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EVALUATION OF ENERGY MANEUVERABILITY PROCEDURES IN AIRCRAFT FLIGHT PATH OPTIMIZATION AND PERFORMANCE ESTIMATION

David T. Johnson

Air Force Flight Dynamics Laboratory
Wright-Patterson Air Force Base, Ohio

November 1972

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TECHNICAL REPORT AFFDL-TR-72-58

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13. ABSTRACT

An assessment is made of the applicability of Energy Maneuverability techniques (EM) to flight path optimization. A series of minimum time and fuel maneuvers using the F-4C aircraft were established to progressively violate the assumptions inherent in the EM program and comparisons were made with the Air Force Flight Dynamics Laboratory's (AFFDL) Three-Degree-of-Freedom Trajectory Optimization Program and a point mass option of the Six-Degree-of-Freedom flight path program. It was found the EM results were always optimistic in the value of the payoff functions with the optimism increasing as the percentage of the maneuver involving constant energy transitions increases. For the minimum time paths the resulting optimism was less than 2% for the maneuvers where the constant energy percentage was less than 35% followed by a rather steeply rising curve approaching in the limit 100% error for paths which are comprised entirely of constant energy transitions. Two new extensions are developed in the report; the first is a varying throttle technique for use on minimum fuel paths and the second a turning analysis that can be applied in conjunction with a Rutowski path. Both extensions were applied to F-4C maneuvers in conjunction with Rutowski's paths generated from the Air Force Armament Laboratory's Energy Maneuverability program. The study findings are that energy methods offer a tool especially useful in the early stages of preliminary design and functional performance studies where rapid results with reasonable accuracy are adequate. If the analyst uses good judgement in its applications to maneuvers the results provide a good qualitative insight for comparative purposes. The paths should not, however, be used as a source of maneuver design or flight schedule without verification especially on relatively dynamic maneuvers where the accuracy and optimality of the method decreases.

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Minimum Fuel/Time Comparisons						

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DAVID T. JOHNSON

FOREWORD

This report was prepared by personnel of the High Speed Aero Performance Branch, Flight Mechanics Division of the Air Force Flight Dynamics Laboratory (AFFDL/FXG), Wright-Patterson Air Force Base, Ohio. The research was accomplished under Project 1431 "Flight Mechanics and Performance Evaluation of Military Vehicles", and Task 143109 "Flight Path Analysis Methodology for Combat Aircraft and Missiles".

The work period extended from July 1967 to August 1970. Acknowledgement is made of the help received from B. R. Benson for overall guidance and R. C. Nash for the optimized flight paths run on AFFDL's Three-Degree-of-Freedom Optimization Program.

This report was submitted by the author May 1972.

This technical report has been reviewed and is approved.

Philip P. Antonatos
PHILIP P. ANTONATOS
Chief, Flight Mechanics Division
AF Flight Dynamics Laboratory

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SYMBOLS

C_L	lift coefficient
D	drag
E	total energy
Em	maneuvering energy
E_s	specific energy
g	acceleration of gravity
h	altitude
K	constant used in section III
L	lift
M	Mach number
n, N	normal load factor
m	mass
P_s	specific excess power
q	dynamic pressure
T_a	thrust available
Tx, Ty	thrust components in wind axis system
t	time
TE	turning efficiency
V	velocity
Vavg	average velocity
W	weight
w_a	fuel weight available for use
w_f	fuel weight
α	angle of attack
ΔX	variable X increment for given change in variable y where x and y can be any two variables

SYMBOLS (CONTINUED)

γ	flight path angle
λ	engine cant angle
ϕ	bank angle
ψ	heading angle

subscripts

f	final condition
i, o	initial condition
p	penalty

dot over symbol indicates differentiations with respect to time

SECTION I

INTRODUCTION

In making a comparison of two aircraft, in evaluating a design, in determining suitable operational maneuvers, and in measuring the performance of an aircraft, optimum flight paths are invaluable as a standard. One of the more accurate ways of determining an optimum path is through the use of a program such as the Air Force Flight Dynamics Laboratory's (AFFDL) Three-Dimensional Trajectory Optimization Program (TOP) (Reference 1). The TOP program uses the method of steepest descent which is an iterative scheme starting with any nonoptimal path and deriving an improved trajectory in each iteration until the payoff function is optimized and all constraints are satisfied. This program provides a complete time history of the optimum path in three dimensions, incorporates complete equations of motion, includes realistic vehicle representation, and can use any integrated variable as a payoff quantity. The disadvantage of the program is that it takes a relatively long time to reach an optimum due to the repeated solution of the equations of motion for each path improvement.

A second technique using the methods of E. S. Rutowski (Reference 2) is based on the use of total energy to provide an approximation to solutions of minimum time or minimum fuel problems. The technique has been extended and developed into a computer program called Energy Maneuverability (EM) by the Air Proving Ground Center (Reference 3). The method is predicated on 1g level flight and lends no insight into such parameters as load factor or pitch angle along the path. However, a solution is obtained very rapidly making the technique attractive where rapid solution time is necessary.

The objective of this study was to determine the applicability of the EM techniques to flight path optimization and to investigate improvements to alleviate some of its restrictions. Section II contains the basic equations used in the study and a discussion of the assumptions and limitations of the EM method. Two new extensions are developed, the first a varying throttle technique for use in generating minimum fuel paths and the second a turning analysis that can be applied in conjunction with a Rutowski path. Both the varying throttle and the turning analysis were designed so they can be applied using the existing EM program and a desk calculator although the methods could also be fully automated. A series of minimum time and fuel maneuvers was then established using the F-4C aircraft and comparisons made with AFFDL's TOP program and a point mass option of the Six-Degree-of-Freedom (SDF) flight path program (Reference 4). For the minimum fuel paths both maximum power and varying throttle paths are used; for the out of plane maneuvers results both with and without the turning analysis are shown. The below table indicates the type of comparisons made, maneuvers selected, and programs used in Sections III and IV

SECTION	MANEUVERS	METHODS		
		Baseline	Flyability/accuracy Comparison	Optimality Comparison
III	Vertical Plane	EM and modified EM	SDF	TOP
IV	Out of plane	modified EM	SDF	TOP

SECTION II

METHODS

1. ENERGY MANEUVERABILITY METHODS

The basic energy maneuverability equations used in this study are outlined in the following paragraphs. A more complete description of energy maneuverability methods may be found in References 2, 3, or 5.

a. Specific Excess Power

The energy state of a vehicle is defined as the sum of its potential and kinetic energy:

$$E = W\,(h + V^2/2g) \qquad (1)$$

and specific energy:

$$E_s = E/W = (h + V^2/2g) \qquad (2)$$

the time derivative of E_s is then:

$$\dot{E}_s = \dot{h} + V\,\dot{V}/g \qquad (3)$$

Writing the equation of motion along the flight path, $\dot{E}_s$ is related to vehicle parameters as follows:

$$W/g\,\dot{V} = T_a - D - W\sin\gamma \qquad (4)$$

$$(T_a - D)/W = \sin\gamma + \dot{V}/g \qquad (5)$$

$$V\,(T_a - D)/W = V\,\sin\gamma + V\,\dot{V}/g = \dot{h} + V\,\dot{V}/g = \dot{E}_s \qquad (6)$$

specific excess power is then:

$$P_s = \dot{E}_s = V(T_a - D)/W \quad (7)$$

Computation of P_s at a given h, V, and n is performed by setting:

$$L = nW \text{ (for level unbanked flight } n = 1) \quad (8)$$

T_a and C_D are obtained from tabular listings of aircraft characteristics at the specified conditions and P_s computed from Equation (7). Contours of P_s may be obtained by repeated solution and interpolations over a mesh of points.

b. Rutowski Paths

Computation of Rutowski paths is based on minimization of the integral:

$$t = \int_{E_{s_1}}^{E_{s_2}} \left[1/dE_s/dt \right] dE_s \quad (9)$$

for a minimum time path from a lower to a higher energy state and

$$w_f = \int_{E_{s_1}}^{E_{s_2}} \left[1/dE_s/dw_f \right] dE_s \quad (10)$$

for a minimum fuel path.

The method of finding the Mach - altitude history for the minimum time path can be shown graphically as the points of tangency between the P_s and E_s contours. Similarly the Mach - altitude history of the minimum fuel path can be shown as the points of tangency between the $P_s/\dot{w}$ and E_s contours. For specific maneuvers initial and final conditions off the Rutowski paths are reached by constant energy dives or zooms. Rutowski paths for selected maneuvers will be shown in Section III.

2. LIMITATIONS AND POSSIBLE EXTENSIONS OF ENERGY CLIMB

a. Thrust Along Flight Path

The usual assumption is that the thrust is directed along the flight path as in Equation (4) when in fact its direction is a function of angle of attack and engine cant angle. A change to include these effects could be incorporated as follows:

Breaking the thrust into components Equation (7) becomes,

$$P_s = V/W\ (T_x - D) \quad \text{where } T_x = T_a \cos(\alpha + \lambda) \tag{11}$$

The corrected lift distribution, in place of Equation (8), would then become,

$$L = nW - T_y \quad \text{where } T_y = T_a \sin(\alpha + \lambda) \tag{12}$$

or

$$C_L qS = nW - T_a \sin(\alpha + \lambda) \tag{13}$$

With tabular aerodynamic coefficient input, both sides of Equation (13) are a function of α (since C_L varies with α) and the equation may be iterated for α at a particular M, h, λ, and n. P_s would then be computed by Equation (11).

As this change in computational procedure would increase running time and complexity of the EM program it was not implemented in this study.

b. Path Flyability and Optimality

Rutowski paths are computed at constant load factors and there is no assurance that the resultant path is flyable as the full dynamics involved are not considered. For example a result of the energy method is that energy can be converted from potential to kinetic or vice versa along lines of constant energy in zero time with zero fuel expended. This is physically not possible so it is necessary to deviate from the path in practice producing a difference in the final result. A major portion of this study effort has been devoted to assessing the flyability and optimality of Rutowski paths. In the next two sections smoothed flight path angle histories from Rutowski paths for specific maneuvers were input to AFFDL's trajectory program (Reference 4) to assess the flyability and realism of the energy generated path. Minimum time paths were then generated on AFFDL's trajectory optimization program (Reference 1) to compare with the above two results.

The theory for the Rutowski path computation is defined for increasing specific energy only, hence, no decreasing energy maneuvers were considered. However, the investigation did include a zoom maneuver as will be shown in the next section. References 3 and 6 suggest the use of a "rule of thumb" to develop corrections to the energy path along the constant energy segments to improve flyability of the paths. This approach was applied to a sample maneuver in the next section.

c. Constant Throttle Setting

Historically Rutowski paths have been computed at a constant throttle setting for both minimum time and fuel. To investigate the gains possible by varying the throttle for minimum fuel paths, three throttle settings were used in this study: military, minimum afterburner (A/B), and maximum A/B. Switching between throttle settings, and corresponding Rutowski minimum fuel paths, was accomplished to minimize the integral of Equation (10). Specific examples are shown in Section III - 2a.

d. Vertical Plane Restriction

The energy climb procedure considers only the changes in energy state of a vehicle and hence does not permit turning maneuvers. However in practice many of the maneuvers of interest to analysts and pilots include varying amounts of turn as well as changes in the vehicles energy state. The following turning procedure was used in this study to obtain an estimate of turning performance in conjunction with a Rutowski path. This method was designed to permit computation of turning maneuvers by any person who has a method of computing Rutowski paths such as the program of Reference 3.

With the assumption of level flight and that the thrust is directed along the flight path we can write using Equation (8):

$$L \cos \phi = W \tag{14}$$

$$\text{and } \phi = \cos^{-1}\left(\frac{1}{n}\right) \tag{15}$$

writing the equation in the lateral direction

$$mV\dot{\psi} = L \sin \phi = nW\left(\sqrt{n^2-1}\right) \tag{16}$$

$$\text{or } \dot{\psi} = g/V\left(\sqrt{n^2-1}\right) \tag{17}$$

Equation (17) was then applied between energy levels along constant load factor Rutowski paths to obtain $\dot{\psi}$. The amount of turn between energy levels at a given load factor was then calculated by:

$$\Delta\dot{\psi} - \dot{\psi}\Delta t \tag{18}$$

where Δt is the time increment to go between energy levels along a Rutowski path calculated by Equations (7), (8), and (9). The time penalty (Δt_p) to operate at a load factor higher than one is then calculated between two energy levels by:

$$\Delta t_p = \Delta t(n > 1) - \Delta t(n = 1). \tag{19}$$

The turning efficiency between energy levels was then defined as:

$$
\begin{aligned}
(TE) &= \Delta\psi / \Delta t_p \qquad \text{for } n > 1 \qquad \text{on minimum time paths} \\
(TE) &= \Delta\psi / \Delta W_p \qquad \text{for } n > 1 \qquad \text{on minimum fuel paths} \\
(TE) &= 0 \qquad \text{for } n = 1.
\end{aligned}
\tag{20}
$$

The turns used in this study were computed at constant values of turning efficiency (TE) and hence are called "constant efficiency turns" in the following sections. Specific examples of the calculation procedure are shown in Section IV.

SECTION III

VERTICAL PLANE MANEUVERS

The maneuver type and the initial and final conditions for the nonturning maneuvers selected are shown in columns 1, 2, and 3 of Table 1. Maneuver 1 is a long climb acceleration with initial and final conditions close to the Rutowski path. Maneuver 2 is a shorter climb acceleration with initial conditions considerably away from the Rutowski path in the h-M plane and at a higher energy level. Maneuver 3 has both initial and final conditions at the same altitude starting at a higher and ending at a lower energy level than Maneuver 2. Maneuver 4 is a zoom to maximize altitude. Column 4 shows the method used, column 5 the time required, and column 6 the fuel expended; these will be discussed in the following paragraphs. Data for the F-4C aircraft used in the simulations was obtained from the manufacturer.

1. MINIMUM TIME PATHS

a. Flyability and Accuracy

The Rutowski minimum time paths were run for the above maneuvers using the program described in Reference 5 at a 1 g load factor. Figure 1 shows the P_s contours and the energy path resulting from connecting the points of tangency between the P_s and E_s contours for the F-4C at maximum A/B power. The Rutowski path changes only slightly with small changes in aircraft weight, thus all of the maneuvers basically are made up of segments of the path shown in Figure 1 together with constant energy segments to connect the path to the desired initial and final points. The maneuver results are shown in Table I as "EM".

To investigate the flyability and accuracy of these EM paths the maneuvers were simulated on option 6 (point mass) of the SDF program (Reference 4). A certain amount of modification to the path was required for simulation on the SDF program as far as smoothing the path history. Since constant energy dives and zooms are not achievable in practice (without setting $T_a = D$) the pushovers were done at low positive lift and the pull-ups at moderate positive load factor.

TABLE I

CATEGORY I - VERTICAL PLANE MANEUVERS

Type	Initial conditions	Final conditions	Procedure	Time (SEC)	Fuel(LBS)	Throttle
			Maneuver 1			
Min Time	$M = .25$ $h = 50$ ft $W = 38400$ lbs $\gamma = 0°$	$M = 2$ $h = 45K$ Ft	EM	315.9	5254	Max A/B
			EM-SDF	320.2	5303	
			TOP	319.2	5098	
Min Fuel			EM	538	4294	MIL-Min A/B-Max A/B
			EM-SDF	539	4578	MIL-Min A/B-Max A/B
			EM	366	4687	Max A/B only
			Maneuver 2			
Min Time	$M = .7$ $h = 30K$ Ft $W = 38400$ lbs $\gamma = 0°$	$M = 2$ $h = 45K$ Ft	EM	260	4100	Max A/B
			EM-SDF	270	4107	
			TOP	262.7	3994	
Min Fuel			EM	379	3521	Min A/B-Max A/B
			EM-SDF	385	3614	MIL-Min A/B-Max A/B
			Maneuver 3			
Min Time	$M = .75$ $h = 35K$ Ft $W = 38400$ lbs $\gamma = 0°$	$M = 1.95$ $h = 35K$ Ft	EM	158.2	2797	Max A/B
			EM-SDF	172.4	2811	
			EM-MOD	169.1	2740	
			TOP	168.9	2684	
Min Fuel			EM	225	2261	Min A/B-Max A/B
			EM-SDF	253	2440	MIL-Min A/B-Max A/B

TABLE I CONCLUDED

Type	Initial conditions	Final conditions	Procedure	Final Altitude (ft)
			Maneuver 4	
Zoom	$M = 2$ h = 50K Ft W = 33238 lbs $\gamma = 0°$	Maximize Altitude	EM	107000
			EM-MOD	85000
			TOP	92000

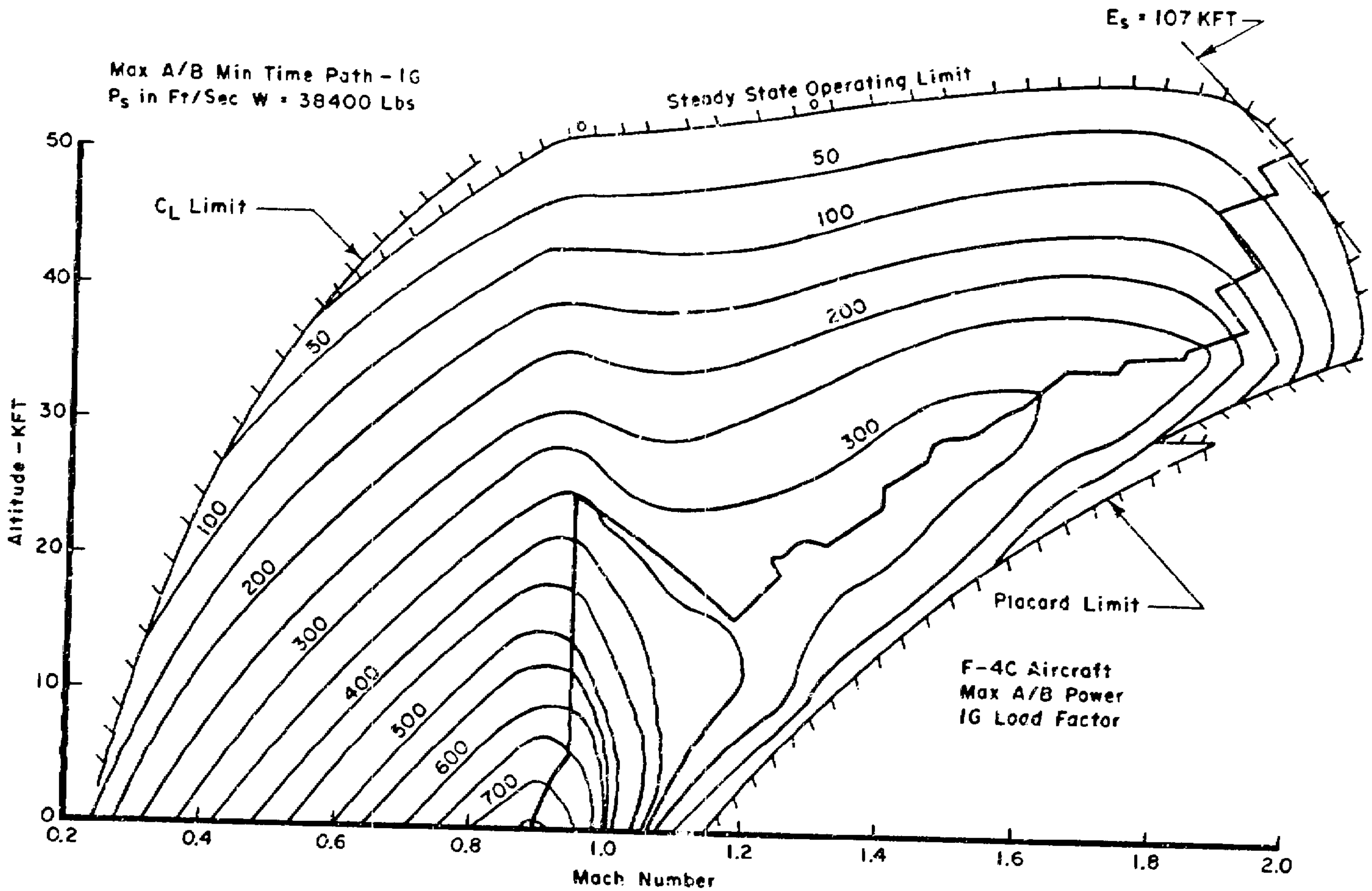

Figure 1. Specific Excess Power (P_S) Contour Map

Two flight plan programmers were used in the SDF program $\gamma = f(h)$ and $n = f(M)$. The flight path angle history was calculated from the EM history by:

$$\gamma = \sin^{-1} \left(\Delta h / \Delta t \; V \right) \qquad (21)$$

and used to control most of the path. The load factor programmer was used primarily to perform climbs at near constant Mach number. Staging of controls was used at various points along the paths to change modes of operation. Table II shows a typical example of the staging and control used in SDF simulation. The example consists of a level acceleration at military power, a subsonic climb at minimum A/B, a pushover/pullout transition at maximum A/B and a supersonic climb/acceleration. This path was split into nine control segments or stages as shown in the Table II to facilitate the SDF simulation. An initial flight path angle of 0° and unconstrained terminal flight path angle was used. SDF paths are listed in Table I as "EM-SDF".

Figure 2 shows the paths for the two computation methods for Maneuver 1 which has starting and ending conditions near the Rutowski path. The figure shows the SDF and EM times are close all along the path with the principle difference in the path profiles occuring in the transonic area where the EM path has a constant energy dive due to the shape of the P_s contours shown in Figure 1. The agreement between methods is quite good with the EM being 4.3 seconds or 1.3% optimistic in time and 49 pounds or 0.9% optimistic in fuel.

Figure 3 shows the paths for Maneuver 2 which uses the same end conditions as Maneuver 1, however, the initial conditions are at a higher energy level and are not close to the Rutowski path. To reach it requires a sizable transition segment. The principle difference in the path profiles occurs in the initial constant energy dive ($.7 < M < 0.95$) and in the high transonic -supersonic area ($0.95 < M < 1.25$). The SDF times lag the EM times by a wider range than in Maneuver 1 over the entire path. On this path the agreement is not quite as good. The EM being 10 seconds or 3.7% optimistic. Agreement in fuel used is quite good (7 pounds or 0.2%) apparently due to the SDF path being higher over the diving portion saving fuel which is later consumed by the longer time required to reach the final conditions.

