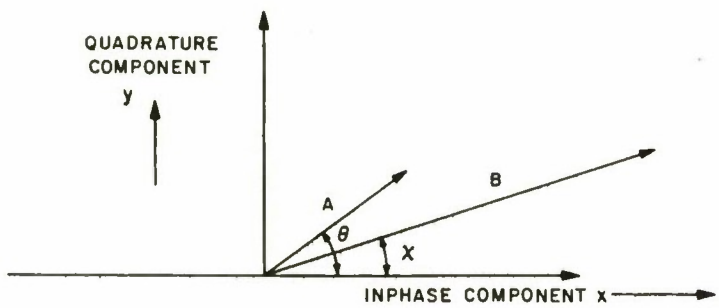

Figure 8 VECTOR DIAGRAM OF BIPHASE MODULATED SIGNAL A AND INTERFERING SINUSOID B

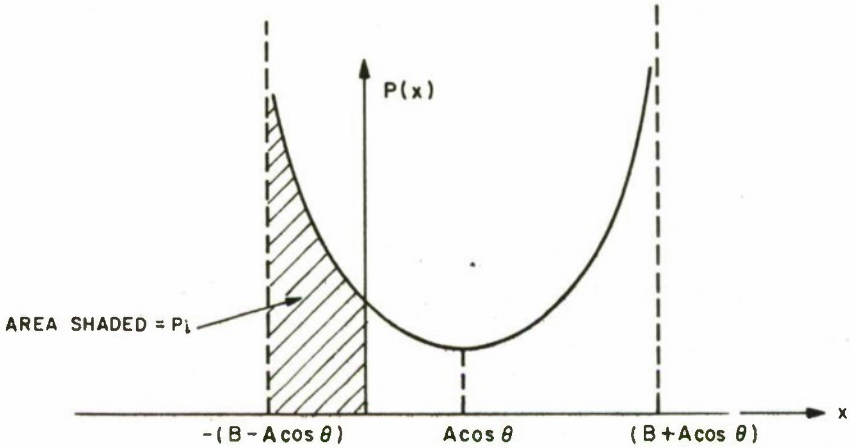

Figure 9 PROBABILITY DENSITY FUNCTION FOR THE INPUT TO THE I CHANNEL A/D CONVERTER

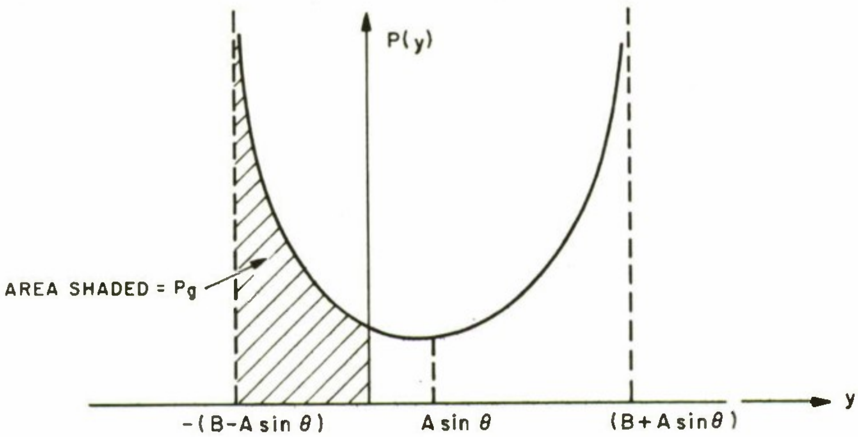

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Figure 10 PROBABILITY DENSITY FUNCTION FOR THE INPUT TO THE Q CHANNEL A/D CONVERTER

For B > A (say two or three times as large)*

$$p_i = \frac{1}{2} - \frac{1}{\pi} \; \frac{A \cos \theta}{B} \tag{1}$$

Similarly the probability of an incorrect digit being entered in the Q channel is

$$p_q = \frac{1}{2} - \frac{1}{\pi} \; \frac{A \sin \theta}{B} \tag{2}$$

Restricting attention to the I channel, the probability that exactly k of the n digits in the shift register (for the in sync condition) is given by the binomial distribution

$$p(k) = \binom{n}{k} (1 - p_i)^k \; p_i^{\,n-k} \tag{3}$$

The digital number of voltage at the output of the I channel shift register is given by

$$\begin{aligned} V_I &= \text{Number of correct digits} - \text{Number of incorrect digits} \\ &= k - (n - k) \\ V_I &= 2k - n \end{aligned} \tag{4}$$

