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The first experimental works which pertain to this problem were conducted by V. V. Baturin and I. A. Shepelev [16], Yu. V. Ivanov [17] and G. S. Shandurov [1]; as a result of these works empirical formulas were recommended for the calculation of the form of the axis of the jet which turned out to be dependent on the relation of dynamic pressures in the initial jet cross-section and incident flow and on the initial angle of slope of the jet.

Fig. 14.

Fig. 15.

Fig. 14. Flow lines in a jet of low velocity which flows out into a flow of high velocity ($m = 27$).

Fig. 15. Flow lines in a liquid being sucked in toward a flat submerged jet which flows out from an opening in the wall.

Subsequently, M. V. Volynskiy and I [1] developed approximate methods for the calculation of the form of the axis of the jet which are confirmed by the data of the indicated experimental works. Similar studies of other authors are also known. Recently two works of T. A. Girshovich [18, 19] have been published in which this problem for a plane jet is solved by a more rigid theoretical method, whereupon it is possible to find not only the form of the axis of the jet but also its boundaries and velocity profiles in different transverse cross sections, i.e., to construct the entire flow as a whole.

The problem is solved in a curvilinear coordinate system, the abscissa of which coincided with the axis of the jet, and the ordinate is orthogonal to the axis of jet. The boundary layer equations are written in this coordinate system for the zone of mixing taking into account the pressure field being created by centrifugal forces and the variable accompanying velocity. For determining the external jet boundary (from the direction of the incident flow) the latter is considered conditionally as the boundary surface of the current obtained from the addition of the incident irrotational flow with the system of the sources arranged on a line parallel to the incident flow and passing through the origin of the jet (Fig. 19), whereupon the distribution of the sources is selected from a supplementary condition which is reduced to the fact that the pressures on the jet boundary and the incident flow are identical.

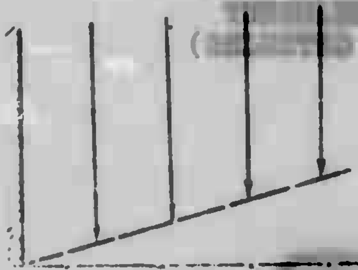

Fig. 16.

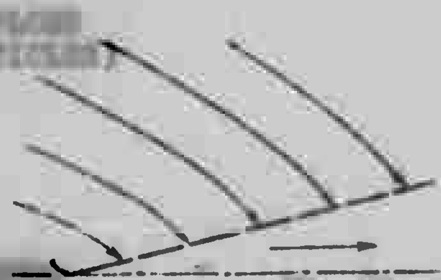

Fig. 17.

Fig. 16. Flow lines in a liquid being sucked in toward an axisymmetric submerged jet which flows out from a slot in a wall.

Fig. 17. Flow lines in a liquid being sucked in toward an axially symmetrical jet which flows out from a nozzle.

The axis of the jet calculated by T. A. Girshovich and its boundary are plotted on Fig. 19 and the corresponding experimental points are given; furthermore, the velocity fields in two orthogonal jet cross-sections are depicted (the experiment was conducted with a plane jet of air which flows out at an angle of 90° to the airflow with ratio of velocities $u_H/u_0 = 0.2$).

Fig. 18.

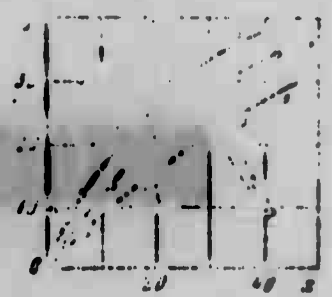

Fig. 19.

Fig. 18. Flow lines in a liquid being sucked in toward an axially symmetrical jet which flows out from a tube which is inserted flush into a plate (the jet flows out on a flat screen).

Fig. 19. Configurations of an airflow which flows out into a lateral airflow at an initial angle of 90° with ratio of velocities $u_H/u_0 = 0.2$ (points - experiment).

Figure 20 gives the theoretical curve and the experimental points of change in velocity along the axis of a jet for the same conditions. G. S. Shandurov indicated a simple method of considering the dissimilarity of the densities of lateral flow and a jet consisting in the fact that with equal values of the relations of dynamic pressures the pictures of the flow coincide.

In certain cases it is necessary to deal with turbulent jets subjected to the action of gravitational forces. If the direction of a jets of gas which has in the initial cross section a density which differs from that in the environment differs from the vertical, the gravity distorts it. In the works of S. N. Syrkin and of D. N. Lyakhovsky [20] the forms of the axis of a jet of heated air which flows out into air of normal temperature are experimentally investigated; the axis turned out to be more distorted the greater the preheating of the air. V. V. Baturin and I. A. Shepelev [21] and G. N. Abramovich [1] developed theoretical methods of calculation of the form of a distorted jet. It turned out that all the experimental data can be placed on a single theoretical curve if we introduce into the calculation Archimedes's criterion

$$Ar = \frac{u^2}{gd}\frac{\Delta T_0}{T_H}$$

(G. A. Abramovich's comparison of experimental points with the calculated curve is given on Fig. 21).

An interesting example of the use of curvilinear nonisothermal jets, called fountains, is the ventilation of the large exhibition pavillion in Sokolniki Park (Moscow). Jets of cold (street) air are fed to the premises from several inclined slotted openings arranged along opposite walls of the pavillion (Fig. 22). Under the action of the initial impulse these jets rise upward, but the air density in cold jets is greater than in the air of the pavillion, in consequence of which the vertical velocity in the jets gradually decreases; at some height the initial impulse is balanced by the Archimedes force directed downward, whereupon the jet, under the action of the latter, begins to drop and finally it comes into the operating zone of the pavillion but, in this case, the jet already manages to warm up - and thus in the working zone where the visitors of the pavillion are located it is possible to create comfortable conditions. The described ventilation system was designed according to I. A. Shepelev's theory.

In recent years theoretical and experimental studies have been conducted of the convective jets which arise near heated horizontal [1] jet of gas [22] surfaces.

There are important results for the turbulent jets in which Lyakhovskiy(flames of combustion) occurs; without dwelling on this question, we will refer the reader to the appropriate literature [1, 23].

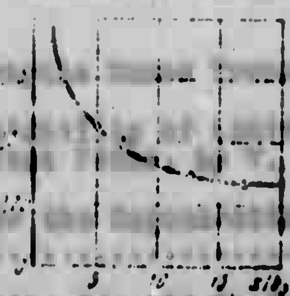

Fig. 20.

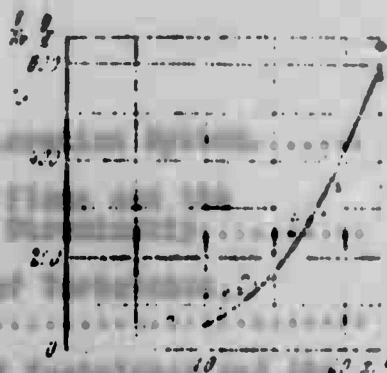

Fig. 21.

Fig. 20. Change in dimensionless velocity along the axis of a jet which flows out into a lateral airflow at an initial angle of 90° with ratio of velocities $u_H/u_0 = 0.2$ (points - experiment).

Fig. 21. The axis of a submerged air jet distorted by gravitational forces (points - experiment).

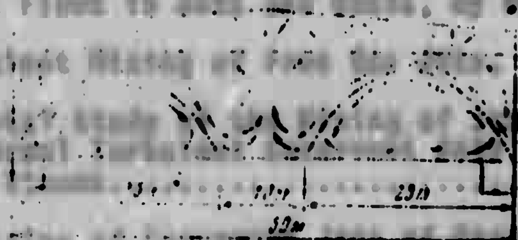

Fig. 22. Fountains of cold air (the axis of the fountain is shown by the dot-dash line).

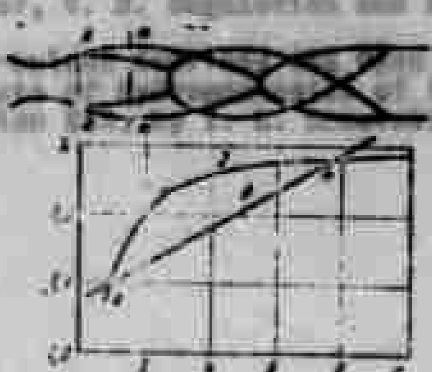

Fig. 23. Diagram of an underexpanded supersonic jet and curves of the dependence of the coefficient of average velocity ($\lambda = u/a_{кр}$) on the relative area of cross section ($f = F/F_a$) calculated from conditions for the conservation of momentum J and the weight-to-mass flow rate of gas G (in the initial and maximum cross sections both conditions are satisfied simultaneously).

A special problem is the supersonic gas jet. In this regard, in the case of the so-called *design conditions of outflow* with which the pressure in the initial section of the supersonic jet equals the ambient pressure the regular laws of development of a jet remain the same in principle as for a subsonic jet of variable density (one should only consider that the density distribution with high velocities is connected with the velocity distribution).

In the case of off-design outflow, i.e., with an initial pressure different from that in the surrounding environment, the form of a jet is modified and requires special study.

The chief characteristic of an off-design supersonic jet is the fact that, beginning from the mouth of the nozzle, a considerable restructuring of the flow appears in the process of which in one or another system of rarefaction waves and shock waves which depend on the outflow conditions the transition occurs from the initial pressure in the jet toward the ambient pressure.

The section of such a transition, which is characterized by a considerable nonuniformity of the pressure field, is called the *gas-dynamic section* and is a subject of special study [5]; we will not dwell on it since turbulent mixing has secondary significance here.

However, beyond the limits of the gas-dynamic section, the jet becomes isobarometric and its subsequent development is determined by the laws of turbulent mixing. In the initial section of the isobarometric section, the velocity profile has a considerable nonuniformity (with a dip close to the axis of the jet) which depends on the form of the gas-dynamic section and changes with the degree of off-design $n = p_a/p_H$ (p_a - pressure at the nozzle edge, p_H - the pressure in the environment) and with the Mach number at the beginning of the jet (M_a).

An interesting study of an off-design supersonic jet was performed by B. A. Zhestkov, M. M. Maximov et al [1]. A simple method for the determination of the form of the jet on the gas-dynamic section was proposed by A. Ya. Cherkez [24]; a detailed experimental and theoretical investigation of the isobarometric section of a supersonic jet was conducted by Chiang Tse-hsing [27]. In A. Ya. Cherkez's mentioned work, the calculation of the gas-dynamic section whose diagram is given in Fig. 23 is conducted by the methods of one-dimensional gas dynamics with the use of equations of conservation and without consideration of the mixing-in of the surrounding environment with the jet.

An important result of the work of B. A. Zhestkov, M. M. Maximov et al is the establishment of the fact that the damping of velocity on a large part of the isobarometric section of the jet is expressed in logarithmic coordinates by parallel straight lines that have the same slope as in the jet of a noncompressible liquid; the effect of the degree of off-design and the initial value of the Mach number is manifested primarily in the shifting of the point where the drop in velocity on the axis of the jet begins, i.e., in a change in the abscissa x_n of the beginning of the main section of the jet (Figs. 24 and 25).

Fig. 24.

Fig. 25.

Fig. 24. Curves of the change in dimensionless velocity along the axis of a supersonic jet with different values of the degree of off-design $n = p_a/p_H$ for $M_a = 1.5$.

Fig. 25. Curves of the change in dimensionless velocity along the axis of a supersonic jet with different values of the degree of off-design $n = p_a/p_H$ for $M_a = 3$.

Everything said pertains to a submerged supersonic jet. With the presence of a cocurrent flow, the picture of the flow becomes complicated and up to now has been little studied.

Work in the field of turbulent gas jets is being conducted at the present time in many domestic and foreign laboratories. One of the important directions of these studies is research on the turbulent microstructure of a jet and the establishment of the direct connection between it and the averaged flow conditions. Only the first steps in this direction thus far have been made in the works of A. S. Ginevskiy et al [25], G. N. Abramovich et al [10] and some studies of foreign authors [26].

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THE PROBLEM OF THE STABILITY OF LAMINAR FLOWS AND THE TRANSITION TO TURBULENT FLOWS

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Summary

At the present time the theory of aerodynamic stability has been converted into a large independent section of science covering a very wide circle of interesting scientific and engineering problems.

Pertaining to this section are the classical problems of flow stability in tubes, ducts, and wing boundary layer, the problem of the flow stability of a viscous liquid in limited volumes, and problems of the stability of zonal winds in the atmosphere and of the stability of heat convection. Pertinent here are interesting problems of the flow stability of the conducting liquid in a magnetic field, the problems of the flow stability of plasma, and many others.

The circle of these questions is so broad that it cannot be illuminated in one report. Thus, we will dwell only on some questions of hydrodynamic stability which have the greatest relationship to the phenomena of the transition of the laminar flow

THE TURBULENT MIXING OF FREE GAS JETS

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(Leningrad)

1. Very considerable attention is given in domestic and foreign literature to the study of the turbulent jets of liquid and gas in recent decades. This is explained first of all by the wealth and diversity of engineering applications of the jet flows. At the same time, their examination makes it possible to obtain valuable information about the mechanism of turbulent motion and turbulent transfer. In this respect it is significant that after a temporary decline, interest increased noticeably again in the detailed study of the pulsating structure of turbulent flows, in particular jet flows. Especially promising for the present are isolated attempts at active influence (mechanical, acoustic, electromagnetic - for a conducting medium and others [1-3]) on the development of jets.

If, at the first stage, during the creation of the theory of turbulent jets, the main thing was the development of general regularities (similarity of flow and, in particular, the self-similarity of jets according to Reynolds number), then at the present time the primary thing becomes the clarification of the finer features of the flow and the role of different, in the first approximation "secondary" factors. In light of the aforesaid, let us examine some general questions of the theory of turbulent jets and, in more detail, the results of one of the attempts at the active influence of a flow [1, 11].

2. The theory and the methods of calculation of turbulent jets are one of the developed sections of the contemporary semi-empirical theory of turbulence. Without going into details, let us note the satisfactory solution, in general, of the basic problem of such theory for a number of jet flows. Drawing on limited empirical information, it is possible to arrive at sufficient agreement (frequently within the limits of the accuracy of experiment) of the calculated and experimental data for the middle flow (in Reynolds' sense). As a rule, this can be achieved in different ways. Actually, different semi-empirical methods of calculation of turbulent jets (based on the scheme of an asymptotic boundary layer or layer of finite thickness, on the integration of differential equations under specific assumptions about "turbulent viscosity" or on the use of integral relationships, on the polynomial representation of the velocity profile or friction stress or on the a priori selection of the profile, etc. [4, 5, 6]) in a number of cases, especially for self-similar jets, lead to a satisfactory description of the middle flow. It is more complex with non-self-similar jet flows; however, even here different interpolation schemes ([4, 6, 7, 8] and others) and especially the method of the equivalent problem of the theory of thermal conductivity [5] prove to be sufficiently effective for engineering calculation in certain cases.

The fundamental problems standing in this area are connected with the expansion of a nevertheless comparatively narrow circle of flows which yield to calculation and to the reduction, to the minimum, of the data borrowed from experience. As concerns the selection of a more effective method of calculation, (for turbulent jets of a noncompressible liquid) this question is not so fundamental. One should, however, indicate certain advantages of the method of the asymptotic layer over the layer of the finite thickness and the necessity for a difference in the nominal thicknesses of the dynamic and thermal layers (in accordance with the value of a "turbulent Prandtl number" on the order of 0.7-0.8 for the thermal and diffusion problems) and others.

Also waiting its turn is the expansion of the investigation of the balance of fluctuating energy for problems of the theory of turbulent jets for which this method, apparently, will be very effective.

3. More complex than for the noncompressible liquid is the matter of the study and calculation of the turbulent jets of a compressible gas. Characteristic of the latter is a certain contradiction in experimental reference data on turbulent mixing. While some authors [4, 9] consider the intensity of turbulent mixing in cocurrent gas jets minimum with the identical values of velocity in the jet and surrounding environment[1] [4, 9], others cite data which testify to the presence, in the conditions of the experiment, of a minimum in the intensity of mixing with approximately identical values of ρu^2 [5, 10]. Finally, assertions are also encountered about the determining role of the value ρu. However the question as a whole is much more complex.

Each of these conclusions is based on experimental data obtained under specific conditions. Therefore, it is difficult to assume that the divergence is caused only by the difference in the procedure for processing the experiment alone (in some works the judgement on the intensity of mixing is based on a comparison of secondary characteristics - the thickness of the mixing region [9], in others - on the direct mixing of the gas from the jet and the cocurrent flow [10]). It is more probable that under the different conditions of

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[1]For the outflow of a jet of denser gas into the cocurrent flow of a less dense gas.

the experiment, the effect of the initial conditions difficult to consider is felt - the velocity profile, the initial level and scale of turbulence, intermittance, and also the value of the Reynolds number. The role of these factors is also insufficiently clarified for the turbulent jets of a noncompressible liquid. It is known, in particular, that with a decrease in number $Re = u_0 d/\nu$ (where u_0 is the exhaust velocity) the intensity of attenuation of a submerged turbulent jet does not fall, which would be natural at first glance, but increases noticeably. Apparently, the mutual application of molecular and molar effects in the initial and transitory sections of the jet has a substantial effect.

With a somewhat increased (to 8-10% and more) initial turbulence, as tests show [5], a unique developed regime of the turbulent flow is accomplished. In this case the profiles of values of ρu^2, etc., in gas jets (and the burning flame [10]) become universal. Under these conditions the leading role of the difference in values of ρu^2 in turbulent mixing (in the range of values of parameters studied in the experiments) is usually exhibited. On the other hand, with the effective suppression of the initial turbulence (for example, with outflow through extremely fine grids which rarely lower the value of the scale of turbulence) the regular laws of molecular (and close to it fine-scale) mixing are more strongly pronounced, especially in the initial section of the jet. These representations need direct experimental check. And although specially posed experiments with the diffusion combustion of a gas showed that the length of flame is noticeably greater (and, consequently, mixing is worse) other conditions being equal if the values of ρu^2 in the jet and cocurrent flow are identical [10]), here too the effect of secondary factors could be felt.

4. For an investigation of the features of the development of a turbulent jet under conditions of an increased (in which regard controlled) initial level of turbulence, experiments were conducted[1]

[1]With the participation of K. Ye. Dzhaygashtin and I. A. Kel'manson.

on a unit with a mechanical vortex generator described in the work. The preliminary results of the examination make it possible to draw a number of conclusions apropos the structure of the middle and fluctuating flows in such a jet. It was made clear that for a field of average velocity the determining criterion is the value of the Strouhal number $Sh = nd_0/u_0$ (where n is the number of revolutions of the disc of the vortex generator, d_0 - the diameter of the nozzle, u_0 - the exhaust velocity). With an increase in the Sh number the intensity of turbulent mixing increases noticeably. Some data which illustrate this are given in Fig. 1 (change of velocity on the axis of the jet at different values of Sh number) and Figs. 2 and 3 (velocity profile - Fig. 2 - and temperature profile - Fig. 3 - in the transverse cross sections of a weakly heated jet). Analogous data were obtained with a considerable variation in the conditions of the experiment (with d = 10, 20 and 40 mm; u_0 = 20-130 m/s; n = 0-250 r/s).

For the characteristic of the total intensifying action of the vortex generator let us note that the air flow rate in the jet was subordinated to a relation of the type (for cross sections with ratio $x/d \le 20$) $m/m_0 = 1 + 0.2\, k\, (Sh)\, x/d$, where k = 1; 1.18; 1.36 and 1.44 respectively for numbers $Sh \cdot 10^2$ = 0; 3.7; 8.0 and 10.5. In this, in all cases number $Pr_{turb} \approx 0.75$.

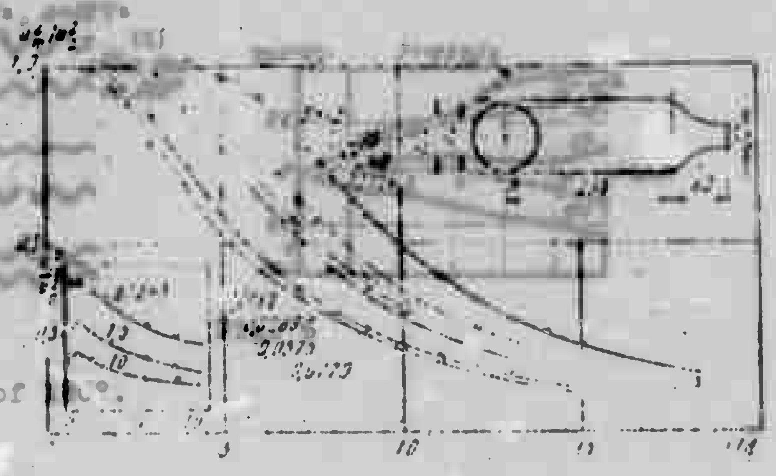

Fig. 1.