TABLE II

CATEGORY I, MANEUVER I - MINIMUM FUEL STAGING EXAMPLE

Stage No	Staging Parameter to next stage	Controls	Remarks	Power Setting
1	$M = .60$	$\gamma \approx 0°$	level flight	Military
2	$h = 17{,}000$ ft	$\gamma = f(h)$	climb/accelerate	Min A/B
3	$\gamma = 18°$	$n = f(M)$	climb/accelerate	Min A/B
4	$h = 40{,}500$	$n = f(M)$	constant Mach climb	Min A/B
5	$M = 1.21$	$\gamma = f(h)$	Pushover at low lift	Max A/B
6	$M = 1.84$	$\gamma = f(h)$	Pullout and Acceleration/climb	Max A/B
7	$h = 47{,}200$	$\gamma = f(h)$	rapid climb at near constant Mach	Max A/B
8	$M = 1.913$	$\gamma = .1°$	near level accelerating flight	Max A/B
9	$M = 2$	$\gamma = f(h)$	Pushover and dive to end conditions	Max A/B

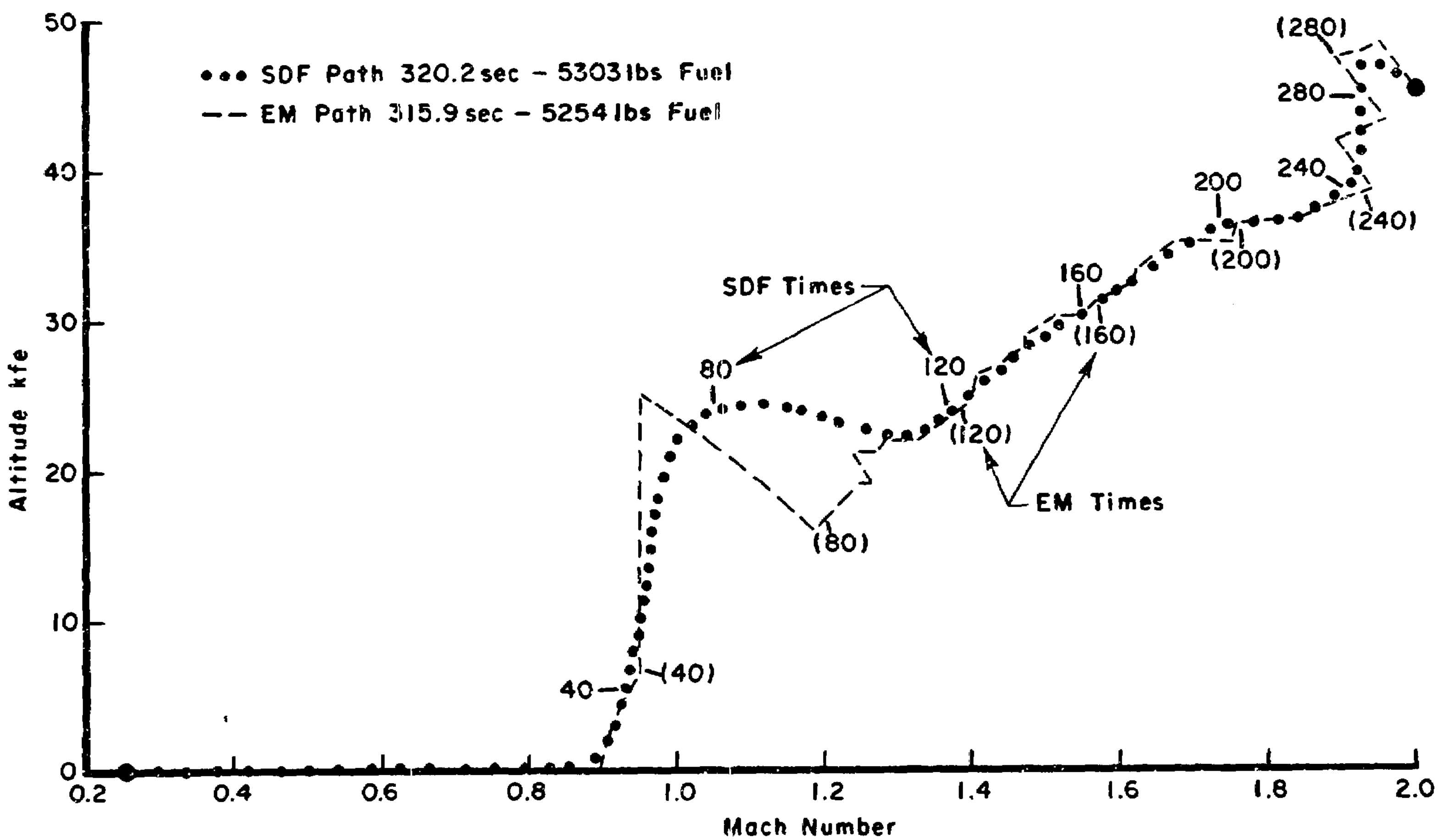

Figure 2. Category I, Maneuver 1, Minimum Time Path-EM and SDF

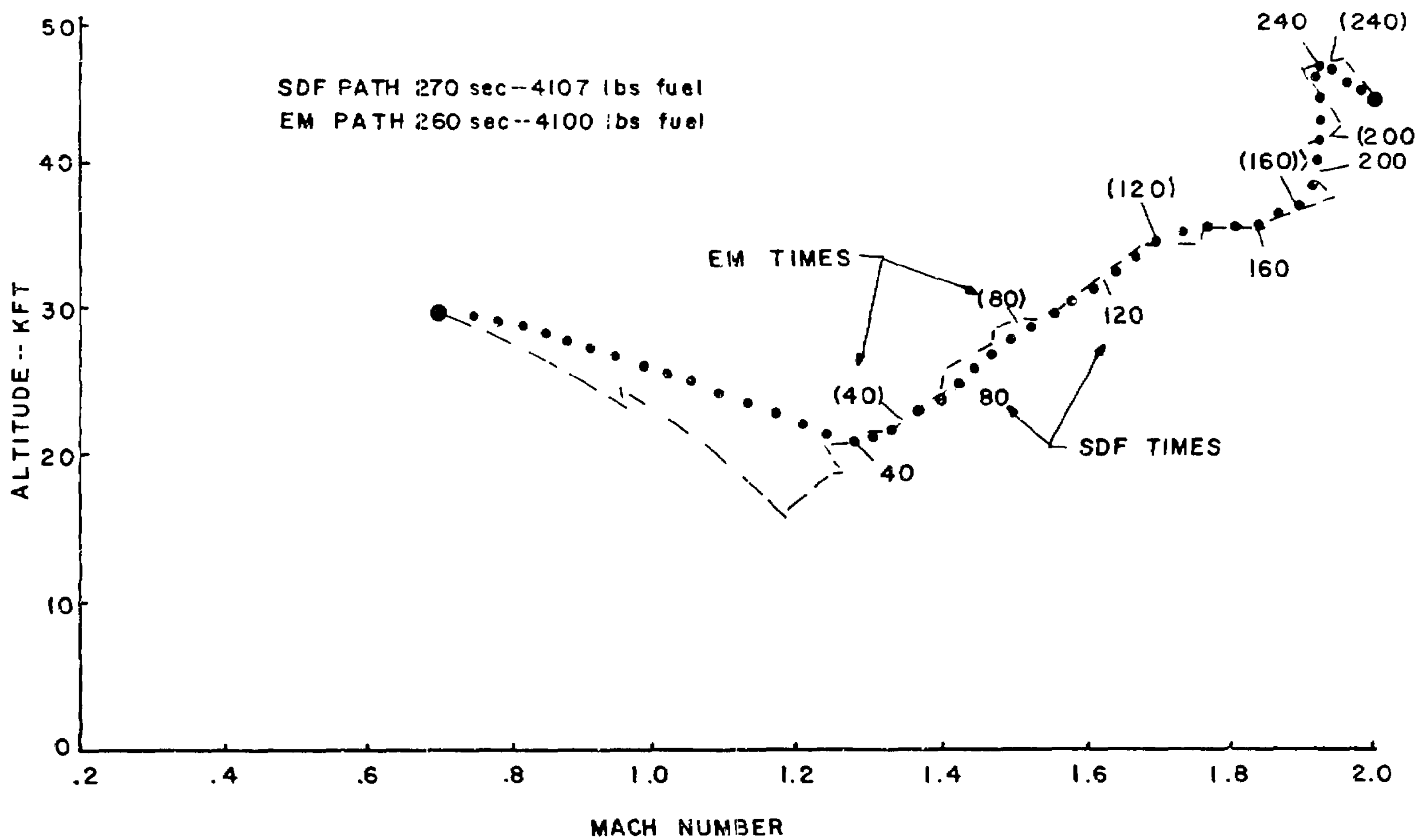

Figure 3. Category I, Maneuver 2, Minimum Time Path-EM and SDF

Maneuver 3, Figure 4, has both starting and ending conditions at 35,000 feet. The path starts at a higher and ends at a lower energy level than Maneuver 2. Again, the initial conditions are well off the Rutowski path. The EM path has a long initial constant energy dive which does not connect to the subsonic but to the supersonic portion of the Rutowski path. This region ($0.75 < M < 1.3$) accounts for the major difference in the paths and the SDF lags the EM times by a larger amount than in the other maneuvers. The final result is that the EM path is 14.2 seconds or 8.2% optimistic in time and 14 pounds or 0.5% optimistic in fuel.

The indications from these three maneuvers are that the EM procedure predicts times with reasonable accuracy ranging from less than 2% optimistic for a 310 + second path with initial and final conditions close to the path to about 8% optimistic for a 150+ second path with initial conditions well off the path. The associated fuel required for the maneuvers showed surprisingly good agreement, being within 1%, regardless of the length of the path or placement of the initial conditions within the above limits. However, from a flyability aspect the Rutowski path contains basic inherent discrepancies because connecting the points of tangency with constant energy segments produces violent gradients in control when attempting to follow the path. A pilot, when attempting to follow such a path, might think he was doing it and even state that this was done when in actuality his smoothing and alterations of the profile would shift it close to the paths marked EM-SDF. References 3 and 6 suggest a "rule of thumb" technique to modify the constant energy segments of the path and improve flyability. The rule is as follows:

Along constant energy segments the specific energy level is to be increased according to the altitude lost or gained in the maneuver by:

$$\Delta E_s = K \Delta h \tag{22}$$

where $K = -1$ if Mach number is increasing (dive)

$K = 2/3$ if Mach number is decreasing (climb).

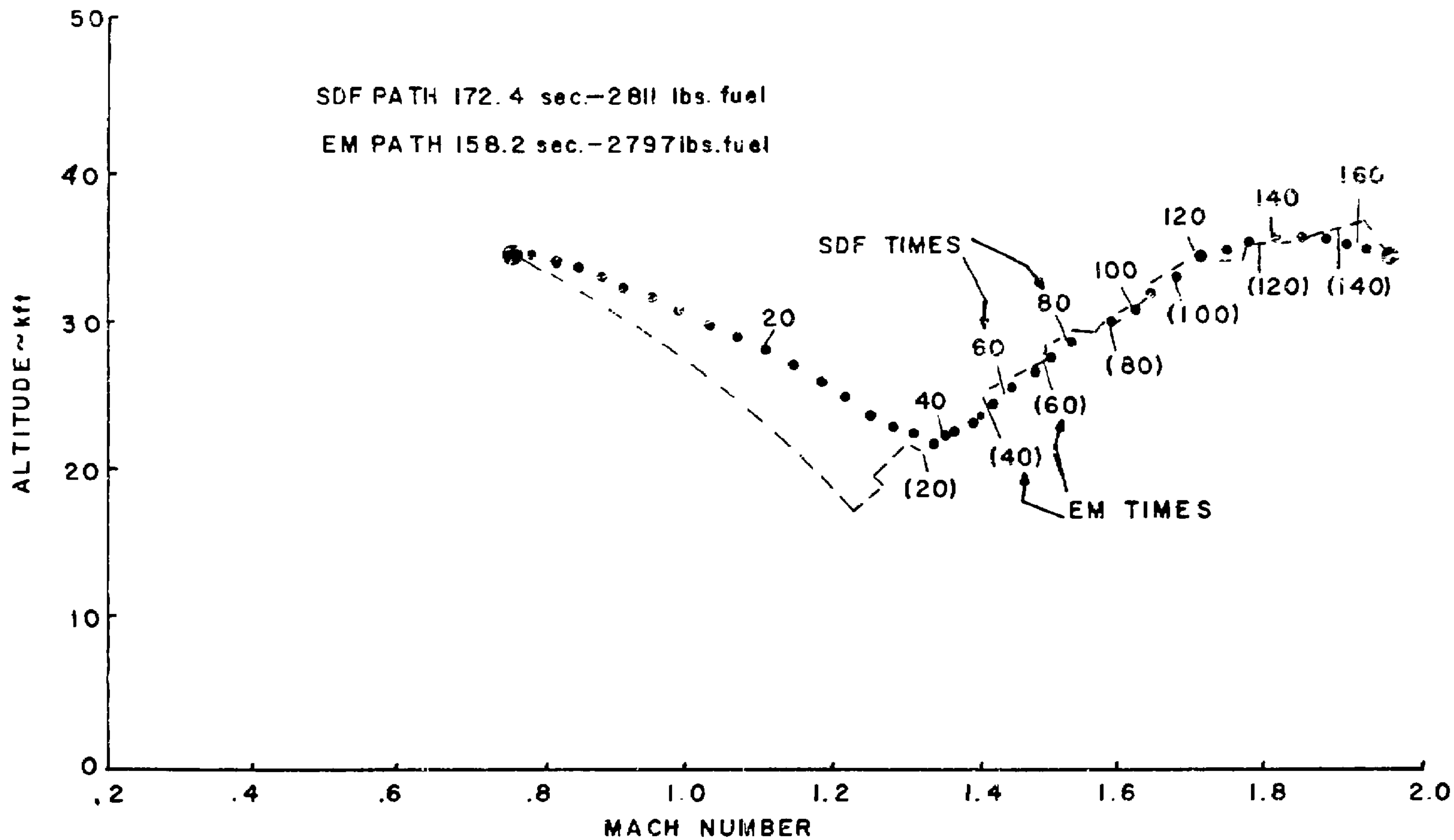

Figure 4. Category I, Maneuver 3, Minimum Time Path-EM and SDF

This approach was applied to the initial and final dives of Maneuver 3 where the biggest difference in results was noted previously. The modified portion of the path is noted by the hatched line shown in Figure 5. The figure indicates the changed path is indeed closer to the SDF path. It is interesting to note that the SDF path undershoots or, in other words, converts potential to kinetic energy faster than the E-M modified path once an appreciable negative flight path angle has been generated. From Equations (9) and (10), in summation form, the net change in the time required for the path can be written as:

$$\Delta t = \sum_{E_{s_1}}^{E_{s_2}} \left(\frac{P_s - P_s^*}{P_s \, P_s^*} \right) \Delta E_s \tag{23}$$

and the change in fuel by:

$$\Delta w_f = \sum_{E_{s_1}}^{E_{s_2}} \left[\frac{(P_s / \dot{w}_f) - (P_s^* / \dot{w}_f^*)}{(P_s / \dot{w}_f)(P_s^* / \dot{w}_f^*)} \right] \Delta E_s \tag{24}$$

where the asterisk indicates the EM modified values.

Using an energy level increment of 2000 feet the time calculation showed a 10.2 second increase in the initial dive and a 0.7 second increase in the terminal dive for a net Δt = 10.9 seconds. The fuel calculation showed a 113 pound decrease in the initial dive and a 56 pound increase in the terminal dive for a net change of Δw_f = - 57 lbs. Totals then are 169.1 seconds for time and 2740 lbs for fuel, these values are shown as "EM - MOD" in Table I. The modification of the maneuver does make the path more flyable and provides a time result closer to the SDF path, however, the fuel correction creates a bigger differential. This correction was not applied to the rest of the maneuvers as the basic program provided good agreement between final results. If the constant energy segments were allowed to increase even more than in Maneuver 3 some type of correcting factor would definitely be required to make the EM more flyable.

b. Optimality

To investigate how closely the EM paths approach the optimum minimum time schedule the three maneuvers were run on AFFDL's Trajectory Optimization Program (TOP). The TOP program provides integrated constrained

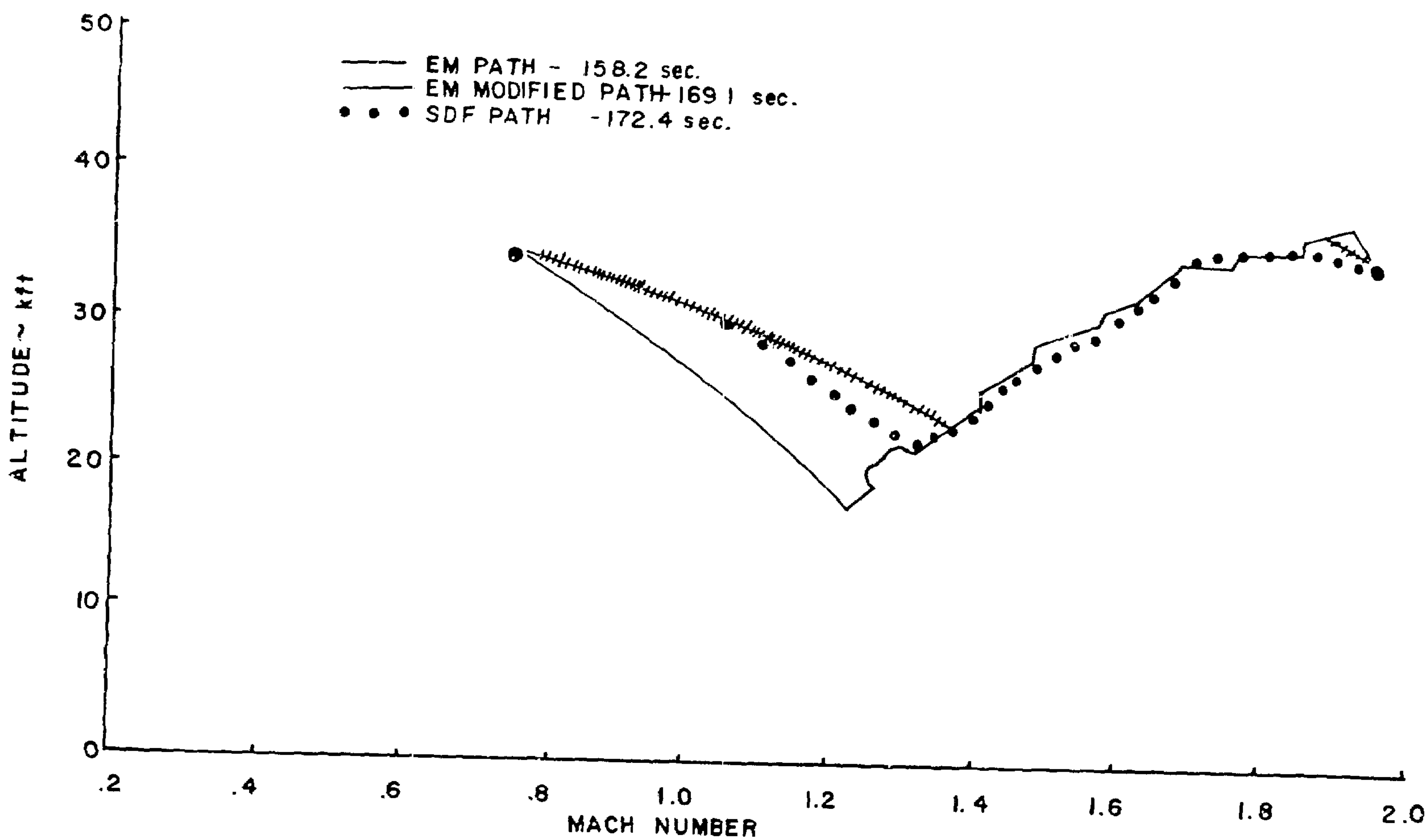

Figure 5. Category I, Maneuver 3, Modified Minimum Time Energy Path

optimum solutions to the maneuvers under consideration. The results are shown in Figures 6 through 8 and are labeled "TOP" in Table I.

Figure 6 shows the comparison between the TOP and EM paths for Maneuver 1. The major elements and differences in the shape of the paths are: (1) both paths show a near level acceleration for $0.25 < M < 0.9$ with the TOP doing a gentle pullup, (2) the TOP path performs a supersonic climb crossing Mach = 1 at about 6000 feet altitude while the EM path performs a constant Mach number subsonic climb at $M = 0.95$, (3) the TOP path does a pushover and almost level acceleration for $1.1 < M < 1.4$ at about 27,500 feet altitude while the EM schedule indicates a dive with pullout at about 17,000 feet altitude and $M = 1.18$ followed by a climb, (4) from $1.4 < M < 2$ the paths are very similar with both showing the hook in the very end due to the poor level acceleration capability of the F-4C at high speed and altitude. Although the paths differ considerably in the transonic and low supersonic range the agreement in final results was good with the EM method being 3.3 seconds or about 1% optimistic in time and 156 pounds or 3% pessimistic in fuel consumption. The fact that these results are in such good agreement, although the paths differ, indicates a fairly wide region in the transonic - low supersonic area where there is low sensitivity of the payoff (time) to changes in the flight profile. The EM path is generally lower in altitude than the TOP accounting for its higher fuel consumption prediction although shorter time.

Figure 7 shows a comparison between the TOP and EM for Maneuver 2. The major elements in the shape of the path are: (1) the EM path has an initial diving constant energy segment to the subsonic Rutowski path at $M = 0.95$ followed by another diving constant energy transition to the supersonic path with pullout at about 17,000 feet and $M = 1.18$. The TOP has a continuous dive with pullout occuring at about 20,000 feet and Mach = 1.35, (2) for Mach numbers greater than 1.5 both paths are about the same as in Maneuver 1 with both exhibiting the hook in the end for $1.9 < M < 2$. Agreement in results is again quite good with the EM method being 2.7 seconds or 1% optimistic in time and 6 pounds or 0.2% pessimistic in fuel.

