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ARRAY PROCESSING OF ALASKAN LONG-PERIOD ARRAY DATA

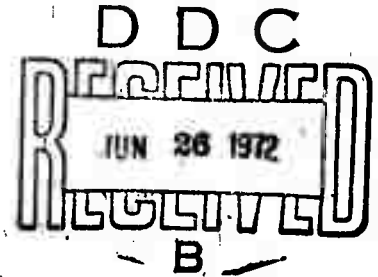
SPECIAL REPORT NO. 3
EXTENDED ARRAY EVALUATION PROGRAM

Prepared by
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SECTION I

INTRODUCTION

The purpose of this study was to evaluate the performance of multi-channel filter (MCF) processors on ALPA data, and to compare their performance with that of the much simpler beamsteer processor. The types of MCF processors tested included two different frequency-domain optimal filters, a time-domain optimal filter, and a time-domain adaptive filter. In addition, the effectiveness of the weighted beamsteer technique was investigated. The work was performed using the vertical component of the nine-site subarray which was available during 1970 and early 1971. The results are discussed in the following sections.

SECTION II

COMPARISON OF MCF AND BEAMSTEER PROCESSORS

At the present time MCF processors for ALPA are designed in the frequency domain. The time series data used to estimate the noise crosspower spectral density matrices (CPS) are divided into 256 second segments. These data are resampled to a two second sampling rate. The strong rejection by the ALPA system response above 0.25 Hz permits resampling without prior anti-alias filtering. After taking the discrete Fourier transform (DFT) of the data in each segment, the CPS are formed for each segment. Smoothed CPS are formed by averaging the matrices over segments. The signal CPS are generated theoretically, to represent a plane wave signal traversing the array. The signal autopower at each site and frequency is set equal to four times the average noise autopower at that frequency. The signal crosspowers are assigned phase delays corresponding to those of a plane wave coming from the designated source direction with a surface-wave velocity. At each frequency, after designing the filters, an estimate of the design noise MCF output power density is obtained by post- and pre-multiplying the noise CPS by the vector of filter transforms and its conjugate transpose respectively.

$$\begin{aligned}
 \text{NPO}(k) &= \{F_1^*(k) \quad F_2^*(k) \quad \dots \quad F_{\text{NCH}}^*(k)\} \cdot \\
 &\quad \frac{1}{M} \left[\begin{Bmatrix} N_1^{1*}(k) \\ N_2^{1*}(k) \\ \vdots \\ N_{\text{NCH}}^{1*}(k) \end{Bmatrix} \{N_1^1(k) \quad N_2^1(k) \quad \dots \quad N_{\text{NCH}}^1(k)\} \begin{Bmatrix} N_1^M(k) \\ N_2^M(k) \\ \vdots \\ N_{\text{NCH}}^M(k) \end{Bmatrix} \{N_1^M(k) \quad N_2^M(k) \quad \dots \quad N_{\text{NCH}}^M(k)\} \right] \\
 &\quad + \dots + \begin{Bmatrix} N_1^{M*}(k) \\ N_2^{M*}(k) \\ \vdots \\ N_{\text{NCH}}^{M*}(k) \end{Bmatrix} \{N_1^M(k) \quad N_2^M(k) \quad \dots \quad N_{\text{NCH}}^M(k)\} \\
 &\quad \cdot \begin{Bmatrix} F_1(k) \\ F_2(k) \\ \vdots \\ F_{\text{NCH}}(k) \end{Bmatrix}
 \end{aligned} \tag{1}$$

where: $NPO(k)$ is the MCF output power density for design noise at frequency index k ,

$F_i(k)$ is the filter transform for channel i at frequency index k ,

$N_i^j(k)$ is the transform of the design noise for channel i at frequency index k from segment j ,

* indicates complex conjugation.

Matrix equation (1) implies the following procedure. Within each segment at each DFT frequency the data and filter transforms are multiplied for each channel, and the resultant products are summed to yield the MCF output transform. The product of this output transform with its complex conjugate gives the power density of the MCF output for this segment and frequency. Finally these power densities are averaged over the segments of the design noise gate. As shown in appendix A this procedure implies a channel-by-channel circular convolution of the time-domain data and filter impulse responses within each segment. The resultant $NPO(k)$ will be referred to as the matrix multiply spectra. It is important to note that this results in an indirect estimate of the spectral content of the design noise after processing with the MCF.

