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PRESENT AND ADVANCED GUIDANCE TECHNIQUES

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Aerospace Corporation

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Present and Advanced Guidance Techniques

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Engineering Science Operations
THE AEROSPACE CORPORATION

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13. ABSTRACT

Seven guidance equation techniques, both present and advanced, are discussed. Sufficient detail is given so that the basic philosophy and equation development for each technique may be understood. The techniques described are: Q, Delta, Explicit, Linear Tangent, "Optimal," Numerical Integration, and Parameter Optimization. Both normal and fast reaction targeting concepts are discussed. Comparison is made between the seven techniques with respect to computer storage and speed requirements, targeting requirements, and mission flexibility. It is noted that Q and Delta guidance concepts require the least computer storage but also offer the least flexibility. The Explicit and Linear Tangent techniques require exact integration of approximate dynamics equations. Optimal and Numerical Integration guidance both require numerical integration of more exact dynamics equations. Parameter Optimization guidance has the severest computational requirements, but offers many significant advantages. It is hoped that this review will motivate new programs now in the planning stages to consider the advanced guidance concepts.

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FOREWORD

This report was prepared in support of the Air Force Space and Missile Test Center (SAMTEC) Inertially Designated Tracker Study and is published by The Aerospace Corporation, El Segundo, California, under Air Force Contract F-4701-72-C-0073.

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

WARD J. LANTIER, Lt Col, USAF
Chief, Missile Guidance and Technology
Deputy for Technology

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I. INTRODUCTION

This report provides a discussion of seven guidance techniques. Some of these techniques are considered to be advanced in that their targeting requirements are minimal. It is believed that these advanced techniques are potential candidates for future space or weapon system guidance programs, and that guidance software for advanced vehicle systems should not be restricted to current guidance schemes. The advanced guidance techniques included in this report require a higher speed, greater capability, and generally a more versatile guidance computer. Crude estimates of computer speed and capacity are provided in Table 1 for each of the techniques discussed in the report.

The seven guidance techniques are:

1. Q
2. Delta (or Δ)
3. Explicit
4. Numerical Integration
5. Linear Tangent
6. "Optimal"
7. Parameter Optimization

A summary of the features of these guidance techniques is given in Table 1. Note that Q guidance is included for completeness, although this technique is obsolete.

It has been noted that there is considerable variation in the computer requirements for each of these techniques. The requirements are minimal for Q and Δ guidance; in fact, these techniques were developed for computers that were not equipped with a hardware divide instruction. Their computer technology is of the 1956 era. As a result, these techniques were extremely difficult to target (i.e., to obtain all guidance constants).

Table 1. Guidance Technique Features

Guidance Technique	Status	Direct Optimization Routine	Crude Estimates Computational Requirements[1]		Targeting Comments
			Memory Capacity[2]	Add Time (μsec)	
Q	Obsolete	No	200	1000 with analogue integration	Very difficult; requires large least square fitting routines to determine polynomial coefficients
Δ	Currently used	No	500	100	Same as above
Explicit	Currently used	No	700 to 1500	10 to 100	Moderately difficult; often needs aim points for single stage formulations
Numerical Integration	Currently used	No	10,000	2	Easy to target; only a pitch program for flight through the atmosphere is required
Linear Tangent	Not yet used	Yes	5,000	10	Same as above
Optimal	Not yet used	Yes	7,000	5	Same as above
Parameter Optimization	Not yet used	Yes	20 to 40,000	0.1 to 1	Self-targeting

[1] The efforts of W. Sturm and R. Erilane are acknowledged for helping in preparing the estimates of computational complexity

[2] Words of storage

As time progressed, computers became available that were more versatile. Then the "explicit" guidance techniques became practical and were devised by several contractors. Targeting routines were still required in order to obtain the gravity-turn pitch program for flight through the atmosphere. However, a complicated least squares computation program to obtain polynomial expansion coefficients was not required as it was for the Q and Δ mechanizations.

The computer requirements for Numerical Integration guidance are quite large (20,000 words, 1 μsec). The targeting requirements are minimal since only a booster pitch program is required.

The Linear Tangent Guidance Program uses an approximate integration of the rocket equations of motion. The program is as easy to target as the Numerical Integration technique, since a multistage capability is included. In addition, the computer requirements are not much greater than those for explicit guidance programs.

The "Optimal" Guidance Program is based on a calculus of variations solution that requires machine computation (iteration and integration). The computation complexity is therefore somewhere between the two previous techniques. This technique also requires only an input pitch program and is therefore easy to target.

Parameter Optimization guidance is self-targeting and also has the greatest computation requirements on speed and capacity. It is equipped with an optimum capability for guided flight through the atmosphere and has the greatest capability for fast reaction of any of the previous techniques. With the ever advancing computer technology, it is possible that an airborne digital computer will one day be designed to permit usage of this technique for the weapon systems.

II. PURPOSE OF GUIDANCE EQUATIONS

The purpose of the vehicle guidance system is to utilize certain measurements of the vehicle's state and to alter the course of the vehicle in order to achieve the desired end conditions in the presence of disturbances, which include but are not necessarily limited to the following:

- Winds
- Non-standard propulsion parameters (thrust, specific impulse, engine misalignments, centering, etc.)
- Drag and li . uncertainties
- Weight errors
- Control system dispersions

The basic guidance system may be either radio or inertial. When a radio guidance system is used, the measurements consist of range, azimuth and elevation angle (R, A, E). In some cases, rate measurements may also be obtained. These measurements are then processed in a Kalman filter to obtain the vehicle navigation data, as shown in Fig. 1. All computations shown in Fig. 1 are performed in the ground guidance computer. The steering commands are sent to the vehicle over the guidance data link, decoded, and fed to the vehicle control system.

When an inertial system is employed, the sensed measurements are obtained from the vehicle accelerometers. These measurements are then processed on board the vehicle in the Flight Digital Computer as shown in Fig. 2. Note that the mechanization shown in Fig. 2 assumes that the accelerometers are mounted on an inertially stabilized platform. The navigation loop would appear different when a strapdown inertial system is employed.

The guidance equations blocks for either the radio or the inertial methods may be quite similar. The role of guidance is (either implicitly or explicitly) to specify the corrective action such that the actual vehicle state will become the desired vehicle state at the burnout point (or at orbital insertion).

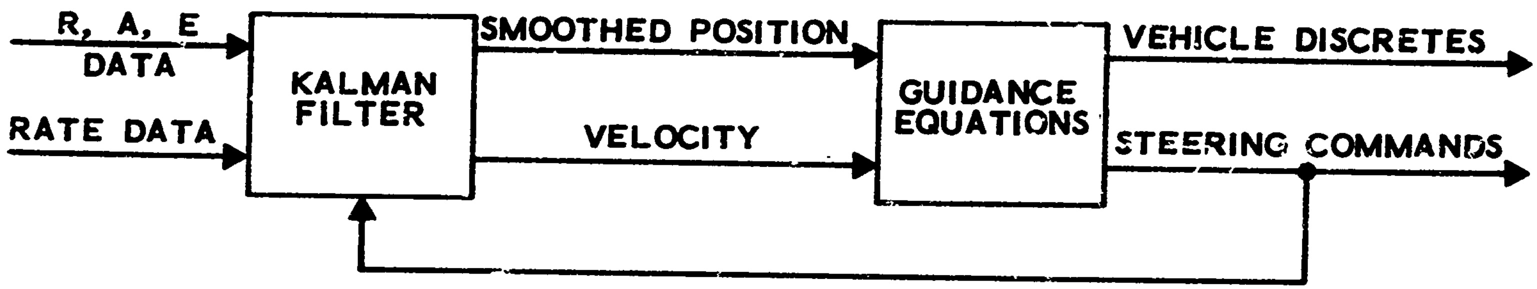


Figure 1. Guidance Mechanization Used with Radio Guidance

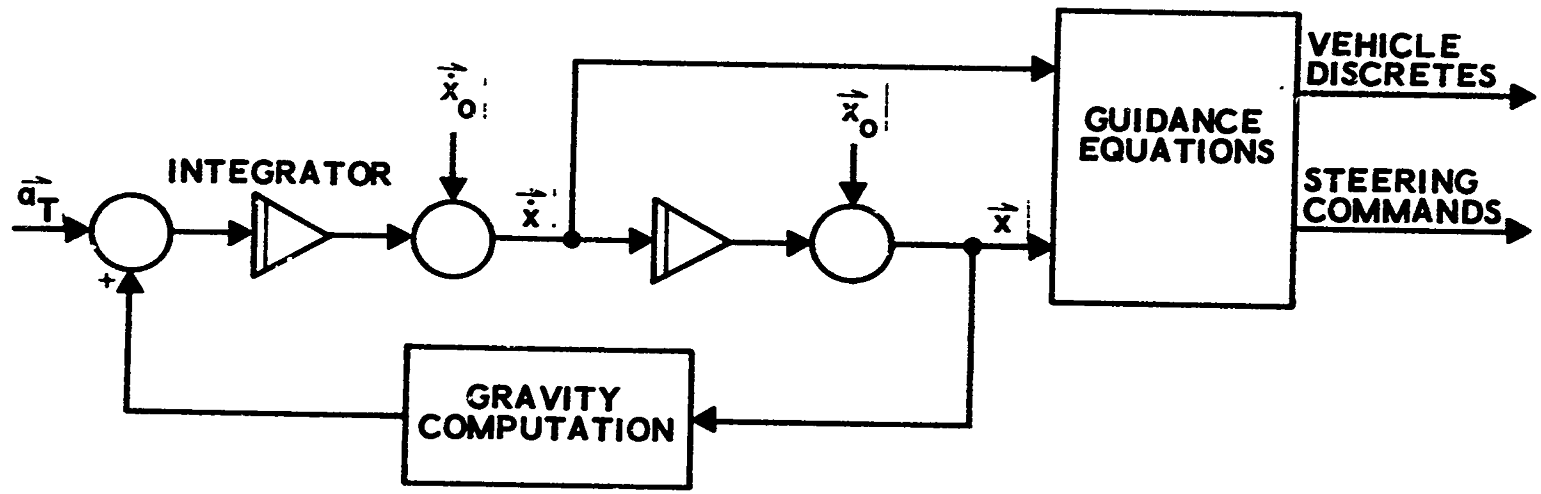


Figure 2. Guidance Mechanization Used with Inertial Guidance (IMU Assumed)

This report discusses only the philosophy of the guidance equations. Some insight into the derivations is given. In general, the airborne digital computer used with inertial guidance systems may contain many functions in addition to what has been referred to as explicit guidance. A possible listing of some of these functions and the computer words of storage and timing estimates are given in Table 2.

In addition, Appendix A is included to show the detailed equations associated with an actual guidance computer program. The block flow diagrams apply to the Radio Guided Gemini launches using the Titan launch vehicle. The total computer size (Burroughs A1 at ETR) was 3200 words including an auxiliary memory unit. Thus, one may get a feeling for the size, complexity, and logic loops when the computer permits storage in excess of 10,000 words. Some filtering is included in the time-to-go and steering command computation blocks; however, this is easy to recognize and may be ignored for inertial guidance applications.

Table 2. Typical Missile-Borne Computer Programs*

	Words	Timing (tps)
Executive Program	500	10,000
Navigation	500	2,500
Explicit Guidance	1500	2,500
Atmospheric Steering	200	200
Vehicle Sequencing	200	100
Coast Equations	200	100
Ground C/0 and Calibration	2000	--
Backup Modes	100	100
Digital Flight Control	5000	50,000
Digital Attitude Control	1000	30,000
	11,200	60,000 Worst Path

*Estimate by J. Shaul, The Aerospace Corporation

III. TARGETING

An important figure of merit of any set of guidance equations is the time required to target the equations. At the present time, a wide disparity exists for the various techniques. This figure includes the time required by all computers involved to compute all input constants required by the guidance equations in order to "fly" the mission. It also includes the time to transmit, insert, and verify these constants into the launch guidance computer. Thus, it is clear that the targeting time may be the vehicle flight reaction time. Of course, in some cases it is possible for vehicle or guidance hardware flexibility limitations to be the pacing item in some fast reaction launch situations.

The normal method for vehicle targeting (using explicit guidance equations) is to use a large high-speed ground-based computer equipped with a multi-vehicle simulation (MVS) program to generate an open-loop trajectory. (A simplified block diagram of a vehicle simulation is shown in Appendix B.)

This trajectory is tested using dispersion runs to verify acceptability. The trajectory is then used to generate the guidance constants. The vehicle simulation then runs a closed-loop trajectory using the guidance constants. Dispersion runs are again run to verify that the trajectory still satisfies the vehicle constraints. If the closed-loop trajectory fails the test, a recycle of this procedure is required. A block diagram of this procedure is shown in Fig. 3. Targeting times using this technique vary from two weeks to several months.

