ADVANCED THRUST VECTOR CONTROL PRELIMINARY DESIGN COMPUTER PROGRAM

Volume I Program Description

Book 1 Requirements

THIOKOL / WASATCH DIVISION

A DIVISION OF THIOKOL CHEMICAL CORPORATION

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AIR FORCE ROCKET PROPULSION LABORATORY
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FOREWORD

The ASC/TMC Preliminary Design Computer Program for air and surface launched missiles was developed under Contract F04611-71-C-0013 by the Thiokol Chemical Corporation, Wasatch Division, Brigham City, Utah. The program was started on 1 October 1970 and completed on 1 September 1972. The Air Force Project Monitor was Lt Louis Fox, MKCD.

Wasatch Division personnel who contributed significantly to development of the computer program and to preparation of this final report are identified in the following tabulation.

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This technical report has been reviewed and is approved.

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This final report documents work accomplished under contract F04611-71-C-0013. The program discussion describes the intent and capabilities of the computer program which was developed to (1) evaluate preliminary duty cycles for missile systems, (2) develop specifications for the control system being employed, (3) perform preliminary design analysis on any of the control options, and (4) predict the performance capability of a vehicle utilizing the control system characteristics obtained from the program. The seven potential control inputs are liquid injection thrust vector control, hot gas thrust vector control, gimbal ring, ball and socket, flexible seal, jet tabs, and aerodynamic surfaces. The program has the capability to determine the thrust magnitude required to fly any one of the six types of trajectories where thrust vector and thrust magnitude control is required. The nozzle design capability includes two types of fixed nozzles and five types of movable nozzles. The program also incorporates design capability for pintle nozzle single chamber thrust magnitude control with or without thrust vector control. Two material options are available for case design, metal and filament wound glass. A three dimensional six degrees of freedom trajectory routine is available in the program.

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SECTION I

PROGRAM SUMMARY

The preliminary design and evaluation of a thrust vector control (TVC) system previously required the expenditure of considerable effort and time over a period of months.

To alleviate this situation, the Rocket Propulsion Laboratory, Edwards Air Force Base, California, funded the first TVC design program, under Contract AF 04(611)-9720, which was completed by Thiokol in 1964. Subsequent changes in state-of-the-art and a need to expand this capability led to the program entitled "Advanced Thrust Vector Control Preliminary Design Program" developed under Contract AF 04(611)-11647. The program resulting from this contract is described in the "Final Report, Advanced Thrust Vector Control Preliminary Design Computer Program," January 1968, designated Technical Report AFRPL-TR-67-318. Subsequent further changes in the state-of-the-art and the need to expand the TVC design capability to provide advanced steering and thrust magnitude control (ASC/TMC) led to the program described in this report. This additional capability was accomplished through additions and modifications to the previously developed Advanced TVC Preliminary Design Computer Program. The major additions to this program are:

1. Capability to determine the thrust magnitude required to fly any one of six types of trajectories where thrust vector and thrust magnitude control is required; and
2. Design capability for a pintle nozzle single chamber thrust magnitude control concept with or without TVC.

Additionally, data permanently stored within the computer program have been improved and equations have been updated for use in the current subroutines for the design and performance prediction functions. Changes have been made to improve basic accuracy and to extend the range applicability of the design models to include nozzle designs with throat diameters ranging from 0.5 to 100 inches. Included in the models updated are the nozzle, torque and actuation subroutines.

The program also has been modified to improve its efficiency through allowing more flexibility in performing multiple runs, minimizing program input requirements, and improving capability for debugging program input through the use of computer print flags.

A three-dimensional, six degrees of freedom trajectory routine is available in this computer program. Included is the capability to simulate a linear control system using either input or internally calculated control system gains. An associated optimization procedure for flight path shaping is included. The complete discussion of the trajectory routine is contained in subsequent sections of this book. The nozzle design capability provided in this computer program represents comprehensive design calculations for two types of fixed nozzles and five types of movable nozzles: submerged fixed; external fixed; subsonic splitline, hinged, integral, submerged inlet supersonic splitline; and external inlet supersonic splitline. The exit cone design can be either conical or contoured. The output from the nozzle subroutine includes nozzle dimensional characteristics, nozzle performance parameters, and nozzle mass properties information.

Additionally, a pintle nozzle subroutine has been added which is capable of designing a movable pintle for nozzle throat area control. This subroutine has been so designed and integrated that a pintle can be incorporated in any of the nozzle types in which this feature is practical. These types are the subsonic splitline, external fixed hinged, integral and external entry supersonic splitline.

The pintle nozzle subroutine generally applies to the nozzle designs incorporating an external inlet (convergent cone), rather than for those with submerged inlets. In addition, provision has been made to design a pintle for use with integral nozzles which have gimbal ring or flexible seal TVC.

The incorporation of a pintle does not affect the incorporation of TVC in any of these designs. The program includes an election of the preferred material. For throats less than 10 in. in diameter, tungsten is the preferred material. Solid tungsten pintles are designed for throat diameters less than 2 in. in diameter, while tungsten shell pintles are designed for throat diameters between 2 and 10 inches. Ablatives are preferred for pintles larger than 10 in. in diameter. The full range of insulation and structural materials is included in the nozzle routine for the remaining pintle components. Thermal sizing is accomplished with the insulation design section; structural sizing is part of the pintle subroutine. Provision for area balancing of the pintles is incorporated. The pintle will be supported by insulated struts with the number to vary from two to six.

The pintle will be hydraulically actuated by a single, double-acting actuator for small sized and an optional annular actuator for the larger pintles. Fluid is supplied and returned through the passages in the struts. The actuation auxiliary power unit is sized in the actuation and roll control subroutine.

The computer program includes capability for simulation of a nonprogramed thrust profile. Thrust control systems simulated are either a perfect system (i.e., achieved thrust equals commanded thrust), or a pintle controller network. The pintle controller commands the time rate change of throat area from a control law consisting of a chamber pressure error with time rate change feedback. Thrust control modes incorporated include: specific velocity-time profile, axial acceleration proportional to line of sight rates, constant Mach number, thrust proportional to commanded turning rates, minimum velocity during a commanded turn, and dynamic pressure constraints.

The pintle nozzle TMC portion of the program defines performance, structure, insulation, actuation, and system components for both fixed and movable nozzles.

The thrust control system logic used in the trajectory simulation is broken into two separate parts: (1) the command thrust logic, and (2) the controllable motor thrust dynamics. The command thrust logic, or thrust control law chosen, will provide the needed thrust command for the motor to enable the missile system to achieve the desired trajectory condition. The criteria for evaluating the commanded thrust is established by flight performance parameters such as specified velocity history, stipulated Mach number, commanded turning rates, minimum velocity or constrained dynamic pressure.

This final report is organized into three basic volumes:

I Program Description

II User's Manual

III Test Cases

Volume I contains a summary of equations used in formulating the mathematical models used in the program. This volume contains two major books: Book 1 - Requirements, and Book 2 - Hardware. The trajectory documentation is included as a separate section in Book 1, as are the aerodynamic coefficients and the roll control requirements. Book 2 of Volume I includes a complete description of the hardware and component design routines and theory.

In Volume II - User's Manual, instructions for use of the program are given; however, the input portion of the trajectory program write-up is also needed for reference by the user.

Sample test cases are contained in Volume III. The description of each test case is given, along with a listing of the computer input card deck and a reproduction of the complete computer output.

References to the source of the material used in developing this program are included in the particular sections to which they apply.

SECTION II

AERODYNAMIC COEFFICIENTS

The subroutines contained in this section provide the capability of determining the forces and moments generated on a missile by the flow of air over the external surface of an airborne vehicle. The approach used was to separate the forces and moments into two groups: those produced on the missile body (payload, motors, interstages, etc.), and those produced on the lifting surfaces (canards, fins).

The forces are represented by the nondimensional lift coefficient, $C_{N\alpha}$ for bodies, $C_{L\alpha}$ and $C_{L\delta}$ for canards and fins, and the drag coefficients, C_A for bodies, and C_{DO} and C_{DL} for canards and fins. The moments are determined from the forces and the point of application, referred to as centers of pressure. The coefficients are calculated at the Mach numbers listed in Table 1.

Included in the subroutine generated for lifting surfaces is the capability of calculating a weight estimate of the surfaces and the location of their longitudinal and lateral center-of-gravity.

A. BODY AXIAL FORCE COEFFICIENT

This subsection presents the method used for calculating the aerodynamic axial force coefficient of a body of revolution in axial flow which is composed of cones, cone-frustums, and cylindrical sections.

The total aerodynamic axial force of a body is composed of three separate components: forebody pressure forces, viscous forces (skin friction), and the base pressure forces. The axial force coefficient for each of these forces must be calculated separately.

TABLE 1

STORED MACH NUMBER TABLES

First Stage Mach No.	Upper Stage Mach No.
0.00	1.25
0.50	1.50
0.75	2.00
0.95	2.60
1.00	3.00
1.25	4.00
1.50	5.00
2.00	6.00
2.60	8.00
3.00	10.00
4.00	
5.00	
6.00	
8.00	
10.00	

1. FOREBODY PRESSURE FORCES

$$C_{A_{FB}} = C_{P_{FB}} \frac{D_2^{\,2} - D_1^{\,2}}{D^2_{ref}} \tag{1}$$

$$C_{P_{FB}} = C_{P_1} + C_{P_2}$$

$$= 2.1 \sin^2\sigma + \frac{\sin\sigma}{2\sqrt{M^2 - 1}} \tag{2}$$

The forebody axial force coefficient is calculated separately for each section of the body, which is either a cone or cone frustum. Where the first section of a body is a blunted cone and the equivalent nose radius is less than 20 percent of the cone base radius, the section can be considered as a pointed cone. The axial force coefficient, $C_{A_{FB}}$, is calculated for $M = 3.0$; the value at $M = 3.0$ is multiplied by the factor K_{FB} shown in Table 2 . Thus,

$$(C_{A_{FB}})_M = 2.0 = (K_{FB})_M = 2.0 \quad (C_{A_{FB}})_M = 3.0 \tag{3}$$

and so on for values of $M < 3.0$.

The pressure coefficient $C_{P_{FB}}$ is calculated using Equation 2.

2. VISCOUS FORCES (SKIN FRICTION)

$$C_{A_F} = C_F \frac{A_{wet}}{S_{ref}} \tag{4}$$

The axial force coefficient due to the viscous effects of the fluid is a function of Reynold's No., Mach No., and the surface area in contact with the fluid. Typical values of Reynold's No. per foot vs Mach No. are provided in Table 3 for a small ballistic missile, a larger ballistic missile, a large booster type vehicle, and a small air launched missile. To determine the flat plate coefficient, C_F, Reynold's No. vs Mach No. must be calculated in the following manner. Using Table 3 , R_N/ft is determined

TABLE 2

SUBSONIC AND TRANSONIC FOREBODY PRESSURE DRAG COEFFICIENT (K_{FB})

Mach No.	K_{FB}
0.00	0.36
0.50	0.36
0.75	0.53
0.95	1.00
1.00	1.24
1.25	1.71
1.50	1.58
2.00	1.31
2.60	1.09
3.00	1.00
3.00	1.00
3.00	1.00
3.00	1.00
3.00	1.00

TABLE 3

VARIATION OF REYNOLD'S NUMBER PER FOOT VS MACH NUMBER

Mach No.	$(R_N/ft)\ 10^{-6}$*			
	(1)	(2)	(3)	(4)
0.00	3.70	3.30	3.55	1.976
0.50	3.70	3.30	3.55	1.976
0.75	5.25	4.62	4.68	2.964
0.95	6.25	5.55	5.45	3.754
1.00	6.50	5.77	5.55	3.952
1.25	7.47	6.57	6.21	4.940
1.50	8.22	7.13	6.75	5.928
2.00	9.02	7.87	7.40	7.904
2.60	9.20	8.21	7.50	10.275
3.00	8.90	8.15	7.04	11.856
4.00	7.00	6.70	4.52	15.808
5.00	4.75	4.85	2.62	19.760
6.00	3.10	3.40	1.44	23.712
8.00	1.05	1.18	0.48	31.616
10.00	0.45	0.42	0.36	39.520

---

* (1) Small ballistic missile
(2) Large ballistic missile
(3) Large booster vehicle
(4) Small air launched missile

at a particular Mach number. Then $R_N = R_N/ft \times L$, where L is the length of the missile in feet from the nose to the end of the aft skirt. Knowing R_N and Mach No., the appropriate value of C_F can be calculated using Eq 5.

$$C_F = \left[\frac{0.445}{(\log_{10} R_N)^{2.58}}\right] \left[(1 + 0.162\ M^2)^{-0.58}\right] \tag{5}$$

The parameter A_{wet} is the total surface area of the missile excluding the base area. Total wetted area is the sum of the wetted area of the parts; therefore,

$$A_{wet} = A_{wet_{cone}} + A_{wet_{\substack{cone-\\frustums}}} + A_{wet_{cylinders}} \tag{6}$$

C_{A_F} then can be calculated using Eq 4.

3. BASE PRESSURE FORCES

$$C_{A_B} = -C_{P_B} \frac{A_{base}}{S_{ref}} \tag{7}$$

Base pressure coefficient, C_{P_B}, vs Mach No. is given in Table 4. The effective base area, A_{base}, is considered equal to one-half the difference between the base area of the missile and the total nozzle exit area. Using Eq 7, C_{A_B} then is calculated.

4. TOTAL AXIAL FORCE COEFFICIENT

Total axial force coefficient is as shown in Eq 8.

$$C_A = C_{A_{FB}} + C_{A_F} + C_{A_B} \tag{8}$$

5. NOMENCLATURE

Symbol	Definition	Units
D_2	Largest diameter of a frustum of a cone	in.
D_1	Smallest diameter of a frustum of a cone	in.

TABLE 4

BASE PRESSURE COEFFICIENT (C_{PB})

First Stage		Upper Stages	
M	C_{PB}	M	C_{PB}
0.00	-0.800	1.25	-0.289
0.50	-0.483	1.50	-0.206
0.75	-0.417	2.00	-0.152
0.95	-0.466	2.60	-0.116
1.00	-0.516	3.00	-0.098
1.25	-0.289	4.00	-0.067
1.50	-0.206	5.00	-0.047
2.00	-0.152	6.00	-0.032
2.60	-0.116	8.00	-0.013
3.00	-0.098	10.00	-0.000
4.00	-0.067		
5.00	-0.047		
6.00	-0.032		
8.00	-0.013		
10.00	-0.000		

Symbol	Definition	Units
σ	Semivertex angle of a cone or frustum of a cone	deg
K_{FB}	Subsonic and transonic forebody pressure drag factor	dim.
$C_{A_{FB}}$	Forebody axial force coefficient due to pressure	dim.
$C_{P_{FB}}$	Forebody pressure coefficient $\left(C_P = \frac{P-P_a}{q}\right)$	dim.
P	Pressure on local surface	lb/sq ft
P_a	Ambient pressure	lb/sq ft
q	Dynamic pressure	lb/sq ft
D_{ref}	Reference diameter (usually taken as diameter of largest motor)	ft
C_{A_F}	Axial force coefficient due to viscous forces	dim.
R_N	Reynold's No. $\left(R_N = \frac{\rho VL}{\mu} = \frac{VL}{\upsilon}\right)$	dim.
ρ	Density of air	slug/cu ft
V	Velocity	ft/sec
L	Characteristic length (length of missile)	ft
μ	Viscosity of air	slug/ft sec
υ	Kinematic viscosity of air, ρ/μ	sq ft/sec
C_F	Flat plate skin friction coefficient	dim.
A_{wet}	Missile wetted area (surface area in contact with the air)	sq ft
S_{ref}	Reference area $\left(\frac{\pi D_{ref}^2}{4}\right)$	sq ft
C_{A_B}	Axial force coefficient due to base pressure	dim.
C_{P_B}	Base pressure coefficient $\left(C_{P_B} = \frac{P_B - P_a}{q}\right)$	dim.
P_B	Base pressure	lb/sq ft
A_{base}	Effective base area of missile $\left(A_{base} = \frac{1}{2}\right.$ (cross-sectional area of motor base minus nozzle exit area)$\left.\right)$	sq ft

B. BODY SUBSONIC-TRANSONIC NORMAL FORCE COEF[illegible]

This subroutine is used to calculate the initial slope of the normal force coefficient vs angle of attack and the longitudinal position of the normal force coefficient center of pressure over the subsonic-transonic range of Mach numbers. The method used is applicable to bodies of revolution which are composed of conical sections (cones and cone frustums) and cylinders, such as are encountered with most ballistic missile configurations. One restraint must be applied to conical sections: the slope of these sections must be forward facing (no boat-tail sections) at zero angle of attack.

This procedure for determining C_{N_α} and x_{cp} for the subsonic-transonic Mach No. range is limited to the Mach No. range of M = 0 to M = 2.0.

1. NORMAL FORCE COEFFICIENT SLOPE

The initial slope of the curve of normal force coefficient, C_N, versus angle of attack, α, is defined in this procedure by the following equation.

$$C_{N_\alpha} = \left(\frac{dC_N}{d\alpha}\right)_{\alpha = 0^\circ} = K_N \left[\frac{A_B}{S} \sin 2\alpha \cos \alpha/2 + \eta C_{d_c} A_P/S \sin^2 \alpha\right] \qquad (1)$$

where $\alpha = 1^\circ$.

$$\text{Therefore; } C_{N\alpha} = K_N[.0349\ A_B/S + .00031\, \eta\, C_{d_c}\ A_P/S] \qquad (2)$$

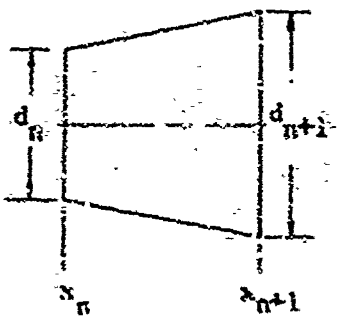
$$= .0349\ K_N\ A_B/S + .00031\ K_N\ C_{d_c}\ \eta\ A_P/S \qquad (2a)$$

$$= K_1\ A_B/S \quad K_2\ \eta\ A_P/S \qquad (2b)$$

The factors K_1 and K_2 are functions of Mach number only and are provided in Figures 1 and 2. The factors S_{ref}, A_B, and A_P, and η are functions of body geometry and are defined below.

S_{REF}	Cross-sectional area of the first stage motor cylindrical section ($S = \frac{\pi}{4} d_{cyl}^2$) and is the reference area on which C_N is based. (ft²)
A_B	Cross-sectional area of the largest cylindrical section if there is no conical frustum on the aft end of the vehicle. If there is a conical frustum on the aft end of the vehicle, A_B is equal to the cross-sectional area of the conical frustum at the point of its largest diameter. (ft^2)
A_P	Planform area of the total vehicle which is the lateral projected area of the body. As the vehicle will be composed of cones, conical frustums, and cylinders, the areas of these sections are as follows: (ft^2)

Cones and Conical Frustums



$$\Delta A_P = \left(\frac{d_n + d_{n+1}}{2}\right)(x_{n+1} - x_n) \qquad (3)$$

NOTE: For a cone $d_n = 0$

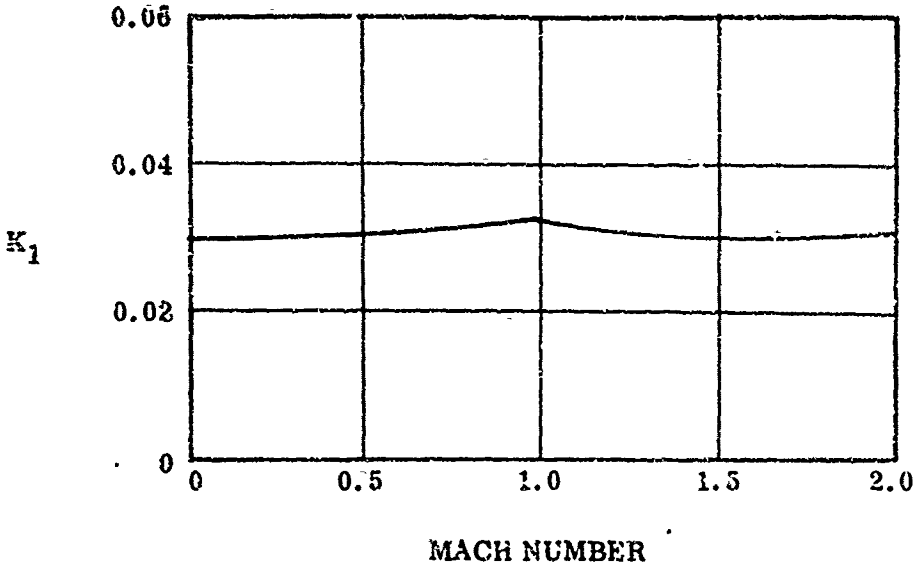


Figure 1. Subsonic-Transonic Normal Force Parameter K_1

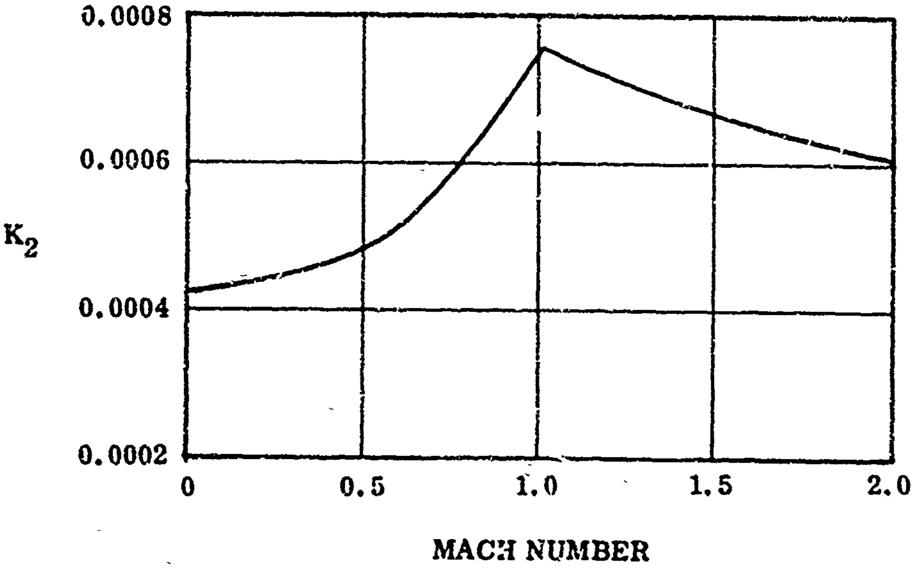
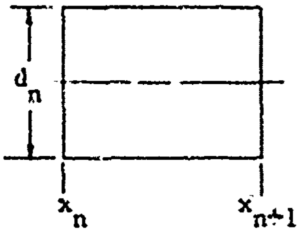


Figure 2. Subsonic-Transonic Normal Force Parameter K_2

Cylinder



$$\Delta A_p = d_n(x_{n+1} - x_n) \tag{4}$$

Therefore:

$$A_p = \Sigma \Delta A_{p_{\text{cones}}} + \Sigma \Delta A_{p_{\substack{\text{conical}\\ \text{frustum}}}} + \Sigma \Delta A_{p_{\text{cylinder}}}$$

L/d	Total body fineness ratio where L is the length of the body, and d is the diameter of the largest cylindrical section. (nondimensional)
η	Factor which is a function of total body fineness ratio. (Figure 3) (nondimensional)
K_1, K_2	Constants which are a function of Mach number. (Figures 1 and 2) (nondimensional)

2. NORMAL FORCE CENTER OF PRESSURE

The normal force center of pressure, x_{cp}, as defined in this procedure, is the location of a point, measured from the nose reference station of the vehicle, at which the total normal force acting on the body could be placed to produce the same moment about the body nose reference station as would the distributed normal force.

The following equation is used to calculate this parameter:

$$x_{cp} = x_1 - \frac{K_x}{C_{N_\alpha}} \left\{ .4188 \left[\frac{Q - A_B L}{S_{ref}} \right] - .00366\ x_c\ \eta\ C_{d_c}\ A_p/S_{ref} \right\} \tag{5}$$

$$= x_1 - \frac{1}{C_{N_\alpha}} \left\{ .4188\, K_x \left[\frac{Q - A_B L}{S_{ref}} \right] - .00366\ K_x\ C_{d_c}\ x_c\ \eta\ A_p/S_{ref} \right\} \tag{5a}$$

$$= x_1 - \frac{1}{C_{N_\alpha}} \left\{ K_3 \left[\frac{Q - A_B L}{S_{ref}} \right] - K_4\ x_c\ \eta\ A_p/S_{ref} \right\} \quad \text{(inches)} \tag{5b}$$

The parameters A_B, S_{ref}, η ; L, A_p, and C_{N_α} are the same as defined for the purpose of calculating the slope of the normal force coefficient, C_{N_α}. The factors K_3 and K_4 are a function of Mach number and are provided in Figures 4 and 5 , while Q and x_c are functions of body geometry and are defined as follows:

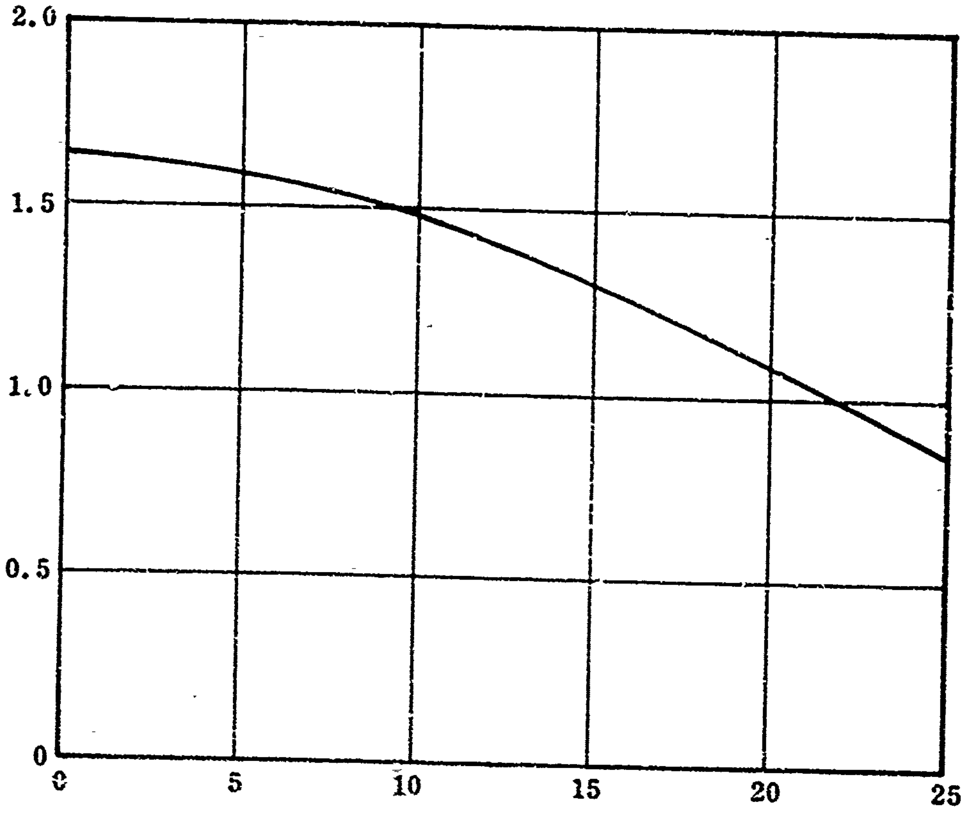


Figure 3. Subsonic-Transonic Normal Force Parameter η

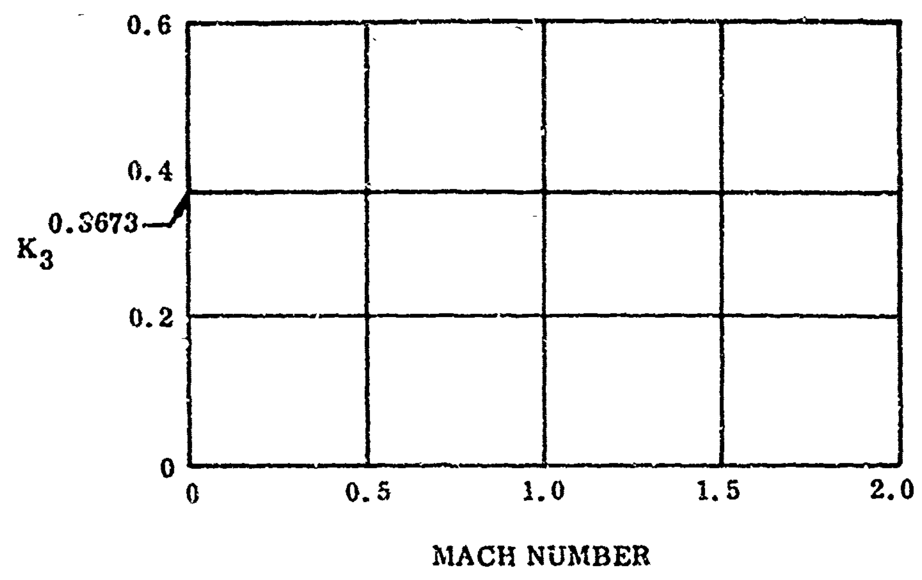


Figure 4. Subsonic-Transonic Center of Pressure Parameter K_3

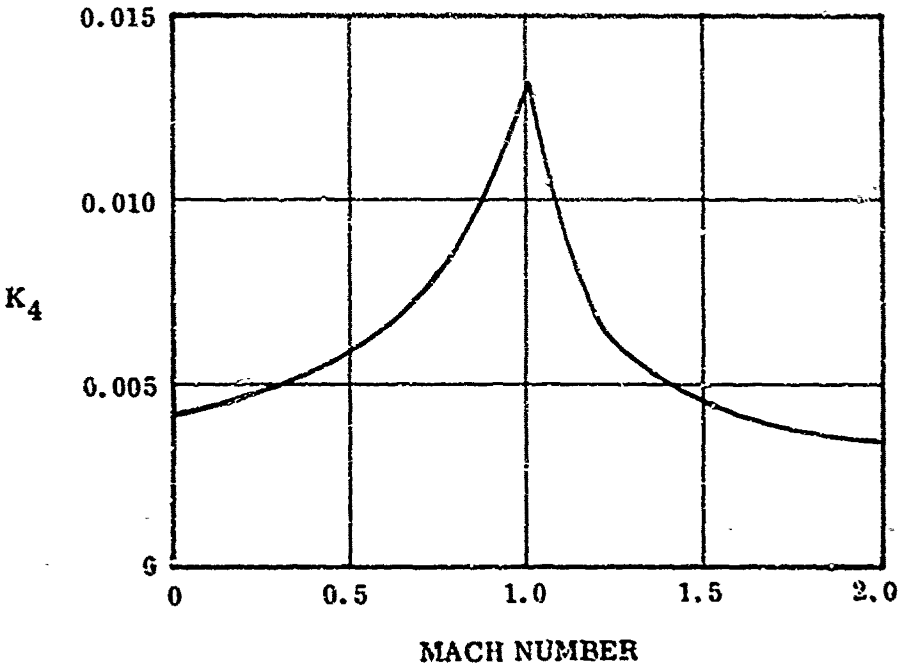
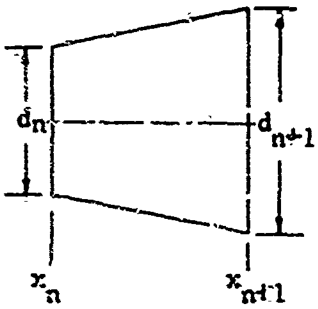


Figure 5. Subsonic-Transonic Center of Pressure Parameter K_4

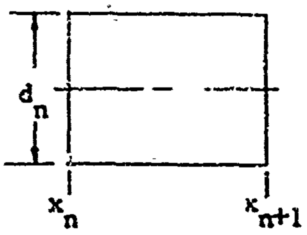
Centroid of Cones and Frustums



$$\bar{x} = x_n + (x_{n+1} - x_n) \left[\frac{3(d_n/2) + 2|(d_{n+1}/2) - (d_n/2)|}{6(d_n/2) + 3|(d_{n+1}/2) - (d_n/2)|} \right] \tag{6}$$

NOTE: For a cone $d_n = 0$

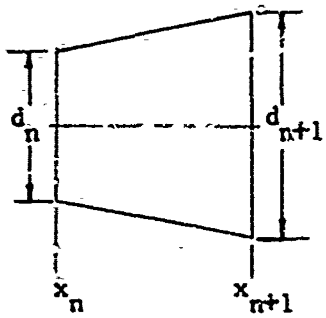
Centroid of Cylinder



$$\bar{x} = x_n + \left(\frac{x_{n+1} - x_n}{2} \right) \tag{7}$$

Q	Body volume of the total vehicle (Figure 6). The equations to be used to calculate the volumes of the cones, conical frustums, and cylinders are as follows: (ft^3)

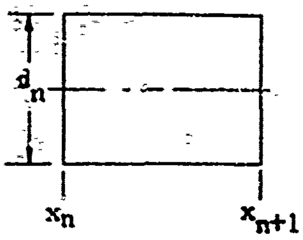
Cones and Conical Frustums



$$\Delta Q = \pi\left(\frac{x_{n+1} - x_n}{3}\right)\left[\left(\frac{d_n}{2}\right)^2 + \left(\frac{d_n}{2}\right)\left(\frac{d_{n+1}}{2}\right) + \left(\frac{d_{n+1}}{2}\right)^2\right] \qquad (8)$$

NOTE: For A cone $d_n = 0$

Cylinder



$$\Delta Q = \pi\left(\frac{d_{n+1}}{2}\right)^2 (x_{n+1} - x_n) \qquad (9)$$

Therefore

$$Q = \Sigma\Delta Q_{\text{cones}} + \Sigma\Delta Q_{\substack{\text{conical}\\ \text{frustum}}} + \Sigma\Delta Q_{\text{cylinder}}$$

L	Length of vehicle (ft)
x_c	Centroid of the total planform area measured from the body nose along the body longitudinal axis. (ft)

$$x_c = \frac{\Sigma \text{ Moment of Area}}{\Sigma \text{ Area}} = \frac{\Sigma\Delta A_p \bar{x}}{\Sigma\Delta A_p} - x_1 \qquad (10)$$

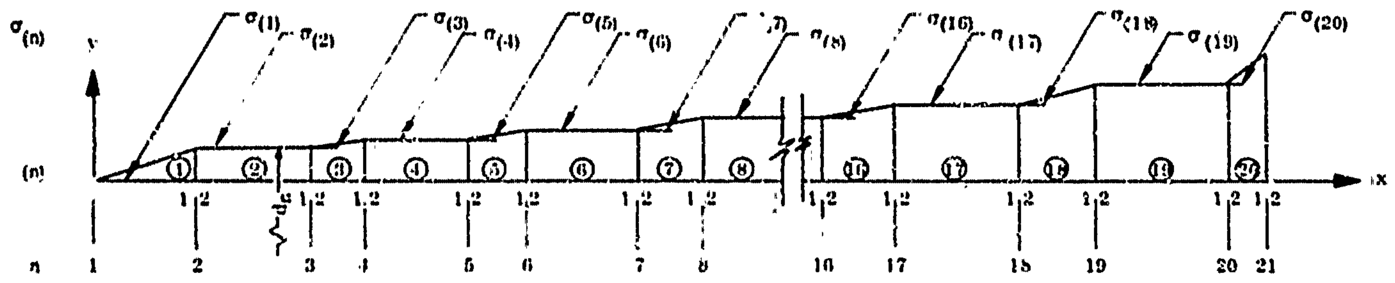


Figure 6. Body Description

C. BODY SUPERSONIC NORMAL FORCE COEFFICIENT

The method described herein has been generated for the purpose of calculating the slope of the normal force coefficient, $C_{N\alpha}$, the normal force center of pressure, x_{cp}, and the normal force loading distribution, $dC_{N\alpha}/dx$, for a pointed nose body of revolution near zero angle of attack in supersonic air flow. The Mach number range for this method is considered to be from 2.60 to 10. This method is based on second-order shock expansion theory as presented in reference 1* and an extension of the method derived in reference 2*.

Though the current approach was intended to produce a means of calculating values of the above parameters for a body of revolution composed of conical and cylindrical sections, bodies with curved profiles can be accommodated by approximating curved sections with straight line sections.

1. NORMAL FORCE COEFFICIENT SLOPE, $C_{N\alpha}$

Beginning with the standard definition for the normal force coefficient, C_N, we have

$$C_N = \frac{N}{q\,S_{ref}} \tag{1}$$

$$= \frac{N}{\frac{\gamma}{2} P_\infty M_\infty^2 S_{ref}} \tag{1a}$$

$$= \frac{2}{\frac{\gamma}{2} P_\infty M_\infty^2 S_{ref}} \int_0^l \int_0^\pi P\,r\cos\varphi\,d\varphi\,dx \tag{1b}$$

$$= \frac{4}{\gamma P_\infty M_\infty^2 S_{ref}} \int_0^l \int_0^\pi P\,r\cos\varphi\,d\varphi\,dx \tag{1c}$$

As the slope of the normal force coefficient is desired, the equation for C_N is differentiated with respect to angle of attack, α. Thus

$$C_{N\alpha} = \frac{dC_N}{d\alpha} = \frac{4}{\gamma P_\infty M_\infty^2 S_{ref}} \int_0^l \int_0^\pi \frac{dP}{d\alpha}\,r\cos\varphi\,d\varphi\,dx \tag{2}$$

---

*These references are listed in Table 5.

TABLE 5

REFERENCES

1. Syvertson, C. A. and Dennis, D. H.: A Second-Order Shock-Expansion Method Applicable to Bodies of Revolution Near Zero Lift. NACA TR-1328.

2. Capiaux, R.: An Extension of Second-Order Shock-Expansion Theory. LMSD 48381, Lockheed Aircraft Corporation.

3. Blick, E. F.: "Similarity Rule Estimation Methods for & Cones," AIAA Journal. Volume 1, Number 10, 2,415-2,416, 1963.

4. Linnell, R. D. and Bailey, J. Z.: "Similarity-Rule Estimation Methods for Cones and Parabolic Noses," Journal Aeronautical Sciences. 23, 796-797 (1956).

5. Sims, J. L.: Tables for Supersonic Flow Around Right Circular Cones at Small Angle of Attack. NASA SP-3007.

Now defining Λ as the non-dimensional loading on a thin disk, the equation for Λ is

$$\Lambda = \frac{2}{\gamma\, P_\infty M_\infty^2 \pi} \int_0^\pi \frac{dP}{d\alpha} \cos \varphi \, d\varphi \tag{3}$$

and the equation for $C_{N\alpha}$, after substitution, becomes

$$C_{N\alpha} = \frac{2\pi}{A_B} \int_0^l \Lambda\, r \, dx \tag{4}$$

where $A_B = \pi r_i^2$ and r_i is the radius at the base of the body section for which $C_{N\alpha}$ is being calculated.

The basic problem at this point is to define the loading parameter, Λ, for each section of the body. As shown by equation (3), to define Λ it is necessary to define $\frac{dP}{d\alpha}$. In reference 1, the pressure at any point on the body is defined as

$$P = P_c - (P_c - P_2)\, e^{-\eta} \tag{5}$$

$$= (1 - e^{-\eta})\, P_c + e^{-\eta}\, P_2 \tag{5a}$$

Differenting with respect to angle of attack, α, gives

$$\frac{dP}{d\alpha} = (1 - e^{-\eta}) \frac{dP_c}{d\alpha} + e^{-\eta} \left[\frac{dP_2}{d\alpha}\right] \tag{6}$$

where

$$\eta = \left(\frac{\partial P_2}{\partial S}\right) \frac{x - x_2}{(P_c - P_2) \cos \sigma_2} \tag{7}$$

Referring to equations (3) and (6), we can define Λ as

$$\Lambda = \Lambda_a + \Lambda_b \tag{8}$$

where

$$\Lambda_a = \frac{2}{\gamma P_\infty M_\infty^2 \pi} \int_0^\pi (1 - e^{-\eta}) \frac{dP_c}{d\alpha} \cos \varphi \, d\varphi \tag{9}$$

$$= (1 - e^{-\eta}) \tan \sigma \, C_{N\alpha_{TC}} \tag{9a}$$

and

$$\Lambda_b = \frac{2}{\gamma P_\infty M_\infty^2 \pi} \int_0^\pi e^{-\eta} \frac{dP_2}{d\alpha} \cos \varphi \, d\varphi \tag{10}$$

$$= e^{-\eta} \Gamma \tag{10a}$$

where

$$\Gamma = \frac{2}{\gamma P_\infty M_\infty^2 \pi} \int_0^\pi \frac{dP_2}{d\alpha} \cos \varphi \, d\varphi \tag{11}$$

Thus

$$\Lambda = (1 - e^{-\eta}) \tan \sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma \tag{12}$$

with

$$\Lambda = \Lambda_a + \Lambda_b \tag{8}$$

then

$$C_{N\alpha} = C_{N\alpha_a} + C_{N\alpha_b} \tag{13}$$

and the value of $C_{N\alpha}$ for each body section can be calculated in two increments:

$$C_{N\alpha_a} = \frac{2\pi}{A_B} \int_0^l (1 - e^{-\eta}) \tan \sigma \, C_{N\alpha_{TC}} \, r \, dx \tag{14}$$

and

$$C_{N\alpha_b} = \frac{2\pi}{A_B} \int_0^l e^{-\eta} \, \Gamma \, r \, dx \tag{15}$$

Two parameters now remain to be defined, $(\partial P/\partial S)_2$ in equation (7) and $dP_2/d\alpha$ in equation (11). Each must be defined separately for convex corners and concave corners.

A convex corner, as used herein, is defined as a corner for which the section downstream of the corner has an angle σ which is smaller than the angle σ of the section upstream of the corner. It follows then that a concave corner is one where the angle σ of the downstream section is larger than the angle σ of the upstream section.

The definitions of $(\partial P/\partial S)_2$, which are contained in reference 1, for convex and concave corners will be used in this method. They are, with substitution of symbols:

<u>Convex Corner</u>

$$\left(\frac{\partial P}{\partial S}\right)_2 = \frac{b_2}{r}\left(\frac{\Omega_1}{\Omega_2} \sin \sigma_1 - \sin \sigma_2\right) + \frac{b_2}{b_1}\frac{\Omega_1}{\Omega_2}\left(\frac{\partial P}{\partial S}\right)_1 \tag{16}$$

where

$$b = \frac{\gamma P M^2}{2(M^2 - 1)} \tag{17}$$

$$\Omega = \frac{1}{M}\left[\frac{1 + \left(\frac{\gamma - 1}{2}\right)M^2}{\frac{\gamma + 1}{2}}\right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \tag{18}$$

$$\left(\frac{\partial p}{\partial s}\right)_1 = \left(\frac{\partial p}{\partial s}\right)_3 = \left(\frac{P_c - P_3}{P_c - P_2}\right)\left(\frac{\partial p}{\partial s}\right)_2 \tag{19}$$

$$= e^{-\eta_3}\left(\frac{\partial p}{\partial s}\right)_2 \tag{19a}$$

Concave Corner

$$\left(\frac{\partial p}{\partial s}\right)_2 = \frac{\tan(\epsilon + \sigma_1 - \sigma_2)}{\tan(\epsilon + \sigma_1 - \sigma_2) + \tan \mu_2}\left\{\frac{2b_2}{r}\left[\frac{\sin \epsilon \sin \sigma_1}{\sin(\epsilon+\sigma_1-\sigma_2)} - \sin \sigma_2\right]\right.$$
$$\left. + \left(\frac{\partial p}{\partial s}\right)_1\left[\frac{b_2}{b_1}\,\frac{\sin \epsilon}{\sin(\epsilon+\sigma_1-\sigma_2)} + \left(\frac{P_2}{P_1} - F\right)\frac{\cos \epsilon \tan \mu_2}{\sin(\epsilon+\sigma_1-\sigma_2)}\right]\right\} \tag{20}$$

where

$$\mu = \sin^{-1}\left(\frac{1}{M}\right) \tag{21}$$

$$b = \frac{\gamma P M^2}{2(M^2 - 1)} \tag{22}$$

$$\left(\frac{\partial p}{\partial s}\right)_1 = \left(\frac{\partial p}{\partial s}\right)_3 = \left(\frac{P_c - P_3}{P_c - P_2}\right)\left(\frac{\partial p}{\partial s}\right)_2 \tag{23}$$

$$= e^{-\eta_3}\left(\frac{\partial p}{\partial s}\right)_2 \tag{23a}$$

$$F \approx \left(\frac{4}{\gamma + 1}\right)\left(1 + \frac{\gamma - 1}{2} M_1^2\right)(\sin \epsilon)\, \zeta \qquad (24)$$

where

$$\zeta = \frac{[(\gamma + 1) \tan(\sigma_2 - \sigma_1) \cos \epsilon - \sin \epsilon]\, M_1^2 \sin^2 \epsilon + \sin \epsilon}{1 + [1 - 2\sin^2 \epsilon + 2\tan(\sigma_2 - \sigma_1) \sin \epsilon \cos \epsilon]\, M_1^2 \sin^2 \epsilon} \qquad (25)$$

To define $dP_2/d\alpha$, the definitions as found in reference 2 for both convex and concave corners are used with only slight changes in nomenclature.

Convex Corner

$$\frac{dP_2}{d\alpha} = \left(\frac{dP_1}{d\alpha}\right)A + \left(\frac{dP_{t1}}{d\alpha}\right)B \qquad (26)$$

where

$$A = \left(\frac{\partial M_1/\partial P_1}{\partial M_2/\partial P_2}\right)\left(\frac{\partial E/\partial M_1}{\partial E/\partial M_2}\right) \qquad (27)$$

$$B = \left(\frac{\partial M_1/\partial P_{t1}}{\partial M_2/\partial P_2}\right)\left(\frac{\partial E/\partial M_1}{\partial E/\partial M_2}\right) - \left(\frac{\partial M_2/\partial P_{t2}}{\partial M_2/\partial P_2}\right)$$

Concave Corner

$$\frac{dP_2}{d\alpha} = \left(\frac{dP_1}{d\alpha}\right)C + \left(\frac{dP_{t1}}{d\alpha}\right)D \qquad (29)$$

$$\frac{dP_{t2}}{d\alpha} = \left(\frac{dP_{t1}}{d\alpha}\right)G + \left(\frac{dP_1}{d\alpha}\right)H \qquad (30)$$

where

$$C = \frac{\partial P_2}{\partial P_1} + \left(\frac{\partial P_2}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_1}\right) - \left(\frac{\partial P_2}{\partial \epsilon}\right)\left(\frac{\partial M_1}{\partial P_1}\right)\left(\frac{\partial \theta}{\partial M_1} \Big/ \frac{\partial \theta}{\partial \epsilon}\right) \qquad (31)$$

$$D = \left(\frac{\partial P_2}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_{t1}}\right) - \left(\frac{\partial P_2}{\partial \epsilon}\right)\left(\frac{\partial \theta}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_{t1}} \Big/ \frac{\partial \theta}{\partial \epsilon}\right) \qquad (32)$$

$$G = \left(\frac{\partial P_{t2}}{\partial P_{t1}}\right) + \left(\frac{\partial P_{t2}}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_{t1}}\right) - \left(\frac{\partial P_{t2}}{\partial \epsilon}\right)\left(\frac{\partial \theta}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_{t1}} \Big/ \frac{\partial \theta}{\partial \epsilon}\right) \tag{33}$$

$$H = \left(\frac{\partial P_{t2}}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_1}\right) \quad \left(\frac{\partial P_{t2}}{\partial \epsilon}\right)\left(\frac{\partial \theta}{\partial M_1}\right)\left(\frac{\partial M_1}{\partial P_1} \Big/ \frac{\partial \theta}{\partial \epsilon}\right) \tag{34}$$

At this point it is desirable to recapitulate in such a manner as to consider the equations previously given as being the general equations of this method and the equations that follow will be those to be used for numerical calculations. This requires modifying the subscript nomenclature to make possible calculations for a multi-sectioned body.

Rewriting equations (4), (5), (7), (9), (10), (11), (12), (13), (14) and (15) gives

$$C_{N\alpha_{(n)}} = \frac{2\pi}{A_{B(n)}} \int_0^l \Lambda_{(n)}\, r_{(n)}\, dx \tag{35}$$

where

$$r_{(n)} = r_n + (x - x_n) \tan \sigma_{(n)} \tag{36}$$

$$P_{(n)} = (1 - e^{-\eta(n)})\, P_{c(n)} + e^{-\eta(n)}\, P_{n2} \tag{37}$$

$$\eta_{(n)} = \frac{\partial P_{n2}}{\partial S} \; \frac{x - x_n}{(P_{c(n)} - P_{n2}) \cos \sigma_{(n)}} \tag{38}$$

$$\Lambda_{a(n)} = (1 - e^{-\eta(n)}) \tan \sigma_{(n)}\, C_{N\alpha_{TC(n)}} \tag{39}$$

$$\Lambda_{b(n)} = e^{-\eta(n)}\, \Gamma_{(n)} \tag{40}$$

$$\Gamma_{(n)} = \frac{2}{\gamma\, P_{n1}\, M_{n1}^2\, \pi} \int_0^{\pi} \left(\frac{dP_{n2}}{d\alpha}\right) \cos \varphi\, d\varphi \tag{41}$$

$$A_{(n)} = (1 - e^{-\eta(n)}) \tan \sigma_{(n)} C_{N\alpha_{TC(n)}} + e^{-\eta(n)} \Gamma_{(n)} \tag{42}$$

$$C_{N\alpha_{(n)}} = C_{N\alpha_{a(n)}} + C_{N\alpha_{b(n)}} \tag{43}$$

$$C_{N\alpha_{a(n)}} = \frac{2\pi}{A_{B(n)}} \int_0^l (1 - e^{-\eta(n)}) \tan \sigma_{(n)} C_{N\alpha_{TC(n)}} r_{(n)} dx \tag{44}$$

$$C_{N\alpha_{b(n)}} = \frac{2\pi}{A_{B(n)}} \int_0^l e^{-\eta(n)} \Gamma_{(n)} r_{(n)} dx \tag{45}$$

The terms $\left(\frac{\partial p_{n2}}{\partial S}\right)$ in equation (38) and $\left(\frac{dP_{n2}}{d\alpha}\right)$ in equation (41), as noted previously, must be defined separately for a convex corner and a concave corner and are as follows;

Convex Corner

Using the definition for $\left(\frac{\partial P}{\partial S}\right)_2$ given in equations (16) through (19) and applying subscripts results in the following:

$$\frac{\partial P_{n2}}{\partial S} = \frac{b_{n2}}{r_n}\left(\frac{\Omega_{n1}}{\Omega_{n2}} \sin \sigma_{(n-1)} - \sin \sigma_{(n)}\right) + \frac{b_{n2}}{b_{n1}} \frac{\Omega_{n1}}{\Omega_{n2}} \left(\frac{\partial P}{\partial S}\right)_{n1} \tag{46}$$

where

$$b_{n2} = \frac{\gamma P_{n2} M_{n2}^2}{2(M_{n2}^2 - 1)} \tag{47}$$

$$b_{n1} = \frac{\gamma P_{n1} M_{n1}^2}{2(M_{n1}^2 - 1)} \tag{48}$$

$$\frac{b_{n2}}{b_{n1}} = \left(\frac{P_{n2}}{P_{n1}}\right)\left(\frac{M_{n2}^2}{M_{n1}^2}\right)\left(\frac{M_{n1}^2 - 1}{M_{n2}^2 - 1}\right) \tag{49}$$

$$\Omega_{n1} = \frac{1}{M_{n1}}\left[\frac{1 + \frac{\gamma - 1}{2} M_{n1}^2}{\frac{\gamma + 1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{50}$$

$$\Omega_{n2} = \frac{1}{M_{n2}}\left[\frac{1 + \frac{\gamma - 1}{2} M_{n2}^2}{\frac{\gamma + 1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{51}$$

$$\frac{\Omega_{n1}}{\Omega_{n2}} = \left(\frac{M_{n2}}{M_{n1}}\right)\left[\frac{1 + \frac{\gamma - 1}{2} M_{n1}^2}{1 + \frac{\gamma - 1}{2} M_{n2}^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{52}$$

$$\left(\frac{\partial P}{\partial S}\right)_{n1} = e^{-\eta_{(n-1)}}\left(\frac{\partial P}{\partial S}\right)_{(n-1)2} \tag{53}$$

where $\left(\frac{\partial P}{\partial S}\right)_{(n-1)2}$ is defined in calculations for body section (n-1)

Concave Corner

Using the definition for $\left(\frac{\partial P}{\partial S}\right)_2$ given in equations (20), (21), (22), and (23) and applying subscripts produces:

$$\left(\frac{\partial P}{\partial S}\right)_{n2} = \frac{\tan[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}]}{\tan[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}] + \tan\mu_{n2}}\left[\frac{.7\, P_{n2} M_{n2}^2}{r_n(M_{n2}^2 - 1)}\left(\frac{\sin\epsilon_{(n)}\sin\sigma_{(n-1)}}{\sin[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}]} - \sin\sigma_{(n)}\right)\right.$$
$$+\left(\frac{\partial P}{\partial S}\right)_{n1}\left(\frac{P_{n2}}{P_{n1}}\frac{M_{n2}^2}{M_{n1}^2}\frac{(M_{n1}^2 - 1)}{(M_{n2}^2 - 1)}\;\frac{\sin\epsilon_{(n)}}{\sin[\epsilon_{(n)} + \sigma_{(n-2)} - \sigma_{(n)}]}\right. \tag{54}$$
$$\left.\left.+\left[\frac{P_{n2}}{P_{n1}} - F\right]\frac{\cos\epsilon_{(n)}\tan\mu_{n2}}{\sin[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}]}\right)\right]$$

where

$$F = \left(\frac{4}{\gamma + 1}\right)\left(1 + \frac{\gamma - 1}{2} M_{n1}^{2}\right)(\sin \epsilon_{(n)}) \zeta \tag{55}$$

and

$$\zeta = \frac{\left((\gamma + 1) \tan[\sigma_{(n)} - \sigma_{(n-1)}] \cos \epsilon_{(n)} - \sin \epsilon_{(n)}\right) M_{n1}^{2} \sin^{2}\epsilon_{(n)} + \sin \epsilon_{(n)}}{1 + \left(1 - 2\sin^{2}\epsilon_{(n)} + 2\tan[\sigma_{(n)} - \sigma_{(n-\ )}] \sin \epsilon_{(n)} \cos \epsilon_{(n)}\right) M_{n1}^{2} \sin^{2}\epsilon_{(n)}} \tag{55a}$$

Using the definitions for $dP_2/d\alpha$, equation (26) for convex corners and equations (29) and (30) for concave corners, and applying subscripts provides the following:

Convex Corner

$$\left(\frac{dP_2}{d\alpha}\right)_n = \left(\frac{dP_1}{d\alpha}\right)_n A_n + \left(\frac{dP_{t1}}{d\alpha}\right)_n B_n \tag{56}$$

Concave Corner

$$\left(\frac{dP_2}{d\alpha}\right)_n = \left(\frac{dP_1}{d\alpha}\right)_n C_n + \left(\frac{dP_{t1}}{d\alpha}\right)_n D_n \tag{57}$$

$$\left(\frac{dP_{t2}}{d\alpha}\right) = \left(\frac{dP_{t1}}{d\alpha}\right) G + \left(\frac{dP_1}{d\alpha}\right) H \tag{58}$$

The subscript n has been omitted on equation (58) because the pressures are a function of the body sections which have preceded body section (n). Consider equation (57) and note that $\left(\frac{dP_2}{d\alpha}\right)_n$ is a function of $\left(\frac{dP_{t1}}{d\alpha}\right)_n$ which is a function of $\left(\frac{dP_{t2}}{d\alpha}\right)_{n-1}$ which in turn is a function of $\left(\frac{dP_{t1}}{d\alpha}\right)_{n-1}$ and so forth upstream until $\left(\frac{dP_{t1}}{d\alpha}\right) = 0$. Thus there is established a recurrence formula which, in the process of defining $\left(\frac{dP_2}{d\alpha}\right)$, also defines the parameter ζ.

Definitions of parameters A, B, C, D, G and H are provided in equations (59) through (37).

The recurrence formulas providing for the evaluation of Γ for convex corners are shown in Table 6 and those for concave corners in Table 7.

TABLE 6

EQUATIONS FOR Γ FOR A CONVEX CORNER

Equation No.	No. of Concave Corners Preceding	$\Gamma_{(n)}$
(1)	nose cone	$A_n \tan \sigma_{(n-1)} C_{N\alpha_{TC(n-1)}}$
(2)	preceded by convex corners only	$A_n \Lambda_{n1}$
(3)	1	Eq (2) + $B_n H_K \Lambda_{K1})$
(4)	2	Eq (3) + $B_n (G_K H_L \Lambda_{L1})$
(5)	3	Eq (4) + $B_n (G_K G_L H_M \Lambda_{M1})$
(6)	4	Eq (5) + $B_n (G_K G_L G_M H_N \Lambda_{N1})$
(7)	5	Eq (6) + $B_n (G_K G_L G_M G_N H_O \Lambda_{O1})$
(8)	6	Eq (7) + $B_n (G_K G_L G_M G_N G_O H_P \Lambda_{P1})$
(9)	7	Eq (8) + $B_n (G_K G_L G_M G_N G_O G_P H_Q \Lambda_{Q1})$
(10)	8	Eq (9) + $B_n (G_K G_L G_M G_N G_O G_P G_Q H_R \Lambda_{R1})$
(11)	9	Eq (10) + $B_n (G_K G_L G_M G_N G_O G_P G_Q G_R H_S \Lambda_{S1})$
(12)	10	Eq (11) + $B_n (G_K G_L G_M G_N G_O G_P G_Q G_R G_S H_T \Lambda_{T1})$

$$\Lambda_{\text{convex corner}} = \Lambda_{(n)} = (1 - e^{-\eta(n)}) \tan \sigma_{(n)} C_{N\alpha_{TC(n)}} + e^{-\eta(n)} \Gamma_{(n)}$$

TABLE 7

EQUATIONS FOR Γ FOR A CONCAVE CORNER

Equation No.	No. of Concave Corners Preceding	$\Gamma_{(n)}$
(1)	nose cone	$C_n \Lambda_{n1}$
(2)	0	$C_n \Lambda_{n1}$ (assumes only nose cone plus convex corners precede)
(3)	1	$C_n \Lambda_{n1} + D_n(H_K \Lambda_{K1})$
(4)	2	Eq (3) + $D_n(G_K H_L \Lambda_{L1})$
(5)	3	Eq (4) + $D_n (G_K G_L H_M \Lambda_{M1})$
(6)	4	Eq (5) + $D_n (G_K G_L G_M H_N \Lambda_{N1})$
(7)	5	Eq (6) + $D_n (G_K G_L G_M G_N H_O \Lambda_{O1})$
(8)	6	Eq (7) + $D_n (G_K G_L G_M G_N G_O H_P \Lambda_{P1})$
(9)	7	Eq (8) + $D_n (G_K G_L G_M G_N G_O G_P H_Q \Lambda_{Q1})$
(10)	8	Eq (9) + $D_n (G_K G_L G_M G_N G_O G_P G_Q H_R \Lambda_{R1})$
(11)	9	Eq (10) + $D_n (G_K G_L G_M G_N G_O G_P G_Q G_R H_S \Lambda_{S1})$
(12)	10	Eq (11) + $D_n (G_K G_L G_M G_N G_O G_P G_Q G_R G_S H_T \Lambda_{T1})$

$$\Lambda_{\text{concave corner}} = \Lambda_{(n)} = (1 - e^{-\eta_{(n)}}) \tan \alpha_{(n)} C_{N\alpha_{TC(n)}} + e^{-\eta_{(n)}} \Gamma_{(n)}$$

TABLE 7 (Cont)

Where the Subscripts* Are

n	Upstream corner of body section being analyzed
K	Upstream corner of nearest concave body section upstream of body section (n)
L	Upstream corner of nearest concave body section upstream of body section (K)
M	Same as L except upstream of body section (L)
N	" " " " " " " " (M)
O	" " " " " " " " (N)
P	" " " " " " " " (O)
Q	" " " " " " " " (P)
R	" " " " " " " " (Q)
S	" " " " " " " " (R)
T	" " " " " " " " (S)
1	When used with a letter subscript (Example K1); refers to conditions on upstream side of corner.

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*These subscripts are applicable to both Tables 6 and 7.

$$A = \left(\frac{\partial M_{n1}/\partial P_{n1}}{\partial M_{n2}/\partial P_{n2}}\right)\left(\frac{\partial E/\partial M_{n1}}{\partial E/\partial M_{n2}}\right) \tag{59}$$

$$B = \left(\frac{\partial M_{n1}/\partial P_{t_{n1}}}{\partial M_{n2}/\partial P_{n2}}\right)\left(\frac{\partial E/\partial M_{n1}}{\partial E/\partial M_{n2}}\right) - \left(\frac{\partial M_{n2}/\partial P_{t_{n2}}}{\partial M_{n2}/\partial P_{n2}}\right) \tag{60}$$

$$C = \frac{\partial P_{n2}}{\partial P_{n1}} + \left(\frac{\partial P_{n2}}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{n1}}\right) - \left(\frac{\partial P_{n2}}{\partial \epsilon_{(n)}}\right)\left(\frac{\partial M_{n1}}{\partial P_{n1}}\right)\left(\frac{\partial \theta_n}{\partial M_{n1}} \Big/ \frac{\partial \theta_n}{\partial \epsilon_{(n)}}\right) \tag{61}$$

$$D = \left(\frac{\partial P_{n2}}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{t_{n1}}}\right) - \left(\frac{\partial P_{n2}}{\partial \epsilon_{(n)}}\right)\left(\frac{\partial \theta_n}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{t_{n1}}} \Big/ \frac{\partial \theta_n}{\partial \epsilon_{(n)}}\right) \tag{62}$$

$$G = \left(\frac{\partial P_{t_{n2}}}{\partial P_{t_{n1}}}\right) + \left(\frac{\partial P_{t_{n2}}}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{t_{n1}}}\right) - \left(\frac{\partial P_{t_{n2}}}{\partial \epsilon_{(n)}}\right)\left(\frac{\partial \theta_n}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{t_{n1}}} \Big/ \frac{\partial \theta_n}{\partial \epsilon_{(n)}}\right) \tag{63}$$

$$H = \left(\frac{\partial P_{t_{n2}}}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{n1}}\right) - \left(\frac{\partial P_{t_{n2}}}{\partial \epsilon_{(n)}}\right)\left(\frac{\partial \theta_n}{\partial M_{n1}}\right)\left(\frac{\partial M_{n1}}{\partial P_{n1}} \Big/ \frac{\partial \theta_n}{\partial \epsilon_{(n)}}\right) \tag{64}$$

$$E = \left\{\cos^{-1}\frac{1}{M} + \tfrac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}}\cos^{-1}\left(1 - \frac{\gamma+1}{1+\frac{\gamma-1}{2}M^2}\right)\right\} \tag{65}$$

$$\frac{\partial M_{n1}}{\partial P_{n1}} = -\frac{1}{\gamma\, M_{n1}\, P_{n1}}\left(\frac{P_{t_{n1}}}{P_{n1}}\right)^{\frac{\gamma-1}{\gamma}} \tag{66}$$

$$\frac{\partial M_{n2}}{\partial P_{n2}} = -\frac{1}{\gamma\, M_{n2}\, P_{n2}}\left(\frac{P_{t_{n2}}}{P_{n2}}\right)^{\frac{\gamma-1}{\gamma}} \tag{67}$$

$$\frac{\partial E}{\partial M_{n1}} = \frac{1}{M_{n1}(M_{n1}^2-1)^{\frac{1}{2}}} - \frac{1}{2}(\gamma+1)^{\frac{1}{2}}(\gamma-1)^{\frac{1}{2}}\left\{\frac{\left(\dfrac{M_{n1}}{1+\frac{\gamma-1}{2}M_{n1}^2}\right)^2}{\left[1-\left(1-\dfrac{\gamma+1}{1+\frac{\gamma-1}{2}M_{n1}^2}\right)^2\right]^{\frac{1}{2}}}\right\} \tag{68}$$

$$\frac{\partial E}{\partial M_{n2}} = \frac{1}{M_{n2}(M_{n2}^2 - 1)^{\frac{1}{2}}} - \frac{1}{2}(\gamma + 1)^{\frac{3}{2}} (\gamma - 1)^{\frac{1}{2}} \left\{ \frac{\dfrac{M_{n2}}{\left(1 + \frac{\gamma - 1}{2} M_{n2}^2\right)^2}}{\left[1 - \left(1 - \dfrac{\gamma + 1}{1 + \frac{\gamma - 1}{2} M_{n2}^2}\right)^2\right]^{\frac{1}{2}}} \right\} \tag{69}$$

$$\frac{\partial M_{n1}}{\partial P_{t_{n1}}} = \frac{1}{\gamma P_{n1} M_{n1}} \left(\frac{P_{t_{n1}}}{P_{n1}}\right)^{-\frac{1}{\gamma}} \tag{70}$$

$$\frac{\partial M_{n2}}{\partial P_{t_{n2}}} = \frac{1}{\gamma P_{n2} M_{n2}} \left(\frac{P_{t_{n2}}}{P_{n2}}\right)^{-\frac{1}{\gamma}} \tag{71}$$

$$\frac{\partial P_{n2}}{\partial P_{n1}} = \frac{2 \gamma M_{n1}^2 \sin^2 \epsilon_{(n)} - (\gamma - 1)}{\gamma + 1} \tag{72}$$

$$\frac{\partial P_{n2}}{\partial M_{n1}} = P_{n1} \left(\frac{4 \gamma}{\gamma + 1}\right) M_{n1} \sin^2 \epsilon_{(n)} \tag{73}$$

$$\frac{\partial P_{n2}}{\partial \epsilon_{(n)}} = \frac{4 \gamma P_{n1}}{\gamma + 1} M_{n1}^2 \sin \epsilon_{(n)} \cos \epsilon_{(n)} \tag{74}$$

$$\frac{\partial \theta_n}{\partial \epsilon_{(n)}} = \frac{[2 + M_{n1}^2(\gamma+1-2 \sin^2 \epsilon_{(n)})][4M_{n1}^2 \cos^2 \epsilon_{(n)} - 2M_{n1}^2 + \frac{2}{\sin^2 \epsilon_{(n)}}]}{[2 + M_{n1}^2(\gamma+1-2 \sin^2 \epsilon_{(n)})]^2 + [2 M_{n1}^2 \sin \epsilon_{(n)} \cos \epsilon_{(n)} - 2\cot \epsilon_{(n)}]^2}$$

$$+ \frac{[2M_{n1}^2 \sin \epsilon_{(n)} \cos \epsilon_{(n)} - 2 \cot \epsilon_{(n)}][4 M_{n1}^2 \sin \epsilon_{(n)} \cos \epsilon_{(n)}]}{[2 + M_{n1}^2(\gamma+1-2 \sin^2 \epsilon_{(n)})]^2 + [2 M_{n2}^2 \sin \epsilon_{(n)} \cos \epsilon_{(n)} - 2\cot \epsilon_{(n)}]^2} \tag{75}$$

$$\frac{\partial\delta_n}{\partial M_{n1}} = \frac{(\gamma - 1)\, M_{n1} \cot\epsilon_{(n)}}{[1 - M_{n1}^2(\sin^2\epsilon_{(n)} - \frac{\gamma + 1}{2})]^2 + [M_{n1}^2 \sin\epsilon_{(n)} \cos\epsilon_{(n)} - \cot\epsilon_{(n)}]^2} \tag{76}$$

$$\frac{\partial P_{t_{n2}}}{\partial P_{t_{n1}}} = \left[\left(\frac{\gamma + 1}{2\gamma}\right)\left[\frac{1}{M_{n1}^2 \sin^2\epsilon_{(n)} - \left(\frac{\gamma - 1}{2\gamma}\right)}\right]\left[\left(\frac{\gamma + 1}{2}\right)\left(\frac{2 M_{n1}^2 \sin^2\epsilon_{(n)}}{2 + (\gamma - 1)M_{n1}^2 \sin^2\epsilon_{(n)}}\right)\right]^{\gamma}\right]^{\frac{1}{\gamma-1}} \tag{77}$$

$$\frac{\partial P_{t_{n2}}}{\partial M_{n1}} = \frac{\partial P_{t_{n1}}}{\partial M_{n1}}\left(\frac{\rho_{n2}}{\rho_{n1}}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{P_{n1}}{P_{n2}}\right)^{\frac{1}{\gamma-1}} + P_{t_{n1}}\left[\left(\frac{P_{n1}}{P_{n2}}\right)^{\frac{1}{\gamma-1}} \frac{\partial\left(\rho_{n2}/\rho_{n1}\right)^{\frac{\gamma}{\gamma-1}}}{\partial M_{n1}} + \left(\frac{\rho_{n2}}{\rho_{n1}}\right)^{\frac{\gamma}{\gamma-1}} \frac{\partial\left(P_{n1}/P_{n2}\right)^{\frac{1}{\gamma-1}}}{\partial M_{n1}}\right] \tag{78}$$

$$\frac{\partial P_{t_{n2}}}{\partial \epsilon_{(n)}} = P_{t_{n1}}\left[\left(\frac{P_{n1}}{P_{n2}}\right)^{\frac{1}{\gamma-1}} \frac{\partial\left(\rho_{n2}/\rho_{n1}\right)^{\frac{\gamma}{\gamma-1}}}{\partial \epsilon_{(n)}} + \left(\frac{\rho_{n2}}{\rho_{n1}}\right)^{\frac{\gamma}{\gamma-1}} \frac{\partial\left(P_{n1}/P_{n2}\right)^{\frac{1}{\gamma-1}}}{\partial \epsilon_{(n)}}\right] \tag{79}$$

$$\frac{\partial\left(P_{n1}/P_{n2}\right)^{\frac{1}{\gamma-1}}}{\partial M_{n1}} = \left(\frac{-1}{\gamma-1}\right)\left[\frac{\gamma + 1}{2\gamma X_{n1} - (\gamma-1)}\right]^{\frac{2-\gamma}{\gamma-1}}\left\{\frac{(\gamma + 1)\,[4\gamma X_{n1}/M_{n1}]}{[2\gamma X_{n1} - (\gamma - 1)]^2}\right\} \tag{80}$$

$$\frac{\partial\left(\rho_{n2}/\rho_{n1}\right)^{\frac{\gamma}{\gamma-1}}}{\partial M_{n1}} = \frac{\gamma}{\gamma - 1}\left[\frac{(\gamma + 1) X_{n1}}{(\gamma - 1) X_{n1} + 2}\right]^{\frac{1}{\gamma-1}}\left\{\frac{4(\gamma + 1) X_{n1}/M_{n1}}{[(\gamma - 1) X_{n1} + 2]^2}\right\} \tag{81}$$

$$\frac{\partial\left(p_{n1}/p_{n2}\right)^{\frac{1}{\gamma-1}}}{\partial \epsilon_{(n)}} = -(4\ \gamma)\left(\frac{\gamma + 1}{\gamma - 1}\right)\left(\frac{p_{n1}}{p_{n2}}\right)^{\frac{2-\gamma}{\gamma-1}} \left\{\frac{X_{n1}\ \cot\ \epsilon_{(n)}}{[2\ \gamma\ X_{n1} - (\gamma - 1)]^2}\right\} \tag{82}$$

$$\frac{\partial\left(\rho_{n2}/\rho_{n1}\right)^{\frac{\gamma}{\gamma-1}}}{\partial \epsilon_{(n)}} = (4\ \gamma)\left(\frac{\gamma + 1}{\gamma - 1}\right)\left(\frac{\rho_{n2}}{\rho_{n1}}\right)^{\frac{1}{\gamma-1}} \left\{\frac{X_{n1}\ \cot\ \epsilon_{(n)}}{[(\gamma - 1)\ X_{n1} + 2]^2}\right\} \tag{83}$$

$$\frac{p_{n1}}{p_{n2}} = \frac{\gamma + 1}{2\ \gamma\ M_{n1}^2\ \sin^2\ \epsilon_{(n)} - (\gamma - 1)} \tag{84}$$

$$\frac{\rho_{n2}}{\rho_{n1}} = \frac{(\gamma + 1)\ M_{n1}^2\ \sin^2\ \epsilon_{(n)}}{(\gamma - 1)\ M_{n1}^2\ \sin^2\ \epsilon_{(n)} + 2} \tag{85}$$

$$X_{n1} = M_{n1}^2\ \sin^2\ \epsilon_{(n)} \tag{86}$$

$$\frac{\partial p_{t_{n1}}}{\partial M_{n1}} = \gamma\ p_{n1}\ M_{n1}\left(1 + \frac{\gamma - 1}{2} M_{n1}^2\right)^{\frac{1}{\gamma-1}} \tag{87}$$

Referring now to equations (5) and (12), it is seen that values of the pressure on a cone, P_c, and the slope of the normal force coefficient for a cone, $C_{N\alpha_{TC}}$, are required. Both of these parameters are a function of Mach number and, therefore, the surface Mach number on a cone is also required. Values of these parameters can be determined from several sources, but to simplify the procedure for machine calculation, equations will be used to calculate the pressure and Mach number on the cone surface. Values from reference 5 will be used for values of $C_{N\alpha_{TC}}$ and these are contained in Table 8.

The Mach number, as derived from reference 3, is

$$M_n = \frac{M_{n1} \cos \sigma_{(n)} \left(1 - \frac{\sin \sigma_{(n)}}{M_{n1}}\right)^{\frac{1}{2}}}{\left[1.0 + 0.35(M_{n1} \sin \sigma_{(n)})^{1.5}\right]^{\frac{1}{2}}} \tag{88}$$

for $0 \leq M_{n1} \sin \sigma_{(n)} \geq 1.0$

and

$$M_{n2} = \frac{M_{n1} \cos \sigma_{(n)} \left(1 - \frac{\sin \sigma_{(n)}}{M_{n1}}\right)^{\frac{1}{2}}}{\left\{\left[1 + \exp\left(-1-1.52\, M_{n1} \sin \sigma_{(n)}\right)\right]\left[1 + \left(\frac{M_{n1} \sin \sigma_{(n)}}{2}\right)^{2}\right]\right\}^{\frac{1}{2}}} \tag{89}$$

for $M_{n1} \sin \sigma_{(n)} \geq 1.0$

The pressure on a cone, P_c, is calculated from the equation for pressure coefficient in references 3 and 4, which is in current nomenclature,

TABLE 8

Values of $C_{N\alpha_{TC}}$ per Degree

M_{n1} \ $\sigma(n)$	0°	2.5°	5.0°	7.5°	10.0°	12.5°	15.0°	17.5°	20.0°	22.5°	25.0°	27.5°	30.0°	35.0°
1.5	.03490	.03450	.03374	.03284	.03190	.03095	.02999	.02902	.02802	.02698	.02592	.02499	.02400	.02260
1.75	.03490	.03443	.03361	.03270	.03180	.03094	.03009	.02922	.02831	.02734	.02631	.02522	.02417	.02176
2.0	.03490	.03436	.03347	.03256	.03173	.03096	.03021	.02942	.02857	.02764	.02664	.02556	.02441	.02200
2.5	.03490	.03421	.03322	.03237	.03169	.03110	.03049	.02981	.02903	.02816	.02717	.02609	.02493	.02240
3.0	.03490	.03405	.03301	.03227	.03176	.03131	.03079	.03017	.02942	.02855	.02757	.02648	.02530	.02272
3.5	.03490	.03390	.03285	.03226	.03190	.03154	.03107	.03047	.02973	.02886	.02786	.02676	.02556	.02298
4.0	.03490	.03375	.03274	.03231	.03207	.03178	.03132	.03073	.02998	.02910	.02809	.02697	.02575	.02317
4.5	.03490	.03362	.03268	.03240	.03224	.03197	.03154	.03094	.03018	.02929	.02826	.02712	.02589	.02329
5.0	.03490	.03349	.03266	.03251	.03241	.03216	.03173	.03112	.03035	.02943	.02839	.02725	.02600	.02335
6.0	.03490	.03327	.03270	.03276	.03272	.03248	.03203	.03139	.03059	.02965	.02859	.02741	.02615	.02344
7.0	.03490	.03311	.03282	.03300	.03299	.03273	.03225	.03158	.03076	.02980	.02871	.02752	.02625	.02351
8.0	.03490	.03299	.03297	.03322	.03320	.03292	.03241	.03172	.03088	.02990	.02880	.02760	.02631	.02356
10.0	.03490	.03290	.03329	.03359	.03353	.03319	.03264	.03191	.03103	.03002	.02891	.02769	.02639	.02360
12.0	.03490	.03294	.03358	.03387	.03376	.03337	.03278	.03202	.03112	.03010	.02897	.02774	.02643	.02363
14.0	.03490	.03306	.03383	.03407	.03391	.03349	.03287	.03209	.03117	.03013	.02901	.02777	.02646	.02368
15.0	.03490	.03312	.03394	.03416	.03398	.03353	.03290	.03211	.03119	.03016	.02902	.02779	.02647	.02369
16.0	.03490	.03320	.03404	.03422	.03402	.03358	.03293	.03212	.03120	.03018	.02903	.02780	.02648	.02370
18.0	.03490	.03337	.03421	.03434	.03410	.03362	.03297	.03216	.03122	.03020	.02905	.02781	.02649	.02373
20.0	.03490	.03351	.03436	.03445	.03417	.03367	.03300	.03219	.03125	.03021	.02906	.02782	.02650	.02373
25.0	.03490	.03382	.03462	.03462	.03429	.03375	.03301	.03223	.03126	.03028	.02909	.02786	.02652	.02374
30.0	.03490	.03413	.03483	.03473	.03434	.03377	.03303	.03230	.03129	.03035	.02912	.02789	.02654	.02374

$$C_p = \frac{(4 \sin^2 \sigma_{(n)})(2.5 + 8\sqrt{M_{n1}^2 - 1}\sin \sigma_{(n)})}{1 + 16\sqrt{M_{n1}^2 - 1}\sin \sigma_{(n)}} \qquad (90)$$

But also

$$C_p = \frac{P/P_\infty - 1}{\frac{\gamma}{2} M_\infty^2} \qquad (91)$$

therefore

$$P_{c(n)} = P_{n1}\left(1 + \frac{\gamma}{2} M_{n1}^2 C_p\right) \qquad (92)$$

$$= P_{n1}\left\{1 + \frac{\gamma}{2} M_{n1}^2 \left[\frac{(4 \sin^2 \sigma_{(n)})(2.5 + 8\sqrt{M_{n1}^2 - 1}\sin \sigma_{(n)})}{1 + 16\sqrt{M_{n1}^2 - 1}\sin \sigma_{(n)}}\right]\right\} \qquad (92a)$$

It has been shown that this method of calculating $C_{N\alpha}$ for a vehicle calculates the incremental contribution of each body section. These incremental contributions are nondimensionalized by the dynamic pressure, q, immediately upstream of the particular body section and the cross-sectional area at the downstream end of the same body section. It is necessary to use common nondimensionalizing factors and these will be the dynamic pressure of the free stream immediately ahead of the vehicle and the cross-sectional area of the largest cylindrical section of the vehicle. These factors are identified by the symbol q_{11} for the free stream dynamic pressure and S_{ref} for the cross-sectional reference area.

Thus

$$C_{N\alpha_t} = \sum_{(1)}^{(n)} \Delta C_{N\alpha_{(n)}} = \sum_{(1)}^{(n)} (C_{N\alpha_{(n)}}) \frac{q_{n1}}{q_{11}} \frac{A_{B(n)}}{S_{ref}} \qquad (93)$$

where

$$q_{n1} = \frac{\gamma}{2} P_{n1} M_{n1}^2 \qquad (94)$$

$$q_{11} = \frac{\gamma}{2} P_{11} M_{11}^2 \qquad (95)$$

$$A_{B(n)} = \pi r^2_{n + 1} \qquad (96)$$

$$S_{ref} = \pi r^2_{ref} \qquad (97)$$

2. UNIT NORMAL FORCE LOADING DISTRIBUTION

The longitudinal loading distribution of the normal force coefficient is expressed by the parameter $dC_{N\alpha}/dx$ and is developed below.

Previously the slope of the normal force coefficient was defined as

$$C_{N\alpha} = \frac{2\pi}{A_B} \int_0^l \Lambda \, r dx \qquad (4)$$

but it can also be defined as

$$C_{N\alpha} = \int_0^l (dC_{N\alpha}/dx) \, dx \qquad (98)$$

therefore

$$(dC_{N\alpha}/dx) = \frac{2\pi}{A_B} \Lambda r \qquad (99)$$

where, as before,

$$\Lambda = \Lambda_a + \Lambda_b \qquad (8)$$

$$= (1 - e^{-\eta}) \tan \sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma \qquad (12)$$

and

$$r = r_1 + x \tan \sigma \qquad (100)$$

Thus

$$(dC_{N\alpha}/dx) = \frac{2\pi}{A_B} \left[(1 - e^{-\eta}) \tan \sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma \right] (r_1 + x \tan \sigma) \qquad (101)$$

The distribution in the above form is referenced to the dynamic pressure immediately upstream of the subject body section and to the cross-sectional area at the downstream end of the same body section. Once again, it is necessary that the incremental values be referenced to the same value of the dynamic pressure q_{11}, and area S_{ref}. The equation for the loading distribution then becomes

$$\left(d\Delta C_{N\alpha}/dx\right) = \frac{2\pi q_{n1}}{12\, q_{11}\, S_{ref}} \left[\left(1 - e^{-\eta}\right)\tan\sigma\, C_{N\alpha_{TC}} + e^{-\eta}\,\Gamma\right](r_1 + x\tan\sigma) \tag{102}$$

The above development for the loading distribution $(d\Delta C_{N\alpha}/dx)$, provides the general equation for this parameter. Adding appropriate subscripts produces the form of the equation to be used for calculating the loading distribution over each body section of a multi-sectioned vehicle. This is

$$\left(d\Delta C_{N\alpha}/dx\right)_{(n)} = \frac{2\pi q_{n1}}{12\, q_{11}\, S_{ref}} \left[\left(1 - e^{-\eta(n)}\right)\tan\sigma_{(n)}\, C_{N\alpha_{TC(n)}} + e^{-\eta(n)}\,\Gamma_{(n)}\right]\left(r_n + x\tan\sigma_{(n)}\right) \tag{103}$$

where $x = x_{ni} - x_n$

For most body sections, the loading distribution should be adequately described by values at six longitudinal body stations. These points will be at the forward end, aft end and at four intermediate locations equally spaced along the body section of the vehicle. This will apply to each body section of the vehicle.

3. NORMAL FORCE CENTER OF PRESSURE

The center of pressure, as used here, will be defined as the longitudinal location, in body station, of the centroid of the normal force loading distribution.

Using the normal definition of a moment of force and the usual sign convention

$$\text{Mom} = -Nx \qquad (104)$$

$$= N(x_{ref} - x_{cp})$$

Now

$$N = C_{N\alpha}\, \alpha\, q\, S_{ref} \qquad (105)$$

and

$$\text{Mom} = C_{m\alpha}\, \alpha\, q\, S_{ref} d \qquad (106)$$

Thus

$$C_{m\alpha}\, \alpha\, q\, S_{ref}\, d = C_{N\alpha}\, \alpha\, q\, S_{ref}(x_{ref} - x_{cp}) \qquad (107)$$

Rearranging

$$x_{cp} = \frac{C_{m\alpha}\, \alpha\, q\, S_{ref}\, d}{C_{N\alpha}\, \alpha\, q\, S_{ref}} \qquad (108)$$

$$= \frac{C_{m\alpha}\, d}{C_{N\alpha}} - x_{ref} \qquad (108a)$$

If x_{ref} is defined as zero body station, then $x_{ref} = 0$ and

$$x_{cp} = \frac{C_{m\alpha}\, d}{C_{N\alpha}} \qquad (109)$$

In reference 1, the definition of $C_{m\alpha}$ is given as

$$C_{m\alpha} = \frac{2\,\pi}{A_B\, d} \int_0^l \Lambda\, rx\, dx \qquad (110)$$

with

$$\Lambda = (1 - e^{-\eta}) \tan \sigma\, C_{N\alpha_{TC}} + e^{-\eta}\, \Gamma \qquad (12)$$

then

$$C_{m\alpha} = \frac{2\,\pi}{A_B\, d} \int_0^l \left[(1 - e^{-\eta}) \tan \sigma\, C_{N\alpha_{TC}} + e^{-\eta}\, \Gamma\right] rx\, dx \qquad (111)$$

and

$$C_{N\alpha} = \frac{2\pi}{A_B} \int_0^l \left[(1 - e^{-\eta}) \tan\sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma\right] r dx \tag{112}$$

and the center of pressure is

$$x_{cp} = \frac{\left\{\frac{2\pi}{A_B d} \int_0^l \left[(1 - e^{-\eta}) \tan\sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma\right] rxdx \right\} d}{\frac{2\pi}{A_B} \int_0^l \left[(1 - e^{-\eta}) \tan\sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma\right] rdx} \tag{113}$$

$$= \frac{\int_0^l \left[(1 - e^{-\eta}) \tan\sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma\right] rxdx}{\int_0^l \left[(1 - e^{-\eta}) \tan\sigma \, C_{N\alpha_{TC}} + e^{-\eta} \Gamma\right] rdx} \tag{113a}$$

Equation (113) is then the general equation for the center of pressure. The same equation with the proper subscripts added will be used for calculations and is

$$x_{cp_{(n)}} = \frac{\int_0^l \left[\left(1 - e^{-\eta_{(n)}}\right) \tan\sigma_{(n)} \, C_{N\alpha_{TC_{(n)}}} + e^{-\eta_{(n)}} \Gamma_{(n)}\right] r_{(n)} x_{(n)} dx}{\int_0^l \left[\left(1 - e^{-\eta_{(n)}}\right) \tan\sigma_{(n)} \, C_{N\alpha_{TC_{(n)}}} + e^{-\eta_{(n)}} \Gamma_{(n)}\right] r_{(n)} dx} \tag{114}$$

where

$$x_{(n)} = (x - x_n) \tag{13}$$

and

$$r_{(n)} = r_n + (x - x_n) \tan\sigma_{(n)} \tag{36}$$

The above equation defines the center of pressure for one particular body section, whereas the center of pressure of the loading over the complete body is desired; therefore, equation (115) defines the summation to be used

$$x_{cp_{total}} = \frac{\sum_{(1)}^{(n)} \int_0^{l} \left[(1 - e^{-\eta_{(n)}}) \tan \sigma_{(n)} C_{N\alpha_{TC(n)}} + e^{-\eta_{(n)}} \Gamma_{(n)} \right] r_{(n)} x_{(n)} \, dx}{\sum_{(1)}^{(n)} \int_0^{l} \left[(1 - e^{-\eta_{(n)}}) \tan \sigma_{(n)} C_{N\alpha_{TC(n)}} + e^{-\eta_{(n)}} \Gamma_{(n)} \right] r_{(n)} \, dx} \qquad (115)$$

To utilize this method for determining the normal force coefficient center of pressure and loading distribution, it is first necessary to calculate the variation along the body of the static and total pressure, Mach number, pressure gradient and the loading parameter F. The following equations, (116-138), are to be used for this purpose.

$$P_{n2} = P_{n1}\left[\frac{2\gamma M_{n1}^2 \sin^2 \epsilon_{(n)} - (\gamma - 1)}{\gamma + 1}\right] \qquad 116$$

$$M_{n2} = \left\{\frac{(\gamma + 1)^2 M_{n1}^4 \sin^2 \epsilon_{(n)} - 4\,(M_{n1}^2 \sin^2 \epsilon_{(n)} - 1)(\gamma M_{n1}^2 \sin^2 \epsilon_{(n)} + 1)}{\left[2\gamma M_{n1}^2 \sin^2 \epsilon_{(n)} - (\gamma - 1)\right]\left[(\gamma - 1) M_{n1}^2 \sin^2 \epsilon_{(n)} + 2\right]}\right\}^{\frac{1}{2}} \qquad 117$$

$$P_{t_{n2}} = P_{t_{n1}}\left[\frac{(\gamma + 1) M_{n1}^2 \sin^2 \epsilon_{(n)}}{(\gamma - 1) M_{n1}^2 \sin^2 \epsilon_{(n)} + 2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma + 1}{2\gamma M_{n1}^2 \sin^2 \epsilon_{(n)} - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}} \qquad 118$$

$$M_{n1} = \left\{\frac{2\left[\cot \epsilon_{(n)} + \mathrm{Tan}\,(\sigma_{(n)} - \sigma_{(n-1)})\right]}{\sin 2\,\epsilon_{(n)} - (\gamma + \cos 2\,\epsilon_{(n)})\,\mathrm{Tan}\,(\sigma_{(n)} - \sigma_{(n-1)})}\right\}^{\frac{1}{2}} \qquad 119$$

$$(\sigma_{(n)} - \sigma_{(n-1)}) = \mathrm{Tan}^{-1}\left[\frac{2(M_{n1}^2 \sin^2 \epsilon_{(n)} - 1)\cot \epsilon_{(n)}}{M_{n1}^2\,(\gamma + 1 - 2\sin^2 \epsilon_{(n)}) + 2}\right] \qquad 120$$

$$\cot(\sigma_{(n)} - \sigma_{(n-1)}) = \mathrm{Tan}\,\epsilon_{(n)}\left[\frac{(\gamma + 1) M_{n1}^2}{2(M_{n1}^2 \sin^2 \epsilon_{(n)} - 1)} - 1\right] \qquad 121$$

$$\sigma_{(n)} - \sigma_{(n-1)} = \left[\cos^{-1}\frac{1}{M_{n2}} + \frac{1}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right)^{\frac{1}{2}}\cos^{-1}\left(1 - \frac{\gamma + 1}{1 + \frac{\gamma - 1}{2}M_{n2}^2}\right)\right] \qquad 122$$

$$- \left[\cos^{-1}\frac{1}{M_{n1}} + \frac{1}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right)^{\frac{1}{2}}\cos^{-1}\left(1 - \frac{\gamma + 1}{1 + \frac{\gamma - 1}{2}M_{n1}^2}\right)\right]$$

$$M_{n1} = \left\{\frac{2}{\gamma - 1}\left[\left(\frac{P_{t_{n1}}}{P_{n1}}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]\right\}^{\frac{1}{2}} \qquad 123$$

$$M_{n2} = \left\{\frac{2}{\gamma - 1}\left[\left(\frac{P_{t_{n2}}}{P_{n2}}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]\right\}^{\frac{1}{2}} \qquad 124$$

$$P_{t_{n1}} = P_{n1}\left(1 + \frac{\gamma - 1}{2} M_{n1}^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad .125$$

$$P_{t_{n2}} = P_{n2}\left(1 + \frac{\gamma - 1}{2} M_{n2}^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad 126$$

$$P_{n1} = \frac{P_{t_{n1}}}{\left(1 + \frac{\gamma - 1}{2} M_{n1}^2\right)^{\frac{\gamma}{\gamma - 1}}} \qquad 127$$

$$P_{n2} = \frac{P_{t_{n2}}}{\left(1 + \frac{\gamma - 1}{2} M_{n2}^2\right)^{\frac{\gamma}{\gamma - 1}}} \qquad 128$$

$$P_{c_{(n)}} = P_{n1}\left(1 + \frac{\gamma}{2} M_{n1}^2 \, C_{P_{c_{(n)}}}\right) \qquad 129$$

$$\left(\frac{\partial P}{\partial S}\right)_{n2} = \frac{b_{n2}}{r_n}\left[\frac{\Omega_{n1}}{\Omega_{n2}} \sin \sigma_{(n-1)} - \sin \sigma_{(n)}\right] + \frac{b_{n2}}{b_{n1}} \frac{\Omega_{n1}}{\Omega_{n2}} \left(\frac{\partial P}{\partial S}\right)_{n1} \qquad 130$$

$$\eta_{(n)} = \left(\frac{\partial P}{\partial S}\right)_{n2} \frac{x - x_n}{(P_{c_{(n)}} - P_{n2}) \cos \sigma_{(n)}} \qquad 131$$

$$P_{nx} = P_{c_{(n)}} - (P_{c_{(n)}} - P_{n2})\, e^{-\eta(n)x} \qquad 132$$

$$\left(\frac{\partial P}{\partial S}\right)_{(n+1)1} = \left(\frac{P_{c_{(n)}} - P_{(n+1)1}}{P_{c_{(n)}} - P_{n2}}\right)\left(\frac{\partial P}{\partial S}\right)_{n2} \qquad 133$$

$$\left(\frac{\partial P}{\partial S}\right)_{n2} = \frac{\tan\left[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}\right]}{\tan\left[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}\right] + \tan \mu_{n2}} \left[\frac{\gamma P_{n2} M_{n2}^2}{r_n (M_{n2}^2 - 1)}\left(\frac{\sin \epsilon_{(n)} \sin \sigma_{(n-1)}}{\sin\left[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}\right]}\right.\right.$$

$$\left. - \sin \sigma_{(n)}\right) + \left(\frac{\partial P}{\partial S}\right)_{n1}\left(\frac{P_{n2}}{P_{n1}} \frac{M_{n2}^2}{M_{n1}^2} \frac{(M_{n1}^2 - 1)}{(M_{n2}^2 - 1)} \frac{\sin \epsilon_{(n)}}{\sin\left[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}\right]}\right.$$

$$\left.\left. + \left[\frac{P_{n2}}{P_{n1}} - F\right] \frac{\cos \epsilon_{(n)} \tan \mu_{n2}}{\sin\left[\epsilon_{(n)} + \sigma_{(n-1)} - \sigma_{(n)}\right]}\right)\right] \qquad 134$$

$$F = \left(\frac{4}{\gamma + 1}\right)\left(1 + \frac{\gamma - 1}{2} M_{n1}^2\right) \sin \epsilon_{(n)} \zeta \qquad 135$$

where

$$\zeta = \frac{[(\gamma + 1) \tan (\sigma_{(n)} - \sigma_{(n-1)}) \cos \epsilon_{(n)} - \sin \epsilon_{(n)}] M_{n1}^2 \sin^2 \epsilon_{(n)} + \sin \epsilon_{(n)}}{1 + [1 - 2 \sin^2 \epsilon_{(n)} + 2 \tan (\sigma_{(n)} - \sigma_{(n-1)}) \sin \epsilon_{(n)} \cos \epsilon_{(n)}] M_{n1}^2 \sin^2 \epsilon_{(n)}}$$

$\underline{0 \leq M_{n1} \sin \sigma_{(n)} \leq 1.0}$

$$M_{n2} = \frac{M_{n1} \cos \sigma_{(n)} \left(1 - \frac{\sin \sigma_{(n)}}{M_{n1}}\right)^{\frac{1}{2}}}{[1.0 + 0.35(M_{n1} \sin \sigma_{(n)})^{1.5}]^{\frac{1}{2}}} \qquad 136$$

$\underline{M_{n1} \sin \sigma_{(n)} \geq 1.0}$

$$M_{n2} = \frac{M_{n1} \cos \sigma_{(n)} \left(1 - \frac{\sin \sigma_{(n)}}{M_{n1}}\right)^{\frac{1}{2}}}{\left\{\left[1 + \exp\left(-1 - 1.52 M_{n1} \sin \sigma_{(n)}\right)\right]\left[1 + \left(\frac{M_{n1} \sin \sigma_{(n)}}{2}\right)^2\right]\right\}^{\frac{1}{2}}} \qquad 137$$

$$C_{p_{C(n)}} = \frac{(4 \sin^2 \sigma_{(n)})(2.5 + 8\sqrt{M_{n1}^2 - 1} \sin \sigma_{(n)})}{(1 + 16\sqrt{M_{n1}^2 - 1} \sin \sigma_{(n)})} \qquad 138$$

4. LIMITS ON PARAMETERS

1. $(\sigma_{(n)} - \sigma_{(n-1)})$

   a. $0° < (\sigma_{(n)} - \sigma_{(n-1)}) \leq 30.0°$

2. M_{n1}

   a. $2.6 \leq M_{n1} \leq 10.0$

3. $\left(\frac{\partial P}{\partial S}\right)_{n2}$ for a convex corner, $(\sigma_{(n)} < \sigma_{(n-1)})$

   a. $\left(\frac{\partial P}{\partial S}\right)_{n2} \geq 0.0$

   b. Must have same sign as the pressure difference $(P_{c(n)} - P_{n2})$

4. $\left(\frac{\partial P}{\partial S}\right)_{n2}$ for a concave corner, $(\sigma_{(n)} > \sigma_{(n-1)})$

   a. $\left(\frac{\partial P}{\partial S}\right)_{n2} \leq 0.0$

   b. Must have same sign as the pressure difference $(P_{c(n)} - P_{n2})$

5. $\eta_{(n)}$ for both convex and concave body sections

   a. Because of 3 and 4 above, it follows that $\eta_{(n)} \geq 0.0$

5. NOMENCLATURE

Symbol	Definition	Units
A	Parameter Defined by Equation (59)	
A_B	Base Area of Body Section (n)	sq ft
b	Parameter Defined by Equation (17)	
B	Parameter Defined by Equation (60)	
C	Parameter Defined by Equation (61)	
$C_{M\alpha}$, $dC_M/d\alpha$	Slope of Pitching Moment Coefficient	1/deg
C_N	Normal Force Coefficient	dim.
C_p	Pressure Coefficient $(= P - P_\infty / \frac{\gamma}{2} P_\infty M_\infty^2)$	dim.
$C_{N\alpha}$, $dC_N/d\alpha$	Slope of Normal Force Coefficient	1/deg
$C_{N\alpha_{TC}}$	Slope of Normal Force Coefficient for a Tangent Cone	1/deg
$dC_{N\alpha}/dx$	Longitudinal Loading Parameter	1/deg-in.
d	Body Diameter	in. ft
D	Parameter Defined by Equation (62)	
e	Base of Naperian Logarithm	dim.
E	Parameter Defined Equation (65)	dim.
F	Parameter Defined by Equation (55) and (55a)	dim.
G	Parameter Defined by Equation (63)	dim.
H	Parameter Defined by Equation (64)	dim.
l	Upper Limit of Integration	ft
M	Mach Number	dim.
Mom	Moment	in-lb
N	Normal Force	lb
P	Pressure	psi psf
P_2	Static Pressure at Downstream Side of a Corner	psi psf
P_c	Cone Surface Static Pressure	psi psf

q	Dynamic Pressure $(\frac{\gamma}{2} PM^2)$	psf
r	Body Radius	in. ft.
S	Distance Along Streamline	in.
S_{ref}	Reference Area (Cross-sectional Area of Largest Cylindrical Body Section)	sq ft
x	Longitudinal Distance Along Body (Body Station)	in. ft
x_{cp}	Normal Force Center of Pressure, Body Station	in.
α	Angle of Attack in Pitch Plane	deg
γ	Ratio of Specific Heats	dim.
Γ	Loading Parameter Defined by Equation (11)	dim.
δ	Shockwave Angle Relative to Body Centerline	deg
ϵ	Shock Wave Angle Relative to Flow Direction Immediately Upstream of Body Corner (Two-Dimensional Flow)	deg
ζ	Parameter Defined by Equation (55a)	
η	Exponent of e	dim.
θ	Flow Turn Angle $(\sigma_{(n)} - \sigma_{(n-1)})$	deg
Λ	Loading Parameter Defined by Equation (3) & (12)	dim.
μ	$\sin^{-1} \frac{1}{M}$	dim.
ρ	Density	Slugs/cu ft
π	Pi = 3.1416	dim.
σ	Body Surface Angle Relative to Body Centerline	deg
φ	Body Meridional Angle	deg
Ω	Parameter Defined by Equation (18)	

6. SUBSCRIPTS

a	First Increment
b	Second Increment
B	Base
c	Cone Value
cp	Center of Pressure
i	i th Section or Location
(n)	Body Section for which Values are being Calculated
n	Upstream Corner of Body Section (n)
n1	Upstream Side of Upstream Corner of Body Section (n)
n2	Downstream Side of Upstream Corner of Body Section (n)
n3	Upstream Side of Downstream Corner of Body Section (n) (Same as (n + 1)1)
K	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Section (n)
L	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner K
M	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner L
N	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner M
O	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner N
P	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner O
Q	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner P
R	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner Q

S	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner R
T	Nearest Body Corner at which a Concave Corner Occurs Upstream of Body Corner S
t	Total
TC	Tangent Cone
ref	Reference
1	Upstream Side of Upstream Corner of Body Section
2	Downstream Side of Upstream Corner of Body Section
3	Upstream Side of Downstream Corner of Body Section
11	Body Free Stream Value
α	Derivative with Respect to Angle of Attack α
∞	Local Free Stream Value

D. LIFTING SURFACES AERODYNAMIC PARAMETERS

This subroutine is designed to provide the capability of predicting the force and moment contribution of lifting surfaces to missile aerodynamics and control. The planforms of the surfaces are restricted to a basic delta planform with an unswept trailing edge and taper ratios in the range of 0.0 to 0.60 ($0.0 \leq \lambda \leq 0.60$).

The surfaces, whether they are canards or tail fins, are considered to occur in sets consisting of two pairs of surfaces in a cruciform configuration. Deflections for pitch and yaw control are possible but only by deflecting the full semi-span surface (i.e. no provisions for partial chord flaps) and each surface of a pair is deflected identically to the other (i.e. no differential deflections).

The basic aerodynamic parameters derived from this procedure are $C_{L\alpha}$, C_{DO}, C_{DL}, and x_{cp} versus Mach number for canards and fins. The force coefficients $C_{L\alpha}$, C_{DO} and C_{DL} are calculated per pair based on the exposed planform area, S_{exp}, of the particular lifting surface. As it is desired to use the values of these parameters as they apply to a missile system, equations are provided to resolve them into normal, side and axial force coefficients, C_N, C_Y, and C_A in the missile axis system and based on S_{ref}, the missile reference area.

In addition, equations are provided for determination of hinge moments produced by the lifting surfaces and a procedure for calculating an estimated weight and center of gravity of the lifting surfaces. The weight calculated is an estimate only of the weight of the exposed surfaces and does not include an estimate for attachment or actuation system weights.

1. BASIC FORCE AND MOMENT EQUATIONS

Assuming a missile system composed of an axisymmetric body with cruciform canards and cruciform fins, the following equations can be written, referenced to the missile axis system.

Static Forces

$N = N_b + N_c + N_f$ (Pitch) (1)

$Y = Y_b + Y_c + Y_f$ (Yaw) (2)

$A = A_b + A_c + A_f$ (Axial) (3)

Where

$$N_b = C_{Nb}\ q\ S_{ref} \qquad (4)$$

$$N_c = C_{Nc}\ q\ S_{ref} \qquad (5)$$

$$N_f = C_{Nf}\ q\ S_{ref} \qquad (6)$$

$$Y_b = C_{Yb}\ q\ S_{ref} \qquad (7)$$

$$Y_c = C_{Yc}\ q\ S_{ref} \qquad (8)$$

$$Y_f = C_{Yf}\ q\ S_{ref} \qquad (9)$$

And

$$C_{Nb} = C_{N\alpha b}\alpha \qquad (10)$$

$$C_{Nc} = C_{N\alpha c}\ \alpha_{effc}\ \frac{S_c}{S_{ref}}$$

$$= \frac{S_c}{S_{ref}} \left(\left[C_{L\alpha c}(\alpha+\delta_{cP})+C_{1c}(\alpha+\delta_{cP})^2\right]\cos\alpha\right.$$

$$\left.+\left\{C_{D0c}+K_{Lc}\left[C_{L\alpha c}(\alpha+\delta_{cp})+C_{1c}(\alpha+\delta_{cp})^2\right]^2\right\}\sin\alpha\right) \qquad (11)$$

$$C_{Nf} = C_{N\alpha f}\alpha_{efff}\ \frac{S_f}{S_{ref}}$$

$$= \frac{S_f}{S_{ref}} \left(\left[C_{L\alpha f}(\alpha+\delta_{fP})+C_{1f}(\alpha+\delta_{fP})^2\right]\cos\alpha\right.$$

$$\left.+\left\{C_{D0f}+K_{Lf}\left[C_{L\alpha f}(\alpha+\delta_{fP})+C_{1f}(\alpha+\delta_{fP})^2\right]^2\right\}\sin\alpha\right) \qquad (12)$$

$$C_{Yb} = C_{N\alpha b}\beta \qquad (13)$$

$$C_{Yc} = C_{N\alpha c}\beta_{effc}\frac{S_c}{S_{ref}}$$

$$= \frac{S_c}{S_{ref}}\left(\left[C_{L\alpha c}(\beta+\delta_{cY})+C_{lc}(\beta+\delta_{cY})^2\right]\cos\beta\right.$$

$$\left.+\left\{C_{D0c}+K_{Lc}\left[C_{L\alpha c}(\beta+\delta_{cY})+C_{lc}(\beta+\delta_{cY})^2\right]^2\right\}\sin\beta\right) \quad (14)$$

$$C_{Yf} = C_{N\alpha f}\beta_{efff}\frac{S_f}{S_{ref}} \quad (15)$$

$$= \frac{S_f}{S_{ref}}\left(\left[C_{L\alpha f}(\beta+\delta_{fY})+C_{lf}(\beta+\delta_{fY})^2\right]\cos\beta\right.$$

$$\left.+\left\{C_{D0f}+K_{Lf}\left[C_{L\alpha f}(\beta+\delta_{fY})+C_{lf}(\beta+\delta_{fY})^2\right]^2\sin\beta\right) \quad (15)$$

For the Axial Force

$$A_b = C_{Ab}\, q\, S_{ref} \quad (16)$$

$$A_c = (C_{AcP}+C_{AcY})\, q\, S_{ref} \quad (17)$$

$$A_f = (C_{AfP}+C_{AfY})\, q\, S_{ref} \quad (18)$$

Where

$$C_{Ab} = C_{Ab} \quad (19)$$

$$C_{AcP} = \left(\left\{C_{D0c}+K_{Lc}\left[C_{L\alpha c}(\alpha+\delta_{cP})+C_{lc}(\alpha+\delta_{cP})^2\right]^2\right\}\cos\alpha\right.$$

$$\left.-\left[C_{L\alpha c}(\alpha+\delta_{cP})+C_{lc}(\alpha+\delta_{cP})+C_{lc}(\alpha+\delta_{cP})^2\right]\sin\alpha\right)\frac{S_c}{S_{ref}} \quad (20)$$

$$C_{AcY} = \left(\left\{C_{D0c}+K_{Lc}\left[C_{L\alpha c}(\beta+\delta_{cY})+C_{1c}(\beta+\delta_{cY})^2\right]^2\right\}\cos\beta - \left[C_{L\alpha c}(\beta+\delta_{cY})+C_{1c}(\beta+\delta_{cY})^2\right]\sin\beta\right)\frac{S_c}{S_{ref}} \quad (21)$$

$$C_{AfP} = \left(\left\{C_{D0f}+K_{Lf}\left[C_{L\alpha f}(\alpha+\delta_{fP})+C_{1f}(\alpha+\delta_{fP})^2\right]^2\right\}\cos\alpha - \left[C_{L\alpha f}(\alpha+\delta_{fP})+C_{1f}(\alpha+\delta_{fP})^2\right]\sin\alpha\right)\frac{S_f}{S_{ref}} \quad (22)$$

$$C_{AfY} = \left(\left\{C_{D0f}+K_{Lf}\left[C_{L\alpha f}(\beta+\delta_{fY})+C_{1f}(\beta+\delta_{fY})^2\right]^2\right\}\cos\beta - \left[C_{L\alpha f}(\beta+\delta_{fY})+C_{1f}(\beta+\delta_{fY})^2\right]\sin\beta\right)\frac{S_f}{S_{ref}} \quad (23)$$

Static Moments

$$Mom = M_b + M_c + M_f \quad (24)$$

Pitch Moment

$$M_y = N_b\,\bar{x}_b + N_c\,\bar{x}_c + N_f\,\bar{x}_f + A_b\bar{z}_b \quad (25)$$

Yaw Moment

$$M_z = Y_b\,\bar{x}_b + Y_c\,\bar{x}_c + Y_f\,\bar{x}_f + A_b\,\bar{y}_b \quad (26)$$

Roll Moment

$$M_x = -N_b\,\bar{y}_b - Y_b\,\bar{z}_b - N_c\,y_{cg} - N_f\,y_{cg} - Y_c\,z_{cg} - Y_f\,z_{cg} \quad (27)$$

$$= -N_b\,\bar{y}_b - Y_b\,\bar{z}_b - N_c\,\bar{y}_b - N_f\,\bar{y}_b - Y_c\,\bar{z}_b - Y_f\,\bar{z}_b \quad (28)$$

$$= -(N_b + N_c + N_f)\,\bar{y}_b - (Y_b + Y_c + Y_f)\,\bar{z}_b \quad (29)$$

The forces N_b, N_c, N_f, A_b, Y_b, Y_c and Y_f have been defined leaving the definition of the moment arms. These are as follows:

$$\bar{x}_b = x_{cg} - x_{cpb} \tag{30}$$

$$\bar{x}_c = x_{cg} - x_{cpc} \tag{31}$$

$$\bar{x}_f = x_{cg} - x_{cpf} \tag{32}$$

$$\bar{z}_b = z_{cg} \tag{33}$$

$$\bar{y}_b = y_{cg} \tag{34}$$

Lifting Surface Hinge Moment

The general equations for the lifting surface hinge moments and moment arms are:

$$(H.M.)_y = N\bar{x}_h - C_A\bar{z}_h \tag{35}$$

$$(H.M.)_z = Y\bar{x}_h + C_A\bar{y}_h \tag{36}$$

Where

$$\bar{x}_h = (x_H - x_{cp}) \cos \delta \tag{37}$$

$$\bar{y}_h = (x_H - x_{cp}) \sin \delta \tag{38}$$

$$\bar{z}_h = (x_H - x_{cp}) \sin \delta \tag{39}$$

Thus, for the canards

$$(H.M.)_{yc} = N_c(x_{Hc} - x_{cpc})\cos \delta_{cP} + C_{AcP} (x_{Hc} - x_{cpc}) \sin \delta_{cP} \tag{40}$$

$$(H.M.)_{zc} = Y_c (x_{Hc} - x_{cpc})\cos \delta_{cY} + C_{AcY} (x_{Hc} - x_{cpc}) \sin\delta_{cY} \tag{41}$$

and similarly for the fins

$$(H.M.)_{yf} = N_f \ (x_{Hf} - x_{cpf}) \cos \delta_{fP} + C_{AfP} \ (x_{Hf} - x_{cpf}) \sin \delta_{fP} \qquad (42)$$

$$(H.M.)_{zf} = Y_f \ (x_{Hf} - x_{cpf}) \cos \delta_{fY} + C_{AfY} \ (x_{Hf} - x_{cpf}) \sin \delta_{fY} \qquad (43)$$

2. INPUT PARAMETERS

The parameters required as input are x_{or}, c_R, ϵ_L, λ, τ, x_H, r_b and d_{ref}. Except for d_{ref}, sets of values of the other parameters must be provided for each set of lifting surfaces, canards and tail fins.

Limitations exist on the range of permissible values for some of these parameters and are defined below. Definitions of all the parameters can be found in the section on nomenclature and sketches defining the axis system; lifting surface geometry and missile geometry are provided in Figures 7 thru 10.

$$24.0° \leq \epsilon_L \leq 70.0°$$

$$0.0 \leq \lambda \leq 0.60$$

$$0.03 \leq \tau \leq 0.12$$

$$x_{orf} \leq x_e - c_{Rf}$$

The above are limitations on the input parameters and must be controlled at that point. Additional limitations exist relative to values of some of the parameters which are calculated. These are:

AR Limits on this parameter are defined in Figure 11 as a function of the limits imposed on ϵ_L and λ.

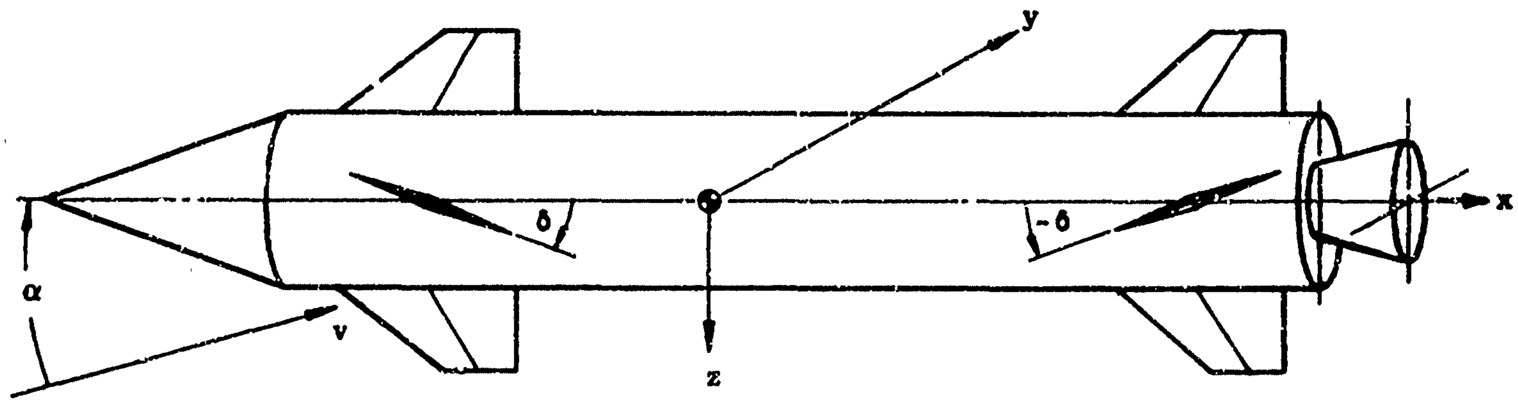
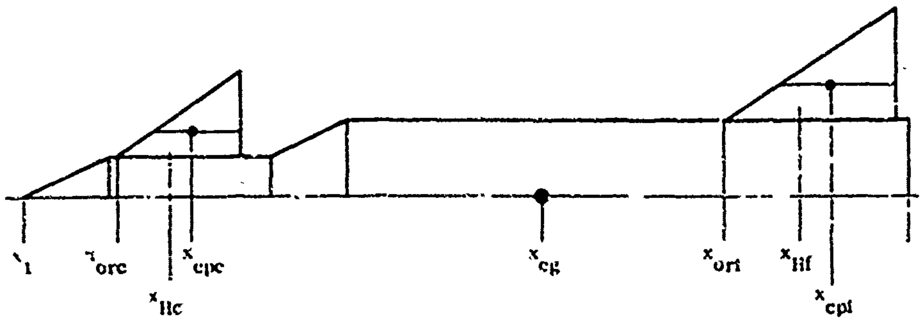
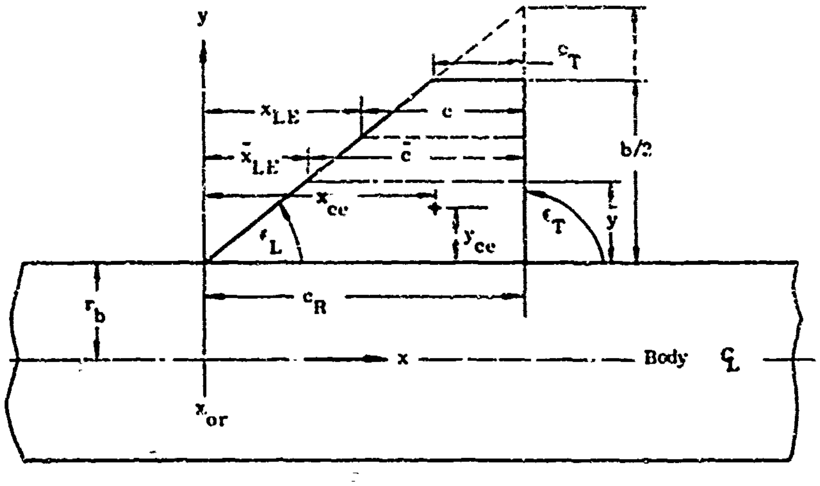


Figure 7. Missile Axis System

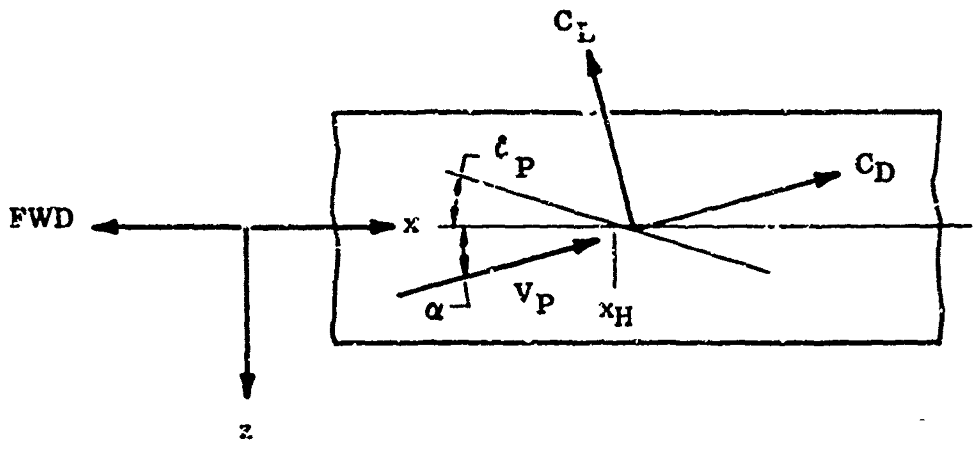


a. MISSILE SYSTEM

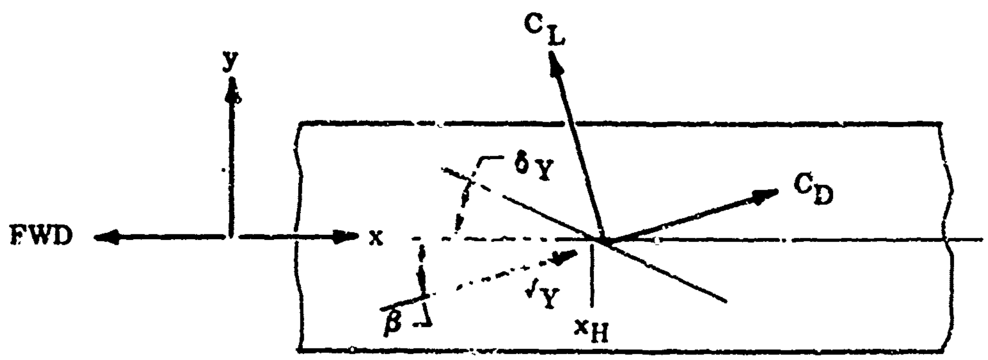


b. LIFTING SURFACE

Figure 8. Lifting Surface Geometry Parameters

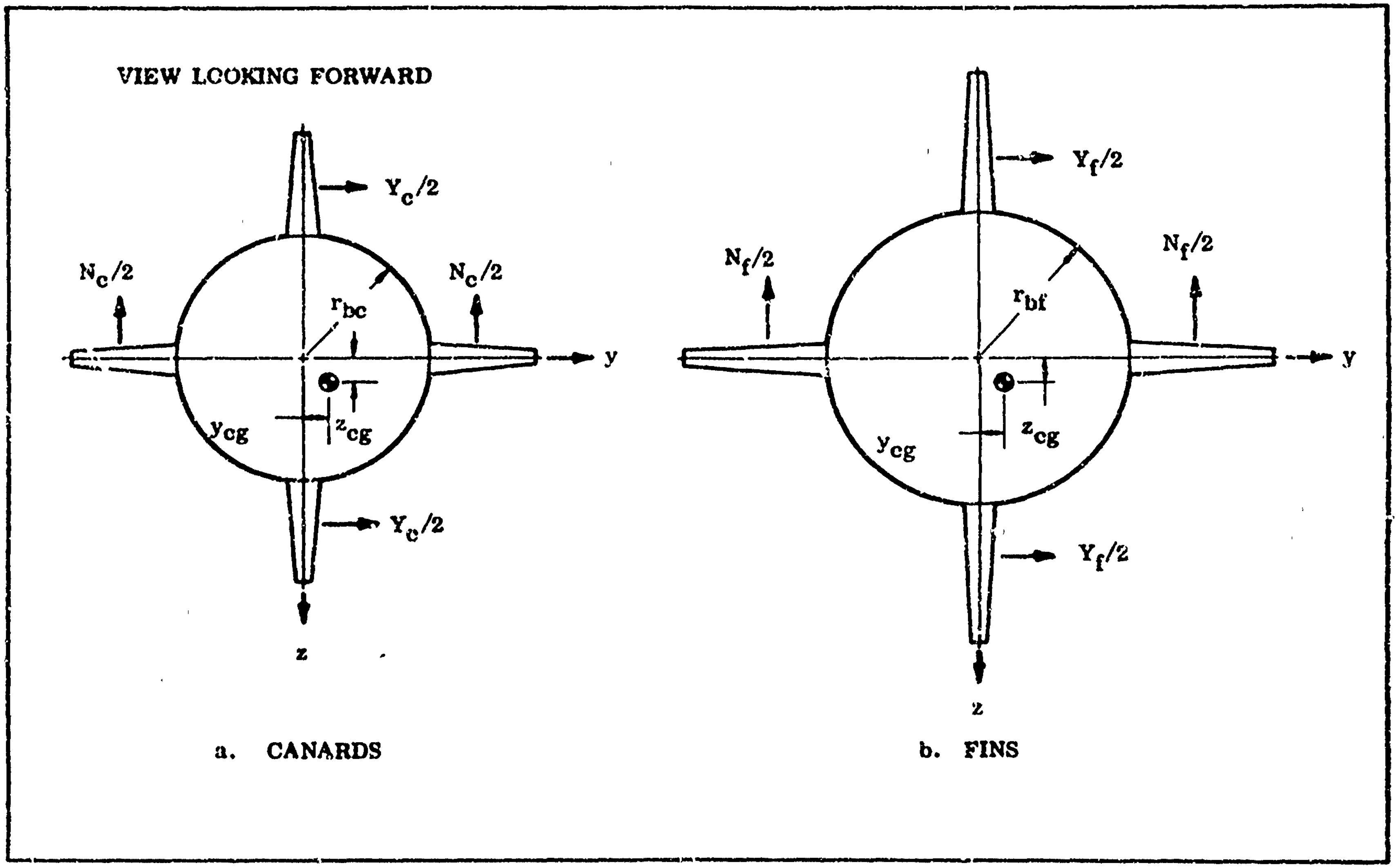


a. PITCH PLANE



b. YAW PLANE

Figure 9. Lifting Surfaces Pitch, Yaw and Deflection Angles and Force Coefficients



a. CANARDS

b. FINS

Figure 10. Cruciform Configuration

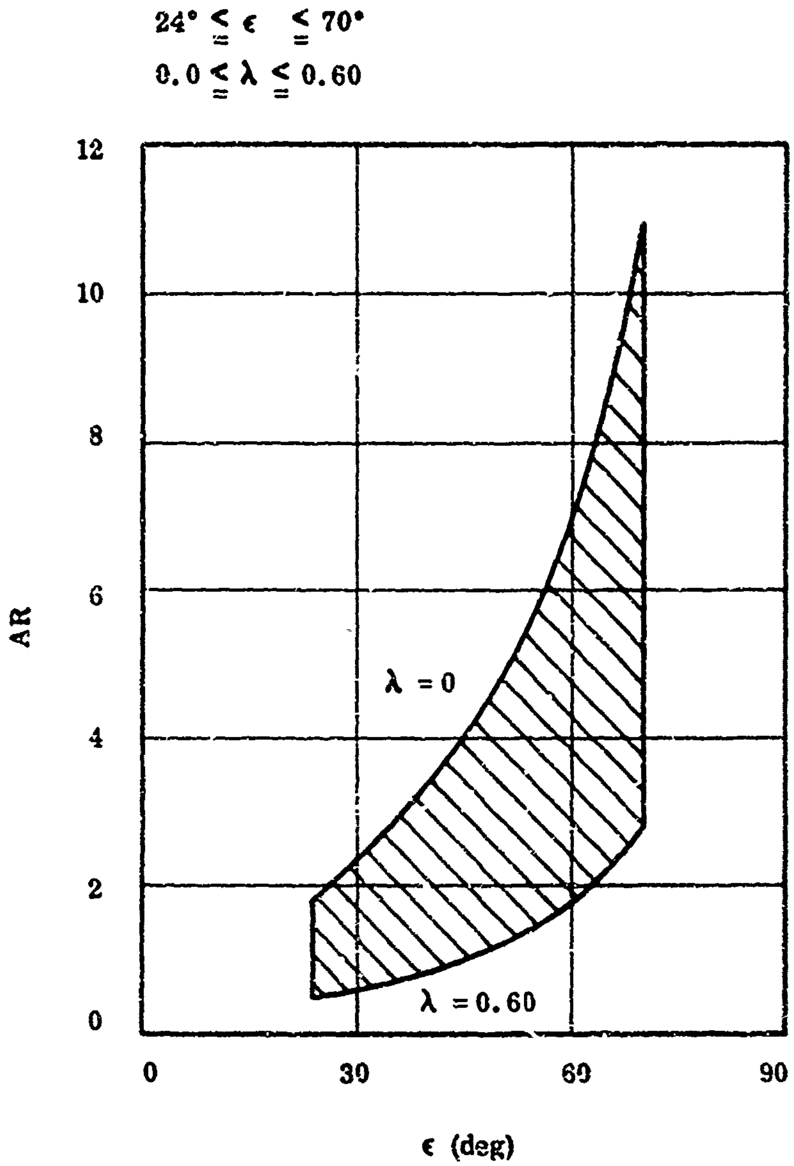


Figure 11. Range of Aspect Ratios Accommodated for Calculation of C_{L_α}

3. DETERMINATION OF PARAMETERS

An examination of the equations which define the static forces and moments will show that values must be determined for the parameters $C_{L\alpha}$, C_1, C_{DO}, K_L, S_{exp} and X_{cp}. Values of the parameters versus Mach number must be determined for each set of surfaces.