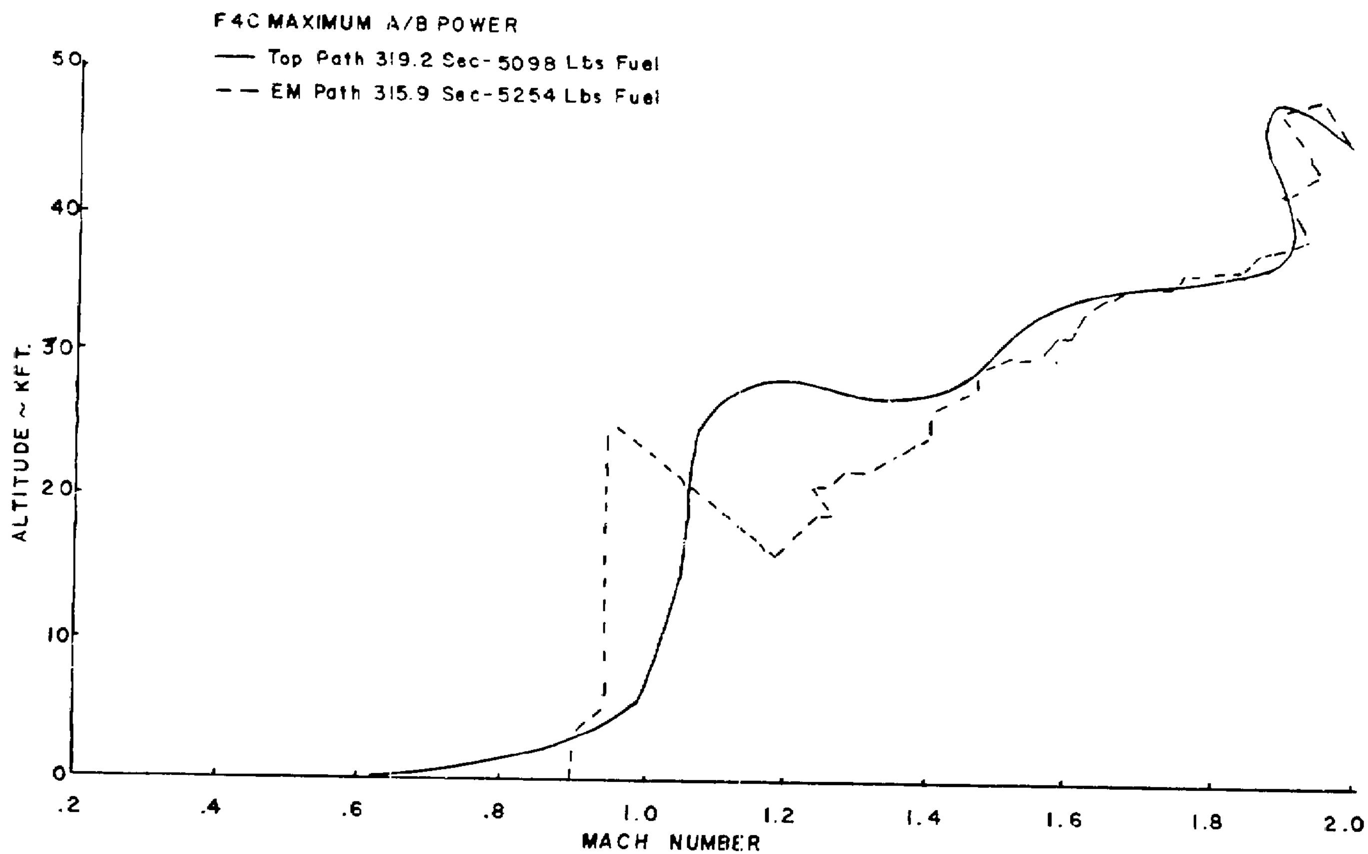

Figure 6. Category I, Maneuver 1, Minimum Time Path-EM and TOP

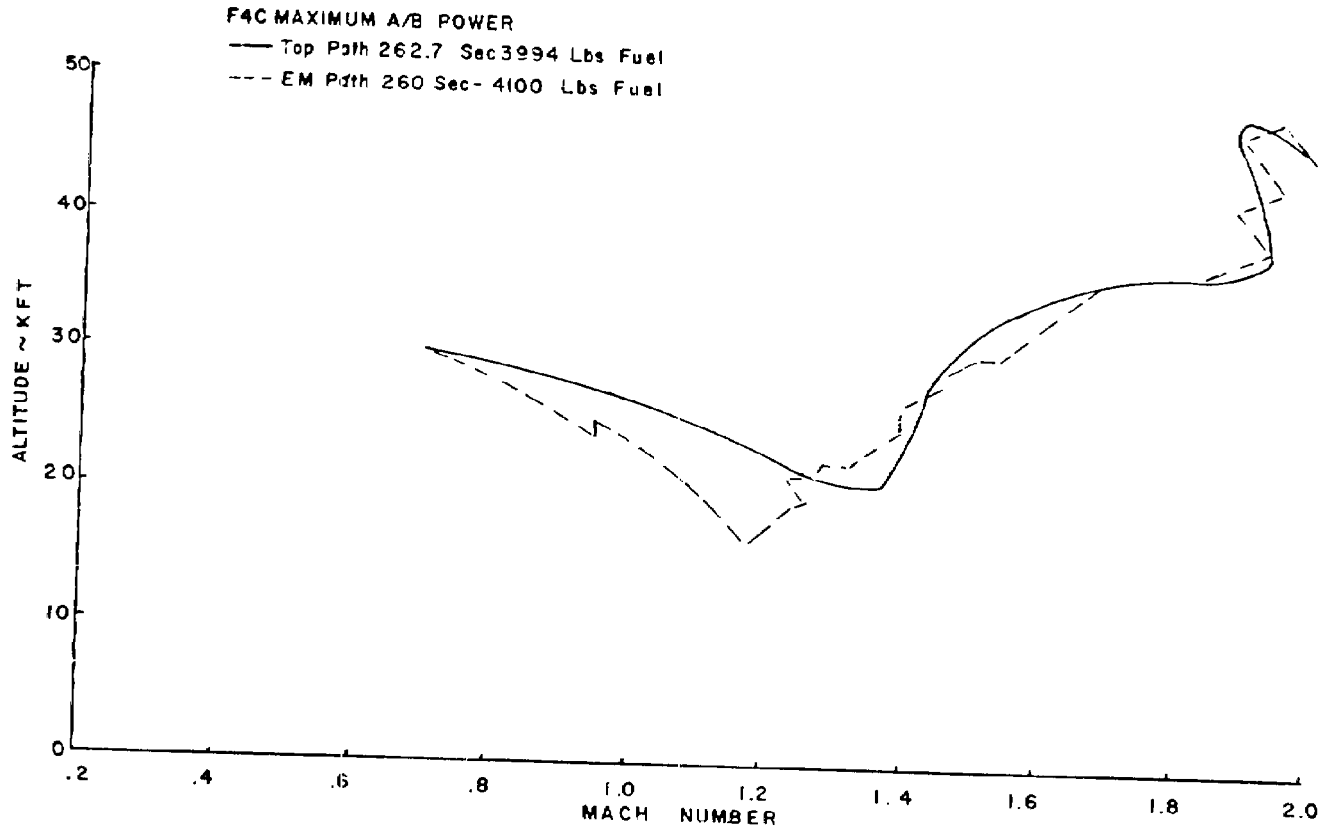

Figure 7. Category I, Maneuver 2, Minimum Time Path-EM and TOP

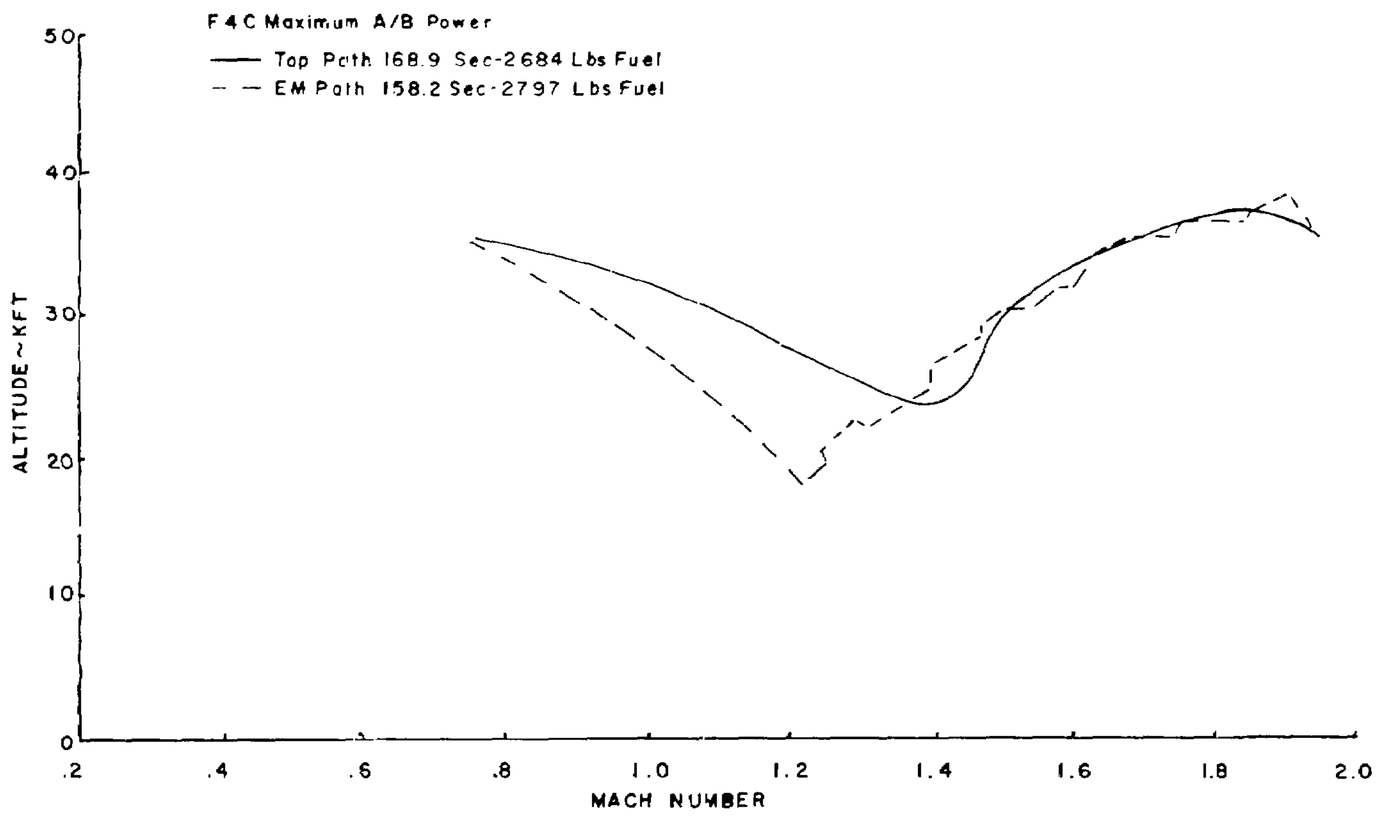

Figure 8. Category I, Maneuver 3, Minimum Time Path-EM and TOP

Figure 8 shows the two paths for maneuver 3. For the EM schedule the initial constant energy transition bypasses completely the subsonic portion of the Rutowski path (being at a higher energy level) and intersects the supersonic portion at about M = 1.22 and 18,000 feet while the TOP dives and pulls out at about 23,000 feet and M= 1.4. For Mach numbers greater than 1.5 the paths are close together and both exhibit a small hook at the end rather than a constant altitude acceleration. Agreement in final results is not as good with the EM being 10.7 seconds or 6% optimistic in time and 113 pounds or 4% pessimistic in fuel.

c. Discussion

Since it is known that for a minimum time problem where kinetic and potential energy are to be exchanged at no net change in energy (pure dive or zoom) the basic EM technique would indicate 0 seconds or 100% error, an additional problem was formulated to provide a data point between the above maneuvers and a pure energy exchange. The problem chosen was to obtain a minimum time path between an initial state of h = 35,000 feet, M = .6, γ = 0°, W = 38,400 lbs and a terminal state of h = 44,000 feet, M = 1.2, γ = free. Figure 9 shows the two paths. The EM path consists of a portion of the Rutowski path connected by constant energy segments to the initial and final conditions. The TOP path dives through the EM path pulling up at about M = 1.4 and h = 20,000 feet. The results for this problem are; EM-79 seconds, TOP-104 seconds. Thus, the EM path is 25 seconds or 24% optimistic. It is interesting to note that although the basic EM path is not close to the TOP in the H - M plane, the form of the "rule of thumb" correction discussed previously would warp the shape of the path in the proper direction.

The percentage difference between the EM and SDF, EM and TOP final times in all four maneuvers is plotted as a function of the percentage of the maneuver involving constant energy segments in Figure 10. This constant energy percentage was calculated by the length* of the constant energy segments

* Scaled directly from Figures 2, 3, 4 and 9.

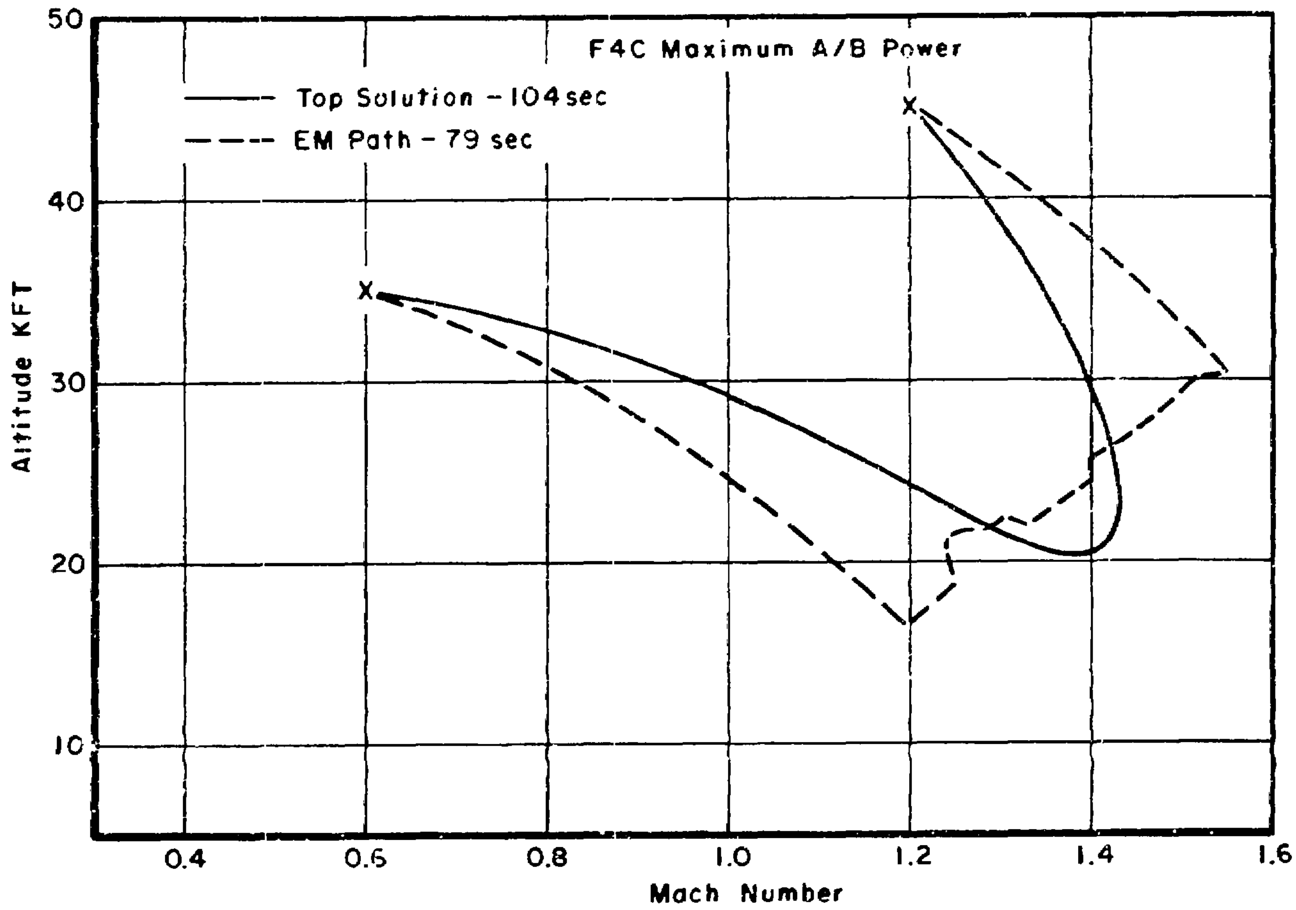

Figure 9. Category I, Short Minimum Time Path-EM and TOP

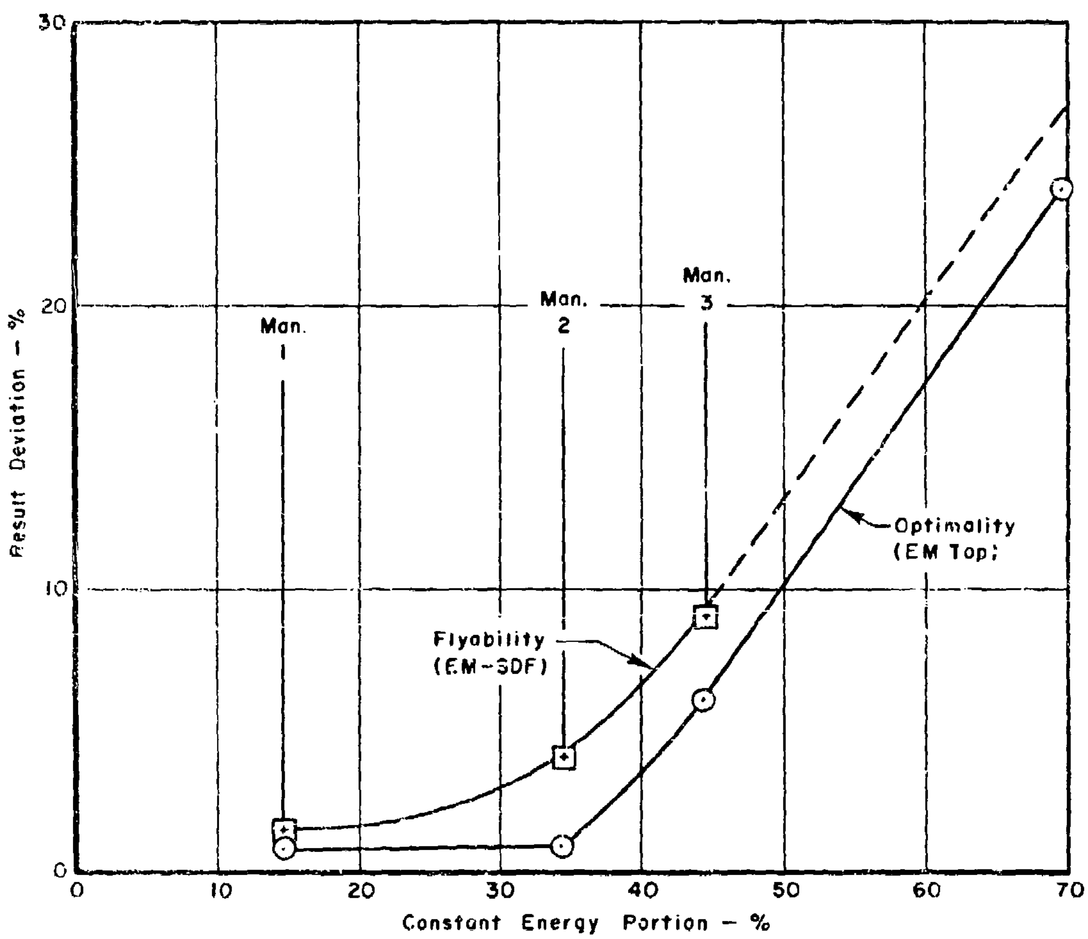

Figure 10. Percent Deviation in EM Results-Category I, Minimum Time Maneuvers

divided by the total path length in the h - M plane. Figure 10 shows that as the percentage of the path involving constant energy exchanges increases, the accuracy of the time predicted by the EM method decreases. The curve is relatively flat and accuracy good where constant energy exchanges are less than 35%. This is followed by a rather steeply rising curve for the higher percentages which, in the limit, approaches 100% result deviation as the constant energy percentage approaches 100%. The curve indicates that for constant energy percentages above 40 to 50% some type of correction or modification to the path is necessary to maintain a reasonable level of accuracy or optimality in the final results.

2. MINIMUM FUEL PATHS

a. Throttle Switching

Contours of maneuvering energy (E_m) are defined by:

$$E_m = (P_s/\dot{w}_f)\, w_a \tag{25}$$

These contours can be calculated in a similar manner as the P_s contours. The minimum fuel path can be computed in an similar manner as the minimum time path discussed previously. Contour plots of E_m and the resultant Rutowski paths for the F-4C are shown in Figures 11, 12, and 13 for maximum A/B, minimum A/B and military power respectively. As the objective of the EM minimum fuel path is to minimize the integral of Equation (10), this is equivalent to minimizing dw/dE_s or $\dot{w}_f/P_s$ along the path. Figure 14 shows the results of plotting $\dot{w}_f/P_s$ versus energy level along 1 g Rutowski minimum fuel paths for the three throttle settings. The energy levels where it is beneficial to change the throttle setting, as shown in the figure, are obtained by noting the crossing points of the curves to minimize the net area under the curve. This approach could be extended to more discrete throttle settings or a continuous throttle variation. However, this study considered only the three listed above. The curve shows a smooth increase in throttle setting with increasing specific energy level changing from military power to minimum A/B at E_s = 32,000 feet and from minimum A/B to maximum A/B at E_s = 52,000 feet.