The targeting procedure may be speeded up by using prestored pitch programs and autopilot gains, as shown in Fig. 4. These are generally functions of the payload weight, altitude of injection, and desired orbital inclination. The reaction time is further decreased by programming the actual vehicle guidance equations into the vehicle simulation and using these directly as the search tool to obtain the closed-loop reference trajectory. In addition, a print denoting satisfaction of the constraints should be furnished. If the

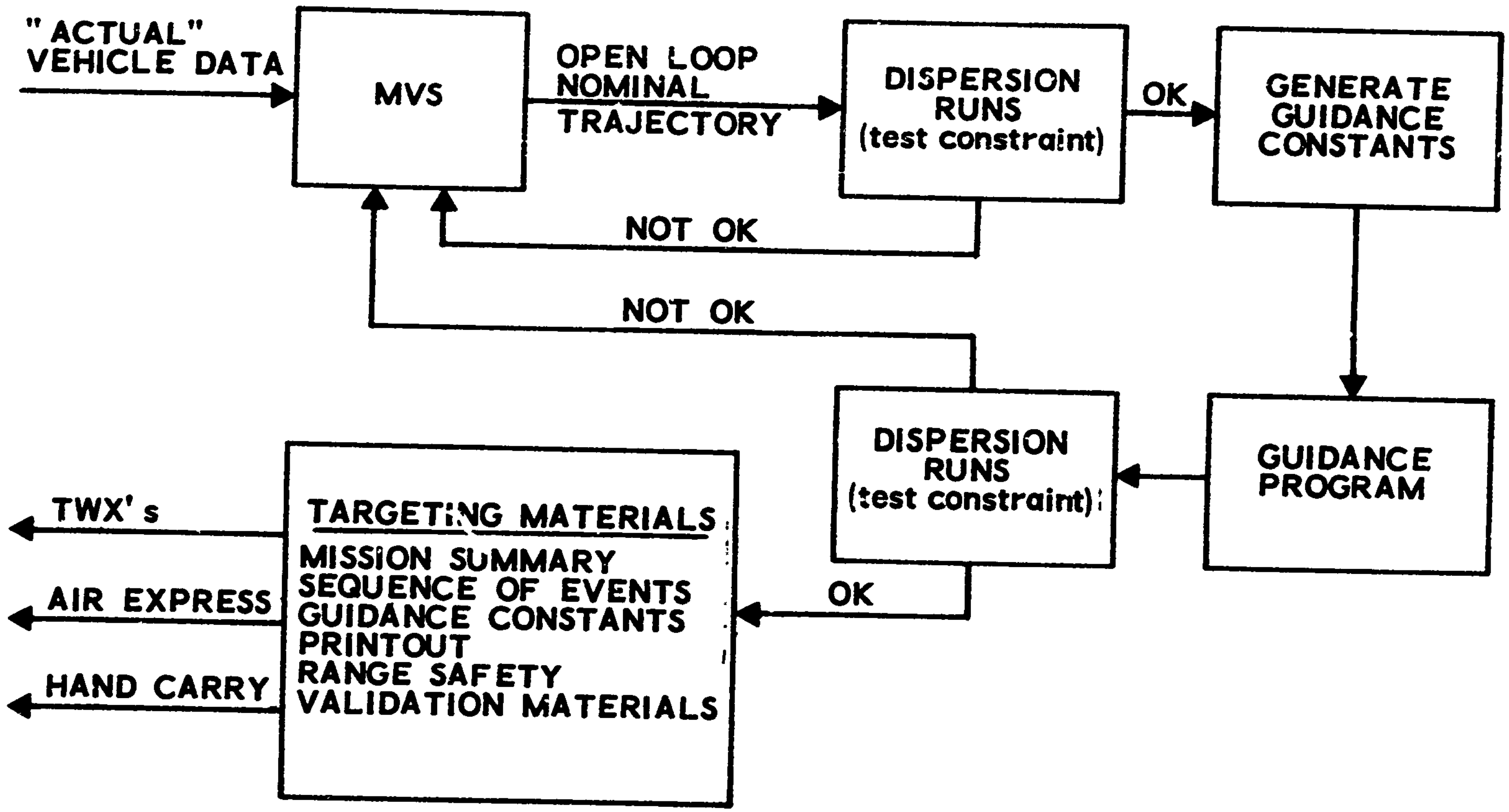


Figure 3. General Techniques for Parameter Selection (Targeting)

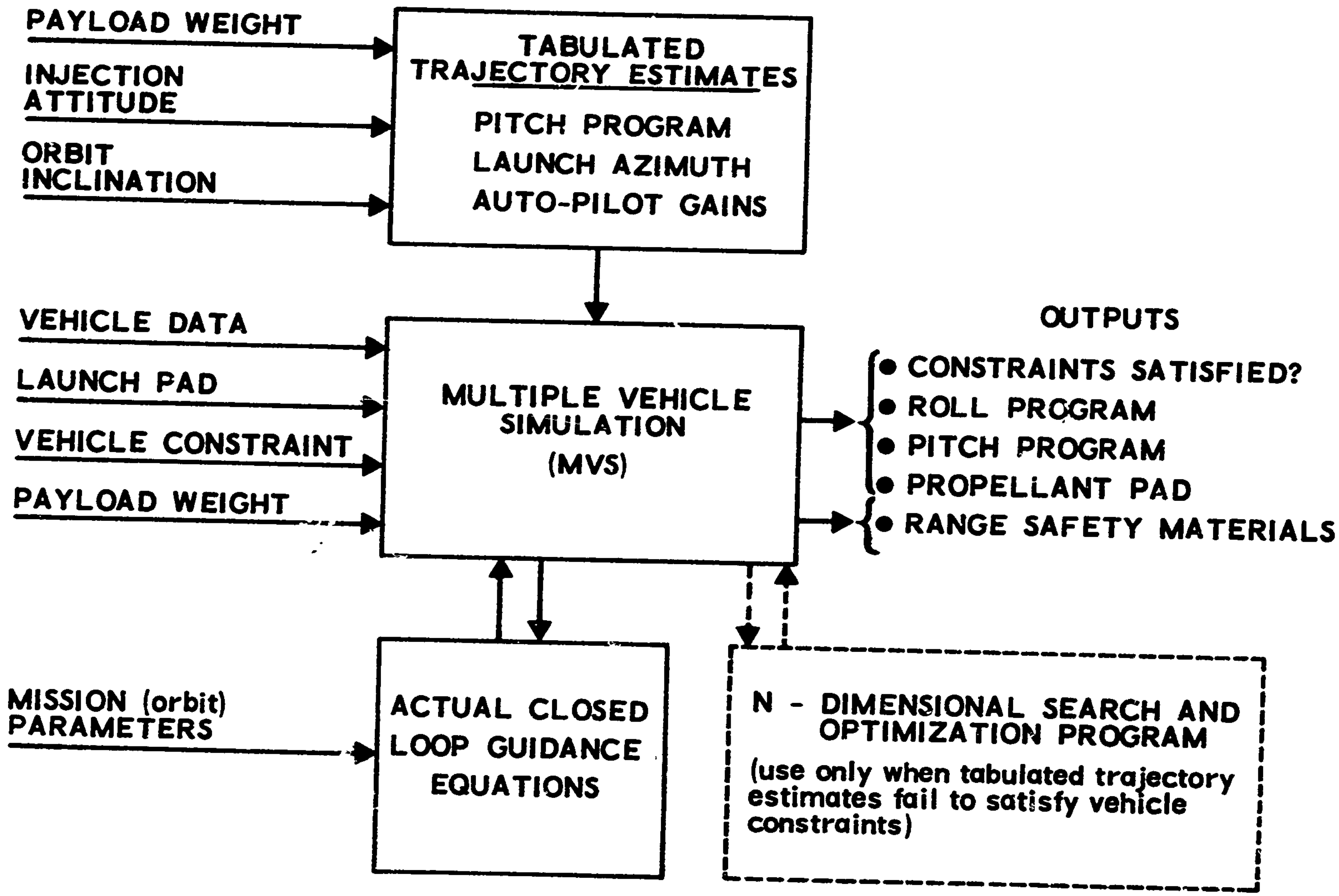


Figure 4. Fast Reaction Targeting Procedure

reference trajectory is not satisfactory, the operator must modify the selection of launch azimuth or injection altitude manually in order to obtain a satisfactory nominal trajectory. In the event that the fast reaction procedure cannot find a satisfactory reference trajectory, it is desirable to use an N-dimensional search and optimization procedure. Mission reaction times of several hours may be obtained when this technique is employed. Even faster reaction times, of the order of minutes, are achievable when dictated by program requirements.

The passage of time and the increase in comput : efficiency has seen guidance equations become more and more sophisticated with the result that targeting requirements have become minimized. An enormous targeting burden was associated with the Q and Δ guidance equation formulations. Each of these techniques may contain 30 to well over several hundred constants that must be determined by the targeting computer. Even the gravity computation contained constants that required specification during the targeting procedure.

Either the general purpose or the fast reaction technique may be used with "explicit," "numerical integration," "linear tangent," and "optimal" guidance techniques. The fast reaction targeting technique works particularly well with the linear tangent guidance equations because of their built-in multi-stage capability. In the case of "Optimal" and "Explicit" guidance techniques, it may be necessary for the fast reaction targeting routine to specify "Aim Points" for each of the exo-atmospheric vehicle stages.

IV. Q-GUIDANCE

Q-guidance is described in Refs. 1 and 2, and has been used successfully in Short-Range Ballistic Missile applications.

Existing formulations of the Δ and Q guidance equation mechanizations involve the concept of the required velocity vector. The required velocity vector, $\vec{V}_R (\vec{x}, t)$ is defined as the velocity that is required at the present position, $\vec{x}$, and time, t, to satisfy the mission requirements assuming an impulse burn to achieve $\vec{V}_R$. The vector $\vec{x}$ is measured from the center of the earth in an inertial coordinate system. The ballistic missile is to be steered such that the velocity to be gained

$$\vec{V}_g = \vec{V}_R - \vec{V}$$

goes to zero at burnout where $\vec{V} = \dot{\vec{x}}$ denotes the instantaneous velocity vector. With this steering philosophy, it is obvious that only a limited number of injection constraints may be satisfied. For example, the three components of the desired velocity vector at burnout can be achieved. If a variable coast period to the target is allowed, then one additional constraint may be satisfied. A complete description of the constraints that may be satisfied is given in Refs. 1 and 2.

The flexibility offered by the impulsive burn definition of the required velocity vector is completely satisfactory for the minimum energy type of trajectory. In this type of mission, it is only required to hit a target. The dispersions in the velocity vector when the target is hit are of no concern. This approach is generally not sufficient for satellite injection purposes where both the altitude and velocity must be steered to the desired values. A typical altitude dispersion that would result from use of the impulsive $\vec{V}_R$ approach is several miles.

The guidance equations further limit the specification of $\vec{V}_R(\vec{x}, t)$. This vector must be defined such that any freely falling particle for which $\vec{V} = \vec{V}_R$ initially will continue to have $\vec{V} = \vec{V}_R$ at all subsequent positions and time.

From the definition of $\vec{V}_R$, it follows in general that

$$\frac{d\vec{V}_R}{dt} = \sum_{R=1}^{3} \frac{\partial \vec{V}_R}{\partial x^k} \frac{dx^k}{dt} + \frac{\partial \vec{V}_R}{\partial t} \tag{2}$$

where

$$x^1 = x$$

$$x^2 = y$$

$$x^3 = z$$

and x, y, z denote coordinates in the earth centered inertial computational system.

For the particular class of required velocity vectors acceptable for Q guidance, it may be shown that

$$\vec{g} = \sum_{R=1}^{3} \frac{\partial \vec{V}_R}{\partial x^k} V_R^k + \frac{\partial \vec{V}_R}{\partial t} \tag{3}$$

The rocket equation for ballistic missile flight is written as

$$\vec{g} = \frac{d\vec{V}}{dt} - \vec{a}_T \tag{4}$$

where $\vec{a}_T$ denotes the thrust acceleration. The above equations may be combined to give the Q-guidance equation

$$\dot{\vec{V}}_g = -\vec{a}_T - \sum_{R=1}^{3} \frac{\partial \vec{V}_R}{\partial x^k} V_g^{\,k} \qquad (5)$$

where

$$V_g^1 = V_{gx}, \; V_g^2 = V_{gy}, \text{ and } V_g^3 = V_{gz}$$

Q-guidance has one very desirable feature in that it does not require a complete navigation determination. That is, the missile does not need to know its velocity or position vectors in order to determine $\vec{V}_g$. This quantity is obtained using a single integration of the Q-guidance equation. It is necessary to specify the initial conditions $\vec{V}_g(0)$ and the Q matrix

$$Q_{ij} = \frac{\partial V_R^{\,i}}{\partial x^j} \; (i, j, = 1, 2, 3) \qquad (6)$$

It has been shown that these quantities may be chosen as constants for an IRBM mission. An involved targeting search routine is required in order to provide this determination and still keep the guidance equation injection errors within acceptable bounds. In general, the Q_{ij} matrix coefficients are complicated functions of time involving many empirical constants. These constants must be determined as a result of extensive computer programs.

The steering procedure used with some Q guidance mechanizations requires that the unit thrust vector, $\vec{\xi}$, be directed along $\vec{V}_g$:

$$\vec{\xi} = \frac{\vec{V}_g}{V_g} \qquad (7)$$

This is equivalent to issuing a commanded turning rate, ω_c, to torque the rate integrating gyros in the missile control system as

$$\vec{\omega}_c = \vec{\xi} \times \dot{\vec{\xi}} = \frac{1}{V_g^2} \left(\vec{V}_g \times \dot{\vec{V}}_g \right)$$

For practical reasons, the gain on the vector cross product may be replaced by a simple time varying function to avoid the singularity that occurs when $V_g = 0$. This singularity is particularly troublesome when the thrust misalignments are significant.

In many space missions, it is desirable that the vehicle contain instantaneous position and velocity information. When this information is necessary for other purposes, then the advantage of the Q mechanization is less attractive.

V. DELTA GUIDANCE EQUATIONS

Delta guidance has been effectively used for the past fifteen years and is still in use (Ref. 3). It is a viable method for space missions; however, targeting and the lack of one flight trajectory control variable motivate against its use.