$C_{L\alpha}$

The linear value of lift curve slope for canards and fins is determined versus Mach number. For subsonic values of Mach number, $0 \leq M \leq 1.0$, $C_{L\alpha}$ is determined from Figure 12 as a function of $\sqrt{1-M^2}$ tan ϵ_L and taper ratio, λ; and for supersonic Mach numbers, $M > 1.0$, $C_{L\alpha}$ is determined from Figure 13 and the equation shown on that figure as a function of $\sqrt{M^2-1}$ AR and taper ratio. The parameter determined in this manner is $C_{L\alpha}/AR$ per radian, and to produce $C_{L\alpha}$ per degree use the equation below:

$$C_{L\alpha} = \left(\frac{C_{L\alpha}}{AR}\right)\frac{AR}{57.3} \qquad (44)$$

Data provided for the above will accommodate values of βAR up to 109.34788 which corresponds to a condition of $M = 10.0$, $AR = 10.98988$ and $\lambda = 0$.

C_1

The parameter C_1 is defined as the nonlinear lift curve factor and is obtained from Figure 14 for the appropriate aspect ratio.

C_{DO}

The drag coefficient at zero lift, C_{DO}, is composed of two components, friction drag coefficient, C_{DF}, and form or pressure drag coefficient, C_{DP}. For incompressible flow, the form drag coefficient is small enough relative to the friction drag coefficient, that the parameter C_{DO} for $M<1.0$ is defined as follows:

$$C_{DO} = \frac{1.11\ C_{DF}}{\sqrt{1-M^2}} \qquad M < 1.0 \qquad (45)$$

The above is limited to the case where $M < 1.0$, therefore, another definition for C_{DO} must be used for the case where $M > 1.0$, thus

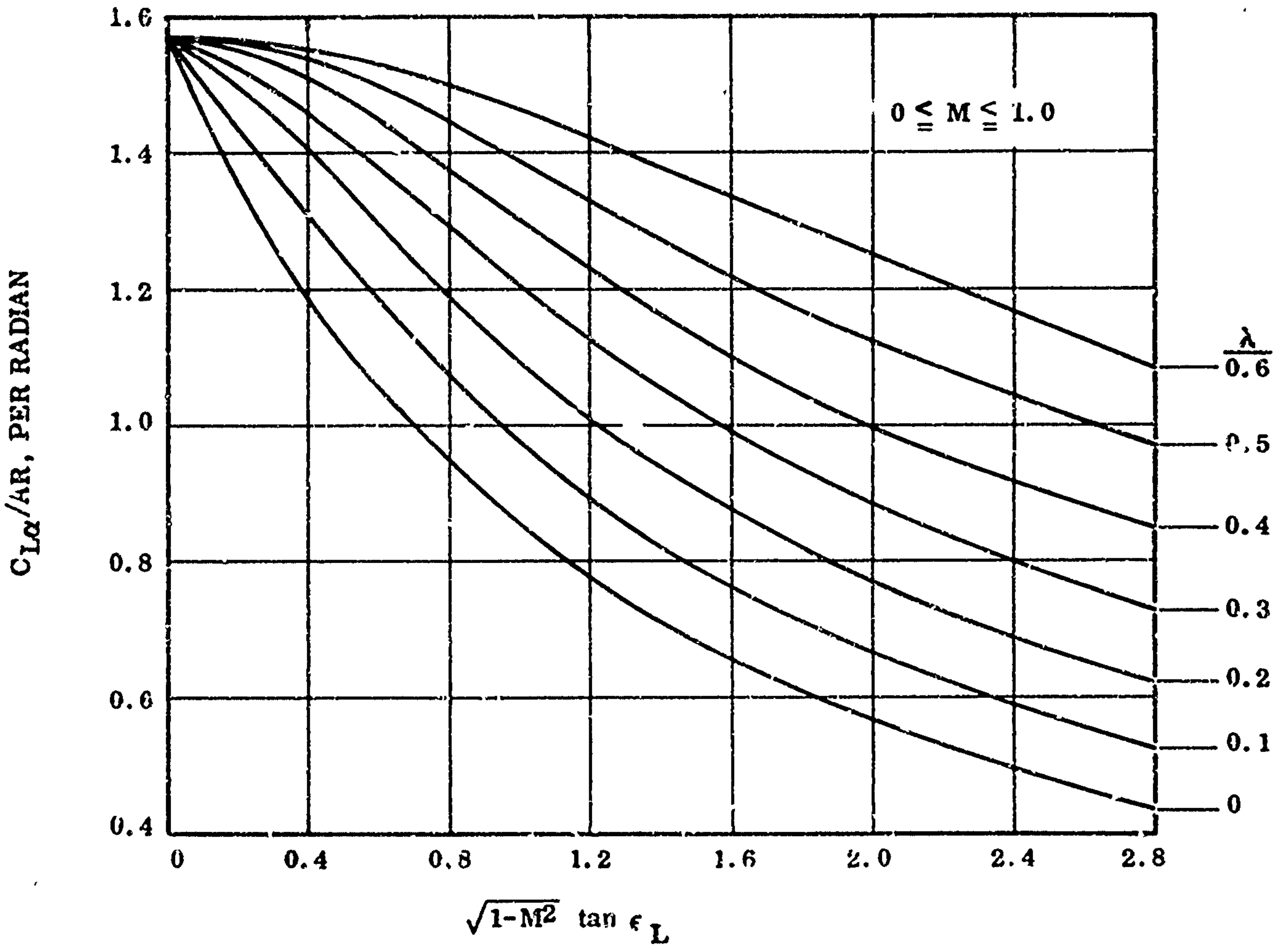


Figure 12. Subsonic Linear Lift Curve Slope

SUPERSONIC LINEAR LIFT CURVE SLOPE

For $\sqrt{M^2-1}$ AR > 4.0 Use

$$(C_{L\alpha}/AR)_i = (C_{L\alpha}/AR)_o (\beta_o/\beta_i)$$

Where

$(C_{L\alpha}/AR)_i$ Is Value When $\sqrt{M^2-1}$ AR > 4.0
$(C_{L\alpha}/AR)_o$ Is Value When $\sqrt{M^2-1}$ AR = 4.0
$\beta_o = \sqrt{M^2-1}$ When $\sqrt{M^2-1}$ AR = 4.0
$\beta_i = \sqrt{M^2-1}$ When $\sqrt{M^2-1}$ AR > 4.0
$(\sqrt{M^2-1}\ AR)_{max} = 102.34788$

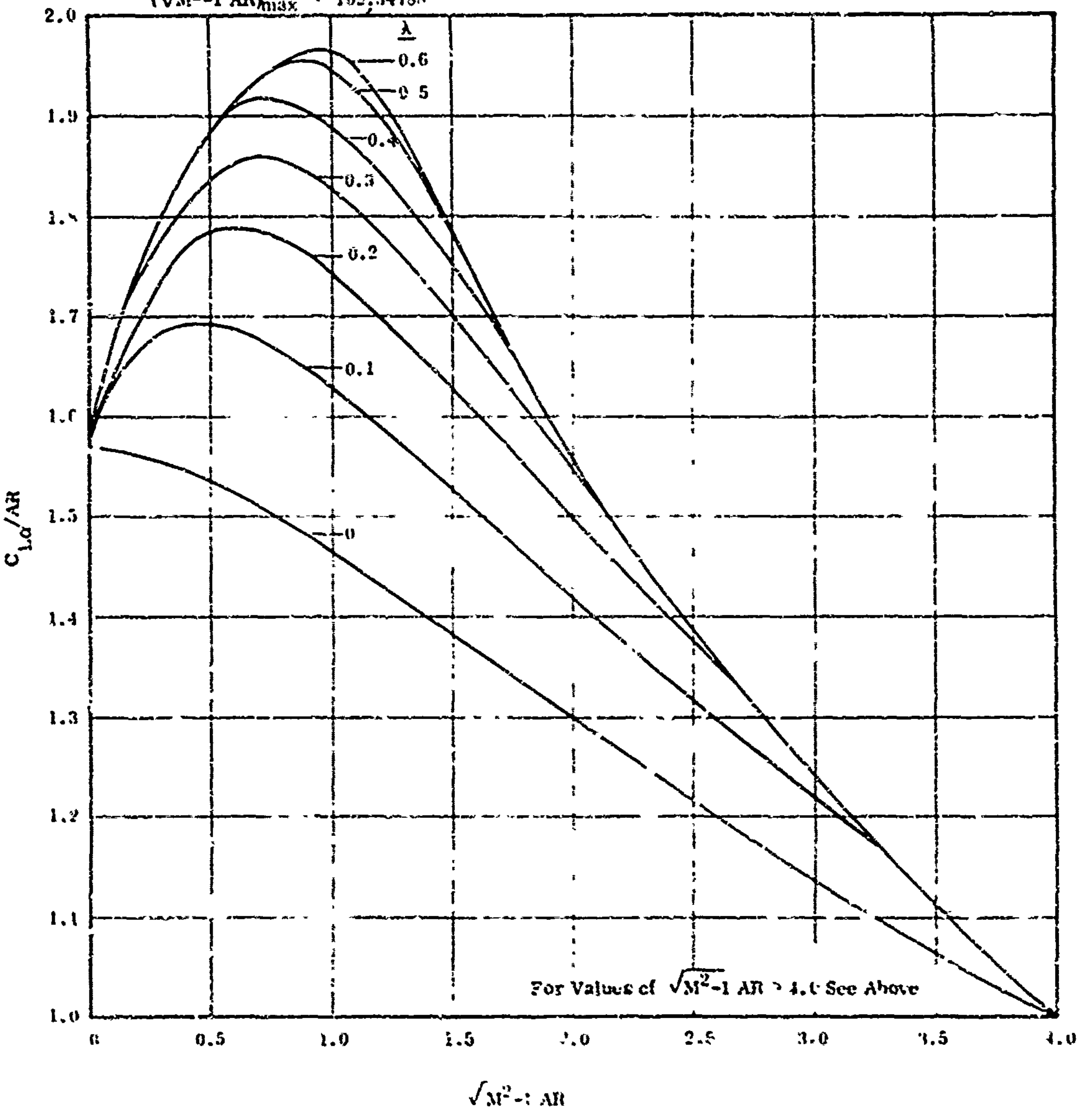


Figure 13. Supersonic Linear Lift Curve Slope

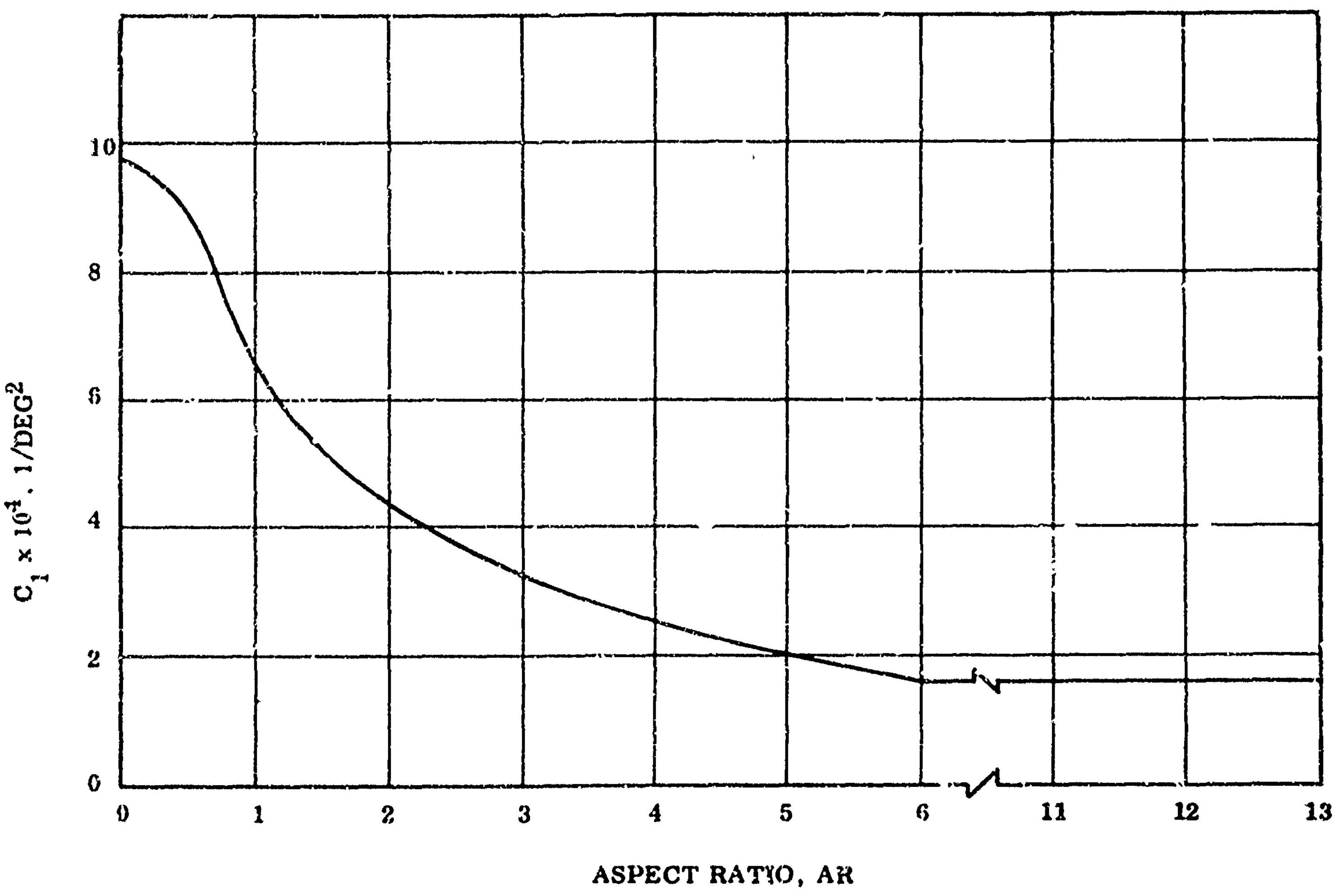


Figure 14. Nonlinear Lift Factor

$$C_{D0} = C_{DF} + C_{DP} \qquad M > 1.0 \tag{46}$$

where

$$C_{DP} = K_D \tau^2 \left\{ B(10)^{CAR^i} \left[(M - 0.9) + DAR^j \right]^{EAR^k} \right\} \tag{46a}$$

with

$B = 0.51393$ $\qquad i = -0.79948$

$C = 0.35801$ $\qquad j = -1.4767$

$D = 1.65$ $\qquad k = -0.1765$

$E = -1.0613$ $\qquad K_D = 5.743$

The friction drag coefficient C_{DF} is a function of Reynolds number and wetted area and is determined in the following manner.

a. For a given Mach number and type of missile, determine R_N/FT from Table 9.

b. Calculate $\bar{c}$ in feet

$$\bar{c} = \frac{c_R}{18} \left(1 + \frac{\lambda^2}{1 + \lambda}\right) \tag{47}$$

where c_R is in inches

c. Calculate R_N

$$R_N = (R_N/FT) \ \bar{c} \tag{48}$$

d. For given M and corresponding R_N, calculate C_f from equation 49

$$C_f = \left[\frac{0.455}{(\log_{10} R_N)^{2.58}}\right] \left[(1 + 0.162\ M^2)^{-0.58}\right] \tag{49}$$

or, for hand calculations, C_f may be obtained from Figure 15.

e. Calculate C_{DF}

$$C_{DF} = C_f \frac{A_{WET}}{S_{Exp}}$$

$$= 2C_f \tag{50}$$

TABLE 9

VARIATION OF REYNOLDS NUMBER PER FOOT VERSUS MACH NUMBER

Mach Number	$(R_N/FT)10^{-6}$*			
	(1)	(2)	(3)	(4)
0.0	3.70	3.30	3.55	1.976
0.50	3.70	3.30	3.55	1.976
0.75	5.25	4.62	4.68	2.964
0.95	6.25	5.55	5.45	3.754
1.00	6.50	5.77	5.55	3.952
1.25	7.47	6.57	6.21	4.940
1.50	8.22	7.13	6.75	5.928
2.00	9.02	7.87	7.40	7.904
2.60	9.20	8.21	7.50	10.275
3.00	8.90	8.15	7.04	11.856
4.00	7.00	6.70	4.52	15.808
5.00	4.75	4.85	2.62	19.760
6.00	3.10	3.40	1.44	23.712
8.00	1.05	1.18	0.48	31.616
10.00	0.45	0.42	0.30	39.520

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*

(1) Small Ballistic Missile

(2) Large Ballistic Missile

(3) Large Booster Vehicle

(4) Small Air Launched Missile

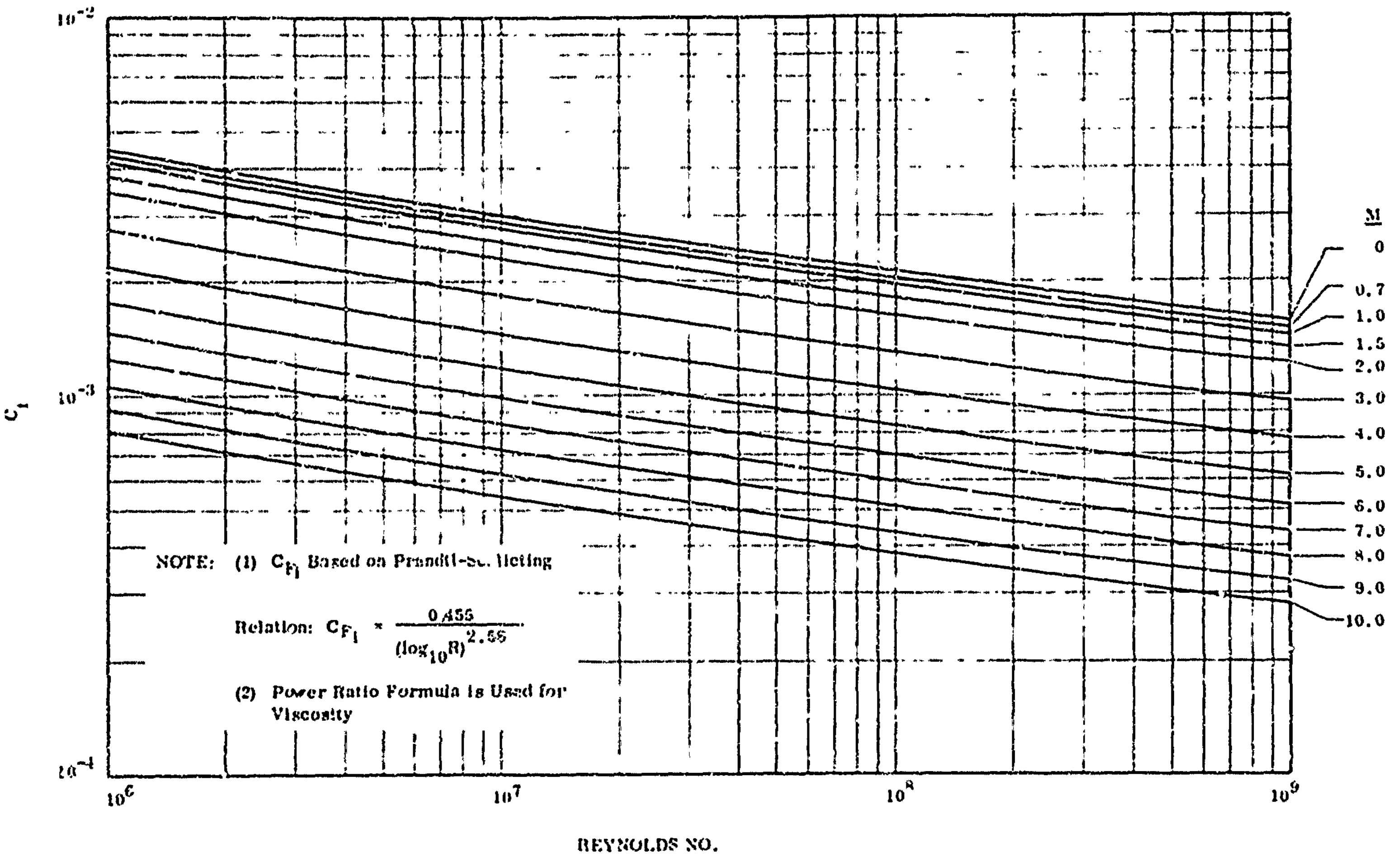


Figure 15. Turbulent Skin Friction Coefficient for Insulated Flat Plate

K_L

The drag due to lift factor, K_L, describes the shape of the drag polar as a function of $C_L{}^2$ and, combined with C_{DO}, provides the drag coefficient of the lifting surface. As used in this procedure, the following definition applies.

$$K_L = \frac{1}{C_{L\alpha}} \tag{51}$$

where $C_{L\alpha}$ is in radian units.

S_{Exp}

The exposed planform area, S_{Exp}, is defined as the planform area of the lifting surface which exists external to the body and is calculated from the equation below.

$$S_{Exp} = (1 - \lambda^2)\ c_R^2 \tan \epsilon_L \tag{52}$$

x_{cp}

The longitudinal location of the lifting surface center of pressure is defined in inches of missile body station and is determined from the equations below as a function of geometric location and Mach number.

$$x_{cp} = x_{or} + \bar{x}_{LE} + K_{cp}\ \bar{c} \tag{53}$$

where

$$\bar{x}_{LE} = \frac{\bar{y}}{\tan \epsilon_L}$$

$$= \frac{(1 + 2\lambda)\ (1 - \lambda)\ c_R}{3\ (1 + \lambda)} \tag{54}$$

and

$$\bar{c} = \frac{2}{3}\ c_R\ (1 + \frac{\lambda^2}{1 + \lambda})$$

and K_{cp} is determined as a function of Mach number from Figure 16.

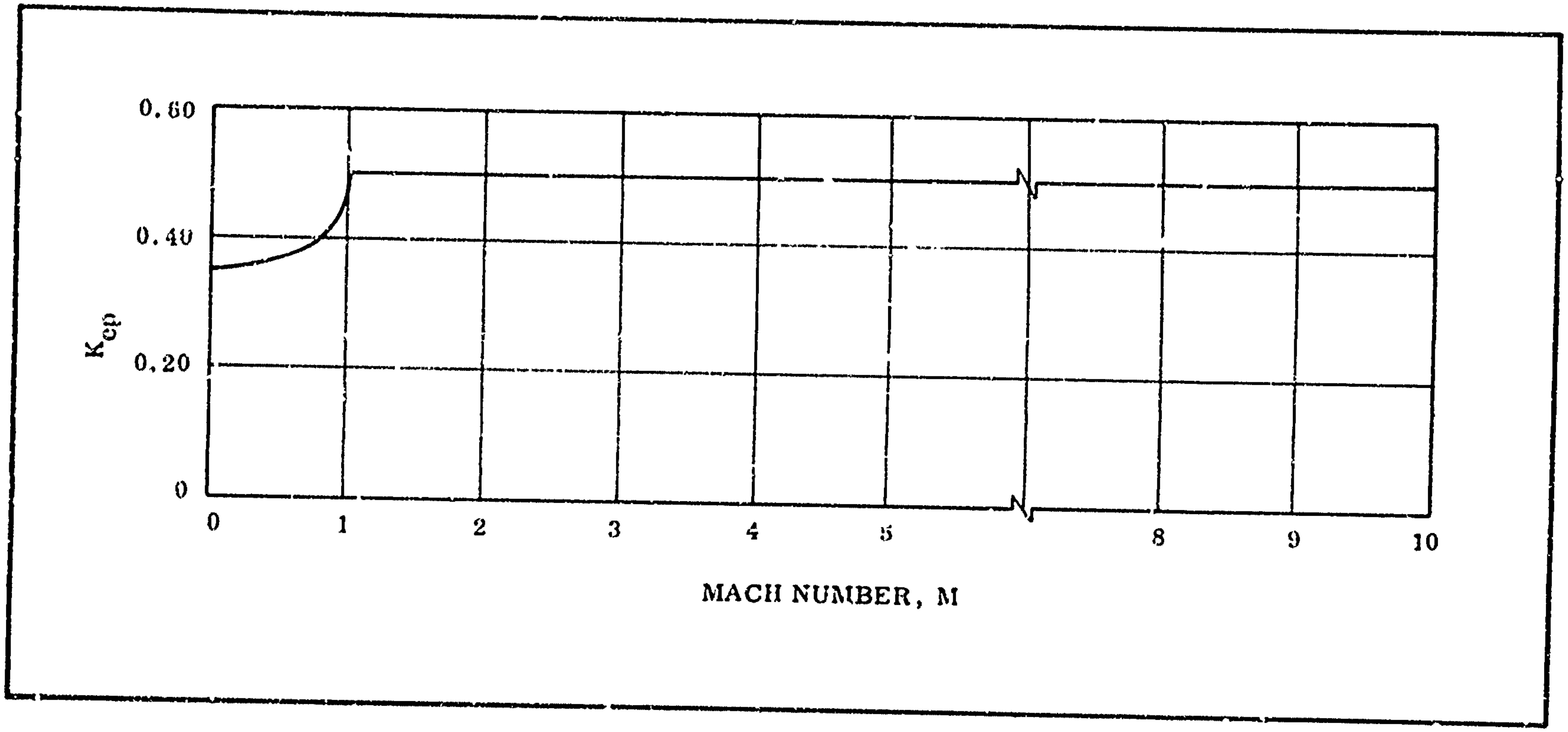


Figure 16. Center of Pressure Factor

y_{cp} and z_{cp}

The spanwise center of pressure is determined from the basic assumption that the spanwise loading distribution over the lifting surfaces is elliptical in shape at all Mach numbers. With this assumption then, the center of pressure is a constant fraction of the lifting surface semi-span, b/2, and the definition below will be used.

$$y_{cp} = r_b + 0.42441\ (b/2) \tag{56}$$

and

$$z_{cp} = r_b + 0.42441\ (b/2) \tag{57}$$

For application in the controls section of the trajectory subroutine, it is desired that the center of pressure and hinge line be referenced from the root chord leading edge in fraction of the root chord. This can be accomplished by using the equations below.

$$\frac{x_{cp}}{c_R} = \frac{x_{LE} + K_{cp}\ \bar{c}}{c_R} \tag{58}$$

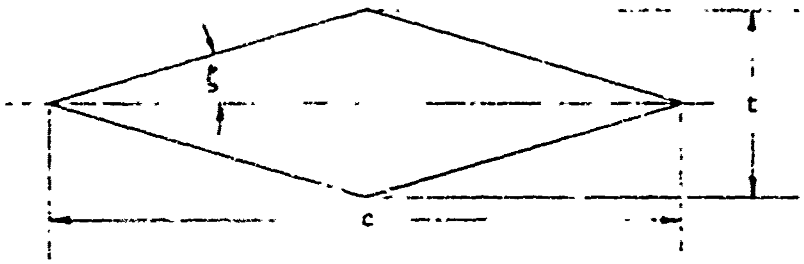
$$\frac{x_H}{c_R} = \frac{x_H - x_{or}}{c_R} \tag{59}$$

4. LIFTING SURFACE WEIGHT AND CENTER-OF-GRAVITY

The weight and center of gravity of the lifting surfaces, as formulated below, will be based on a symmetric double wedge type airfoil section. The volume of material for each fin will be calculated assuming a solid airfoil section. The resulting weight will be calculated from the volume and the density of the material used. An option between two materials, steel and aluminum alloy, is provided.

Using the approach of a solid airfoil section for the lifting surfaces will provide an estimate of what should be a near maximum weight for a set of canards or fins in a cruciform configuration.

a. Lifting Surface Weight



$$t = \tau c \tag{60}$$

$$\mathrm{TAN}\ \zeta = \frac{t}{2} \Big/ \frac{c}{2} = \frac{t}{c} \tag{61}$$

Cross Sectional Area

$$A = (4)\ (\tfrac{1}{2})\ (\tfrac{t}{2})\ (\tfrac{c}{2}) \qquad (62)$$

$$= \tfrac{1}{2}\ tc$$

$$= \tfrac{1}{2}\ \tau c^2$$

Where $\tau = \frac{t}{c}$

$$c = x_{TE} - x_{LE} \qquad (63)$$

$$x_{TE} = x_{or} + c_R \qquad (64)$$

$$x_{LE} = x_{or} + y \cot \epsilon_L \qquad (65)$$

$$c = c_R - y \cot \epsilon_L \qquad (66)$$

$$VOL = 4 \int_0^{b/2} A_{x\text{-}sect} dy$$

$$= 4 \int_0^{b/2} \tfrac{1}{2} \tau c^2 dy$$

$$= 2\tau \int_0^{b/2} (c_R - y \cot \epsilon_L)^2$$

$$= 2\tau \left[c_R^2\, y - 2c_R \cot \epsilon_L \frac{y^2}{2} + \frac{y^3}{3} \cot^2 \epsilon_L \right]_0^{b/2}$$

$$= \tau \left[bc_R^2 = \tfrac{1}{2}\ c_R\ b^2 \cot \epsilon_L + 1/12\ b^3 \cot^2 \epsilon_L \right] \qquad (67)$$

$$WT = (\rho)\ (VOL)$$

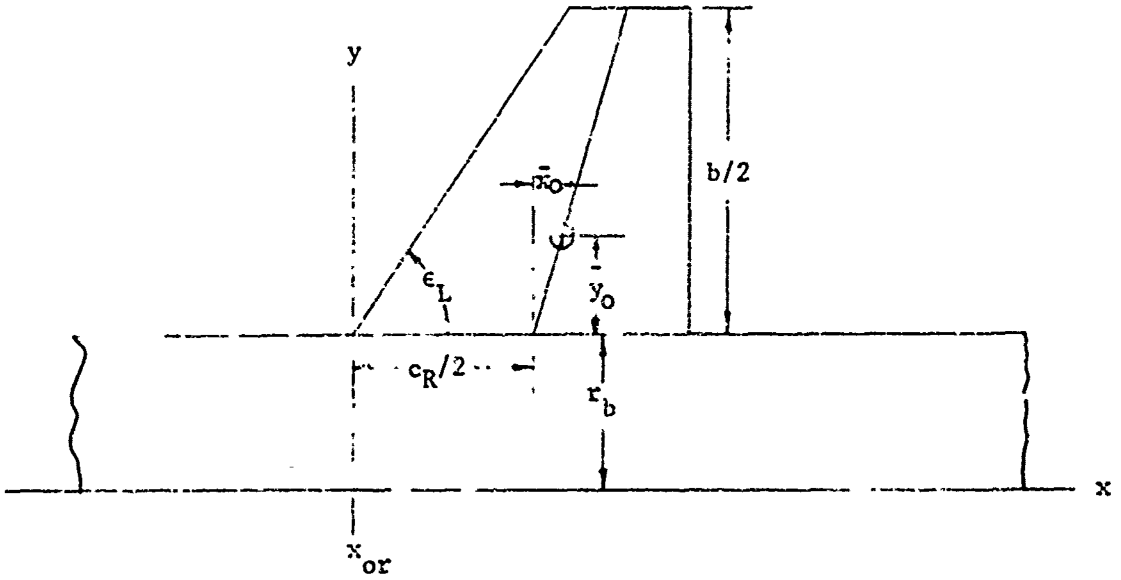
$$= \tau\rho \left[bc_R^2 - \tfrac{1}{2}\ c_R\ b^2 \cot \epsilon_L + 1/12\ b^3 \cot^2 \epsilon_L \right] \qquad (68)$$

Where ρ is determined from

Material	Density ρ lb/in^3
Aluminum Alloy	.101
Steel	.286

b. Lifting Surface Center-of-Gravity

The center of gravity of each lifting surface will be calculated using the same assumption of a solid airfoil section as for the weight calculation. The coordinate system is as shown in the sketch below with the equations following.



Longitudinal Location of Center of Gravity

$$\bar{X} = x_{or} + c_R/2 + \bar{x}_o \qquad (69)$$

$$\bar{x}_o = \frac{\text{Moment of Fin Volume about } c_R/2 \text{ Axis}}{\text{Volume per Fin}} \qquad (70)$$

The volume per fin, as defined for the weight calculation, is

$$\text{Vol} = \int_0^{b/2} A_{x\text{-sect}}\, dy \qquad (71)$$

$$\text{Vol} = \int_0^{b/2} \frac{\tau}{2} (c_R - y \cot \epsilon_L)^2 dy$$

$$= \frac{\tau}{16} [4bc_R^2 - 2b^2 c_R \cot \epsilon_L + \frac{1}{3} b^3 \cot^2 \epsilon_L] \tag{72}$$

and the moment about an axis at $c_R/2$ normal to the x axis is

$$\text{Mom}_x = \text{Vol} \cdot \bar{x}_o \tag{73}$$

$$\bar{x}_o = \frac{\text{Mom}_x}{\text{Vol}} \tag{74}$$

$$\text{Mom}_x = \int_0^{b/2} A_{x\text{-sect}} \bar{x}_i \, dy \tag{75}$$

where $\bar{x}_i = y/2 \cot \epsilon_L$

$$\text{Mom}_x = \int_0^{b/2} \frac{\tau}{2} (c_R - y \cot \epsilon_L)^2 (y/2 \cot \epsilon_L) dy$$

$$= \int_0^{b/2} \frac{\tau}{4} (c_R^2 y \cot \epsilon_L - 2y^2 c_R \cot^2 \epsilon_L + y^3 \cot^3 \epsilon_L) dy$$

$$= \frac{\tau}{32} [b^2 c_R^2 \cot \epsilon_L - \frac{2}{3} b^3 c_R \cot^2 \epsilon_L + \frac{1}{8} b^4 \cot^3 \epsilon_L] \tag{76}$$

thus

$$\bar{x}_o = \frac{[b^2 c_R^2 \cot \epsilon_L - \frac{2}{3} b^3 c_R \cot^2 \epsilon_L + \frac{1}{8} b^4 \cot^3 \epsilon_L]}{2[4bc_R^2 - 2b^2 c_R \cot \epsilon_L + \frac{1}{3} b^3 \cot^2 \epsilon_L]} \tag{77}$$

Lateral Location of Center of Gravity

$$\bar{Y} = r_b + \bar{y}_o \tag{78}$$

$$\bar{y}_o = \frac{\text{Moment of Fin Volume about } c_R}{\text{Volume per Fin}} \tag{79}$$

The volume per fin is as defined before, and the moment about an axis parallel to the x axis and located at the root chord of the surface is

$$\text{Mom}_y = \text{Vol} \cdot \bar{y}_o \tag{80}$$

$$\bar{y}_o = \frac{\text{Mom}_y}{\text{Vol}} \tag{81}$$

$$\text{Mom}_y = \int_0^{b/2} A_{x\text{-sect}} \; \bar{y}_i \; dy \tag{82}$$

where $y_i = y$

Thus

$$\text{Mom}_y = \int_0^{b/2} \frac{\tau}{2} (c_R - y \cot \epsilon_L)^2 y dy$$

$$= \int_0^{b/2} \frac{\tau}{2} (y c_R^2 - 2y^2 c_R \cot \epsilon_L + y^3 \cot^2 \epsilon_L) dy$$

$$= \frac{\tau}{16} [b^2 c_R^2 - \frac{2}{3} b^3 c_R \cot \epsilon_L + \frac{1}{8} b^4 \cot^2 \epsilon_L] \tag{83}$$

and

$$\bar{y}_o = \frac{[b^2 c_R^2 - \frac{2}{3} b^3 c_R \cot \epsilon_L + \frac{1}{8} b^4 \cot^2 \epsilon_L]}{[4bc_R^2 - 2b^2 c_R \cot \epsilon_L + \frac{1}{3} b^3 \cot^2 \epsilon_L]} \tag{84}$$

As the cruciform configuration is symmetrical then $\bar{z}_o$ is numerically equal to $\bar{y}_o$, therefore

$$\bar{z}_o = \bar{y}_o \tag{85}$$

5. EQUATIONS FOR LIFTING SURFACE GEOMETRIC PARAMETERS

$$\epsilon_T = 90.0° \qquad (1A)$$

$$\lambda = \frac{c_T}{c_R} \qquad (2A)$$

$$AR = \frac{4\ (b/2)^2}{S_{Exp}} = \frac{4 \tan \epsilon_L\ (1-\lambda)}{1 + \lambda} \qquad (3A)$$

$$\bar{c} = \frac{2}{3}\ c_R\ (1 + \frac{\lambda^2}{1 + \lambda}) \qquad (4A)$$

$$\bar{y} = \frac{(1+2\lambda)\ (1-\lambda)}{3(1+\lambda)}\ c_R \tan \epsilon_L \qquad (5A)$$

$$\bar{x}_{LE} = \bar{y} \cot \epsilon_L \qquad (6A)$$

$$x_{LE} = y \cot \epsilon_L \qquad (7A)$$

$$x_{TE} = c_R \qquad (8A)$$

$$c = x_{TE} - x_{LE} = c_R - y \cot \epsilon_L \qquad (9A)$$

$$\frac{b}{2} = (1 - \lambda)\ c_R \tan \epsilon_L \qquad (10A)$$

$$S_{Exp} = 2(\frac{b}{2})\ (\frac{c_R + c_T}{2}) = (1 - \lambda^2)\ c_R^2 \tan \epsilon_L \qquad (11A)$$

$$S_{ref} = \frac{\pi}{4}\ d_{ref}^2 \qquad (12A)$$

$$x_{ce} = \frac{[3 - (1 - \lambda)^2]c_R}{3(1 + \lambda)} \qquad (13A)$$

$$y_{ce} = \left[\frac{1 + \lambda - 2\lambda^2}{3(1 + \lambda)}\right]\ c_R\ \mathrm{TAN}\ \epsilon_L \qquad (14A)$$

6. NOMENCLATURE

Symbol	Definition	Units
A	Axial Force in Missile Axis System Also, Area	lb sq in.
A_{WET}	Surface Area	sq in.
$A_{x\text{-sect}}$	Area of Airfoil Cross-Section	sq in.
AR	Lifting Surface Aspect Ratio	dim.
b/2	Lifting Surface Semi-Span (y distance From Body External Contour to Lifting Surface Tip Chord	in.
B	Constant in Equation 46a	dim.
c	Length of Lifting Surface Chord	in.
c_R	Length of Lifting Surface Root Chord	in.
c_T	Lifting Surface Tip Chord	in.
$\bar{c}$	Length of Mean Aerodynamic Chord	in.
C	Constant in Equation 46a	dim.
C_{DF}	Friction Drag Coefficient	dim.
C_{DL}	Lifting Surface Drag Due to Lift Coefficient	dim.
C_{DO}	Lifting Surface Drag Coefficient at Zero Lift	dim.
C_f	Flat Plate Friction Drag Coefficient	dim.
C_A	Axial Force Coefficient in Missile Axis System	dim.
C_D	Drag Coefficient	dim.
C_{DP}	Pressure Drag Coefficient	dim.
C_L	Lift Coefficient	dim.
$C_{L\alpha}$	Slope of Lifting Surface Linear Lift Coefficient Curve	1/deg, 1 RAD

C_N	Normal Force Coefficient in Pitch Plane of the Missile Axis System	dim.
$C_{N\alpha}$	Slope of Normal Force Coefficient	1/deg
C_Y	Normal Force Coefficient in Yaw Plane of the Missile Axis System	dim.
C_1	Non-Linear Lift Coefficient Factor	$1/deg^2$
d_{ref}	Reference Diameter (Diameter of Largest Cylindrical Section of Missile Body	in.
dy	Differential Length in y	in.
D	Constant in Equation 46a	dim.
E	Constant in Equation 46a	dim.
Fwd	Forward	dim.
$(HM)_y$	Hinge Moment in Pitch Plane	in.-lb
$(HM)_z$	Hinge Moment in Yaw Plane	in.-lb
i	Constant in Equation 46a	dim.
j	Constant in Equation 46a	dim.
k	Constant in Equation 46a	dim.
K_{cp}	Factor Locating Lifting Surface Center of Pressure as a Fraction of $\bar{c}$ Measured From the Leading Edge of $\bar{c}$	dim.
K_D	Constant in Equation 46a	dim.
K_L	Drag Due to Lift Factor	rad
M	Mach Number	dim.
M_b	Moment Due to Body Aerodynamic Forces	in.-lb
M_c	Moment Due to Canard Aerodynamic Forces	in.-lb
M_f	Moment Due to Fin Aerodynamic Forces	in.-lb
M_x	Moment About x Axis	in.-lb
M_y	Moment About y Axis	in.-lb

M_z	Moment About z Axis	in.-lb
Mom	Moment	in.-lb
Mom_x	Longitudinal Moment of Lifting Surface Weight About Midpoint of Root Chord	in.-lb
Mom_y	Lateral Moment of Lifting Surface Weight About Root Chord	in.-lb
N	Normal Force in the Pitch Plane of the Missile Axis System	lb
q	Dynamic Pressure $(= \frac{\gamma}{2} P M^2)$	psf
r_b	Average Radius of Missile Body Between the Root Chord Leading and Trailing Edges	in.
R_N	Reynolds Number	dim.
S	Planform Area Per Pair of Lifting Surfaces	sq in./sq ft
S_{Exp}	Area of Two Lifting Surface Semi-Spans Outside the Body External Contour	sq in./sq ft
S_{ref}	Missile System Reference Area $(=\pi d_{ref}^2/4)$	sq in./sq ft
t	Thickness of Airfoil Section (Lifting Surface)	in.
V	Relative Velocity	fps
Vol	Volume of Set of Lifting Surfaces (4 Semi-Spans)	cu in.
Wt	Weight of Set of Lifting Surfaces (4 Semi-Spans)	lb
x	Longitudinal Distance	in.
x_{ce}	Longitudinal Distance the Planform Centroid Lies Aft of Root Chord Leading Edge	in.
x_{cp}	Longitudinal Location of Center of Pressure	in.
x_e	Body Station at End of Aft Skirt	in.
x_{or}	Body Station of Lifting Surface Root Chord Leading Edge	in.
x_L	Longitudinal Location of Hinge Axis (Canards and Fins)	in.
x_{LE}	Distance of Lifting Surface Leading Edge From x_{or}	in.

x_{TE}	Distance of Lifting Surface Trailing Edge From x_{or}	in.
$\bar{x}$	Distance Between Component Center of Pressure and Missile Center of Gravity	in.
x_h	Longitudinal Distance Between Center of Pressure and Hinge Axis	in.
$\bar{x}_o$	Longitudinal Distance of Lifting Surface Center of Gravity from Root Chord Midpoint	in.
$\bar{x}_{LE}$	Distance Lifting Surface Mean Aerodynamic Chord Leading Edge Lies Behind Root Chord Leading Edge	in.
x_1	Body Station of Missile Body Theoretical Nose	in.
$\bar{X}$	Longitudinal Location of Lifting Surface Center of Gravity Relative to Body Station 0.0	in.
y	Lateral Distance	in.
y_{ce}	Lateral Distance the Planform Centroid for one Semi-Span lies Outboard From the Root Chord	in.
y_{cp}	Lateral Location of Lifting Surface Center of Pressure Relative to Vehicle Centerline Axis	in.
Y	Normal Force in the Yaw Plane of the Missile Axis System	lb
$\bar{y}$	Lateral Distance of Mean Aerodynamic Chord From Root Chord	in.
$\bar{y}_o$	Lateral Distance of Lifting Surface Center of Gravity from Root Chord	in.
$\bar{y}_b$	y Distance of Missile cg From Body Centerline	in.
$\bar{y}_h$	Lateral Distance Between Center of Pressure and Hinge Axis	in.
$\bar{Y}$	Lateral Location of Lifting Surface Center of Gravity Relative to Vehicle Centerline Axis	in.
z	Vertical Distance	in.
z_{cp}	Vertical Location of Lifting Surface Center of Pressure Relative to Vehicle Centerline Axis	in.
$\bar{z}_b$	z Distance of Missile cg From Body Centerline	in.
$\bar{z}_h$	Vertical Distance Between Center of Pressure and Hinge Axis	in.

$\bar{z}_o$	Vertical Distance of Lifting Surface Center of Gravity From Root Chord	in.
$\bar{z}$	Vertical Location of Lifting Surface Center of Gravity Relative to Vehicle Centerline Axis	in.
℄	Centerline	dim.

α	Angle of Attack in Pitch Plane	deg
α_{eff}	Effective Angle of Attach in Pitch Plane (Canards and Fins)	deg
β	Angle of Attack in Yaw Plane	deg
	$\sqrt{M^2-1}$ (Where $1.0 < M \leq 10.0$)	dim.
β_{eff}	Effective Angle of Attack in Yaw Plane (Canards and Fins)	deg
γ	Ratio of Specific Heats	dim.
δ	Deflection Angle of Lifting Surface	deg
ϵ_L	Complement of Lifting Surface Leading Edge Sweep-Back Angle	deg
ϵ_T	Complement of Lifting Surface Trailing Edge Sweep-Back Angle	deg
ζ	Surface Angle of Airfoil Section Relative to Chord Plane (Double Wedge Section)	deg
λ	Lifting Surface Taper Ratio ($=c_T/c_R$)	dim.
π	3.1416	dim.
ρ	Density	lb/cu in.
τ	Thickness Ratio of Airfoil Section ($=t/c$)	dim.

7. SUBSCRIPTS

b	Body
c	Canard
cg	Center of Gravity
cP	Canard for Pitch Control
cY	Canard for Yaw Control
f	Fin
fP	Fin for Pitch Control
fY	Fin For Yaw Control
P	In Pitch Plane
Y	In Yaw Plane

SECTION III

SIX DEGREES OF FREEDOM TRAJECTORY PROGRAM

A. INTRODUCTION

This document describes a versatile digital trajectory program developed by the Wasatch Division of Thiokol Chemical Corporation. The program was developed, as are most programs, in an evolutionary manner. As a result, most of the general capabilities are the result of past experience, thus making the program well adapted to most forms of missile system flight simulation.

The equations and logic permit the simulation of missile flight in three dimensions with an additional three degrees of freedom possible, i.e., the vehicle can pitch, yaw, and roll about its center-of-gravity. A spherical rotating earth model is utilized for missile location. The gravitational forces are calculated from an oblate earth model.

The program has three general capabilities in three dimensions: (a) flight simulation of a rigid body point mass missile system, (b) flight simulation of a rigid body missile system with angular momentum considered plus attitude stability provided by a closed loop control system, and (c) flight simulation of a rigid body point mass missile system with a quasi-attitude control system which exactly balances the disturbing pitching moments. These three capabilities are primarily used in (1) performance requirements for analysis, (2) vehicle control requirements determination, and (3) vehicle stability, control and loads analysis, respectively.

Vehicle launch can occur at any azimuth and altitude and from any latitude and longitude. The vehicle is commanded with respect to an inertial system which does not rotate with the earth. Flight path through the atmosphere can be commanded by (1) input turning rates, (2) gravity turn equations, (3) steering

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equations whose coefficients are determined internally, (4) rail launch equations, (5) constant altitude equations, (6) constant normal load factor equations, (7) intercept guidance equations, and (8) homing guidance equations. These different methods of flight path control can be changed for different parts of the flight, thus allowing wide variations in flight path determination and accurate results with relative ease by the user.

Program input is minimal, the input complexity being dictated by the sophistication the user wishes to employ. Mandatory input includes the missile launch conditions, data describing the flight path desired, and data describing missile characteristics. Missile characteristics include weight, thrust history, specific impulse, exit area, and aerodynamic coefficients versus Mach number. Up to four discrete propulsion stages can be simulated.

Missile thrust is computed from an input thrust history table and atmospheric back pressure. Instantaneous weight is a function of stage initial weight, weight flow, thrust, and propellant vacuum specific impulse. The capability of simulating simultaneous operation of motors having different characteristics is available by the use of separate tabular input.

A pintle nozzle thrust modulation control motor can be simulated. This thrust control system logic is broken into two separate parts: (1) the command thrust logic and (2) the controllable motor thrust dynamics. The command thrust logic or thrust control law provides the needed thrust command for the motor so the missile system will achieve the desired trajectory conditions. The criteria for evaluating the commanded thrust is established by flight performance parameters such as a specified velocity history, stipulated Mach number, commanded turning rates, minimum velocity, constrained dynamic pressure or line of sight rate.

The actual motor thrust dynamic is based on the solution of the time rate of change of chamber pressure equation. This equation is a function of nozzle throat area, internal gas properties, free volume, burning surface area, and propellant burning rate characteristics. The instantaneous vacuum thrust is evaluated from the pintle throat area and the motor chamber pressure. The motor ballistic characteristics are evaluated from one of the three input options: (a) thrust time table, (b) chamber pressure table, or (c) surface area-web depth fraction table.

The effective thrust gimbal point can be located at any place about the missile. Pitch, yaw, and roll moments of inertia are specified as functions of instantaneous vehicle weight.

Pitch and yaw thrust vectoring are obtained from first or second order transfer functions or from an infinite autopilot gain model. Roll control results from the operation of a system separate from the main propulsive one. A first-order transfer function is used to simulate a bang-bang with deadband or proportional roll control system. Pitch, yaw, and roll control system commands can be generated by vehicle attitude errors, rates, and steady state errors.

The controlling pitch, yaw and roll moments are determined for each computed interval. The required thrust vector slew rate, duty cycle, and other thrust vector control requirements are evaluated for the TVC design stage.

Aerodynamic forces are functions of Mach number, input aerodynamic coefficients, and angle of attack. Up to a third degree normal force slope coefficient can be used. The angles of attack are modified so that flight with values up to ±180 degrees can be simulated with axial forces going to zero at ± 90 degrees and normal forces zero at 0 and 180 degrees. Normal force center of pressure is a function of Mach number. Aerodynamic damping forces and moments are determined from input coefficients, rate change of angle of attack, pitch rate, and dynamic pressure.

Movable aerodynamic control fins can be simulated. Fin aerodynamic characteristics are determined from input fin lift and drag coefficient plus center of pressure and hinge location.

Dynamic forces include gravity, thrust, jet damping, and aerodynamic. The dynamics include overturning moments created by differences in thrust level of individual motors in a clustered solid propellant rocket motor stage. These moments are determined by statistical techniques from the cluster geometry and single motor variances in burning time and total impulse.

Program options which provide utility to the users are special print capabilities, flexibility in inputting thrust time curves, variable methods of terminating a propulsion stage, and an efficient and highly sophisticated hunting procedure. The hunting procedure can maximize, minimize or isolate any dependent variable subject to a maximum of seven constraint conditions.

1. DIAGRAMS

The more important symbols with geometric interpretation are shown on Figures 17 thru 29.

2. DEFINITIONS

Input, output, and internal program parameters are designated by symbols defined in this section.

a. General Comments--Parameter units required by program logic or for input are abbreviated as follows:

dbi	Determined by input	lb	Pound
deg	Degree	min	Minutes
dim	Dimensionless	nm	Nautical mile
ft	Feet	rad	Radian
in.	Inch	sec	Second

Multipliers are associated with many of the input parameters. Since only the unit of the product of the parameter and multiplier is specified by program logic, the units of the parameter, and multiplier are an option of the program user. Dimensions are designated dbi for parameters which have optional units. Units of the

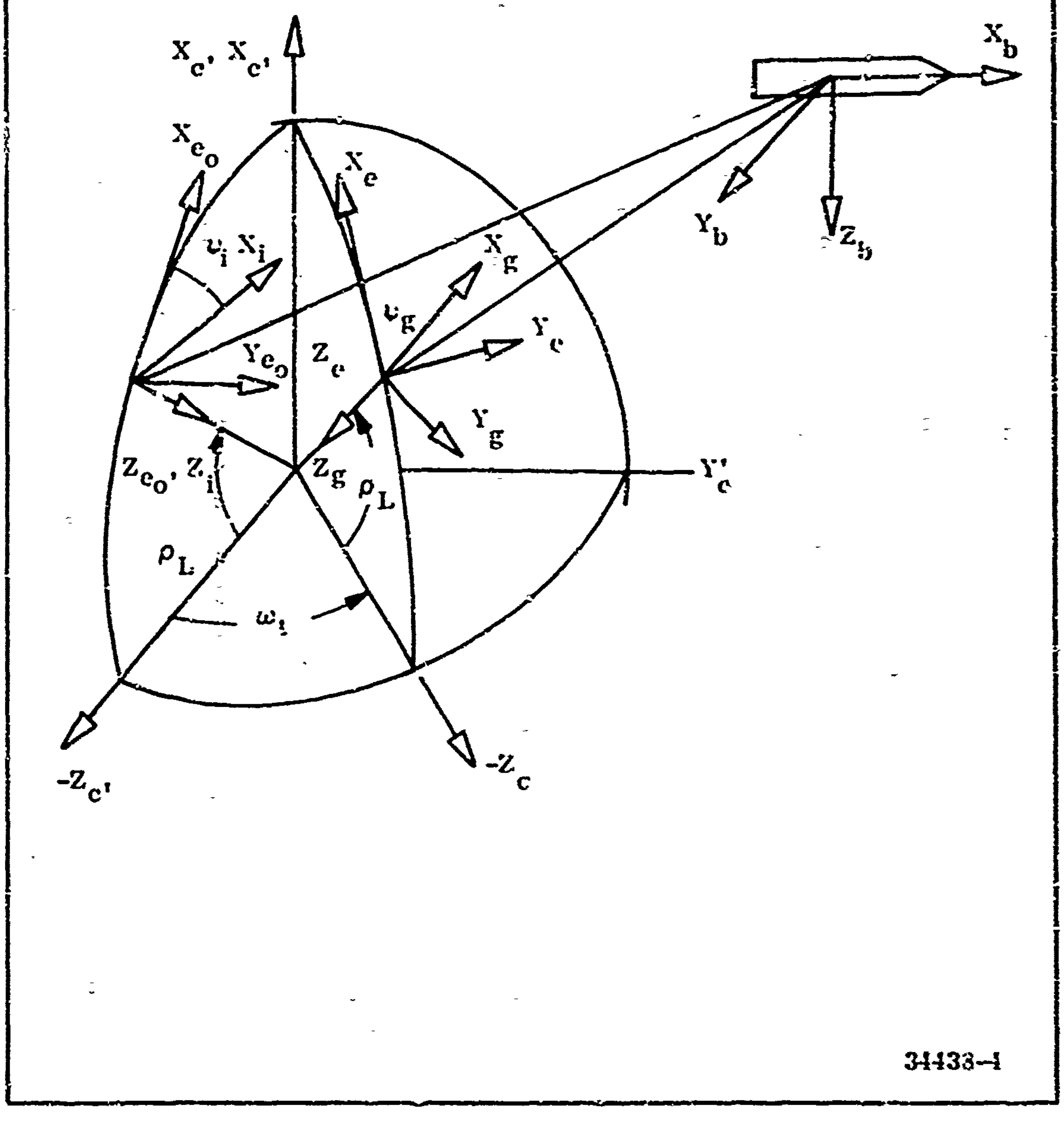

Figure 17. 3-D Coordinate Systems

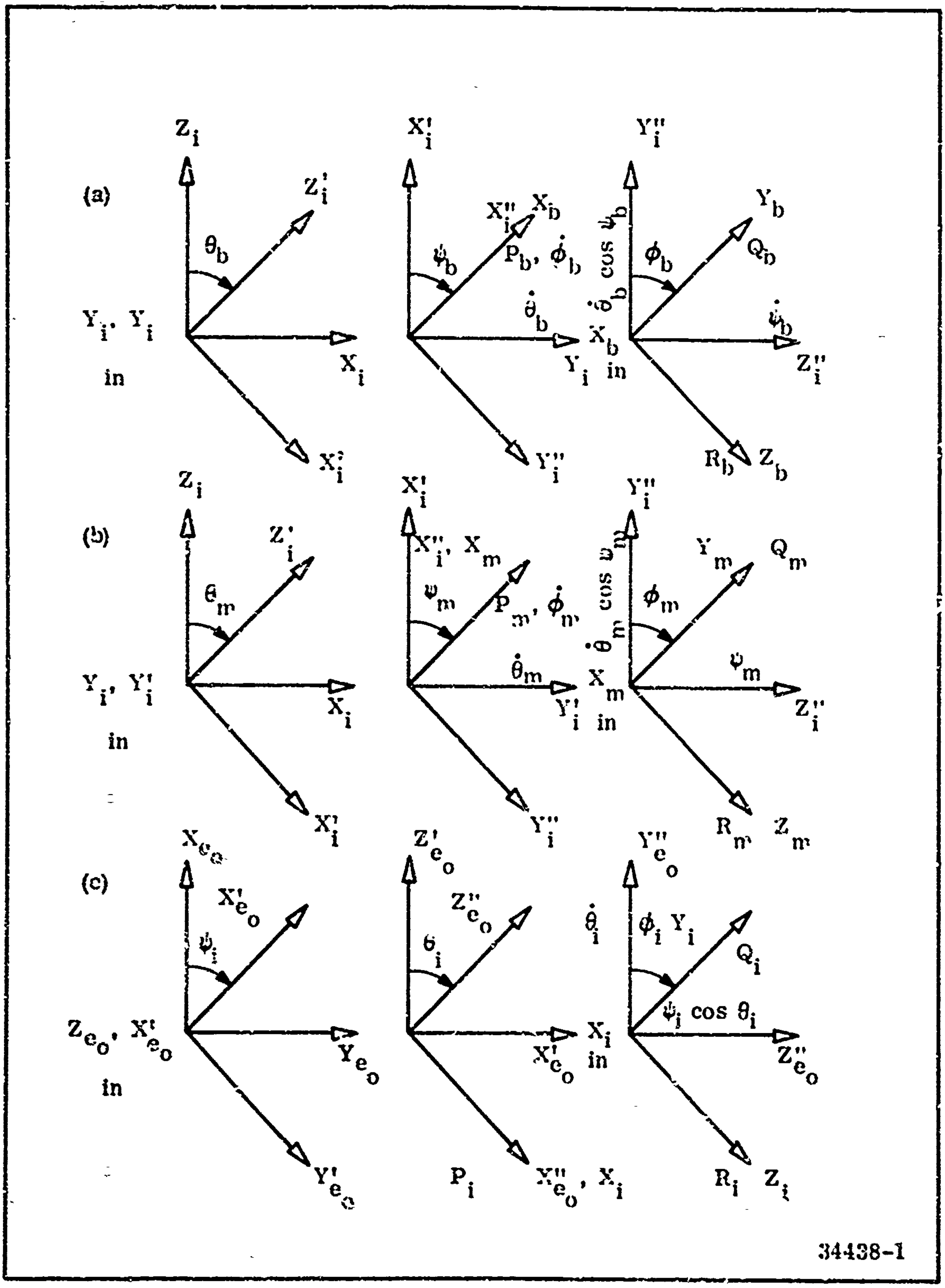

Figure 18. Euler Angles

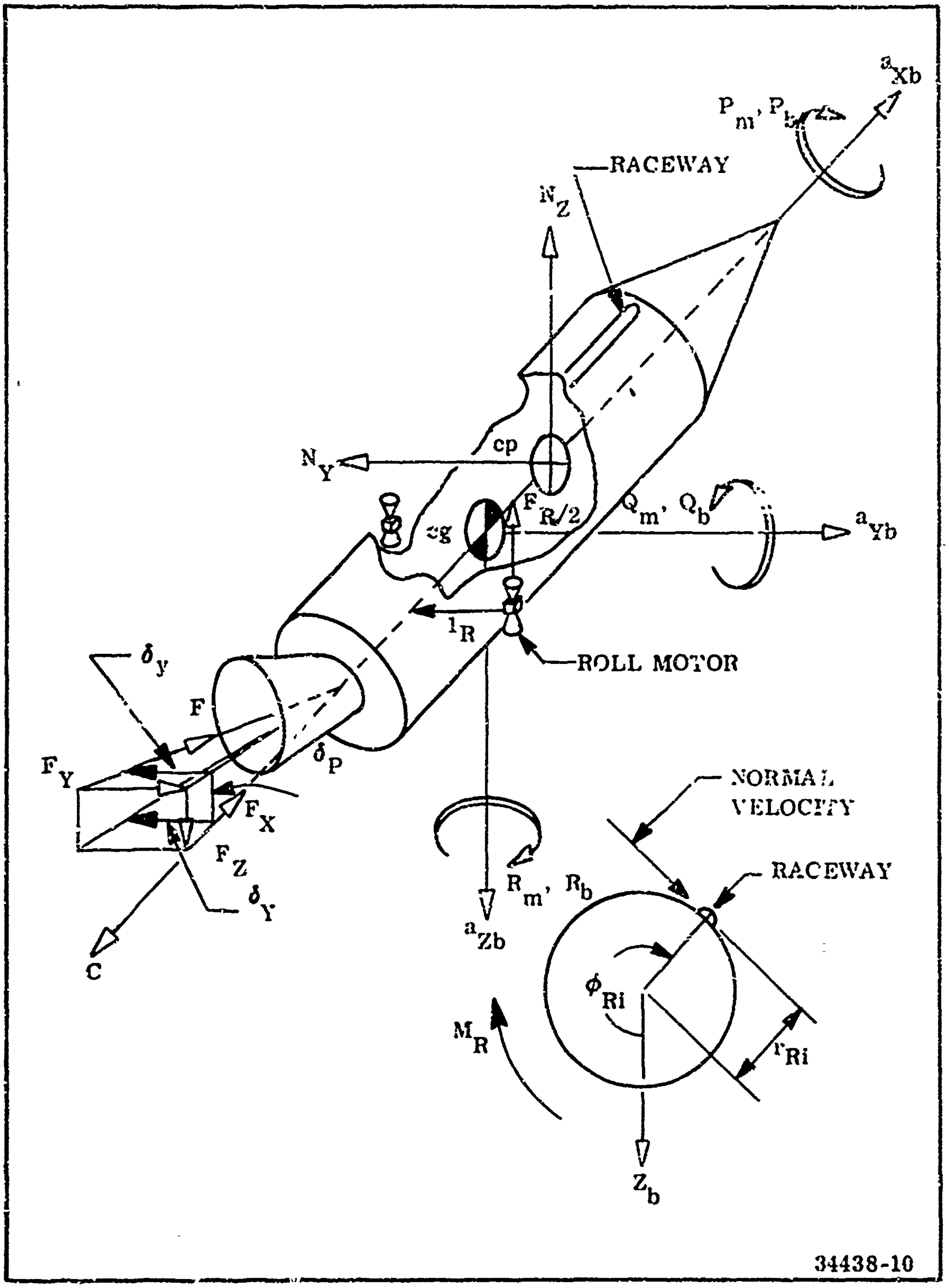

Figure 19. Forces and Rates

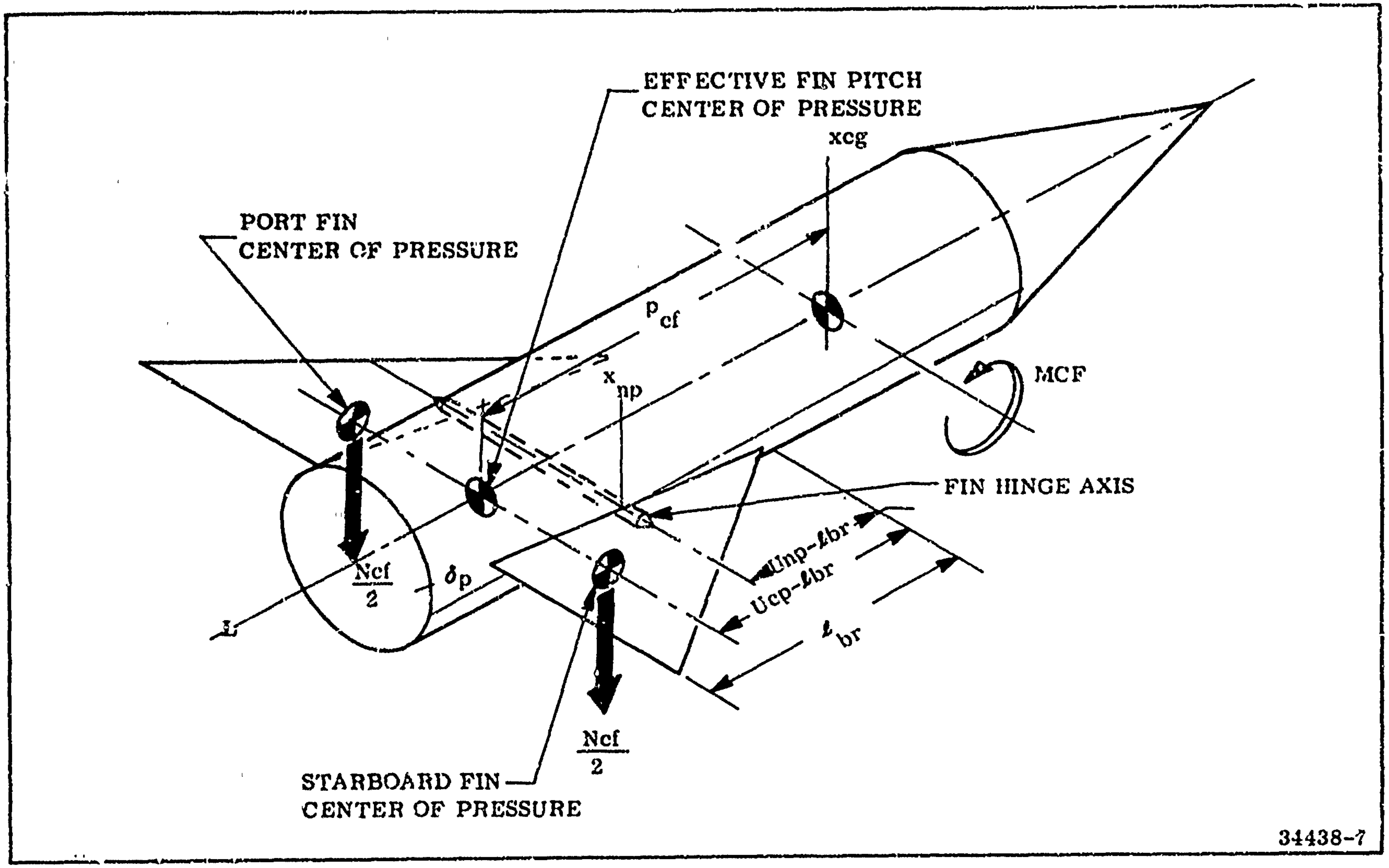

Figure 20. Aerodynamic Control Fin Schematic

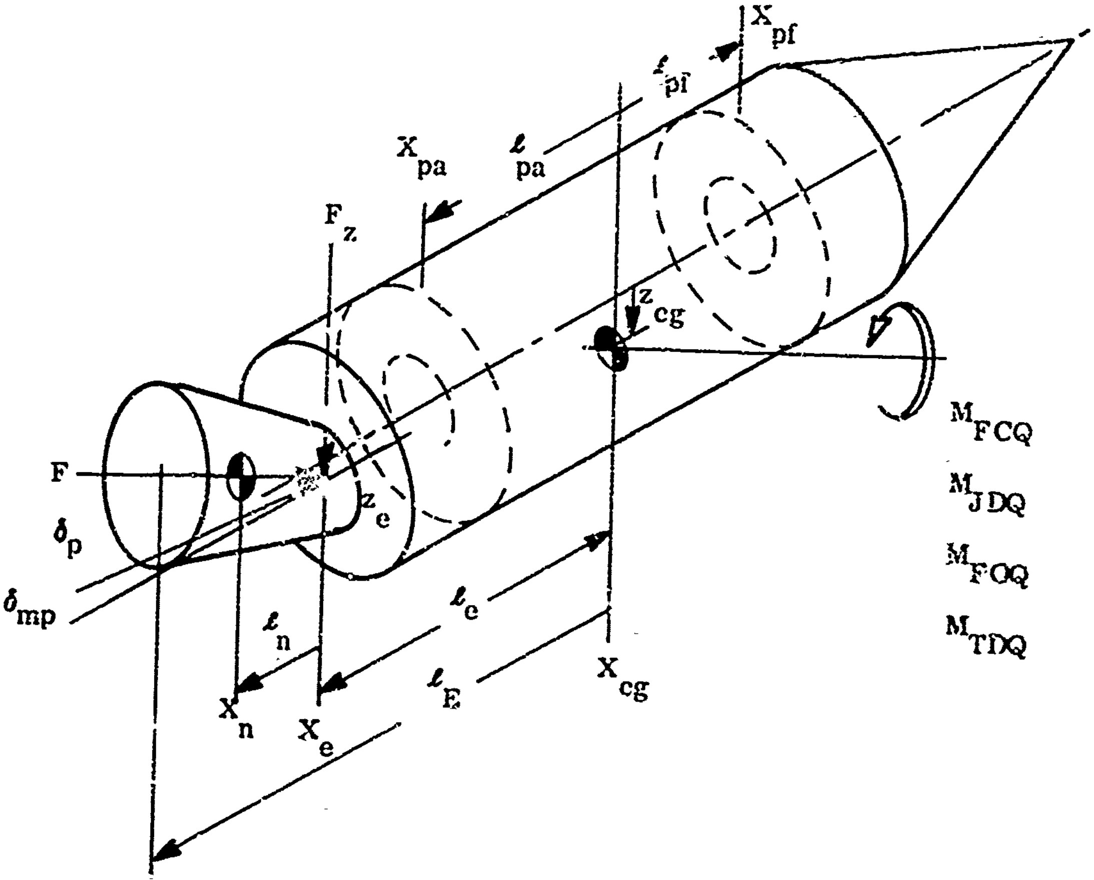

Figure 21. Thrust Vector Control Schematic

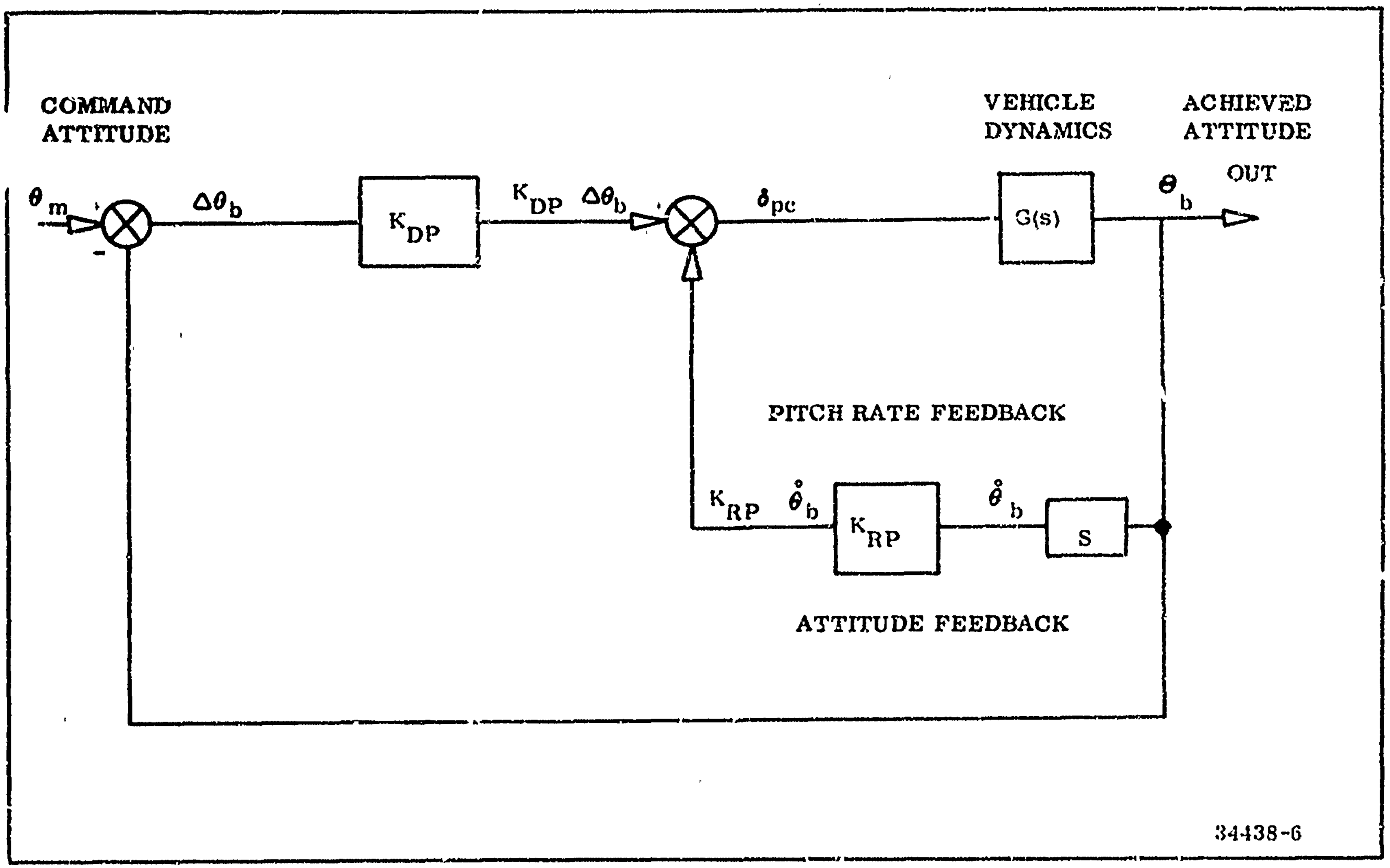

Figure 22. Control Loop Diagram

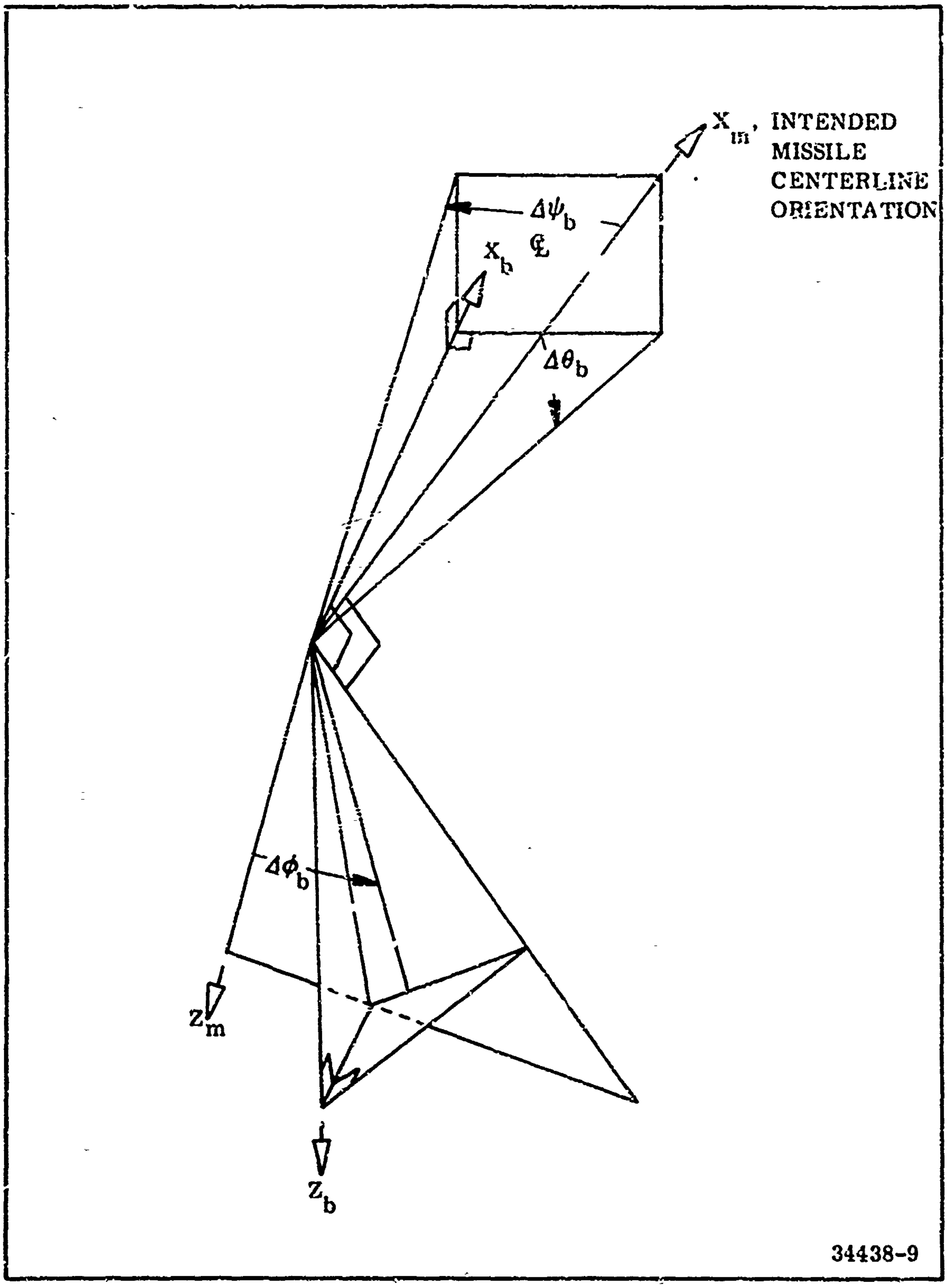

Figure 23. Vehicle Attitude Errors

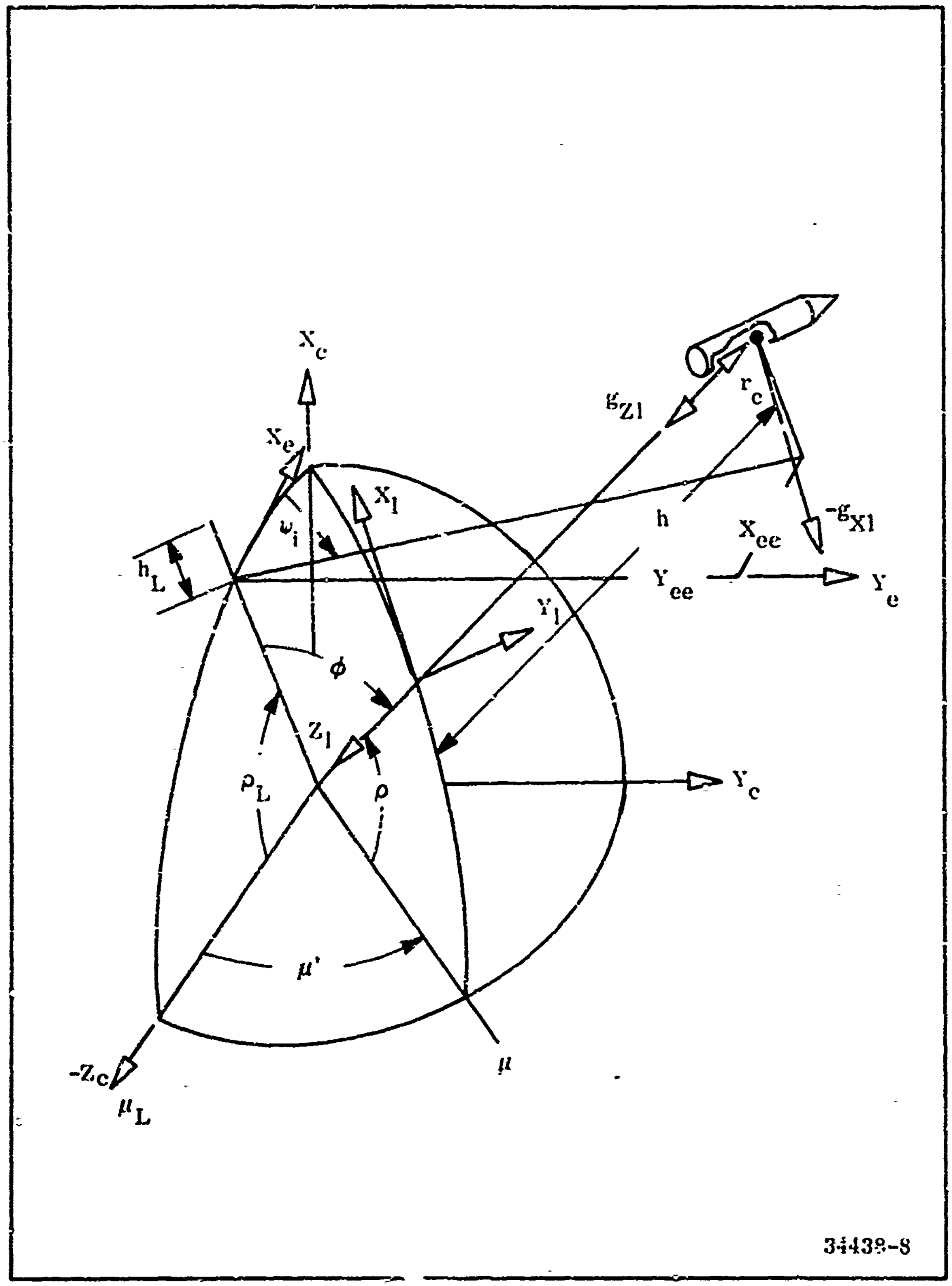

Figure 24. 3-D Missile Location

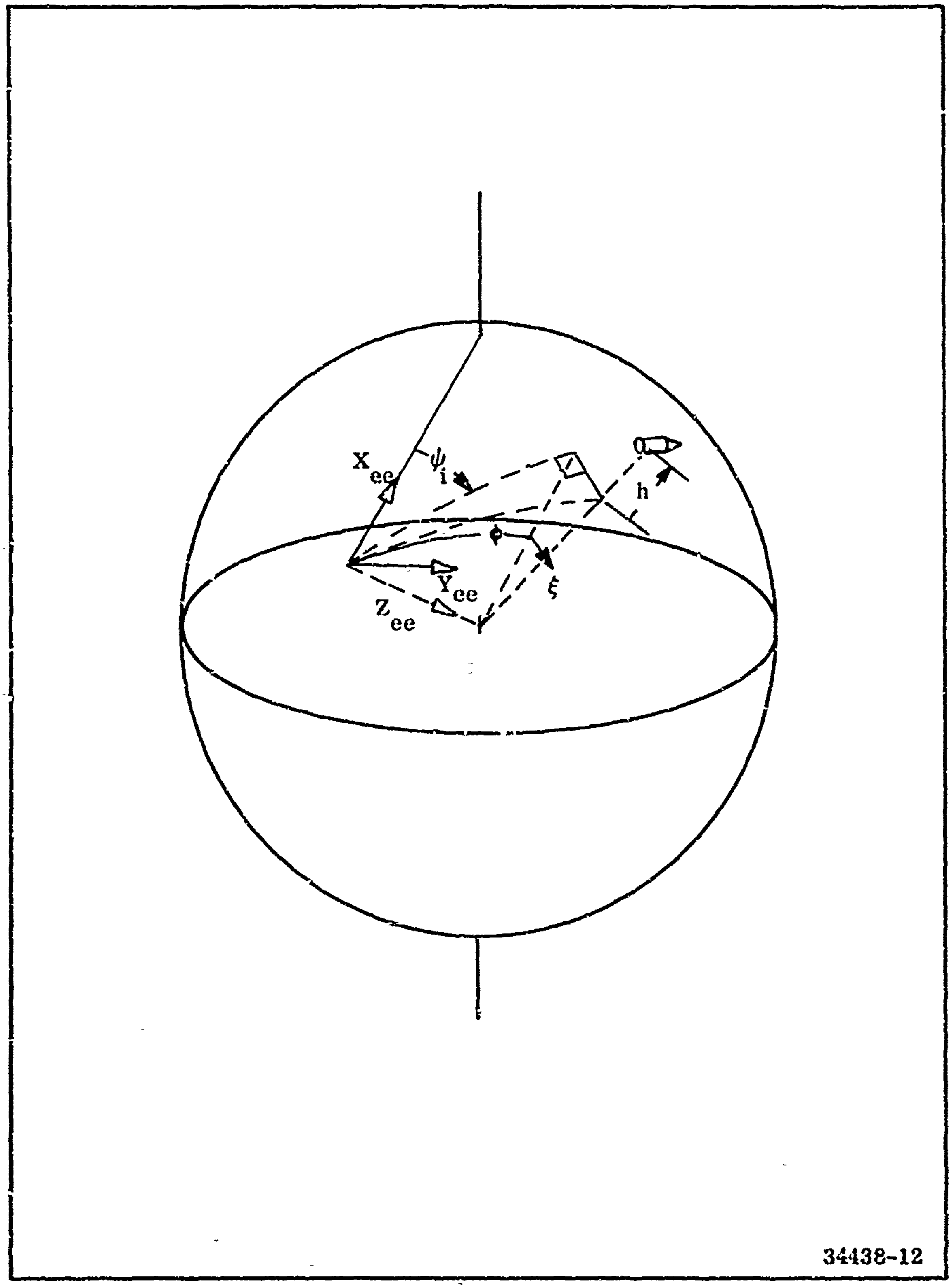

Figure 25. 3-D Down Range-Cross Range Coordinates

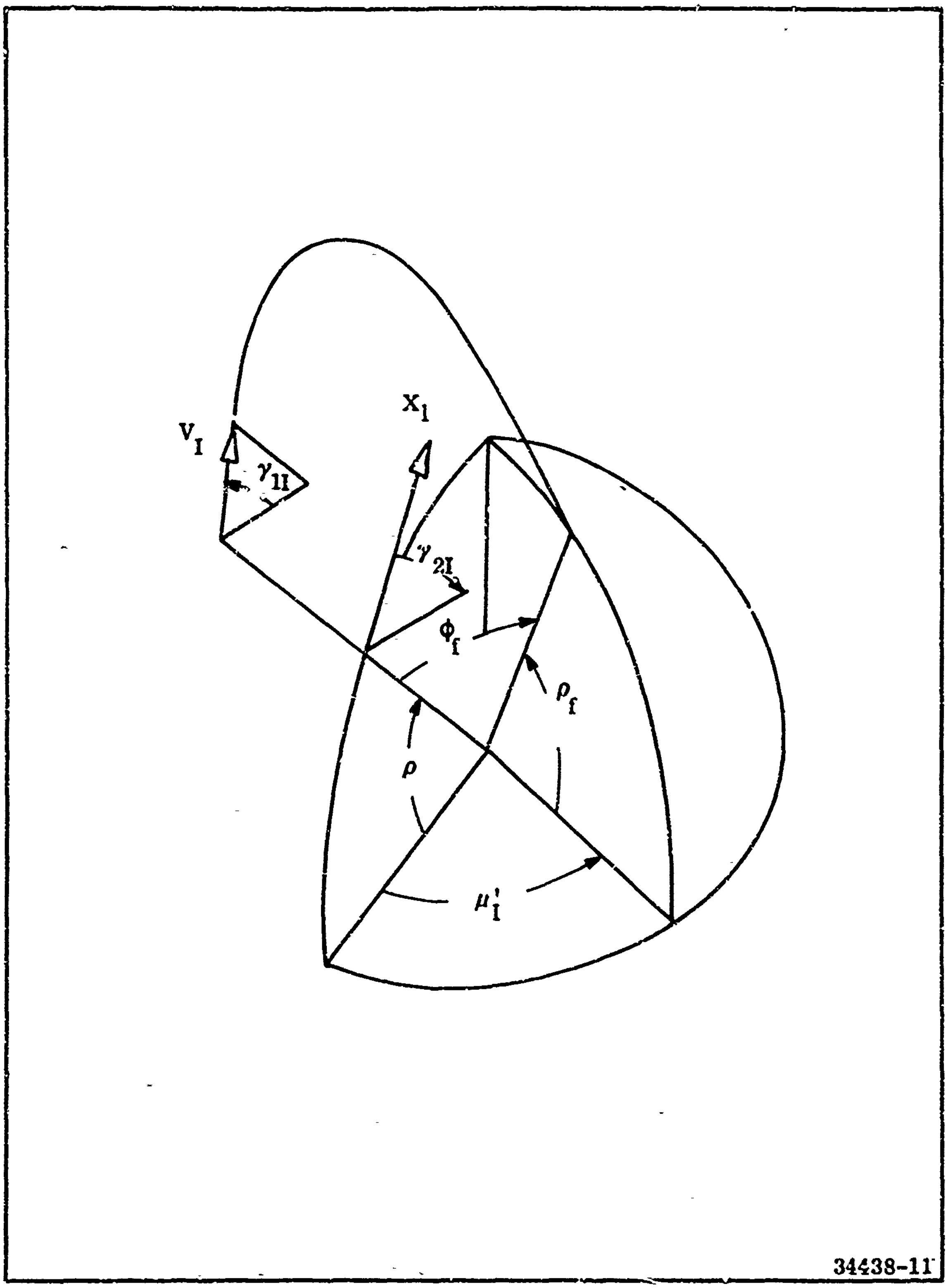

Figure 26. Impact Parameters

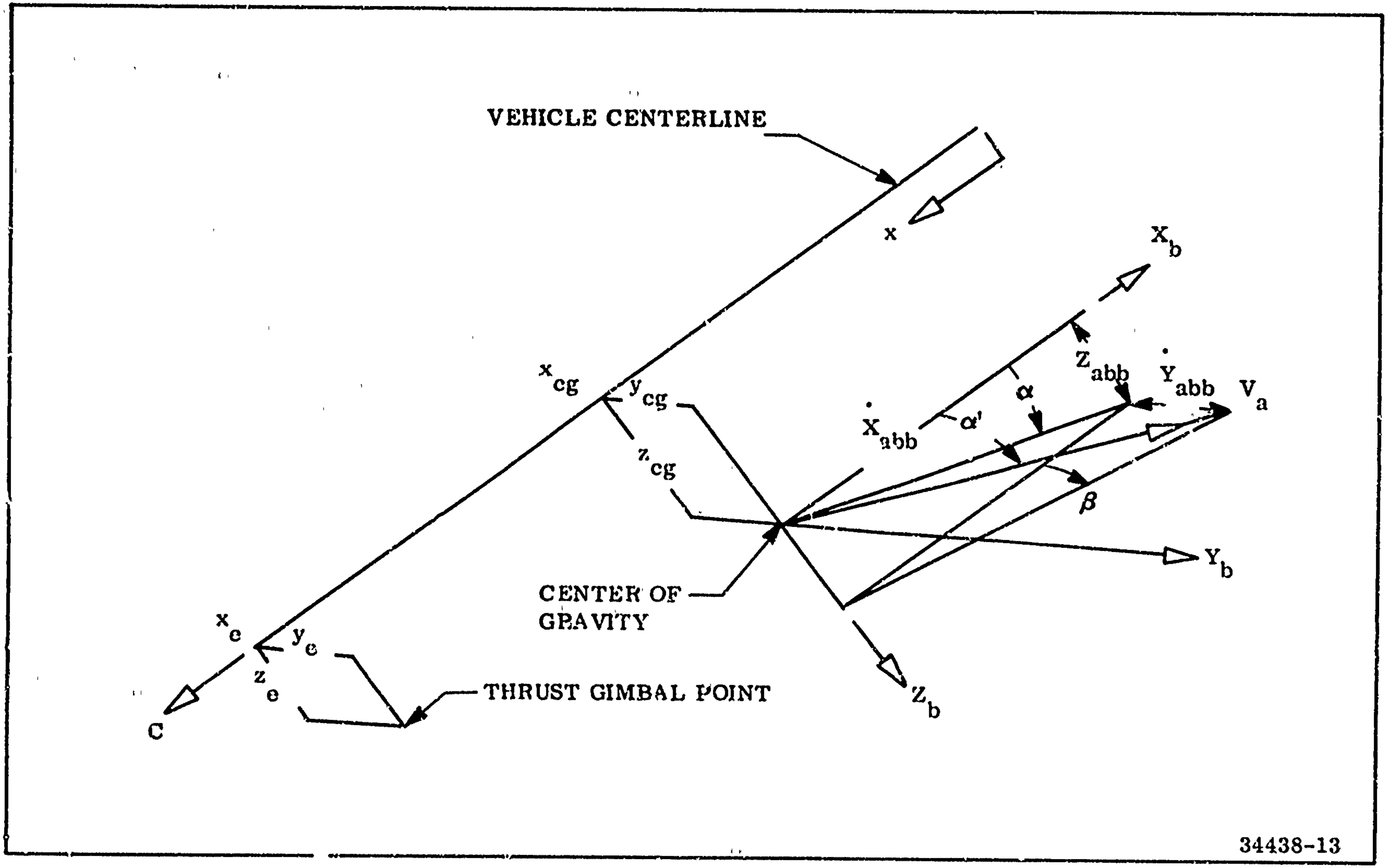

Figure 27. 3-D Angles of Attack

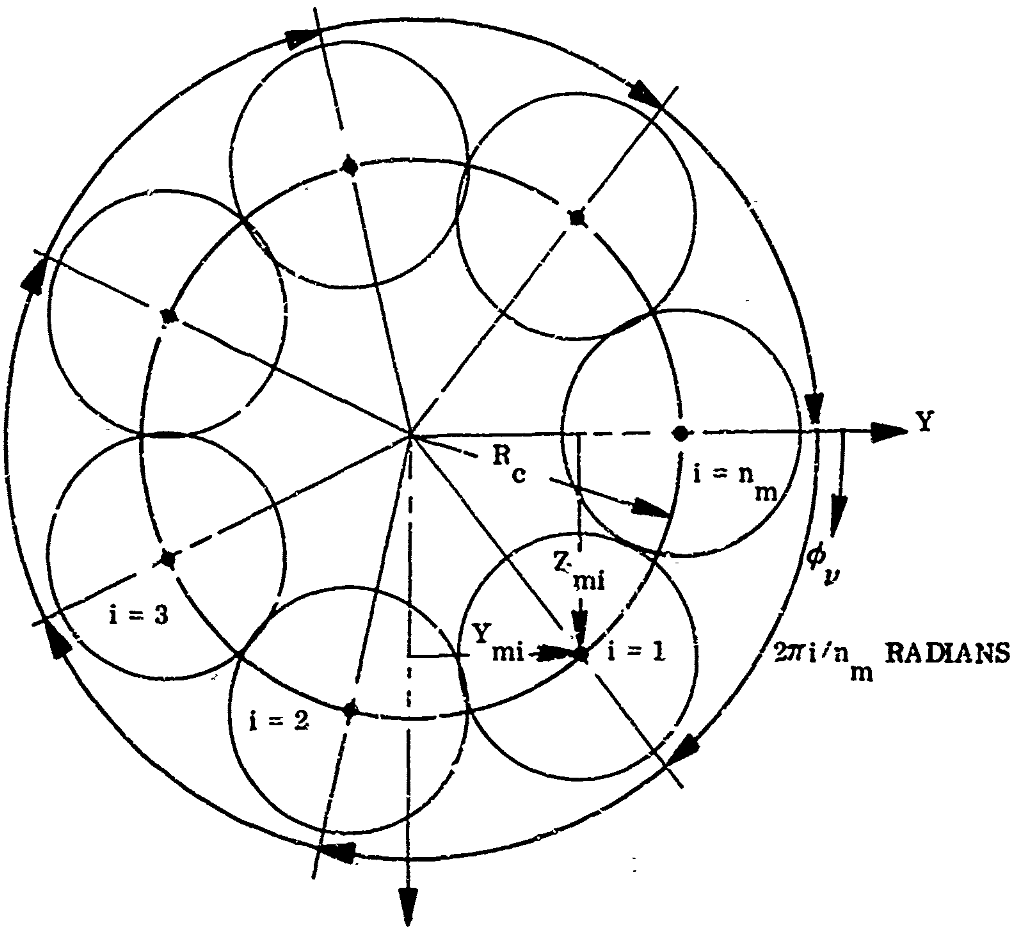

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Figure 28. Clustered Motor Configuration

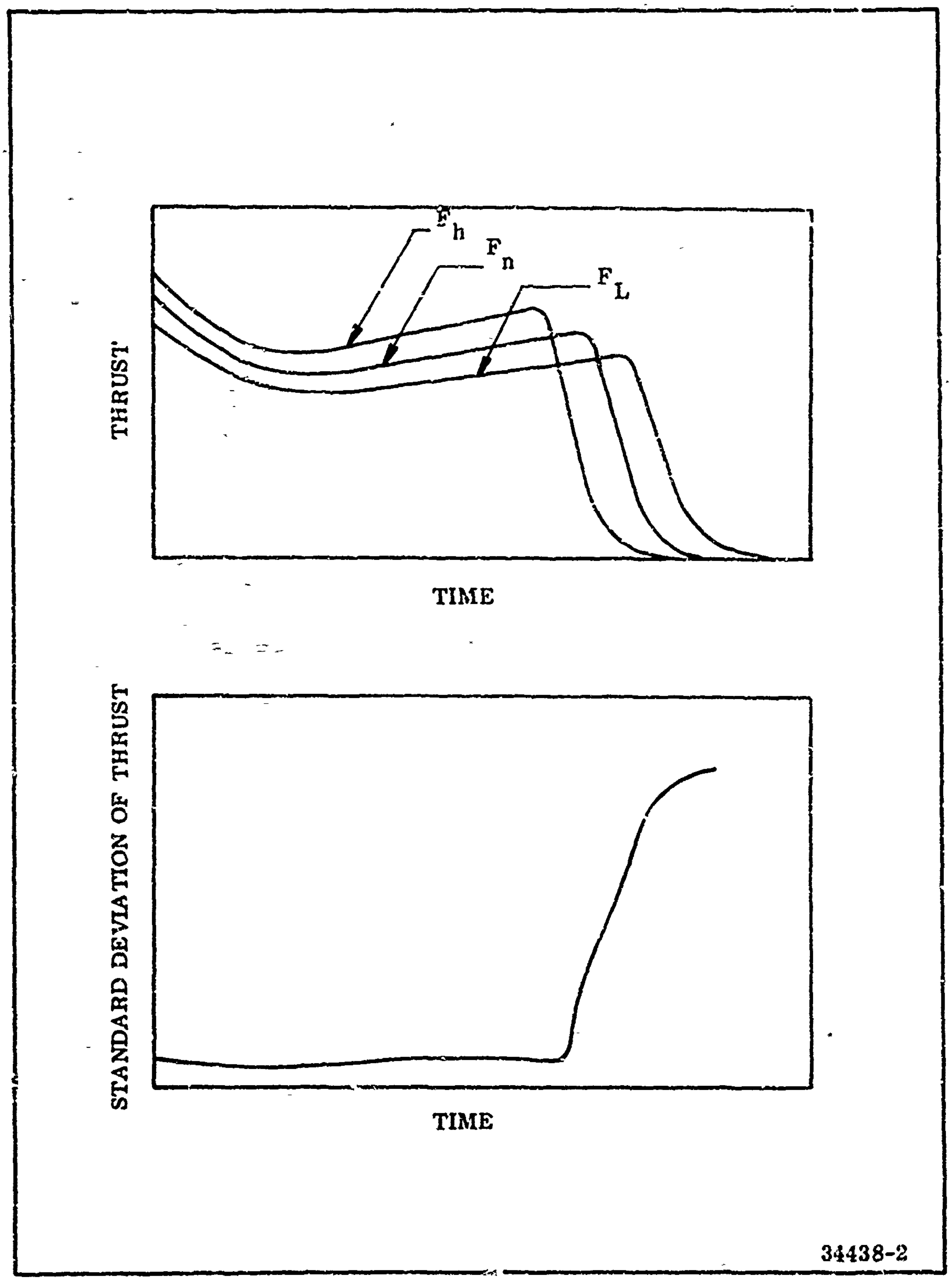

Figure 29. Typical Variation of Thrust History

parameters used by the program should be noted so that the product units are the same.

Because of the four-stage capability of the program, many parameters are applicable for each stage. When further clarification is necessary, the subscript k is shown with the symbol of the parameter.

Two tables, for a main stage and a complementary stage, are available for inputting thrust, weight, and weight flow data. Parameters which apply to one or the other table are referenced accordingly.

b. Coordinate System--Coordinates are referenced to right-handed Cartesian systems having X, Y, and Z with lower-case subscripts as the coordinate axis. For example, the e system has coordinates axis $\vec{X}_{ee}$, e.g., X_{ee} represents a value along the appropriate coordinate axis. In some instances, vector and matrix notation are used to simplify the writing of equation; i.e.,

$$\vec{X}_{ee} = \vec{X}_{ee}\, i + Y_{ee}\, \vec{j} + Z_{ee}\, \vec{k} = \begin{Bmatrix} X_{ee} \\ Y_{ee} \\ Z_{ee} \end{Bmatrix}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vector parallel to the coordinate axis X_{ee}, Y_{ee}, and Z_{ee}, respectively.

The coordinate systems discussed below have coincident origins; i.e., no translation is involved in their relations with other systems. However, defining them with their origins at the conceptual locations rather than all simply rotated about one another is convenient. Furthermore, velocity and acceleration components would be affected only by rotation and not translation. The coordinate systems are shown in the illustrations appearing in Section III.A.1 and are defined as follows:

Systems	Definitions
b	Missile orientation coordinates. Origin is fixed at the vehicle center-of-gravity with X_b parallel to the vehicle centerline, positive forward. As seen from the rear of the vehicle Z_b is positive down.
e	Earth fixed system whose origin is located at sea level at the launch latitude and longitude. The X axis is positive north, the Y_e axis is positive east, and the Z_e axis is along the launch vertical, positive down.
e_o	Initially coincident with the e system, but does not rotate with the earth.
g	Earth fixed, launch centered, azimuth oriented coordinate system. The orientation of this system about the e system is specified by the input Euler angles ψ_g, θ_g, and ϕ_g.
i	Origin is coincident with the e_o system. The orientation of this system is specified by input Euler angles ψ_i, θ_i, and ϕ_i.
ee	$\vec{X}_{ee}$ and $\vec{\dot{X}}_{ee}$ are the components of missile acceleration and velocity in a right-handed Cartesian coordinate system whose origin is fixed at sea level at the input launch latitude, ρ_L and longitude μ_L. The X_e axis is positive north, Y_e axis positive east, and Z_e, the vertical axis, positive down. This coordinate system is the one in which the equations of linear motion are integrated. $\vec{X}_{ee}$ is defined in section B.1 as the linear momenta equations. $\vec{X}_{ee} = \vec{X}_{eeo} + \int_{t_o}^{t} \vec{\ddot{X}}_{ee} \, dt$ $\vec{X}_{ee} = \vec{X}_{eeo} + \int_{t_o}^{t} \vec{\dot{X}}_{ee} \, dt$
eo	$\dot{X}_{eo}$ are the components of velocity which are initially coincident with the e system, but do not rotate with the earth. Components in this system are not calculated directly because the eo system is an intermediate system in evaluating the missile achieved and commanded components. $\vec{\dot{X}}_{eo} = [A_{eo}] \vec{\dot{X}}_{ee}$

Systems	Definitions
ii	Origin is coincident with the eo system. The orientation of this system about the eo system is specified by input Euler angles ψ_i, θ_i, and φ_i. The position, velocity, and acceleration in this axis system represent the components aligned with the inertial platform. Components in this system are not calculated directly because the ii system is an intermediate system in evaluating the missile achieved and commanded components. $\vec{X}_{ii} = [A_i] \vec{X}_{eo}$
bb	$\vec{X}_{bb}$, $\dot{\vec{X}}_{bb}$ are the earth fixed launch centered components of missile position and velocity, respectively. Position, forward along missile body station axis: $\dot{\vec{X}}_{bb} = [D]^{-1} \dot{\vec{X}}_{ee} + [\dot{D}]^{-1} \vec{X}_{ee}$ $\vec{X}_{bb} = [D]^{-1} \vec{X}_{ee}$
mm	Missile orientation coordinates with axis coincident with those of the b system, if control system operation is not being simulated. For simulated control operation, the coordinate axes orientation is specified by commanded turning rates or specified value evaluated by the type of flight. Components in this system are not calculated directly because the angles relatively between ii and the mm system are those of interest. $\vec{X}_{mm} = [D]^{-1} \vec{X}_{ee}$
gg	Earth fixed, launch centered, azimuth oriented coordinate system. The orientation of this system about the eo system is specified by the input Euler angles ψ_g, θ_g, and φ_g. The velocity components of this system are used in the velocity steering logic. $\vec{X}_{gg} = [A_g]\vec{X}_{ee}$ $\dot{\vec{X}}_{gg} = [A_g]\dot{\vec{X}}_{ee}$ $\ddot{\vec{X}}_{gg} = [A_g]\ddot{\vec{X}}_{ee}$
11	$\dot{X}_{11}$ and $\ddot{X}_{11}$ are the earth fixed velocity and acceleration components in the local system (the i^{11} system). X_{11} is positive in the local north direction, Y_{11} is positive in the local easterly direction and

Systems Definitions

$\dot{Z}_{11}$ is positive toward the center of the earth. Note this system travels along with the missile center.

$$\dot{\vec{X}}_{11} = [A_1]^{-1}\dot{\vec{X}}_{ee}$$

$$\ddot{\vec{X}}_{11} = [A_1]^{-1}\ddot{X}_{ee} + [\dot{A}_1]^{-1}\dot{\vec{X}}_{ee}$$

wee

$\dot{\vec{X}}_{wee}$ is the wind velocity component in the ee system.

$$\dot{\vec{X}}_{wee} = [A_1]\dot{\vec{X}}_{w11}$$

w11

$\dot{\vec{X}}_{w11}$ and $\ddot{\vec{X}}_{w11}$ are the Cartesian components of wind velocity and acceleration in the local coordinate system (the 11 system).

$$\dot{\vec{X}}_{w11} = -\nu_w \begin{Bmatrix} \cos\psi_w \\ \sin\psi_w \\ 0 \end{Bmatrix}$$

$$\ddot{\vec{X}}_{w11} = -\dot{\nu}_w \begin{Bmatrix} \cos\psi_w \\ \sin\psi_w \\ 0 \end{Bmatrix} - (\pi/180)\,\dot{\psi}_w\,\nu_w \begin{Bmatrix} -\sin\psi_w \\ \cos\psi_w \\ 0 \end{Bmatrix}$$

cc

$\vec{X}_{cc}$ and $\dot{\vec{X}}_{cc}$ are the earth geocentric coordinate position and velocity with the origin at the center of the earth. The X_{cc} is positive toward the earth north pole, Y_{cc} is positive 90 degree longitude eastward from earth surface launcher point and Z_{cc} is negative through the equator at the launcher longitude.

$$\vec{X}_{cc} = \begin{bmatrix} \cos\rho_L & 0 & -\sin\rho_L \\ 0 & 1 & 0 \\ \sin\rho_L & 0 & \cos\rho_L \end{bmatrix} \vec{X}_{ee} + \gamma_e \begin{Bmatrix} \sin\rho_L \\ 0 \\ -\cos\rho_L \end{Bmatrix}$$

Systems **Definitions**

$$\vec{\dot{X}}_{cc} = \begin{bmatrix} \cos\rho_L & 0 & -\sin\rho_L \\ 0 & 1 & 0 \\ \sin\rho_L & 0 & \cos\rho_L \end{bmatrix} \vec{\dot{X}}_{ee}$$

abb $\vec{\dot{X}}_{abb}$ and $\vec{\ddot{X}}_{abb}$ are the components of missile velocity and acceleration in the missile oriented coordinate (the b system).

$$\vec{\dot{X}}_{abb} = [D]^{-1} \dot{X}_{aee}$$

$$\vec{\ddot{X}}_{abb} = [D]^{-1} \ddot{X}_{ee} + [\dot{D}]^{-1} \vec{\dot{X}}_{ee} - [D]^{-1} [A_1] \vec{\ddot{X}}_{w11}$$

$$- [D]^{-1} [\dot{A}_1] \vec{X}_{w11} - [D]^{-1} [A_1] \vec{\dot{X}}_{w11}$$

Systems	Definitions
aee	$\dot{\vec{X}}_{aee}$ are the earth fixed launch centered velocity components with respect to the local ambient air. $\dot{\vec{X}}_{aee} = \dot{\vec{X}}_{ee} = \dot{\vec{X}}_{wee}$

c. Rotation Matrices--The Euler angle rotation matrices have the properties of being a uniorthogonal array. Their determinate in unity and A-inverse ($[A]^{-1}$) is equal to A-transpose ($[A]^T$). (The transpose of a matrix is obtained by interchanging rows and columns.)

The Euler angle rotation matrices used in the trajectory program are defined as follows:

Systems	Definitions
$[A_{eo}]$	The Euler angle rotation matrix which rotates the e system to the e_o system thru the angles ρ_L at (2), ωt at (1), and ρ_L to (2). This rotation accounts for the effect of the launcher point rotating with the earth. The matrix is defined in section B.1.c (D Matrix).
$[A_i]$	The Euler angle rotation matrix which rotates the i system to the e_o system thru the angles ϕ_i at (1), θ_i at (2), and ψ_i at (3). These Euler angles represent the inertial platform alignment angles such that ψ_i defines the reference azimuth, θ_i defines the reference elevation and ϕ_i defines the reference roll attitude. This matrix is defined in section B.1.c. (D Matrix).
$[A_b]$	The Euler angle rotation matrix which rotates the b system to the i system thru the angles φ_b at (1), ψ_b at (3), and θ_b at (2). These angles are the achieved vehicle attitude angles.
$[D]$	The Euler angle rotation matrix which rotates the b system to the e system as $[D] = [A_{eo}]\ [A_i]\ [A_b]$
A_m	The Euler angle rotation matrix which rotates the m system to the i system thru the angles ϕ_m at (1), ψ_m at (3), and θ_m at (2). These angles are the command vehicle attitude angles.

Systems	Definitions
$[K]$	The Euler angle rotation matrix which rotates the m system to the b system. The pitch $(\Delta\theta_b)$, yaw $(\Delta\psi_b)$, and roll $(\Delta\varphi_b)$ attitude error angles are the direction cosines between the achieved and desired vehicle attitude and are defined as elements of this matrix. $[K] = [A_m]^{-1} [A_b]$
$[A_1]$	The Euler angle rotation matrix which rotates the e system to the 1 system thru the angles $-\rho$ at (2), μ' at (1), and ρ_L at (2). These angles are the launcher latitude, missile instantaneous change in longitude and missile instaneous latitude. This rotation basically rotates the earth fixed launcher coordinate to the local earth fixed coordinates. Winds and gravity are defined in local coordinates then transformed to the e system for inclusion in the equations of motion.
$[A_g]$	The Euler angle rotation matrix which rotates the e system to the g system thru the generalized coordinate orientation angles ϕ_g at (1), ψ_g at (3), and θ_g at (2). This rotation is used in velocity steering logic.
$[A_{TM}]$	The Euler angle rotation matrix which rotates the e system to the TM system thru the angle ζ at (1), ϕ at (2), and ψ at (3). These angles are the firing azimuth angle, down range angle, and cross range angle. This rotation matrix is used in evaluation of the missile-target coordinates.
$[A_c]$	The Euler angle rotation matrix which rotates the vehicle velocity vector to the m system thru the angles φ_c at (1), α_c at (2), and β'_c at (3). This rotation is used in commanding the desired vehicle attitude.
$[A_{\gamma 1}]$	The Euler angle rotation matrix which rotates the local azimuth oriented velocity vector to the missile velocity vector thru the elevation flight path angle.
$[A_{\gamma 2}]$	The Euler angle rotation matrix which rotates the local velocity vector in the 11 system to the local azimuth orient velocity vector thru the azimuth flight path angle. $\begin{Bmatrix} V_e \\ 0 \\ 0 \end{Bmatrix} = [A_{\gamma 1}] [A_{\gamma 2}] \vec{\dot{X}}_{11}$

Systems	Definitions
$[A_{\bar{\alpha}}]$	The Euler angle rotation matrix which rotates the roll and yaw compensated no-wind velocity vector to the b system thru the no-wind angle of attack $\bar{\alpha}$.
$[A_{\bar{\beta}}']$	The Euler angle rotation matrix which rotates the roll oriented no-wind velocity vector to the roll and yaw compensated no-wind velocity vector thru the angle $\bar{\beta}'$.
$[\varphi]$	The Euler angle rotation matrix which rotates the no-wind velocity vector such that the z axis passes thru the center of the earth. This rotation is used in evaluating the local vehicle back angle attitide.
k	Stage number. For example, one value of the parameter $\bar{I}_Y$ is needed in each stage, and $\bar{I}_{YK}$ appears in many equations as the particular value of $\bar{I}_Y$ applicable to the current stage.
o	An initial condition, e.g., the trajectory start time, t_o.
f	The value of a parameter at impact or intercept.
J	The final value in a table. Several functions are input in tabular form, and often it is unnecessary to fill one entire table to define a function. Thus, in some equations involving tabular functions, statements like "if $W > W_J$ . . ." appear.
•	First derivative with respect to time, e.g., the pitch thrust vector deflection angular rate $\dot{\delta}$ p is $d\delta p/d_t$.
••	Second derivative with respect to time, e.g., the vehicle pitch angular acceleration $\ddot{\theta}_b$ is $d^2\theta b/dt^2$
'	Input modified by the program, e.g., the input radius of the earth r_e' if input zero is set to 20,926,400 feet.