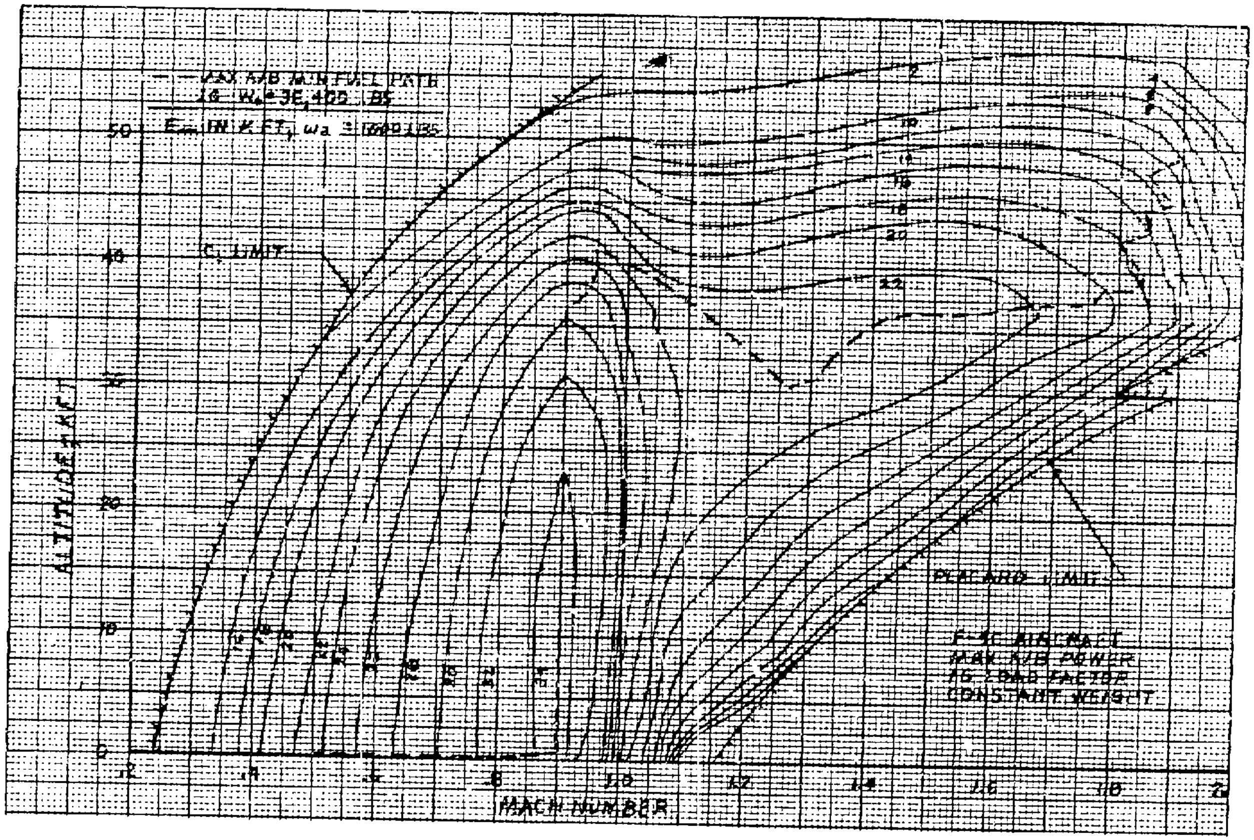

Figure 11. Maneuvering Energy (E_m) Contour Map-Maximum A/B Power

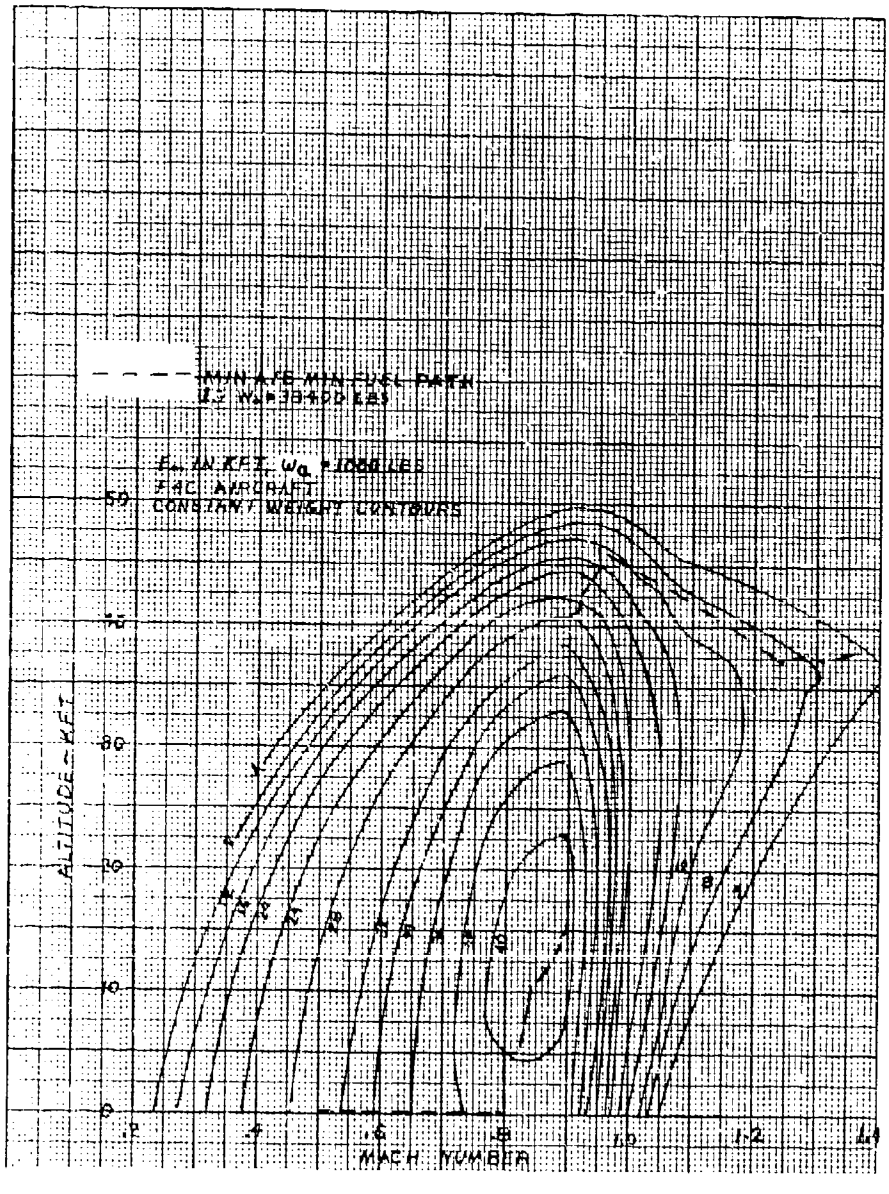

Figure 12. Maneuvering Energy (E_m) Contour Map- Minimum A/B Power

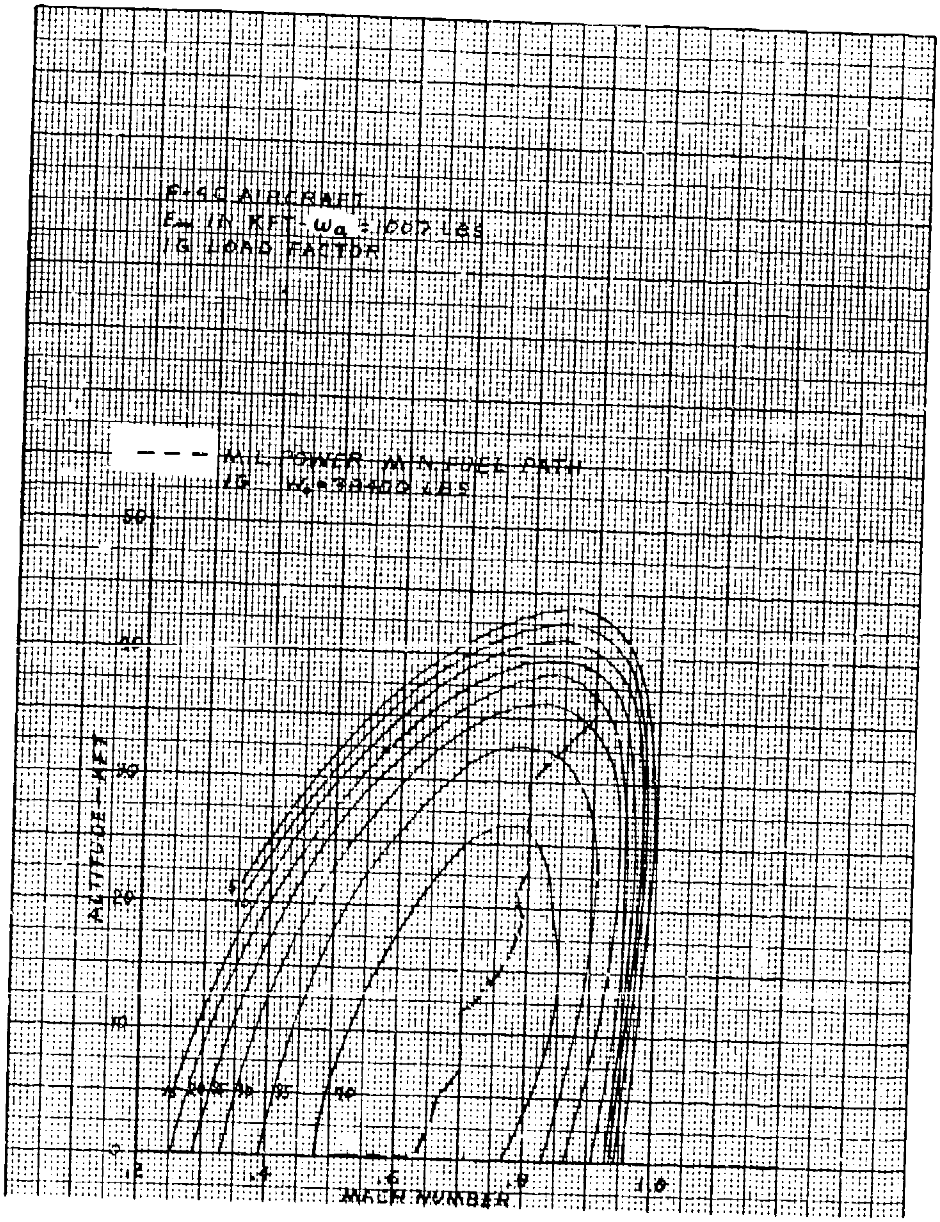

Figure 13. Maneuvering Energy (E_m) Contour Map-Military Power

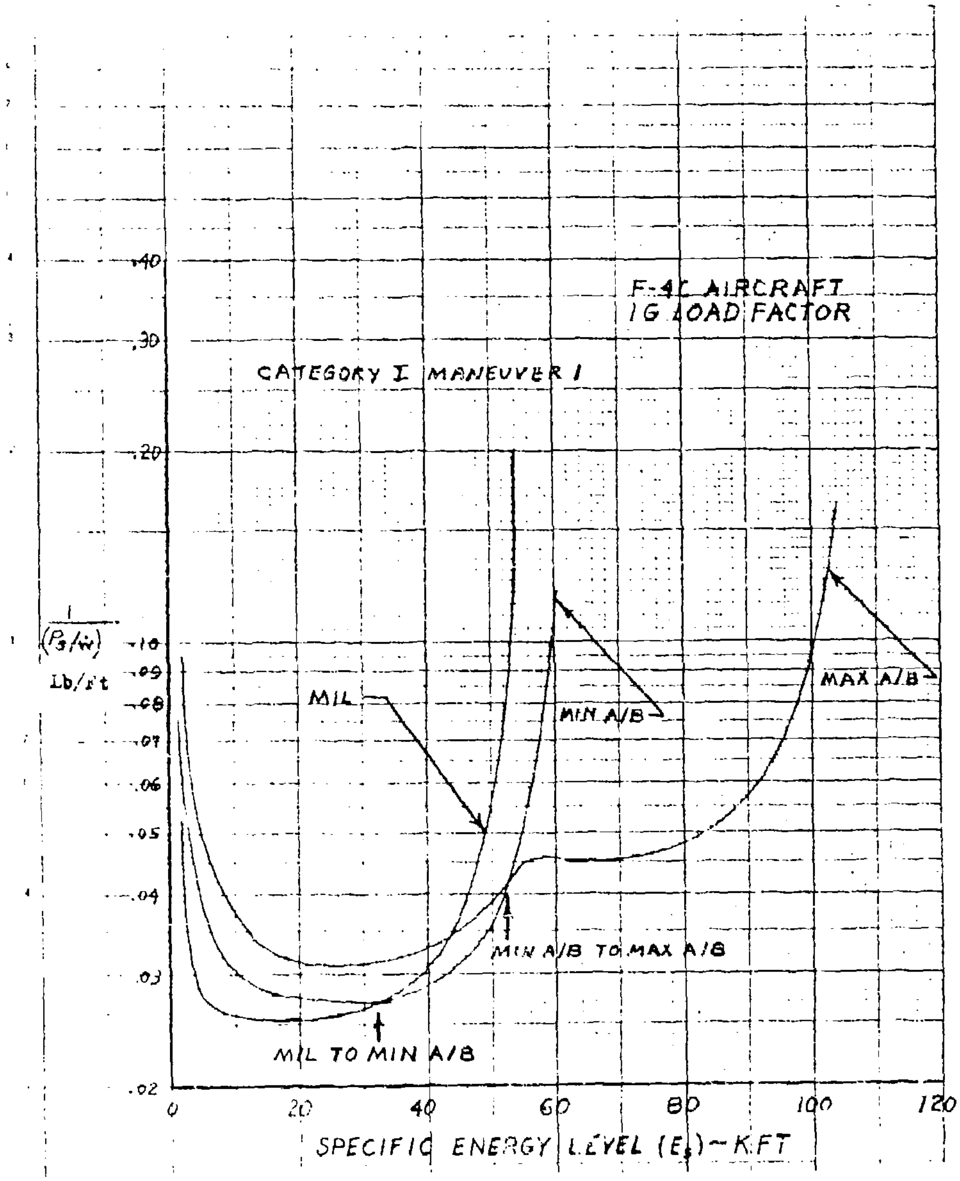

Figure 14. Throttle Efficiencies Along Minimum Fuel Paths

If the contours shown in Figures 11, 12, and 13 are overlayed on each other, contours of relative advantage, or best $P_s/\dot{w}_f$, of one throttle setting over another at a 1 g load factor may be generated. Figure 15 indicates the regions of relative advantage obtained by the procedure. For example, in the region marked MIL POWER the $P_s/\dot{w}_f$ values are higher in military power than they would be in either of the other two throttle settings. This type of plot would indicate to a pilot the regions in which the various power settings are most efficient in terms of fuel required for increasing energy. The dotted path shown in the figure is the Rutowski path for maximum A/B power and the dashed line represents the three throttle minimum fuel path obtained as described in the previous paragraph.

The figure shows the three throttle path contains constant energy transitions at the two throttle switching points. In the SDF simulation of the EM paths the dives were started slightly before the switching energy level and the throttle was changed at the boundaries of the advantage regions of Figure 15 rather than at the energy levels of Figure 14. For the basic EM computation these constant energy transitions at the switching points take place in zero time so the throttle increment points were at the energy level as indicated by the dashed path.

b. Maneuver Results

Figures 16, 17, and 18 show the paths for Maneuvers 1, 2, and 3 and the end results are tabulated in Table I. In Maneuver 1, Figure 16, the SDF and EM paths are close together with the biggest differences occuring at the throttle switching points and in the transonic constant energy segment. For comparison the maximum A/B path is also shown on the figure. The EM max A/B path indicates 4687 pounds of fuel required while the three throttle path indicated 4294 lbs, a saving of 393 pounds or over 8% for the varying throttle analysis. The total time required increased considerably from 366 seconds for the max A/B to 538 seconds for the three throttle method. Comparing the SDF to the EM three throttle path the EM fuel prediction was 284 lbs or 6.2% optimistic, the time required for the paths was 538 and 539 seconds respectively.