In this mechanization, the $\vec{V}_R$ vector is represented by expansions of the type

$$V_{Rx} = V_{Rxn} + A(x - x_n) + B(y - y_n) + C(z - z_n) + D(t - t_n) \tag{9}$$

plus second order terms as required. Similar expansions are used for V_{Ry} and V_{Rz}. Variations in this specification exist, but the general philosophy is the same.

As in the Q-guidance problem, the question of constant determination (A, B, C, D, ...) may require many hours on a large capacity computer. The targeting procedure involves the determination of many possible burnout points and least squares fitting of this data to the selected polynomial expansions.

This method of guidance has been used in the past in small capacity airborne computers that did not have built-in divide and square root instructions.

The Δ mechanization does require a navigation loop to determine position and velocity. The gravity computation is also an expansion with one formulation of this system. That is, because of computer limitations, $\vec{g}$ is computed using an expansion of the form

$$g_x = K(x - x_0)\left[C_0 + C_1\,x + C_2\,z + C_3\,x^2 + C_4\,z^2 + C_5\,x\,z\right] \tag{10}$$

with similar terms for g_y, g_z rather than the explicit form

$$\vec{g} = \frac{K}{r^3} \vec{x}$$

$$r = \sqrt{x^2 + y^2 z^2}$$

Steering with the Δ guidance mechanization is discussed in detail in Ref. 3. In one system, the unit thrust vector is directed as

$$\vec{\xi} = \frac{K}{a_T} \vec{V}_g^* + \frac{L}{a_T} \int_{t_o}^{t} \vec{V}_g^* \, dt$$

where the * on $\vec{V}_g$ denotes that the pitch (ρ) and yaw (y) components of V_g have been modified by adding on the tangential component, as follows:

$$V_{gp}^* = V_{gp} + K_1 V_{gt}$$

$$V_{gy}^* = V_{gy} + K_2 V_{gt}$$

where K_1 and K_2 are determined by the procedure to minimize vehicle propellant losses which occur during the closed-loop-g idance phase of flight. Note that V_{gp}^* and V_{gy}^* still vanish at final stage burnout, since this event is commanded when $V_{gt} = 0$. The integral terms in the steering equation have been found to compensate well for thrust misalignments.

The Δ guidance equation mechanization has been tested with success for flights as complex as the synchronous equatorial mission. The procedure essentially required that $\vec{V}_R$ values be specified to hit space target (position) vectors. Prediction polynomials were then used, also in the form of expansions, to compute velocity components at the end of coast periods. The prediction polynomials were also used to calculate the time of flight during the coast periods. Note that this mechanization requires that verniers be used to cut off each stage precisely at burnout. This follows from the fact that an assumption is made in the equations that the target (position) vector is actually hit. That is, it is assumed that $\vec{V}_g = 0$ at orbital insertion. Obviously, the target would not be hit if a large velocity error were made in the equations. The above problem is probably solvable, but the machine time and complexity of equations would increase several times in the event that a vernier was not used for orbital insertion, especially when a long coast period followed main engine burnout. This problem area would probably be even more critical and difficult to cope with for the Q-guidance mechanization.

VI. EXPLICIT GUIDANCE EQUATIONS

The explicit guidance equation mechanization has been used on a number of space missions. This technique is consistently effective and will probably be used for several more years, especially with all-inertial systems. Details on explicit guidance are given in Refs. 4 through 8. The derivation of the explicit guidance (and steering) procedure is generally credited to Duncan MacPherson.

The principal use of explicit guidance mechanizations has been for guiding a single rocket stage. When additional stages are used, the concept of Aim Points has been used. That is, the equations are designed to cause each stage to pass through a pre-designated aiming point. The Aim Points are obtained during the targeting procedure.

Explicit guidance equation formulations imply that closed form solutions to Newton's equations of motion are employed in the computations. At Aerospace, the explicit concept implies that closed form solutions are also used in the computation of commanded thrust attitude or thrust attitude rate. Thus, the Aerospace explicit guidance equation mechanization takes into account the finite thrusting characteristics of engines (as opposed to the impulsive burn assumptions of the Q and Δ mechanizations). This procedure allows additional constraints to be satisfied at burnout.

Five constraints can be satisfied with the Aerospace explicit guidance equation mechanization. These include not only the three components of the velocity vector, but also the two lateral components of the position vector. The tangential component of vehicle position cannot be constrained without excessive loss in performance because the vehicle is not equipped with engine thrust magnitude control.

The explicit solution of the rocket equations is computed in the pitch plane using three controls: (1) the initial (constant) value of thrust attitude,

(2) the thrust attitude rate, and (3) the final (engine cut-off) time. The lateral position and velocity are also controlled by using corresponding attitude degrees of freedom in the yaw plane. A summary explanation of the explicit guidance technique is given below for motion in the pitch plane.

The differential equations of motion for the thrusting vehicle in the atmosphere are given as:

$$(r^2\dot{\theta})^{\cdot} = r a_T \cos\psi + (D, L)$$
$$\ddot{r} = r\dot{\theta}^2 - \frac{GM}{r^2} + a_T \sin\psi + (D, L) \qquad (14)$$

where

r = distance from center of earth to missile

θ = angular displacement between the present position to a future position measured from the center of the earth

a_T = thrust acceleration

(D, L) = small aerodynamic drag and lift terms

ψ = thrust attitude measured from the local horizontal

Equations (9-14) may be written in terms of V, γ where

V = inertial velocity magnitude

γ = elevation angle of the velocity vector with respect to the local horizontal using the transformation

$$\dot{r} = V \sin\gamma$$

$$r\dot{\theta} = V \cos\gamma$$

The above equations then become

$$\dot{V} = a_T \cos(\psi - \gamma) - \frac{GM}{r^2} \sin\gamma + (D, L)$$

$$\ddot{r} = a_T \sin\psi + \frac{V^2 \cos^2\gamma}{r} - \frac{GM}{r^2} + (D, L) \qquad (15)$$

These equations may be approximately written as

$$\dot{V} = a_T + \text{Small Terms}$$

$$\ddot{r} = a_T\psi + \text{Small Terms} \qquad (16)$$

It has been shown that choosing the pitch or yaw thrust attitude as a linear time function will result in efficient flight. This assumes that the thrust vector is oriented such that it is nearly normal (within ≈ 10 deg) to the local gravity vector. Then

$$\psi = \psi_o + \dot{\psi}_o t \qquad (17)$$

The form of a_T is

$$a_T = \frac{a_{TO}}{1 - \frac{a_{TO} t}{C^*}} \qquad (18)$$

Thus, the first of these equations may be readily integrated to obtain an approximate time-to-go, t_g, until the stage burns out and the desired value of the final velocity is achieved.

When the approximate value of t_g is obtained, the second equation may be integrated, neglecting the small terms, as

$$\dot{r}_f = \dot{r}_o + \int_o^{t_g} a_T(\tau) \left[\psi_o + \dot{\psi}_o \tau \right] d\tau$$

$$r_f = r_o + \dot{r}_o t_g + \int_o^{t_g} (t_g - \tau) \, a_T(\tau) \left[\psi_o + \dot{\psi}_o \tau \right] d\tau \qquad (19)$$

Equations (9-19) are then solved for ψ_o and $\dot{\psi}_o$, such that the mission altitude and flight path angle requirements will be approximately satisfied. The effect of the small terms is then evaluated as a perturbation solution and a small adjustment in time-to-go, attitude, and attitude rate is made.

The use of the Aerospace explicit guidance equations is not wasteful of propellant for exo-atmospheric flight. This is evidenced by the fact that this procedure has been used effectively to generate nominal trajectories at Aerospace. It is even possible to develop a targeting routine by flying the first stage using a gravity (zero lift) turn through the atmosphere. The explicit guidance technique is then used for controlling the upper stages. This targeting technique requires iteration on the gravity turn and also on Aim Points if several upper stages are required. However, propellant waste is not a serious problem. The explicit technique allows the satisfaction of additional constraints as required for satellite injection where both the lateral position and velocity must be constrained at burnout.

A second principal advantage of this mechanization arises in the area of targeting. Only a minimum number of trajectory dependent constants is required. No additional constants to provide optimum trajectory shaping need be determined.

There is a third advantage of explicit guidance techniques. This method explicitly solves for the required initial attitude and attitude rate required to accomplish the end conditions at burnout. The Q and Δ steering formulations command error quantities to the control system that are functions of $\vec{V}_g$. Then $\vec{V}_g \rightarrow 0$ as a result of a control system damping procedure. That is, in the Q and Δ mechanizations, the lateral components of $\vec{V}_g$ will go to zero as the solution of a first, second, or third order system (whichever is the case), exhibiting the transient response characteristics of these damped systems. Hence, system stability is of primary concern with systems of this nature. This type of solution behavior is not obtained with the highly predictive explicit mechanizations. In this mechanization, the approach to the desired end conditions is monotonic without overshoot, undershoot, or any significant damped oscillation. This follows from the fact that the system is not designed to null small departures from some prespecified nominal trajectory.

VII. NUMERICAL INTEGRATION GUIDANCE

This guidance technique has been used to guide large rocket vehicles using radar data (R, A, E) measurements. As a result, a high-speed, large (32 k) digital computer is employed to perform all the computations.

Attitude profiles are devised as shown in Fig. 5 for both pitch and yaw. Note that there are three controls in pitch, θ_o, $\dot{\theta}_o$ and θ_f as well as a similar number in yaw plus a final engine cut-off control T. Hence, seven parameters at orbital insertion can be steered with this technique. These include

h = attitude

γ = flight path angle

θ_p = final pitch attitude

y = sideways position

$\dot{y}$ = sideways velocity

θ_y = final yaw attitude

V = velocity magnitude

In order to illustrate the operation of the guidance technique, it is only necessary to consider the controls and constraints in the pitch plane. Hence, we assume the following controls and corresponding constraints:

$\theta_o \longleftrightarrow \gamma$

$\dot{\theta}_o \longleftrightarrow h$

$\dot{\theta}_f \longleftrightarrow \theta_f$

$T \longleftrightarrow V$

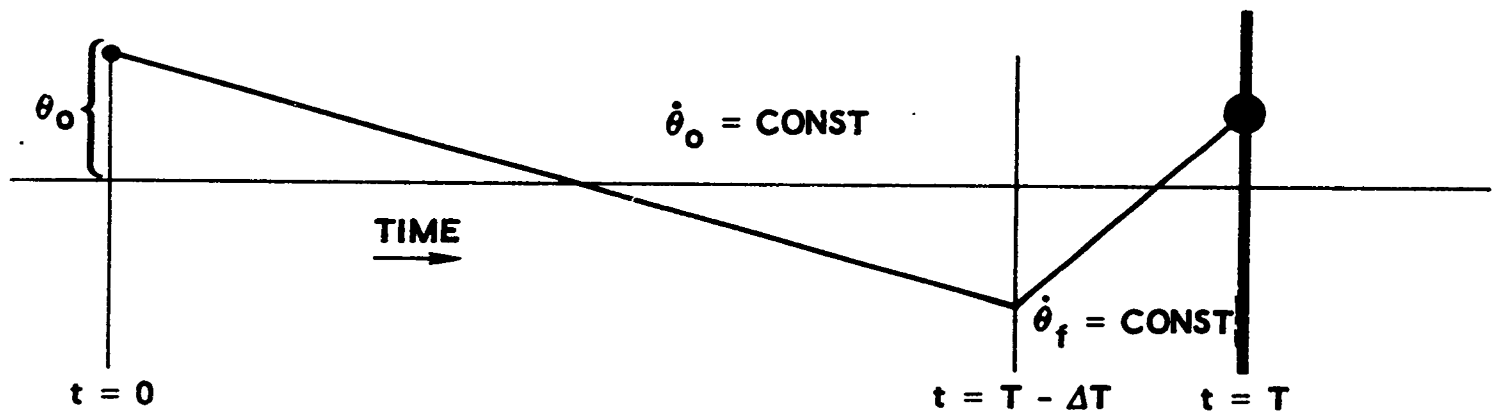


Figure 5. Numerical Integration Guidance Attitude Profile

It is now convenient to consider a large powered flight simulation, with all powered stages contained in the guidance computer. A nominal run is made using the best estimate of θ_o, $\dot{\theta}_o$, $\dot{\theta}_f$, and T in the computer. Four dispersed runs are then made in which each of the four control parameters is varied one at a time. Note that, when the final attitude is constrained, there is essentially one less control parameter because the following approximate relation exists between θ_o, θ_f and T:

$$\theta_o + \dot{\theta}_o (T - \Delta T) + \dot{\theta}_f \Delta T = \theta_f^* \tag{20}$$

Thus, when final attitude is controlled, the following three equations are obtained:

$$\frac{\partial V_f}{\partial \dot{\theta}_o} \Delta \dot{\theta}_o + \frac{\partial V_f}{\partial \dot{\theta}_f} \Delta \dot{\theta}_f + \frac{\partial V_f}{\partial T} \Delta T = \Delta V_D \tag{21}$$

$$\frac{\partial h_f}{\partial \dot{\theta}_o} \Delta \dot{\theta}_o + \frac{\partial h_f}{\partial \dot{\theta}_f} \Delta \dot{\theta}_f + \frac{\partial h_f}{\partial T} \Delta T = \Delta h_D \tag{22}$$

$$\frac{\partial \gamma_f}{\partial \dot{\theta}_o} \Delta \dot{\theta}_o + \frac{\partial \gamma_f}{\partial \dot{\theta}_f} \Delta \dot{\theta}_f + \frac{\partial \gamma_f}{\partial T} \Delta T = \Delta \gamma_D \tag{23}$$

where the partial derivatives are obtained by numerical differencing of the four dispersed runs from the nominal. The deviations on the right-hand side of the equations are the deviations of the nominal from the desired burn-out conditions. The set of three equations is solved for $\Delta\dot{\theta}_o$, $\Delta\dot{\theta}_f$, and ΔT. The values for θ_o, $\dot{\theta}_o$, $\dot{\theta}_f$ and T are then obtained using the nominal values and the previous attitude constraint relation.