Symbols

Symbols appearing in this document are defined on the following pages. Those symbols which are input to the program or output from the program have their associated "L-numbers" following the description. Input data have L-numbers less than 5,000, output data have L-numbers greater than (or equal) to 5,000.

a, A

Symbol	Definition	Units
a_f	Input value of the dependent variable to be isolated by the hunting procedure (P1) (L0082)	(dbi)
a'_s	Input nozzle separation polynominal coefficient used in the separated flow nozzle thrust equation. If input zero is set to 0. 3 (Lk017)	(dim)
a_{TC}	Target transverse acceleration (L5446)	(g's)
a_{TCj}	Input target earth reference acceleration crosswise to the target velocity vector for the j-th period of the target dynamical condition table (L0643, 647, etc)	(g's)
a_{Tj}	Input transformation constant used in the simultaneous hunting procedure (P2) j - 1, 2, ... 7 (L0099, 108, etc)	(dbi)
a_{TN}	Target normal to its velocity vector acceleration (L5445)	(g's)
a_{TNj}	Input target earth reference acceleration normal to the target velocity vector for the j-th period of the target dynamical condition table (L0642, 646, etc)	(g's)
a_{TT}	Target tangential acceleration (L5444)	(g's)
a_{TTj}	Input target earth reference acceleration tangential to the target velocity vector for the j-th period of the target dynamical condition table (L0641, 645, etc)	(g's)
a_{Xb}	Component of vehicle acceleration due to total thrust and aerodynamic forces. Positive in the direction of the coordinate axes of b system (L5504)	(g's)

a, A

Symbol	Definition	Units
$a_{Xb(B1)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage I (L5911)	(g's)
$a_{Xb(B2)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage II (L5936)	(g's)
$a_{Xb(B3)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage III (L5961)	(g's)
$a_{Xb(B4)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage IV (L5986)	(g's)
a_{Yb}	Component of vehicle acceleration due to total thrust and aerodynamic forces. Positive in the direction of the coordinate Y axes of the b system (L5505)	(g's)
a_{Zb}	Components of vehicle acceleration due to total thrust and aerodynamic forces. Positive in the direction of the coordinate Z axes of b system (L5506)	(g's)
$a_{o,jk}$	Input constants used in the Z_{gg} and constant components nontarget dependent pitch steering equations for the k-th stage (Lk391-392)	(rad)
A_{ax}	Earth fixed axial acceleration, constrain to be greater than zero, during rail launch type of flight.	(ft/sec^2)
A_{eC}	Input complementary thrust-weight table stage nozzle exit area (Lk104)	(ft^2.
A_{Eij}	Elements of the (A_E) matrix which define the commanded turning rates for type 2 flight.	(deg/sec)
A'_{eM}	Input main thrust-weight table input stage nozzle exit area (Lk011)	(ft^2)

a, A

Symbol	Definition	Units
A'_{FW1}, A_{FW1}	Input and output respectively Stage I vacuum thrust to liftoff weight used in the vehicle characteristics pertinent to roll requirement (L0675)	(g's)
A_L	Input amplitude of limit cycle for the k -th stage. (Lk386)	(deg)
A'_{ri}	Input and utilized relative integration tolerance, j = 1, 2, . . . , 7 (L0281, 283, etc)	(dbi)
A_{SI}	Burn surface area (L5079).	(in^2)
A_t	Pintle nozzle throat area. Used in TMC logic (L5735).	(in^2)
$\dot{a}_t$	Time rate change of pintle throat area (L5785).	(in^2/sec)
A'_t	Input absolute allowable break-up tolerance of time (L0306).	(dim)
A_{tcc}	Commanded throat area (L5080).	(in^2)
A'_x	Input absolute allowable break-up tolerance of target value (L0308).	(sec)
A_{XI}	Propellant extinguishment throat area (L5085).	(in^2)

b, B

Symbol	Definition	Units
b_{jk}	Input constants used in the $\ddot{X}_{gg}$, X^2_{gg}, and $\dot{X}^3_{gg}$ components of the nontarget dependent pitch steering equation for the k-th stage, j = 1, 2, 3 (Lk393-395).	(rad-sec/ft, rad-sec^2/ft and rad-sec)
b'_s	Input nozzle separation polynominal coefficient used in the separated flow nozzle thrust equation if input zero is set to 0.7 (Lk018).	(dim)
BD	Input basic deck number (L0000).	(dim)
B'_t	Input relative allowable break-up tolerance of time (L0307).	(dim)
B'_x	Input relative allowable break-up tolerance of target value (L0309).	(dim)

c, C

Symbol	Definition	Units
c'_s	Input nozzle separation polynominal coefficient used in the separated flow nozzle thrust equation. If input zero is set to 0.884 (Lk019).	(dim)
C	Instantaneous aerodynamic axial force (L5164).	(lb)
$\bar{C}'$	Input stage axial force control function and multiplier. If input zero, the multiplier is set to one; if nonzero , the axial force is determined from input and multiplied by $\bar{C}$ (Lk186).	(dim)
C_a	Speed of sound at the missile (L5066).	(ft/sec)
C_A	Instantaneous aerodynamic axial force coefficient (L5573).	(dim)
C_{Aj}	Input and instantaneous (with Mach number M_j) aerodynamic axial force coefficients, respectively, where j = 1, 2, . . . , 15 per stage (Lk189, 191, etc).	(dim)
C_{BN}	Added aerodynamic base drag coefficient due to nozzles not thrusting (L5574).	(dim)
C'_D	Input and calculated nozzle efficiency coefficient used in the separated flow nozzle thrust equation (Lk015).	(dim)
C_{Dz}	Instantaneous aerodynamic pitch movable fin drag coefficient (L5566).	(dim)
$\bar{C}'_{Dz}$	Input aerodynamic pitch fin drag coefficient multiplier. If zero set equal to one (Lk708).	(dim)
$\hat{C}_{Dz}$	Instantaneous aerodynamic pitch movable fin total drag coefficient (L5579).	(dim)
C_{Dzj}	Input aerodynamic pitch fin drag coefficient j = 1, 2, . . . , 15 per stage (Lk713, 719, etc).	(dim)

c, C

Symbol	Definition	Units
C_{Fyj}	Input thrust system proportionality system gain of the j-th type TMC (L0804, 814, etc).	(dim)
C_{FI}	Thrust coefficient (Lk5081).	(dim)
C_{FO}	Thrust coefficient at optimum expansion ($P_e = P_a$) (L5082).	(dim)
$\hat{C}_{Ly}$	Instantaneous aerodynamic yaw movable fin total lift coefficient (L5577).	(dim)
$\bar{C}'_{lz}$	Input aerodynamic nonlinear pitch fin drag coefficient multiplier. If input zero is set to 1.0 (Lk707).	(dim)
C_{lz}	Instantaneous aerodynamic pitch movable fin nonlinear fin lift coefficient (L5565).	$(1/\mathrm{deg}^2)$
C_{Lz}	Instantaneous aerodynamic pitch movable fins linear fin lift coefficient (L5564).	$(1/\mathrm{deg}^2)$
C_{lzj}	Input and calculated aerodynamic pitch fin nonlinear lift coefficient j = 1, 2, ..., 15 per stage (Lk712, 718, etc)	$(1/\mathrm{deg}^2)$
$\bar{C}'_{Lz}$	Input aerodynamic linear pitch fin lift coefficient multiplier. If input zero is set to 1.0 (Lk706).	(dim)
$\hat{C}_{Lz}$	Instantaneous aerodynamic pitch movable fin total lift coefficient (L5578).	(dim)
C_{Lzj}	Input and calculated aerodynamic pitch fin nonlinear lift coefficient j = 1, 2, ..., 15 per stage (Lk711, 717, etc).	$(1/\mathrm{deg}^2)$
C_{MQ}	Calculated and unadjusted for translation aerodynamic pitch damping moment due to pitch rate coefficient (L5575).	(1/deg)
C_{MQj}	Input aerodynamic pitch damping moment due to pitch rate coefficient where j = 1, 2, ..., 15 per stage (Lk306, 309, etc).	(1/deg)

c, C

Symbol	Definition	Units
$C_{M\dot{\alpha}}$	Calculated and unadjusted for translation aerodynamic pitch damping moment due to rate change of angle of attack coefficient (L5576).	(1/deg)
$C_{M\dot{\alpha}j}$	Input aerodynamic pitch damping moment due to rate change of angle of attach coefficient where j = 1, 2, . . . , 15 per stage (Lk307, 310, etc).	(1/deg)
C_N	Instantaneous aerodynamic normal force coefficient (L5569).	(dim)
CN_1 CN_2 CN_3	Instantaneous first, second, and third derivatives, respectively, of C_N with respect to the angle of attack (L5570, 5571, 5572).	(1/deg $1/deg^2$ $1/deg^3$)
$C_{N1,2,3j}$	Input values of $C_{N1,2,3}$ respectively, corresponding to M_j where j = 1, 2, . . . , 15 per stage (Lk224-226, 229-231).	$(deg/deg^2/deg^3)$
C_{RRj}	Input raceway aerodynamic force coefficient corresponding to the M_{Rj}, j = 1, 2, . . . , 10 (Lk674, 676).	(dim)
$C_{\delta z}$	Aerodynamic pitch fin axial force (L5155).	(lb)

d, D

Symbol	Definition	Units
dC_a/dh	Partial derivative of the speed of sound with altitude (L5067).	(1/sec)
dP_a/dh	Partial derivative of ambient pressure with altitude (L5069).	(lb/ft)
D_b	Propellant burn depth. Used in TMC logic (L5733).	(in)
D_B	Output TVC duty cycle stage case diameter from axial force reference area.	(in)
D_i	Input roll control system hysteresis for the i-th zone, i = 1, 2, or 3 (Lk640, 650, etc).	(dim)
D_p	Input diameter of propellant (Lk001).	(in)
$D_{r\ell}$	Rail launch friction drag (L5154).	(lb)
D_{RN}	Input aerodynamic reference diameter (Lk298).	(ft)
D_y	Input initial or restart discontinuity print flag (L0009).	(dim)

e, E

Symbol	Definition	Units
e	Eccentricity of the missile path during the glide phase (L5112).	(dim)
E/m	Total missile energy per unit mass during the glide phase. Potential energy at the launcher is taken as zero (L5109).	(ft^2-sec^2)

f, F

Symbol	Definition	Units
f_D	Input and flag specifying maximization and isolation of the same parameter used in hunt procedure (P2). If f_D is nonzero, the function f is maximized relative to the independent variables $x_2, x_3, \ldots, x_n$ and is isolated to a value f_D by varying x_1 (L0089).	(dbi)
f_{Gik}	Input attitude control system gain control flag. If equal to zero, input gains are utilized; if not equal to zero, the automatic gains are utilized for the j-th control zone, i = 1, 2, or 3; and k-th stage, k = 1, 2, 3, or 4 (Lk456, 465, etc).	(dim)
F	Total instantaneous thrust acting along missile centerline. Positive when thrust vector points forward along missile centerline (L5122).	(lb)
$\bar{F}$	Instantaneous delivered thrust per motor during the TVC design stage.	(lb)
F_C	Instantaneous complementary thrust (L5127).	(lb)
$\hat{F}_c$	Thrust required to maintain V_e; i.e., retarding axial force used in TMC logic (L5134).	(lb)
F_{CALOS}	Command thrust to provide acceleration proportional to LOS rate used in TMC logic (L5137).	(lb)
F_{cclos}	Command thrust to provide a minimum missile to target closing rate used in TMC logic (L5138).	(lb)

f, F

Symbol	Definition	Units
F_{Cj}	Input value of the complementary vacuum thrust to be used during $t_{Cj} \leq t \leq t_{C(j+1)}$, where j = 1, 2,..., 25 per stage (Lk112, 115).	(lb)
F_{com}	Commanded altitude thrust used in TMC logic (L5130).	(lb)
F_{cqmax}	Maximum thrust so that the vehicle will not exceed the maximum dynamic pressure used in TMC logic (L5136).	(lb)
F_{cqmin}	Require thrust so that the vehicle will maintain the minimum dynamic pressure used in TMC logic (L5135).	(lb)
F_{CV}	Instantaneous complementary vacuum thrust (L5128).	(lb)
$\dot{F}_{CV}$	Instantaneous complementary vacuum thrust time rate (L5129).	(lb/sec)
F_{CV}	Commanded vacuum thrust used in TMC logic (L5131).	(lb)
F_D	Instantaneous roll control system phase plane signal (L5449).	(deg)
F_h	Vacuum thrust of the σ_{tb} short-time trace.	(lb)
F_{JFy}	Jet damping yawing transverse force (L5149).	(lb)
F_{JDz}	Jet damping pitching transverse force (L5149).	(lb)
F_L	Vacuum thrust of the σ_{tb} long-time trace.	(lb)
F_M	Instantaneous main thrust (L5124).	(lb)
F_{Mj}	Input value of the main vacuum thrust to be used during $t_{Mj} \leq t \leq t_{Mj+1}$, where j = 1, 2, ..., 25 per stage (Lk021, 024, etc).	(lb)

f, F

Symbol	Definition	Units
F_{MV}	Instantaneous main vacuum thrust (L5125).	(lb)
$\dot{F}_{MV}$	Instantaneous main vacuum thrust rate (L5126).	(lb)
F_n	Nominal vacuum thrust of each motor.	(lb)
F_N	Nominal altitude thrust used in TMC logic.	(lb)
$\bar{F}_q$	Delivered motor thrust ($\bar{F}$) at t_{Bq} during TVC design stage.	(lb)
F_R	Instantaneous roll control thrust. Positive if the vehicle is intended to rotate clockwise as seen from the rear of the vehicle. (L5714).	(lb)
$\dot{F}_R$	Time rate change of roll control thrust (L5769).	(lb/sec)
F_{Rc}	Instantaneous roll control system thrust command signal (L5721).	(lb)
F_{RO}	Input per stage initial roll thrust (Lk424).	(lb)
F_{TDy}	Movable nozzle tail-wag-dog force in yaw, positive to the right (L5147).	(lb)
F_{TDz}	Movable nozzle tail-wag-dog force in pitch, positive down (L5148).	(lb)
F_v	Instantaneous total vacuum thrust (L5123).	(lb)
F_{VN}	Nominal vacuum thrust used in TMC logic (L5132).	(lb)
$\lvert F/\dot{W} \rvert$	Instantaneous effective specific impulse (L5115).	(sec)
FW1	Input Stage I vacuum thrust to liftoff weight used in the vehicle characteristics pertinent to roll requirement (L0675).	(g's)
F_x	Components of total vehicle thrust parallel to the coordinate axes of the b system (L5141).	(lb)

f, F

Symbol	Definition	Units
F_X	Thrust force along nozzle centerline (L5144).	(lb)
F_y	Components of total vehicle thrust parallel to the coordinate axes of the b system (L5142).	(lb)
F_Y	Component of total vehicle thrust moved to nozzle centerline, positive right (L5145).	(lb)
F_{yj}	Input thrust control law code for the j-th type of TMC table (L6800, 810, etc).	(dim)
F_z	Components of total vehicle thrust parallel to the coordinate axes of the b system (L5143).	(lb)
F_Z	Component of total vehicle thrust normal to nozzle centerline, positive down (L5146).	(lb)
$F_{\Delta i}$	Input maximum roll control thrust for the i-th zone, i = 1, 2, or 3 (Lk641, 651, 661).	(lb)
F_{vave}	Output TVC duty cycle stage average vacuum thrust.	(lb)
F_{vac}	Nominal input vacuum thrust time curve	(lb)
$\bar{F}_{vac}$	Instantaneous vacuum motor thrust	(lb)
$\bar{F}_{vacq}$	Output vacuum motor thrust ($\bar{F}_{vac}$) during the TVC design stage at t_{Bq}.	(lb)

g, G

Symbol	Definition	Units
g_e	Gravitational force at the missile.	(ft/sec^2)
$\tilde{g}_e$	Input mass conversion gravity. If input zero, set equal to 32.174 (L0024).	(ft/sec^2)
g'_e	Input gravitational acceleration at the surface of the reference body. If input zero, set equal to 32.14625 (L0025).	(ft/sec^2)
g_i	Lagranian constraining function of the i-th dependent variables y_i used in the hunting procedure (P2).	(dbi)
g_{PI}	Fraction of propellant removed (L5084).	(dim)
g_{WI}	Percent web (L5077).	(dim)
g_{xe}	Launch centered earth fixed northernly component of gravity (L5041).	(ft/sec^2)
g_{xl}	Local northernly component of gravity (L5038).	(ft/sec^2)
g_{ye}	Launch centered earth fixed easternly component of gravity (L5042).	(ft/sec^2)
g_{yl}	Local easternly component of gravity (L5039).	(ft/sec^2)
g_{ze}	Launch centered earth fixed downward component of gravity (L5043).	(ft/sec^2)
g_{zl}	Local downward component of gravity (L5040).	(ft/sec^2)
G_Z	Partial derivatives of altitude acceleration to vehicle altitude (L5455).	(ft/sec^2-deg)
G	Lagrangian constraining function maximized or minimized to provide an external of the function f subject to one or more constraints used in the hunting procedure (P2).	(dim)

h, H

Symbol	Definition	Units
h	Missile geometric altitude. Distance between the surface of the reference body and the missile measured along the local vertical. Positive away from the reference body (L5028)	(ft)
$\dot{h}$	Time rate change of missile geometric altitude rate between the surface of the reference body and missile measured along the local vertical positive away from the reference body (L5029)	(ft/sec)
$\ddot{h}$	Instantaneous altitude acceleration (L5030)	(ft/sec^2)
h_a	Apogee altitude of the missile during the glide phase (L5031)	(nm)
h_{ab}	Altitude above launcher (L5034)	(ft)
$h_{a(B1)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage I (L5915)	(nm)
$h_{a(B2)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage II (L5940)	(nm)
$h_{a(B3)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage III (L5965)	(nm)
$h_{a(B4)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage IV (L5990)	(nm)
h_{ap}	Height of apogee + paragee (L5033)	(nm)
$h_{(B1)}$	Missile geometric altitude at the termination of stage I (L5902)	(ft)
$h_{(B2)}$	Missile geometric altitude at the termination of stage II (L5927)	(ft)
$h_{(B3)}$	Missile geometric altitude at the termination of stage III (L5952)	(ft)

h, H

Symbol	Definition	Units
$h_{(B4)}$	Missile geometric altitude at the termination of stage IV (L5977)	(ft)
h_{BI}	Base geopotential altitude associated with the atmosphere representation	(ft)
h_{cj}	Input commanded altitude used in the constant altitude type of flight (Ty=8) (L0313, 320, etc)	(ft)
h_e	Input altitude of atmospheric entry (L0042)	(ft)
h_E	Input altitude above which ambient pressure and aerodynamic forces are zero and speed of sound is 1,000 ft/sec. If input zero is set to 300,000 ft (L0021)	(ft)
h_f	Input altitude at the termination of the glide phase (L0039)	(ft)
h_j	Input wind velocity altitude associated with v_{wj} and ψ_{wj} where j = 1, 2, ..., 30 (L0410, 413, etc)	(ft)
h_L	Input launcher altitude (L0019)	(ft)
h_{MI}	Estimated altitude at target intercept (L5442)	(ft)
h_{mxwd}	Input altitude of maximum wind velocity. Also flag to set up wind table per MMRBM wind shear criteria (L0682)	(ft)
h_p	Perigee altitude of the missile during the glide phase (L5032)	(nm)
$h_{p(B1)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage I (L5914)	(nm)
$h_{p(B2)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage II (L5939)	(nm)
$h_{p(B3)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage III (L5964)	(nm)

h, H

Symbol	Definition	Units
$h_{p(B4)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage IV (L5989)	(nm)
h_T	Target altitude (L5730)	(ft)
h_{To}	Input target initial altitude at the start of the target maneuvering (L0634)	(ft)
h_α	Input final attitude of maximum wind shear used in TVC duty cycle slew rate calculations (L0672)	(ft)
h_β	Input initial altitude of maximum wind shear used in TVC duty cycle slew rate calculations (L0673)	(ft)
h_o	Input missile altitude at the trajectory start time (L0012)	(ft)
H_e	Heating parameter. Integral of qv from stage initiation to the time being printed (L5747)	(lb/ft)
$\dot{h}_T$	Target altitude rate (L5780)	(ft/sec)

i, I

Symbol	Definition	Units
i	Orbital inclination angle (L5111)	(deg)
I	Total missile impulse measured from stage initiation to the time being printed (L5737)	(lb-sec)
$\bar{I}$	Output motor thrust impulse for TVC duty cycle stage	(lb-sec)
$I_{FC(1)}$	Calculated complementary table nozzle back pressure impulse for stage I (L5847)	(lb-sec)
$I_{FC(2)}$	Calculated complementary table nozzle back pressure impulse for stage II (L5862)	(lb-sec)
$I_{FC(3)}$	Calculated complementary table nozzle back pressure impulse for stage III (L5877)	(lb-sec)
$I_{FC(4)}$	Calculated complementary table nozzle back pressure impulse for stage IV (L5892)	(lb-sec)
$I_{FM(1)}$	Calculated main table nozzle back pressure impulse for stage I (L5841)	(lb-sec)
$I_{FM(2)}$	Calculated main table nozzle back pressure impulse for stage II (L5856)	(lb-sec)
$I_{FM(3)}$	Calculated main table nozzle back pressure impulse for stage III (L5871)	(lb-sec)
$I_{FM(4)}$	Calculated main table nozzle back pressure impulse for stage IV (L5886)	(lb-sec)
I_n	Input stage movable portion nozzle movement of inertia abcut the gimbal point (Lk483)	(slug-ft^2)
I_{PRD}	Pitch inertia rotation damping moment integral (L5193)	(ft-lb-sec)
I_P	Pitch control thrust impulse from stage initiation to the time being printed (L5739)	(lb-sec)
$\bar{I}_P$	Output pitch control thrust impulse per control motor from TVC duty cycle initiation to stage termination	(lb-sec)
I_R	Auxiliary roll control system delivered total impulse (L5745)	(lb-sec)

i, I

Symbol	Definition	Units
I_{RRD}	Roll inertia rotation damping moment integral (L5195)	(ft-lb-sec)
I_{spaug}	Input and output estimated TVC system caused specific impulse augmentation (positive) or degradation (negative). Used in trajectory TVC Design program refly (L0679)	(sec)
I_{spC}	Input complementary specific impulse used to compute vehicle weight flow. If zero weight flow is determined from input weight flow [illegible]	(sec)
$I_{spC(1)}$	Calculated complementary table adjusted to vacuum specific impulse for stage I (L5852)	(sec)
$I_{spC(2)}$	Calculated complementary table adjusted to vacuum specific impulse for stage II (L5867)	(sec)
$I_{spC(3)}$	Calculated complementary table adjusted to vacuum specific impulse for stage III (L5882)	(sec)
$I_{spC(4)}$	Calculated complementary table adjusted to vacuum specific impulse for stage IV (L5897)	(sec)
I_{spM}	Input main specific impulse used to compute vehicle weight flow. If zero, weight flow is determined from the input weight flow. Also output in the TVC duty cycle (Lk010)	(sec)
$I_{spM(1)}$	Calculated main table adjusted to vacuum specific impulse for stage I (L5846)	(sec)
$I_{spM(2)}$	Calculated main table adjusted to vacuum specific impulse for stage II (L5861)	(sec)
$I_{spM(3)}$	Calculated main table adjusted to vacuum specific impulse for stage III (L5876)	(sec)
$I_{spM(4)}$	Calculated main table adjusted to vacuum specific impulse for stage IV (L5891)	(sec)
I_{SPRi}	Input roll control motor specific impulse for the i-th zone, i = 1, 2, or 3 (Lk646, 656, 666)	(sec)
I_V	Total missile vacuum impulse, measured from stage initiation to the time being printed (L5738)	(lb-sec)

i, I

Symbol	Definition	Units
$I_{V(B1)}$	Total missile vacuum impulse for stage I (L5913)	(lb-sec)
$I_{V(B2)}$	Total missile vacuum impulse for stage II (L5938)	(lb-sec)
$I_{V(B3)}$	Total missile vacuum impulse for stage III (L5963)	(lb-sec)
$I_{V(B4)}$	Total missile vacuum impulse for stage IV (L5988)	(lb-sec)
$\bar{I}_v$	Output motor vacuum thrust impulse for TVC duty cycle stage	(lb-sec)
I'_{vC}	Input complementary stage total vacuum impulse (Lk106)	(lb-sec)
I''_{vC}	Vacuum impulse under input complementary thrust curve (L5105)	(lb-sec)
$I_{vC(1)}$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage I (L5849)	(lb-sec)
$I_{vC(2)}$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage II (L5863)	(lb-sec)
$I_{vC(3)}$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage III (L5879)	(lb-sec)
$I_{vC(4)}$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage IV (L5893)	(lb-sec)
$\hat{I}_{vC(1)}$	Calculated complementary table input total impulse adjusted to vacuum condition for stage I (L5849)	(lb-sec)
$\hat{I}_{vC(2)}$	Calculated complementary table input total impulse adjusted to vacuum condition for stage II (L5863)	(lb-sec)
$\hat{I}_{vC(3)}$	Calculated complementary table input total impulse adjusted to vacuum conditions for stage III (L5878)	(lb-sec)
$\hat{I}_{vC(4)}$	Calculated complementary table input total impulse adjusted to vacuum conditions for stage IV (L5893)	(lb-sec)

i, I

Symbol	Definition	Units
$I^*_{vC(1)}$	Calculated complementary table vacuum adjusted thrust integral for stage I (L5850)	(lb-sec)
$I^*_{vC(2)}$	Calculated complementary table vacuum adjusted thrust integral for stage II (L5863)	(lb-sec)
$I^*_{vC(3)}$	Calculated complementary table vacuum adjusted thrust integral for stage III (L5879)	(lb-sec)
$I^*_{vC(4)}$	Calculated complementary table vacuum adjusted thrust integral for stage IV (L5893)	(lb-sec)
I'_{vM}	Input main stage total vacuum impulse (Lk005)	(lb-sec)
I''_{vM}	Vacuum impulse under input main thrust curve (L5104)	(lb-sec)
$I_{vM(1)}$	Calculated input vacuum corrected main table time adjusted thrust integral for stage I (L5843)	(lb-sec)
$I_{vM(2)}$	Calculated input vacuum corrected main table time adjusted thrust integral for stage II (L5858)	(lb-sec)
$I_{vM(3)}$	Calculated input vacuum corrected main table time adjusted thrust integral for stage III (L5873)	(lb-sec)
$I_{vM(4)}$	Calculated input vacuum corrected main table time adjusted thrust integral for stage IV (L5888)	(lb-sec)
$\hat{I}_{vM(1)}$	Calculated main table input total impulse adjusted to vacuum conditions for stage I (L5842)	(lb-sec)
$\hat{I}_{vM(2)}$	Calculated main table input total impulse adjusted to vacuum conditions for stage II (L5857)	(lb-sec)
$\hat{I}_{vM(3)}$	Calculated main table input total impulse adjusted to vacuum conditions for stage III (L5872)	(lb-sec)
$\hat{I}_{vM(4)}$	Calculated main table input total impulse adjusted to vacuum conditions for stage IV (L5887)	(lb-sec)
$I^*_{vM(1)}$	Calculated main table vacuum adjusted thrust integral for stage I (L5844)	(lb-sec)
$I^*_{vM(2)}$	Calculated main table vacuum adjusted thrust integral for stage II (L5859)	(lb-sec)

i, I

Symbol	Definition	Units
$I^*_{vM(3)}$	Calculated main table vacuum adjusted thrust integral for stage III (L5874)	(lb-sec)
$I^*_{vM(4)}$	Calculated main table vacuum adjusted thrust integral for stage IV (L5889)	(lb-sec)
I'_{vT}	Input total of the main and complementary vacuum impulse (Lk004)	(lb-sec)
$I_{vT(1)}$	Calculated input vacuum corrected main and complementary impulse corrected for stage I (L5855)	(lb-sec)
$I_{vT(2)}$	Calculated input vacuum corrected main and complementary impulse time corrected for stage II (L5870)	(lb-sec)
$I_{vT(3)}$	Calculated input vacuum corrected main and complementary impulse time corrected for stage III (L5885)	(lb-sec)
$I_{vT(4)}$	Calculated input vacuum corrected main and complementary impulse time corrected for stage IV (L5900)	(lb-sec)
$\hat{I}_{vT(1)}$	Calculated input main and complementary impulse corrected to vacuum condition for stage I (L5853)	(lb-sec)
$\hat{I}_{vT(2)}$	Calculated input main and complementary impulse corrected to vacuum conditions for stage II (L5868)	(lb-sec)
$\hat{I}_{vT(3)}$	Calculated input main and complementary impulse corrected to vacuum conditions for stage III (L5883)	(lb-sec)
$\hat{I}_{vT(4)}$	Calculated input main and complementary impulse corrected to vacuum conditions for stage IV (L5898)	(lb-sec)
$I^*_{vT(1)}$	Calculated main and complementary vacuum adjusted thrust integral for stage I (L5854)	(lb-sec)
$I^*_{vT(2)}$	Calculated main and complementary vacuum adjusted thrust integral for stage II (L5869)	(lb-sec)
$I^*_{vT(3)}$	Calculated main and complementary vacuum adjusted thrust integral for stage III (L5884)	(lb-sec)

i, I

Symbol	Definition	Units
$I^*_{vT(4)}$	Calculated main and complementary vacuum adjusted thrust integral for stage IV (L5899)	(lb-sec)
I_{vT}''	Output integral of the stage main plus complementary thrust tables as adjusted by the calculated multipliers	(lb-sec)
$\bar{I}_X$	Input roll moment of inertia multipler (Lk477)	(dbi)
I_{Xj}	Input total vehicle roll moment of inertia weight input Wj, where j = 1, 2, . . . , 15 per stage (Lk494, 504)	(slug ft^2)
I_{XX}	Roll moment of inertia about vehicle center-of-gravity (L5175)	(ft-lb-sec^2)
$\dot{I}_{XX}$	Time rate change of roll moment of inertia (L5184)	(ft-lb-sec)
I_{XY}	Roll-yaw product of inertia about vehicle center-of-gravity (L5176)	(ft-lb-sec^2)
$\dot{I}_{XY}$	Time rate change of roll-yaw product of inertia (L5185)	(ft-lb-sec)
I_{XYj}	Input total vehicle roll-yaw product of inertia corresponding to the total weight input Wj, where j = 1, 2, . . . , 15 per stage (Lk495, 505)	(slug ft^2)
I_{XZ}	Roll-pitch product of inertia about vehicle center-of-gravity (L5177)	(ft-lb-sec^2)
$\dot{I}_{XZ}$	Time rate change of roll-pitch product of inertia (L5186)	(ft-lb-sec)
I_Y	Yaw control thrust impulse from stage initiation to the time being printed (L5740)	(lb-sec)
$\bar{I}_Y$	Input moment of inertia multiplier (Lk475)	(dbi)
I_{Yj}	Input total vehicle pitch moment of inertia corresponding to the total vehicle weight input Wj where j = 1, 2, . . . , 15 per stage (Lk490, 500)	(dbi)
$\bar{I}_y$	Output yaw control thrust impulse per control motor from TVC stage initiation to stage termination	(lb-sec)

I, I

Symbol	Definition	Units
$I^*_{vT(4)}$	Calculated main and complementary vacuum adjusted thrust integral for stage IV (L5809)	(lb-sec)
I_{vT}''	Output integral of the stage main plus complementary thrust tables as adjusted by the calculated multipliers	(lb-sec)
$\bar{I}_X$	Input roll moment of inertia multipler (Lk477)	(dbi)
I_{Xj}	Input total vehicle roll moment of inertia weight input Wj, where j = 1, 2, ..., 15 per stage (Lk494, 504)	(slug ft^2)
I_{XX}	Roll moment of inertia about vehicle center-of-gravity (L5175)	(ft-lb-sec^2)
$\dot{I}_{XX}$	Time rate change of roll moment of inertia (L5184)	(ft-lb-sec)
I_{XY}	Roll-yaw product of inertia about vehicle center-of-gravity (L5176)	(ft-lb-sec^2)
$\dot{I}_{XY}$	Time rate change of roll-yaw product of inertia (L5185)	(ft-lb-sec)
I_{XYj}	Input total vehicle roll-yaw product of inertia corresponding to the total weight input Wj, where j = 1, 2, ..., 15 per stage (Lk495, 505)	(slug ft^2)
I_{XZ}	Roll-pitch product of inertia about vehicle center-of-gravity (L5177)	(ft-lb-sec^2)
$\dot{I}_{XZ}$	Time rate change of roll-pitch product of inertia (L5186)	(ft-lb-sec)
I_Y	Yaw control thrust impulse from stage initiation to the time being printed (L5740)	(lb-sec)
$\bar{I}_Y$	Input moment of inertia multiplier (Lk478)	(dbi)
I_{Yj}	Input total-vehicle pitch moment of inertia corresponding to the total vehicle weight input Wj where j = 1, 2, ..., 15 per stage (Lk490, 500)	(dbi)
$\bar{I}_Y$	Output yaw control thrust impulse per control motor from TVC stage initiation to stage termination	(lb-sec)

i, I

Symbol	Definition	Units
I_{YRD}	Yaw inertia rotation damping moment integral (L5194)	(ft-lb-sec)
I_{YY}	Pitch moment of inertia about vehicle center-of-gravity (L5179)	(ft-lb-sec^2)
$\dot{I}_{YY}$	Time rate change of pitch moment of inertia (L5188)	(ft-lb-sec)
I_{YZ}	Yaw-pitch product of inertia about vehicle center-of-gravity (L5180)	(ft-lb-sec^2)
$\dot{I}_{YZ}$	Time rate change of yaw-pitch product of inertia (L5189)	(ft-lb-sec)
I_{YZj}	Input total vehicle yaw pitch product of inertia corresponding to the total vehicle weight input Wj, where j = 1, 2, ..., 15 per stage (Lk496, 506)	(slug ft^2)
$\bar{I}_Z$	Input yaw moment of inertia multiplier (Lk476)	(dbi)
I_{Zj}	Input total vehicle yaw moment of inertia corresponding to the total vehicle weight input W, where j = 1, 2, ..., 15 per stage (Lk492, 502)	(slug ft^2)
I_{ZXj}	Input total vehicle pitch-roll product of inertia corresponding to the total vehicle weight input Wj, where j = 1, 2, ..., 15 per stage (Lk497, 507)	(slug ft^2)
I_{ZZ}	Yaw moment of inertia about vehicle center-of-gravity (L5183)	(ft-lb-sec^2)
$\dot{I}_{ZZ}$	Time rate change of yaw moment of inertia (L5192)	(ft-lb-sec)
$I_{\dot{\delta}P}$	Sum of pitch angular thrust vectoring velocities from stage initiation to the time being printed (L5741)	(deg)
$\bar{I}_{\dot{\delta}p}$	Sum of pitch angular thrust vectoring velocities from stage initiation to the time being printed corrected for dither (L5116)	(deg)
$I_{\dot{\delta}Y}$	Sum of yaw angular thrust vectoring velocities from stage initiation to the time being printed (L5742)	(deg)
$\bar{I}_{\dot{\delta}y}$	Sum of yaw angular thrust vectoring velocities from stage initiation to the time being printed corrected for dither (L5117)	(deg)

i, J

Symbol	Definition	Units
J	Input gravitational value which accounts for the earth's oblateness (L0023)	(dim)

k, K

Symbol	Definition	Units
K_3	Input quantity which determines stage termination (Lk003)	(dbi)
K_a	Input isolation-maximization control function. If zero, isolation is specified and if nonzero, maximization of the dependent variables will occur used in hunting procedure (Pl) (L0076)	(dim)
K_{BD}	Input flag which stipulates that the main nozzle exit area will be used in the base drag calculations when splitting main and complementary tables to allow for up to 47 thrust time points (Lk108)	(dim)
K_{cv}	Commanded thrust velocity error gain used in TMC (L5452)	(lb-sec/ft)
K_{cq}	Commanded thrust dynamic pressure error gain used in TMC (L5453)	(ft^2)
$K_{c1,2}$	Input lower and upper limits, respectively, for computation of orbital elements and impact determination computations (L0037, 38)	(dbi)
K_{dc}	Input and output stage number of TVC duty cycle stage (L0671)	(dim)
K_{DP}	Instantaneous control system pitch attitude error gain (L5461)	(dim)
K_{DPj}	Input control system pitch attitude error gain for the j-th control region j = 1, 2, or 3 (Lk450, 459, etc)	(dim)
K_{DR}	Instantaneous control system roll attitude error gain · (L5463)	(dim)
K_{DRi}	Input roll control system attitude error gain for the i-th zone, i = 1, 2, or 3 (Lk643, 653, 663)	(dim)
K_{DY}	Instantaneous control system yaw attitude error gain (L5462)	(dim)
K_{DYj}	Input control system yaw attitude error gain for the j-th control region j = 1, 2, or 3 (Lk451, 460, etc)	(dim)

k, K

Symbol	Definition	Units
K'_{FC}	Input multiplier of the complementary vacuum thrust. If input zero for the k-th stage and I'_{vT} = I'_{vT} = 0, the complementary thrust for the k-th stage are zero (Lk100)	(dim)
K''_{FC}	Calculated vacuum scale factor for back pressure term for complementary thrust time table	(dim)
$K^*_{FC(1)}$	Calculated complementary table thrust multiplier for stage I (L5851)	(dim)
$K^*_{FC(2)}$	Calculated complementary table thrust multiplier for stage II (L5866)	(dim)
$K^*_{FC(3)}$	Calculated complementary table thrust multiplier for stage III (L5881)	(dim)
$K^*_{FC(4)}$	Calculated complementary table thrust multiplier for stage IV (L5896)	(dim)
K_{fj}	Input limit of the j-th type of flight where j = 1, 2, ..., or 16 (L0312, 319, etc)	(dbi)
K_{FM}	Input multipliers of the main vacuum thrust (Lk007)	(dim)
K''_{FM}	Calculated vacuum scale factor for back pressure term for main thrust time table	(dim)
$K^*_{FM(1)}$	Calculated main table thrust multiplier for stage I (L5845)	(dim)
$K^*_{FM(2)}$	Calculated main table thrust multiplier for stage II (L5860)	(dim)
$K^*_{FM(3)}$	Calculated main table thrust multiplier for stage III (L5875)	(dim)
$K^*_{FM(4)}$	Calculated main table thrust multiplier for stage IV (L5890)	(dim)
K_{Fyj}	Input limit of the j-th type TMC (L0802, 812, etc)	(dbi)
K_{Gik}	Input attitude control system gain zone limits (i = 1, 2, or 3) and the k-th stage k = 1, 2, 3, or 4 (Lk458, 467)	(dbi)

k, K

Symbol	Definition	Units
K_{g1k}	Input quantity which designates the start of the acquisition zone for evaluation of the steering equation coefficient for the k-th stage, k = 1, 2, 3, or 4 (Lk398)	(dbi)
K_{g2k}	Input quantity which designates the end of the acquisition zone for evaluation of the steering equation coefficient for the k-th stage, k = 1, 2, 3, or 4 (Lk400)	(dbi)
K_h	Input wind altitude multiplier and flag. If zero, no wind effects are considered (L0407)	(dbi)
K_{HGj}	Input navigation constant used in the Homing Guidance (Ty=11) (L0313, 320, etc)	(dim)
K_{ij}	i-th row, j-th column of the [K] Euler angle rotation matrix which rotates the m system to the b system. These elements are the direction cosines between the achieved and desired vehicle attitude, ie, the error angles.	
K_{IP}	Instantaneous control system angle of attack gain (L5469)	(dim)
K_{IPj}	Input pitch angle of attack gain for the j-th control region j = 1, 2, or 3 (Lk454, 463, etc)	(dim)
K_{L_R}	Roll control attitude bias gain (L5466)	(1/sec)
K_{IY}	Instantaneous control system angle of side slip gain (L5465)	(dim)
K_{IYj}	Input yaw angle of slide slip gain for the j-th control region j = 1, 2, or 3 (Lk455, 464)	(dim)
K_{Jj}	Input value of that parameter at which the weight W_{JTj} is to be jettisoned where j = 1, 2, ... or 8 (L0051, 54, etc)	(dbi)
K'_k	Input stage start control function. If 1, 2, 3, or 4, the run starts at the initiation of the first, second, third, or fourth stage, respectively if input zero set equal to 1 (L0003)	(dim)

k, K

Symbol	Definition	Units
K_{Lz}	Instantaneous aerodynamic pitch movable fin drag due to lift factor (L5567)	(dim)
$\bar{K}'_{Lz}$	Input aerodynamic pitch fin drag due to lift multiplier. If zero set equal to 1 (Lk709)	(dim)
K_{Lzj}	Input aerodynamic pitch fin drag due to lift factor (Lk714, 720, etc.)	(rad)
K_{Mj}	Input quantity which designates the limit of the j-th mode type end where $j = 1, 2, \ldots, 10$ (L062, 605, etc)	(dim)
K_{NOk}	Input complementary thrust-weight table weight carryover flag for the k-th stage. If K_{Ok} and K_{NOk} are nonzero, separation has occurred with regards to the complementary weight. If K_{Ok} is nonzero and and K_{NOk} is zero, the total vehicle weight at the termination of the k-1 stage is used as the initial weight of the k-th stage (Lk107)	(dim)
K_{Ok}	Input main thrust-weight table weight carryover flag for the k-th stage. If zero, separation has occurred with regards to the main and complementary weights. If nonzero, the main weight at the termination of the k-1 stage is used as the initial main weight of the k-th stage (Lk012)	(dim)
K_p	Pressure rate gain used in pintle area control low in the TMC (L5451)	(sec)
K_{cf}	Input aerodynamic pitch fin deflection angle multiplier (Lk705)	(dim)
K_Q	Input aerodynamic pitch damping moment due to pitch rate multiplier (Lk301)	(dim)
K_{RC}	Input roll control system flag. If equal 1, an auxiliary roll thruster system is simulated. If equal 2, aerodynamic central fins are used (Lk405)	(dim)
K_{RP}	Instantaneous control system pitch rate gain (L5467)	(sec)

k, K

Symbol	Definition	Units
K_{RPj}	Input control system pitch attitude rate gain for j-th control region, j = 1, 2, or 3 (Lk452, 461, etc)	(sec)
K_{RR}	Instantaneous control system roll rate gain (L5469)	(sec)
K_{RRi}	Input roll control system attitude rate gain for the i-th zone, i = 1, 2, or 3 (Lk645, 655, 665)	(sec)
K_{RY}	Instantaneous control system yaw rate gain (L5468)	(sec)
K_{RYj}	Input control system yaw attitude rate gain for the j-th control region, j = 1, 2, or 3 (Lk453, 462, etc.)	(sec)
K_S	Pressure error gain used in pintle area control low in the TMC (L5450)	(in.^{4}/lb-sec)
K_S	Input stage print control function. A nonzero value is required to print the trajectory at the termination of each stage during the hunting procedure (L0074)	(dim)
K_{sh}	Input shaper control flag where: if it equals zero, ignore routine 1. maximize range, 2. maximize payload to a given range, or 3. determine payload to a circular orbit (L0598)	(dim)
K'_{tc}	Input complementary switching time multiplier. If zero, the program assumes a value of 1 (Lk102)	(dim)
K_{tj}	Input value when a trajectory printout is desired where j = 1, 2, ..., 8 (L0237, 239, etc)	(dbi)
K'_{tM}	Input main switching time multipliers. If zero, the program assumes a value of 1. If σ_s is designated at t_B, then K_3 is multiplied by K'_{tm}. (Lk009)	(dbi)
K_{tM}	Output for the TVC duty cycle stage, the main switching time multiplier	(dim)
K_{TPF}	Input titled print flag (L0683)	(dim)

k, K

Symbol	Definition	Units
K_{Tto}	Input quantity which designates the start of target maneuvering (L0631)	(dbi)
K_v	Input wind speed multiplier (L0408)	(dbi)
K'_{WC}	Input complementary weight flow multiplier. If input zero is set to 1.0 (Lk101)	(dim)
K_{WM}	Input mainweight flow multiplier. If input zero is set to 1.0 (Lk008)	(dim)
K_{XXX}	Commanded thrust system gain. Set equal to K_{CPR} if Fy=3 and $K_{A\,LOS}$ if Fy=6 (L5454)	(dim)
K'_{ycf}	Input aerodynamic yaw fin deflection angle multiplier (Lk704)	(dim)
K_{yk}	Input gain constant in the nontarget dependent yaw steering equation for the k-th stage (Lk401)	(dim)
K_Z	Altitude error gain used in type 8 flight (L5456)	(deg/ft)
$K_{\dot{Z}}$	Altitude rate gain used in type 8 flight (L5457)	(deg-sec/ft)
$K_{\ddot{Z}}$	Altitude acceleration gain used in type 8 flight (L5458)	(deg-sec^2/ft)
$K_{\dot{\alpha}}$	Input aerodynamic pitch damping moment due to time rate change of angle of attack multiplier (Lk303)	(dim)
K_γ	Input glide phase termination control function. A value of plus 1 will specify impact after apogee, while a minus 1 will specify impact before apogee (L0040)	(dim)
K_δ	Input thrust control flag. If zero, control thrust is determined from instantaneous vehicle thrust. If 1, the control thrust is obtained from the instantaneous main stage thrust, if 2 control thrust is nonexistent (Lk434)	(dim)

k, K

Symbol	Definition	Units
K'_{Δ}	Input side impulse multiplier. If input zero is set to 1 (Lk387)	(dim)
$K_{\ddot{\theta}}$	Command attitude pitch attitude angular acceleration gain used in type of flight 8 and 9 (L5459)	(sec^2)
K_{ψ}	Input wind azimuth multiplier (L0409)	(dim)
$K_{\ddot{\psi}}$	Command attitude, yaw attitude angular acceleration gain used in type of flight 8 and 9 (L5460)	(sec^2)

l, L

Symbol	Definition	Units
ℓ_{bz}	Input pitch fin base root length (Lk703)	(ft)
ℓ_{cp}	Vehicle center-of-gravity to aerodynamic center-of-pressure distance (L5596)	(ft)
ℓ_{e}	Gimbal point to vehicle center-of-gravity distance (L5593)	(ft)
ℓ_{E}	Nozzle exit to center-of-gravity distance (L5594)	(ft)
ℓ_{hz}	Pitch movable control fin center-of-pressure to hinge axis lever arm (L5602)	(ft)
ℓ_{r}	Missile travel distance on the rail launcher used in ground launch tape of flight (Ty=6) (L5113)	(ft)
ℓ_{n}	Movable portion of the nozzle center-of-gravity to gimbal point distance (L5595)	(ft)
ℓ_{N}	Vehicle center-of-gravity to stop-start motor thrust point distance (L5597)	(ft)
ℓ_{Pa}	Aft end of propellant grain to center-of-gravity distance (L5599)	(ft)
ℓ_{Pf}	Forward end of propellant grain to center-of-gravity distance (L5598)	(ft)
ℓ_{Ri}	Input thruster roll control lever arm for the i-th center zone (Lk638, 648, 658)	(ft)
$\ell_{\delta R}$	Input roll fin radial center-of-pressure to missile centerline distance (Lk404)	(ft)
$\ell_{\delta y}$	Yaw movable control fin center-of-pressure to vehicle center-of-gravity lever arm (L5600)	(ft)
$\ell_{\delta z}$	Pitch movable control fin center-of-pressure to vehicle center-of-gravity lever arm (L5601)	(ft)
L_D	Drag velocity loss from stage ignition (L5746)	(ft/sec)
$L_{D(B1)}$	Drag velocity loss for stage I (L5907)	(ft/sec)

l, L

Symbol	Definition	Units
$L_{D(B2)}$	Drag velocity loss for stage II (L5932)	(ft/sec)
$L_{D(B3)}$	Drag velocity loss due to back pressure for stage III (L5957)	(ft/sec)
$L_{D(B4)}$	Drag velocity loss for stage IV (L5982)	(ft/sec)
L_F	Output total velocity loss due to back pressure from stage initiation to the time being printed (L5743)	(ft/sec)
$L_{F(B1)}$	Total thrust velocity loss due to back pressure for stage I (L5906)	(ft/sec)
$L_{F(B2)}$	Total thrust velocity loss due to back pressure for stage II (L5931)	(ft/sec)
$L_{F(B3)}$	Total thrust velocity loss due to back pressure for stage III (L5956)	(ft/sec)
$L_{F(B4)}$	Total thrust velocity loss due to back pressure for stage IV (L5981)	(ft/sec)
L_g	Gravity losses, from trajectory initiation to the time being printed (L5748)	(ft/sec)
$L_{g(B1)}$	Gravity velocity loss for stage I (L5908)	(ft/sec)
$L_{g(B2)}$	Gravity velocity loss for stage II (L5933)	(ft/sec)
$L_{g(B3)}$	Gravity velocity loss for stage III (L5918)	(ft/sec)
$L_{g(B4)}$	Gravity velocity loss for stage IV (L5983)	(ft/sec)
L_j	Input control system where j = 1, 2, ..., 10 per stage. If zero, a limit is not applied; otherwise, L_j limits the following parameters where the statement number is j (1) $K_{DP}\ \Delta\theta_b$, (2) δ_{Pc}, (3) δ_P, (4) $\dot{\delta}_P$ and (5) $\ddot{\delta}_P$, (6) $K_{DY}\ \Delta\psi_b$, (7) δ_{YC}, (9) $\dot{\delta}_Y$ (10) $\ddot{\delta}_Y$ (Lk440-449)	(deg)

l, L

Symbol	Definition	Units
L_{RCi}	Input maximum roll control thrust for the i-th zone, i = 1, 2, or 3 (Lk642, 652, 662)	(lb)
L_V	Output ideal velocity vectoring losses (L5118)	(ft/sec)
$L_{V(B1)}$	Vectoring velocity loss for stage I (L5909)	(ft/sec)
$L_{V(B2)}$	Vectoring velocity loss for stage II (L5934)	(ft/sec)
$L_{V(B3)}$	Vectoring velocity loss for stage III (L5959)	(ft/sec)
$L_{V(B4)}$	Vectoring velocity loss for stage IV (L5984)	(ft/sec)

m, M

Symbol	Definition	Units
m	Instantaneous missile mass (L5092)	(lb-sec^2/ft)
M	Missile Mach number (L5063)	(dim)
M_{Aj}	Input Mach number for aerodynamic axial representation where j = 1, 2, ..., 15 per stage (Lk188, 190, etc)	(dim)
M_{CP}	Controlling moment about vehicle center-of-gravity in roll (L5210)	(ft-lb)
M_{CQ}	Controlling moment about vehicle center-of-gravity in pitch (L5208)	(ft-lb)
M_{CR}	Controlling moment about vehicle center-of-gravity in yaw (L5209)	(ft-lb)
M_{CYG}	Aerodynamic axial force center-of-gravity offset yawing moment (L5229)	(ft-lb)
M_{CZG}	Aerodynamic axial force center-of-gravity offset pitching moment (L5230)	(ft-lb)
M_{Dj}	Input Mach number for aerodynamic representation where j = 1, 2, ..., 15 per stage (Lk305, 308, etc)	(dim)
M_{DP}	Perturbing moment about vehicle center-of-gravity in roll (L5207)	(ft-lb)
M_{DQ}	Perturbing moment about vehicle center-of-gravity in pitch (L5205)	(ft-lb)
M_{DR}	Perturbing moment about vehicle center-of-gravity in yaw (L5206)	(ft-lb)
M_{FCP}	Auxiliary roll thrust control moment (L5225)	(ft-lb)
M_{FCQ}	Thrust vector control pitching moment (L5223)	(ft-lb)
M_{FCR}	Thrust vector control yawing moment (L5224)	(ft-lb)

n, M

Symbol	Definition	Units
MF_j	Input Mach number for aerodynamic fin representation where j = 1, 2, . . . , 15 per stage (Lk710, 716, etc)	(dim)
M_{FOP}	Thrust offset rolling moment due to pitch and yaw TVC (L5216)	(ft-lb)
M_{FOQ}	Thrust offset pitching moment (L5214)	(ft-lb)
M_{FOR}	Thrust offset yawing moment (L5215)	(ft-lb)
M_{FVP}	Rolling moment about vehicle center-of-gravity due to vortexing effect of axial gas flow through the nozzle (L5219)	(ft-lb)
M_{Hy}	Torque about the yaw fin hinge axis (L5247)	(ft-lb)
M_{Hz}	Torque about the pitch fin hinge axis (L5248)	(ft-lb)
MIN_j	Input minimum velocity or constant Mach number of the j-th type of TMC (L0806, 816, etc)	(dim or ft/sec)
M_{JDQ}	Jet damping pitching moment (L5220)	(ft-lb)
M_{JDR}	Jet damping yawing moment (L5221)	(ft-lb)
M_{IP}	Unbalanced roll moment about vehicle center-of-gravity (L5204)	(ft-lb)
M_{IQ}	Unbalanced pitching moment about vehicle center-of-gravity (L5202)	(ft-lb)
M_{IR}	Unbalanced yaw moment about vehicle center-of-gravity (L5203)	(ft-lb)
M_{hymax}	Output maximum of the absolute value yaw fin hinge torque for the TVC design stage	(ft-lb)
M_{hzmax}	Output maximum of the absolute value pitch fin hinge torque for the TVC design stage	(ft-lb)

m, M

Symbol	Definition	Units
M_{NDQ}	Aerodynamic damping moment about vehicle center-of-gravity in pitch (L5235)	(ft-lb)
M_{NDR}	Aerodynamic damping moment about vehiche center-of-gravity in yaw (L5236)	(ft-lb)
M_{Nj}	Input Mach number for aerodynamic normal force coefficient representation where j = 1, 2, . . . , 15 per stage (Lk223, 228, etc)	(dim)
M_{NP}	Aerodynamic rolling moment about vehicle center-of-gravity (L5213)	(ft-lb)
M_{NQ}	Aerodynamic yawing moment about vehicle center-of-gravity (L5211)	(ft-lb)
M_{NR}	Aerodynamic yawing moment about vehicle center-of-gravity (L5212)	(ft-lb)
M_{NSQ}	Aerodynamic static pitching moment about vehicle center-of-gravity (L5332)	(ft-lb)
M_{NSR}	Aerodyanmic static yawing moment about vehicle center-of-gravity (L5233)	(ft-lb)
M_{PAC}	Pitch aerodynamic control moment per radian angle of attack (L5241)	(ft-lb)
M_{PAD}	Pitch aerodynamic disturbing moment per radian angle of attack (L5244)	(ft-lb)
M_{PCD}	Total pitch control moment per radian deflection angle (L5251)	(ft-lb)
M_{PDA}	Total pitch disturbing moment per radian angle of attack (L5250)	(ft-lb)
M_{PMC}	Pitch main thrust control moment per radial TVC deflection angle (L5234)	(ft-lb)
M_{PRR}	Pitch inertial rotation reaction moment used in the automatic gain logic (L5235)	(ft-lb)

m, M

Symbol	Definition	Units
M_{PTC}	Pitch total thrust control moment per radian TVC deflection angle (L5231)	(ft-lb)
$\overline{M}_Q$	Input aerodynamic pitch damping moment due to pitch rate multiplier (Lk299)	(dim)
$M_{q\alpha}$	Output Mach number at maximum $q\alpha$ during the TVC duty cycle stage	(dim)
M_{RAC}	Roll aerodynamic control moment per radian fin deflection angle (L5243)	(ft-lb)
M_{RAP}	Aerodynamic rolling moment induced by raceways (L5222)	(ft-lb)
M'_{Rj}	Input Mach number for aerodynamic rolling moment, j = 1, 2, ..., 10 (Lk673, 675, etc)	(dim)
M_{RRR}	Roll Rotation Reaction moment used in the automatic gain logic (L5240)	(ft-lb)
M_{TDQ}	Movable nozzle tail-wag-dog moment about vehicle center-of-gravity in pitch (L5217)	(ft-lb)
M_{TDR}	Movable nozzle tail-wag-dog moment about vehicle center-of-gravity in yaw (L5218)	(ft-lb)
M_y	Mode type	(dim)
M'_y	Input initial or restart type of mode control flag (L0007)	(dim)
M_{YAC}	Yaw aerodynamic control moment per radian angle of side slip (L5242)	(ft-lb)
M_{YAD}	Yaw aerodynamic disturbing moment per radian angle of side slip (L5245)	(ft-lb)
M_{yj}	Input and output mode type control where j = 1, 2, ..., 10. If 1 rigid body with controls, and 2 rigid body with controls (L0600, 603, etc)	(dim)

m, M

Symbol	Definition	Units
M_{YRR}	Yaw inertial rotation reaction moment used in the automatic altitude gain logic (L5239)	(ft-lb)
$\overline{M}_{\dot{\alpha}}$	Input aerodynamic pitch damping moment due to rate change of angle of attack multiplier (Lk300)	(dim)
$M_{\delta P}$	Rolling moment due to the aerodynamic control force (L5228)	(ft-lb)
$M_{\delta Q}$	Pitching moment due to the aerodynamic control force (L5226)	(ft-lb)
$M_{\delta R}$	Yawing moment due to the aerodynamic control force (L5227)	(ft-lb)

n, N

Symbol	Definition	Units
n_{dc}	Output of number TVC duty cycle t_B data points	(dim)
n_g	Output number of velocity data points used in calculating the coefficients for the pitch steering equations	(dim)
n_i	Output iteration number of the hunting procedure	(dim)
n	Burning rate exponent used in internal ballistic evaluation is set to 0.6 if not input (Lk097)	(dim)
n'_c	Input and output internally calculated number of control nozzles for the cluster motor logic (Lk381)	(dim)
n'_m	Input and output internally calculated number of motors in the stage cluster (Lk381)	(dim)
n_{t1}	Input trajectory number limit. No more than n trajectories will be computed during the hunting procedure (P1) by varying X (L0075)	(dim)
n_{t2}	Input specified maximum number of hunt predictions (P2) beyond the initial array. If n_{t2} is input a negative number, the hunt will restart after $\lvert n_{t2} \rvert$ iterations (L0090)	(dim)
$\tilde{N}_k$	Input normal force control function and normal force multiplier. If input zero, the multiplier is set to 1 and if nonzero, the normal force is determined from input and multiplied by N_k where k = 1, 2, 3, 4 (Lk221)	(dim)
N_{NVA}	Force normal to velocity vector per radian angle of attack (L5161)	(lb)
N_{PAC}	Pitch aerodynamic control normal force per radian fin deflection angle (L5158)	(lb)

n, N

Symbol	Definition	Units
N_{PAD}	Pitch aerodynamic disturbing normal force per radian angle of attach (L5159)	(lb)
N_{PCD}	Total pitch control normal force per radian deflection angle (L5162)	(lb)
N_{PDA}	Total pitch disturbing normal force per radian angle of attack (L5160)	(lb)
N_{PEA}	Pitch trim normal force per radian angle of attack (L5163)	(lb)
N_{PY}	Aerodynamic force due to damping in yaw (L5167)	(lb)
N_{PZ}	Aerodynamic force due to damping and pitch (L5168)	(lb)
	Instantaneous yaw aerodynamic axial normal forces directed opposite to the direction of the Y_b axis (L5165)	(lb)
N_Z	Instantaneous pitch aerodynamic normal forces directed opposite to the direction of the Z_b axes (L5166)	(lb)
$N_{\delta y}$	Aerodynamic yaw fin normal force (L5156)	(lb)
$N_{\delta z}$	Aerodynamic pitch fin normal force (L5157)	(lb)

p, P

Symbol	Definition	Units
P	Glide phase orbital period (L5014)	(min)
P_a	Ambient pressure at the missile (L5068)	(lb/ft^2)
P^*	Main motor nozzle critical pressure used in separated flow equations (L5072)	(lb/in^2)
P_{arC}	Input complementary table thrust reference atmospheric pressure (Lk109)	(lb/ft^2)
P_{aRM}	Input main table thrust reference atmospheric pressure (Lk020)	(lb/ft^2)
P_b	Instantaneous vehicle angular roll velocity, roll clockwise is positive (L5709)	(deg/sec)
$\dot{P}_b$	Instantaneous vehicle angular roll acceleration, roll clockwise positive (L5759)	(deg/sec^2)
P_{bo}	Input initial vehicle roll rate (L0033)	(deg/sec)
P_c	Main motor chamber pressure used in separated flow equation (L5734)	$(lb/in.^2)$
$\dot{P}_C$	Time rate change of chamber pressure (L5784)	$(lb/in.^2-sec)$
P_{ca}	Output action time average motor chamber pressure for the TVC design duty cycle stage	$(lb/in.^2)$
P_{cc}	Commanded chamber pressure used in pintle motor control logic (L5071)	$(lb/in.^2)$
P_e	Main motor exit pressure used in separated flow equations (L5074)	(lb/ft^2)
PL-DA GB, K, KB, N, O	Input flag where nonzero values are required if printlines DA, GB, K, KB, N, O are desired (L0200-209)	(dim)
P_d	Base pressure	$(lb/in.^2)$
P_m	Instantaneous desired roll turning rate (L5403)	(deg/sec)

p, P

Symbol	Definition	Units
P_{max}	Maximum allowable chamber pressure used in internal ballistic evaluation (Lk099)	(lb/in.^{2})
P_{Mj}	Input vehicle roll turning rate positive if the vehicle is intended to roll clockwise looking at if from the aft end (Ty=1) (L0315, 322, etc)	(deg/sec)
P_s	Main motor nozzle separation pressured used in separated flow equations (L5073)	(lb/in.^{2})
P_1	Input flag to specify hunt procedure (P1) (L0073)	(dim)
P_2	Input flag to specify hunt procedure (P2). If $P_2 = 0$, by-pass hunt procedure 2; $P_2 = 1$, use a linear response; $\lvert P_2 \rvert = 2$, use a quadratic response surface where +2 maximizes and -2 minimizes; $\lvert P_2 \rvert = 3$, use an incomplete quadratic response surface where +3 maximizes and -3 minimizes. (L0084)	(dim)

q, Q

Symbol	Definition	Units
$q_q\alpha'$	Output dynamic pressure at maximum qα' during the TVC duty cycle stage	(lb/ft)
$q\alpha'_{max}$	Output product of the maximum absolute value of the dynamic pressure-angle of attack for the TVC design stage	(lb-deg/ft^2)
q	Missile dynamic pressure (L5070)	(lb/ft^2)
q_{maxj}	Input maximum allowable dynamic pressure of the j-th type of TMC (L807, 817, etc)	(lb/ft^2)
q_{min}	Input minimum allowable dynamic pressure of the j-th type of TMC (L0808, 818, etc)	(lb/ft^2)
$q\alpha'$	Product of total angle of attack and dynamic pressure (L5110)	(lb/deg/ft^2)
$q_{(B1)}$	Missile dynamic pressure at the termination of stage I (L5912)	(lb/ft^2)
$q_{(B2)}$	Missile dynamic pressure at the termination of stage II (L5937,	(lb/ft^2)
$q_{(B3)}$	Missile dynamic pressure at the termination of stage III (L5962,	(lb/ft^2)
$q_{(B4)}$	Missile dynamic pressure at the termination of stage IV (L5987)	(lb/ft^2)
Q_b	Instantaneous vehicle angular pitch velocity. Pitch up is positive (L5707)	(deg/sec)
$\dot{Q}_b$	Instantaneous vehicle angular pitch acceleration. Pitch up positive (L5757)	(deg/sec^2)
Q_{bo}	Input initial vehicle pitch rate (L0031)	(deg/sec)
Q_m	Instantaneous desired pitch turning rate (L5401)	(deg/sec)

q, Q

Symbol	Definition	Units
Q_{mj}	Input vehicle pitch turning rate. Positive if the vehicle is intended to pitch up (Ty=1) (L0313, 320, etc)	(deg/sec)
Q_{mo}	Input initial command pitch attitude (L0028)	(deg)

r, R

Symbol	Definition	Units
r_a	Distance between the center of the reference body and apogee during the glide phase	(ft)
r_b	Propellant burn rate (L5783)	(m/sec)
r'_{b1000}	Input burning rate of propellant at 1,000 lb/in^2 chamber pressure and flag to determine evaluation option (Lk002)	(in./sec)
r_c	Instantaneous distance between the center of the reference body and the missile (L5035)	(ft)
r'_c	Value of r_c at the beginning of the glide phase	(ft)
r_{co}	Initial missile radius from center of earth	(ft)
r_e	Radius of the earth	(ft)
r'_e	Input geometric radius of the earth. If input zero, set equal to 20,926,400 (L0026)	(ft)
r_f	Radial distance at the termination of the glide phase	(ft)
r_p	Distance between the center of the reference body and perigee during the glide phase	(ft)
r_{Ri}	Input distance from the vehicle center-line to the i-th raceway center of pressure, i = 1, 2 (Lk668, 671)	(in)
R	Altitude associated geopotential earth radius used in atmosphere model	(ft)
R^*	Universal gas constant	(joules/ kilogram ^{o}K)
R_b	Instantaneous vehicle angular yaw velocity. Yaw right is positive (L5708)	(deg/sec)

r, R

Symbol	Definition	Units
$\dot{R}_b$	Instantaneous vehicle angular yaw acceleration. Yaw right positive (L5758)	(deg/sec^2)
R_{bo}	Input initial vehicle yaw rate (L0032)	(deg/sec)
R_c	Input radius of cluster for the k-th stage (Lk382)	(ft)
R_m	Instantaneous desired yaw turning rate (L5402)	(deg/sec)
R_{MI}	Estimated range to target intercept (L5400)	(ft)
R_{mj}	Input vehicle yaw turning rate positive if the vehicle is intended to turn right (Ty=1) (L0314, 321, etc)	(deg/sec)
R_{MT}	Missile to target range distance (L5437)	(ft)
$\dot{R}_{MT}$	Time rate change of missile to target distance (L5438)	(ft/sec)
R_{PFV}	Output ratio of motor chamber pressure to vacuum thrust of the main thrust table of the TVC design stage	(1/in.^{2})
RR	Input reference run number (L0001)	(dim)
RUN	Input run number (L0002)	(dim)

s, S

Symbol	Definition	Units
S	Missile ground range. Distance along the surface of the earth measured clockwise from the launch vertical to the local vertical down range (L5018)	(ft)
$\dot{S}$	Time rate change of missile down range (L5019)	(ft/sec)
S_a	Total missile ground range at flight apogee (L5022)	(nm)
S_c	Missile cross ground range. Distance along the surface of the earth measured clockwise from launch azimuth (L5028)	(ft)
$\dot{S}_c$	Time rate change of missile cross range (L5021)	(ft/sec)
S_{CMI}	Estimated earth surface cross range at target intercept (L5443)	(ft)
S_{co}	Input initial earth surface cross-range at trajectory start time (L0043)	(nm)
S_E	Total missile ground range to atmospheric entry (L5023)	(nm)
S_f	Total missile ground range at the termination of the glide phase (L5024)	(nm)
S_{Fz}	Input aerodynamic pitch fin lift and drag coefficient reference area (Lk700)	(ft^2)
$S_{f(B1)}$	Total missile ground range at the termination of the glide phase if the powered flight were to end at the termination of stage I (L5905)	(nm)
$S_{f(B2)}$	Total missile ground range at the termination of the glide phase of the powered flight were to end at the termination of stage II (L5930)	(nm)

s, S

Symbol	Definition	Units
$S_{f(B3)}$	Total missile ground range at the termination of the glide phase if the powered flight were to end at the termination of stage III (L5955)	(nm)
$S_{f(B4)}$	Total missile ground range at the termination of the glide phase if the powered flight were to end at the termination of stage IV (L5980)	(nm)
S_{MI}	Estimated earth surface down range at target intercept (L5441)	(ft)
S_{PF}	Input missile platform area used in calculating tumbling aerodynamic axial force coefficients (Lk218)	(ft^2)
S_{RC}	Input aerodynamic chord force coefficient reference areas (Lk165)	(ft^2)
S_{RN}	Input aerodynamic normal force coefficient reference area (Lk220)	(ft^2)
S_{RRi}	Input reference area of the i-th raceway, i = 1, 2 (Lk667, 670)	(ft^2)
S_s	Missile slant ground range. Distance along earth surface from the launch vertical to the local vertical slantwise (L5025)	(ft)
S_{sh}	Input target range or orbital altitude (L0599)	(nm)
SST_2	Start time stage II (L5006)	(sec)
SST_3	Start time stage III (L5007)	(sec)
SST_4	Start time stage IV (L5008)	(sec)
S_T	Target down range (L5731)	(ft)
S_{TC}	Target cross range (L5732)	(ft)
S_{TCO}	Input initial target position cross range (L0637)	(nm)

s, S

Symbol	Definition	Units
S_{TO}	Input initial target position down range (L0636)	(nm)
S_y	Input initial or restart special print flag (L0008)	(dim)
S_o	Input missile ground range at the trajectory start time (L0013)	(ft)
$\dot{S}_T$	Target down range rate (L5781)	(ft/sec)
$\dot{S}_{TC}$	Target cross range rate (L5782)	(ft/sec)

t, T

Symbol	Definition	Units
t	Instantaneous time (L5000)	(sec)
t_a	Total flight time to the glide phase apogee altitude (L5012)	(sec)
t_{Ba}	Output TVC duty cycle stage time	(sec)
t_B	Time from the current stage initiation (L5001)	(sec)
t_{B2}	Time from stage II initiation zero if I-II staging has not occurred (L5002)	(sec)
t_{B3}	Time from stage III initiation zero if II-III staging has not occurred (L5003)	(sec)
t_{B4}	Time from stage IV initiation zero if III-IV staging has not occurred (L5004)	(sec)
t_{Bf}	Trajectory burnout time for the stage below (L5009)	(sec)
t_{Bqj}	Output stage time which TVC duty points occur	(sec)
t_{cj}	Input limit of the computing interval Δt_{Cj} where j = 1, 2, ..., 8 (L0169, 171, etc)	(sec)
t'_{Cj}	Input complementary thrust-weight switching time from stage initiation where j = 1, 2, 3, ..., 25 per stage (Lk111, 114, etc)	(sec)
t_E	Total flight time to atmospheric entry (L5013)	(sec)
t_f	Total flight time to the termination of the glide phase (L5014)	(sec)
$t_{f(B1)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage I (L5916)	(sec)

t, T

Symbol	Definition	Units
$t_{f(B2)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage II (L5941)	(sec)
$t_{f(B3)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage III (L5966)	(sec)
$t_{f(B4)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage IV (L5991)	(sec)
t_k	Stage start time (L5005)	(sec)
t'_k	Input start time with the initial stage K_k (L0005)	(sec)
t_{MI}	Estimated time to intercept (L5439)	(ft)
t'_{Mj}	Input main thrust-weight switching time from stage initiation where j = 2, 3, ..., or 25 per stage (Lk023, 026, etc)	(sec)
t_{pj}	Input limit of the main printline print interval where j = 1, 2, ..., 8 (L0185, 187, etc)	(sec)
t_{Pj}	Input limit of the auxiliary printline print interval where j = 1, 2, ..., 11 (L0221, 223, etc)	(sec)
$t_{q\alpha'}$	Output TVC duty cycle stage time at maximum $q\alpha'$	(sec)
t'_{Ri}	Input operating time from stage initiation of the roll control function for the i-th zone, i = 2, 3 (Lk647, 667)	(sec)
t_T	Time from target maneuvering initiation (L5010)	(sec)
t_{Tj}	Input target time terminating the j-th target acceleration value of dynamical condition table (L0640, 644, etc)	(sec)

t, T

Symbol	Definition	Units
t_{TS}	Target start time (L5011)	(sec)
t_{VPj}	Input time value for specific velocity - time profile used in Fy=1 TMC j = 1, 2, ..., 15 (L0870, 872, etc)	(sec)
$t_{\eta P}$	Output stage time at which maximum magnitude pitch thrust vector deflection angle occurs during the TVC design stage (δ_{Pmax})	(sec)
t_o	Input trajectory start time (L0004)	(sec)
T_{Bj}	Input staging values flag, j = 1, 2, ..., 5 (L0270, 272, etc)	(dim)
T_{mj}	Input maximum print region flag (L0260, 262, etc)	(dim)
T_{Tj}	Input transformation flag used in hunting procedure (P2) j = 1, 2, ..., 7 (L0098, 107, etc)	(dim)
TMC_j	Input thrust dynamic mode of the j-th type TMC (L0803, 813, etc)	(dim)
T_M	Instantaneous molecular scale temperature	(^oK)
T_{MB}, T_{MBj}	Base molecular scale temperature and tabular values associated with the atmosphere representation	(^oK)
T_{Tj}	Input transformation flag used in hunting procedure (P2) j = 1, 2, ..., 7 (L0098, 107, ..., 152)	(dim)
T_y	Type of flight	(dim)
T'_{yk}	Input initial or restart type of flight control flag (L0006)	(dim)
T_{yj}	Input and output type of flight control flag where j = 1, 2, ..., or 16 (L0310, 317 etc)	(dim)

u, U

Symbol	Definition	Units
u_j	Elements (j = 1, 2, ..., 5) of the vector containing the numeric value of the independent variables used in the generation of steering coefficients	(dbi)
$[U_1]$	Matrix containing specified elements u_j for use in the steering equation coefficient acquisition	(dbi)
$[U_2]$	A vector containing specified elements u_j for use in the steering equation coefficient acquisition	(dbi)
U_{cz}	Output pitch aerodynamic control fin center of pressure as a ratio of fin chord length (L5568)	(dim)
U_{czj}	Input pitch aerodynamic control fin center of pressure as a ratio of fin chord length (Lk715, Lk721)	(dim)
$[U_g]$	A vector which contains a scaled version of the steering equation steering coefficients	(dim)
U_{gi}	Elements of the $[U_g]$ vector which are the scaled version of the steering equation steering coefficients i = 1, 2, 3, or 4	(dim)
U_{go}	Mean of the regressed least squares fit for use in the steering equation coefficient acquisition	(dim)
U_{hz}	Input pitch movable control fin hinge axis to the leading fin base root location distance to the pitch fin base root length ratio (Lk702)	(dim)

v, V

Symbol	Definition	Units
v_{TO}	Input target initial velocity at start of start of target maneuvering (L0632)	(ft/sec)
v_w	Instantaneous wind speed change (L5056)	(ft/sec)
$\dot{v}_w$	Instantaneous wind speed time rate change (L5057)	(ft/sec)
v_{wj}	Input (with altitude h) wind speeds where $j = 1, 2, \ldots, 30$ (L0411, 414, etc)	(ft/sec)
V_a	Missile velocity with respect to air (L5049)	(ft/sec)
V_{aE}	Velocity with respect to the ambient air at entry (L5058)	(ft/sec)
V_C	Target transverse velocity (L5727)	(ft/sec)
V_{cI}	Chamber volume (L5078)	$(in.^3)$
V_e	Missile earth referenced velocity (L5046)	(ft/sec)
$\dot{V}_e$	Time rate change of missile earth reference velocity (L5047)	(ft/sec^2)
$\dot{V}_{ecq}$	Command acceleration to constrain dynamic pressure used in the TMC command logic (L5062)	(ft/sec^2)
V_{ek}	Earth fixed velocity at stages (L5048)	(ft/sec)
V_{eo}	Input missile velocity at the trajectory start time (L0010)	(ft/sec)
V_I	Missile intertial velocity (L5050)	(ft/sec)
V_{Ia}	Missile inertial velocity at apogee if powered flight ends at the time being printed (L5041)	(ft/sec)
$V_{I(B1)}$	Missile inertial velocity at the termination of stage I (L5901)	(ft/sec)
$V_{I(B2)}$	Missile inertial velocity at the termination of stage II (L5926.	(ft/sec)

v, V

Symbol	Definition	Units
$V_{I(B3)}$	Missile inertial velocity at the termination of stage III (L5951)	(ft/sec)
$V_{I(B4)}$	Missile inertial velocity at the termination of stage IV (L5976)	(ft/sec)
V_{IE}	Inertial velocity at entry conditions (L5059)	(ft/sec)
V_{If}	Missile inertial velocity at apogee and impact of intercept, respectively, if powered flight and the atmosphere end at the time being printed (L5042)	(ft/sec)
$V_{If(B1)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage I (L5918)	(ft/sec)
$V_{If(B2)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage II (L5945)	(ft/sec)
$V_{If(B3)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage III (L5968)	(ft/sec)
$V_{If(B4)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage IV (L5993)	(ft/sec)
V_N	Target normal velocity (L5726)	(ft/sec)
V_T	Target tangential velocity (L5725)	(ft/sec)
V_{VPj}	Input earth reference velocity for the specific velocity - time profile used in Fy=1, TMC j = 1, 2, ..., 15 (L0871, 873, etc)	(ft/sec)
V_{xxx}	Command velocity used in the TMC command logic. V_{ecv} if Fy = 1, V_{ecm} if Fy = 4 (L5060)	(ft/sec^2)
$\dot{V}_{xxx}$	Command acceleration used in the TMC command logic. $\dot{V}_{ecv}$ if Fy = 1, $\dot{V}_{ecm}$ if Fy = 2, and zero if Fy = 1 and Fy = 2 (L5061)	(ft/sec^2)

v, V

Symbol	Definition	Units
$\dot{V}_{Cj}$	Target transverse acceleration (L5777)	(ft/sec)
$\dot{V}_{Nj}$	Target normal acceleration (L5776)	(ft/sec)
$\dot{V}_{Tj}$	Target tangential acceleration (L5775)	(ft/sec)

w, W

Symbol	Definition	Units
W	Total instantaneous missile weight (L5093)	(lb)
$\dot{W}$	Total expended instantaneous missile mass flow weight (L5094)	(lb/sec)
$\bar{W}$	Input stage weight multiplier. If input zero, is set internally to one and if nonzero, the input weight used in the mass properties table W_j (j = 1, 2, ..., 15) is multiplied by W (Lk478)	(dim)
$\dot{\bar{W}}$	Instantaneous total motor weight flow	(lb/sec)
W_B	Instantaneous gross vehicle weight minus the useful load (L5095)	(lb)
$W_{(B1)}$	Total missile weight at the termination of stage I (L5904)	(lb)
$W_{(B2)}$	Total instantaneous missile weight at the termination of stage II (L5929)	(lb)
$W_{(B3)}$	Total instantaneous missile weight at the termination of stage III (L5954)	(lb)
$W_{(B4)}$	Total instantaneous missile weight at the termination of stage IV (L5979)	(lb)
W_C	Total instantaneous expended complementary weight (L5099)	(lb)
$\dot{W}_C$	Total instantaneous expended complementary weight flow rate (L5100)	(lb/sec)
$\hat{\dot{W}}_C$	Total complementary weight flow at t_{Cj}	(lb/sec)
$\dot{W}_{Cj}$	Input complementary weight flow at t_{Cj} where j = 1, 2, ..., 25 per stage (Lk113, 116, etc)	(lb/sec)
W_{Cok}	Input initial complementary weight for the k-th stage (Lk105)	(lb)
W_{exi}	Input estimated weight of the TVC system expend weight during the TVC design stage during the original vehicle flight (L0678)	(lb)

w, W

Symbol	Definition	Units
W_{JT}	Total weight jettisoned (L5096)	(lb)
W_{JTj}	Input weight to be jettisoned when σ_j parameter had K_j value (L0049, 52, etc)	(lb)
W_j	Input vehicle weight to relate the input moment of inertia values j = 1, 2, ..., 15 per stage (Lk488-498)	(lb)
W_{kdc}	Output main motor stage weight for the design stage	(lb)
W_M	Total instantaneous expended main weight (L5097)	(lb)
$\dot{W}_M$	Total instantaneous expended main weight flow rate (L5098)	(lb/sec)
$\dot{W}_{Mj}$	Input main weight flow at t_{Mj} where j = 1, 2, ..., 25 per stage (Lk022, 025, etc)	(lb/sec)
$\hat{\dot{W}}_{Mj}$	Total instantaneous main weight flow at t_{Mj} where j = 1, 2, ..., or 25	(lb/sec)
W_{MO}	Input initial main weight for the k-th stage (Lk006)	(lb)
$\dot{W}_{MP}$	Mass flow rate of gases thru pintle nozzle throat (L5786)	(lb/sec)
W_n	Input stage movable portion nozzle weight (Lk482)	(lb)
W_{PL}	Input payload weight (L0020)	(lb)
W_{pr}	Weight of propellant removed. Used in TMC logic (L5736)	(lb)
$\bar{\dot{W}}_q$	Output motor weight flow ($\dot{W}$) at t_{Bq} the TVC duty cycle point	(lb/sec)
W_R	Instantaneous expended weight due to roll control motor operation (L5720)	(lb)
$\dot{W}_R$	Roll control system mass flow rate (L5770)	(lb/sec)

w, W

Symbol	Definition	Units
W_S	Stage weight (L5101)	(lb)
W_{SC}	Initial weight of complementing motor (L5103)	(lb)
W_{SM}	Initial weight of main motor (L5102)	(lb)
W_{TVC}	Input and output estimated TVC system fixed weight. Used in TVC design stage for the refly option (L0677)	(lb)
W_{ol}	Input stage I liftoff weight used in roll control requirements (L0676)	(lb)
W_o	Output TVC duty cycle stage liftoff weight used in the roll control requirements	(lb)

x, X

Symbol	Definition	Units
$\dot{X}_{abb}$	X component of missile velocity with respect to the ambient air in the b system (L5507)	(ft/sec)
$\ddot{X}_{abb}$	x component of missile acceleration with respect to ambient air in the b system (L5510)	(ft/sec^2)
$\dot{X}_{bb}$	Inertial component of missile velocity along x_b axis (L5537)	(ft/sec)
$\ddot{X}_b$	Missile acceleration along vehicle body axes, position forward (L5540)	(ft/sec^2)
X_{cc}	Earth centered missile position northern axis component (L5513)	(ft)
$\dot{X}_{cc}$	Northern component of missile velocity in earth centered coordinates (L5516)	(ft/sec)
$\ddot{X}_{cc}$	Northern component of missile acceleration in earth centered coordinates (L5519)	(ft/sec^2)
x_{cg}	Instantaneous center-of-gravity body station numbers (L5584)	(ft)
x_{cgj}	Input instantaneous (with total vehicle weight, W_j) center-of-gravity body station numbers, respectively, where j = 1, 2, ..., 15 per stage (Lk489, 499, etc)	(dbi & ft)
X_{cgq}	Output vehicle center-of-gravity at t_{Bq}; the TVC duty cycle point	(ft)
x_{cp}	Input and instantaneous (with Mach number M) aerodynamic normal force center of pressure body station numbers, respectively, where j = 1, 2, ..., 15 per stage respectively (L5583)	(ft & dbi)
x_{cpj}	Input and output instantaneous (with Mach number M) aerodynamic normal force center of pressure body station numbers, respectively, where j = 1, 2, ..., 15 per stage (Lk227, 232, etc)	(dbi & ft)
x'_e	Input stage thrust gimbal body station numbers, also output for the TVC duty cycle (Lk431)	(ft)

x, X

Symbol	Definition	Units
x'_E	Input and computed stage nozzle exit body station (Lk419)	(ft)
X_{gg}	Instantaneous component of vehicle position in the generalized coordinates down range from launcher (L5522)	(ft)
$\dot{X}_{gg}$	Instantaneous component of vehicle velocity in the generalized coordinates down range from launcher (L5525)	(ft/sec)
$\ddot{X}_{gg}$	Instantaneous component of vehicle acceleration in the generalized coordinates down range from launcher (L5528)	(ft/sec^2)
x_{hz}	Input missile body station of the pitch fin hinge axis (Lk701)	(ft)
x_{ij}	Input initial array reference independent variable designated by code input that is used in hunting procedure (P2) (L0092, 101, etc)	(dbi)
x_j	Independent variable designated by code input that is varied during hunting procedure (P2) where j = 1, 2, ..., or 7	(dbi)
x_{Lj}	Input lower limit that an independent variable may assume a value in hunt procedure (P2) where j = 1, 2, ..., 7 (L0161-167)	(dbi)
x_n	Input stage movable portion of nozzle center-of-gravity body station (Lk484)	(dbi)
x_{nf}	Input body station of nozzle flange. Used in the TVC design program (L0680)	(ft)
x_{Pa}	Computed stage aft end propellant grain body station (L5588)	(ft)
x'_{Pa}	Input stage aft end of propellant grain body station (Lk418)	(ft)

x, X

Symbol	Definition	Units
x_{Pc}	Input missile body station of the centroid of the platform area (Lk219)	(ft)
x_{pf}	Computed stage forward end of propellant grain body station (L5587)	(ft)
x'_{Pf}	Input stage forward end of propellant grain body station (Lk417)	(ft)
x'_{RQ}	Input and calculated pitch damping moment due to pitch rate reference moment point body station (Lk302)	(dbi & ft)
$x'^{\bullet}_{R\alpha}$	Input and calculated pitch damping moment due to rate change of angle of attack reference moment point body station (Lk304)	(dbi & ft)
$x_{\tilde{U}j}$	Input upper limit that an independent variable may assume a value used in hunt procedure (P2) where j = 1, 2, ..., 7 (L0154-160)	(dbi)
$\dot{X}_{wee}$	Transformed launcher northerly component of wind velocity at missile location (L5549)	(ft/sec)
$\dot{X}_{wll}$	Local northerly component of wind velocity (L5531)	(ft/sec)
$\ddot{X}_{wll}$	Local time rate change of northern component of wind velocity (L5534)	(ft/sec)
$\dot{X}_{ll}$	Local northerly component of missile velocity (L5534)	(ft/sec)
$\ddot{X}_{ll}$	Local northerly component of missile acceleration (L5546)	(ft/sec^2)
$\dot{X}_{llI}$	Local northerly component of missile inertial velocity (L5552)	(ft/sec)
X	Output independent variable designated by code input that is varied during the hunting procedure (P1)	(dbi)
X_{cco}	Initial earth centered missile northern axis component	(ft)

x, X

Symbol	Definition	Units
X_{cg}	Input stage center-of-gravity position multiplier (Lk479)	(dbi)
$\bar{X}_{cp}$	Input stage aerodynamic normal force center of pressure multiplier (Lk222)	(dbi)
$\bar{X}_{e}$	Input stage thrust gimbal position multiplier (Lk430)	(dbi)
$\dot{X}_{ee}$	Instantaneous components of vehicle velocity, north of the launcher (L5701)	(ft/sec)
X_{ee}	Instantaneous component of vehicle position north of launcher (L5704)	(ft)
$\ddot{X}_{ee}$	Instantaneous northerly component of vehicle acceleration at launcher (L5751)	(ft/sec^2)
X^*_{ee}	Northern component along launcher of missile position	(ft)
X_{eeo}	Initial component of vehicle position north of launcher	(ft)
$\dot{X}_{eeo}$	Initial component of vehicle velocity north of launcher	(ft/sec)
X_{i}	Input value of the first guess of X used in hunting procedure (L0080)	(dbi)
X_{j}	Output independent variable used in hunting procedure (P2) where j = 1, 2, ..., or 7	(dbi)
$\bar{X}_{j}$	Computed value of X during the hunting procedure (P1) where j = 1, 2, ..., 4	(dbi)

y, Y

Symbol	Definition	Units
$\dot{Y}_{abb}$	Y component of the missile velocity with respect to the ambient air in the b system (L5508)	(ft/sec)
$\ddot{Y}_{abb}$	y component of missile acceleration with respect to the ambient air in the b system (L5511)	(ft/sec)
$\dot{Y}_{bb}$	Inertial component of missile velocity along y_b axis (L5538)	(ft/sec)
$\ddot{Y}_{b}$	Inertial component of missile acceleration y_b axes (L5541)	(ft/sec)
Y_{cc}	Earth centered missile position east from launcher (L5514)	(ft)
$\dot{Y}_{cc}$	East from launcher component of missile velocity in earth centered coordinate (L5517)	(ft/sec)
$\ddot{Y}_{cc}$	East from launcher component of missile acceleration in earth centered coordinates (L5520)	(ft/sec^2)
y_{cg}	Center-of-gravity offset bias distance positive in the Z_b direction (L5585)	(ft)
y'_{cg}	Input center-of-gravity offset bias distance in yaw, positive to the right (Lk481)	(dbi)

y, Y

Symbol	Definition	Units
y_{cgj}	Input instantaneous (with total vehicle weight, W) center of gravity butt line number, where j = 1, 2, ..., 15 per stage (Lk493, 503, etc)	(dbi and ft)
y_c	Thrust gimbal yaw point eccentricity position in the Y_b axis direction (L5589)	(ft)
y'_c	Input stage thrust gimbal yaw eccentricities. Positive in the Y_b axis direction (Lk432)	(ft)
Y_{gg}	Instantaneous component of vehicle position in the generalized coordinates cross range from launcher (L5523)	(ft)
$\dot{Y}_{gg}$	Instantaneous component of vehicle velocity in the generalized coordinates crosswise from launcher (L5526)	(ft/sec)
$\ddot{Y}_{gg}$	Instantaneous component of vehicle acceleration in the generalized coordinates crosswise from launcher (L5529)	(ft/sec^2)
y_i	Dependent variable used in hunting procedure (P2) where j = 1, 2, ..., or 7	(dbi)
y_{Li}	Input desired dependent variable or lower constraint boundary of the dependent variable designated by code input that is used in hunting procedure (P2) where j = 1, 2, ..., 7 (L0095, 104, etc)	(dbi)

y, Y

Symbol	Definition	Units
y_{Ui}	Input upper constraint boundary of the dependent variable designated by code input that is used in hunting procedure (P2) where j - 1, 2, ..., 7 (L0096, 105, etc)	(dbi)
$\dot{Y}_{wee}$	Transformed launcher easterly component of wind velocity at missile location (L5550)	(ft/sec)
$\dot{Y}_{wll}$	Local easterly component of wind velocity (L5532)	(ft/sec)
$\ddot{Y}_{wll}$	Local time rate change of easterly component of wind velocity (L5535)	(ft/sec^2)
$\dot{Y}_{ll}$	Local easterly component of missile velocity (L5544)	(ft/sec)
$\ddot{Y}_{ll}$	Local easterly component of missile acceleration (L5547)	(ft/sec^2)
$\dot{Y}_{lil}$	Local easterly component of missile inertial velocity (L5553)	(ft/sec)
Y_{cco}	Initial earth centered missile position east from launcher	(ft)
Y_{ee}	Instantaneous component of vehicle position east of launcher (L5705)	(ft)

y, Y

Symbol	Definition	Units
$\dot{Y}_{ee}$	Instantaneous components of vehicle velocity, east of the launcher (L5703)	(ft/sec)
$\ddot{Y}_{ee}$	Instantaneous easternly component of vehicle acceleration at launcher (L5752)	(ft/sec^2)
Y^*_{ee}	Eastern component along launcher of missile position	(ft)
Y_{eeo}	Initial component of vehicle position east of launcher	(ft)
$\dot{Y}_{eeo}$	Initial component of vehicle velocities east of launcher	(ft)

z, Z

Symbol	Definition	Units
z	Input dependent variable to be maximized or minimized designated by code input used in the hunting procedure (P2)	(dbi)
z_{cg}	Center-of-gravity offset bias distance, positive down (L5586)	(dbi)
z'_{cg}	Input center-of-gravity offset bias distance, in pitch, positive down (Lk480)	(dbi)
z_{cgj}	Input instantaneous (with total vehicle weight, W_j) center-of-gravity offsets, respectively, where j = 1, 2, ...,15 per stage. Positive in the Z_b axis direction (Lk491, 501, etc)	(dbi and ft)
z_e	Thrust gimbal pitch point position in the Z_b axis direction (L5590)	(ft)
z'_e	Input stage thrust gimbal pitch point eccentricities, respectively. Positive in the Z_b axis direction (Lk433)	(dbi and ft)
$\dot{Z}_{abb}$	z component of missile velocity with respect to the ambient air in the b system (L5509)	(ft/sec)
$\ddot{Z}_{abb}$	z component of missile acceleration with respect to the ambient air in the b system (L5512)	(ft/sec^2)
$\dot{Z}_{bb}$	Inertial component of missile velocity along z_b axis (L5539)	(ft/sec)

z, Z

Symbol	Definition	Units
$\ddot{Z}_b$	Inertial components of missile acceleration along z_b axis (L5542)	(ft/sec^2)
Z	Output primary dependent variable which is to be maximized or minimized used in hunting procedure (P2)	(dbi)
Z_j	Output secondary or constrained dependent variables which are used in hunting procedure (P2), $j = 1, 2, \ldots, 7$	(dbi)
Z_{cc}	Earth centered missile position away from earth axis at launcher longitude (L5515)	(ft)
$\dot{Z}_{cc}$	Away from launcher longitude component of missile velocity in earth centered coordinates (L5518)	(ft/sec)
$\ddot{Z}_{cc}$	Away from launcher longitude component of missile acceleration in earth centered coordinates (L5521)	(ft/sec^2)
Z_{cco}	Initial earth centered missile position away from earth axis at launcher longitude	(ft)
Z_{ee}	Instantaneous component of vehicle position negative up from sea level launcher latitude (L5706)	(ft)
$\dot{Z}_{ee}$	Instantaneous component of vehicle velocity negative up from sea level launcher latitude (L5703)	(ft/sec)
$\ddot{Z}_{ee}$	Instantaneous downward component of vehicle acceleration at launcher (L5753)	(ft/sec^2)

z, Z

Symbol	Definition	Units
Z^*_{ee}	Downward component along launcher of missile position	(ft)
Z_{eeo}	Initial component of vehicle position negative up from sea level launcher	(ft)
$\dot{Z}_{eeo}$	Initial component of vehicle velocity negative up from sea level launcher	(ft/sec)
Z_{gg}	Instantaneous component of vehicle positive vertical from launcher (L5524)	(ft)
$\dot{Z}_{gg}$	Instantaneous component of vehicle velocity in the generalized coordinates vertical from launcher (L5527)	(ft/sec)
$\ddot{Z}_{gg}$	Instantaneous component of vehicle acceleration in the generalized coordinates vertical from launcher (L5530)	(ft/sec^2)
$\dot{Z}_{wee}$	Transformed launcher downward component of wind velocity at missile location (L5551)	(ft/sec)
$\dot{Z}_{wll}$	Local downward component of wind velocity (L5533)	(ft/sec)
$\ddot{Z}_{wll}$	Local time rate change of downward component of wind velocity (L5536)	(ft/sec^2)
$\dot{Z}_{ll}$	Local downward component of missile velocity (L5545)	(ft/sec)
$\ddot{Z}_{ll}$	Local downward component of missile acceleration (L5548)	(ft/sec^2)

z, Z

Symbol	Definition	Units
$\dot{Z}_{1iI}$	Local downward component of missile inertial velocity (L5554)	(ft/sec)

α

Symbol	Definition	Units
α	Instantaneous pitch angle of attack; positive if the vehicle centerline is above the air velocity vector (L5301)	(deg)
$\dot{\alpha}$	Time rate change of angle of attack (L5302)	(deg/sec)
α'	Total vehicle angle of attack; angle between the centerline of the vehicle and the missile air velocity vector; always positive (L5309)	(deg)
$\bar{\alpha}$	Still wind angle of attack (L5310)	(deg)
$\bar{\alpha}'$	Still air total angle of attack (L5305)	(deg)
$\dot{\bar{\alpha}}$	Time rate change of still wind angle of attack (L5307)	(deg/sec)
α_{c_j}	Input command angle of attack (L5303)	(deg)
α_c	Commanded angle of attack	(deg)
α_{c_j}	Input commanded angle of attack used in the constant angle of attack and angle of side slip (Ty = 2) (L0313, 320, etc)	(deg)
α'_d	Input nozzle half angle used in the separated flow nozzle thrust equation (L[illegible]16)	(deg)
α_E	Effective pitch angle of attack used to compute the aerodynamic normal force (L5311)	(deg)
α'_E	Effective total angle of attack used to compute the aerodynamic normal force (L5304)	(deg)
α_ℓ	Angle of attack rotated to command vertical	(deg)
α_{m1}	Commanded angle of attack for constant angle of attack flight (Ty = 2) (L5312)	(deg)
$\alpha_{\max_j}$	Input limit of angle of attack during the j-th type of flight (L0315, 322, etc)	(deg)

α

Symbol	Definition	Units
α_p	Propellant diffusivity used in internal ballistic evaluation is set to 0.00027 if not input (Lk09b)	(in.^{2}/sec)

β

Symbol	Definition	Units
β	Angle of side slip. Positive if the vehicle centerline is left of the air velocity vector when viewed from the rear of the vehicle (L5315)	(deg)
$\dot{\beta}$	Time rate change of angle of side slip (L5316)	(deg/sec)
$\bar{\beta}$	Still wind angle of side slip (L5318)	(deg)
$\dot{\bar{\beta}}$	Time rate change of still wind angle of side slip (L5308)	(deg/sec)
$\breve{\beta}'$	Still wind angle of side slip in the commanded coordinate system used to evaluate the local bank angle (L5319)	(deg)
β_c	Commanded angle of side slip (L5317)	(deg)
β_{c_j}	Input commanded angle of side slip used in the constant angle of attack and angle of side slip (Ty = 2) (L6314, 321, etc)	(deg)
β_E	Effective yaw angle of side slip used to compute the aerodynamic normal face (L5320)	(deg)
$\beta_{\ell f}$	Angle of side slip rotated to the commanded horizontal	(deg)

γ, Γ

Symbol	Definition	Units
γ'_d	Input and output calculated ratio of specific heats of the rocket motor exhaust gases. If input zero 1.18 is used. Used in separated flow nozzle thrust equations (Lk034)	(dim)
γ_G	Calculated local angle of velocity to be gained (L5335)	(deg)
γ_M	Relative azimuthal velocity vector angle in missile-target coordinates (L5336)	(deg)
$\dot{\gamma}_M$	Relative azimuthal velocity vector angular rate in missile-target coordinates (L5337)	(deg)
γ_R	Output required velocity flight path angle at the missile instantaneous position (L5334)	(deg)
γ_T	Target pitch flight path angle (L5728)	(deg)
γ_{TO}	Input initial target flight path angle at start of target maneuvering (L0633)	(deg)
γ_1	Pitch flight path angle. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (L5321)	(deg)
$\dot{\gamma}_1$	Pitch flight path angular rate. Positive up (L5322)	(deg/sec)
γ_{1E}	Pitch flight path angle with respect to the ambient air at entry conditions (L5327)	(deg)
γ_{1I}	Inertial pitch flight path angle. Angle between the inertial velocity vector and the local tangent plane. Positive away from the earth (L5330)	(deg)
γ_{1IE}	Entry conditions inertial pitch flight path angles, if powered flight and the atmospheric end at the time being printed (L5328)	(deg)
γ_{1If}	Impact or intercept inertial pitch flight path angle, if powered flight and the atmosphere end at the time being printed (L5332)	(deg)

γ, Γ

Symbol	Definition	Units
γ_1 (B1)	Pitch flight path angle at the termination of Stage I angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (L5903)	(deg)
γ_1 (B2)	Pitch flight path angle at the termination of Stage II. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (L5928)	(deg)
γ_1 (B3)	Pitch flight path angle at the termination of Stage III. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (L5953)	(deg)
γ_1 (B4)	Pitch flight path angle at the termination of Stage IV. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (L5978)	(deg)
γ_{1If} (B1)	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of Stage I (L5917)	(deg)
γ_{1If} (B2)	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of Stage II (L5942)	(deg)
γ_{1If} (B3)	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of Stage III (L5967)	(deg)
γ_{1If} (B4)	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of Stage IV (L5992)	(sec)
γ_{10}	Input flight path angle at the trajectory start time, $-180° < \gamma_{10} \leq 180°$ (L0011)	(deg)
γ_2	Azimuthal flight path angle. Angle between the horizontal projection of the earth reference velocity vector and the local north. Positive clockwise from north (L5323)	(deg)

γ, Γ

Symbol	Definition	Units
$\dot{\gamma}_2$	Azimuthal flight path angular rate (L5324)	(deg/sec)
γ_{2I}	Inertial azimuth flight path angle. Angle between local north clockwise to the projection of the inertial velocity vector on the local tangent plane (L5331)	(deg)
γ_{2Ia}	Inertial azimuth flight path angle at apogee (L5326)	(deg)
γ_{2IE}	Entry conditions inertial yaw flight path azimuth angle, if powered flight and the atmosphere end at the time being printed (L5329)	(deg)
γ_{2If}	Impact or intercept inertial yaw flight path azimuthal angle, if powered flight and the atmosphere end at the time being printed (L5333)	(deg)
γ_{20}	Input azimuthal flight path angle at trajectory start time (L0044)	(deg)
Γ_d	Ratio of specific heats functional constants used in separated flow thrust equations	(dim)
$\dot{\gamma}_\Gamma$	Target pitch flight path angular rate (L5778)	(deg/sec)

δ, Δ

Symbol	Definition	Units
δ_{ave}	Output average TVC deflection angle per control motor for the TVC design stage	(deg)
δ_{me}	Input maximum vector angle design limit also output in TVC design duty cycle (L0681)	(deg)
δ_{MI}	Azimuth flight path error to intercept. Used in type 10 flight (L5338)	(deg)
δ_{MR}	Input roll system fin misalignment angle (Lk403)	(deg)
δ_{MP}	Input nozzle misalignment angle in pitch (Lk389)	(deg)
δ_{MT}	Seeker yaw look angle (L5351)	(deg)
$\dot{\delta}_{MT}$	Seeker yaw look angular rate (L5352)	(deg/sec)
δ_{MY}	Input nozzle misalignment angle in yaw (Lk390)	(deg)
δ_p	Pitch thrust deflection angle. Positive up (L5716)	(deg)
$\dot{\delta}_p$	Pitch thrust deflection angular rate positive up (L5713)	(deg/sec)
$\ddot{\delta}_p$	Pitch thrust deflection angular acceleration angle positive up (L5763)	(deg/sec^2)
$\bar{\delta}_P$	Modified pitch thrust deflection angle to include limit cycle and misalignment angle (L5342)	(deg)
δ_{Pc}	Pitch plane thrust deflection commands (L5339)	(deg)
$\delta_{PO}, \dot{\delta}_{PO}$	Input per stage initial pitch thrust vector deflection angle and angular rate at the trajectory initiation or stage initiation (Lk420-421)	(deg and deg/sec)
δ_{PH}	Pitch nozzle deflection angle at final altitude of maximum wind shear	(deg)
δ_{PL}	Nozzle deflection angle at initial altitude of maximum wind shear	(deg)

Symbol	Definition	Units
$\bar{\delta}_{Pmax}$	Output maximum magnitude pitch thrust vector deflection angles respectively, per control motor for the TVC design stage	(deg)
δ'_{Pmax}	Maximum pitch deflection outside region of maximum wind shear	(deg)
$\dot{\delta}_{Pmax}$	Output maximum pitch thrust vector deflection angular rate, respectively for the TVC design stage	(deg/sec)
δ_{Pq}	Pitch thrust deflection angle at t_{Bj}	(deg)
$\bar{\delta}_{Pq}$	Output modified the TVC design duty cycle points pitch thrust deflection angle at t_{Bj}	(deg)
δ_{PW}	Maximum pitch within region of maximum wind shear	(deg)
$\bar{\delta}_S$	Output pitch slew angle for TVC design stage	(deg)
δ_S	TVC design stage vehicle slew angle	(deg)
$\bar{\dot{\delta}}_S$	Output control system design slew rate for TVC design stage	(deg/sec)
δ_R	Aerodynamic roll fins deflection angle (L5718)	(deg)
$\dot{\delta}_R$	Aerodynamic roll fins deflection angular rate (L5715)	(deg/sec)
δ_{Rc}	Commanded roll control fin deflection angle (L5341)	(deg)
δ_y	Yaw thrust deflection angle, positive left (L5717)	(deg)
$\dot{\delta}_Y$	Yaw thrust deflection angular rate, positive left (L5714)	(deg/sec)
$\ddot{\delta}_y$	Yaw thrust deflection angular acceleration angle positive left	(deg/sec^2)
$\bar{\delta}_Y$	Modified yaw thrust deflection angle to include limit cycle and misalignment angles (L5343)	(deg)
δ_{Yc}	Yaw plane thrust deflection commands (L5340)	(deg)

δ, Δ

Symbol	Definition	Units
$\delta_{YO}, \dot{\delta}_{YO}$	Input per stage initial yaw thrust vector deflection angle and angular rate at the trajectory initiation or stage initiation (Lk422-423)	(deg and deg/sec)

δ, Δ

Symbol	Definition	Units
Δ_a	Input hunting procedure (P1) values that "a" should be computed within the isolation or maximization routine (L0083)	(dbi)
Δ'_a	Pitch flare-in angle (L5349)	(deg)
Δ_{ak}	Pitch flare in constant used in evaluation Δ'_{ak} for restart (L5347)	(deg)
Δ'_b	Yaw flare-in angle (L5350)	(deg)
Δ_{bk}	Flare-in constant used in evaluating Δ'_{bk} for restart (L5348)	(deg)
Δ_{dc}	Input value of number of desired duty cycle points (100 maximum) if input zero, set equal to 50 (L0670)	(dim)
Δ_h	Calculated altitude difference of the target and missile (L5435)	(ft)
$\dot{\Delta}_h$	Calculated time rate change of the altitude difference of the target and missile (L5436)	(ft/sec)
ΔQ_b	Input stage pitch attitude reaction angular impulse; i.e., added to θ_b at staging (Lk428)	(deg)
ΔR_b	Input stage yaw attitude reaction angular impulse; i.e., added to ψ_b at staging (Lk429)	(deg)
Δ_S	Calculated earth surface down range difference of the target and missile (L5431)	(ft)
$\dot{\Delta}_S$	Calculated time rate change of the earth surface down range difference of the target and missile (L5432)	(ft/sec)
ΔS_c	Calculated earth surface cross range difference of the target and missile (L5433)	(ft)
$\dot{\Delta} S_c$	Calculated time rate change of the earth surface cross range difference of the target and missile (L5434)	(ft/sec)

δ, Δ

Symbol	Definition	Units
Δt_{cj}	Input computing interval during $t_{cj-1} \leq t \leq t_{cj}$ where J = 1, 2, ..., 8 (L0168, 170, etc)	(sec)
Δt_{pj}	Input main printing interval during $t_{p\,(j-1)} \leq t \leq t_{pj}$ where J = 1, 2, ..., 8 (L0184, 186, etc)	(sec)
Δt_{Pj}	Input auxiliary printline interval during $t_{p\,(j-1)}$ $\leq t \leq t_{pj}$ where j = 1, 2, ..., 8 (L0220, 222, etc)	(sec)
Δ_V	Ideal missile velocity resulting from achieved thrust (L5744)	(ft/sec)
$\Delta V_{(B1)}$	Ideal missile velocity for Stage I (L5910)	(ft/sec)
$\Delta V_{(B2)}$	Ideal missile velocity for Stage II (L5935)	(ft/sec)
$\Delta V_{(B3)}$	Ideal missile velocity for Stage III (L5960)	(ft/sec)
$\Delta V_{(B4)}$	Ideal missile velocity for Stage IV (L5985)	(ft/sec)
ΔX	Input increments that X is incremented during the hunting procedure (P1) (L0081)	(dbi)
Δx_{ij}	Input increment of x_{ij} used in incrementing during hunting procedure (P2), J = 1, 2, ..., 7 (L0093, 102, etc)	(dbi)
$\Delta \theta_b$	Vehicle pitch attitude error angle (L5344)	(deg)
$\Delta \phi_b$	Vehicle roll attitude error angle (L5348)	(deg)
$\Delta \psi_b$	Vehicle yaw attitude error angle (L5345)	(deg)

Symbol	Definition	Units
ϵ	Total angle of attack roll orientation angle. Angle between total angle of attack plane and yaw axis. Measured counterclockwise (L5353)	(deg)
$\bar{\epsilon}$	No wind total angle of attack roll orientation angle (L5354)	(deg)
ϵ_{cj}	Input tolerance on the j-th condition of constraint used in hunt procedure (P2) were $J = 1, 2, \ldots, 7$ (L0097, 106, etc)	(dbi)
ϵ_d	Input and output nozzle expansion ratio used in the separated flow nozzle thrust equations (Lk013)	(dim)
ϵ_m	Input flag to specify model error used in hunt procedure (P2). If $\epsilon_m < 0$, the model will be iterated until an extremal solution has a 95 percent probable model error. If $\epsilon_m = 0$, the extremal solution is obtained without regard to probable model error (L0087)	(dim)
ϵ_{MI}	Flight path error to estimated intercept (L5355)	(deg)
ϵ_{MT}	Seeker pitch look angle (L5356)	(deg)
$\dot{\epsilon}_{MT}$	Seeker pitch look angular rate (L5357)	(deg/sec)
ϵ_s	Main motor nozzle separation expansion ratio used in separated flow equation (L5075)	(dim)
ϵ_z	Input value specifying the tolerance of the predicted maximization parameter (z). Used in the hunt procedure (P2) (L0086)	(dbi)

ζ

Symbol	Definition	Units
ζ	Cross range angle. Angle between local vertical and vertical on firing azimuth down range location. Positive if positive is left of firing azimuth (L5358)	(deg)
$\dot{\zeta}$	Cross range angular rate (L5359)	(deg/sec)
ζ_c	Input stage pitch control systems damping ratio for the thrust vector deflection second-order transfer function (Lk436)	(dim)
ζ'_v	Input vehicle controlled damping ratio (Lk438)	(dim)
ζ_T	Target azimuthal flight path angle (L5729)	(deg)
ζ_{TO}	Input initial target azimuthal flight path angle at start of target maneuvering (L0635)	(deg)
ζ_{zj}	Input constant altitude ($T_v = 8$) control damping ratio where J = 1, 2, ..., or 16 (L0315, 322, etc)	(dim)
ζ_o	Initial cross range angle	(deg)
$\dot{\zeta}_T$	Target azimuthal path angular rate (L5779)	(deg/sec)

η

Symbol	Definition	Units
η	Transformed value of the independent variable x used in hunt procedure (P2)	(dbi)
η_{ba}	Acceleration load factor along velocity vector (L5169)	(g's)
η_{bn}	Acceleration load factor normal to the velocity vector (L5171)	(g's)
η_{bt}	Acceleration load factor transverse to the velocity vector (L5170)	(g's)
η_{cn}	Commanded load normal factor	(g's)
η_{cj}	Input command normal load factor for constant load factor type of flight where J = 1, 2, ..., or 13 (Ty = 9) (L0313, 320, etc)	(g's)
η_{ct}	Commanded load crosswire factor	(g's)
η_{ctj}	Input command load factor crosswise to the velocity vector used for constant load factor type of flight where j = 1, 2, ... (Ty = 9) (L0314, 321, etc)	(g's)
$\eta_{\ell n}$	Achieved load factor normal to the velocity vector in the commanded local roll coordinates	(g's)
$\eta_{\ell t}$	Achieved load factor crosswise to the velocity vector in the command local roll coordinates	(g's)
η_{vr}	Input disturbing roll nozzle vortex multiplier. If not input is set to 0.00363 (Lk406)	(ft)

θ

Symbol	Definition	Units
θ_b	Achieved missile Euler angle pitch attitude (L5710)	(deg)
$\dot{\theta}_b$	Achieved vehicle Euler angle pitch rate (L5760)	(deg/sec)
θ_{bo}	Input pitch orientation angle at the trajectory start time, $-180° < \theta_{bo} \leq 180°$ (L0034)	(deg)
θ_i	Input inertial elevation axis Euler angle relating the i and e_o systems (L0015)	(deg)
θ_g	Input generalized coordinate orientation for velocity steering. First rotation angle (about the Y_e -axis) of the set θ_g, ψ_g, and ϕ_g (L0047)	(deg)
θ_m	Desired missile attitude Euler angle relating the m and i system (L5722)	(deg)
$\dot{\theta}_m$	Desired vehicle pitch Euler angular rate (L5772)	(deg/sec)

λ

Symbol	Definition	Units
λ_{MT}	Angle of missile to target line projection on horizontal and firing azimuth (L5360)	(deg/sec)
$\dot{\lambda}_{MT}$	Angular rate of missile to target line projection or horizontal and firing azimuth (L5361)	(deg/sec)
λ_i	Lagrange multiplier used in the simultaneous hunt (P2) for the i-th constraint function	(dbi)
λ_a	Apogee longitude	(deg)
λ_d	Nozzle half angle momentum correction coefficient	(dim)

μ

Symbol	Definition	Units
μ	Instantaneous vehicle longitude. Value is positive or negative west or east of Greenwich, England, respectively (L5362)	(deg)
μ'	Instantaneous vehicle change of longitude from launch longitude. Value is positive east (L5363)	(deg)
$\dot{\mu}'$	Vehicle longitude time rate change (L5364)	(deg/sec)
μ_a	Vehicle apogee longitude if powered flight and the atmosphere end at the time being printed (L5365)	(deg)
μ_f	Missile impact or intercept longitude if powered flight and the atmosphere end at the time being printed (L5366)	(deg)
μ_L	Input launcher longitude (L0012)	(deg)
μ'_o	Initial change in longitude from launch longitude	(deg)

Ξ

Symbol	Definition	Units
Ξ	Ratio of specific heats function constant used in separated flow thrust equation	(dim)

ρ

Symbol	Definition	Units
ρ	Instantaneous vehicle latitude positive north of the equator $-90^\circ \leqq \rho \geqq 90^\circ$ (L5367)	(deg)
$\dot{\rho}$	Vehicle latitude time rate change (L5368)	(deg/sec)
ρ_a	Vehicle apogee latitude if powered flight and the atmosphere end at the time being printed (L5369)	(deg)
ρ_f	Missile impact or intercept latitude if powered flight and the atmosphere end at the time being printed (L5370)	(deg)
ρ_L	Input launch latitude, $-90^\circ \leqq \rho \geqq 90^\circ$ (L0017)	(deg)
ρ_p	Density of propellant used in internal ballistic evaluation is set to 0.065 if not input (Lk095)	(lb/in.^{3})
ρ_o	Initial vehicle latitude	(deg)

σ

Symbol	Definition	Units
σ_a	Input code which designates the dependent variable in the hunting procedure (P1) (L0078)	(dim)
σ_{Aj}	Input code which designates the integration tolerance parameters where J = 1, 2, . . . , or 7 (L0280, 283, etc)	(dbi)
σ_{Bj}	Input code which designates the quantities whose values at staging are to be available to the hunting procedure, j = 1, 2, . . . , 5 (L0271, 273, etc)	(dim)
σ_{BX1}	Values which are designated by input, at staging which are available to the hunting procedure (L5496-500)	(dbi)
σ_{BX2}		
σ_{BX3}		
σ_{BX4}		
σ_{BX5}		
σ_c	Input code which designates the quantity that determines the flight region when the orbital elements and impact determination are desired (L0041)	(dim)
σ_{Dj}	Input code which designates the quantity that determines when to print a discontinuity where j = 1, 2, . . . , 8 (L0252-259)	(dim)
σ_f	Standard of vacuum thrust to nominal vacuum thrust at any nominal time point	(dim)
σ_{fj}	Input code which designates the quantity that determines when the j-th type of flight ends where j = 1, 2, . . . , 16 (L0311, 318, etc)	(dim)
σ_{Fyj}	Input code which designates the quantity that determines when the j-th type of TMC ends (L0801, 811, etc)	(dim)
σ_{Gj}	Input code which designates attitude control system gain zone limits (j = 1, 2, or 3) (Lk457, 466)	(dim)

σ

Symbol	Definition	Units
σ_{g1k}	Input code which designates the start of the acquisition zone for evaluation of the steering equations coefficients for the k-th stage, k = 1, 2, 3, or 4 (Lk397)	(dbi)
σ_{g2k}	Input quantity which designates the end of the acquisition zone for evaluation of the steering equation coefficient for the k-th stage, k = 1, 2, 3, or 4 (Lk399)	(dbi)
σ_{It}	Input standard deviation of the ratio of total impulse to nominal total impulse for the k-th stage (Lk383)	(dim)
σ_{Jj}	Input code which designates the quantity that determines when the W_{JT} weight is to be jettisoned where j = 1, 2, ..., or 8 (L0050, 53, etc)	(dim)
σ_{MI}	Local flight path angle to estimated target, intercept (L5373)	(deg)
σ_{mj}	Input code which designates the quantity whose maximum value is to be printed following each stage time where j = 1, 2, ..., 5 (L0261, 263, etc)	(dbi)
σ_{Mj}	Input code which designates the quantity that determines when the j-th mode type ends where j = 1, 2, ..., 10 (L0601, 609, etc)	(dim)
σ_{MT}	Angle of missile to target line and local horizontal (L5371)	(deg)
$\dot{\sigma}_{MT}$	Angular rate of missile to target line and local horizontal (L5372)	(deg/sec)
σ_{mx1}	Values which are designated by input, as quantity whose maximum value is to be printed following each stage time (L5491, 495)	(dbi)
σ_{mx2}		
σ_{mx3}		
σ_{mx4}		
σ_{mx5}		

σ

Symbol	Definition	Units
σ_{Pj}	Input code designates the quantity to be printed in printline Z where j = 1, 2, ..., 8 (L0209-219)	(sec)
σ_{Sk}	Input code which designates the quantity that determines when a stage is terminated where k = 1, 2, 3, 4 (Lk000)	(dim)
σ_{tb}	Input standard deviation of the ratio of web burntime to nominal burntime for the k-th stage (Lk384)	(dim)
σ_{tj}	Input code which designates the quantity that determines when to print a special time where j = 1, 2, ..., 8 (L0236, 238, etc)	(dim)
σ_{Tto}	Input code designating start of target manuevering (L0630)	(dim)
σ_X	Input code which designates the independent variable in the hunting procedure (P1) (L0077)	(dim)
σ_{xj}	Input code which designates the independent variables used in hunting procedure (P2) where j = 1, 2, ..., 7 (L0091, 100, etc)	(dim)
σ_{yj}	Input code which designates the dependent variables used in hunting procedure (P2) where j = 1, 2, ..., 7 (L0094, 103, etc)	(dim)
σ_z	Input code. The σ_z identifies the dependent variable being maximized or minimized. Used in hunt procedure (P2). If $\sigma_z < 0$, the values of $x_1, x_2, \ldots, x_{in}$ (L0085)	(dim)

T

Symbol	Definition	Units
τ_c	Input stage pitch and yaw control systems time constant for the thrust vector deflection first order transfer function (Lk435)	(sec)
τ_{cf}	First order lead constant which will be set proportional to equivalent first order lag of controllable motor used in the TMC command logic	(sec)
τ_{fk}	Input pitch flare-in time constant for k-th stage used in velocity steering type of flight (Ty = 4) (Lk396)	(sec)
τ_{Fyj}	Input control system time constant of the j-th type TMC (L0805, 815, etc)	(sec)
τ_{HGj}	Input pitch and yaw flare-in factor used in the Homing Guidance (Ty = 11) (L0314, 321, etc)	(sec)
τ_{IGj}	Input pitch and yaw flare-in factor used in the intercept guidance (Ty = 10) (L0314, 321, etc)	(sec)
τ_{MIj}	Input guidance associated first order intercept guidance controller time constant used in type 10 flight where j = 1, 2, ..., or 13 (L0313, 320, etc)	(sec)
τ_{Ri}	Input stage roll control system time constant, for the first order transfer (Lk639, 649, 659)	(sec)
τ_w	Web fraction used in internal ballistic evaluation is set to 0. 8 if not input (Lk096)	(dim)
τ_{yk}	Input time constant in the nontarget dependent yaw steering equation for the k-th stage (Lk402)	(sec)
$\tau_{\eta j}$	Input flare-in time constant for constant load factor type of flight (Ty = 9) (L0315, 322, etc)	(g's)

τ

Symbol	Definition	Units
τ_{MI}	Azimuthal angle to target intercept (L5374)	(deg)

ϕ, φ

Symbol	Definition	Units
ϕ	Instantaneous down range angle. Angle between the launch vertical and the local vertical on the down range azimuth point. Positive as shown in Figure 1. 1, $\infty < \varphi < \infty$ (L5375)	(deg)
$\dot{\phi}$	Instantaneous range angular rate. Positive down range (L5376)	(deg/sec)
ϕ_a	Glide range angle to the apogee vertical (L5378)	(deg)
$\phi_a a_4$	Glide range angle to the apogee vertical (L5114)	(deg)
ϕ_b	Achieved missile Euler angle roll attitude (L5712)	(deg)
ϕ_b	Achieved vehicle Euler angle roll rate (L5762)	(deg/sec)
ϕ_{bo}	Input roll orientation angle at the trajectory start time $-180° < \varphi_{bc} \leq 180°$ (L0036)	(deg)
ϕ_f	Glide range angle to glide phase termination vertical used in Keperian impact predictions (L5381)	(deg)
ϕ_g	Input generalized coordinate orientation angle. Final rotation angle (about the X_g axis) of the set θ_g, ψ_g, and ϕg	(deg)
ϕ_i	Input inertial meridional axis Euler angle relating the i and e_o systems (L0016)	(deg)
ϕ_m	Desired missile attitude Euler angle relating the m and i systems (L5724).	(deg)
ϕ_m	Desired vehicle roll Euler angular rate (L5774)	(deg/sec)
ϕ_{mo}	Input initial commanded roll attitude (L0030)	(deg)
ϕ_{Ri}	Input bank angle location of the i-th raceway where $i = 1, 2$. As seen from the rear of the vehicle, a positive angle is measured clockwise from the Z_k axis direction (Lk669, 672)	(deg)

ϕ, φ

Symbol	Definition	Units
ϕ_s	Instantaneous slant range angle. Angle between the launch vertical and the local vertical (L5379)	(deg)
ϕ_v	Input bivariant consideration axis for the k-th stage (Lk385)	(deg)
ϕ_o	Initial down range angle	(deg)
φ	Local bank angle (L5380)	(deg)
φ_c	Commanded bank angle (L5377)	(deg)
φ_{cj}	Input command roll attitude used in the constant angle of attack and angle of side slip type of flight (Ty = 7) or the constant load factor type of flight where j = 1, 2, ..., or 13 (Ty = 9) (L0315, 322, etc)	(deg)

ψ

Symbol	Definition	Units
ψ	Vehicle azimuth in the launch horizontal plane (L5382)	(deg)
ψ_b	Achieved missile Euler angle yaw attitude (L5711)	(deg)
$\dot{\psi}_b$	Achieved vehicle Euler angle yaw rate (L5761)	(deg/sec)
ψ_{bo}	Input yaw orientation angle at the trajectory start time $-180° < \psi_{bo} \leq 180°$ (L0035)	(deg)
ψ_g	Input generalized coordinate orientation angle. Second rotation angle (about the resulting Z axis after rotating through θ_g, ψ_g, and φ_g (L0046)	(deg)
ψ_i	Input flight plane azimuth angle. Angle measured north to the flight plane, $-180° \leq \psi \leq 180°$ (L0014)	(ft)
ψ_m	Desired missile attitude Euler angle relating the m and i systems (L5723)	(deg)
ψ_m	Desired vehicle yaw Euler angular rate (L5773)	(deg/sec)
ψ_{mo}	Input initial commanded yaw attitude (L0029)	(deg)
ψ_w	Instantaneous wind azimuth angles, measured in a plane parallel to the local tangent plane where j = 1, 2, ..., 30. Angle measured clockwise from north to the direction from which the wind is coming (L5383)	(deg)
ψ_{wj}	Input instantaneous (with altitude h) wind azimuth angles, measured in a plane parallel to the local tangent plane where j = 1, 2, ..., 30. Angle measured clockwise from north to the direction from which the wind is coming. (L0417, 415, etc)	(deg and dbi)

ω, Ω

Symbol	Definition	Units
ω	Input magnitude of the earth's angular velocity. If input greater than 0.5, set equal to 7.29211 E-5 (L0027)	(rad/sec)
ω_c	Input stage pitch control systems forcing frequency for the thrust vector deflection second-order transfer function (Lk437)	(rad/sec)
$\tilde{\omega}_c$	Output slew frequency used in the TVC design stage slew rate calculations	(rad/sec)
ω_L	Input frequency of limit cycle for the k-th stage	(rad/sec)
ω_p	Pintle control frequency (L5083)	(rad/sec)
ω_s	Input slew frequency used in the TVC design stage slew rate calculations (L0674)	(rad/sec)
ω'_v	Input vehicle controlled frequency for the k-th stage (Lk439)	(rad/sec)
ω_{zj}	Input attitude control frequency used in constant attitude type of flight where j = 1, 2, ..., 16 (Ty = 8) (L0314, 321, etc)	(rad/sec)
Ω_d	Ratio of specific heats functional constant. Used in separated flow thrust equations	(dim)

B. EQUATIONS OF MOTION

The equations and logic given in this section establish the trajectory program which simulates missile flight in three dimensions with an additional three degrees of freedom possible, i.e., the vehicle can pitch, yaw, and roll about its center of gravity. A spherical rotating earth mode is utilized for missile location; gravitational forces from an oblate earth can be included.