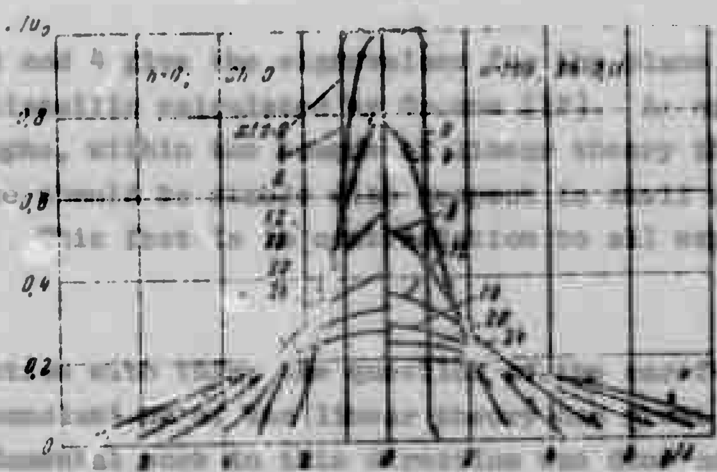

Fig. 2.

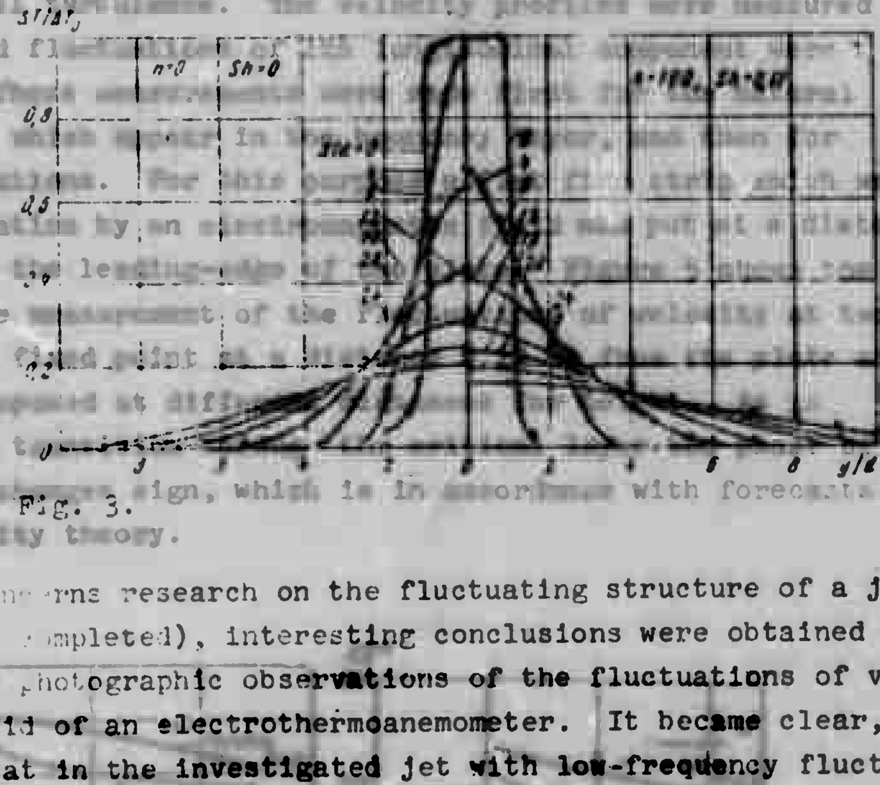

Fig. 3.

As concerns research on the fluctuating structure of a jet (still not completed), interesting conclusions were obtained with visual and photographic observations of the fluctuations of velocity with the aid of an electrothermoanemometer. It became clear, first of all, that in the investigated jet with low-frequency fluctuations superimposed with the aid of the vortex generator in general one should distinguish three characteristic regions of flow. In the first of them, adjacent to the mouth of the nozzle, the flow is quasi-regular; the oscillation frequency is equal to double the number of revolutions of the disc. In the second - transitory - developing turbulent fluctuations are superimposed on the basic forced oscillations. Finally, in the third the characteristic irregular turbulent spectrum of fluctuations is observed. As concerns the dimensions of the regions occupied by each type of flow, they depend on the value of Sh number. With Sh number $\geq$ 0.1-0.12 practically the entire jet is a region of developed turbulent motion, whereas with small values of Sh number there is a distinctly expressed section of a "fluctuating" quasi-monochromatic jet in it. The qualitative picture of the flow in a jet with Sh number $\approx$ 0.005 is presented on Fig. 4, where the boundaries of characteristic zones and also the profiles of average speed are shown. The same figure presents the characteristic oscillograms of fluctuations for all three zones (and for comparison with number Sh = 0).

As concerns the value of the intensity of fluctuations, with a sufficiently large Sh number it is higher than in a usual jet. Thus, with number Sh = 0 at the nozzle edge $\xi_u = \bar{u}'/u_0 = 0.5-1.0\%$, and with x/d = 10 on the axis of jet $\xi_u = 18\%$. With number Sh $\geq$ 0.03 the initial value $\xi_u = 10-12\%$, and the maximum (on the axis) is shifted to x/d = 4 and is equal to $\xi_u = 32\%$.

Most complex is the nature of the change of value ξ_u (and also of the entire picture of the flow) in the first region of the flow - with quasi-regular fluctuations, for the interpretation of which additional measurements will be required (specifically, frequency characteristics).

The obtained data as a whole make it possible to assume that with the further examination of a jet with forced fluctuations, along with the obvious applied results, additional information can be obtained about the structure of a free turbulent flow. Specifically, this pertains to the picture of the transition from the initial, approximately monochromatic, low-frequency fluctuations, given by the vortex generator to the complex turbulent spectrum.

is necessary for both. The difference in critical Reynolds numbers is almost two orders — this is already too much. The transition region turned out to be unexpectedly [illegible] from Fig. 2, actually lacking [illegible] and [illegible] this entire transition [illegible] turbulent fluctuations [illegible] are practically absent, with the exception of the terminal portion of this zone.

As follows from the material of [illegible], the question of the experimental confirmation of linear theory cannot be considered finally [illegible]. At the present time it is [illegible] even more thorough experimental [illegible] in the problem of the [illegible] of linear theory. These experiments can be conducted only in special low-turbulence wind tunnels.

It should be noted that generally the analysis of experimental research on the structure of the transition region has great significance. Indeed it is necessary to welcome such work in every way possible. In recent years, a number of investigations have been accomplished in this direction. Discovered in Emmons' classical experiments [26] was the sudden appearance of small turbulent regions (turbulent spots) of irregular shape which increased in dimensions during their movement downstream. [illegible]

Fig. 9

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... measurement of the coefficient of intermittence at different points along the length of the tube for different Reynolds numbers. With small Reynolds numbers the transitional region turned out to be large, being measured in thousands of diameters of the tubes.

Fig. 10

Fig. 11

Thus, turbulent spots and turbulent plugs are an important phenomenon for understanding the transition process. In the experiments of Schubauer and Skramstad with a plate (with a small degree of turbulence of flow) and in the experiments of Rotta in a tube with small Reynolds numbers (i.e., with slight perturbations in the initial section of a tube) the length of the transitional region turned out to be very large. In a large part of this zone, the flow fluctuations are practically absent and turbulent spots

EXPERIMENTAL STUDY OF THE MIXING OF ISOTHERMAL AND NONISOTHERMAL TURBULENT COCURRENT JETS IN NOZZLES

V. L. [illegible]

(Moscow)

The study of turbulence characteristics in flows in ducts of variable cross section is an important problem of the aerodynamics of viscous flow. Sufficiently complete information about the turbulent parameters over a wide range of flow conditions is necessary for the creation of methods for the calculation of flows in ducts which rest on the real representations of the process and use physical parameters for the description.

Such a parameter which describes diffusion in a transverse direction is the mean square deviation of liquid particles (dispersion) from the middle lines of current. The use of dispersion averaged over cross section forms the basis of the diffusion model of a free turbulent jet [1].

The use of dispersion for the description of the mixing process in ducts of variable cross section is complicated by the fact that unlike the case of the free jets its change is determined not only by the processes of mixing, but also by the deformation of the flow with a change in the area of the cross section.

Let us examine the picture of the flow in the unit which was used in the experiments (Fig. 1). A heated jet which consists of the combustion products of gasoline rarefied by air was partially mixed with a cocurrent flow of cold air and it fell into a narrowing nozzle. Experiments were carried out with the identical velocities of the jet and the cocurrent flow. With the flow of a nonisothermal flow in the nozzle the shift of longitudinal velocities is developed. This is connected with the fact that, as is established by experiment, the total pressure on the nozzle edge is constant, i.e., in the process of flow in the nozzle a transition proceeds from a flow with constant velocity to a flow with constant velocity coefficients in the cross sections.

The characteristic determined by the mixing process is the difference in the dispersion of a real flow and a hypothetical flow under the assumption of the absence of mixing on the investigated section of the nozzle. In this case, as will be shown below, for the description of the process in the nozzles it is more convenient to use dispersion $\Sigma_c^2(x) = [r_H^2/r_c^2(x)]\sigma_c^2(x)$, reduced to the radius of a cylindrical chamber, and to characterize the mixing by the difference $\Sigma^2(x) = \Sigma_c^2(x) - \Sigma_{co}^2(x)$.

The expression for determining the change $\Sigma_{co}^2(x)$ in the absence of mixing in the nozzle can be obtained from the condition for the retention of surplus enthalpy. Taking the form of the profile of the stagnation temperature in the form of Gaussian curves (i.e., considering real profiles in the entrance mouth of the nozzle sufficiently broad and the diameter of the chamber so large that an increase in temperature at the walls due to mixing is insignificant), the extreme value of dispersion Σ_{co}^2 attained in cross sections with constant velocity coefficients can be found from the expression (with constant velocity in the initial cross section of the nozzle)

$$\Sigma_{co}^2 = \frac{1}{2}\ln\frac{T_1}{T_2}\left[\left(\frac{T_1}{T_2}\right)^{1/2} - 1\right]^{-1} r_H^2. \quad (1)$$

where σ^2_H is the dispersion at the end of the cylindrical chamber and T_1, T_2 - the maximum and minimum stagnation temperature in the entrance mouth of the nozzle.

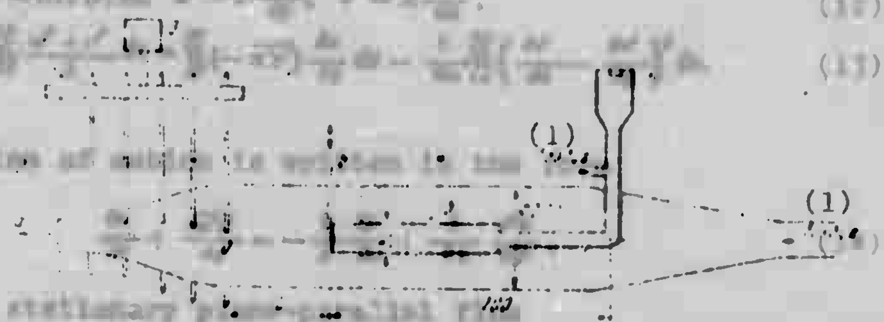

Fig. 1. Experimental unit: 1 - precombustion chamber; 2 - differential thermocouples; 3 - electric air traversing equipment; 4 - equalizing grid (distance between centers of holes 9 mm), diameter of holes 6 mm); 5 - asbestos heat insulation; 6 - movable thermocouple for the measurement of temperature of jet.
KEY: (1) Air.

In the region of transition from a flow with constant velocity to a flow with the constant velocity coefficients, the value $\Sigma^2_{co}(x)$ is described by an approximate expression valid with small distances from the entrance mouth of the nozzle:

$$\Sigma^2_{co}(x) = \ln\frac{T_1}{T_2}\left[\ln\frac{T_1}{T_2}\frac{[1-\pi_c(x)-\pi_k]}{2[1-\pi_c(x)]} + \left(\frac{T_1}{T_2}-1\right)\frac{[\pi_k-\pi_c(x)]}{2[1-\pi_c(x)]}\right]^{-1}\sigma^2_H \quad (2)$$

where $\pi(x) = \left[1 - \frac{\kappa - 1}{\kappa + 1}\lambda^2(x)\right]^{\kappa/(\kappa-1)}$, λ is the velocity coefficient in the nozzle with isothermal flow. In the case of a constant velocity coefficient at the end of the cylindrical chamber or with an isothermal one-dimensional flow $\Sigma^2_{co}(x) = \sigma^2_H$.

In the extreme case of small overheatings, when the longitudinal velocity component in each section is constant, the mixing is determined by the behavior of the coefficient of turbulent diffusion along the nozzle, whereupon under the assumption of the equidistance of all averaged lines of flow (in dimensionless coordinates)

$r_x/r_c(x)$ = const parameter $\Sigma^2(x)$ is connected with the value of the transverse diffusion coefficient by the relationship

$$\frac{d\Sigma^2}{dx} = 2\,\frac{r_x^2}{r_c^2(x)}\,\frac{D(x)}{u(x)}. \qquad (3)$$

where u is longitudinal velocity. An analogous relationship for dispersion σ^2 is more cumbersome and takes the following form [2]:

$$\frac{d\sigma^2}{dx} + \frac{2}{r_c}\,\frac{dr_c}{dx}\,\sigma^2 = 2\,\frac{D}{u}.$$

The measurements of the quoted dispersion $\Sigma^2(x)$ with small overheatings permit, in accordance with relationship (3), investigating the behavior of the coefficient of turbulent diffusion with deformation of the flow. Figure 2 shows experimental values of $\Sigma^2(x)$ with the mixing of the weakly heated jet in a Vitashinskiy nozzle (length 400 mm, diameters 200 mm and 85 mm, pressure gradient critical) and the results of the calculations (curves 1 and 2).

The available results of the theoretical investigation of the rapid deformation of turbulence obtained in [3] make it possible to determine a change in the longitudinal and transverse mean-square velocity component along the nozzle.

V. M. Iyevlev obtained formulas for the change in the scales of turbulence with rapid axisymmetric deformation, which made it possible to obtain relationships for the diffusion coefficient. Apart from this, V. M. Iyevlev examined the case of the gradual deformation with which the isotropy of turbulence manages to be established.

The calculation of the transverse diffusion coefficient under the assumption of rapid deformation shows its increase in the first quarter of the length of the nozzle up to the limiting value of $\sqrt{4/3}$ times, and with gradual deformation the coefficient of turbulent diffusion does not change. Function $\Sigma^2(x)$ for rapid (curve 1) and gradual (curve 2) deformations was found from relationship (3). Used in the calculations was the experimental value of the coefficient of turbulent diffusion in the chamber which was found by means of the measurements of profiles in the cylindrical compartment established between the nozzle and the chamber (a straight line approximating the experiments is shown by the dotted line on Fig. 2).

The measurements of the temperature profiles were conducted with differential chromel-copel thermocouples, the movable joints of which were raised simultaneously by electric air transversing equipment and could freely emerge from the nozzle if necessary. In the analysis of the experiments, the surplus stagnation temperature was taken as the parameter which coincides with the concentration.

It is possible to show that if the change in the quoted dispersion $\sum^2(x)$ in the isothermal flow is equal in the nozzle and the cylinder, then in both cases mixing occurs equally, i.e., a drop in the axial concentration of the jet (or the maximum excess overheating) occurs equally and the profiles of concentration coincide with an affine increase in the cross section of the nozzle up to the dimensions of the cylindrical chamber. In particular, for the noncompressible liquid ($r_c^2 u$ = const) when D = const the behavior of $\sum^2$ for the nozzle and the cylinder should be identical. The compressibility effect leads to the fact that with the narrowing of the duct due to a decrease in the density, the velocity increases faster than for the noncompressible liquid. This leads, with the constant diffusion coefficient, to the retarding of mixing as compared with the mixing in a cylinder (Fig. 2, curve 2).

Within the accuracy of the experiment, the mixing in a narrowing nozzle and cylindrical tube occurs equally. This is also confirmed by the absence of the stratification of the relative maximum excess stagnation temperatures (Fig. 3). (In constructing the graph, the results of the measurements were corrected in accordance with the values of the recovery factor found experimentally in the cross sections.)

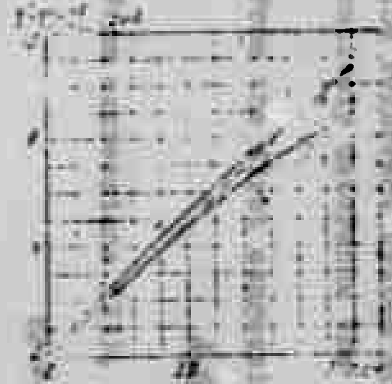

Fig. 2.

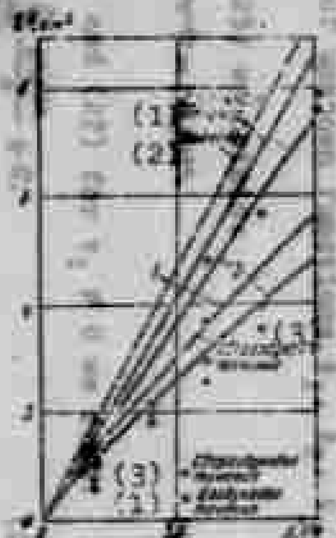

Fig. 4.

Fig. 3.

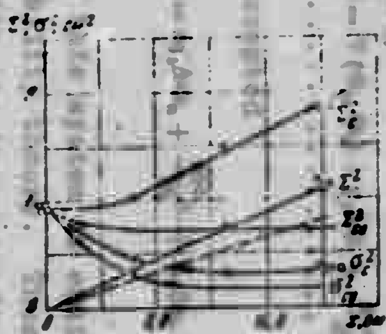

Fig. 5.

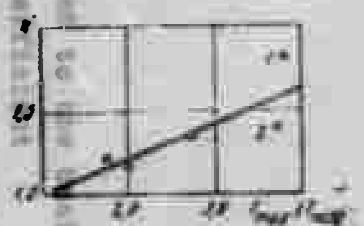

Fig. 6.

Fig. 2. The mixing of a weakly heated jet in a narrowing nozzle: 1 - calculation for rapid deformation; 2 - calculation for gradual deformation; points - experiment.

Fig. 3. Change of maximum stagnation temperatures with mixing in the cylinder and nozzle.
KEY: (1) Cylinder; (2) Nozzle.

Fig. 4. Mixing in the expanded part of a Laval nozzle: 1 - calculation for rapid deformation; 2 - calculation for gradual deformation.
KEY: (1) Subsonic flow; (2) Cylinder; (3) Supersonic flow.

Fig. 5. Mixing of a nonisothermal jet (T_{max}/T_{min} = 2.7) in a narrowing nozzle (broken line - the results of experiments in a cylinder).

Fig. 6. Effect of the nonuniformity of temperatures on mixing in a narrowing nozzle: 1 - nozzle; L = 200 mm; 2 - nozzle; L = 400 mm; points - experiment.

The experiments carried out in the expanded part of a Laval nozzle (cone 400 mm long; diameters of entrance and exit 85.3 mm and 90.0 mm; subsonic part a with a length of 200 mm, Vitashinskiy profile) with pressures in the chamber which ensure supersonic ($M = 1.0$-1.4) and subsonic ($M = 0.79$-0.62) conditions showed that in this case the retarding of mixing due to compressibility is more considerable, especially for the supersonic conditions (Fig. 4). As follows from the graph, the experimental points lie nearer to the curves calculated under the assumption of gradual deformation (D = const). (To explain the actual mechanism for the deformation of turbulence with flow in the nozzles, it is necessary to perform the measurements of the fluctuating characteristics of velocity which behave substantially differently in the case of rapid and gradual deformation.)