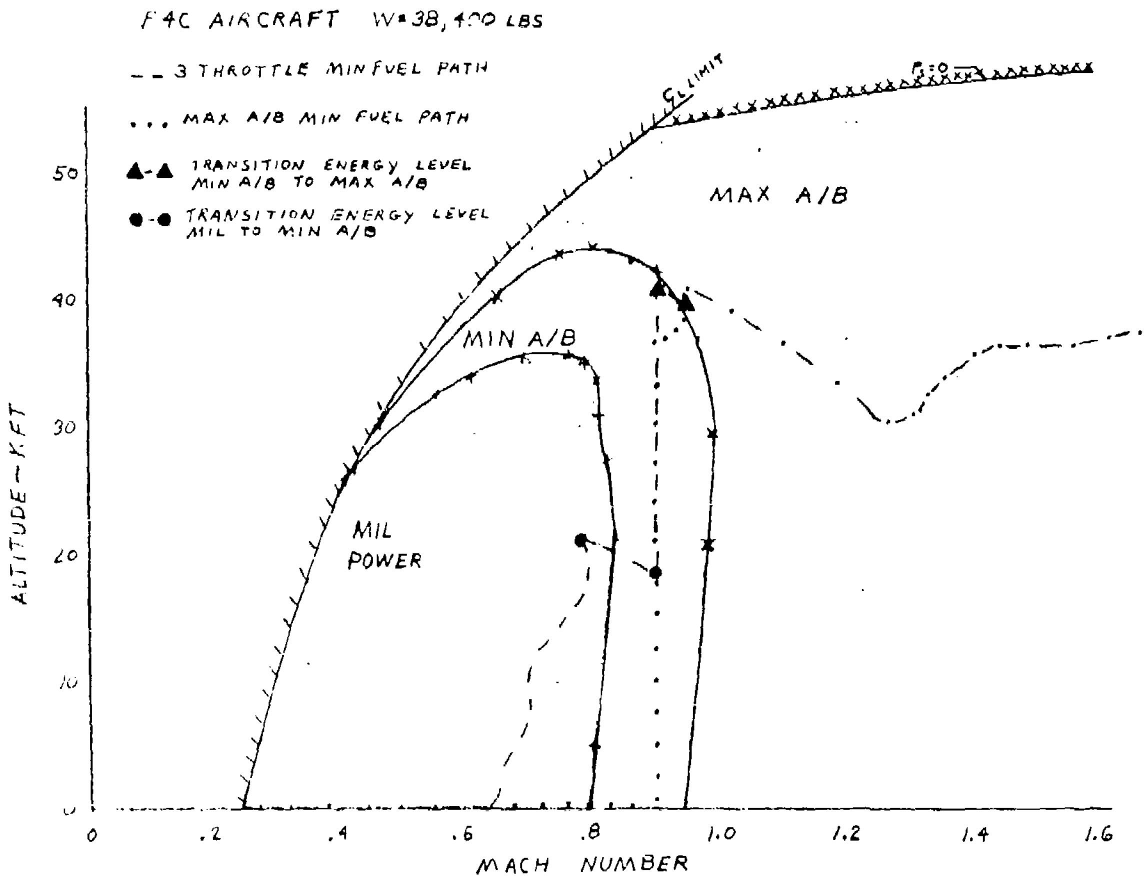

Figure 15. Regions for Best Fuel Efficiency-3 Throttle Analysis

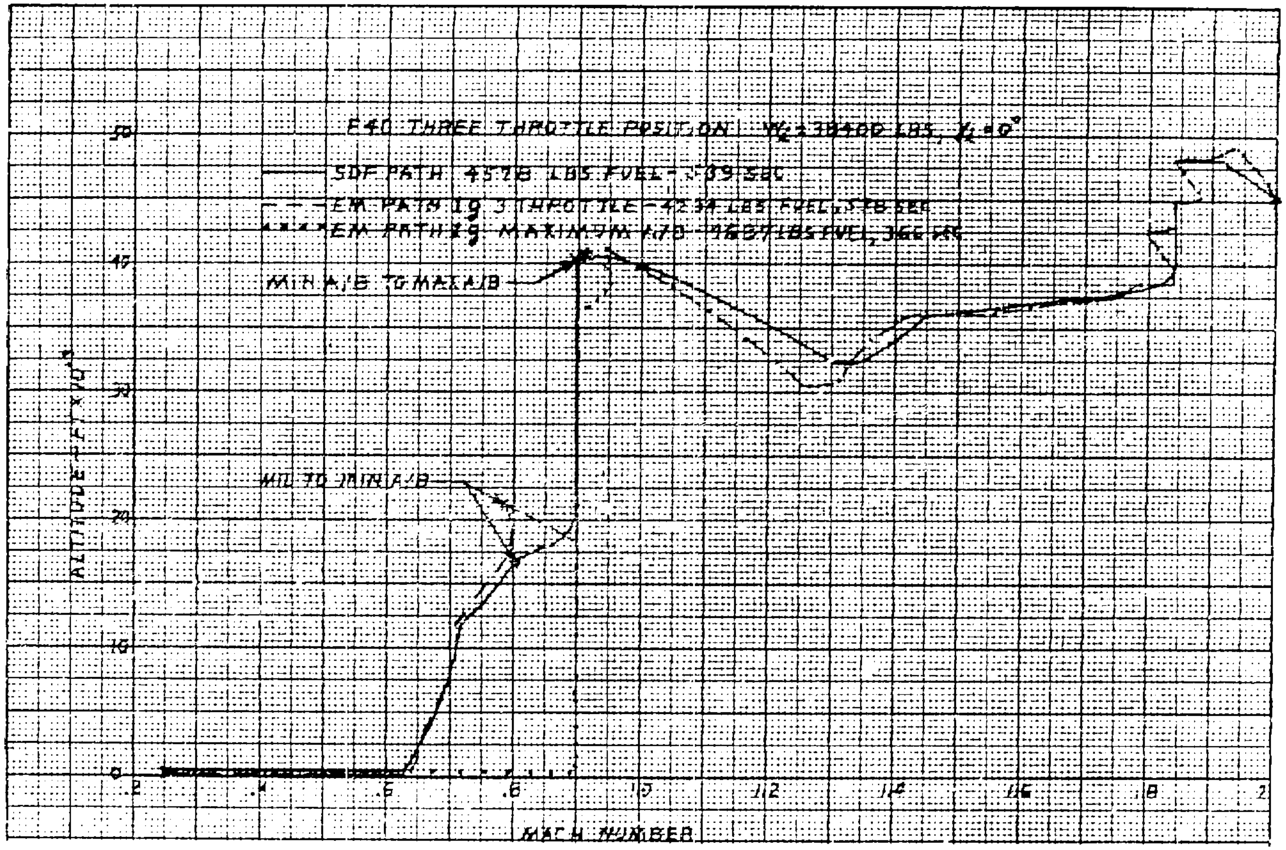

Figure 16. Category I. Maneuver 1, Minimum Fuel Path-EM and SDF

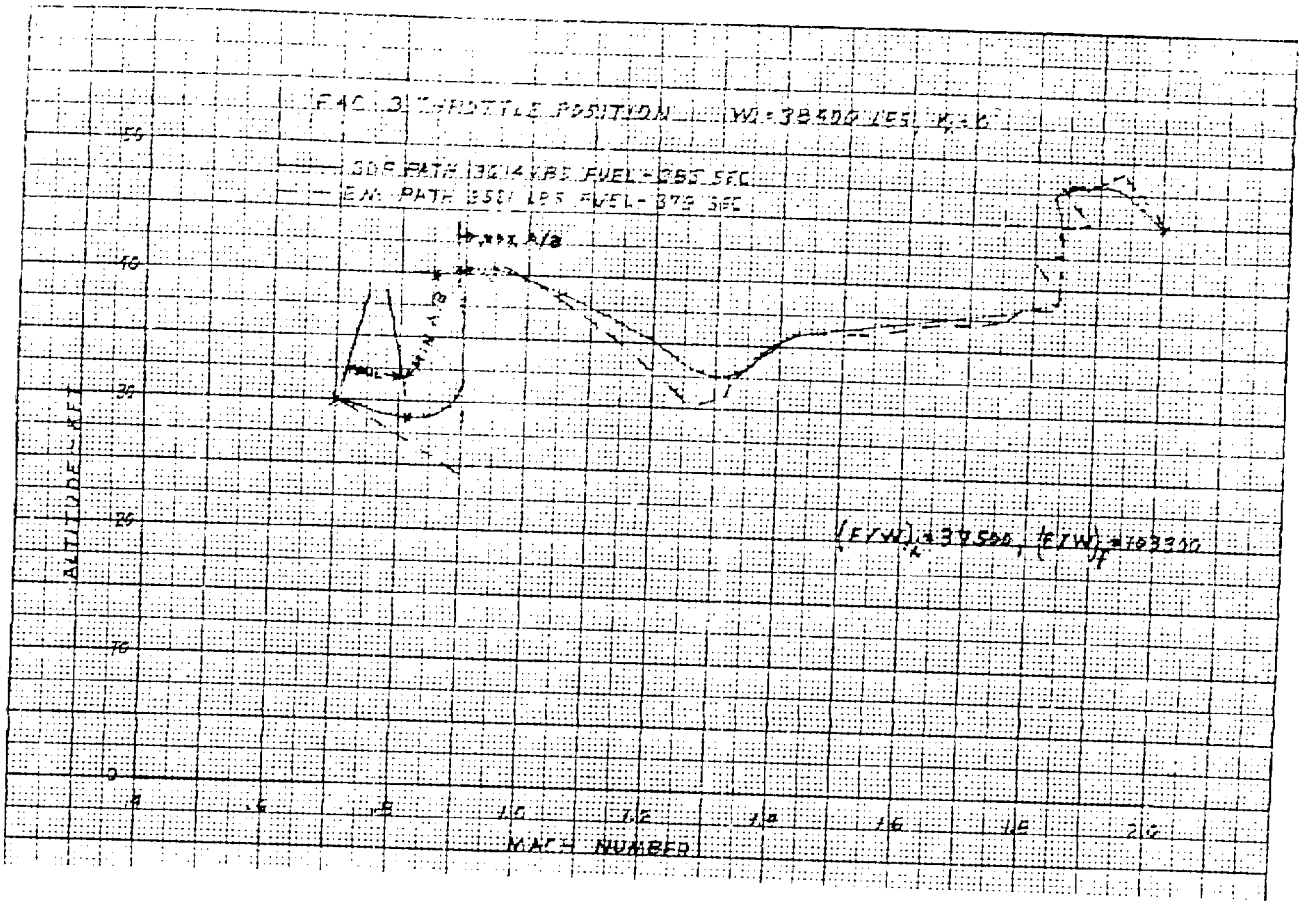

Figure 17. Category I, Maneuver 2, Minimum Fuel Path-EM and SDF

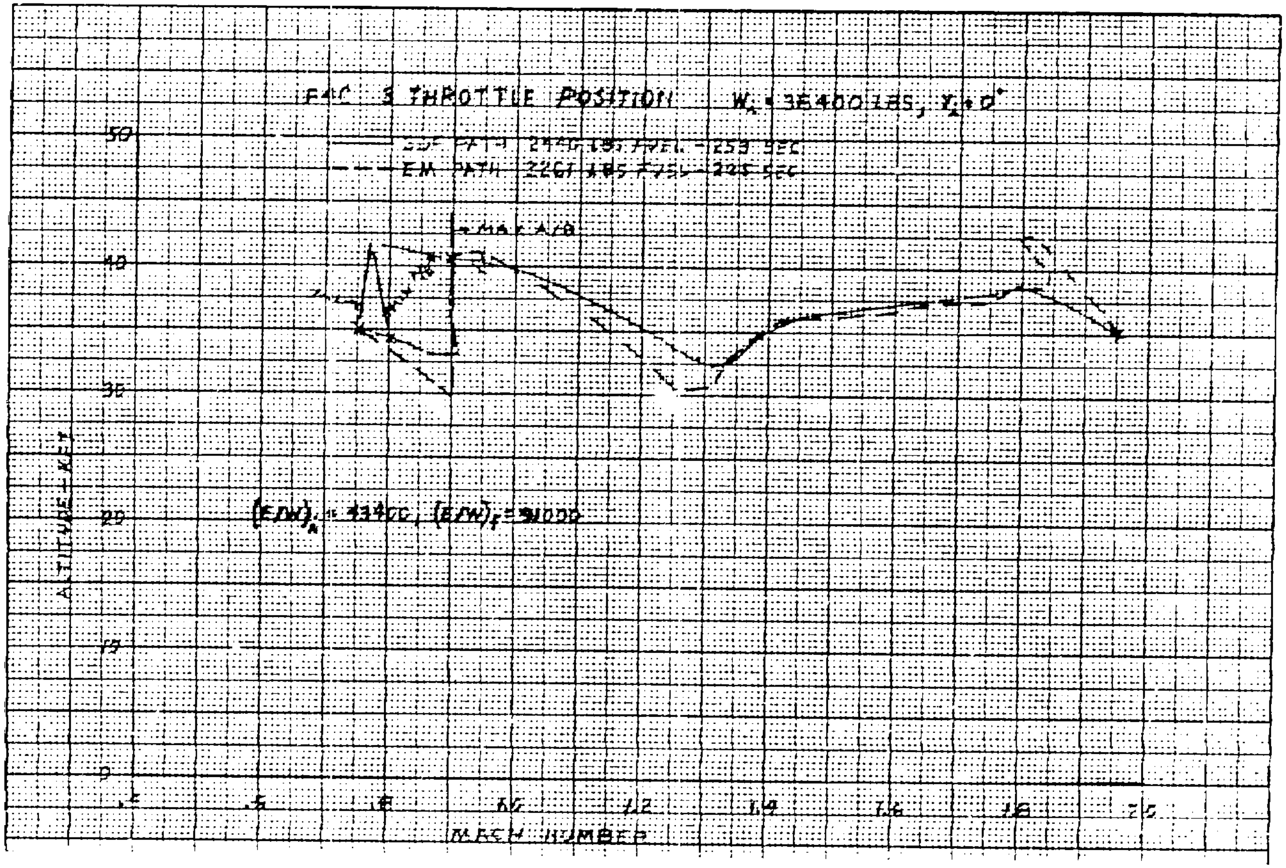

Figure 18. Category I, Maneuver 3, Minimum Fuel Path-EM and SDF

The EM and SDF Maneuver 2 paths are shown on Figure 17. Since the initial conditions fall in an area of Figure 15 where MIL power is more efficient the SDF simulation employed all three throttle settings as indicated on the figure while the EM used only min and max A/B. Again agreement in the shape of the paths was good except in the throttle switching and transonic areas. Good agreement was obtained in the fuel required with the EM being 93 pounds or 2.6% optimistic. The time for the SDF path was 6 seconds longer than the EM due probably to the initial military power segment.

The Maneuver 3 paths are shown in Figure 18 with good path agreement except in the constant energy portions. As in Maneuver 2 the SDF path includes a short initial military power segment while the EM path does not. As in the minimum time case this maneuver showed the poorest correlation of results of the three basic maneuvers with the EM being 179 pounds or 7.9% optimistic. Again the SDF path was longer, taking 28 more seconds than the EM.

c. Discussion

Use of a variable throttle showed a definite advantage over using only maximum A/B power for obtaining the minimum fuel path for all three maneuvers studied. The EM technique shows reasonable accuracy where constant energy exchanges between kinetic and potential comprise less than 50% as shown in the following table.

Maneuver	Constant E/W percentage	%difference in fuel EM - SDF
1	23	6.6
2	37	2.6
3	47	7.9

As in the minimum time case, it is known that in a pure energy exchange the EM technique indicates 0 lbs fuel required or a 100% error. Therefore, again some correction to the EM technique would be necessary for higher constant E/W percentages to achieve reasonable accuracies. The path differences between the EM and SDF were not as pronounced in the minimum fuel paths as they were in the minimum time cases due to the higher altitudes, lower flight path angles, and lower accelerations during the climb portions of the path.

3. ZOOM MANEUVER

The final vertical plane maneuver selected was a zoom to maximize altitude. Figure 1 indicates that the highest energy level that can be obtained within the steady state operating envelope is about 107,000 feet. In a pure energy exchange, converting all the energy to potential energy, 107,000 feet would, therefore, be the forecast maximum altitude. This forecast does not account for any losses in the pull-up to initiate the zoom nor does it indicate the best pull-up angle to employ. Straight up or $\gamma = 90°$ would be required to convert all the kinetic energy to potential energy. The starting conditions selected for this problem were Mach = 2, 50,000 feet altitude, and level flight ($E_s \approx$ 107,000 feet).

To investigate how much of the energy could be converted to altitude and find the most efficient means of doing it, the problem was first run on the TOP program. The resulting path is shown in Figure 19 and shows a rather surprising result. The TOP solution is a dive with a pullout at the engine placard limits before starting the actual zoom. The maximum load factor for the path was 4 g's occuring at the pull-up altitude. Maximum altitude attained was 92,000 feet leaving about 2,000 feet of kinetic energy uncoverted.

A series of immediate pull-up zooms was then run on the SDF program to investigate the effect of a pull-up in lieu of the the initial dive generated by the TOP program. Constant, increasing, and decreasing load factors were used. The highest altitude attained was 88,680 feet using a decreasing load factor schedule with an initial value of 2.9.

Finally the rule of thumb correction for energy exchanges of Equation (22) was applied to the starting conditions. This correction, as noted earlier, provided a set amount of energy loss for each foot of climb altitude. The result of this calculation produces the path shown in Figure 19 as "EM modified" and provides a forecast of 85,000 feet for the maximum altitude.

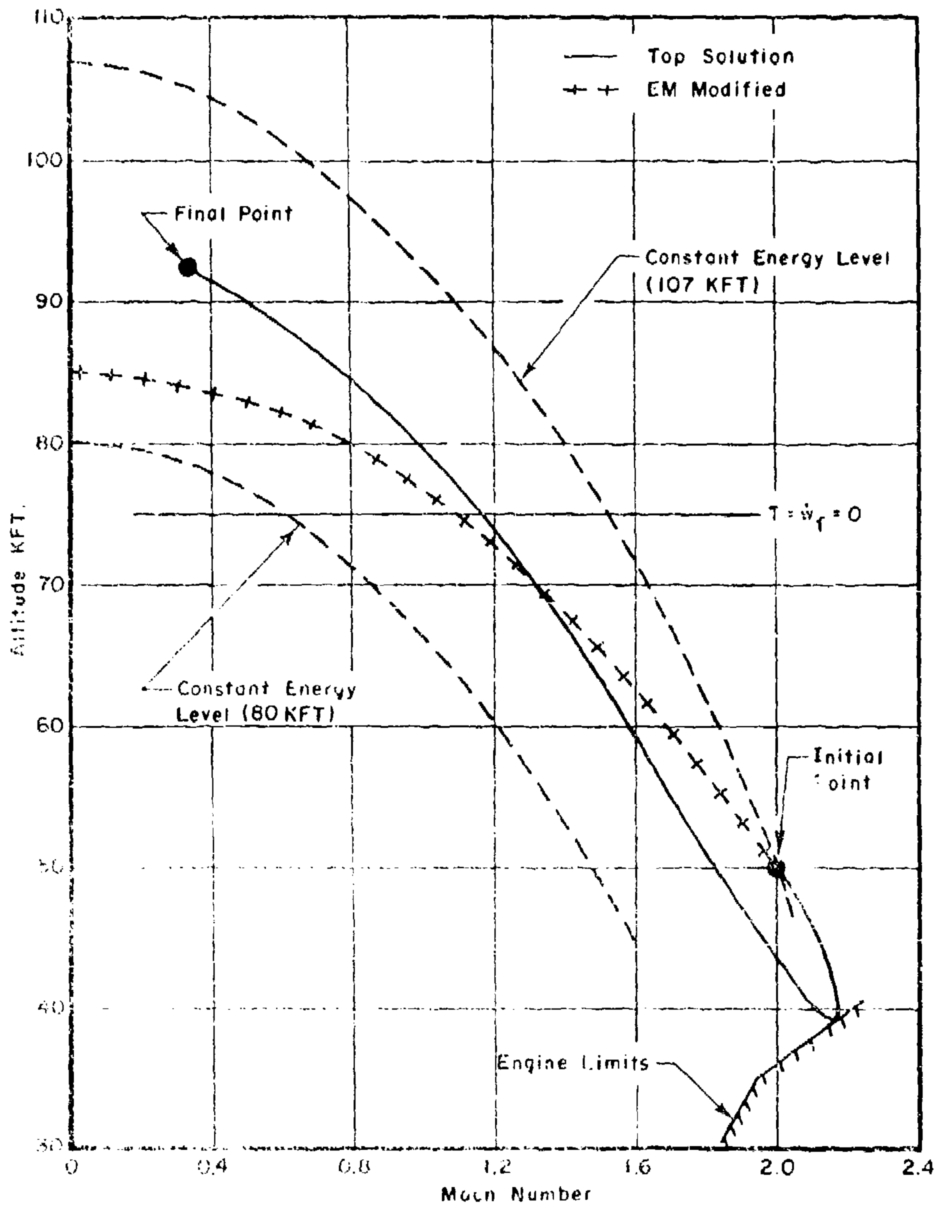

Figure 19. Zoom Maneuver to Maximize Altitude

A comparison of the results of the above calculations shows that a 107,000 foot altitude (all energy converted to potential) is not achievable. Application of the rule of thumb gives too low an estimate and this rule provides a set amount of loss for each foot of climb whereas the integrated programs (TOP and SDF) indicate most of the losses are in the first 10,000 feet of the zoom. The optimized path produced by the TOP program showed that, in this instance, an increase in final altitude would be achieved by an initial diving maneuver before starting the zoom.

SECTION IV

OUT OF PLANE MANEUVERS

The initial and final conditions for the two maneuvers selected are shown in columns 2 and 3 of Table III, both maneuvers are climb accelerations with a 180° turn. Column 4 shows the method used and columns 5 and 6 the results which will be discussed in the following paragraphs.

1. MINIMUM TIME PATHS

a. Turning Charts

Implementing the method outlined in Section II, Equation (17) was plotted for selected values of load factor and is shown in Figure 20. Rutowski paths for the F-4C at maximum A/B power were then run for the two maneuvers at 1g and the load factors plotted on Figure 20. Breaking the constant load factor paths down into ΔE/W increments of 5000 feet each, d ψ/dt was then read from Figure 20 for the average velocity between energy levels. Table IV shows a sample calculation for a load factor of 1.5 - column 1 shows the mean specific energy level of the segments (here line 1 is the average between E/W = 5000 and 10,000) column 2 is the average velocity of the Rutowski path between the initial and final energy level of the segment (line 1 shows 670 fps) and column 3 shows the turning rate as read from Figure 20 (reading Figure 20 for line 1 the load factor 1.5 line intersects a velocity of 670 fps at a turning rate of 3.05 deg/sec). $\Delta\psi$ is calculated by Equation (18) as follows: the time required to go between the energy levels along a 1.5 g Rutowski path is shown in column 4 (line 1 shows a time of 9.64 seconds was required to go between E/W = 5000 and E/W = 10,000), $\Delta\psi$ is then column 3 multiplied by column 4 and is shown in column 5. The time penalty to operate at a load factor of 1.5 instead of 1.0 is calculated by Equation (19) as follows: Column 6 shows the time required to go between energy levels along a 1 g Rutowski path (line 1 shows 9.40 sec) and column 7 the penalty time, column 4 minus column 6, which is to be plotted in Figure 21 (line one shows 9.64 - 9.40 = 0.24 sec). The turning efficiency, Equation (20), is then column 5 divided by column 7 and is shown in column 8 (line 1 shows 29.4/.24 = 122° turn/sec penalty). Figure 21 shows the penalty time for the

TABLE III

CATEGORY II - OUT OF PLANE MANEUVERS

Type	Initial Conditions	Final Conditions	Procedure	Time	Fuel
			Maneuver 1		
Min Time	M = 0.5	M = 1.95	EM-No Turn	219	3954
	h = 500 Ft	h = 35K Ft	EM-turn	220.8	--
	W = 38400 LBS	ψ = 180°	SDF-No Turn	229	4079
	γ = 0°		SDF-Turn	231	4178
	ψ = 0°		TOP-turn	230.8	4056
Min Fuel			EM-No turn	422	3216
			EM-turn	--	3224
			SDF-No turn	428	3352
			SDF-turn	429	3363
			Maneuver 2		
Min Time	M = 0.7	M = 1.95	EM-No turn	228	3720
	h = 30K Ft	h = 45K Ft	EM-turn	234.9	--
	W = 38400 LBS	ψ = 180°	SDF-No turn	240.4	3767
	γ = 0°		SDF-Turn	247.4	3897
	ψ = 0°		TOP-turn	240.3	3810
Min Fuel			EM-No turn	330	3183
			EM-turn	--	3243
			SDF-No turn	355	3301
			SDF-turn	359	3351

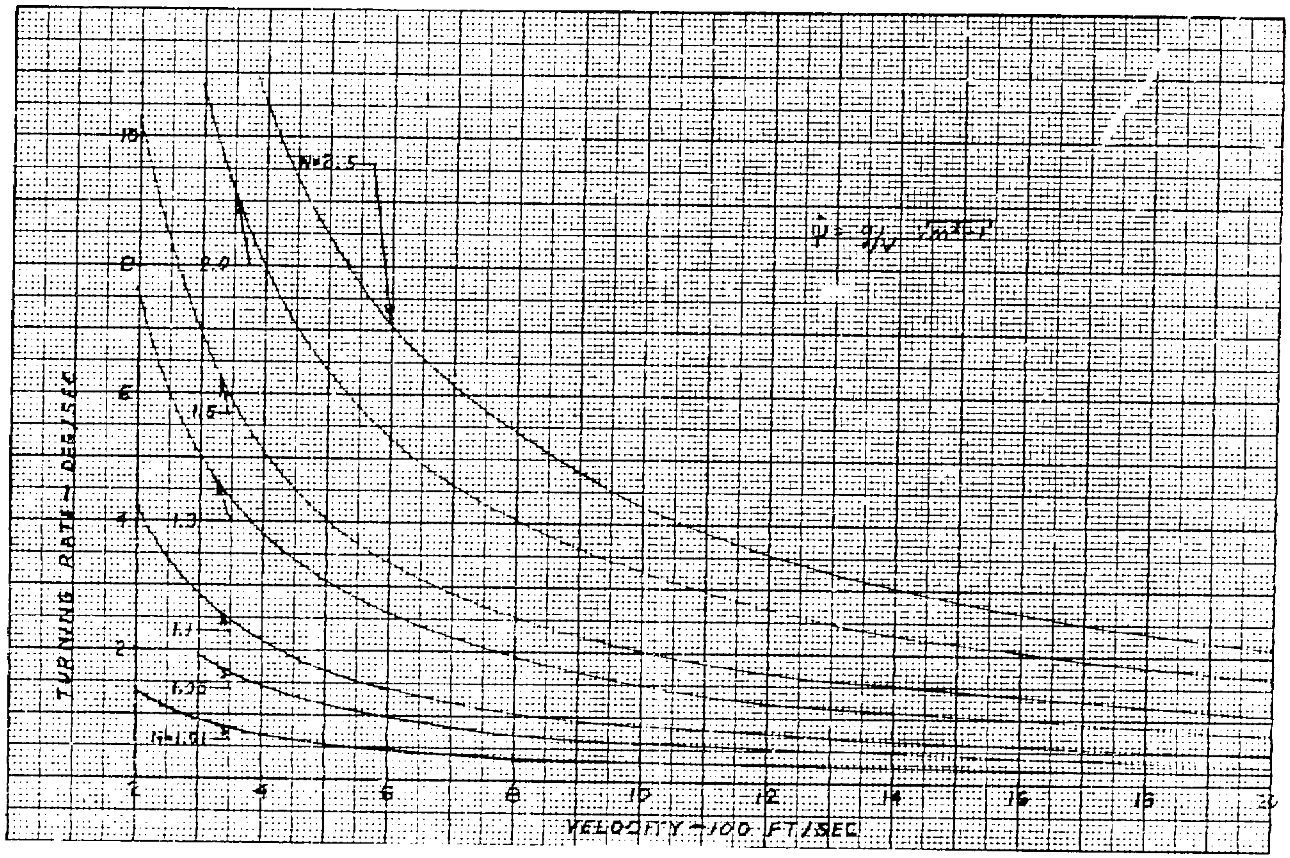

Figure 26. Horizontal Turning Rates

TABLE IV

MINIMUM TIME PATH TURNING EFFICIENCY EXAMPLE - 1.5g

①	②	③	④	⑤	⑥	⑦	⑧
Mean E/W(ft)	Vavg(fps) (from 1.5 g path)	$\Delta\Psi/dt(\frac{deg}{sec})$ (from Fig 20)	Δt 1.5g(Sec) (from 1.5g path)	$\Delta\Psi$(deg) ③x④	Δt 1.0 g(Sec) (from 1.0 path)	Δt_p(Sec) ④-⑥ plotted fig 21	TE$(\frac{deg}{sec})$ ⑤/⑦ plotted fig 22
7500	670	3.05	9.64	29.4	9.40	0.24	122
12500	892	2.35	7.20	16.9	7.10	0.10	169
17500	997	2.02	6.87	13.9	6.77	0.10	139
22500	1020	2.00	7.74	15.5	7.60	0.14	110

Δ E/W increment = 5000 ft

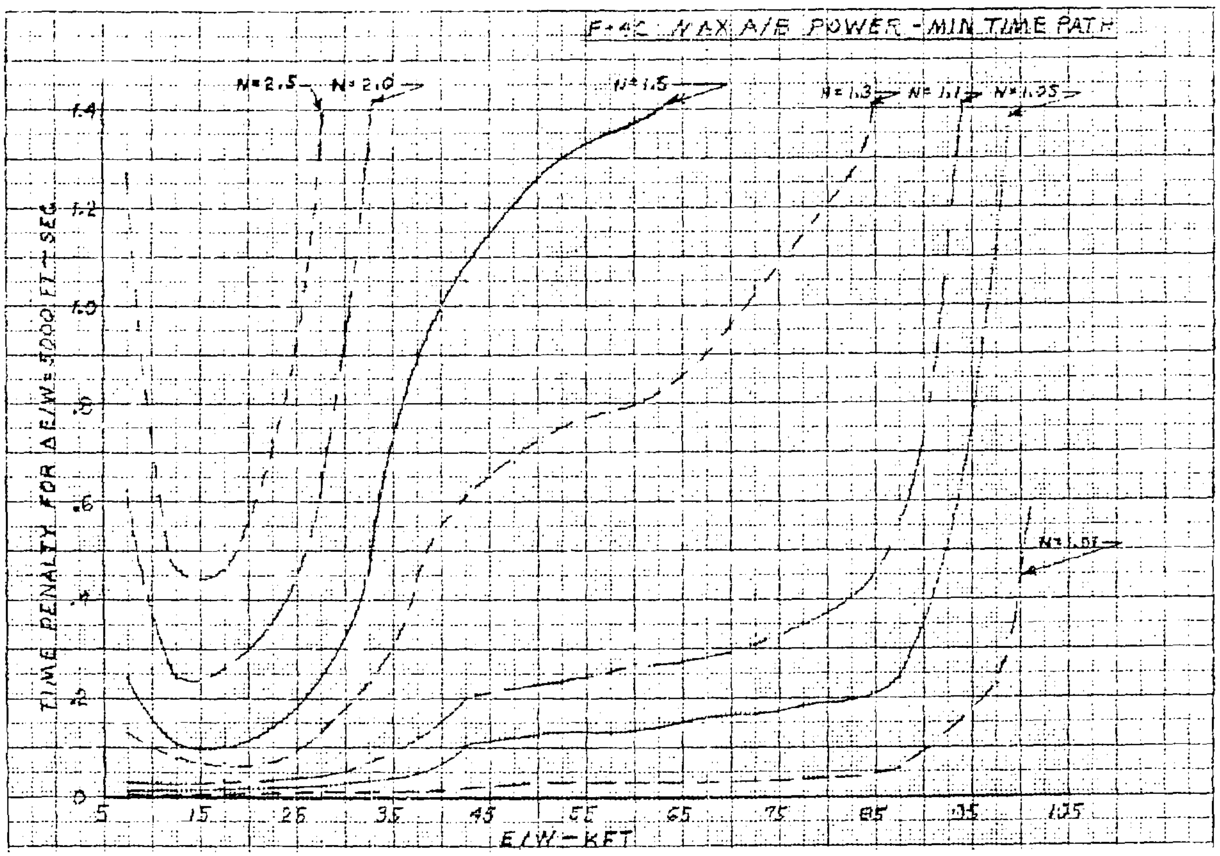

Figure 21. Acceleration Time Penalties Along Minimum Time Paths

series of load factors. For the sample calculation the time penalty of 0.24 seconds is plotted at a load factor of 1.5 and specific energy level of 7500 feet. Figure 22 shows the turning efficiency for the series of load factors. For the sample calculation TE = 122 is plotted at load factor = 1.5 and E/W = 7500 feet.

Figure 22 shows, as would be expected, the highest values of turning efficiency are at low speeds and low load factors. At the very low energy levels there is a reduction in efficiency due to the low level of excess power available for acceleration causing the hump in the curves. The "constant efficiency" turns used correspond to horizontal lines across this figure producing a table of load factor versus energy level. From this load factor table Figure 21 can be read to obtain the penalty time and the amount of turn calculated by Equation 20.