Differential equations requiring numerical integration are as follows:

1. Linear momenta equations
2. Angular momenta equations
3. Desired missile attitude angular velocities
4. Achieved missile attitude angular velocities
5. Thrust vectoring equations

To yield an essentially point mass solution, items 2, 4, and 5 are ignored if $M_y = 1$ or 4.

1. Linear Momenta Equations

The linear momenta equations which yield the three-dimensional missile acceleration components are computed from the real forces acting on the missile and the accelerations arising from the earth rotation. The equations are:

$$\begin{Bmatrix} \ddot{X}_{ee} \\ \ddot{Y}_{ee} \\ \ddot{Z}_{ee} \end{Bmatrix} = [D] \begin{Bmatrix} a_{Xb}\bar{g}_e \\ a_{Yb}\bar{g}_e \\ a_{Zb}\bar{g}_e \end{Bmatrix} - \begin{Bmatrix} 2\omega\dot{Y}_{ee} \sin \rho_L \\ -2\omega(\dot{X}_{ee}\sin \rho_L + \dot{Z}_{ee}\cos \rho_L) \\ 2\omega\dot{Y}_{ee} \cos \rho_L \end{Bmatrix} - \begin{Bmatrix} g_{Xe} \\ g_{Ye} \\ g_{Ze} \end{Bmatrix}$$

The above parameters are defined as follows:

$\ddot{X}_{ee}$ and $\dot{X}_{ee}$ are the components of missile acceleration and velocity in a right handed Cartesian coordinate system whose origin is fixed at sea level at the input launch latitude, ρ_L. The X_e axis is positive north, Y_e-axis positive east, and Z_e, the vertical axis, is positive down.

a. Initial Condition

The initial condition position and velocity vector are as follows:

Down Range Angle

$$\Phi_o = (180/\pi)\ S_o/r_e$$

Cross Range Angle

$$\zeta_o = (180/\pi)\ S_{co}/r_e$$

Radius From Center of Earth

$$r_{c_o} = h_o + r_e$$

Launch Centered Coordinates

$$X_{eeo} = r_{co}\ (\cos \vartheta_i \cos \zeta_o \sin \Phi_o - \sin \vartheta_i \sin \zeta_o)$$

$$Y_{eeo} = r_{co}\ (\sin \vartheta_i \cos \zeta_o \sin \Phi_o + \cos \vartheta_i \sin \zeta_o)$$

$$Z_{eeo} = r_e - r_{co} \cos \zeta_o \cos \Phi_o$$

Initial Earth Geocentric Coordinates

$$X_{cco} = X_{eeo} \cos \rho_L - Z_{eeo} \sin \rho_L + r_e \sin \rho_L$$

$$Y_{cco} = Y_{eeo}$$

$$Z_{cco} = X_{eeo} \sin \rho_L + Z_{eeo} \cos \rho_L - r_e \cos \rho_L$$

Initial Latitude and Differential Lognitude

$$\sin \rho_o = X_{cco}/r_{co}$$

$$\cos \rho_o = (Y^2_{cco} + Z^2_{cco})^{\frac{1}{2}}/r_{co}$$

$$\sin \mu'_o = Y_{cco}/(Y^2_{cc} + Z^2_{cc})^{\frac{1}{2}}$$

$$\cos \mu'_o = -Z_{cc}/(Y^2_{cc} + Z^2_{cc})^{\frac{1}{2}}$$

Initial Launch Centered Velocity Coordinates

$$\begin{bmatrix} \dot{X}_{eeo} \\ \dot{Y}_{eeo} \\ \dot{Z}_{eeo} \end{bmatrix} = \begin{bmatrix} \cos \rho_L & 0 & \sin \rho_L \\ 0 & 1 & 0 \\ -\sin \rho_L & 0 & \cos \rho_L \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \mu'_o & -\sin \mu'_o \\ 0 & \sin \mu'_o & \cos \mu'_o \end{bmatrix}$$

$$\times \begin{bmatrix} \cos \rho_o & 0 & -\sin \rho_o \\ 0 & 1 & 0 \\ \sin \rho_o & 0 & \cos \rho_o \end{bmatrix} \begin{bmatrix} V_{eo} \cos \gamma_{1o} \cos \gamma_{2o} \\ V_{eo} \cos \gamma_{1o} \sin \gamma_{2o} \\ -V_e \sin \gamma_{1o} \end{bmatrix}$$

b. **D Matrix**

The matrix [D], which rotates components from the b system to the e system, is computed from:

$$[D] = [A_{ec}][A_i][A_b]$$

where ω is input, $[A_r]$ rotates components from the b system to the i system, $[A_i]$ from the i system to the e_o system, and $[A_{eo}]$ from the eo system to the e system. The coordinate axes are discussed in Section A.2.c and shown in Figure 17. These rotation matrices are defined as follows:

$$[A_{eo}] = \begin{bmatrix} \cos \rho_L & 0 & \sin \rho_L \\ 0 & 1 & 0 \\ -\sin \rho_L & 0 & \cos \rho_L \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega t & \sin \omega t \\ 0 & -\sin \omega t & \cos \omega t \end{bmatrix} \begin{bmatrix} \cos \rho_L & 0 & -\sin \rho_L \\ 0 & 1 & 0 \\ \sin \rho_L & 0 & \cos \rho_L \end{bmatrix}$$

$$[A_i] = \begin{bmatrix} \cos \psi_i & -\sin \psi_i & 0 \\ \sin \psi_i & \cos \psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_i & 0 & \sin \theta_i \\ 0 & 1 & 0 \\ -\sin \theta_i & 0 & \cos \theta_i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi_i & -\sin \Phi_i \\ 0 & \sin \Phi_i & \cos \Phi_i \end{bmatrix}$$

with

$$[A_m] = \begin{bmatrix} \cos \theta_m & 0 & \sin \theta_m \\ 0 & 1 & 0 \\ -\sin \theta_m & 0 & \cos \theta_m \end{bmatrix} \begin{bmatrix} \cos \psi_m & -\sin \psi_m & 0 \\ \sin \psi_m & \cos \psi_m & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi_m & -\sin \Phi_m \\ 0 & \sin \Phi_m & \cos \Phi_m \end{bmatrix}$$

$$[A_b] = \begin{bmatrix} \cos \theta_b & 0 & \sin \theta_b \\ 0 & 1 & 0 \\ -\sin \theta_b & 0 & \cos \theta_b \end{bmatrix} \begin{bmatrix} \cos \psi_b & -\sin \psi_b & 0 \\ \sin \psi_b & \cos \psi_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi_b & -\sin \Phi_b \\ 0 & \sin \Phi_b & \cos \Phi_b \end{bmatrix}$$

where t is the instantaneous time, θ_i, ψ_i, Φ_i the input Euler angles defining the relation between i and e_o systems, and the subscript m and b Euler angles are computed from the integration of their rate equations. The angles are shown in Figure 17 and Figure 18.

c. Inertial Acceleration Components

The components of acceleration arising from the thrust and aerodynamic forces acting on the missile are:

$$a_{Xb} = \left[F_x - C - C_{\delta z} - D_{r\ell} \ \mathrm{sgn}\ (\dot{X}_{bb})\right]/W$$

$$a_{Yb} = (F_y + F_{JDy} + F_{TDy} + N_{Py} - N_Y + N_{\delta Y})/W$$

$$a_{Zb} = (F_z + F_{JDz} + F_{TDz} + N_{Pz} - N_Z + N_{\delta Z})/W$$

where $F_{x,y,z}$ are the components of missile thrust, main and/or complementary; F_{JDy} and F_{JDz} are the jet damping forces in yaw and pitch respectively; N_{Py} and N_{Pz} are the aerodynamic damping forces in yaw and pitch respectively; F_{TDy} and F_{TDz} are the movable nozzle tail-wag-dog force in yaw and pitch respectively; and C, N_Y and N_Z are the aerodynamic force components. $N_{\delta Y}$ and $N_{\delta Z}$ are the control fins normal force in yaw and pitch respectively; and $C_{\delta Z}$ is the control fin axial force. $D_{r\ell}$ is the input rail drag. The mass conversion gravity value, $\tilde{g}_e$, is input.

The second and third terms of the linear momenta equations result from Coriolis and gravitional accelerations.

Some of these forces are depicted on Figures 19 and 20.

d. Gravity

The gravitational force is specified by the two components shown in Figure 24; one directed down from the missile towards the earth center and the other perpendicular to the preceding component, directed towards the equatorial plane. Both components are functions of vehicle geocentric latitude and radial distances and include the accelerations due to mass attraction and centrifugal force.

The local components of gravity are:

$$g_{X1} = -(2r_e^4\, g_e\, J/r_c^4 + \omega^2\, r_c)\cos\rho\,\sin\rho$$

$$g_{Y1} = 0$$

$$g_{Z1} = r_e^2\, g_e\, \left[1 - J\, r_e^2\,(3\sin^2\rho - 1)/r_c^2 - \omega^2\, r_c\cos^2\rho\right]$$

where r_e, g_e, J, and ω are input and r_c and ρ are the instantaneous missile earth center distance and latitude, respectively.

The components in the e system are

$$\vec{g}_e = [A_1]\,\vec{g}_1$$

where the matrix $[A_1]$ is defined in Section F.2.b.

The energy per unit mass is:

$$E/m = g_e\, r_c - g_e\, r_e^2/r_c + V_I^2/2$$

2. Angular Momenta Equations

The angular momenta equations which yield the angular acceleration components about the vehicle center of gravity are defined as follows:

$$\begin{bmatrix} I_{XX} & -I_{XY} & -I_{XZ} \\ -I_{XZ} & I_{YY} & I_{YZ} \\ -I_{XZ} & -I_{YZ} & I_{ZZ} \end{bmatrix} \begin{Bmatrix} \dot{P}_b \\ \dot{Q}_b \\ \dot{R}_b \end{Bmatrix} + \begin{bmatrix} \dot{I}_{XX} & -\dot{I}_{XY} & -\dot{I}_{XZ} \\ -\dot{I}_{XY} & \dot{I}_{YY} & -\dot{I}_{YZ} \\ -\dot{I}_{XZ} & -\dot{I}_{YZ} & \dot{I}_{ZZ} \end{bmatrix} \begin{Bmatrix} P_b \\ Q_b \\ R_b \end{Bmatrix}$$

$$+ \begin{bmatrix} 0 & -R_b & Q_b \\ R_b & 0 & -P_b \\ -Q_b & P_b & 0 \end{bmatrix} \begin{bmatrix} I_{XX} & -I_{XY} & -I_{XZ} \\ -I_{XY} & I_{YY} & -I_{YZ} \\ -I_{XZ} & -I_{YZ} & I_{ZZ} \end{bmatrix} \begin{Bmatrix} P_b \\ Q_b \\ R_b \end{Bmatrix} = \begin{Bmatrix} M_{IP} \\ M_{IQ} \\ M \end{Bmatrix}$$

where P_b, Q_b and R_b are the roll, pitch and yaw turning rates about the missile center of gravity (Figure 21), I_{XX}, I_{YY} and I_{ZZ} are the roll, pitch and yaw moments of inertia respectively; I_{XY}, I_{XZ}, and I_{YZ} are the products of inertia; and M_{IP}, M_{IQ} and M_{IR} are the roll, pitch, and yaw unbalance moments respectively.

a. Initial Conditions, Angles, and Angular Rates

Initial values of θ_b, ψ_b, and Φ_b, at the trajectory start time, t_o, are obtained from input and thereafter, at the initiation of the mode are equated to the values existing at the end of the previous mode segment. At staging, the input stage attitude reaction angular impulse, ΔQ_b is added to Q_b and ΔR_b is added to R_b.

b. Angular Moments

Angular moments are defined as follows.

Perturbing Pitching Moments

$$M_{DQ} = M_{NQ} + M_{FOQ} + M_{TDQ} + M_{JDQ}$$

where M_{NQ} is the aerodynamic pitching moment, M_{FOQ} is the thrust offset pitching moment, M_{TDQ} is the movable nozzle tail-wag-dog pitching moment, and M_{JDQ} is the jet damping pitching moment.

Controlling Pitching Moments

$$M_{CQ} = M_{FCQ} + M_{\delta Q}$$

where M_{FCQ} is the thrust vector control pitching moment and $M_{\delta Q}$ is the movable pitch control fin moment.

Unbalance Pitching Moment

$$M_{IQ} = M_{DQ} + M_{CQ}$$

Perturbing Yaw Moments

$$M_{DR} = M_{NR} + M_{FOR} + M_{TDR} + M_{JDR}$$

where M_{NR} is the aerodynamic yawing moment, M_{FOR} is the thrust offset yawing moment, M_{TDR} is the movable nozzle tail-wag-dog yawing moment, and M_{JDR} is the jet damping yawing moment.

Controlling Yawing Moment

$$M_{CR} = M_{FCR} + M_{\delta R}$$

where M_{FCR} is the thrust vector control yawing moment and $M_{\delta R}$ is the movable yaw control fin moment.

Unbalance Yawing Moment

$$M_{IR} = M_{DR} + M_{CR}$$

Perturbing Roll Moments

$$M_{DP} = M_{NP} + M_{FOP} + M_{FVP} + M_{RAP}$$

where M_{NP} is the aerodynamic normal force offset rolling moment, M_{FOP} is the thrust offset rolling moment, M_{FVP} is the thrust vortex rolling moment, and M_{RAP} the raceway aerodynamic rolling moment.

Controlling Rolling Moment

$$M_{CP} = M_{FCP} + M_{\delta R}$$

where M_{FCP} is the auxilliary roll control rolling moment and $M_{\delta R}$ is the aerodynamic fin rolling control moment.

Unbalance Rolling Moment

$$M_{IP} = M_{DP} + M_{CP}$$

C. CONTROL TABLES

Three different control tables regulate the type of trajectory simulation. The mode control table stipulates the degrees of freedom to be simulated, the attitude control table dictates the type of flight, and the thrust modulation control table specifies the thrust control law.

1. Mode Control Table

The mode table controls the degree of sophistication that is obtained in the simulation of missile flight. Options such as rigid body with and without thrust vector control are controlled by the mode table. The desired option or mode is determined by the type of mode input M_y.

If the input σ_{M1} is zero, all of the logic given below is ignored and only the Mode 1 option of a rigid body without controls is applicable throughout the run.

The following logic applies to the mode regions:

Inputting a maximum of ten mode regions per trajectory is possible. Let σ_{Mj} $(j = 1, 2, \ldots, 10)$ be the achieved value of a parameter designated by code input and let K_{Mj} be the input of the parameter; then the j-th mode region will apply until $\sigma_{Mj} = K_{Mj}$, then mode is switched to next mode type.

The time when equality in the above occurs will be determined and is defined as t_{Mj}. The K_{Mj} criteria are checked monotonically with respect to the j index. The trajectory is terminated if the final t_{Mj} is reached. The applicable region at the beginning of a trajectory run is determined from the region start flag, M'_{yj}, as follows:

1. The j-th ($j = 1, 2, \ldots, 10$) region applies if $M'_{yj} = j \neq 0$

2. If $M'_{yj} = 0$, the first margin applies.

At the initiation of Mode 4, the initial thrust vector angles and rates are obtained from input unless they are not input; then, they are equated to the values at the end of the previous mode region.

The modes have the following meaning:

a. Point mass simulation ($M_y = 4$)
   Mode 4 simulates in three dimensions the vehicle motion as a rigid body without controls (point mass).

b. Rigid body simulation ($M_y = 5$)
   Mode 5 simulates in three dimensions the vehicle motion as a rigid body with controls.

The trajectory will be terminated with an error message and dump if any mode other than 4 or 5 is encountered.

2. Flight Control Table

Desired missile attitude (the relation between the m and i systems) as shown in Figure 18 will be computed by on of the many types of flight methods given below. The m system is the basic coordinates that the autopilot controls. The command m system is calculated for desired angles of attack, angle of sideslip and back angle for certain types of flight. The type of attitude control desired will be determined by the "type of flight" input T_y according to the following logic:

If $T_y = 1$

programmed flight is desired. Missile attitude is determined from the input turning rates.

If $T_y = 2$

stipulated angle of attack and angle of sideslip for a given back angle is desired. Commanded pitch, yaw, and roll attitudes are determined from the commanded relative attitude from achieved earth reference velocity vector.

If $T_y = 4$

nontarget dependent guided flight is desired. Missile attitude is determined from velocity steering equations.

If $T_y = 6$

rail launch dynamic is desired. Missile attitude is fixed in pitch and yaw but the roll attitude is allowed to be commanded from input roll rate.

If $T_y = 8$

constant attitude is specified. Missile attitude will be controlled from a command law specifying angle of attack and angle of side slip. Roll attitude will be specified from command back angle.

If $T_y = 9$

constant normal load factor flight is desired. Missile attitude will be controlled from command law specifying angle of attack, and angle of side slip. Roll attitude will be specified from command back angle.

If $T_y = 10$

intercept guidance flight is desired. Missile attitude is commanded so as to fly the missile on a collision course to intersect with the input moving target coordinates.

If $T_y = 11$

Homing Guidance flight is desired. Missile attitude is calculated to home on a target by so called proportional navigational steering.

The following logic applies to all types of flight:

A maximum of 13 types of flight per trajectory can be input. Let σ_{fj} (j = 1, 2, ..., 13) be the achieved value of a parameter designated by code input and let K_{fj} be the input value of the parameter; then, the j-th type of flight will apply for as long as σ_{fj} does not equal K_{fj}. An error will occur if σ_{fj} and K_{fj} are equal at the beginning of the j = the type of flight.

The j-th type of flight will end and the trajectory will be computed when equality occurs. Criterion for ending the (j+1)-type of flight is not checked until the j-th type of flight ends. Once the j-th type of flight ends, it cannot be re-entered during the trajectory.

a. Programmed Flight ($T_y = 1$)

When $T_y = 1$ during the j-th type of flight, the pitch, yaw and roll turning rates are obtained from input and the desired vehicle attitude is determined from:

$$\dot{\theta}_m = (Q_{mj} \cos \phi_m - R_{mj} \sin \phi_m)/\cos \psi_m$$

$$\dot{\psi}_m = Q_{mj} \sin \phi_m + R_{mj} \cos \phi_m$$

$$\dot{\phi}_m = P_{mj} - \dot{\theta}_m \sin \psi_m$$

where Q_{mj}, R_{mj}, and P_{mj} are the input turning rates in the attitude control table whose positive directions are shown in Figure 19.

The initial values of θ_m, ϕ_m, and ψ_m are obtained from input if $t = t_o$, or from the last computed values from the previous type of flight.

b. Constant Angle of Attack and Angle of Side Slip ($T_y = 2$)

When $T_y = 2$ during the j-th type of flight, the desired vehicle attitude is determined from:

$$\theta_m = \arctan\,[(-A_{m31})/(A_{m11})] \qquad -180^\circ < \theta_m \leq 180^\circ$$

$$\psi_m = \arcsin\,[A_{m21}] \qquad -90^\circ \leq \psi_m \leq 90^\circ$$

$$\phi_m = \arctan\,[(-A_{m23})/(A_{m22})] \qquad -180^\circ < \phi_m \leq 180^\circ$$

where

$$[A_m] = [A_i]^{-1}\,[A_{eo}]^{-1}\,[A_1]\,[A_{\gamma_2}]^{-1}\,[A_{\gamma_1}]^{-1}\,[A_c]^{-1}$$

where

$$[A_c] = \begin{bmatrix} \cos\alpha_c & 0 & -\sin\alpha_c \\ 0 & 1 & 0 \\ \sin\alpha_c & 0 & \cos\alpha_c \end{bmatrix} \begin{bmatrix} \cos\beta_c' & -\sin\beta_c' & 0 \\ \sin\beta_c' & \cos\beta_c' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_c & \sin\varphi_c \\ 0 & -\sin\varphi_c & \cos\varphi_c \end{bmatrix}$$

where

$$\beta_c' = \arctan [(\tan \beta_c)/(\sec \alpha_c)]$$

The values of α_c, β_c and φ_c are input in the flight control table in the Q_m, R_m, and $\underline{P}_m$ columns.

$$\begin{aligned}[\dot{A}_m]^{-1} = [A_c] \ ([\dot{A}_{\gamma_1}] \ [A_{\gamma_2}] \ [A_1]^{-1} \ [A_{eo}] \\ + [A_{\gamma_1}] \ [\dot{A}_{\gamma_2}] \ [A_1]^{-1} \ [A_{eo}] \\ + [A_{\gamma_1}] \ [A_{\gamma_2}] \ [\dot{A}_1]^{-1} \ [A_{eo}] \\ + [A_{\gamma_1}] \ [A_{\gamma_2}] \ [A_1]^{-1} \ [\dot{A}_{eo}]) \ [A_1]\end{aligned}$$

$$[\dot{A}_{eo}] = \omega \begin{bmatrix} 0 & -\sin\rho_L & 0 \\ \sin\rho_L & 0 & \cos\rho_L \\ 0 & -\cos\rho_L & 0 \end{bmatrix} [A_{eo}]$$

$$[A_E] = [\dot{A}_m]^{-1} \ [A_m]$$

$$P_m = A_{E23}$$

$$Q_m = A_{E31}$$

$$R_m = A_{E11}$$

and $\dot{\theta}_m$, $\dot{\psi}_m$, and $\dot{\phi}_m$ are determined as in $T_y = 1$.

c. Pitch Steering (Ty=4)

When Ty=4 during the j-th segment of flight, the vehicle attitude and rate are obtained from instantaneous missile position and velocity components as follows:

$$\theta_m = (180/\pi)[a_{1k}\dot{Z}_{gg} + a_{0k} + b_{1k}\dot{X}_{gg} + b_{2k}\dot{X}^2_{gg} + b_{3k}\dot{X}^3_{gg} + \Delta_{ak}]$$

$$\dot{\theta}_m = (180/\pi)[a_{1k}\ddot{Z}_{gg} + b_{1k}\ddot{X}_{gg} + 2b_{2k}\ddot{X}_{gg}\ddot{X}_{gg} + 3b_{3k}\dot{X}^2_{gg}\ddot{X}_{gg} + \dot{\Delta}_{ak}]$$

The constants a_{1k}, a_{0k}, b_{1k}, b_{2k}, b_{3k} ($k = 1, 2, 3$, or 4) are input per stage, computed under acquisition of coefficients for steering equation logic delineates in section L.3.a, or obtained from previous run if $\sigma_{g1k} < 0$. Also,

$$\Delta_{ak} = \Delta'_{ak} \exp[(t_{Bf} - t)/\tau_{fk}]$$

$$\dot{\Delta}_{ak} = -(\Delta_{bk}/\tau_{fk}) \exp[(t_{Bf} = t)/\tau_{fk}]$$

$$\Delta'_{ak} = [(\pi/180)\theta_{mf} - a_{1k}\dot{Z}_{gf} - a_{0k} - b_{1k}\dot{X}_{gf} - b_{2k}\dot{X}^2_{gf} - b_{3k}\dot{X}^3_{gf}]$$

where values of the vehicle attitude angle and velocity components with subscript 'f' are determined at the beginning of this segment or at staging, whichever is the last occurring event, t is the time, t_{Bfk} is the value of time when Ty=4 flight begins or is zero at staging, and τ_{fk} is the input pitch flare-in time constant for the k-th stage.

(1) Commanded Yaw Attitude

$$\psi_m = - (180/\pi) K_{yk} [\tau_{yk} \dot{Y}_{gg} + Y_{gg}]$$

$$\dot{\psi}_m = - (180/\pi) K_{yk} [\tau_{yk} \ddot{Y}_{gg} + \dot{Y}_{gg}]$$

(2) Command Roll Attitude

$$\phi_m = \dot{\phi}_m = 0$$

d. Rail Launch ($T_Y = 6$)

When $T_y = 6$ during the j-th segment of flight, the vehicle flight path will simulate flying down a launcher rail or tube.

The linear momenta equations are set as

$$\begin{bmatrix} \ddot{X}_{ee} \\ \ddot{Y}_{ee} \\ \ddot{Z}_{ee} \end{bmatrix} = [D] \begin{bmatrix} A_{ax} \\ 0 \\ 0 \end{bmatrix}$$

where

$$A_{ax} = \text{MAX} \begin{bmatrix} 0 \\ a_{xo}\, \bar{g}_e - 2\omega\, \dot{Y}_{ee} \sin \rho_L - g_{xe} \end{bmatrix}$$

where the quantities are delineated in section B.1.

set

$$\theta_b = \theta_m$$

$$\psi_b = \psi_m$$

$$\begin{Bmatrix} P_m \\ Q_m \\ R_m \end{Bmatrix} = [D] \begin{Bmatrix} \omega \\ 0 \\ 0 \end{Bmatrix}$$

$$\dot{\theta}_m = \frac{(Q_m \cos \phi_m - R_m \sin \phi_m)}{\cos \psi_m}$$

$$\dot{\psi}_m = Q_m \sin \phi_m + R_m \cos \phi_m$$

$$\dot{\phi}_m = P_m - \dot{\theta}_m \sin \psi_m$$

The distance traveled along the rail is

$$l_{lr} = [(X_{ee} - X^*_{ee})^2 + (Y_{ee} - Y^*_{ee})^2 + (Z_{ee} - Z^*_{ee})^2]^{\frac{1}{2}}$$

where l_{lr} is assigned a "P" number 5113 and

$$X^*_{ee} = X_{eeo} + \dot{X}_{eeo}(t - t_o)$$

$$Y^*_{ee} = Y_{eeo} + \dot{Y}_{eeo} \, (t-t_o)$$

$$Z^*_{ee} = Z_{eeo} + \dot{Z}_{eeo} \, (t-t_o)$$

where X_{eeo}, Y_{eeo}, Z_{eeo}, and t_o are the initial values of X_{ee}, and Y_{ee}, Z_{ee}, and t at the beginning of the segment of flight.

e. Constant Altitude ($T_y = 8$)

The constant altitude type of flight will command the vehicle so as to fly the missile to a specified altitude and then maintain that altitude. The directed flight path azimuth will be down the generalized coordinates $X_{gg} - Z_{gg}$ plane. The transition of the vehicle initial altitude to the desired altitude is accomplished by a second order transfer function characterized by an input altitude control frequency and damping ratio.

When $T_y = 8$ during the j-th segment of flight, the vehicle attitude is obtained from instantaneous values of no-wind total angle of attack, roll angle of total angle of attack orientation, bank angle, altitude distance, rate, body angular attitude, and acceleration, as well as from input values, command altitude, altitude control frequency, and damping ratio.

The commanded angle of attack and angle of side slip is defined as follows:

(1) Commanded Angle of Attack

$$\alpha_c = \begin{cases} 45° + \Delta_{ak} & \text{If } \alpha_c^* > 45° \\ -45° + \Delta_{ak} & \text{If } \alpha_c^* < -45° \\ \alpha_c^* + \Delta_{ak} & \text{Otherwise} \end{cases}$$

where

$$\begin{aligned} \alpha_c^* = \alpha_l &+ [(h_c - h) K_z - \dot{h} K_{\dot{z}} - \ddot{h} K_{\ddot{z}}] \cos \varphi_c \\ &+ [Y_{gg} K_z + \dot{Y}_{gg} \dot{K}_z + \dot{Y}_{gg} K_{\ddot{z}}] \sin \varphi_c \\ &- K_{\ddot{\theta}} Q_b \cos (\varphi - \varphi_c) - K_{\ddot{\psi}} \dot{R}_b \sin (\varphi - \varphi_c) \end{aligned}$$

and the angle of attack exponential flair-in factor is determined as follows:

$$\Delta_{ak} = \begin{cases} \Delta'_{ak} \exp \{ \quad (t_{Bf} - t) \quad [\omega_z/(2\ \zeta_z)] & \text{If } \omega_z > 0 \\ 0 & \text{Otherwise} \end{cases}$$

where

$$\Delta'_{ak} = [\alpha^*_{cf} - \alpha_{lf}]$$

and the angle of attack rotated to the commanded vertical is

$$\alpha_\ell = \arctan \{[\tan \tilde{\alpha}']/[\sec (\varphi - \varphi_c + \bar{\epsilon})]\}$$

where the values of the vehicle trajectory parameters with subscript "f" are determined at the beginning of this segment of flight type control.

(2) Commanded Angle of Side Slip

$$\beta_c = \begin{cases} 45 + \Delta_{bk} & \text{If } \beta^*_c > 45° \\ -45 + \Delta_{bk} & \text{If } \beta_c < -45 \\ \beta^*_c + \Delta_{bk} & \text{Otherwise} \end{cases}$$

where

$$\begin{aligned} \beta^*_c = \beta_\ell &+ [Y_{gg}\ K_z + \dot{Y}_{gg}\ K_{\dot{z}} + \ddot{Y}_{gg}\ K_{\ddot{z}}] \cos \varphi_c \\ &- [(h_c - h)\ K_z - \dot{h}\ K_{\dot{z}} - \ddot{h}\ K_{\ddot{z}}] \sin \varphi_c \\ &+ K_{\ddot{\theta}}\ Q_b \sin (\varphi - \varphi_c) - K_{\ddot{\psi}}\ R_b \cos (\varphi - \varphi_c) \end{aligned}$$

and the angle of side slip exponential flare-in factor is determined as follows:

$$\Delta_{bk} = \begin{cases} \Delta'_{bk} \exp \{ (t_{Bf} - t) \quad [\omega_z/(2 \zeta_z)] & \text{If } \omega_z > 0 \\ 0 & \text{Otherwise} \end{cases}$$

and the angle of sideslip in the horizontal is:

$$\beta_\ell = \arctan \{[(\tan \bar{\alpha}')]/[\csc (\varphi - \varphi_c + \bar{\epsilon})]\}$$

where

$$\Delta'_{bk} = [\beta_{\ell f} - \beta^*_{cf}]$$

where the values of the vehicle trajectory parameters with subscript "f" are determined at the beginning of this segment of flight type control.

The above gain are defined as:

$$K_z = \omega_z^2/G_z$$

$$K_{\dot{z}} = 2\zeta_z|\omega_z|G_z$$

$$K_{\ddot{z}} = 1/G_z$$

where the values of h_c, ω_z, and ζ_z are input in the attitude control table in the locations of Q_{mj}, R_{mj}, and P_{mj}, respectively for the j-th type of flight row and the variable gain factor is determined as:

$$G_z = (\pi/180)[\ddot{a}_{Xb}\,\bar{g}_e + N_{PEA}\,g_e/W]$$

$$K_{\ddot{\theta}} = \begin{cases} 0 \text{ if } M_y = 1 \text{ or } 4 \\ N_{PCD}\,I_{YY}/(N_{NVA}M_{PCD} + N_{PCD} \cdot M_{PDA}) & \text{Otherwise} \end{cases}$$

Pitch aerodynamic control normal force per radian fin deflection angle

$$N_{PAC} = (180/\pi)\;C_{Lz}\;q\;S_{Fz}$$

Pitch aerodynamic disturbing normal force per radian angle of attack

$$N_{PAD} = (180/\pi)\;C_{N1}\;q\;S_{RN}\;\bar{\tilde{N}}$$

Pitch total thrust control moment per radian TVC deflection angle

$$M_{PTC} = F\,\ell_e$$

Pitch main thrust control moment per radian TVC deflection angle

$$M_{PMC} = F_H\;\ell_e$$

Pitch aerodynamic control moment per radian fin deflection angle

$$M_{PAC} = N_{PAC} \cdot \ell_{\delta}$$

Pitch aerodynamic disturbing moment per radian angle of attack

$$M_{PAD} = N_{PAD} \cdot \ell_{cp}$$

Total pitch control moment per radian deflection angle

$$M_{PCD} = \begin{cases} K_{cf} \cdot M_{PAC} + M_{PTC} & \text{if } K_{\delta} = 0 \text{ or } K_{\delta} > 2 \\ K_{cf} \cdot M_{PAC} + M_{PMC} & \text{if } K_{\delta} = 1 \\ M_{PAC} & \text{if } K_{\delta} = 2 \end{cases}$$

Total pitch disturbing moment per radian angle of attack

$$M_{PDA} = M_{PAD} - M_{PAC}$$

Total pitch control normal force per radian deflection angle

$$N_{PCD} = \begin{cases} K_{cf} \; N_{PAC} + F & \text{if } K_{\delta} = 0 \\ K_{cf} \; N_{PAC} + F_{M} & \text{if } K_{\delta} = 1 \\ N_{PAC} & \text{if } K_{\delta} = 2 \end{cases}$$

Total pitch disturbing normal force per radian angle of attack

$$N_{PDA} = N_{PAD} + N_{PAC}$$

Pitch trim normal force per radian angle of attack

$$N_{PEA} = \begin{cases} N_{PDA} & \text{if } M_{y} = 1 \text{ or } 4 \\ N_{PDA} + (M_{PDA}/M_{PCD})N_{PCD} & \text{Otherwise} \end{cases}$$

Force normal to velocity vector per radian angle of attack

$$N_{NVA} = a_{Xb} W + N_{PDA}$$

$$K_{\ddot{\psi}} = \left(I_{ZZ}/I_{YY}\right) K_{\ddot{\theta}}$$

Commanded bank angle

$$\varphi_c = 0$$

f. Constant Normal Load Factor ($T_y = 9$)

The programmed normal load factor type of flight will command the vehicle attitude so as to fly the missile with a specified acceleration in g's normal to the earth reference velocity vector. The transition from the initial load factor to the desired load factor is accomplished by a first order transfer function characterized by an input load factor flare-in constant.

The values of η_{cn}, η_{ct}, and φ_c are input in the attitude control table in the Q_{mj}, R_{mj}, and P_{mj} locations.

Defining the achieved load factor along the velocity vector is:

$$\begin{Bmatrix} \eta_{ba} \\ -\eta_{bt} \\ -\eta_{bn} \end{Bmatrix} = [A_\beta]^{-1} [A_\alpha]^{-1} \{\vec{\ddot{X}}_b\} \; 1/\bar{g}_e$$

where η_{ba} is the acceleration along the velocity vector, η_{bt} is the acceleration transverse to the velocity vector, and η_{bn} is the acceleration normal to the velocity vector.

The achieved load factor along the velocity vector is: (in commanded local roll coordinates).

$$\eta_{lt} = \eta_{bt} \cos (\varphi - \varphi_c) - \eta_{bn} \sin (\varphi - \varphi_c)$$

$$\eta_{ln} = \eta_{bt} \sin (\varphi - \varphi_c) + \eta_{bn} \cos (\varphi - \varphi_c)$$

The commanded angle of attack

$$\alpha_c = \begin{cases} 45° + \Delta_{ak} & \text{If } \alpha_c^* > 45° \\ -45° + \Delta_{ak} & \text{If } \alpha_c^* < -45° \\ \alpha_c^* + \Delta_{ak} & \text{Otherwise} \end{cases}$$

where

$${}^{*}_{c} = \alpha_\ell + (\eta_{cn} - \eta_{\ell n}) K_\eta$$
$$- K_{\ddot\theta} Q_b \cos (\varphi - \varphi_c) - K_{\ddot\psi} R_b \sin (\varphi - \varphi_c)$$

and the angle of attack experimental flare-in factor is determined as follows:

$$\Delta_{ak} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ \Delta'_{ak} \exp \; [(t_{Bf} - t)/\tau_f] & \text{Otherwise} \end{cases}$$

$$\Delta'_{ak} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ [\alpha_{\ell f} - \alpha^*_{cf}) & \text{Otherwise} \end{cases}$$

The commanded angle of sideslip

$$\beta_c = \begin{cases} 45° + \Delta_{bk} & \text{If } \beta_c^* > 45° \\ -45° + \Delta_{bk} & \text{If } \alpha_c^* < -45° \\ \alpha_c^* + \Delta_{bk} & \text{Otherwise} \end{cases}$$

where

$$\beta_c^* = \beta_\ell - (\eta_{ct} - \eta_{\ell n}) K_\eta$$
$$+ K_{\ddot\theta} Q_b \sin (\varphi - \varphi_c) + K_{\ddot\psi} R_b \cos (\varphi - \varphi_c)$$

and the angle of side slip exponential flare-in factor is determined as follows:

$$\Delta_{bk} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ \Delta'_{bk} \exp \quad (t'_{Bf} - t) & \text{Otherwise} \end{cases}$$

$$\Delta'_{bk} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ (\beta_{\ell f} - \beta^*_{cf}) & \text{Otherwise} \end{cases}$$

g. Intercept Guidance ($T_y = 10$)

The intercept guidance type of flight will command the vehicle so as to fly the missile on a collision course to intersect with the input moving target coordinates. This type of flight represents a radio command guidance system where the signals are generated by an external facility which keeps tract of the missile and target dynamical characteristics.

When $T_y = 10$ during the j-th segment of flight, the vehicle attitude is obtained from instantaneous values of flight path angle (γ_1), the flight path error to intercept angle (ϵ_{MI}), the flight path azimuth error to intercept (σ_{MI}), the partial derivative of the linear inertial acceleration normal to the velocity vector to the vehicle angle of attack (G_z), the earth gravity and rotation acceleration (g_{Z1}), and vehicle velocity (V_e) as well as the guidance associated first order intercept guidance controller time constant (τ_{MI}), and maximum angle of attack (α_{max}) input in the attitude control table in the Q_{mj} and R_{mj} locations.

The command angles of attack angle of sideslip are defined as follows:

$$\alpha_c = \begin{cases} \alpha'_{IG} + \Delta_{ak} & \text{If } -\alpha_{max} < \alpha'_{IG} < \alpha_{max} \text{ or } \alpha_{max} = 0 \\ \alpha_{max} + \Delta_{ak} & \text{If } \alpha'_{IG} > \alpha_{max} \\ -\alpha_{max} + \Delta_{ak} & \text{If } \alpha_{max} > \alpha'_{IG} \end{cases}$$

where

Commanded angle of attack

$$\alpha^*_c = \begin{cases} 0 & \text{if } G_Z = 0 \\ \epsilon_{MI} + \dfrac{g_{Z1} \cos \gamma_1}{G_Z} & \text{if } \tau_{MI} = 0 \\ \dfrac{(\pi/180)\, V_e\left(\dfrac{\epsilon_{MI}}{\tau_{MI}} + \dot{\sigma}_{MT}\right) + g_{Z1} \cos \gamma_1}{G_Z} & \text{if } \tau_{MI} \neq 0 \end{cases}$$

$$\beta_c = \begin{cases} \beta'_{IG} + \Delta_{bk} & \text{If } -\alpha_{max} < \beta^*_{IG} < \alpha_{max} \text{ or } \alpha_{max} = 0 \\ \alpha_{max} + \Delta_{bk} & \text{If } \beta^*_{IG} > \alpha_{max} \\ -\alpha_{max} + \Delta_{bk} & \text{If} -\alpha_{max} > \beta^*_{IG} \end{cases}$$

Commanded angle of side slip

$$\beta^*_c = \begin{cases} 0 & \text{if } G_Z = 0 \\ -\delta_{MI} & \text{if } \tau_{MI} = 0 \\ \dfrac{-(\pi/180)\, V_e \left(\dfrac{\delta_{MI}}{\tau_{MI}} + \dot{\lambda}_{MI}\right)}{G_Z} & \text{if } \tau_{MI} \neq 0 \end{cases}$$

where

G_z is formulated in section C.2.e (Type 8 flight), ϵ_{MI}, $\dot{\sigma}_{MI}$, δ_{MI}, and λ_{MI} are formulated in section J.4.c, "Missile Target Coordinates"

The exponential flare-in factor is determined as follows:

$$\Delta_{ak} = \begin{cases} 0 & \text{If } \tau_{IG} = 0 \\ \Delta'_{ak} \exp\ [(t_{Bf} - t)/\tau_{IG}] & \text{Otherwise} \end{cases}$$

where

$$\Delta'_{ak} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ [\alpha'_{IGf} - \alpha_{If}] & \text{Otherwise} \end{cases}$$

$$\Delta'_{bk} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ \Delta_{bk} \exp\ [(t_{Bf} - t)/\tau_{IG}] & \text{Otherwise} \end{cases}$$

$$\Delta^{2}_{tk} = \begin{cases} 0 & \text{If } \tau_f = 0 \\ (\beta_{\ell f} - \beta^{*}_{cf}) & \text{Otherwise} \end{cases}$$

where the values of the vehicle trajectory parameters with subscript "f" are determined at the beginning of the segment of flight type control.

h. Homing Guidance ($T_y = 11$)

The homing guidance type of flight will command the vehicle so as to fly the missile to a specified moving target. This type of flight is also called proportional navigation steering which commands the time rate change of the missile flight path angle proportional to the turning rate of the look angle between the missile seeker and the target.

When $T_y = 11$ during the j-th segment of flight the vehicle attitude is obtained from instantaneous values of flight path angle (γ_1); time rate change of the angle to target in pitch ($\dot{\alpha}_{MT}$), time rate change of the angle to target in yaw ($\dot{\lambda}_{MT}$) partial derivative of the linear inertial acceleration normal to the velocity vector to the vehicle angle of attack (G_z), the earth gravity and rotation acceleration (R_1), and vehicle velocity (V_e) as well as the navigation constant (K_{HG}), flare-in time factor (τ_{HG}), and maximum angle of attack (α_{max}) input in the attitude control table in the Q_{mj}, R_{mj}, and P_{mj} locations.

Commanded Angle of Attack

$$\alpha_c = \begin{cases} \alpha'_{HG} + \Delta_{ak} & \text{If } -\alpha_{max} < \alpha_c^* < \alpha_{max} \text{ or } \alpha_{max} = 0 \\ \alpha_{max} + \Delta_{ak} & \text{If } \alpha_c^* > \alpha_{max} \\ -\alpha_{max} + \Delta_{ak} & \text{If } \alpha_{max} > \alpha_c^* \end{cases}$$

where

$$\alpha_c^* = \begin{cases} 0 & \text{If } G_z = 0 \\ [K_{HG}\ (\pi/180)\ \dot{\alpha}_{MT}\ V_e + (g_{Z_1} \cos \gamma_1)]/G_z & \end{cases}$$

$$\beta_c = \left[\begin{array}{ll} \beta_c^* + \Delta_{ck} & \text{If } -\alpha_{max} < \beta_c^* < \alpha_{max} \text{ or } \alpha_{max} = 0 \\ \alpha_{max} + \Delta_{ck} & \text{If } \beta_c^* > \alpha_{max} \\ -\alpha_{max} + \Delta_{ck} & \text{If } \alpha_{max} > \beta_c^* \end{array}\right.$$

$$\beta_C^* = \left[\begin{array}{ll} 0 & \text{If } G_z = 0 \\ -\left[K_{HG}\ (\pi/180)\ \dot{\lambda}_{MT} V_e / G_z\right] & \text{Otherwise} \end{array}\right.$$

where

G_z is formulated in paragraph C.2.e, $\dot{\alpha}_{MT}$ and $\dot{\lambda}_{MT}$ are formulated in section J.4.c, "Missile to Target Angles."

The exponential flare-in factor is determined as follows:

$$\Delta_{ak} = \left[\begin{array}{ll} 0 & \text{If } \tau_f = 0 \\ \Delta'_{ak} \exp\ \left[(t_{Bf} - t)\ /\tau_f\right] & \text{Otherwise} \end{array}\right.$$

where

$$\Delta'_{ak} = \left[\begin{array}{ll} 0 & \text{If } \tau_{HG} = 0 \\ (\alpha'_{HG\ell f} - \alpha_{\ell f}) & \text{Otherwise} \end{array}\right.$$

$$\Delta_{bk} = \left[\begin{array}{ll} 0 & \text{If } \tau_f = 0 \\ \Delta'_{bk} \exp\ \left[(t_{Bf} - t)\ /\tau_f\right] & \text{Otherwise} \end{array}\right.$$

where

$$\Delta'_{bk} = \left[\begin{array}{ll} 0 & \text{If } \tau_f = 0 \\ (\beta_c^* - \beta_{\ell f}) & \text{Otherwise} \end{array}\right.$$

where the values of the vehicle trajectory parameters with subscript "f" are determined at the beginning of the segment of flight type control.

3. Thrust Modulation Control Table

The control law type shall be determined from a sequential input table. Each row in this table contains the following ten control variables.

1. F_y thrust control law mode - dim

$F_y = 1$ Specific velocity-time profile

$F_y = 2$ Constant Mach number-time profile

$F_y = 3$ Proportional-to-commanded turning rate profile

$F_y = 4$ Minimum velocity during commanded turn profile

$F_y = 5$ Constrained dynamic pressure profile

$F_y = 6$ Axial acceleration proportional to line-of-sight rate

2.	σ_{Fy}	Code which designates the quanity that determines when the j-th type of TMC ends	dim
3.	κ_{Fy}	Limit of the j-th type of TMC	dbi
4.	TMC	Thrust dynamics mode	dim
	TMC = 0.	Achieved thrust equals commanded thrust	
	TMC = 1.	First order response system (solve $\dot{P}_c$ equation)	
5.	C_{Fy}	Thrust system proportionality system gain	dim
6.	τ_{Fy}	Control system time constant	sec
7.	MIN	Minimum velocity or constant Mach number	dim/ft/sec
8	q_{max}	Maximum allowable dynamic pressure	lb/ft^2

9.	q_{min}	Minimum allowable dynamic pressure	lb/ft^2
10.		Open	

Inputting a maximum of seven segments of thrust mode control per trajectory is possible. Let σ_{Fyj} (j = 1, 2, ..., 7) be the achieved value of a parameter designated by input and let K_{Fy}, be the input value of the parameter, then the j-th segment of thrust modulation control will apply for:

$$\sigma_{Fyj} \neq K_{Fyj}$$

The time when equality in the above occurs will be determined and is defined as t_{Fyj}.When the criteria for leaving a segment has been satisfied, that segment is not re-entered during the trajectory.

The K_{Fyj} criteria are checked monotonically with respect to the j-index. The table is input in the L800-L869 region.

The following discussion provides a description of the control laws which will be used to determine the control thrust command, F_c. These control laws are based upon the commanded thrust being proportional to the error in the control variable modified with a signal proportional to the control variable rate of change. The control gains will be determined internally within the program unless input by the program user.

a. Specific Velocity-Time Profile

If Fy = 1, the flight for a preprogramed velocity-time profile will be established from specifying the command velocity history in tabular form. This tabular input will be in the L-number region L870-L899. The time coordinate t_{VPj} will be input in L870, L872, ..., L898 and the velocity V_{Pj} in L871, L873, ..., L899. This corresponds to a maximum of 15 points. If the time is less than or greater than the first or last points in the table, the corresponding end point velocity are used. The points in the input velocity-time table are fit with a third order quadratic function, and the velocity and its time derivative are evaluated as V_{ecv} and $\dot{V}_{ecv}$ respectively.

To minimize the collective system error and provide the desired velocity history, the following control equation for the thrust required to provide the desired acceleration is:

$$F_c = \hat{F}_c + K_{cv} (V_{ecv} - V_c) + m \dot{V}_{ecv}$$

where V_{ecv} is the command velocity, V_e is the achieved velocity, K_{cv} is a velocity error gain, $\hat{F}_c$ is the thrust required to maintain V_e a constant value, m is the missile mass and $\dot{V}_{ecv}$ is the time rate change of the command velocity. The vehicle mass divided by the velocity error gain magnitude represents the time required to eliminate the accumulated velocity error. The automatic control of this gain will be

$$K_{cv} = m/\tau_{CF}$$

where m is the missile mass and τ_{CF} represents the first order lead constant which will be set proportional to the equivalent first order lag of the controllable motor or to the input value.

$$\tau_{CF} = \begin{cases} 5/\omega_p & \text{If } \tau_{Fy} = 0 \\ \tau_{Fy} & \text{Otherwise} \end{cases}$$

where ω_p is the pintle control frequency and τ_{Fy} is control system time constant input in the thrust modulation control table.

The thrust required to maintain V_e, i.e., retarding force is:

$$\hat{F}_c = C + C_{\delta z} - (N_{Py} - N_Y - N_{\delta y}) \sin \bar{\beta}/\cos \bar{\alpha}'$$

$$- (N_{Pz} - N_Z - N_{\delta Z}) \sin \bar{\alpha}/\cos \bar{\alpha}'$$

$$+ g_{z1} \sin \gamma_1 m/\cos \bar{\alpha}'$$

where C, N_Y and N_Z are the aerodynamic force components, $N_{\delta Y}$ and $N_{\delta Z}$ are the control fins normal force in yaw and pitch respectively. $C_{\delta Z}$ is the control fin axial force, $\bar{\alpha}$ is the still wind angle of attack, $\bar{\beta}$ is the still wind angle of yaw, and $\bar{\alpha}'$ is the still wind total angle of attack. N_{Py} and N_{Pz} are the aerodynamic normal forces due to yaw and pitch damping.

b. Constant Mach Number Flight

If Fy = 2, a constant Mach number flight path is utilized in a cruise type vehicle such as SRAM. Flying constant Mach number can be used to maximize the vehicle lift to drag for a maximum range glide trajectory.

The command acceleration for constant Mach number flight is:

$$\dot{V}_{ecm} = V_e M \frac{d\ Ca}{dh} \sin \gamma_1$$

where V_e is the achieved missile velocity, M is the achieved missile Mach number, d Ca/dh is the rate change of the speed of sound with change in altitude, and γ_1 is the flight path angle. The command velocity is:

$$V_{ecm} = M_c C_a$$

where M_c is the command Mach number input in the thrust modulation control table as MIN and C_a is the instantaneous speed of sound.

To provide the transition to achieve initially required steady state Mach number and to minimize the collective system error, the control law presented in Fy = 1 is used, i.e.,

$$F_c = \hat{F}_c + K_{cv} (V_{ecm} - V_e) + m \dot{V}_{ecm}$$

c. Proportional-to-Commanded Turning Rate

If Fy = 3, the thrust level will be commanded proportional to the attitude command turning rate. For air-to-air missiles, the guidance and control logic is benefited by a propulsion system in which the time rate change of flight path is proportional to the seeker look angle rate. The trajectory control forces to provide the time rate change of flight path angle are those normal to the missile flight path angle. Current air-to-air missile control the angular rate change by pulling angle of attack, which provides an aerodynamic lift force.

A different, and possibly more desirable, method of providing the forces to change the flight path angle is by TVC system and propulsion thrust, or a combination of lift and TVC. The thrust magnitude proportional-to-commanded turning rate can be used in simulation of advanced air-to-air missile TVC and TMC propulsion systems.

The control equation is as follows:

$$F_c = (\pi/180)\, K_{CPR}\, F_N\, (\dot{\theta}_m^2 + \dot{\psi}_m^2)^{\frac{1}{2}}$$

where $\dot{\theta}_m$ is the commanded pitch rate, $\dot{\psi}_m$ is the commanded yaw rate, F_N is the nominal delivered thrust and K_{CPR} is the thrust magnitude proportional to commanded turning rate system gain. This gain will be input in the thrust mode control table as C_{Fyj}.

d. Minimum Velocity

If Fy = 4, the trajectory will use the thrust time history input into the trajectory program when the value of velocity is greater than the minimum specified value; if the input thrust produces velocity less than the specified value, the thrust will be modulated to a value adequate to maintain the missile specified minimum velocity. The following logic will be used to perform this control. The command thrust is

$$F_c = \begin{cases} F_N & \text{If } V_{cv} < V_e \\ \hat{F}_c + K_{cv}\,(V_{cv} - V_e) & \text{If } V_{cv} \geq V_e \end{cases}$$

The control law presented in Fy = 1 is that used for V_{cv}, ≥ V_e, and V_{cv} is the imput minimum velocity value set in the TMC table as MIN. F_N is the nominal thrust.

e. Constrained Dynamic Pressure Profile

If Fy = 5, the thrust shall be commanded to constrain the dynamic pressure between an upper and lower value input in the TMC cable as q_{min} and q_{max}. The missile structural loads are established by the forces and moment on the body during flight in the atmosphere. These forces result from thrust and aerodynamics. One of the severest structural loads is the bending moments caused by aerodynamic lift which is proportional to the dynamic pressure, and the angle of attack. Thus, if the dynamic pressure were to be constrained to be less than some specified value by using thrust magnitude controls, the missile critical loads could be controlled. For air-to-air missiles depending on aerodynamic surfaces for lift and for control; a desirable property of the propulsion system is that it be capable of providing enough thrust to keep the vehicle from "stalling out". A thrust magnitude control to guarantee a minimum dynamic pressure will prevent this stall condition. The control equations for the specified thrust for a constrained dynamic pressure are:

$$F_{cqmin} = \hat{F}_c + K_{cq} (q_{min} - q) + m \dot{V}_{ccq}$$

$$F_{cqmax} = \hat{F}_c + K_{cq} (q_{max} - q) + m V_{ccq}$$

$$F_c = \begin{cases} F_N & \text{If } q_{min} \leqq q \text{ and } F_N \geqq F_{cqmin} \\ & \text{or } q_{max} \geqq q \quad \text{and } F_N \leqq F_{cqmax} \\ F_{cqmin} & \text{If } q < q_{min} \text{ or } F_N < F_{cqmin} \\ F_{cqmax} & \text{If } q > q_{max} \text{ or } F_N > F_{cqmax} \end{cases}$$

The command acceleration to constrain dynamic pressure is:

$$\dot{V}_{ecq} = \left(\frac{d C_a}{dh} M - \frac{d P_a}{dh} \frac{V_e}{2P_a}\right) V_e \sin \gamma_1$$

where $d\,C_a/dh$ is the change in the speed of sound with change in altitude, $d\,P_a/dh$ is the change in atmospheric pressure with change in altitude; both are a function of altitude and are evaluated in the Atmosphere Parameter (F-1) routine. M is the instantaneous Mach number, P_a is the instantaneous atmospheric pressure, V_e is the instantaneous velocity and γ_1 is the instantaneous flight path angle, where $\hat{F}_c$ is the thrust required to maintain V_e delineated in Fy = 1 logic, F_N is the nominal delivered thrust and K_{cq} is the dynamic pressure error gain.

This gain error term will null the residual dynamic pressure error. The dynamic pressure error gain is:

$$K_{cq} = \frac{V_e\, m}{2\, q\, \tau_{cq}}$$

the time constant is:

$$\tau_{cq} = \begin{cases} 5/\omega p & \text{If } \tau_{Fy} = 0 \\ \tau_{Fy} & \text{Otherwise} \end{cases}$$

ω_p is the control frequency of the controllable motor and τ_{Fy} is the control system gain input in the TMC table.

f. Axial Acceleration Proportional to Line-of-Sight Rate

If $F_y = 6$, the thrust shall be commanded so that the axial acceleration is proportional to the line-of-sight rate between the attacking missile and its target. In addition, the missile-to-target distance rate shall be maintained less than a specified value. For an air-to-air missile, complex guidance and control logic is greatly enhanced by a propulsion system in which the axial acceleration is proportional to the line-of-sight rate between the missile and the target, subject to the constraint of minimum closing rate and constrained dynamic pressure bond,

The basic philosophy of this mode of TMC is that the flight path changes are needed only when changes in the missile-target collision flight path are needed. Thus, if the missile is coasting towards the target, no thrust is needed except that to maintain closing rate. The trajectory control forces that provide time rate change of flight path angle are those normal to the missile flight path angle. Thus, when the target performs evasive maneuvers, power must be applied to change the missile flight path. Current air-to-air missiles control the angular rate change by pulling angle of attack, which provides an aerodynamic lift force. Advanced systems will provide the side force by attitude changes with TVC systems. Both systems are greatly improved by increased thrust during the turn. The constraint of dynamic pressure will assure, on the lower limit, that enough aerodynamic control force is available to provide control and the upper limit, that the aerodynamic bias on the vehicle will not cause structural failure. This type of thrust modulation control is essentially a mixture of (3) proportional-to-commanded rate profile, (4) minimum velocity during command turn profile, ad (5) constrained dynamic pressure profile.

The commanded thrust to provide acceleration proportional to LOS rate is

$$F_{cALOS} \text{ [illegible] } V_c \, m \, (\pi/180) \, (\dot{\sigma}_{MT}^{2} + \dot{\lambda}_{MT}^{2})^{1/2} + \hat{F}_c$$

where K_{ALOS} is the system gain set equal to C_{Fyj} in the thrust modulation control table. V_c is the instantaneous missile velocity, m is the missile mass, $\dot{\sigma}_{MT}$ and $\dot{\lambda}_{MT}$ are the LOS rates in pitch and yaw respectively, and $\hat{F}_c$ is the thrust required to maintain constant velocity delineated in Fy = 1.

The command thrust to provide a minimum missile to target closing rate is:

$$F_{cclos} = \hat{F}_c + K_{cv} \, (\dot{R}_{MTmin} - \dot{R}_{MT})$$

where K_{cv} is the velocity error gain delineated in Fy = 1, $\dot{R}_{MTmin}$ is the minimum closing velocity input in the thrust modulation control table as MIN, and $\dot{R}_{MT}$ is the time rate change of the missile to target distance.

The commanded thrust to maintain the dynamic pressure is:

$$F_{cqmin} = \hat{F}_c + K_{cq} \, (q_{min} - q) + m \, \dot{V}_{ccq}$$

and the commanded thrust to maintain the dynamic pressure below the upper constraint is:

$$F_{cqmax} = \hat{F}_c + K_{cq} \, (q_{max} - q) + m \, \dot{V}_{ccq}$$

where the control laws and therms are the same as those given in Fy = 5, constrained dynamic pressure thrust profile.

The commanded thrust for Fy = 6 is as follows:

$$F_{clos} = \begin{cases} F_{cALOS} & \text{If } F_{cALOS} > F_{cclos} \\ F_{cclos} & \text{Otherwise} \end{cases}$$

$$F_c = \begin{cases} F_{cqmin} & \text{If } F_{cqmin} > F_{clos} \\ F_{cqmax} & \text{If } F_{cqmax} < F_{clos} \\ F_{clos} & \text{Otherwise} \end{cases}$$

D. ATTITUDE CONTROL SYSTEM

For point mass systems (My = 1 or 4) the achieved attitude is identical to the commanded attitude, or

$$\theta_b = \theta_m$$

$$\psi_b = \psi_m$$

$$\phi_b = \dot{\phi}_m$$

and all other attitude control system calculations are ignored.

For finite-mass systems (My = 2 or 5), the achieved missile attitude (the relation between the b and i systems as shown in Figure 18) is a function of the angular velocities about the missile center-of-gravity:

$$\dot{\theta}_b = (Q_b \cos \phi_b - R_b \sin \phi_b) / \cos \psi_b$$

$$\dot{\psi}_b = Q_b \sin \phi_b + R_b \cos \phi_b$$

$$\dot{}_b = P_b - \dot{\theta}_b \sin \psi_b$$

Initial values of θ_b, ψ_b and ϕ_b are obtained from input.

If My equals 1 or 4 and Ty = 8, 9, 10, or 11 at the end of the compute interval, set:

$$\dot{\theta}_m = (\theta_{m(j+1)} - \theta_{mj}) / \Delta t_{cj}$$

$$\dot{\psi}_m = (\psi_{m(j+1)} - \psi_{mj}) / \Delta t_{cj}$$

$$\dot{\phi}_m = (\phi_{m(j+1)} - \phi_{mj}) / \Delta t_{cj}$$

where Δt_{cj} is the compute interval

$$P_m = \dot{\phi}_m + \dot{\theta}_m \sin \psi_m$$

$$Q_m = \dot{\theta}_m \cos \psi_m \cos \phi_m + \dot{\psi}_m \sin \phi_m$$

$$R_m = \dot{\psi}_m \cos \phi_m - \dot{\theta}_m \cos \psi_m \sin \phi_m$$

$$P_b = P_m$$

$$Q_b = Q_m$$

$$R_b = R_m$$

1. PITCH AND YAW THRUST DEFLECTIONS

Thrust deflections are functions of missile attitude errors, angular velocities, and steady state errors and whether the simulated thrust vector control system is for a first or second order transfer function.

a. Attitude Error Angles--The pitch ($\Delta\theta_b$) and yaw ($\Delta\psi_b$) attitude errors shown in Figure23 are the difference between the achieved and desired vehicle attitude and are formulated as follows:

$$\Delta\theta_b = \arcsin (-K_{13})$$

$$\Delta\psi_b = \arcsin K_{12}$$

with

$$-90° \leqq \Delta\theta_b, \Delta\psi_b \leqq 90°$$

where K_{13} and K_{12} are the elements of the K-matrix defined by:

$$\begin{bmatrix} K \end{bmatrix} = \begin{bmatrix} A_m \end{bmatrix}^{-1} \begin{bmatrix} A_t \end{bmatrix}$$

b. Autopilot Control Law--The control loop diagram depicting those control laws are shown on Figure 22.

The command deflection signals are determined from

$$\delta_{Pc} = K_{DP}\ \Delta\theta_b - K_{RP}Q_b - K_{IP}\,\alpha$$

and

$$\delta_{Yc} = -K_{DY}\ \Delta\psi_b + K_{RY}R_b + K_{IY}\,\beta$$

where $K_{DP,\ Y}$ and $K_{RP,\ Y}$ are the attitude error and rate gains, respectively.

c. TVC Deflection Angle Achieved to Commanded Transfer Function--The thrust vector control system can be represented by a zero order, first order, or a second order transfer function. The second order transfer function is needed if the tail-wag-dog effect of a gimbal nozzle is to be simulated. If $\bar{I}_y = 0$ or $\bar{I}_z = 0$, quasi-finite mass calculations are used for δ_P and δ_Y respectively.

The following differential equations and logic are employed

1. Zero order

$$\left(\omega_c = \tau_c = 0\right)$$

$$\delta_P = \delta_{Pc},\ \ddot{\delta}_P = 0\ (\bar{I}_Y \neq 0)$$

$$\delta_Y = \delta_{Yc},\ \ddot{\delta}_Y = 0\ (\bar{I}_Z \neq 0)$$

$\dot{\delta}_P$ and $\dot{\delta}_Y$ are obtained by numerically differentiating δ_P and δ_Y

2. First order

$$\left(\omega_c = 0 \text{ and } \tau_c \neq 0\right)$$

$$\tau_c\ \dot{\delta}_P + \delta_P = \delta_{Pc},\ \text{and}\ \ddot{\delta}_P = 0\ (\bar{I}_Y \neq 0)$$

$$\tau_c\ \dot{\delta}_Y + \delta_Y = \delta_{Yc},\ \text{and}\ \ddot{\delta}_Y = 0\ (\bar{I}_Z \neq 0)$$

3. Second order

$(\omega_c \neq 0 \text{ and } I_n = 0)$

$$\ddot{\delta}_P + 2.0\, \xi_c\, \omega_c\, \dot{\delta}_P + \omega_c^2\, \delta_P = \omega_c^2\, \delta_{Pc}\ (\bar{I}_Y \neq 0)$$

$$\ddot{\delta}_Y + 2.0\, \xi_c\, \omega_c\, \dot{\delta}_Y + \omega_c^2\, \delta_Y = \omega_c^2\, \delta_{Yc}\ (\bar{I}_Z \neq 0)$$

4. Second order, including tail-wag-dog effect $(\omega_c \neq 0,\ I_n \neq 0)$

$$\ddot{\delta}_P + 2.0\, \xi_c\, \omega_c\, \delta_P + \omega_c^2\, \delta_P = \omega_c^2\, \delta_{Pc} + Q_b \left|1.0 + [W_n\, \ell_e\, \ell_n/(I_n\, \bar{g}_e)]\right|$$

$$+ W_n\, \ell_n\, Z_{bb}\, (180/\pi)/(g_e\, I_n)\ \ (\bar{I}_Y \neq 0)$$

$$\ddot{\delta}_Y + 2.0\, \xi_c\, \omega_c\, \dot{\delta}_Y + \omega_c^2\, \delta_Y = \omega_c^2\, \delta_{Yc} + \dot{R}_b \left|1.0 + [W_n\, \ell_e\, \ell_n/(I_n\, \ddot{g}_e)]\right|$$

$$+ W_n\, \ell_n\, \ddot{Y}_{bb}\, (180/\pi)/(\bar{g}_e\, I_n)\ (\bar{I}_Z \neq 0)$$

Values of δ_{Po} and δ_{Yo} and $\dot{\delta}_{Po}$ and $\dot{\delta}_{Yo}$, the initial values of the pitch and yaw thrust vector deflection angles and angular rates, are obtained from input per stage or equated to the values existing at the end of the previous mode segment.

If $\bar{I}_Y = 0$, the logic and equations used to determine δ_P are:

$\ddot{\delta}_P = 0 \qquad \dot{\delta}_P = 0$

$$\delta_P = (-\delta_{MP}) + (180/\pi) \left[\begin{array}{ll} M_{DQ}/M_{PTC} & \text{If } K_\delta = 0 \text{ and } K_{Pcf} = 0 \\ M_{DQ}/M_{PMC} & \text{If } K_\delta = 1 \text{ and } K_{Pcf} = 0 \\ 0 & \text{If } K_\delta = 2 \text{ and } K_{Pcf} = 0 \\ M_{DQ}/(M_{PTC} + M_{PAC} \cdot K_{Pcf}) & \text{If } K_\delta = 0 \text{ and } K_{Pcf} \neq 0 \\ M_{DQ}/(M_{PMC} + M_{PAC} \cdot K_{Pcf}) & \text{If } K_\delta = 1 \text{ and } K_{Pcf} \neq 0 \\ M_{DQ}/M_{PAC} & \text{If } K_\delta = 2 \text{ and } K_{Pcf} \neq 0 \\ & \text{and } h \leq h_E \text{ and } h_E \neq -1 \\ 0 & \text{If } K_\delta = 2 \text{ and } K_{Pcf} \neq 0 \\ & \text{and outside the atmosphere} \end{array} \right.$$

If $\bar{I}_Z = 0$ and My = 5, the logic and equations used for δ_Y are:

$\ddot{\delta}_Y = 0, \qquad \dot{\delta}_Y = 0$

$$\delta_Y = (-\delta_{MY}) + (180/\pi) \begin{cases} M_{DR}/M_{PTC} & \text{If } K_\delta = 0 \text{ and } K_{Ycf} = 0 \\ M_{DR}/M_{PMC} & \text{If } K_\delta = 1 \text{ and } K_{Ycf} = 0 \\ G & \text{If } K_\delta = 2 \text{ and } K_{Ycf} = 0 \\ M_{DR}/(M_{PTC} + M_{YAC} \cdot K_{Ycf}) & \text{If } K_\delta = 0 \text{ and } K_{Ycf} \neq 0 \\ M_{DR}/(M_{PMC} + M_{YAC} - K_{Ycf}) & \text{If } K_\delta = 1 \text{ and } K_{Ycf} \neq 0 \\ M_{DR}/M_{YAC} & \text{If } K_\delta = 2 \text{ and } K_{Ycf} \neq 0 \\ & \text{and } h \leq h_E \text{ and } h_E \neq -1 \\ 0 & \text{If } K_\delta = 2 \text{ and } K_{Ycf} \neq 0 \\ & \text{and outside the atmosphere} \end{cases}$$

Where ω_c is the input stage control system forcing frequency for the thrust vector deflection second order transfer function, τ_c is the input stage control system time constant for the thrust vector deflection first-order transfer function, ζ_c is the input stage control system damping ratio for the thrust vector deflection second order transfer function, δ_{Pc} and δ_{Yc} are the pitch and yaw thrust deflection angle commands, δ_P & δ_Y, $\dot{\delta}_P$ & $\dot{\delta}_Y$, and $\ddot{\delta}_P$ & $\ddot{\delta}_Y$ are the pitch and yaw thrust deflection angles, angular rates, and angular accelerations, respectively, W_n is the input stage movable portion nozzle weight, l_n is the movable portion nozzle center of gravity gimbal point distance, I_n is the input stage movable portion nozzle moment of inertia about the gimbal point, $\dot{Q}_b$ and $\dot{R}_b$ are the pitch and yaw missile attitude angular acceleration, $\ddot{Y}_{bb}$ and $\ddot{Z}_{bb}$ are the inertial missile acceleration positive along the pitch axis and yaw axis, [illegible] the missile gimbal point to center of gravity distance.

d. Autopilot Gains--Three gain zones are available for the control system of each stage. Zone durations are as follows:

from stage initiation until $\sigma_{G1} = K_{G1}$	Zone 1
until $\sigma_{G2} = K_{G2}$	Zone 2
thereafter to stage termination	Zone 3

where σ_{Gi} (i = 1, 2) are the achieved values of quantities designated [illegible] code input and K_{Gi} (i = 1, 2) are the input zone limits. The time [illegible]n equality occurs is always computed.

The control system gains per gain zone and stage are as follows:

Pitch Rotation Reaction Moment

$$M_{PRR} = I_{YY}\,\omega_v^2 + |M_{PAD} - M_{PAC}|$$

Yaw Rotation Reaction Moment

$$M_{YRR} = I_{ZZ}\,\omega_v^2 + |M_{YAD} - M_{YAC}|$$

Pitch Rotation Damping Moment Integral

$$I_{PRD} = 2\,\zeta_v\,I_{YY}\,\omega_v$$

Yaw Rotation Damping Moment Integral

$$I_{YRD} = 2\,\zeta_v\,I_{ZZ}\,\omega_v$$

Pitch and Yaw Total Thrust Control Moment per Radian TVC Deflection Angle

$$M_{PTC} = F \ell_e$$

Pitch and Yaw Main Thrust Control Moment per Radian TVC Deflection Angle

$$M_{PMC} = F_M \ell_e$$

Pitch Aerodynamic Control Moment per Radian Fin Deflection Angle

$$M_{PAC} = (180/\pi) C_{Lz} q S_{Fz} \ell_\delta$$

Yaw Aerodynamic Control Moment per Radian Fin Deflection Angle

$$M_{YAC} = M_{PAC}$$

Pitch Aerodynamic Disturbing Moment per Radian Angle of Attack

$$M_{PAD} = (180/\pi) C_{N1} q S_{RN} \bar{N} \ell_{cp}$$

Yaw Aerodynamic Disturbing Moment per Radian Angle of Attack

$$M_{YAD} = M_{PAD}$$

Pitch Attitude Error Gain

$$K_{DP} = \left[\begin{array}{ll} 0 & \text{If } \bar{I}_Y = 0 \\ M_{PRR}/M_{PTC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Pcf} = 0 \\ M_{PRR}/M_{PMC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Pcf} = 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Pcf} = 0 \\ M_{PRR}/(M_{PTC} + M_{PAC} \cdot K_{Pcf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Pcf} \neq 0 \\ M_{PRR}/(M_{PMC} + M_{PAC} \cdot K_{Pcf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Pcf} \neq 0 \\ M_{PRR}/M_{PAC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Pcf} \neq 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Pcf} \neq 0 \\ & \text{and } V_a = 0 \\ K'_{DPi} & \text{Otherwise} \end{array}\right.$$

Yaw Attitude Error Gain

$$K_{DY} = \left[\begin{array}{ll} 0 & \text{If } \bar{I}_Z = 0 \\ M_{YRR}/M_{PTC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Ycf} = 0 \\ M_{YRR}/M_{PMC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Ycf} = 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Ycf} = 0 \\ M_{YRR}/(M_{PTC} + M_{YAC} \cdot K_{Ycf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Ycf} \neq 0 \\ M_{YRR}/(M_{PMC} + M_{YAC} \cdot K_{Ycf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Ycf} \neq 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Ycf} \neq 0 \\ & \text{and } V_a = 0 \\ M_{YRR}/M_{YAC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Ycf} \neq 0 \\ K'_{DYi} & \text{Otherwise} \end{array}\right.$$

Pitch Angle of Attack Gain

$$K_{IP} = \begin{cases} 0 & \text{If } \bar{I}_Y = 0 \text{ or } f_{Gi} = 1 \\ (M_{PAD} - M_{PAC})/(M_{PTC} + M_{PAC} \cdot K_{Pcf}) & \text{If } f_{Gi} = 2 \text{ and } K_\delta = 0 \\ (M_{PAD} - M_{PAC})/(M_{PMC} + M_{PAC} \cdot K_{Pcf}) & \text{If } f_{Gi} = 2 \text{ and } K_\delta = 1 \\ 0 & \text{If } f_{Gi} = 2 \text{ and } K_\delta = 2 \text{ and } V_a = 0 \\ (M_{PAD} - M_{PAC})/(M_{PAC}) & \text{If } f_{Gi} = 2 \text{ and } K_\delta = 2 \\ K'_{IPi} & \text{If } f_{Gi} = 0 \end{cases}$$

Yaw Angle of Side Slip Gain

$$K_{IY} = \begin{cases} 0 & \text{If } \bar{I}_z = 0 \text{ or } f_{Gi} = 1 \\ (M_{YAD} - M_{YAC})/(M_{PTC} + M_{YAC} \cdot K_{Ycf}) & \text{If } f_{Gi} = 2 \text{ and } K_\delta = 0 \\ (M_{YAD} - M_{YAC})/(M_{PMC} + M_{YAC} \cdot K_{Ycf}) & \text{If } f_{Gi} = 2 \text{ and } K_\delta = 1 \\ 0 & \text{If } f_{Gi} = 2 \text{ and } K\delta = 2 \text{ and } V_a = 0 \\ (M_{YAD} - M_{YAC})/(M_{YAC}) & \text{If } f_{Gi} = 2 \text{ and } K\delta = 2 \\ K'_{IYi} & \text{If } f_{Gi} = 0 \end{cases}$$

Pitch Attitude Rate Gain

$$K_{RP} = \begin{cases} 0 & \text{If } \bar{I}_Y = 0 \\ I_{PRD}/M_{PTC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Pcf} = 0 \\ I_{PRD}/M_{PMC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Pcf} = 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Pcf} = 0 \\ I_{PRD}/(M_{PTC} + M_{PAC} \cdot K_{Pcf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Pcf} \neq 0 \\ I_{PRD}/(M_{PMC} + M_{PAC} \cdot K_{Pcf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Pcf} \neq 0 \\ 0 & \text{If } f_{GI} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Pcf} \neq 0 \\ & \text{and } V_a = 0 \\ I_{PRD}/M_{PAC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Pcf} \neq 0 \\ K'_{RPi} & \text{Otherwise} \end{cases}$$

Yaw Attitude Rate Gain

$$K_{RY} = \begin{cases} 0 & \text{If } \bar{I}_z = 0 \\ I_{YRD}/M_{PTC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Ycf} = 0 \\ I_{YRD}/M_{PMC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Ycf} = 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Ycf} = 0 \\ I_{YRD}/(M_{PTC} + M_{YAC} \cdot K_{Ycf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 0 \text{ and } K_{Ycf} \neq 0 \\ I_{YRD}/(M_{PMC} + M_{YAC} \cdot K_{Ycf}) & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 1 \text{ and } K_{Ycf} \neq 0 \\ 0 & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Ycf} \neq 0 \\ & \text{and } V_a = 0 \\ I_{YRD}/M_{YAC} & \text{If } f_{Gi} \neq 0 \text{ and } K_\delta = 2 \text{ and } K_{Ycf} \neq 0 \\ K'_{RYi} & \text{Otherwise} \end{cases}$$

where the criteria for evaluating vehicle damping ratio and the vehicle control frequency are given in section D.1.e.

The following parameters are limited by input:

$$|K_{DP}\Delta\theta_b| \leqq L_1 \qquad\qquad |K_{DY}\Delta\psi_b| \leqq L_6$$

$$|\delta_{Pc}| \leqq L_2 \qquad\qquad |\delta_{Yc}| \leqq L_7$$

$$|\delta_P| \leqq L_3 \qquad\qquad |\delta_Y| \leqq L_8$$

$$|\dot{\delta}_P| \leqq L_4 \qquad\qquad |\dot{\delta}_Y| \leqq L_9$$

$$|\ddot{\delta}_P| \leqq L_5 \qquad\qquad |\ddot{\delta}_Y| \leqq L_{10}$$

where L_j ($j = 1, 2, \ldots, 10$) are input for each stage. If a limit is zero, then no limit of the parameter will be made.

The angular velocities, $\dot{\delta}_P$ and $\dot{\delta}_Y$ are further limited by the following:

If

$$\delta_P = L_3, \text{ then } \dot{\delta}_P = 0$$

or

$$\delta_Y = L_8, \text{ then } \dot{\delta}_Y = 0$$

e. Vehicle Control Frequency and Damping Ratio

$$\zeta_v = \begin{cases} \zeta_v' & \text{If } \zeta_v' \neq 0 \\ 0.7 & \text{Otherwise} \end{cases}$$

$$\omega_v = \begin{cases} \omega_v' & \text{If } \omega_v' \neq 0 \\ \text{anti } \ell_n \, [10.132079 - 1.382972 \, (\ell n \, W_{01}) \\ + 0.0624466 \, (\ell n \, W_{01})^2 - 0.00093655891 \, (\ell n \, W_{01})^3 \,] \end{cases}$$

where ζ_v' and ω_v' are input per stage

where the liftoff weight is

The stage I liftoff weight is determined as follows:

$$W_{01} = \begin{cases} W_{01}' & \text{If } W_{01}' > 0 \\ 1 \times \hat{W}_{01} & \text{If } K_k < 2 \\ 3 \times \hat{W}_{02} & \text{If } K_k = 2 \\ 7 \times \hat{W}_{03} & \text{If } K_k = 3 \\ 11 \times \hat{W}_{04} & \text{If } K_k = 4 \end{cases}$$

where $\hat{W}_{0k}$ is the vehicle initial weight of the K_k-th stage.

2. ROLL CONTROL

The simulated roll control system discussed below assumes either a power source from the main propulsive one (Figure 19), or aerodynamic control fins. If $K_{RC} = 0$, a roll system does not exist. If $K_{RC} = 1$, an auxiliary thrust control system is operating and if $K_{RC} = 2$ aerodynamic fins provide roll control. A bang-bang system with dead band is simulated if K_{DR} is zero and a proportional system is simulated if K_{DR} is nonzero. The force signal is a function of vehicle roll attitude error, angular velocity, and steady state error. The roll control equations are ignored if either $M_y \neq 5$ or I_x is zero

a. Roll Attitude Error Angle

The roll attitude error shown in Figure 23 is determined from

$$\Delta\phi_b = \text{archtan}\ (K_{23} - K_{32})/(K_{22} + K_{33})$$

with

$$-180° < \Delta\phi_b \leq 180°$$

and the roll attitude error rate (not integrated) is

$$\dot{\Delta\phi}_b = P_m - P_b + (Q_m K_{21} + R_m K_{31} - Q_b K_{12} - R_b K_{13})/(1 + K_{11})$$

where K_{DR} and K_{IR} are the attitude error and bias gains determined from input, K_{ij}'s are the elements of the K matrix and P_m, Q_m, and R_m are the current input roll, pitch, and yaw commands, respectively.

b. Auxiliary Roll Thrusters

The roll command signal is determined from one of the following if $K_{RC} = 1$

Zero Order Thrusters

If $K_{DR} \neq 0$ and $K_{IR} = 0$, then $F_c = K_{DR}\Delta\phi_b - K_{RR}P_b$

First Order Thrusters

If $K_{DR} \neq 0$ and $K_{IR} \neq 0$, then $\dot{F}_c = K_{DR} \Delta\phi_b - K_{RR}\dot{P}_b + K_{IR} \Delta\phi_b$

and F_c is the integral of $\dot{F}_c$.

Bang-Bang Thrusters ($K_{DR} = 0$)

$$F_c = \frac{L_{Rc}}{2} \left[1 \cdot \text{sign} \left| F_D - F_\Delta \left[1 - D \left(1 + 1 \cdot \text{sign } \dot{F}_D \right) / 2 \right] \right| \right.$$

$$\left. + 1 \cdot \text{sign} \left| F_D + F_\Delta \left[1 + D \left(1 + 1 \cdot \text{sign } \dot{F}_D \right) / 2 \right] \right| \right]$$

where the rate gain, K_{RR}, the dead band control quantity, F_Δ, and the hysteresis, D, are determined from input and if F_c is integrated to obtain F_c, then its initial value at stage initiation is obtained from input. The phase plane signal and derivative (not integrated) are

$$F_D = \Delta\phi_b - K_{RR}P_b$$

$$\dot{F}_D = \Delta\dot{\phi}_b - K_{RR}\dot{P}_b$$

The roll command signal is limited if $L_{Rc} \neq 0$ such that

$$|F_c| \leq L_{Rc}$$

where L_{Rc} is determined from input.

The achieved roll thrust level is computed from

$$\dot{F}_R = (F_c - F_R)/\tau_R$$

where the roll control time constant, τ_R is determined from input and the initial value at stage initiation is obtained from input.

Three sets of roll control system parameters can be input for each stage. Values are determined as follows:

$$\tau_R, F_\Delta, D, L_{Rc}, K'_{PR_i}, K'_{RR_i}, K'_{IR_i} \qquad \begin{array}{l} \tau_{R1} \ \ldots\ldots \text{ if } t_B \leq t'_{R2} \\ \tau_{R2} \ \ldots\ldots \text{ if } t'_{R2} \leq t_B \leq t'_{R3} \\ \tau_{R3} \ \ldots\ldots \text{ if } t_B \geq t'_{R3} \end{array}$$

where τ_{Rj}, $F_{\Delta j}$, D_j, L_{Rcj}, $K_{D,R,IRj}$, and ℓ_{Rj} and $t'_{R2,3}$ are input for each stage and where ℓ_R is the roll control motor lever arm shown in Figure 1.5 and used in the angular momenta equation.

Thrust Control Moment

$$M_{FCP} = F_R \ell_R$$

c. Roll Control Aerodynamic Fins

The roll command signal for aerodynamic control fins is as follows:

If $\bar{I}_R \neq 0$ and $K_{Rc} = 2$ the following differential equations and logic are employed:

1. Zero Order ($\omega_c = \tau_c = 0$)

$$\delta_R = \delta_{Rc}, \ \dot{\delta}_R = \dot{\delta}_{Rc} \text{ and } \ddot{\delta}_R = 0$$

2. First Order ($\omega_c = 0$ and $\tau_c \neq 0$)

$$\tau_c \dot{\delta}_R + \delta_R = \delta_{Rc}, \text{ and } \ddot{\delta}_R = 0$$

3. Second Order ($\omega_c \neq 0$)

$$\ddot{\delta}_R + 2.0\ \zeta_c \omega_c \dot{\delta}_R + \omega_c^2\ \delta_R = \omega_c^2\ \delta_{Rc}$$

If $\bar{I}_R = 0$, quasi-finite mass equations are used:

$$\ddot{\delta}_R = 0,\ \dot{\delta}_R = 0 \text{ and}$$

$$\delta_R = (-\delta_{MR}) + (180/\pi)\ M_{DP}/M_{RAC}$$

d. <u>Roll Control Gains</u>

Roll Aerodynamic Control Moment per Radian Fin Deflection Angle

$$M_{RAC} = 2\ (180/\pi)\ q\ S_{Fz}\ C_{Lz}\ \ell_{\delta r}$$

Roll Rotation Reaction Moment

$$M_{RRR} = I_{xx}\ \omega_v^2$$

Roll Rotation Damping Moment Integral

$$I_{RRD} = 2\ \zeta_v\ I_{xx}\ \omega_v$$

Roll Attitude Error Gain

$$K_{DR} = \begin{cases} 0 & \text{If } \bar{I}_X = 0 \\ M_{RRR}/M_{RAC} & \text{If } K'_{DRi} = 0\ \&\ K_{RC} \neq 0 \\ K'_{DRi} & \text{Otherwise} \end{cases}$$

Roll Attitude Rate Gain

$$K_{RR} = \left[\begin{array}{ll} 0 & \text{If } \bar{I}_X = 0 \\ I_{RRD}/M_{RAC} & \text{If } K'_{RRi} = 0 \text{ and } K_{RC} \neq 0 \\ K'_{RRi} & \text{Otherwise} \end{array}\right.$$

$$K_{IR} = K'_{IR_i}$$

Roll control commanded Aerodynamic Fin Angle

$$\delta_{RC} = K_{DR} \Delta\phi_b - K_{RR} P_b$$

Z. MISSILE LOCATION

The vehicle location with respect to a spherical earth is defined. Parameters are shown in Figure 24.

1. Altitude and Earth Centered Coordinates

Vehicle altitude and rate change are determined from position components defined in an earth geocentric coordinate system as follows:

$$X_{cc} = X_{ee} \cos \rho_L - Z_{ee} \sin \rho_L + r_e \sin \rho_L$$

$$Y_{cc} = Y_{ee}$$

$$Z_{cc} = X_{ee} \sin \rho_L + Z_{ee} \cos \rho_L - r_e \cos \rho_L$$

$$\dot{X}_{cc} = \dot{X}_{ee} \cos \rho_L - \dot{Z}_{ee} \sin \rho_L$$

$$\dot{Y}_{cc} = \dot{Y}_{ee}$$

$$\dot{Z}_{cc} = \dot{X}_{ee} \sin \rho_L + \dot{Z}_{ee} \cos \rho_L$$

where the launch latitude, ρ_L, and the earth radius, r_e, are input and position components X_{ee}, Y_{ee}, and Z_{ee} are obtained from the integration of the linear momenta equations.

Vehicle distance from the earth center is

$$r_c = (X^2_{cc} + Y^2_{cc} + Z^2_{cc})^{\frac{1}{2}}$$

$$\dot{r}_c = (\dot{X}_{cc} X_{cc} + \dot{Y}_{cc} Y_{cc} + \dot{Z}_{cc} Z_{cc})/r_c$$

and the vehicle altitude and rate change are

$$h = r_c - r_e$$

$$\dot{h} = \dot{r}_c$$

2. Heading Azimuth, Latitude and Longitude

Vehicle azimuth in the launch horizontal plane is

$$\psi = \begin{cases} \psi_i & \text{If } \dot{Y}_{ee} = \dot{X}_{ee} = 0 \\ \arctan \dot{Y}_{ee}/\dot{X}_{ee} & \text{Otherwise} \end{cases}$$

with

$$0 \leq \psi < 360°$$

Latitude and Longitude

The vehicle latitude is

$$\rho = \arcsin X_{cc}/r_c$$

with

$$-90° \leq \rho \leq 90°$$

and the change in longitude from launch is

$$\mu' = \arctan Y_{cc}/-Z_{cc}$$

with

$$0 \leq \mu' < 360°$$

The longitude is defined so that west of Greenwich, England is positive and east is negative. The longitude is

$$\mu = \begin{cases} \mu_L - \mu' & \text{If } -180° < \mu_L - \mu' \\ 360° + \mu_L - \mu' & \text{Otherwise} \end{cases}$$

where the launch longitude, μ_L, is input.

3. Ground and Slant Ranges

The down range-cross range coordinates are shown in Figure 25.

The range angle is

$$\phi_s = \begin{cases} \arcsin \, (X_{ee}^2 + Y_{ee}^2)^{\frac{1}{2}}/r_c & \text{If } \sin \phi \leq 0.7 \\ \arccos \, (r_e - Z_{ee})/r_c & \text{Otherwise} \end{cases}$$

with

$$0 \leq \phi_s < 180°$$

and the ground range is

$$S_s = (\pi/180) r_e \, \phi_s$$

The cross range angle is

$$\zeta = \arcsin \, [(Y_{ee} \cos \psi_i - X_{ee} \sin \psi_i)/r_c]$$

$$\dot{\zeta} = \frac{(180/\pi)(\dot{Y}_{ee} \cos \psi_i - \dot{X}_{ee} \sin \psi_i - \dot{r}_c \sin \zeta)}{r_c \cos \zeta}$$

where

$$-90° < \zeta \leq 90°$$

the cross range is

$$S_c = (\pi/180) \, r_e \, \zeta$$

$$\dot{S}_c = (\pi/180) \, r_e \, \dot{\zeta}$$

the down range angle is

$$\phi = \arctan \, [(X_{ee} \cos \psi_i + Y_{ee} \sin \psi_i)/(r_e - Z_{ee})]$$

$$\dot{\Phi} = (180/\pi)\ [(\dot{X}_{ee} \cos \Psi_i + \dot{Y}_{ee} \sin \Psi_i) \cos^2 \Phi \quad \dot{Z}_{cc} \sin \Phi \cos \Phi]/ (r_e - Z_{ee})$$

where

$$-180° < \Phi \leq 180°$$

the down range is

$$S = (\pi/180)\ r_e\ \Phi$$

$$\dot{S} = (\pi/180)\ r_e\ \dot{\Phi}$$

F. FLIGHT ENVIRONMENT AND FORCES

Parameters which define the vehicle flight environment and aerodynamic forces are given below.

1. Atmosphereic Parameters

The ambient pressure, P_a, the speed of sound, C_a, the partial derivative of ambient pressure with altitude, $\partial P_a/\partial h$, and, the partial derivative of the speed of sound with altitude, $\partial C_a/\partial h$, are functions of the mixxile altitude, h, and the formulas and constants contained in National Aeronautics and Space Administration, "U.S. Standard Atmosphere, 1962".

a. Speed of Sound, ft/sec

$$C_a = \begin{cases} 1,177.7301 & \text{If } h \leq -16,391.307 \\ 3,417.9390 & \text{If } h \geq 2,296,587.9 \\ \sqrt{G_{RZ}\ T_M} & \text{Otherwise} \end{cases}$$

where

$$G_{RZ} = 2403.1756$$

Temperature Gradient, °R/ft

$$S_B = \begin{cases} 0.0 & \text{If } h \leq -16,391.307 \\ 0.0 & \text{If } h \geq 2,296,587.9 \\ \dfrac{T_{MB(i+1)} - T_{MBi}}{h_{B(i+1)} - h_{Bi}} & \text{Otherwise} \end{cases}$$

Temperature, °R

$$T_M = T_{MBi} + S_B\ (h - h_{Bi})$$

The values of T_{MB} and h_B are given in the table at the end of this section. The index value i is set such that $h_{Bi} \leq h < h_{B(i+1)}$.

b. Partial Derivative of the Speed of Sound with Altitude, (1/sec)

$$\frac{\partial C_a}{\partial h} = \begin{cases} 0.0 & \text{If } h \leq -16{,}391.307 \\ 0.0 & \text{If } h \geq 2{,}296{,}587.9 \\ \dfrac{G_{RZ}\ S_B}{2\ C_a} & \text{Otherwise} \end{cases}$$

c. Ambient Pressure, lb/ft^2

$$P_a = \begin{cases} 3711.0756 & \text{If } h \leq -16{,}391.307 \\ 2.4891 \times 10^{-9} & \text{If } h \geq 2{,}296{,}587.9 \\ P_B\, e^{-\dfrac{Z_{MGM}\,(h_B - h)}{R^*\,(S_B R_B - T_{MB})\,R\,R_B}} & \\ \text{X} \left[\dfrac{R_B\,(S_B\,(h - h_B) + T_{MB})}{R\,T_{MB}} \right]^{-\dfrac{Z_{MGM}\,S_B}{R^*\,(S_B R_B - T_{MB})^2}} & \end{cases}$$

where

$R = h + R_Z$

$R_B = h_B + R_Z$

$Z_{MXGM} = 40.80696 \times 10^{16}$

$R_Z = 20{,}925{,}780.$

$R^* = 49718.9585$

and

$$Z_{MGM} = Z_{MXGM} \times Z_K$$

d. Partial Derivative of Ambient Pressure with Altitude (lb/ft^3)

$$\frac{\partial P_a}{\partial h} = \begin{cases} 0.0 & \text{If } h \leq -16,391.307 \\ 0.0 & \text{If } h \geq 2,296,587.9 \\ P_a \left[\dfrac{Z_{MGM}}{R^* \; (S_B R_B - T_{MB}) R^2} - \dfrac{Z_{MGM}}{R^* \; (S_B R_B - T_{MB})^2} \left(\dfrac{S_B}{S_B \; (h - h_B) + T_{MB}} - \dfrac{1}{R} \right) \right] & \\ & \text{Otherwise} \end{cases}$$

e. U. S. Standard Atmosphere Altitude - Temperature - Pressure Table

Altitude, h_B, ft	Temperature T_{MB}, °R	Pressure, P_B, lb/ft^2	Correction Z_K, dim
-16,391.307	577.17	3711.0756	1.00000969
0.0	518.67	2116.22	1.00006922
36,151.798	389.97	472.68122	0.99997683
65,823.897	389.97	114.3456	0.99994861
105,518.46	411.57	18.12897	0.99988460
155,348.08	487.17	2.3163292	0.99994118
172,010.76	487.17	1.2322622	0.99994953
202,070.31	454.77	0.38032587	1.00007811
262,447.98	325.17	2.1673089×10^{-2}	0.99990452
295,275.59	325.17	3.433025×10^{-3}	0.99988145
328,083.99	379.17	6.28115×10^{-4}	0.99987195
360,892.39	469.17	1.5360×10^{-4}	0.99984532
393,700.79	649.17	5.2667×10^{-5}	0.99983723
492,125.98	1729.17	1.0572×10^{-5}	0.99994038
524,934.38	1999.17	7.7157×10^{-6}	0.99979481
557,742.78	2179.17	5.8325×10^{-6}	0.99978217
623,359.58	2431.17	3.5197×10^{-6}	0.99978447
754,593.18	2791.17	1.4537×10^{-6}	0.99968987
984,251.97	3295.17	3.9345×10^{-7}	0.99959327
1,312,336.0	3889.17	8.4177×10^{-8}	0.99944931
1,640,419.9	4357.17	2.2885×10^{-8}	0.99941075
1,968,503.9	4663.17	7.2059×10^{-9}	0.99921789
2,296,587.9	4861.17	2.4891×10^{-9}	

2. Winds

a. Wind Speed and Azimuth

The wind velocity is a function of vehicle altitude and input wind speed, v_w, and azimuth ψ_w. The wind direction is directed parallel to the local horizontal with ψ_w being the clockwise angle from north to the direction the wind is coming from.

If the input altitude multiplier, K_n, is zero, or the vehicle is outside the atmosphere

$$v_w = \dot{v}_w = \psi_w = \dot{\psi}_w = 0 \text{ and}$$

$$\dot{\vec{X}}_{aee} = \dot{\vec{X}}_{ee}$$

where $\dot{\vec{X}}_{aee}$ are the vehicle velocity components with respect to the air as measured in the 3-system and $\dot{\vec{X}}_{ee}$ the still air components.

For

$$K_h \neq 0 \text{ and } K_h h_j \leq h \leq K_h h_{j+1}$$

$$v_w = K_v[K_h(h_j v_{w(j+1)} - h_{j+1} v_{wj}) + h(v_{wj} - v_{w(j+1)})]/(h_j - h_{j+1})K_h$$

$$\psi_w = K[K_h(h_j \psi_{w\ (j+1)} - h_{j+1}\ \psi_{wj}) + h(\psi_{wj} - \psi_{w(j+1)})]/(h_j - h_{j+1})K_h$$

If $h > K_h h_J$ (J denoting the last input where $J = 1, 2, \ldots, 32$),
then $v_w = K_v v_{wJ}$ and $\psi_w = K_\psi \psi_{wJ}$

If $h < K_h h_1$, then $v_w = K_v v_{w1}$ and $\psi_w = K_\psi \psi_{w1}$

The derivatives are calculated as

$$\dot{v}_w = (dv_w/dh)\dot{h}$$

$$\dot{\psi} = (d\psi_w/dh)\dot{h}$$

with

$$0 \le \psi_w < 360°$$

$$dv_w/dh = \begin{cases} 0 & \text{If } h < K_h h_1 \text{ or } h > K_h h_J \\ K_v(v_{w(j+1)} - v_{wj})/[K_h(h_{j+1} - h_j)] & \text{Otherwise} \end{cases}$$

$$d\psi_w/dh = \begin{cases} 0 & \text{If } h < K_h h_1 \text{ or } h > K_h h_J \\ K_\psi(\psi_{w(j+1)} - \psi_{wj})/[K_h(h_{j+1} - h_j)] & \text{Otherwise} \end{cases}$$

where v_{wj}, ψ_{wj}, and h_j ($j = 1, 2, \ldots, 32$), K_v, and K_ψ are input.

b. Wind Local and Launch Centered Cartesian Coordinate

The Cartesian components of wind velocity and acceleration in the local coordinate system (the l-system) are

$$\dot{X}_{wll} = -v_w \cos \psi_w$$

$$\dot{Y}_{wll} = -v_w \sin \psi_w$$

$$\dot{Z}_{wll} = 0$$

$$\ddot{X}_{wll} = -\dot{v}_w \cos \psi_w + (\pi/180)\, \dot{\psi}_w v_w \sin \psi_w$$

$$\ddot{Y}_{wll} = -v_w \sin \psi_w - (\pi/180)\, \dot{\psi}_w v_w \cos \psi_w$$

$$\ddot{Z}_{wll} = 0$$

where $\dot{X}_{wll}$ is positive north, $\dot{Y}_{wll}$ east, and $\dot{Z}_{wll}$ along the vector connecting the missile and earth center, positive down.

From Figure 24, the wind velocity components in the e-system are

$$\vec{\dot{X}}_{wee} = [A_1] \vec{\dot{X}}_{wll}$$

with

$$[A_1] = \begin{bmatrix} \cos \rho_L & 0 & \sin \rho_L \\ 0 & 1 & 0 \\ -\sin \rho_L & 0 & \cos \rho_L \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \mu' & -\sin \mu' \\ 0 & \sin \mu' & \cos \mu' \end{bmatrix} \begin{bmatrix} \cos \rho & 0 & -\sin \rho \\ 0 & 1 & 0 \\ \sin \rho & 0 & \cos \rho \end{bmatrix}$$

where ρ_L is the input latitude and ρ and μ' the instantaneous vehicle latitude and longitude change, respectively.

c. Artificial Wind Profile

The MMRBM TVC design wind profile is constructed from the input specified peak wind altitude, hmxwd (L0682). This artificial altitude-wind velocity table is set into the wind profile table. The final and initial altitude of maximum wind shear used in the TVC duty cycle slow rate calculations are also set.

If $h_{mxwd} > 0$ the wind profile is established as

$$h_{mxwd} = \begin{cases} 240{,}000 & \text{If } h_{mxwd} > 240{,}000 \\ h_{mxwd} & \text{Otherwise} \end{cases}$$

Altitudes of maximum wind shear

$$h_{\alpha} = h_{mxwd} + 3000$$

$$h_{\beta} = \begin{cases} 0 & \text{If } h_{mxwd} < 3000 \\ h_{mxwd} - 3000 & \text{Otherwise} \end{cases}$$

Velocity of wind shear

$$V_{w1k} = V_{wmax} - 1{,}000 \quad S_{1k}$$

$$V_{w3k} = V_{wmax} - 3{,}000 \quad S_{3k}$$

$$V_{w5k} = V_{wmax} - 5{,}000 \quad S_{5k}$$

where V_{wmax} is the maximum wind velocity and S_{1k}, S_{3k}, and S_{5k} are the wind shear for 1,000, 3,000, & 5,000 feet depths respectively, all of which are function of the peak wind altitude.

If $0 < h_{mxwd} \le 1,000$

$h_1 = 0$ $V_{w1} = [V_{w1k}\ h_{mxwd} - V_{wmax}\ (h_{mxwd} - 1,000)]/1000$

$h_2 = h_{mxwd}$ $V_{w2} = V_{wmax}$

$h_3 = h_{mxwd} + 1,000$ $V_{w3} = V_{w1k}$

$h_4 = h_{mxwd} + 3,000$ $V_{w4} = V_{w3k}$

$h_5 = h_{mxwd} + 5,000$ $V_{w5} = V_{w5k}$

$h_6 = 80,000$ $V_{w6} = 75$

$h_7 = 100,000$ $V_{w7} = 90$

$h_8 = 300,000$ $V_{w8} = 90$

If $1,000 < h_{mxwd} \le 3,000$

$h_1 = 0$ $V_{w1} = [V_{w3k}\ (h_{mxwd} - 1,000) - V_{w1k}\ (h_{mxwd} - 3000)]/2,000$

$h_2 = h_{mxwd} - 1,000$ $V_{w2} = V_{w1k}$

$h_3 = h_{mxwd}$ $V_{w3} = V_{wmax}$

$h_4 = h_{mxwd} + 1,000$ $V_{w4} = V_{w1k}$

$h_5 = h_{mxwd} + 3,000$ $V_{w5} = V_{w3k}$

$h_6 = h_{mxwd} + 5,000$ $V_{w6} = V_{w5k}$

$h_7 = 80{,}000$ $\quad V_{w7} = 75$

$h_8 = 100{,}000$ $\quad V_{w8} = 90$

$h_9 = 300{,}000$ $\quad V_{w9} = 90$

If $3{,}000 < h_{mxwd} \leq 5{,}000$

$h_1 = 0$ $\quad V_{w1} = [V_{w5k} (h_{mxwd} - 3000) - V_{w3k} (h_{mxwd} - 5000)]/2{,}000$

$h_2 = h_{mxwd} - 3{,}000$ $\quad V_{w2} = V_{w3k}$

$h_3 = h_{mxwd} - 1{,}000$ $\quad V_{w3} = V_{w1k}$

$h_4 = h_{mxwd}$ $\quad V_{w4} = V_{wmax}$

$h_5 = h_{mxwd} + 1{,}000$ $\quad V_{w5} = V_{w1k}$

$h_6 = h_{mxwd} + 3{,}000$ $\quad V_{w6} = V_{w3k}$

$h_7 = h_{mxwd} + 5{,}000$ $\quad V_{w7} = V_{w5k}$

$h_8 = 80{,}000$ $\quad V_{w8} = 75$

$h_9 = 100{,}000$ $\quad V_{w9} = 90$

$h_{10} = 300{,}000$ $\quad V_{w10} = 90$

If $5{,}000 < h_{mxwd} \leq 10{,}000$

$h_1 = 0$ $\quad V_{w1} = 50 - .003\ h_{mxwd}$

$h_2 = h_{mxwd} - 5,000$ $V_{w2} = V_{w5k}$

$h_3 = h_{mxwd} - 3,000$ $V_{w3} = V_{w3k}$

$h_4 = h_{mxwd} - 1,000$ $V_{w4} = V_{w1k}$

$h_5 = h_{mxwd}$ $V_{w5} = V_{wmax}$

$h_6 = h_{mxwd} + 1,000$ $V_{w6} = V_{w1k}$

$h_7 = h_{mxwd} + 3,000$ $V_{w7} = V_{w3k}$

$h_8 = h_{mxwd} + 5,000$ $V_{w8} = V_{w5k}$

$h_9 = 30,000$ $V_{w9} = 75$

$h_{10} = 100,000$ $V_{w10} = 90$

$h_{11} = 300,000$ $V_{w11} = 90$

IF $10,000 < h_{mxwd} \leq 32,000$

$h_1 = 0$ $V_{w1} = 20$

$h_2 = h_{mxwd} - 5,000$ $V_{w2} = V_{w5k}$

$h_3 = h_{mxwd} - 3,000$ $V_{w3} = V_{w3k}$

$h_4 = h_{mxwd} - 1,000$ $V_{w4} = V_{w1k}$

$h_5 = h_{mxwd}$ $V_{w5} = V_{wmax}$

$h_6 = h_{mxwd} + 1,000$ $V_{w6} = V_{w1k}$

$h_7 = h_{mxwd} + 3,000$ $V_{w7} = V_{w3k}$

$h_8 = h_{mxwd} + 5,000$ $V_{w8} = V_{w5k}$

$h_9 = 80,000$ $V_{w9} = 75$

$h_{10} = 100,000$ $V_{w10} = 90$

$h_{11} = 300,000$ $V_{w11} = 90$

If $32,000 < h_{mxwd} \leq 42,000$

$h_1 = 0$ $V_{w1} = 20$

$h_2 = 10,000$ $V_{w2} = 50$

$h_3 = h_{mxwd} - 5,000$ $V_{w3} = V_{w5k}$

$h_4 = h_{mxwd} - 3,000$ $V_{w4} = V_{w3k}$

$h_5 = h_{mxwd} - 1,000$ $V_{w5} = V_{w1k}$

$h_6 = h_{mxwd}$ $V_{w6} = V_{wmax}$

$h_7 = h_{mxwd} + 1,000$ $V_{w7} = V_{w1k}$

$h_8 = h_{mxwd} + 3,000$ $V_{w8} = V_{w5k}$

$h_9 = h_{mxwd} + 5,000$ $V_{w9} = V_{w5k}$

$h_{10} = 80,000$ $V_{w10} = 75$

$h_{11} = 100,000$ $V_{w11} = 90$

$h_{12} = 300,000$ $V_{w12} = 90$

If $h_{mxwd} > 42,000$

$h_1 = 0 \qquad V_{w1} = 20$

$h_2 = 10,000 \qquad V_{w2} = 50$

$h_3 = h_{mxwd} - 5,000 \qquad V_{w3} = V_{w5k}$

$h_4 = h_{mxwd} - 3,000 \qquad V_{w4} = V_{w3k}$

$h_5 = h_{mxwd} - 1,000 \qquad V_{w5} = V_{w1k}$

$h_6 = h_{mxwd} \qquad V_{w6} = V_{wmax}$

$h_7 = h_{mxwd} + 1,000 \qquad V_{w7} = V_{w1k}$

$h_8 = h_{mxwd} + 3,000 \qquad V_{w8} = V_{w3k}$

$h_9 = h_{mxwd} + 5,000 \qquad V_{w9} = V_{w5k}$

$h_{10} = h_{mxwd} + 38,000 \qquad V_{w10} = 75$

$h_{11} = h_{mxwd} + 58,000 \qquad V_{w11} = 90$

$h_{12} = 300,000 \qquad V_{w12} = 90$

Maximum Wind Velocity

The maximum wind velocity is a tabular function of the input altitude of peak winds. The table is linearly interpolated.

Peak Wind Altitude h_{mxwd}, feet	Maximum Wind Velocity V_{wmax}, ft/sec
0	92
19,000	210
32,000	348
42,000	348
61,500	197
90,000	197
196,000	590
∞	590

Wind Shears

The wind envelope shears for incremental altitude of 1,000, 3,000, and 5,000 feet from the input design wind altitude (L682). Tables for the shears are listed at 1,000 feet intervals, starting at zero feet altitude and ending at 90,000 feet altitude. If Peak Wind Altitude exceeds 90,000 feet values at 90,000 feet are used. The table is linearly interpolated.

Wind Shear Table

Peak Wind Altitude	Shear Depth		
	1,000 feet	3,000 feet	5,000 feet
h_{mxwd}, feet	S_{1k}, 1/sec x 10^3	S_{3k}, 1/sec x 10^3	S_{5k}, 1/sec x 10^3
0	.0425	.0325	.0145
1,000	.0409	.0305	.0153
2,000	.0393	.0288	.0161
3,000	.0378	.0272	.0168
4,000	.0362	.0258	.0173
5,000	.0348	.0247	.0177
6,000	.0348	.0238	.0179
7,000	.0321	.0232	.0178
8,000	.0308	.0227	.0175
9,000	.0296	.0224	.0171
10,000	.0284	.0222	.0167

Wind Shear Table (Cont'd)

Peak Wind Altitude		Shear Depth	
	1,000 feet	3,000 feet	5,000 feet
h_{mxwd}, feet	S_{1k}, 1/sec x 10^3	S_{3k}, 1/sec x 10^3	S_{5k}, 1/sec x 10^3
11,000	.0273	.0220	.0164
12,000	.0263	.0219	.0161
13,000	.0253	.0216	.0158
14,000	.0244	.0210	.0156
15,000	.0237	.0203	.0158
16,000	.0230	.0198	.0163
17,000	.0226	.0194	.0169
18,000	.0226	.0192	.0176
19,000	.0232	.0193	.0183
20,000	.0226	.0200	.0190
21,000	.0271	.0208	.0197
22,000	.0300	.0217	.0203
23,000	.0311	.0227	.0210
24,000	.0318	.0236	.0216
25,000	.0324	.0247	.0223
26,000	.0331	.0258	.0230
27,000	.0341	.0274	.0236
28,000	.0360	.0295	.0245
29,000	.0487	.0318	.0258
30,000	.0550	.0347	.0285
31,000	.0588	.0384	.0308
32,000	.0631	.0432	.0321
33,000	.0683	.0478	.0328
34,000	.0758	.0501	.0332
35,000	.0764	.0503	.0334
36,000	.0759	.0482	.0333
37,000	.0744	.0386	.0330
38,000	.0707	.0356	.0325
39,000	.0651	.0337	.0316
40,000	.0562	.0325	.0300
41,000	.0487	.0316	.0277

Wind Shear Table (Cont'd)

Peak Wind Altitude h_{mxwd}, feet	Shear Depth 1,000 feet S_{1k}, 1/sec x 10^3	3,000 feet S_{3k}, 1/sec x 10^3	5,000 feet S_{5k}, 1/sec x 10^3
42,000	.0442	.0308	.0256
43,000	.0407	.0303	.0243
44,000	.0379	.0299	.0235
45,000	.0359	.0296	.0228
46,000	.0346	.0295	.0222
47,000	.0338	.0294	.0217
48,000	.0336	.0292	.0212
49,000	.0342	.0291	.0207
50,000	.0363	.0288	.0202
51,000	.0397	.0287	.0198
52,000	.0428	.0286	.0195
53,000	.0440	.0283	.0191
54,000	.0442	.0280	.0187
55,000	.0432	.0277	.0183
56,000	.0428	.0272	.0179
57,000	.0411	.0267	.0175
58,000	.0389	.0259	.0171
59,000	.0368	.0252	.0168
60,000	.0348	.0243	.0164
61,000	.0327	.0233	.0160
62,000	.0308	.0223	.0157
63,000	.0291	.0213	.0153
64,000	.0275	.0203	.0149
65,000	.0262	.0193	.0146
66,000	.0249	.0186	.0143
67,000	.0239	.0178	.0140
68,000	.0231	.0172	.0137
69,000	.0223	.0166	.0134
70,000	.0217	.0160	.0131
71,000	.0213	.0156	.0128
72,000	.0209	.0152	.0126

Wind Shear Table (Cont'd)

Peak Wind Altitude h_{mxwd}, feet	Shear Depth		
	1,000 feet S_{1k}, 1/sec x 10^3	3,000 feet S_{3k}, 1/sec x 10^3	5,000 feet S_{5k}, 1/sec x 10^3
73,000	.0206	.0148	.0124
74,000	.0205	.0145	.0122
75,000	.0204	.0142	.0120
76,000	.0204	.0139	.0118
77,000	.0203	.0136	.0116
78,000	.0202	.0133	.0114
79,000	.0201	.0131	.0112
80,000	.0200	.0128	.0110
81,000	.0198	.0126	.0109
82,000	.0197	.0124	.0107
83,000	.0193	.0122	.0105
84,000	.0191	.0119	.0103
85,000	.0188	.0117	.0102
86,000	.0185	.0113	.0100
87,000	.0182	.0110	.0098
88,000	.0179	.0107	.0096
89,000	.0176	.0103	.0094
90,000	.0173	.0100	.0093
∞	.0173	.0100	.0093

3. Angles of Attack and Side Slip

The angles of attack and side slip are shown in Figure 27.

a. Still Air Angles of Attack

The still air angles of attack are functions of the component of missile velocity in the b system.

$$\vec{\dot{X}}_{bb} = [D]^{-1}\vec{\dot{X}}_{ee} + [\dot{D}]^{-1}\vec{X}_{ee}$$

The still air angle of attack is

$$\bar{\alpha} = \begin{cases} 0 & \text{If } \dot{X}_{bb} = \dot{Z}_{bb} = 0 \\ \arctan(-\dot{Z}_{bb}/\dot{X}_{bb}) & \text{Otherwise} \end{cases}$$

$$\bar{\beta} = \begin{cases} 0 & \text{If } \dot{X}_{bb} = \dot{Y}_{bb} = 0 \\ \arctan(\dot{Y}_{bb}/\dot{X}_{bb}) & \text{Otherwise} \end{cases}$$

$$\bar{\alpha}' = \begin{cases} 0 & \text{If } V_e = 0 \\ \arctan \{[(\dot{Z}^2_{bb} + \dot{x}^2_{bb})^{\frac{1}{2}}]/\dot{X}_{bb}\} & \text{Otherwise} \end{cases}$$

$$\epsilon = \begin{cases} 0 & \text{If } \dot{Y}_{bb} = \dot{Z}_{bb} = 0 \\ \arctan (\dot{Y}_{bb}/\dot{Z}_{bb}) & \text{Otherwise} \end{cases}$$

$$\beta = \arctan \left[\dot{Y}_{bb} (\dot{X}_{bb}{}^2 + \dot{Z}_{bb}{}^2)^{-1/2}\right]$$

The vehicle velocity components with respect to the air are

$$\dot{\vec{X}}_{aee} = \dot{\vec{X}}_{ee} - \dot{\vec{X}}_{we};$$

and the total velocity is

$$V_a = (\dot{X}^2_{aee} + \dot{Y}^2_{aee} + \dot{Z}^2_{aee})^{\frac{1}{2}}$$

b. Angles of Attack with Winds

Angles of attack are a function of missile velocity with respect to air and missile attitude. The components of missile velocity in the b-system are

$$\dot{\vec{X}}_{abb} = [D]^{-1} \dot{\vec{X}}_{aee}$$

where the matrix [D] is defined in Section B.1.b

The component angles of attack (see Figure 27) are

$$\alpha = \begin{cases} \tilde{\alpha} & \text{If outside atmosphere or } \dot{X}_{abb} = \dot{Z}_{abb} = 0 \\ \arctan (\dot{Z}_{abb}/\dot{X}_{abb}) & \end{cases}$$

$$\beta = \begin{cases} \bar{\beta} & \text{If outside atmosphere or } \dot{X}_{abb} = \dot{Y}_{abb} = 0 \\ \arctan (\dot{Y}_{abb}/\dot{X}_{abb}) & \end{cases}$$

$$\epsilon = \begin{cases} \bar{\epsilon} & \text{If outside atmosphere or } \dot{Y}_{abb} = \dot{Z}_{abb} = 0 \\ \arctan (\dot{Y}_{abb}/\dot{Z}_{abb}) & \end{cases}$$

with

$$-180° < \alpha,\ \beta \leq 180°$$

and the total angle of attack is

$$\alpha' = \begin{cases} \bar{\alpha} & \text{If outside atmosphere} \\ \arctan\ (\dot{Z}^2_{abb} + \dot{Y}^2_{abb})^{\frac{1}{2}}/\dot{X}_{abb} & \text{Otherwise} \end{cases}$$

The effective angles of attack used to compute the aerodynamic normal forces are given below. These values account for the reduction in the forces as the angles of attack move from ± 90 degrees to ± 180 degrees. The values are

$$\alpha_E = \begin{cases} \alpha & \text{If } -90° \leq \alpha \leq 90° \\ 180° - \alpha & \text{If } 90° < \alpha \leq 180° \\ -180° - \alpha & \text{If } -180° < \alpha < -90° \end{cases}$$

and

$$\beta_E = \begin{cases} \beta & \text{If } -90° \leq \beta \leq 90° \\ 180° - \beta & \text{If } 90° < \beta \leq 180° \\ -180° - \beta & \text{If } -180° < \beta < -90° \end{cases}$$

and

$$\alpha'_E = \begin{cases} \alpha' & \text{If } 0 \leq \alpha \leq 90° \\ 180° - \alpha & \text{Otherwise} \end{cases}$$

c. Time Rate Change of Angle of Attack

The time rate change of angle of attack shall be calculated if the input multiplier, $\bar{M}_{\dot{\alpha}} \neq 0$ and the mode type $M_y = 5$.