*In practice, an exact implementation may be used for the attitude constraint relation.

Rate steering commands in pitch are then sent to torque the vehicle control system rate integrating gyros as follows:

$$\omega_c = k \frac{\theta - \theta_o}{\Delta t} + \dot{\theta}_o$$

where

θ = actual vehicle pitch altitude estimated by the Kalman filter

Δt = steering command computation interval (≈ 0.5 sec)

k = gain factor chosen to provide system stability ($k \approx 1$).

A similar computation is made for yaw steering commands. In general, ω_c will be limited to some value, generally less than 2 or 3 deg/sec. Thus, when the computed value of ω_c exceeds the limited value, then the limited value is used. When $T - \Delta T < t < T$, it follows that one of the controls is lost. Generally closed-loop attitude steering may be terminated at this time and the constant values for $\dot{\theta}_o$ and $\dot{\theta}_f$ employed on an open-loop basis. The cut-off computation to obtain updated values for T is still performed approximately every 0.5 sec until cut-off countdown.

Several variations of this general guidance technique may be used.

It is often very wasteful of propellant to steer the final insertion attitude as a control variable. In addition, it is often unnecessary for mission success to control final attitude. In many missions, the payload can perform its desired function regardless of the insertion attitude. In this case the numerical integration guidance technique may be employed eliminating $\dot{\theta}_f$ as a control (i.e., set $\Delta T = 0$). The attitude constraint equation will not be applicable for this case. The three controls (in the pitch plane) are then

$$\theta_o \longleftrightarrow \gamma$$

$$\dot{\theta}_o \longleftrightarrow h$$

$$T \longleftrightarrow V$$

and the operation proceeds in a manner similar to the previous description.

The advantage of numerical integration guidance is that it is flexible in adapting to vehicle configuration constraints. The disadvantage is the requirement for a high-speed large capacity computer.

VIII. LINEAR TANGENT GUIDANCE

Linear tangent guidance was devised at Aerospace by F. M. Perkins and is described in Refs. 9 through 11. Although the technique has not been employed for guidance of any launch vehicle, it has been used in several targeting simulations. One problem with this guidance formulation is that a "flat-earth" gravity model is used in the derivation. Correction terms for this approximation are subsequently added on. A principal advantage of these equations is that they have a multi-stage capability without the need for Aim Points as is generally the case with the "explicit" guidance equations.

The simple derivation for the basis of the linear tangent formulation given herein differs considerably from that given by Perkins.

It may be assumed that it is desired to obtain the thrust vector steering profile that will minimize the expended thrust velocity. Then a suitable cost function is obtained as

$$J = \int_0^T a_T(\tau)\,dt. \tag{25}$$

However, the purpose of guidance is to steer the vehicle such that the position, $\vec{x}_D$, velocity, $\dot{\vec{x}}_D$, and possibly vehicle thrust attitude, $\ddot{\vec{x}}_D$, are constrained. The augmented cost function may then be formed as

$$J^* = \int_0^T a_T(\tau)\,dt + \vec{\lambda} \cdot (\vec{x}_f - \vec{x}_o) + \vec{\mu} \cdot (\dot{\vec{x}}_f - \dot{\vec{x}}_o) + \vec{\eta}\,(\ddot{\vec{x}}_f - \ddot{\vec{x}}_C) \tag{26}$$

where $\vec{\lambda}$, $\vec{\mu}$, $\vec{\eta}$ are constant Lagrange multiplier vectors, and the subscript f denotes the value at t = T. The flat earth rocket dynamics equations are then

$$\ddot{\vec{x}} = A_T \vec{\xi} + \vec{g} \tag{27}$$

where g is assumed to be constant. This equation may be integrated to obtain position and velocity as

$$\dot{\vec{x}}_f = \dot{\vec{x}}_o + \vec{g}T + \int_0^T a_T(\tau)\vec{\xi}(\tau)\,d\tau \tag{28}$$

and

$$\vec{x}_f = \vec{x}_o + \dot{\vec{x}}_o T + \tfrac{1}{2}\vec{g}T^2 + \int_0^T a_T(\tau)(T-\tau)\vec{\xi}(\tau)\,d\tau \tag{29}$$

The result of substituting into the augmented cost function Eq. (26) is

$$J^* = \int_0^T a_T \left[1 + \vec{\lambda}\cdot\vec{\xi}(\tau) + \vec{\mu}\cdot\vec{\xi}(\tau)(T-\tau) + \vec{\eta}\cdot\vec{\xi}(\tau)\,\delta(T-\tau)\right] dt \tag{30}$$
$$+ \text{ fixed terms}$$

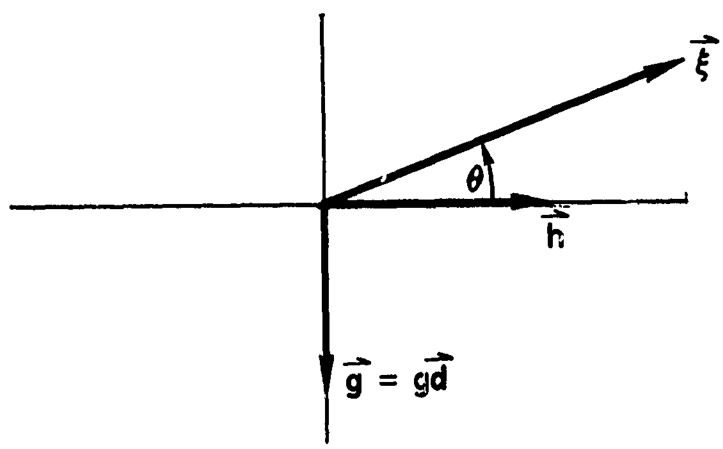
where δ (T-t) denotes the Dirac Delta Function. It is then clear that the augmented cost function is minimized by choosing

$$\vec{\xi}(t) \sim \vec{\lambda} + \vec{\mu}(T-t) + \vec{\eta}\,\delta(T-t) \tag{31}$$

where the symbol $\sim$ denotes parallelism. Since $\vec{\xi}$ is constrained to be a unit vector in the direction of the thrust acceleration, this equation must be normalized.

The derived form of the vector $\vec{\xi}(t)$ is subject to some interpretation. Note that the guidance constants are the Lagrange multipliers $\vec{\lambda}$, $\vec{\mu}$, and $\vec{\eta}$. Further, it follows that thrust velocity is minimized by using constant attitude steering when it is desired to control only the final velocity vector. If it is also desired to control the final position, then $\mu \neq 0$ and an attitude rate is required to achieve optimality. It also follows that, in the event that the final attitude is constrained, the vehicle should change its attitude to the desired value of attitude and shut off engines simultaneously. Since an attitude rate maneuver such as this would generally break up the vehicle, several seconds (≈10) must generally be allowed for this maneuver. Note that this was done in the case with numerical integration guidance, where $\Delta T \approx 10$ seconds.

It is easy to demonstrate the relation of the above derivation to the linear tangent guidance formulation. Consider the figure below:



The figure shows the gravity vector acting down along the unit vector $\vec{d}$. The thrust attitude vector can then be decomposed along the local horizontal unit vector, h, and d as

$$= \frac{\left[\lambda_h + \mu_h (T-t)\right] \vec{h} + \left[\lambda_d + \mu_d (T-t)\right] \vec{d}}{N(t)} \tag{32}$$

where N(t) is the normalization factor required to cause $\vec{\xi}$ to be a unit vector. Then it follows that

$$\tan\,\theta = -\frac{\lambda_d + \mu_d\,(T-t)}{\lambda_h + \mu_h\,(T-t)} \tag{33}$$

Thus, it is shown that the local tangent vector, as also derived in Ref. 9, is a bilinear function of time.

A principal result of the Perkins linear tangent guidance program is that the vacuum flight rocket equations with constant gravity are exactly integrable. Hence, Perkins includes a Newton-Raphson technique for obtaining the Lagrange multipliers λ and μ. The values for the partial derivations are also included.

The advantages of the linear tangent technique are many. It has a multistage capability, provides fast reaction, and is readily targetable. The disadvantages arise from the approximate computation of gravity. However, Perkins includes a technique that has been extensively simulated and works well.

IX. OPTIMAL GUIDANCE

A guidance technique generally referred to as "Optimal Guidance" is described in Refs. 12 through 18. Although this technique has not been used to guide the flight of a rocket vehicle, the program has been simulated for use with good results.

The basis for this guidance technique is best illustrated by the Pontryagin Maximum Principle. A brief description of this principle is given in Appendix C.

Using this procedure, the augmented cost function is obtained as

$$J = \phi(x(T), T) + \int_0^T \left\{ 1 + \vec{\lambda} \cdot (\vec{V} - \dot{\vec{x}}) + \vec{\mu} \cdot \left[\vec{g}(x) + a_T \vec{\xi} - \dot{\vec{V}} \right] \right\} dt \qquad (34)$$

where ϕ (x(T), T) denote end contraints to be satisfied. The rocket flight dynamics constraints are used as

$$\dot{\vec{x}} = \vec{V}$$

$$\dot{\vec{V}} = \vec{g}(x) + a_T \vec{\xi}$$

with $\vec{x}_o$, $\vec{V}_o$ given, where $\vec{x}$ denotes the inertial vector from the center of the earth to the rocket vehicle. As before, a_T denotes the vehicle thrust acceleration magnitude and $\vec{\xi}$ its direction.

The Hamiltonian is then obtained as

$$H = 1 + \vec{\lambda} \cdot \vec{V} + \vec{\mu} \cdot \left[\vec{g}(x) + a_T \vec{\xi} \right] \qquad (35)$$

It is now necessary to choose the value for $\vec{\xi}$ which minimizes the Hamiltonian. It is clear that this value is obtained as

$$\vec{\xi} = \frac{-\vec{\mu}}{|\mu|} \tag{36}$$

When this value for $\vec{\xi}$ is substituted into the Hamiltonian, the reduced Hamiltonian H^* is obtained as

$$H = 1 + \vec{\lambda} \cdot \vec{V} + \vec{\mu} \cdot \vec{g}(x) - a_T \mu \tag{37}$$

Application of the Pontryagin equations then gives:

$$\frac{\partial H^*}{\partial \vec{x}} = -\dot{\vec{\lambda}} = \frac{\vec{\mu} \cdot \partial \vec{g}(x)}{\partial \vec{x}} \tag{38}$$

and

$$\frac{\partial H^*}{\partial \vec{v}} = -\dot{\vec{\mu}} = \vec{\lambda} \tag{39}$$

Thus, the following result is obtained:

$$\ddot{\vec{\mu}} = \frac{\vec{\mu} \cdot \partial \vec{g}(x)}{\partial \vec{x}} \tag{40}$$

and the optimal direction of the thrust vector is specified as

$$\vec{\xi} = \frac{-\vec{\mu}}{|\mu|} \tag{41}$$

Note that in general there are six parameters to be specified, e.g.,

$$\vec{\mu}_o \text{ and } \dot{\vec{\mu}}_o$$

except that these reduce to five when the constraint that $\vec{\xi}$ be a unit vector is applied. Note that this is exactly the number of injection parameters that may be specified at engine burnout (orbital insertion) when final vehicle attitude is not controlled.

In the case where $\vec{g}$ is approximated as a constant (as in linear tangent guidance), the following result is obtained:

$$\ddot{\vec{\mu}} = 0 \tag{42}$$

or

$$\vec{\mu} \sim \vec{\mu}_o + \dot{\vec{\mu}}_o \, t \tag{43}$$

which corresponds exactly to the result obtained in the previous section.

When the customary round earth expression for gravity is used

$$\vec{g} = \frac{-GM}{r^3} \vec{x} \tag{44}$$

$$\ddot{\mu}^i = -\frac{GM}{r^3} \sum_{j=1}^{3} \left[r^2 \delta_{ij} - 3 x^i x^j \right] \tag{45}$$

where μ^i, $i = 1, 2, 3$, denote the three components of the vector $\vec{\mu}$, x^i are the components of the vector $\vec{x}$, and δ_{ij} denotes the Knonecker Delta.

The above program has been developed at IBM to include many cases for orbital insertion alternatives. For example, velocity, altitude, and orbital inclination can be controlled as well as many other possibilities.

The advantages of this approach are that gravity is handled directly in an optimal fashion. In addition, it is reported that the program size and speed requirements permit use in an on-board digital computer.

The disadvantages of the technique are that it is difficult to include the effects of atmospheric (drag and lift) forces in the optimization. In addition, it is difficult to compensate for other path constraints on staging angle of attack, heating vehicle rate limiting, etc. Even so, the technique is a very powerful one and may find wide usage in future rocket flight applications as well as targeting software systems.

X. PARAMETER OPTIMIZATION GUIDANCE

This technique has never been used to guide a launch vehicle. The computational speed requirements exceed those of the other techniques described by possibly an order of magnitude. It is required that a highly modeled powered flight vehicle simulation be run one or more times each guidance computation cycle (≈ 1 sec). The technique described herein has been used effectively as a general vehicle targeting technique. However, the guidance application requires a program that will run in real time. It is possible that this is achievable at the present time for radio guidance applications using a large ground-based guidance computer. However, it is felt that some computational speed enhancement will be required before the technique can be used with inertial systems.