The actual application of the filters to the data is accomplished in the time domain. The inverse DFT of the filter transform yields the filter impulse responses which are convolved with the data to yield the desired time-domain output. A second estimate of the MCF output power density is computed from the DFT of this MCF output time series. This estimate will be referred to as the direct spectra. It provides the actual spectral content of the noise after processing with the MCF. Since the procedures to estimate the two types of spectra are not algebraically equivalent it is not certain that the resultant spectra will be in agreement. More important, the procedures used to obtain the matrix multiply spectra are implicit in the MCF design algorithm. Thus the filters are optimal for this type of application but are sub-optimal for actual time-domain application.

These procedures were implemented on a suite of 23 independent one-hour noise samples. Table II-1 gives the data start time and the number of

TABLE II-1
LIST OF ONE-HOUR NOISE SAMPLES

Sample No.	Date	Start Time	MCF Look Direction	No. of Sites
1	5/27/70	03:09:08	327°	3
2	7/03/70	02:52:17	350°	6
3	7/28/70	06:12:00	277°	8
4	7/30/70	11:13:09	-19°	6
5	8/02/70	12:09:36	-61°	6
6	8/08/70	09:49:02	323°	5
7	8/12/70	23:56:54	321°	6
8	8/13/70	03:18:04	-55°	9
9	8/13/70	20:47:16	-89°	7
10	8/19/70	08:51:06	-90°	7
11	8/20/70	10:56:30	-17°	8
12	8/22/70	09:59:10	-90°	9
13	8/25/70	23:16:31	-95°	9
14	8/28/70	00:08:58	-35°	6
15	8/29/70	07:54:35	10°	9
16	8/29/70	14:02:56	-72°	6
17	8/30/70	05:48:23	-89°	9
18	8/30/70	14:59:43	-19°	6
19	8/31/70	11:40:00	-9°	7
20	9/14/70	20:51:56	270°	5
21	10/14/70	12:40:00	-45°	7
22	12/17/70	05:58:28	328°	6
23	12/23/70	05:46:45	343°	7