The number k is binomially distributed with a mean value $\overline{k}$ given by

$$\overline{k} = n \, (1 - p_i) \tag{5}$$

---

* Approximating the sine of an angle by the angle measured in radians only leads to an error of approximately 5% for an angle even as large as 30°. This ensures that the approximation used will always have an error less than 5% if B > 2A.

and variance

$$\sigma_k^2 = n\ (1 - p_i)\ p_i \tag{6}$$

Using equations 4 and 5, the mean value $\overline{V}_I$ is given by

$$\begin{aligned} \overline{V}_I &= 2\overline{k} - n \\ &= 2n\ (1 - p_i) - n \\ \overline{V}_I &= n\ (1 - 2p_i) \end{aligned} \tag{7}$$

Substituting in equation 7 for p_i from equation 1, we have

$$\overline{V}_i = \frac{2n}{\pi} \frac{A \cos \theta}{B} \tag{8}$$

Also the variance $\sigma_{V_I}^2$ is given by

$$\sigma_{V_I}^2 = (2\sigma_K)^2 \tag{9}$$

Using equations 6 and 9 we have

$$\sigma_{V_I}^2 = 4n\ (1 - p_i)\ p_i \tag{10}$$

Substituting for p_i from equation 1 in equation 10, we have

$$\begin{aligned} \sigma_{V_I}^2 &= 4n\ \left(\frac{1}{2} + \frac{1}{\pi}\ \frac{A \cos \theta}{B}\right)\ \left(\frac{1}{2} - \frac{1}{\pi}\ \frac{A \cos \theta}{B}\right) \\ &= n \left[1 - \left(\frac{2}{\pi}\ \frac{A \cos \theta}{B}\right)^2\right] \\ &\approx n \quad \text{for } \frac{A}{B} << 1 \end{aligned} \tag{11}$$

Similarly, the mean and variance of the output voltage from the Q channel shift register is given by

$$\overline{V}_q \approx \frac{2n}{\pi} \; \frac{A \sin \theta}{B} \tag{12}$$

and

$$\sigma_q^2 \approx n \tag{13}$$

Following the procedure covered in Appendix A, and in particular equations 16 and 17, we have that the probability density function for the output Z of the square root circuit is given by

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{1}{2n} \; \{Z^2 + \frac{4n^2}{\pi^2} \; \frac{A^2}{B^2}\} \right] \; I_o \; (\frac{Z}{n} \, \frac{2n}{\pi} \, \frac{A}{B}) \tag{14}$$

Since

$$\frac{A^2}{B^2} = (\frac{S}{J}) = \frac{\text{Signal Power}}{\text{Interference Power}} \tag{15}$$

equation 14 may be rewritten as

$$p(Z) = \frac{Z}{n} \; \exp - \left[\frac{1}{2n} \{Z^2 + \frac{4n^2}{\pi^2} \; (\frac{S}{J})\} \right] \; I_o \left\{ \frac{Z}{n} \sqrt{\frac{4n^2}{\pi^2} \; (\frac{S}{J})} \right\} \tag{16}$$

APPENDIX C

ANALYSIS OF VARIOUS ASPECTS OF DMF PERFORMANCE AGAINST FIXED PHASE SINUSOIDAL INTERFERENCE

C.1 Introduction

In this appendix we consider some aspects of the performance of the Digital Matched Filter Receiver against fixed phase sinusoidal interference. In some respects this work is only of theoretical interest since it is unlikely that the receiver would be subjected to this type of interference. For the case of an enemy jammer using an inband jamming sinusoid it would be very difficult for the jammer to remain exactly on frequency and in phase with the communication signal for a period of time corresponding to the transmission of many bits of information. This inability to remain on frequency and in fixed phase relation to the communication signal is due to frequency instabilities in the communication transmitter and receiver together with a possible doppler frequency due to one or both terminals being airborne. One possible case of an on frequency fixed phase interfering signal is that of a strong multipath reflection of the communication signal into the receiving antenna.