This guidance technique employs an n dimensional search and optimization procedure to determine all the discrete parameters of a rate steering profile such as the one shown in Fig. 6. A three-stage rocket vehicle is depicted in this figure. Note that ten attitude turning rates are shown (five in Stage I, two in Stage II, and three in Stage III). Experience indicates that this is an adequate number of parameters to implement even the most exotic mission requirements. In addition, the final engine shutoff time, T_f, is shown as a control. The problem now is to find the values of all the control rates and T_f such that all vehicle constraints are satisfied and a cost function (related to maximum payload or minimum propellant wasted) is minimized.

Vehicle constraints can be quite general. The following inequality constraints are typical:

1. Dynamic pressure: $q < q_{max}$
2. Angle of attack: $\alpha < \alpha_o$ at q_{max} and $\alpha < \alpha_s$ at Staging
3. Heating indicator: $\int qV < H_o$
4. Radar and telemetry antenna look angle constraints
5. Vehicle turning rates: $|\omega| < \omega_{max}$

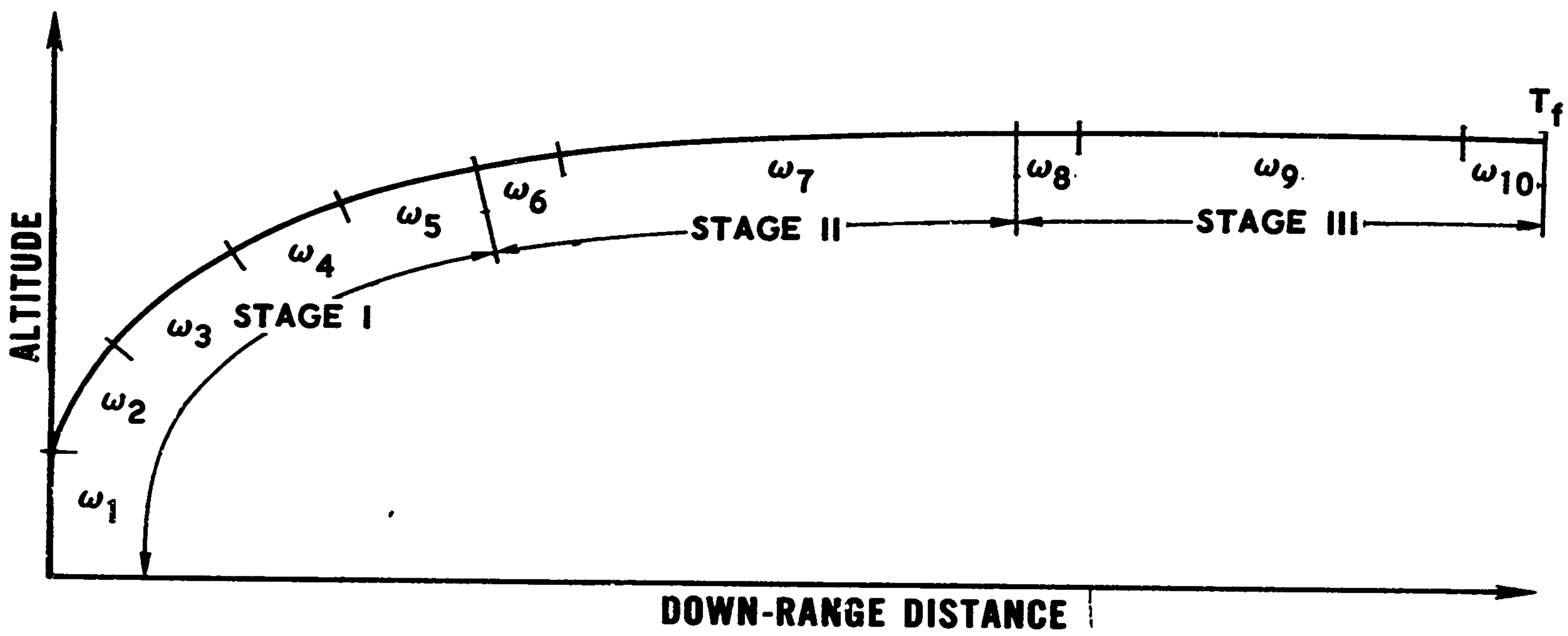


Figure 6. Discrete Parameter Profile Associated with Parameter Optimization Guidance

In addition, the mission equality constraints may be listed:

1. Velocity magnitude
2. Flight path angle
3. Velocity azimuth
4. Attitude
5. Out-of-plane position

Constraints on the final attitude may be added if required. It should also be noted that the five orbital insertion constraints defined above could be written in many different coordinate systems, such as ECI, launch pad inertial, or even orbital parameters such as apogee, perigee, inclination period, etc.

When all the prespecified vehicle turning rates are obtained by means of the general n-dimensional search and optimization procedure (NDSOP), it is only necessary to command the vehicle to follow the specified rates. The time to shut down the engines is obtained as T_f.

It is clear that the key to a successful parameter optimization guidance program depends upon the computational speed associated with the NDSOP. The core of the program is a steamlined vehicle simulation of sufficient accuracy for guidance purposes. It is felt that the simulation can be simplified in several areas and still be accurate enough for guidance. For example, it is probably not necessary that the thrust model be altitude dependent. In addition, the breaktimes associated with the discrete rate parameter specification may be fixed rather than variable.

The operation of the n-dimensional search and optimization procedure will be briefly explained. Consider a cost function

$$J = \phi (u^{\alpha}) \tag{46}$$

to be minimized, where u^{α} denotes the control vector. In the example, there are $\alpha = 1, \ldots, 11$ controls denoting $\omega_1 \ldots, \omega_{10}, T_f$. The augmented cost function is formed by using Lagrange multipliers to adjoin the constraints as

$$J^* (u) = \phi (u^\alpha) + \lambda_p \cdot \left[G^p (u^\alpha) - G_D^p\right] \tag{47}$$

where $G^p(u^\alpha)$ and G_D^p denote the actual and desired constraints. Index notation employing the summation convention for twice repeated indices will be used. Greek indices range over the number of control variables, Arabic indices range over the number of equality and inequality constraints. Inequality constraints that are violated are treated as active equality constraints. Thus, the number of constraints may change from iteration to iteration, and program control logic must be included to perform this bookkeeping and counting function.

It is assumed that an initial estimate of the values of the control variables, u_o^α and λ_o^p, is available. It is now desired to find Δu^α and $\Delta\lambda_p$ such that the augmented cost function will be minimized. Then,

$$J^* = \phi (u_o^\alpha + \Delta u^\alpha) + (\lambda_o^p + \Delta\lambda_p) \left[G^p(u_o^\alpha + \Delta u^\alpha) - G_D^p\right] \tag{48}$$

This expression for J* may be expanded to obtain

$$J^* = J_o + \Delta u^\alpha \left[\phi_\alpha + \lambda_o^p G_\alpha^p\right] + \frac{1}{2} \Delta u^\alpha \Delta u^\beta \left[\phi_{\alpha\beta} + \lambda_o^p G_{\alpha\beta}^p\right]$$

$$+ \Delta\lambda_p \Delta G^p (u_o^\alpha) + G_\alpha^p \Delta_p^\lambda \Delta u^\alpha \tag{49}$$

where $J_o = J^* (u_o^\alpha)$ and $\Delta G^p = G^p(u_o^\alpha) - G_D^p$.

The cost function may be minimized with respect to Δu^{α} by differentiating to obtain

$$\frac{\partial J^*}{\partial(\Delta u^{\alpha})} = M_{\alpha} + G_{\alpha}^{p}\Delta\lambda_{p} + \Delta u^{\beta} M_{\alpha\beta} = 0 \qquad (50)$$

where

$$M_{\alpha} = \phi_{\alpha} + \lambda_{o}^{p} G_{\alpha}^{p}$$

$$M_{\alpha\beta} = \phi_{\alpha\beta} + \lambda_{o}^{p} G_{\alpha\beta}^{p}$$

The cost function may be minimized with respect to the Lagrange multiplier by differentiating to obtain

$$\frac{\partial J^*}{\partial(\Delta\lambda b)} = \Delta G^{p} + G_{\alpha}^{p}\Delta u^{\alpha} = 0 \qquad (51)$$

It is now necessary to solve these two equations for Δu^{α} and $\Delta\lambda^{q}$. The solution may readily be shown to be

$$\begin{aligned} \Delta\lambda^{q} &= \left[\Delta G^{p} - G_{\alpha}^{p} M_{\alpha\beta}^{-1} M_{\beta}\right]\left[G_{\alpha}^{p} M_{\alpha\beta}^{-1} G_{\beta}^{q}\right]^{-1} \\ \Delta u^{\beta} &= -M_{\alpha\beta}^{-1}\left[M_{\alpha} + G_{\alpha}^{p}\Delta\lambda^{p}\right] \end{aligned} \qquad (52)$$

These correction values are then added to λ_{o}^{q} and u_{o}^{β}, respectively, to obtain an improved trajectory. It is felt that the procedure should converge within several iterations (<10) to obtain an optimum trajectory prior to launch. It is then conjectured that a single computation of the procedure every 1 to 10 sec during the total powered flight operation would be adequate for guidance. (The 10-sec computation time interval is probably achievable at the present time; however, the 1-sec interval is not.)

The parameter optimization approach has been simulated as a trajectory optimization and a targeting tool. It has been noted that a very accurate computation of the gradients, ϕ_α and G^p_α, is essential. In order to obtain the desired speed, it is recommended that the simulation compute the gradients analytically each time a run is made rather than to numerically difference the runs. It has also been shown that inaccurate second derivatives, $\phi_{\alpha\beta}$ and $G^p_{\alpha\beta}$, are quite acceptable. These can probably be precomputed prior to launch.

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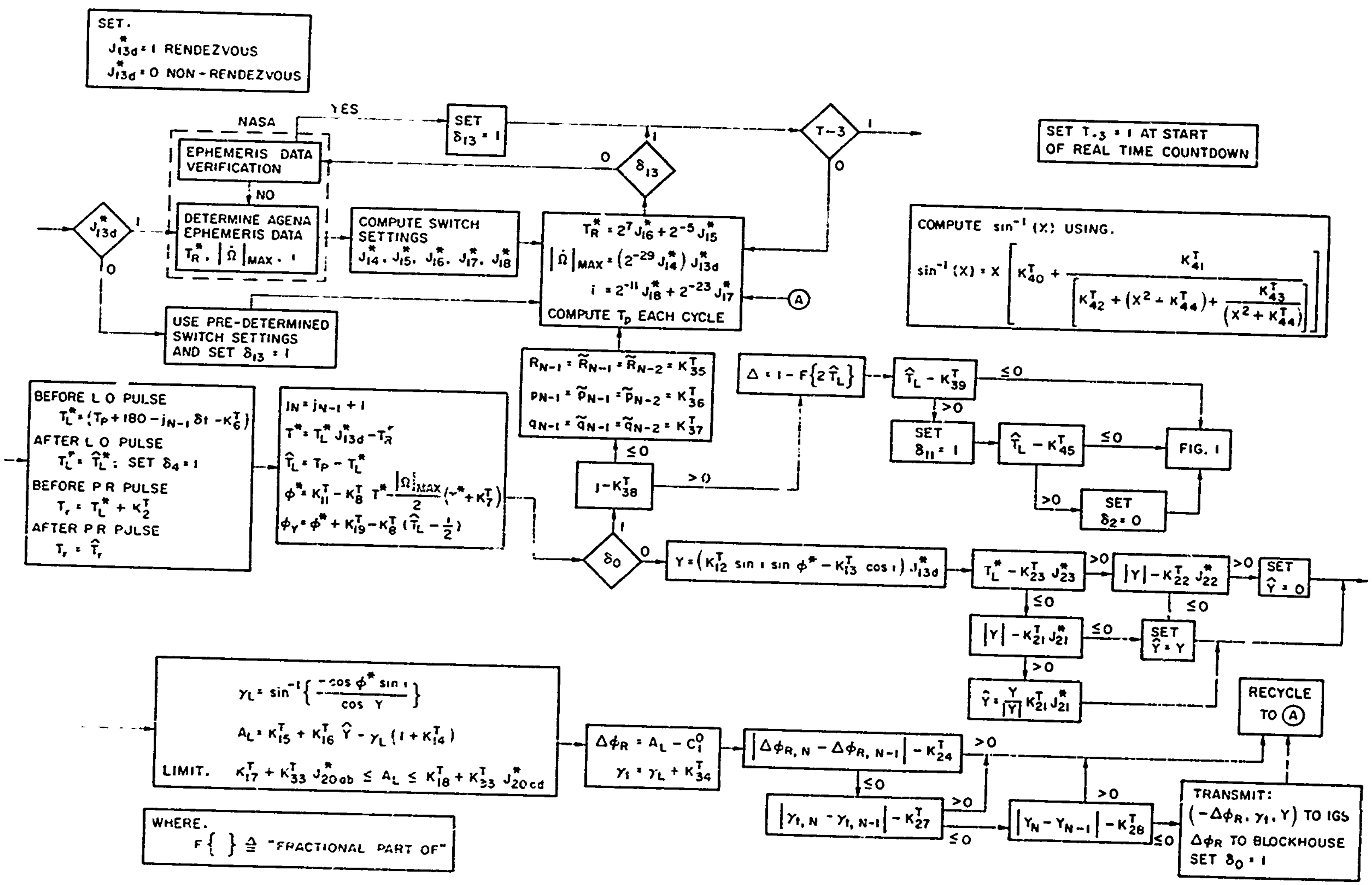
APPENDIX A