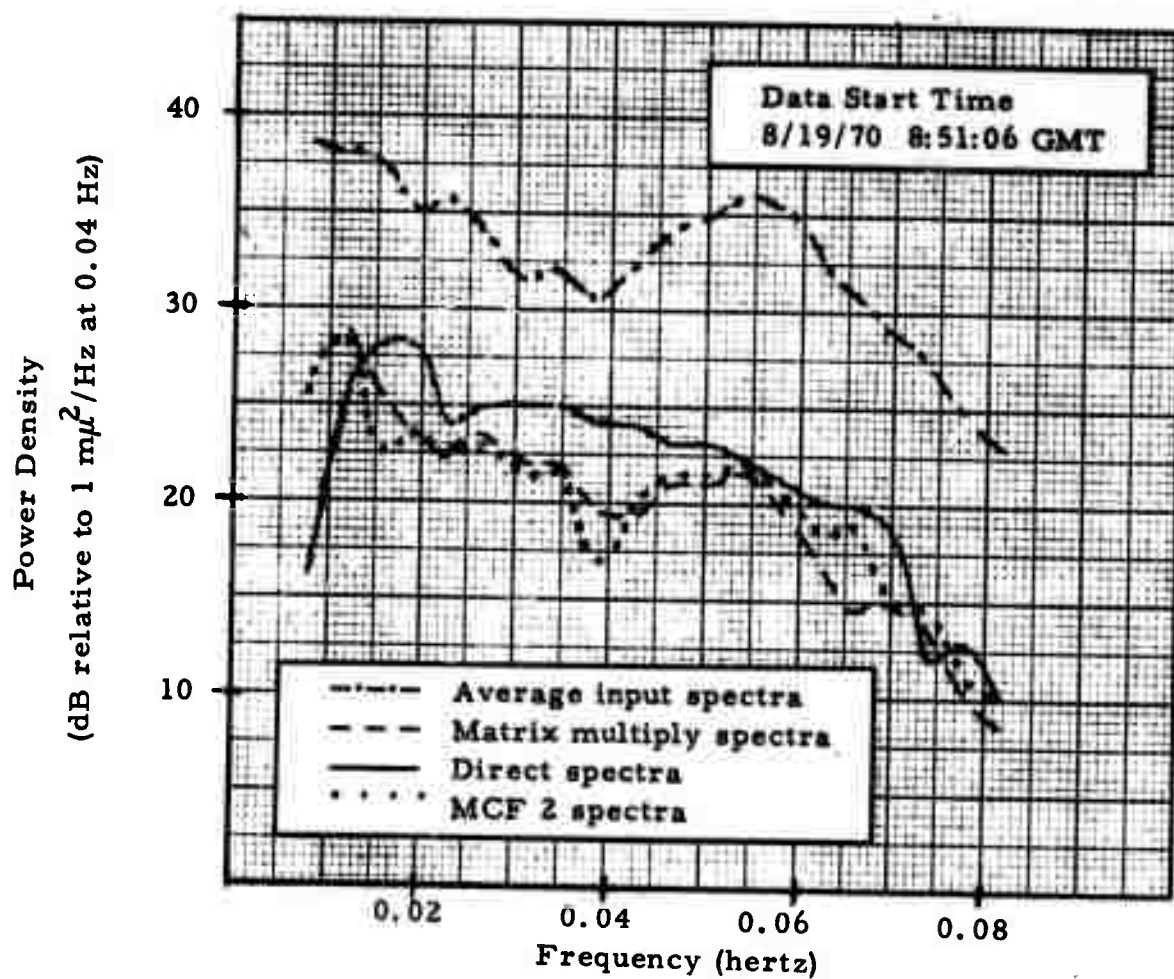


FIGURE II-1
TYPICAL SEVEN-SITE MCF OUTPUT SPECTRA

channels used by the MCF for each of these samples. The vertical component only was used in this study. In each case both the matrix multiply and the direct spectra were estimated. Figure II-1 shows the various power density spectra for a typical sample. In the approximate signal band (0.02 to 0.06 Hz) the matrix multiply spectra is always lower than the direct spectra. The power in the band 0.02 to 0.06 Hz was chosen as a measure for comparison. For each of the 23 samples this power computed from the matrix multiply spectra was lower than the power computed from the direct spectra. The average ratio of these two powers was 2.5 dB and in one case the ratio was 4.8 dB. These results suggest that the suboptimality of the filters leads to significant performance degradation.

As shown in appendix A, there is an alternate procedure for design and application which is not subject to this difficulty. In this case both the design and application are done in the frequency domain. The final output transform is then inverse transformed to get the desired time-domain output. This type of design was performed on 14 of the 23 samples mentioned above. Figure II-1 also includes an output obtained from this type of MCF labelled MCF2. This provides the actual spectral content of the noise after processing with the alternate MCF. This spectra is similar to the matrix multiply spectra and superior to the direct spectra. In all 14 cases the broadband power (0.02 to 0.06 Hz) obtained using the new technique was lower than that obtained from the direct spectra. The largest ratio of these two powers was 3.9 dB and the average was 2.2 dB.

These results indicate that the new technique is superior to that currently in use. It must be noted, however, that this conclusion is based on their performances on design noise. The crucial question is their relative performances on off-design noise. In four of the 23 cases above it was possible to locate an off-design noise sample in reasonable time proximity to the design noise. In each case both the old and new MCF processors as well as a beam-steer processor were applied to the off-design noise. The noise suppressions in the 0.02 to 0.06 Hz band for each of these processors are given in Table II-2. The corresponding MCF results for design noise also are given. In this table

MCF1 refers to the old technique and MCF2 refers to the new technique. The time intervals between design and off-design noise are: NS1 - 1.12 hours; NS2 - 0.77 hours; NS3 - 3.18 hours; NS4 - 2.05 hours. In the ideal case the design-noise gate would immediately precede the time gate of the signal to be extracted. Frequently, however, the presence of arrivals from other events or of data dropouts precludes the selection of a design-noise gate immediately preceding the desired signal. Thus the separations between design and off-design noise for these four events are not atypical. The data of table II-2 indicates that the new MCF loses most of its superiority over the old MCF when working on off-design noise. Neither MCF shows marked superiority over the beamsteer processor for off-design noise. Note that in these four cases the

TABLE II-2
ARRAY PROCESSOR NOISE SUPPRESSION

	Noise In/Noise Out (dB)						
	Off-Design Noise			Design Noise		Number of Sites (n)	$\sqrt{n}$ Improvement
	B/S	MCF1	MCF2	MCF1	MCF2		
NS1	8.7	10.2	10.6	12.5	14.1	6	7.8
NS2	10.8	8.4	10.0	9.6	12.5	8	9.0
NS3	7.8	7.5	7.8	9.6	13.2	8	9.0
NS4	9.3	8.2	8.0	12.8	14.2	8	9.0

beamsteer suppression is within two dB of the $\sqrt{n}$ figure.