We note that the mixing in a cylindrical duct corresponds to a value of the coefficient of turbulent diffusion several times larger than with established tube turbulence (the dotted lines in Figs. 2 and 4 correspond to $D/2r_{\text{н}}u_{\text{н}} = 0.006$ while for the established tube turbulence $D/2r_{\text{н}}u_{\text{н}} = 0.0009$ with $Re > 10^5$ [4]). This, apparently, is connected, apart from the large blocking of the flow (pylons, grids, etc.), with the presence of a boundary layer on the feeding tube which intensifies the mixing process of the jet.

During the analysis of the experiments, the source of the heated gas cannot be considered as a point source. In this case, it is possible to show that with a one-dimensional flow the distribution of concentration (excess stagnation temperature) over the cross section is described by the solution of the diffusion equation for a circular source which, with the insignificant effect of the walls on the concentration, has the following form:

[illegible] (4)

where $a(x) = a_0 r_\sigma(x)/r_{\text{н}}$; $a(x)$ is the radius of the jet in the current cross section of the nozzle and a_0 - the initial radius of the jet.

For determining the value of dispersion from the profile described by the P-function, from the equation which follows

$$\Delta T_{max} = \Delta T_0 [1 - \exp[-a(x)/2\sigma(x)]], \quad (5)$$

directly from (4), the ratio a/σ was found according to maximum temperature. The value σ was determined from the width of profile b with $\Delta T = \Delta T_{max}/2$. Used for this was the dependence of b/σ on a/σ which was obtained from the data of [5], where the P-function is tabulated. For the isothermal flow, dispersion can also be found directly from (5) if we make use of the expression for a(x).

For investigating the mixing of an nonisothermal jet, a unit was used which differs from heat depicted in Fig. 1 in the fact that the precombustion chamber was installed coaxially within the cylindrical chamber and the tube with length ≈0.3 m for the feeding of the jet was smooth (without heat insulation). This led to the fact that the slope of the straight line $\Sigma^2(x)$ with mixing in the cylinder corresponded almost half as much to the value of the coefficient of turbulent diffusion than in the preceding experiments.

The temperature of the jet was about 2000°K, and nonisothermicity at the nozzle entry was controlled by a change in the distance between the beginning of the jet and the nozzle entry (in the experiments: L = 170 mm, 300 mm, 450 mm). Experiments were carried out both in Vitoshinskiy nozzles with a length of 200 mm and 400 mm, with the above indicated cross sections with a critical pressure gradient and in a cylindrical compartment. For an example, Fig. 5 gives the results of experiments in a nozzle with a length of 200 mm with L = 300 mm. The amount of dispersion of the experimental profiles of the excess stagnation temperature which were approximated by the P-function was found from the width of the profile and the maximum temperature, as indicated above. A change in the dispersion along the nozzle in the absence of mixing was found from formulas (1) and (2); in this, the profile at the nozzle entry was replaced by Gaussian curve with some effective parameters a_* and σ_* - such that the maximum excess temperature and the width of profile would coincide respectively. From the parameters for Gaussian profiles calculated along the nozzle the dispersion σ^2_{co} and $\sum^2_{co}$ was determined for the profiles described by the P-function and having Gaussian width and maximum temperature.

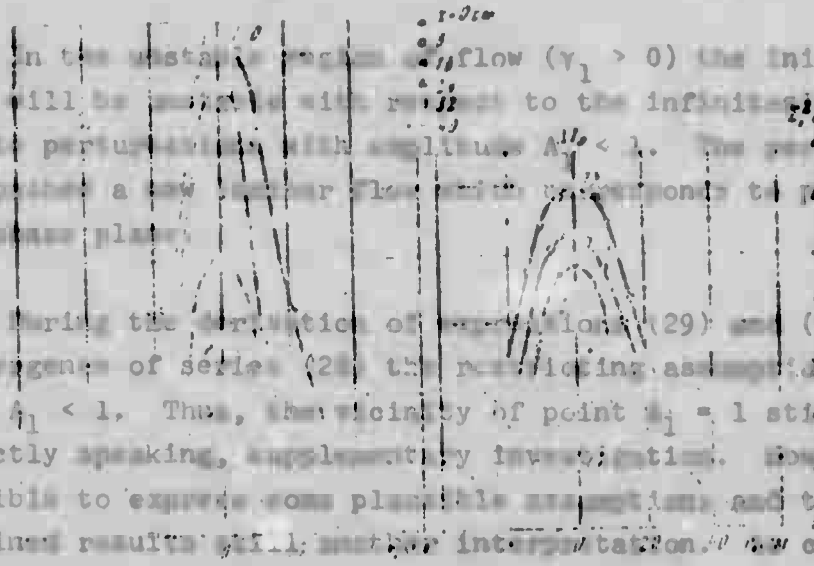

Fig. 7.

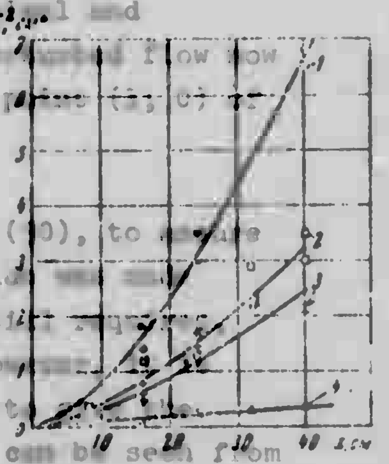

Fig. 8.

Fig. 7. Temperature profiles with mixing in a narrowing nozzle.

Fig. 8. The dependence of the parameters of mixing with a low level of initial turbulence:

$$\left.\begin{array}{l} 1 - \Delta T_0 = 1200^\circ;\ \Delta T_{max}(0) = 720^\circ \\ 2 - \Delta T_0 = 600^\circ;\ \Delta T_{max}(0) = 240^\circ \\ 3 - \Delta T_0 = 160^\circ;\ \Delta T_{max}(0) = 75^\circ \end{array}\right\} \text{Experiments in the nozzle; } \textstyle\sum^2 \text{ was determined from b and } \Delta T_{max}$$

$$\left.4 - \Delta T_0 = 160^\circ;\ \Delta T_{max}(0) = 76^\circ\right\} \text{Experiments in the cylinder; } \textstyle\sum^2 \text{ was determined from } \Delta T_{max}$$

As follows from Fig. 5, despite the presence of mixing $[\sigma^2_c(x) > \sigma^2_{co}(x)]$, the absolute value of dispersion in the nozzle decreases. It is interesting to note that the experimental values of $\sum^2(x)$ practically lie on a straight line. Figure 6 gives the dependence of the relation of the tangents of the angles of slope of straight lines approximating experimental values of $\sum^2$ in the nozzle and cylinder on the relation of the maximum and minimum temperatures at the entry to the nozzle. From the graph it follows that with an increase in the nonuniformity of temperatures the angle of slope increases, which indicates additional turbulence with flow in the nozzle.

Experiments were also carried out in the study of the process of the intensification of mixing with a nonisothermal flow in a nozzle with a reduced level of initial turbulence. For this purpose, on the unit depicted on Fig. 1, the entire flow before the entry nozzle was passed through a honeycomb having ducts with a diaemter of 2.5 mm and prepared from a long corrugated metal strip 0.5 mm thick and 40 mm wide. Figure 7 gives as an example the temperature profiles measured by differential thermocouples in a nozzle with a length of 400 mm with the initial overheating of the jet $\Delta T_0 = 160°$ and $\Delta T_0 = 1200°$ (on the nozzle edge M = 0.87; overheating was controlled by changing the flow rate of the combustion products from the precombustion chamber and air being mixed). From the drawing one can see that with great overheating the temperature falls more intensely. Figure 8 gives values of Σ^2 with mixing in a cylinder and nozzle with different overheatings; the great effect of the nonisothermicity of the flow on the mixing is evident. It is interesting to note that even in the case of small overheating in the nozzle the relatively small agitation of the flow takes place with damped turbulence which is imperceptible with large levels of turbulence (Fig. 2). This, apparently, is connected not only with the nonuniformity of velocities (which can attain 5-7%) due to nonisothermicity but also with the longitudinal velocity shift with the deviation of actual flow in the nozzle from a one-dimensional one.

The author thanks L. D. Kulikova who took a large part in the conduct of the experiments.

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ON THE MIXING OF COCURRENT JETS OF GASES OF DIFFERENT AND IDENTICAL DENSITY

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(Moscow)

The presently-existing semiempirical theories of free turbulent flows expounded in [1-3] are, in essence, differential methods of processing experimental data. Furthermore, measurements of the turbulence structure do not confirm the models forming the basis of these theories. Therefore, they cannot be used in cases not first investigated experimentally. Some information about free turbulent flows can be obtained using a dimensional analysis, considerations of symmetry, laws of conservation of mass and momentum, and also the experimental data on the structure of turbulence. But the full picture of the mixing of two flows can be obtained only on the basis of a correctly posed experiment. In view of the large number of parameters which affect mixing in a specific experiment and their simultaneous change, the isolation of dependence on a specific parameter, for example on the relationship of velocities, is not always possible. In many works on the jet flows changes in the initial conditions are not considered, which leads to different forms of the dependences being obtained.

Figure 1 gives the diagram of a working section. Central nozzles with a diameter of 40 mm and 30 mm and thickness of the edge of 0.4 mm were fastened to two pylons in tubes with a diameter of 300 mm and 120 mm respectively. For the deturbulence of the flows and change in conditions, at the entrance in the cocurrent flow and in the central nozzle fine-pored grids with the dimension of mesh from 0.07 mm to 0.3 mm and porosity from 0.35 to 0.5 respectively were installed. The measurements of the coefficient of turbulent diffusion D_T in the air flow behind such grids according to the measurements of the expansion of the thermal trace behind a heated wire with a diameter of 0.03 mm with the air speed of 15 m/s and normal temperature showed that it is close in value to the molecular coefficient of diffusion.

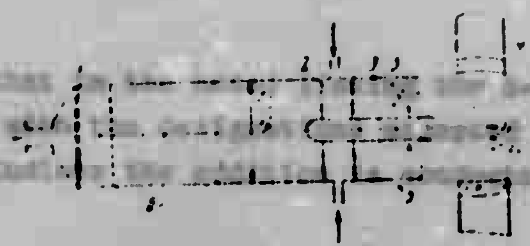

Fig. 1. Diagram of the working part of the unit. 1 – grid and felt, 2 – pylon, 3 – central nozzle, 4 – shadow instrument IAB-451, 5 – grid on the edge, 6 – fine-pored grid.

The setting of the general grid (or package of two-three grids) at the nozzle edge of the working section (see Fig. 1) effectively equalized the velocity profiles in the boundary layers on the edge. Setting the grids at different distances from the edge, it is possible to obtain the different dimensions of the boundary layers on the edge which determine the initial conditions of the experiment.

The geometric characteristics of the flow – the width of the zone of mixing and the length of the "nucleus" – were determined from the measurements of the temperature profiles and dynamic pressure. Taken as the boundary of the zone of mixing[1] were points of the profiles in which the relative excess of temperature ΔT or dynamic pressure $\Delta(\rho u^2/2)$, or velocity Δu equal 0.97 and 0.03. For the measurement of the temperature profile one of the flows was heated. The temperature drop between the flows was not more than 50°C. The measurements of the temperature fields in the flow were carried out with the use of chromel-copel thermocouples assembled in a comb of 30 pieces with the distance between them of 3 mm. The diameter of the joint of the thermocouple was 0.1 mm. The readings of the thermocouples were recorded on our EPP-09 recording potentiometer. The time for reading of one profile was 15 s. The profiles of the dynamic pressure were measured by a comb of 25 total pressure tubes whose readings were taken on an inclined multitube pressure gauge. Furthermore, on the nozzle edge the gas currents of different density were photographed by the shadow method or with the use of a Toepler tube. Velocities were changed in a range of 10-50 m/s. Air, helium and freon-12 were used as the working media.

The boundary layer effect can be illustrated by Fig. 2, where the dependence of the ordinate of the internal boundary of the zone of mixing y_1 (according to the relative velocity $\Delta u = 0.03$) on the relation of velocities m of the cocurrent flows of air at distances from the nozzle edge $x/d = 7.5$ is given (this value is different at different distances from the edge and increases with distance from it). In the first case the grid on the edge was absent and, on the external wall of the central nozzle, the boundary layer at a length of 250 mm grew. In the second case, on the edge a grid was installed with the dimensions of mesh 0.07 mm and a porosity of 0.35. As can be seen from the graph, in the absence of a grid at the edge, mixing stops depending on the relationship of velocities, beginning with $m = 0.5$.

[1]In each case, the value according to which the boundary of the zone of mixing is determined is specially stipulated.

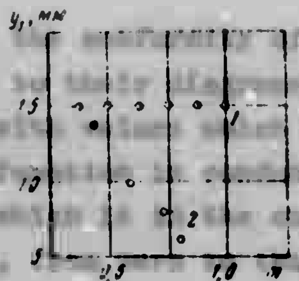

Fig. 2. Dependence of y_1 on the relation of the velocities of two jets with constant velocity of the central jet equal to 15 m/s; x/d = 7.5. 1 – without grid at the edge; 2 – with grid.

For homogeneous gases, the dependence of the width of the zone of mixing on the relation of velocities with a weak effect of initial conditions was investigated with grids installed on the nozzle edge. The boundary layer thickness in this case comprised less than 1 mm from each side of the edge. The corresponding dependence of the width of the zone of mixing determined from the relative excess velocity on the relationship of the velocities is given in Fig. 3. As is evident, the experimental results are well described by the linear dependence up to m = 0.95. In obtaining this dependence, it was considered that on a considerable part of the initial section of the jet the flow bears either a clearly expressed periodic character, or a transitional character to a turbulent one – this section was eliminated from examination. Its length was determined from the instantaneous photographs of the flow.

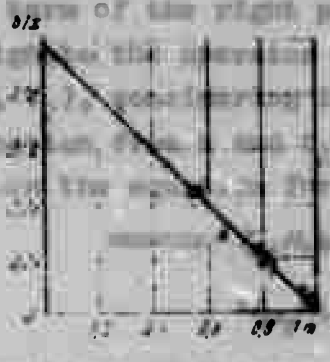

Fig. 3. The dependence of the relation b/x on $m = u_1/u_2$ with $n = \rho_2/\rho_1 = 1$ [$b/x = \phi(n)$ $(1 - m)$, where $(1) = 0.2$, and $b = y_1 + y_2$].

In connection with the above indicated strong influence of the initial conditions (to which the relative misalignment of the two flows should also pertain) and absence in different works of the necessary information about them, it is difficult to conduct a comparison of different experimental data. In some works a number of dependences of the width of the zone of mixing on relations of velocities m and densities n obtained under different assumptions relative to the model of the flow in the layer of mixing is proposed. Thus, in works [1] and [4] it is assumed that the relation of the width of the zone of mixing to the distance from the edge is proportional to the relation of the two velocities: the transverse displacement is proportional to the modulus of the difference in the velocities of the two flows $\Delta u = |u_1 - u_2|$, and longitudinal – to some average velocity for the zone of mixing determined according to the formula

$$u^* = \frac{\int \rho u u\, dy}{\int \rho u\, dy}.$$

Then we have: $b/x \sim \Delta u/u^*$. With the mixing of homogeneous flows this leads to the relation $b/x \sim (1 - m)/(1 + m)$ which is not confirmed in the experiments of the authors of this work. Also not confirmed is the dependence of the width of the zone of mixing on the density ratio for the submerged jets which follows from this relationship. From it it follows that with the mixing of a jet in the atmosphere of a lighter gas, the length of the "nucleus" cannot increase as compared with the mixing of homogeneous flows more than two-fold, and the width of the zone of mixing of the jet in an atmosphere of heavier gas exceeds considerably that obtained in the experiments. In the experiments of the authors the length of the "nucleus" of the submerged jet of freon in an atmosphere of air increases approximately 3.5 times.

From the above indicated assumptions it follows that the dependence of mixing on n is different for different m, i.e., with equal Δu but with different absolute velocities. However, it is more reasonable to assume that the features of the interaction of the two flows will be determined only by their relative motion if, of course, mixing is not influenced by th initial nonuniformities of velocity and initial turbulence. This assumption, taking into account the experimental results, makes the foregoing dependences unacceptable.

If one assumes that the dependence of mixing on the density ratio with an equal difference in velocities does not depend on the absolute values of velocity, then for the width of the zone of mixing we obtain:

$$b = (x - x_0)(1 - m)\varphi(n).$$

where the x_0 is the effective origin of the coordinates determined by the conditions of transition to the turbulent flow and $f(m)$ and $\phi(n)$ are some functions which depend respectively only on m and n.

The linear dependence of the width of a flat zone of displacement on x is obtained from dimensional analysis, but dimensional analysis provides no information relative to the type of function $f(m)$ and $\phi(n)$. However, if one assumed that the mixing is entirely determined by the difference in the velocities, and the lesser of them accomplishes a shift of the picture of flow in the flow direction, then we obtain:

$$b = (x - x_0) f(m)\varphi(n).$$

In the derivation of this dependence it was assumed that with an identical difference in velocities the dimensions of the zones of mixing with flows with different absolute velocities are equal with the same values of coordinate $x - x_0$, in which the longitudinal mixings of unperturbed particles of one flow are equal relative to the unperturbed particles of the other. The linear dependence of the width of the zone of mixing on the relation of velocities obtained under such assumptions is confirmed well experimentally with n = 1 (see Fig. 3). From these assumptions it follows that the dependence of the width of the zone of mixing on the density ratio is identical with any relationship of velocities. Therefore, the experimental determination of the function $\phi(n)$ can be conducted with any relation of velocities, including in submerged jets ($m = 0$), which was also done by the authors.

The study was carried out in the range of the density ratio $1/n$ from 1/30 (jet of freon in an atmosphere of helium) to 30 (jet of helium in an atmosphere of freon). The central jet was heated, and the width of the zone of shift was determined from the temperature profile. For a decrease in the heat exchange and temperature boundary layers on the dividing edge the flows were heat-insulated with the aid of a thermal insulation case slipped over the central nozzle. Measurements in each cross section were carried out on the edge of the tube. In individual experiments the value of m was somewhat more than zero, but it never exceeded 0.05.

As can be seen from Fig. 4, the intensity of the mixing of a heavy jet in the atmosphere of a light gas decreases sharply in comparison with the intensity of the mixing of homogeneous flows ($n = 1$), and with a decrease in the jet density as compared with the cocurrent flow the intensity of mixing increases very weakly. This can be explained by the fact that the intensity of mixing is determined by processes on the interface between turbulent and nonturbulent liquids, on which the density ratio is changed sharply with the blowing-in of a heavy jet into the atmosphere of a lighter gas and weakly in the opposite case.

Figure 5 presents the profiles of the relative dynamic pressure and relative temperature for a jet of freon in an atmosphere of air. Clearly, it can be seen that the profile of dynamic pressure is narrower than the temperature profile in the

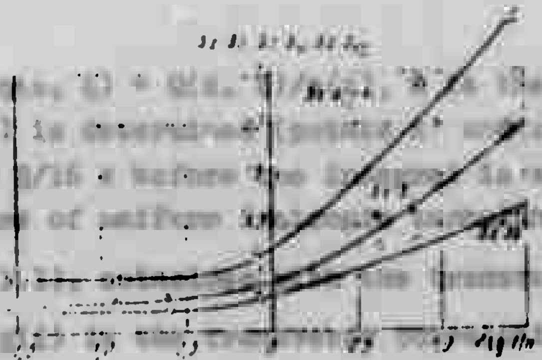

Fig. 4. Dependence of the width of the zone of mixing on the density ratio $= \rho_{атм}/\rho_{jet} =$ with m = 0.