In summary, although it appears that the case of an on-frequency fixed phase interfering sinusoid is unlikely it should be considered since it will represent an upper bound on performance against a sinusoidal interfering signal.

C2. Signal Suppression Zones

In this section we consider the conditions under which it is possible for a fixed phase sinusoidal interfering signal to completely suppress the inphase or quadrature components of the DMF output or both.

Figure 11 shows the vector diagram for the signal of amplitude A and phase θ and the jamming sinusoid amplitude B and phase χ.

The input x to the I channel A/D converter of the DMF (see Fig. 11) will be

$$x = \pm A \cos \theta + B \cos \chi \tag{1}$$

and the input y to the Q channel A/D converter will be

$$y = \pm A \sin \theta + B \sin \chi \tag{2}$$

Considering equation 1 we see that the sign of output of the I channel A/D converter is completely determined by the interfering signal B if

$$|B \cos \chi| \geq |A \cos \theta| \tag{3}$$

and by the communication signal A if

$$|A \cos \theta| > |B \cos \chi| \tag{4}$$

Let the value of χ (considering only the first quadrant for the moment) at which this change over occurs be denoted by χ_i, where

$$\chi_i = \cos^{-1} \left(\frac{A}{B} \cos \theta\right) \tag{5}$$

Similarly, there is a critical value of χ at which the sign of the Q channel A/D converter output is determined either by the interfering signal or the communications signal. Using equation 2 this angle is given by