The degradation in MCF performance on off-design noise is due either to overdesign or to nonstationarity of the noise. If the noise is stationary the problem can be overcome by using a longer noise gate for computing the noise crosspower matrices. Two five-hour noise samples were selected for processing with the new MCF. The first 12288 seconds of each sample was designated as design noise. This is three times as much noise as was used in the designs discussed above. The next 4096 seconds was designated as off-design noise. Initially, for control in each case, an MCF was designed from

TABLE II-3
NOISE SUPPRESSION FOR TWO LONG NOISE SAMPLES

	Noise In/Noise Out (dB)					
	MCF Designed From 4096 Seconds of Noise		MCF Designed From 12288 Seconds of Noise		Beamsteer on Off- Design Noise	$\sqrt{n}$ Improvement
	Design Noise	Off-Design Noise	Design Noise	Off-Design Noise		
NS5	13.6	8.7	12.1	9.3	7.8	8.5
NS6	13.3	7.1	11.2	8.4	7.6	8.5

the last 4096 seconds of the design noise gate. These filters were then applied back to the 4096 seconds of design noise and to the off-design noise. The resultant noise suppressions are given in table II-3. In these two cases the differences between design and off-design noise suppression were 4.9 dB and 6.2 dB. These results are consistent with those of the four events in table II-2. The filters were next designed using the full 12288 seconds of design noise in each case. Again they were applied to the last 4096 seconds of design noise and to the off-design noise. The data of table II-3 shows that the noise suppressions on design noise degraded by 1.5 dB and 2.1 dB in these two cases. As expected the filters designed from the full 12288 seconds were not as highly tuned to the last 4096 seconds of design noise. Also as expected the suppressions of off-design noise improved by 0.6 dB and 1.3 dB. The differences between off-design and design noise suppression, however, are still 2.8 dB in both cases. Also as shown in the table, beamsteer processors operating on the off-design noise were only 1.5 dB and 0.8 dB inferior to the MCF.

In several of the cases discussed in this section the various MCF and beamsteer processors were applied to associated large Rayleigh-wave signals to evaluate signal preservation. In none of these cases was there significant signal degradation.

In summary it appears that the new MCF design technique is somewhat superior to the old technique for the suppression of design noise. When the MCF's are applied to off-design noise the new MCF loses much of its superiority over the old MCF and performs about as well as a beamsteer processor. Both the new MCF designed from 4096 seconds of data and a beamsteer processor were compared on six different noise samples. The ratio of MCF to beamsteer broadband (0.02 to 0.06 Hz) noise suppression ranged from 1.9 dB in favor of the MCF to 1.3 dB in favor of the beamsteer and had an average of 0.2 dB in favor of the MCF. In two of these cases the MCF was re-designed using 12288 seconds of design noise. The performances on off-design noise improved slightly but the MCF's were still only 1.5 dB and 0.8 dB superior to the beamsteer.

These results suggest that MCF's designed from three hours or less of design noise and applied to a different data gate containing a presumed signal show little improvement over the much simpler beamsteer processor. Very rarely is it possible to locate more than three hours of pure design noise in time proximity to a desired signal. It is concluded, therefore, that the design of MCF's from a design noise gate not including the desired signal is not a useful technique for ALPA data. This conclusion is based on the foregoing results obtained with array processors employing nine sites or less from the southern half of ALPA. It does not appear likely that the addition of the remaining ten sites will add significantly to the multiple coherence of the noise. Little change in these results is expected, therefore, for the full nineteen-site array. It is possible that the noise structure at other arrays such as NORSAR may lead to different conclusions.

There is a possible alternate method of MCF processing. It may be practical to use a noise gate which includes the desired signal for estimating

the noise crosspower matrices. In this case the filters are applied back to the data from which they were designed and the off-design problem is nonexistent. This procedure is discussed in section III.