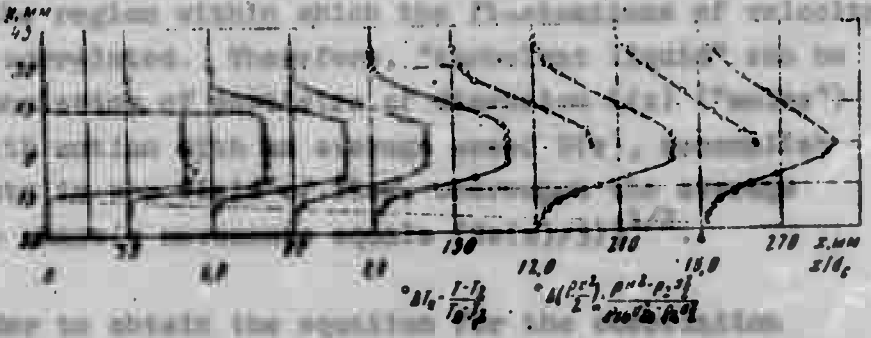

Fig. 5. Profiles of relative temperature ΔT and relative dynamic pressure $\Delta(\rho u^2/2)$ on the cross section of a submerged jet of freon in the air at different distances from the nozzle edge (u_{10} = = 10.4 m/s, T_{10} = 310°K).

exterior portion of the zone of mixing, whereas in the inner part of their boundaries they coincide, i.e., the velocity profile is narrower than the temperature profile whose boundaries coincide with the boundaries of the profile of concentrations. Consequently, in the case of different densities the velocity profile is narrower than the profile of concentration, too. This leads to the nonmonotonic change of the profile of dynamic pressure in the zone of the mixing of two flows of different density when the difference in dynamic pressures is small and the dynamic pressure in a light gas is greater than in a heavy gas. Figure 6 gives the profiles of dynamic pressure and temperature in the zone of mixing of a jet of helium with a cocurrent flow of air with equal dynamic pressures. These results confirm the agreement of the boundaries of the profiles of temperature and density.

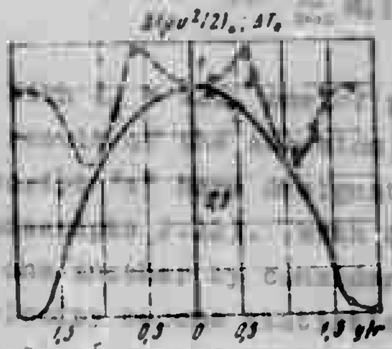

Fig. 6. Profiles of dynamic pressure and temperature with equal dynamic pressures of the central jet of helium and cocurrent flow of air ($x = 102$ mm, $u_{air} = 14.2$ m/s).

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DIFFUSION AND VORTEX MODELS OF A TURBULENT JET

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In the practice of engineering calculations we widely use the so-called diffusion model of a jet which, in individual details, is close to the semiempirical models of mixing in turbulent jets [1, 2], while in its mathematical basis it uses known ideas and representations of contemporary statistical theory of turbulent diffusion [3-5]. The following parameters are introduced in the diffusion model of a jet [6, 7]: the dispersion $\sigma^2(x)$ of the liquid particle of the jet which consists of the dispersion of purely convective transfer $\sigma_T^2(x)$, and dispersion of gradient (molecular) diffusion $\sigma_c^2(x)$, and the mean trajectory of the boundary stream a_T. Introduced further is the concept of the probability of appearance P_2 of the substance of the jet (cocurrent flow) which satisfies the equation of diffusion with the corresponding boundary and initial conditions. Similar parameters $\sigma_v^2(x)$ and a_v are also introduced for the velocity field. The given are assumed to be dispersion $\sigma^2(x)$, Pr number ($Pr = \sigma_v^2/\sigma_T^2$) and the degree of homogeneity of mixing $N = \sigma_c/\sigma$. Parameters a_T and a_v are determined from the integral laws of conservation of mass and momentum [6].

Let us present for an example the formulas for the calculation of the layer of mixing between two plane-parallel nonisothermal flows. Let the density, velocity and temperature of the first and second flows equal respectively ρ_{01}, v_1, T_1 and ρ_{02}, v_2, T_2. The average partial densities are equal to $\bar\rho_1$ and $\bar\rho_2$.

With the usual assumptions, the mixing is described by the equation of turbulent diffusion of the form

$$u\frac{\partial P_1}{\partial x} + v\frac{\partial P_1}{\partial y} = \frac{dz^2}{2dx}\frac{\partial^2 P_1}{\partial y^2}\bar{u}_{cp}, \qquad (1)$$

(where $\bar u_{cp}$ – the average longitudinal velocity in the layer of mixing), whose solution with initial conditions:

$$x = 0: \quad P_1 = 1 \text{ with } y < 0, \qquad P_1 = 0 \text{ with } y > 0$$

and boundary conditions:

$$y = -\infty: \quad P_1 = 1, \qquad y = \infty: \quad P_1 = 0$$

has the form

$$P_1 = \frac{1}{2}\left[1 - \Phi\left(\frac{y}{a_1 x}\right)\right]. \quad [illegible] \qquad (2)$$

By analogy, the velocity profile is written in the form

$$\frac{u}{u_1} = \frac{1}{2}\left[1 - \Phi\left(\frac{y}{a_2 x}\right)\right] \ [illegible] \ \frac{2}{\sqrt{\pi}}\int [illegible]\, dt. \qquad (3)$$

The integral laws of conservation of the flows of substance and momentum written on the assumption that the transverse velocity when y = -∞ is equal to zero are the following:

$$\int_{-\infty}^{\infty} u P_2\, dy = 0. \qquad (4)$$

$$\int_{-\infty}^{0} [illegible]\, dy + \int_{0}^{\infty} [illegible]\, dy = 0 \qquad (5)$$

$$(\rho = \bar\rho_1 + \bar\rho_2 = \rho_{01}P_1 + \rho_{02}P_2).$$

They make it possible to establish the following connections between the diffusion parameters:

$$\frac{a_T}{\sqrt{\sigma^2+\sigma_0^2}} - (1-m)\Psi(x) = 0, \tag{6}$$

$$\Psi(x) = \frac{ny + (1-m)(1-n)\Psi(y) - mn\sqrt{\dfrac{p_T}{\pi\left(1+\dfrac{\sigma_T^2}{\sigma^2}\right)}}}{\sqrt{\dfrac{1+p_T}{1+\dfrac{\sigma_T^2}{\sigma^2}}}\,(1-2m+mn)}, \tag{7}$$

where $m = v_1/v_2$, $n = \rho_{01}/\rho_{02}$. Furthermore, the designations are introduced:

$$\Psi(x) = \frac{x}{2}[1+\Phi(x)] + \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}, \quad x = \frac{a_T - \bar{a}_T}{\sqrt{\sigma^2+\sigma_0^2}}, \quad \sigma_T = \frac{e^{1/\pi}\sigma_T}{1+\Phi\left(\frac{1}{\sqrt{\pi}}\right)},$$

$$y = x\sqrt{\frac{1+p_T}{1+\sigma_T^2/\sigma^2}} + \sqrt{\frac{p_T}{\pi(1+\sigma_T^2/\sigma^2)}}.$$

Formulas (6) and (7) in the particular case $n = 1$ give the simple relationship:

$$a_0 = \frac{1-m}{\sqrt{\pi}}\sigma_0 \tag{7'}$$

For the determination of the temperature profile the following "two-stage" model is used.[1] First the quasi-laminar temperature profile with uniform mixing with dispersion σ_c^2 is found:

$$w_{2D} = \frac{T_c - T_1}{T_2 - T_1}\frac{\rho_{2c}}{\rho_{1c}+\rho_{2c}} = \frac{P_{2c}}{nP_{1c}+P_{2c}}.$$

[1]Its introduction is necessary during the study of the mixing of jets of substantially different densities and temperatures, and also to account for the nonuniformity of mixing during the calculations of combustion and radiation in jets.

Then the true average profile is found on the assumption that the quasi-laminar profile is transferred randomly according to the normal law with dispersion $\sigma_T^2 = \sigma^2 - \sigma_C^2$:

$$\frac{\bar{T}-T_1}{T_2-T_1}=\int_{-\infty}^{\infty}\frac{T_L-T_1}{T_2-T_1}\,P(y,y_m)\,dy_m \qquad (8)$$

where

$$P(y,y_m)=\frac{1}{\sqrt{2\pi}\,\sigma_T}\,e^{-\frac{(y-y_m)^2}{2\sigma_T^2}}.$$

Analogously it is possible to obtain the expression for the mean-square fluctuation of temperature:

$$\frac{\overline{(T-\bar{T})^2}}{(T_2-T_1)^2}=\int_{-\infty}^{\infty}\left[\frac{T_L-T_1}{T_2-T_1}-\frac{\bar{T}-T_1}{T_2-T_1}\right]^2 P(y,y_m)\,dy_m$$

The comparison of the results of the calculation with the experimental data for a series of problems (circular jet, plane jet at the wall, distributed blowing-in from the walls, pressure behind an offset with the turning of a supersonic flow) is presented on Figs. 1-5. A diffusion model was used also for the calculation of diffusion with the flow of a jet in a nozzle, the Coanda effect, diffusion flames and other technical problems with the same satisfactory agreement with the experiment.

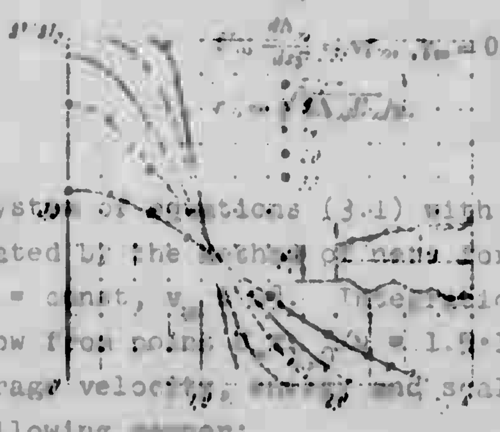

Fig. 1. Change of temperature in a cocurrent circular jet; I. B. Palatnik's experiments (continuous lines – calculation; $\Delta T_0 = 25°K$).

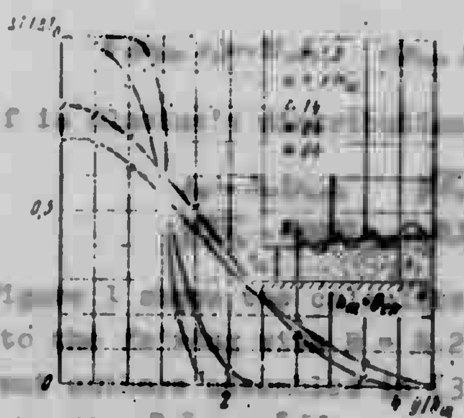

Fig. 2.

Fig. 3.

Fig. 2. Change of temperature in a cocurrent jet at a wall; V. Ya. Borodachev's experiments (continuous lines – calculation; $h_ш = 5$ mm, $\Delta T_0 = 690°K$).

Fig. 3. Axial velocity in a circular submerged jet. The dotted line – experimental data of Korsin for an air jet; the small circles are Forstoll's experimental data for a water jet; the continuous line – calculation with $\sigma/a = 0.095(x/d - 2)$, Pr = 0.5).

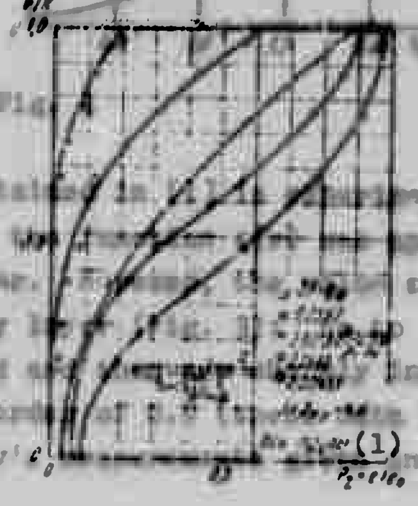

Fig. 4. Profiles of concentration for various blow-ins; experiments of I. V. Bespalov and A. M. Gubertov (continuous lines – calculation). KEY: (1) Axis of tube.

Everywhere the turbulent number Pr was taken as Pr = 0.5. The degree of uniformity of mixing N, determined for the first time for gases by N. A. Zamyatina [8, 9] with the aid of the optical-diffusion method, was taken as equal to N = 0.4 (gas - gas) and N = 0 (gas - liquid). The only determining parameter was the dispersion of the jet.

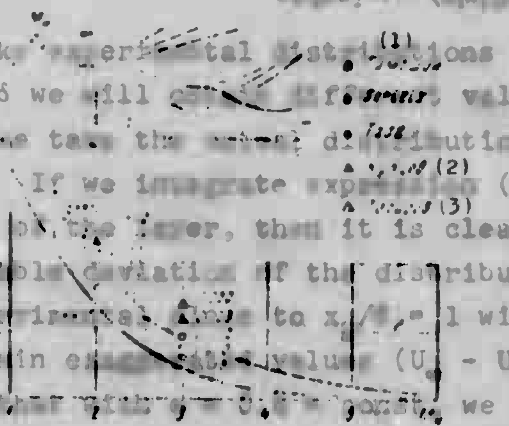

Fig. 5. Base pressure behind a flat offset (dotted line - Korst's calculation: $\sigma^*/h = 0$; continuous - S. Kh. Oganesyan's calculation according to the diffusion model of mixing: $\delta^{**}/h = 0$). KEY: (1) Timoshin; (2) Kuptsov; (3) Tagirov.

The dispersion of a jet on the basic section depends only on relationship of velocities (the parameter of cocurrence m) and for all the above-enumerated problems proves to be identical with the identical value of parameter m with an accuracy up to the length of the initial section. All other factors - difference in densities, initial turbulence, twisting of the jet, boundary layer thickness, acoustic effects, etc., - affect basically only the length of the initial section of the jet. For example, for the subsonic velocities dispersion on the basic section satisfies the following calculated relationship:

$$\sigma^2 = \sigma_0^2 + k^2(x - x_0)^2. \qquad (9)$$

Here σ_0 is dispersion with m = 1; $\sigma_0^2 = \varepsilon_1^2 x^2$ with $x < 2x'$; $\sigma_0^2 = 2D_1/v_1 \, x(x - x')$ with $x \geq 2x'$; where $x' = D_1/2v_1\varepsilon_1^2$; $D_1/v_1 d_1 \approx 0.0009$, and ε_1, D_1, v_1 - the level of turbulence, the diffusion coefficient and velocity of the external flow respectively; $x_0 \approx -d_0$ for n = 1 and the initial tube profile of velocity; d_1 - the diameter of the tube; d_0 - the diameter of the jet, and the values of k can be approximated by the dependences: $k = k_T(1 - m)$, $k_T = 0.09$ for $m < 1$ and $k = k_T(1 - m)$, $k_T = 0.06$ for $1 < m \leq 2$.

However, in the diffusion model, as in all semiempirical models, the characteristics of mixing are used which are borrowed from the experiment; this model does not make it possible to disclose the physical nature of the diffusion of a jet and, consequently, in principle it does not make it possible to theoretically determine the dispersion of a jet. Furthermore, this model does not provide a satisfactory description of the velocity field in the examination of semibounded jets and the boundary layer and is completely unsuitable, just as any other semiempirical models, for the "closing" of the equation of the conservation of energy (with the calculation of losses of total pressure).

Therefore, more promising (for the development of semiempirical theories of the "old type") it seems to us, is another, so-called vortex model. We note that recently a deeper penetration into the vortex nature of turbulent flows with a transverse shift is characteristic of many investigators [10, 11].

Figure 6a presents a diagram of a proposed vortex model for a free turbulent layer. The wave amplitude of the perturbation of a tangential velocity discontinuity increases up to the value of the order of the radius of the vortex [$\zeta(t) \approx l_B$], whereupon the wave is rolled up into the vortex. The nature of the change in ζ in the course of time is presented in Fig. 6b; the simplest approximation of the dependence $\zeta(t)$ is given. A more precise approximation, taking into account nonlinearity according to Landau [12], will not change the essence of subsequent reasonings. On curve $\zeta = \zeta(t)$ (Fig. 6b) it is possible to note two characteristic scales: the lifetime of the discontinuity τ_p and the time of the vortex formation $\tau_{o.B.}$, which satisfy the following relationships:

$$\tau_p = \frac{1}{\lambda_\tau}\ln\left(\frac{l_B}{\zeta_0}\right) \text{ and } \tau_{o.B} = \frac{1}{\lambda_\tau}, \qquad (10)$$

where λ_τ is the increment of an increase in the disturbance wave in the course of time in the coordinate system connected with the wave. The transition to the corresponding scales of length in a motionless coordinate system is evident: $x_p = v_\phi\tau_p$ and $x_{o.B} = v_\phi\tau_{o.B}$ (v_ϕ – the phase velocity of a wave of a "drift"-type disturbance, ζ_0 – the amplitude of the initial disturbance of the discontinuity surface). The value of ζ_0, just as the scale l_B, is a random function on the strength of which the scale τ_p is also a random function. We will obtain its average value, substituting the expression ζ_0 given in [12] through the fluctuations of velocity on the initial section and averaging:

$$x_p = \frac{1}{\lambda_x}\ln\left(2\sqrt{1+\frac{\Delta u^2}{\varepsilon_1^2u_1^2+\varepsilon_2^2u_2^2}}\right). \qquad (11)$$

where λ_x is the increment of an increase in the disturbance wave of the "drift" type. Experiments [16] show that the maximum level of "noises" under normal conditions does not exceed $\varepsilon_1 = \varepsilon_2 = \varepsilon_{max} = 0.25$, so that $x_p/x_{o.B} \approx 3$.

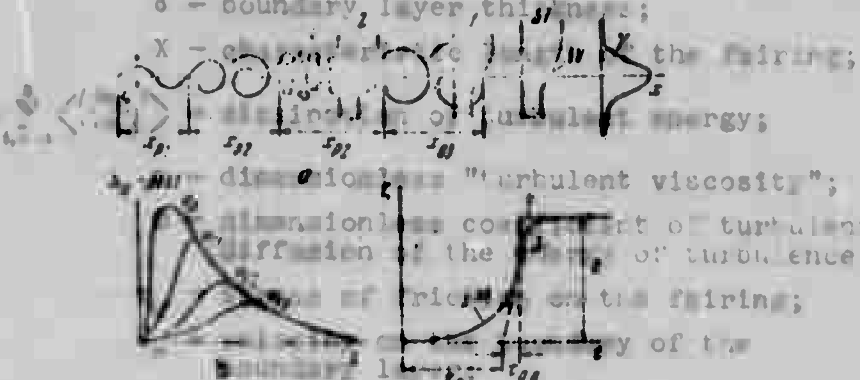

Fig. 6. Vortex model (free layer of mixing). 1 – velocity profile in a large vortex; 2 – velocity profile in a trace of breakdown; 3 – increase in amplitude of perturbation of the discontinuity surface; 4 – model approximation.

The vortex as a single liquid volume (mole) is separated from the remaining flow by an unstable discontinuity surface; for a vortex there are two characteristic scales: the lifetime of the vortex $\tau_{\text{в}}$ and the time of its breakdown $\tau_{\text{р.в}}$ and, correspondingly, scales $x_{\text{в}} = v_{\text{в}}\tau_{\text{в}}$ and $x_{\text{р.в}} = v_{\text{в}}\tau_{\text{р.в}}$ ($v_{\text{в}}$ - the speed of movement of the center of mass of the vortex). By virtue of conditions (10) the vortex formation occurs almost instantly, which is equivalent to the inelastic collision of two liquid volumes. The laws of conservation of moment, momentum and the position of the center of mass for a vortex up to and after "collision" provide for a free boundary layer (center of the disturbance wave lies on the x-axis, see Fig. 6a):

$$\omega = \frac{\rho_1\rho_2 l_{\text{в}} \Delta u}{\rho_{cp}^2 4 l_1^2} \ (a), \qquad v_{\text{в}} = \frac{\rho_1 u_1 + \rho_2 u_2}{\rho_1 + \rho_2} \ (b), \qquad y_c = \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)\frac{l_{\text{в}}}{2} \ (c), \tag{12}$$

where $\rho_{cp} = \rho_1 + \rho_2/2$, l_1 is the radius of inertia of the vortex and ω is the angular velocity of the vortex.

For the near-wall layer (center of vortex lies at distance $l_{\text{в}}$ from wall):

$$\omega = \frac{l_2}{l_1}\frac{v_{\text{в}}}{2l_1} \ (a), \qquad v_{\text{в}} = \frac{\rho_1 v_1}{\rho_1 + \left(\frac{l_2}{2l_{\text{в}} - l_2}\right)\rho_2} \ (b). \qquad y_c = l_{\text{в}} \ (c), \tag{13}$$

where ρ_1 and v_1 are respectively the density and the flow rate far from the wall, ρ_2 - density near the wall, l_2 - the momentum thickness in the boundary layer up to the formation of a large vortex, and ω - the angular velocity of the vortex.