$$\chi_q = \sin^{-1} \left(\frac{A \sin \theta}{B}\right) \tag{6}$$

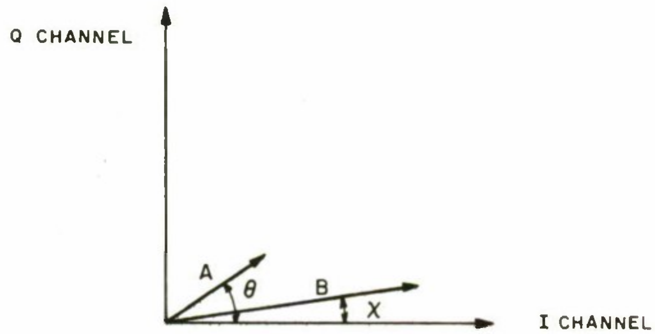

Figure 11 VECTOR DIAGRAM OF THE BI-PHASE SIGNAL A AND THE INTERFERING SINUSOID B

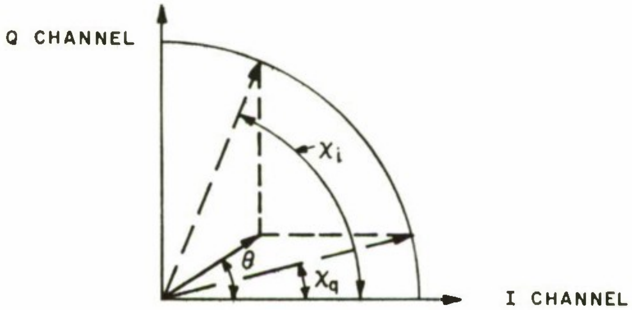

Figure 12 SHOWING HOW THE FIRST QUADRANT IS DIVIDED INTO THREE ANGULAR ZONES

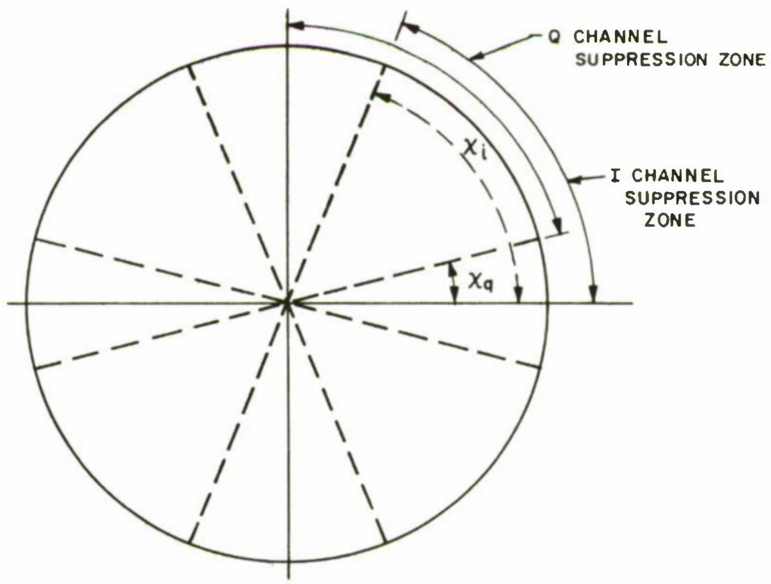

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Figure 13 SHOWING HOW THE SIGNAL SUPPRESSION ZONES ARE REPEATED IN EACH QUADRANT

Figure 12 shows how χ_i and χ_q divide the first quadrant into three angular zones such that for

$\chi < \chi_i$ the I channel is suppressed

$\chi > \chi_q$ the Q channel is suppressed

If χ falls in the zone

$$\chi_q < \chi < \chi_i$$

then both the I and Q channel components at the output of the DMF are suppressed. As the phase angle χ of the jamming sinusoid varies between 0° and 2π the suppression zones for the I and Q channels are repeated in each quadrant as shown in Figure 13. We see that depending on the value of χ that either the I or Q channel output has an output n units large and the other a noise component of rms value $\sqrt{n}$ or that both the I and Q channels have a noise component of rms value $\sqrt{n}$ with no signal component. It is also obvious that the change from a signal component in the DMF output Z to all noise is very sharp and occurs when ever χ moves into the double suppression zone (i.e., I and Q both suppressed).

We see therefore that unlike the previous cases considered (i.e., Gaussian noise or a random phase sinusoid) in appendices A and B it is possible to get complete signal suppression if the phase of the interfering sinusoid lies in the double suppression zone.

C.3 Probability of Double Suppression

We saw in the previous section that if the phase angle of the jamming sinusoid χ falls in the angular zone

$$\chi_q < \chi < \chi_i$$

then both the I and Q channel outputs are suppressed and the DMF output is just noise. It is obvious that the probability of the jamming phase angle falling in or remaining in a double signal suppression zone increases as the angular zone between $\chi = \chi_i$ and $\chi = \chi_q$ increases. It is therefore of interest to determine the size of the double suppression zone and then relate it to the probability of complete signal suppression.

Let $\omega = (\chi_i - \chi_q)$ = angular width of the double suppression zone.

From equations 5 and 6 we have

$$\chi_i = \cos^{-1} \left(\frac{A}{B} \cos \theta\right) \tag{7}$$

and

$$\chi_q = \sin^{-1} \left(\frac{A}{B} \sin \theta\right) \tag{8}$$

Since

$$\cos \omega = \cos (\chi_i - \chi_q) = \cos \chi_i \cos \chi_q + \sin \chi_i \sin \chi_q$$

we have using equations 7 and 8 that

$$\cos \omega = \frac{A \cos \theta}{B} \frac{\sqrt{B^2 - A^2 \sin^2 \theta}}{B} + \frac{\sqrt{B^2 - A^2 \cos^2 \theta}}{B} \frac{A \sin \theta}{B}$$

$$= \frac{A}{B} \left[\cos \theta \sqrt{(1 - \frac{A^2}{B^2} \sin^2 \theta)} + \sin \theta \sqrt{(1 - \frac{A^2}{B^2} \cos^2 \theta)}\right]$$

$$= \frac{A}{B} \sqrt{2 - \frac{A^2}{B^2}} \cos (\theta - \alpha) \tag{9}$$

where

$$\alpha = \tan^{-1} \sqrt{\frac{1 - \frac{A^2}{B^2}\cos^2\theta}{1 - \frac{A^2}{B^2}\sin^2\theta}}$$

If B is considerably larger than A, as would be the case for effective jamming $\alpha \rightarrow \frac{\pi}{4}$ so that using equation 9 we have

$$\cos\omega \approx \frac{A}{B}\sqrt{2}\;\cos\left(\theta - \frac{\pi}{4}\right) \qquad (10)$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