Pitch

$$\dot{\alpha} = \begin{cases} 0 & \text{If } \dot{Z}_{abb} = \dot{X}_{abb} = 0 \\ (180/\pi)(\dot{X}_{abb}\ddot{Z}_{abb} - \dot{Z}_{abb}\ddot{X}_{abb})/(\dot{X}^2_{abb} + \dot{Z}^2_{abb}) & \text{Otherwise} \end{cases}$$

Yaw

$$\dot{\beta} = \begin{cases} 0 & \text{If } \dot{Y}_{abb} = \dot{X}_{abb} = 0 \\ (180/\pi)(\dot{X}_{abb}\ddot{Y}_{abb} - \dot{Y}_{abb}\ddot{X}_{abb})/(\dot{X}^2_{abb} + \dot{Y}^2_{abb}) & \text{Otherwise} \end{cases}$$

where $\dot{\vec{X}}_{abb}$, the missile velocity components with respect to the surrounding air in the b-coordinate directions, are given in Section F. 3. a and the corresponding components of the missile acceleration are:

$$\ddot{\vec{X}}_{abb} = [D]^{-1} \ddot{\vec{X}}_{ee} + [\dot{D}]^{-1} \dot{\vec{X}}_{ee} - [D]^{-1} [A_1] \ddot{\vec{X}}_{w11} - [D]^{-1} [\dot{A}_1] \dot{\vec{X}}_{w11} - [D]^{-1} [A_1] \dot{\vec{X}}_{w11}$$

where $\dot{\vec{X}}_{ee}$ and $\ddot{\vec{X}}_{ee}$ are the components of missile velocity and acceleration in a right handed Cartesian coordinate system located at the launcher; $\ddot{\vec{X}}_{w11}$ and $\dot{\vec{X}}_{w11}$ are the components of wind acceleration and velocity in the local coordinate system (the 1 system; the matrix D is defined in section B. 1. b and the time derivative of the inverse is

$$[\dot{D}]^{-1} = [A_b]^{-1} [A_i]^{-1} [\dot{A}_{eo}]^{-1} + [\dot{A}_b]^{-1} [A_i]^{-1} [A_{eo}]^{-1}$$

where the matrix $[A_i]$ which rotates the inertial axes coordinate components from the i-system to the eo-system is defined in section B. 1. b; the matrix $[A_{eo}]$ which rotates the inertial axes coordinate components from the eo-system to the e-system is defined in section B. 1. b and the time derivative of the inverse is

$$[\dot{A}_{eo}]^{-1} = \omega \begin{bmatrix} 0 & \sin\rho_L & 0 \\ -\sin\rho_L & 0 & -\cos\rho_L \\ 0 & \cos\rho_L & 0 \end{bmatrix} [A_{eo}]^{-1}$$

where ω is the input magnitude of the earth's angular velocity and ρ_L is the input launcher latitude.

The matrix $[A_b]$ which rotates the inertial axes coordinate components from the b-system to the i-system is defined in Section B. 1. b and the time derivative of the inverse is

$$[\dot{A}_b]^{-1} = \begin{bmatrix} 0 & +R_b & -Q_b \\ -R_b & 0 & +P_b \\ Q_b & -P_b & 0 \end{bmatrix} [A_b]^{-1}$$

where P_b, Q_b and R_b are the instantaneous vehicle angular velocities in roll, pitch, and yaw respectively.

The matrix $[A_1]$ which rotates the axes components from the 1-system to the 3-system is defined in Section F. 2. b and the time derivative is

$$[\dot{A}_1](\pi/180) = \left\{ \dot{\mu}' \begin{bmatrix} 0 & \sin\rho_L & 0 \\ -\sin\rho_L & 0 & -\cos\rho_L \\ 0 & \cos\rho_L & 0 \end{bmatrix} + \dot{\rho} \begin{bmatrix} 0 & \cos\rho_L \sin\mu' & -\cos\mu' \\ -\cos\rho_L \sin\mu' & 0 & \sin\rho_L \sin\mu' \\ \cos\mu' & -\sin\rho_L \sin\mu' & 0 \end{bmatrix} \right\} [A_1]$$

where time rate change of vehicle longitude change from the launcher and vehicle latitude are

$$\dot{\mu}' = (180/\pi)(\dot{Z}_{cc} Y_{cc} - \dot{Y}_{cc} Z_{cc})/(Y^2_{cc} + Z^2_{cc})$$

$$\dot{\rho} = (180/\pi)[\dot{X}_{cc} (Y^2_{cc} + Z^2_{cc}) - \dot{Y}_{cc} X_{cc} Y_{cc} - \dot{Z}_{cc} X_{cc} Z_{cc}]/[r^2_c (Y^2_{cc} + Z^2_{cc})^{\frac{1}{2}}]$$

where $\vec{X}_{cc}$ and $\dot{\vec{X}}_{cc}$ are the earth's geocentric coordinate system vector and time derivative respectively.

The rate change of the missile velocity components is:

$$\vec{\ddot{X}}_{bb} = [D]^{-1} \vec{\ddot{X}}_{ee} + [\dot{D}]^{-1} \vec{\dot{X}}_{ee}$$

thus

Pitch

$$\dot{\alpha} = \begin{cases} 0 & \text{if } \dot{Z}_{bb} = \dot{X}_{bb} = 0 \\ (180/\pi)\ (\dot{X}_{bb}\ \ddot{Z}_{bb} - \dot{Z}_{bb}\ \ddot{X}_{bb}) \ / \ (\dot{X}^2_{bb} + \dot{Z}^2_{bb}) & \end{cases}$$

Yaw

$$\dot{\beta} = \begin{cases} 0 & \text{if } Y_{bb} = X_{bb} = 0 \\ (180/\pi)\ (\dot{Y}_{bb}\ddot{Y}_{bb} - \dot{Y}_{bb}\ddot{X}_{bb}) \ / \ (\dot{X}^2_{bb} + \dot{Y}^2_{bb}) & \end{cases}$$

These are calculated only if $T_y = 10$ or 11.

4. Mach Number and Dynamic Pressure

The Mach number is

$M = V_a/C_a$

and dynamic pressure is

$q = (0.7)\ P_a M^2$

G. AERODYNAMIC CHARACTERISTICS

The aerodynamic force and moments are contained here. The aerodynamic representative is applicable only to a symmetrical body about the longitudinal axis.

1. Aerodynamic Stages

There are four aerodynamic stages. All aerodynamic characteristics are input as functions of Mach number. The Mach numbers are input monotonically increasing. The following logic and comments apply to all parameters:

If $|M| \leq M_1$

where M_1 is the first input Mach number, then the coefficients are held at the first input values.

If $|M| \geq M_J$

where M_J (J = 1, 2, ..., 15) is the last input Mach number, then the coefficients are held at the last input values.

a. Body Axial Force

A maximum of fifteen axial force coefficients can be input per stage. The axial force multiplier is

$$\bar{C}_k = \left[\begin{array}{ll} 1.0 & \text{If } \bar{C}'_k = 0 \\ \\ \bar{C}'_k & \text{Otherwise} \end{array}\right.$$

The axial force during the current stage is

$$C = S_{RC}\, q\, C_A\, \bar{C}_k\, |\cos \alpha'|\, \cos \alpha'$$

Where S_{RC} and $\bar{C}'_k$ are input.

The axial force coefficient for

$$M_j \leq |M| \leq M_{j+1}$$

is

$$C_A = \left[\begin{array}{ll} \{M_j\, C_{A(j+1)} - M_{j+1}\, C_{Aj} + [C_{Aj} - C_{A(j+1)}]M\}/ & \\ (M_j - M_{j+1}) + C_{BN} & \text{if } |\alpha| \leq 90 \\ C_{CA} & \text{Otherwise} \end{array}\right.$$

where C_{Aj} and its corresponding Mach number M_j (not necessarily the same Mach numbers as those for the normal force) are input.

The added aerodynamic base drag coefficient due to nozzles not thrusting is

$$C_{BN} = \begin{cases} 0 & \text{if } K_{BD} = 1 \text{ or } 2 \text{ and} \\ & F_M > 0 \text{ or } F_C > 0 \\ (C_{BAM} + C_{BAC}) \quad C_{BD} /2.0 & \text{Otherwise} \end{cases}$$

Area coefficient for main motor

$$C_{BAM} = \begin{cases} A_{eM}/S_{RC} & \text{If } F_M = 0 \text{ and } K_{BD} \neq 1 \\ 0.0 & \text{Otherwise} \end{cases}$$

Area coefficient for complementary motor

$$C_{BAC} = \begin{cases} A_{eC}/S_{RC} & \text{If } F_C = 0 \text{ and } K_{BD} \neq 2 \\ 0.0 & \text{Otherwise} \end{cases}$$

Generalized base drag coefficient

$$C_{BD} = \begin{cases} 0.79996568 + 0.28892727 \ |M| - 10.60569 \ M^2 + 105.77183 \ |M|^3 & \\ -486.47910 \ M^4 + 943.6044 \ |M|^5 - 647.72025 \ M^6 & \text{if } |M| < 0.5 \\ & \\ 3.4709 - 26.428040 \ |M| + 102.16591 \ M^2 - 213.47669 \ |M|^3 & \\ +247.78677 \ M^4 - 150.51554 \ |M|^5 + 37.510830 \ M^6 & \text{if } 0.5 \leq |M| < 1.0 \\ & \\ 41.875203 - 153.36892 \ |M| + 238.07985 \ M^2 - 198.19504 \ |M|^3 & \\ + 92.924624 \ M^4 - 23.211652 \ |M|^5 + 2.4099132 \ M^6 & \text{if } 1.0 \leq |M| < 2.0 \\ & \\ 0.34063761 - 0.95438446 \ |M| - 0.01990222 \ M^2 + 0.15989115 \ |M|^3 & \\ -0.0034737133 \ M^4 + 0.00033548493 \ |M|^5 - 0.000012362510 \ M^6 & \\ & \text{if } 2.0 \leq |M| < 7.0 \\ & \\ 0.007 - 0.007 \ |M| & \text{if } 7.0 \leq |M| < 10.0 \\ & \\ 0.0 & \text{if } 10.0 \leq |M| \end{cases}$$

The drag coefficient at 180° angle of attack

$$C_{CA} = \begin{cases} 0.85 + 0.02076766 \ |M| & \text{if } |M| < 0.60 \\ 1.7337905 - 2.8023463 \ |M| + 2.2502164 \ M^2 & \text{if } 0.060 \leq |M| < 1.00 \\ -1.8550591 + 5.1110871 \ |M| - 2.4796399 \ M^2 & \\ + 0.40418455 \ |M|^3 & \text{if } 1.00 \leq |M| < 2.14 \\ 1.688 & \text{if } 2.14 \leq |M| \end{cases}$$

where A_{eM}, and A_{eC} are the input nozzle exit areas for the main motor and complementary motor, respectively, F_M and F_C are the delivered thrusts of the main motor and complementary motor respectively, S_{RC} is the input aerodynamic axial force coefficient reference, M is the Mach number, and K_{BD} is the input base drag suppression flag input in Lk108.

b. Body Normal Forces Due to Angles of Attack

A maximum of fifteen normal force coefficients can be input per stage.

The normal force multiplier is

$$\bar{N}_k = \begin{cases} 1.0 & \text{If } \bar{N}'_k = 0 \\ \bar{N}'_k & \text{Otherwise} \end{cases}$$

The normal force for the current stage is:

$$N_Y = S_{RNg} \ C_N \ \bar{N} \ \sin \epsilon$$

$$N_Z = S_{RNg} \ C_N \ \bar{N} \ \cos \epsilon$$

where ϵ is the bank angle.

where S_{RN} is input and C_N is defined as follows

$$C_N = \begin{cases} C_{N1}\,\alpha'_E + C_{N2}\alpha'_E\,|\alpha'_E| + C_{N3}\,\alpha'^{3}_E & \text{if } S_{PF} = 0 \\ C_{N1}\,\alpha'_E + [(S_{PF}/S_{RN})C_{CN} - (180/\pi)(\pi/2)C_{N1}]\sin\alpha'_E \cdot |\sin\alpha'_E| & \text{Otherwise} \end{cases}$$

The coefficients for

$$M_j \leq |M| \leq M_{j+1}$$

are

$$C_{N1} = \{M_j\,C_{N1(j+1)} - M_{j+1}\,C_{N1j} + [C_{N1j} - C_{N1(j+1)}]M\}/(M_j - M_{j+1})$$

$$C_{N2} = \{M_j\,C_{N2(j+1)} - M_{j+1}\,C_{N2j} + [C_{N2j} - C_{N2(j+1)}]M\}/(M_j - M_{j+1})$$

$$C_{N3} = \{M_j\,C_{N3(j+1)} - M_{j+1}\,C_{N3j} + [C_{N3j} - C_{N3(j+1)}]M\}/(M_j - M_{j+1})$$

where C_{N1j}, C_{N2j}, and C_{N3j} and their corresponding Mach number M_j ($j = 1, 2, \ldots, 15$) are input.

The following logic apply:

If $C_{N21} = C_{N22} = 0$, then

$C_{N2} = 0$ for all M

and if $C_{N31} = C_{N32} = 0$, then

$C_{N3} = 0$ for all M

where C_{N21}, C_{N22}, C_{N31}, and C_{N32} are input.

where the normal force coefficient at 90° angle of attack

$$C_{CN} = \begin{cases} 0.84584966 + 0.32629796\ |M| - 3.1030112\ |M|^2 + 11.086155\ |M|^3 \\ \quad - 16.513306\ |M|^4 + 9.3484993\ |M|^5 \text{ if } |M| < 0.9 \\ -628.76609 + 2{,}827.6544\ |M| - 5{,}085.6157\ |M|^2 + 4{,}584.0631\ |M|^3 \\ \quad - 2{,}070.4406\ |M|^4 + 374.70503\ |M|^5 \text{ if } 0.9 < |M| < 1.2 \\ 4.3744073 - 5.5002559\ |M| + 4.1332796\ |M|^2 - 1.6453339\ |M|^3 \\ \quad + 0.33668120\ |M|^4 - 0.026740990\ |M|^5 \text{ if } 1.2 \le |M| < 2.2 \\ 1.268 \quad \text{if } |M| \ge 2.2 \end{cases}$$

c. Aerodynamic Normal Force Due to Damping

The transverse force, positive down or to the left, attributable to the vehicle pitching and yawing and to rate change of angle of attack is:

$$N_{Py} = \begin{cases} 0 & \text{If } V_a = 0 \\ q\ S_{RN}\ C_{N1}\ \ell_{cp}\ (\bar{M}_Q\ R_b + \bar{M}_{\dot{\alpha}}\ \dot{\beta})/(2V_a) \end{cases}$$

$$N_{Pz} = \begin{cases} 0 & \text{If } V_a = 0 \\ q\ S_{RN}\ C_{N1}\ \ell_{cp}\ (\bar{M}_Q\ Q_b + \bar{M}_{\dot{\alpha}}\ \dot{\alpha})/(2V_a) \end{cases}$$

where V_a is the missile velocity with respect to the air, Q_b and R_b are the missile angular pitch and yaw rates respectively, $\dot{\alpha}$ and $\dot{\beta}$ are the angular angle of attack rate in pitch and yaw respectively, q is the dynamic pressure ℓ_{cp} is the center of gravity to center of pressure distance, C_{N1} is the linear normal force coefficient, and S_{RN}, $\bar{M}_Q$ and $\bar{M}_{\dot{\alpha}}$ are input.

d. Aerodynamic Fin Force Coefficient

A maximum of fifteen aerodynamic pitch movable fin linear fin lift coefficients, C_{Lz}, nonlinear fin lift coefficients, C_{lz}, drag coefficient, C_{Dz}, and drag due to lift factor, K_{Lz} can be input per stage.

the coefficient for

$$M_j \leq |M| \leq M_{(j+1)}$$

let

$$K_{int} = \frac{M_{(j+1)} - M}{M_{(j+1)} - M_j} \quad \text{and linearly interpolate}$$

$$C_{Lz} = \bar{C}_{Lz} [K_{int} C_{Lz(j+1)} + (1 - K_{int}) C_{Lzj}]$$

$$C_{lz} = \bar{C}_{lz} [K_{int} C_{lz(j+1)} + (1 - K_{int}) C_{lzj}]$$

$$C_{Dz} = \bar{C}_{Dz} [K_{int} C_{Dz(j+1)} + (1 - K_{int}) C_{Dzj}]$$

$$K_{Lz} = \bar{K}_{Lz} [K_{int} K_{Lz(j+1)} + (1 - K_{int}) K_{Lzj}]$$

where C_{Lzj}, C_{lzj}, C_{Dzj}, and K_{Lzj} and their corresponding Mach numbers M_j ($j = 1, 2, \ldots, 15$) are input in the same table with the pitch aerodynamic control fin center of pressure data.

First and last input logic for the aerodynamic pitch movable fin force coefficients are

$$C_{Lz} = \begin{cases} \bar{C}_{Lz}\ C_{Lz1} \text{ if } |M| < M_1 \\ \bar{C}_{Lz}\ C_{LzJ} \text{ if } |M| > M_J \end{cases}$$

$$C_{lz} = \begin{cases} \bar{C}_{lz}\ C_{lz1} \text{ if } |M| < M_1 \\ \bar{C}_{lz}\ C_{lzJ} \text{ if } |M| > M_J \end{cases}$$

$$C_{Dz} = \begin{cases} \bar{C}_{Dz}\ C_{Dz1} \text{ if } |M| < M_1 \\ \bar{C}_{Dz}\ C_{DzJ} \text{ if } |M| > M_J \end{cases}$$

$$K_{Lz} = \begin{cases} \bar{K}_{Lz}\ K_{Lz1} \text{ if } |M| < M_1 \\ \bar{K}_{Lz}\ K_{LzJ} \text{ if } |M| > M_J \end{cases}$$

where the subscript 1 refers to the first table input coefficient and the subscript J refers to the last table entry.

The coefficients multipliers

$$\bar{C}_{Lz} = \begin{cases} 1.0 \text{ if } \bar{C}_{Lz}' = 0 \\ \bar{C}_{Lz}' \text{ otherwise} \end{cases}$$

$$\bar{C}_{lz} = \begin{cases} 1.0 \text{ if } \bar{C}_{Lz}' = 0 \\ \bar{C}_{lz}' \text{ otherwise} \end{cases}$$

$$\bar{C}_{Dz} = \begin{cases} 1.0 \text{ if } \bar{C}_{Dz}' = 0 \\ \bar{C}_{Dz}' \text{ otherwise} \end{cases}$$

$$\bar{K}_{Lz} = \begin{cases} 1.0 \text{ if } \bar{K}_{Dz}' = 0 \\ \bar{K}_{Dz}' \text{ otherwise} \end{cases}$$

where $\bar{C}_{Lz}'$, $\bar{C}_{lz}'$, $\bar{C}_{Dz}'$, and $\bar{K}_{Dz}'$ are input

e. Aerodynamic Fin Forces

Fin angle of Attack

$$\alpha_{cf} = \alpha_E - K_{cf}\ \delta_P$$

$$\alpha_{cf1} = \alpha_{cf} + \delta_R + \delta_{MR} - \alpha_{he}$$

$$\alpha_{cf2} = \alpha_{cf} - \delta_R - \delta_{MR} - \alpha_{he}$$

$$\beta_{cf} = \beta_E - K_{cf}\ \delta_y$$

$$\beta_{cf1} = \beta_{cf} + \delta_R + \delta_{MR} + \alpha_{he}$$

$$\beta_{cf} = \beta_{cf} - \delta_R - \delta_{MR} - \alpha_{he}$$

where the Helix Angle of attack is: $\alpha_{he} = \arctan \{[P_b\ (\pi/180)\ (\ell_{\delta r})/\ [V_e]\}$

where the lift coefficient is:

$$\hat{C}_{Lz1} = C_{LZ}\ \alpha_{cf1} + C_{1Z}\ \alpha_{cf1}\ |\alpha_{cf1}|$$

$$\hat{C}_{Lz} = C_{LZ}\ \alpha_{cf2} + C_{1Z}\ \alpha_{cf2}\ |\alpha_{cf2}|$$

$$\hat{C}_{Lz} = (\hat{C}_{Lz1} + \hat{C}_{Lz2})/2$$

$$\hat{C}_{Ly} = C_{LZ}\ \beta_{cf} + C_{1Z}\ \beta_{cf1}\ |\beta_{cf1}|$$

$$\hat{C}_{Ly} = C_{LZ}\ \beta_{cf2} + C_{1Z}\ \beta_{cf2}\ |\beta_{cf2}|$$

$$\hat{C}_{Ly} = (\hat{C}_{Ly1} + \hat{C}_{Ly2})/2$$

the drag coefficient is:

$$\hat{C}_{DZ1} + C_{DZ} + K_{LZ}\ \hat{C}^2_{LZ1}$$

$$\hat{C}_{DZ2} = C_{DZ} + K_{LZ}\ \hat{C}^2_{LZ2}$$

$$\hat{C}_{DZ} = (\hat{C}_{DZ1} + \hat{C}_{DZ2})/2.0$$

$$\hat{C}_{Dy1} = C_{DZ} + K_{LZ} \hat{C}^2_{Ly1}$$

$$\hat{C}_{Dy2} = C_{DZ} + K_{LZ} \hat{C}^2_{Ly2}$$

$$\hat{C}_{Dy} = (\hat{C}_{Dy1} + \hat{C}_{Dy2})/2.0$$

Normal Force

Pitch

$$N_{\delta z} = -S_{Fz}\ q\ (\hat{C}_{Lz} \cos \alpha_E + \hat{C}_{DZ} \sin \alpha_E)$$

Yaw

$$N_{\delta y} = -S_{Fz}\ q\ (\hat{C}_{Ly} \cos \beta_E + \hat{C}_{Dy} \sin \beta_E)$$

Drag

$$C_{\delta z} = S_{FZ}\ q\ (\hat{C}_{DZ} \cos \alpha_E + \hat{C}_{Dy} \cos \beta_E - \hat{C}_{Lz} \sin \alpha_E - \hat{C}_{Ly} \sin \beta_E)$$

2. Aerodynamic Moments

a. Body Center of Pressure

A maximum of fifteen total center of pressure body stations can be input.

The center of pressure location for

$$M_j \leq M \leq M_{j+1}$$

is

$$x_{cp} = \begin{cases} \hat{x}_{cp} & \text{if } x_{pc} = 0 \\ x_{pc}\ (\sin \alpha')^2 + \hat{x}_{cp}\ (\cos \alpha')^2 & \text{if } |\alpha'| < 90° \ \& \ x_{pc} \neq 0 \\ x_{pc}\ (\sin \alpha')^2 + (0.8\ x_e + 0.2\ x_{pc})\ (\cos \alpha')^2 & \text{if } |\alpha'| > 90° \ \& \ x_{pc} \neq 0 \end{cases}$$

where x_{pc} is the input plan form area centroid body station, x_e is the nozzle exit body station, and α is the angle of attack.

$$\text{where } \hat{x}_{cp} = \bar{X}_{cp}\ \{M_j x_{cp(j+1)} - M_{j+1} x_{cpj} + [x_{cpj} - x_{cp(j+1)}]\ M\}/(M_j - M_{j+1})$$

where x_{cpj} and M_j ($j = 1, 2, \ldots, 15$) and $\bar{X}_{cp}$ are input.

$$\bar{X}_{cp} = \begin{cases} \bar{X}_{cp} & \text{If } \bar{X}_{cp} = 0 \\ 1.0 & \text{Otherwise} \end{cases}$$

The center of pressure and normal force coefficient values are input in the same table. The Mach numbers are input monotonically increasing so the $M_1 < M_2 \ . \ , \ .$

First and last input logic for the center of pressure is

$$\bar{X}_{cp} = \begin{cases} \bar{X}_{cp} x_{cp1} & \text{If } M < M_1 \\ \bar{X}_{cp} x_{cpJ} & \text{If } M > M_J \end{cases}$$

where the x_{cp1} and M_1 and the x_{cpJ} and M_J (J = 1, 2, ..., or 15) are the first and last input values, respectively.

b. Pitching and Yawing Damping Coefficients

The aerodynamic pitch damping moment coefficients are

$$C_{MQ} = \begin{cases} 0 \text{ if } D_{RN} = 0 \\ \hat{C}_{MQ} + K_Q \; (2C_{N1}/D_{RN}^2)[2x_{cp}(x_{RQ} - x_{cg}) - x_{cg}^2 + x_{RQ}^2] \text{ otherwise} \end{cases}$$

$$C_{M\dot{\alpha}} = \begin{cases} 0 \text{ if } D_{RN} = 0 \\ \hat{C}_{M\dot{\alpha}} + K_{\dot{\alpha}} \; (D_{RN}^2)[2x_{cp}(x_{R\dot{\alpha}} - x_{cg}) - x_{cg}^2 + x_{R\dot{\alpha}}^2] \text{ otherwise} \end{cases}$$

where C_{MQ} and $\hat{C}_{M\dot{\alpha}}$ are the unadjusted aerodynamic pitch damping moment coefficients, C_{N1} is the linear normal force coefficient, x_{cp} is the aerodynamic center of pressure body station, x_{cg} is the vehicle center of gravity body station, x_{RQ} and $x_{R\dot{\alpha}}$ are the reference aerodynamic pitch damping moment body stations, and K_Q, $K_{\dot{\alpha}}$, D_{RN} are input.

Reference Pitch Damping Body Stations

The reference aerodynamic pitch damping moment body stations are

$$x_{RQ} = \bar{X}_{cp} \, x'_{RQ}$$

$$x_{R\dot{\alpha}} = \bar{X}_{cp} \, x_{R\dot{\alpha}}$$

where x'_{RQ}, and $x'_{R\dot{\alpha}}$ are input

c. Unadjusted Aerodynamic Pitch Damping Moment Coefficient

A maximum of ten total unadjusted aerodynamic pitch damping moment coefficients can be input.

The coefficients for $M_j \leq |M| \leq M_{j+1}$ are

$$\hat{C}_{MQ} = M_j C_{MQ(j+1)} - M_{j+1} C_{MQj} + [C_{MQj} - C_{MQ(j+1)}] \;/ (M_j - M_{j+1})$$

$$\hat{C}_{M\dot{\alpha}} = M_j C_{M\dot{\alpha}(j+1)} - M_{j+1} C_{M\dot{\alpha}j} + [C_{M\dot{\alpha}j} - C_{M\dot{\alpha}(j+1)}] \;/ (M_j - M_{j+1})$$

The unadjusted aerodynamic pitch damping moments coefficient for pitch rate and angle of attack rate are input in the same table. The Mach numbers are input monotonically increasing so that $M_1 < M_2 \ldots$

First and last input logic for the unadjusted aerodynamic pitch damping moment coefficients are

$$\hat{C}_{MQ} = \left[\begin{array}{l} C_{MQ1} \text{ if } |M| < M_1 \\ \\ C_{MQJ} \text{ if } |M| < M_J \end{array}\right.$$

$$\hat{C}_{M\dot{\alpha}} \quad \left[\begin{array}{l} C_{M\dot{\alpha}1} \text{ if } |M| < M_1 \\ \\ C_{M\dot{\alpha}J} \text{ if } |M| > M_J \end{array}\right.$$

where the C_{MQ1}, $C_{M\dot{\alpha}1}$, and M_1 and the C_{MQJ}, $C_{M\dot{\alpha}J}$, and M_J (J = 1, 2, ..., or 15) are the first and last input values respectively.

d. Aerodynamic Control Fin Center of Pressure

A maximum of fifteen pitch aerodynamic control fin center of pressure ratio of the input pitch fin base root length can be input.

The pitch aerodynamic control fin center of pressure location for

$$M_j \geq |M| \leq M_{j+1}$$

is

$$U_{cz} = \{M_j \, U_{cz(j+1)} - M_{(j+1)} \, U_{czj} - U_{cz(j+1)} \, |M\}$$

$$/ \; (M_j - M_{(j+1)}$$

where U_{czj} and M_j ($j = 1, 2, \ldots, 15$) are input

The pitch aerodynamic control fin center of pressure, and force ceofficients are input in the same table. The Mach numbers are input monotonically increasing so that $M_1 < M_2 \ldots$

First and last input logic for the aerodynamic control surface center of pressure is

$$U_{cz} = \begin{cases} U_{cz1} \text{ if } |M| < M_1 \\ U_{czJ} \text{ if } |M| > M_J \end{cases}$$

where the U_{cz1} and the U_{czJ} and M_J ($J = 1, 2, \ldots,$ or 15) are the first and last input values, respectively.

e. Aerodynamic Disturbing Pitch and Yaw Moments

The moments due to the aerodynamic normal forces are:

$$M_{CZG} = C \quad z_{cg}$$

$$M_{NSQ} = N_Z \, l_{cp}$$

$$M_{NDQ} = \begin{cases} 0 & \text{If } V_a = 0 \\ [q \; S_{RN} \; D^2_{RN}/(2V_a)]\,[\bar{M}_Q C_{MQ} Q_b + \bar{M}_{\dot{\alpha}} C_{M\dot{\alpha}} \dot{\alpha}] & \end{cases}$$

$$M_{NQ} = M_{CZG} + M_{NSQ} + M_{NDQ}$$

$$M_{CYG} = -C \quad y_{cg}$$

$$M_{NSR} = -N_y \, \ell_{cp}$$

$$M_{NDR} = \begin{cases} 0 & \text{If } V_a = 0 \\ [q \; S_{RN} \; D^2_{RN}/ \; (2V_a)]\,[\bar{M}_Q C_{NR} R_b + \bar{M}_{\dot{\alpha}} C_{M\dot{\alpha}} \dot{\beta}] & \end{cases}$$

$$M_{NR} = M_{CYG} + M_{NSR} + M_{NDR}$$

$$\bar{M}_Q = \begin{cases} \bar{M}'_Q & \text{If } \bar{M}'_Q \neq 0 \\ 1.0 & \text{Otherwise} \end{cases}$$

$$\bar{M}_{\dot{\alpha}} = \begin{cases} \bar{M}'_{\dot{\alpha}} & \text{If } \bar{M}'_{\dot{\alpha}} \neq 0 \\ 1.0 & \text{Otherwise} \end{cases}$$

$$M_{NP} = N_z \, y_{cg} \, -N_y \, z_{cg}$$

where N_z and N_Z are the aerodynamic static normal forces, ℓ_{cp} is the center of pressure lever arm, q is the dynamic pressure, V_a is the missile velocity with respect to the air, C_{MQ} and $C_{M\dot{\alpha}}$ are the aerodynamic pitch damping moment derivative, Q_b and R_b are the missile angular pitch and yaw rates respectively, $\dot{\alpha}$ and $\dot{\beta}$ are the angular angle of attack rates, and S_{RN}, D_{RN}, $\bar{M}'_Q$ and $\bar{M}'_{\dot{\alpha}}$ are the input.

f. Disturbing Aerodynamic Rolling Moment

An aerodynamic rolling moment may occur when the missile is not a body of revolution. The equations given below are developed considering raceways, but can be applied to other protuberances as well. Two raceways for each stage can be defined. Their locations (see Figure 19) and reference areas are specified by input. The aerodynamic characteristics of both raceways are specified by a single set of normal force coefficients. A maximum of 10 normal force coefficients versus Mach number can be input for each stage.

If the input roll force reference area, S_{RR1}, is zero for the current stage or there is no atmosphere, then the rolling moment equations are ignored. Otherwise, the rolling moment is:

$$M_{RAP} = (\gamma_a/2)P_a \, (|M_{R1}| \, M_{R1} \, C_{RR1} S_{RR1} \, r_{R1} + |M_{R2}| \, M_{R2} \, C_{RR2} S_{RR2} \, r_{R2})$$

where the reference areas, S_{RRi}, and the center of pressure radial locations from the missile centerline, r_{Ri} (i = 1, 2), are input and the other parameters are computed as follows:

The Mach number due to the velocity normal to the i-th raceway is:

$$M_{Ri} = (-Z_{abb} \sin \Phi_{Ri} - \dot{Y}_{abb} \cos \Phi_{Ri})/C_a$$

where the bank angles of the raceways, Φ_{Ri}, are input.

The instantaneous normal force coefficient for $M_j \leq |M_{Ri}| \leq M_{j+1}$ is:

$$C_{RR} = [M_j \, C_{RR(j+1)} - M_{j+1} \, C_{RRj} + (C_{RRj} - C_{RR(j+1)}) \, |M_{Ri}|] / (M_j - M_{j+1})$$

where C_{RRj} (j = 1, 2, ..., 10) and its corresponding Mach number, M_j (not necessarily the same as in the above sections), are input.

g. Pitch, Yaw and Roll Control Moments

The pitching moment due to the aerodynamic control force is:

$$M_{\delta Q} = N_{\delta z} \cdot \ell_{\delta z}$$

where $N_{\delta z}$ is the pitch fin normal force and $\ell_{\delta z}$ is the pitch movable control fin center of pressure to center of gravity distance.

The yawing moment due to the aerodynamic control force is:

$$M_{\delta R} = N_{\delta y} \cdot \ell_{\delta y}$$

where $N_{\delta y}$ is the yaw fin normal force and $\ell_{\delta y}$ is the yaw movable control fin center of pressure to center of gravity distance.

The rolling moment due to the aerodynamic control force is:

$$M_{\delta P} = \frac{\ell_{\delta r} \, S_{Fz} \, q}{2.0} \left[(\hat{C}_{LZ1} - \hat{C}_{LZ2}) \cos \alpha_E + (\hat{C}_{DZ1} - \hat{C}_{DZ2}) \sin \alpha_E \right.$$

$$\left. + (\hat{C}_{Ly1} - \hat{C}_{Ly2}) \cos \beta_E + (\hat{C}_{Dy1} - \hat{C}_{Dy2}) \sin \beta_E \right]$$

where $\ell_{\delta r}$ is the fin radial center of pressure to missile centerline, S_{Fz} is the input fin reference area, q is the dynamic pressure, α_E is the effective angle of attack, β_E is the effective side slip angle, $\hat{C}_{Lz1}$ and $\hat{C}_{Lz2}$ are the pitch fin lift coefficients, $\hat{C}_{Ly1}$ and $\hat{C}_{Ly2}$ are the yaw fin lift coefficient, $\hat{C}_{Dz1}$ and $\hat{C}_{Dz2}$ are the pitch fin drag coefficient and $\hat{C}_{Dy1}$ and $\hat{C}_{Dy2}$ are the yaw fin drag coefficients.

The torque about the pitch fin hinge axis is:

$$M_{hz} = S_{fz}\, q\, \hat{C}_{LZ} \cdot \ell_{hz}$$

where ℓ_{hz} is the pitch movable control fin center of pressure to hinge axis distance

$$M_{hy} = S_{Fz}\, q\, \hat{C}_{Ly} \cdot \ell_{hz}$$

where ℓ_{hy} is the yaw movable control fin center of pressure to hinge axis location.

H. MISSILE THRUST AND WEIGHT

1. THRUST-WEIGHT STAGES

There are four thrust-weight stages. Missile thrust is computed either from input vacuum thrust, nozzle exit area, and atmospheric pressure or from internal ballistics. Instantaneous weight is a function of input stage weight, weight flow, and propellant vacuum specific impulse. There are two thrust-weight tables for each stage: the main and complementary thrust-weight tables. These two tables allow the capability of simulating the simultaneous operation of motors having different characteristics or they can be used in sequence. Twenty-five data points can be input in each table. Under the internal ballistics option, the input main thrust-weight table is replaced by a thrust vs time, a pressure vs time or a burning area vs web table.

Total axially directed thrust, F, is computed from main thrust, F_M, and complementary thrust, F_C, as follows:

$$F = F_M + F_C$$

a. Noninternal Ballistics Evaluation of Main Thrust Weight Table--The main thrust-weight table consists of the following inputs: stage termination control parameters, σ_S, k_3: total vacuum impulse quantities, I_{vT}, I_{vM}; main stage weight, W_{MO}; thrust, weight flow and time perturbation factors, K_{FM}, K_{WM}, K_{tM}; specific impulse, I_{spM}; nozzle exit area, A_{eM}; weight carryover flag, K_O; nozzle separated flow parameters, ϵ_d, γ_d', C_d', α_d', a_s', c_s', and a maximum of 25, $j = 1, 2, \ldots, 25$, monotonically increasing main thrust weight switching times, $t_M(j)$, with corresponding vacuum thrust, $F_{M(j)}$, and weight flow, $\dot{W}_{M(j)}$.

An adjusted main thrust-weight table of 6 parameters and a maximum of 25 rows is generated as follows:

(1) Adjusted time switching points

$$t_{M(j)} = K_{tM} t_{M(j)}$$

(2) Adjusted vacuum thrust points

$$F_{M(j)} = K_{FM} F_{M(j)}$$

(3) Adjusted total main weight flow points

$$W_{M(j)} = \begin{cases} 0 & \text{if } \hat{t}_{M(j+1)} = \hat{t}_{Mj} \text{ or } j = J \text{ and } W'_{MO} < 0 \\ (W_{Mj} - W_{M(j+1)})/t_{M(j+1} - \hat{t}_{Mj)} & \text{if } \hat{t}_{M(j+1)} \neq \hat{t}_{Mj} \text{ and } j \neq J \text{ and } W'_{MO} < 0 \\ K_{WM} W_{M(j)} & \text{if } I_{spM} = 0 \text{ and } W'_{MO} \geqq 0 \\ K_{WM} W_{M(j)} + \hat{F}_{M(j)}/I_{spM} & \text{Otherwise} \end{cases}$$

(4) Adjusted time rate change of vacuum thrust points

$$\hat{\dot{F}}_{M(j)} = \begin{cases} 0 & \text{if } \hat{t}_{M(j)} \geq t_{M(j+1)} \\ [\hat{F}_{M(j+1)} - \hat{F}_{M(j)}]/[\hat{t}_{M(j+1)} - \hat{t}_{M(j)}] & \text{Otherwise} \end{cases}$$

(5) Adjusted time rate change of weight flow points

$$\hat{\ddot{W}}_{M(j)} = \begin{cases} 0 & \text{if } t_{M(j)} \geq t_{M(j+1)} \text{ or } W'_{MO} < 0 \\ [\hat{\dot{W}}_{M(j+1)} - \hat{\dot{W}}_{M(j)}]/[\hat{t}_{M(j+1)} - \hat{t}_{M(j+1)}] & \text{Otherwise} \end{cases}$$

(6) Adjusted main table expended weight points

$$\hat{W}_{M(j)} = \begin{cases} 0 & \text{if } j = 1 \\ W_{M2} - W_{Mj} & \text{if } W'_{MO} < 0 \\ \sum_{i=1}^{j} 0.5\,[\hat{\dot{W}}_{M(i)} + \hat{\dot{W}}_{M(i-1)}][\hat{t}_{M(i)} - \hat{t}_{M(i-1)}] & \text{Otherwise} \end{cases}$$

Weight, weight flow and thrust are evaluated as:

weight expended for main motor

$$W_M = \hat{W}_{M(j)} + \hat{\dot{W}}_{M(j)} \Delta t_M + 0.5 \hat{\ddot{W}}_{M(j)} \Delta t_M^2$$

weight flow rate

$$\dot{W}_M = \hat{\dot{W}}_{M(j)} + \hat{\ddot{W}}_{M(j)} \Delta t_M$$

vacuum thrust

$$F_{Mv} = \hat{F}_{M(j)} + \hat{\dot{F}}_{M(j)} \Delta t_M$$

where j is such that

$$\hat{t}_{M(j)} \leq t_B \leq \hat{t}_{M(j+1)}$$

and

$$\Delta t_M = t_B - \hat{t}_{M(j)}$$

b. <u>Noninternal Ballistics Evaluation of Motor Chamber Pressure and Nozzle Vacuum Pressure Ratio</u>--If the input propellant diameter (D_p) is less than or equal to zero and the nozzle expansion ratio ϵ_d is greater than 1.0, the following logic is used to establish the motor chamber pressure (P_c) and the nozzle vacuum pressure ratio (P_c/P_e).

Nozzle half angle momentum correction coefficient

$$\lambda_d = (1 \cos \alpha_d)/2.$$

Nozzle half angle

$$\alpha_d = \begin{cases} 15^\circ & \text{if } \alpha_d' = 0 \\ \alpha_d' & \text{Otherwise} \end{cases}$$

Where α_d' is the input nozzle half angle.

Optimum thrust coefficient $(P_e = P_d)$

$$C_{FO} = \Omega_d \, [1 - (P_c/P_e)^{-\Gamma_d}]^{\frac{1}{2}}$$

Chamber pressure

$$P_c = F_{Mv} \epsilon_d / \left\{ \left[C_D \lambda_d C_{FO} + \epsilon_d/(P_c/P_e) \right] \Lambda_{em} \times 144. \right\}$$

Nozzle efficiency coefficient

$$C_D = \begin{cases} 0.96 & \text{If } C_D' = 0 \\ C_D' & \text{Otherwise} \end{cases}$$

Where C_D' is the input nozzle efficiency coefficient

Ratio of specific heats functional constants

$$\Gamma_d = (\gamma_d - 1)/\gamma_d$$

$$\Xi_d = [(\gamma_d - 1)/(\gamma_d + 1)]^{\frac{1}{2}} [2/(\gamma_d + 1)]^{[1/(\gamma_d - 1)]}$$

$$\Omega_d = \gamma_d \, [2/(\gamma_d - 1)]^{\frac{1}{2}} [2/(\gamma_d + 1)]^{\{(\gamma_d + 1)/[2(\gamma_d - 1)]\}}$$

$$\gamma_d = \begin{cases} 1.18 & \text{if } \gamma_d' = 0 \\ \gamma_d' & \text{Otherwise} \end{cases}$$

Where γ_d' is the input exhaust gas ratio of specific heats.

Nozzle expansion ratio

$$\epsilon = \epsilon_d$$

Pressure ratio

Solve for (P_c/P_e) by Newton-Raphson iteration method

$$\epsilon_d = \Xi_d (P_c/P_e)^{(1/\gamma_d)} [1 - (P_c/P_e)^{-\Gamma_d}]^{-\frac{1}{2}}$$

Let $[P_c/P_e]_{(n)} = R_{(n)}$

$$R_o = (0.3953 + 2.785\ \gamma_d)\epsilon_d^{(0.28563 + 0.8631\ \gamma_d)}$$

Algorithm

$$R_{(n+1)} = R_{(n)} - F[R_{(n)}]F'[R_{(n)}] \quad n = 0, 1, 2, \ldots$$

Where

$$F[R_{(n)}] = \Xi_d (R_{(n)}]^{(1/\gamma_d)} \{ 1 - [R_{(n)}]^{-\Gamma_d}\}^{-\frac{1}{2}} - \epsilon_d$$

$$F'[R_{(n)}] = \Xi_d\ (1/\gamma_d)\ [R_{(n)}]^{-\Gamma_d} (1 - [R_{(n)}]^{-\Gamma_d} \}^{-\frac{1}{2}}$$

$$- (1/2)\ \Xi_d\ \Gamma_d\ [R_{(n)}]^{-(2\ \Gamma_d)} \{ 1 - [R_{(n)}]^{-\Gamma_d}\}^{-3/2}$$

Terminate iteration if

$$F[R_{(n)}]/\epsilon_d < 0.000001$$

Then set

$$(P_c/P_e) = R_{(n)}$$

Where ϵ_d • input nozzle expansion ratio

c. <u>Internal Ballistic Evaluation of Main Thrust Weight Table</u>--If the input propellant diameter (D_p) is greater than zero, the following equations and logic are used to simulate the operation of a single chamber controllable motor.

L-Number	Symbol	Definition	Default
L-k001	D_p	Diameter of propellant, in.	0.0
L-k002	r'_{b1000}	Burning rate of propellant at 1000 lb/sq in. chamber pressure and flag to determine evaluation option, in./sec	0.0
K-k013	ϵ_d	Nozzle expansion ratio, dim	0.0
L-k014	γ_d	Ratio of specific heats of the rocket motor exhaust gases, dim	1.18
L-k015	C_D	Nozzle efficiency coefficient, dim	0.96
K-k016	α_d	Nozzle effective half angle, deg	15
L-k095	ρ_p	Density of propellant, lb/cu in.	0.065
L-k096	τ_w	Web fraction, dim	0.8
L-k097	n	Burn rate exponent, dim	0.6
L-k098	α_p	Propellant diffusivity, sq in./sec	0.00027
L-k099	P_{max}	Maximum allowable chamber pressure, lb/sq in.	0.0
L-k407	ω_{PC}	Pressure control frequency, rad/sec	0.0

The following logic will initialize the following tables depending on the input options: (a) Vacuum Thrust-Time Table, i.e., if $r'_{b1000} = 0$. (b) Chamber Pressure-Time, i.e., if $r'_{b1000} < 0$, (c) Surface Area-Web Depth Fraction, i.e., if $r'_{b1000} > 0$.

1	$\hat{t}_{Mj}$	Time, sec
2	$\hat{F}_{Mj}$	Vacuum thrust, lb
3	P_{cj}	Chamber pressure, lb/sq in.
4	$\dot{P}_{cj}$	Time change of chamber pressure, lb/sq in.-sec
5	g_{wj}	Web depth fraction, dim
6	V_{cj}	Chamber volume, cu in.

7	g_{Pj}	Expended propellant fraction, dim
8	A_{Sj}	Surface area, sq in.
9	A_x	Throat extinguishment area, sq in.

In addition to these tabular values, the following constants are also calculated:

C^*, A_{tref}, r_{b1000}, f_{wt}, W_p, I_{TM}, a, C_{AS1}, C_{AS2} and C_{AS3}

A_{xmax}, A_{xmin}, P_{cmxa} and $\overline{P}_{cref}$

Thrust coefficient route (dim)

$$C_f = f\,(\gamma_d, \epsilon_d, \alpha_d, C_d)$$

Ratio of specific heats functional constants

$$\Gamma_d = (\gamma_d - 1)/\gamma_d$$

$$\Xi_d = \left[(\gamma_d - 1)/(\gamma_d + 1)\right]^{1/2} \left[2/(\gamma_d + 1)\right] \left[1/(\gamma_d - 1)\right]$$

$$\Omega_d = \gamma_d \left[2/(\gamma_d - 1)\right]^{1/2} \left[2/(\gamma_d + 1)\right] \left\{(\gamma_d + 1)/\left[2(\gamma_d - 1)\right]\right\}$$

Pressure ratio

Solve for (P_c/P_e) by Newton-Raphson iteration method

$$\epsilon_d = \Xi_d\,(P_c/P_e)^{(1/\gamma_d)} \left[1 - (P_c/P_e) - \Gamma_d\right]^{-1/2}$$

let $\left[P_c/P_e\right]_{(\ell)} = R_{(\ell)}$

$$R_o = (0.3953 + 2.785\,\gamma_d)\ \epsilon_d\ (0.28563 + 0.8631\ \gamma_d)$$

Algorithm

$$R_{(\ell + 1)} = R_{(\ell)} - F\left[R_{(\ell)}\right] / F'\left[R_{(\ell)}\right] \quad l = 0, 1, 2, \ldots$$

Where

$$F\left[R_{(\ell)}\right] = \Xi_d \left[R_{(\ell)}\right]^{(1/\gamma_d)} \quad 1 - \left[R_{(\ell)}\right]^{-\Gamma_d} \quad ^{-1/2} - \epsilon_d$$

$$F'\left[R_{(\ell)}\right] = \Xi_d \,(1/\gamma_d) \left[R_{(\ell)}\right]^{-\Gamma_d} \left\{1 - \left[R_{(\ell)}\right]^{-\Gamma_d}\right\}^{-1/2}$$

$$- (1/2)\, \Xi_d\, \Gamma_d \left[R_{(\ell)}\right]^{-(2\Gamma_d)} \left\{1 - \left[R_{(\ell)}\right]^{-\Gamma_d}\right\}^{-3/2}$$

Terminate iteration if

$$\left| F\left[R_{(\ell)}\right] / \epsilon_d \right| < 0.000001 \text{ or } \ell = 10$$

then set

$$(P_c/P_e = R_{(\ell)}$$

Optimum thrust coefficient $(P_e = P_d)$

$$C_{FO} = \Omega_d \left[1 - (P_c/P_e)^{-\Gamma_d}\right]^{1/2}$$

Vacuum thrust coefficient $(P_d = 0)$

$$C_f = C_D\, \lambda_d\, C_{FO} + \epsilon \frac{P_c}{P_c}$$

Characteristic Velocity (ft/sec)

$$C^* = \frac{I_{spH}\, \bar{g}_c}{C_f}$$

Reference Throat Area (in^2)

$$A_{tref} = \frac{A_{eM} \times 144}{\epsilon_d}$$

Web Thickness (in)

$$f_{wt} = \tau_w \, D_p/2$$

Number of points in table (dim)

Set J such that

$$t'_{MJ} > t'_{M(J+1)}$$

or when table is full J = 25

For option using input thrust versus time

if $r'_{b1000} = 0$

the thrust multiplier logic is used to evaluate the adjusted vacuum thrust time points $\hat{F}_M$, and $\hat{t}_{Mj}$

Chamber Pressure (lb/in^2)

$$P_{cj} = \hat{F}_{Mj}/(A_{tref} \cdot C_f)$$

$$j = 1, \ldots, J$$

The total impulse (lb-sec)

$$I_T = 0.5 \sum_{j=1}^{J-1} (\hat{F}_{M(j+1)} + \hat{F}_{Mj})(\hat{t}_{M(j+1)} - \hat{t}_{Mj})$$

Burn Rate Coefficient ([in/sec][in²/lb]ⁿ)

$$a = \frac{f_{wt}}{I_r}$$

Where

$$I_{rb(jh)} = \begin{cases} \dfrac{[P_{c(j+1)}^{\;n+1} - P_{cj}^{\;n+1}]\,[\hat{t}_{M(j+1)} - \hat{t}_{Mj}]}{(n+1)\,[P_{c(j+1)} - P_{cj}]} & \text{If } P_{c(j+1)} \neq P_{cj} \\ \\ P_{cj}^{\;n}\,[\hat{t}_{M(j+1)} - \hat{t}_{Mj}] & \text{Otherwise} \end{cases}$$

$$I_r = \sum_{j=2}^{J} I_{rbj}$$

Reference Burn Rate at 1000 psi (in/sec)

$$r_{bref} = a(1000)^n$$

Fraction Web Burned at Thrust Time Points (dim)

$$g_{w1} = 0$$

$$g_{wj} = g_{w(j-1)} + \frac{a}{f_{wt}}\,I_{rbj}$$

$j = 2, \ldots J$

Fraction Propellant at Thrust Time Points (dim)

$$g_{p1} = 0$$

$$g_{pj} = g_{p(j-1)} + \frac{[\hat{F}_{M(j)} + \hat{F}_{M(j-1)}]\,[\hat{t}_{Mj} - \hat{t}_{M(j-1)}]}{2 * I_{TM}}$$

$j = 2, \ldots J$

Total Propellant Weight (lb)

$$W_p = I_{TM}/I_{SPM}$$

Initial Chamber Volume (in^3)

$$V_{c1} = \frac{W_p}{\rho_p}\left(\frac{(1-\tau_w)^2}{1 - (1-\tau_w)^2}\right)$$

Chamber Volume at Thrust Time Points (in^3)

$$V_{cj} = V_{c1} + \frac{W_p}{\rho_p}\, g_{pj} \qquad \cdot\ 2, \ldots J$$

Time Rate Change of Chamber Pressure Between Thrust Time Points (lb/in^2- sec)

$$\dot{P}_c \begin{cases} [P_{c(j+1)} - P_c]/[\hat{t}_{M(j+1)} - \hat{t}_{Mj}] & \text{If } \hat{t}_{M(j+1)} \neq \hat{t}_{Mj} \\ [P_{c(j+1)} - P_{cj}]/.0001 & \text{Otherwise} \end{cases}$$

$j = 1, \ldots, J-1$

Last point

$$\dot{P}_{cJ} = 0$$

Burn Surface Coefficients (1/in), (in^2/sec),([lb/in^2-sec][in^2/lb]^n)

$$C_{AS1} = \frac{\bar{g}_e \left(\frac{\gamma_d+1}{2}\right)^{\frac{\gamma_d+1}{\gamma_d-1}}}{\gamma_d C^{*2} \times 12}$$

$$C_{AS2} = \frac{A_{tref} \bar{g}_e}{C^*}$$

$$C_{AS3} = \rho_p a$$

Surface Areas (in^2)

$$A_{S1} = (CAS2/CAS3) P_{c1}^{1-n}$$

$$A_{Sj} = \frac{C_{AS1} \dot{P}_{cj} V_{cj} + C_{AS2} P_{cj}}{C_{AS3} P_{cj}^{n}}$$

$j = 2, \ldots, J$

If $A_{Sj} < 0$, A_{Sj} is set to zero

For option using input chamber pressure versus time

If $r'_{b1000} < 0$

Chamber Pressure (lb/in^2)

$P_{cj} = F'_{Mj}$ $j = 1, \ldots, J$

Vacuum Thrust (lb)

$$\hat{F}_{Mj} = P_{cj} A_{tref} C_f \qquad j = 1, \ldots, J$$

Total Impulse (lb-sec)

$$I_{TM} = 0.5 K_t \sum_{j=1}^{J-1} (\hat{F}_{M(j+1)} + \hat{F}_{Mj})(t'_{M(j+1)} - t'_{Mj})$$

The initialization parameters are set as:

$$I'_{VT} = 0$$

$$I'_{VM} = I_{TM}$$

$$P'_{arm} = 0$$

The thrust multiplier logic is used to evaluate the adjusted vacuum thrust time points $\hat{F}_{Cj}$ and $\hat{t}_{cj}$ and the logic in the option using the input thrust versus time is used to evaluate the ballistic parameters.

For option using input A_{Sj} versus g_{wj}

If $r_{b1000} > 0$

Set

$$g_{wj} = 0$$

$$g_{wj} = t'_{Mj} \qquad j = 2, \ldots, J$$

$$A_{Sj} = F'_{Mj} \qquad j = 1, \ldots, J$$

Burn Rate Coefficient ([in/sec][in²/lb]^n)

$$a = r'_{b1000}/(1000)^n$$

Burn Surface Area Coefficients (1/in),(in²/sec),([lb/in²-sec][in²/lb]^n)

$$C_{AS1} = \frac{\tilde{g}_e \left(\frac{\gamma_d+1}{2}\right)^{\frac{\gamma_d+1}{\gamma_d-1}}}{12\ \gamma_d\ C^{*2}}$$

$$C_{AS2} = \frac{A_{tref}\ \tilde{g}_e}{C^*}$$

$$C_{AS3} = \rho_p\ a$$

Chamber Pressure Coefficients

$$C_{PC1} = \frac{C_{AS1}\ a}{(n+1)\ C_{AS2}}$$

$$C_{PC2} = \frac{C_{AS1}}{2\ \rho_p}$$

$$C_{PC3} = \frac{C_{AS3}}{C_{AS2}}$$

Initial Chamber Pressure (lb/in²)

$$P_{c1} = (C_{PC3}\ A_{S1})^{\frac{1}{1-n}}$$

Evaluating V_{cj} , P_{cj}, $\dot{P}_{cj}$, $\ddot{t}_{Mj}$ at g_{wj} points

The following calculations are performed for $j = 1, \ldots, J$

Incremental Burn Distance

$$\Delta r_{bj} = (g_{wj+1} - g_{wj})\, f_{wt}$$

Iteration for Chamber Pressure, i.e., $P_{c(j+1)}$

let

k = iteration number

$$F(P_{ck}) = P_{ck} + C_{PC1}\, \frac{V_{c(j-1)}}{\Delta r_{bj}} \left(P_{ck}^{\,n+1} - P_{cj-1}^{\,n+1}\right)$$

$$+ \, C_{PC2}\, (P_{ck}^{\,2} - P_{cj-1}^{\,2}) - C_{PC3}\, A_{S\,(j)}\, P_{ck}^{\,n}$$

$$F'(P_{ck}) = 1 + C_{PC1}\, \frac{V_{c(j-1)}}{\Delta r_{bj}}\, (n+1)\, P_{ck}^{\,n}$$

$$+ \, 2\, C_{PC2}\, P_{ck} - nC_{PC3}\, A_{S(j)}\, P_{ck}^{\,n-1}$$

Newton-Raphson Iteration Method

Algorithm

$$P_{(k+1)} = P_{ck} - F\,(P_{ck})F'\,(P_{ck}) \qquad k = 1, 2, \ldots$$

Terminate iteration if:

$$\left(\frac{P_{c(k+1)} - P_{ck}}{\left|P_{ck}\right|}\right) < 1.E-10$$

or $k = 50$

Set

$$P_{c\,(j)} = P_{c\,(k+1)}$$

Time rate change of chamber pressure

$$\dot{P}_{cj} = \frac{P_{c(j)} - P_{c(j-1)}}{\tau} = \frac{a}{\Delta r_{bj(n+1)}} \left(P_{c(j)}^{\;n+1} - P_{cj-1}^{\;n+1}\right)$$

Initial chamber volume

$$V_{c1} = \frac{(1-\tau_w)^2}{1-(1-\tau_w)^2} \; \frac{fwt}{2} \; \sum_{j=1}^{J-1} \left(A_{S\,(j+1)} + A_{Sj}\right)\left(g_{w(j+1)} - g_{wi}\right)$$

Chamber volume

$$V_{c(j+1)} = V_{cj} + \frac{C_{AS2}}{2\,\rho} \; \tau_j \left(P_{c(j+1)} + P_{cj}\right)$$

Incremental burn time (sec)

$$\tau_j = \frac{\Delta r_{bj}\,(n+1)}{a} \left[\frac{P_{c(j+1)} - P_{cj}}{P_{c(j+1)}^{\;n+1} - P_{cj}^{\;n+1}}\right]$$

Time

$$\hat{t}_{M1} = 0$$

$$\hat{t}_{Mj} = \sum_{2}^{j} \hat{t}_{M(i-1)} \, \tau_{i-1}$$

Vacuum Thrust (lb)

$$\hat{F}_{Mj} = P_{cj} \, C_f \, A_{tref} \qquad j = 1, \ldots, J$$

Total Impulse (lb-sec)

$$I_{TM} = 0.5 \sum_{j=1}^{j_{max-1}} (\hat{F}_{M(j+1)} + \hat{F}_{Mj})(\hat{t}_{M(j+1)} - \hat{t}_{Mj})$$

Total Propellant Weight (lb)

$$W_p = I_{TM}/I_{SPM}$$

Fraction Expended Propellant at Thrust Time Points for j = 1, 2, ..., J

$$\varepsilon_{p1} = 0$$

$$\varepsilon_{pj} = \varepsilon_{p(j-1)} + \frac{[\hat{F}_{M(j)} + \hat{F}_{M(j-1)}][\hat{t}_{Mj} - \hat{t}_{M(j-1)}]}{2 \times I_{TM}}$$

The initialization parameters are set as:

$$I'_{VT} = 0$$

$$I'_{VM} = I_{TM}$$

$$P_{arm} = 0$$

The thrust multiplier logic is used to evaluate the adjusted complementary tube vacuum thrust time points $\hat{F}_{Cj}$ and $\hat{t}_{Cj}$.

The following logic is applicable to all options when $D_p \neq 0$

Extinguishment Area

$$A_{xj} = 1.1 \; \frac{AS_j \; C_{AS3} \; C^*}{\bar{g}_c} \left[\frac{a}{n\alpha_p} \; \frac{V_{Cj}}{AS_j} \; \frac{C_{AS1}}{\rho_p} \right]^{\frac{1-n}{1+n}}$$

Maximum ExtinguishmentArea

$$A_{xmax} = MAX \; (A_{xj} \quad j = 1, \ldots, j_{max})$$

Average Reference Chamber Pressure

$$\bar{P}_{cref} = \frac{I_{TM}}{A_{tref} \; C_f \; \hat{t}_{MJ}}$$

Maximum Reference Chamber Pressure

$$P_{cmax} = MAX \; (P_{cj} \quad j = 1, \ldots, J)$$

Maximum Allowable Chamber Pressure

$$P_{CMXA} \left[\begin{array}{ll} 1.E\;30 & P_{max} = 0 \\ P_{max} & P_{max} > 0 \\ -P_{max} \; P_{cmax} & P_{max} < 0 \end{array} \right.$$

Maximum Surface Area

$$A_{SMAX} = MAX \; (A_{Sj} \; j = 1, \ldots, J)$$

Minimum Throat Area

$$A_{xmin} = A_{SMAX} \; P_{CMXA}^{\;n-1} \; CAS3 \; \frac{C^*}{g_e}$$

d. Controllable Motor Dynamics

The following logic and equations evaluate the dynamic internal ballistics of a pintle controlled single chamber motor.

Burn Rate

$$r_b = \begin{cases} 0 & \text{If } A_t > A_{XI} \\ a\ P_c^{\ n} & \text{Otherwise} \end{cases}$$

Burn Depth

$$D_b = {}_o\int^t r_b \, dt$$

Percent Web

$$g_{wI} = D_b / f_{wt}$$

<u>Interpolation Formulas</u>

Index Table

j is such that

$$g_{wj} \leq g_{wI} < g_{w(j+1)}$$

Incremental Burn Distance

$$\Delta r_b = (g_{wI} - g_{wj}) \ f_{wt}$$

Interpolation Time

$$\tau = \begin{cases} \dfrac{\left[(n+1)\ \dot{P}_{cj}\ \dfrac{\Delta r_b}{a} + (P_{cj})^{n+1}\right]^{1/(n+1)} - P_{cj}}{\dot{P}_{cj}} & \text{If } P_{cj} \neq P_{c(j+1)} \\ \\ \dfrac{\Delta r_b}{a}\ P_{cj}^{-n} & \text{Otherwise} \end{cases}$$

Chamber Volume

$$V_{cI} = V_{cj} + \frac{A_{tref} \cdot \bar{g}_e}{\rho_p\ C^*}\ (P_{cj}\ \tau + \tfrac{1}{2}\ \dot{P}_{cj}\ \tau^2)$$

Nominal Vacuum Thrust

$$F_{VN} = \hat{F}_{M_j} + \frac{\left(\hat{F}_{M(j+1)} - \hat{F}_{Mj}\right)}{t_{M(j+1)} - t_{Mj}}\ \tau$$

Nominal Delivered Thrust

$$F_N = F_{VN} - P_a A_{eM}$$

Burn Surface Areas

$$A_{SI} = \frac{C_{AS1}\ \dot{P}_{cj}\ V_{CI} + C_{AS2}\ (P_{cj} + \dot{P}_{cj}\ \tau)}{C_{AS3}\ (P_{cj} + \dot{P}_{cj}\ \tau)^n}$$

Time Change of Chamber Pressure

$$\dot{P}_c = \begin{cases} (C_{AS3}\ A_{SI} P_c^{\,n} - \dfrac{A_t}{A_{tref}}\ C_{AS2}\ P_c)/(C_{AS1}\ V_{cI}) & \text{If } A_t < A_X \\ \\ -\dfrac{A_t}{A_{tref}}\ C_{AS2}\ P_c/C_{AS1}\ V_{cI} & \text{Otherwise} \end{cases}$$

Equilibrium

$$P_{ceq} = \begin{cases} \left[\dfrac{C_{AS3} A_{tref} A_{SI}}{A_t C_{AS2}} \right]^{\frac{1}{1-n}} & \text{If } A_t < A_x \\ 0 & \text{Otherwise} \end{cases}$$

Chamber Pressure

$$P_c = \begin{cases} P_{co} + \int_{t_{Bk} = 0}^{t} \dot{P}_c \, dt & \text{If TMC} \neq 0 \\ P_{ceq} & \text{Otherwise} \end{cases}$$

where TMC is the thrust dynamic mode, if = 0, the achieved thrust equals command thrust and, if = 1, the first order response system is solved.

The initial chamber pressure is set at $P_{co} = P_{c1}$ ($\dot{P}_c$ equation)

Expansion Ratio

$$\epsilon = \epsilon_d A_{tref}/A_t$$

Thrust Coefficient

$$C_{FI} = f\ (\gamma_d, \epsilon, \alpha_d, C_d)$$

Thrust

$$F_{Mv} = P_c A_t C_{FI}$$

Mass Weight Flow Rate

$$\dot{W}_{MP} = \frac{P_c\ A_t\ \bar{g}_e}{C^*}$$

Mass Propellant Removed

$$W_{pr} = \int_{t_{Bk=0}}^{t_{Bk}} \dot{W}_{MP}\ dt$$

Fraction of Propellant Removed

$$g_{PI} = \frac{W_{pr}}{W_p}$$

Extinguishment Area

$$A_{XI} = \frac{A_{SI}\ C_{AS3}\ C^*}{\bar{g}_e} \left[\frac{a}{n\alpha_p}\ \frac{V_{CI}}{A_{SI}}\ \frac{C_{AS1}}{\rho_p} \right]^{\frac{1-n}{1+n}}$$

The following logic evaluates the commanded chamber pressure and equilibrium throat area for a given commanded thrust.

Commanded Vacuum [illegible]t (lb)

$$F_{vcom} = F_c + P_a A_{eM}$$

Interate for R_{ATCC}

$$R_{ATCCO} = 1$$

$$\epsilon_{cci} = \epsilon_d / R_{ATcci}$$

$$C_{fi} = f\ (\gamma_d, \epsilon_{cci}, \alpha_d, C_d)$$

$$R_{ATcc(i+1)} = \left[\frac{C_{AS3}}{C_{AS2}} \ A_{sI} \right]^{\frac{1}{n}} \left(\frac{A_{tref}\ R_{ATcci}\ C_{fi}}{F_{vcom}} \right)^{\frac{1-n}{n}}$$

Converged if

$$\frac{\left| R_{ATcc(i+1)} - R_{ATcci} \right|}{R_{ATcci}} < 1. \times 10^{-6}$$

$$A_{tcc} = A_{tref}\ R_{ATcci}$$

Command Chamber Pressure (lb/in^2)

$$P_{cc} = \frac{F_{cv}}{A_{tcc}\ C_{fi}}$$

Pintle Area Control Law

Rate Change of Throat Area

$$\dot{A}_t = K_s \left(P_c - P_{cc} + K_p \dot{P}_c\right)$$

where

the pintle control frequency (rad/sec)

$$\omega_p = \frac{C_{AS2}\ (1-n)}{C_{AS1}\ V_{CI}}$$

the dynamic constant $(\mathrm{lb/in^4\text{-}sec})$

$$\lambda_p = \frac{C_{AS2}}{A_{tref}\ C_{AS1}\ V_{CI}} \left[\frac{C_{AS3}\ A_{SI}}{C_{AS2}}\right]^{\frac{1}{1-n}}$$

Controller Frequency

$$\omega_{pc} = \begin{bmatrix} \omega_p \text{ if } \omega'_{pc} = 0 \\ \omega_{pc} \text{ otherwise} \end{bmatrix}$$

Pressure Error Gain $(\mathrm{in^4/lb\text{-}sec})$

$$K_S = \omega_{pc}^2 / \lambda_p$$

Pressure Rate Gain (sec)

$$K_p = \frac{.4}{\omega_{pc}}$$

Throat Area

$$A_{tp} = \begin{cases} A_{t0} + \int_{t_{Bk} = 0}^{t} \dot{A}_t dt & \text{If TMC} \neq 0 \\ A_{tcc} & \text{Otherwise} \end{cases}$$

$$A_t = \begin{cases} A_{xmax} & \text{If } A_{tp} > A_{xmax} \\ A_{xmin} & \text{If } A_{tp} < A_{xmin} \\ A_{tp} & \text{Otherwise} \end{cases}$$

Initial Throat Area

$$A_{to} = A_{tref}$$

If $A_{tp} > A_{xmin}$ or $A_{tp} < A_{xmin}$

Set

$$\dot{A}_t = 0$$

e. Main Table Delivered Thrust

Thrust equation to include flow separation in the nozzle:

$$F_M = \left[\begin{array}{ll} 0 & \text{if } F_{Mv} = 0 \\ F_{Mv} - P_d A_{eM} & \text{if } \epsilon_d = 0 \text{ or } P_s \leqq P_e \\ 0 & \text{if } \epsilon_d \neq 0 \text{ and } P_c \leqq P_d \\ \multicolumn{2}{l}{\left| \lambda_d \, C_D \Omega_d \, \left[1 - (P_s/P_c)\Gamma d\right]^{1/2} + (P_s - P_d)\,(\epsilon_s/P_c) \right|} \\ \left[(P_c A_{eM}/\epsilon) \quad (1/144)\right] & \text{Otherwise,} \end{array} \right.$$

Where F_{Mv} is obtained from the main vacuum thrust-weight table, A_{eM} the input area, and P_d the static base pressure.

Nozzle exit pressure (lb/in.^{2})

$$P_e = P_c/(P_c/P_e).$$

Nozzle separation pressure (lb/in.^{2})

$$P_s = \left[\begin{array}{ll} \multicolumn{2}{l}{P_d \, (a_s + b_s) \left| \left[2/(\gamma_d + 1)\right]^{\left[\gamma_d/(\gamma_d - 1)\right]} (P_c/P_d) \right|^{-c_s}} \\ & \text{It } P_c > P^* \\ P_d & \text{Otherwise} \end{array} \right.$$

Static base pressure (lb/in.^{2})

$$P_d = P_a/144.0$$

Where P_a is the ambient pressure.

Where

$$a_s = \begin{cases} 0.3 & \text{If } a_s' = b_s' = c_s' = 0 \\ a_s' & \text{Otherwise} \end{cases}$$

$$b_s = \begin{cases} 0.7 & \text{If } a_s' = b_s' = c_s' = 0 \\ b_s' & \text{Otherwise} \end{cases}$$

$$c_s = \begin{cases} 0.884 & \text{If } a_s' = b_s' = c_s' = 0 \\ c_s' & \text{Otherwise} \end{cases}$$

Where a_s', b_s', and c_s' are input nozzle separation polynomial coefficients.

Nozzle separation expansion ratio

$$\epsilon_s = \begin{cases} \Xi_d \ (P_s/P_c)^{-(1/\gamma_d)}[1 - (P_s/P_c)^{\Gamma_d - \frac{1}{2}}] & \text{If } P_c > P^* \\ 1 & \text{Otherwise} \end{cases}$$

Nozzle critical pressure

$$P^* = P_c[2/(\gamma_d + 1)]^{[\gamma_d/(\gamma_\alpha - 1)]}$$

f. Complementary Thrust-Weight

The complementary thrust-weight table consists of the following input: thrust, weight flow and time perturbation factors, K'_{FC}, K_{WC}, K_{tC}; specific impulse, I_{spC}; nozzle exit area, A_{eC}; complementary stage weight, $\dot{W}_{CO}$; total vacuum impulse quantities, I'_{VC}; complementary weight carryover flag, K_{NO}; and a maximum of 25, j = 1, 2, ..., 25, monotonically increasing complementary thrust weight switching times, $t_{C(j)}$, and weight flow, $\dot{W}_{C(j)}$.

The adjusted complementary thrust-weight table is as follows: At stage initiation set up a 6 parameter table with a maximum of 25 rows such that the:

Adjusted time switching points are:

$$\hat{t}_{C(j)} = K_{tC}\, t_{C(j)}$$

Adjusted vacuum thrust points are:

$$\hat{F}_{C(j)} = K_{FC}\, F_{M(j)}$$

Adjusted total complementary weight flow points are

$$\hat{\dot{W}}_{C(j)} = \begin{cases} 0 & \text{If } \hat{t}_{C(j+1)} = \hat{t}_{Cj} \text{ or } j - J \text{ and } W_{Co} < 0 \\ (W_{Cj} - W_{C(j+1)})/(\hat{t}_{C(j+1)} - \hat{t}_{Mj}) & \\ & \text{If } \hat{t}_{C(j+1)} \neq \hat{t}_{Cj} \text{ and } j \neq J \text{ and } W'_{Co} < 0 \\ K_{WC}\, \dot{W}_{C(j)} & \text{If } I_{spC} = 0 \text{ and } W'_{Co} \geq 0 \\ K_{WC}\, \dot{W}_{C(j)} = \hat{F}_{C(j)}/I_{spC} & \\ & \text{Otherwise} \end{cases}$$

Adjusted time rate change of vacuum thrust points is

$$\dot{\hat{F}}_{C(j)} = \begin{cases} 0 & \text{If } \hat{t}_{C(j)} \geq t_{C(j+1)} \\ [\hat{F}_{C(j+1)} - \hat{F}_{C(j)}]/[\hat{t}_{C(j+1)} - \hat{t}_{C(j)}] & \text{Otherwise} \end{cases}$$

Adjusted time rate change of weight flow points is

$$\hat{\ddot{W}}_{C(j)} = \begin{cases} 0 & \text{If } \hat{t}_{C(j)} \geq \hat{t}_{C(j+1)} \text{ or } W'_{CO} < 0 \\ [\hat{\dot{W}}_{C(j+1)} - \hat{\dot{W}}_{C(j)}]/[\hat{t}_{C(j\pm1)} - \hat{t}_{C(j)} & \text{Otherwise} \end{cases}$$

Adjusted complementary table expended weight points are

$$\hat{W}_{C(j)} = \begin{cases} 0 & \text{If } j = 1 \\ W_{C1} - W_{Cj} & \text{If } W'_{CO} < 0 \\ \sum_{i=1}^{j} 0.5\,[\dot{W}_{C(i)} = \dot{W}_{C(i-1)}]\,[\hat{t}_{C(i)} - \hat{t}_{C(i-1)}] & \text{Otherwise} \end{cases}$$

For table interpolation:

Weight expended for complementary motor

$$W_C = \hat{W}_{C(j)} + \dot{W}_{C(j)}\,\Delta t_C + 0.5\,\hat{\ddot{W}}_{C(j)}\,\Delta t_C^2$$

Weight flow rate

$$\dot{W}_C = \hat{\dot{W}}_{C(j)} + \hat{\ddot{W}}_{C(j)} \Delta t_C$$

Vacuum thrust

$$F_{Cv} = \hat{F}_{C(j)} + \dot{F}_{C(j)} \Delta t_C$$

Where j is such that

$$\hat{t}_{C(j)} \leq t_B < \hat{t}_{C(j+1)} \text{ or } t_{C(j+1)} < \hat{t}_{C(j)} < t_B$$

And

$$\Delta t_C = t_B - \hat{t}_B - \hat{t}_{C(j)}$$

Thrust equation for the complementary thrust

$$F_C = \begin{cases} 0 & \text{If } F_{Cv} = 0 \\ F_{Cv} - A_{eC} P_a & \text{Otherwise} \end{cases}$$

Where F_{Cv} is the complementary thrust weight table vacuum thrust, A_{eC} is the input complementary exit area, and P_a is the atmospheric pressure.

g. **Thrust Multipliers**

The logic and equations presented in this section will accept the input thrust time tables to correspond to any reference altitudes and convert them to vacuum conditions so that the trajectory program may use them in the standard manner. The reference altitude pressure, input in pounds per square foot will stipulate that the input thrust time points will be for that atmospheric pressure and similarly designate that the input specific impulse and input total impulse be applicable at that altitude pressure.

Main table time multiplier

$$K_{tM} = \begin{cases} 1.0 & \text{If } K'_{tM} = 0.0 \\ K_{tM} & \text{Otherwise} \end{cases}$$

Main table reference atmospheric pressure

$$P_{arM} = \begin{cases} 2116 & \text{If } P'_{arM} < 0 \\ P'_{arM} & \text{Otherwise} \end{cases}$$

Calculate the back pressure impulse as

$$I_{PM} = P_{arM} A_{eM} K_{tM} \sum_{j=1}^{J_M-1} S_{IMj} \left[t_{M(j+1)} - t_{Mj}\right]$$

Where

$$\delta_{IMj} = \begin{cases} 0 & \text{If } F'_{mj} = F'_{M(j+1)} = 0 \\ 1.0 & \text{Otherwise} \end{cases}$$

The input main total impulse adjusted to vacuum total impulse is:

$$\hat{I}_{vM} = \begin{cases} 0 & \text{If } I'_{vM} = 0 \\ I'_{vM} + I_{FM} & \text{Otherwise} \end{cases}$$

Main table time adjusted input thrust integral is:

$$I_{vM} = K_{tM} \sum_{j=1}^{J_M-1} [F'_{M(j)} + F'_{M(j+1)}][t_{M(j+1)} - t_{M(j)}]/2$$

The main table time multiplier, K_{tM} is input in Lk009

The main table thrust reference atmospheric pressure, P'_{arM}, is input in Lk020

The main table nozzle exit area, A_{eM}, is input in Lk011

The main table tabular time points, t_{Mj}, are input in Lk0XX, where XX = 17 + 3j and $t_{M1} = 0$

The main table tabular thrust points F'_{Mj} are input in Lk0YY, where YY = 18 + 3j

The main table total impulse, I'_{vM}, is input in Lk005

The subscript j refers to the j-th row of the thrust weight table

J_M refers to the last input row of the main thrust weight table. Such that $t_{M(j+1)} \geq t_{M(j)}$ or $j = 25$

Complementary table time multiplier:

$$K_{tC} = \begin{cases} 1.0 & \text{If } K'_{tM} = 0.0 \\ K'_{tM} & \text{Otherwise} \end{cases}$$

Complementary table reference atmospheric pressure:

$$P_{arC} = \begin{cases} 2116 & \text{If } P'_{arC} < 0 \\ P'_{arC} & \text{Otherwise} \end{cases}$$

Calculate the back pressure impulse as

$$I_{FC} = P_{arC} A_{eC} K_{tC} \sum_{j=1}^{J_C-1} \delta_{ICj} [t_{C(j+1)} - t_{Cj}]$$

Where

$$\delta_{ICj} = \begin{cases} 0 & \text{If } F'_{Cj} = F'_{C(j+1)} = 0 \\ 1.0 & \text{Otherwise} \end{cases}$$

The input complementary total impulse adjusted to vacuum total impulse is:

$$\ddot{I}_{vC} = \begin{cases} 0 & \text{If } I'_{vC} = 0 \\ I'_{vC} + I_{FC} & \text{Otherwise} \end{cases}$$

Complementary table adjusted input thrust integral

$$I_{vC} = K_{tC} \sum_{j=1}^{J_C-1} [F'_{C(j)} + F'_{C(j+1)}] [t_{C(j+1)} - t_{C(j)}]/2$$

The complementary table time multiplier, K'_{tC}, is input in Lk102

The complementary table thrust reference atmospheric pressure, P'_{arC}, is input in Lk110

The complementary table nozzle exit area, A_{eC}, is input in Lk104

The complementary table tabular time points, t_{Cj}, are input in Lk1XX, where XX = 7 + 3j and $t_{C1} = 0$

The complementary tabular thrust points, F'_{Cj}, are input in Lk1YY, where YY = 8 + 3j

The complementary table total impulse, I'_{vC}, is input in Lk106

The subscript j refers to the j-th row of the thrust weight table, such that $t_{C(j+1)} \geq t_{C(j)}$ or j = 25

J_C refers to the last input row of the complementary thrust weight table

Total main and complementary impulses corrected to vacuum

$$\hat{I}_{vT} = \begin{cases} 0 & \text{If } I'_{vT} = 0 \\ I'_{vT} + I_{FM} + I_{FC} & \text{Otherwise} \end{cases}$$

The vacuum scale factors for the main and complementary stage thrust-time table, are determined as follows:

$$K''_{FM} = \begin{cases} I'_{vT}/(I_{vM} + I_{vC}) & \text{If } I'_{vT} \neq 0 \\ I'_{vT}/I_{vM} & \text{If } I'_{vT} = 0 \text{ and } I'_{vM} \neq 0 \\ K'_{FM} & \text{If } I'_{vT} = I'_{vM} = 0 \\ 1.0 & \text{If } I'_{vT} = I'_{vM} = K'_{FM} = 0 \end{cases}$$

$$K''_{FC} = \begin{cases} I'_{vT}/(I_{vM} + I_{vC}) & \text{If } I'_{vT} \neq 0 \\ I'_{vT}/I_{vC} & \text{If } I'_{vT} = 0 \text{ and } I'_{vC} \neq 0 \\ K'_{FC} & \text{If } I'_{vT} = I'_{vC} = 0 \\ 1.0 & \text{If } I'_{vT} = I'_{vC} = K'_{FC} = 0 \end{cases}$$

Main table vacuum adjusted thrust integral is:

$$I^*_{VM} = I_{vM} + I_{FM} / K''_{FM}$$

The main table adjusted to vacuum thrust points are:

$$F_{Mj} = F'_{Mj} + P_{arM} A_{eM} \delta_{FMj}$$

Where

$$\delta_{FMj} = \begin{cases} 0 & \text{If } F'_{Mj} = 0 \\ 1.0 / K''_{FM} & \text{Otherwise} \end{cases}$$

Complementary table vacuum adjusted thrust integral

$$I^*_{vC} = I_{vC} + I_{FC} / K''_{FC}$$

The complementary table adjusted to vacuum thrust points are:

$$F_{Cj} = F'_{Cj} + P_{arC} \, A_{eC} \quad \delta_{FCj}$$

Where

$$\delta_{FCj} = \begin{cases} 0 & \text{If } F'_C = 0 \\ 1.0 / K''_{FC} & \text{Otherwise} \end{cases}$$

The total of the main and complementary impulse, I'_{vT}, is input in LK004.

K_{FM} and K_{FC}, the multipliers of the main and complementary stage vacuum thrusts, respectively, are determined as follows:

$$K_{FM} = \begin{cases} \hat{I}_{vT} / (I^*_{vM} + I^*_{vC}) & \text{If } I'_{vT} \neq 0 \\ \hat{I}_{vM} / I^*_{vM} & \text{If } I'_{vT} = 0 \text{ and } I'_{vM} \neq 0 \\ K'_{FM} & \text{If } I'_{vT} = I'_{vM} = 0 \\ 1.0 & \text{If } I'_{vT} = I'_{vM} = K'_{FM} = 0 \end{cases}$$

$$K_{FC} = \begin{cases} \hat{I}_{vT} / (I^*_{vM} = I^*_{vC}) & \text{If } I'_{vT} \neq 0 \\ \hat{I}_{vC} / I^*_{vM} & \text{If } I'_{vT} = 0 \text{ and } I'_{vC} \neq 0 \\ K'_{FC} & \text{If } I'_{vT} = I'_{vM} = 0 \\ 1.0 & \text{If } I_{vT} = I'_{vM} = K'_{FC} = 0 \end{cases}$$

Where the main table thrust multiplier, K'_{FM}, is input in Lk007

The complementary table thrust multiplier, K'_{FC}, is input in Lk100

The main table adjusted to vacuum specific impulse is:

$$I_{spM} = I'_{spM} I^*_{vM} K_{FM}/(I^*_{vM} K_{FM} - I_{FM})$$

Where

The main table specific impulse, I'_{spM} is input in Lk010

The complementary table adjusted to vacuum specific impulse is:

$$I_{spC} = I'_{spC} I^*_{vC} K_{FC}/(I^*_{vC} K_{FC} - I_{FC})$$

Where

The complementary table specific impulse, I'_{spC}, is input in Lk103

The total main and complementary tables vacuum adjusted thrust integral

$$I^*_{vT} = I^*_{vM} + I^*_{vC}$$

Total main and complementary calculated total vacuum impulse

$$I_{vT} = I^*_{vM} K_{FM} + I^*_{vC} K_{FC}$$

h. Auxiliary Roll Control System

The weight expended for roll control is determined by integrating the roll control weight flow determined from:

$$\dot{W}_R = \begin{cases} 0 & \text{If } I_{spR} = 0 \\ |F_R|/I_{spR} & \text{Otherwise} \end{cases}$$

where F_R is the instantaneous roll control thrust and:

$$I_{spR} = \begin{cases} I_{spR1} \text{ for } t_B \leq t'_{R2} \\ I_{spR2} \text{ for } t'_{R2} \leq t_B \leq t'_{R3} \\ I_{spR3} \text{ for } t_B \geq t'_{R3} \end{cases}$$

with I_{spRj} ($j = 1, 2, 3$) input for each stage with the roll control data and t_B the instantaneous current stage time.

At the initiation of the current stage, the expended roll control weight is:

$$W_R = \begin{cases} 0 & \text{If } K_{0k} = 0 \\ W_R\ (t_{S(k-1)}) & \text{If } K_{0k} \neq 0 \end{cases}$$

where $W_R(t_{S(k-1)})$ is the expended roll control weight at the end of the previous stage.

The roll thrust, F_R, formulation is given in section D.2.b.

2. Thrust Forces and Moments

Thrust components and moments required in the linear and angular momenta equations during the K-th stage are as follows:

a. Axial Forces

$$F_x = \begin{cases} F/[1.0 + \tan^2 (\delta_P + \delta_{MP}) + \tan^2 (\delta_Y + \delta_{MY})]^{\frac{1}{2}} & \text{If } K_\delta = 0 \\ \{F_M/[1.0 + \tan^2 (\delta_P + \delta_{MP}) + \tan^2 (\delta_Y + \delta_{MY})]^{\frac{1}{2}}\} & \\ + F_C & \text{If } K_\delta = 1 \\ F/[1.0 + \tan^2 \delta_{MP} + \tan^2 \delta_{MY}]^{\frac{1}{2}} & \text{If } K_\delta = 2 \end{cases}$$

$$F_X = \begin{cases} F/[1.0 + \tan^2 \delta_P + \tan^2 \delta_Y]^{\frac{1}{2}} & \text{If } K_\delta = 0 \\ F_M/[1.0 + \tan^2 \delta_P + \tan^2 \delta_Y]^{\frac{1}{2}} & \text{If } K_\delta = 1 \\ F & \text{If } K_\delta = 2 \end{cases}$$

b. Lateral

$$F_y = \begin{cases} [F \tan (\delta_Y + \delta_{MY})]/[1.0 + \tan^2 (\delta_P + \delta_{MP}) + \tan^2 (\delta_Y + \delta_{MY})]^{\frac{1}{2}} & \text{If } K_\delta = 0 \\ \{[F_M \tan (\delta_Y + \delta_{MY})]/[1.0 + \tan^2 (\delta_P + \delta_{MP}) + \tan^2 (\delta_Y + \delta_{MY})]^{\frac{1}{2}} & \text{If } K_\delta = 1 \\ [F \tan \delta_{MY}]/[1.0 + \tan^2 \delta_{MP} + \tan^2 \delta_{MY}]^{\frac{1}{2}} & \text{If } K_\delta = 2 \end{cases}$$

$$F_Y = \begin{cases} [F \tan \delta_Y]/[1.0 + \tan^2 \delta_P + \tan^2 \delta_Y]^{\frac{1}{2}} & \text{If } K_\delta = 0 \\ [F_M \tan \delta_Y]/[1.0 + \tan^2 \delta_P + \tan^2 \delta_Y]^{\frac{1}{2}} & \text{If } K_\delta = 1 \\ 0 & \text{If } K_\delta = 2 \end{cases}$$

c. Transverse

$$F_Z = \begin{cases} [F \tan (\delta_P + \delta_{MP})]/[1.0 + \tan^2 (\delta_P + \delta_{MP}) + \tan^2 (\delta_Y + \delta_{MY})]^{\frac{1}{2}} & \text{If } K_\delta = 0 \\ \{[F_M \tan (\delta_P + \delta_{MP})]/[1.0 + \tan^2 (\delta_P + \delta_{MP}) + \tan^2 (\delta_Y + \delta_{MY})]^{\frac{1}{2}} & \text{If } K_\delta = 1 \\ [F \tan \delta_{MP}]/[1.0 + \tan^2 \delta_{MP} + \tan^2 \delta_{MY}]^{\frac{1}{2}} & \text{If } K_\delta = 2 \end{cases}$$

$$F_Z = \begin{cases} [F \tan \delta_P]/[1.0 + \tan^2 \delta_P + \tan^2 \delta_Y]^{\frac{1}{2}} & \text{If } K_\delta = 0 \\ [F_M \tan \delta_P]/[1.0 + \tan^2 \delta_P + \tan^2 \delta_Y]^{\frac{1}{2}} & \text{If } K_\delta = 1 \\ 0 & \text{If } K_\delta = 2 \end{cases}$$

Where δ_P and δ_Y are the pitch and yaw thrust deflections, respectively, δ_{MP} and δ_{MY} are the stage input thrust vector misalignment angles, K_δ is the input per stage control flag, F is the total thrust, and F_M is the main thrust, F_C is the complementary thrust.

d. Thrust Moments

Pitch

Offset

$M_{FOQ} = F_x (z_e - z_{cg})$

Control

$M_{FCQ} = F_z \ell_e$

Yaw

Offset

$M_{FOR} = -F_x (y_e - y_{cg})$

Control

$M_{FCR} = -F_y \ell_e$

Roll

Offset

$M_{FOP} = F_z (y_e - y_{cg}) - F_y (z_e - z_{cg})$

Vortex

$M_{FVP} = \eta_{vr} F$ Where $\eta_{vr} = 0.00363$
unless otherwise input

e. Jet Damping

The jet damping forces and moments on a missile having an internally burning fuel arise from the reaction to the Coriolis force exerted on the exhaust gas as it moves along the missile x_b axis toward the nozzle exit plane in the presence of missile angular rotation.

a. Force

$$F_{JDy} = (\dot{W}/\bar{g}_e)(2\ell_E - \ell_{Pf} - \ell_{Pa})\, R_b\ (\pi/180)$$

$$F_{JDz} = (\dot{W}/\bar{g}_e)(2\ell_E - \ell_{Pf} - \ell_{Pa})\, Q_b\ (\pi/180)$$

b. Moment

$$M_{JDQ} = -(\dot{W}/\bar{g}_e)[\ell_E^2 - (\ell_{Pf}^2 + \ell_{Pf}\ell_{Pa} + \ell_{Pa}^2)/3]\ Q_b\ (\pi/180)$$

$$M_{JDR} = -(\dot{W}/\bar{g}_e)[\ell_E^2 - (\ell_{Pf}^2 + \ell_{Pf}\ell_{Pa} + \ell_{Pa}^2)/3]\ R_b\ (\pi/180)$$

where $\dot{W}$ is the total instantaneous missile weight flow, $\bar{g}_e$ is the input mass conversion gravity, Q_b and R_b are the instantaneous vehicle angular pitch and yaw velocities, and ℓ_E, ℓ_{Pf}, and ℓ_{Pa} are the nozzle exit, forward and aft propellant grain lever arms defined in Section I.3.

f. Propellant Grain and Nozzle Exit Location

The values are computed as follows:

(1) Nozzle Exit

$$x_E = \begin{cases} x_e & \text{If } x_E' = 0 \\ \bar{X}_e x_E' & \text{Otherwise} \end{cases}$$

(2) Aft End of Propellant

$$x_{Pa} = \begin{cases} x_{cg} & \text{If } x_{Pa}' = 0 \\ \bar{X}_e x_{Pa}' & \text{Otherwise} \end{cases}$$

(3) Forward End of Propellant

$$x_{Pf} = \begin{cases} x_{cg} & \text{If } x_{Pf}' = 0 \\ \bar{X}_e x_{Pf}' & \text{Otherwise} \end{cases}$$

where x_E', x_{Pf}', and x_{Pa}' are input per stage, x_e is the thrust gimbal location, and x_{cg} is the vehicle instantaneous center of gravity.

g. Thrust Gimbal Location

The thrust gimbal location is specified in the left-handed Cartesian coordinate system shown in Figure 27. Thrust location components are x_e, the missile body station measured along the centerline, and y_e and z_e are measured from the vehicle centerline positive to the right and down, respectively. The gimbal components are input for each stage as constants. The Y and Z components represent a thrust eccentricity which yields moments during powered flight. The clustered motor configuration is shown in Figure 28 and the effect of typical variation of thrust history is pictured in Figure 29. The effect of clustered motors is simulated as follows as a thrust vector point offset.

$$x_e = \bar{X}_e x'_e$$

$$n'_m = 0 \text{ or } 1 \quad \left[\begin{array}{l} y_e = \bar{X}_e \; y'_e \\ \\ z_e = \bar{X}_e \; z'_e \end{array}\right.$$

$$n'_m = 2 \quad \left[\begin{array}{l} y_e = [\sigma_f \; R_c/\sqrt{2}] \cos \Phi_\nu + \bar{X}_e \; y'_e \\ \\ z_e = [\sigma_f \; R_c/\sqrt{2}] \sin \Phi_\nu + \bar{X}_e \; z'_e \end{array}\right.$$

$$n'_m > 2 = \quad \left[\begin{array}{l} y_e = [\sigma_f \; R_c/\sqrt{2} \; n_m] \cos \Phi_\nu + \bar{X}_e \; y'_e \\ \\ z_e = [\sigma_f \; R_c/\sqrt{2} \; n_m] \sin \Phi_\nu + \bar{X}_e \; z'_e \end{array}\right.$$

$$\sigma_f = [4.5 \; (F_h^2 + F_n^2 + F_L^2)/(F_h + F_n + F_L)^2 - 1.5]^{\frac{1}{2}}$$

and

$$F_L = \{1/(1 + \{[\sigma_{tb}^2 + \sigma_{It}^2]/2)^{\frac{1}{2}}] \; \{f[F_{vac} \; (1 - \sigma_{tb}) \; t]\}$$

$$F_h = [1 + \{[\sigma_{tb}^2 + \sigma_{It}^2]/2\}^{\frac{1}{2}}] \{f[F_{vac}(1 + \sigma_{tb})t]\}$$

$$F_n = F_{vac}$$

The following are input to this part of the program:

$\bar{X}_e$ X_e', y_e', z_e', n_m': R_c, Φ_ν, σ_{tb}, σ_{It}, and $F(t)$.

The following are required output from this analysis:

$$y_e = f(t)$$

$$z_e = f(t)$$

The internally calculated number of motors in a cluster (n_m) and the number of control nozzles in that cluster (n_c) are defined.

$$n_m = \begin{cases} 1 & \text{If } n_m' = 0 \\ n_m' & \text{Otherwise} \end{cases}$$

$$n_c = \begin{cases} n_m' & \text{If } n_c = 0 \\ n_c' & \text{Otherwise} \end{cases}$$

where n_m' is the input number of motors for the k th stage, n_c' is the input number of control motors for the k th stage.

h. Movable Nozzle Tail-Wag-Dog Forces Moments

Pitch

$$F_{TDz} = \begin{cases} 0 \text{ if } I_n = 0 \text{ or } \omega_c = 0 \text{ or } M_y \neq 2, \text{ or } 5 \\ (\pi/180)\, W_n\, \ell_n\, \ddot{\delta}_P/W & \text{Otherwise} \end{cases}$$

$$M_{TDQ} = \begin{cases} 0 \text{ if } I_n = 0 \text{ or } \omega_c = 0 \text{ or } M_y \neq 2, \text{ or } 5 \\ (\pi/180)(I_n + W_n\, \ell_e \ell_n/\bar{g}_e)\, \ddot{\delta}_P & \text{Otherwise} \end{cases}$$

Yaw

$$F_{TDy} = \begin{cases} 0 \text{ if } I_n = 0 \text{ or } \omega_c = 0 \text{ or } M_y \neq 5 \\ (\pi/180) W_n \ell_n \ddot{\delta}_Y/W & \text{Otherwise} \end{cases}$$

$$M_{TDR} = \begin{cases} 0 \text{ if } I_n = 0 \text{ or } \omega_c = 0 \text{ or } M_y \neq 5 \\ (\pi/180)(I_n + W_n \ell_e \ell_n/\bar{g}_e)\ddot{\delta}_Y & \text{Otherwise} \end{cases}$$

where I_n is the movable portion of the nozzle mass moment of inertia, W_n is the weight of the movable portion of the nozzle, ℓ_e is the vehicle center of gravity to nozzle gimbal point distance, ℓ_n is the movable portion of the nozzle center of gravity to the gimbal point distance, ω_c is the input second order TVC transfer function flag, $\bar{g}_e$ is the mass to weight conversion gravity, and $\ddot{\delta}_P$, $\ddot{\delta}_Y$ are the nozzle angular deflection acceleration in pitch and yaw respectively.

3. Vehicle Weight

Instantaneous missile weight, W, is computed from the unadjusted staging weight, W_S, the expended main weight, W_M, the expended complementary weight, W_C, the expended roll control weight, W_R, and the jettisoned weight, W_{JT}, respectively, as follows:

$$W = W_S - W_M - W_C - W_R - W_{JT}$$

Main and complementary weights are functions of input initial stage weights, weight flows, vacuum thrust and specific impulse, and multipliers. In addition, the main weight is dependent upon input weights at each thrust-weight switching time. Roll control and auxiliary motor expended weights are functions of thrust and specific impulse. If $W \leq 0$, the run is halted and "WEIGHT HAS GONE TO ZERO" is printed.

Missile mass is computed as follows:

$$m = W/\bar{g}_e$$

where

$$\bar{g}_e = \begin{cases} g_e & \text{If } \bar{g}_e' \text{ is input 1.4} \\ 32.174 & \text{If } \bar{g}_e' \text{ is input zero} \\ \bar{g}_e' & \text{Otherwise} \end{cases}$$

The unadjusted staging weight defined as follows:

$$W_{S(k)} = \left[\begin{array}{ll} W_{PL} + \sum_{p=k}^{4} [W_{MO(p)} + W_{CO(p)}] & \text{if } k=K_k \text{ or } K_{O(k-1)} = \\ & 0 \text{ and } K_{NO(k-1)} = 0 \\ W_{(g)} & \\ W_{PL} - W_{C(g)} + W_{C)(k-1)} + \sum_{p=k}^{4} [W_{MO(p)} + W_{CO(p)}] & \\ & \text{if } K_{NO(k-1)} \neq 0 \text{ and} \\ & K_{O(k-1)} = 0 \end{array} \right.$$

Where k is the current stage number; K_k is the input stage start control function; $K_{O(k)}$ and $K_{NO(k)}$ are the input weight carryover flag and input complementary thrust-weight table carryover flag respectively; $W_{(g)}$ and $W_{C(g)}$ are the vehicle weight, and expended complementary weight at the termination of the (k-1)th stage respectively, $W_{MO(k)}$ and $W_{CO(k)}$ are the main and complementary initial stage weight for the k-th stage, and W_{PL} is the input payload weight.

Main and complementary weights are functions of input initial stage weights, weight flows, vacuum thrust and specific impulse, and multipliers. In addition, the main weight is dependent upon input weights at each thrust-weight switching time. If $W \leq 0$, the run is halted and the statement "WEIGHT HAS GONE TO ZERO" is printed.

Total weight flow is

$$\dot{W} = \dot{W}_M + \dot{W}_C + \dot{W}_R$$

a. Initial Stage Weight Calculations--Initial stage weights are calculated as follows:

If the input main stage weight W_{MO} is input negative, the stage weight will be calculated as a function of stage time and the input values in the thrust weight table. The stage weight-history values are input in the column normally used for the input weight flow values. Linear interpolation between these input table weight points will be used to evaluate the instantaneous stage weight.

Similar logic holds for the complementary weight if W_{CO} is input negative.

Main Table Weight

$$W_{MO} = \begin{cases} W_{M1} & \text{if } W'_{MO} < 0 \\ W'_{MO} & \text{Otherwise} \end{cases}$$

Where W'_{MO} is the input main stage weight input in Lk006 and W_{M1} is the first input main table weight point input in Lk022.

Complementary table weight

$$W_{CO} = \begin{cases} W_{C1} & \text{if } W'_{CO} < 0 \\ W'_{CO} & \text{Otherwise} \end{cases}$$

Where W'_{CO} is the input complementary stage weight input in Lk105 and W_{C1} is the first input complementary table weight point input in Lk012.

b. Jettison Weight Calculations

The weight lost to jettisoning is

$$W_{JT} = \sum_{i=1}^{8} W_{JT(j)}$$

Where

$$W_{JT(j)} = \begin{cases} 0 \text{ before } \sigma_{J(j)} = K_{J(j)} \\ W_{JT(j)} \text{ after } \sigma_{J(j)} = K_{J(j)} \end{cases}$$

with $\sigma_{J(j)}$ ($j = 1, 2, \ldots, 8$) the achieved value of the parameter designated by code input and $K_{J(j)}$ the input value of that parameter at which the weight, $W_{JT(j)}$, is to be jettisoned. Logically the jettisoning must be satisfied sequentially.

I. MASS PROPERTIES

1. Center-of-Gravity

The center of-gravity location is specified in the same left-handed coordinate system as the gimbal location. The Y and Z components represent a center-of-gravity offset which yields moments during powered and atmospheric flight. The center-of-gravity location will be input for each stage as a linear function of the total instantaneous vehicle weight.

a. Center-of-Gravity Body Station

Fifteen center of gravity body stations (x_{cg}) can be input for each stage. The body station is computed from input during the current stage as follows:

For $W_j \geq W \geq W_{j+1}$

$$x_{cg} = \bar{X}_{cg} [W_j x_{cg(j+1)} - W_{j+1} x_{cgj} + (x_{cgj} - x_{cg(j+1)})W]/$$

$$(W_j - W_{j+1})$$

where $\bar{x}_{cg}$, W_j, and x_{cgj} ($j = 1, 2, \ldots, 15$) are input for each stage with the moment of inertia data, and W is the instantaneous vehicle weight. The W_j are input monotonically decreasing so that $W_1 > W_2 \ldots$

The first and last input logic for the center of gravity is:

If $W > W_1$, then

$$x_{cg} = \bar{X}_{cg} x_{cg1}$$

or if $W < W_J$, then

$$x_{cg} = \bar{X}_{cg} x_{cgJ}.$$

where the W_1 and x_{cg1} and the W_J ($J = 1, 2, \ldots$ or 15) and x_{cgJ} are the first and last input values respectively.

b. Center-of-Gravity Offset

Fifteen offsets can be input for each stage. The Y and Z offsets for the current stage are as follows:

$$y_{cg} = z_{cg} = 0$$

Otherwise for $W_j \geq W \geq W_{j+1}$

$$y_{cg} = \bar{X}_{cg} \; [W_j \; y_{cg(j+1)} - W_{j+1} \; y_{cgi} + (y_{cgi} - y_{cg(i+1)})W/(W_i - W_{i+1})$$

$$z_{cg} = \bar{X}_{cg} \; [W_j \; a_{cg(j+1)} - W_{j+1} \; z_{cgi} + (z_{cgj} - z_{cg(j+1)}) \; W/(W_i - W_{j+1})$$

where $j = 1, 2, \ldots, 15$ and $@_j$, y_{cgj}, and z_{cgj} are input for each stage in a table with the body station values. The W_j's are input monotonically decreasing.

The first and last input logic given for the body station center of gravity are applicable for the offset data.

2. Moments of Inertia

Fifteen pitch, I_{YY}, yaw, I_{ZZ}, and roll, I_{XX}, moment of inertia values can be input per stage. The moments of inertia are linear functions of total instantaneous vehicle weight and are computed as follows:

For $W_j \geq W \geq W_{j+1}$

$$I_{XX} = \bar{I}_{Xk} [W_j \; I_{X(j+1)} - W_{j+1} \; I_{Xj} + (I_{Xj} - I_{X(j+1)})W]/(W_j - W_{j+1})$$

$$I_{YY} = \bar{I}_{Yk} [W_j \; I_{Y(j+1)} - W_{j+1} \; I_{Yj} + (I_{Yj} - I_{Y(j+1)})W]/(W_j - W_{j+1})$$

$$I_{ZZ} = \bar{I}_{Zk} [W_j \; I_{Z(j+1)} - W_{j+1} \; I_{Zj} + (I_{Zj} - I_{Z(j+1)})W]/(W_j - W_{j+1})$$

$$I_{XY} = \bar{I}_X [W_j \; I_{XY(j+1)} - W_{j+1} \; I_{XYj} + (I_{XYj} - I_{XY(j+1)})W]/(W_j - W_{j+1})$$

$$I_{XZ} = \bar{I}_X [W_j \; I_{XZ(j+1)} - W_{j+1} \; I_{XZj} + (I_{XZj} - I_{XZ(j+1)} \; W]/W_j - W_{j+1})$$

$$I_{YZ} = \bar{I}_X [W_j \; I_{YZ(j+1)} - W_{j+1} \; I_{YZj} + (I_{YZj} - I_{YZ(j+1)}W]/(W_j - W_{j+1})$$

where $\bar{I}_{X,\,Y,\,Z}$, W_j, and I_X, I_Y, I_Z, I_{XY}, I_{XZ}, I_{YZ} $(j = 1, 2, \ldots, 15)$ are input for each stage with the body station center of gravity and W is the instantaneous vehicle weight. The W_j's are input monotonically decreasing.

The time derivatives of the moments of inertia required in the angular momenta equations are:

For $W_j \geq W \geq W_{j+1}$

$$\dot{I}_{XX} = \begin{cases} 0 & \text{If } \bar{I}_{Xk} = 0 \\ -\dot{W}\bar{I}_X (I_{Xj} - I_{X(j+1)})/(W_j - W_{j+1}) & \text{Otherwise} \end{cases}$$

$$\dot{I}_{YY} = -\dot{W}\bar{I}_Y(I_{Yj} - I_{Y(j+1)})/(W_j - W_{j+1}) \quad \text{Otherwise}$$

$$\dot{I}_{ZZ} = \begin{cases} -\dot{W}\bar{I}_Z(I_{Zj} - I_{Z(j+1)})/(W_j - W_{j+1}) & \text{If } \bar{I}_{Zk} \neq 0 \\ \dot{I}_{YY} & \text{Otherwise} \end{cases}$$

$$\dot{I}_{XY} = \begin{cases} 0 & \text{If } \bar{I}_X = 0 \\ -\dot{W}\bar{I}_X(I_{XYj} - I_{XY(j+1)})/(W_j - W_{j+1}) & \text{Otherwise} \end{cases}$$

$$\dot{I}_{XZ} = \begin{cases} 0 & \text{If } \bar{I}_X = 0 \\ -\dot{W}\bar{I}_X(I_{XZj} - I_{XZ(j+1)})/(W_j - W_{j+1}) & \text{Otherwise} \end{cases}$$

$$\dot{I}_{YZ} = \begin{cases} 0 & \text{If } \bar{I}_X = 0 \\ -\dot{W}\bar{I}_X(I_{YZj} - I_{YZ(j+1)})/(W_j - W_{j+1}) & \text{Otherwise} \end{cases}$$

where W is the total instantaneous vehicle weight flow.

The first and last input logic for the moments of inertia and their derivatives are:

$$I_{XX} = \begin{cases} \bar{I}_X I_{X1} & \text{If } W > W_1 \\ \bar{I}_X I_{XJ} & \text{If } W < W_J \end{cases}$$

$$I_{YY} = \begin{cases} \bar{I}_Y I_{Y1} & \text{If } W > W_1 \\ \bar{I}_Y I_{YJ} & \text{If } W < W_J \end{cases}$$

$$I_{ZZ} = \begin{cases} I_{YY} & \text{If } \bar{I}_Z = 0 \\ \bar{I}_Z I_{Z1} & \text{If } \bar{I}_Z \neq 0 \text{ and } W > W_1 \\ \bar{I}_Z I_{ZJ} & \text{If } \bar{I}_Z \neq 0 \text{ and } W < W_J \end{cases}$$

$$I_{XY} = \begin{cases} \bar{I}_X \ I_{XY1} & \text{If } W > W_1 \\ \bar{I}_X \ I_{XYJ} & \text{If } W < W_J \end{cases}$$

$$I_{XZ} = \begin{cases} \bar{I}_X \ I_{XZ1} & \text{If } W > W_1 \\ \bar{I}_X \ I_{XZJ} & \text{If } W < W_J \end{cases}$$

$$I_{YZ} = \begin{cases} \bar{I}_X \ I_{YZ1} & \text{If } W > W_1 \\ \bar{I}_X \ I_{YZJ} & \text{If } W < W_J \end{cases}$$

and if $W > W_1$ or $W < W_J$

where the W_1 and $I_{X,Y,Z1}$ and the W_J and $I_{X,Y,ZJ}$ $(J = 1, 2, \ldots, 15)$ are the first and last input values respectively.

3. Lever Arms

Significant distances shown in Figures 20 and 21 are given below:

a. Gimbal Point to Center-of-Gravity

$$\ell_e = x_e - x_{cg}$$

b. Movable Portion of Nozzle Center-of-Gravity to Gimbal Point

$$\ell_n = (x_n' - x_e') \bar{X}_e$$

c. Center-of-Gravity to Center of Pressure Distance

$$\ell_{cp} = x_{cg} - x_{cp}$$

d. Nozzle Exit to Center-of-Gravity

$$\ell_E = x_E - x_{cg}$$

e. Forward End of Propellant Grain to Center-of-Gravity

$$\ell_{Pf} = x_{Pf} - x_{cg}$$

f. Aft End of Propellant Grain to Center-of-Gravity

$$\ell_{Pa} = x_{Pa} - x_{cg}$$

g. Pitch Movable Control Fin Center of Pressure to Hinge Axis

$$\ell_{hz} = (U_{cz} - U_{hz}) \ell_{bz}$$

h. Pitch Movable Control Fin Center of Pressure to Center-of-Gravity

$$\ell_{\delta z} = x_{hz} - x_{cg} + \ell_{hz} \cos (\delta_p * K_{cf})$$

$$\ell_{\delta} = x_{hz} - x_{cg} + \ell_{hz}$$

i. Yaw Movable Control Fin Center of Pressure to Center-of-Gravity

$$\ell_{\delta y} = x_{hz} - x_{cg} + \ell_{hz} \cos (\delta_y * K_{cf})$$

where x_e is the gimbal point body station, x_n is the movable portion of nozzle center of gravity body station, x_{cp} is the aerodynamic center of pressure body station x_{cg} is the vehicle center-of-gravity body station, x_E is the nozzle exit body station, x_{Pf} is the forward end of propellant grain body station, x_{Pa} is the aft end of propellant grain body station. U_{cz} is the pitch movable control fin center of pressure to the leading pitch fin base root location distance to the pitch fin base root length ration, U_{hz} is the pitch movable control fin hinge axis to the leading fin base root location distance to the pitch fin base root length ratio, ℓ_{bz} is the input pitch fin base root length, x_{hz} is the input missile body station of the pitch fin hinge axis, δ_p is the instantaneous pitch deflection angle , δ_y is the instantaneous yaw deflection angle and K_{cf} is the fin control multiplier.

J. TRAJECTORY PARAMETERS

Parameters which have no effect on the solution of the momenta equations are formulated. The parameters are introduced because of their usefulness as switching functions and/or the additional trajectory characteristics they furnish. A few of the parameters that can be represented geometrically are shown in Figure 26.

1. Orbital Elements and Impact Determination

The equations in this section are computed if

$K_{c1} \leq \sigma_c \leq K_{c2}$ or at termination of the final stage

where σ_c is the achieved value of a quantity designated by code input and K_{c1} and K_{c2} are input limits.

If the quantities in this section are involved as switching functions, e.g., flight segment initiation or termination, then the proper values of $K_{c1,2}$ and σ_c must be input. However, the equations are always computed at cut-off, but cannot be used as cut-off switching function unless the σ_c-logic is satisfied. The print is always given at cut-off.

Once $\sigma_c \geq K_{c1}$ or $\sigma_c > K_{c2}$, the possibility that at some later time $\sigma_c < K_{c1}$ or $\sigma_c \leq K_{c2}$ is ignored. Obtaining the limit values by interpolation is not done.

a. Orbital Elements

If the above criteria apply, the following are computed:

The parameters

$$a_1 = r_c V_I^2 / g_e r_e^2$$

$$a_2 = a_1 \cos^2 \gamma_{1I}$$

$$a_3 = a_1 \sin \gamma_{1I} \cos \gamma_{1I}$$

where r_c, V_I, and γ_{1I} are current values

The eccentricity

$$e = [a_3^2 + (1 - a_2)^2]^{\frac{1}{2}}$$

and the perigee altitude

$$h_p = (r_p - r_e)/6076.1033$$

where r_e is input and $r_p = a_2 r_c/(1 + e)$

b. Apogee and Impact

If $0 < e < 1$, compute:

The apogee altitude

$$h_a = (r_a - r_e)/6076.10333$$

where $r_a = a_2 r_c/(1 - e)$

The velocity at apogee

$$V_{Ia} = \left| (r_c V_I \cos \gamma_{1I})/r_a \right|$$

The angle to apogee

$$\phi_a = \arctan [a_3 a_4/(1 - a_2)]$$

where $0 \leq \phi_a < 360°$ and

$$a_4 = \begin{cases} +1 & \text{If } |\gamma_{1I}| < 90° \\ -1 & \text{Otherwise} \end{cases}$$

The flight time to apogee

$$\tau_a = (r_c^2 V_I \sin \gamma_{1I})/GM\ (2 - a_1)$$
$$+ 2[r_c^3/GM\ (2 - a_1)^3]^{\frac{1}{2}}\ (\pi/180) \arctan \{[(1 + e)/(1 - e)]^{\frac{1}{2}}/$$
$$(1/\tan \tfrac{1}{2} \phi_a)\}$$

where $0 \leq \arctan \{\} < 180$, t is the current time, and $\tau_a = 0$ if $\tan \frac{1}{2} \phi_a = 0$.

The total flight time to apogee

$$t_a = t + \tau_a$$

The orbital period

$$P = \pi[(r_a + r_p)/2]^{3/2}/30\sqrt{GM}$$

and the terminal radial distance

$$r_f = r_e + h_f$$

where h_f is input

Besides the above criteria, the following must be satisfied in order to compute the equations given below:

1. If $K_\gamma = -1$, then, $\gamma_{1I} > 0$, $r_a > r_f$, and $h < h_f$
2. If $K_\gamma = +1$, then, $r_a > r_f$, $r_p < r_f$, and if $\gamma_{1I} < 0$, then, $h > h_f$

where K_γ is input as plus or minus one, so that

$K_\gamma = +1$ if impact is to occur at an altitude of h_f
$K_\gamma = -1$ if a space intercept is to occur at an altitude of h_f

If the above are satisfied, compute:

The glide range angle

$$\phi_f' = \phi_a + K_\gamma \arctan \ \{[e^2 - (1 - a_2 r_c/r_f)^2]^{\frac{1}{2}}/(1 - a_2 r_c/r_f)\}$$

where $0 \leq \arctan \{\} < 180°$

The modified range angle is

$$\phi_f = \begin{cases} \phi_f' & \text{If } \phi_f' < 360° \\ \phi_f' - 360° & \text{Otherwise} \end{cases}$$

If $K_\gamma = 1$, $r_a > r_f$, and $r_p \geq r_f$, compute the modified range angle as follows:

$$\phi_f = (180/\pi) \quad \pi\{(r_c/r_f)a_1 + 1 + 2[1 - (r_f/r_c)]/[1 - (r_c^2/r_f^2) \cos^2 \gamma_{1I}]\}$$

$$+ (\pi/180) \arctan \ \{2a_4[r_c/r_f) - 1] \tan \gamma_{1I}\}/\{(r_c/r_f - 1]^2 -$$

$$\tan^2 \gamma_{1I}\}$$

where $-180° < \arctan \ \{2a_4 \ [(r_c/r_f) - 1] \tan \gamma_{1I}\}/\{[r_c/r_f - 1]^2 - \tan^2 \gamma_{1I}\} \leq 180°$

The glide time is

$$\tau_f = r_f[(1 - a_2) \sin \phi_f + a_3 a_4 \ (-\cos \phi_f + r_c/r_f)]/(2 - a_1)V_I |\cos \gamma_{1I}|$$

$$+ 2[r_c^3/GM(2 - a_1)^3]^{\frac{1}{2}}(\pi/180) \arctan \{[(-1 + 2/a_1)^{\frac{1}{2}} \sin (\phi_f/2)]/$$

$$\cos (a_5 + \phi_f/2)\} + \tau_p$$

where $0 \leq \arctan \{\} \leq 180°$ and

$$a_5 = \begin{cases} \gamma_{1I} & \text{If } a_4 > 0 \\ -180 - \gamma_{1I} & \text{If } a_4 < 0 \text{ and } \gamma_{1I} < 0 \\ 180 - \gamma_{1I} & \text{Otherwise} \end{cases}$$

and

$$\tau_P = \begin{cases} 0 & \text{If } \phi_f < 360^\circ \\ 60\ P & \text{Otherwise} \end{cases}$$

Total flight time is

$$t_f = t + \tau_f$$

where t is the current time.

The total range angle is

$$\Phi_f = \phi + a_4\ \phi_f - (180/\pi)\ \bar{\omega} t_f$$

where ϕ is the current range angle and $\bar{\omega}$ is input.

The ground range is

$$S_f = (\pi/180)\ \Phi_f r_e/6076.10333$$

The instantaneous latitude, longitudinal, and azimuthal flight path angles are:

$$\rho = \text{arc sin } (\sin\ \rho_L \cos \phi + \cos \rho_L \sin \phi \cos \vartheta_i)$$

where

$$-90^\circ \leq \rho \leq 90^\circ$$

$$\mu' = \arctan\ [(\sin \phi \sin \vartheta_i \cos \rho_L)/(\cos \phi - \sin \rho_L \sin \rho)]$$

where

$$-180° < \mu \leq 180°$$

$$\mu = \mu_L - \mu'$$

$$-180° < \mu \leq 180°$$

$$\gamma_{21} = \arctan \{[V_e \cos \gamma_1 \cos \rho_L \sin \psi_i]/[V_e \cos \gamma_1 \cos \rho (\cos \psi_i \cos \mu' - \sin \psi_i \sin \mu' \sin \rho_L) + \omega r_c \cos^2 \rho]\}$$

$$0° \leq \gamma_{21} < 360°$$

ρ_f and μ_f are calculated as follows:

Impact latitude is

$$\rho_f = \text{arc sin} (\sin \rho \cos |\phi_f| + \cos \rho \sin |\phi_f| \cos \gamma_{21})$$

with

$$-90° \leq \rho_f \leq 90°$$

where ρ and γ_{21} are the instantaneous latitude and azimuthal flight path angle, respectively.

Change is inertial longitude, positive in eastward direction, $(0 \leq \arctan [\,] \leq 360°)$ $\mu_i' = (\text{sign } \phi_f)[\text{sign } (\sin \gamma_{21})] \arctan [(\sin |\phi_f| \; |\sin \gamma_{21}| \cos \rho)/(\cos |\phi_f| - \sin \rho_f \sin \rho)]$

Impact Longitude is

$$\mu_f = \begin{cases} \mu - \mu_I' + (180/\pi)\omega\tau_f & \text{If } -180° < \mu - \mu_I' + (180/-)\omega\tau_f \leq +180 \\ 360° + \mu - \mu_I' + (180/\pi)\omega\tau_f & \text{If } -180 \leq \mu - \mu_I' + (180/\pi)\omega\tau_f \\ -360° + \mu - \mu_I' + (180/\pi)\omega\tau_f & \text{If } 180 < \mu - \mu_I' + (180/\omega\tau_f \end{cases}$$

where ω is the input angular velocity of the earth.