SECTION III

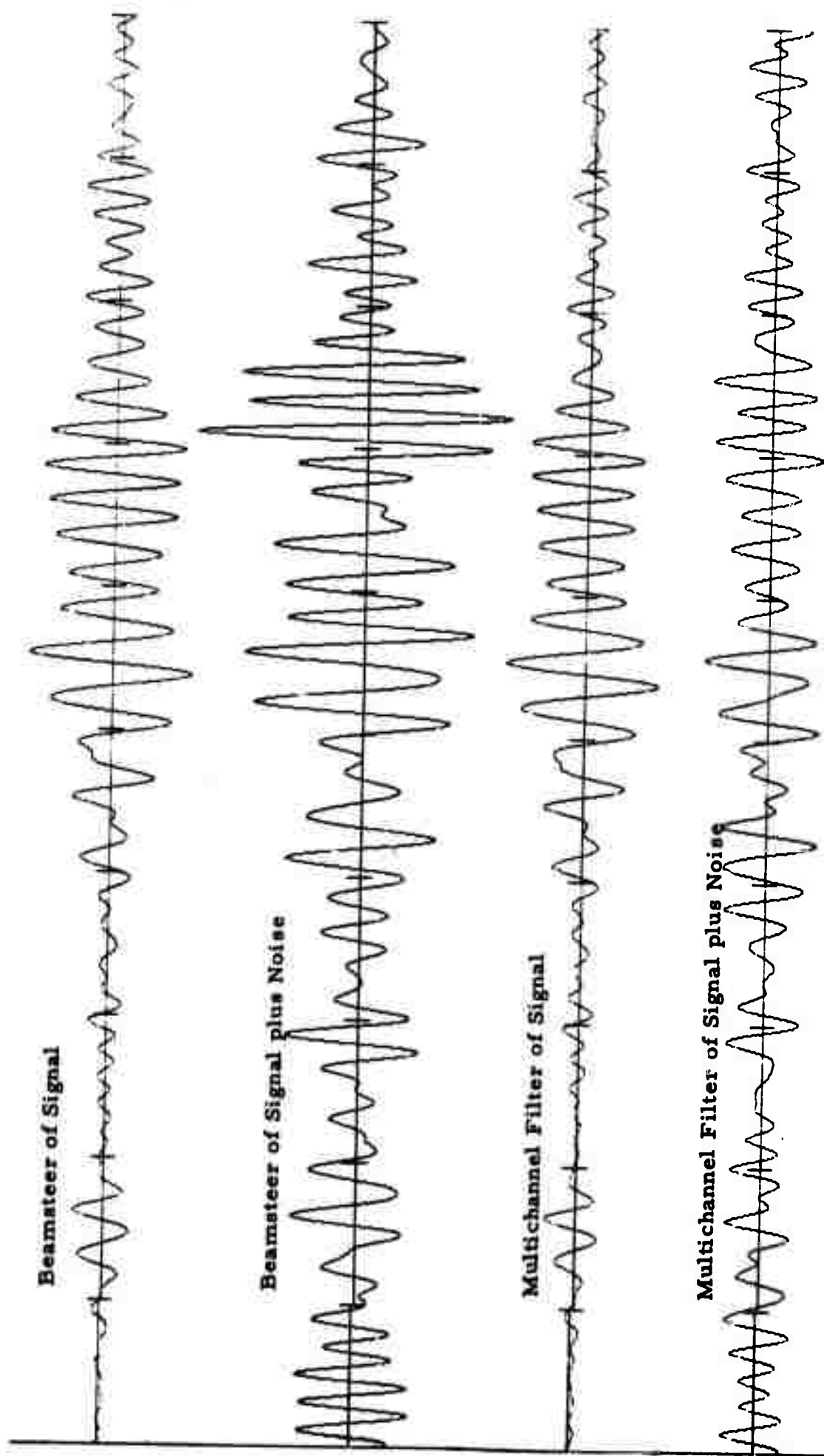
DESIGN OF MCF PROCESSORS ON THE SIGNAL GATE

The loss in noise suppression resulting from the failure of the design noise to accurately characterize the noise in the off-design gate can be avoided by using the signal gate itself to estimate the noise crosspower densities. In so doing, however, one is treating whatever signal is actually present as part of the ambient noise field. It might be hoped that in the practical case where the signal to be extracted is small in comparison with the ambient noise, the resultant perturbation of the noise crosspower densities will be insignificant. The results of an experimental study of this approach are presented here.

The use of small events in such a study is not suitable since one has no way of knowing the true signal and consequently no way of measuring signal preservation by the processor. To circumvent this difficulty, large signals were scaled down and "buried" in segments of ambient noise. In this way one not only knows the true signal, but is able to beamsteer or MCF the pure noise, the pure signal, and the composite of signal plus noise.

The first such composite was formed by scaling down a magnitude 5.4 event from Eastern Russia by a factor of 50. After scaling, the ratio of average single site RMS noise in the band 0.025 to 0.055 Hz, to the largest zero-to-peak signal value was 1.6. Six good sites were available for processing. The beamsteer of the pure signal as well as that of the composite is shown in Figure III-1. The amplitude scales in this and subsequent figures were chosen so that if there is no distortion of the signal it should appear with the same amplitude in each trace. The beamsteer of the signal alone is assumed to have no distortion and hence to represent the true signal waveform. There is distortion in the beamsteer of the composite, and the peak values are considerably larger than those of the true signal. The noise passed by the beamsteer cancels some of the signal peaks and reinforces others.