Since

$$\frac{A^2}{B^2} = \frac{\text{Signal Power}}{\text{Jamming Power}} = \left(\frac{S}{J}\right)$$

Equation 10 may be written as

$$\cos\omega \approx \sqrt{2\left(\frac{S}{J}\right)}\;\cos\left(\theta - \frac{\pi}{4}\right) \qquad (11)$$

For fixed $\left(\frac{S}{J}\right)$ the double suppression zone has its minimum size when the phase angle θ of the received signal is $\left(\frac{\pi}{4}\right)$, i.e., $\cos\left(\theta - \frac{\pi}{4}\right) = 1$ so that $\cos\omega$ has its maximum value and hence ω its minimum value. The suppression zone has its maximum size when $\theta = 0$ or $\frac{\pi}{2}$.

If we assume that the phase of the jamming sinusoid has a uniform distribution bit to bit and remains essentially constant during an information bit the probability of complete signal suppression is given by the size of the angular suppression zone divided by $\frac{\pi}{2}$, i.e.,

$$\text{Prob (Signal Suppression)} = P_s = \frac{\omega}{\pi/2}$$

using equation 11 we have

$$Ps = \frac{2}{\pi} \cos^{-1}\left[\sqrt{2\left(\frac{S}{J}\right)} \cos\left(\theta - \frac{\pi}{4}\right)\right] \qquad (12)$$

The cosine term in equation 12 always has a value between .7071 and 1.0 so that a reasonable approximation for Ps is

$$Ps = \frac{2}{\pi} \cos^{-1}\left(\sqrt{2\left(\frac{S}{J}\right)}\right) \qquad (13)$$

for $\left(\frac{S}{J}\right) << 1$

C.4 Probability Density Function for the DMF Output Voltage (Single Suppression Zone)

In this section we determine the PDF for the output voltage from the DMF when the phase angle of the jamming sinusoid is in one of the single suppression zones.

The probability density function for the output Z of the square root circuit of the receiver (see Fig. 2) when χ is not in the double suppression zone is given by the envelope of a constant amplitude inphase component ($\overline{V}_i$ = n and $\sigma^2_{V_i}$ = 0) plus a quadrature Gaussian noise component ($\overline{V}_q$ = 0 and $\sigma^2_{V_q}$ = n).

Figure 14 shows a sketch of the normal PDF for the quadrature output V_q from the DMF located in the plane V_i = n. The probability that the output Z of the DMF has a value between Z and Z + dZ is equal to the shaded area under the normal distribution as shown. That is