From the law of conservation of energy it is also possible to determine total pressure losses in a large vortex and trace of breakdown. From relationships (12) and (13) it can be seen that in a free layer a large vortex moves practically without slipping (the profile of longitudinal velocity takes the form shown on Fig. 6a), and in the boundary layer (since $l_1 \sim l_{\text{в}} >> l_2$) the vortex moves with slippage, forming a laminar sublayer from below and from above. The profile of the volumetric concentration in this case coincides with the profile of longitudinal velocity (Prandtl turbulence number in the vortex equals unit: $Pr_в \approx 1$; Fig. 7a).

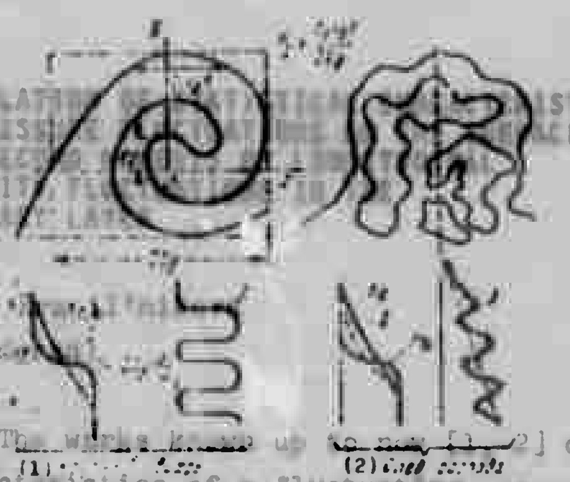

Fig. 7. Diagram of mixing in a large vortex and trace of breakdown. 1a - the distribution of volumetric concentrations; 1b - the distribution of longitudinal velocities; 2 - mathematical expectation of intermediate compositions; 3 - field of instantaneous concentrations.
KEY: (1) Large vortex; (2) Trace of breakdown.

The cascade breakdown of a large vortex into a series of fine vortices with uniform and isotropic distribution (zero moment relative to the center of mass) provides the so-called trace of breakdown (turbulent spot) of the same scale $l_в$. The problem of the determination of the properties of the turbulent spot in many ways is identical to the problem of the breakdown of turbulence behind a grid. Therefore, the mean-square rate of fluctuations in the turbulent spot $\sqrt{v_p'^2}$ in time t after the breakdown of a large vortex and the coefficient of microturbulent viscosity (diffusion) in the turbulent spot $\nu_T \approx D_T$ can be taken from the experiments [11, 13]:

$$[illegible] \qquad (14)$$

($\bar{v}_p$ - the average velocity of the motion of the trace of breakdown).

The profile of the average concentration (temperature) in the trace of breakdown is not reconstructed but continues to expand with time under the action of microturbulent gradient diffusion with coefficient D_{MT}. The profile of longitudinal velocity can be reconstructed so that according to the laws of conservation of the quantity and moment of momentum the upper part of the volume of a large vortex which has broken down will move at the velocity of the upper flow ($v_{p1} \approx V_2$), and the lower – with the velocity of lower flow ($v_{p2} \approx V_1$), i.e., the field of longitudinal velocities after breakdown in principle can return to the initial state (curve 1b on Fig. 7b). The average profiles of concentration (temperature) and velocity at an arbitrary point of a turbulent layer are determined by the type of the function $p(l)$ – the density of distribution of the probability of scales l, and also by the probability of the appearance of a large vortex in an arbitrary cross section x of layer P_B, or by the probability of the appearance of a trace of breakdown P_p. Function $P_B(x, Re_\Delta)$ [or $P_p(x, Re_\Delta)$] with small x is sufficiently complex. However with an increase in x, where the number $Re_\Delta = \Delta u l/\nu$ will become sufficiently large, and the probability of the appearance of laminar spots is sufficiently small, these functions take the form

$$P_B = \frac{\bar{z}_{Bl}}{\bar{z}_{pl} + \bar{z}_{Bd}}, \qquad P_p = \frac{\bar{z}_{pl}}{\bar{z}_{pl} + \bar{z}_{Bd}}. \tag{15}$$

Estimates according to a number of indirect experimental observations and also according to some considerations which follow from linear theory, permit assuming that P_p and P_B are values of one order, i.e.,

$$P_p = P_B \approx 0.5. \tag{15a}$$

The function $p(l)$ in type is close to a Maxwellian distribution; it is either proportional to the function of the increment of an increase in the perturbations of the "drift" type of wavelength L (i.e., $p(l_{s}) = \lambda_{x}(a \cdot L)$ – a coefficient of the order of unity; preliminary estimates according to [16] give $a \approx 0.3$-0.5), or it is a function of this increment (depending on the type of spectrum of initial perturbations). The features of curve $p(l)$ are such that the greatest probability will be with scale l_{m} with the increment close to λ_{max} which decreases with an increase in the microturbulent viscosity for the envelope λ_{x} = [illegible] – the curve of the increment for an infinitely fine surface of a tangential discontinuity (see Fig. 6c). By virtue of this the mean value of the scale $\bar{l} \approx l_{m}$ for a free layer satisfies the following relationship:

$$[illegible] \qquad (16)$$

whereupon $\Delta v \bar{l}_{n}(x)/\nu_{\tau} \approx 250$-$290$ – according to the data of [13]; $Re_{\lambda} = \Delta u l_{n+1}/\nu_{\tau\, n} \approx 380$ – according to data for processing solutions [14] for the case $n = 1$; $m \approx 0$ and the laminar thickness of the discontinuity surface; $P_{p} = 0.5$ according to condition (15a); $x_{s} \sim 1/\lambda_{max}$ according to condition (11). In this relationship, the the dependence on n is considerable only for the first large vortices ($x \leq x_{p1}$); during subsequent formations of large vortices from the traces of breakdown the value of n is close to unity by virtue of sufficient uniformity of the composition in the trace ($\rho_{1} \approx \rho_{2} \approx \rho_{cp}$).*

We will dwell now on the physical interpretation of the parameters of a diffusion model from the viewpoint of the vortex model. The scale of the width of the profile of concentration is not changed from the breakdown of a large vortex and is equal to the scale of large vortices, so that always $c \sim \bar{l}$. The scale of the field of longitudinal velocities changes with the breakdown of the vortex from scale $\bar{l}$ to scale $l_{p} \ll \bar{l}$ (l_{p} – the scale of vortices in the trace of breakdown, it is the scale of the thickness of the new "discontinuity surface"), so that

*[Translator's note: cp = average].

$$z_a \approx l P_a - l_p P_p \approx l_a P_a \qquad \text{with } l_p \ll \bar{l}$$

$$\text{and } P_f \approx P_a^2 + 2 P_a P_p \frac{l_p}{l_a} \approx 0{,}5 \qquad \text{with } \Delta l_p \ll l_a. \tag{17}$$

The degree of uniformity of mixing N is not the same, either: within the limit ($\tau_{o.p}/\tau_B \ll 1$ and $l_p \ll \bar{l}$) it is close to zero and unity respectively in a large vortex and trace of breakdown (see Fig 7b), so that

$$\bar{N} = N_a P_a + N_p P_p \approx P_p N_p \approx 0{,}5 \text{ with } N_p \approx 1. \tag{18}$$

The average shift of the field of concentrations along the **y-axis (parameter a_T) is explained by the average stagnation** of **the volumes of a more rapid flow; the shift of the field** of longitudinal velocities (parameter a_V) is caused by the fact that the centers of vortices y_c and **the** sections of discontinuity do not lie on the **x-axis; their** mean **position** is determined **by** the integral law of conservation of the impulse flow.

In conclusion, let us illustrate the analytical possibilities of the model with its agreement with the experiment. The field of average velocities is determined by the relationship

$$\bar{u} = P_a \int_0^\infty \bar{u}_a \frac{l}{\bar{l}} p(l)\, dl + P_p \int_0^\infty \bar{u}_p \frac{l_p}{\bar{l}_p} p(l_p)\, dl_p, \tag{19}$$

where

$$p(l)\,dl = \lim_{N_0 \to \infty} \frac{n(l)\,dl}{\int_0^\infty n(l)\,dl} \approx \dots\, dl;$$

$N_0 = \int_0^\infty n(l)\,dl$ is the overall number of vortices which pass through **cross section x during infinitely large time t; $\bar{l} = \int_0^\infty l p(l)\, dl$; $\frac{l}{\bar{l}} = \frac{2l/r_a}{2\bar{l}/r_a}$** – **the time** (relative) of the **passage** of a vortex **with dimension** $2l$ through the cross section x; $\bar{u}_a(y - y_c)$ – the profile of the longitudinal average velocity in a large vortex; u_p – in the trace of breakdown.

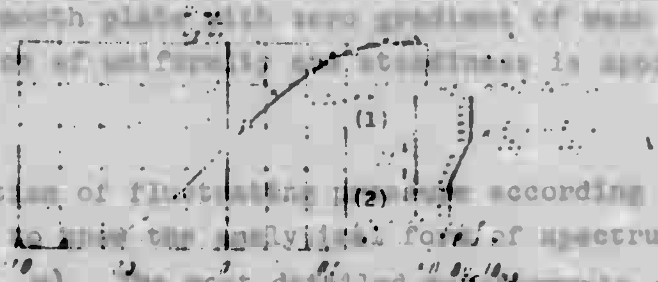

Fig. 8. Layer of mixing: experiments of Zhestkov and Albertson ($2a_0$ = 25.4 mm; v_2 = 44 m/s; σ_v = = 0.75 $\sqrt{t}$ = 4.7)
KEY: (1) Calculation; (2) Velocity distribution in the vortex.

Figure 8 presents the results of the calculation of the free layer of mixing and the experimental points [15]. Figure 9 gives the experimental profile of the average velocity at the wall [10] and the simplest calculated version of the model (absence of the slippage of the vortex with the maximum slippage of the trace of breakdown)

$$\frac{u}{u_0} = P_p \left\{ \sqrt{\frac{\pi}{8}} \frac{y}{2\sigma_p} \left[1 - \Phi\left(\frac{y}{2\sigma_p}\right) \right] + 1 - e^{-\frac{y^2}{8\sigma_p^2}} \right\} + (1 - P_p) \left\{ \sqrt{\frac{\pi}{8}} \frac{y}{2\sigma_t} \left[1 - \Phi\left(\frac{y}{2\sigma_t}\right) \right] + 1 - e^{-\frac{y^2}{8\sigma_t^2}} \right\}$$

(x = 76 cm, u_0 = 1280 cm/s; σ_p = 0,605 mm, σ_t = 6,3 mm, P_p = 0,67).

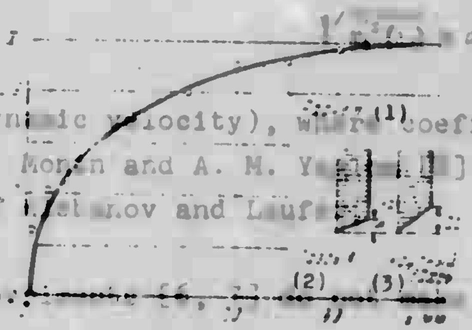

Fig. 9. Turbulent boundary layer; A. A. Townsend's experiment.
KEY: (1) Calculation; (2) Discontinuity; (3) Large vortex.

Consideration of the discontinuity (more precisely, the laminar sublayers on the lower and uppers boundaries of the vortex and trace of breakdown) provides profile $\overline{u}$ with the characteristic inflection point which disappears with an increase in the dispersion of scales l for x.

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STUDIES OF TURBULENT CONFINED AND OPEN FLOWS ACCOMPLISHED IN THE INSTITUTE OF HYDROMECHANICS OF THE ACADEMY OF SCIENCES OF THE UKRAINIAN SSR

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Review

In the Institute of Hydromechanics of the Academy of Sciences (AN) of the Ukrainian SSR, for a number of years studies of turbulent flows with a free surface and in delivery pipes have been accomplished, and studies of a turbulent boundary layer are also developing. Considerable attention is given to the experimental studies. In connection with the fact that at the present time in the USSR, unfortunately, there is no sufficient practice in the reliable use of instruments of the hot-wire anemometer type for dropping liquids, during the studies the following were basically used: 1) the method of visual study of a flow by means of the introduction, into the flow, of solid or liquid particles - indicators with cinematographic photography of the flow and subsequent statistical processing of the photographs (subsequently, we will use for it the shortened names: photographic and cinema method); 2) one- and two-component velocity sensors based on the principle of the dynamic effect of a flow on bodies introduced into it.

The photographic method was developed in the USSR by M. A. Velikanov, N. P. Zrelov, B. A. Fidman and I. K. Nikitin. I. K. Nikitin [1] proposed using solid particles as indicators — balls from a mixture of paraffin with whiting that has identical volume weight to water, and defatted fine aluminum powder with particle dimensions of 10-100 μ. Illumination during photography is accomplished at specific time intervals by a flash bulb with power supply from a pulse generator. With photographing in mutually perpendicular planes, it is possible to obtain a three-dimensional picture of the flow.

In order to determine possible errors with the photographic method with the use of aluminum powder as the indicators, E. V. Zalutsky accomplished an analysis of the structure of turbulent flows with a shift by the following scheme. Following Townsend [2], the turbulent flow was considered as the superposition, on the main flow, of simplest vortex structures of different scales and intensities. Using Laufer's data [3] on the energy spectrum of a turbulent flow with a shift, it was possible to obtain the representation of scales and velocities of different vortices. Then, having made use of Favre's works [4], it was possible to determine the possible deviation of the trajectories of solid particles of different dimensions from the trajectories of liquid particles.

The analysis showed that for conditions under which laboratory investigations of water flows with velocities up to 1 m/s and linear dimensions (depths) on the order of 0.3-0.5 m (which corresponds to Reynolds numbers on the order of $1 \cdot 10^5 - 5 \cdot 10^5$) are usually accomplished, with the use of aluminum powder it is possible to catch vortex structures which also approach turbulence for size and microscale.

The advantages of the visual method are the absence of noticeable distortions of flow, simplicity of equipment, the possibility of measurements in flows with a high intensity of turbulence, and also the possibility of making measurements in the immediate proximity of the walls. The main deficiency of the method – the laborious processing of the results. The question of the automation of the processing of the results of the photographic method and direct input of information into a computer thus far does not have a satisfactory solution and it should be the subject of further research.

One of the instruments based on the dynamic effect of flow is a ball two-component velocity sensor, investigated in detail in the Institute of Hydromechanics of the AS of the Ukrainian SSR by B. M. Yegidis [5]. A diagram of the instrument, which makes it possible to simultaneously measure two velocity components at one point, is shown in Fig. 1. Investigations showed that the force which acts on the ball in a nonstationary turbulent flow differs from that in a corresponding stationary flow as a result of: a) the virtual mass effect and b) the phenomenon of "hydromechanical inertia" which possesses a vortex zone behind the ball. Because of the latter circumstance, with the acceleration of the flow the resistance coefficient of the ball is relatively less, and with deceleration – greater than in a stationary flow.

Fig. 1. Diagram of a two-component sensor. 1, 2 – mutually perpendicular plates with wire strain gauges glued on them; 3 – balls which perceive the dynamic pressure of the flow.

Furthermore, as a result of some instability of the position of the line of separation of the boundary-layer, behind the sphere the fluctuation of the drag force is observed (and when a gradient of average speed is present – also lift) even in a nonturbulent external flow.

The intensity of these apparent fluctuations of velocity being recorded by the instrument attains 5-7% of the forward velocity. Therefore, the ball sensor is recommended for use only for studying comparatively large vortex structures in flows with a high intensity of turbulence where the errors indicated above will play a comparatively small role.

1. Studies of uniform flows in water conduits with smooth and rough walls. One of the most important in practice is the steady flow in pipes and open ducts of constant cross section uniform in the direction of main flow (uniform flow). The distribution over the cross section of the averaged (for time) velocities for the simplest flows of such a type (circular pipe, wide rectangular delivery duct) has been studied in sufficient detail, and the turbulent structure - considerably less. The velocity field near a solid wall is also insufficiently investigated, especially in the case of rough walls.

The detailed investigations in confined and open ducts of rectangular cross section were executed under the guidance of I. K. Nikitin over a wide range of change in the Reynolds numbers and roughness of the walls. These studies made it possible to obtain the distribution of average velocities, single-point moments for the components of the fluctuations of velocity and, in a number of cases - also two-point correlations and one-dimensional energy spectra of the fluctuations of velocity.

For the processing and theoretical explanation of the results of the experiments, I. K. Nikitin proposed using a linear parameter - the thickness of the wall layer δ, with aid of which he managed to obtain a "two-layered" distribution curve of velocities, general for the cases of smooth, "slightly rough," and completely rough walls, and a general law of resistance for all cases. The physical conception of I. K. Nikitin consists in the following. In rough channels, similar to smooth ones, there is a region, in which the specific properties of a flow which flows around the projections of roughness are exhibited. The unique structure of vortices, caused by the detached flow around the roughness elements, creates specific additional "viscosity" whose effect disappears only at a specific distance from the apexes of the projections of roughness. Furthermore, if we perform a three-dimensional averaging of the longitudinal velocity components between the projections of roughness, then the distribution along the vertical of such "averaged velocities" turns out to be similar to the linear (analogy with linear velocity distribution in a viscous sublayer).

The presented considerations allowed I. K. Nikitin to propose the following formula, for the distribution of average velocities along the vertical of a plane turbulent flow

$$\frac{\bar{u}}{u_*} = 5.6\left[1.13\lg\frac{y}{\delta} + 1.5 - 0.5\frac{\delta}{y}\right], \quad (1.1)$$

where $\bar{u}_1$ is the averaged velocity at the given point; u_* – dynamic velocity; y – distance from the wall; δ – the thickness of the wall layer, for a smooth wall equal to the thickness of the viscous sublayer and for a completely rough wall – equal to the average height of the projections of roughness.

For a slightly rough channel, the boundary of the wall layer passes higher than the projections of roughness. The curve provides a smooth coupling with the straight line of velocity distribution in the sublayer. (This is discussed in greater detail in the report of I. K. Nikitin contained in this collection.) Some deficiency in the curve is its "two-layer quality" and velocity gradient different from zero on the axis of flow (at the free surface of an open duct).

We note that good agreement with the data of numerous experiments conducted in the Institute of Hydromechanics of the Academy of Sciences of the Ukrainian SSR was shown by the curve of Pai Shih-i [6], suitable for the entire flow in a smooth channel including the viscous sublayer (single-layer scheme). However, unfortunately, the coefficients which enter this formula depend on the Reynolds number in a sufficiently complex manner.

Having a sufficiently precise distribution curve of average velocities and knowing the law of resistance, it is possible to calculate the Reynolds stress of friction $\overline{u_1' u_2'}$ at any point according to the depth of the plane flow and to completely examine the energy balance of averaged motion. In particular, it is possible to find the depth distribution of the rate of the generation of the energy of turbulence and energy losses of the averaged motion for different Reynolds numbers and walls of different roughness [7]. The results of one such calculation are shown on Fig. 2, on which the distance from the wall is laid off along the ordinate referred to the half-width h of the channel. It is interesting to note that in a smooth channel the generation of turbulence and viscous dissipation (i.e., what in hydraulics is customarily called the energy losses) is concentrated in a relatively thin region at the wall, while for rough walls this region embraces a considerably greater part of the flow.

As a result of the generalization of the experimental data, I. K. Nikitin proposed empirical formulas for determining the distribution in the plane flow of longitudinal and vertical mean square fluctuating velocities $\sqrt{\overline{u_1'^2}}$ and $\sqrt{\overline{u_2'^2}}$, referred to u_* [1]. With the aid of these formulas it is possible, with some degree of approximation, to find the distribution along the vertical of the averaged value of the kinetic energy of turbulence $\overline{q}$. If we, further take the well-known expression $\nu_t = l\sqrt{\overline{q}}$ for the coefficient of turbulent viscosity, then hence it is possible to calculate the distribution for the cross section of scale l. Such an analysis was performed by Ye. V. Yeremenko (see the report contained in this collection).