Noise crosspower densities were estimated from the composite data and used in the design of an MCF. In all cases in this section the length of the



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Figure III-1

Array Processing of Eastern Russia Rayleigh Wave

composite was 4096 seconds of which the signal occupied about 500 seconds. This MCF was applied to both the pure signal and to the composite with the results shown in Figure III-1. All of the data of this figure have been filtered with an 0.025 to 0.055 Hz bandpass filter. The result of applying the MCF to the pure signal is not greatly different from the beamsteer of the pure signal. This suggests that the response of the MCF to plane wave energy coming from the designated source direction is essentially unity. When the MCF is applied to the composite, however, the output signal is a poor replica of the true signal. It would appear that while the MCF passes plane wave energy coming from the designated source direction with little distortion, it operates on the noise so as to cause it to cancel the signal. Since the actual signal and the noise are both included in the data used to estimate the noise crosspower densities, the filters "know" to some degree the behavior of these two components and can effect the partial cancellation. The theoretical possibility of this hypothesis can be illustrated with a simple example.

Consider the problem of designing a two-channel MCF where the noise crosspower densities are estimated from data which include both the noise and the signal. Assume that on channel one the noise happens to completely cancel the signal, but that such cancellation does not occur on channel two. Then the resultant noise crosspower matrix and the theoretical signal crosspower matrix can be represented as:

$$\begin{bmatrix} 0 & 0 \\ 0 & N_{22} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Then the MCF design equation from the appendix will be:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & (N_{22} + S_{22}) \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} S_{11} \\ S_{21} \end{Bmatrix}$$

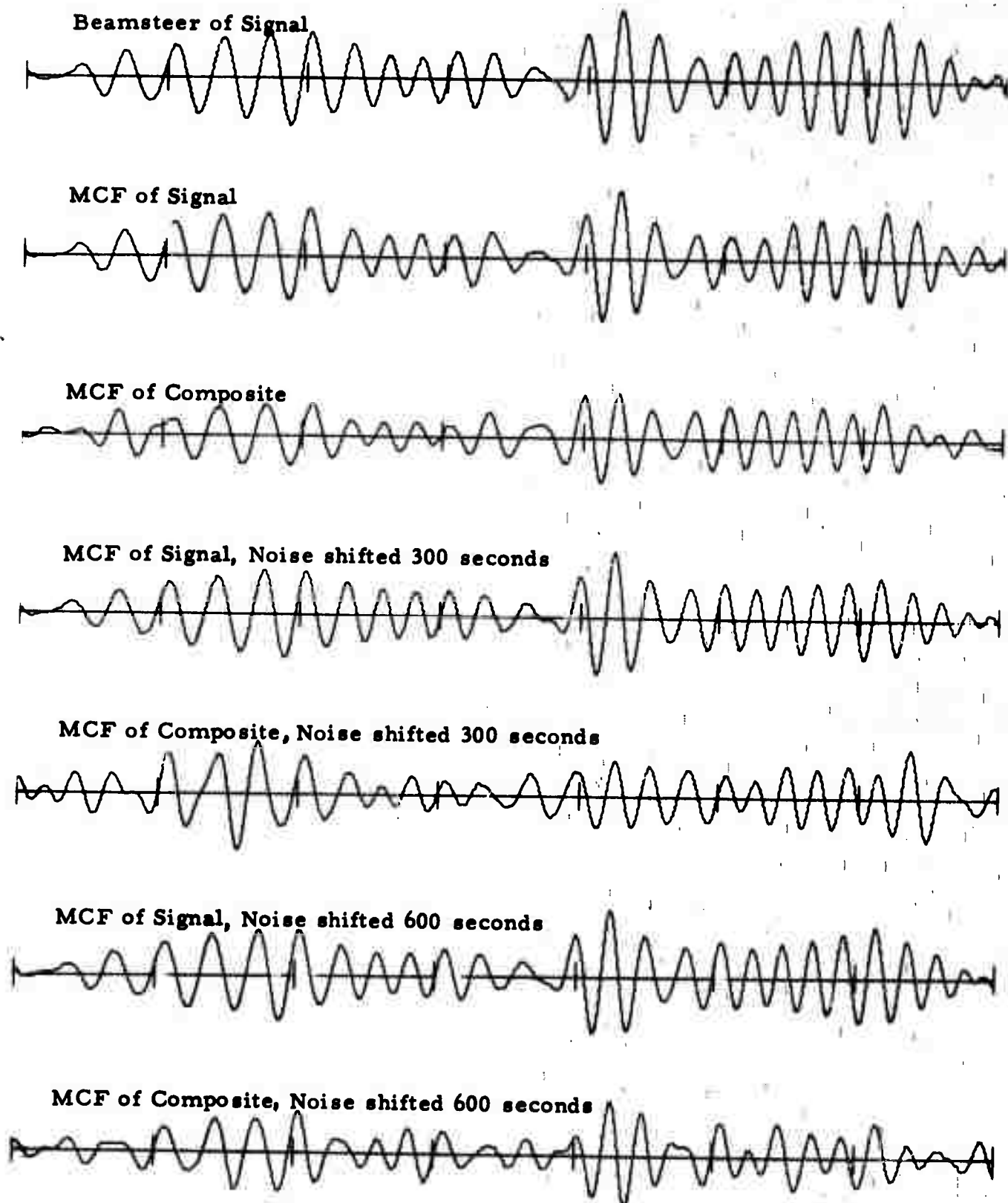
The solution of this equation is $F_1=1$, $F_2=0$. If the theoretical signal matrix is post- and pre-multiplied by this filter vector and its conjugate transpose respectively, it is seen that the desired signal on channel one is reproduced exactly. On the other hand if the filter is applied to the actual data, represented by the noise crosspower matrix above, the output power is zero. Thus while the filters have the desired response to the signal model, they are able to work on the signal and