GEMINI GUIDANCE EQUATIONS

GEMINI GUIDANCE EQUATIONS



TARGETING

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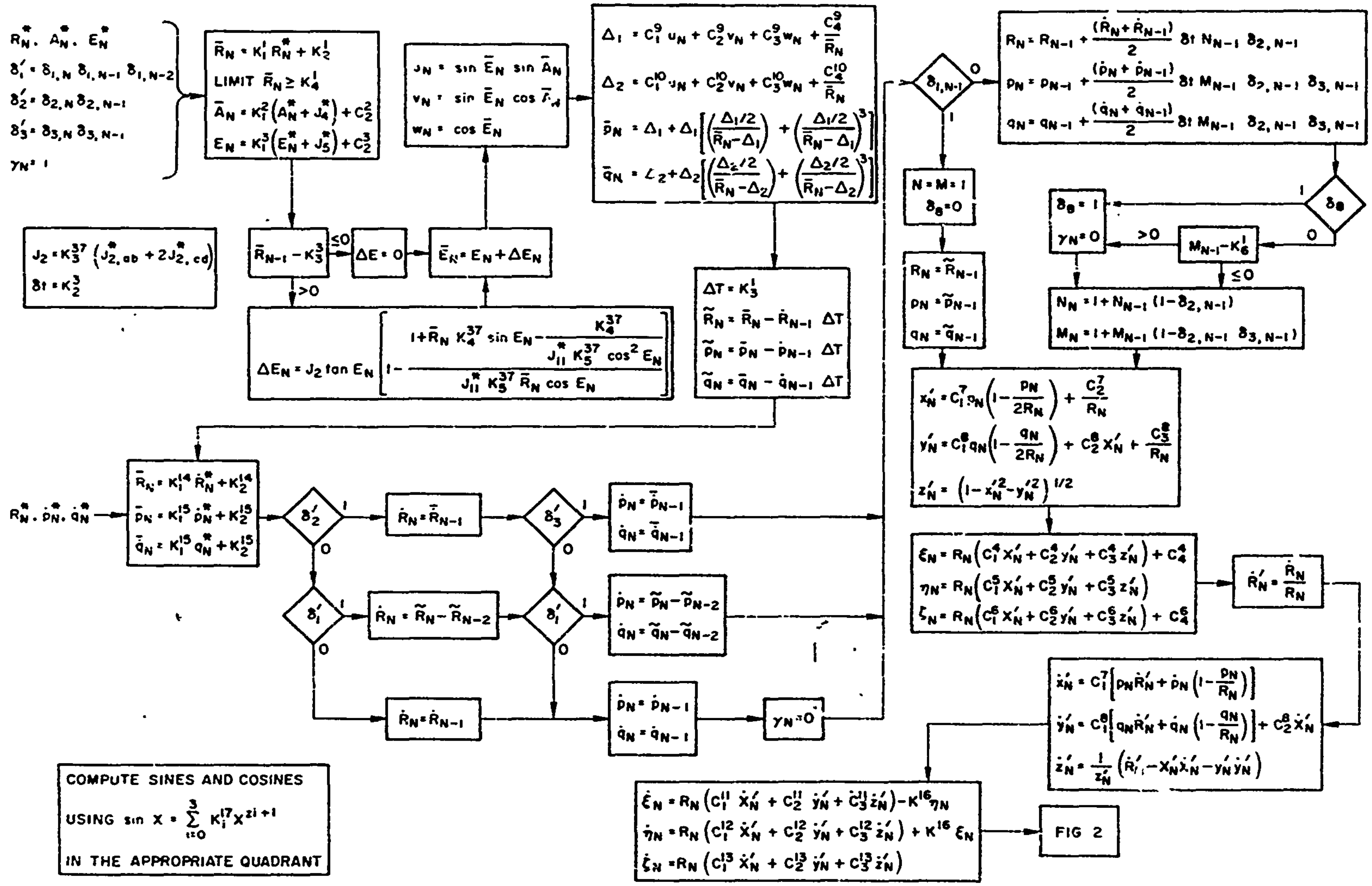


TRANSFORMATION AND CONVERSION

$$C^* = K_1^{50} + 2^{-14} J_7^*$$

$$V_f = K_2^{50} + 2^{-15} J_8^*$$

$$V_{pf} = K_3^{50} + 2^{-15} J_{10}^*$$

$$r_f = K_4^{50} + 2^{-18} J_9^*$$

$$\Gamma_{pf} = \frac{V_{pf}}{V_f}$$

$$\xi_Y = \sin i \sin \phi_Y$$

$$\eta_Y = -\sin i \cos \phi_Y$$

$$\zeta_Y = -\cos i$$

$$r = (\xi^2 + \eta^2 + \zeta^2)^{1/2}$$

$$V = (\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2)^{1/2}$$

$$V_p = \frac{\xi \dot{\xi} + \eta \dot{\eta} + \zeta \dot{\zeta}}{r}$$

$$V_y = \dot{\xi}\, \xi_Y + \dot{\eta}\, \eta_Y + \dot{\zeta}\, \zeta_Y$$

$$\widetilde{V} = \left[V_N (K_5^{50} + \Delta) - V_{N-1} (K_6^{50} + \Delta) \right] \delta_{11}$$

$$\Gamma_p = \frac{V_p}{V}$$

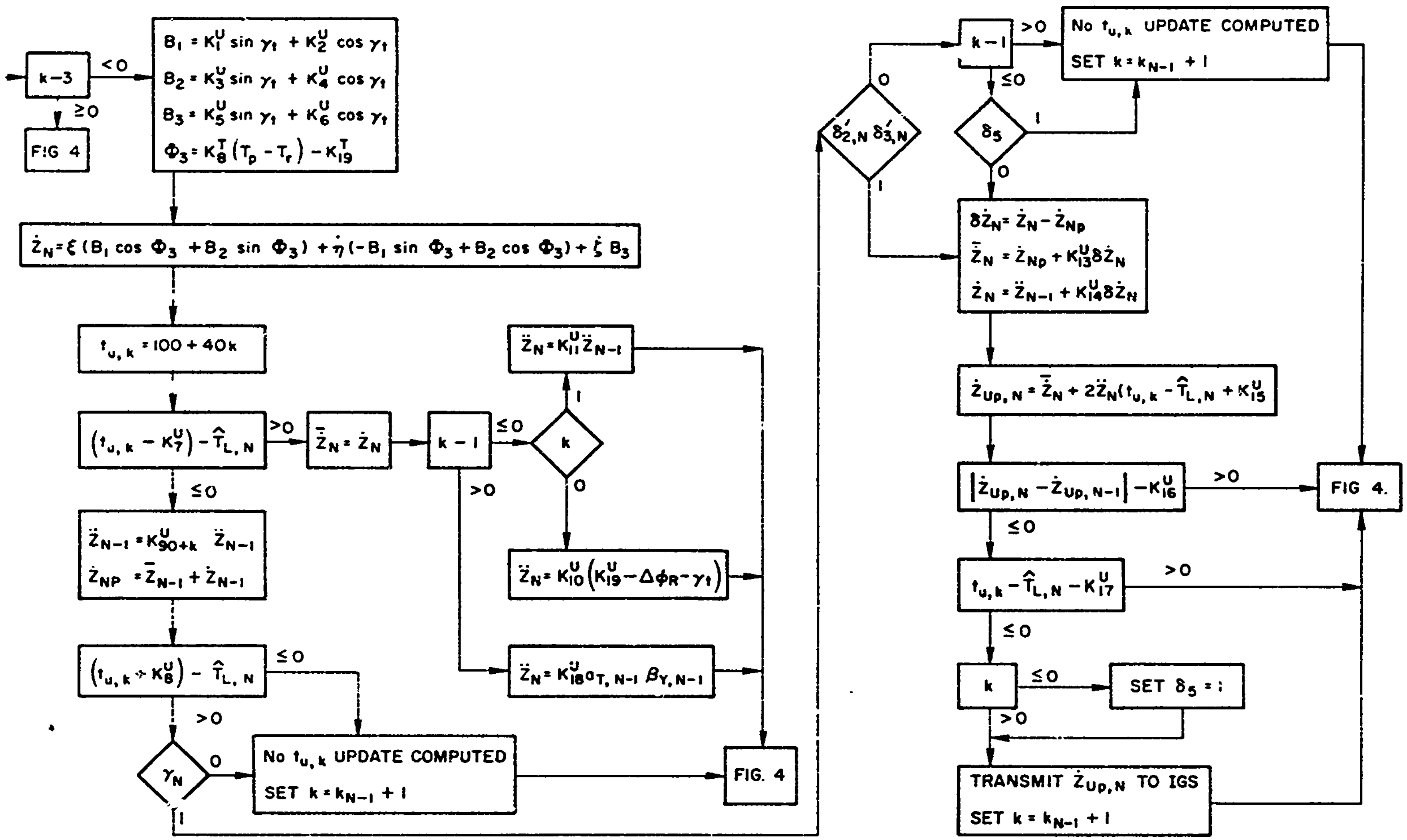
$$\widetilde{\Gamma}_p = \left[\Gamma_p (K_5^{50} + \Delta) - \Gamma_{p,\,N-1} (K_6^{50} + \Delta) \right] \delta_{11}$$