For Maneuver 1 a 180° turn between energy states $(E/W)_i$, = 5,000 feet and $(E/W)_f$ = 91,000 feet is required. To start the calculation a turning efficiency is assumed, say 103°/sec penalty. Figure 22 is then read at the various energy levels with the results shown in Table V, column 2. The turn was considered complete by an (E/W) = 62,500 ft as the load factor dropped below 1.01 g's which was the lowest considered in the analysis. Figure 21 is then read to produce the penalty times for each segment shown in column 3 (line one shows 0.40 seconds penalty for a load factor of 1.73 at an E/W of 7500). The load factors can then be converted to bank angles through the use of Equation 15 and this is shown in column 4 (line 1 shows bank = $\cos^{-1} (^1/1.73)$ = 54.7°). Summing column 3 the total penalty time is 1.74 seconds and the corresponding amount of turn achieved through Equation 20 is 103 x 1.74 - 179.2°. In general the first guess at the turning efficiency would not be this close and a graph such as Figure 23 could be made by plotting the results of several calculations. The solid line shows the amount of turn achieved between specific energy levels of 5,000 and 91,000 feet for varying values of turning efficiency. The dashed line shows the corresponding time penalty. Our sample calculation is shown by the

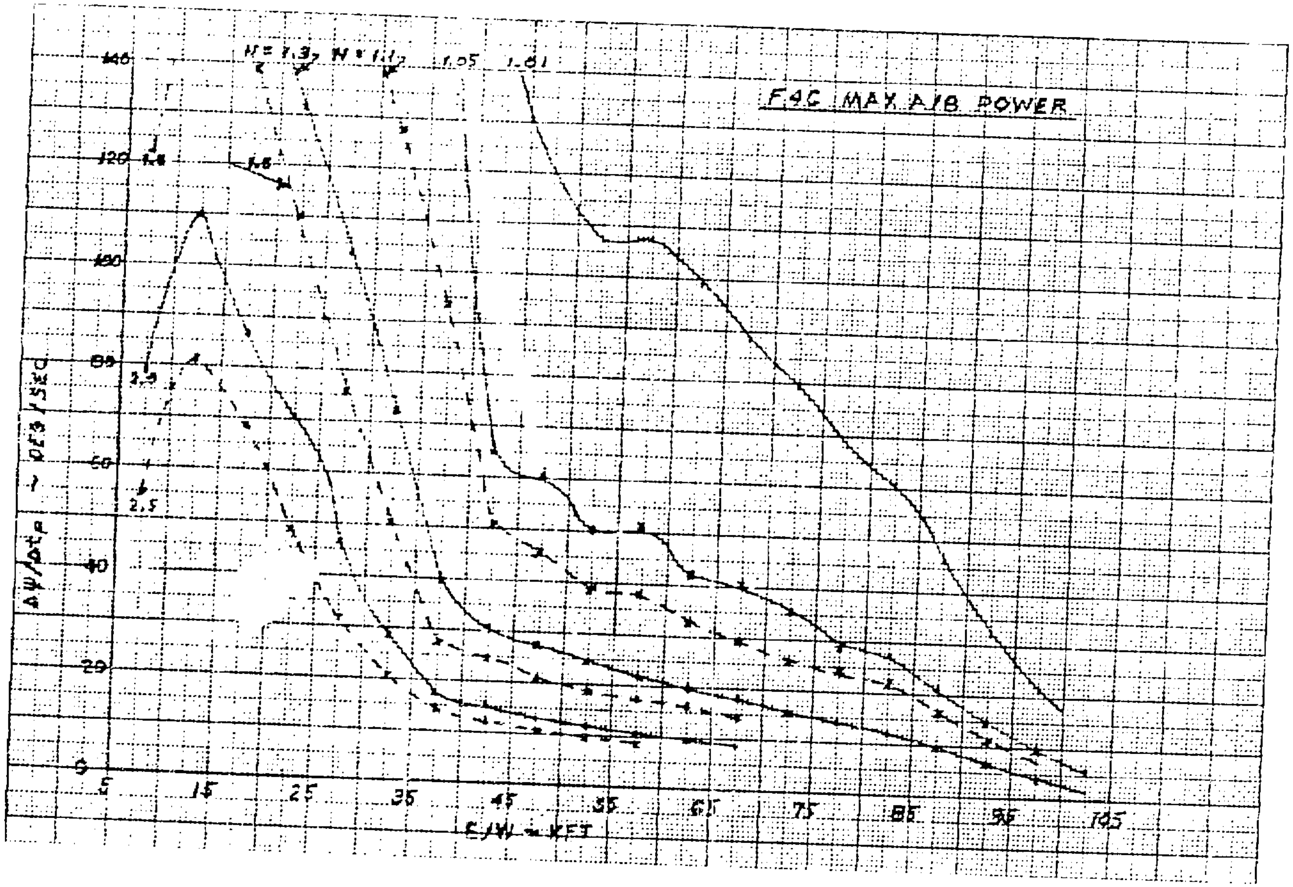

Figure 22. Turning Efficiency Along Minimum Time Paths

TABLE V

MINIMUM TIME CONSTANT EFFICIENCY TURN EXAMPLE

CATEGORY II MANEUVER 1

$\Delta\psi/\Delta t_p$ = 103 deg/sec penalty

Δ E/W increment = 5000 ft.

(1.) Mean E/W (ft)	(2.) n (g's) from fig 22	(3.) t_p (sec) from fig 21	(4.) Bank(deg) Eqn 15
7500	1.73	.40	54.7
12500	2.10	.31	61.5
17500	1.85	.20	57.3
22500	1.58	.19	50.7
27500	1.3	.17	39.7
32500	1.19	.15	32.9
37500	1.085	.10	22.9
42500	1.033	.07	14.8
47500	1.022	.06	12.1
52500	1.014	.04	10.0
57500	1.011	.03	8.9
62500	1.009	.02	7.7

Total Δt_p = 1.74 sec

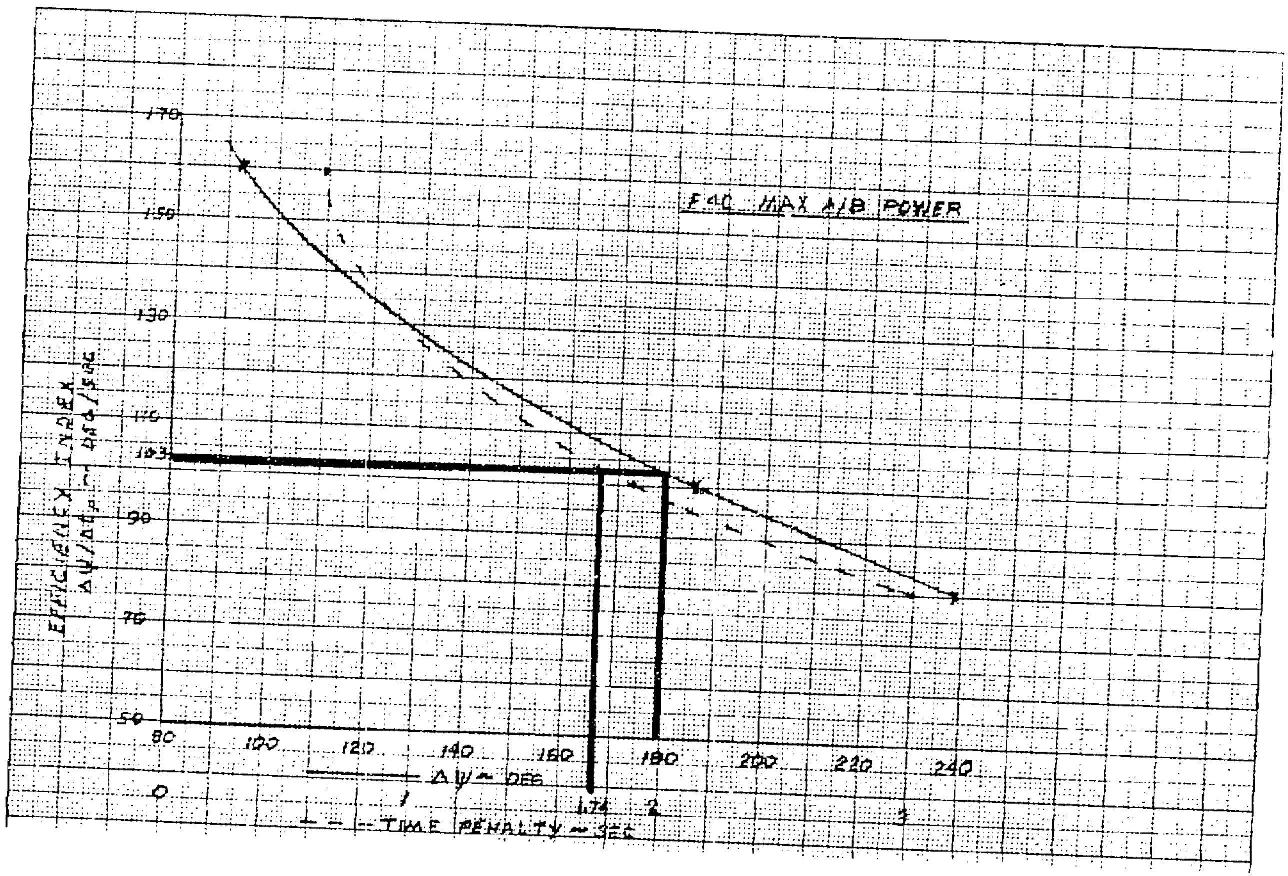

Figure 23. Category II, Maneuver 1, Minimum Time Turning Efficiency

heavy lines. The family of bank curves generated by varying values of turning efficiency for this maneuver is shown in Figure 24.

b. Flyability and Accuracy

The procedure used to investigate the accuracy and flyability of these maneuvers was the same as the method of the previous section, with the addition of the bank angle. The SDF program was used first to simulate the no turn (zero bank) Rutowski path and then the bank angle arrived at by the EM turning procedure was inserted as a function of time and the SDF program run again using the same angle of attack control. Figure 25 shows the resultant paths for Maneuver 1 and the results are tabulated in columns 5 and 6 of Table III. The EM path with no turn was 219 seconds, with turn 220.8 seconds. The SDF path with no turn was 229 seconds and with the turn 231 seconds. This result indicates that the turning penalty calculated is a realistic value and also that the EM path underpredicted the time required for the maneuver by 10.2 seconds or 4%. The EM path underpredicted the fuel required on the no turn path by 97 lbs or 2%. The point where the 180° turn was completed on the SDF program, marked on Figures 24 and 25, was reached slightly before the 1.01 g load factor prediction point. At this point the bank angle in the SDF program was set to zero for the rest of the path.

The calculations for Maneuver 2, which starts and ends at a higher energy level, were done in the same manner as Maneuver 1 using Figures 20, 21, and 22. The family of curves shown in Figure 26 were generated for this maneuver. As shown in the figure the efficiency index for a 180° turn was down to 26° turn/second penalty in contrast to the efficiency of 103° turn/second used in Maneuver 1. This decrease in turning efficiency is due to the much higher starting energy level for Maneuver 2. Note in Figure 26 that an efficiency of 100 for this maneuver produces a turn of only 40° heading change. For the SDF simulation when the minimum bank of 7.5° was reached this value was held until the turn was completed. The bank angle was then set to zero for the rest of the path. The path results are shown in Figure 27 and tabulation in Table III. The EM path with no turn was 228 seconds and with the turn correction 234.9 seconds; a penalty of 180/26 or 6.9 seconds. The SDF path for the maneuver with no turn took 240.4 seconds, with the bank schedule for the turn

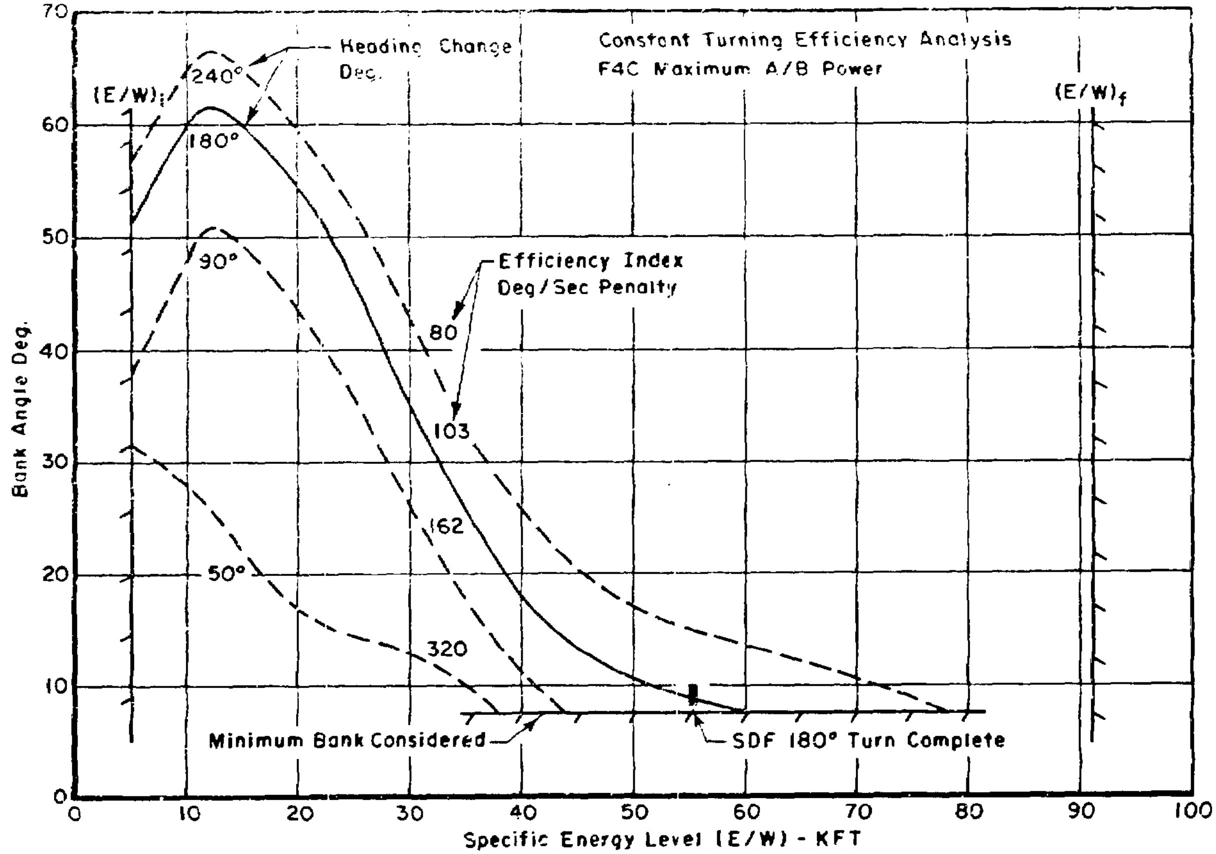

Figure 24. Category II, Maneuver 1, Minimum Time-Bank Schedule Family

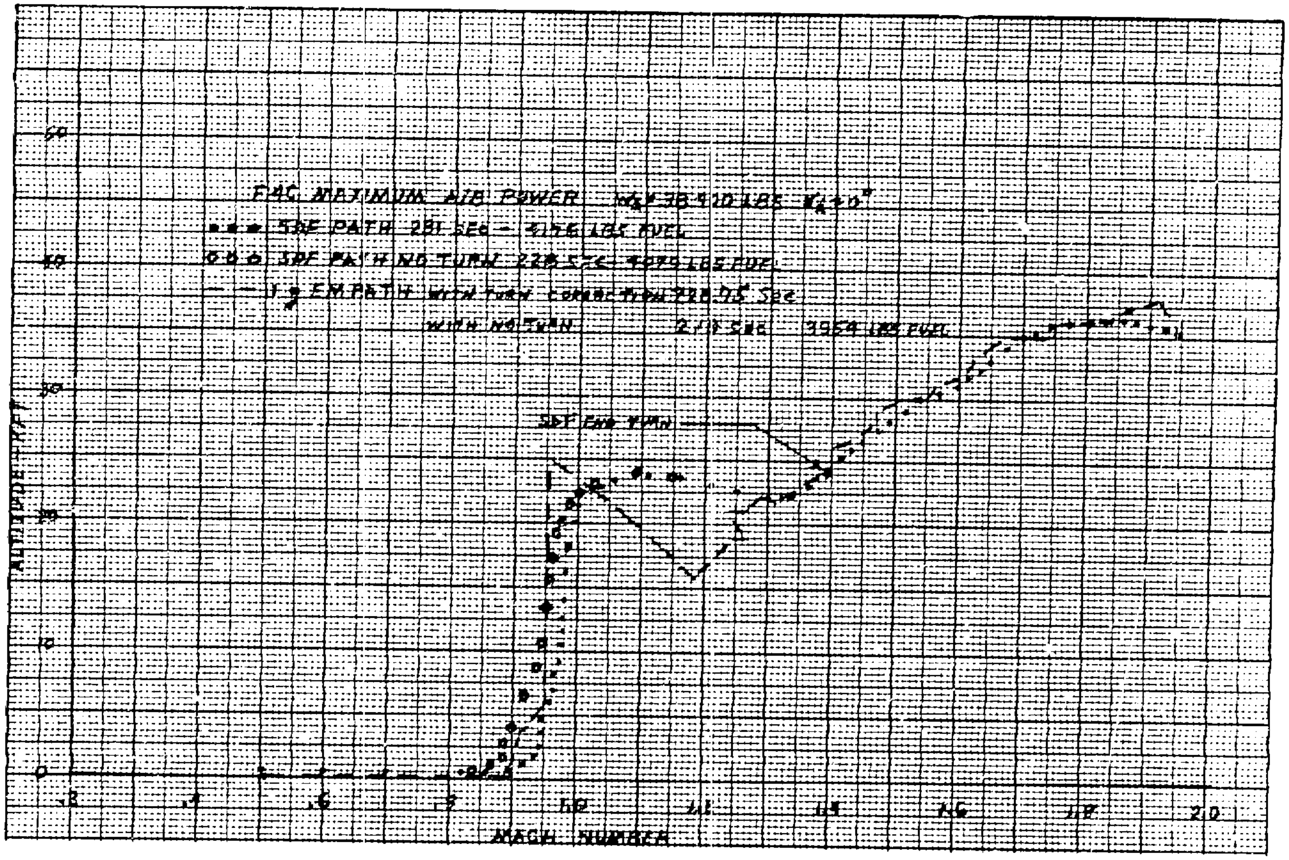

Figure 25. Category II, Maneuver 1, Minimum Time Path-EM and SDF

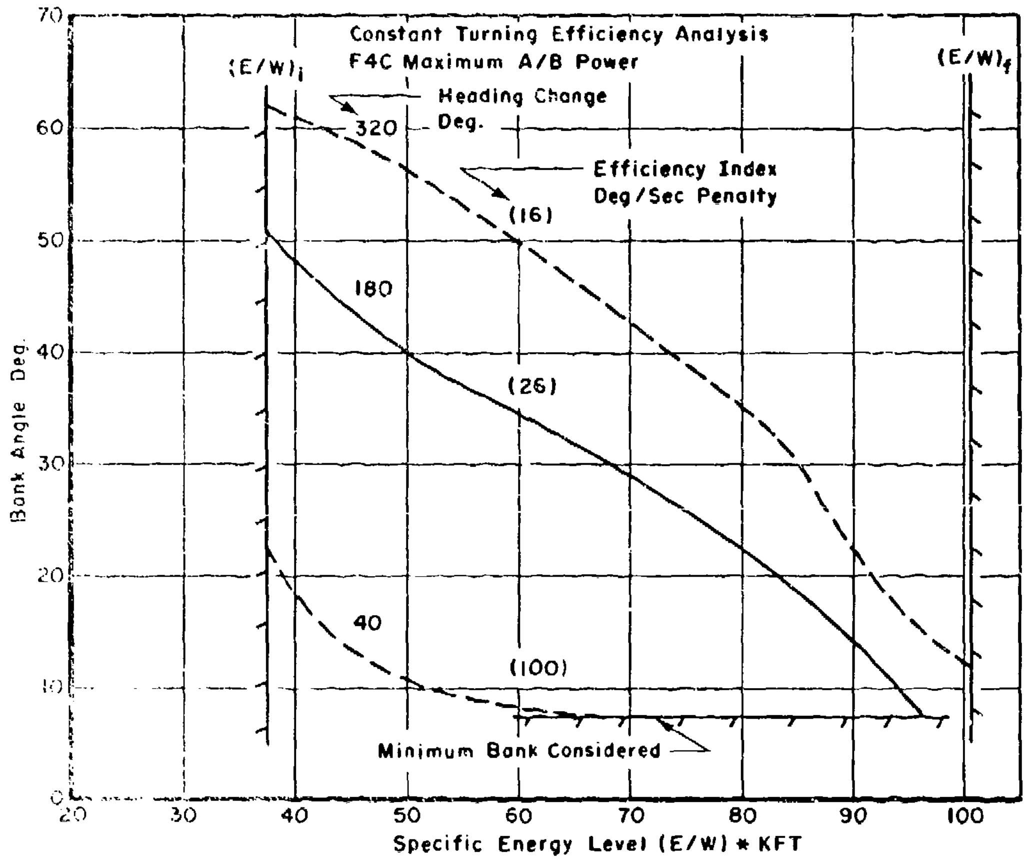

Figure 26. Category II, Maneuver 2, Minimum Time-Bank Schedule Family

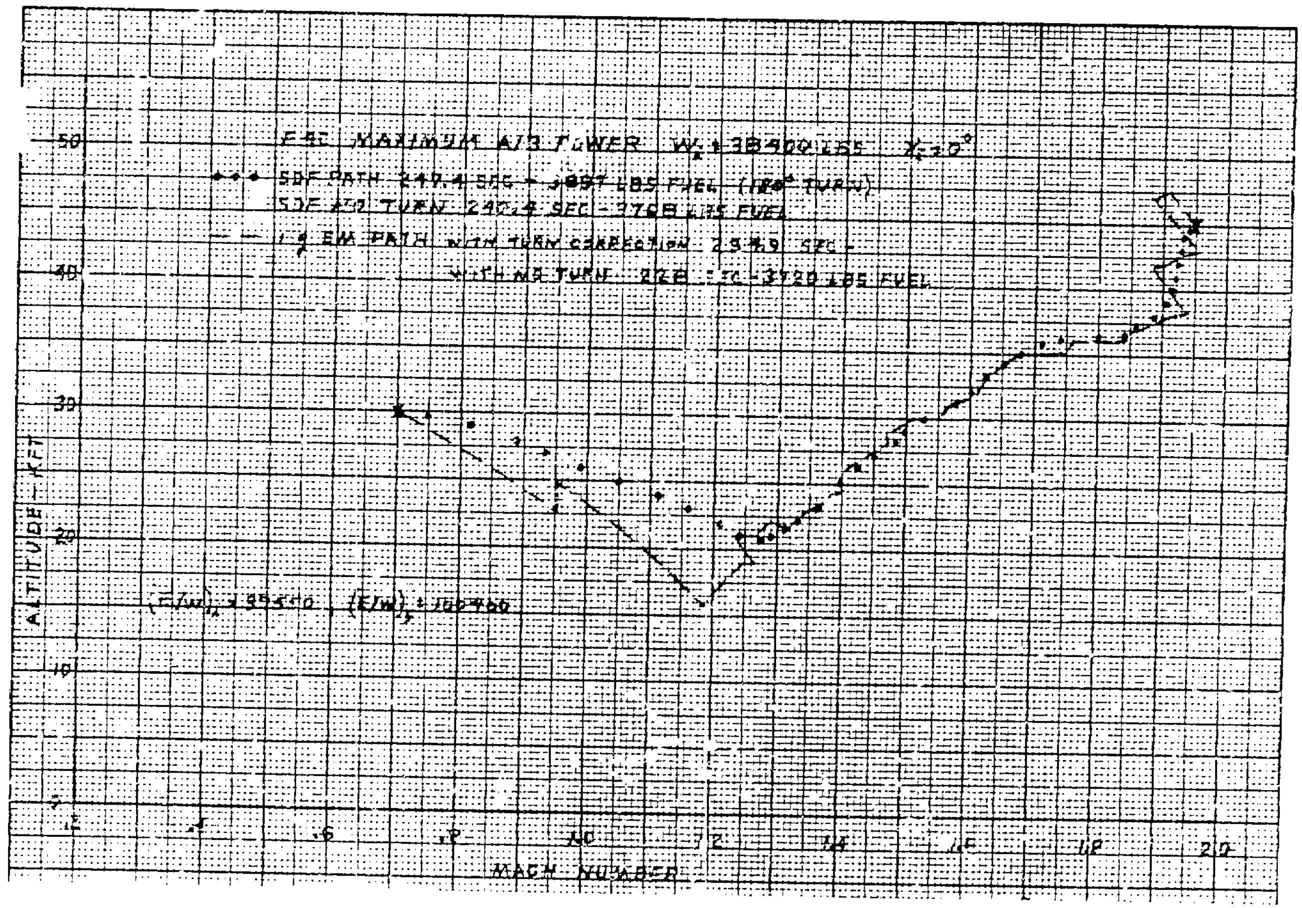

Figure 27. Category II, Maneuver 2, Minimum Time Path-FM and SDF

247.4 seconds, and a turning time penalty of 7 seconds. As in Maneuver 1 the turning penalty calculated by the use of the charts (6.9 seconds) appears to be a reasonable value when compared to the SDF result of 7 seconds. Again the EM path underpredicted the total time required for the maneuver this time by 12.5 seconds, or 5%. The fuel required on the no turn case again showed good agreement with the EM procedure, underpredicting by 48 lbs or a little more than 1%.

c. Optimality

To investigate how closely the flight paths and results of the turning procedure approach the optimum minimum time maneuvers these maneuvers were run on the TOP program. The results are labeled "TOP" in Table III. The solid line in Figure 28 shows the TOP path for Maneuver 1 and the dashed line shows the EM path for comparison. As in Category I, Maneuver 1, the principle differences in the paths in the h - M plane are that the TOP path performs a supersonic climb while the EM is subsonic and that the TOP path does a pushover and almost level acceleration between 1.1 $<$ M $<$ 1.4 while the EM path shows a pronounced dive and pullup. The EM path with turn correction as noted previously predicted 220.8 seconds while the constrained optimum as computed by the TOP program was 230.8, the EM underpredicting the time required by 10 seconds or 4%. Figure 29 shows a bank angle comparison between the two methods. Although the bank schedules are considerably different, in both cases nearly all of the turn is completed in the first 80 seconds of the maneuver.