$$p(Z)\, dZ = 2p\,(V_q)\, dV_q$$

or

$$p(Z) = 2p\,(V_q) \frac{dV_q}{dZ} \qquad (14)$$

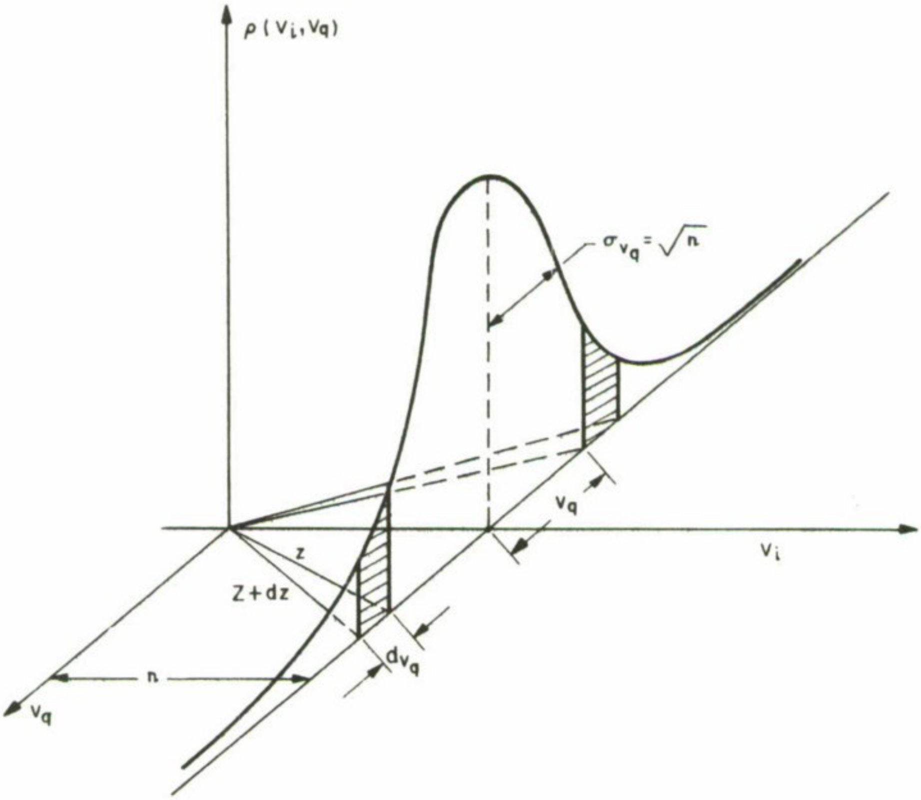

Figure 14 SKETCH OF NORMAL PROBABILITY DENSITY FUNCTION RESULTING FROM ANGLE χ IN ONE OF THE SINGLE SUPPRESSION ZONES

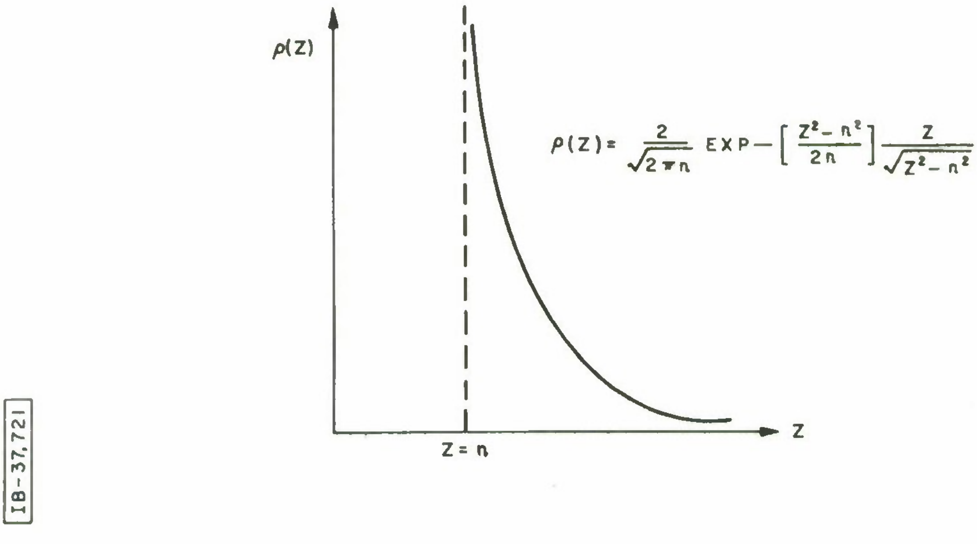

Figure 15 PROBABILITY DENSITY FUNCTION FOR OUTPUT VOLTAGE Z FROM DMF FOR JAMMER PHASE IN ONE OF THE SINGLE SUPPRESSION ZONES

Since $Z^2 = V_q^{\ 2} + V_i^{\ 2}$, and $V_i = n$, we have

$$Z^2 = V_q^{\ 2} + n^2 \tag{15}$$

Differentiating equation 15 we have

$$ZdZ = V_q dV_q \tag{16}$$

or

$$\frac{dV_q}{dZ} = \frac{Z}{V_q}$$

Substituting for V_q from equation 15 we have

$$\frac{dV_q}{dZ} = \frac{Z}{\sqrt{Z^2 - n^2}} \tag{17}$$

Since

$$p(V_q) = \frac{1}{\sqrt{2\pi}} \quad \sigma_{V_q} \quad \exp - \left(\frac{V_q}{\sqrt{2}} \quad \sigma_{V_q}\right)^2$$

and

$$\sigma_{V_q}^{\ 2} = n$$

we have

$$p(V_q) = \frac{1}{\sqrt{2\pi n}} \quad \exp - \left(\frac{V_q}{2n}\right)^2 \tag{18}$$