$$a_g = \frac{K_1^{41}}{r^2}$$

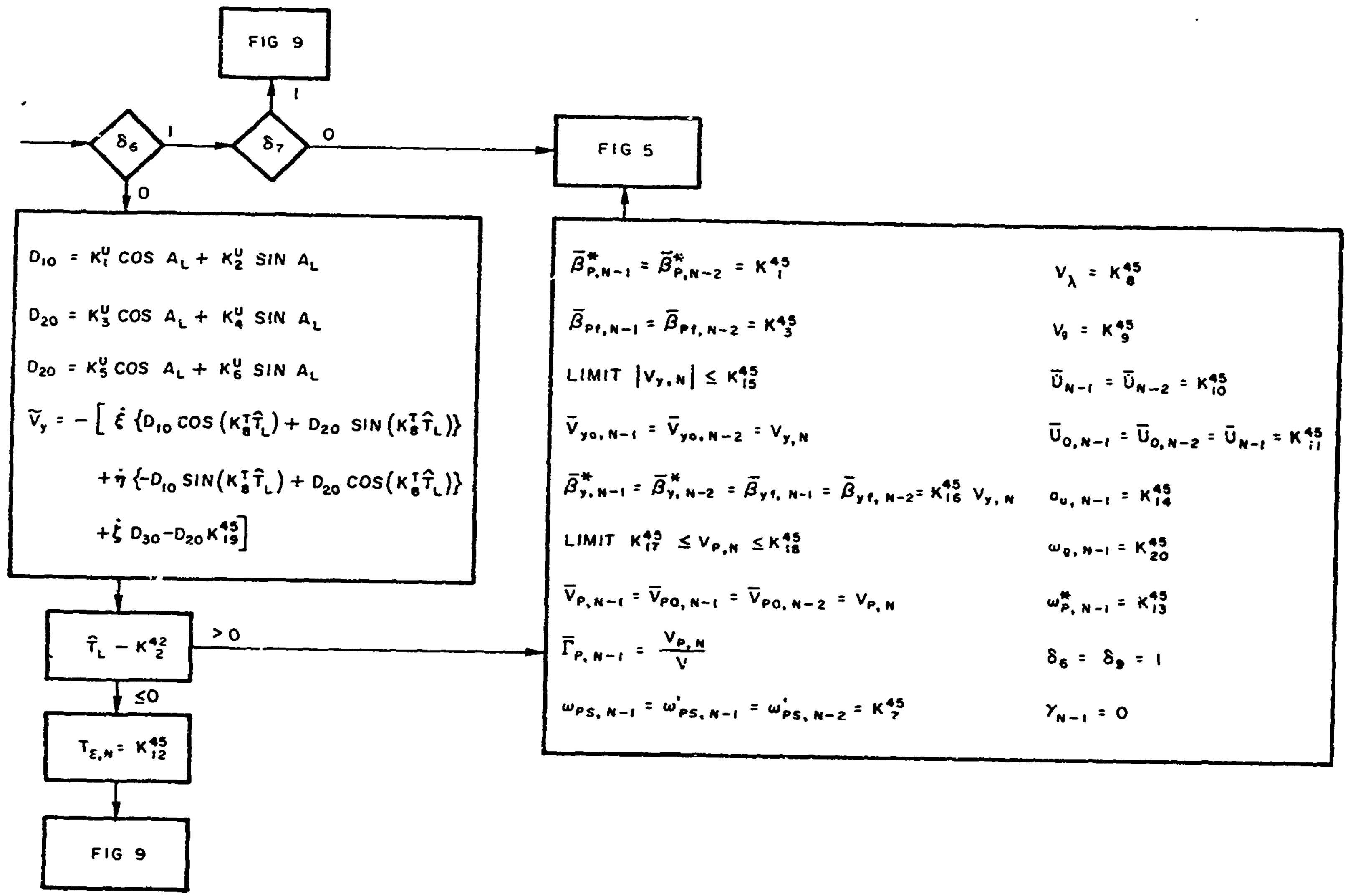
$$a_g^* = r a_g$$

FIG. 3

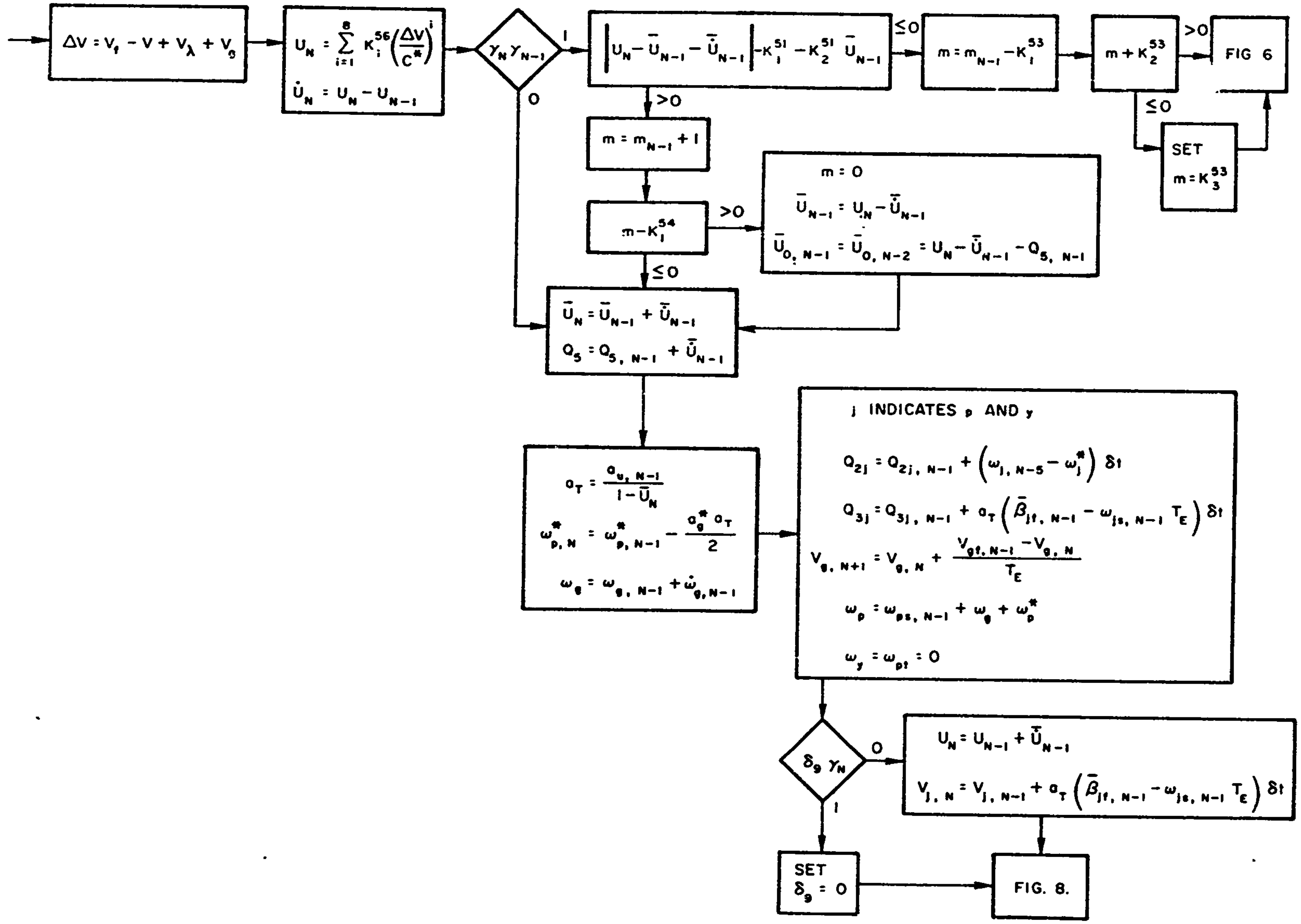
DESIRED FINAL VALUES


k−3
< 0
≥ 0
FIG 4
B₁ = K₁ᵁ sin γₜ + K₂ᵁ cos γₜ
B₂ = K₃ᵁ sin γₜ + K₄ᵁ cos γₜ
B₃ = K₅ᵁ sin γₜ + K₆ᵁ cos γₜ
Φ₃ = K₈ᵀ (Tₚ − Tᵣ) − K₁₉ᵀ
Ż_N = ξ (B₁ cos Φ₃ + B₂ sin Φ₃) + η̇ (−B₁ sin Φ₃ + B₂ cos Φ₃) + ζ̇ B₃
t_u,k = 100 + 40k
(t_u,k − K₇ᵁ) − T̂_L,N
> 0
≤ 0
Z̄_N = Ż_N
k − 1
k
Z̈_N = K₁₁ᵁ Z̈_N−1
Z̈_N = K₁₀ᵁ (K₁₉ᵁ − Δφ_R − γₜ)
Z̈_N = K₁₈ᵁ a_T,N−1 β_Y,N−1
Z̈_N−1 = K₉₀₊ₖᵁ Z̈_N−1
Ż_NP = Z̄_N−1 + Ż_N−1
(t_u,k + K₈ᵁ) − T̂_L,N
γ_N
No t_u,k UPDATE COMPUTED
SET k = k_N−1 + 1
FIG. 4
δ′₂,N δ′₃,N
k − 1
δ₅
No t_u,k UPDATE COMPUTED
SET k = k_N−1 + 1
δŻ_N = Ż_N − Ż_Np
Z̄_N = Ż_Np + K₁₃ᵁ δŻ_N
Ż_N = Z̈_N−1 + K₁₄ᵁ δŻ_N
Ż_Up,N = Z̄_N + 2Z̈_N (t_u,k − T̂_L,N + K₁₅ᵁ)
|Ż_Up,N − Ż_Up,N−1| − K₁₆ᵁ
FIG 4.
t_u,k − T̂_L,N − K₁₇ᵁ
k
SET δ₅ = 1
TRANSMIT Ż_Up,N TO IGS
SET k = k_N−1 + 1