The solid line in Figure 30 shows the TOP path for Maneuver 2 and the dashed line the EM path for comparison. The principle difference in the paths in the h-M plane, as in Category I Maneuver 2, was in the initial diving portions. The EM path with turn correction predicted 234.9 seconds while the optimum as computed by TOP was 240.3 seconds, the EM underpredicting the optimum time by 5.4 seconds or 2%. Figure 31 shows the bank schedules from the two methods which are again considerably different. The TOP result again shows most of the turn completed in the first 80 seconds while the constant

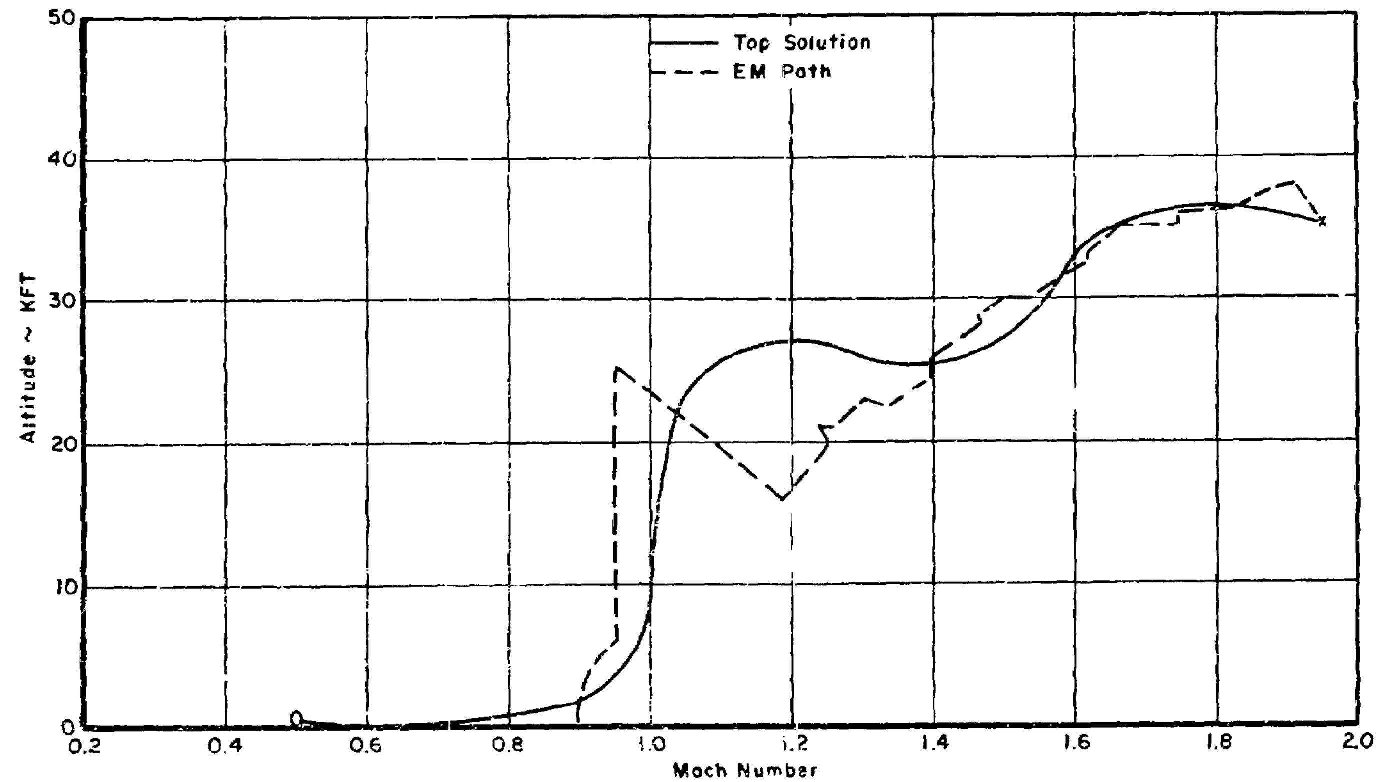

Figure 28. Category II, Maneuver 1, Minimum Time Path-EM and TOP

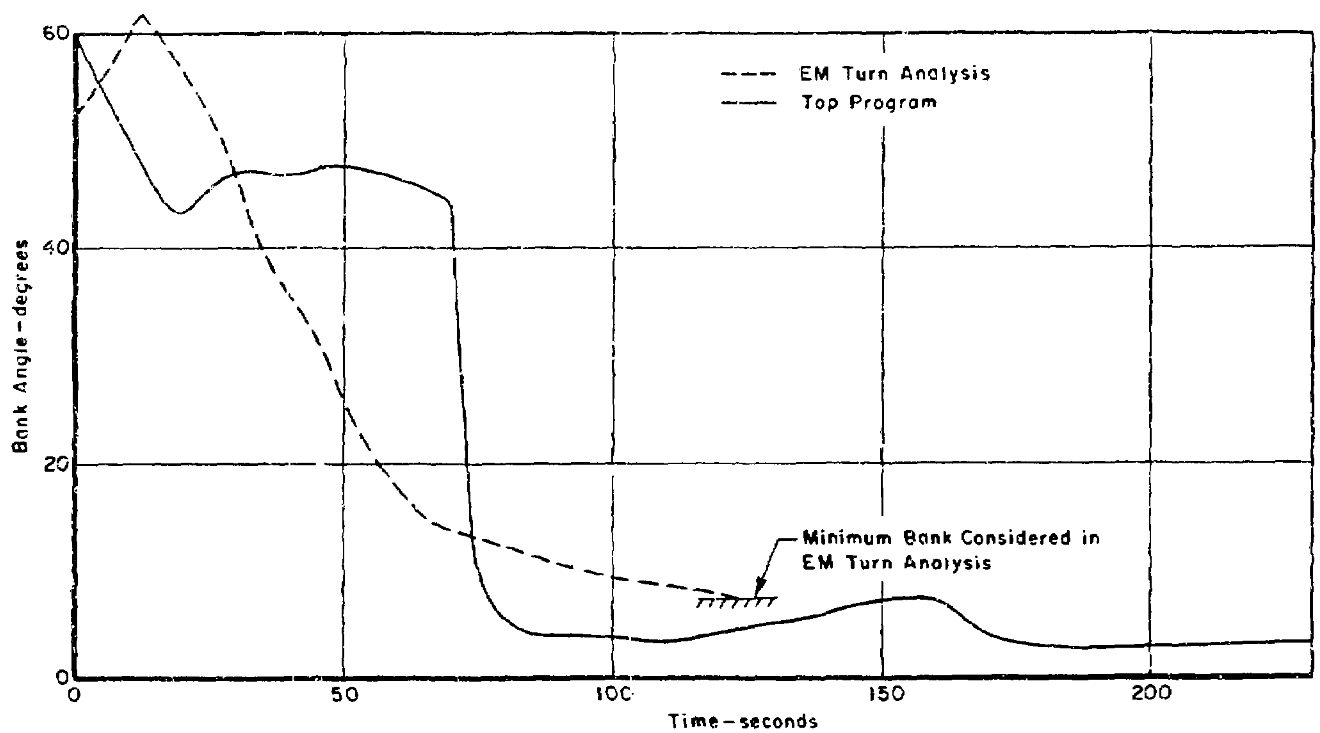

Figure 29. Category II, Maneuver 1, Minimum Time-EM Turn/TOP Bank Angles

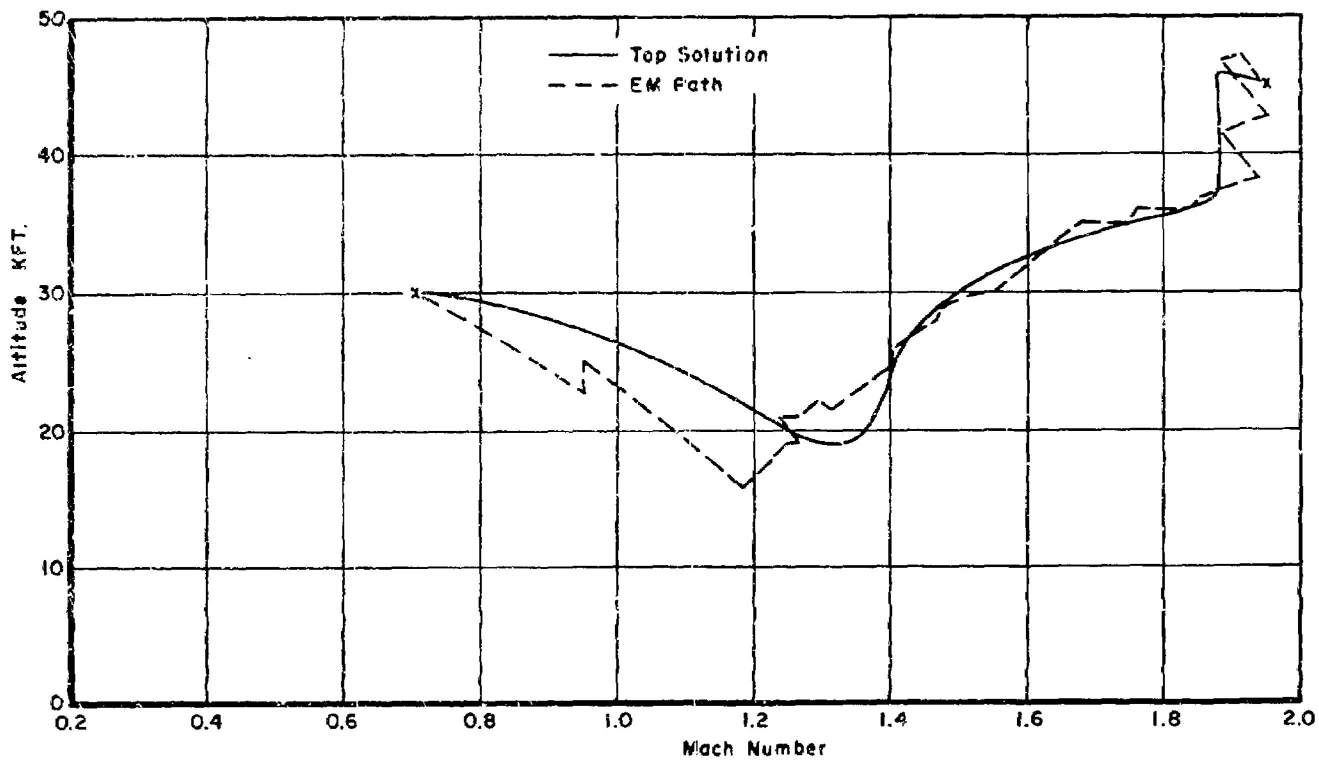

Figure 30. Category II, Maneuver 2, Minimum Time Path-EM and TOP

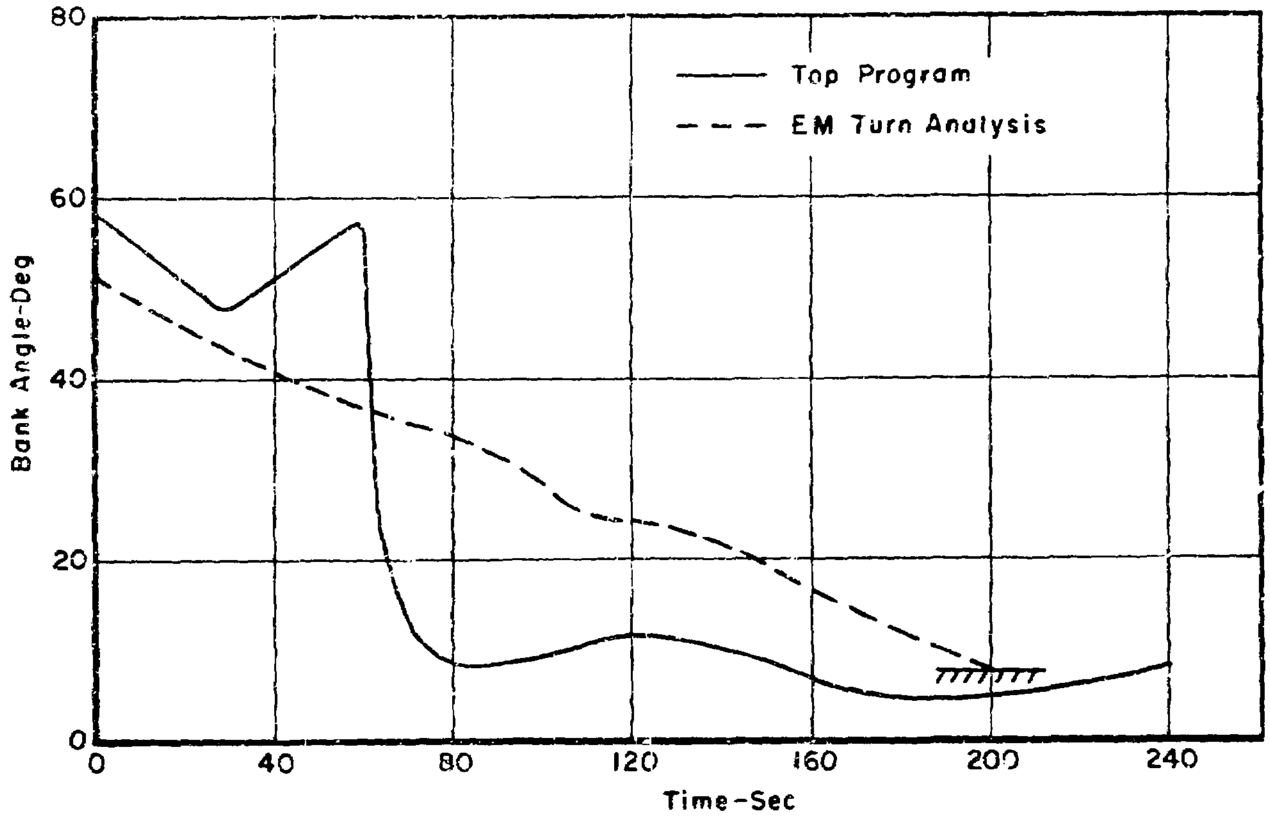

Figure 31. Category II, Maneuver 2, Minimum Time-EM Turn/TOP Bank Angles

efficiency turn uses bank angles greater than 20° for the first 140 seconds of the maneuver.

d. Discussion

In the two turning maneuvers studied good agreement in the final results was obtained between the EM, SDF, and TOP simulations. The same comments on the basic nonturning paths as were made in Section III, 1.c apply here. The turning procedure appears to give realistic turning penalties when compared to the SDF simulation in these maneuvers. The bank angle history was quite different from that obtained by the TOP program yet there was good agreement in the final results indicating the results were not extremely sensitive to the exact optimum bank schedule.

2. MINIMUM FUEL PATHS

a. Turning Charts

The minimum fuel turns were treated in a similar manner to the minimum time turns described previously using minimum fuel Rutowski paths. However, the procedure is somewhat longer when several throttle positions are considered. Minimum fuel Rutowski paths were run for military, minimum A/B, and maximum A/B, at the load factors plotted on Figure 20 up through 1.5 g's. The throttle switching points between power settings were determined by the method of Section III, 2.a.

Table VI shows a sample calculation to develop the minimum fuel turning efficiency charts for a 1.1 g load factor. Column 1 in the table shows power setting, column 2 the energy segment considered, column 3 the average velocity of the 1.1 g Rutowski path between the initial and final energy level of the segment, column 4 is the turning rate as read from Figure 20. $\Delta\psi$ is then calculated by Equation (18) as follows: column 5 is the time required to go between the energy levels of the segment along a 1.1 g Rutowski path $\Delta\psi$ is then colunn 4 multiplied by column 5 and is shown in column 6. The fuel penalty to operate at a load factor of 1.1 instead of 1.0 is calculated by Equation 19 as follows: column 7 shows the fuel required for the Rutowski path segment at a

TABLE VI

MINIMUM FUEL PATH TURNING EFFICIENCY EXAMPLE - 1.1g

(1) Throttle	(2) K ft E/W increment	(3) ft/sec V_{avg}	(4) deg/sec $d\Psi/dt$ (fig 20)	(5) sec Δt 1.1g	(6) deg $\Delta\Psi$ (4)×(5)	(7) lbs W.1.1g	(8) lbs W 1g	(9) lbs W_p (7)-(8)	(10) deg/lb $\Delta\Psi/\Delta W_p$ (6)/(9)
MIL	5-10	629	1.32	22.77	30.05	133.43	132.15	1.28	23.47
	10-15	735	1.12	23.99	26.86	130.78	129.52	1.26	21.31
	15-20	766	1.07	26.43	28.28	130.13	128.56	1.57	18.01
	20-25	795	1.03	29.62	30.50	131.88	129.50	2.38	12.81
	25-29.5	817	1.00	30.80	32.15	135.65	132.54	3.11	10.33
Min A/B	29.5-30	948	.870	1.56					
	30-35	936	.875	17.36	15.19	138.60	136.80	1.80	8.44
	35-40	915	.900	21.96	19.76	143.30	141.39	2.41	8.19
	40-45	894	.920	29.56	27.19	155.01	149.71	5.30	5.13
	45-49.5	878	.936	38.81	38.87	182.46	168.72	13.74	2.82
Max A/B	49.5-50	905	.910	2.80					
	50-55	1080	.770	23.29	17.93	228.63	212.60	16.03	1.11

load factor of 1.1, column 8 the fuel required at a load factor of 1.0, and column 9 is the fuel penalty to operate at a load factor of 1.1 instead of 1.0 calculated by column 7 minus column 8. The fuel turning efficiency, Equation (20), is then column 6 divided by column 9. Figure 32 shows the fuel penalty time calculations for the series of load factors up through 1.5. Figure 33 shows the turning efficiency in degrees of turn per pound of fuel penalty.

Figure 33 shows, as in the time case, that the best efficiencies are at the lower load factors and speeds. The crosshatched lines show the throttle switching boundaries for best efficiency along the Rutowski paths. As before, the "constant efficiency" turns correspond to horizontal lines across this figure, each efficiency level producing a table of load factors versus energy levels which, by using Figure 32 and Equation (20), can be resolved into the fuel penalty and amount of turn. In general, the more turn that is required between two energy levels the lower the efficiency level must be and the greater is the fuel penalty.