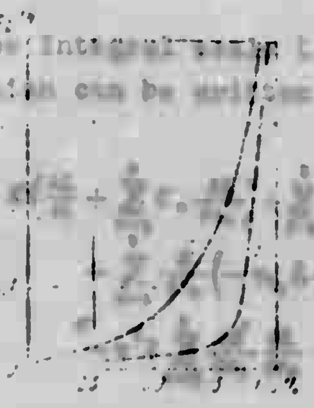

Fig. 2. Integral curve of the distribution for depth of the total losses of energy of averaged motion (in percentages of the full magnitude of losses) in plane flows with smooth (1) and rough (2) bottom; Re = = 20,000.

The sufficiently precise knowledge of the law of resistance and distribution of velocity and the kinetic energy of turbulence makes it possible to calculate with some approximation the distribution over the cross section (excluding the bottom region) of the rate of dissipation of energy of turbulence ε and correspondingly the value of the scale of dissipation $\lambda_{д}$ [8]. For this it is possible to use the formula

$$\varepsilon = C_1 \frac{\overline{v'^3}}{l} = C_1 \frac{\overline{v'^3}}{\lambda_{д}} \qquad (1.2)$$

The experimental value of coefficient C_1 was found by applying the equation of balance of fluctuating energy to that point of flow in which, from rough estimates, the diffusion of the energy of turbulence is absent and the dissipation of turbulent energy is equal to its generation. Thus the value $C_1 = 0.077$ was obtained sufficiently close to those found by other authors.

With the use of such calculations the distribution of the amount of velocity of dissipation was obtained for the cross section and then the values of the scale of the dissipation $\lambda_{д}$ and the Kolmogorov scale η were found.

The graph of the dimensionless scale

$$\eta' = \frac{\eta}{h}\left(\frac{u_* h}{\nu}\right)^{3/4} \qquad (1.3)$$

is given in Fig. 3 (here h is the half-width of the duct, u_* – dynamic velocity). The comparison of the calculations with the direct measurements of the scale of dissipation made by D. Laufer in a flat duct with smooth walls gave satisfactory agreement.

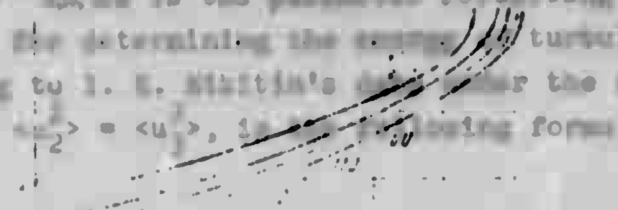

Fig. 3. Graph of dimensionless Kolmogorov scale η'.

I. K. Nikitin used the data on the velocity distribution and value of the friction drag in a uniform flow obtained by him for the calculation of the turbulent boundary layer on a flat plate with a rough surface, and also for the calculation of the boundary-layer and heat-mass transfer through a free water surface covered with waves [9, 10].

2. Studies of nonuniform turbulent flows. If the structure of uniform turbulent flows and the laws of resistance in them are investigated sufficiently well at the present time, then nonuniform turbulent flows (to which, for example, pertain the flow in diffusers and converging nozzles of pressure systems, flows with curves of backwater effect and fall-offs in open ducts and others) are studied very little in this respect.

The studies of nonuniform flows in open chutes were accomplished in the Institute of Hydromechanics the AS of the Ukrainian SSR by E. V. Zalutskiy with the aid of the photographic method [11]. The experiments were preceded by an analysis of possible systematic errors connected with the use of this method for three-dimensional-nonuniform flows and the determination of necessary corrections. The distribution of the averaged and fluctuating velocities in open flows which expand and narrow along the length under conditions of the flow close to plane-parallel was further investigated. The angle of divergence and convergence comprised approximately 2° and the value of the Reynolds number changed within limits of 5000-20,000. The results of experiments in the form of the energy-distribution curves of turbulence referred to the square of the average velocity for cross section are shown on Fig. 4.

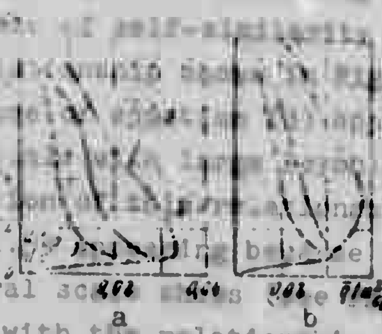

Fig. 4. The energy distribution of turbulence along the vertical with a nonuniform flow in smooth (a) and rough (b) channels (Δ — the height of projection of roughness).

For the central region of flow (nuclei) an already well-known result was obtained: with acceleration of the flow the diagram of average velocities becomes more complete, and with deceleration — completeness of the diagrams decreases. However, detailed photography of the distribution curve of velocities close to the bottom showed that the velocity gradient expressed in the corresponding dimensionless quantities not only does not increase with acceleration but even decreases.

A study of fluctuating velocities showed the following (Fig. 4). In the wall region, no considerable difference between fluctuating velocities [illegible] and [illegible] in the cases of uniform and nonuniform flows was observed; the flow rapidly "adapted itself" to new conditions. In the nucleus of the flow, with the acceleration of flow a pronounced decrease in the level of turbulence (relations | and |) was observed in comparison with the level for a uniform flow, and with deceleration - an increase in this value.

According to data of the investigations, the energy balance of the averaged motion in a nonuniform flow was calculated and the coefficients of resistance C_J and energy losses λ were determined. Let us note that in a nonuniform flow, unlike a uniform flow, these values do not coincide basically because of the gradient of normal turbulent (Reynolds) stresses.

The results of the calculations led to somewhat paradoxical conclusions. The coefficients of resistance C_J and energy losses λ both in a decelerating as well as in an accelerating flow prove to be less than under corresponding conditions in a uniform flow. This conclusion correlates with the data presented above about the reduction in the velocity gradient close to the bottom in an accelerating flow. Physically this phenomenon can be explained by the fact that in an accelerating flow, where the average level of turbulence for the cross section is considerably lower than in a uniform flow, the main source of hydraulic losses - the generation of turbulence $\overline{u_1'u_2'}(d\bar{u}_1/dx_2)$ - decreases due to a decrease in $\overline{u_1'u_2'}$ and in a decelerating flow - decreases due to a decrease in the velocity gradient $d\bar{u}_1/dx_2$ in that part of the flow, where $\overline{u_1'u_2'}$ has a significant value.

The presented considerations are purely qualitative. The studies of these interesting cases of flow in the Institute of Hydromechanics of the AS of the Ukrainian SSR continue.

3. Studies of nonstationary turbulent flows in delivery pipes and open ducts. For the solution of a number of practical problems, of great interest are nonstationary turbulent flows in which the average statistical characteristics of the flow are a function of time (random). Examples of such problems are unsteady motion in open channels and pressure systems and calculations

of a turbulent boundary layer and resistances during the unsteady motion of a body in a liquid.

First of all, it is necessary to determine the possible methods of studying such flows. If the time scale of change in the averaged characteristics of a turbulent flow is considerably greater than the corresponding characteristic scale of fluctuations of velocity, then the averaging operation does not cause difficulties. Different approximate methods of the analysis of nonstationary random functions were proposed by A. N. Patrashev [12] and V. S. Pugachev [13].

As is known, the only strict method of obtaining the characteristics of turbulent flow in the most general case is the method of statistical averaging or averaging on ensemble. In the works of the Institute of Hydromechanics of the AS of the Ukrainian SSR an experimental method which corresponds in theory to statistical averaging was also developed. The experiment was repeated many times under invariable initial and boundary conditions and, in a certain phase of flow (time coordinate) photography of the flow was conducted with a sufficiently small exposure. Thus, for each time coordinate it was possible to obtain a sufficiently long statistical series of the values of the interesting quantities.

Detailed studies of three cases of flow were accomplished by the described method: 1) discontinuous wave in an open flow, 2) smoothly changing unsteady stream in an open flow 3) unsteady motion in a delivery pipe.

Extremely curious results were obtained by Ye. V. Yeremenko during the study of the discontinuous wave which is formed in an open duct with the sudden opening of the gate [14]. Apropos the structure of such a wave, different opinions have been voiced in literature: 1) the flow is accomplished according to a pattern of two layers, whereupon the upper layer moves considerably faster than the lower with the formation of an interface between them, 2) the surface layer leaking in "extrudes" the liquid ahead of itself and forces the flow to move accelerated along its entire depth.

The results of the detailed experimental studies of this case of flow in the form of curves of the change of the components of average velocity and moments of fluctuating velocities for different cross sections of flow are shown on Fig. 5. As is evident, in actuality the flow according to the "water on water" diagram is very clearly realized, whereupon the separation boundary is the intense source of powerful fluctuations of velocity. Therefore, one ought, in essence, to approach the analysis of such motion from the positions of the theory of free turbulent flows, which up to now, has not been done.

The studies of a smoothly changing unsteady turbulent flow in an open channel by the method described above were performed by A. N. Shabrin [15]. The results of these studies in some respects are analogous to the results given above for nonuniform motion. It was precisely here that the phenomenon of the "inertia of turbulence" was observed, i.e., the lagging of a change in the level of turbulence behind a change in the velocities of the averaged flow. The diagram of the averaged velocities changed comparatively little.

The unsteady flow in open channels is distinguished by great complexity: both local and convection accelerations take place, in which regard, often of different signs. Furthermore, in the process of flow the flow geometry (free surface) changes. In order to investigate this phenomenon in a purer form, it was decided to pose the experimental studies of unsteady flows which are accelerated and decelerated with time in a rectangular delivery duct under conditions close to the conditions of a plane flow.

Introducing expressions (8), (9), (11), and (14) into equations (1) and (2), considering that these expressions and coefficients [illegible] are universal, adding the continuity equation and relationship [illegible] presented in Fig. 1, we will obtain a closed system for the calculation of the unsteady motion of the [illegible] flow.

Introducing the dimensionless quantities (designated by the line on top)

$$[illegible]$$

(g_s - the projection of gravity acceleration on the [illegible] axis, U_0 - average velocity at the initial moment, [illegible], h_0 - the transverse dimension of the flow at the initial moment, Re - the Reynolds number [illegible]) and having made the necessary conversions, [illegible] the initial system in the form

$$[illegible] \qquad (15)$$

(lines above the dimensionless quantities are omitted). Coefficient $k = h_0$ [illegible] of [illegible] flow is a function of the longitudinal coordinate x_1 and time t.

Fig. 5. [Caption to Fig. 5 appears on the following page.]

The system of equations (15) can be solved numerically with the aid of the local-one-dimensional method A. A. Samarskiy proposed [14] for equations of the form

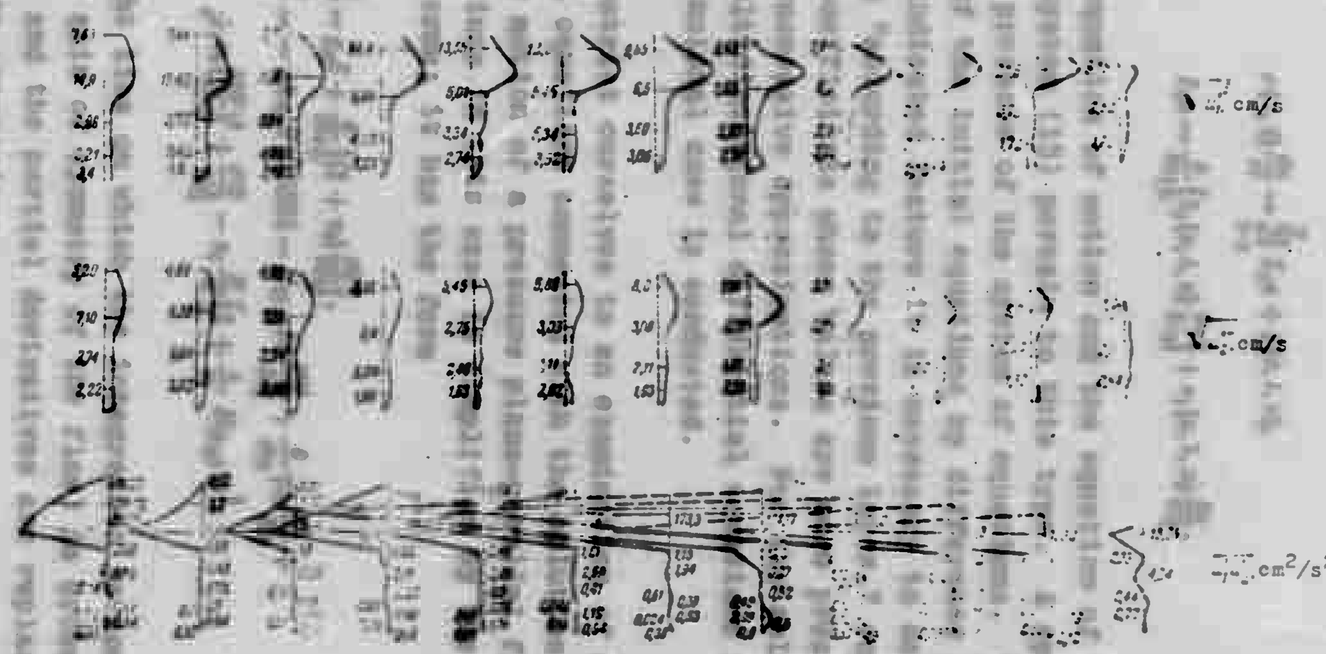

Fig. [illegible]. Distribution of kinematic characteristics in a discontinuous wave ($\bar{u}_1$, $\bar{u}_2$ – the longitudinal and vertical components of averaged velocity, u_1', u_2' – corresponding fluctuation in velocities).

Such studies are being accomplished in the Institute of Hydromechanics of the AS of the Ukrainian SSR by S. B. Markov. They are still unfinished, but some results have already been obtained.

The studies were accomplished in a tube with a cross section of 10 × 38 cm. The flow rates changed within limits of from 0.16 to 0.72 m/s with acceleration on the order of 0.1 m/s^2; the maximum value of Reynolds number was 72,000. The results of the experiments showed that with acceleration of the flow in the mass of the flow the completeness of the velocity curve increases. The reverse picture is observed during the deceleration of the flow. The measurement of fluctuating velocities also showed in this case the lagging of the change in intensity of fluctuations behind a change in the field of averaged flow rates.

4. Approaches to calculations of nonuniform and unsteady turbulent flows. A semiempirical method with the use of a system of Reynolds equations and the equation of the balance of turbulent energy was used for the calculation of the field of averaged and fluctuating velocities in nonuniform and unsteady flows. This method was successfully used by A. S. Monin, I. Rotta, G. S. Glushko, and V. B. Levin for the calculation of uniform flows and flows in the turbulent boundary layer. As is known, the basic idea of the method consists of the closure of a system of equations by using a series of approximate dependences for determining turbulent viscosity and the dissipation of energy and diffusion terms in the equation for the balance of energy. Such basic approximating dependences are:

a) the expression given above for the coefficient of turbulent viscosity:

$$\nu_t = l\sqrt{\bar{q}} \qquad (4.1)$$

b) the expression for the dissipation of the energy of turbulence:

$$\varepsilon = \nu C_2 \frac{\bar{q}}{l^2} + C_1 \frac{\bar{q}^{3/2}}{l}, \tag{4.2}$$

where the addend corresponds to small Reynolds numbers, and the augend — large; with sufficiently large Reynolds numbers the addend can be disregarded and then we will obtain expression (1.2);

c) the expression for the diffusion of the energy of turbulence; considering that transfer of the energy of turbulence in a transverse direction is gradient type diffusion, it is possible to express the diffusion terms by the dependence

$$[illegible] \tag{4.3}$$

Furthermore, it is necessary to have an expression for scale l. As is known, G. S. Glushko [16], on the strength of a series of experimental data, obtained for l a sufficiently complex dependence (graphically it is a broken line).

Thus, a closed system of equations is obtained with some empirical coefficients for finding the components of averaged velocity and the kinetic energy of turbulence. This procedure was improved somewhat by Ye. V. Yeremenko and is used for the calculation of smoothly changing motions in wide open and delivery ducts, the flow in which is close to plane.

Using the experimental data of I. K. Nikitin and the curve of Pai Shih-I which provide good agreements with the experimental data close to the axis of a plane confined flow or close to the free surface of an open flow, the distribution of scale l over the cross section was refined.

Analysis showed that with sufficiently large Reynolds numbers the distribution of scale l stops depending on this number. Furthermore, unlike the data of G. S. Glushko, on approaching the axis of flow (or the free surface of an open flow) the value of l does not decrease, but it increases monotonically and it approaches a constant value.

A substantial improvement of the method was made with consideration of the diffusion of the energy of fluctuations of pressure. As is known, the fluctuation of pressure at a given point of flow is determined by the velocity field in the entire flow and is expressed through this field by the known adiabatic equation. Consequently, the diffusion of the kinetic energy of turbulence and the diffusion of the energy of the pressure fluctuations are different phenomena. The data of D. Laufer and I. K. Nikitin show that the nature of a change in these values over a cross section is completely different, and for order of magnitude they are close to each other; therefore, neglect of the diffusion of pressure energy in general is not justified.

Ye. V. Yeremenko, using the equation which connects the fluctuations of pressure and velocity, obtained an expression for the value of the diffusion of energy through the gradient of averaged velocity and distribution of scales over the cross section and composed a closed system of equations which makes it possible to perform calculations of nonuniform and unsteady turbulent flows. The calculations were carried out to the end for the case of a pressure plane flow and satisfactory agreement with the experiments of S. B. Markov was obtained (see the report of Ye. V. Yeremenko contained in this collection).

It appears that this method can be used successfully for the calculations of unsteady and nonuniform open flows and, in particular, it will make it possible to obtain sufficiently precise data on resistances with these forms of motion.

5. Studies of the structure of turbulent flows with interfaces. In connection with the solution of the practical problem –

the expansion of the flow which issues from the opening of a structure, – with the aid of the photographic method and of a dynamic sensor the structure of such a flow was investigated in detail in the area of the interface between a forward moving jet and the closed vortex regions surrounding it. This case differs from the usual turbulent jet by the effect of the bottom and the walls which restrict the flow (the three-dimensional problem).

As a result of the studies, a large amount of experimental material has been accumulated about the values of fluctuating velocities and their single-point covariances [17]. The experiments showed an increase in the intensity of turbulence in proportion to the flow expansion. The maximum of turbulent tangential stress is noted immediately close to the interface where the greatest gradient of the averaged velocities and the maximum value of the velocity fluctuations occur. The interfaces between the forward moving liquid and the vortex zones are a powerful source of turbulence of flow. Great fluctuations of velocity and pressure cause considerable dynamic loads on the structures enclosing the flow, and they also sharply increase the capability of the flow to wash out the ground behind the structure.

On the basis of experimental curves of fluctuations of velocity recorded by the sensor, the autocorrelation functions of the fluctuations of velocity were constructed and by means of their transformation – frequency spectra. The autocorrelation function in the region of the interface close to the opening has the expressed character of a sinusoid with attenuated amplitude and only at a sufficient distance from inlets takes the form characteristic for the correlation of the random variables.

In accordance with the aforesaid about correlation functions, the spectral curve (dependence of amplitude on frequency) directly behind the opening has a pronounced maximum in a specific frequency region, which tells of the periodic separation of the vortices

(Karman street type). In proportion to the distance from the opening, the maximums are alleviated, moving into the region of lower frequencies, and the spectral curve takes a form usual for turbulent flows. This tells about the gradual transition from regular perturbations to a purely random turbulent structure.

6. **Studies of turbulent flows which carry solid particles.** For a number of years, in the Institute of Hydromechanics of the AS of the Ukrainian SSR experimental studies were conducted of the kinematic structure and dynamic characteristics of flows which carry suspended solid particles in comparatively large quantities (from 3 to 25% by volume).

The studies were conducted over a wide range of change in the basic parameters which characterize the flow. The diameters of conduits changed from 100 to 900 mm, the flow rate — from the minimum to 8 m/s, the size of particles — from 0.1 to 40 mm, and the density of solid material — from 1.4 to 4.5 t/m^3. Sand, coal, gravel, waste products from iron-ore combines, etc., were used as the material being transported. Very great experimental material has been accumulated which still requires theoretical interpretation. We will dwell here very briefly on some of the results [18, 19].

Highly-saturated suspension-carrying flows are characterized by great nonuniformity in the distribution of the concentration of solid particles along the vertical. The degree of this nonuniformity increases with a decrease in the average velocity and with an increase in the size and density of the particles.