noise together so as to yield a zero output. While the situation hypothesized here is rather extreme, it does indicate the theoretical possibility of the type of result observed above with real data.

The first portions of the composite traces of Figure III-1 indicate that the noise rejection of the MCF is superior to that of the beamsteer. When the two processors were applied to the noise alone the output RMS values in the 0.025 to 0.055 Hz bandwidth were 2.59 millimicrons for the beamsteer and 1.97 millimicrons for the MCF; a difference of 2.4 dB. The peak values of the composite output signals for the two processors differed by 8.5 dB in favor of the beamsteer. While M_s estimates obtained from the output of either processor would be in error, the ease of implementation of the beamsteer would suggest its use rather than the MCF designed on the signal gate.

It should be noted that these MCF's were designed using a signal model which was much larger than the signal which is actually present. The intent of the author has been to use the MCF's to discriminate between signal and noise purely on the basis of differences in their vector velocities. Noise rejection based on frequency content is accomplished by bandpass filtering the MCF output. The end result desired is to preserve all plane wave energy propagating with the vector velocity of the signal and falling within the range of the bandpass filter. The question of how these results would change if the true signal-to-noise ratio was used in the MCF design was not investigated. It is recognized that the approach used here constitutes a departure from the classical least-mean-square-error criterion, and this departure was intentional. It is believed that our results are similar to those which would be achieved by a maximum likelihood MCF followed by a bandpass filter.

A second sample was formed by scaling an $m_b = 5.7$ Iran event with an 0.0005 scale factor and adding it to noise. After scaling, the ratio of the RMS noise to the zero-to-peak signal amplitude was 1.9. In this case seven channels were available for processing. The beamsteered signal and several MCF'ed signal and composite traces are shown in Figure III-2. These processed outputs have been bandpassed with the 0.025 to 0.055 Hz filter. Again the signal in the MCF of the



100 seconds

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Figure III-2

Array Processing of Iran Rayleigh Wave

composite is smaller than that obtained when the MCF is applied to the pure signal, suggesting that while the MCF response to energy coming from the designated source direction is essentially unity, the MCF is able to work the noise against the signal to produce a smaller signal in the composite output. To determine whether this results from a unique time alignment of the signal and noise in the composite, two new composites were formed. In the first of these the noise was shifted 300 seconds with respect to the signal before adding them together. In the second the shift was 600 seconds. As seen in Figure III-2 despite the new time alignments of signal and noise in the composites, the signal in the MCF'ed composite is smaller than when the MCF's are applied to the pure signal.

It is difficult to assign a quantitative measure of signal distortion to these results since the distortion varies as a function of time along the signal. In an effort to obtain such a measure, the beamsteer of the pure signal and the beamsteer and MCF of each of the composites were filtered with a chirp filter. In each case the largest zero-to-peak value in the chirp filtered output was taken as the measure of signal amplitude and that value for the beamsteered pure signal was considered to be the true signal amplitude. Table III-1 gives the ratio of signal amplitude in each of the composite outputs to that of the true signal. In each case the beamsteered composite yields a high estimate while the MCF leads to a small estimate.