The components of inertial velocity and total inertial velocity at the terminal altitude are

$$\dot{r}_f = \{V_I \cos \gamma_{1I} [a_3 a_4 \cos \Phi_f - (1 - a_2) \sin \Phi_f]/a_2\} a_r$$

$$r_f \dot{\Phi}_f = (180/\pi)(r_c V_I \cos \gamma_{1I})/r_f$$

$$V_{If} = [\dot{r}_f^2 + (\pi/180)^2 (r_f \dot{\Phi}_f)^2]^{\frac{1}{2}}$$

and the terminal inertial flight path angle is

$$\gamma_{1If} = \arctan \dot{r}_f / r_f \dot{\Phi}_f \ (\pi/180)$$

γ_{2If}, ρ_a, μ_a, S_a, γ_{2Ia}, and i are calculated as follows:

$$\gamma_{21f} = \arctan [\cos \rho \sin \gamma_{21} \sin \Phi_f / (\cos \Phi_f \sin \rho_f - \sin \rho)]$$

with

$$-90° \leq \gamma_{1If} \leq 90°$$

and

$$0 \leq \gamma_{2If} < 360°$$

Apogee longitude is

$$\mu_a = \begin{cases} \mu - \mu'_{Ia} + (180/\pi)\omega\tau_a & \text{If } -180° < \mu - \mu'_{Ia} + (180/\pi)\omega\tau_a \leq 180° \\ 360° + \mu - \mu'_{Ia} + (180/\pi)\omega\tau_a & \text{If } -180° \geq \mu - \mu'_{Ia} + (180/\pi)\omega\tau_a \\ -360° + \mu - \mu'_{Ia} + (180/\pi)\omega\tau_a & \text{If } 180° < \mu - \mu'_{Ia} + (180/\pi)\omega\tau_a \end{cases}$$

The total ground range to apogee is

$$S_a = \frac{(\pi/180) r_e}{6076.10333} \begin{bmatrix} \arccos(\cos\rho_L \cos\rho_a \cos\Delta_a + \sin\rho_L \sin\rho_a) \\ \text{If } 0 \leq \Lambda_a < 180° \\ 360° - \arccos(\cos\rho_L \cos\rho_a \cos\Delta_a + \sin\rho_L \sin\rho_a) \\ \text{If } 180° \leq \Lambda_a < 360° \end{bmatrix}$$

where $0 \leq \arccos[\,] \leq 180°$

$$\Lambda_a = |\mu_L - \mu_a|$$

Inertial azimuthal flight path angle at apogee is

$$\gamma_{2Ia} = \arctan[(\cos\rho \sin\gamma_{2I} \sin\Phi_a)/(\cos\Phi_a \sin\rho_a - \sin\rho)]$$

where $0 \leq \gamma_{2Ia} < 360°$

Inclination angle is

$$i = \arccos(\cos\rho \sin\gamma_{2I})$$

where $0 \leq i \leq 180°$

Calculate the atmospheric entry condition if the following criteria are met:

1. $h_e \neq 0$ where h_e is input
2. $r_a > r_E$
3. $r_E = r_e + h_e$. If $\gamma_{1I} < 0$, then $h > h_e$

If the above are satisfied, compute the glide range to entry angle

$$\phi_E = \left[\begin{array}{l} \phi_a + \arctan \{[e^2 - (1 - a_2 r_c/r_E)^2]^{\frac{1}{2}}/(1 - a_2 r_c/r_E) \\ \qquad \text{If } r_p < r_E \\ \text{where } 0 \leq \arctan [\,] < 180° \\ (180/\pi) \left[\pi\{(r_c/r_E)a_1 + 1 + 2 \,[1 - (r_E/r_c)]/[1 - r_c^2/r_E^2) \right. \\ \cos^2 \gamma_{1I}]\} + (\pi/180) \arctan \left(\{2a_4[(r_c/r_E) - 1] \tan \right. \\ \gamma_{1I}\}/\{[r_c/r_E - 1)]^2 - \tan^2 \gamma_{1I}\}\Big)\Big] \\ \qquad \text{If } r_p \geq r_E \\ \\ \text{where } 0 \leq \arctan (\,) < 180° \end{array} \right.$$

$$-360° < \phi_E < 360°$$

The glide time is

$$\tau_E = r_E \, [(1-a_2) \sin \phi_E + a_3 a_4 \, (-\cos \phi_E + r_c/r_E)]/(2-a_1)V_I |\cos \gamma_{1I}|$$
$$+ 2[r_c^3/GM(2-a_1)^3]^{\frac{1}{2}}(\pi/180) \arctan \{[(-1+2/a_1)^{\frac{1}{2}} \sin (\phi_E/2)]/$$
$$\cos (a_5 + \phi_E)]$$

where $0 < \arctan \{\} \leq 180°$ and

Total flight time to entry is

$$t_E = t + \tau_E$$

where t is the current time.

The total range angle to entry is

$$\phi_E = \phi + a_4 \;\; \phi_E - (180/\pi)\bar{\omega}\tau_E$$

where ϕ is the current range angle and $\bar{\omega}$ is the quasi earth rotation role.

The ground range is

$$S_E = (\pi/180)\phi_E r_e/6076.10333$$

Entry latitude is

$$\rho_E = \arcsin\,(\sin\rho\,\cos|\phi_E| + \cos\rho\,\sin|\phi_E|\cos\gamma_{2I})$$

with

$$-90° \leq \rho_E \leq 90°$$

where ρ and γ_{2I} are the instantaneous latitude and azimuthal flight path angle, respectively.

The components of inertial velocity and total inertial velocity at the entry altitude are

$$\dot{r}_E = \{V_I \cos\gamma_{1I}[a_3 a_4 \cos\phi_E - (1-a_2)\sin\phi_E]/a_2\}a_4$$
$$r_F\dot{\phi}_E = (180/\pi)(r_c V_I \cos\gamma_{1I})/r_E$$
$$V_{IE} = [\dot{r}_E^2 + (\pi/180)^2\,(r_E\dot{\phi}_E)^2]^{\frac{1}{2}}$$

and the entry inertial flight angle is

$$\gamma_{1IE} = \arctan \dot{r}_E/[r_E\phi_E(\pi/180)]$$

where $-180° < \gamma_{IE} \leq 180°$

$$\gamma_{2IE} = \arctan\,[\cos\rho\,\sin\gamma_{2I}\,\sin\phi_E/(\cos\phi_E\,\sin\rho_E - \sin\rho)]$$

with

$$-90° \leq \gamma_{1IE} \leq 90°$$

and

$$0 \leq \gamma_{2IE} < 360°$$

The velocity and flight path angle with respect to the ambient air at entry conditions are

$$V_{aE} = (V_{IE}^2 - 2\omega V_{IE} r_e \sin \gamma_{2IE} \cos \rho_E \cos \gamma_{1IE} + r_E^2 \omega^2 \cos^2 \rho_E)^{\frac{1}{2}}$$

$$\gamma_{1E} = \arcsin (V_{IE} \sin \gamma_{1IE}/V_{aE}$$

$$-90° \leq \gamma_{1E} \leq 90°$$

2. Integrals

a. Motor Impulse

The vacuum impulse is

$$I_V = \int_{t_k}^{t} F_V \, dt$$

and total impulse during the k-th stage is

$$I = \int_{t_k}^{t} F \, dt$$

where t_k is the time the k-th stage begins and t is the current time.

b. Control Impulse

The control system related integrals are

$$I_P = (1 + K_\Delta) \int_{t_k}^{t} |F_Z| dt$$

$$I_Y = (1 + K_\Delta) \int_{t_k}^{t} |F_Y| dt$$

where

$$K_\Delta = \begin{cases} K'_\Delta & \text{If } K'_\Delta \neq 0 \\ 1.0 & \text{Otherwise} \end{cases}$$

A_L is the input amplitude of the limit cycle, ω_L is the input frequency of the limit cycle, t_B is stage time, K'_Δ is the input side impulse multiplier.

$$I_{\dot{\delta}P} = 2 \, t_B \, A_L \, \omega_L / \pi + \int_{t_k}^{t} |\dot{\delta}_r| \, dt$$

$$I_{\dot{\delta}Y} = 2\ t_B\ A_L\ \omega_L/\pi + \int_{t_k}^{t} + \ |\dot{\delta}_Y|dt$$

$$\bar{I}_{\dot{\delta}P} = (n'_m/n'_c)\ I_{\dot{\delta}P}$$

$$\bar{I}_{\dot{\delta}Y} = (n'_m/n'_c)\ I_{\dot{\delta}Y}.$$

where

If τ_c and $\omega_c \neq 0$

$$\dot{\delta}_P = (\delta_{Pj} - \delta_{P(j-1)}/\Delta t_c)$$

$$\dot{\delta}_Y = (\delta_{Yj} - \delta_{Y(j-1)}/\Delta t_c)$$

where I_P and I_Y are the pitch and yaw side impulse, respectively; and $I_{\dot{\delta}P}$ and $I_{\dot{\delta}Y}$ are the sum of angular thrust vectoring velocities. The integrals are ignored if $M_y \neq 5$.

The roll control expended impulse is

$$I_R = \int_{t_k}^{t} F_R\ dt$$

c. Aerodynamic Heating

The heat integral is

$$H_e = \int_{t_o}^{t} q\ V_a\ dt$$

where t_o is the trajectory start time, q the missile dynamic pressure and V_a the total vehicle velocity with respect to the air.

d. Delta Velocities

The following are computed merely for print purposes.

The thrust-to-weight-flow ratio

$$|F/\dot{W}| = \begin{cases} 0 & \text{If } \dot{W} = 0 \\ |F/\dot{W}| & \text{Otherwise} \end{cases}$$

where F and $\dot{W}$ are total instantaneous values.

The gravity loss from trajectory initiation

$$L_g = \int_{t_o}^{t} g_{Z1} \sin \gamma_1 \, dt$$

where t_o is the trajectory start time, G_{Z1} is the gravity acceleration components and γ_1 the instantaneous flight path angle.

The ideal velocity from stage initiation

$$\Delta V = \bar{g}_e \int_{t_k}^{t} (F/\dot{W}) \, dt$$

where $\bar{g}_e$ is gravity at the reference body surface and t_k is the stage initiation time.

The drag loss from stage initiation is

$$L_D = \bar{g}_e \int_{t_k}^{t} [(C + C_{\delta z}) \text{ COS } \bar{\alpha}' + (N_Z - N_{Pz} - N_{\delta z}) \text{ Sin } \bar{\alpha} + (N_Y - N_{Py} - N_{\delta y}) \text{ Sin} \bar{\beta}]/W \, dt$$

where C is the instantaneous aerodynamic axial force.

Instantaneous back pressure loss is

$$L_F = g_e \int_{t_k}^{t} (F_V - F)/W \, dt$$

where F_V is the instantaneous total vacuum thrust, F_V is the instantaneous delivered thrust g_e is the mass gravity and W is the instantaneous vehicle weight. Vector velocity loss from stage initiation is

$$L_V = \Delta V - L_F - L_g - L_D - V_e + V_{eok}$$

where V is the ideal velocity from stage initiation; L_F is the back pressure velocity loss. L_g is the gravity velocity loss, L_D is the drag velocity loss, V_e is the instantaneous missile earth reference velocity, and V_{eok} is the missile earth reference velocity at state initiation.

3. Target Position

The target position and velocities are determined from (1) input code (q_{Tto}), and value (K_{Tto}) which designates the start of the target maneuvering; (2) input initial velocity (V_{To}), flight path angle (γ_{To}), differential range azimuth (ζ_{To}), altitude (h_{To}), down range (S_{To}) and cross range (S_{TCo}) and; (3) tabular input target earth reference accelerations in "g's" tangential (a_{TTj}) normal (a_{TNj}) and transverse (a_{TCj}) to the target velocity vector and the target time terminating the j-th acceleration value (t_{Tj}).

a. **Target Start Time**

The target start time is set, i.e., ($t_{TS} = t$) when the specific value, K_{Tto}, delineated by the code, α_{Tto}, is achieved. If the value has not been achieved set:

$$t_{To} = 0 \qquad a_{TT} = 0$$

$$V_T = 0 \qquad a_{TN} = 0$$

$$\gamma_T = \gamma_{To} \qquad a_{TC} = 0$$

$$\zeta_T = \zeta_{To} \qquad \dot{h}_T = 0$$

$$S_T = S_{To} \qquad \dot{S}_T = 0$$

$$S_{TC} = S_{TCo} \qquad S_{TC} = 0$$

$$h_T = h_{To}$$

b. **Target Coordinates**

The target coordinates are as follows.

Acceleration (ft/sec)

$$\dot{V}_{Tj} = a_{TTj}\,\bar{g}_e$$

$$\dot{V}_{Nj} = a_{TNj}\,\bar{g}_e$$

$$\dot{V}_{Cj} = a_{TCj}\,\bar{g}_e$$

where j is such that $t_{T(j-1)} \leq t - t_{TS} < t_{Tj}$

If $t - t_{TS} \geqq t_{TJ}$ set $j = J$

where J is the number of input time points such that
$t_{T(k-1)} < t_{Tj}$

The following differentials (target coordinates) are numerically integrated with respect to time.

Flight Path Angle Rate

$$\dot{\gamma}_T = (180/\pi)\, \dot{V}_N / V_T$$

Azimuthal Path Angle Rate

$$\dot{\zeta}_T = (180/\pi)\, \dot{V}_c / (V_T \cos \gamma_T)$$

Altitude Rate

$$\dot{h}_T = V_T \sin \gamma_T$$

Down Range Rate

$$\dot{S}_T = V_T \cos \gamma_T \cos \zeta_T$$

Cross Range Rate

$$\dot{S}_{TC} = V_T \cos \gamma_T \sin \zeta_T$$

4. Missile-Target Coordinates

The relation between the target and missile is used in Intercept Guidance ($T_y = 10$) and Homing Guidance ($T_y = 11$) types of flight to direct the attacking missile to the target. The following equations and logic delineate missile to target; differential altitude and earth surface down range and cross range distance (Δh, ΔS, and ΔS_c) distance rates ($\Delta\dot{h}$, $\Delta\dot{S}$, and $\Delta\dot{S}_c$) and total range distance and distance rate (R_{MT} and $\dot{R}_{MT}$), angle and angular rate of the missile to target line and local horizontal (α_{MT} and $\dot{\alpha}_{MT}$), angle and angular rate of the missile to target line differential azimuth (λ_{MT} and $\dot{\lambda}_{MT}$); seeker look angle and angular rate in pitch (ϵ_{MT} and $\dot{\epsilon}_{MT}$) and yaw (δ_{MT} and $\dot{\delta}_{MT}$); time to intercept (t_{MI}); range to target intercept (R_{MI}); intercept coordinates (S_{MI}, S_{MCI}, and h_{MI}), local flight path angle to intercept (α_{MI}) and differential flight path azimuth to intercept (γ_{MI}); flight path error to intercept (ε_{MI}); and flight path azimuth error to intercept (δ_{MI}).

a. Missile to Target Distance

Earth surface difference and distance rate

Down Range

$$\Delta S = S_T - S$$

$$\Delta\dot{S} = \dot{S}_T - \dot{S}$$

Cross Range

$$\Delta S_c = S_{TC} - S_c$$

$$\Delta\dot{S}_c = \dot{S}_{TC} - \dot{S}_c$$

Altitude Difference Distance and Distance Rate

$$\Delta h = h_T - h$$
$$\Delta\dot{h} = \dot{h}_T - \dot{h}$$

Missile to Target Distance and Distance Rate

$$R_{MT} = (\Delta S^2 + \Delta S_c^2 + \Delta h^2)^{\frac{1}{2}}$$
$$\dot{R}_{MT} = (\Delta S \Delta\dot{S} + \Delta S_c \Delta\dot{S}_c + \Delta h \Delta\dot{h})/R_{MT}$$

b. Missile to Target Angles

Elevation

$$\sigma_{MT} = \arcsin\,(\Delta h/R_{MT}) \qquad -90 \leq \sigma_{MT} \leq 90$$
$$\dot{\sigma}_{MT} = (180/\pi)(\Delta\dot{h}R_{MT} - \dot{R}_{MT}\Delta h)/(R_{MT}^2 \cos\sigma_{MT})$$

Differential Azimuth

$$\lambda_{MT} = \arctan\,(\Delta S_c/\Delta S) \qquad -180° < \lambda_{MT} \leq 180°$$
$$\dot{\lambda}_{MT} = (180/\pi)(\Delta\dot{S}_c \Delta S - \Delta S_c \Delta\dot{S}) \cos^2\lambda_{MT}/\Delta S^2$$

Seeker Look Angle and Angular Rate

Relative Azimuthal velocity vector component

$$\gamma_M = \arctan\,(\dot{Y}_{TM}/\dot{X}_{TM}) \qquad -180° < \gamma_M \leq 180°$$

$$\dot{\gamma}_M = \begin{cases} 0 & \text{If } \dot{X}_{TM} = 0 \\ (180/\pi)\cos^2\gamma_M\,[(\ddot{Y}_{TM}\dot{X}_{TM} - \dot{Y}_{TM}\ddot{X}_{TM})/\dot{X}_{TM}^2] & \end{cases}$$

$$[\dot{A}_{TM}] = \left\{ \dot{\zeta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} + \dot{\phi} \begin{bmatrix} 0 & \sin\zeta & \cos\zeta \\ -\sin\zeta & 0 & 0 \\ -\cos\zeta & 0 & 0 \end{bmatrix} \right\} [A_{TM}]$$

where

$$[A_{TM}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & \sin\zeta \\ 0 & -\sin\zeta & \cos\zeta \end{bmatrix} \begin{bmatrix} \cos\Phi & 0 & \sin\Phi \\ 0 & 1 & 0 \\ -\sin\Phi & 0 & \cos\Phi \end{bmatrix} \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{\dot{X}}_{TM} = [A_{TM}]\,\vec{\dot{X}}_{ee}$$

$$\vec{\ddot{X}}_{TM} = [A_{TM}]\,\vec{\ddot{X}}_{ee} + [\dot{A}_{TM}]\,\vec{\dot{X}}_{ee}$$

$$\epsilon_{MT} = (\alpha_{MT} - \gamma_1)\cos\varphi + (\lambda_{MT} - \gamma_M)\sin\varphi - \bar{\alpha}$$

$$\dot{\epsilon}_{MT} = (\dot{\alpha}_{MT} - \dot{\gamma}_1)\cos\varphi + (\dot{\lambda}_{MT} - \dot{\gamma}_M)\sin\varphi - \dot{\bar{\alpha}}$$

$$+ \dot{\varphi}\,[(\lambda_{MT} - \gamma_M)\cos\varphi - (\alpha_{MT} - \gamma_1)\sin\varphi]$$

$$\delta_{MT} = (\alpha_{MT} - \gamma_1)\sin\varphi + (\lambda_{MT} - \gamma_M)\cos\varphi + \bar{\beta}$$

$$\dot{\delta}_{MT} = (\dot{\alpha}_{MT} - \dot{\gamma}_1)\sin\varphi + (\dot{\lambda}_{MT} - \dot{\gamma}_M)\cos\varphi + \dot{\bar{\beta}}$$

$$+ \dot{\varphi}\,[(\alpha_{MT} - \gamma_1)\cos\varphi + (\lambda_{MT} - \gamma_M)\sin\varphi]$$

c. Target Intercept Parameters

Time to Target Intercept

$$t_{MI} = R_{MT}/\{[V_c^2 - V_T^2 \sin^2 \epsilon_{MI}^*]^{\frac{1}{2}} - V_T \cos \epsilon_{MI}^*\}$$

where

$$\epsilon_{MI}^* = \arccos\,[\cos\alpha_T \cos\gamma_T \cos(\lambda_{MT} - \gamma_M) + \sin\alpha_T \sin\gamma_T]$$

Range to Target Intercept

$$R_{MI} = V_e \, t_{MI}$$

Intercept Coordinates

$$S_{MI} = S_T + \dot{S}_T \, t_{MI}$$

$$h_{MI} = h_T + \dot{h}_T \, t_{MI}$$

$$S_{CMI} = S_{CT} + \dot{S}_{CT} \, t_{MI}$$

Angle to Target Intercept

Elevation

$$\sigma_{MI} = \text{arc sin } [(h_{MI} - h)/R_{MI}]$$

$$- 90 \leq \sigma_{MI} \leq 90°$$

Azimuth

$$\tau_{MI} = \text{arc tan } [(S_{CMI} - S_C)/(S_{MI} - S)]$$

$$- 180° < \tau_{MI} \leq 180°$$

Flight Path Error to Intercept

Elevation

$$\epsilon_{MI} = \sigma_{MI} - \gamma_1 \qquad -180 < \epsilon_{MI} \leq 180°$$

Azimuth

$$\delta_{MI} = \tau_{MI} - \gamma_M \qquad -180 < \delta_{MI} < 180°$$

5. General

Because the following parameters can be involved as switching functions, their solutions are necessary after each integration step.

a. Stage Data

The instantaneous current stage time and main weight are

$$t_B = t - t_k$$

and

$$W_B = W_{gjk} - \Delta W_M$$

where t is the instantaneous time, t_k the stage start time, W_{gjk} the main step weight for the current stage at time t_j, and W_M the main weight lost between t_j and t.

b. Earth Referenced Values

The total vehicle velocity with respect to the launch site is

$$V_e = (\dot{X}^2_{ee} + \dot{Y}^2_{ee} + \dot{Z}^2_{ee})^{\frac{1}{2}}$$

where $\dot{\vec{X}}_{ee}$ is obtained from the integration of the linear momenta equations.

The total vehicle acceleration along the flight path with respect to the launcher is

$$\dot{V}_e = \begin{cases} (\ddot{X}^2_{ee} + \ddot{Y}^2_{ee} + \ddot{Z}^2_{ee})^{\frac{1}{2}} & \text{If } V_e = 0 \\ \dfrac{\dot{X}_{ee}\ddot{X}_{ee} + \dot{Y}_{ee}\ddot{Y}_{ee} + \dot{Z}_{ee}\ddot{Z}_{ee}}{V_e} & \text{Otherwise} \end{cases}$$

where $\ddot{\vec{X}}_{ee}$ is obtained from the linear momenta equations.

The local pitch flight path angle is

$$\gamma_1 = \begin{cases} 0 & \text{If } V_e = 0 \\ \arcsin\,(-\dot{Z}_{11}/V_e) & \text{Otherwise} \end{cases}$$

with

$$-\,90° \leqq \gamma_1 \leqq 90°$$

$$\dot{\gamma}_1 = \begin{cases} 0 & \text{If } V_e = 0 \\ (\dot{Z}_{11}\dot{V}_e - \ddot{Z}_{11}V_e)/(V_e^2 \cos\gamma_1) & \text{Otherwise} \end{cases}$$

and the local azimuthal angle is

$$\gamma_2 = \begin{cases} 0 & \text{If } V_e = 0 \\ \arctan\,(\dot{Y}_{11}/\dot{X}_{11}) & \text{Otherwise} \end{cases}$$

where $0 \leqq \gamma_2 < 360°$

$$\dot{\gamma}_2 = \begin{cases} 0 & \text{If } \dot{X}_{11} = 0 \\ \dfrac{(\ddot{Y}_{11}\dot{X}_{11} - \ddot{X}_{11}\dot{Y}_{11})\cos^2\gamma_2}{\dot{X}_{11}^2} & \text{Otherwise} \end{cases}$$

where the velocity and acceleration components in the 1 system are

$$\dot{\vec{X}}_{11} = [A_1]^{-1}\,\dot{\vec{X}}_{ee}$$

$$\ddot{\vec{X}}_{11} = [A_1]^{-1}\,\ddot{\vec{X}}_{ee} + [\dot{A}_1]^{-1}\,\dot{\vec{X}}_{ee}$$

c. Local Bank Attitude

Transformation from the local, north, east, and down system to the missile axis system for a commanded angle of attack, angle of sideslip, and back angle is accomplished by

$$X_{bb} = [A_{\alpha}]\ [A'_{\beta}]\ [A_{\phi}]\ [A_{\gamma_1}]\,[A_{\gamma_2}]\ \dot{X}_{11}$$

$$\dot{X}_{bb} = [D]^{-1}\ [A]\ \dot{X}_{11}$$

where the azimuth velocity vector transformation is

$$[A_{\gamma_2}^{-1}] = \begin{bmatrix} \cos\gamma_2 & -\sin\gamma_2 & 0 \\ \sin\gamma_2 & \cos\gamma_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$[\dot{A}_{\gamma_2}^{-1}] = \dot{\gamma}_2 \begin{bmatrix} -\sin\gamma_2 & -\cos\gamma_2 & 0 \\ \cos\gamma_2 & -\sin\gamma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\gamma}_2 & 0 \\ \dot{\gamma}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} [A_{\gamma_2}^{-1}]$$

where the elevation velocity vector transformation is

$$[A_{\gamma_1}^{-1}] = \begin{bmatrix} \cos\gamma_1 & 0 & \sin\gamma_1 \\ 0 & 1 & 0 \\ \sin\gamma_1 & 0 & \cos\gamma_1 \end{bmatrix}$$

and

$$[\dot{A}_{\gamma_1}^{-1}] = \dot{\gamma}_1 \begin{bmatrix} -\sin\gamma_1 & 0 & \cos\gamma_1 \\ 0 & 0 & 0 \\ \cos\gamma_1 & 0 & -\sin\gamma_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dot{\gamma}_1 \\ 0 & 0 & 0 \\ -\dot{\gamma}_1 & 0 & 0 \end{bmatrix} [A_{\gamma_1}^{-1}]$$

The bank angle transformation is

$$[A_\varphi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix}$$

The angle of attack transformation is

$$[A_\alpha]^{-1} = \begin{bmatrix} \cos\bar{\alpha} & 0 & +\sin\bar{\alpha} \\ 0 & 1 & 0 \\ -\sin\bar{\alpha} & 0 & \cos\bar{\alpha} \end{bmatrix}$$

$$[\dot{A}_\alpha]^{-1} = \alpha \begin{bmatrix} -\sin\alpha & 0 & \cos\alpha \\ 0 & 0 & 0 \\ -\cos\alpha & 0 & -\sin\alpha \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha & 0 & 0 \end{bmatrix} [A_\alpha]^{-1}$$

The side slip angle transformation is

$$[A_\beta{}']^{-1} = \begin{bmatrix} \cos\bar{\beta}' & +\sin\bar{\beta}' & 0 \\ -\sin\bar{\beta}' & \cos\bar{\beta}' & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\bar{\beta}' = \arctan\left[(\tan\bar{\beta})/(\sec\bar{\alpha})\right]$

$$[\dot{A}_\beta{}']^{-1} = \dot{\bar{\beta}}' \begin{bmatrix} -\sin\bar{\beta}' & \cos\bar{\beta}' & 0 \\ -\cos\bar{\beta}' & -\sin\bar{\beta}' & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \dot{\bar{\beta}}' & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} [A_\beta{}']^{-1}$$

where

$$\dot{\bar{\beta}} = \begin{cases} 0 & \text{If } \cos\beta = 0 \\ \dfrac{\cos^2\bar{\beta}'}{\cos^2\bar{\beta}} \left[\dot{\bar{\beta}}\cos\bar{\alpha} + \dot{\bar{\alpha}}\sin\bar{\beta}\cos\bar{\beta}\sin\bar{\alpha}\right] & \text{otherwise} \end{cases}$$

The instantaneous bank angle is

$$\varphi = \arctan\left[a_{\varphi_{23}}/a_{\varphi_{22}}\right]$$

$$[A_{\varphi}] = [A_{\alpha}]^{-1} [A'_{\beta}]^{-1} [D]^{-1} [A_1] [A_{\gamma 2}]^{-1} [A_{\gamma 1}]^{-1}$$

where

$$\begin{aligned}
[\dot{A}_{\varphi}] = \; & [\dot{A}'_{\beta}]^{-1} [A_{\alpha}]^{-1} [D]^{-1} [A_1] [A_{\gamma 2}]^{-1} [A_{\gamma 1}]^{-1} \\
& + [A'_{\beta}]^{-1} [\dot{A}_{\alpha}]^{-1} [D]^{-1} [A_1] [A_{\gamma 2}]^{-1} [A_{\gamma 1}]^{-1} \\
& + [A'_{\beta}]^{-1} [A_{\alpha}]^{-1} [\dot{D}]^{-1} [A_1] [A_{\gamma 2}]^{-1} [A_{\gamma 1}]^{-1} \\
& + [A'_{\beta}]^{-1} [A_{\alpha}]^{-1} [D]^{-1} [\dot{A}_1] [A_{\gamma 2}]^{-1} [A_{\gamma 1}]^{-1} \\
& + [A'_{\beta}]^{-1} [A_{\alpha}]^{-1} [D]^{-1} [A_1] [\dot{A}_{\gamma 2}]^{-1} [A_{\gamma 1}]^{-1} \\
& + [A'_{\beta}]^{-1} [A_{\alpha}]^{-1} [D]^{-1} [A_1] [A_{\gamma 2}]^{-1} [\dot{A}_{\gamma 1}]^{-1}
\end{aligned}$$

$$\dot{\varphi} = (180/\pi) \quad [a_{\varphi 22} \, \dot{a}_{\varphi 23} - a_{\varphi 23} \, \dot{a}_{\varphi 22}]$$

J. Inertial Values

The inertial values are referenced to a system fixed in, but not rotating with the earth.

The missile energy per unit mass is

$$E/m = GM/r_e - GM/r_c + V_I^2/2$$

where zero potential energy is assumed at the surface of the reference body, r_e is the input reference body radius, GM is computed from input, and r_c and VI are the missile distance from earth center and inertial velocity respectively.

The inertial velocity components are

$$\dot{X}_{11I} = \dot{X}_{11}$$
$$\dot{Y}_{11I} = \dot{Y}_{11} + \omega\, r_c \cos \rho$$
$$\dot{Z}_{11I} = \dot{Z}_{11}$$

$$V_I = (\dot{X}^2_{11} + \dot{Y}^2_{11I} + \dot{Z}^2_{11})^{\frac{1}{2}}$$

The flight path angles are

$$\gamma_{1I} = \begin{cases} 0 & \text{If } V_I = 0 \\ \arcsin\,(-\dot{Z}_{11}/V_I) & \text{Otherwise} \end{cases}$$

and

$$\gamma_{2I} = \begin{cases} \psi_{io} & \text{If } \dot{Y}_{11I} = \dot{X}_{11} = 0 \\ \arctan\,(\dot{Y}_{11I}/\dot{X}_{11}) & \text{Otherwise} \end{cases}$$

with

$$-90^\circ \leq \gamma_{1I} < 90^\circ$$

$$0 \leq \gamma_{2I} < 360^\circ$$

where ψ_{io} is the input azimuth angle.

e. Generalized Coordinates

The g system, introduced below, can be used to obtain vehicle position and velocity components in a coordinate system oriented about the e system. The system also is a part of the guidance system model. The orientation of the g system is specified by input Euler angles which perform the following rotations: θ_g about the Y_e-axis, ψ_g about the resulting Z-axis, and ϕ_g about the X-axis. The components are computed from the following if the input "PL-KB" is nonzero or if the type of flight, T_y, is four or five or the current stage σ_{glk} is nonzero.

$$X_{gg} = [A_g]\ X_{ee}$$

$$\dot{X}_{gg} = [A_g]\ \dot{X}_{ee}$$

$$\ddot{X}_{gg} = [A_g]\ \ddot{X}_{ee}$$

where

$$[A_g] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_g & \sin\phi_g \\ 0 & -\sin\phi_g & \cos\phi_g \end{bmatrix} \begin{bmatrix} \cos\psi_g & \sin\psi_g & 0 \\ -\sin\psi_g & \cos\psi_g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_g & 0 & -\sin\theta_g \\ 0 & 1 & 0 \\ \sin\theta_g & 0 & \cos\theta_g \end{bmatrix}$$

The orientation of the g system is shown in Figure 17 for ϕ_g and ψ_g equal to zero.

6. TVC Requirements and Duty Cycle

The following parameters are calculated and stored in the L-7000-7999 block, if $K_{dc} \neq 0$ (L-671).

L-No.	Symbol	Definition	Units
7000	$\bar{I}$	Motor thrust impulse for TVC duty cycle stage.	lb-sec
7001	$\bar{I}_v$	Motor vacuum thrust impulse for TVC duty cycle stage.	lb-sec
7002	x_e	TVC duty cycle stage thrust vector point body station.	ft
7003	x_{nf}	Body station of nozzle flange of the TVC design stage. Input in L-680.	ft
7004	δ_{me}	Design maximum vector angle for TVC duty cycle stage. Input in L-681.	deg
7005	I_{SPM}	Main table specific impulse for the TVC duty cycle stage. Input in L-(K_{dc}) 010.	sec
7006	$\bar{\omega}_c$	Slew frequency used in the TVC design stage slew rate calculation.	rad/sec
7007	$\dot{\delta}_S$	Control system design slew rate for TVC design stage.	deg/sec
7008	$\bar{\delta}_S$	Slew angle for TVC design stage.	deg
7009	$\bar{I}_P$	Pitch control thrust impulse per control motor from TVC duty cycle initiation to stage termination.	lb-sec
7010	$\bar{I}_{\dot{\delta}P}$	Integral of the pitch angular thrust vectoring velocities from TVC duty cycle stage initiation to stage termination.	deg
7011	$t_{\eta P}$	Stage time at which maximum magnitude pitch thrust vector deflection angle occurs during the TVC design stage $(\bar{\delta}_{Pmax})$.	sec
7012	η_P	Ratio of the delivered thrust (F) to vacuum thrust (Fvac) at maximum magnitude pitch TVC deflection angle $(\bar{\delta}_{Pmax})$.	dim
7013	$\bar{\dot{\delta}}_{Pmax}$	Maximum pitch thrust vector deflection angular rate, for the TVC design stage.	deg/sec
7014	$\bar{\delta}_{Pmax}$	Maximum magnitude pitch thrust vector deflection angle, per control motor for the TVC design stage.	deg

L-No.	Symbol	Definition	Units
7015	$\bar{I}_y$	Yaw control thrust impulse per control motor from TVC stage initiation to stage termination.	lb-sec
7016	$\bar{I}_{\dot{\delta} Y}$	Yaw deflection rate integral per vehicle control motor.	deg
7017	$t_{\eta Y}$	Stage time at which maximum magnitude yaw thrust vector deflection angle occurs during the TVC design stage (δ_{Ymax}).	sec
7018	η_Y	Ratio of the delivered thrust (F) to vacuum thrust (F_{vac}) at maximum magnitude yaw TVC deflection angle (δ_{Ymax}).	dim
7019	$\dot{\delta}_{Ymax}$	Maximum yaw thrust vector deflection angular rate, for the TVC design stage.	deg/sec
7020	δ_{Ymax}	Maximum magnitude yaw thrust vector deflection angle, per control motor for the TVC design stage.	deg
7021	$q\alpha'_{max}$	Product of the maximum absolute value of the dynamic pressure-angle of attack for the TVC design stage.	lb-deg/sq ft
7022	$q_{q\alpha'}$	Dynamic pressure at maximum $q\alpha'$ during the TVC duty cycle stage.	lb/sq ft
7023	$t_{q\alpha}$	TVC duty cycle stage time at maximum $q\alpha'$.	sec
7024	$C_{N\alpha q}$	Aerodynamic normal force coefficient at maximum $q\alpha'$ during the TVC duty cycle stage.	1/deg
7025	$M_{q\alpha}$	Mach number at maximum $q\alpha'$ during the TVC duty cycle stage.	dim.
7026	$\bar{\delta}_{ave}$	Average TVC deflection angle per control motor for the TVC design stage.	deg
7027	K_{dc}	Stage number of the TVC duty cycle stage. Input in L-671.	dim.
7028	D_B	TVC duty cycle stage case diameter from axial force reference area.	in.
7029	A_{FW1}	Stage I vacuum thrust to liftoff weight used in the vehicle characteristics pertinent to roll requirements.	g's
7030	t_B	TVC duty cycle stage time.	sec

L-No.	Symbol	Definition	Units
7031	F_{vave}	TVC duty cycle stage average vacuum thrust.	lb
7032	W_o	TVC duty cycle stage liftoff weight used in the roll control requirements.	lb
7033	K_{PFV}	Ratio of motor chamber pressure to vacuum thrust of the main thrust table of the TVC design stage.	1/sq in.
7034	$\bar{\epsilon}_d$	Nozzle average expansion ratio for TVC design stage. Input in L-(K_{dc}) 013.	dim.
7035	$\bar{A}_t$	Nozzle throat area for the main motor of the TVC design stage.	sq in.
7036	$\bar{\gamma}_d$	Ratio of specific heats of the rocket motor exhaust gases of the TVC design stage.	dim.
7037	P_{ca}	Action time average motor chamber pressure for the TVC design duty cycle stage.	lb/sq in.
7038	C^*	Rocket motor propellant characteristic velocity for the TVC design duty cycle stage.	ft/sec
7039	W_{TVC}	Estimated TVC system fixed weight. Used in TVC design stage for the refly option. Input in L-677.	lb
7040	W_{exi}	Estimated weight of the TVC system expended weight during the TVC design stage during the original vehicle flight. Input in L-678.	lb
7041	I_{spaug}	Estimated TVC system caused specific impulse augmentation (positive) or degradation (negative). Used in trajectory TVC design program refly. Input in L-679.	sec
7044	n_m	Number of motors in the stage cluster	dim.
7045	n_c	Number of control nozzles for the cluster motor logic.	dim.
7046	Δ_{dc}	Number TVC duty cycle t_B data points.	dim.
7047	M_{hzmax}	Maximum of the absolute value pitch fin hinge torque for the TVC design stage.	ft-lb

L-No.	Symbol	Definition	Units
7048	M_{hymax}	Maximum of the absolute value yaw fin hinge torque forthe TVC design stage.	ft-lb
7052	$h_{q\alpha'}$	Altitude of TVC duty cycle stage maximum $q\alpha'$.	ft
7053	$P_{q\alpha'}$	Atmospheric pressure of TVC duty cycle stage maximum $q\alpha'$.	lb/sq ft
7054	$\bar{I}_{VM}$	Integral of the vacuum thrust of the input main thrust table in the TVC duty cycle stage. Input in L-(K_{dc}) 005.	lb-sec
7055	$\bar{W}_{MO}$	Initial main weight for the TVC duty cycle stage. Input in L-(K_{dc}) 006.	lb
7056	$\bar{K}_{tM}$	TVC duty cycle stage, the main switching time multiplier. Input in L-(K_{dc}) 009.	dim.
7057	$\alpha_{q\alpha'}$	Angle of attack in pitch at TVC design stage maximum $q\alpha'$.	deg
7058	$\beta_{q\alpha'}$	Angle of side slip in yaw at TVC design stage maximum $q\alpha'$.	deg
7059	P_{cmax}	Maximum main motor chamber pressure in TVC duty cycle stage.	lb/sq ft
7060	$\dot{A}_{tmax}$	Pintle System required throat area rate.	in^2/sec
7061	I_{At}	Integral of the absolute value of the pintle throat area rate.	in
7062	F_{max}	Pintle motor maximum vacuum thrust	lb
7063	P_{cmax}	Pintle motor chamber pressure at maximum vacuum thrust	lb/in.^{2}
7064	ϵ_{max}	Pintle motor expansion ratio at maximum vacuum thrust	dim.
7065	A_{tmin}	Pintle motor throat area at maximum vacuum thrust	in.^{2}

L-No.	Symbol	Definition	Units
7066	C_{fmax}	Pintle motor vacuum thr coefficient at maximum vacuum thrust	dim.
7067	F_{min}	Pintle motor minimum vacuum thrust	lb
7068	P_{cmin}	Pintle motor chamber pressure at minimum vacuum thrust	lb/in.^{2}
7069	ϵ_{min}	Pintle motor expansion ratio at minimum vacuum thrust	dim.
7070	A_{tmax}	Pintle motor throat area at minimum vacuum thrust	in.^{2}
7071	C_{fmin}	Pintle motor vacuum thrust coefficient at minimum vacuum thrust	dim.
7072	A_{xmax}	Motor extinguishment throat area	in.^{2}
7073	A_{tmn}	Pintle motor duty cycle minimum throat area	in.^{2}
7074	A_{tmx}	Pintle motor duty cycle maximum throat area	in.^{2}
7100 7199	t_{Bq}	Stage time at which TVC duty points occur.	sec
7200 7299	$\bar{\delta}_{Pq}$	Pitch thrust deflection angle at t_{Bq}.	deg
7300 7399	$\bar{\delta}_{Yq}$	Output modified yaw thrust deflection angle at t_{Bq} for TVC design stage.	deg
7400 7499	F_q	Delivered motor thrust (F) at t_{Bq} during TVC design stage.	lb
7500 7599	X_{cgq}	Vehicle center-of-gravity at t_{Bq}; the TVC duty cycle point.	ft
7600	F_{vacq}	Vacuum motor thrust during the TVC design stage ($\bar{F}_{vac}$) at t_{Bq}.	lb
7700	$\dot{W}_q$	Output motor weight flow (W) at t_{Bq} the	lb/sec

a. Slew Rate

The calculation of maximum TVC slew rate is computed as follows:

$$\dot{\delta}_S = 0.22797\ \bar{\omega}_c\ \bar{\delta}_S$$

where $\bar{\dot{\delta}}_S$ is the design slew rate, $\bar{\delta}_S$ is the pitch slew angle. The 0.45594 $(\omega_c/2)$ factor is created by the maximum velocity of the response to a unit input step function at one-half band width frequency. If the input does not have ω_c, then the following logic is used for evaluating the actuator system band width frequency.

Slew Frequency

$$\bar{\omega}_c = \begin{cases} \omega_s & \text{If } \omega_s \neq 0 \\ \omega_c & \text{If } \omega_s = 0 \text{ and } \omega_c \neq 0 \\ \text{anti } \ell n\ [12.146979 - 1.3829872\ (\ell n\ W_{01})\ ] & \\ + 0.062446656\ (\ln W_{01})^2 - 0.00093655891\ (\ell n\ W_{01})^3] & \\ & \text{Otherwise} \end{cases}$$

where W_{01} is the stage I liftoff weight, ω_s is the input slew frequency and ω_c is the input second order TVC frequency.

Slew Angle Amplitude

The deflection terms of the slew rate are computed as follows:

$$\delta'_{Pmax} = \left[\begin{array}{l}\text{The maximum magnitude pitch deflection in the region where} \\ \\ h_o < h < h_\beta \quad \text{and } h_\alpha < h\end{array}\right.$$

and

$$\delta_{PW} = \left[\begin{array}{l}|\delta_{PM} - \delta_{PL}| \\ \text{or} \\ |\delta_{PM} - \delta_{PH}| \text{ , whichever is greater}\end{array}\right.$$

$$\delta_{YW} = \left[\begin{array}{l}|\delta_{YM} - \delta_{YL}| \\ \text{or} \\ |\delta_{YM} - \delta_{YH}| \text{ , whichever is greater}\end{array}\right.$$

where δ_{PM} and δ_{YM} are the maximum magnitude pitch and yaw deflection angle in the region where $h_\beta \leq h \leq h_\alpha$ and δ_{PL}, δ_{PM}, δ_{YL}, and δ_{YH} are the pitch and yaw deflections at the input altitudes h_β and h_α.

If h_β falls outside the TVC design stage altitude regime, set $\delta_{PL} = \delta_{YL} = 0$.

$$\delta_S = \text{MAX} \; (\; |\delta'_{pmax}|, \; \delta_{PW}, \text{ and } \delta_{YW})$$

$$\delta_S = (n_m/n_c) \; \delta_S$$

where n_m and n_c are the internally calculated number of motors in a cluster and the number of control nozzles in that cluster respectively.

It is suggested that the altitudes h_β and h_α correspond to the altitude at which the wind shears start and stop, respectively. Using these altitudes will assure that the maximum thrust vector deflection used in evaluating design slew rates will be approximately compatible with the actual system requirements for near step function inputs.

b. Duty Cycle Storage Logic

The values of $\bar{\delta}_{Pq}$, $\bar{\delta}_{Yq}$, x_{cgq}, F_q, F_{vacq}, $\dot{W}_q$ at t_{Bq} are stored for the k_{dc}-th stage for use in the hardware design subroutine.

Where

$\bar{\delta}_{Pq}$ is the modified pitch thrust deflection angle ($\bar{\delta}_P$) at t_{Bq} (deg),

$\bar{\delta}_{Yq}$ is the modified yaw thrust deflection angle ($\bar{\delta}_Y$) at t_{Bq} (deg),

x_{cgq} is the instantaneous vehicle center-of-gravity at t_{Bq} (ft),

F_q is the delivered motor thrust ($\bar{F}$) at t_{Bq} (lb),

F_{vacq} is the vacuum motor thrust ($\bar{F}_{vac}$) at t_{Bq} (lb),

$\dot{W}_q$ is motor weight flow ($\dot{W}$) at t_{Bq} (lb/sec)

Where

t_{Bq} is the stage compute time just greater than or equal to Jt'_f/Δ_{dc} ($J = 0, 1, 2, \ldots, \Delta_{dc}$) and t_B at stage termination. If any of these conditions coincide, one value shall be stored. The number of t_B data points shall be counted and stored as n_{dc}.

Where

t'_f is greatest, $t'_j K_t$ or $t'_{cj} K_{tc}$ ($j = 1, 2,$ or 10) value on the input thrust time values for the TVC duty cycle stage,

Δ_{dc} is the input value of number of desired duty cycle points (99 maximum) if input zero, set equal to 50,

K_{dc} is the input stage number of the TVC duty cycle stage,

K_t and K_{tc} are the switching time multipliers for the main and complementary thrust tables.

c. Modified Thrust Deflection Angle--The modified thrust deflection angle is

$$\bar{\delta}_P = [\delta_P + A_L \sin (\omega_L [180/\pi] [t - t_k])] (n_m/n_c)$$

$$\bar{\delta}_Y = [\delta_Y + A_L \sin (\omega_L [180/\pi[[t - t_k])] (n_m/n_c)$$

$$\bar{\delta}_P = [\delta_P + A_L \omega_L \cos (\omega_L [180/\pi] [t - t_k])] (n_m/n_c)$$

$$\bar{\delta}_Y = [\delta_Y + A_L \omega_L \cos (\omega_L [180/\pi] [t - t_k])] (n_m/n_c)$$

Where

$\delta_{P,Y}$ are the pitch and yaw thrust deflection angles (deg),

$\dot{\delta}_{P,Y}$ are the pitch and yaw thrust deflection angular rates (deg/sec)

A_L is the amplitude of the limit cycle (deg),

t is the time (sec) and t_K is stage initiation time (sec),

ω_L is the frequency of the limit cycle (rad/sec),

n_m is internally calculated number of motors in a cluster (dim),

n_c is the number of control motors in the cluster (dim),

d. Modified Duty Cycle Thrust Vector Integrals and Weights Flow for a Single Motor

The thrust vector integrals and weight flow per motor of a cluster configuration is

$$\bar{F} = F/n_m$$

$$\bar{F}_{vac} = F_{vac}/n_m$$

$$\dot{\bar{W}} = \dot{W}/n_m$$

$$\bar{I}_v = I_v/n_m$$

$$\bar{I} = I/n_m$$

$$\bar{I}_p = I_P/n_c$$

$$\bar{I}_y = I_Y/n_c$$

$$\bar{I}_{\dot{\delta}P} = (n_m/n_c)\ \bar{I}_{\dot{\delta}P}$$

$$I_{\dot{\delta}Y} = (n_m/n_c)\ I_{\dot{\delta}Y}$$

Where

$\bar{F}$ is the instantaneous motor thrust (lb),

$\bar{F}_{vac}$ is the instantaneous vacuum motor thrust (lb),

$\dot{\bar{W}}$ is the instantaneous motor weight flow (lb/sec),

$\bar{I}_v$ is the motor vacuum thrust impulse from TVC duty cycle stage initiation to stage termination (lb-sec),

I_v is the total vehicle vacuum thrust impulse from TVC duty cycle stage initiation to stage termination (lb-sec),

I is the total vehicle delivered thrust impulse from TVC duty cycle stage initiation to stage termination (lb-sec).

$\bar{I}_p$ is the pitch control thrust impulse per control motor from TVC duty cycle stage initiation to stage termination (lb-sec),

$\bar{I}_{\delta p}$ is the pitch deflection rate integral per vehicle control motor (deg),

$I_{\dot{\delta} p}$ is the integral of pitch angular thrusting vectoring velocity from stage initiation (deg).

Vehicle Characteristics Pertinent to Roll Requirement

Average Stage Vacuum Thrust

$$F_{vave} = I_v / t_B$$

where I_v and t_B are the TVC duty cycle stage vacuum impulse and stage time respectively at stage termination.

TVC Stage Liftoff Weight

$W_o = W_{o(Kdc)}$ is the vehicle weight at TVC duty cycle stage initiation.

Stage I Vacuum Thrust to Weight

$$A_{FW1} = \begin{cases} A_{FW1}' & \text{If } A_{FW1}' \neq 0 \\ (I_v/t_{B1})/W_{o1} & \text{If } K_k = 1 \\ 3.0 & \text{Otherwise} \end{cases}$$

Base Diameter

$$D_B = 24.0 \, (S_{RC_{kdc}} / \pi)^{\frac{1}{2}}$$

Maximum Dynamic Pressure - Angle of Attack

The maximum q ' and the corresponding normal force coefficient and stage time are saved for the TVC duty cycle stage.

$$C_{N\alpha q} = \left| N_z \right| / [S_{RN} \; (q\alpha')] \text{ at } q\alpha' \text{ max}$$

where N_z is the normal force, S_{RN} is the aerodynamic normal reference area and $q\alpha'$ is the dynamic pressure-angle of attack product.

$q\alpha'_{max}$ = $q\alpha'$ at maximum $q\alpha'$

$t_{q\alpha}$ = t_B at maximum $q\alpha'$ (stage time)

$M_{q\alpha}$ = Mach number at maximum $q\alpha'$

$q'_{q\alpha}$ = Dynamic pressure at maximum $q\alpha'$

$h'_{q\alpha}$ = Altitude at maximum $q\alpha'$

$\alpha_{q\alpha'}$ = Angle of attack at $q\alpha'$

$\beta_{q\alpha'}$ = Angle of side slip at $q\alpha'$

Nozzle Parameter

The parameters input of internally calculated in the program germane to the nozzle design are R_{PFV}, ϵ_d, $\bar{A}_t$, I_{spM}, P_{ca}, C*, and γ_d for the TVC design stage. ϵ_d and I_{spM} are input and γ_d is specified in Main Thrust Weight logic.

Motor Chamber Pressure to Vacuum Thrust Ratio

$$R_{PFV} = \begin{cases} 0 & \text{If } \epsilon_d = 0 \\ \text{PCOFMV} \cdot n_m/144 & \text{Otherwise} \end{cases}$$

where PCOFMV is the ratio of chamber pressure to vehicle main vacuum thrust calculated in the nozzle separated flow logic, n_m is the number of motors in a cluster and ϵ_d is the input nozzle expansion ratio.

Motor Nozzle Throat Area

$$\bar{A}_t = \begin{cases} 0 & \text{If } \epsilon_d = 0 \\ 144\, A_{eM}/(\epsilon_d n_m) & \text{Otherwise} \end{cases}$$

where A_{eM} is the input nozzle exit area.

Action Time Average Motor Chamber Pressure

$$P_{ca} = \bar{I}_v \cdot R_{PFV}/t_B$$

where $\bar{I}_v$ is the motor vacuum thrust impulse from TVC duty cycle stage initiation to stage termination, and t_B is the TVC duty cycle stage time.

Rocket Motor Propellant Characteristic Velocity

$$C^* = \begin{cases} 0 & \text{If } \epsilon_d = 0 \\ PCOFMV \cdot A_e M \cdot \bar{g}_e \cdot I_{spM}/\epsilon_d & \text{Otherwise} \end{cases}$$

where I_{spM} is the input vacuum specific impulse for the main thrust weight table.

Average Thrust Vector Deflection Angle

$$\bar{\delta}_{ave} = \text{arc sin } [(\bar{I}_P + \bar{I}_Y)/\bar{I}]$$

where $\bar{I}_P$ and $\bar{I}_y$ and $\bar{I}$ are the pitch control thrust impulse, yaw control thrust impulse delivered thrust impulse per control motor for the TVC duty cycle stage.

e. Thrust Modulation Control Related Parameter

If $D_p \neq 0$ for the TVC design stage the following parameters are calculated and stored.

Pintle System Required Throat Area Rate (in^2/sec)

$$\dot{A}_{tmax} = 0.45594 \quad \omega_p \; (A_{xmax} - A_{xmin})$$

where ω_p is the pintle control frequency (rad/sec), A_{xmax} is the maximum extinguishment throat area and A_{xmin} is the minimum throat area and 0.45594 is the factor created by the maximum velocity of the response to a unit input step function at a band width frequency ω_p and amplitude of $(A_{xmax} - A_{xmin})$.

Integral of Pintle area rate (in)

$$I_{At} = \int_o^{t_B} |\dot{A}_t| \quad dt$$

where $\dot{A}_t$ is the rate change of throat area and t_B is the stage time.

Minimum Throat Conditions

The minimum throat area value is evaluated from each compute interval and the corresponding values of thrust coefficient, chamber pressure, vacuum thrust, and nozzle expansion ratio are also saved.

$$A_{tmin} = A_t$$

$$C_{fmax} = C_{FI}$$

$$F_{max} = F_{MV}$$

$$\epsilon_{max} = \epsilon$$

Minimum Throat Conditions

The minimum throat area value is evaluated from each computer interval and the corresponding values of thrust coefficient chamber pressure, vaccum thrust, and nozzle expansion ratio are also saved.

$$A_{tmin} = A_T$$

$$C_{fmax} = C_{FI}$$

$$F_{max} = F_{MV}$$

$$\epsilon_{max} = \epsilon$$

Maximum Throat Conditions

The maximum throat area value is evaluated from each compute interval and the corresponding values of thrust coefficient, chamber pressure, vacuum thrust and nozzle expansion ratio are also saved.

$A_{tmax} = A_T$

$C_{fmin} = C_{FI}$

$F_{min} = F_{MV}$

$\epsilon_{max} = \epsilon$

K. TRAJECTORY LOGIC

Logic governing stage initiation and termination, numerical integration step size, and the times at which trajectory values are printed are defined.

1. Initiation - Termination

a Trajectory Initiation

It is possible to start a trajectory at any stage initiation. Besides the initial values required for the differential equations and the values for vehicle characteristics, the following additional data are required input: (1) the stage designation parameter. K_k ($k = 1, 2, 3,$ or 4), (2) the trajectory start time. t_o. (3) the type of flight segment, T'_{yj} ($j = 1, 2, \ldots,$ or 16), and (4) the mode segment. M'_{oj} ($j = 1, 2, \ldots,$ or 10). There are exceptions to the requirement of inputting the last two items. These are explained in Mode and Flight Control Sections.

b. Stage Termination

Each of the four stages will end when

$$\Delta_k = 0 \ (k = 1, 2, 3, 4)$$

with

$$\Delta_k = \sigma_{Sk} - k_{3k}$$

where σ_{Sk} is the achieved value of a quantity designated by code input, k_{3k} is input. If σ_{Sk} is designated as stage time. t_B or time, t, then the input k_{3k} is multiplied by the input k_{tk},

when $\Delta_k = 0$, let $t = t_{Sk}$

If $\sigma_{S1} = 0$, halt the run and print the reason.

If $\sigma_{S2} = 0$, only one stage is desired and cutoff occurs

when $\Delta_1 = 0$

If $\sigma_{S3} = 0$, two-stage operation is desired and cutoff occurs

when $\Delta_2 = 0$

If $\sigma_{S4} = 0$, three-stage operation is desired and cutoff occurs

when $\Delta_3 = 0$

If σ_{S1}, σ_{S2}, σ_{S3}, and $\sigma_{S4} \neq 0$, four-stage operation is desired and cutoff occurs

when $\Delta_4 = 0$

Unless other logic applies, the run ends when cutoff occurs. When the criterion for leaving a stage has been satisfied, that stage is not re-entered during the trajectory.

c. Staging Values

The values of specified values at the termination of the k-th stage are made available to the hunting procedure control by specifying input τ_{Bj} and σ_{Bj} ($j = 1, 2, ..., 5$). The σ_{Bj} specifies the achieved value of the parameter designated by code input at the termination of the input τ_{Bj} stage. Certain selected trajectory parameters, i.e., V_f, h, γ_1, W, S_f, L_F, L_D, L_g, L_v, Δv, a_{Xb}, q, I_v, h_p, h_a, t_f, γ_{1If}, and V_{If} for stages 1, 2, 3 and 4, are also available to the hunting procedure.

d. Trajectory Halts

Certain computations and parameter values result in program logic terminating the run. The error which causes the abort and the location of the error are identified by the printout. The printout format is discussed in the final section of this volume.

2. Compute Time

The following are computed for all modes:

1. The main and complementary thrust-weight switching times.
2. The times when the trajectory begins and each stage ends.
3. The times each segment of flight in the altitude control table and mode region ends.
4. The time when the special print is satisfied.
5. The times when weight jettison occur.
6. The times when TVC gains change.
7. The time when the main print occur.
8. The time when the auxiliary print occur.
9. If the wind table altitude multiplier K_h is nonzero at the time when the input wind altitude occurs.
10. The time each segment of target dynamical switch conditions occurs.
11. Time of lift off.
12. The time each segment of the TMC table ends.
13. The time each segment of the roll control values tables ends.

a. Variable Compute Interval with Accuracy Check

The numerical integration method utilized in the trajectory program is a fifth-order Runge-Kitta-Merson with a fourth-order/fifth-order accuracy check routine. Generally this routine functions as follows.

Let y be the vector to be integrated (σ code 5701 thru 5750) and let y' by the derivative vector (5751 thru 5800). The differential equation to solve is

$$y' = f\ (y,\ t)$$

Following standard Runge-Kutta notation, define

$$k_1 = hf\ [y_n,\ t_n]\ /3$$
$$k_2 = hf\ [y_n + k_1,\ \ t_n + h/3]\ /3$$
$$k_3 = hf\ [y_n\ + (k_1 + k_2)/2,\ t_n + h/3]\ /3$$
$$k_4 = hf\ [y_n + 3k_1/8\ + 9k_3/8\ .\ t_n + h/2]\ \ /3$$
$$k_5 - hf\ [y_n + 3k_1/2 - 9k_3/2 + 6k_4,\ t_n + h]\ /3$$

Then two independent estimates of y are given by

$$y_{n+1} = y_{n+} 3k_1/2 - 9k_3/2 + 6\ k_4 + 0\ (h^4)$$

and

$$y_{n+1} = + (k_1 + 4\,k_4 + k_5)/2 + 0\ \ (h^5)$$

The second of these is used in this program. An estimate of the error is the difference between the two, or

$$e = \left| k_1 - 9\,k_3/2 + 4k_4 - k_{5/2} \right|$$

The error index is calculated as

$$E_i = \frac{e_i}{R_i + R_i \left| Y_i \right|} \qquad i = 1, 2, \cdots 50$$

where the R_i are error tolerances, described below.

Code	Parameter	R	Code	Parameter	R
5701	XDEE	.01	5726	VN	.001
5702	YDEE	.01	5727	VC	.001
5703	ZDEE	.01	5728	GAMT	.001
5704	XEE	.001	5729	ZETAT	.001
5705	YEE	.001	5730	HT	.001
5706	ZEE	.001	5731	ST	.001
5707	QB	.1	5732	STC	.001
5708	RB	.1	5733	DB	.01
5709	PB	.1	5734	PC	.1
5710	THETAB	.01	5735	AT	.1
5711	PSIB	.01	5736	WPR	.01
5712	PHIB	.01	5737	IMP	.01
5713	DELDP	.01	5738	IV	.001
5714	DELDY	.01	5739	IP	.1
5715	DELDR	.01	5740	IY	.1
5716	DELP	.001	5741	IAT	1.0
5717	DELY	.001	5742	Open	
5718	DELR	.001	5743	LF	1.0
5719	FR	.01	5744	DELTAV	1.0
5720	WR	.1	5745	IR	1.0
5721	FRC	.1	5746	LD	1.0
5722	THETAM	.01	5747	LGG	.1
5723	PSIM	.01	5748	HE	10.0
5724	PHIM	.01	5749	Open	
5725	VT	.001	5750	Open	

The error tolerances may be modified by input. In input locations L280 thru L303, up to 12 pairs of index/multiplier may be entered. For any index in the range of 1 thru 50, the corresponding values of Ri are multiplied by the input multiplier A'r.

The maximum Ei (EMAX) is employed to control the integration step size:

If EMAX $\geq$ 1.0, halve the step size

If EMAX $\leq$ 0.031, triple the step size

5. Specified Maximum Compute Interval

If the compute interval is input in the compute interval table, the interval is limited as follows:

Interval	Maximum Compute Interval
$t \leq t_{c1}$	Δt_{c1}
$t_{c1} < t \leq t_{cz}$	Δt_{cz}
• • •	•
$t_{c7} < t \leq t_{c8}$	Δt_{c8}

t_{cj} and Δt_{cj} ($j = 1, 2, \ldots, 8$) are input and t is the instantaneous time.

If $t > t_{cJ}$ where the subscript J refers to the last input point in the compute interval table, the compute interval is unlimited.

c. **Discontinuity Print**

The discontinuity print will occur when the input σ_{Dj} specifies the first "L" number of an input table row.

TABLE	CODE PARAMETER
Jettison Weight	W_{JTj} (j = 1, 2, ..., 8)
Special Print	σ_{tj} (j = 1, 2, ..., 8)
Attitude Control	$\dot{\ }_{yj}$ (j = 1, 2, ..., 16)
Wind Profile	h_j (j = 1, 2, ..., 30)
Mode Control	M_{yj} (j = 1, 2, ..., 10)
Main Thrust-Weight	t'_j (j = 2, 3, ..., 25) Back side
Complementary Thrust-Weight	t'_{Cj} (j = 2, 3, ..., 25) Back side
Gains	K_{DPj} (j = 1, or 2)
Target dynamical condition	t_{Tj} (j = 1, 2, ..., 10)
TMC	Fyj (j = 1, 2, ..., 7)
Roll Control	K_{DRj} (j = 1, or 2)

d. **Variable Break-up Tolerances**

Compute internal break ups are used to exactly compute discontinuities for the following conditions:

1. Attitude Control Table
2. Gains (TVC) Table
3. Weight Jettison Table
4. Special Print Table
5. Wind Profile Table
6. Mode Control Table
7. Staging
8. Target Dynamical Conditions Table
9. Main Thrust-Weight Table
10. Complementary Thrust-Weight Table
11. Lift off
12. TMC Table
13. Target Dynamical Condition Table
14. Roll Control Table

A break-up will occur whenever either delta time or delta x becomes less than the corresponding tolerance, DT_{min} or DX_{min}, where delta x is the difference between a break-up variable and its target value, and

$$DT_{min} = A_t + B_t \ TIME$$

$$DX_{min} = A_x + B_x \quad |X_{target}|$$

The default values, which are used unless over ridden by positive input values, are

A_t = 2.E-4

B_t = 1.E-5

A_x = 1.E-9

B_x = 1.E-5

Experience has shown these values to be reasonable most of the time. There is probably little danger in using values that are too small; the principal effect would be an increase in running time although it is possible to cause the integration routine to fail (with appropriate messages and a dump) if the tolerances are too tight. Looser tolerances allow a more random flight, and could cause the hunting procedures to fail.

Any given break-up point will depend almost entirely on only one of the four break-up tolerance parameters. In general, the following rules apply:

A. If the break-up variable is changing slowly with time, DX_{min} will control

   1. if $|X_{target}|$ is small, A_x will control

   2. if $|X_{target}|$ is large, B_x will control

B. If the break-up parameter is changing rapidly, DT_{min} will control

1. if TIME is small, A_t will control

2. if TIME is large, B_t will control

3. Print

Trajectory print times are discussed. Two blocks of printout are featured, i.e., the main and auxiliary prints. The main print contains printlines A through [illegible] and succeeding printlines that are given when special criteria are met. The auxiliary print is given in two lines which are labeled Printline Z and ZZ. Print times of the main and auxiliary prints are controlled by the logic given below.

a. Main Print Interval

Printout of the main print occurs when the following times are achieved:

Interval	Print Time
$t \leqq t_{p1}$	$t = n \Delta t_{p1}$
$t_{p1} < t \leqq t_{p2}$	$t = t_{p1} + n \Delta t_{p2}$
$t_{p7} < t \leqq t_{p8}$	$t = t_{p7} + n \Delta t_{p8}$

where t_{pj} and Δt_{pj} ($j = 1, 2, \ldots, 8$) are input.

If $\Delta t_{pj} < 0$, every compute point is output.

b. Special Print

The main print is given when the logic given below is satisfied.

With σ_{tj} ($j = 1, 2, \ldots, 8$) the achieved value of a parameter designated by code input and K_{tj} the input value of that parameter, then, the main print is printed when $\delta_{tj} = K_{tj}$.

c. Max Print

The maximum print and hunting procedure control quantities are specified by input T_{mj} and σ_{mj} $(j = 1, 2, ..., 5)$. The $|\sigma_{mj}|$ specifies the maximum or minimum achieved value of the parameter designated by the code input. Input $T_{mj} < 0$ specifies a minimum value; input $\sigma_{mj} = 0$ specifies ignore the max print logic for the j-th input; input $\sigma_{mj} > 0$ specifies a maximum value; and input $\sigma_{mj} < 0$ specifies the maximum absolute value regardless of the sign of T_{mj}.

The flag $|T_{mj}|$ directs the maximum control region. If $|T_{mj}| = 0$, the maximum print logic for the σ_{mj} parameter will be computed for each stage and the max print will appear after the termination of that stage. If $|T_{mj}| = k$ $(k = 1, 2, 3, \text{ or } 4)$, the max print logic for the σ_{mj} parameter will be computed for the k-th stage and the max print will appear after the termination of the k-th stage only. If $|T_{mj}| = 5$, the max print logic for the σ_{mj} parameter will be computed for the entire flight duration and the max print will appear after the termination of the last stage. When more than one parameter is designated, the prints occur in the order of the j-th index.

d. Auxiliary Print

The auxiliary print consists of time and eleven parameters designated by the code input σ_{Pj} $(j = 1, 2, ..., 11)$. The print will occur if the input, σ_{P1} is non-zero and when

Interval	Print Time
$t \leq t_{P1}$	$t = n \Delta t_{P1}$
. . .	. . ..

Interval	Print Time
$t_{P1} < t \leq t_{P2}$	$t = t_{P1} + n \, \Delta t_{P2}$
$t_{P7} < t \leq t_{P8}$	$t = t_{P7} + n \, \Delta t_{P8}$

where t_{Pj} and Δt_{Pj} $(j = 1, 2, \ldots, 8)$ are input.

L. HUNTING PROCEDURE

Two methods of isolating variables and obtaining extremals are provided in the program. The first method incorporates an incremental hunting procedure (P1) which isolates and maximizes the dependent variable by varying the independent parameter. The second method incorporates a simultaneous hunting procedure (P2) which isolates and obtains extremals (maximum or minimum values) by varying up to seven parameters simultaneously. Either or both methods may be specified. If both methods are specified simultaneously, procedure 1 is used as a subroutine within procedure 2. (Experience has demonstrate that some problems will converge more rapidly if range is maximized by P1 while other parameters are simultaneously hunted using P2. Experience with the program will expose other areas where joint utilization of the hunting procedure is advantageous.)

The hunting procedures vary independent variables which are initially specified by input to obtain extremal and isolation solutions for dependent variables evaluated at cutoff.

Another option delineated in this section is one that allows the coefficients utilized in the steering equations to be automatically computed.

1. Incremental Hunting Procedure (P1)

The hunting procedure logic and equations for this option apply if the input P1 is non-zero.

The hunting procedure makes it possible to obtain a trajectory in which a specified quantity "a" (the dependent variable), can be found (isolated) or may be maximized with a single controller independent variable. The value of X (the independent variable) and the resulting value of "a" (the dependent variable) are designated by inputing the proper sigma code for σ_X and σ_a, respectively.

Isolation or maximization is an iterative process involving trajectory computations and hunting procedure logic. Dependent values for the first trajectory are computed with the input guess of the independent variable X_i. Then the value of X is computed from their previous values and the input increment ΔX until the isolation or maximization region is bracketed. Thereafter, the independent variables are found by interpolation using quadratic equations.

If ΔX is input zero, the error message "DELTA X INPUT ZERO" is output and the run is terminated. If Δa is input zero, the error message "INTERNAL TOLERANCE INPUT ZERO" is output and the run is terminated. If the quadratic fit is singular, the error message "SINGULARITY IN QUADRIT FIT" is output and the run is terminated. If for the first two iterations, the dependent quantity does not vary, the error message "DEPENDENT PARAMETER NOT VARYING" is output and the run is terminated.

a. Isolation

The isolation procedure in this section will apply if $K_a = 0$.

Desired value of the dependent variable is the input a_f. The achieved dependent values resulting from trajectories computed with the dependent values $\bar{X}_j$ (j = 1, 2, 3, or 4) are $\bar{a}_j$.

(1) Isolation Incrementing Routine

The dependent variable $\bar{a}_j$ is computed using the input starting value X_i then this point set $(\bar{a}_1, \bar{X}_1)$ is transfered to the low point set $(\bar{a}_3, \bar{X}_3)$. The center point set $(\bar{a}_2, \bar{X}_2)$ is computed using $\bar{X}_2 = \bar{X}_3 + \Delta X$.

If the desired dependent parameter is not bracketed but progressing in the correction direction, i.e., $\bar{a}_3 < \bar{a}_2 < a_f$ or $\bar{a}_3 > \bar{a}_2 > a_f$, then $\bar{X}_2$ is determined by incrementing, i.e., $X_1 = X_2 + \Delta X$ and the

$\bar{a}_1$ is evaluated by running a trajectory. The incremental process is continued until the desired dependent parameter is bracketed. Then the three data point sets are arranged such that the dependent variables are in descending order then sent to the quadratic interpolation routine. If after the incremental direction is established, the trend of the dependent variable reverse, i.e., diverges from the desired value a_f, then the error message "IMPOSSIBLE REGION EXISTS" is output and the run terminated.

If the desired dependent parameter is not bracketed but progressing in the wrong direction, i.e., $\bar{a}_2 < \bar{a}_3 < a_f$ or $\bar{a}_2 > \bar{a}_3 > a_f$, the second and third data sets are interchanged, the incremental direction changed, i.e., $\Delta X = - \Delta X$, and then the incremental process as described above is utilized.

(2) Isolation Linear Interpolation Routing

If the desired dependent parameter is bracketed then the two data sets are arranged in the first and third position such that $\bar{a}_1 > \bar{a}_3$, then $\bar{X}_2$ is determined by linear interpolation using a_f and then a_2 is evaluated by running a trajectory.

If the predicted dependent value is within the bracketed zone, then the three data points sets are such that the dependent variable are descending order and then sent to the quadratic interpolation routine. If not, the one point of the original point set and the newly predicted point which span the desired dependent variable are singled out and the process is repeated.

(3) Isolation Quadratic Interpolation Routine

When the dependent variable span the desired value, and are arranged in descending order, i.e., $\bar{a}_1 > a_f > \bar{a}_3$ and $\bar{a}_1 < \bar{a}_2 < \bar{a}_3$, then the predicted independent variable X_4 is determined by quadratic interpolation using a_f and the predicted dependent variable $\bar{a}_4$ is evaluated by running a trajectory.

If the predicted dependent variable $\bar{a}_4$ is within the bracketed zone, then the bracket point sets and predicted point set are arranged such that the dependent variables are descending order, i.e., $\bar{a}_1 > \bar{a}_2 > \bar{a}_3$, and then transfer back to the quadratic interpolation routine.

If the predicted dependent variable $\bar{a}_4$ is outside the bracketed zone, then the predicted point set and the one point set of the two bracketed point set which span the desired dependent variable are singled out and transfer back to the linear interpolate routine.

(4) Isolation Convergence Criteria

If at any place in the logic the value of the dependent variable out of the trajectory routine is within the input Δ_a distance of the desired value a_f, then the hunting procedure 1 isolation has converged.

b. Maximization-Minimization

The maximization procedure in this section will apply if $K_a \neq 0$.

The achieved dependent values resulting from trajectories computed with the dependent values, $\bar{X}_j$ 9 j = 1, 2, 3, or 4) are a_j. If the input flag P1 is greater than zero, the function corresponding to σ_a is to be maximized. If the input flag P1 is less than zero, the function corresponding to σ_a is to be minimized.

(1) Maximization-Minimization Incrementing Routine

The dependent variable $\bar{a}_1$ is computed using the input starting value $\bar{X}_1$, then this point set $(\bar{a}_1, \bar{X}_1)$ is transferred to the low point set $(\bar{a}_3, \bar{X}_3)$. The center point set $(\bar{a}_2, \bar{X}_2)$ is computed using $\bar{X}_2 = \bar{X}_3 + \Delta X$, where ΔX is the input increment.

If the dependent variables are increasing for maximization or decreasing for minimization, i.e., $\bar{a}_2 > \bar{a}_3$ and $pl > 0$ or $a_2 < 0$, $\bar{X}_1$ is determined by incrementing, i.e., $\bar{X}_1 = \bar{X}_2 + \Delta X$, and $\bar{a}_1$ is evaluated by running a trajectory. This incremental process is continued in a hill climber fashion until the peak has been crossed; that is, the dependent variable has become less than for maximization or greater than for minimization, the previous value, i.e., $\bar{a}_1 < \bar{a}_2$ and $Pl > 0$ or $\bar{a}_1 > \bar{a}_2$ and $Pl < 0$.

These three point sets which form a peak or a valley are then sent to the quadratic routine. If the dependent parameter is progressing in the wrong direction; that is, decreasing for maximization or increasing for minization, i.e., $\bar{a}_2 < \bar{a}_3$ and $Pl > 0$ or $\bar{a}_2 > \bar{a}_3$ and $Pl < 0$, the second and third data sets are interchanged, the incremental direction changed, i.e., $\Delta X = -\Delta X$, and then the incremental process as described above utilized.

2) Maximization-Minimization Quadratic Routine and Convergence Criteria

When the dependent variables describe a peak for maximization or a valley for minimization, i.e., $\bar{a}_1 < \bar{a}_2$ and $\bar{a}_3 < \bar{a}_2$ and $Pl > 0$ or $\bar{a}_1 > \bar{a}_2$ and $a_3 > a_2$ and $Pl < 0$, then the predicted independent variable X_4 is determined at the inflection point by a quadratic equation, the corresponding curve fit dependent variable set to a_f, and the predicted dependent variable a_4 evaluated by running a trajectory. If the predicted dependent value is within the input convergence tolerance, distance of the curve fit dependent variable a_f, then the [illegible] procedure in maximization-minimization has converged.

If the predicted dependent variable $\bar{a}_4$ is progressing towards the optimum value; that is, the predicted dependent variable is greater than for maximization or less than for minimization, i.e.,

$\bar{a}_4 > \bar{a}_2$ and P1 > 0 or $\bar{a}_4 < \bar{a}_2$ and P1 < 0, then the points sets are arranged such that the predicted point set $(\bar{a}_4, \bar{X}_4)$ are in the center data set $(\bar{a}_2, \bar{X}_2)$ and the two independent variables; X, closest to but one greater than and one less than the predicted independent variable X are transfer to the high $(\bar{a}_1, \bar{X}_1)$ and low $(\bar{a}_3, \bar{X}_3)$ point sets where high and low correspond to the value of the independent variable X. These three point sets which form a peak or valley are then sent to the quadratic routine.

If the predicted variable $\bar{a}_4$ is not progressing towards the local optimum; that is, the predicted dependent variable is less than for maximization or greater than for minimization, the center set dependent variable, $\bar{a}_2$, i.e., $\bar{a}_4 < \bar{a}_2$ and P1 > 0 or $\bar{a}_4 > \bar{a}_2$ and P1 > 0, then another point set in which the dependent variable is half way between the two independent variables not spanning the predicted independent variable $\bar{X}_a$, is calculated by running another trajectory. Three point sets of the five inclusive of the center point set, $(\bar{a}_2, \bar{X}_2)$, in which the dependent variables form a peak for maximization or a valley for minimization with the independent values in numerical order are selected and then arranged such that the independent variable, X, are in descending order. These three data sets are then sent back to the quadratic routine.

2. Simultaneous Hunting Procedure (P2)

The hunting procedure logic and equations for this option apply if the input P2 is non-zero.

The logic for this section is contained in the OPTMZ subroutine writeup*, **which describes in detail a generalized hunting procedure. This hunting procedure uses one of three empirical models whose coefficients are estimated by least square fit

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*Brimhall, R. K., "Design Optimization Using Model Estimation Programming", Thiokol Chemical Corporation No. 1166-13,63 dated 23 January 1957

**Olson, R. A., "Determining Extremals and/or Constraints Using the Method of Lagrangers Multipliers", Thiokol Chemical Corporation Subroutine 9117 dated 4 November 1965.

The input code word P2 determines which model will be used.

P2 =
+1 Linear Model Isolation Only
+2 Complete Quadratic--Maximization
-2 Complete Quadratic--Minimization
+3 Incomplete Quadratic--Maximization
-3 Incomplete Quadratic--Minimization

These three models will now be discussed to facilitate trajectory program input.

a) Linear Model

The logic in this section applies if P2 equals to one. The linear model is only used to isolate input dependent parameters. The linear empirical model consists of a system of polynomial equations equating the independent variables $x_1, y_2, \ldots, x_n$ to each of the dependent variables $y_1, y_2, \ldots, y_n$.

$$\begin{array}{lllllll}
y_1 = a_{10} & + a_{11}x_1 & + a_{12}x_2 & + a_{13}x_3 & + & \ldots & + a_{1n}x_n \\
y_2 = a_{20} & + a_{21}x_1 & + a_{22}x_2 & + a_{23}x_3 & + & \ldots & + a_{2n}x_n \\
y_3 = a_{30} & + a_{31}x_1 & + a_{32}x_2 & + a_{33}x_3 & + & \ldots & + a_{3n}x_n \\
\cdot & \cdot & \cdot & \cdot & \cdot & \ldots & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \ldots & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \ldots & \cdot \\
y_n = a_{n0} & + a_{n1}x_1 & + a_{n2}x_2 & + a_{n3}x_3 & + & \ldots & + a_{nn}x_n
\end{array} \qquad (1)$$

where n may vary from one to seven.

The generalized hunting procedure logic systematically varies the values of the independent variables (x's) from which the trajectory program evaluates the corresponding dependent variables (y's). The resulting dependent and independent variables are submitted to a multiple regression scheme to determine the coefficients (a's) of the above equation by at least square technique.

The desired value of the dependent variables (y_U's) specified by input are equated to their polynomial equations and this system of linear equations is solved for the independent variables (x's). These predicted independent variables (x's) are input in the trajectory program to evaluate the dependent variables (y's). Because the polynomial equations do not exactly match the true functions, the model determined dependent variables (y's) will yield some error; this resulting error between the predicted and the achieved is further reduced to an acceptable increment specified by input by an incremented process involving the differential of the model system of equations.

The required input for this option is σ_{x1}, σ_{x2}, ..., σ_{xn} to identify each independent variable; x_{i1}, x_{i2}, ..., x_{in} to specify the initial value for each of the corresponding independent variables (x's); Δx_{i1}, Δx_{i2}, ..., Δx_{in} to establish the model array size and also to establish the largest allowable change in the independent variables (y's) during an iteration; σ_{y1}, σ_{y2}, ..., σ_{yn} to identify the dependent variable being isolated; y_{U1}, y_{U2}, ..., y_{Un} to specify the desired value of the dependent variables; ϵ_{c1}, ϵ_{c2}, ..., ϵ_{cn} to stipulate a tolerance of convergence upon the dependent variables (y's); x_{L1}, x_{L2}, ..., x_{Ln} to establish a lower restraint limit on the independent variables (x's); and x_{U1}, x_{U2}, ..., x_{Un} to establish an upper restraint limit on the independent variables (x's).

An example of utilizing the linear model isolation simultaneous hunting procedure is as follows:

Problem -- 100 nm circular orbital injection

Model P2 = 1

Independent variables

$x_1 = Q_2$ (Pitch rate at kickover)

$x_2 = Q_4$ (Terminal pitch rate)

$x_3 = W_{PL}$ (Payload weight)

Set

$\sigma_{x1} = 320$ $x_{i1} = 10$ $\Delta x_{i1} = 1$ $x_{L1} = -15$ $x_{U1} = -5$

$\sigma_{x2} = 333$ $x_{i2} = -0.05$ $\Delta x_{i2} = 0.01$ $x_{L2} = -0.15$ $x_{U2} = 0.05$

$\sigma_{x3} = 20$ $x_{i3} = 40{,}000$ $\Delta x_{i3} = 5{,}000$ $x_{L3} = 20{,}000$ $x_{U3} = 60{,}000$

Dependent variables

$y_1 = V_I$ (circular orbital injection inertial velocity)

$y_2 = \gamma_{1_I}$ (circular orbital injection inertial flight path angle)

$y_3 = h$ (circular orbital altitude)

$\sigma_{y1} = 5029$ $y_{U1} = 25{,}570$ $\epsilon_{c1} = 5$

$\sigma_{x2} = 5255$ $y_{U2} = 0$ $\epsilon_{c2} = 0.01$

$\sigma_{y3} = 5501$ $y_{U3} = 607{,}610$ $\epsilon_{c3} = 100$

The generalized hunting routine uses the technique of Lagrange to maximize or minimize the function f subject to the constraint functions $g_1, g_2, \ldots, g_m$. Thus

$$G = f(x_1, x_2, \ldots, x_n) + \sum_{i=1}^{i=m} \lambda_i \left[g_i\,(x_1, x_2, \ldots, x_n) - L_i\right]$$

where L_i is the value of the constraint, λ_i is the Lagrangian multiplier and G is the Lagrangian function. The Lagrangian function G is differentiated with respect to each x and each λ, then the partial derivatives of G are each equated to zero, and the system of $n + m$ equations are solved for the $n + m$ unknowns; thus yielding the required values of $x_1, x_2, \ldots, x_n$ which maximizes or minimizes the Lagrangian function G.

The generalized hunting routine is designed so that only maximums, as specified by P2 = +2, or minimums, as specified by P2 = -2, will be obtained. There is no guarantee that the solution obtained will not be a local maxima or minima. Saddle point solutions will not be obtained.

The functions f and g are represented by a quadratic response surface of the form:

function f

$$\begin{aligned} z &= a_{00} \\ &+ a_{10}x_1 + a_{20}x_1^2 \\ &+ a_{30}x_2 + a_{40}x_1x_2 + a_{50}x_2^2 \\ &+ a_{60}x_3 + a_{70}x_1x_3 + a_{80}x_2x_3 + a_{90}x_3^2 \end{aligned}$$

function g

$$\begin{aligned} y_i &= a_{0i} \\ &+ a_{1i}x_1 + a_{2i}x_1^2 \\ &+ a_{3i}x_2 + a_{4i}x_1x_2 + a_{5i}x_2^2 \\ &+ a_{6i}x_3 + a_{7i}x_1x_3 + a_{8i}x_2x_3 + a_{9i}x_3^2 \\ &+ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned}$$

where i = 1, 2, ..., m

The generalized hunt procedure will determine the coefficients (a's) in the same manner as outlined for the linear model.

b. Complete Quadratic Response Surface

The logic in this section applies if P2 equals positive or negative two. The complete quadratic response surface model is used (1) to maximize or minimize a function subject to constraint or (2) to isolate a particular solution from a family of maximized or minimized functions subject to constraints.

If the input f_D equals zero, the generalized hunting procedure will maximize or minimize the primary dependent variables called z where $z = f(x_1, x_2, \ldots, x_n)$, subject to constraints in m* secondary dependent variables called $y_1, y_2, \ldots, y_m$ where $y_i = g_i (x_1, x_2, \ldots, x_n)$. The number of constraint functions may vary from zero to n where n is the number of independent variables (x's) and n cannot exceed seven.

If the input f_D equals non-zero, the generalized hunting procedure will isolate a particular solution from a family of maximized or minimized functions subject to constraints. This family relates the independent variable x_1 as function of the maximized or minimized dependent variable z with respect to the independent variables $x_2, x_3, \ldots, x_n$ subject to constraints in m dependent variables $y_1, y_2. \ldots, y_m$ and m cannot exceed n-1. Isolation involves selecting the independent variables x_i which yields the maximized or minimized dependent variable z specified by the input value f_D.

The required input for this solution is σ_z, identify the dependent variable for which the extremal solution is desired; $\sigma_{x1}, \sigma_{x2}, \ldots, \sigma_{xn}$, to identify the independent variables; $x_{i1}, x_{i2}, \ldots, x_{in}$, to specify the initial values of the independent variables; $\Delta x_{i1}, \Delta x_{i2}, \ldots, \Delta x_{in}$, to specify the model array size and to specify the maximum change of an independent variable during a given iteration; $\sigma_{y1}, \sigma_{y2}, \ldots, \sigma_{ym}$, to identify the dependent variables being constrained; $y_{L1}, y_{L2}, \ldots, y_{Lm}$, to establish the lower constraint boundary; $y_{U1}, y_{U2}, \ldots, y_{Um}$, to establish the upper constraint boundary; $\epsilon_{c1}, \epsilon_{c2}, \ldots, \epsilon_{cm}$, to establish tolerances or the constraint solution; x_{L1}, x_{L2}, ..., x_{Ln}, to establish a lower restraining limit on the independent variables; and x_{U1}, x_{U2}, ..., x_{Un}, to establish an upper restraining limit on the independent variable.

An example of utilizing the complete quadratic response surface model to maximize a function subject to constraints is as follows:

Problem -- maximize range with fixed payload subject to
the constraint that the maximum dynamic pressure
be less than 1000 psf

Model P2 = +2

Dependent Variables

$x_1 = Q_2$ (Pitch rate at kickover)

$x_2 = Q_4$ (Terminal pitch rate)

Set

$\sigma_{x1} = 319$ $x_{i1} = -12$ $\Delta x_{i1} = 2$ $x_{L1} = -20$ $x_{U1} = -4$

$\sigma_{x2} = 331$ $x_{i2} = -1$ $\Delta x_{i2} = 0.1$ $x_{L2} = -10$ $x_{U2} = 2$

Independent variables

$\sigma_z = S_f$ (range)

σ_{y1} = qmax (maximum dynamic pressure)

Set

$\sigma_z = 5013$ $y_{L1} = 0$ $y_{U1} = 1000$ $\epsilon_z = 0.5$

$\sigma_{y1} = 271$ $\epsilon_{c1} = 1$

$\sigma_{m1} = 5037$ $T_m = 5$

c. Incomplete Quadratic Response Surface

This model is used if P2 = +3 to specify maximization, or P2 = -3 to specify a minimization.

This model is used in identically the same manner as the complete quadratic response surface discussed in Section L.2.b. The relative merits between the two models are discussed in section L.2.c. The model used to estimate the dependent variables z and y_i is of the form:

function f

$$z = a_{00} + a_{10}x_1 + a_{20}x_1^2 + a_{30}x_2 + a_{40}x_2^2 + \ldots + a_{(2n-1)_0}x_n + a_{(2n)0}x_n^2$$

function g

$$y_i = a_{0i} + a_{1i}x_1 + a_{2i}x_1^2 + a_{3i}x_2 + a_{4i}x_2^2 + \ldots + a_{(2n-1)i}x_n + a_{(2n)}$$

where $i = 1, 2, \ldots, m^*$.

The required input is the same as that described in Section L.2.b.

An example of utilizing the incomplete quadratic response surface model to isolate a particular solution from a family of maximized functions subject to constraints is as follows:

Problem -- determine the payload consistant with a maximum range trajectory for a range of 5500 nm subject to the constraint that the maximum dynamic pressure be less than 1000 psf.

Model P2 = +3

Independent variables

$x_1 = W_{PL}$ (payload weight)
$x_2 = Q_2$ (pitch rate at kickover)
$x_3 = Q_4$ (terminal pitch rate)

Set

$\sigma_{x1} = 20$ $x_{i1} = 10{,}000$ $\Delta x_{i1} = 1{,}000$ $x_{L1} = 0$ $x_{U1} = 20{,}000$
$\sigma_{x2} = 320$ $x_{i2} = -8$ $\Delta x_{i2} = 1$ $x_{L2} = -12$ $x_{U2} = 3$
$\sigma_{x3} = 333$ $_{i3} = -0.5$ $\Delta x_{i3} = 0.05$ $x_{L3} = -5$ $x_{U3} = 1$

Dependent variables

$z = S_f$ (range)
y_1 = qmax (maximum dynamic pressure)

Set

$\sigma_z = 5024$ $y_{L1} = 0$ $y_{U1} = 1000$ $\epsilon_z = 0.5$
$\sigma_{y1} = 261$ $\epsilon_{c1} = 1$
$\sigma_{m1} = 5070$ $T_m = 5$ $f_D = 5500$

d. Generalized Hunt Characteristics

The data described in this section is applicable to that described for each model.

In the extremal problem the variance of the model is evaluated at the apparent solution point. If the input parameter ϵ_m is nonzero and if the probably is greater than 0.05 that the model error is greater than ϵ_m, the entire maximization (or minimization) will be repeated.

The independent variables may be transformed to aid the convergence of the model. A transform code input is available for each independent variable and is input $T_{T1}, T_{T2}, \ldots, T_{Tn}$. A constant of transformation $a_{T1}, a_{T2}, \ldots, a_{Tn}$ is input for each transformation. The available transformations are:

$T_T = 0$ $\quad \eta = x + a_T$
$T_T = 1$ $\quad \eta = |x|^{a_T}(x)/|x|$
$T_T = 2$ $\quad \eta = \ln a_T x$
$T_T = 3$ $\quad \eta = a_T/x$
$T_T = 4$ $\quad \eta = e^{a_T x}$

where η is the transformed independent variable.

The relative merits of using the complete quadratic response surface as opposed to the incomplete quadratic response surface is in running time. If ϵ_m is non-zero and if it is the same value for either method, the same answer should be eventually obtained. Advantage to using the incomplete quadratic response surface model will probably not materialize until n = 6 or 7. As a guide to choice and an aid to estimation of running time, the following table describes the number of trajectories necessary to estimate the coefficients required to make the first optimal prediction.

n	Complete Quadratic		Incomplete Quadratic	
	Trajectories Simulated	Coefficients Estimated	Trajectories Simulated	Coefficients Estimated
1	3	3	5	3
2	6	6	9	5
3	15	10	13	7
4	25	15	17	9
5	27	21	21	11
6	45	28	25	13
7	79	36	29	15

The complete quadratic response surface model will always make better predictions for any given iteration; therefore, the incomplete quadratic response surface model should require more iterations. Experience will dictate the most efficient method for solving the hunting procedure problem.

The maximum number of hunt predictions beyond the initial array must be specified in n_{t2}. If n_{t2} is input, a negative number, the hunt will restart after n_{t2}-th iterations.

3. Special Routines

a. Acquisition of Coefficients for Steering Equations

The coefficients for the pitch steering equations are obtained by a least square curve fit of flight path parameters. This option automates the acquisition of these coefficients. The portion of the trajectory to be fit will be specified by sigma codes contained in the switching table.

If σ_{g1k} is zero, this option is by passed for the k-th stage. If $\sigma_{g1k} > 0$, the data locations containing the summation matrix and the observation counter n_g will be initially set to zero at the beginning of each stage. Over the portion of the trajectory which is to be fit, a summation of data will be generated in the following form for each integration step. Data will be processed over the open interval $K_{g1k} \leq \sigma_{g1k}$ to $\sigma_{g2k} \leq K_{g2k}$.

$$n_g = \Sigma\, 1.0$$

let

$$u_1 = \dot{Z}_{gg}$$
$$u_2 = \dot{X}_{gg}$$
$$u_3 = \dot{X}^2_{gg}$$
$$u_4 = \dot{X}^3_{gg}$$
$$u_5 = \theta_m \text{ (radians)}$$

Then the summation matrix appears as follows:

$$\begin{array}{ccccc}
\Sigma u_1 & \Sigma u_2 & \Sigma u_3 & \Sigma u_4 & \Sigma u_5 \\
\Sigma (u_1)^2 & \Sigma u_1 u_2 & \Sigma u_1 u_3 & \Sigma u_1 u_4 & \Sigma u_1 u_5 \\
\Sigma u_2 u_1 & \Sigma\, (u_2)^2 & \Sigma u_2 u_3 & \Sigma u_2 u_4 & \Sigma u_2 u_5 \\
\Sigma u_3 u_1 & \Sigma u_3 u_2 & \Sigma (u_3)^2 & \Sigma u_3 u_4 & \Sigma u_3 u_5 \\
\Sigma u_4 u_1 & \Sigma u_4 u_2 & \Sigma u_4 u_3 & \Sigma (u_4)^2 & \Sigma u_4 u_5
\end{array}$$

3. Special Routines

a. Acquisition of Coefficients for Steering Equations

The coefficients for the pitch steering equations are obtained by a least square curve fit of flight path parameters. This option automates the acquisition of these coefficients. The portion of the trajectory to be fit will be specified by sigma codes contained in the switching table.

If σ_{g1k} is zero, this option is by passed for the k-th stage. If $\sigma_{g1k} > 0$, the data locations containing the summation matrix and the observation counter n_g will be initially set to zero at the beginning of each stage. Over the portion of the trajectory which is to be fit, a summation of data will be generated in the following form for each integration step. Data will be processed over the open interval $K_{g1k} \leq \sigma_{g1k}$ to $\sigma_{g2k} \leq K_{g2k}$.

$$n_g = \Sigma 1.0$$

let

$$u_1 = \dot{Z}_{gg}$$
$$u_2 = \dot{X}_{gg}$$
$$u_3 = \dot{X}^2_{gg}$$
$$u_4 = \dot{X}^3_{gg}$$
$$u_5 = \theta_m \text{ (radians)}$$

Then the summation matrix appears as follows:

$$\begin{array}{lllll}
\Sigma u_1 & \Sigma u_2 & \Sigma u_3 & \Sigma u_4 & \Sigma u_5 \\
\Sigma (u_1)^2 & \Sigma u_1 u_2 & \Sigma u_1 u_3 & \Sigma u_1 u_4 & \Sigma u_1 u_5 \\
\Sigma u_2 u_1 & \Sigma\ (u_2)^2 & \Sigma u_2 u_3 & \Sigma u_2 u_4 & \Sigma u_2 u_5 \\
\Sigma u_3 u_1 & \Sigma u_3 u_2 & \Sigma (u_3)^2 & \Sigma u_3 u_4 & \Sigma u_3 u_5 \\
\Sigma u_4 u_1 & \Sigma u_4 u_2 & \Sigma u_4 u_3 & \Sigma (u_4)^2 & \Sigma u_4 u_5
\end{array}$$

At the end of staging, after the printing of "TERMINATION OF STAGE K" before the printing of the note "MAXIMUM VALUES" the data generated should be processed.

If $\sigma_{glk} > 0$ and $n_g < 5$, a note should be printed as follows, "STEERING COEFFICIENTS NOT CALCULATED." The trajectory should continue as though this option were not called for.

If σ_{glk} is non-zero and $n_g \geq 5$, then data has been generated and should be processed as follows:

The summation of the product terms, $\Sigma u_i u_j$, is scaled by and corrected for the mean by multiplying by n_g^2, dividing by $\Sigma u_i \Sigma u_j$ and subtracting n_g. The corrected terms will then be of the form

$$\frac{n_g^2 \, \Sigma u_i u_j}{\Sigma u_i \Sigma u_j} - n_g$$

The corrected matrix then is the following 4 X 5 matrix:

$$\begin{array}{ccccc}
\frac{n_g^2 \Sigma(u_1)^2}{(\Sigma u_1)2} - n_g & \frac{n_g^2 \Sigma u_1 u_2}{\Sigma u_1 \Sigma u_2} - n_g & \frac{n_g^2 \Sigma u_1 u_3}{\Sigma u_1 \Sigma u_3} - n_g & \frac{n_g^2 \Sigma u_1 u_4}{\Sigma u_1 \Sigma u_4} - n_g & \frac{n_g^2 \Sigma u_1 u_5}{\Sigma u_1 \Sigma u_5} - n_g \\
\frac{n_g^2 \Sigma u_2 u_1}{\Sigma u_2 \Sigma u_1} - n_g & \frac{n_g^2 \Sigma(u_2)^2}{(\Sigma u_2)^2} - n_g & \frac{n_g^2 \Sigma u_2 u_3}{\Sigma u_2 \Sigma u_3} - n_g & \frac{n_g^2 \Sigma u_2 u_4}{\Sigma u_2 \Sigma u_4} - n_g & \frac{n_g^2 \Sigma u_2 u_5}{\Sigma u_2 \Sigma u_5} - n_g \\
\frac{n_g^2 \Sigma u_3 u_1}{\Sigma u_3 \Sigma u_1} - n_g & \frac{n_g^2 \Sigma u_3 u_2}{\Sigma u_3 \Sigma u_2} - n_g & \frac{n_g^2 \Sigma(u_3)^2}{(\Sigma u_3)^2} - n_g & \frac{n_g^2 \Sigma u_3 u_4}{\Sigma u_3 \Sigma u_4} - n_g & \frac{n_g^2 \Sigma u_3 u_5}{\Sigma u_3 \Sigma u_5} - n_g \\
\frac{n_g^2 \Sigma u_4 u_1}{\Sigma u_4 \Sigma u_1} - n_g & \frac{n_g^2 \Sigma u_4 u_2}{\Sigma u_4 \Sigma u_2} - n_g & \frac{n_g^2 \Sigma u_4 u_3}{\Sigma u_4 \Sigma u_3} - n_g & \frac{n_g^2 \Sigma(u_4)^2}{(\Sigma u_4)^2} - n_g & \frac{n_g^2 \Sigma u_4 u_5}{\Sigma u_4 \Sigma u_5} - n_g
\end{array}$$

where the first four columns are the matrix U_j and the last column is the vector U_2.

Then the coefficient vector U_g is determined by

$$\{U_g\} = [U_1]^{-1} \{U_2\}$$

with the constant term defined as

$$U_{g0} = (1 - U_{g1} - U_{g2} - U_{g3} - U_{h4}) (\Sigma u_5)/n_g$$

The steering coefficients for the k-th stage will then be as follows:

$$a_{0k} = U_{g0}$$

$$a_{1k} = U_{g1} (\Sigma u_5)/(\Sigma u_1)$$

$$b_{1k} = U_{g2} (\Sigma u_5)/(\Sigma u_2)$$

$$b_{2k} = U_{g3} (\Sigma u_5)/(\Sigma u_3)$$

$$b_{3k} = U_{g4} (\Sigma u_5)/(\Sigma u_4)$$

The steering equation coefficients are saved to be available on the next run.

b. Shaper Subroutine

(1). Introduction

This routine utilizes a generalized attitude control table and sets up the hunting procedure to (1) maximize range for a given payload, (2) maximize payload to a given range or (3) isolate the payload weight for a given circular orbit. The generalized attitude control table specifies the following flight path.

1. The vehicle will fly vertically off the pad until a velocity of 170.0 feet per second has been obtained before initiating pitchover. This ascent will assure that the vehicle has cleared the ground handling equipment and launching facilities.

2. The vehicle will pitch down at a rate consistent with available TVC angle and reasonable stress levels in the interstage. This rate is established empirically as a function of the vehicle liftoff weight. The pitchover is terminated when a specific pitch attitude, θ_{m2}, has been obtained.

3. The vehicle is maintained at this specific attitude until the velocity vector angle coincides with the vehicle centerline, i.e., at zero degrees angle of attack.

4. The final and major portion of flight is programmed as a gravity turn path such that the vehicle is thrusting along the velocity vector, i.e., at zero degrees angle of attack. This flight profile will be utilized for both the ballistic range trajectory and the orbital trajectory.

The prime inputs to the routine are the input control flag which specifies the mission, the input target range or orbital attitude, the stage rocket motor ballistic characteristics, as specified by the thrust-weight input and the input trajectory initial conditions such as the launch azimuth, initial altitude, velocity, etc. Inherent in the hunting procedure are a starting point and a maximum walking distance. The hunting procedure in the trajectory program requires that the initial and incremental values of the independent parameters be specified. These initial values of the independent variables are determined as continuous functions of the vehicle ideal velocity, stage burn times, and initial conditions.

This program will simplify the input to the trajectory program when doing performance trajectory work.

(2) Shaper Equations and Logic

Input Control Flags

$$K_{sh} = \begin{cases} 0 \text{ ignore routine} \\ 1 \text{ maximize range} \\ 2 \text{ maximize payload to a given range} \\ 3 \text{ determine payload to a circular orbit} \end{cases}$$

S_{sh} = target range or orbital altitude (nmi)

Terminal Stage Number

n_k = largest stage number containing a non-zero input, σ_s

Calculate impulses and thrust multipliers

K_{FM}, K_{FC}, K_{tM}, K_{tC}, K_{WM}, K_{WC}, I_{vM}, I_{vC}, and I_{vT}

The input main weight flow integral:

$$\Delta W_{Mi} = K_{WM} K_{tM} \sum_{j=1}^{J_M-1} (\dot{W}_{M(j+1)} + \dot{W}_{M(j)}) (t_{M(j+1)} - t_{M(j)})/2.$$

The expended main propellant weight:

$$\Delta W_{Mf} = \begin{cases} 0 & \text{if } I_{spM} = 0.0 \\ K_{FM}K_{tM}I_{vM}/I_{spM} & \text{Otherwise} \end{cases}$$

The complementary weight flow integral:

$$\Delta W_{Ci} = K_{WC}K_{tC} \sum_{j=1}^{J_C-1} (\dot{W}_{C(j+1)} + \dot{W}_{C(j)}) (t_{C(j+1)} - t_{C(.)})/2$$

The expended complementary propellant weight

$$\Delta W_{Cf} = \begin{cases} 0 & \text{if } I_{spC} = 0.0 \\ K_{FC}K_{tC}I_{vC}/I_{spC}, & \text{otherwise} \end{cases}$$

Total expended weight:

$$\Delta W = \Delta W_{Mi} + \Delta W_{Mf} + \Delta W_{Ci} + \Delta W_{Cf}$$

Stage effective specific impulse

$$I_{speff} = I_{vT}/\Delta W$$

Carryover weight:

$$W_{OV} = \begin{cases} 0 & \text{If } k - 1 \text{ or } K_{O(k-1)} = K_{NO(k-1)} = 0.0 \\ W_{B(k-1)} - \Delta W_{(k-1)} & \text{If } K_{O(k-1)} \neq 0.0 \\ W_{CO(k-1)} - \Delta W_{Ci(k-1)} - \Delta W_{Cf(k-1)} & \text{If } K_{NO(k-1)} \neq 0.0 \\ & \text{and } K_{O(k-1)} \neq 0.0 \end{cases}$$

Stage Weight:

$$W_B = W_{MO} + W_{CO} + W_{OV}$$

Stage Weight Ratio:

$$\upsilon = W_B/W_{B1}$$

Stage duration:

$$t_B = t_{B(J_M)} K_{tM}$$

Earth rotation factor:

$$\tilde{\omega}/\omega = \cos \rho_L \sin \Psi_i$$

Propulsion system liftoff weight

$$W_{PLO_k} = W_{B_k} + \sum_{j=k+1}^{n_k} (W_{MO(j)} + W_{CO(j)})$$

Maximum Range:

If $K_{sh} = 1$, apply the following logic and equations

Ideal Velocity Equation:

$$\Delta V = \bar{g}_e \sum_{k=k_k}^{n_k} I_{speff_k} \ln \left[\frac{W_{PL} + W_{PLO_k}}{W_{PL} + W_{PLO_k} - \Delta W_k} \right]$$

Vehicle Liftoff Weight:

$$W_T = W_{PL} + \sum_{k=k_k}^{n_k} W_M)_k + W_{CO_k}$$

Velocity at Pitch Over:

$$V_{PO} = \begin{cases} 170.0 & \text{If } V_{eo} \leq 160 \\ V_{eo} + 10 & \text{Otherwise} \end{cases}$$

Pitch Over Pitch Rate:

$$Q_{m2} = \{2,186.428 + \ln (W_T) [-218.16888 + 5,4511537 \ln (W_T)]\}/W_T^{1/3}$$

Pitch Over Angle:

$$\begin{array}{llll}
\theta_{n2} = -19.33383 & + .0031310 & \Delta V & \text{(ft/sec)} \\
 & - .00000012854 & \Delta V^2 & \\
 & + .81426 & t_{B1} & \text{(sec)} \\
 & + .00082444 & t_{B1}^2 & \\
 & + .13693 & t_{B2} & \text{(sec)} \\
 & + .036519 & t_{B3} & \text{(sec)} \\
 & .043040 & V_0 & \text{(ft/sec)} \\
 & - .00022205 & h_0 & \text{(ft)} \\
 & - .0000000085475 & h_0^2 & \\
 & - 5.13440 & \bar{\omega}/\omega & \text{(dim)} \\
 & + 23.95712 & \nu_2 & \text{(dim)} \\
 & + 47.53343 & \nu_3 & \text{(dim)} \\
 & - 1.45158 & Q_{m2} & \text{(deg/sec)} \\
 & - .013984 & (Q_{mq})^2 &
\end{array}$$

Initial Conditions

L11	90.0	Initial flight path angle	(deg)
L28	90.0	Initial command pitch attitude	(deg)

Hunting Procedure P1

L73	1.0	Hunting procedure P1 flag	(dim)
L74	0.0	Stage print flag	(dim)
L75	20.0	Trajectory limit number	(dim)
L76	1.0	Isolation-maximization flag	(dim)
L77	319.0	Independent variable code [K_{f2}]	(dim)
L78	5024.0	Dependent variable code [S_f]	(dim)
L80	$\hat{\theta}m2$	Initial value of independent variable	(deg)
L81	2.0	Incremental value of independent variable	(deg)
L83	1.0	Convergence accuracy of dependent variable	(nmi)

Hunting Procedure P2

L84	0.0	Hunting procedure P2 flag	(dim)

Main Print

L184	10.0	Main print time interval	(sec)
L183	2000.0	Main print time limit	(sec)

Discontinuity Print

L252	310.0	Code for discontinuity print at end of vertical rise	(dim)
L253	317.0	Code for discontinuity print at end of pitch over	(dim)
L254	324.0	Code for discontinuity print at start of gravity turn	(dim)

Attitude Control

L310	1.0	Type of flight region 1 [command rate]	(dim)
L311	5046.0	Parameter code for termination of region 1 [V_e]	(dim)
L312	V_{PO}	Velocity to terminate region 1	(ft/sec)
L313	0.0	Pitch rate for region 1	(deg/sec)
L316	1.0	Type of flight region 2 [command rate]	(dim)
L317	5722.0	Parameter code for termination of region 2 [θ_m]	(dim)
L318	$\hat{\theta}$ m2	Command attitude to terminate region 2	(deg)
L319	Qm2	Pitch rate for region 2	(deg/sec)
L322	1.0	Type of flight region 3 [command rate]	(dim)
L323	5310.0	Parameter code for termination of region 3 [α]	(dim)
L324	0.0	Angle of attack to terminate region 3	(deg)
L325	0.0	Pitch rate for region 3	(deg/sec)
L328	2.0	Type of flight region 4 [gravity turn]	(dim)
L329	5000.0	Parameter code for termination of region 4 [t]	(dim)
L330	2000.0	Time to terminate region 4	(sec)
L331	0.0	Angle of attack for region 4	(deg)

(3) Maximun Payload to a Given Range

If $K_{sh} = 2$, apply the following logic and equations:

Estimated required Ideal Velocity for Range $\leq$ 8000 Nautical Miles:

$V =$	$- .0041972$		
	$- 1.46207$	V_0	(ft/sec)
	$- 25,15275$	t_{B1}	(sec)
	$+ .33069$	t_{B1}^2	
	$+ 10.9855$	t_{B2}	(sec)
	$+ 4.7776$	t_{B3}	(sec)
	$- 1152.8991$	$\bar{\omega}/\omega$	(dim)
	$- .04378$	h_0	(ft)
	$+ .00000074999$	H_0^2	
	$+ 33642.902$	$(S_F/1000)$	(nmi)
	$- 23126.67$	$(S_F/1000)^2$	
	$+ 10625.982$	$(S_F/1000)^3$	
	$- 2570.1949$	$(S_F/1000)^4$	
	$+ 355.47505$	$(S_F/1000)^5$	
	$- 26.22204$	$(S_F/1000)^6$	
	$+ .80102$	$(S_F/1000)^7$	

For Range > 8000 Nautical Miles·

$V = 29000$

Payload Iteration Procedure

Ideal Velocity Equation

$$\Delta V^{\ell} = \bar{g}_e \sum_{k=k_k}^{n_k} I_{speff}\, \ell n \left[\frac{W_{PL}^{\ell} + W_{PLO_k}}{W_{PL}^{\ell} + W_{PLO_k} - \Delta W_k} \right]$$

where ℓ is the iteration counter.

Initial guess for V_{PL}

For $\ell = 1$ $$W_{PL}^{\ell=1} = W_{PLO_{k-k_k}} /30.0$$

For $\ell = 2$ $$W_{PL}^{\ell=2} = 1.01\, W_{PL}^{\ell=1}$$

For $\ell \geq 3$ $$W_{PL}^{\ell\geq 3} = W_{PL}^{\ell-1} - \frac{(\Delta\hat{V} - \Delta V^{\ell-1})(W_{PL}^{\ell-1} - W_{PL}^{\ell-2})}{(\Delta V^{\ell-2} - \Delta V^{\ell-1})}$$

If $W_{PL}^{\ell} < 0$, then set $W_{PL}^{\ell} = W_{PL}^{\ell-1}/2.0$

If $\frac{\Delta\hat{V} - \Delta V^{\ell}}{\Delta\hat{V}} < 0.00004$, terminate iteration

If $\ell = 100$, terminate run and print

"PAYLOAD WEIGHT ITERATION DID NOT CONVERGE"

At the termination of the payload weight iteration,

set $W_{PL} = W_{PL}^{\ell}$

Vehicle Liftoff Weight

$$W_T = W_{PL} + \sum_{k=k_k}^{n_k} W_{M)k} + W_{CCk}$$

Velocity at Pitch Over

$$V_{PO} = \begin{cases} 170.0 & \text{If } V_{eo} \leq 160.0 \\ V_{eo} + 10.0 & \text{Otherwise} \end{cases}$$

(4) **Pitch Over Pitch Rate**

$$Q_{m2} = - \{2,186.428 + \ln (W_T) [-218.16888 + 6.4511537 \ln (W_T)]\}/W_T^{1/3}$$

Pitch Over Angle

$$\begin{array}{llll}
\hat{\theta}_{m2} = -23,42023 & + .86 & t_{B1} & \text{(sec)} \\
 & + .094735 & t_{B2} & \\
 & + .022857 & t_{B3} & \\
 & - .040341 & V_0 & \text{(ft/sec)} \\
 & - .00033343 & h_0 & \text{(ft)} \\
 & - .0000000070714 & h_0^2 & \\
 & - .88686 & \bar{\omega}/\omega & \text{(dim)} \\
 & - 1.2008 & (\bar{\omega}/\omega)^2 & \\
 & + 33.3329 & \nu_2 & \text{(dim)} \\
 & + 50.30273 & \nu_3 & \text{(dim)} \\
 & - 3.91364 & Q_{m2} & \text{(deg/sec)} \\
 & - .14096 & (Q_{m2})^2 & \\
 & - .0027139 & S_F & \text{nm}
\end{array}$$

Initial Conditions

L11	90.0	Initial flight path angle	(deg)
L28	90.0	Initial command pitch attitude	(deg)

Hunting Procedure P1

L73	0.0	Hunting procedure P1 flag	(dim)

Hunting Procedure P2

L84	3.0	Hunting procedure P2 flag [incomplete quadratic]	(dim)
L85	5024.0	Maximized dependent variable code [S_F]	(dim)
L86	1.0	Tolerance of maximization parameter	(nm)
L87	0.0	Model accuracy	(dim)
L88	0.0	Run Time	(min)
L89	S_{sh}	Isolated-maximized parameter [S_f]	(nm)
L90	20.0	Trajectory limit number beyond base array	(dim)
L91	20.0	Isolation independent variable code [W_{PL}]	(dim)
L92	W_{PL}	Initial value of isolation independent variable	(lb)
L93	$0.1 \times W_{PL}$	Incremental value of isolation independent variable	(lb)
L98	0.0	Transformation flag, first independent variable	(dim)
L99	0.0	Transformation constant, first independent variable	(lb)
L100	319.0	Second independent variable code [K_{f2}]	(dim)
L101	$\hat{\theta}_{m2}$	Initial value of independent variable	(deg)
L102	2.0	Incremental value of independent variable	(deg)
L107	0.0	Transformation flag, second independent variable	(dim)
L108	0.0	Transformation constant, second independent variable	(deg)
L154	$0.5\ W_T$	Upper limit first independent variable	(lb)

L155	90.0	Upper limit second independent variable	(deg)
L161	0.0	Lower limit first independent variable	(lb)
L162	20.0	Lower limit second independent variable	(deg)

Main Print

L184	10.0	Main print time interval	(sec)
L185	2000.0	Main print time limit	(sec)

Discontinuity Print

L252	310.0	Code for discontinuity print at end of vertical rise	(dim)
L253	317.0	Code for discontinuity print at end of pitch over	(dim)
L254	324.0	Code for discontinuity print at start of gravity turn	(dim)

Attitude Control

L310	1.0	Type of flight region 1 [command rate]	(dim)
L311	5046.0	Parameter code for termination of region 1 [V_e]	(dim)
L312	V_{PO}	Velocity to terminate region 1	(ft/sec)
L313	0.0	Pitch rate for region 1	(deg/sec)
L316	1.0	Type of flight region 2 [command rate]	(dim)
L317	5722.0	Parameter code for termination of region 2 [θ_m]	(dim)
L318	$\hat{\theta}_{m2}$	Command attitude to terminate region 2	(deg)
L319	Q_{m2}	Pitch rate for region 2	(deg/sec)
L322	1.0	Type of flight region 3 [command rate]	(dim)

L323	5310.0	Parameter code for termination region 3	(dim)
L324	0.0	Angle of attack to terminate region 3	(deg)
L325	0.0	Pitch rate for region 3	(deg/sec)
L328	2.0	Type of flight region 4 [gravity turn]	(dim)
L329	5000.0	Parameter code for termination of region 4 [t]	(dim)
L330	2000.0	Time to terminate region 4	(sec)
L331	0.0	Angle of attack for region 4	(deg)

(5) Payload to Circular Orbit

If $K_{sh} = 3$, apply the following logic and equations:

Estimated required Ideal Velocity

$$
\begin{array}{llll}
\Delta V = 24753.331 & + 16.17 & h_{c0} & \text{(nmi)} \\
 & - 8.8850495 & t_{B1} & \text{(sec)} \\
 & + .38179208 & t_{B1}^{2} & \\
 & + 9.4 & t_{B2} & \text{(sec)} \\
 & - .52 & t_{B3} & \text{(sec)} \\
 & - 856.07602 & \nu_2 & \text{(dim)} \\
 & + 9742.4243 & \nu_3 & \text{(dim)} \\
 & - 1585.8688 & \bar{\omega}/\omega & \text{(dim)} \\
 & - .046233604 & h_0 & \text{(ft)} \\
 & - 2.6770574 & V_0 & \text{(ft/sec)} \\
 & + .0020621782 & V_0^{2} &
\end{array}
$$

Payload Iteration Procedure

Ideal Velocity Equation

$$\Delta V^{\ell} = \bar{g}_e \sum_{k=k_k}^{n_k} I_{speff} \ln \frac{W_{PL}^{\ell} + W_{PLO_k}}{W_{PL}^{\ell} + W_{PLO_k} - \Delta W_k}$$

Initial guess for W_{PL}

For $\ell = 1$ $\quad W_{PL}^{\ell=1} = W_{PLO_{k=k_k}} / 30.0$

For $\ell = 2$ $\quad W_{PL}^{\ell=2} = 1.01\, W_{PL}^{\ell}$

For $\ell = 3$
$$W_{PL}^{\ell=3} = W_{PL}^{\ell-1} - \frac{(\hat{\Delta V} - \Delta V^{\ell-1})(W_{PL}^{\ell-1} - W_{PL}^{\ell-2})}{(\Delta V^{\ell-2} - \Delta V^{\ell-1})}$$

If $W_{PL}^{\ell} < 0$, then set $W_{PL}^{\ell} = W_{PL}^{\ell-1}/2.0$

If $\frac{\hat{\Delta V} - \Delta V^{\ell}}{\Delta V} < 0.00004$, terminate iteration

If $\ell = 100$, terminate run and print

"PAYLOAD WEIGHT ITERATION DID NOT CONVERGE"

At the termination of the payload weight iteration,

set $W_{PL} = W_{PL}^{\ell}$

Vehicle Liftoff Weight

$$W_T = W_{PL} + \sum_{k=k_k}^{n_k} W_{MOk} + W_{COk}$$

Velocity at Pitch Over

$$V_{PO} = \begin{cases} 170.0 & \text{If } V_{eo} = 160.0 \\ V_{eo} + 10.0 & \text{Otherwise} \end{cases}$$

Circular Orbital Altitude

$$h_{CO} = S_{sh} \times 6076.10333$$

Inertial Circular Orbital Velocity

$$V_{ICO} = \left[\frac{g_e r_e^2}{(h_{CO} + r_e)}\right]^{\frac{1}{2}}$$

where

the gravitational constant is:

$$V_{CO} = \begin{cases} 32.14625 & \text{If } g_e' = 0 \\ g_e' & \text{Otherwise} \end{cases}$$

and earth radius is:

$$r_e = \begin{cases} 20{,}926{,}400.0 & \text{If } r_e' = 0 \\ r_e' & \text{Otherwise} \end{cases}$$

Pitch Over Pitch Rate

$$Q_{m2} = - \{2{,}186.428 + \ln (W_T) [-218.16888 + 6.4511537 \ln (W_T)]\}/W_T^{1/3}$$

Pitch Over Angle

θ_{m2} for $V_0 \leq 160$ feet per second:

$$\begin{aligned} \theta_{m2} = -20.618013 &+ .1013901 && h_{CO} && \text{(nmi)} \\ &- .000083344619 && h_{CO}^2 && \\ &+ .7863 && t_{B1} && \text{(sec)} \\ &+ .0752 && t_{B2} && \text{(sec)} \\ &- .0032 && t_{B3} && \text{(sec)} \\ &+ 19.666608 && \nu_2 && \text{(dim)} \\ &+ 92.727271 && \nu_3 && \text{(dim)} \\ &- .33013554 && \bar{\omega}/\omega && \text{(dim)} \\ &- .00020876696 && h_0 && \text{(ft)} \\ &- .011349661 && V_0 && \text{(ft/sec)} \\ &+ 33.876497 && Q_{m2}^{-1} && \text{(deg/sec)} \end{aligned}$$

(6) Initial flight path angle to terminate next to last stage

$$
\begin{aligned}
\hat{\gamma}_{1g} = -4.3479152 \quad & + .0014201471 \quad h_{CO} && \text{(nmi)} \\
& + .000014699632 \quad h_{CO}^{2} && \\
& - .010439863 \quad t_{B1} && \text{(sec)} \\
& + .00024116553 \quad t_{B1}^{2} && \\
& + .0047000003 \quad t_{B2} && \text{(sec)} \\
& + .075800000 \quad t_{B3} && \text{(sec)} \\
& - 3.1327416 \quad \nu_{2} && \text{(dim)} \\
& + 20.151515 \quad \nu_{3} && \text{(dim)} \\
& - .84360441 \quad \tilde{\omega}/\omega && \text{(dim)} \\
& - .000020950557 \quad h_{0} && \text{(ft)} \\
& - .00128745 \quad V_{0} && \text{(ft/sec)} \\
& + .0000013688082 \quad V_{0}^{2} &&
\end{aligned}
$$

Initial Conditions

L11	90.0	Initial flight path angle	(deg)
L28	90.0	Initial command attitude	(deg)

Hunting Procedure P1

L73	0.0	Hunting procedure P1 flag	(dim)

Hunting Procedure P2

L84	1.0	Hunting procedure P2 flag [linear model]	(dim)
L87	0.0	Model accuracy	(dim)
L88	0.0	Run time	(min)
L89	0.0	Isolation-maximized parameter [undefined]	(dbi)
L90	20.0	Trajectory limit number beyond base array	(dim)
L91	20.0	First independent variable code [W_{PL}]	(dim)
L92	W_{PL}	Initial value of first independent variable	(lb)
L93	$0.1 \times W_{PL}$	Incremental value of first independent variable	(lb)
L94	5050.0	First dependent variable code [V_I]	(dim)
L96	V_{ICO}	First dependent variable isolation value	(ft/sec)
L97	5.0	Tolerance of first isolated variable	(ft/sec)
L98	0.0	Transformation flag, first independent variable	(dim)
L99	0.0	Transformation constant, first independent variable	(lb)
L100	319.0	Second independent variable code [K_{f2}]	(dim)
L101	$\hat{\theta}_{m2}$	Initial value of second independent variable	(deg)
L102	2.0	Incremental value of second independent variable	(deg)

Hunting Procedure P2 (Continued)

L103	5330.0	Second dependent variable code γ_{1I}	(dim)
L105	0.0	Second dependent variable isolation value	(deg)
L106	0.01	Tolerance of second isolated variable	(deg)
L107	0.0	Transformation flag, second independent variable	(dim)
L108	0.0	Transformation constant, second independent variable	(deg)
L109	$(n_{k-1})1000 + 3$	Third independent variable code $[k_{3(n_{k-1})}]$	(dim).
L110	$\hat{\gamma}Ig$	Initial value ot third independent variable	(deg)
L111	2.0	Incremental value of third independent variable	(deg)
L112	5028.0	Third dependent variable code [h]	(dim)
L114	h_{CO}	Third dependent variable isolation value	(ft)
L115	100.0	Tolerance of third isolated variable	(ft)
L116	0.0	Transformation flag, third independent variable	(dim)
L117	0.0	Transformation constant, third independent variable	(ft)
L154	$0.2\ W_T$	Upper limit first independent variable	(lb)
L155	90.0	Upper limit second independent variable	(deg)
L156	20.0	Upper limit third independent variable	(deg)
L161	0.0	Lower limit first independent variable	(lb)
L162	20.0	Lower limit second independent variable	(deg)
L163	0.0	Lower limit third independendant variable	(deg)

Main Print

L184	10.0	Main print time interval	(sec)
L185	2000.0	Main print time limit	(sec)

Discontinuity Print

L252	310.0	Code for discontinuity print at end of vertical rise	(dim)
L253	317.0	Code for discontinuity print at end of pitchover	(dim)
L254	324.0	Code for discontinuity print at start of gravity turn	(dim)

Attitude Control

L310	1.0	Type of flight region 1 [command rate]	(dim)
L311	5046.0	Parameter code for termination of region 1 [V_e]	(dim)
L312	V_{PO}	Velocity to terminate region 1	(ft/sec)
L313	0.0	Pitch rate for region 1	(deg/sec)
L316	1.0	Type of flight region 2 [command rate]	(dim)
L317	5722.0	Parameter code for termination of region 2 [θ_m]	(dim)
L318	$\hat{\theta}_{m2}$	Command attitude to terminate region 2	(deg)
L319	Q_{m2}	Pitch rate for region 2	(deg/sec)
L322	1.0	Type of flight region 3 [command rate]	(dim)
L323	5310.0	Parameter code for termination region 3	(deg)
L324	0.0	Angle of attack to terminate region 3	(deg)
L325	0.0	Pitch rate for region 3	(deg/sec)
L328	2.0	Type of flight region 4 [gravity turn]	(dim)
L329	5000.0	Parameter code for termination of region 4 [ft]	(dim)
L330	2000.0	Time to terminate region 4	(sec)
L331	0.0	Angle of attack for region 4	(deg)

Termination of Next to Last Stage

$L_{(n_{k-1})}1000 + 0$	5330.0	Stage termination variable code [γII]	(dim)
$L_{(n_{k-1})}1000 + 1$	0.0	Termination constant 1	(dim)
$L_{(n_{k-1})}1000 + 2$	0.0	Termination constant 2	(dim)
$L_{(n_{k-1})}1000 + 3$	$\hat{\gamma}$Ig	Termination isolation value	(deg)

(7) Shaper Subroutine Nomenclature

Symbol	Definition	Units
GH	Gravitational Constant (32.14625)	ft/sec**2
HGO	Circular orbital altitude	nm
HO	Initial altitude	ft
ISPC	Comp specific impulse used to find vehicle weight flow	sec
ISPFFF	Stage effective specific impulse	sec
ISPM	Main specific impulse used to fine vehicle weight flow	sec
IVC	Integral of the complementary vacuum thrust table	lb-sec
IVM	Integral of the main vacuum thrust table	lb-sec
IVT	Integral of the main and complementary vacuum thrust table	lb-sec
JC	Number of data in complementary thrust-weight table	--
JM	Number of data in main thrust-weight table	--
KEC	Multiplier of the complementary stage vacuum thrust	--
KEM	Multiplier of the main stage vacuum thrust	--
KK	Stage number	--
KNOK	Complementary thrust-weight table wt carry-over flag	--
KOK	Main thrust-weight table weight carry-over flag	--
KSH	Control Flag indicating type of flight	--
KTC	Time multiplier for the complementary thrust table	--
KTM	Time multiplier for the main thrust table	--
KWC	Input complementary weight flow multiplier	--
KWM	Input main weight flow multiplier	--
NK	Largest stage number containing a non-zero input sigma	--
QMP	Pitch over pitch rate	deg/sec

Symbol	Definition	Units
RE	Radius of the earth (20,926,400)	ft
SSH	Target range or orbital altitude	nm
TR	Stage duration	sec
TRJM	Stage time of last main thrust-weight point	sec
TCJ	Complementary thrust-weight switching times	sec
TMJ	Main thrust-weight switching times	sec
TICJ	Complementary thrust-weight switching times (stage start)	sec
TIMJ	Main thrust-weight switching time from stage initiation	sec
VEO	Missile velocity at trajectory start time	ft/sec
VICO	Inertial circular orbital velocity	ft/sec
VO	Initial velocity of vehicle	ft/sec
VPO	Velocity at pitchover	ft/sec
WB	Stage weight	lb
WBK	Total missile weight at the termination of Kth stage	lb
WCO	Initial complementary weight	lb
WCOK	Initial complementary weight for the Kth stage	lb
WDOT CJ	Complementary weight flow at T'CJ, J = 1, ..., 25 per stage	lb/sec
WDOT MJ	Main weight flow at T'MJ, J = 1, ..., 25 per stage	lb/sec
WMO	Initial main weight	lb
WOV	Carry-over weight	lb
WPL	Weight of payload	lb
WPLOK	Propulsion system liftoff weight of Kth stage	lb
WT	Vehicle liftoff weight	lb
GAMA IG	Inertial flight path angle to termination next to last stage	deg
DELTA V	Total ideal velocity of missile	ft/sec
DELTA WMI	Integral of the main weight	lb

Symbol	Definition	Units
DELTA W	Total expended weight	lb
DELTA WCF	Expended complementary propellant weight	lb
DELTA WCFK	Expended complementary propellant weight of Kth stage	lb
DELTA WCI	Complementary weight flow integral	lb
DELTA WCIK	Complementary weight flow integral of the Kth stage	lb
DELTA WK	Total expended weight of Kth stage	lb
DELTA WMF	Expended main propellant weight	lb
THETA M2	Pitch over angle between launch horizondat and command control line	deg
THETA HATM2	Initial value of theta M2	deg
NU 2	Ratio of stage two weight to stage one weight, initially	--
NU 3	Ratio of stage 3 weight to stage one weight, initially	--
OMEG/OMEG	Earth rotation factor	--

M. INPUT

Load sheets are provided to facilitate data input and are shown at the end of this section. Utilization of the load sheets are explained below.