Using Maneuver 2 as an example an efficiency level of 3 degree turn/pound penalty was assumed. Table VII shows the results of the calculations. Column 1 in the table shows the energy segment considered, column 2 the load factor read from Figure 33 (line 1 shows a load factor of 1.4 read from Figure 33 at a mean energy level of 40,000 ft, turning efficiency of 3), Column 3 shows the fuel penalty for operation at the load factor of column 2 (line one shows 18 pounds penalty for the segment to operate at a load factor of 1.4 instead of 1.0 - read from Figure 32 at E/W = 40,000 feet and load factor of 1.4), Column 4 shows the bank angle computed by Equation (15) from the load factors of column 2 (line 1 was calculated by $\cos^{-1}(^1/1.4) = 44.4°$). Summing column 3 the total fuel penalty is 60.3 lbs and the corresponding amount of turn achieved is calculated by Equation 20 as 3 x 60.3° = 180.9°. To generate a family of curves with different amounts of turn the above calculation could be performed for several efficiency levels and a chart similar to Figure 23 prepared.

b. Maneuver Results

The pro edure used to evaluate the results was substantially the same as in Section III, 2.b with the SDF program used first to simulate the no turn

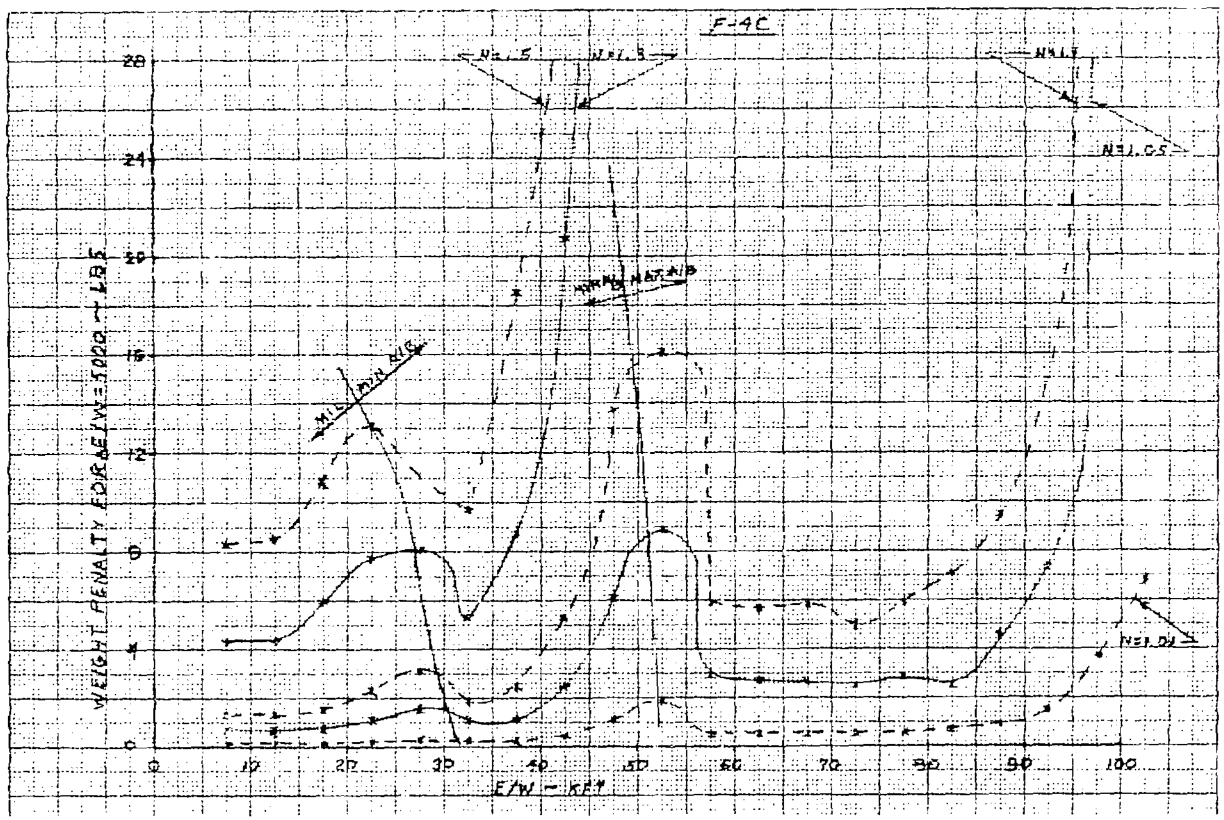

Figure 32. Acceleration Fuel Penalties Along Minimum Fuel Paths

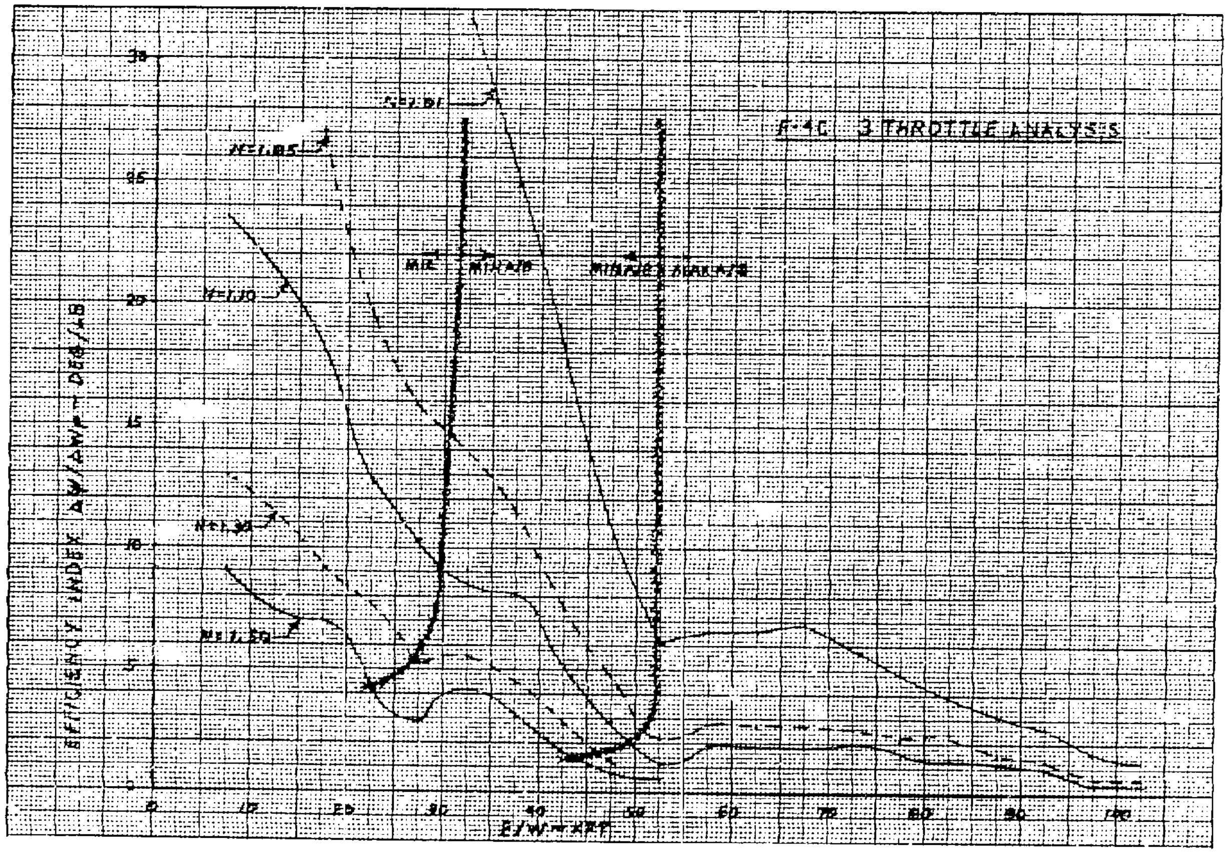

Figure 33. Turning Efficiency Along Minimum Fuel Paths

TABLE VII

MINIMUM FUEL CONSTANT EFFICIENCY TURN EXAMPLE

CATEGORY II MANEUVER 2

$\Delta\psi / \Delta W_p$ = 3 deg/lb penalty

①	②	③	④
E/W(KFT)	n(g's) from fig 33	ΔW_p (lbs) from fig 32	Bank (deg) Eqn 15
37.5-42.5	1.4	18	44.4
42.5-47.5	1.16	14	30.5
47.5-52.5	1.05	8.4	17.8
52.5-57.5	1.045	7.0	16.9
57.5-62.5	1.048	2.5	17.4
62.5-67.5	1.048	2.4	17.4
67.5-72.5	1.046	2.2	17.1
72.5-77.5	1.043	2.0	16.5
77.5-82.5	1.035	1.6	14.9
82.5-87.5	1.015	1.2	9.9
87.5-92.5	1.009	1.0	7.6

Total ΔW_p = 60.3 lbs

Rutowski minimum fuel path then again using the bank schedule from the turning procedure. Figure 34 shows the resultant paths for Maneuver 1 and the results are tabulated in Table III. The EM no turn path required 3216 pounds of fuel and with the turn 3224 pounds a difference of only 8 pounds of fuel required to do a 180° turn in conjunction with the increase in energy state. The SDF no turn path required 3352 pounds of fuel and with the turn 3363 pounds a difference of 11 pounds required for the turn. Comparing the end result the EM procedure was 139 pounds or 4% optimistic for the maneuver. Figure 35 shows the bank angle schedule used for Maneuvers 1 and 2. Maneuver 1 uses steadily decreasing and relatively low bank angles throughout the maneuver completing the turn before reaching supersonic speeds. Maneuver 2 starts at a higher bank and is turning throughout most of the maneuver, the bank angle calculation was performed in the last section and is also shown in Table VII. Figure 36 shows the results for Maneuver 2 and again the paths are quite close together. The SDF path contains a short military power segment as noted on the figure as the initial point was off the Rutowski path falling in the military power advantage region of Figure 15. The EM path starts in min A/B power since the analysis permits an instantaneous constant energy transition to the min A/B Rutowski minimum fuel path. For this maneuver the EM no turn path indicated 3183 pounds of fuel and with the turn 3243 pounds a difference of 60 pounds of fuel needed for the 180° turn. The SDF simulation showed that 3301 pounds of fuel was required in the no turn case and 3351 for the turn, a difference of 50 pounds required for the 180° turn. Comparing the end results the EM procedure was 108 pounds or 3% optimistic for Maneuver 2.

As a further check on the utility of the varying throttle technique the EM analysis of the maneuvers without the turns were also run at maximum A/B power. The results for Maneuver 1 were 3514 pounds fuel and 264 seconds time representing a savings of 290 pounds fuel or 8% for use of the varying throttle technique instead of maximum A/B power only. For Maneuver 2 the result was 3235 pounds fuel and 277 seconds time showing a savings of only 52 pounds or less than 2% due to the relatively short time a power setting other than maximum A/B was used in the varying throttle analysis. The turning analysis was not performed for the maximum power only paths, however, it would be expected considerably more fuel would be required for the turns due to the shorter length of time the vehicles would operate in the most efficient turning area of the charts.

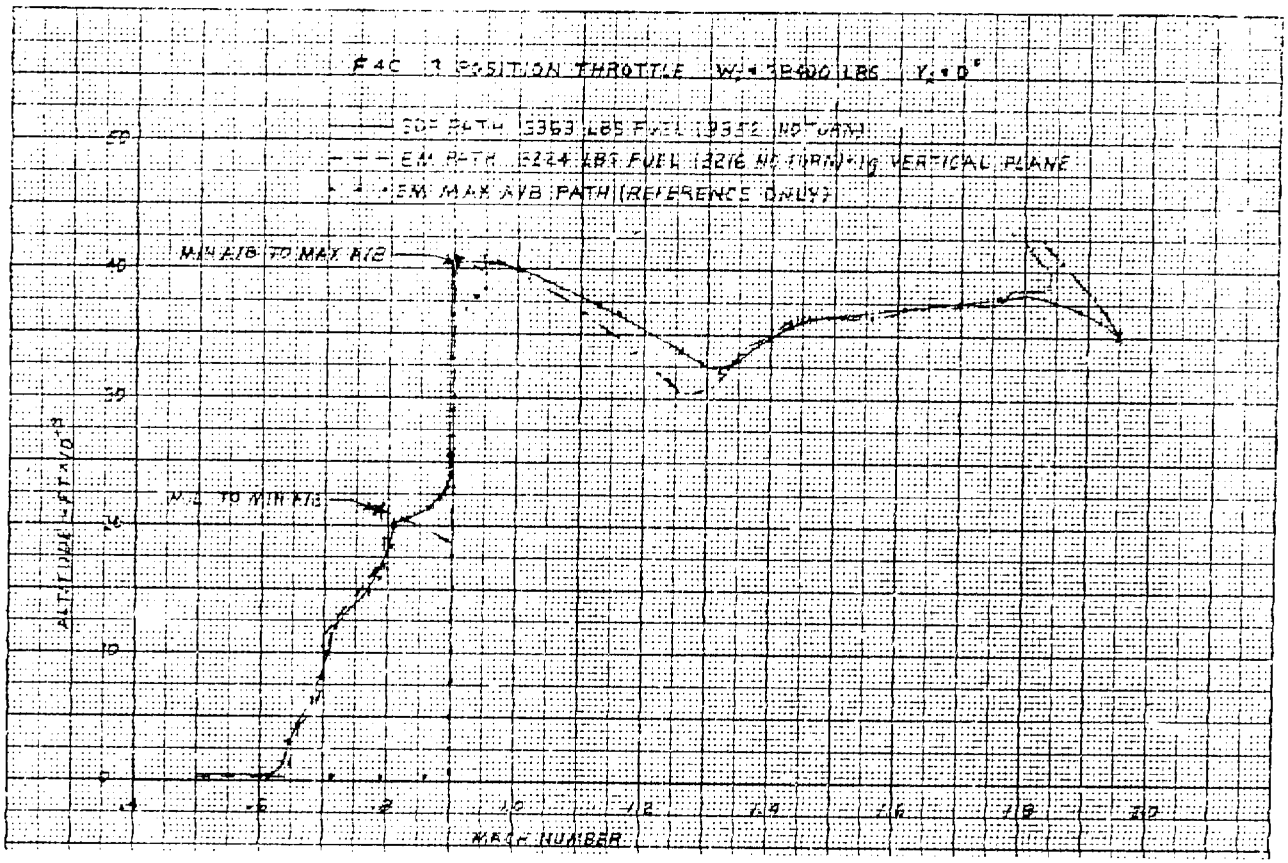

Figure 34. Category II, Maneuver 1, Minimum Fuel Path-EM and SDF

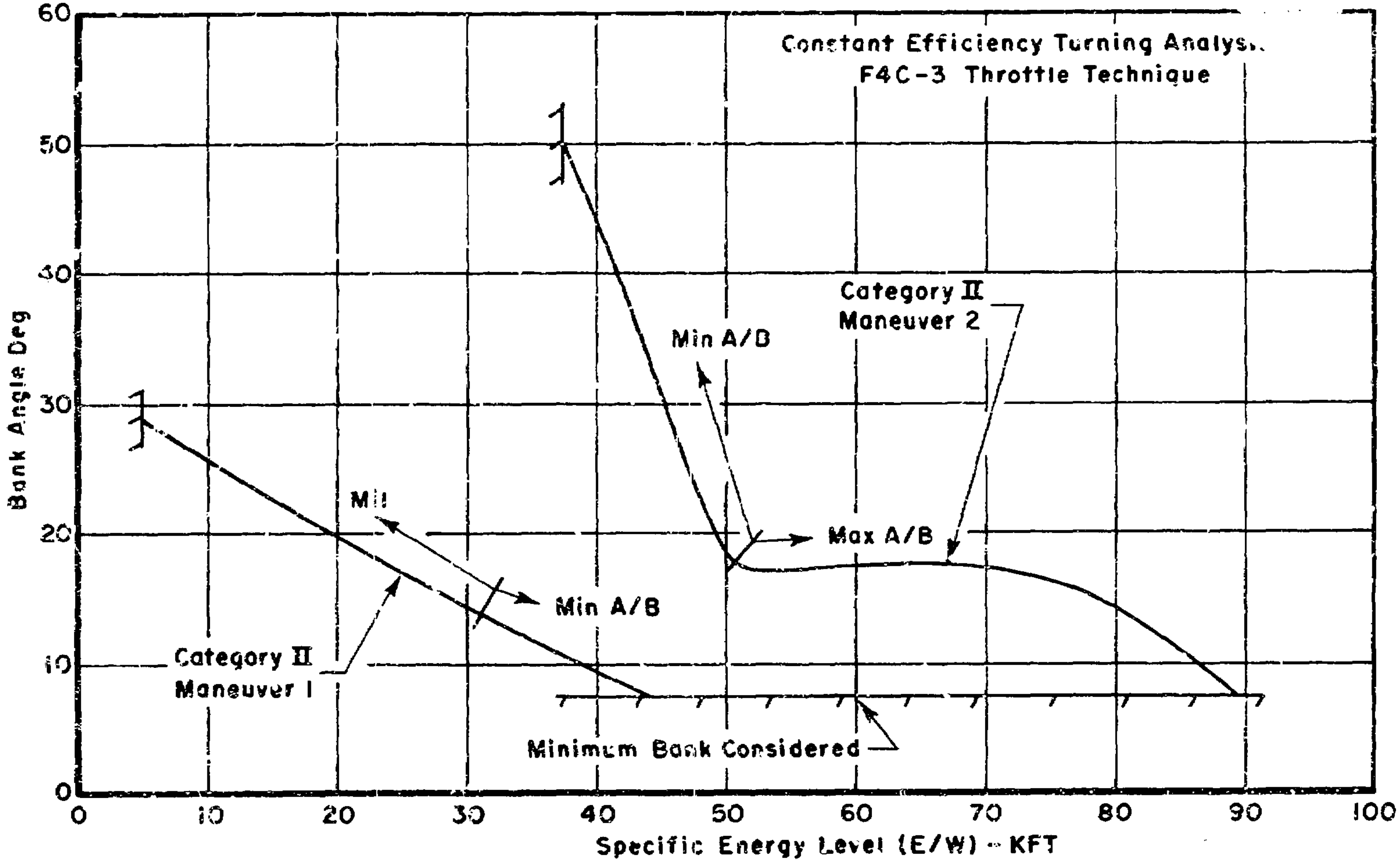

Figure 35. Bank Schedule for the Minimum Fuel Maneuvers

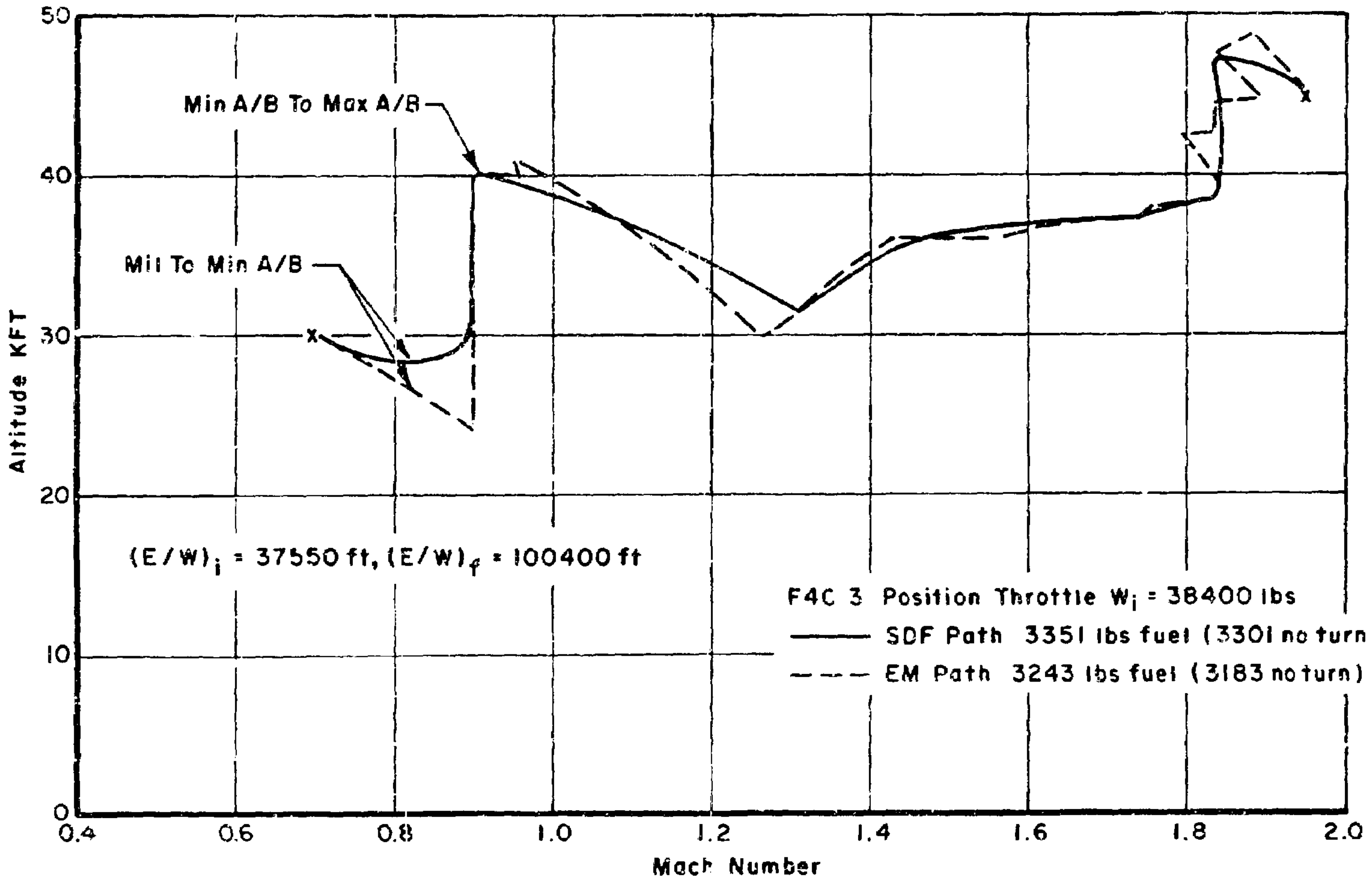

Figure 36. Category II, Maneuver 2, Minimum Fuel Path-EM and SDF

SECTION V

CONCLUSIONS

The comparison of the energy based maneuvers to the TOP and SDF solutions showed the energy paths are always optimistic in the results which can be achieved. This result optimism increased as the percentage of the path involving constant energy dives and zooms in the altitude-Mach plane was increased. For the minimum time paths the result optimism was less than 2% for the maneuvers where the constant energy percentage was less than 35% followed by a rather steeply rising curve approaching, in the limit, 100% error for paths which are comprised entirely of a constant energy transition.

It was observed during the course of the study that the paths run on the TOP program, with the exception of the zoom, tended to dive towards the Rutowski path, follow the path fairly closely, and then perform a zoom or a dive to reach the prescribed end conditions. This trend was followed even when the accuracy of the EM method, as measured by the final result, had deteriorated to 24% error. It follows, therefore, that knowledge of the basic climb path, whether this path is run using EM methods or by an integrated program, can provide a pilot or analyst the approximate path and some indication of the results for a large number of maneuvers. To obtain as much information through parametrics or by separate optimization for each maneuver would require a large number of runs. It should be noted that when computing a Rutowski path, speed and altitude should be limited only by physical limits such as engine placard or buffet boundaries leaving the technique free to produce constant energy dives and zooms. An artificial limit say on the altitude not to exceed the altitude of the desired end condition would force a level acceleration at this altitude if the true Rutowski path was above it, producing a result which could be far from the optimum.

The simulation of the energy paths on the SDF program showed that all paths required some modification to make them flyable. Use of a rule of thumb type correction increased the flyability of the paths. However, there is no guarantee that the results from use of the correction will be any closer to the optimum than the unmodified path particularly on dynamic paths involving large flight path angles. This was evidenced by the zoom maneuver in which application of the rule of thumb changed the result from optimistic to conservative. The TOP program provides flyable optimum solutions which become increasingly important for the more dynamic maneuvers.

The use of a variable throttle setting for the minimum fuel maneuvers showed a clear advantage to use of partial power settings in the regions where they are most effective rather than maximum power only. The results for the F-4C showed that in an energy climb starting at a low energy level this savings was about 393 pounds. This corresponds to an 8% savings in a long climb to near maximum energy or a savings of over 20% if the path is terminated before reaching supersonic speeds.

The turning analysis showed in the maneuvers investigated that turns may be made in conjunction with a Rutowski path without suffering a severe penalty in accuracy. The technique has the advantage that once the turning charts are developed for an aircraft, penalties and bank schedules can be rapidly calculated for a large number of maneuvers. The combination of the varying throttle and the turning analysis provides the analyst quick insight into difficult two or three control variable problems involving pitch angle, bank angle, and throttle setting. The resultant solution can be used as a first estimate and as a nominal for a more accurate integrated program to shorten the running time for the optimization process.

The findings of this study are that energy methods offer a tool especially useful in the early stages of preliminary design and functional performance studies where rapid solutions are needed and reasonable accuracy is acceptable. If the analyst uses good judgement in applying the methods to maneuvers the results provide a good qualitative insight for comparative purposes. The path, however, should not be used as a source of maneuver design or flight schedule in its entirety without verification, especially on relatively dynamic maneuvers where the optimality and accuracy of the energy maneuverability procedure decreases. The determination of what method or combination of methods is best suited for a particular application involves trading off the high speed and limited accuracy of the EM techniques against the longer running but more accurate progams such as TOP.

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