Using equations 14, 15, 17 and 18 we finally obtain

$$p(Z) = \frac{2}{\sqrt{2\pi n}} \exp - \left(\frac{Z^2 - n^2}{2n}\right) \frac{Z}{\sqrt{Z^2 - n^2}}$$

or

$$p(Z)dZ = \frac{2}{\sqrt{2\pi n}} \exp - \left(\frac{Z^2 - n^2}{2n}\right) \frac{Z}{\sqrt{Z^2 - n^2}} \, dZ \qquad (19)$$

Equation 19 gives the probability density function for the DMF output voltage when the phase angle of the jamming sinusoid is in one of the single signal suppression zones and is sketched in Figure 15.

C.5 Cumulative Distribution

In work on synchronization it will be necessary to have the cumulative distribution for the DMF output. Using equation 19 we have

$$p(Z_\alpha) = \frac{2}{\sqrt{2\pi n}} \int_n^{Z_\alpha} \exp - \left(\frac{Z^2 - n^2}{2n}\right) \frac{Z}{\sqrt{Z^2 - n^2}} \, dZ \qquad (20)$$

Let

$$\frac{Z^2 - n^2}{2n} = x^2$$

then $ZdZ = 2nx\, dx$ so that equation 20 becomes

$$P(Z_\alpha) = \frac{2}{\sqrt{2\pi n}} \int_0^{\sqrt{\frac{Z_\alpha^2 - n^2}{2n}}} \exp -(x^2) \, dx \sqrt{2n}$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{Z_\alpha^2 - n^2}{2n}}} \exp - x^2 \, dx$$

$$\therefore \quad P(Z_\alpha) = \operatorname{erf} \left(\sqrt{\frac{Z_\alpha^2 - n^2}{2n}} \right) \qquad (21)$$

C.6 Bit Error Probability (Single Suppression Zone)

For binary transmission two orthogonal PN sequences are used. The receiver would contain two digital matched filters, one matched to each of the two possible sequences. A bit error would occur if the voltage from the DMF to which the received sequence was not matched exceeded that from the matched filter.

The PDF for the output voltage of the DMF matched to the incoming sequence is given by equation 19, i.e.,

$$p(Z)\,dZ = \frac{2}{\sqrt{2\pi n}} \exp - \left[\frac{Z^2 - n^2}{2n} \right] \frac{Z}{\sqrt{Z^2 - n^2}} \, dZ \qquad (22)$$

The PDF for the output voltage Z_1 of the DMF orthogonal to the received sequence is given by the Rayleigh distribution, i.e.,

$$p(Z_1) = \frac{Z_1}{n} \exp \left(\frac{-Z_1^2}{2n} \right) \qquad (23)$$

The probability p_e that Z_1 exceeds the output Z from the DMF matched to the received sequence is given by

$$p_e = \text{Prob}\ (Z_1 > Z)$$

or,

$$p_e = \int_{Z=n}^{\infty} p(Z) \left[\int_{Z_1=Z}^{\infty} p(Z)\ dZ_1 \right] dZ \qquad (24)$$

The inner integral is given by

$$I = \int_{Z}^{\infty} \frac{Z_1}{n} \exp - \left(\frac{Z_1^{\,2}}{2n} \right) dZ_1$$

$$= \exp - \left(\frac{Z^2}{2n} \right)$$

so that equation 24 reduces to

$$p_e = \int_{Z=n}^{\infty} p\ (Z) \left[\exp \frac{-Z^2}{2n} \right] dZ$$

Substituting for p(Z) from equation 22 we obtain

$$p_e = \int_{Z=n}^{\infty} \frac{2}{\sqrt{2\pi n}} \exp - \left[\frac{Z^2 - n^2}{2n} \right] \frac{Z}{\sqrt{Z^2 - n^2}} \exp - \left[\frac{Z^2}{2n} \right] dZ \qquad (25)$$