1. GENERAL INPUT

It is possible to submit any load sheet as the first or last page of the basic deck or run. Any load sheet not required may be deleted from the basic deck or run. Columns one through seventy-two are available for program input. Because of program logic regarding the basic deck system, a minus zero is never input.

Unless a quantity is input in the basic deck or run, that quantity will be zero unless special program logic apply.

The decimal point will be input with the number in the load sheet. If the decimal point is not specified for an input quantity, the decimal will be assumed after the last input integer of that quantity.

If the value of the desired input is too large or too small to satisfactorily indicate it in the spaces provided, write the number, the letter "E" and the power of ten the number is to multiplied by. If no decimal is specified for the number, the decimal is assumed after the last integer before the "E".

The first entry for all tables must appear at the top of the table. The independent variables specified in the load sheets must be filled out monotonically, but the last value may be input before the table ends.

Load sheet pages 11 and beyond can be used for first, second, third, or fourth stage. The desired stage is designated by inserting the stage number in the box following the "L".

Inputs in L0, L1, and L2 are the identification basic deck, reference run, and run. The basic deck and the run number must be nonzero in order for the trajectory to run. These identification numbers are printed at the top of each page of output.

2. INPUT L-NUMBER LIST

L-number	Symbol	Definition	Units
L0000	BD	Input basic deck number	dim
L0001	RR	Input reference run number	dim
L0002	RUN	Input run number	
L0003	K'_k	Input stage start control function. If 1, 2, 3, or 4, the run starts at the initiation of the first, second, third, or fourth stage, respectively if input zero set is equal to 1	dim
L0004	t_o	Input trajectory start time	sec
L0005	t'_k	Input start time with the initial stage K_k	sec
L0006	T'_{yk}	Input initial or restart type of flight control flag	dim
L0007	M'_y	Input initial or restart type of mode control flag	dim
L0008	S_y	Input initial or restart special print flag	dim
L0009	D_y	Input initial or restart discontinuity print flag	dim
L0010	V_{eo}	Input missile velocity at the trajectory start time	ft/sec
L0011	γ_{10}	Input flight path angle at the trajectory start time, $-180° < \gamma_{10} \le 180°$	deg
L0012	h_o	Input missile altitude at the trajectory start time	ft
L0013	S_o	Input missile ground range at the trajectory start time	ft
L0014	ψ_i	Input flight plane azimuth angle. Angle measured north to the flight plane, $-180° \le \psi \le 180°$	ft
L0015	θ_i	Input inertial elevation axis Euler angle relating the i and e_o systems	deg
L0016	ϕ_i	Input inertial meridional axis Euler angle relating the i and e_o systems	deg

L-number	Symbol	Definition	Units
L0017	ρ_L	Input launch latitude, $-90° \leq \rho \leq 90°$	deg
L0018	μ_L	Input launcher longitude	deg
L0019	h_L	Input launcher altitude	ft
L0020	W_{PL}	Input payload weight	lb
L0021	h_E	Input altitude above which ambient pressure and aerodynamic forces are zero and speed of sound is 1,000 ft/sec. If input zero is set to 300,000 ft	ft
L0022		Open	
L0023	J	Input gravitational value which accounts for the earth's oblateness	dim
L0024	$\bar{g}_e$	Input mass conversion gravity. If input zero, set equal to 32.174	ft/sec^2
L0025	g'_e	Input gravitational acceleration at the surface of the reference body. If input zero, set equal to 32.14625	ft/sec^2
L0026	r'_e	Input geometric radius of the earth. If input zero, set equal to 20,926,400	ft
L0027	ω	Input magnitude of the earth's angular velocity. If input greater than 0.5, set equal to 7.29211 E-5	rad/sec
L0028	Q_{mo}	Input initial command pitch attitude	deg
L0029	ψ_{mo}	Input initial commanded yaw attitude	deg
L0030	ϕ_{mo}	Input initial commanded roll attitude	deg
L0031	Q_{bo}	Input initial vehicle pitch rate	deg/sec
L0032	R_{bo}	Input initial vehicle yaw rate	deg/sec
L0033	P_{bo}	Input initial vehicle roll rate	deg/sec
L0034	θ_{bo}	Input pitch orientation angle at the trajectory start time, $-180° < \theta_{bo} \leq 180°$	deg
L0035	ψ_{bo}	Input yaw orientation angle at the trajectory start time $-180° < \psi_{bo} < 180°$	deg

L-number	Symbol	Definitions	Units
L0036	ϕ_{bo}	Input roll orientation angle at the trajectory start time $-180° < \phi_{bo} \leq 180°$	deg
L0037	$K_{cl,2}$	Input lower and upper limits, respectively, for computation of orbital elements and impact determination computations	dbi
L0039	h_f	Input altitude at the termination of the glide phase	ft
L0040	K_γ	Input glide phase termination control function. A value of plus one will specify impact after apogee, while a minus one will specify impact before apogee	dim
L0041	σ_c	Input code which designates the quantity that determines the flight region when the orbital elements and impact determination are desired	dim
L0042	h_e	Input altitude of atmospheric entry	ft
L0043	S_{co}	Input initial earth surface cross-range at trajectory start time	n.m.
L0044	γ_{20}	Input azimuthal flight path angle at trajectory start time	deg
L0045		Open	
L0046	ψ_g	Input generalized coordinate orientation angle. Second rotation angle (about the resulting Z axis after rotating through θ_g, ψ_g, and φ_g	deg
L0047	θ_g	Input generalized coordinate orientation for velocity steering. First rotation angle (about the Y_e-axis) of the set θ_g, ψ_g, and φ_g.	deg
L0048	ϕ_g	Input generalized coordinate orientation angle. Final rotation angle (about the X_g axis) of the set θ_g, ψ_g, and φ_g.	deg
L0049, 52, etc.	W_{JTj}	Input weight to be jettisoned when σ_j parameter had K_j value	lb
L0050, 53, etc.	σ_{Jj}	Input code which designates the quantity that determines when the W_{JT} weight is to be jettisoned where j - 1, 2, ..., or 8	dim
L0051, 54, etc.	K_{Jj}	Input value of that parameter at which the weight W_{JTj} is to be jettisoned where j = 1, 2, ..., or 8	dbi
L0073	P_1	Input flag so specify hunt procedure (P1)	dim
L0074	K_S	Input stage print control function. A non-zero value is required to print the trajectory at the termination of each stage during the hunting procedure	dim

L-number	Symbol	Definition	Units
L0075	n_{tl}	Input trajectory number limit. No more than n trajectories will be computed during the hunting procedure (P1) by varying X	dim
L0076	K_a	Input isolation-maximization control function. If zero, isolation is specified and if nonzero, maximization of the dependent variables will occur used in hunting procedure (P1)	dim
L0077	σ_X	Input code which designates the independent variable in the hunting procedure (P1)	dim
L0078	σ_a	Input code which designates the dependent variable in the hunting procedure (P1)	dim
L0080	X_i	Input value of the first guess of X used in hunting procedure	dbi
L0081	ΔX	Input increments that X is incremented during the hunting procedure (P1)	dbi
L0082	a_f	Input value of the dependent variable to be isolated by the hunting procedure (P1)	dbi
L0083	Δa	Input hunting procedure (P1) values that "a" should be computed within the isolation or maximization routine	dbi
L0084	P_2	Input flag to specify hunt procedure (P2). If P_2 = 0, by-pass hunt procedure 2; P_2 = 1, use a linear response; $\|P_2\|$ = 2, use a quadratic response surface where +2 maximizes and -2 minimizes; $\|P_2\|$ = 3, use an incomplete quadratic response surface where +3 maximizes and -3 minimizes	dim
L0085	σ_z	Input code. The σ_z identifies the dependent variable being maximized or minimized. Used in hunt procedure (P2). If $\sigma_z < 0$, the values of $x_1, x_2, \ldots, x_{in}$	dim
L0086	ϵ_z	Input value specifying the tolerance of the predicted maximization parameter (z). Used in the hunt procedure (P2)	dbi
L0087	ϵ_m	Input flag to specify model error used in hunt procedure (P2). If $\epsilon_m < 0$, the model will be iterated until an extremal soluation has a 95 percent probable model error. If $\epsilon_m = 0$, the extremal solution is obtained without regard to probable model error	dim

L-number	Symbol	Definition	Units
L0088		Open	
L0089	f_D	Input and flag specifying maximization and isolation of the same parameter used in hunt procedure (P2). If f_D is nonzero, the function f is maximized relative to the independent variables $x_2, x_3, \ldots, x_n$ and is isolated to a value f_D by varying x_1	dbi
L0090	n_{t2}	Input specified maximum number of hunt predictions (P2) beyond the initial array. If n_{t2} is input a negative number, the hunt will restart after $\|n_{t2}\|$ iterations	dim
L0091, 100, etc.	σ_{xj}	Input code which designates the independent variables used in hunting procedure (P2) where j = 1, 2, ..., 7	dim
L0092, 101, etc.	x_{ij}	Input initial array reference independent variable designated by code input that is used in hunting procedure (P2)	dbi
L0093, 102, etc.	Δx_{ij}	Input increment of x_{ij} used in incrementing during hunting procedure (P2), j = 1, 2, ..., 7	dbi
L0094, 103, etc.	σ_{yj}	Input code which designates the dependent variables used in hunting procedure (P2) where j - 1, 2, ..., 7	dim
L0095, 104, etc.	y_{Li}	Input desired dependent variable or lower constraint boundary of the dependent variable designated by code input that is used in hunting procedure(P2) where j - 1, 2, ..., 7	dbi
L0096 105, etc.	y_{Ui}	Input upper constraint boundary of the dependent variable designated by code input that is used in hunting procedure (P2) where j - 1, 2, ..., 7	dbi
L0097, 106, etc.	ϵ_{cj}	Input tolerance on the j-th condition of constraint used in hunt procedure (P2) where j - 1, 2, ..., 7	dbi
L0098 107, etc.	τ_{Tj}	Input transformation flag used in hunting procedure (P2) j = 1, 2, ..., 7	dim
L0099 108, etc.	a_{Tj}	Input transformation constant used in the simultaneous hunting procedure (P2) j - 1, 2, ..., 7	dbi

L-number	Symbol	Definition	Units
L0154-160	x_{Uj}	Input upper limit that an independent variable may assume a value used in hunt procedure (P2) where j - 1, 2, ..., 7	dbi
L0161-167	x_{Lj}	Input lower limit that an independent variable may assume a value in hunt procedure (P2) where j - 1, 2, ..., 7	dbi
L0168, 170, etc.	Δt_{cj}	Input computing interval during $t_{cj-1} \leq t \leq t_{cj}$ where j - 1, 2, ..., 8	sec
L0169, 171, etc.	t_{cj}	Input limit of the computing interval Δt_{cj} where j - 1, 2, ..., 8	sec
L0184, 186, etc.	Δt_{pj}	Input main printing interval during $t_{p(j-1)} \leq t \leq t_{pj}$ where j = 1, 2, ..., 8	sec
L0185, 187, etc.	t_{pj}	Input limit of the main printline print interval where j = 1, 2, ..., 8	sec
L0200-205	PL-DA GB,K,KB, N,O	Input flag where nonzero values are required if printlines DA, GB, K, KB, N, O are desired	dim
L0206-208		Open	
L0209-219	σ_{Pj}	Input code designates the quantity to be printed in printline Z where j - 1, 2, ..., 8	sec
L0220, 222, etc.	Δt_{Pj}	Input auxiliary printline interval during $t_{P(j-1)} \leq t \leq t_{Pj}$ where j = 1, 2, ..., 8	sec
L0221, 223, etc.	t_{Pj}	Input limit of the auxiliary printline print interval where j - 1, 2, ..., 11	sec
L0236, 238, etc.	σ_{tj}	Input code which designates the quantity that determines when to print a special time where j = 1, 2, ..., 8	dim
L0237, 239, etc.	K_{tj}	Input value when a trajectory printout is desired where j = 1, 2, ..., 8	dbi
L0252-259	σ_{Dj}	Input code which designates the quantity that determines when to print a discontinuity where j = 1, 2, ..., 8	dim
L0260, 262, etc.	T_{mj}	Input max print region flag	dim

L-number	Symbol	Definition	Units
L0261, 263, etc.	σ_{mj}	Input code which designates the quantity whose maximum value is to be printed following each stage time where j = 1, 2, ..., 5	dbi
L0270, 272, etc.	T_{Bj}	Input staging values flag, j = 1, 2, ..., 5	dim
L0271, 273, etc.	σ_{Bj}	Input code which designates the quantities whose values at staging are to be available to the hunting procedure, j = 1, 2, ..., 5	dim
L0280, 282, etc.	σ_{Aj}	Input code which designates the integration tolerance parameters where j = 1, 2, ..., or 7	dbi
L0281, 283, etc.	A'_{ri}	Input and utilized relative integration tolerance, j = 1, 2, ..., 7	dbi
L0304-305		Open	
L0306	A'_t	Input absolute allowable break-up tolerance of time	dim
L0307	B'_t	Input relative allowable break-up tolerance of time	dim
L0308	A'_x	Input absolute allowable break-up tolerance of target value	sec
L0309	B'_x	Input relative allowable break-up tolerance of target value	dim
L0310 317, etc.	T_{yj}	Input and output type of flight control flag where j = 1, 2, ..., or 16	dim
L0311, 318, etc.	σ_{fj}	Input code which designates the quantity that determines when the j-th type of flight ends where j = 1, 2, ..., 16	dim
L0312, 319, etc.	K_{fj}	Input limit of the j-th type of flight where j = 1, 2, ..., or 16	dbi
L0313, 320, etc.	Q_{mj}	Input vehicle pitch turning rate. Positive if the vehicle is intended to pitch up (Ty=1)	deg/sec
	α_c	Input commanded angle of attack used in the constant angle of attack and angle of side slip (Ty=2)	deg
	h_{cj}	Input commanded altitude used in the constant altitude type of flight (Ty=8)	ft

L-number	Symbol	Definition	Units
	η_{cj}	Input command load factor for constant load factor type of flight where j = 1, 2, ..., or 13 (Ty=9)	g's
	τ_{MIj}	Input guidance associated first order intercept guidance controller time constant used in type 10 flight where j = 1, 2, ..., or 13	sec
	K_{HGj}	Input navigation constant used in the Homing Guidance (Ty=11)	dim
L0314, 321, etc.	R_{mj}	Input vehicle yaw turning rate positive if the vehicle is intended to turn right (Ty=1)	deg/sec
	β_{cj}	Input commanded angle of side slip used in the constnant angle of attack and angle of side slip (Ty=2)	deg
	ω_{zj}	Input attitude control frequency used in constant attitude type of flight where j = 1, 2, ..., 16 (Ty=8)	rad/sec
	η_{ctj}	Input command load factor crosswise to the velocity vector used for constant load factor type of flight where j = 1, 2, ..., (Ty=9)	g's
	τ_{IGj}	Input pitch and yaw flare-in factor used in the Intercept Guidance (Ty=10)	sec
	τ_{HGj}	Input pitch and yaw flare-in factor used in the Homing Guidance (Ty=11)	sec
L0315, 322, etc.	P_{Mj}	Input vehicle roll turning rate positive if the vehicle is intended to roll clockwise looking at if from the aft end (Ty=1)	deg/sec
	φ_{cj}	Input command roll attitude used in the constant angle of attack and angle of side slip type of flight (Ty=7)	deg
	ζ_{zj}	Input constant attitude (Ty=8) attitude control damping ratio where j = 1, 2, ..., or 16	dim
	φ_{cj}	Input command roll attitude used in the constant load factor type of flight where j = 1, 2, ..., or 13 (Ty=9)	
	α_{maxj}	Input limit of angle of attack during the j-th type of flight	deg

L-number	Symbol	Definition	Units
L0316, 323, etc.	$\tau_{\eta j}$	Input flare-in time constant for constant load factor type of flight (Ty=9)	g's
L0401-406		Open	
L0407	K_h	Input wind altitude multiplier and flag. If zero, no wind effects are considered	dbi
L0408	K_v	Input wind speed multiplier	dbi
L0409	K_ψ	Input wind azimuth multiplier	dim
L0410, 413, etc.	h_j	Input wind velocity altitude associated with v_{wj} and ψ_{wj} where j = 1, 2, ..., 30	dbi dbi
L0411, 414, etc.	v_{wj}	Input (with altitude h) wind speeds where j = 1, 2, ..., 30	ft/sec
L0412, 415, etc.	ψ_{wj}	Input instantaneous (with altitude h_j) wind azimuth angles, measured in a plane parallel to the local tangent plane where j = 1, 2, ..., 30. Angle measured clockwise from north to the direction from which the wind is coming.	deg and dbi
L0500-597		Reserved for Aerodynamic Coefficient Calculation Routine	
L0598	K_{sh}	Input shaper control flag where: if it equals 0, ignore routine 1. maximize range 2. maximize payload to a given range or, 3. determine payload to a circular orbit	
L0599	S_{sh}	Input target range or orbital altitude	n.m.
L0600, 603, etc.	M_{yj}	Input and output mode type control where j = 1, 2, ..., 10. If 1 rigid body with controls, and 2 rigid body with controls	dim
L0601, 604, etc.	ς_{Mj}	Input code which designates the quantity that determines when the J-th mode type ends where j = 1, 2, ..., 10	dim
L0602, 605, etc	K_{Mj}	Input quantity which designates the limit of the j-th mode type end where j = 1, 2, ..., 10	dim
L0630	ς_{Tto}	Input code designating start of target maneuvering	dim
L0631	K_{Tto}	Input quantity which designates the start of target maneuvering	dbi

L-number	Symbol	Definitions	Units
L0632	v_{TO}	Input target initial velocity at start of target maneuvering	ft/sec
L0633	γ_{TO}	Input initial target flight path angle at start of target maneuvering	deg
L0634	h_{TO}	Input target initial altitude at the start of the target maneuvering	ft
L0635	ζ_{TO}	Input initial target azimuthal flight path angle at start of target maneuvering	deg
L0636	S_{TO}	Input initial target position down range	n.m.
L0637	S_{TCO}	Input initial target position cross range	n.m.
L0638-639		Open	
L0640 644, etc.	t_{Tj}	Input target time terminating the j-th target acceleration value of dynamical condition table	
L0641, 645, etc.	a_{TTj}	Input target earth reference acceleration tangential to the target velocity vector for the j-th period of the target dynamical condition table	g's
L0642, 646, etc.	a_{TNj}	Input target earth reference acceleration normal to the target velocity vector for the j-th period of the target dynamical condition table	g's
L0643, 647, etc.	a_{TCj}	Input target earth reference acceleration crosswise to the target velocity vector for the j-th period of the target dynamical condition table	g's
L0668-669		Open	
L0670	Δ_{dc}	Input value of number of desired duty cycle points (100 maximum) if input zero, set equal to 50	dim
L0671	K_{dc}	Input and output stage number of TVC duty cycle stage	dim
L0672	h_{α}	Input final attitude of maximum wind shear used in TVC duty cycle slew rate calculations	ft
L0673	h_{β}	Input initial altitude of maximum wind shear used in TVC duty cycle slew rate calculations	ft
L0674	ω_{s}	Input slew frequency used in the TVC design stage slew rate calculations	rad/sec

L-number	Symbol	Definitions	Units
L0675	FW1	Input stage I vacuum thrust to liftoff weight used in the vehicle characteristics pertinent to roll requirement	g's
L0676	W_{ol}	Input stage I liftoff weight used in roll control requirements	lb
L0677	W_{TVC}	Input and output estimated TVC system fixed weight. Used in TVC design stage for the refly option	lb
L0678	W_{exi}	Input estimated weight of the TVC system expend weight during the TVC design stage during the original vehicle flight	[illegible]
L0679	I_{spaug}	Input and output estimated TVC system caused specific impulse augmentation (positive) or degradation (negative). Used in trajectory TVC Design program refly	sec
L0680	x_{nf}	Input body station of nozzle flange. Used in the TVC design program	ft
L0681	δ_{me}	Input maximum vector angle design limit also output in TVC design duty cycle	deg
L0682	h_{mxwd}	Input altitude of maximum wind velocity. Also flag to set up wind table per MMRBM wind shear criteria	ft
L0683	K_{TPF}	Input titled print flag	dim
L0684	799	Reserved for plotter option	

L-number	Symbol	Definition	Units
L0800, 810, etc.	F_{yj}	Input thrust control law code for the j-th type of TMC table	dim
L0801, 811, etc.	σ_{Fyj}	Input code which designates the quantity that determines when the j-th type of TMC ends	dim
L0802, 812, etc.	K_{Fyj}	Input limit of the j-th type TMC	dim
L0803, 813, etc.	TMC_j	Input thrust dynamic mode of the j-th type TMC	dim
L0804, 814, etc.	C_{Fyj}	Input thrust system proportionality system gain of the j-th type TMC	dim
L0805, 815, etc.	τ_{Fyj}	Input control system time constant of the j-th type TMC	sec
L0806, 816, etc.	MIN_j	Input minimum velocity or constant Mach number of the j-th type of TMC	dim/ft/sec
L807, 817, etc.	q_{maxj}	Input maximum allowable dynamic pressure of the j-th type of TMC	lb/ft^2
L0808, 818, etc.	q_{min}	Input minimum allowable dynamic pressure of the j-th type of TMC	lb/ft^2
L0809, 819, etc.		Open	
L0870, 872, etc.	t_{VPj}	Input time value for Specific Velocity - Time Profile used in Fy=1 TMC j = 1, 2, ..., 15	sec
L0871, 873, etc.	V_{VPj}	Input earth reference velocity for the Specific Velocity - Time Profile used in Fy=1 TMC j = 1, 2, ..., 15	ft/sec
L0900-999		Special Coding	

The following parameters are input for each stage. The first digit of the L-number indicates the stage number.

L-number	Symbol	Definition	Units
K000*	σ_{Sk}	Input code which designates the quantity that determines when a stage is terminated where k = 1, 2, 3, 4	dim
K001	D_p	Input diameter of propellant	in
K002	r'_{b1000}	Input burnrate of propellant at 1000 lb/in^2 chamber pressure and flag to determine evaluation option	in/sec
K003	K_3	Input quantity which determines stage termination	dbi
K004	I'_{vT}	Input total of the main and complementary vacuum impulse	lb/sec
K005	I'_{vM}	Input main stage total vacuum impulse	lb/sec
K006	W_{MO}	Input initial main weight for the k-th stage	lb
K007	K_{FM}	Input multipliers of the main vacuum thrust	dim
K008	K_{WM}	Input main weight flow multiplier. If input zero is set to 1.0	dim
K009	K'_{tM}	Input main switching time multipliers. If zero, the program assumes a value of one. If σ_s is designated at t_B, then K_3 is multiplied by K'_{tm}.	dbi
K010	I_{spM}	Input main specific impulse used to compute vehicle weight flow. If zero, weight flow is determined from theinput weightflow. Also output in the TVC duty cycle.	sec
K011	A'_{eM}	Input main thrust-weight table input stage nozzle exit area	ft^2
K012	K_{Ok}	Input main thrust-weight table weight carry-over flag for the k-th stage. If zero, separation has occurred with regards to the main and complementary weights. If non-zero, the main weight at the termination of the k-1 stage is used as the initial main weight of the k-th stage	dim
K013	ϵ_d	Input and output nozzle expansion ratio used in the separated flow nozzle thrust equations	dim

The following parameters are input for each stage. The first digit of the L-number indicates the stage number.

L-number	Symbol	Definition	Units
K000*	σ_{Sk}	Input code which designates the quantity that determines when a stage is terminated where k = 1, 2, 3, 4	dim
K001	D_p	Input diameter of propellant	in
K002	r'_{b1000}	Input burnrate of propellant at 1000 lb/in^2 chamber pressure and flag to determine evaluation option	in/sec
K003	K_3	Input quantity which determines stage termination	dbi
K004	I'_{vT}	Input total of the main and complementary vacuum impulse	lb/sec
K005	I'_{vM}	Input main stage total vacuum impulse	lb/sec
K006	W_{MO}	Input initial main weight for the k-th stage	lb
K007	K_{FM}	Input multipliers of the main vacuum thrust	dim
K008	K_{WM}	Input mainweight flow multiplier. If input zero is set to 1.0	dim
K009	K'_{tM}	Input main switching time multipliers. If zero, the program assumes a value of one. If σ_S is designated at t_B, then K_3 is multiplied by K'_{tm}.	dbi
K010	I_{spM}	Input main specific impulse used to compute vehicle weight flow. If zero, weight flow is determined from theinput weightflow. Also output in the TVC duty cycle.	sec
K011	A'_{eM}	Input main thrust-weight table input stage nozzle exit area	ft^2
K012	K_{0k}	Input main thrust-weight table weight carry-over flag for the k-th stage. If zero, separation has occurred with regards to the main and complementary weights. If non-zero, the main weight at the termination of the k-1 stage is used as the initial main weight of the k-th stage	dim
K013	ϵ_d	Input and output nozzle expansion ratio used in the separated flow nozzle thrust equations	dim

L-number	Symbol	Definition	Units
K014	γ'_d	Input and output calculated ratio of specific heats of the rocket motor exhaust gases. If input zero 1.18 is used. Used in separated flow nozzle thrust equations	dim
K015	C'_D	Input and calculated nozzle efficiency coefficient used in the separated flow nozzle thrust equation	dim
K016	α'_d	Input nozzle half angle used in the separated flow nozzle thrust equation.	deg
K017	a'_s	Input nozzle separation polynominal coefficient used in the separated flow nozzle thrust equation. If input zero is set to 0.3	dim
K018	b'_s	Input nozzle separation polynominal coefficient used in the separated flow nozzle thrust equation if input zero is set to 0.7	dim
K019	c'_s	Input nozzle separation polynominal coefficient used in the separated flow nozzle thrust equation. If input zero is set to 0.884	dim
K020	P_{aRM}	Input main table thrust reference atmospheric pressure	lb/ft^2
K021, 024, etc.	F_{Mj}	Input value of the main vacuum thrust to be used during $t_{Mj} \leq t < t_{Mj+1}$ where j = 1, 2, ..., 25 per stage	lb
K022, 025, etc.	$\dot{W}_{Mj}$	Input main weight flow at t'_{Mj} where j = 1, 2, ..., 25 per stage	lb/sec
K023, 026, etc.	t'_{Mj}	Input main thrust-weight switching time from stage initiation where j = 2, 3, ..., or 25 per stage	sec
K095	ρ_p	Density of propellant used in Internal Ballistic evaluation is set to 0.063 if not input	lb/in^3
K096	τ_w	Web fraction used in internal ballistic evaluation is set to 0.8 if not input	dim
K097	n	Burnrate exponent used in internal ballistic evaluation is set to 0.6 if not input	dim
K098	α_p	Propellant diffusivity used in internal ballistic evaluation is set to 0.00027 if not input	in^2/sec
K099	P_{max}	Maximum allowable chamber pressure used in internal ballistic evaluation	lb/in^2

N-number	Symbol	Definition	Units
K100	K'_{FC}	Input multiplier of the complementary vacuum thrust. If input zero for the k-th stage and $I'_{VT} = I'_{VT} = 0$, the complementary thrust for the k-th stage are zero	dim
K101	K'_{WC}	Input complementary weight flow multiplier. If input zero is set to 1.0	dim
K102	K'_{tc}	Input complementary switching time multiplier. If zero, the program assumes a value of one.	dim
K103	I_{spC}	Input complementary specific impulse used to compute vehicle weight flow. If zero weight flow is determined from input weight flow.	sec
K104	A_{eC}	Input complementary thrust-weight table stage nozzle exit area	ft^2
K105	W_{Cok}	Input initial complementary weight for the k-th stage	lb
K106	I'_{vC}	Input complementary stage total vacuum impulse	lb-sec
K107	K_{NOk}	Input complementary thrust-weight table weight carryover flag for the k-th stage. If K_{Ok} and K_{NOk} are non-zero, separation has occurred with regards to the complementary weight. If K_{Ok} is non-zero and K_{NOk} is zero, the total vehicle weight at the termination of the k-1 stage is used as the initial weight of the k-th stage	dim
K108	K_{BD}	Input flag which stipulates that the main nozzle exit area will be used in the base drag calculations when splitting main and complementary tables to allow for up to 47 thrust time points	dim
K109	P_{arC}	Input complementary table thrust reference atmospheric pressure	lb/ft^2
K111, 114 etc	t'_{Cj}	Input complementary thrust-weight switching time from stage initiation where j = 1, 2, 3, ..., 25 per stage	sec
K112, 115, etc	F_{Cj}	Input value of the complementary vacuum thrust to be used during $t_{Cj} \leq t \leq t_{C(j+1)}$ where j = 1, 2, ..., 25 per stage	lb
K113, 116 etc	$\dot{W}_{Cj}$	Input complementary weight flow at t'_{Cj} where j = 1, 2, ..., 25 per stage	lb/sec
K185	S_{RC}	Input aerodynamic chord force coefficient reference areas	sq ft

L-number	Symbol	Definition	Units
K186	$\bar{C}'$	Input stage axial force control function and multiplier. If input zero, the multiplier is set to one; if non-zero, the axial force is determined from input and multiplied by $\bar{C}$	dim
K187		Open	
K188, 190, etc.	M_{Aj}	Input Mach number for aerodynamic axial representation where j = 1, 2, ..., 15 per stage	dim
K189, 191, etc.	C_{Aj}	Input and instantaneous (with Mach number M_j) aerodynamic axial force coefficients, respectively, where j = 1, 2, ..., 15 per stage	dim
K218	S_{PF}	Input missile platform area used in calculating tumbling aerodynamic axial force coefficients	sq ft
K219	x_{Pc}	Input missile body station of the centroid of the platform area	ft
K220	S_{RN}	Input aerodynamic normal force coefficient reference area	sq ft
K221	$\bar{N}_k$	Input normal force control function and normal force multiplier. If input zero, the multiplier is set to one and if non-zero, the normal force is determined from input and multiplied by N_k where k = 1, 2, 3, 4	dim
K222	$\bar{X}_{cp}$	Input stage aerodynamic normal force center of pressure multiplier	dbi
K223, 228, etc.	M_{Nj}	Input Mach number for aerodynamic normal force coefficient representation where j = 1, 2, ..., 15 per stage	dim
K224-226	$C_{N1,2,3j}$	Input values of $C_{N1,2,3}$, respectively, corresponding to M_j where j = 1, 2, ..., 15 per stage	deg/deg^2/deg^3
K227, 232, etc.	x_{cpj}	Input and output instantaneous (with Mach number M) aerodynamic normal force center of pressure body station numbers, respectively, where j = 1, 2, ..., 15 per stage	dbi and ft
K298	D_{RN}	Input aerodynamic reference diameter	ft
K299	$\bar{M}_Q$	Input aerodynamic pitch damping moment due to pitch rate multiplier	dim
K300	$\bar{M}_{\dot{\alpha}}$	Input aerodynamic pitch damping moment due to rate change of angle of attack multiplier	dim

L-number	Symbol	Definition	Units
K301	K_Q	Input aerodynamic pitch damping moment due to pitch rate multiplier	dim
K302	x'_{RQ}	Input and calculated pitch damping moment due to pitch rate reference moment point body station	dbi and ft
K303	$K_{\dot{\alpha}}$	Input aerodynamic pitch damping moment due to time rate change of angle of attack multiplier	dim
K304	$x'_{R\dot{\alpha}}$	Input and calculated pitch damping moment due to rate change of angle of attack reference moment point body station	dbi and ft
K305,308, etc	M_{Dj}	Input Mach number for aerodynamic representation where j = 1, 2, ..., 15 per stage	dim
K306,309 etc	C_{MQj}	Input aerodynamic pitch damping moment due to pitch rate coefficient where j = 1, 2, ..., 15 per stage	/deg
K307,310 etc	$C_{M\dot{\alpha}j}$	Input aerodynamic pitch damping moment due to rate change of angle of attack coefficient where j = 1, 2, ..., 15 per stage	/deg
K350-379		Open	
K380	n'_m	Input and output internally calculated number of motors in the stage cluster	dim
K381	n'_c	Input and output internally calculated number of control nozzles for the cluster motor logic	dim
K382	R_c	Input radius of cluster for the k-th stage	ft
K383	σ_{It}	Input standard deviation of the ratio of total impulse to nominal total impulse for the k-th stage	dim
K384	σ_{tb}	Input standard deviation of the ratio of web burntime to nominal burntime for the k-th stage	dim
K385	ϕ_v	Input bivariant consideration axis for the k-th stage	deg
K386	Δ_1	Input amplitude of limit cycle for the k-th stage	
K387	K'_{Δ}	Input side impulse multiplier. If input zero is set to one	dim
K388	ω_L	Input frequency of limit cycle for the k-th stage	rad/sec

L-number	Symbol	Definition	Units
K389	δ_{MP}	Input nozzle misalignment angle in pitch	deg
K390	δ_{MY}	Input nozzle misalignment angle in yaw	deg
K391-392	$a_{o,jk}$	Input constants used in the Z_{gg} and constant components nontarget dependent pitch steering equations for the k-th stage	rad
K393-395	b_{jk}	Input constants used in the $\ddot{X}_{gg}$, X^2_{gg}, and $\dot{X}^3_{gg}$ components of the nontarget dependent pitch steering equation for the k-th stage j = 1, 2, 3	rad-sec/ft rad-sec^2/ft and rad-sec
K396	τ_{fk}	Input pitch flare-in time constant for k-th stage used in velocity steering type of flight (Ty=4)	sec
K397	σ_{g1k}	Input code which designates the start of the acquisition zone for evaluation of the steering equations coefficients for the k-th stage k = 1, 2, 3, or 4	dbi
K398	κ_{g1k}	Input quantity which designates the start of the acquisition zone for evaluation of the steering equation coefficient for the k-th stage k = 1, 2, 3, or 4	dbi
K399	σ_{g2k}	Input quantity which designates the end of the acquisition zone for evaluation of the steering equation coefficient for the k-th stage, k = 1, 2, 3, or 4	dbi
K400	κ_{g2k}	Input quantity which designates the end of the acquisition zone for evaluation of the steering equation coefficient for the k-th stage, k = 1 2, 3, or 4	dbi
K401	κ_{yk}	Input gain constant in the nontarget dependent yaw steering equation for the k-th stage	dim
K402	τ_{yk}	Input time constant in the nontarget dependent yaw steering equation for the k-th stage	sec
K403	δ_{MR}	Input roll system fin misalignment angle	deg
K404	$\ell_{\delta R}$	Input roll fin radial center of pressure to missile centerline distance	ft
K405	κ_{RC}	Input roll control system flag. If equal 1, an auxiliary roll thruster system is simulated. If equal 2, aerodynamic centeral fins are used	dim

L-number	Symbol	Definition	Units
K406	n_{vr}	Input disturbing roll nozzle vortex multiplier. If not input is set to 0.00363	ft
K407	ω_{Pc}	Pintle control frequency	rad/sec
K408-416		Open	
K417	x'_{Pf}	Input stage forward end of propellant grain body station	ft
K418	x'_{Pa}	Input stage aft end of propellant grain body station	ft
K419	x'_E	Input and computed stage nozzle exit body station	ft
K420-421	δ_{PO}, $\dot{\delta}_{PO}$	Input per stage initial pitch thrust vector deflection angle and angular rate at the trajectory initiation or stage initiation	deg and deg/sec
K422-423	δ_{YO}, $\dot{\delta}_{YO}$	Input per stage initial yaw thrust vector deflection angle and angular rate at the trajectory initiation or stage initiation	deg and deg/sec
K424	F_{RO}	Input per stage initial roll thrust	lb
K425-427		Open	
K428	ΔQ_b	Input stage pitch attitude reaction angular impulse; i.e., added to θ_b at staging	deg
K429	ΔR_b	Input stage yaw attitude reaction angular impulse; i.e., added to ψ_b at staging	deg
K430	$\bar{X}_e$	Input stage thrust gimbal position multiplier	dbi
K431	X'_e	Input stage thrust gimbal position body station	ft
K432	y'_e	Input stage thrust gimbal yaw eccentricities. Positive in the Y_b axis direction.	ft
K433	z_e	Input stage thrust gimbal pitch point eccentricities, respectively. Positive in the Z_b axis direction	dbi and ft
K434	K_δ	Input thrust control flag. If zero, control thrust is determined from instantaneous vehicle thrust. If one, the control thrust is obtained from the instantaneous main stage thrust, if two control thrust is nonexistent	dim
K435	τ_c	Input stage pitch and yaw control systems time constant for the thrust vector deflection first order transfer function	sec

L-number	Symbol	Definition	Units
K436	ζ_c	Input stage pitch control systems damping ratio for the thrust vector deflection second order transfer function	dim
K437	ω_c	Input stage pitch control systems forcing frequency for the thrust vector deflection second order transfer function	rad/sec
K438	ζ'_v	Input vehicle controlled damping ratio	dim
K439	ω'_v	Input vehicle controlled frequency for the k-th stage	rad/sec
K440-449	L_j	Input control system where j = 1, 2, ..., 10 per stage. If zero, a limit is not applied; otherwise, L_j limits the following parameters where the statement number is j (1) $K_{DP}\Delta\theta_b$, (2) δ_{Pc}, (3)δ_P, (4) δ_p and (5) $\ddot{\delta}_P$ (6) $K_{DY}\Delta\psi_b$, (7) δ_{YC}, (8) δ_Y, (9) $\dot{\delta}_Y$ (10) $\ddot{\delta}_Y$	deg deg deg deg/sec deg/sec^2
K450,459 etc	K_{DPj}	Input control system pitch attitude error gain for the j-th control region j = 1, 2, or 3	dim
K451,460 etc	K_{DYj}	Input control system yaw attitude error gain for the j-th control region j = 1, 2, or 3	dim
K452,461 etc	K_{RPj}	Input control system pitch attitude rate gain for the j-th control region j = 1, 2, or 3	sec
K453, 462 etc	K_{RYj}	Input control system yaw attitude rate gain for the j-th control region j = 1, 2, or 3	sec
K454, 463 etc	K_{IPj}	Input pitch angle of attack gain for the j-th control region j = 1, 2, or 3.	dim
K455, 464 etc	K_{IYD}	Input your angle of side slip gain for the j-th control region j=1, 2, or 3	
K456, 465 etc	f_{Gik}	Input attitude control system gain control flag. If equal to zero, input gains are utilized; if not equal to zero, the automatic gains are utilized for the j-th control zone i = 1, 2, or 3; and k-th stage, k = 1, 2, 3, or 4	dim
K457, 466	σ_{Gj}	Input code which designates attitude control system gain zone limits (j = 1, 2, or 3)	dim
K458, 467	K_{Gik}	Input attitude control system gain zone limits (i = 1, 2, or 3) and the k-th stage k = 1, 2, 3, or 4	dbi
K475	$\hat{I}_Y$	Input moment of inertial multiplier	dbi

L-number	Symbol	Definition	Units
K476	$\bar{I}_Z$	Input yaw moment of inertia multiplier	dbi
K477	$\bar{I}_X$	Input roll moment of inertia multiplier	dbi
K478	$\bar{W}$	Input stage weight multiplier. If input zero, is set internally to one and if nonzero, the input weight used in the mass properties table W_j (j = 1, 2, ..., 15) is multiplied by W	dim
K479	X_{cg}	Input stage center of gravity position multiplier	dbi
K480	z'_{cg}	Input center-of-gravity offset bias distance, in pitch, positive down	dbi
K481	y'_{cg}	Input center-of-gravity offset bias distance in yaw, positive to the right	dbi
K482	W_n	Input stage movable portion nozzle weight	lb
K483	I_n	Input stage movable portion nozzle moment of inertia about the gimbal point	slug-ft^2
K484	x_n	Input stage movable portion of nozzle center-of-gravity body station	dbi
K485-487		Open	
K488- 498 etc	W_j	Input vehicle weight to relate the input moment of inertia values j = 1, 2, ..., 15 per stage	lb
K489, 499 etc	x_{cgj}	Input instantaneous (with total vehicle weight, W_j) center-of-gravity body station numbers, respectively, where j = 1, 2, ..., 15 per stage	dbi and ft
K490, 500 etc	I_{Yj}	Input total vehicle pitch moment of inertia corresponding to the total vehicle weight input W_j where j = 1, 2, ..., 15 per stage	dbi
K491, 501 etc	z_{cgj}	Input instantaneous (with total vehicle weight, W_j) center of gravity offsets, respectively, where j = 1, 2, ..., 15 per stage. Positive in the Z_b axis direction	dbi and ft
K492, 502	I_{Zj}	Input total vehicle yaw moment of inertia corresponding to the total vehicle weight input W, where j = 1, 2, ..., 15 per stage	slug-ft^2
K493, 503	y_{cyj}	Input instantaneous (with total vehicle weight, W) center of gravity butt line number, where j = 1, 2, ..., 15 per stage	dbi and ft

L-number	Symbol	Definition	Units
K494, 504	I_{Xj}	Input total vehicle roll moment of inertia weight input Wj, where j = 1, 2, ..., 15 per stage	slug ft^2
K495, 505	I_{XYj}	Input total vehicle roll-yaw product of inertia corresponding to the total weight input Wj, where j - 1, 2, ..., 15 per stage	slug ft^2
K496, 506	I_{YZj}	Input total vehicle yaw pitch product of inertia corresponding to the total vehicle weight input Wj, where j = 1, 2, ..., 15 per stage	slug ft^2
K497, 507	I_{ZXj}	Input total vehicle pitch-roll product of inertia corresponding to the total vehicle weight input Wj, where j = 1, 2, ..., 15 per stage	slug ft^2
K638, 648 658	ℓ_{Ri}	Input thruster roll control lever arm for the i-th center zone	ft
K639, 649 659	τ_{Ri}	Input stage roll control system time constant, for the first order transfer	sec
K640, 650 660	D_i	Input roll control system hysteresis for the i-th zone, i = 1, 2, or 3	dim
K641, 651, 661	$F_{\Delta i}$	Input maximum roll control thrust for the i-th zone, i = 1, 2, or 3	lb
K642, 652, 662	L_{RCi}	Input maximum roll control thrust for the i-th zone, i = 1, 2, or 3	lb
K643, 653 663	^{-}DRi	Input roll control system attitude error gain for the i-th zone, i = 1, 2, or 3	dim
K644, 654 664		Open	
K645, 655 665	K_{RRi}	Input roll control system attitude rate gain for the i-th zone, i = 1, 2, or 3	sec
K646, 656 666	I_{SPRi}	Input roll control motor specific impulse for the i-th zone, i = 1, 2, or 3	sec
K647, 667	t'_{Ri}	Input operating time from stage initiation of the roll control function for the i-th zone, i = 2, 3	sec
K667, 670	S_{RRi}	Input reference area of the i-th raceway, i = 1, 2	ft^2

L-number	Symbol	Definition	Units
K668, 671	r_{Ri}	Input distance from the vehicle centerline to the i-th raceway center of pressure i = 1, 2	ft
K669, 672	ϕ_{Ri}	Input bank angle location of the i-th raceway where i = 1, 2. As seen from the rear of the vehicle, a positive angle is measured clockwise from the Z_b axis direction	deg
K673, 675, etc	M'_{Rj}	Input Mach number for aerodynamic rolling moment, j = 1, 2, ..., 10	dim
K674, 676, etc	C_{RRj}	Input raceway aerodynamic force coefficient corresponding to the M_{Rj}, j = 1, 2, ..., 10	dim
K693-699		Open	
K700	S_{Fz}	Input aerodynamic pitch fin lift and drag coefficient reference area	sq ft
K701	x_{hz}	Input missile body station of the pitch fin hinge axis	ft
K702	U_{hz}	Input pitch movable control fin hinge axis to the leading fin base root location distance to the pitch fin base root length ratio	dim
K703	ℓ_{bz}	Input pitch fin base root length	ft
K704	K_{ycf}	Input aerodynamic yaw fin deflection angle multiplier	dim
K705	K_{pcf}	Input aerodynamic pitch fin deflection angle multiplier	dim
K706	$\bar{C}'_{Lz}$	Input aerodyanmic linear pitch fin lift coefficient multiplier. If input zero is set to 1.0	dim
K707	$\bar{C}'_{lz}$	Input aerodynamic nonlinear pitch fin drag coefficient multiplier. If input zero is set to 1.0	dim
K708	$\bar{C}'_{Dz}$	Input aerodynamic pitch fin drag coefficient multiplier. If zero set equal to one	dim
K709	$\bar{K}'_{Lz}$	Input aerodynamic pitch fin drag due to lift multiplier. If zero set equal to one	dim
K710, 716, etc	MF_j	Input mach number for aerodynamic fin representation where j = 1, 2, ..., 15 per stage	dim
K711, 717, etc	C_{Lzj}	Input and calculated aerodynamic pitch fin nonlinear lift coefficient j = 1, 2, ..., 15 per stage	2/deg

L-number	Symbol	Definition	Units
K712, 718 etc	C_{lzj}	Input and calculated aerodynamic pitch fin nonlinear lift coefficient j = 1, 2,..., 15 per stage	1/deg ^{2}
K713, 719, etc	C_{Dzj}	Input aerodynamic pitch fin drag coefficient j = 1, 2, ..., 15 per stage	dim
K714, 720, etc	K_{Lzj}	Input aerodynamic pitch fin drag due to lift factor	rad
K715, 721, etc	U_{czj}	Input pitch aerodynamic control fin center of pressure as a ratio of fin chord length	dim
K800-999		Open	

3. SWITCHING CODE

Quantities which can be involved as switching functions or in the hunting procedure are assigned a code number shown in succeeding pages. The code number is input in the appropriate space on the load sheet and the program determines the parameter which corresponds to the code number. The parameter and not the code input is used in program equations and logic.

The input parameter code is designated by inputting the L-number (delete the"L" of the parameters). These parameters can be used only as independent variables in the hunting procedure.

If the sigma code number is input negatively, the absolute value of the parameter is then utilized in the program equations and logic .

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5000	T	t	Instantaneous time (sec)
5001	TB	t_B	Time from the current stage initiation (sec)
5002	TB2	t_{B2}	Time from Stage II initiation zero if I-II staging has not occurred (sec)
5003	TB3	t_{B3}	Time from Stage III initiation zero if II-III Staging has not occurred (sec)
5004	TB4	t_{B4}	Time from Stage IV initiation zero if III-IV staging has not occurred (sec)
5005	SST	t_k	Stage start time (sec)
5006	SST2	SST_2	Start time stage II (sec)
5007	SST3	SST_3	Start time stage III (sec)
5008	SST4	SST_4	Start time stage IV (sec)
5009	TBF	t_{Bf}	Trajectory burnout time for the stage below (sec)
5010	TT	t_T	Time from target maneuvering initiation (sec)
5011	TTS	t_{TS}	Target start time (sec)
5012	TA	t_a	Total flight time to the glide phase apogee altitude (sec)
5013	TE	t_E	Total flight time to atmospheric entry (sec)
5014	TF	t_f	Total flight time to the termination of the glide phase (sec)
5014	PERIOD	P	Glide phase orbital period (min)
5016-5017			Open
5018	RANGE	S	Missile ground range. Distance along the surface of the earth measured clockwise from the launch vertical to the local vertical down range (ft)
5019	SD	$\dot{S}$	Time rate change of missile down range (ft/sec)
5020	SC	S_c	Missile cross ground rage. Distance along earth surface from the launch vertical to the local vertical crosswise from launch azimuth (ft)
5021	SCD	$\dot{S}_c$	Time rate change of missile cross range (ft/sec)
5022	SA	S_a	Total missile ground range at flight apogee (nm)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5023	SE	S_E	Total missile ground range to atmospheric entry (nm)
5024	SF	S_f	Total missile ground range at the termination of the glide phase (nm)
5025	SS	S_s	Missile slant ground range. Distance along earth surface from the launch vertical to the local vertical slantwise (ft)
5026-5027			Open
5028	H	h	Missile geometric altitude. Distance between the surface of the reference body and the missile measured along the local vertical. Positive away from the reference body (ft)
5029	HD	$\dot{h}$	Time rate change of missile geometric altitude. Rate between the surface of the reference body and missile measured along the local vertical positive away from the reference body (ft/sec)
5030	HDD	$\ddot{h}$	Instantaneous altitude acceleration (ft/sec^2)
5031	HA	H_a	Apogee altitude of the missile during the glide phase (nm)
5032	HP	h_p	Perigee altitude of the missile during the glide phase (nm)
5033	HAP	h_{ap}	Height of apogee + perigee (nm)
5034	HAB	h_{ab}	Altitude above launcer (ft)
5035	RC	r_c	Instantaneous distance between the center of the reference body and the missile (ft)
5036-5037			Open
5038	GXl	g_{xl}	Local northernly component of gravity (ft/sec^2)
5039	GYl	g_{yl}	Local easternly component of gravity (ft/sec^2)
5040	GZl	g_{zl}	Local downward component of gravity (ft/sec^2)
5041	GXE	g_{xe}	Launch centered earth fixed northernly component of gravity (ft/sec^2)
5042	GYE	g_{ye}	Launch centered earth fixed easternly component of gravity (ft/sec^2)
5043	GZE	g_{xe}	Launch centered earth fixed downward component of gravity (ft/sec^2)
5044-5045			Open

L-number	FORTRAN Symbol	Engineer's Symbol	Definitions
5046	VE	V_e	Missile earth referenced velocity (ft/sec)
5047	VDE	$\dot{V}_e$	Time rate charge of missile earth reference velocity (ft/sec^2)
5048	VEK	V_{ek}	Earth fixed velocity at stages (ft/sec)
5049	VA	V_a	Missile velocity with respect to air (ft/sec)
5050	VI	V_I	Missile inertial velocity (ft/sec)
5051	VIA	V_{Ia}	Missile inertial velocity at apogee if powered flight ends at the time being printed (ft/sec)
5052	VIF	V_{If}	Missile inertial velocity at apogee and impact of intercept, respectively, if powered flight and the atmosphere end at the time being printed (ft/sec)
5053-5055			Open
5056	VW	v_w	Instantaneous wind speeds and time rate change respectively (ft/sec and ft/sec^2)
5057	VDW	$\dot{v}_w$	Open
5058	VAE	V_{aE}	Velocity with respect to the ambient air at entry (ft/sec)
5059	VIE	V_{IE}	Inertial velocity at entry conditions (ft/sec)
5060	VXXX	V_{xxx}	Command velocity used in the TMC command logic. V_{ecv} if $Fy = 1$, V_{ecm} if $Fy = 4$ (ft/sec^2)
5061	VDXXX	$\dot{V}_{xxx}$	Command acceleration used in the TMC command logic. $\dot{V}_{ecv}$ if $Fy = 1$, $\dot{V}_{ecm}$ if $Fy = 2$, and zero if $Fy = 1$ and $Fy = 2$ (ft/sec^2)
5062	VDECQ	$\dot{V}_{ecq}$	Command acceleration to constrain dynamic pressure used in the TMC command logic (ft/sec^2)
5063	*MACH	M	Missile Mach number (dim)
5064-5065			Open
5066	CAV	C_a	Speed of sound at the missile (ft/sec)
5067	DCAVDH	dC_a/dh	Partial derivative of the speed of sound with altitude (1/sec)
5068	PA	P_a	Ambient pressure at the missile (lb/ft^2)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5069	DPADH	dP_a/dh	Partial derivative of ambient pressure with altitude (lb/ft^3)
5070	Q	q	Missile dynamic pressure (lb/ft^2)
5071	PCC	P_{cc}	Commanded chamber pressure used in pintle motor control logic (lb/in^2)
5072	PSTAR	$P*$	Main motor nozzle critical pressure used in separated flow equations (lb/in^2)
5073	PS	P_s	Main motor nozzle separation pressured used in separated flow equations (lb/in^2)
5074	PE	P_e	Main motor exit pressure used in separated flow equations (lb/in^2)
5075	EPSILS	ϵ_s	Main motor nozzle separation expansion ratio used in separated flow equation (dim)
5076			Open
5077	GWI	g_{WI}	Percent web (dim)
5078	VCI	V_{cI}	Chamber volume (in^3)
5079	ASI	A_{SI}	Burn surface area (in^2)
5080	ATCC	A_{tcc}	Commanded throat area (in^2)
5081	CF	C_{FI}	Thrust coefficient (dim)
5082	CFO	C_{FO}	Thrust coefficient at optimum expansion ($P_e = P_a$) (dim)
5083	OMEGAP	ω_p	Pintle control frequency (rad/sec)
5084	GPI	g_{PI}	Fraction of propellant removed (dim)
5085	AXI	A_{XI}	Propellant extinguishment throat area (in^2)
5086-5091			Open
5092	*MASS	m	Instantaneous missile mass ($lb-sec^2/ft$)
5093 5094	W WD	W $\dot{W}$	Total instantaneous missile weight and total expended instantaneous missile flow respectively (lb and lb/sec)
5095	WB	W_B	Instantaneous gross vehicle weight minus the useful load (lb)

L-number	FORTRAN Symbol	Engineer's Symbol	Definitions
5096	WJT	W_{JT}	Total weight jettisoned
5097 5098	WM WDM	W_M $\dot{W}_M$	Total instantaneous expended main weight and weight flow respectively (lb and lb/sec)
5099 5100	WC WDC	W_C $\dot{W}_C$	Total instantaneous expended main weight and weight flow respectivel (lb and lb/sec)
5101	WS	W_S	Stage weight (lb)
5102	WSMAIN	W_{SM}	Initial weight of main motor (lb)
5103	WSCOMP	W_{SC}	Initial weight of complementing motor (lb)
5104	PIVM	I''_{vM}	Vacuum impulse under input main thrust curve (lb-sec)
5105	PIVC	I''_{vC}	Vacuum impulse under input complement any thrust curve (lb-sec)
5106-5108			Open
5109	EOM	E/m	Total missile energy per unit mass during the glide phase. Potential energy at the launcher is taken as zero (ft^2 sec^2)
5110	QAP	q_{α}'	Product of total angle of attack and dynamic pressure (lb/deg/ft^2)
5111	INCL	i	Orbital inclination angle (deg) time rate change respectively
5112	E	e	Eccentricity of the missile path during the glide phase (dim)
5113	LLR	ℓ_r	Missile travel distance on the rail launcher used in ground launch tape of flight (Ty=6) (ft)
5114	PHIA4	$\Phi_{a}{}^{a}{}_{4}$	Glide range angle to the apogee vertical (deg)
5115	FOWD	$\lvert F/\dot{W} \rvert$	Instantaneous effective specific impulse (sec)
5116	IBDDP	$\bar{I}_{\dot{\delta}p}$	Sum of pitch angular thrust vectoring velocities from stage initiation to the time being printed corrected for dither (deg)
5117	IBDDY	$\bar{I}_{\dot{\delta}y}$	Sum of yaw angular thrust vectoring velocities from stage initiation to the time being printed corrected for dither (deg)
5118	LV	L_v	Output ideal velocity vectoring losses (ft/sec)
5119-5121			Open

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5122	F	F	Total instantaneous thrust acting along missile centerline. Positive when thrust vector points forward along missile centerline(lb)
5123	FVAC	F_v	Instantaneous total vacuum thrust (lb)
5124	FM	F_M	Instantaneous main thrust (lb)
5125	FMV	F_{MV}	Instantaneous main vacuum thrust and thrust
5126	FDMV	$\dot{F}_{MV}$	rate respectively (lb and lb/sec)
5127	FC	F_C	Instantaneous complementary thrust (lb)
5128	FCV	F_{CV}	Instantaneous complementary vacuum thrust and
5129	FDCV	$\dot{F}_{CV}$	time rate respectively (lb and lb/sec)
5130	FCOM	F_{com}	Commanded altitude thrust used in TMC logic (lb)
5131	FVCOM	F_{vcom}	Commanded vacuum thrust used in TMC logic (lb)
5132	FVN	F_{VN}	Nominal vacuum thrust used in TMC logic (lb)
5133	FN	F_N	Nominal altitude thrust used in TMC Logic (lb)
5134	FHATC	$\hat{F}_c$	Thrust required to maintain V_e; i.e., retarding axial force used in TMC logic (lb)
5135	FCQMIN	F_{cqmin}	Require thrust so that the vehicle will maintain the minimum dynamic pressure used in TMC logic (lb)
5136	FCQMAX	F_{cqmax}	Maximum thrust so that the vehicle will not exceed the maximum dynamic pressured used in TMC logic (lb)
5137	FCALOS	F_{CALOS}	Command thrust to provide acceleration proportional to LOS rated used in TMC logic (lb)
5138	FCCLOS	$F_{cc\ell os}$	Command thrust to provide a minimum missile to target closing rate used in TMC logic (lb)
5139-5140			Open
5141	FX	F_x	Components of total vehicle thrust parallel to the coordinate axes of the b system (lb)
5142	FY	F_y	Components of total vehicle thrust parallel to the coordinate axes of the b system (lb)
5143	FZ	F_z	Components of total vehicle thrust parallel to the coordinate axes of the b system (lb)
5144	FXX	F_X	Thrust force along nozzle centerline (lb)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5145	FYY	F_Y	Component of total vehicle thrust moved to nozzle centerline; positive right (lb)
5146	FZZ	F_Z	Component of total vehicle thrust normal to nozzle centerline; positive down (lb)
5147	FTDY	F_{TDy}	Movable nozzle tail-wag-dog force in yaw positive to the right (lb)
5148	FTDZ	F_{TDz}	Movable nozzle tail-wag-dog force in pitch positive to the down (lb)
5149	FJDY	F_{JFy}	Jet damping yawing transverse force (lb)
5150	FJDZ	F_{JDz}	Jet damping pitching transverse force (lb)
5151-5153			Open
5154	DRL	$D_{r\ell}$	Rail launch friction drag (lb)
5133	CDELZ	$C_{\delta z}$	Aerodynamic pitch fin axial force (lb)
5156	NDELY	$N_{\delta y}$	Aerodynamic yaw fin normal force (lb)
5157	NDELZ	$N_{\delta z}$	Aerodynamic pitch fin normal force (lb)
5158	NPAC	N_{PAC}	Pitch aerodynamic control normal force per radian fin deflection angle (lb)
5159	NPAD	N_{PAD}	Pitch aerodynamic disturbing normal force per radian angle of attack (lb)
5160	NPDA	N_{PDA}	Total pitch disturbing normal force per radian angle of attack (lb)
5161	NNVA	N_{NVA}	Force normal to velocity vector per radian angle of attack (lb)
5162	NPCD	N_{PCD}	Total pitch control normal force per radian deflection angle (lb)
5163	NPEA	N_{PEA}	Pitch trim normal force per radian angle of attack (lb)
5164	C	C	Instantaneous aerodynamic axial force (dbi)
5165	NY	N_Y	Instantaneous yaw aerodynamic axial normal forces directed opposite to the direction of the Y_b axis
5166	NZ	N_Z	Instantaneous pitch aerodynamic normal forces directed opposite to the direction of the Z_b -axes (lb)
5167	NPY	N_{FY}	Aerodynamic force due to damping in yaw (lb)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5168	NPZ	N_{PZ}	Aerodynamic force due to damping and pitch (lb)
5169	ETABA	η_{ba}	Acceleration load factor along velocity vector (g's)
5170	ETABT	η_{bt}	Acceleration load factor transverse to the velocity vector (g's)
5171	ETABN	η_{bn}	Acceleration load factor normal to the velocity vector (g's)
5172-5174			Open
5175	IXX	I_{XX}	Roll moment of inertia about vehicle center-of-gravity (ft-lb-sec^2)
5176	IXY	I_{XY}	Roll-yaw product of inertia about vehicle center-of-gravity (ft-lb-sec^2)
5177	IXZ	I_{XZ}	Roll-pitch product of inertia about vehicle center-of-gravity (ft-lb-sec^2)
5178			Open
5179	IYY	I_{YY}	Pitch moment of inertia about vehicle center-of-gravity (ft-lb-sec^2)
5180	IYZ	I_{YZ}	Yaw-pitch product of inertia about vehicle center-of-gravity (ft-lb-sec^2)
5181-5182			Open
5183	IZZ	I_{ZZ}	Yaw moment of inertia about vehicle center-of-gravity (ft-lb-sec^2)
5184	IDXX	$\dot{I}_{XX}$	Time rate change of roll moment of inertia (ft-lb-sec)
5185	IDXY	$\dot{I}_{XY}$	Time rate change of roll-yaw product of inertia (ft-lb-sec)
5186	IDXZ	$\dot{I}_{XZ}$	Time rate change of roll-pitch product of inertia (ft-lb-sec)
5187			Open
5188	IDYY	$\dot{I}_{YY}$	Time rate change of pitch moment of inertia (ft-lb-sec)
5189	IDYZ	$\dot{I}_{YZ}$	Time rate change of yaw-pitch product of inertia (ft-lb-sec)
5190-5191			Open

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5192	IDZZ	$\dot{I}_{ZZ}$	Time rate change of yaw moment of inertia (ft-lb-sec)
. 193	IPRD	I_{PRD}	Pitch inertia rotation damping moment integral (ft-lb-sec)
5194	IYRD	I_{YRD}	Yaw inertia rotation damping moment integral (ft-lb-sec)
5195	IRRD	I_{RRD}	Roll inertia rotation damping moment integral (ft/sec)
5196-5201			Open
5202	MIQ	M_{IQ}	Unbalanced pitching moment about vehicle center-of-gravity (ft-lb)
5203	MIR	M_{IR}	Unbalanced yaw moment about vehicle center-of-gravity (ft-lb)
5204	MIP	M_{IP}	Unbalanced roll moment about vehicle center-of-gravity (ft-lb)
5205	MDQ	M_{DQ}	Perturbing moment about vehicle center-of-gravity in pitch (ft-lb)
5206	MDR	M_{DR}	Perturbing moment about vehicle center-of-gravity in yaw (ft-lb)
5207	MDP	M_{DP}	Perturbing moment about vehicle center-of-gravity in roll (ft-lb)
5208	MCQ	M_{CQ}	Controlling moment about vehicle center-of-gravity in pitch (ft-lb)
5209	MCR	M_{CR}	Controlling moment about vehicle center-of-gravity in yaw (ft-lb)
5210	MCP	M_{CP}	Controlling moment about vehicle center-of-gravity in roll (ft-lb)
5211	MNQ	M_{NQ}	Aerodynamic yawing moment about vehicle center-of-gravity (ft-lb)
5212	MNR	M_{NR}	Aerodynamic yawing moment about vehicle center-of-gravity (ft-lb)
5213	MNP	M_{NP}	Aerodynamic rolling moment about vehicle center-of-gravity (ft-lb)
5214	MFOQ	M_{FOQ}	Thrust offset pitching moment (ft-lb)
5215	MFOR	M_{FOR}	Thrust offset yawing moment (ft-lb)
5216	MFOP	M_{FOP}	Thrust offset rolling moment due to pitch and yaw TVC (ft-lb)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5217	MTDQ	M_{TDQ}	Movable nozzle tail-wag-dog moment about vehicle center-of-gravity in pitch (ft-lb)
5218	MTDR	M_{TDR}	Movable nozzle tail-wag-dog moment about vehicle center-of-gravity in yaw (ft-lb)
5219	MFVP	M_{FVP}	Rolling moment about vehicle center-of-gravity due to vortexing effect of axial gas flow through the nozzle (ft-lb)
5220	MJDQ	M_{JDQ}	Jet damping pitching moment (ft-lb)
5221	MJDR	M_{JDR}	Jet damping yawing moment (ft-lb)
5222	MRAP	M_{RAP}	Aerodynamic rolling moment induced by raceways (ft-lb)
5223	MFCQ	M_{FCQ}	Thrust vector control pitching moment (ft-lb)
5224	MFCR	M_{FCR}	Thrust vector control yawing moment (ft-lb)
5225	MFCP	M_{FCP}	Auxiliary roll thrust control moment (ft-lb)
5226	MDELQ	$M_{\delta Q}$	Pitching moment due to the aerodynamic control force (ft-lb)
5227	MDELR	$M_{\delta R}$	Yawing moment due to the aerodynamic control force (ft-lb)
5228	MDELP	$M_{\delta P}$	Rolling moment due to the aerodynamic control force (ft-lb)
5229	MCYQ	M_{CYG}	Aerodynamic axial force center-of-gravity offset yawing moment (ft-lb)
5230	MCZG	M_{CZG}	Aerodynamic axial force center-of-gravity offset pitching moment (ft-lb)
5231	MPTC	M_{PTC}	Pitch total thrust control moment per radian TVC deflection angle (ft-lb)
5332	MNSQ	M_{NSQ}	Aerodynamic static pitching moment about vehicle center-of-gravity (ft-lb)
5233	MNSR	M_{NSR}	Aerodyanmic static yawing moment about vehicle center-of-gravity (ft-lb)
5234	MPMC	M_{PMC}	Pitch main thrust control moment per radial TVC deflection angle (ft-lb)
5235	MNDQ	M_{NDQ}	Aerodynamic damping moment about vehicle center-of-gravity in pitch (ft-lb)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5236	MNDR	M_{NDR}	Aerodynamic damping moment about vehicle center-of-gravity in yaw (ft-lb)
5237			Open
5238	MPRR	M_{PRR}	Pitch inertial rotation reaction moment used in the automatic gain logic (ft-lb)
5239	MYRR	M_{YRR}	Yaw inertial rotation reaction moment used in the automatic altitude gain logic (ft-lb)
5240	MRRR	M_{RRR}	Roll Rotation reaction moment used in the automatic gain logic (ft-lb)
5241	MPAC	M_{FAC}	Pitch aerodynamic control moment per radian angle of attack (ft-lb)
5242	MYAC	M_{YAC}	Yaw aerodynamic control moment per radian angle of side slip (ft-lb)
5243	MARC	M_{RAC}	Roll aerodynamic control moment per radian fin deflection angle (ft-lb)
5244	MPAD	M_{PAD}	Pitch aerodynamic disturbing moment per radian angle of attack (ft-lb)
5245	MYAD	M_{YAD}	Yaw aerodynamic disturbing moment per radian angle of side slip (ft-lb)
5246	MRAD	M_{RAD}	Open
5247	MHY	M_{Hy}	Torque about the yaw fin hinge axis (ft-lb)
5248	MHZ	M_{Hz}	Torque about the pitch fin hinge axis (ft-lb)
5249			Open
5250	MPDA	M_{PDA}	Total pitch disturbing moment per radian angle of attack (ft-lb)
5251	MPCD	M_{PCD}	Total pitch control moment per radian deflection angle (ft-lb)
5252-5300			Open
5301	ALPHA	α	Instantaneous pitch angle of attack. Positive if the vehicle centerline is above the air velocity vector (deg)
5302	ALPHAD	$\dot{\alpha}$	Time rate change of angle of attack (deg/sec)
5303	ALPHAC	α_c	Command angle of attack (deg)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5304	ALPPE	α'_E	Effective total angle of attack used to compute the aerodynamic normal force (deg)
5305	ALBAR	$\bar{\alpha}'$	Still air total angle of attack (deg)
5306			Open
5307	ALPHBD	$\dot{\bar{\alpha}}$	Time rate change of still wind angle of attack (deg/sec)
5308	BETABD	$\dot{\bar{\beta}}$	Time rate change of still wind angle of side slip (deg/sec)
5309	APHRIM	α'	Total vehicle angle of attack. Angle between the centerline of the vehicle and the missile air velocity vector. Always positive (deg)
5310	ALBAR	$\bar{\alpha}$	Still wind angle of attack (deg)
5311	ALPHAE	α_E	Effective pitch angle of attack used to compute the aerodynamic normal force (deg)
5312	ALPHAM	α_m	Commanded angle of attack for constant angle of attack flight (Ty-2) (deg)
5313-5314			Open
5315	BETA	β	Angle of side slip. Positive if the vehicle centerline is left of the air velocity vector when viewed from the rear of the vehicle (deg)
5316	BETAD	$\dot{\beta}$	Time rate change of angle of side slip (deg/sec)
5317	BETAC	β_c	Commanded angle of side slip (deg)
5318	BETBAR	$\bar{\beta}$	Still wind angle of side slip
5319	BTPBAR	$\bar{\beta}'$	Still wind angle of side slip in the commanded coordinate system used to evaluate the local bank angle
5320	BETAE	β_E	Effective yaw angle of side slip used to compute the aerodynamic normal face (deg)
5321	GAM1	γ_1	Pitch flight path angle. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (deg)
5322	GAMD1	$\dot{\gamma}_1$	Pitch flight path angular rate. Positive up (deg/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5323	GAM2	γ_2	Azimuthal flight path angle. Angle between the horizontal projection of the earth reference velocity vector and the local north. Positive clockwise from north (deg)
5324	GAMD2	$\dot{\gamma}_2$	Azimuthal flight path angular rate (deg/sec)
5325			Open
5326	GAM2IA	γ_{2Ia}	Inertial azimuth flight path angle at apogee (deg)
5327	GAM1E	γ_{1E}	Pitch flight path angle with respect to the ambient air at entry conditions (deg)
5328	GAM1IE	γ_{1IE}	Entry conditions inertial pitch flight path angles, if powered flight and the atmospheric end at the time being printed (deg)
5329	GAM2IE	γ_{2IE}	Entry conditions inertial yaw flight path azimuth angle, if powered flight and the atmosphere end at the time being printed (deg)
5330	GAM1I	γ_{1I}	Inertial pitch flight path angle. Angle between the inertial velocity vector and the local tangent plane. Positive away from the earth (deg)
5331	GAM2I	γ_{2I}	Inertial azimuth flight path angle. Angle between local north clockwise to the projection of the inertial velocity vector on the local tangent plane (deg)
5332	GAM1IF	γ_{1If}	Impact or intercept inertial pitch flight path angle, if powered flight and the atmosphere end at the time being printed (deg)
5333	GAM2IF	γ_{2If}	Impact or intercept inertial yaw flight path azimuthal angle, if powered flight and the atmosphere end at the time being printed (deg)
5334	GAMMAR	γ_R	Output required velocity flight path angle at the missile instantaneous position (deg)
5335	GAMMAG	γ_G	Calculated local angle of velocity to be gained (deg)
5336	GAMM	γ_M	Relative azimuthal velocity vector angle in missile-target coordinates
5337	GAMDM	$\dot{\gamma}_M$	Relative azimuthal velocity vector angular rate in missile-target coordinates

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5338	DELMI	δ_{MI}	Azimuth flight path error to intercept. Used in type 10 flight (deg)
5339	DELPC	δ_{Pc}	Pitch plane thrust deflection commands (deg)
5340	DELYC	δ_{Yc}	Yaw plane thrust deflection commands (deg)
5341	DELRC	δ_{Rc}	Commanded roll control fin deflection angle (deg)
5342	DELBRP	$\bar{\tilde{\delta}}_P$	Modified pitch thrust deflection angle to include limit cycle and misalignment angle (deg)
5343	DELBRY	$\bar{\tilde{\delta}}_Y$	Modified yaw thrust deflection angle to include limit cycle and misalignment angles (deg)
5344	DELTHB	$\Delta\theta_b$	Vehicle pitch attitude error angle (deg)
5345	DELPSB	$\Delta\psi_b$	Vehicle yaw attitude error angle (deg)
5346	DELPHB	$\Delta\Phi_b$	Vehicle roll attitude error angle (deg)
5347	DELAK	Δ_{ak}	Pitch flare in constant used in evaluation Δ'_{ak} for restart (deg)
5348	DELBK	Δ_{bk}	Flare-in constant used in evaluating Δ'_{bk} for restart (deg)
5349	DELAP	Δ'_a	Pitch flare-in angle (deg)
5350	DELBP	Δ'_b	Yaw flare-in angle (deg)
5351	DELMT	δ_{MT}	Seeker yaw look angle (deg)
5352	DELDMT	$\dot{\delta}_{MT}$	Seeker yaw look angular rate (deg/sec)
5353	EPSILN	ϵ	Total angle of attack roll orientation angle. Angle between total angle of attack plane and yaw axis. Measured counterclockwise (deg)
5354	EPSBAR	$\bar{\epsilon}$	No-wind total angle of attack roll orientation angle (deg)
5355	EPSMI	ϵ_{MI}	Flight path error to estimated intercept (deg)
5356	EPSMT	ϵ_{MT}	Seeker pitch look angle (deg)
5357	EPSDMT	$\dot{\epsilon}_{MT}$	Seeker pitch look angular rate (deg/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5358	ZETA	ζ	Cross range angle. Angle between local vertical and vertical on firing azimuth down range location. Positive if positive is left of firing azimuth (deg)
5359	ZETAD	$\dot{\zeta}$	Cross range angular rate (deg/sec)
5360	LAMMT	λ_{MT}	Angle of missile to target line projection on horizontal and firing azimuth (deg)
5361	LAMDMT	$\dot{\lambda}_{MT}$	Angular rate of missile to target line projection or horizontal and firing azimuth (deg/sec)
5362	MU	μ	Instantaneous vehicle longitude. Value is positive or negative west or east of Greenwich, England, respectively (deg)
5363	MUP	μ'	Instantaneous vehicle change of longitude from launch longitude. Value is positive east (deg)
5364	MUPD	$\dot{\mu}'$	Vehicle longitude time rate change (deg/sec)
5365	MUA	μ_a	Vehicle apogee longitude if powered flight and the atmoshpere end at the time being printed (deg)
5366	MUF	μ_f	Missile impact or intercept longitude if powered flight and the atmosphere end at the time being printed (deg)
5367	RHO	ρ	Instantaneous vehicle latitude positive north of the equation $-90° \leq \rho \geq 90°$
5368	RHOD	$\dot{\rho}$	Vehicle latitude time rate change (deg/sec)
5369	RHOA	ρ_a	Vehicle apogee latitude if powered flight and the atmosphere end at the time being printed (deg)
5370	RHOF	ρ_f	Missile impact or intercept latitude if powered flight and the atmosphere end at the time being printed (deg)
5371	SIGMT	σ_{MT}	Angle of missile to target line and local horizatonal (deg)
5372	SIGDMT	$\dot{\sigma}_{MT}$	Angular rate of missile to target line and local horizontal
5373	SIGMI	σ_{MI}	Local flight path angle to estimated target, intercept (deg)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5374	UPSMI	Ψ_{MI}	Azimuthal angle to target intercept (deg)
5375	PHI	Φ	Instantaneous down range angle. Angle between the launch vertical and the local vertical on the down range azimuth point. Positive as shown in Figure 1.1, $\infty < \phi < \infty$ (deg)
5376	PHID	$\dot{\Phi}$	Instantaneous range angular rate. Positive down range (deg/sec)
5377			Open
5378	PHIA	Φ_a	Glide range angle to the apogee vertical (deg)
5379	PHIS	Φ_s	Instantaneous slant range angle. Angle between the launch vertical and the local vertical (deg)
5380	SCRPHI	φ	Local bank angle (deg)
5381	PHIF	Φ_f	Glide range angle to glide phase termination vertical used in Keperian impact predictions (deg)
5382	PSI	ψ	Vehicle azimuth in the launch horizontal plane (deg)
5383	PSIW	ψ_w	Instantaneous wind azimuth angles, measured in a plane parallel to the local tangent plane where j = 1, 2, ..., 30. Angle measured clockwise from north to the direction from which the wind is coming (deg)
5384-5400			Open
5401	QM	Q_m	Instantaneous desired pitch turning rate (deg/sec)
5402	RM	R_m	Instantaneous desired yaw turning rate (deg/sec)
5403	PM	P_m	Instantaneous desired roll turning rate (deg/sec)
5404-5430			Open
5431	DELS	Δ_S	Calculated earth surface down range difference of the target and missile (ft)
5432	DELSD	$\dot{\Delta}_S$	Calculated time rate change of the earth surface down range difference of the target and missile (ft/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5433	DELSC	ΔS_c	Calculated earth surface cross range difference of the target and missile (ft)
5434	DELSDC	$\dot{\Delta S}_c$	Calculated time rate change of the earth surface cross range difference of the target and missile (ft/sec)
5435	DELH	Δ_h	Calculated altitude difference of the target and missile (ft)
5436	DELHD	$\dot{\Delta}_h$	Calculated time rate change of the altitude difference of the target and missile (ft/sec)
5437	RMT	R_{MT}	Missile to target range distance (ft)
5438	RDMT	$\dot{R}_{MT}$	Time rate change of missile to target distance (ft/sec)
5439	TMI	t_{MI}	Estimated time to intercept (ft)
5440	RMI	R_{MI}	Estimated range to target intercept (ft)
5441	SMI	S_{MI}	Estimated earth surface down range at target intercept (ft)
5442	HMI	h_{MI}	Estimated altitude at target intercept (ft)
5443	SCMI	S_{CMI}	Estimated earth surface cross range at target intercept (ft)
5444	ATT	a_{TT}	Target tangential acceleration (g's)
5445	ATN	a_{TN}	Target normal to its velocity vector acceleration (g's)
5446	ATC	a_{TC}	Target transverse acceleration (g's)
5447-5448			Open
5449	FD	F_D	Instantaneous roll control system phase plane signal (deg)
5450	KS	K_S	Pressure error gain used in pintle area control low in the TMC (in^4/lb-sec)
5451	KP	K_P	Pressure rate gain used in pintle area control low in the TMC (sec)
5452	KCV	K_{cv}	Commanded thrust velocity error gain used in TMC (lb-sec/ft)
5453	KCQ	K_{cq}	Commanded thrust dynamic pressure error gain used in TMC (ft^2)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5454	KXXX	K_{XXX}	Commanded thrust system gain. Set equal to K_{CPR} if Fy=3 and K_{ALOS} if Fy=6 (dim)
5455	GZ	G_Z	Partial derivatives of altitude acceleration to vehicle altitude (ft/sec^2-deg)
5456	KZ	K_Z	Altitude error gain used in type 3 flight (deg/ft)
5457	KZD	$K_{\dot{Z}}$	Altitude rate gain used in type 8 flight (deg-sec/ft)
5458	KZDD	$K_{\ddot{Z}}$	Altitude acceleration gain used in type 8 flight (deg-sec^2/ft)
5459	KTHDD	$K_{\ddot{\theta}}$	Command attitude pitch attitude angular accleration gain used in type of flight 8 and 9 (sec^2)
5460	KPSDD	$K_{\ddot{\psi}}$	Command attitude, yaw attitude angular acceleration gain used in type of flight 8 and 9 (sec^2)
5461	KDP	K_{DP}	Instantaneous control system pitch attitude error gain (dim)
5462	KDY	K_{DY}	Instantaneous control system yaw attitude error gain (dim)
5463	KDR	K_{DR}	Instantaneous control system roll attitude error gain (dim)
5464	KIP	K_{IP}	Instantaneous control system angle of attack gain (dim)
5465	KIY	K_{IY}	Instantaneous control system angle of side slip gain (dim)
5466	KIR	K_{IR}	Roll control attitude bias gain (1/sec)
5467	KRP	K_{RP}	Instantaneous control system pitch rate gain (sec)
5468	KRY	K_{RY}	Instantaneous control system yaw rate gain (sec)
5469	KRR	K_{RR}	Instantaneous control system roll rate gain (sec)
5470-5490			Open

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5491	MAXVAL(1)	σ_{mx1}	Values which are designated by input, as quantity whose maximum value is to be printed following each stage time (dbi)
5492	MAXVAL(2)	σ_{mx2}	
5493	MAXVAL(3)	σ_{mx3}	
5494	MAXVAL(4)	σ_{mx4}	
5495	MAXVAL(5)	σ_{mx5}	
5496	STGVAL(1)	σ_{BX1}	Values which are designated by input, at staging which are available to the hunting procedure (dbi)
5497	STGVAL(2)	σ_{BX2}	
5498	STGVAL(3)	σ_{BX3}	
5499	STGVAL(4)	σ_{BX4}	
5500	STGVAL(5)	σ_{BX5}	
5501-5503			Open
5504	AXB	a_{Xb}	Component of vehicle acceleration due to total thrust and aerodynamic forces. Positive in the direction of the coordinate axes of b system (g's)
5505	AYB	a_{Yb}	Component of vehicle acceleration due to total thrust and aerodynamic forces. Positive in the direction of the coordinate Y axes of the b system (g's)
5506	AZB	a_{Zb}	Components of vehicle acceleration due to total thrust and aerodynamic forces. Positive in direction of the coordinate Z axes of b system (g's)
5507	XDABB	$\dot{X}_{abb}$	X component of missile velocity with respect to the ambient air in the b system (ft/sec)
5508	YDABB	$\dot{Y}_{abb}$	Y component of the missile velocity with respect to the ambient air in the b system (ft/sec)
5509	ZDABB	$\dot{Z}_{abb}$	z component of missile velocity with respect to the ambient air in the b system (ft/sec)
5510	XDDZBB	$\ddot{X}_{abb}$	x component of missile acceleration with respect to ambient air in the b system (ft/sec^2)
5511	YDDABB	$\ddot{Y}_{abb}$	y component of missile acceleration with respect to the ambient air in the b system (ft/sec^2)
5512	ZDDABB	$\ddot{Z}_{abb}$	z component of missile acceleration with respect to the ambient air in the b system (ft/sec^2)
5513	XCC	X_{cc}	Earth centered missile position northern axis component (ft)
5514	YCC	Y_{cc}	Earth centered missile position east from launcher (ft)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5515	ZCC	Z_{cc}	Earth centered missile position away from earth axis at launcher longitude (ft)
5516	XDCC	$\dot{X}_{cc}$	Northern component of missile velocity in earth centered coordinates (ft/sec)
5517	YDCC	$\dot{Y}_{cc}$	East from launcher component of missile velocity in earth centered coordinate (ft/sec)
5518	ZDCC	$\dot{Z}_{cc}$	Away from launcher longitude component of missile velocity in earth centered coordinates (ft/sec)
5519	XDDCC	$\ddot{X}_{cc}$	Northern component of missile acceleration in earth centered coordinates (ft/sec^2)
5520	YDDCC	$\ddot{Y}_{cc}$	East from launcher component of missile acceleration in earth centered coordinates (ft/sec^2)
5521	ZDDCC	$\ddot{Z}_{cc}$	Away from launcher longitude component of missile acceleration in earth centered coordinates (ft/sec^2)
5522	XGG	X_{gg}	Instantaneous component of vehicle position in the generalized coordinates down range from launcher (ft)
5523	YGG	Y_{gg}	Instantaneous component of vehicle position in the generalized coordinates cross range from launcher (ft)
5524	ZGG	Z_{gg}	Instantaneous component of vehicle positive vertical from launcher (ft)
5525	XDGG	$\dot{X}_{gg}$	Instantaneous component of vehicle velocity in the generalized coordinates down range from launcher (ft/sec)
5526	YDGG	$\dot{Y}_{gg}$	Instantaneous component of vehicle velocity in the generalized coordinates crosswise from launcher (ft/sec)
5527	ZDGG	$\dot{Z}_{gg}$	Instantaneous component of vehicle velocity in the generalized coordinates vertical from launcher (ft/sec)
5528	XDDGG	$\ddot{X}_{gg}$	Instantaneous component of vehicle acceleration in the generalized coordinates down range from launcher (ft/sec^2)
5529	YDDGG	$\ddot{Y}_{gg}$	Instantaneous component of vehicle acceleration in the generalized coordinates crosswise from launcher (ft/sec^2)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5530	ZDDGG	$\ddot{z}_{gg}$	Instantaneous component of vehicle acceleration in the generalized coordinates vertical from launcher (ft/sec^2)
5531	XDW11	$\dot{X}_{w11}$	Local northernly component of wind velocity (ft/sec)
5532	YDW11	$\dot{Y}_{w11}$	Local easternly component of wind velocity (ft/sec)
5533	ZDW11	$\dot{z}_{w11}$	Local downward component of wind velocity (ft/sec)
5534	XDDW11	$\ddot{X}_{w11}$	Local time rate change of northern component of wind velocity (ft/sec^2)
5535	YDDW11	$\ddot{Y}_{w11}$	Local time rate change of easternly component of wind velocity (ft/sec^2)
5536	ZDDW11	$\ddot{Z}_{w11}$	Local time rate change of downward component of wind velocity (ft/sec^2)
5537	XDBB	$\dot{X}_{bb}$	Inertial component of missile velocity along x_b axis (ft/sec)
5538	[illegible]BB	$\dot{Y}_{bb}$	Inertial component of missile velocity along y_b axis (ft/sec)
5539	ZDBB	$\dot{z}_{bb}$	Inertial component of missile velocity along z_b axis (ft/sec)
5540	XDDBB	$\ddot{X}_b$	Missile acceleration along vehicle body axis, position forward (ft/sec^2)
5541	YDDBB	$\ddot{Y}_b$	Inertial component of missile acceleration y_b axes (ft/sec^2)
5542	ZDDBB	$\ddot{Z}_b$	Inertial components of missile acceleration along z_b axis (ft/sec^2)
5543	XD11	$\dot{X}_{11}$	Local northernly component of missile velocity (ft/sec)
5544	YD11	$\dot{Y}_{11}$	Local easternly component of missile velocity (ft/sec)
5545	ZD11	$\dot{Z}_{11}$	Local downward component of missile velocity (ft/sec)
5546	XDD11	$\ddot{X}_{11}$	Local northernly component of missile acceleration (ft/sec^2)
5547	YDD11	$\ddot{Y}_{11}$	Local easternly component of missile acceleration (ft/sec^2)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5548	ZDD11	$\ddot{Z}_{11}$	Local downward component of missile acceleration (ft/sec^2)
5549	XDWEE	$\dot{X}_{wee}$	Transformed launcher northernly component of wind velocity at missile location (ft/sec)
5550	YDWEE	$\dot{Y}_{wee}$	Transformed launcher easternly component of wind velocity at missile location (ft/sec)
5551	ZDWEE	$\dot{Z}_{wee}$	Transformed launcher downward component of wind velocity at missile location (ft/sec)
5552	XD11I	$\dot{X}_{11I}$	Local northernly component of missile inertial velocity (ft/sec)
5553	YD11I	$\dot{Y}_{11I}$	Local easternly component of missile inertial velocity (ft/sec)
5554	ZD11I	$\dot{Z}_{11I}$	Local downward component of missile inertial velocity (ft/sec)
5555-5563			Open
5564	CLZ	C_{Lz}	Instantaneous aerodynamic pitch movable fins linear fin lift coefficient (1/deg^2)
5565	ClZ	C_{lz}	Instantaneous aerodynamic pitch movable fin nonlinear fin lift coefficient (1/deg^2)
5566	CDZ	C_{Dz}	Instantaneous aerodynamic pitch movable fin drag coefficient (dim)
5567	KLZ	K_{Lz}	Instantaneous aerodynamic pitch movable fin drag due to lift factor (dim)
5568	UCZ	U_{cz}	Pitch aerodynamic control fin center of pressure ratio relative to the root chord (dim)
5569	CN	C_N	Instantaneous aerodynamic normal force coefficient (dim)
5570	CN1	CN_1	Instantaneous first, second, and third derivatives, respectively, of C_N with respect to the angle of attack, (/deg, /deg^2, /deg^3, respectively)
5571	CN2	CN_2	
5572	CN3	CN_3	
5573	CA	C_A	Instantaneous aerodynamic axial force coefficient
5574	CBN	C_{BN}	Added aerodynamic base drag coefficient due to nozzles not thrusting (dim)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5575	CMQ	C_{MQ}	Calculated and unadjusted for translation aerodynamic pitch damping moment due to pitch rate coefficient (1/deg)
5576	CMAD	$C_{M\alpha}$	Calculated and unadjusted for translation aerodynamic pitch damping moment due to rate change of angle of attack coefficient (1/deg)
5577	CHLY	$\hat{C}_{Ly}$	Instantaneous aerodynamic yaw movable fin total lift coefficient (dim)
5578	CHLZ	$\hat{C}_{Lz}$	Instantaneous aerodynamic pitch movable fin total lift coefficient (dim)
5579	CHDZ	$\hat{C}_{Dz}$	Instantaneous aerodynamic pitch movable fin total drag coefficient (dim)
5580-5582			Open
5583	XCP	x_{cp}	Input and instantaneous (with Mach number M) aerodynamic normal force center of pressure body station numbers, respectively, where j = 1, 2, ..., 15 per stage (ft and dbi, respectively)
5584	XCG	x_{cg}	Instantaneous center of gravity body station numbers (ft)
5585	YCG	y_{cg}	Center of gravity offset bias distance positive in the Z_b direction (ft)
5586	ZCG	z_{cg}	Center of gravity offset bias distance positive down (dbi)
5587	XPF	x_{pf}	Computed stage forward end of propellant grain body station (ft)
5588	XPA	x_{Pa}	Computed stage aft end propellant grain body station (ft)
5589	YE	y_e	Thrust gimbal yaw point eccentricity position in the Y_b axis direction (ft)
5590	ZE	z_e	Thrust gimbal pitch point position in the Z_b axis direction (ft)
5591-5592			Open
5593	LE	l_e	Gimbal point to vehicle center of gravity distance (ft)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5594	LEE	ℓ_E	Nozzle exit to center of gravity distance (ft)
5595	LN	ℓ_n	Movable portion of the nozzle center of gravity to gimbal point distance (ft)
5596	LCP	ℓ_{cp}	Vehicle center of gravity to aerodynamic center of pressure distance (ft)
5597	LNN	ℓ_N	Vehicle center of gravity to stop-start motor thrust point distance (ft)
5598	LPF	ℓ_{Pf}	Forward end of propellang grain to center of gravity distance (ft)
5599	LPA	ℓ_{Pa}	Aft end of propellant grain to center of gravity distance (ft)
5600	LDELY	$\ell_{\delta y}$	Yaw movable control fin center of pressure to vehicle center of gravity lever arm (ft)
5601	LDELZ	$\ell_{\delta z}$	Pitch movable control fin center of pressure to vehicle center of gravity lever arm (ft)
5602	LHZ	ℓ_{hz}	Pitch movable control fin center of pressure to hinge axis lever arm (ft)
5603-5700			Open
5701	XDEE	$\dot{X}_{ee}$	Instantaneous components of vehicle velocity, north of the launcher (ft/sec)
5703	YDEE	$\dot{Y}_{ee}$	Instantaneous components of vehicle velocity, east of the launcher (ft/sec)
5703	ZDEE	$\dot{Z}_{ee}$	Instantaneous component of vehicle velocity, negative up from sea level launcher latitude (ft/sec)
5704	XEE	X_{ee}	Instantaneous component of vehicle position north of launcher (ft)
5705	YEE	Y_{ee}	Instantaneous component of vehicle position east of launcher (ft)
5706	ZEE	Z_{ee}	Instantaneous component of vehicle position negative up from sea level launcher latitude (ft)
5707	QB	Q_b	Instantaneous vehicle angular pitch velocity. Pitch up is positive (deg/sec)
5708	RB	R_b	Instantaneous vehicle angular yaw velocity. Yaw right is positive (deg/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5709	PB	P_b	Instantaneous vehicle angular roll velocity. Roll Clockwise is positive (deg/sec)
5710	THETAB	θ_b	Achieved missile Euler angle pitch attitude (deg)
5711	PSIB	ψ_b	Achieved missile Euler angle yaw attitude (deg)
5712	PHIB	Φ_b	Achieved missile Euler angle roll attitude (deg)
5713	DELDP	$\dot{\delta}_p$	Pitch thrust deflection angular rate positive up (deg/sec)
5714	DELDY	$\dot{\delta}_Y$	Yaw thrust deflection agular rate, positive left (deg/sec)
5715	DELDR	$\dot{\delta}_R$	Aerodynamic roll fins deflection angular rate (deg/sec)
5716	DELP	δ_p	Pitch thrust deflection angle. Positive up (deg)
5717	DELY	δ_y	Yaw thrust deflection angle. Positive left (deg)
5718	DELR	δ_R	Aerodynamic roll fins deflection angle (deg)
5719	FR	F_R	Instantaneous roll control thrust. Positive if the vehicle is intended to rotate clockwise as seen from the rear of the vehicle (lb)
5720	WR	W_R	Instantaneous expended weight due to roll control motor operation (lb)
5721	FRC	F_{Rc}	Instantaneous roll control system thrust command signal (lb)
5722	THETAM	θ_m	Desired missile attitude Euler angle relating the m and i system (deg)
5723	PSIM	ψ_m	Desired missile attitude Euler angle relating the m and i systems (deg)
5724	PHIM	Φ_m	Desired missile attitude Euler angle relating the m and i systems (deg)
5725	VT	V_T	Target tangential velocity (ft/sec)
5726	VN	V_N	Target normal velocity (ft/sec)
5727	VC	V_C	Target transverse velocity (ft/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5728	GAMT	γ_T	Target pitch flight path angle (deg)
5729	ZETAT	ζ_T	Target azimuthal flight path angle (deg)
5730	HT	h_T	Target altitude (ft)
5731	ST	S_T	Target down range (ft)
5732	STC	S_{TC}	Target cross range (ft)
5733	DB	D_b	Propellant burn depth. Used in TMC logic (in)
5734	PC	P_c	Main motor chamber pressure used in separated flow equation (lb/in^2)
5735	AT	A_t	Pintle nozzle throat area. Used in TMC logic (in^2)
5736	WPR	W_{pr}	Weight of propellant removed. Used in TMC logic (lb)
5737 5738	IMP IV	I I_V	Total missile impulse and vacuum impulse, respectively, measured from stage initiation to the time being printed (lb-sec)
5739	IP	I_P	Pitch control thrust impulse from stage initiation to the time being printed (lb-sec)
5740	IY	I_Y	Yaw control thrust impulse from stage initiation to the time being printed (lb-sec)
5741	IDELDP	$I_{\delta P}$	Sum of pitch angular thrust vectoring velocities from stage initiation to the time being printed (deg)
5742	IDELDY	$I_{\delta Y}$	Sum of yaw angular thrust vectoring velocities from stage initiation to the time being printed (deg)
5743	LF	L_F	Output total velocity loss due to back pressure from stage initiation to time being printed (lb-sec)
5744	DELTAV	Δ_V	Ideal missile velocity resulting from achieved thrust (ft/sec)
5745	IR	I_R	Auxiliary roll control system delivered total impulse (lb-sec)
5746	LD	L_D	Drag velocity loss from stage ignition (ft/sec)
5747	HE	H_e	Heating parameter. Integral of qv from stage initiation to the time being printed (lb/ft)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5748	LGG	L_g	Gravity losses, from trajectory initiation to the time being printed (ft/sec)
5749-5750			Open
5751	XDDEE	$\ddot{X}_{ee}$	Instantaneous northernly component of vehicle acceleration at launcher (ft/sec^2)
5752	YDDEE	$\ddot{Y}_{ee}$	Instantaneous easternly component of vehicle acceleration at launcher (ft/sec^2)
5753	ZDDEE	$\ddot{Z}_{ee}$	Instantaneous downward component of vehicle acceleration at launcher (ft/sec^2)
5754-5756			Open
5757	QDB	$\dot{Q}_b$	Instantaneous vehicle angular pitch acceleration. Pitch up positive (deg/sec^2)
5758	RDB	$\dot{R}_b$	Instantaneous vehicle angular yaw acceleration. Yaw right positive (deg/sec^2)
5759	PDB	$\dot{P}_b$	Instantaneous vehicle angular roll acceleration, roll clockwise positive (deg/sec^2)
5760	THETDB	$\dot{\theta}_b$	Achieved vehicle Euler angle pitch rate (deg/sec)
5761	PSIDB	$\dot{\psi}_b$	Achieved vehicle Euler angle yaw rate (deg/sec)
5762	PHIDB	$\dot{\phi}_b$	Achieved vehicle Euler angle roll rate (deg/sec)
5763	DELDDP	$\ddot{\delta}_p$	Pitch thrust deflection angular acceleration angle positive up (deg/sec^2)
5764	DELDDY	$\ddot{\delta}_y$	Yaw thrust deflection angular acceleration angle positive left (deg/sec^2)
5765-5768			Open
5769	FDR	$\dot{F}_R$	Time rate change of roll control thrust (lb/sec)
5770	WDR	$\dot{W}_R$	Roll control system mass flow rate (lb/sec)
5771			Open
5772	THETDM	$\dot{\theta}_m$	Desired vehicle pitch Euler angular rate (deg/sec)
5773	PSIDM	$\dot{\psi}_m$	Desired vehicle yaw Euler angular rate (deg/sec)
5774	PHIDM	$\dot{\phi}_m$	Desired vehicle roll Euler angular rate (deg/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5775	VTD	$\dot{V}_{Tj}$	Target tangential acceleration (ft/sec)
5776	VND	$\dot{V}_{Nj}$	Target normal acceleration (ft/sec)
5777	VCD	$\dot{V}_{Cj}$	Target transverse acceleration (ft/sec)
5778	GAMMTD	$\dot{\gamma}_T$	Target pitch flight path angular rate (deg/sec)
5779	ZETATD	$\dot{\zeta}_T$	Target azimuthal path angular rate (deg/sec)
5780	HTD	$\dot{h}_T$	Target altitude rate (ft/sec)
5781	STD	$\dot{S}_T$	Target down range rate (ft/sec)
5782	STCD	$\dot{S}_{TC}$	Target cross range rate (ft/sec)
5783	RBB	r_b	Propellant burn rate (in. /sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5784	PDC	$\dot{P}_C$	Time rate change of chamber pressure (lb/in^2-sec)
5785	ADT	$\dot{a}_t$	Time rate change of pintle throat area (in^2-sec)
5786	WDMT	$\dot{W}_{MT}$	Mass flow rate of gases three pintle nozzle throat (lb/sec)
5787-5800			Open
5801-5840	BLOCK(1)-(40)		Begin reserved for coding output block (dbi)
5841	Z(1,1)	$I_{FM}(1)$	Calculated main table nozzle back pressure impulse for stage I (lb-sec)
5842	Z(2,1)	$\hat{I}_{vM}(1)$	Calculated main table input total impulse adjusted to vacuum conditions for stage I (lb-sec)
5843	Z(3,1)	$I_{vM}(1)$	Calculated input vacuum corrected main table time adjusted thrust integral for stage I (lb-sec)
5844	Z(4,1)	$I^*_{vM}(1)$	Calculated main table vacuum adjusted thrust integral for stage I (lb-sec)
5845	Z(5,1)	$K^*_{FM}(1)$	Calculated main table thrust multiplier for stage I (lb-sec)
5846	Z(6,1)	$I_{spM}(1)$	Calculated main table adjusted to vacuum specific impulse for stage I (sec)
5847	Z(7,1)	$I_{FC}(1)$	Calculated complementary table nozzle back pressure impulse for stage I (lb-sec)
5848	Z(8,1)	$\hat{I}_{vC}(1)$	Calculated complementary table input total impulse adjusted to vacuum condition for stage I (lb-sec)
5849	Z(9,1)	$I_{vC}(1)$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage I (lb-sec)
5850	Z(10,1)	$I^*_{vC}(1)$	Calculated complementary table vacuum adjusted thrust integral for stage I (lb-sec)
5851	Z(11,1)	$K^*_{FC}(1)$	Calculated complementary table thrust multiplier for stage I (dim)
5852	Z(12,1)	$I_{spC}(1)$	Calculated complementary table adjusted to vacuum specific impulse for stage I (sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5853	Z(13,1)	$\hat{I}_{vT(1)}$	Calculated input main and complementary impulse corrected to vacuum condition for stage I (lb-sec)
5854	Z(14,1)	$I^*_{vT}(1)$	Calculated main and complementary vacuum adjusted thrust integral for Stage I (lb-sec)
5855	Z(15,1)	$I_{vT}(1)$	Calculated input vacuum corrected main and complementary impulse corrected for stage I (lb-sec)
5856	Z(1,2)	$I_{FM}(2)$	Calculated main table nozzle back pressure impulse for stage II (lb-sec)
5857	Z(2,2)	$\hat{I}_{vM}(2)$	Calculated main table input total impulse adjusted to vacuum conditions for stage II (lb-sec)
5358	Z(3,2)	$I_{vm}(2)$	Calculated input vacuum corrected main table time adjusted thrust integral for stage II (lb-sec)
5859	Z(4,2)	$I^*_{vM}(2)$	Calculated main table vacuum adjusted thrust integral for stage II (lb-sec)
5860	Z(5,2)	$K^*_{FM}(2)$	Calculated main table thrust multiplier for stage II (dim)
5861	Z(6,2)	$I_{spM}(2)$	Calculated main table adjusted to vacuum specific impulse for stage II (sec)
5862	Z(7,2)	$I_{FC}(2)$	Calculated complementary table nozzle back pressure impulse for stage II (lb-sec)
5863	Z(8,2)	$\hat{I}_{vC}(2)$	Calculated complementary table input total impulse adjusted to vacuum condition for stage II (lb-sec)
5864	Z(9,2)	$I_{vC}(2)$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage II (lb-sec)
5865	Z(10,2)	$I^*_{vC}(2)$	Calculated complementary table vacuum adjusted thrust integral for stage II (lb-sec)
5866	Z(11,2)	$K^*_{FC}(2)$	Calculated complementary table thrust multiplier for stage II(Dim)
5867	Z(12,2)	$I_{spC}(2)$	Calculated complementary table adjusted to vacuum specific impulse for stage II (sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5868	Z(13,2)	$\hat{I}_{vT}(2)$	Calculated input main and complementary impulse corrected to vacuum conditions for stage II (lb-sec)
5869	Z(14,2)	$I^*_{vT}(2)$	Calculated main and complementary vacuum adjusted thrust integral for Stage II (lb-sec)
5870	Z(15,2)	$I_{vT}(2)$	Calculated input vacuum corrected main and complementary impulse time corrected for stage II (lb-sec)
5871	Z(1,3)	$I_{FM}(3)$	Calculated main table nozzle back pressure impulse for stage III (lb-sec)
5872	Z(2,3)	$\hat{I}_{vM}(3)$	Calculated main table input total impulse adjusted to vacuum conditions for stage III (lb-sec)
5873	Z(3,3)	$I_{vm}(3)$	Calculated input vacuum corrected main table time adjusted thrust integral for stage III (lb-sec)
5874	Z(4,3)	$I^*_{vM}(3)$	Calculated main table vacuum adjusted thrust integral for stage III (lb-sec)
5875	Z(5,3)	$K^*_{FM}(3)$	Calculated main table thrust multiplier for stage III (dim)
5876	Z(6,3)	$I_{spM}(3)$	Calculated main table adjusted to vacuum specific impulse for stage III (sec)
5877	Z(7,3)	$I_{FC}(3)$	Calculated complementary table nozzle back pressure impulse for stage III (lb-sec)
5878	Z(8,3)	$\hat{I}_{vC}(3)$	Calculated complementary table input total impulse adjusted to vacuum conditions for stage III (lb-sec)
5879	Z(9,3)	$I_{vC}(3)$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage III (lb-sec)
5880	Z(10,3)	$I^*_{vC}(3)$	Calculated complementary table vacuum adjusted thrust integral for stage III (lb-sec)
5881	Z(11,3)	K^*_{FC}	Calculated complementary table thrust multipler for stage III (dim)
5882	Z(12,3)	$I_{spC}(3)$	Calculated complementary table adjusted to vacuum specific impulse for stage III (sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5883	Z(13,3)	$\hat{I}_{vT}(3)$	Calculated input main and complementary impulse corrected to vacuum conditions for stage III (lb-sec)
5884	Z(14,3)	$I^*_{vT}(3)$	Calculated main and complementary vacuum adjusted thrust integral for stage III (lb-sec)
5885	Z(15,3)	$I_{vT}(3)$	Calculated input vacuum corrected main and complementary impulse time corrected for stage III (lb-sec)
5886	Z(1,4)	$I_{FM}(4)$	Calculated main table nozzle back pressure impulse for stage IV(lb-sec)
5887	Z(2,4)	$\hat{I}_{vM}(4)$	Calculated main table input total impulse adjusted to vacuum conditions for Stage IV (lb-sec)
5888	Z(3,4)	$I_{vM}(4)$	Calculated input vacuum corrected main table time adjusted thrust integral for stage IV (lb-sec)
5889	Z(4,4)	$I^*_{vM}(4)$	Calculated main table vacuum adjusted thrust integral for stage IV (lb-sec)
5890	Z(5,4)	$K^*_{FM}(4)$	Calculated main table thrust multiplier for stage IV (dim)
5891	Z(6,4)	$I_{spM}(4)$	Calculated main table adjusted to vacuum specific impulse for stage IV (sec)
5892	Z(7,4)	$I_{FC}(4)$	Calculated complementary table nozzle back pressure impulse for stage IV (lb-sec)
5893	Z(8,4)	$\hat{I}_{vC}(4)$	Calculated complementary table input total impulse adjusted to vacuum conditions for stage IV (lb-sec)
5894	Z(9,4)	$I_{vC}(4)$	Calculated input vacuum corrected complementary table time adjusted thrust integral for stage IV (lb-sec)
5895	Z(10,4)	$I^*_{vC}(4)$	Calculated complementary table vacuum adjusted thrust integral for stage IV (lb-sec)
5896	Z(11,4)	$K^*_{FC}(4)$	Calculated complementary table thrust multiplier for stage IV (dim)
5897	Z(12,4)	$I_{spC}(4)$	Calculated complementary table adjusted to vacuum specific impulse for stage IV (sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5898	Z(13,4)	$\hat{I}_{vT(4)}$	Calculated input main and complementary impulse corrected to vacuum conditions for stage IV (lb-sec)
5899	Z(14,4)	$I^{*}_{vT(4)}$	Calculated main and complementary vacuum adjusted thrust integral for stage IV (lb-sec)
5900	Z(15,4)	$I_{vT(4)}$	Calculated input vacuum corrected main and complementary inpulse time corrected for stage IV (lb-sec)
5901	SAVED(1,1)	$V_{I(B1)}$	Missile inertial velocity at the termination of stage I (ft/sec)
5902	SAVED(2,1)	$h_{(B1)}$	Missile geometric altitude at the termination of stage I (ft)
*5903	SAVED(3,1)	$\dot{\gamma}_{1(B1)}$	Pitch flight path angle at the termination of stage I angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (deg)
5904	SAVED(4,1)	$W_{(B1)}$	Total missile weight at the termination of stage I (lb)
5905	SAVED(5,1)	$S_{f(B1)}$	Total missile ground range at the termination of the glide phase if the powered flight were to end at the termination of stage I (nm)
5906	SAVED(6,1)	$L_{F(B1)}$	Total thrust velocity loss due to back pressure for stage I (ft/sec)
5907	SAVED(7,1)	$L_{D(B1)}$	Drag velocity loss for stage I (ft/sec)
5908	SAVED (8,1)	$L_{g(B1)}$	Gravity velocity loss for stage I (Ft/sec)
5909	SAVED(9,1)	$L_{V(B1)}$	Vectoring velocity loss for stage I (ft/sec)
5910	SAVED(10,1)	$\Delta V_{(B1)}$	Ideal missile velocity for stage I (ft/sec)
5911	SAVED(11,1)	$a_{Xb(B1)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage I (g's)
5912	SAVED(12,1)	$q_{(B1)}$	Missile dynamic pressure at the termination of stage I (lb/ft^2)
5913	SAVED(13,1)	$I_{V(B1)}$	Total missile vacuum impulse for stage I (lb-sec)
5914	SAVED(14,1)	$h_{p(B1)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage I (nm)

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*Stored values of saved angles are in degrees

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5915	SAVED(15,1)	$h_{a(B1)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage I (nm)
5916	SAVED(16,1)	$t_{f(B1)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage I (sec)
*5917	SAVED(17,1)	$\gamma_{1If(B1)}$	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of stage I (deg)
5918	SAVED(18,1)	$V_{If(B1)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage I (ft/sec)
5919-5925			Open
5926	SAVED(1,2)	$V_{I(B2)}$	Missile inertial velocity at the termination of stage II (ft/sec)
5927	SAVED(2,2)	$h_{(B2)}$	Missile geometric altitude at the termination of stage II (ft)
*5928	SAVED(3,2)	$\gamma_{1(B2)}$	Pitch flight path angle at the termination of stage II. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth. (deg)
5929	SAVED(4,2)	$W(_{B2})$	Total instantaneous missile weight at the termination of stage II (lb)
5930	SAVED (5,2)	$S_{f(B2)}$	Total missile ground range at the termination of the glide phase of the powered flight were to end at the termination of stage II (nm)
5931	SAVED(6,2)	$L_{F(B2)}$	Total thrust velocity loss due to back pressure for stage II (ft/sec)
5932	SAVED(7,2)	$L_{D(B2)}$	Drag velocity loss for stage II (ft/sec)
5933	SAVED(8,2)	$L_{g(B2)}$	Gravity velocity loss for stage II (ft/sec)
5934	SAVED(9,2)	$L_{V(B2)}$	Vectoring velocity loss for stage II (ft/sec)
5935	SAVED(10,2)	$\Delta V(_{B2})$	Ideal missile velocity for stage II (ft/sec)
5936	SAVED(11,2)	$a_{Xb(B2)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage II (g's)

(Stored values of saved angles are in degrees)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5937	SAVED(12,2)	$q_{(B2)}$	Missile dynamic pressure at the termination of stage II (lb/ft)
5938	SAVED(13,2)	$I_{V(B2)}$	Total missile vacuum impulse for stage II (lb-sec)
5939	SAVED(14,2)	$h_{p(B2)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage II (nm)
5940	SAVED(15,2)	$h_{a(B2)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage II (nm)
5941	SAVED(16,2)	$t_{f(B2)}$	Total flight time to the termination of the flight phase if the powered flight were to end at the termination of stage II (sec)
5942	SAVED(17,2)	$\gamma_{1If(B2)}$	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of stage II (deg)
5943	SAVED(18,2)	$V_{If(B2)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage II (ft/sec)
5944-5950			Open
5951	SAVED(1,3)	$V_{I(B3)}$	Missile inertial velocity at the termination of stage III (ft/sec)
5952	SAVED(2,3)	$h_{(B3)}$	Missile geometric altitude at the termination of stage III (ft)
5953	SAVED(3,3)	$\gamma_{1(B3)}$	Pitch flight path angle at the termination of stage III. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (deg)
5954	SAVED(4,3)	$W_{(B3)}$	Total instantaneous missile weight at the termination of stage III (lb)
5955	SAVED(5,3)	$S_{f(B3)}$	Total missile ground range at the termination of the glide phase if the powered flight were to end at the termination of stage III (nm)
5956	SAVED(6,3)	$L_{F(B3)}$	Total thrust velocity loss due to back pressure for stage III (ft/sec)
5957	SAVED(7,3)	$L_{D(B3)}$	Drag velocity loss due to back pressure for stage III (ft/sec)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5958	SAVED(8,3)	$L_{g(B3)}$	Gravity velocity loss for stage III (ft/sec)
5959	SAVED(9,3)	$L_{V(B3)}$	Vectoring velocity loss for stage III (ft/sec)
5960	SAVED(10,3)	$\Delta_{V(B3)}$	Ideal missile velocity for stage III (ft/sec)
5961	SAVED(11,3)	$a_{Xb(B3)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage III (g's)
5962	SAVED(12,3)	$q_{(B3)}$	Missile dynamic pressure at the termination of stage III (lb/ft^2)
5963	SAVED(13,3)	$I_{V(B3)}$	Total missile vacuum impulse for stage III (lb-sec)
5964	SAVED(14,3)	$h_{p(B3)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage III (nm)
5965	SAVED(15,3)	$h_{a(B3)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage III (nm)
5966	SAVED(16,3)	$t_{f(B3)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage III (sec)
5967	SAVED(17,3)	$\gamma_{IIf(B3)}$	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of stage III (deg)
5968	SAVED(18,3)	$V_{If(B3)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage III (ft/sec)
5969-5975			Open
5976	SAVED(1,4)	$V_{I(B4)}$	Missile inertial velocity at the termination of stage IV (ft/sec)
5977	SAVED(2,4)	$h_{(B4)}$	Missile geometric altitude at the termination of stage IV (ft)
5978	SAVED(3,4)	$\gamma_{I(B4)}$	Pitch flight path angle at the termination of stage IV. Angle between the earth referenced velocity vector and the local tangent plane. Positive away from the earth (deg)
5979	SAVED(4,4)	$W_{(B4)}$	Total instantaneous missile weight at the termination of stage IV (lb)

L-number	FORTRAN Symbol	Engineer's Symbol	Definition
5980	SAVED(5,4)	$S_{f(B4)}$	Total missile ground range at the termination of the glide phase if the powered flight were to end at the termination of stage IV (nm)
5981	SAVED(6,4)	$L_{F(B4)}$	Total thrust velocity loss due to back pressure for stage IV (ft/sec)
5982	SAVED(7,4)	$L_{D(B4)}$	Drag velocity loss for stage IV (ft/sec)
5983	SAVED(8,4)	$L_{g(B4)}$	Gravity velocity loss for stage IV (ft/sec)
5984	SAVED(9,4)	$L_{V(B4)}$	Vectoring velocity loss for stage IV (ft/sec)
5985	SAVED(10,4)	$\Delta V_{(B4)}$	Ideal missile velocity for stage IV (ft/sec)
5986	SAVED(11,4)	$a_{Xb(B4)}$	Vehicle acceleration due to total thrust and aerodynamic force at the termination of stage IV (g's)
5987	SAVED(12,4)	$q_{(B4)}$	Missile dynamic pressure at the termination of stage IV (lb/ft^2)
5988	SAVED(13,4)	$I_{V(B4)}$	Total missile vacuum impulse for stage IV (lb/sec)
5989	SAVED(14,4)	$h_{p(B4)}$	Perigee altitude of the glide phase if the powered flight were to end at the termination of stage IV (nm)
5990	SAVED(15,4)	$h_{a(B4)}$	Apogee altitude of the glide phase if the powered flight were to end at the termination of stage IV (nm)
5991	SAVED(16,4)	$t_{f(B4)}$	Total flight time to the termination of the glide phase if the powered flight were to end at the termination of stage IV (sec)
5992	SAVED(17,4)	$\gamma_{IIf(B4)}$	Impact or intercept inertial pitch flight path angle if the powered flight were to end at the termination of stage IV (sec)
5993	SAVED(18,4)	$V_{If(B4)}$	Missile inertial velocity at the termination of the glide phase if the powered flight were to end at the termination of stage IV (ft/sec)
5994-5999			Open

4. Load Sheets

The load sheets are shown in the following pages.