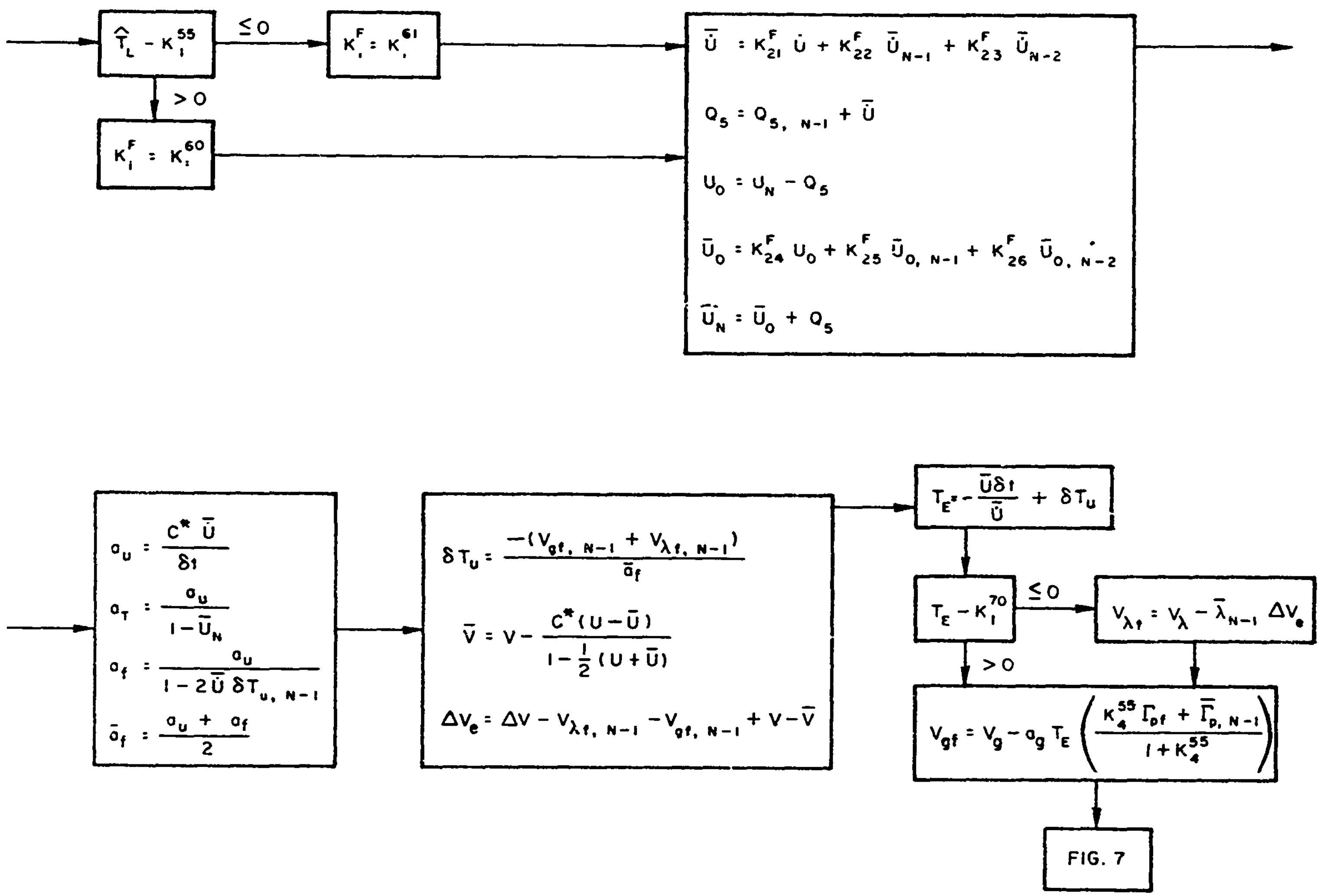
IGS UPDATE



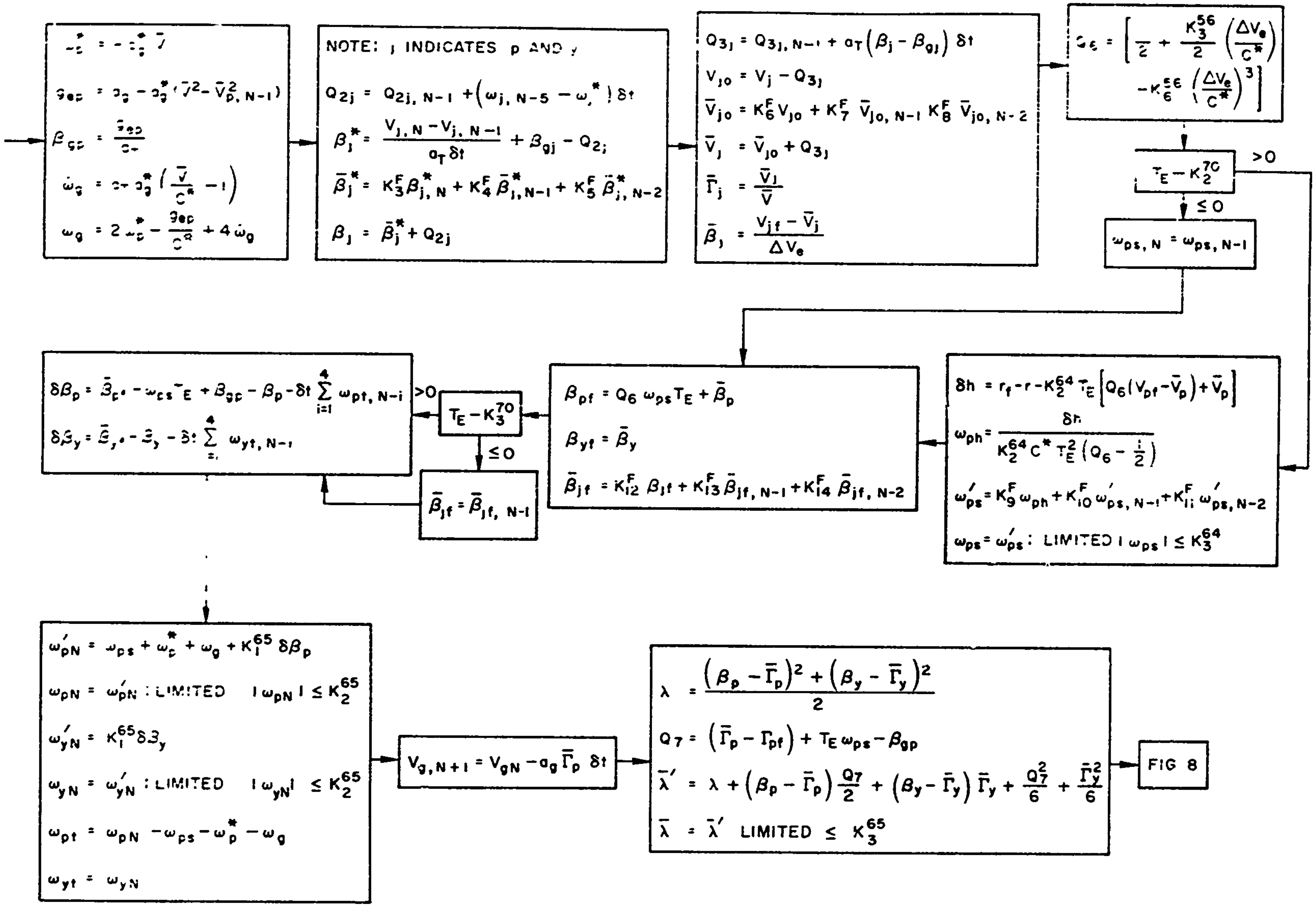
INITIALIZATION



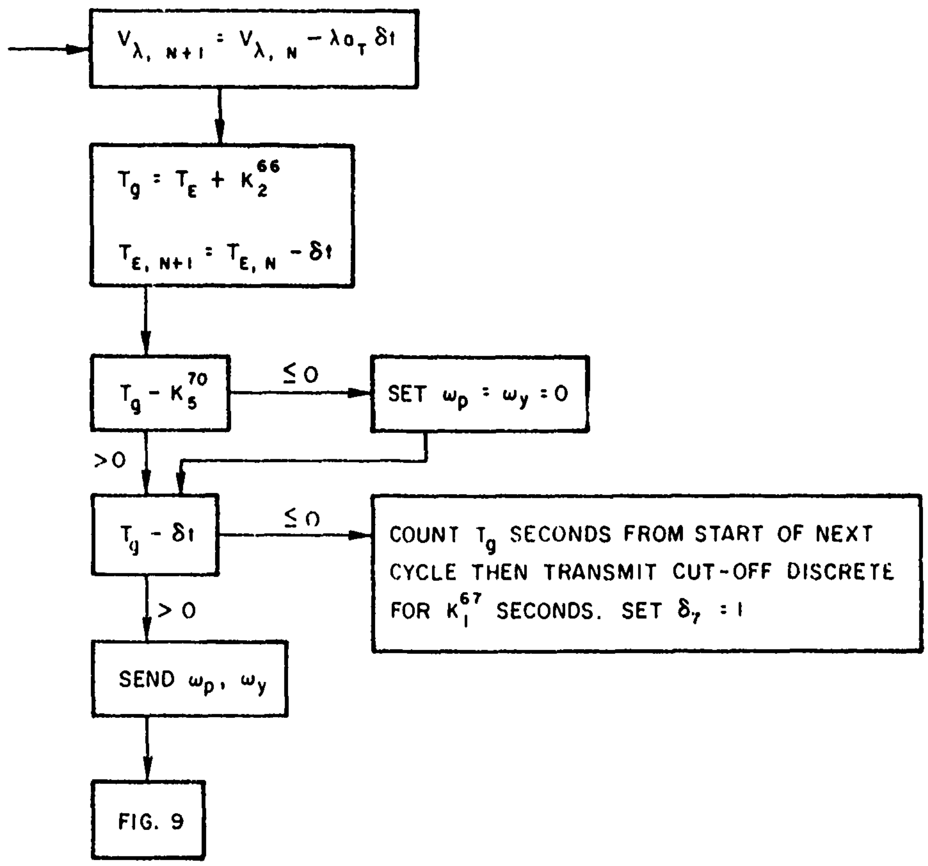
DATA EXTRAPOLATION


$\hat{T}_L - K_1^{55}$
≤ 0
$K_1^F = K_1^{61}$
> 0
$K_1^F = K_1^{60}$
$\bar{U} = K_{21}^F \dot{U} + K_{22}^F \bar{U}_{N-1} + K_{23}^F \bar{U}_{N-2}$
$Q_5 = Q_{5,\,N-1} + \bar{U}$
$U_0 = U_N - Q_5$
$\bar{U}_0 = K_{24}^F U_0 + K_{25}^F \bar{U}_{0,\,N-1} + K_{26}^F \bar{U}_{0,\,N-2}$
$\bar{U}_N = \bar{U}_0 + Q_5$
$a_u = \frac{C^* \bar{U}}{\delta t}$
$a_T = \frac{a_u}{1 - \bar{U}_N}$
$a_f = \frac{a_u}{1 - 2\bar{U}\,\delta T_{u,\,N-1}}$
$\bar{a}_f = \frac{a_u + a_f}{2}$
$\delta T_u = \frac{-(V_{gf,\,N-1} + V_{\lambda f,\,N-1})}{\bar{a}_f}$
$\bar{V} = V - \frac{C^*(U - \bar{U})}{1 - \frac{1}{2}(U + \bar{U})}$
$\Delta V_e = \Delta V - V_{\lambda f,\,N-1} - V_{gf,\,N-1} + V - \bar{V}$
$T_E = -\frac{\bar{U}\delta t}{\bar{U}} + \delta T_u$
$T_E - K_1^{70}$
≤ 0
$V_{\lambda f} = V_\lambda - \bar{\lambda}_{N-1}\,\Delta V_e$
> 0
$V_{gf} = V_g - a_g T_E \left(\frac{K_4^{55} \Gamma_{pf} + \bar{\Gamma}_{p,\,N-1}}{1 + K_4^{55}} \right)$
FIG. 7

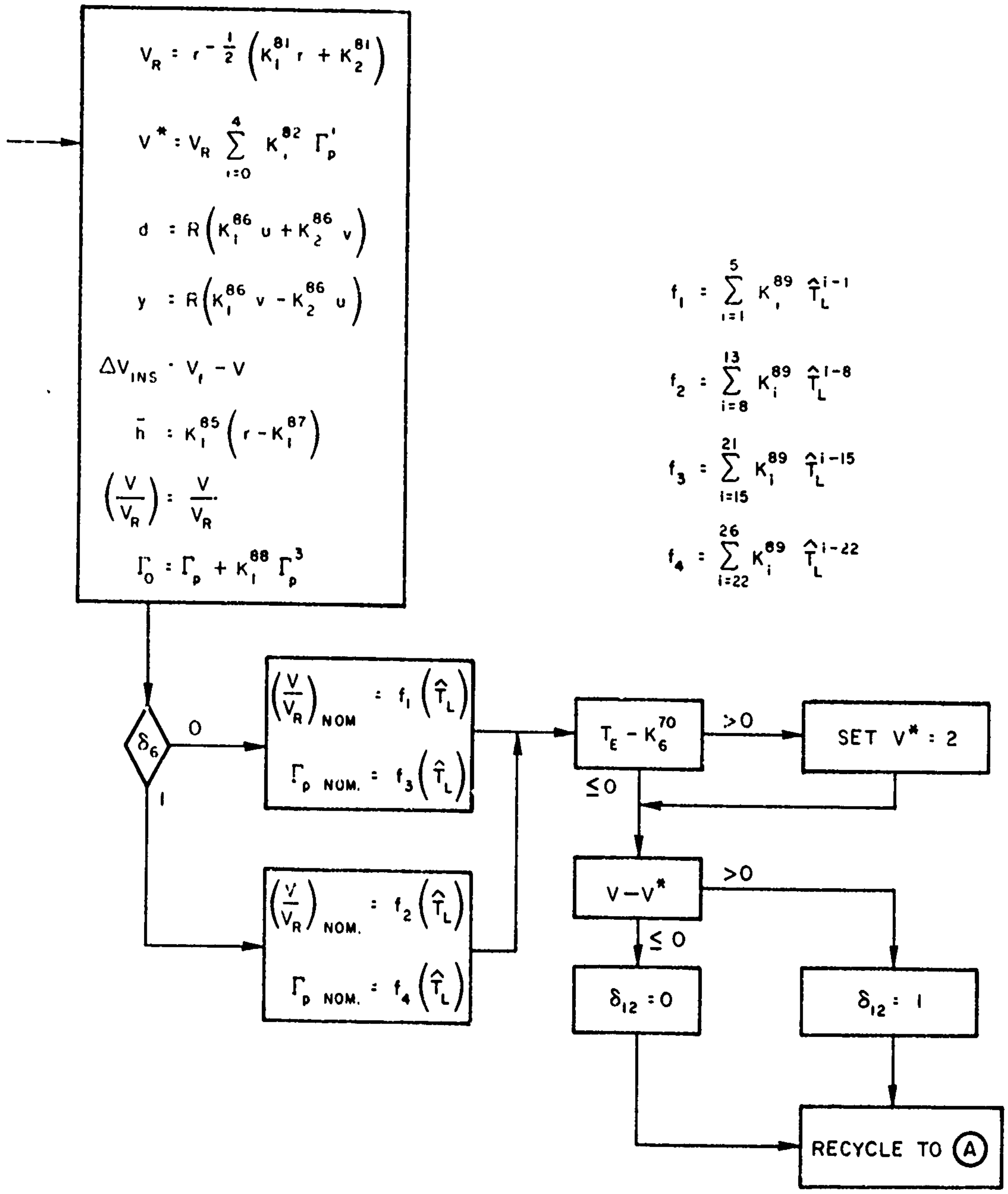
<u>TIME TO GO</u>



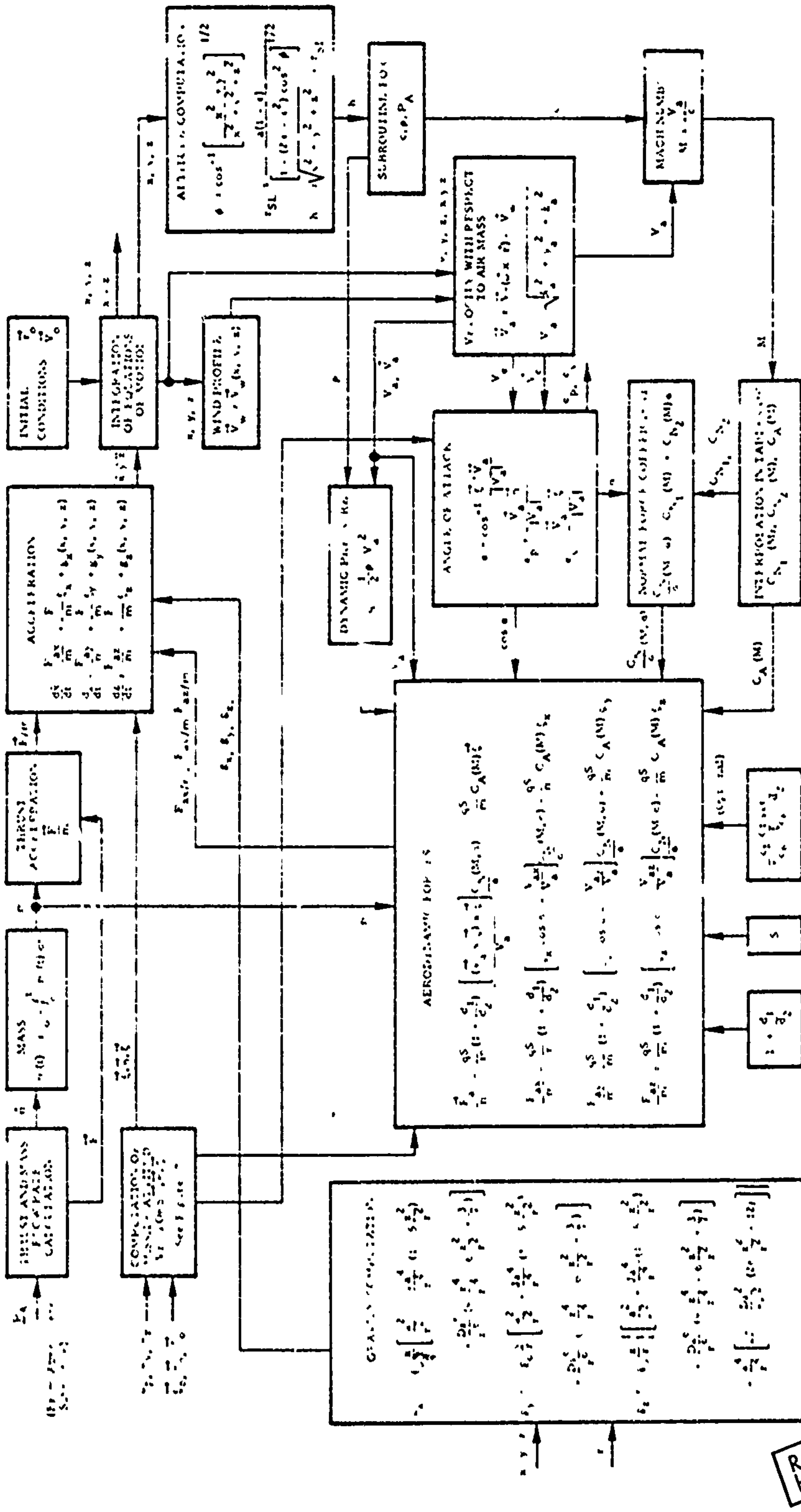
STEERING COMMANDS


$V_{\lambda, N+1} = V_{\lambda, N} - \lambda a_T \delta t$
$T_g = T_E + K_2^{66}$
$T_{E, N+1} = T_{E, N} - \delta t$
$T_g - K_5^{70}$
≤ 0
SET $\omega_p = \omega_y = 0$
> 0
$T_g - \delta t$
≤ 0
COUNT T_g SECONDS FROM START OF NEXT CYCLE THEN TRANSMIT CUT-OFF DISCRETE FOR K_1^{67} SECONDS. SET $\delta_7 = 1$
> 0
SEND ω_p, ω_y
FIG. 9

CUT-OFF



NASA REMOTES



VEHICLE SIMULATION - FUNCTIONAL BLOCK DIAGRAM

APPENDIX C. PONTRYAGIN'S MAXIMUM PRINCIPLE

Find absolute minimum of

$$J = \phi[x(T), T] + \int_o^T L dt \quad \text{subject to} \begin{cases} x = f(x, u, t) \\ x_o = \text{given} \end{cases}$$

With constraints, the augmented cost function is:

$$J^* = \Phi[x(T), T] + \int_o^T \left\{ L + p^T \cdot (f - x) \right\} dt$$

$$\text{Define } H \equiv L + p^T f(x, u, t)$$

Three steps:

1. Find u that minimizes H and substitute to obtain H*

2. Adjoint Equations $\frac{\partial H^*}{\partial x} = -\dot{p}, \left(\frac{\partial H}{\partial p} = \dot{x} \right)$

3. Apply boundary conditions:

$$\left[d\phi(x(T), T) + H^* dt - p^T dx \right]^{\text{End pt}} = 0, \text{ for all differentials}$$

END
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