Let

$$\frac{Z^2 - n^2}{2n} = x^2$$

so that

$$ZdZ = 2nx\ dx$$

Equation 25 may now be written as

$$P_e = \frac{2}{\sqrt{2\pi n}} \int_0^\infty \exp - x^2 \quad 2n \exp - (x^2 + \frac{n}{2}) \, dx$$

$$= \frac{2}{\sqrt{\pi}} \exp \left[- \frac{n}{2} \right] \int_0^\infty \exp - 2x^2 \, dx$$

$$= \frac{2}{\sqrt{\pi}} \exp \left[- \frac{n}{2} \right] \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

$$P_e = \frac{1}{\sqrt{2}} \exp \left(- \frac{n}{2} \right) \tag{26}$$

From equation 26 we see that for reasonably large processing gains (i.e., $n > 50$) the error rate due to the phase angle of the jamming sinusoid being in one of the single suppression zones is small. The controlling term in the error rate for this case would be receiver noise and not the jamming signal.

APPENDIX D

EVALUATION OF AN INTEGRAL RELATED TO ERROR RATE CALCULATIONS

In determining the binary error rate for a matched filter receiving a bi-phase signal together with Gaussian noise or random phase sinusoidal interference, evaluation of the probability that one random variable Z_1 exceeds a second random variable Z is necessary. The two random variables have probability density functions of the form

$$p(Z_1) = \frac{Z_1}{n} \exp \left[\frac{-Z_1^2}{2n}\right] \tag{D1}$$

and

$$p(Z) = \frac{Z}{n} \exp - \left[\frac{Z^2 + c^2}{2n}\right] I_o \left(\frac{Z}{n} c\right) \tag{D2}$$

where c is a known constant.

We therefore require

$$P_e = \text{Prob } (Z_1 > Z)$$

We have therefore that

$$P_e = \int_{Z=0}^{\infty} p(Z) \left[\int_{Z_1=Z}^{\infty} p(Z_1)\, dZ_1\right] dZ \tag{D3}$$

Evaluating the integral containing Z_1 only, we have, using equation D1 that

$$\int_{Z_1=Z}^{\infty} \frac{Z_1}{n} \exp - \frac{Z_1^2}{2n}\, dZ_1 = \exp - \left(\frac{Z^2}{2n}\right) \tag{D4}$$

Substituting in equation D3 for the value of the inner integral we obtain

$$P_e = \int_{Z=0}^{\infty} \frac{Z}{n} \exp - \left[\frac{2Z^2 + c^2}{2n}\right] I_o \left(\frac{Z}{n} c\right) dZ \qquad \text{(D5)}$$

$$= \frac{1}{n} \exp - \frac{c^2}{2n} \int_{Z=0}^{\infty} Z \exp - \left[\frac{Z^2}{n}\right] I_o \left(\frac{Z}{n} c\right) dZ \qquad \text{(D6)}$$

An integral similar to that in equation D6 is evaluated in [6] that is

$$\int_{0}^{\infty} Z \exp (-p^2 Z^2) J_o (aZ) \quad \frac{1}{2p^2} \exp \left(\frac{-a^2}{4p^2}\right) \qquad \text{(D7)}$$

Since

$$J_o [(\sqrt{-1}\, a) Z] = I_o (aZ) \qquad \text{(D8)}$$

we have

$$\int_{0}^{\infty} Z \exp (-p^2 Z^2) I_o (aZ) \, dZ = \frac{1}{2p^2} \exp \left(\frac{a^2}{4p^2}\right) \qquad \text{(D9)}$$

Using the result in equation D9 and letting

$$p^2 = \frac{1}{n} \quad \text{and } a = \frac{c}{n}$$

equation D6 reduces to

$$P_e = \frac{1}{2} \exp \left(- \frac{c^2}{4n}\right) \qquad \text{(D10)}$$

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13. ABSTRACT

This MTR contains a summary of some of the more important theoretical results related to the performance of a digital matched filter receiver designed to receive a biphase modulated signal in the presence of interference. Both the synchronization and data performance of the receiver are evaluated when the signal is received in the presence of a) Gaussian noise, b) Random phase sinusoidal interference, c) Fixed phase sinusoidal interference. Specific attention is paid to the effect of processing the received signal using one bit quantization.

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