The large concentration of solid particles in the lower layers of the horizontal flow leads to a decrease in the velocity of motion in these layers. Because of this, the distribution of the averaged longitudinal velocities along the vertical becomes asymmetric; the maximum velocity is situated higher than the geometrical axis of the flow. With identical average velocity (identical flow rate of the liquid phase) the maximum velocity and the velocity gradients along the vertical become greater than for pure water. Thus, the solid components significantly affect the kinematic flow structure.

The resistance to the motion of a suspension-carrying flow in all cases was considerably greater than for pure water and, as a rule, it increased with an increase in the concentration. However, the latter was observed to a definite limit. With velocities of motion similar to the so-called critical velocities, at which the deposition of particles begins, resistance can decrease with an increase in concentration and, besides, rather considerably. With an increase in the size of the particles the resistance increases, however, only to a definite limit, after which it stops, dependingon size.

The measurement of the fluctuation motion characteristics of liquid and solid particles showed that the energy of turbulence of liquid particles in the middle part of the flow proves to be greater than in the flow of a homogeneous liquid, and only close to the lower wall of the pipe, in the region of an extremely considerable concentration of solid particles, is a decrease in the energy of turbulence observed. On the average, over the cross section the energy of turbulence not only does not decrease, as is frequently accepted, but it increases rather considerably. This is discussed in greater detail in the report of N. A. Silin and V. F. Ocheret'ko (see this collection).

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PRESSURE F[illegible] BOUNDARY OF A TU[illegible]

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1. Appl[illegible] operation to the Navier-Stokes equations, the [illegible] fluctuations of pressure is obtained:

$$-\frac{1}{\rho}\sum\frac{\partial^2 p}{\partial x_j^2}=\underbrace{\sum\sum\frac{\partial^2 u_j u_k}{\partial x_j\partial x_k}-\sum\sum\frac{\partial^2\langle u_j u_k\rangle}{\partial x_j\partial x_k}+\sum\sum\frac{\partial^2(U_j u_k+U_k u_j)}{\partial x_j\partial x_k}}_{f}, \quad (1)$$

where U_j, U_k are the averaged values of the velocity component and u_j, u_k – the fluctuating velocity components; angular brackets $\langle\rangle$ are the sign of averaging for probability.

The right side of equation (1), called the *kinematic function*, is assumed to be known in the sense which is usually kept in mind when they speak about the space-time random function.

The boundary conditions for equation (1) in the case of a motionless flat impenetrable boundary, obtained from the Navier-Stokes equations, takes the following form:

$$\frac{1}{\rho}\frac{\partial p}{\partial y}=\nu\frac{\partial^2 v}{\partial y^2}, \quad (2)$$

v is the component of fluctuating velocity along the y-axis (the y-axis is directed along the normal to the wall).

On the free surface of the flow the fluctuation of pressure is taken as equal to zero:

$$p = 0 \tag{3}$$

or is given in the same way as is usually done in the study of waves on the surface of the water.[1]

In many instances, for example in slightly nonuniform flows on a smooth bottom, boundary condition (2) can be replaced by a simpler one - uniform:

$$\partial p/\partial y = 0. \tag{4}$$

With such a boundary condition equation (1) was investigated for the first time, apparently, by Kraichnan in 1956 [2].

The possibility of the replacement of condition (2) by relationship (4) can be substantiated, for example, by Townsend's estimates ([13], p. 275), according to which in the boundary layer

$$\langle(\partial p/\partial y)^2\rangle|_{y\to 0} \sim 3\cdot 10^{-6}, \quad \langle(\partial p/\partial x)^2\rangle|_{y\to 0} \sim 2\cdot 10^{-6},$$

so that the fluctuation gradient of the pressure at the wall along the flow is considerably greater than across the flow.

If we assume that the turbulent perturbations at the wall are planar, then, using the theory developed in [4], we obtain:

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[1] The recording of the boundary condition taking into account the action of waves and also some other possible boundary conditions are given in [1].

$$\langle(\partial p/\partial x)^2\rangle = \iint\limits_{-\infty}^{\infty} |C(\omega,\varkappa)/\varkappa|^2 S_{\tau_0}(\omega,\varkappa)\,d\omega\,d\varkappa,$$

$$\left\langle\left(\partial p/\partial y\right)^2\right\rangle = \iint\limits_{-\infty}^{\infty} \varkappa^2 S_{\tau_0}(\omega,\varkappa)\,d\omega\,d\varkappa = \left\langle\left(\partial\tau_0/\partial x\right)^2\right\rangle,$$

where $C(\omega, \varkappa)$ is some universal function of frequency ω and wave number [illegible] $(\omega/U_*|$ [illegible] $)$, τ_0 is the fluctuating component of tangential stress on the wall, and $S_{\tau_0}(\omega,\varkappa) = \langle \tau_0^* \bar{\tau}_0^* \rangle$ - the spectrum of the tangential stress on the wall (the asterisk above signifies a Fourier transform, the line above the variable - a complex conjugate value).

Using the empirical data on the spectrum of the tangential stress on the wall, it is possible to ascertain that $\langle(\partial p/\partial y)^2\rangle_{y=-1} < 1/100\,\langle(\partial p/\partial x)^2\rangle$, and condition (4) becomes even more reliable.

2. In the plane flow with a free surface and slightly changing depth, the solution of equation (1) with boundary conditions (3) and (4) for points on the bottom of the flow has the following form [5]:

$$p(x,t) = -\int\limits_{-1}^{0} d\eta \int\limits_{-\infty}^{\infty} G(\xi,\eta)\, f(x-\xi,\eta)\,d\xi. \tag{5}$$

The function of weight

$$G(\xi,\eta) = \frac{1}{2\pi} \ln \frac{\operatorname{ch}\frac{\pi\xi}{2} + \sin\frac{\pi\eta}{2}}{\operatorname{ch}\frac{\pi\xi}{2} - \sin\frac{\pi\eta}{2}} \tag{6}$$

decreases rapidly with withdrawal from the point of bottom in question ($y = -1$, $\eta = -1$; $x = x_0$, $\xi = 0$) into the depth of the flow ($0 > \eta > -1$), downward ($\xi > 0$) or upward ($\xi < 0$) along the flow (Fig. 1).[1]

[1] In formulas (5)-(7) the depth of flow is taken as equal to unity; axes y and η are directed upward, the reference point - at the surface of the water.

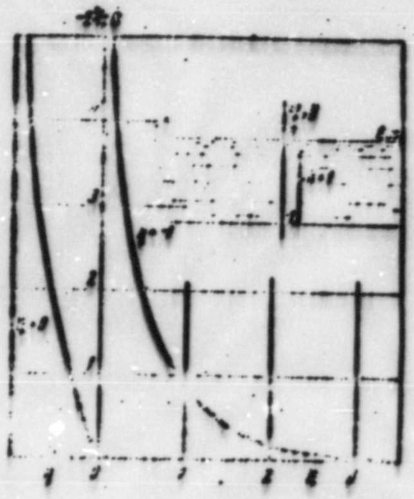

Fig. 1.

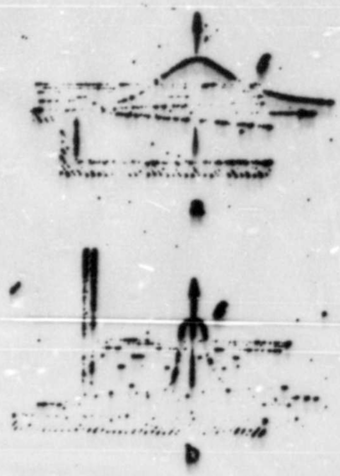

Fig. 2.

The longitudinal spectrum of the fluctuations of pressure on the bottom of a uniform plane flow can be simply calculated if the mutual longitudinal spectrum of the kinematic function in different layers of flow $S_f(\omega, \eta, \eta')$ is known:

$$S_p(\omega) = \langle p^2 \rangle = \frac{1}{\varkappa^2 \operatorname{ch}^2 \varkappa} \int_{-1}^{0} \int_{-1}^{0} S_f(\varkappa, \eta, \eta') \operatorname{ch} \varkappa\eta \cdot \operatorname{ch} \varkappa\eta' \, d\eta \, d\eta'. \qquad (7)$$

Further calculations of integral (7) can be accomplished by using experimental data and plausible hypotheses on the structure of the kinematic function. The results of such a semi-empirical calculation of the pressure spectrum agree with the data of direct measurements [6].

3. For hydraulic engineering practice more interesting is the case of sharply nonuniform motion. Just as in a uniform flow, the basic difficulty which impedes the theoretical calculation consists of the insufficient study of the velocity field. In connection with this, it is necessary to use the approximate representations which rest on indirect observations, comparing the results of the calculations with the data of the direct measurements of pressure fluctuations.

The calculation of pressure fluctuations on the boundary of the flow with the presence of an interface and narrow "zone of mixing" with increased eddying (Fig. 2) can be simplified, assuming the dispersion of hte kinematic function nonzero only within the limits of this "zone of mixing."

The use of this condition leads to the following relationship for the space-time correlation of pressure on the bottom of the flow:

$$R_p(x,x',\tau) = \langle p(x,t)\, p(x',t+\tau)\rangle =$$

$$= \iint b(\xi)\, b(\xi')\, G(x,\xi)\, G(x',\xi')\, R_f(\xi,\xi',\tau)\, d\xi\, d\xi'. \qquad (8)$$

Here b is the width of the "zone of mixing," $G(x, \xi)$ - the value of the weight function in the "zone of mixing," $R_f(\xi, \xi', \tau)$ - the longitudinal time correlation of the kinematic function in the "zone of mixing."

Further calculations with the unchanged form of the correlations of the kinematic function R_f lead to different results depending on whether the weight function along the layer of mixing changes slowly or rapidly. If the zone of mixing is situated at the surface and the weight function changes relatively slowly (Fig. 2a), then the space-time correlation of the pressure fluctuations on the bottom differs significantly from the correlation of the kinematic function. Specifically, if the proper "frozen turbulence" is traced in the kinematic function, then in the pressure fluctuations on the bottom it may not appear in practice.

If, on the contrary, the weight function changes rapidly along the length of the zone of mixing (Fig. 2b), then all features of the kinematic function are brightly reflected in the properties of the pressure fluctuations on the bottom.

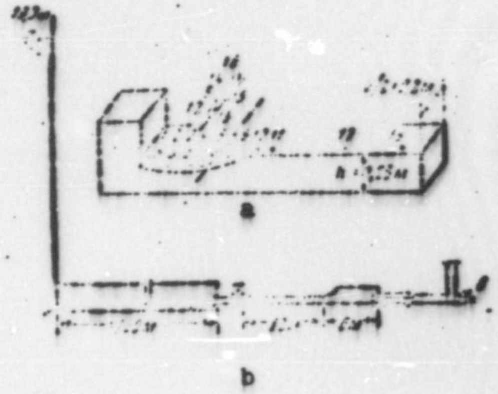

Fig. 3.

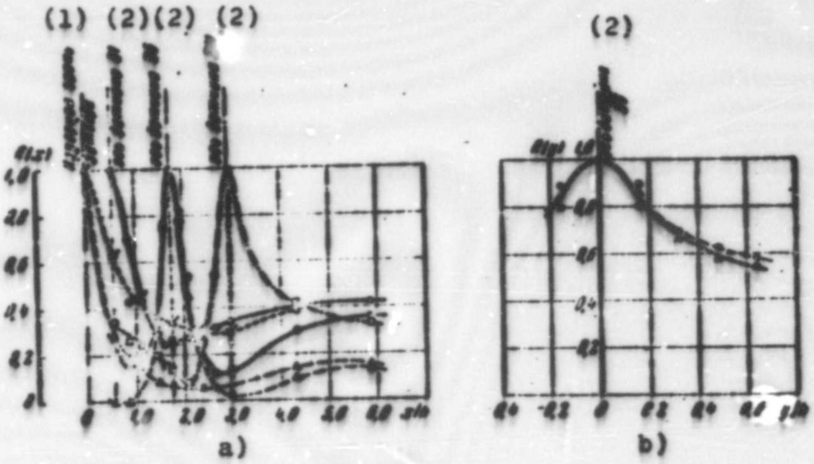

Fig. 4. The spatial correlations of fluctuations of pressure along the flow a) and across flow b).
KEY: (1) Pressure boundary of gate; (2) Axis of sensor.

This conclusion was confirmed, in particular, in experiments on a special high-pressure unit created by the Scientific Research Department of the All-Union Planning, Surveying, and Scientific Research Institute im. S. Ya. Zhuk (NIS) in the territory of the Istrinskiy hydraulic power system.

Figure 3 shows a diagram of this unit (b) and a diagram of the model of the spillway (a) on which the distribution of the correlations and standards of the pressure fluctuations in the breakaway zone was studied.

Figure 4 gives the results of the measurements of the correlations of the pressure fluctuations on the ceiling of the spillway with different locations of the base measuring point. As can be seen from Fig. 4, for the different points shown by different designations (besides triangles), a high correlation is observed at great distances between points. This means the presence, in the pressure system in question, of long-wave fluctuations not directly connected with turbulence. It was possible to suppress these fluctuations to some measure having constructed a pneumatic surge tank before the exit gate of the system. The correlation of pressure at far distances in this case was substantially reduced (triangles on Fig. 4).

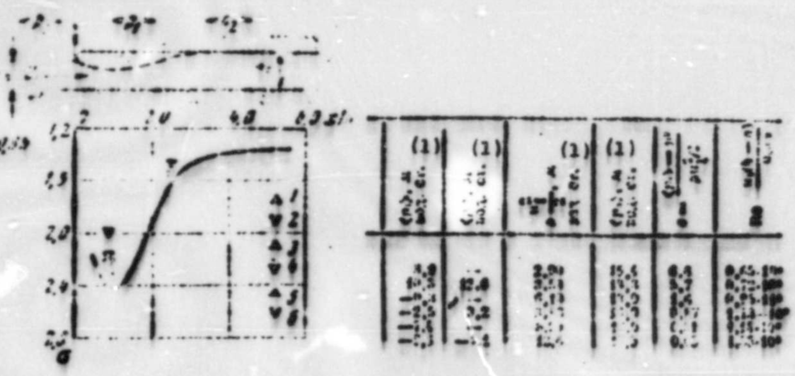

Fig. 5. Distribution of averaged pressures $\frac{\langle p_i\rangle - \langle p\rangle}{\rho v^2/2}$ on the ceiling of spillway in the breakaway zone.
KEY: (1) Water column.

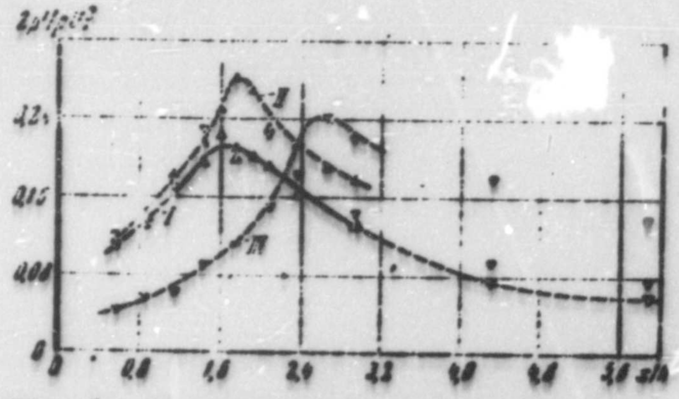

Fig. 6. The distribution of the standards of the pressure fluctuation on the ceiling of a spillway in the breakaway zone: I - in the absence of phase transitions, II - internal aeration of the flow, III - developed cavitation. The designations of the points are the same as on Fig. 5.

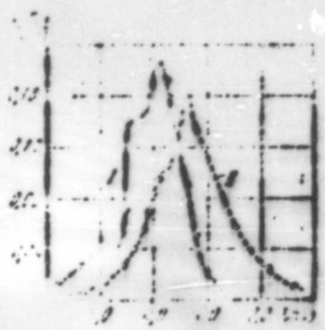

Fig. 7. The spectra of pressure fluctuations at point b (Fig. 3) in the absence of cavitation (I - σ = 6.6) and in the presence of cavitation (II - σ = 0.31).

The pressure fluctuations on the ceiling of the spillway were studied at different absolute pressures in a flow [with different cavitation numbers $\sigma = (p_0 - p_t)/(\rho U_1^2/2)$]. In this case it was detected that in proportion to a decrease in σ, beginning with some value of it, the standard of the pressure fluctuations first sharply increases and then falls rapidly (Figs. 5 and 6). An analogous effect was recently described by Naudascher and Locher [7]. This is explained by the qualitative change in the structure of the fluctuations. Under conditions of the experiment with cavitation numbers less than 0.5 the intense liberation of air from the flow began (internal aeration of the flow). The amplitude of the pressure fluctuations in this case sharply increased. It was possible to note the appearance of high-frequency fluctuations directly on the oscillogram. A further reduction in the cavitation number causes a decrease in the standard of fluctuations because of the "cutting off" of declines lower than the cavitation threshold. In this case, however, the form of the spectrum of the pressure fluctuations is sharply changed - the spectrum is shifted in the direction of higher frequencies (Fig. 7). With respect to the effect on the structural elements, this change in the spectrum can be considerably more important than some decrease in the standard of fluctuations. Actually, the stresses in the plate which comprise part of the boundary of the flow and had the natural vibration frequency of about 200 Hz, with the presence of a cavitizing flow, were approximately 2.5 times greater than in the absence of cavitation (with the recalculation of the experimental data on the very same dynamic pressure). Analysis of the oscillograms shows that these changes are connected mainly with the excitation of the high-frequency natural oscillations of construction (the resonant build-up of the construction).

These effects are frequently the direct cause of the failure of the facings of construction under the action of a cavitizing flow.

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TURBULENT FLOWS IN JETS AND DUCTS

TURBULENT JETS

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Review

It is necessary to deal with turbulent jets of liquid, gas and a non-single phase medium in many areas of technology: aviation, rocketry, energy, metallurgical, ventilation, and others. In connection with this, in the USSR and abroad many works are being conducted which are devoted to the study of the turbulent jets and all possible jet apparatuses.

It does not appear possible to make any complete survey of the most important works on turbulent jets in one report. But there is no need for this either, since many problems of this field (jets of a noncompressible liquid, numerous applications of the theory of jets, and others) are examined in detail in monographs [1-3] as well as in well-known books and articles. Therefore, the report gives a survey of works only on turbulent jets of variable density which recently acquired an especially important significance but are still insufficiently studied.

The first work on the theory of the turbulent jet of a compressible gas was published in 1939 [4]. In this work, obtained from the general equations of motion, energy and continuity is the system of Reynolds equations for the turbulent flow of variable density (the terms reflecting the effect of molecular viscosity were disregarded). The transformation of equations is conducted in the usual way, whereupon each of the variable values (components of velocity, density, temperature, pressure) is replaced by the sum of its value average for time and the fluctuating addition so that in averaging for the final time interval the latter equals zero. To simplify the equations, moments of the third order are rejected and the terms which contain the derivatives of the fluctuating values along the axis parallel to the flow which are small in comparison with the derivatives of the same values along the transverse axis are disregarded. Furthermore, in accordance with Prandtl's well-known ideas, the fluctuating values are expressed by the mixing length and the gradients of average values ($u' \sim v' \sim l\partial\bar{u}/\partial y$, $\Delta T' \sim l\partial\bar{T}/\partial y$, etc.).

The obtained Reynolds equations in the general form are not integrated; therefore the analytical solution is given for the case of relatively small compressibility effect (the jet velocity is less than the speed of sound, the relative value of the difference in the temperatures in the jet and in the environment $\Delta T_O/T_H$ is not more than 20%). In this case, the compressibility effect is considered with the use of one small parameter ($S = \Delta T_O/T_H$), on the value of which the form of the jet boundary layer and the distribution of the velocity, temperature, and density in its cross section depends.

In 1960 a new analytical solution of the problem [1] was published, valid for a large degree of compressibility of gas; in this solution the system of equations is simplified by the use of Van Driest's method of averaging parameters in which, unlike the method described above, the density fluctuation of the current is assumed (the products of the gas density times the velocity, for example, $(\rho u)' \sim l\partial\rho\bar{u}/\partial y$) a combined value (instead of the fluctuating of velocity).

In both described methods for the calculation of the turbulent jet of a compressible gas, the equation of state for a perfect gas is used.