INPUT SHEET

NAME

BASIC INPUT

DATE
$J = 0.001625$ $\bar{g}_e = 32.174$
$g_e = 32.14625$ $r_e = 20,926,400$
$\omega = 7.29211 E - 5$

PAGE

LOCATION	Basic Deck	Ref Run	Run				
L0	+	+	+				
	Stage, K_L	Time, t_0	Stage Time, t_k'	Att Start, T_y'	Mode Start, I_y	Initial	Conditions
L3	+	+	+	+	+		
	Sp Pt Start, S_y	Disc Pt Start, D_y					
L8	+	+					
	Velocity, V_{eo}	Flight Path, γ_{1o}	Altitude, h_o	Range, S_o			
L1 0	+	+	+	+			
	Launch, ψ_i	Launch, θ_i	Launch, ϕ_i	Launch Lat, ρ_L	Launch Long, μ_L	Launcher Alt, h_L	Launcher
L1 4	+	+	+	+	+	+	
	Payload, W_{PL}	Atmos End, h_E					
L2 0	+	+					
		Mass Grav, $\bar{g}_e$	Grav Accel, g_e	Radius, r_e	Rotation, ω		Earth
L2 3	+	+	+	+	+		
	Desired, θ_{mo}	Desired ψ_{mo}	Desired, ϕ_{mo}				
L2 8	+	+	+				
	Miss Rat, Q_{bo}	Miss Rat, R_{bo}	Miss Rat, P_{bo}	Achieved, θ_{bo}	Achieved ψ_{bo}	Achieved ϕ_{bo}	
L3 1	+	+	+	+	+	+	
	From k_{c1}	To k_{c2}	Impact Alt, h_f	K_γ σ_c	Entry Altitude, h_e	Orbital Element Values	
L3 7	+	+	+	1 +	+		
	Cross Range S_{co}	Azimuth Flight Path					
L4 3	+	+					
	Gen Coord, γ_g	Gen Coord, θ_g	Gen Coord, γ_g				Gen'l Coord
L4 6	+	+	+				
	W_{JT1}	σ_{J1}	K_{J1}	W_{JT2}	σ_{J2}	K_{J2}	Jettison Weight
L4 9	+	+	+	+	+	+	
	W_{JT3}	σ_{J3}	K_{J3}	W_{JT4}	σ_{J4}	K_{J4}	
L5 5	+	+	+	+	+	+	
	W_{JT5}	σ_{JT5}	K_{J5}	W_{JT6}	σ_{J6}	K_{J6}	
L6 1	+	+	+	+	+	+	
	W_{JT7}	σ_{J7}	K_{J7}	W_{JT8}	σ_{J8}	K_{J8}	
7	+	+	+	+	+	+	

SHEET NAME HUNTING PROCEDURES DATE PAGE

L73	P1	K_s	h_{t1}	K_a	σ_X	σ_a			Procedure 1
L80	Initial X, X_i	X Increment, ΔX	Final a, a_f	a Accuracy, Δ_a					
L84	P2	σ_z	z Accuracy, ϵ_z	Model Acc, ϵ_m	Run Time, R_t	Des Max, f_D	h_{t2}		Procedure 2
L91	σ_{x1}	Initial X, x_{i1}	X Incr, Δx_{i1}	σ_{y1}	Low y, y_{L1}	Upper y, y_{U1}	Constr Err, ϵ_{c1}	T_{T1}	Transf Const, a_{T1}
L100	σ_{x2}								
L109	σ_{x3}								
L118	σ_{x4}								
L127	σ_{x5}								
L136	σ_{x6}								
L145	σ_{x7}	x_{i7}	Δx_{i7}	σ_{y7}	y_{L7}	y_{U7}	ϵ_{c7}	T_{T7}	a_{T7}
L154	x_{U1}	x_{U2}	x_{U3}	x_{U4}	x_{U5}	x_{U6}	x_{U7}	Upper Lim of Indepen Var	
L161	x_{L1}	x_{L2}	x_{L3}	x_{L4}	x_{L5}	x_{L6}	x_{L7}	Lower Lim of Indepen Var	

INPUT SHEET

NAME COMPUTE AND PRINT TIMES

DATE

PAGE

L168	Δt_{c1}	t_{c1}	Δt_{c2}	t_{c2}	Δt_{c3}	t_{c3}	Δt_{c4}	t_{c4}				Compute Interva
L176	Δt_{c5}	t_{c5}	Δt_{c6}	t_{c6}	Δt_{c7}	t_{c7}	Δt_{c8}	t_{c8}				
L184	Δt_{p1}	t_{p1}	Δt_{p2}	t_{p2}	Δt_{p3}	t_{p3}	Δt_{p4}	t_{p4}				Main Print
L192	Δt_{p5}	t_{p5}	Δt_{p6}	t_{p6}	Δt_{p7}	t_{p7}	Δt_{p8}	t_{p8}				
L200	PL-DA	PL-H	PL-K	PL-KB	PL-N	PL-O	PL-IC					
L209	σ_{p1}	σ_{p2}	σ_{p3}	σ_{p4}	σ_{p5}	σ_{p6}	σ_{p7}	σ_{p8}	σ_{p9}	σ_{p10}	σ_{p11}	
L220	Δt_{p1}	t_{p1}	Δt_{p2}	t_{p2}	Δt_{p3}	t_{p3}	Δt_{p4}	t_{p4}				
L228	Δt_{p5}	t_{p5}	Δt_{p6}	t_{p6}	Δt_{p7}	t_{p7}	Δt_{p8}	t_{p8}				
L236	σ_{t1}	K_{t1}	σ_{t2}	K_{c2}	σ_{t3}	K_{t3}	σ_{t4}	K_{t4}				Special Print
L244	σ_{t5}	K_{t5}	σ_{t6}	K_{t6}	σ_{t7}	K_{t7}	σ_{t8}	K_{t8}				
L252	σ_{D1}	σ_{D2}	σ_{D3}	σ_{D4}	σ_{D5}	σ_{D6}	σ_{D7}	σ_{D8}				Discontinuity Print
L260	T_{m1}	σ_{m1}	T_{m2}	σ_{m2}	T_{m3}	σ_{m3}	T_{m4}	σ_{m4}	T_{m5}	σ_{m5}		Maximum Print
L270	T_{B1}	σ_{B1}	T_{B2}	σ_{B2}	T_{B3}	σ_{B3}	T_{B4}	σ_{B4}	T_{B5}	σ_{B5}		Stage Values

INPUT SHEET

NAME

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

σ_{A1} A'_{r1}

L 2 8 0 + +

σ_{A2} A'_{r2}

L 2 8 2 + +

σ_{A3} A'_{r3}

L 2 8 4 + +

σ_{A4} A'_{r4}

L 2 8 6 + +

σ_{A5} A'_{r5}

L 2 8 8 + +

σ_{A6} A'_{r6}

L 2 9 0 + +

σ_{A7} A'_{r7}

L 2 9 2 + +

σ_{A8} A'_{r8}

L 2 9 4 + +

σ_{A9} A'_{r9}

L 2 9 6 + +

σ_{A10} A'_{r10}

L 2 9 8 + +

σ_{A11} A'_{r11}

L 3 0 0 + +

σ_{A12} A'_{r12}

L 3 0 2 + +

Breakup Tolerance

A_t B_t A_x B_x

L 3 0 6 + + + +

INPUT SHEET
FORM TC NO 285

NAME
ATTITUDE CONTROL

DATE

PAGE

1 2 3 4 5 6	7 8 9	10 11 12 13 14	15 16 17 18 19 20 21 22 23 24 25 26	27 28 29 30 31 32 33 34 35 36	37 38 39 40 41 42 43 44 45 46 47	48 49 50 51 52 53 54 55 56 57 58	59 60 61 62 63 64 65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
	T_{y1}	σ_{f1}	to K_{f1}	$Q_{m1}, k_{D1}, h_{c1}, \eta_{cj}$	$R_{m1}, k_{R1}, \omega_z, \tau_{nj}$	$P_{m1}, \alpha_{max1}, r_{z1}$	τ_f	
L 3 1 0	+	+	+					
		σ_{f2}						
L 3 1 7	+	+	+					
		σ_{f3}						
L 3 2 4	+	+	+					
		σ_{f4}						
L 3 3 1	+	+	+					
		σ_{f5}						
L 3 3 8	+	+	+					
		σ_{f6}						
L 3 4 5	+	+	+					
		σ_{f7}						
L 3 5 2	+	+	+					
		σ_{f8}						
L 3 5 9	+	+	+					
		σ_{f9}						
L 3 6 6	+	+	+					
		σ_{f10}						
L 3 7 3	+	+	+					
		σ_{f11}						
L 3 8 0			+					
		σ_{f12}						
L 3 8 7	+	+	+					
		σ_{f13}						
L 3 9 4	+	+	+					

INPUT SH

NAME WIND PROFILE

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78

	Alt Mult, K_h	Vel Mult, K_v'	Angle Mult, K_ψ'			
L 407	+	+	+			
	Altitude, h_1	Wind Speed, v_{w1}	Wind Angle, ψ_{w1}	h_2	v_{w2}	ψ_{w2}
L 410	+	+	+	+	+	+
	h_3	v_{w3}	ψ_{w3}	h_4	v_{w4}	ψ_{w4}
L 416	+	+	+	+	+	+
	h_5	v_{w5}	ψ_{w5}	h_6	v_{w6}	ψ_{w6}
L 422	+	+	+	+	+	+
	h_7	v_{w7}	ψ_{w7}	h_8	v_{w8}	ψ_{w8}
L 428	+	+	+	+	+	+
	h_9	v_{w9}	ψ_{w9}	h_{10}	v_{w10}	ψ_{w10}
L 434	+	+	+	+	+	+
	h_{11}	v_{w11}	ψ_{w11}	h_{12}	v_{w12}	ψ_{w11}
L 440	+	+	+	+	+	+
	h_{13}	v_{w13}	ψ_{w13}	h_{14}	v_{w14}	ψ_{w14}
L 446	+	+	+	+	+	+
	h_{15}	v_{w15}	ψ_{w15}	h_{16}	v_{w16}	ψ_{w16}
L 452	+	+	+	+	+	+
	h_{17}	v_{w17}	ψ_{w17}	h_{18}	v_{w18}	ψ_{w18}
L 458	+	+	+	+	+	+
	h_{19}	v_{w19}	ψ_{w19}	h_{20}	v_{w20}	ψ_{w20}
L 464	+	+	+	+	+	+
	h_{21}	v_{w21}	ψ_{w21}	h_{22}	v_{w22}	ψ_{w22}
L 470	+	+	+	+	+	+
	h_{23}	v_{w23}	ψ_{w23}	h_{24}	v_{w24}	ψ_{w24}
L 476	+	+	+	+	+	+
	h_{25}	v_{w25}	ψ_{w25}	h_{26}	v_{w26}	ψ_{w26}
L 482	+	+	+	+	+	+
	h_{27}	v_{w27}	ψ_{w27}	h_{28}	v_{w28}	ψ_{w28}
L 488	+	+	+	+	+	+
	h_{29}	v_{w29}	ψ_{w29}	h_{30}	v_{w30}	ψ_{w30}
		+	+		+	+

INPUT SHEET
FORM TC NO 239

NAME SHAPER

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

Control flag, K_{sh}

Target range or orbital Alt, S_{sh}

L0598 + +

INPUT SHE
FORM TC NO

NAME

MODE CONTROL TABLE

DATE

AGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79

	M_{y1}	σ_{M1}	to K_{M1}	Mode Control Tab
L600	+	+	+	
		σ_{M2}		
L603	+	+	+	
		σ_{M3}		
L606	+	+	+	
		σ_{M4}		
L609	+	+	+	
		σ_{M5}		
L612	+	+	+	
		σ_{M6}		
L615	+	+	+	
		σ_{M7}		
L618	+	+	+	
		σ_{M8}		
L621	+	+	+	
		σ_{M9}		
L624	+	+	+	
		σ_{M10}		
L627	+	+	+	

INPUT -
FORM NO.

TARGET DYNAMICAL CONDITIONS

DATE

PAGE 9

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79

L630	a_{Tto} +	K_{Tto} +	Velocity, V_{Tto} +	Flight Path A, γ_{To} +	+	B_{to} +	Initial Co
L636	S_{to} +	STCO +					
L638	Time +	t_{T1}	Tang. Acc, a_{TT1} +	Normal Acc, a_{TN1} +	Tran Acc +	A_{tci}	
L644	+	t_{T2}	+	+	+		
L648	+	t_{T3}	+	+	+		
L652	+	t_{T4}	+	+			
L656	+	t_{T5}	+	+	+		
L660	+	t_{T6}	+	+	+		
L664		t_{T7}	+	+	+		
L668	+	t_{T8}	+	+	+		

INPUT SHEET
FORM NO.

NAME

DATE

PAGE

TVC DUTY CYCLE AND SLEW RATE CRITERIA

L670	No Duty Pt, Δ_{dc}	Duty Stage, K_{dc}	Final wind h, h_{α}	Start Wind h, h_{β}	
L674	Slew Freq, ω_s	Stage 1 F/W A_{FW1}	Vehicle Wt, W_{o1}		
L677	W_{TVC}	W_{exi}	I_{spaug}		Refly
L680	Noz Flange, x_{nf}	δ-design, δ_{me}	Alt Max Wind, h_{mxwd}	Titled Print Flag, K_{TPF}	

INPUT SHEET
FORM [illegible]

NAME THRUST MODULATION CONTROL

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

	Fy1	σ_{Fy1}	K_{Fy1}	TMCL	C_{fy1}	Fyl	MIN1	q_{max1}	q_{min1}
L 800	+	+		+ +		+		+	+
L 810	+	σ_{Fy2} +		+ +		+		+	+
L 820	+	σ_{Fy3} +		+ +		+		+	+
L 830	+	σ_{Fy4} +		+ +		+		+	+
L 840	+	σ_{Fy5} +		+ +	+	+		+	+
L 850	+	σ_{Fy6} +		+ +		+		+	+
L 860	+	σ_{Fy7} +		+ +	+	+		+	+

INPUT SHEET

NAME

DATE

PAGE

SPECIFIC VELOCITY TIME PROFILE

	Time,	Velocity,
L 8 7 0	t_{vp1}	V_{pj}
L 8 7 2	t_{vp2}	
L 8 7 4	t_{vp3}	
L 8 7 6	t_{vp4}	
L 8 7 8	t_{vp5}	
L 8 8 0	t_{vp6}	
L 8 8 2	t_{vp7}	
L 8 8 4	t_{vp8}	
L 8 8 6	t_{vp9}	
L 8 8 8	t_{vp10}	
L 8 9 0	t_{vp11}	
L 8 9 2	t_{vp12}	
L 8 9 4	t_{vp13}	
L 8 9 6	t_{vp14}	
L 8 9 8	t_{vp15}	

INPUT SHEET
FORM TC NO 300

NAME
MAIN THRUST-WEIGHT TABLE

DATE

PAGE

L 000	c_S	D_p	r'_{b1000}	k_3	Tot Impulse, I'_{vT}	Main [illegible] e, I'_{vM}	Weight, K_{Mo}
07	Thrust Pert, K'_{FM}	Weight Pert, K'_{WM}	Time Pert, K'_t	Specific Imp, I'_{spM}	Exit Area, A'_{eM}	W. Carry Over, K_O	
13	ϵ_d	γ'_d	C'_D	α'_d	a'_s	b'_s	c'_s
20	Parm						
21	Time from Ign ▨	Thrust, F_{M1}	Weight Flow, $\dot{W}_{M1}$				
23	t'_{M2}	F_{M2}	$\dot{W}_{M2}$				
26	t'_{M3}						
29	t'_{M4}						
32	t'_{M5}						
35	t'_{M6}						
38	t'_{M7}						
41	t'_{M8}						
44	t'_{M9}						
47	t'_{M10}						
50	t'_{M11}						
53	t'_{M12}						

INPUT SHEET
FORM TC NO. 230

NAME
MAIN THRUST-WEIGHT TABLE (continued)

DATE

PAGE

1	056	t_{M13}	F_{M13}	$\dot{w}_{M13}$		
	59	t_{M14}				
	62	t'_{M15}				
	65	t'_{M16}				
	68	t'_{M17}				
	71	t'_{M18}				
	74	t'_{M19}				
	77	t'_{M20}				
	80	t'_{M21}				
	83	t'_{M22}				
	86	t'_{M23}				
	89	t'_{M24}				
	92	t'_{M25}				
	95	ρ_p	τ_w	η	α_p	P_{max}

INPUT SHEET
FORM TC NO [illegible]

NAME

COMPLEMENTARY THRUST-WEIGHT TABLE

DATE

PAGE

1 ☐ 1 00	Thrust Pert, K'_{FC} +	Weight Pert, K'_{WC} +	Time Pert, K'_{tC} +	I_{Sp} Imp, I'_{spC} +	Exit Area, A_{eC} +	Initial Wt, W_{Co} +
06	Comp Impulse, I'_{YC} +	No Wt Carry, K_{NO} +	K_{BD} +	P_{arC} +		
11	Time ▨	Thrust, F_{C1} +	Weight Flow, W_{C1} +			
13	t'_{C2} +	F_{C2} +	W_{C2} +			
16	t'_{C3} +	+	+			
19	t'_{C4} +	+	+			
22	t'_{C5} +	+	+			
25	t'_{C6} +	+	+			
28	t'_{C7} +	+	+			
31	t'_{C8} +	+	+			
34	t'_{C9} +	+	+			
37	t'_{C10} +	+	+			
40	t'_{C11} +	+	+			
43	t'_{C12} +	+	+			
46	t'_{C13} +	+	+			
49	t'_{C14} +	+	+			

INPUT SHEET

NAME

DATE

PAGE

COMPLEMENTARY THRUST-WEIGHT TABLE [illegible]ntinued)

1□152	+ t_{C15}	+ F_{C15}	+ W_{C15}
55	+ t_{C16}	+	+
58	+ t_{C17}	+	+
61	+ t_{C18}	+	+
64	+ t_{C19}	+	+
67	+ t_{C20}	+	+
70	+ t_{C21}	+	+
73	+ t_{C22}	+	+
76	+ t_{C23}	+	+
79	+ t_{C24}	+	+
82	+ t_{C25}	+	+

INPUT SHEET
FORM TC NO 298

NAME TOTAL AXIAL DRAG COEFFICIENT

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

L 185	Reference Area, S_{RC}	Perturbation, C'						
188	Mach No, M_{A1}	C_{A1}	M_{A2}	C_{A2}	M_{A3}	C_{A3}	M_{A4}	C_{A4}
196	M_{A5}	C_{A5}	M_{A6}	C_{A6}	M_{A7}	C_{A7}	M_{A8}	C_{A8}
204	M_{A9}	C_{A9}	M_{A10}	C_{A10}	M_{A11}	C_{A11}	M_{A12}	C_{A12}
V 212	M_{A13}	C_{A13}	M_{A14}	C_{A14}	M_{A15}	C_{A15}		
L 185	Reference Area, S_{RC}	Perturbation, C'						
188	M_{A1}	C_{A1}	M_{A2}	C_{A2}	M_{A3}	C_{A3}	M_{A4}	C_{A4}
196	M_{A5}	C_{A5}	M_{A6}	C_{A6}	M_{A7}	C_{A7}	M_{A8}	C_{A8}
204	M_{A9}	C_{A9}	M_{A10}	C_{A10}	M_{A11}	C_{A11}	M_{A12}	C_{A12}
V 212	M_{A13}	C_{A13}	M_{A14}	C_{A14}	M_{15}	C_{A15}		
L 185	Reference Area, S_{RC}	Perturbation, $\bar{C}'$						
188	M_{A1}	C_{A1}	M_{A2}	C_{A2}	M_{A3}	C_{A3}	M_{A4}	C_{A4}
196	M_{A5}	C_{A5}	M_{A6}	C_{A6}	M_{A7}	C_{A7}	M_{A8}	C_{A8}
204	M_{A9}	C_{A9}	M_{A19}	C_{A10}	M_{A11}	C_{A11}	M_{A12}	C_{A12}
V 212	M_{A13}	C_{A13}	M_{A14}	C_{A14}	M_{A15}	C_{A15}		
218	Plan Form Area, S_{PF}	Centroid Plan Form Area, $\bar{x}_{PC}$						

INPUT SHEET

NAME

DATE

PAGE

BODY AERODYNAMIC NORMAL FORCE COEFFICIENTS

1 2 2 0	Reference Area, S_{RN}	Perturbation, N'	cp Mult, X_{cp}		
2 3	Mach No, M_{N1}	C_{N11}	C_{N21}	C_{N31}	x_{cp1}
2 8	M_{N2}				
3 3	M_{N3}				
3 8	M_{N4}				
4 3	M_{N5}				
4 8	M_{N6}				
5 3	M_{N7}				
5 8	M_{N8}				
6 3	M_{N9}				
6 8	M_{N10}				
7 3	M_{N11}				
7 8	M_{N12}				
8 3	M_{N13}				
8 8	M_{N14}				
9 3	M_{N15}				

INPUT SHEET

NAME

DATE

PAGE

AERODYNAMIC PITCH DAMPING COEFFICIENT

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

L 298	+ Ref Dia, D_{RN}	+ Pitch Pert, M_Q	+ Alpha Pert, $M_{\dot{\alpha}}$	
301	+ Pitch Tran Mult, K_Q	+ Pitch Mom Pt, x'_{RQ}	+ Alpha Tran Mult, $K_{\dot{\alpha}}$	+ Alpha Mom Pt, $x'_{R\dot{\alpha}}$
305	+ Mach No, M_{D1}	- C_{MQ1}	- $C_{M\dot{\alpha}1}$	
308	+ M_{D2}	-	-	
311	+ M_{D3}	-	-	
314	+ M_{D4}	-	-	
317	+ M_{D5}	-	-	
320	+ M_{D6}	-	-	
323	+ M_{D7}	-	-	
326	+ M_{D8}	-	-	
329	+ M_{D9}	-	-	
332	+ M_{D10}	-	-	
335	+ M_{D11}	-	-	
338	+ M_{D12}	-	-	
341	+ M_{D13}	-	-	
344	M_{D14}	-	-	

INPUT SHE

NAME

AERODYNAMIC PITCH DAMPING COEFFICIENTS (continued)

DATE

GE

M_{D15}	C_{MQ15}	$C_{M\dot{A}15}$
L ☐ 3 4 7 +	-	-

GUIDANCE DATA

DATE

PAGE

L 380 No Mtrs, n'_m | n'_c | Clus Rad, R_c | Imp Var, $\varphi_{\Delta t}$ | t_b Var. σ_{tb} | Bivar Ax, $\hat{}_y$ | Cluster Characteristics

L 386 Dit Amp, A_L | Steer Fac, $K_?$ | Dit Freq, ω_L | Pit Mis, δ_{MP} | Yaw Mis δ_{MY} | Side Impulse

Steering Coefficients, Pitch

L 391 C ...t, a_{01} | z Coeff, a_{11} | x Coeff, b_{11} | x^2 Coeff b_{21} | x^3 Coeff, b_{31} | Flow Time, τ_{f1}

L 397 ...t Par, σ_{g11} | From K_{g11} | Second Par, σ_{g21} | To K_{g21} | K_{ye} | τ_{yk}

L 403 δ_{mc} | ℓ_{sr} | K_{rc} | η_{vr}

L 417 Forward Prop, x'_{pf} | Aft Prop, x'_{pa} | Noz Exit, x'_E | Jet Damping

L 420 Pitch, δ_{po} | Rate, $\dot{\delta}_{po}$ | Init Cond

L 428 ΔQ_o | ΔR_o | Gain and Thrust Data

L 430 V.P. Mult, X_e | Body Station, x'_e | y Offset, y'_e | z Offset, z'_e | Thrust Flag, K_δ | F Pt and Mag

L 435 t Const, τ_c | Dam Rat, ζ_c | Freq, ω_c | Veh Dam, ζ_v | Veh Freq, ω_v | Control and Vehicle Characterist:

L 440 $K_{DP\Delta/b}$ L, L_1 | δ_{Pc} Lim, L_2 | δ_p Lim, L_3 | δ_p Lim, L_4 | δ_p Lim, L_5 | Pitch Limits

L 445 $K_{DY\Delta/b}$ L, L_6 | δ_{Yc} Lim, L_7 | δ_Y Lim, L_8 | δ_Y Lim, L_9 | δ_Y Lim, L_{10}

INPUT SHEET

NAME

GAINS

DATE

PAGE

		Att, K_{DP1}	Att, K_{DY1}	Rate, K_{RP1}	Rate, K_{RY1}	Bias, K_{IP1}	Bias, K_{IY1}	f_{G1}	o_{G1}	K_{G1}	Gains
L	450	K_{DP2}	K_{DY2}	K_{RP2}	K_{RY2}	K_{IP2}	K_{IY2}	f_{G2}	o_{G2}	K_{G2}	
L	459	K_{DP3}	K_{DY3}	K_{RP3}	K_{RY3}	K_{IP3}	K_{IY3}	f_{G3}			
L	468										

INPUT SHEET

NAME CENTER-OF-GRAVITY AND INERTIA DATA

DATE Note: $W_1 > W_2 > \ldots$

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

475 Y Mult, I_Y

478 Weight Mult. W | x_{cg} Mult, X_{cg} | x_{cg} Bias, z_{cg} | y'_{cg} Bias, y'_{cg}

482 Nozzle Wt, W_n | Nozzle I_n | Nozzle C.G., x_n

488 Total Wt, W_1 | Miss cg, x_{cg1} | Pit Inertia, I_{Y1} | z offset, z_{cg1}

498 W_2

508 W_3

518 W_4

528 W_5

538 W_6

548 W_7

558 W_8

568 W_9

578 W_{10}

588 W_{11}

[illegible] W_{12}

INPUT SHEET

NAME

CENTER-OF-GRAVITY AND INERTIA DATA (continued)

DATE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

		Total Wt, W_{13}	Miss cg, x_{cg13}	Pit Inertia, I_{Y13}	Offset, z_{cg13}
L	608	+	+	+	+
		W_{14}			
	618	+	+	+	+
		W_{15}			
V	628	+	+	+	+

INPUT SHEET
FORM FE NO 210

NAME

ROLL VALUES

DATE

PAGE

1 2 3 4 5 6	7 8 9 10 11 12 13 14	15 16 17 18 19 20 21 22	23 24 25 26 27 28 29 30	31 32 33 34 35 36 37 38	39 40 41 42 43 44 45 46	47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
	Lev Arm, I_{R1}	t Const, τ_{R1}	Hyst, D_1	Dead Ba, $F_{\Delta 1}$	Max F_c, L_{Rc1}		Roll Values
L 638	+	+	+	+	+		
	Att Ga, K_{DR1}		Rate Ga, K_{RR1}	I_{spR1}	To time, t_{R2}		
643	+	+	+	+	+		
	I_{R2}	τ_{R2}	D_2	$F_{\Delta 2}$	L_{Rc2}		
648	+	+	+	+	+		
	K_{DR2}		K_{RR2}	I_{spR2}	t_{R3}		
653	+	+	+	+	+		
	I_{R3}	τ_{R3}	D_3	$F_{\Delta 3}$	L_{Rc3}		
658	+	+	+	+	+		
	K_{DR3}		K_{RR3}	I_{spR3}			
663	+	+	+	+			

INPUT SHEET

NAME **AERODYNAMIC ROLLING MOMENT**

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

	Ref Area 1, S_{RR1}	cp Rad Loc 1, r_{R1}	Roll Ang 1, φ_{R1}	Ref Area 2, S_{RR2}	cp Rad Loc 2, r_{R2}	Roll Ang 2, φ_{R2}	
L 667	+		+	+	+	+	
73	Mach No, M_1 +						
75	M_2 +						
77	M_3 +						
79	M4 +						
81	M_5 +						
83	M_6 +						
85	M_7 +						
87	M_8 +						
89	M_9 +						
91	M_{10} +						

INPUT SHEET
FORM TE NO 503

NAME

AERODYNAMIC MOVABLE PITCH CONTROL FINS COEFFICIENTS

DATE

PAGE

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

	Fin Area, S_{Fz}	Hinge Pt B.S. x_{hz}	Fin Hinge Pt, U_{hz}	Root Chord length, l_{hz}	K_{pef}	K_{yof}
L 700	+	+	+	+		
	Fin Con. Flag K'_{cf}	Lin. Lift Mult $\bar{C}_{lz}$	Non Lift Mult C'_{lz}	Drag Mult C'_{Dz}	Drag D.T. Lift Mult, K'_{lz}	
705	+	+	+	+	+	
	Mach No. M_1	C_{Lz1}	C_{lz1}	C_{Dz1}	K_{Lz1}	U_{cz1}
710	+	+	+	+	+	+
	M_2					
716	+	+	+	+	+	+
	M_3					
722	+	+	+	+	+	+
	M_4					
728	+	+	+	+	+	+
	M_5					
734	+	+	+	+	+	+
	M_6					
740	+	+	+	+	+	+
	M_7					
746	+	+	+	+	+	+
	M_8					
752	+	+	+	+	+	+
	M_9					
758	+	+	+	+	+	+
	M_{10}					
764	+	+	+	+	+	+
	M_{11}					
770	+	+	+	+	+	+
	M_{12}					
776	+	+	+	+	+	+
	M_{13}					
782	+	+	+	+	+	+
	M_{14}					
788		+	+			+

INPUT SHEET
FORM TC NO

NAME

AERODYNAMIC MOVABLE PITCH CONTROL F[illegible] COEFFICIENTS (continued)

DATE

GE

1 2 3 4 5 6	7 8 9 10 11 12 13 14 15 16 17	18 19 20 21 22 23 24 25 26 27 28	29 30 31 32 33 34 35 36 37 38 39	40 41 42 43 44 45 46 47 48 49 50	51 52 53 54 55 56 57 58 59 60 61	62 63 64 65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
	N_{15}	C_{Lz15}	C_{lz15}	C_{Dz15}	K_{Lz15}	U_{cz15}	
1 7 9 4	+	+	+	+	+	+	

N. PRINTOUT

Printout formats for the input and output prints are given in this section. The X's and O's shown in the formats represent spaces for integers and minus signs, respectively. The numbers and letters appearing at the left side of the printlines are always printed with their respective printlines.

The basic deck (BD), reference run (RR), and run (RUN) numbers and page number appear on every printout page. The date is also printed.

1. Input Print

Input is printed as shown in the following pages.

If all the parameters in a line are zero, the line is deleted. If the lines in a titled section contain all zeros, the title is deleted.

The T-card message is printed after the words "INPUT FOR".

INPUT PRINT

```
BD  XXXX.  RR XXXX.  RUN XXXX.               DATE XXX XX XXXX  PAGE X
INPUT FOR (MESSAGE IN TRAJECTORY RUN CARD, COLUMNS 2 THRU 57)

        INITIAL CONDITIONS
     K_k              t_o           t'_k        T'_y  M'_y      S_y   D_y
 3   OX.           OXXXXX.XXXX  OXXXXX.XXXX  OXX.  OX.       OXX.  OXX.
       V_o           γ_o           h_o          S_o
10   OXXXXX.XXX  OXXXX.XXXXX  OXXXXXXXXX.  OXXXXXXXXX.
       ψ_i           θ_i           φ_i
14   OXXX.XXXXXX  OXXX.XXXXXX  OXXX.XXXXXX
       ρ_L           μ_L           h_l
17   OXX.XXXXXX  OXX.XXXXXX  OXX.XXXXXX
       W_PL          h_E           .
20   OXXXXXXX.XX  OXXXXXXX.  OXXXXXXXXX.
       J             ḡ_e           g_e           r_e             ω
23   OXXX.XXXXXX  OXXX.XXXXXX  OXXX.XXXXXX  OXXXXXXXXXX.X  OXX.XXXXXXXX
       θ_mo          ψ_mo          φ_mo
28   OXXX.XXXXX  OXXX.XXXXX  OXXX.XXXXX
       Q_bo          K_bo          P_bo
31   OXXX.XXXX  OXXX.XXXXX  OXXX.XXXXX
       θ_bo          ψ_bo          φ_bo
34   OXXX.XXXXX  OXXX.XXXXX  OXXX.XXXXX

     ORBITAL ELEMENT VALUES
       k_c1             k_c2             h_f          K_γ   σ_c    h_e
37   OX.XXXXXXEOXX  OX.XXXXXXEOXX  OXXXXXXX.  OX.   XXXX   OXXXXXXX

      TRANSVERSE INITIAL CONDITIONS
         S_co                       γ_20
43   OXXXXXXXXXX.              OXXXXX. XXXXX      OXXXXX. XXX

     GUIDANCE ALIGNMENT
       ψ_g           θ_g           φ_g
46   OXXX.XXXXX  OXXX.XXXXX  OXXX.XXXXX
```

JETTISON WEIGHT

	W_{JT1}	σ_{J1}	K_{J1}
49	OX.XXXXXXEOXX	XXXX	OX.XXXXXXEOXX
	.	.	.
	.	.	.
	W_{JT8}	σ_{J8}	K_{J8}
70	OX.XXXXXXEOXX	XXXX	OX.XXXXXXEOXX

HUNT PROCEDURE 1

	P_1	K_x	n_{t1}	K_a	σ_X	σ_a	K_b
73	OX.	OX.	OXXX.	OX.	OXXXX	OXXXX	OXXX.

	X_i	ΔX	a_f	Δ_a
80	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OXXXX.XXXXX

HUNT PROCEDURE 2

	P2	σ_z	ϵ_z	ϵ_m	R_t	f_D	n_{t2}
84	OX.	OXXXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.X

	σ_{x1}	x_{i1}	Δx_{i1}	σ_{y1}	y_{L1}	y_{U1}
91	OXXXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OXXXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX
		ϵ_{c1}		T_{T1}	a_{T1}	
97		OX.XXXXXXEOXX		OXXX	OX.XXXXXXEOXX	
	.	.	.	.	.	.
	.	.	.	.	.	.
	σ_{x7}	x_{i7}	Δx_{i7}	σ_{y7}	y_{L7}	y_{U7}
145	OXXXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OXXXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX
		ϵ_{c7}		T_{T7}	a_{T7}	
151		OX.XXXXXXEOXX		OXXX	OX.XXXXXXEOXX	

UPPER LIMIT P-2

	x_{U1}	x_{U2}	x_{U3}	x_{U4}
154	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX
	x_{U5}	x_{U6}	x_{U7}	
158	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	

LOWER LIMIT P-2

	x_{L1}	x_{L2}	x_{L3}	x_{L4}
161	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX	OX.XXXXXXEOXX

x_{L5} x_{L6} x_{L7}

165 OX.XXXXXXEOXX OX.XXXXXXEOXX OX.XXXXXXEOXX OX.XXXXXXEOXX

COMPUTE INTERVAL

Δt_{c1} t_{c1} Δt_{c2} t_{c2}

168 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{c3} t_{c3} Δt_{c4} t_{c4}

172 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{c5} t_{c5} Δt_{c6} t_{c6}

176 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{c7} t_{c7} Δt_{c8} t_{c8}

180 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

MAIN PRINT

Δt_{p1} t_{p1} Δt_{p2} t_{p2}

184 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{p3} t_{p3} Δt_{p4} t_{p4}

188 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{p5} t_{p5} Δt_{p6} t_{p6}

192 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{p7} t_{p7} Δt_{p8} t_{p8}

196 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

PRINT FLAGS

PL-DA PL-GB PL-K PL-KB PL-N PL-O

200 OX. OX. OX. OX. OX. OX. OX. OX. OX.

AUXILIARY PRINT

σ_{P1} σ_{P2} σ_{P3} σ_{P4} σ_{P5} σ_{P6} σ_{P7} σ_{P8} σ_{P9} σ_{P10} σ_{P11}

209 OXXXX OXXXX OXXXX OXXXX OXXXX OXXXX OXXXX OXXXX OXXXX OXXXX OXXXX

Δt_{P1} t_{P1} Δt_{P2} t_{P2}

220 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{P3} t_{P3} Δt_{P4} t_{P4}

224 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

Δt_{P5} t_{P5} Δt_{P6} t_{P6}

228 OXXXX.XXXXX OXXXXX.XXXX OXXXX.XXXXX OXXXXX.XXXX

	Δt_{P7}	t_{P7}	Δt_{P8}	t_{P8}
232	OXXXX.XXXXX	OXXXXX.XXXX	OXXXX.XXXXX	OXXXXX.XXXX

SPECIAL PRINT

	σ_{t1}	K_{t1}	σ_{t2}	K_{t2}
236	OXXXX	OX.XXXXXXEOXX	OXXXX	OX.XXXXXXEOXX
	σ_{t3}	K_{t3}	σ_{t4}	K_{t4}
240	OXXXX	OX.XXXXXXEOXX	OXXXX	OX.XXXXXXEOXX
	σ_{t5}	K_{t5}	σ_{t6}	K_{t6}
244	OXXXX	OX.XXXXXXEOXX	OXXXX	OX.XXXXXXEOXX
	σ_{t7}	K_{t7}	σ_{t8}	K_{t8}
248	OXXXX	OX.XXXXXXEOXX	OXXXX	OX.XXXXXXEOXX

DISCONTINUITY PRINT

	σ_{D1}	σ_{D2}	σ_{D3}	σ_{D4}	σ_{D5}	σ_{D6}	σ_{D7}	σ_{D8}
252	OXXXX	OXXXX	OXXXX	OXXXX	OXXXX	OXXXX	OXXXX	OXXXX

MAXIMUM PRINT

	T_{m1}	σ_{m1}	T_{m2}	σ_{m2}	T_{m3}	σ_{m3}	T_{m4}	σ_{m4}	T_{m5}	σ_{m5}
260	OXXX.	OXXXX	OXXX.	OXXXX	OXXX.	OXXXX	OXXX.	OXXXX	OXXX.	OXXXX

STAGING VALUES

	T_{B1}	σ_{B1}	T_{B2}	σ_{B2}	T_{B3}	σ_{B3}	T_{B4}	σ_{B4}	T_{B5}	σ_{B5}
270	OX.	OXXXX	OX.	OXXXX	OX.	OXXXX	OX.	OXXXX	OX.	OXXXX

INTEGRATION TOLERANCES

	σ_{A1}	A'_{r1}
280	OXXXX	OX.XXXXXXEOXX
	σ_{A2}	A'_{r2}
282	OXXXX	OX.XXXXXXEOXX
	σ_{A12}	A'_{r8}
302	OXXXX	OX.XXXXXXEOXX

BREAK UP TOLERANCES

	A_t	B_t	A_x	B_x
L306	±0.XXXXXXXXE±XX	±0.XXXXXXE±XX	±0.XXXXXXE±XX	±0.XXXXXXXXE±XX

ATTITUDE CONTROL

	T_{y1}	σ_{f1}	K_{f1}	$Q_{m16}, \alpha_{i1}, \kappa_{D1}$	$R_{m1}, \alpha_{i1}, \kappa_{R1}$	P_{m1}, α_{max1}	
310	0X.	0XXXX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0XXX.XXXXXX	
	.	.	.	.	.	.	
	.	.	.	.	.	.	
	T_{y13}	σ_{f13}	K_{f16}	$Q_{m16}, \alpha_{i16}, \kappa_{D16}$	$R_{m16}, \alpha_{i16}, \kappa_{R13}$	P_{m13}, α_{max13}	τ_{P13}
394	0X.	0XXXX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0XXX.XXXXXX	0XXXX.XXXX
401	0XXXX.XXXX	0XXXX. XXXX	0XXXX. XXX	0XXXX.XXXX	0XXXX.XXXX		

WIND PROFILE

	K_h	K_v	$K_{\cdot}$
407	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX
	h_1	v_{w1}	$\cdot_{w1}$
410	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0XXX.XX
	.	.	.
	.	.	.
	h_{30}	v_{w30}	$\cdot_{w30}$
497	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0XXX.XX

MODE

	M_{y1}	σ_{M1}	K_{M1}
600	0X.	0XXXX	0X.XXXXXXE0XX
	.	.	.
	.	.	.
	M_{y10}	σ_{M10}	K_{M10}
627	0X.	0XXXX	0X.XXXXXXE0XX

TARGET CYNAMICAL CONDITIONS

	σ_{Tto}	K_{Tto}				
630	0XXXX	0X.XXXXXXXXE0XX				
	V_{TO}	γ_{TO}	h_T	ζ_{TO}	S_{TOO}	S_{TCO}
632	0XXXXXX.XXX	0XXXX.XXXXX	0XXXX XXX.	0XXXXXXXX	0XXXXXXXXX.	0XXXXXXXX
633	0XXXX.XXXX	0XXXX XXXX				

t_{T1} a_{TT1} a_{TN1} a_{TC1}

640 OXXXXX.XXXX OXXX.XXXXX OXXX.XXXXX OXXX.XXXXX

. . .

. . .

t_{T7} a_{TT7} a_{TN7} a_{TC7}

664 OXXXXX.XXXX OXXX.XXXXX OXXX.XXXXX OXXX.XXXXX

668 OXXXX.XXXX OXXXX.XXXX

DUTY CYCLE AND SLEW RATE GRIT

Δ_{dc} k_{dc} h_{α} h_{β}

670 OXX OX OXXXXXX.XX OXXXXXX.XX

ω_{S} A_{FW1} W_{01}

674 XXXX.XXXXX OXX.XXXXXX XXXXXXXXX.

TVC PROPERTIES

W_{TVC} W_{exi} I_{spaug}

677 OX.XXXXXXEOXX OX.XXXXXXEOXX OXXX.XXXXXX

x_{nf} δ_{me} h_{mxwd} k_{tpf}

680 OXXXX.XXXXX OXX.XXXXXX OXXXXXX.XX OXX.

TITLED PRINT FLAG

K_{TPF} K_{TIF}

683 OXX. OXX

THRUST MODULATION CONTROL

	F_{y1}	σ_{Fy1}	K_{Fy1}	T_{MC1}	C_{Fy1}	τ_{Fy1}
800	OXX.	OXXXX.	OX.XXXXXXEOXX	OXX.	OX.XXXXXXEOXX	OXXXXX.XXXXX
	MIN_1	$q_{max\ 1}$	$q_{min\ 1}$			
806	OX.XXXXXXEOXX	OXXXXX.XXXXX	OXXXXX.XXXXX	OXXXXX.XXXXX		
	.	.	.	.		
	.	.	.	.		
	.	.	.	.		
	F_{y7}	σ_{Fy7}	K_{Fy7}			
860	OXX	OXXXX.	OX.XXXXXXEOXX	OXX.	OX.XXXXXXEOXX	OXXXXX.XXXXX
	MIN_7	$q_{max\ 7}$	$q_{min\ 7}$			
866	OX.XXXXXXEOXX	OXXXXX.XXXXX	OXXXXX.XXXX	OXXXXX.XXXXX		

SPECIFIC VELOCITY TIME PROFILE

	t_{VP1}	V_{P1}
L870	OXXXXX.XXXXX	OXXXXX.XXXXX
	.	.
	.	.
	.	.
	t_{VP15}	V_{p15}
L898	OXXXXX.XXXXX	OXXXXXX.XXXXXX

INPUT PRINT

STAGE ONE

MAIN THRUST-WEIGHT

σ_S D_P r'_{b1000} k_3

1000 0XXXX 0X.XXXXXXE0XX 0X.XXXXXXE0XX 0X.XXXXXXE0XX

I'_{vT} I'_{vM} W

1004 0.XXXXXXXXX.X 0XXXXXXXX.X 0X.XXXXXXE0XX

K'_F K_W K_t I_{spM} A'_{eM} K_0

1007 0X.XXXXXXXX 0X.XXXXXXXX 0X.XXXXXXXX 0XXX.XXXXXX 0XXX.XXXXXX 0X.

ϵ_d γ_d C'_D α_d

1013 0XXXX.XXX 0XXXX.XXX 0XXXX.XXX 0XXXX.XXX

a_s b_s c_s P_{arM}

1017 0XXXX.XXX 0XXXX.XXX 0XXXX.XXX 0XXXX.XXX

t'_1 F_1 $\dot{W}_{25}$

1020 0XXXXX.XXXX 0X.XXXXXXE0XX 0X.XXXXXXE0XX

. . .

. . .

t'_{25} F_{25} $\dot{W}_{25}$

1092 0XXXXX.XXXX 0X.XXXXXXE0XX 0X.XXXXXXE0XX

MAIN THRUST MOTOR BALLISTICS

```
K095  X.XXXXX           X.XXXXXX          X.XXXXXX          ±O XXXXXXD±XX     XXXXX.XXX
      CSTAR             ATREF             RB1000            FWT               WP
      XXXXX.XXX         XXXX.XXXX         XX.XXXXXX         XXXX.XXX          XXXXXXXX.XXX
      ITM               A                 CAS1              CAS2              CAS3
      ±O.XXXXXXD±XX     X.XXXXXXXX        ±O.XXXXXXD±XX     ±O.XXXXXXD±XX     ±O.XXXXXXD±XX
      AXMAX             AXMIN             PCMXA             PCBREF
      XXXXXX.XXX        XXXXXX.XXX        XXXXX.XXX         XXXXX.XXX
J     TIME              VAC THRUST        CH.PRES           PCDOT             PROP FRACT.
II    XXX.XXXXX         ±O.XXXXXXD±XX     XXXXX.XXX         ±O.XXXXXXD±XX     X.XXXXXX
.         .                 .                 .                 .                 .
.         .                 .                 .                 .                 .
IJ    XXX.XXXXX         ±O.XXXXXXD±XX     XXXXX.XXX         ±O.XXXXXXD±XX     X.XXXXXX

J     TIME              WEB               SURFACE           VOLUME            EX. AREA
      XXX.XXXXX         X.XXXXXX          ±O.XXXXXXD±XX     ±O.XXXXXXD±XX     XXXXXX.XXX
.         .                 .                 .                 .                 .
.         .                 .                 .                 .                 .
IJ    XXX.XXXXX         X.XXXXXX          ±O.XXXXXXD±XX     ±O.XXXXXXD±XX     XXXXXX.XXX
```

COMPLEMENTARY THRUST-WEIGHT

	K'_{FC}	K_{WC}	K_{tC}	I_{spC}	A_{eC}
1100	0X.XXXXXXXX	0X.XXXXXXXX	0X.XXXXXXXX	0XXX.XXXXXX	0XXX.XXXXXX

	W_{Co}	I'_{vC}	$\ddot{x}_{NO}$	K_{BD}	P_{arC}
1105	0XXXXXXXX.X	0XXXXXXXX.X	0X.X	0X.X	0XXXX.XXXXX

	t'_{C1}	F_{C1}	$\dot{W}_{C1}$
1110	0XXXXX.XXXX	0X.XXXXXXE0XX	0X.XXXXXXE0XX
	.	.	.
	.	.	.
	t'_{25}	F_{C25}	$\dot{W}_{C25}$
1187	0XXXXX.XXXX	0X.XXXXXXE0XX	0X.XXXXXXE0XX

AXIAL FORCE COEFFICIENTS

S_{RC} $\bar{C}'$
1185 0XXXX.XXXXX 0XX.XXXXXXX 0XXXXX.XXXXX

M_1 C_{A1}
1186 0XXXX.XXXXX 0XX.XXXXXXX

. .

. .

M_{15} C_{A15}
1216 0XXXX.XXXXX 0XX.XXXXXXX

PLAN FORM

S_{PF} x_{PC}
1218 0XXXXX.XXXXX 0XXXX.XXXXX

NORMAL FORCE COEFFICIENTS

S_{RN} $\bar{N}'$ $\bar{X}_{cp}$
1220 0XXXX.XXXXX 0XX.XXXXXXX 0XX.XXXXXXX

M_1 C_{N11} C_{N21} C_{N31} x_{cp1}
1223 0XXX.XXXXXX 0X.XXXXXXE0XX0X.XXXXXXE0XX0X.XXXXXXE0XX 0XXXX.XXXXX

. . . . .

. . . . .

M_{15} C_{N115} C_{N215} C_{N315} x_{cp15}
1293 0XXX.XXXXXX 0X.XXXXXXE0XX0X.XXXXXXE0XX0X.XXXXXXE0XX 0XXXX.XXXXX

AERODYNAMIC PITCH DAMPING COEFFICIENTS

D_{RN} $\bar{M}_Q$ $\bar{M}_{\dot{\alpha}}$
1298 XXXX.XXXXX 0XX.XXXXXX 0XX.XXXXXX

K_Q x'_{RQ} $K_{\dot{\alpha}}$ $x'_{R\dot{\alpha}}$
1301 0XX.XXXXXX XXXX.XXXXX 0XX.XXXXXX XXXX.XXXXX

M_1 C_{MQ1} $C_{M\dot{\alpha}1}$
1305 XXX.XXXXXX 0X.XXXXXXXE0XX 0X.XXXXXXXE0XX

. . .

. . .

M_{15} C_{MQ15} $C_{M\dot{\alpha}15}$
1347 XXX.XXXXXX 0X.XXXXXXXE0XX 0X.XXXXXXXE0XX

CLUSTER CHARACTERISTICS AND SIDE IMPULSE

	n_m	n_c	R_c	σ_{It}	σ_{tb}	ϕ_ν
1380	XXXX	XXXX	0XX.XXXX	0XX.XXXX	0XX.XXXX	0XX.XXXX

TVC ANOMALIES

	A_L	K'_Δ	ω_L	δ_{MP}	δ_{MY}
1386	0XX.XXX	0XX.XXXX	0XX.XXXX	0XX.XXXX	0XX.XXXX

PITCH STEERING

	a_{01}	a_{11}
1391	0XXX.XXXXXXXX	0X.XXXXXXXXE0XX

	b_{11}	b_{21}	b_{31}	τ_{f1}
1393	0X.XXXXXXXXE0XX	0X.XXXXXXXXE0XX	0X.XXXXXXXXE0XX	0XXX.XXXXXXXX

STEERING COEFFICIENT EVALUATION

	σ_{g11}	K_{g11}	σ_{g21}	K_{g21}
1397	0XXXX	0X.XXXXXXX	0XXXX	0X.XXXXXXE0XX

YAW STEERING

	K_{yk}	τ_{yk}
1401	0X.XXXXXXXXE0XX	0XXXXX.XXXXX

ROLL ANOMALIES

	δ_{MR}	SR	K_{RC}	η_{Vr}
1403	0XX.XXXXX	XXXXX.XXXXX	0XX.	0XXXXX.XXXXX

JET DAMPING

	x'_{Pf}	x'_{Pa}	x'_E
1417	0XXXX.XXXX	0XXXX.XXXX	0XXXX.XXXX

INITIAL TVC CONDITIONS

	δ_{Po}	$\dot{\delta}_{Po}$	δ_{Yo}	$\dot{\delta}_{Yo}$	F_{Ro}
1420	0XX.XXXXXX	0XXX.XXXXXX	0XX.XXXXXX	0XXX.XXXXXX	0X.XXXXXXXXE0XX
1425	0XXXXX.XXXXX	0XXXXX.XXXXX	0XXXXX.XXXXX		

STAGE ATTITUDE REACTION

	ΔQ_b	ΔR_b
1428	0XXX.XXXXXX	0XXX.XXXXXX

GIMBAL LOCATIONS

	$\bar{X}_e$	x'_e	y'_e	z'_e	K_5
1430	XX.XXXXXXX	XXXX.XXXXX	XXX.XXXXXX	XXX.XXXXXX	XXXX.XXXXX

TVC CHARACTERISTICS

	τ_c	ζ_c	ω_c	ζ_v	ω_v
1435	XXXX.XXXXXXX	XX.XXXXXXX	XXX.XXXXXXX	XX.XXXXXXX	XXX.XXXXXXX

LIMITS

	L_1	L_2	L_3	L_4	L_5
1440	XXXXX.XXXX	XXX.XXXX	XX.XXXX	XXXX.XXX	XX.XXXX

	L_6	L_7	L_8	L_9	L_{10}
1445	XXXXX.XXXX	XXX.XXXX	XX.XXXX	XXXX.XXX	XX.XXXX

GAINS

	K_{DP1}	K_{DY1}	K_{RP1}	K_{RY1}
1450	XXX.XXXXX	XXX.XXXXX	XX.XXXXX	XX.XXXXXX

	K_{IP1}	K_{IY1}	f_{G1}	σ_{G1}	K_{G1}
1454	XXX.XXXXXX	XXX.XXXXXX	0X.	0XXXX	0X.XXXXXXXXE0XX

	K_{DP2}	K_{DY2}	K_{RP2}	K_{RY2}
1459	XXX.XXXXX	XXX.XXXXX	XX.XXXXX	XX.XXXXXX

	K_{IP2}	K_{IY2}	f_{G2}	σ_{G2}	K_{G2}
1463	XXX.XXXXXX	XXX.XXXXXX	0X.	0XXXX	0X.XXXXXXXXE0XX

	K_{DP3}	K_{DY3}	K_{RP3}	K_{RY3}
1468	XXX.XXXXX	XXX.XXXXX	XX.XXXXX	XX.XXXXXX

	K_{IP3}	K_{IY3}	f_{G3}
1472	XXX.XXXXXX	XXX.XXXXXX	0X.

MASS PROPERTIES

	$\bar{I}_Y$	$\bar{I}_Z$	$\bar{I}_X$
1475	OXX.XXXXXXX	OXX.XXXXXXX	OXX.XXXXXXX

	$\dot{W}$	$\bar{x}_{cg}$	z'_{cg}	y'_{cg}
1478	OX.XXXXXEOXX	OXX.XXXXXXX	OXX.XXXXXXX	OXX.XXXXXXX

	W_n	I_n	x_n
1482	OXXXXXXX.XX	OXXXXXXX.XX	OXXXX.XXXXX

1485	OXXXXX.XXXXX	OXXXXX.XXXXX	OXXXXX.XXXXX

	W_1	X_{cg1}	I_{y1}	Z_{cg1}	I_{z1}
1488	OXXXXXXXXX.X	OXXXX.XXXXX	OXXXXXXXX.XX	OXXXX.XXXXXX	OXXXXXXXX.XX

	I_{Z1}	Y_{cg1}	I_{X1}
1492	OXXXXXXX.XX	OXXX.XXXXX	OXXXXXXX.XX

	I_{XY1}	I_{YZ1}	I_{ZX1}
1495	OXXXXXX.XXX	OXXXXXX.XXX	OXXXXXX.XXX

	W_{15}	X_{cg15}	I_{Y15}	Z_{cg15}
1628	OXXXXXXXX.X	OXXXX.XXXXX	OXXXXXXX.XX	OXXX.XXXXXX

	I_{Z15}	Y_{cg15}	I_{X15}
1692	OXXXXXXX.XX	OXXX.XXXXX	OXXXXXXX.XX

	I_{XY15}	I_{XZ15}	I_{ZX15}
1695	OXXXXXX.XXX	OXXXXXX.XXX	OXXXXXXX.XXX

ROLL CONTROL

	ι_{R1}	τ_{R1}	D_1	$F_{\Delta 1}$	L_{Rc1}
1638	XXXXX.XXXX	XX.XXXXXXXXX	X.XXXXXXXX	XXX.XXXXXX	XXXXX.XXXXX
	K_{DR1}		K_{RR1}	I_{spR1}	t'_{R2}
1643	XXXXX.XXXX	XX.XXXXXXXXX	X.XXXXXXXX	XXX.XXXXXX	XXXXX.XXXXX
	ι_{R2}	τ_{R2}	D_2	$F_{\Delta 2}$	L_{Rc2}
1648	XXXXX.XXXX	XXXX.XXXXX	XX.XXXXXX	XXX.XXXXX	XXXXX.XXXXX
	K_{DR2}		K_{RR2}	I_{spR2}	t'_{R3}
1653	XXXX.XXXX	XXX.XXXXXXXXX	XXX.XXXXXXXX	XXXX.XXXXXX	XXXX.XXXXX
	ι_{R3}	τ_{R3}	D_3	$F_{\Delta 3}$	L_{Rc3}
1658	XXXXX.XXXXX	XXXX.XXXXX	XX.XXXXXX	XXX.XXXXX	XXXX.XXXXX
	K_{DR3}		K_{RR3}	I_{spR3}	
1663	XXXX.XXXX	XXX.XXXXXXXXX	XXX.XXXXXXXX	XXXX.XXXXXX	

AERODYNAMIC ROLLING MOMENT

	S_{RR1}	γ_{R1}	φ_{R1}
1667	OXXXXX.XXXX	OXXXXX.XXXX	OXXXXX.XXXX
	S_{RR2}	γ_{R2}	φ_{R2}
1670	OXXXXX.XXXX	OXXXXX.XXXX	OXXXXX.XXXX
	M_1	C_1	
1673	OXXXXXXX.XX	OXXXXXXX.XX	
	.	.	
	.	.	
	M_{10}	C_{10}	
1691	OXXXXXXX.XX	OXXXXXXX.XX	

AERODYNAMIC MOVABLE PITCH CONTROL FINS COEFFICIENTS

	S_{Fz}	x_{nz}	U_{bz}	l_{bz}	K'_{Ycf}	
1700	0XXXX.XXXXX	0XXXX.XXXXX	0XX.XXXXXXX	0XXXX.XXXXX	0XXXX.XXXXX	
	K'_{cf}	$\bar{C}'_{Lz}$	$\bar{C}_{lz}$	$\bar{C}'_{Dz}$	$\bar{K}'_{Lz}$	
1705	0XXXX.XXXXX	0XXX.XXXXX	0XXX,XXXXXX	0XXX.XXXXX	0XXX.XXXXX	
	M_1	C_{Lz15}	C_{lz15}	C_{Dz15}	K_{Lz15}	U_{cz15}
1710	0XX.XXXX	0XXX.XXXXXXX	0X.XXXXXXE0XX	0X.XXXXXX	0XX.XXXXXX	0X.XXXXXX
	.	.	.	.	.	.
	.	.	.	.	.	.
	M_{15}	C_{Lz15}	C_{lz15}	C_{Dz15}	K_{Lz15}	U_{cz15}
1794	0XX.XXXX	0XXX.XXXXXXX	0X.XXXXXXE0XX	0X.XXXXXX	0XX.XXXXXX	0X.XXXXXX

STAGE TWO

2000

.

.

.

2999

STAGE THREE

3000

.

.

.

3999

STAGE FOUR

4000

.

.

.

4999

2. OUTPUT PRINT

The output print contains the parameters which are computed during the trajectory. The output print is separated into two formats. Format I contains stage identified time oriented blocks consisting of printlines A thru P, the main print; Y thru YD, the hunt print; Z thru ZB, the auxiliary print; X thru XA, the steering coefficients print; and V, the maximum print.

Format II consists of the TVC duty cycle print blocks consisting of printlines WA thru WO. Format II follows the entire trajectory Format I.

a. Format I Output Print

Format I output print is given following the input print defined in section 9.1 or the hunt print if it exists. Each stage is titled as specified in the "output stage identification" logic. The printline blocks in the main print, hunt print, and auxiliary print, are time sequenced. The steering coefficients print block are given at stage termination. The main print, hunt print, auxiliary print, and steering coefficients print arrangements are pictured in this section.

(1) Output Stage Identification

The start of each stage is identified by the message:

INITIATION OF STAGE K_k

where K_k is the stage number, i.e., 1, 2, 3, or 4
After the stage termination printout type:
Following the time oriented print, one of these three messages will appear:

1. Normal stage termination

TERMINATION OF STAGE K_k

where K_k is the stage number, i.e., 1, 2, 3, or 4

2. End of flight control table

   TRAJECTORY HALTED-END OF FLIGHT CONTROL TABLE

3. End of mode control table

   TRAJECTORY HALTED-END OF MODE CONTROL TABLE

(2) Main Print

The main print contains printlines A, B, A, and succeeding printlines when the following special criteria are met. The main print is given when criteria of paragraph K.3.a, K.3.b, and K.3.c are satisfied.

Printline	Obtained
A	Always
AA	If My = 4 or 5
B	Always
BA	If My = 4 or 5
BB	If My = 4 or 5
C	Always
CA	If My = 2 or 5
CB	If My = 5
CC	If $\epsilon_d \neq 0$
CD	If My = 5 & $I_{xx} \neq 0$
CE	If $D_p \neq 0$ for kth stage
CF	If $D_p \neq 0$ for kth stage
CG	If $D_p \neq 0$ for kth stage
CH	If $D_p \neq 0$ for kth stage
CI	If $F_y \neq 0$
CJ	If F_y = 5 or 6

D	If $h_E \neq - 1$ or $h < h_E$
DA	If $D_{RN} \neq 0$ or $K_h \neq 0$ or PL - DA $\neq 0$
DB	If ($D_{RN} \neq 0$) or (My = 4 or 5 and Ty = 8, 9, 10 or 11) or (PL - DA $\neq$ 0)
DC	If $h_E \neq - 1$ or $h < h_E$ and My = 2 or 5
DD	If $h_E \neq - 1$ or $h < h_E$ and My = 5
DE	If $h_E \neq - 1$ or $h < h_E$ and $S_{fz} \neq 0$
DF	If $h_E \neq - 1$ or $h < h_E$ and $S_{fz} \neq 0$ and My = 4 or 5
E	If My = 2 or 5
ED	If My = 5
EB	If My = 5 and $I_{xx} \neq 0$
F	If My = 2 or 5
FA	If My = 5
FB	If My - 2 or 5
FC	If My = 5
FD	If My = 5 and $I_{xx} \neq 0$
G	If $t_{T1} \neq 0$
GA	If $t_{T1} \neq 0$ and My = 4 or 5
GB	If PL - GB $\neq$ 0 or Ty = 10 or 11
GC	If (PL - GB $\neq$ 0 or Ty = 10 or 11) and My = 4 or 5
GD	If PL = GB $\neq$ 0 or Ty = 10 or 11
GE	If (PL - GB $\neq$ 0 or Ty = 10 or 11) and My = 4 or 5
IE	If Ty = 6
J	If $K_{cl} < \sigma_e < K_{cr}$ or at the stage termination of final stage and the orbital criteria are satisfied.

Printline	Obtained
J	If $K_{c1} < \sigma_c < K_{c2}$ or at the stage termination of final stage and the orbital criteria are satisfied.
JA	If $K_{c1} < \sigma_c < K_{c2}$ or at the stage termination of the final stage and the impact criteria are satisfied.
JB	If $K_{c1} < \sigma_c < K_{c2}$ or at the stage termination of the final stage and the impact criteria are satisfied and $h_E \neq 0$
JC	If $K_{c1} < \sigma_c < K_{c2}$ or at the stage termination of the final stage and the impact or orbital criteria are satisfied.
K	If PL - K $\neq 0$ or Ty = 4 or $\sigma_{gg} \neq 0$
KA	If (PL - K $\neq 0$ or Ty = 0 or $\sigma_{g1} \neq 0$) and My = 4 or 5
KB	If PL - KB $\neq 0$ or Ty = 4 or $\sigma_{g1} \neq 0$
N	At stage termination or PL - N $\neq 0$
O	At stage termination or PL - O $\neq 0$
P	If $t_B = 0$
PA	If $P_{arm} \neq 0$ or $P_{avc} \neq 0$ and if $t_B = 0$
PB	If $P_{arm} \neq 0$ or $P_{arc} \neq 0$ and if $t_B = 0$

Output Print Format I

Main Print

t S h V_e V_I a_{xb} M_y T_y
A XXXX.XXXX XXXXXXXXX. XXXXXXXX. XXXXX.XXX XXXXX.XXX 0XX.XXXXX XX XX

S_s S_c δ ζ
AA XXXXXXXXX. XXXXXXXXX. 0XXX.XXXX 0XXX.XXX

θ_m $\dot{\theta}_m$ α γ_1 γ_{1I} a_{zb} $\dot{V}_e$
B 0XX.XXXX 0XX.XXXXX 0XXX.XXXX 0XXX.XXXX 0XXX.XXXX 0XX.XXXXX 0XXXX.XX

ψ_m $\dot{\psi}_m$ β γ_2 γ_{2I} a_{Yb} η_{bt}
BA 0XXX.XXXX 0XX.XXXXX 0XXX.XXXX XXX.XXXX 0XXX.XXXX 0XX.XXXXX 0XX.XXX

ϕ_m $\dot{\phi}_m$ P_m Q_m R_m μ ρ
BB 0XXX.XXXX 0XX.XXXXX 0XX.XXXXX 0XX.XXXXX 0XX.XXXXX 0XXX.XXXX 0XX.XXXX

W $\dot{W}$ F F_V $[F/\dot{W}]$ F_x η_{bn}
C XXXXXXXXX. XXXXXX.XX 0XXXXXXXX. 0XXXXXXXX. XXXX.XXXX 0XXXXXXXX. 0XX.XXX

$\dot{I}_{YY}$ M_{DQ} M_{cQ} M_{FCQ} M_{JDQ} F_{JDz} M_{IQ}
CA 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXX

$\dot{I}_{ZZ}$ M_{DR} M_{CR} M_{FCR} M_{JDC} F_{JDy} M_{IR}
CB 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXX

P_c P_e $P*$ P_s ϵ_s
CC 0XXXXXXX.XX 0XXXXXX.XX 0XXXXXXX.XX 0XXXXXXX.XX 0XXX.XXXXX

$\dot{I}_{xx}$ M_{DP} M_{CP} M_{FCP} M_{FVP} M_{FOP} M_{IP}
CD 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXXX. 0XXXXXXX

r_b D_b g_{WI} V_{C1} A_{SI} ϵ C_{FI}
CE XX.XXXXXX XXXX.XXXX X.XXXXX XXXXXXXXX. XXXXXXX.X XXXX.XXXX X.XXXXXXX

$\dot{P}_c$ P_c P_{cc} F_{com} F_{Vcom} F_{VN} F_N
CF ±XXXXXXXX. XXXXX.XXX XXXXX.XXX XXXXXXXXX. XXXXXXXX. XXXXXXXX.XXXXXXXX

$\dot{A}_t$ A_t A_{tcc} K_s K_p ω_p
CG ±XXXXXX.X XXXX.XXXX XXXX.XXXX XXXX.XXXX XXXX.XXXX XXXX.XXXX

$\dot{W}_{MP}$ W_{pr} g_{PI} A_{xI}
CH XXXXXX.XX XXXXXXXX. X.XXXXX XXXX.XXXX

OUTPUT PRINT FORMAT I

MAIN PRINT

	$\hat{F}_c$	K_{cv}	V_{XXX}	$\dot{V}_{XXX}$	K_{XXX}	
CI	XXXXXXXX.	XXXX.XXXX	XXXXX.XXX	OXXXX.XX	XXXX.XXXX	
	F_{cqmin}	F_{cqmax}	K_{cq}	$\dot{V}_{ecq}$	F_{cALOS}	F_{cclos}
CJ	XXXXXXXX.	XXXXXXXXXX.	XXXX.XXXX	OXXXX.XX	XXXXXXXX.	XXXXXXXX.

where:

$$V_{XXX} = \begin{cases} V_{ecv} & \text{If } F_y = 1 \\ V_{ecm} & \text{If } F_y = 2 \\ V_{cv} & \text{If } F_y = 4 \\ 0 & \text{Otherwise} \end{cases}$$

$$\dot{V}_{XXX} = \begin{cases} \dot{V}_{ecv} & \text{If } F_y = 1 \\ \dot{V}_{ecm} & \text{If } F_y = 2 \\ 0 & \text{Otherwise} \end{cases}$$

$$K_{XXX} = \begin{cases} K_{CPR} & \text{If } F_y = 3 \\ K_{ALOS} & \text{If } F_y = 6 \\ 0 & \text{Otherwise} \end{cases}$$

	M	q	V_a	C	N_Z	$q\alpha'$	Pa
D	XX.XXXXX	XXXXX.XX	0XXXXX.XX	XXXXXX.XX	0XXXXXXXX	XXXXXX.X	XXXX.XX
	$\dot{\alpha}$	$\bar{\alpha}$	$\mathring{\bar{\alpha}}$	α:	ψ_w	v_w	$\dot{v}_w$
DA	0XXXX.XXX	0XXX.XXXX	0XXX.XXXX	0XXX.XXXX	XXX.XXXXX	0XXX.XXX	0XXXX.XXX
	$\dot{\beta}$	$\bar{\beta}$	$\mathring{\bar{\beta}}$	$\bar{\alpha}'$	ϵ	ϕ	φ_c
DB	0XXXX.XXX	0XXX.XXXX	0XXX.XXXXX	0XXX.XXXX	XXX.XXXXX	0XXX.XXX	0XXX.XXX
	x_{cp}	M_{NQ}	l_{cp}	M_{NSQ}	M_{CZG}	M_{NDQ}	N_{Pz}
DC	0XXXX.XXXX	0XXXXXXXX.	0XXX.XXXX	0XXXXXXXX.	0XXXXXXXX.	0XXXXXXXX.	0XXXXXXXX.
	M_{NP}	M_{NR}	Nr	M_{NSR}	M_{CYG}	M_{NDR}	N_{Py}
DD	0XXXXXXX.	0XXXXXXXX.	0XXXXXXX.	0XXXXXXXX.	0XXXXXXX.	0XXXXXXXX.	0XXXXXXXX.
	$M_{\delta Q}$	$l_{\delta z}$	$N_{\bar{\delta}z}$	$C_{\delta z}$	M_{hz}	l_{hz}	U_{cz}
DE	0XXXXXXXX.	0XXXX.XXXX	0XXXXXXXX.	0XXXXXX.XX	0XXXXXXXX.	0XXXX.XXXX	0XXXXXX.
	$M_{\delta R}$	$l_{\delta y}$	$N_{\delta y}$	M_{hy}			
DF	0XXXXXXXX.	0XXXX.XXX	0XXXXXXXX.	0XXXXXXXX.			
	I_{YY}	z_e	x_{cg}	z_{cg}	θ_b	Q_b	$\dot{Q}_b$
E	XXXXXXXXX.	0XX.XXXX	XXXX.XXXX	0XXX.XXXX	0XXX.XXXX	0XX.XXXX	0XXX.XXXX
	I_{ZZ}	y_e	y_{cg}	ψ_b	R_b	$\dot{R}_b$	
EA	XXXXXXXXX.	XXXX.XXXX	0XXX.XXXX	0XXX.XXXX	0XX.XXXX	0XXX.XXX	
	I_{XX}	$\dot{\phi}_b$	ϕ_b	P_b	$\dot{P}_b$	M_{RAP}	
EB	XXXXXXXXX.	0XX.XXXXX	0XXX.XXXX	0XX.XXXX	0XXX.XXX	0XXXXXXX.	
	δ_{Pc}	δ_P	$\dot{\delta}_P$	$\ddot{\delta}_P$	F_z	K_{DP}	K_{RP}
F	0XX.XXXXX	0XX.XXXXX	0XXX.XXXXX	0XXXX.XX	0XXXXXX.XX	XXXX.XXX	XXX.XXXX
	δ_{Yc}	δ_Y	$\dot{\delta}_Y$	$\ddot{\delta}_Y$	F_y	K_{DY}	K_{RY}
FA	0XX.XXXXX	0XX.XXXXX	0XXX.XXXXX	0XXXX.XX	0XXXXXX.XX	XXXX.XXX	XXX.XXXX
	$\bar{\delta}_P$	I_P	$I_{\delta P}$	M_{FOQ}	$\Delta\theta_b$	M_{TDQ}	F_{TDz}
FB	0XX.XXXXX	XXXXXXXXX.	XXXXX.XX	0XXXXXXXX.	0XX.XXXX	0XXXXXXXX.	0XXXXXX.
	$\bar{\delta}_Y$	I_Y	$I_{\dot{\delta}Y}$	M_{FOQ}	$\Delta\psi_b$	M_{TDR}	F_{TDy}
FC	0XX.XXXXX	XXXXXXXXX.	XXXXX.XX	0XXXXXXXX.	0XXX.XXXX	0XXXXXXXX	0XXXXXXX
	F_c	I_R	F_D	$\Delta\phi_b$	F_R	$\dot{F}_R$	$\dot{W}_R$
FD	0XXXXXX.X	XXXXXXXXX	0X.XXXXXX	0XXX.XXXX	0XXXXXX.X	0XXXXX.XX	0XXXXX.X
	t_T	S_T	h_T	V_T	γ_T	a_{TT}	a_{TN}
G	XXXX.XXXX	XXXXXXXXX.	XXXXXXXX.	XXXXX.XXX	0XX.XXXX	0XX.XXXXX	0XX.XXXXX
	S_{TC}	γ_M	$\dot{\gamma}_M$	ζ_T	a_{TC}		
GA	XXXXXXXXX.	0XXX.XXXXX	0XXXXX.XXXX	0XX.XXXXX	0XX.XXXXX		

	t_{MI}	S_{MI}	h_{MI}	R_{MI}	σ_{MI}	ϵ_{MI}	
GB	XXXX.XXXX	XXXXXXXXX.	XXXXXXXX.	XXXXXXXX.	0XXX.XXXX	0XXX.XXXX	
	S_{CMI}	ϵ^*_{MI}	T_{MI}	σ_{MI}			
GC	XXXXXXXX.	0XXX.XXX	0XXX.XXXX	0XXX.XXXX			
	ϵ_{MT}	$\dot{\epsilon}_{MT}$	$\dot{R}_{MT}$	R_{MT}	σ_{MT}	$\dot{\sigma}_{MT}$	
GD	0XXX.XXXX	0XXXX.XXXX	0XXXXX.XX	XXXXXXXX.	0XXX.XXXX	0XXXX.XXXXXX	
	δ_{MT}	$\dot{\delta}_{MT}$	λ_{MT}	$\dot{\lambda}_{MT}$			
GE	0XXX.XXXX	0XXXX.XXXX	0XXX.XXXX	0XXXX,XXXXXX			
	l_{lr}						
IE	0X.XXXXXXE0XX						
	e	n_p	h_a	i	t_a	$\phi_a a_4$	P
J	XXX.XXXXX	XXXXX.XXX	XXXXX.XXX	XXX.XXXXX	XXXXXX.XX	XXXX.XXXX	XXXX.XXXX
	t_f	S_f	μ_f	ρ_f	V_{If}	γ_{1If}	γ_{2If}
JA	0XXXX.XXX	XXXXX.XXX	0XXX.XXXX	0XX.XXXXX	XXXXXX.XX	0XX.XXXXX	XXX.XXXX
	t_E	S_E	V_{aE}	γ_{1E}	V_{IE}	γ_{1IE}	γ_{2IE}
JB	0XXXX.XXX	XXXXX.XXX	0XXXXX.XX	0XX.XXXXX	0XXXXX.XX	0XX.XXXXX	XXX.XXXX
	S_a	γ_{2Ia}	μ_a	ρ_a	V_{Ia}		
JC	0XXXXX.XX	XXX.XXXXX	0XXX.XXXX	0XXX.XXXX	XXXXX.XX		
	X_{ee}	$\dot{X}_{ee}$	$\ddot{X}_{ee}$	Z_{ee}	$\dot{Z}_{ee}$	$\ddot{Z}_{ee}$	
K	0XXXXXXXX.	0XXXXX.XX	0XXXX.XXX	0XXXXXXXX.	0XXXXX.XX	0XXXX.XXX	
	Y_{ee}	$\dot{Y}_{ee}$	$\ddot{Y}_{ee}$	Y_{gg}	$\dot{Y}_{gg}$	$\ddot{Y}_{gg}$	
KA	0XXXXXXXX.	0XXXXX.XX	0XXXX.XXX	0XXXXXXXX.	0XXXXX.XX	0XXXX.XXX	
	X_{gg}	$\dot{X}_{gg}$	$\ddot{X}_{gg}$	Z_{gg}	$\dot{Z}_{gg}$	$\ddot{Z}_{gg}$	
KB	0XXXXXXXX.	0XXXXX.XX	0XXXX.XXX	0XXXXXXXX.	0XXXXX.XX	0XXXX.XXX	
	L_F	L_D	L_g	L_v	ΔV	E/M	
N	XXXXXX.XX	XXXXXX.XX	XXXXXX.XX	0XXXXXX.XX	XXXXXX.XX	XXXXXXX.	
	I	I_v	H_e				
O	0X.XXXXXXXE0XX	0X.XXXXXXXE0XX	0X.XXXXXXXE0XX				
	K^*_{FM}	K^*_{FC}	I_{vT}	I_{vM}	I_{vC}		
P	0X.XXXXXXE0XX	0X.XXXXXXE0X	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX		
	I_{FM}	I_{FC}	$\hat{I}_{vT}$	$\hat{I}_{vM}$	$\hat{I}_{vC}$		
PA	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX		
	I_{spM}	I_{spC}	I^*_{vT}	I^*_{vM}	I^*_{vC}		
PB	0XXXX.XXXXX	0XXXX.XXXXX	0X.XXXXXXE0XX	0X.XXXXXXE0XX	0X.XXXXXXE0XX		

(3) Hunt Print

Procedure 1

If P1 is non-zero, the printline Y is printed at trajectory termination. If the input flag K_s is non-zero, the stipulated main print is output, i.e., as described for P1 equal P2 equal zero, for each trajectory run during this hunting procedure. The title for this hunt print is set at "ITERATION" and the number of iteration is output as n_i.

After convergence occurs the trajectory is printed as described for P1 equal to P2 equal zero.

Procedure 2

If P2 is non-zero, printlines YA through YD are printed at end of each trajectory iteration. The hunt print printout continues until procedure 2 convergence occurs, then the trajectory is printed as described for P1 equal P2 equal zero. The hunt print title should be "ARRAY", "BASE CASE", or "ITERATION" for the indicated run. The number of trajectory runs is output as n .

(4) Auxiliary Print

Printlines Z, ZA (if σ_{P5}, σ_{P6}, σ_{P7}, or σ_{P8} is non-zero), and ZB (if σ_{P9}, σ_{P10}, or σ_{P11} is non-zero) when the auxiliary printline criteria of paragraph K.3.d are satisfied.

(5) Steering Coefficients Print

If $\sigma_{g1} > 0$ and $n_g > 5$, then the steering coefficients print is printed after the note "TERMINATION OF STAGE k" and before the note "MAXIMUM VALUES".

If $\sigma_{g1} > 0$ and $n_g < 5$, the note "STEERING COEFFICIENTS NOT CALCULATED" is printed after the note "TERMINATION OF STAGE k" and before the note "MAXIMUM VALUES".

OUTPUT PRINT FORMAT I (Continued)

Hunt Print

```
                   ni        title
     HUNT PRINT   XXX      HHHHHHHHHH

           X                a
Y    0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
           X1               X2               X3               X4
YA   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
           X5               X6               X7
YB   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX

           Z                Z1               Z2               Z3
YC   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
           Z4               Z5               Z6               Z7
YD   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
```

Auxiliary Print

```
          t             σP1              σP2              σP3              σP4
Z    XXXX.XXXX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
          σP5              σP6              σP7              σP8
ZA   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
          σP9              σP10             σP11
ZB   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX   0X.XXXXXXXE0XX
```

Steering Coefficient Print

```
                        STEERING COEFFICIENTS

           a0                 a1
X    0X.XXXXXXXXE0XX   0X.XXXXXXXXE0XX
           b1                 b2                 b3              ng
XA   0X.XXXXXXXXE0XX   0X.XXXXXXXXE0XX   0X.XXXXXXXXE0XX   XXXXX.
```

Maximum Print

```
                        MAXIMUM VALUES

          t        σmj (value)   σmj(code)   Tmi
V    XXXX.XXXX   0X.XXXXXXX      0XXX        0X
```

(6) Maximum Print

If $\sigma_{mj} \neq T_{mj} = k_k$, or $T_{mj} = 1$ with $k_k = 4$, the maximum print (printline V) including the note "MAXIMUM VALUE" is printed after the steering coefficients print of the k-th stage.

(7) Optional Titled Output

If K_{TPF} the titled print flag input in L683 is non-zero the following format is printed at the top of each page of the output print format I.

```
BD  XXX.    RR  XX.    RUN  XXX.
T CARD TITLE MESSAGE IS PRINTED ON THIS LINE

A       TIME     GROUND   GEOMETRIC     EARTH   INERTIAL    AXIAL   MODE TYPE
                  RANGE    ALTITUDE  VELOCITY   VELOCITY    ACCEL

B  ATTITUDE   ATTITUDE       ATTACK    FLIGHT   INERTIAL    TRANS       EARTH
      ANGLE       RATE        ANGLE      PATH     FLIGHT    ACCEL    VELOCITY
                                        ANGLE       PATH                 RATE

C     WEIGHT    WEIGHT        TOTAL    VACUUM   SPECIFIC    AXIAL     VEHICLE
                  FLOW       THRUST    THRUST    IMPULSE   THRUST        LOAD
                                                                      FACTOR

D       MACH   DYNAMIC          AIR     AXIAL     NORMAL        Q     AMBIENT
      NUMBER  PRESSURE     VELOCITY     FORCE      FORCE    ALPHA    PRESSURE
-----------------------------------------------------------------------------
```

b. Format II Output Print

Format II output print is given following the entire Format I output print defined in section N1. The printline block in the duty cycle print is sequenced to present general duty cycle parameters first, then the time dependent duty cycle parameters. The duty cycle print arrangement is pictured in this section.

(1) Duty Cycle Print

If K_{DC} is non-zero, the duty cycle print for the "K_{DC}"-th stage shall be given following the flexible body print.

OUTPUT PRINT FORMAT II

DUTY CYCLE PRINT

$\bar{I}$ $\bar{I}_v$ x_e x_{nf} δ_{me}
I-DEL I-VAC X-E X-NF D-DESIGN
WA OX.XXXXXXXXEOXX OX XXXXXXXXEOXX OXXXX.XXXX OXXXX.XXXX OXX.XXXX

I_{spm} $h_{q\alpha}$ $P_{q\alpha}$ ω_c $\bar{\delta}_S$ $\bar{\delta}_S$
ISPVACM H-Q ALPHA P-Q ALPHA CONT FREQ SLEW D-SLEW
WB OXXXX.XXXX OXXXXXXX OXXXX.XXXX OXXXX.XXXX OXXX.XXXX OXX.XXXX

$\bar{I}_P$ $\bar{I}_{\dot{\delta}p}$ $t_{\eta P}$ η_P δ_{pmax} $\bar{\delta}_{pmax}$
I-P I-DDOTP T-NP ETA-P DDOT-PMAX D-PMAX
WC OX.XXXXXXXXEOXX OXXXXX.XX OXXXX.XXXX OX.XXXXXX OXXXX.XXXX OXX.XXXX

$\bar{I}_y$ $\bar{I}_{\dot{\delta}y}$ $t_{\eta y}$ η_y $\dot{\delta}_{ymax}$ $\bar{\delta}_{ymax}$
I-Y I-DDOTP T-NY ETA-Y DDOT-YMAX D-YMAX
WD OX.XXXXXXXXEOXX OXXXXX.XX OXXXX.XXXX OX.XXXXXX OXXXX.XXXX OXX.XXXX

$q\alpha'_{max}$ $q_{q\alpha'}$ $t_{q\alpha}$ $C_{n\alpha}$ $M_{q\alpha'}$ $\bar{\delta}_{ave}$
Q-ALPHA Q-Q APLHA T-Q ALPHA NORM COEF MACH Q A D-AVE
WE OXXXXXXX.XX OXXXXX.XX OXXXX.XXXX OX.XXXXXXX OXX.XXXX OXX.XXXX

K_{dc} D_B A_{FWI} t_{Ba} F_{vave} W_o
STG CASE DIA VEH VTHR/WT STG TIME STG VAC THRUST STG LIFTOFF WT
WF X OXXXX.XXXX OXXX.XXXXX OXXXX.XXXX OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX

R_{PFV} ϵ_d $\bar{A}_t$ α'_d P_{ce} $C*$
PC/FMV EXP RAT AT GAMMA PCA C-STAR
WG OX.XXXXXXXXEOXX OXXXX.XXXX AXXXXX.XXX OX.XXXXX OX.XXX.XXX OXXXXX.XX

W_{TVC} W_{exi} I_{spavg} P_{cmax}
WTVC W-EXI ISP-AUG PC-MAX
WH OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX

n_m n_c n_{dc} M_{hzmax} M_{hymax}
N-M N-C N-DC M-HINGE-P M-HINGE-Y
WI XXX XXX XXX OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX

I_{vM} W_{MO} K_{tM}
IVT-MAIN WGT-MAIN XT-MAIN
WJ OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX OX.XXXXXXXXEOXX

	$\mathring{A}_{tmax}$	I_{at}	A_{xmax}
	ADTMAX	IAT	AXMAX
WK	±0.XXXXXXEIXX	±0.XXXXXXEIXX	±XXXXX.XXXX

	F_{max}	P_{cmax}	ϵ_{max}	A_{tmin}	C_{fmax}
	FMAX	PCMAX	EPSMAX	ATMIN	CFMAX
WL	±XXXXXXXX.XX	±XXXXX.XXXX	±XXXX.XXXX	±XXXXX.XXXX	±X.XXXXXX
	F_{min}	P_{cmin}	ϵ_{min}	A_{tmax}	C_{fmin}
	FMIN	PCMIN	EPSMIN	ATMAX	CFMIN
WM	±XXXXXXXX.XX	±XXXXX.XXXX	±XXXX.XXXX	±XXXXX.XXXX	±X.XXXXXX

	t_{Bq}	δ_{pq}	δyq	F_q	X_{cgq}	$\vec{P}_{vacq}$	$\bar{W}_q$
	TIME	D-PITCH	D-YAW	F-DEL	X-CG	F-VAC	W-DOT
WO	XXXX.XXX	0X.XXXXX	0XX,XXXXX	XXXXXXXX.X	XXXX.XXXXX	XXXXXXXX.X	XXXXXX.XX
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
WO	XXXX.XXX	0XX.XXXXX	0XX.XXXXX	XXXXXXXX.X	XXXX.XXXXX	XXXXXXXX.X	XXXXXX.XX

(2) Error Code

Certain computations and parameter values result in program logic terminating the run. The error which causes the abort is identified by printout. The printout occurs after the main print is given for the trajectory computation up to the time of error.

(a) Common Error

Verbal descriptions of common errors are as follows:

1. *Undefined type of flight - check input L310, L317, etc.
2. *Undefined mode - check input L600, L603, etc.
3. Undefined type of hunt - check input L84
4. Invalid sigma Z or Y - check input L85, L94, L103, etc
5. Invalid sigma Y - check input L77
6. Input card error
7. Dependent parameter not varying - Hunt Procedure one
8. Impossible region exist - Hunt Procedure one
9. Singularity in quadratic fit - Hunt Procedure one
10. Delta X input zero - check L81
11. Internal tolerance input zero - check L83
12. Invalid sigma A - check input L78
13. *The missile weight has gone to zero - dump follows

*These messages are followed by a formated dump

(b) Integration Routine Errors

In the event of an integration routine error, the following information is provided:

INTEGRATION ERROR TYPE X

Type 1 indicates the initial integration interval is too small: this error should seldom occur.

Type 2 indicates the CNTRL or MAXPRT routine has cut the interval too far in attempting to meet accuracy criteria.

Type 3 indicates the integration routine has cut the interval too far.

The minimum allowable interval is 2 (t +1)x 10^{-8}.

AT TIME = XXXX - the time of the failure

DELT = XXXX - the integration interval

CONTROL COUNTER. XXXX- the number of passes through the CNTRL routine; i.e., the number of time intervals integrated. Zero here indicates the trajectory cannot get started for some reason.

BREAKUP COUNTER. XXXX- The number of passes through the CNTRL routine in attempting the most recent breakup. An integer here and error type 2 indicates trouble with one or more of the following:

1. Attitude control Table
2. Gains Table
3. Special Print Table
4. Weight Jettison Table
5. Wind Profile Table
6. Mode Control Table
7. Staging
8. Target Dynamical Conditions Table
9. Lift off
10. Thrust Modulation Control Table

MAXPRT COUNTER XXXX- The number of passes through the MAXPRT routine: a positive integer here and error type 2 indicates trouble finding a maximum print valve. A value of -1 indicates no maximum prints have been requested.

INTEGRATION COUNTER XXXX- The number of times the integration interval has been cut. A positive integer here and error type 3 indicates trouble in maintaining integration accuracy.

TIME (START) - Time at call to integration routine

DELT (START)-Initial integration interval.

EMAX-- Maximum value of E_i

JMAX - Index i of E_i

Following the above there is a table of 5 columns of 50 items each, labeled Y (start), HK1, HK3, HK4, .nc. Y (start) is just that, and HK1, HK3. and HK4 are the h_1 . h_3 . and h_4 of the Range-Kutter-Menson integration method (k2 is not saved and k5 can be found as D_y in locations 751-800 of the P-region). INC is a logical variable indicating whether the particular equation is included in the integration vector. This information is of use only with type 3 errors.

(c) Formatted Dump

Certain error conditions cause the trajectory to terminate with a formatted dump consisting of the following sections:

P REGION (1000 locations) - this is the section described in switching code; i.e., the output variables identified by L5000-L5999.

GN REGION (100 locations) - this area contains real variables and constants.

JN REGION (100 locations) - this area contains integer variables, constants, counters and flags.

LG REGION (100 locations) - this area contains logical flags

WGT TABLES (150 locations) - this is the WGTBLS common region. It contains supplemental main and complementary thrust-weight tables, each of which are 3 x 25 arrays. The table entries are the derivative of thrust, second derivative of weight, and weight.

STAGE TABLES (75 locations) - this is a 15 x 5 array, containing 15 values each of XOLD, XTEMP, XTARG, DET, and BRKTM. These values are used by the routines CNTRL and BKKPS to perform breakups. $XOLD_i$ is the past value of variable i, $XTEMP_i$ is the most recent value and XTARG is the target value. DET_i is either the estimated time to achieve $XTARG_i$, or -10^8 if $XTARG_i$ has been achieved. $BRKTM_i$ is the time a previous breakup occurred and is used whenever MAXPRT requires repeated passes over the same interval to locate and extremal. The i index relates the type of breakup involved, as follows:

i = 1. Attitude (flight) Control Table
2. Gains (TVC) Table
3. Weight Jettison Table
4. Special Print Table
5. Wind Profile Table
6. Mode Control Table
7. Staging
8. Target Dynamical Conditions Table
9. Liftoff
10. Thrust Modulation Control Table

The remaining locations (i = 9-15) are not used.

STAGE FLAGS (30 locations) - this is a 15 x 2 array of integers; 15 values each of LMP and KDP. These are used with the values above. LMP_i is the L-number, less 5000, of the variable X; for example, LMP = 0 means a breakup on the variable t, or LMP = 270 means a breakup on α. KDP_i is the L-number that identifies the breakup; for example, KDP = 31 means that the breakup of the second line of the attitude control table was in progress. KDP = 0 means that breakup point has not yet been found. The index i is as above. A negative value for LMP_i indicates the corresponding table is not involved in the breakup process.

TVC REGION (800 locations) - this is an array of 100 x 8, consisting of 100 each of the values printed on line WO, followed by the values printed on lines WA through WJ of the Duty Cycle Print.

(d) Simultaneous Hunt Errors

The following message is output if the hunting procedure 2 fails:

```
ERROR HAS OCCURRED IN OPTM2 ROUTINE - DUMP FOLLOWS
```

c. Trajectory Shaper Output Print

The following output format is used in presenting the trajectory Shaper parameters

```
                (OUTPUT FORMAT)

                  TRAJECTORY SHAPER SUBROUTINE

               INPUT

          CONTROL FLAG
          K
L598      X sh          DIM        Message 1
                                   HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH
                                                                    ----------↑----------
          TARGET CONDITION                                           Delete if K_sh = 1
          S                        Message 2
L599 OXXXXXX.XX sh   NM            HHHHHHHHHHHHHHHHHH
                                                                    ----------↓----------
               OUTPUT

          NO STG.   INITIAL VEL.    INITIAL ALT.    EARTH ROTATION
          ,DIM      ,FT/SEC         ,FT             ,DIM
          N                         h               ω/W
          X k       OXXXXXX.XXX     OXXXXXX.XXX     OX.XXXXXXXX
                      Veo             o

               STAGE I         STAGE II         STAGE III        STAGE IV
                 t B1            t B2             t B3             t B4
ACTION TIME OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 ,SEC            I vT1           I vT2            I vT3            I vT4
VAC.IMPULSE OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 ,LB-SEC         I speff1        I speff2         I speff3         I speff4
EFF. ISP    OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 ,SEC            ΔW1             ΔW2              ΔW3              ΔW4
EXP. WEIGHT OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 ,LB             W B1            W B2             W B3             W B4
STG. WEIGHT OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 ,LB             ΔW1/WB1         ΔW2/WB2          ΔW3/WB3          ΔW4/WB3
STG. MASS   OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 FRACT.. DIM     W B1/WB1        W B2/WBL         W B3/WB1         W 34/WB1
STG. RATIO  OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 .DIM            W PLO1          W PLO2           W PLO3           W PLO4
PROPULSION  OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX OX.XXXXXXXEOXX
 WEIGHT, LB

          IDEAL VELOCITY        PAYLOAD WEIGHT          VEHICLE LIFTOFF
           ,FT/SEC               .LB                     WEIGHT, LB
             ΔV                    WPL                      WT
         OXXXXXX.XXX          OXXXXXXXX.XXXX         OXXXXXXXX.XXXX
          PITCH OVER           PITCH OVER ANGULAR      VEHICLE VELOCITY AT
           ANGLE, DEG           RATE, DEG SEC           PITCH OVER, FT SEC
             θM2                      Qu2               Veo
         OXXX.XXXXXX             OXXXX XXXXXX           OXXXX.XXXXXX
                                                                        ------↑------
          CIRCULAR ORBITAL     TERMINAL STAGE  GNITION  CIRCULAR ORBITAL   Delete
           VELOCITY  FT SEC     FLIGHT PATH ANGLE, DEG   ALTITUDE, FT        if
             VICO                      γ 1T               hco
         OXXXXXX.XXX               OXX.XXXXXX           OXXXXXXXX.XXX      K_sh ≠ 3
                                                                        ------↓------
```

Message 1

MAXIMIZE RANGE WITH GIVEN PAYLOAD	If $K_{sh} = 1$
MAXIMIZE PAYLOAD TO A GIVEN RANGE	If $K_{sh} = 2$
DETERMINE PAYLOAD TO A CIRCULAR ORBIT	If $K_{sh} = 3$

Message 2

TARGET RANGE	If $K_{sh} = 2$
ORBITAL ALTITUDE	If $K_{sh} = 3$

SECTION IV

ROLL CONTROL REQUIREMENTS

The roll control requirements are determined from estimates of the roll disturbing moments: the offset roll torque, the vortex roll torque, and the aerodynamic roll torque. The offset roll torque is caused by the transverse thrust vector and the distance between the TVC vector point and the offset center-of-gravity centerline. The vortex roll torque is caused by the spiralling exhaust gases leaving the nozzle. The aerodynamic roll torque is caused by the coupled aerodynamic normal force and the distance between the aerodynamic center of pressure and missile center-of-gravity centerline offsets.

A. LINKAGE WITH TRAJECTORY ROUTINE

The roll control requirement routine can operate with data from the trajectory output duty cycle or can operate independently of the trajectory program by inputting mandatory data. Options are available to input variables and constants or have them calculated internally.

The logic to use data from the TVC duty cycle is as follows.

1. If the mandatory data are not input, i.e., K_k, D_B, F_{vac}, W_o, or F_{vac1}/W_{o1}, then data on print-line WF of the TVC duty cycle are used.
2. If the control flag K_{tr} is nonzero, the parameters $C_{N\alpha'}$, $q\alpha'_m$, and δ'_{mx} are obtained from the TVC duty cycle parameter $C_{N\alpha q}$, $q\alpha'_{max}$, and $\bar{\delta}_{p_{max}}$, respectively.

B. PROGRAM INSTRUCTIONS

1. ROLL TORQUE

$$M_{rtk} = \begin{cases} M'_{rtk} & \text{if } M'_{rtk} \text{ is input} \\ (M^2_{osk} + M^2_{vrk} + M^2_{ark})^{\frac{1}{2}} & \text{otherwise} \end{cases} \qquad \text{(in-lbf)}$$

Where M'_{rtk} is the input roll torque (in-lbf), M_{osk} is the offset roll torque (in-lbf), M_{vrk} is the vortex roll torque (in-lbf) and M_{ark} is the aerodynamic roll torque (in-lbf).

a. Offset Roll Torque

$$M_{osk} = \begin{cases} 0 \text{ if } M'_{rtk} \text{ is input} \\ M'_{osk} \text{ if } M'_{osk} \text{ is input} \\ K_{osk}\ (\pi/180)\ F_{vack}\ \delta_{mxk}\epsilon_{osk} \text{ otherwise} \end{cases} \qquad \text{(in-lbf)}$$

Where M'_{osk} is the input offset roll torque (in-lbf), K_{osk} is the offset roll torque multiplier. F_{vack} is the input average webtime vacuum thrust (lbf), δ_{mxk} is the maximum thrust vector deflection angle (deg) and ϵ_{osk} is the effective offset distance (in).

(1) Offset Roll Torque Multiplier

$$K_{osk} = \begin{cases} K'_{osk} \text{ if } K'_{osk} \text{ is the input} \\ 1.0 \text{ otherwise} \end{cases} \qquad \text{(dim)}$$

Where K'_{osk} is the input offset torque multiplier (dim)

b. Vortex Roll Torque

$$M_{vrk} = \begin{cases} 0 \text{ if } M'_{rtk} \text{ is input} \\ M'_{vrk} \text{ if } M'_{vrk} \text{ is input} \\ K_{vrk}\ \eta_{vrk}\ F_{vack} \end{cases} \qquad \text{(in-lbf)}$$

Where K_{vrk} is the vortex roll torque multiplier (dim), η_{vrk} is the vortex roll torque per pound of thrust factor (in).

(1) Vortex Roll Torque Multiplier

$$K_{vrk} = \begin{cases} K'_{vrk} \text{ if } K'_{vrk} \text{ is input} \\ 1.0 \quad \text{otherwise} \end{cases} \quad \text{(dim.)}$$

Where K'_{vrk} is the input vortex roll torque multiplier (dim.)

(2) Vortex Roll Torque Per Pound of Thrust Factor

$$\eta_{vrk} = \begin{cases} \eta'_{vrk} \text{ if } \eta'_{vrk} \text{ is input} \\ 0.00363 \quad \text{otherwise} \end{cases} \quad \text{(in)}$$

Where η'_{vrk} is the input vortex roll torque per pound of thrust factor (in)

(3) Maximum Deflection Angle

$$\delta_{mxk} = \begin{cases} \delta'_{mxk} \text{ if } \delta'_{mxk} \text{ is input} \\ K_{\delta mk} \left[B_{\delta ok} + B_{\delta 1k} (\ln W_{ok}) + B_{\delta 2K} (\ln W_{ok})^2\right] \quad \text{otherwise} \end{cases} \quad \text{(deg)}$$

Where

Stage I

$B_{\delta 01} = 15.729698$

$B_{\delta 11} = -1.5695603$

$B_{321} = 0.046411178$

$K_{\delta m1} = 1.0$

Stage II

$B_{\delta 02} = 10.480564$

$B_{\delta 12} = -1.072774$

$B_{\delta 22} = 0.032548762$

$K_{\delta m2} = 1.0$

Stage III

$B_{\delta 03} = 5.7204757$

$B_{\delta 13} = -0.58573616$

$B_{\delta 23} = 0.018170142$

$K_{\delta m3} = 1.0$

Stage IV

$B_{\delta 04} = 5.7204757$

$B_{\delta 14} = -0.58573616$

$B_{\delta 24} = 0.018170142$

$K_{\delta m4} = 1.0$

Unless

$B'_{\delta jk}$ is input, then set

$B_{\delta jk} = B'_{\delta ik}$

$j = 0,1,$ or $2 \quad k = 1,2,3,$ or 4

or

$K'_{\delta mk}$ is input, then set

$K_{\delta mk} = K'_{\delta mk}$

$k = 1,2,3,$ or 4

Where W_{ok} is the input k-th stage liftoff weight (lb), $B_{\delta jk}$ are coefficients and $K_{\delta mk}$ is the maximum deflection angle multiplier. The maximum deflection angle versus stage liftoff weight is shown in Figure 30.

(4) Effective Offset Distance

$$\epsilon_{osk} = \begin{cases} \epsilon'_{osk} \text{ if } \epsilon'_{osk} \text{ is input} \\ \\ K_{odk} \left(B_{\epsilon 1} + B_{\epsilon 1} D_{Bk} + B_{\epsilon 2} D^2_{Bk} + B_{\epsilon 3} D^3_{Bk}\right) \end{cases} \quad \text{(in.)}$$

$B_{\epsilon o} = 0.51640019 \times 10^{-1}$

$B_{\epsilon 1} = 0.34855505 \times 10^{-2}$

$B_{\epsilon 2} = -0.21364594 \times 10^{-4}$

$B_{\epsilon 3} = 0.28005590 \times 10^{-6}$

Unless $B'_{\epsilon j}$ is input, then set

$B_{\epsilon j} = B'_{\epsilon j}$

$j = 0,1,2,$ or 3

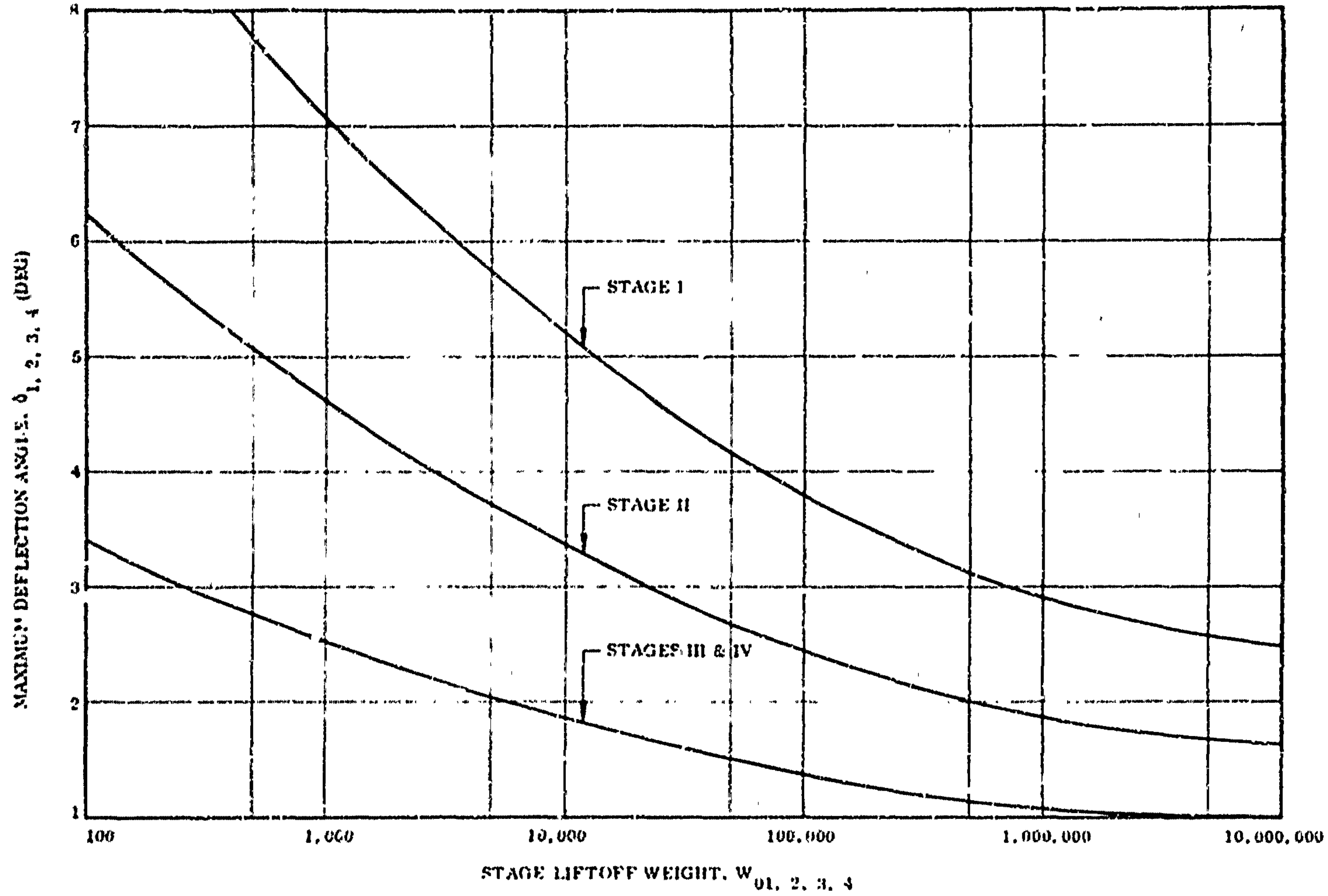

Figure 30. Maximum TVC Deflection Angle vs Stage Liftoff Weight

$$K_{odk} = \begin{cases} K'_{odk} \text{ if } K'_{odk} \text{ is input} \\ \\ 1.0 \quad \text{otherwise} \end{cases} \qquad \text{(dim.)}$$

Where D_{Bk} is the input k-th stage case diameter(in.), B_ϵ are coefficients and K_{odk} is the effective offset distance multiplier (dim.).

c. Aerodynamic Roll Torque

$$M_{ark} = \begin{cases} 0 \text{ if } M'_{rt} \text{ is input} \\ M'_{ark} \text{ if } M'_{ark} \text{ is input} \\ \\ K_{ark}\ (\pi/576)\ C_{N\alpha k}\ D^2_{Bk}\ q\alpha'_{mxk}\ \epsilon_{csk} \end{cases}$$

Where M'_{ark} is the input aerodynamic roll torque (in-lbf), K_{ark} is the aerodynamic roll torque multiplier (dim), $C_{N\alpha}$ is the normal force coefficient (1/deg), D_B is the input case base diameter (in), $q\alpha'_{mxk}$ is the maximum dynamic pressure-angle of attack product (lbf-deg/ft^2). The offset distance versus case diameter is given in Figure 31.

(1) Aerodynamic Roll Torque Multiplier

$$K_{ark} = \begin{cases} K'_{ark} \text{ if } K'_{ark} \text{ is input} \\ \\ 1.0 \quad \text{otherwise} \end{cases} \qquad \text{(dim.)}$$

Where K'_{ark} is the input aerodynamic roll torque multiplier (dim.).

(2) Normal Force Coefficient

$$C_{N\alpha k} = \begin{cases} C'_{N\alpha k} \text{ if } C'_{N\alpha k} \text{ is input} \\ \\ 0.05 \quad \text{otherwise} \end{cases} \qquad \text{(1/deg)}$$

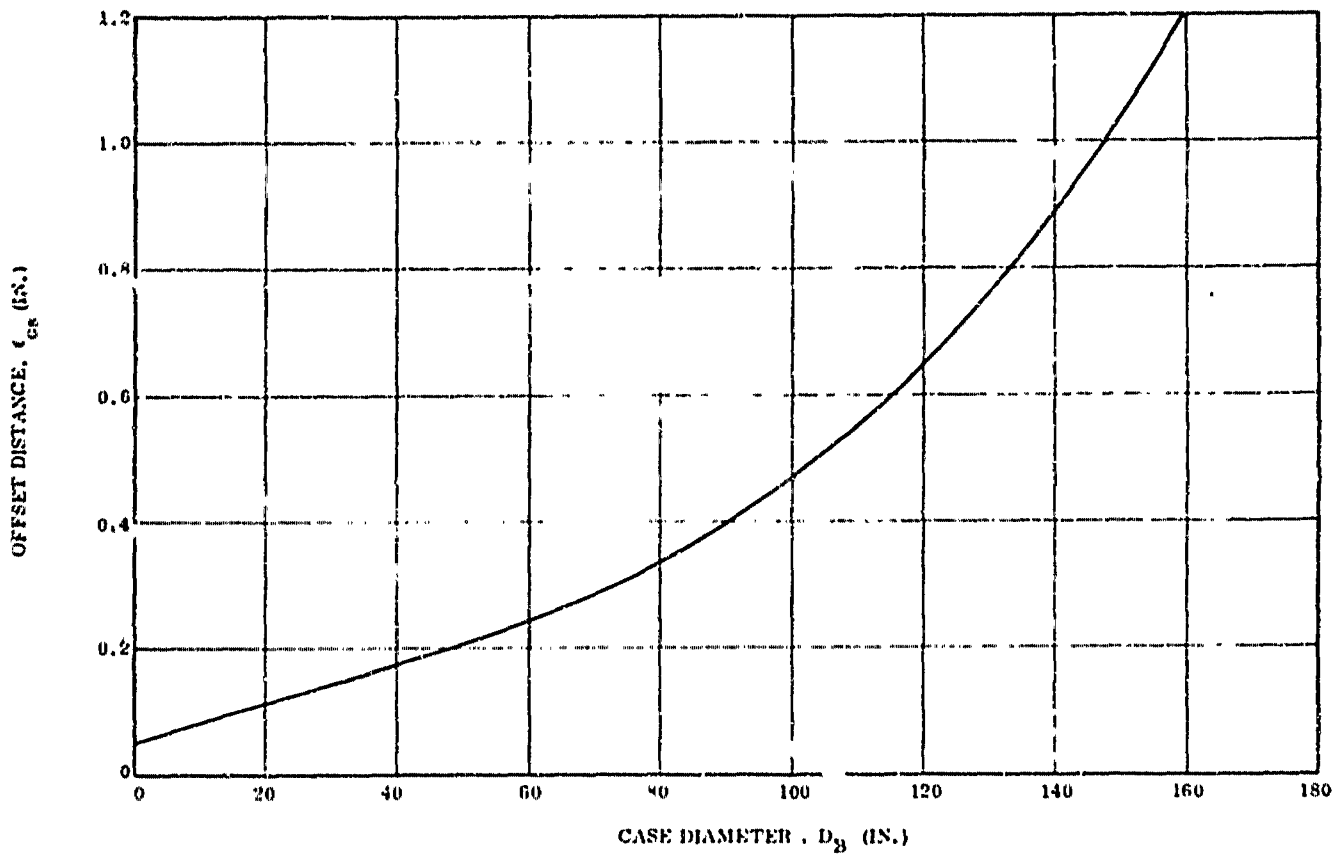

Figure 31. Offset Distance vs Case Diameter

(3) **Maximum Dynamic Pressure-Angle of Attack**

STAGE I

$$q\alpha_{mx1} = \begin{cases} q\alpha'_{mx1} \text{ if } q\alpha'_{mx1} \text{ is input} & \\ & (\text{lbf-deg/ft}^2) \\ K_{q\alpha 1}[B_{q01} + B_{q11} (F_{vac1}/W_{01}) + B_{q21} (F_{vac1}/W_{01})^2 + B_{q31} & \\ \quad (F_{vac1}/W_{01})^3] & \end{cases}$$

Where

$B_{q01} = -1167.3282$

$B_{q11} = 5817.9004$

$B_{q21} = 868.14226$

$B_{q31} = -209.07636$

unless B'_{qj1} is input, then set

$B_{qj1} = B'_{qj1}$

$j = 0,1,2, \text{ or } 3$

$$K_{q\alpha 1} = \begin{cases} K'_{q\alpha 1} \text{ if } K'_{q\alpha 1} \text{ is input} & \\ & (\text{dim}) \\ 1.0 \text{ otherwise} & \end{cases}$$

STAGE II

$$q\alpha_{mx2} = \begin{cases} q\alpha'_{mx2} \text{ if } q\alpha'_{mx2} \text{ is input} & \\ & (\text{lbf deg/ft}^2) \\ K_{q\alpha 2} [B_{q02} = B_{q12} (F_{vac1}/W_{01}) + B_{q22} (F_{vac1}/W_{01})^2 + & \\ \quad B_{q32} (F_{vac1}/W_{01})^3] & \end{cases}$$

Where

$B_{q02} = 2564.6454$

$B_{q12} = -3653.9365$

$B_{q22} = 1477.6228$

$B_{q32} = -60.327994$

unless B'_{qj} is input, then set

$B'_{qj2} = B'_{qj2}$

$i = 0,1,2, \text{ or } 3$

$$K_{q\alpha 2} = \begin{cases} K'_{q\alpha 2} \text{ if } K'_{q\alpha 2} \text{ is input} \\ 1.0 \quad \text{otherwise} \end{cases} \qquad \text{(dim)}$$

STAGE III

$$q\alpha_{mx3} = \begin{cases} q\alpha'_{mx3} \text{ if } q\alpha'_{mx3} \text{ is input} \\ 0 \quad \text{otherwise} \end{cases} \qquad (\text{lbf deg/ft}^2)$$

STAGE IV

$$q_{mx4} = \begin{cases} q'_{mx4} \text{ if } q_{mx4} \text{ is input} \\ 0 \quad \text{otherwise} \end{cases} \qquad (\text{lbf deg/ft}^2)$$

Where B_{qjk} are coefficients (dim), $K_{q\alpha k}$ are the maximum dynamic pressure-angle of attach multipliers (dim), F_{vac1} is the Stage I input average webtime vacuum thrust (lb and W_{01} Stage I liftoff weight (lbm). The maximum qα versus vacuum thrust to weight is given in Figure 32.

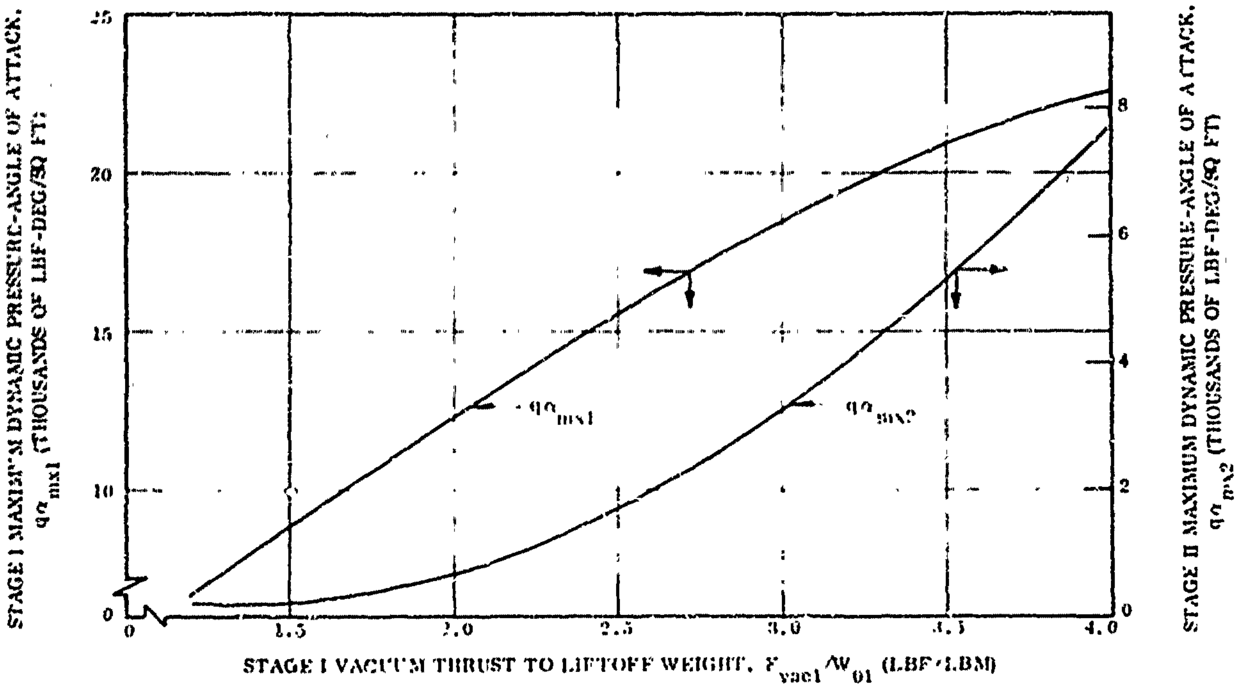

Figure 32. Maximum qα vs Vacuum Thrust to Weight

2. ROLL CONTROL SUBROUTINE NOMENCLATURE

SYMBOL	DEFINITION	UNITS
B DELTA IK	FACTOR OF MAX THRUST VECT DEFLTN ANGLE OF KTH STAGE	—
B EPSI K	CONSTANTS USED TO COMPUTE EFFECTIVE OFFSET DISTANCE	—
BQ JK	COEFFICIENTS FOR COMPUTING MAX DYN PRESS-ANGLE OF ATTK	—
CNALPHA	INPUT NORMAL FORCE COEFFICIENT	(1/DEG)
CNALPHAQ	AERODYNAMIC NORMAL FORCE COEFFICIENT AT MAX QALPHA	(1/DEG)
DB	OUTPUT TVC DUTY CYCLE STAGE CASE DIAMETER	IN
DELTAMXK	MAX THRUST VECTOR DEFLECTION ANGLE	DEG
DELTAPMXK	INPUT MAX THRUST VECTOR DEFLECTION ANGLE	DEG
DELTBARPMX	MAX MAGNITUDE PITCH VECTOR DEFLECTION ANGLE	DEG
EPSILN OSK	EFFECTIVE OFFSET DISTANCE	IN
ETA VRK	VORTEX ROLL TORQUE PER POUND OF THRUST FACTOR	IN
ETA'VRK	INPUT VORTEX ROLL TORQUE PER POUND OF THRUST FACTOR	IN
FVAC	NOMINAL INPUT VACUUM THRUST-TIME CURVE	LB
FVACK	INPUT AVERAGE WEBTIME VACUUM THRUST	LBF
KARK	AERODYNAMIC ROLL TORQUE MULTIPLIER	—
KDELTA MK	FACTOR OF MAX THRUST VECTOR DEFLECTION ANGLE	—
KK	STAGE NUMBER	—
KODK	EFFECTIVE OFFSET DISTANCE MULTIPLIER	—
KOSK	OFFSET ROLL TORQUE MULTIPLIER	—
KQ ALPHA K	FACTOR OF MAX DYNAMIC PRESSURE-ANGLE OF ATTACK PRODUCT	--
KVRK	VORTEX ROLL TORQUE MULTIPLIER	—
K'OSK	INPUT OFFSET TORQUE MULTIPLIER	—
MARK	AERODYNAMIC ROLL TORQUE	IN-LBF
MOSK	OFFSET ROLL TORQUE	IN-LBF
MRTK	ROLL TORQUE	IN-LBF
MVRK	VORTEX ROLL TORQUE	IN-LBF
M'OSK	INPUT OFFSET ROLL TORQUE	IN-LBF
M'RTK	INPUT ROLL TORQUE	IN-LBF
M'VRK	INPUT VORTEX ROLL TORQUE	IN-LBF
QALPHA M	DYNAMIC PRESSURE	LB-FT**2
QALPHA MXK	MAXIMUM DYNAMIC PRESSURE-ANGLE OF ATTACK PRODUCT	LBF-DEG/FT**2
WO	STAGE LIFTOFF WEIGHT	LB