Fig. 1. Dimensionless profiles of excessive temperature in the cross sections of the boundary layer of cryogenic (dot-dash line), air (dotted line) and plasma (continuous line) jets (R_0 - radius of the initial cross section of the jet).
KEY: (1) Calculation.

In 1961, V. A. Golubev [6] developed the theory of the turbulent jet in the case of very high gas temperature taking into account in the special form of the equation of state such factors as the dissociation and ionization of the gas; whereupon for a number of specific cases he managed to obtain the analytical solution of the system of equations. V. A. Golubev's experiments carried out with the plasma jet of water vapor flowing out into the air at temperatures of the latter up to $2 \cdot 10^{4}$ °K confirm the theoretical solution obtained by him, which can be judged from Fig. 1, on which the experimental points and the calculated distribution curves of temperature in submerged streams of air and plasma are plotted.

Fig. 2. Curves of the critical state of Freon-22 at different temperatures.
KEY: (1) Pressure, atm;
(2) Molar freedom of Freon-22 in mixture with nitrogen.

In 1961, on the strength of the same positions, V. I. Bakulev developed the theory of the turbulent jet of a cryogenic substance [6, 7] which flows out into the same medium, but which remains in a gaseous state. The author selected the analytical form of the equation of state which is in good agreement with the tabular data of the thermodynamic calculation and suitable for a gas over a wide range of conditions from the temperature of liquefaction to several hundred degrees.

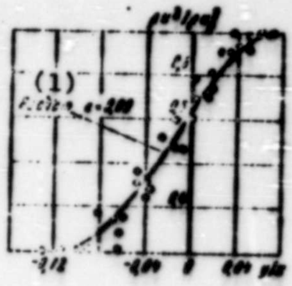

Fig. 3. Profile of the dynamic pressure in a jet of Freon-22 which flows out into gaseous nitrogen with K. A. Malinovskiy's supercritical conditions (the circle denotes experimental data).
KEY: (1) Calculation.

The experimental data obtained by V. I. Bakulev, I. S. Makarov and B. G. Khudenko [7] for a jet of liquid nitrogen which flows out into a space filled with gaseous nitrogen at a temperature of 250-420°K and pressure higher than critical confirmed the theoretical calculations of V. I. Bakulev (see dot-dash curve on Fig. 1 and the corresponding experimental points). It should be noted that at supercritical pressures in these experiments, the liquid nitrogen behaved like a gas (in view of the absence of surface tension) and its mixing with the surrounding heated nitrogen bore the same character as in a single-phase medium.

In 1967, K. A. Malinovskiy [8] refined V. I. Bakulev's theory taking into account that the phase state of the substance is determined not only by pressure but also by temperature, and he constructed diagrams of the state of the mixture of nitrogen and Freon-22; such a diagram is given in Fig. 2 (the parameter for such a curve is temperature expressed in degrees centigrade). In the region lying within each given curve the substance is in a two-phased state (wet steam), outside the curve - in a single-phase state (in the lower portion of the diagram - gas, in the upper - liquid).

K. A. Malinovskiy selected the analytical expression for the equation of state of Freon-22 similar in form to the equation used by V. I. Bakulev for nitrogen and air and conducted an experimental study of the propagation of the cryogenic jet of Freon-22 in an atmosphere of gaseous nitrogen, whereupon the basic experiments were posed with a supercritical state of Freon-22 when the mixture of this gas with gaseous nitrogen did not contain drops of Freon-22.

Using the same equations of motion and energy which were used by V. I. Bakulev and his equation of state, K. A. Malinovskiy calculated the fields of dynamic pressure in a cryogenic jet of Freon-22 which mixes with motionless gaseous nitrogen. The results of the calculation and experiment agree with each other satisfactorily, which is evidenced by Fig. 3.

It is interesting to note that a considerable increase in the width of the zone of the mixing of the initial section of the jet with an increase in the relation of densities in the external flow and in the jet ($n = \rho_H/\rho_0$), which is evident in Fig. 1 follows from theoretical calculations. It is not at all necessary to change Tollmin's constant of turbulence a which is introduced to bring experimental and theoretical profiles into conformity with the transition from a cryogenic jet to an isothermal air jet ($a = 0.09$) and only in the case of the plasma jet does it increase somewhat ($a = 0.14$); from this, it follows that in the theory of a plasma turbulent jet the compressibility effect is a little "under-considered."

In recent years, experimental and theoretical calculation work has been conducted on turbulent mixing with subsonic velocities of heterogeneous jets composed of the following pairs of gases: helium - air, carbon dioxide - air, heated air - cold air, Freon-22 - air. The effect of the relationship of velocities, densities, temperature, and also initial conditions (degree of turbulence, relative thickness of the wall boundary layer before the beginning of mixing) on the development of the zone of mixing of the jet and of the cocurrent flow was explained. The work was carried out by O. V. Yakovlevskiy, I. P. Smirnova, A. N. Sekundov, and S. Yu. Krasheninnikov under the direction of G. N. Abramovich [9, 10]. Schlieren photographs were obtained of the jets being mixed, from which it can be seen that in general the zone of the mixing of the jets consists of three regions.

In the first, adjacent to the nozzle, the flow bears the nature of a laminar flow (with the laminar boundary layer on the nozzle walls); in the second regular vortices are formed whose size is comparable with the thickness the zone of mixing, in which regard these vortices increase in the direction of flow; in the third, the turbulent flow regime is established (large vortices disintegrate into finer ones which move chaotically in the zone of mixing). With an increase in Reynolds number ($Re = u_0 d/\nu$, where u_0 is the velocity at the beginning of the jet, d - the diameter of the initial cross section, ν - the kinematic viscosity) the first and second regions are reduced; when $Re \approx 10^3$ the length of the wave region (in the submerged jet) exceeds three diameters of the jet, the length of the region of regular vortices comprises several jet diameters; when $Re \approx 10^4$ the length of the first region decreases to 0.25-0.5 d, and the second region - to 1.0-1.5 d.

Figure 4 presents photographs of a jet of carbon dioxide which is propagated in stationary air with two values of Reynolds numbers calculated according to the initial diameter of the jet ($Re_d = 2 \cdot 10^3$, $Re_d = 5 \cdot 10^3$); with the comparison of these photographs reduction in the wave and vortex regions is distinctly evident with an increase in the value of Re.

In the indicated work it has been established that the profiles of the dimensionless excess values of velocity, temperature, and impurity concentration are universal and can be expressed by the very same curve (Fig. 5). At the same time, it was clarified that the diffusion, thermal, and dynamic zones of turbulent mixing have a different thickness. If we take as the scale of thickness the distance from the axis (or the inner boundary of the mixing zone - for the initial section) to the place in which excess velocity (or correspondingly excess temperature, or excess concentration) is half that on the axis and designate it by r_u (for velocity), r_T (for temperature) and r_K (for concentration), the relationship of these thicknesses does not depend on relationship of the velocities but changes with the relationship of densities on the axis (or on the inner boundary - for the initial section) and on the outer boundary of the mixing zone:

[illegible]

[illegible]

These relations whose graphs are shown in Fig. 6, are suitable both for the main and for the initial section of the jets. In some works it is pointed out that instead of the universal distribution curve of velocity and temperature (or concentration) it is convenient to use the universal profile of dynamic pressure. The described experiments show that under conditions of a jet of variable density this hypothesis is not justified. The data of Sh. A. Yershin and L. P. Yarin definitely show that in the combustion flame (submerged jet of burning gas) the universality of the velocity profile is observed, and the dimensionless profiles of dynamic pressure with combustion and without combustion substantially differ from each other.

The same result is obtained from a study of heterogeneous jets; on Fig. 7 is plotted the profile of dynamic pressure obtained in the zone of mixing of a jet of Freon-12 with an air current with the relationship of dynamic pressures of two flows close to unity ($\rho u_r^2/\rho u_B^2 \approx 8/7$). Nonmonotony of the curve of distribution of dynamic pressures is explained by the fact that the profile of the concentration in the transverse cross-section of the jet (and, consequently, of the density) turned out to be wider than the velocity profile. If, from the measured profiles of dynamic pressure and density we construct the velocity profile, it proves to be monotonic and corresponds to the curves presented in Fig. 5.

The thickness of the zone of mixing of the initial section of the jet, as the experimental data show, in general depends both on the relationship of m velocities and on the relationship of n densities in the external flow and in the jet. However, if the jet and the external flow have the very same velocity ($m = 1$), the effect of n on the dimensionless thickness of the jet $b^0 = b/x$ practically ceases; the greatest compressibility effect is exhibited in the submerged jet, i.e. with $m = 0$ (Fig. 8).

The curve of the change, along the length of the jet, of the excess values of velocity and of the weight concentration (in logarithmic scale) with different relationships of velocities $m = u_H/u_0$ in the external flow and in the initial section for a jet of Freon-12 which is propagated in the airflow are depicted in Figs. 9 and 10. It is characteristic that for each of these values (Δu_m^0 and c_m) the very same picture of "attenuation" is obtained. On the basic section, the curves of drop in the corresponding value (in logarithmic scale) are practically parallel; the origin of the main section in each case can be considered the point of intersection of this curve with the line of the initial value of the corresponding quantity (for example, $\Delta \bar{u}_m = (u_m - u_H)/(u_0 - u_H) = 1$), in which regard the abscissa of this point x_n depends on values $m = u_H/u_0$ and $n = \rho_H/\rho_0$.

The experiments showed that the dimensionless abscissa of the origin of the main section $\bar{x}_n = x_n/d$ and the end of the initial section $\bar{l} = l/d$ have maximum values with identical velocities in the jet and in the cocurrent flow ($m = 1$) but decrease with an increase the density ratios in the cocurrent flow and in the jet (Fig. 11).

(1) Струя CO_2 в воздухе

Fig. 4. Schlieren photograph of a jet of carbon dioxide which flows out into the air with two values of Reynolds number.
KEY: (1) Jet of CO_2 in the air. GRAPHIC NOT REPRODUCIBLE .

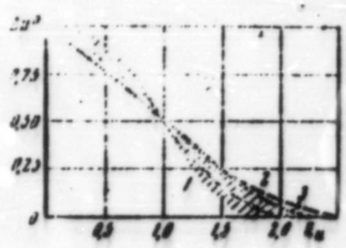

Fig. 5. Calculated profiles of the dimensionless excess values of velocities taken from different theoretical works, and the region (shaded) of the experimental values of dimensionless excess values of velocity, temperature and impurity concentration in heterogeneous jets:

1 - $0.5 (1 + \cos \frac{\pi}{2}\xi$,

2 - $\exp (-\xi_u^2 \ln 2)$,

3 - $[1 - (0.44\ \xi_u)^{3/2}]^2$

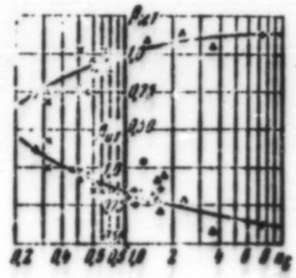

Fig. 6. The relation of ratios of thicknesses of diffusion and thermal ($\beta_{\kappa T}$), as well as dynamic and thermal (β_{uT}) boundary layers of a heterogeneous jet (curves are drawn from experimental points for different relationships of velocities m in jets being mixed).

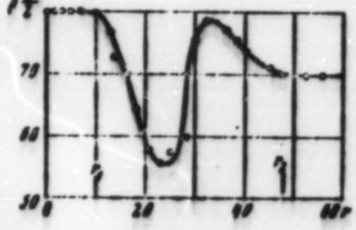

Fig. 7. Profile of dynamic pressure in the boundary layer of a jet of Freon-12 which is propagated in the cocurrent flow of air (r_1 - radius of the inner boundary of the mixing zone, r_2 - radius of the outer boundary).

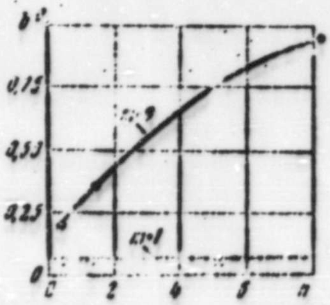

Fig. 8.

Fig. 9.

Fig. 8. The dependence of the relative thickness of a jet boundary layer on the density ratio in the cocurrent flow and jet with m = 0 (submerged jet) and m = 1.

Fig. 9. Change in dimensionless excess velocity along the axis of a jet of Freon-12 which is propagated in the cocurrent flow of air with m = var (n = 0.27).

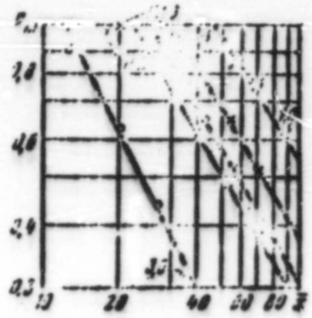

Fig. 10. Change in the dimensionless weight concentration of Freon-22 along the axis of a jet which is propagated in the cocurrent flow of air with m = var (n = 0.27).

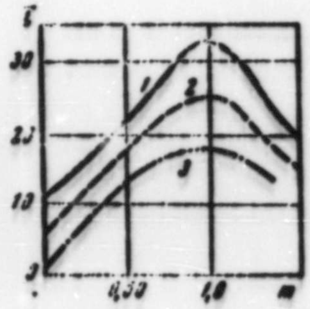

Fig. 11.

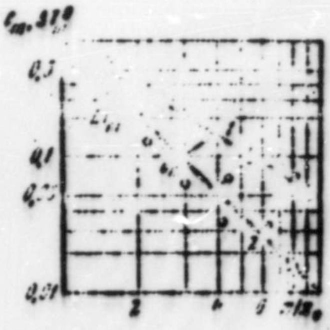

Fig. 12.

Fig. 11. Dependence of the length of the initial section of a heterogeneous jet on ratio of velocities in the cocurrent flow and in a jet with the density ratios: 1 - n = 0.27; 2 - n = 1; 3 - n = 7.25.

Fig. 12. Change in dimensionless values of excess temperature and gas concentration in the air along the axis of a heterogeneous jet (depending on the presented length) in two cases: the cocurrent flow is contained in a duct of constant cross section (1) or it is a submerged jet (2).

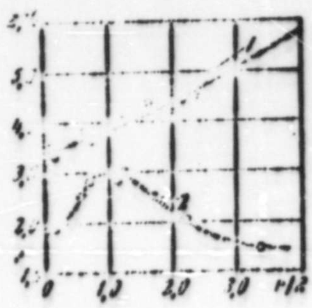

Fig. 13. Change in the degree of turbulence along the cross section of a heterogeneous jet which is propagated in a cocurrent flow in a duct (1) and in a free jet (2): ε - degree of turbulence, r - the current radius, R - the radius of the internal jet.

Curves of attenuation of the values:

$$\Delta\bar{u}_m = \frac{u_m - u_H}{u_0 - u_H}, \quad \Delta\bar{T}_m = \frac{T_m - T_H}{T_0 - T_H}, \quad \Delta\bar{\varkappa}_m = \frac{\varkappa_m - \varkappa_H}{\varkappa_0 - \varkappa_H}$$

along the length of the jets for different pairs of gases which constitute a jet and the external flow can be expressed with the aid of the following monomials:

$$\Delta\bar{u}_m = (x^*)^{-k_u}, \qquad \Delta\bar{T}_m = (x^*)^{-k_T},$$
$$\Delta\bar{\varkappa}_m = (x^*)^{-k_\varkappa}, \qquad \Delta c_m = (x^*)^{-k_c},$$

where $x^* = x/x_n$, and the exponent k_1 is a value close to unity (for weight concentration $k_c \approx 1$, for temperature $k_T \approx 1.3$; for velocity the exponent turned out to be variable in the range $0.85 \leq k_u \leq 1.25$ with an increase in n from 0.27 to 7.2). The presented values of k_1 correspond to the case of the propagation of a turbulent jet in a cocurrent flow limited by solid walls (in a tube of constant cross-section) and not subjected to preliminary artificial agitation.

A different picture is observed with the spreading of the jet within a coaxial jet of larger diameter which has a free outer boundary. In this case the high degree of turbulence in the zone of mixing of the external jet with the surrounding air is the source of the perturbations which are transmitted in a transverse direction, reaching the internal jet, and they intensify its mixing with the surrounding flow. For comparison, Fig. 12 gives attenuation curves of weight concentration c_m along the axis of the internal jet (helium in the air) and excess temperature ΔT^0_m (air in the air), taken in two cases: the upper curve - with a cocurrent flow contained in a duct of constant cross section; lower curve - with an external flow with a free boundary (submerged jet). Figure 13 depicts distribution curves, in both cases, of the degree of turbulence across the cross section of the flow: on the lower curve corresponding to the flow in the duct, seen in the rise in the degree of turbulence in the zone of the mixing of the internal jet; on the upper curve (cocurrent flow - the submerged jet) and the degree of turbulence above and it can be seen that it is given by action from without, in which regard in the zone of mixing of the internal jet there is no "burst" of the degree of turbulence.

Monograph [1] pointed out one possible feature of the mixing of a jet with a cocurrent flow of considerably greater velocity ($n = u_H/u_0 \gg 1$), which consists in the fact that possessing a high "ejecting capability," such a cocurrent flow intensely sucks in particles from the internal jet and, if the gas flow rate in the latter is insufficient for the "feeding" of the cocurrent flow, then directly behind that place where the internal jet "runs out," a zone of closed circulation is formed from which the "trickle feeding" of the external flow is accomplished and to which the excess of the mass then returns. Experiments confirmed the correctness of such a mixing scheme with $m \gg 1$.

As an example, Fig. 14 presents the flow lines obtained in an experiment with the mixing of an air jet with a cocurrent flow of air of the same temperature whose velocity exceeds by 27 times the jet velocity ($u_H \approx 100$ m/s, $u_0' \approx 3.7$ m/s). The boundary of the circulation zone is depicted by a continuous line; the boundary of the region of reverse current is shown by the dotted line; plotted on Fig. 14 are the experimental points from which the flow lines are drawn.

The described special case of the formation of the internal separation of the flow (not from the wall but with the presence of a tangential velocity discontinuity), but already with supersonic speed, was also encountered by American researchers [11], who work under the guidance of A. Ferry, and about which the latter reported in the USSR in the spring of 1966.

The turbulent jet which is extended in a motionless medium (submerged jet), captures (ejects) the particles of this medium and because of this excites the general relatively slow movement of the liquid (irrotational flow) toward its boundaries.

In the works of L. D. Landau and Ye. [illegible] Lifschitz [12], V. V. Pavlovskiy[13], O. V. Yakovlevskiy and A. [illegible] Sekundov [14] and A. S. Ginevskiy [15] theoretical and experimental studies were conducted of the irrotational flow of a submerged jet in the case of a flat axially symmetrical jet and with different positions of the outer boundaries of the medium embracing the jet.

The theoretical solution of the problem of external irrotational flow is obtained according to the distribution of the transverse velocity component on the jet boundary known from calculation. Three examples of irrotational flow outside the jet are illustrated by Figs. 15-17 taken from work [14]. Figure 15 gives the flow line in the case of a plane jet which flows out from a slot in the wall perpendicular to the jet direction; a characteristic feature of this flow is the fact that the direction of its flow lines at the jet boundary is opposite to the jet direction.

Figure 16 presents the flow lines for an analogous case of the outflow of an axially symmetrical jet. Figure 17 depicts the flow lines outside the axially symmetrical jet flowing out from a nozzle into unlimited space (near the nozzle exit there are no enclosing walls). The results of the theoretical calculations and visual-quantitative experiments (outside the jet streams of smoke are photographed which are arranged along the flow lines) agree well with each other. Knowledge of the flow which arises outside the jet is very essential to evaluate the aerodynamic forces which act on the bodies arranged beyond the limits of the jet. For example, in this way it is possible to determine the supplementary aerodynamic force which acts on a jet airplane during vertical takeoff (the force is caused by the intense sucking-in of air to the jet stream which spreads over the surface of earth - Fig. 18).

In a number of technical devices (furnaces of boiler units, the combustion chamber of gas-turbine engines, vertical takeoff jet aircraft moving near the surface of the earth, ventilation air screens, etc.) it is necessary to deal with a turbulent jet being blown off by a lateral flow. The axis of such a jet is distorted, the boundaries of a mixing zone are asymmetric relative to the axis, and the law of velocity change along the axis differs significantly from an analogous law for the case of a jet with a straight-line axis.