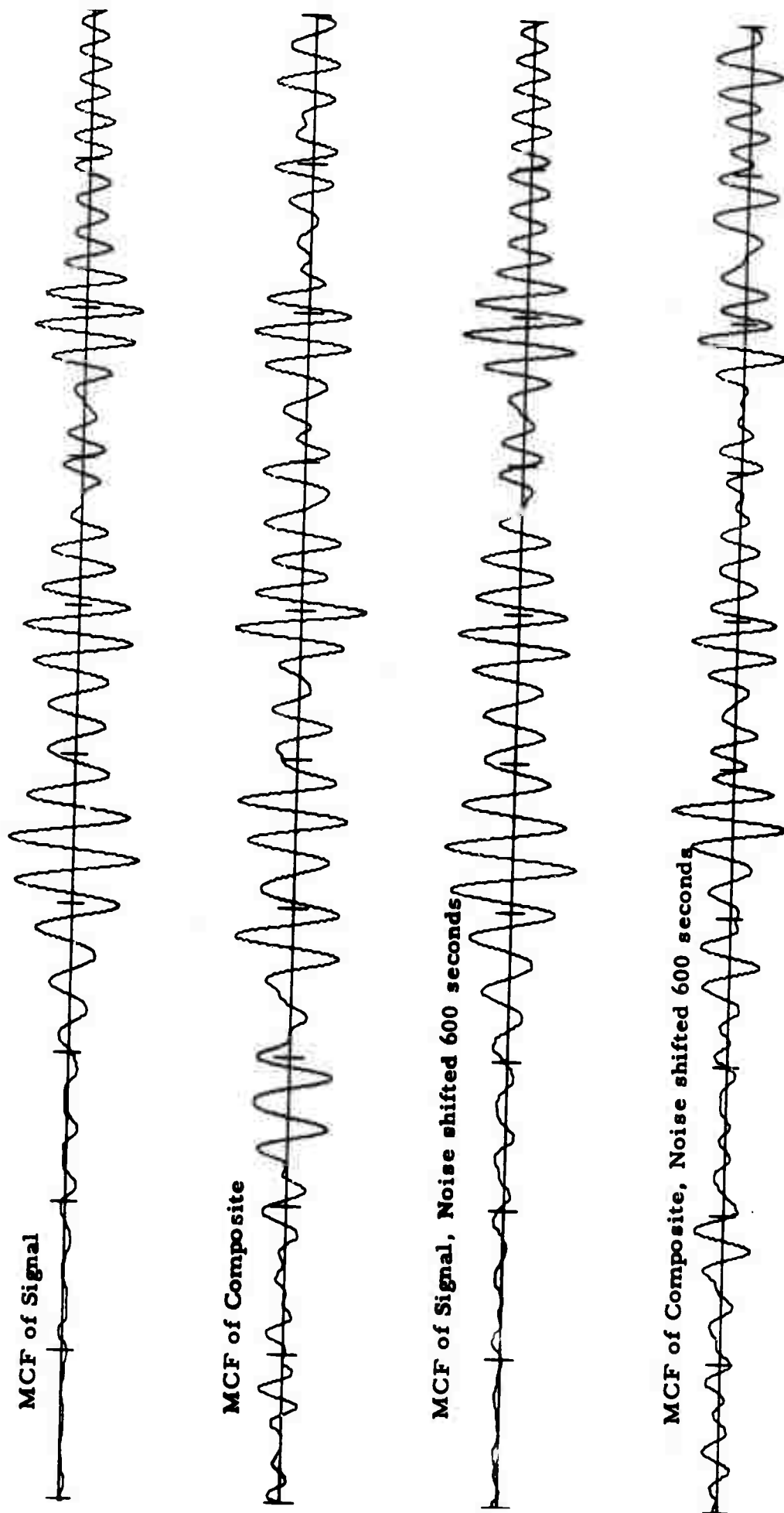
TABLE III-1
IRAN SIGNAL AMPLITUDE IN PROCESSED COMPOSITE/TRUE
SIGNAL AMPLITUDE (dB)

	Relative Alignment of Signal and Noise in Composite (seconds)		
	0	300	600
Beamsteer	2.0	3.5	1.2
MCF	-2.9	-2.7	-3.2

It may appear surprising that the beamsteered and chirp filtered composite would yield a peak value larger than does the similarly processed pure signal. One would expect the noise in the composite to accentuate some signal peaks and suppress others. The chirp filter would presumably average out these effects and provide a peak value similar to that observed for the pure signal. Since the chirp is a linear filter, however, the chirped composite can be regarded as the summation of the chirped noise and chirped signal. The chirp in general is not a perfect match for the signal. As a result instead of having one clearly defined peak, the chirped pure signal may have two or three peaks of almost identical amplitude. The chirped noise also has a series of peaks and troughs, not systematically related to those of the signal. In the summation the noise peaks and troughs tend to cancel some of the signal peaks and accentuate others. In measuring the signal peak value of the composite, the largest peak is chosen. The considerations above indicate that this value will be biased high.

Beamsteering the pure noise used in forming the composites results in an output noise level of $3.28 \text{ m}\mu$ RMS, while each of the MCF processors gave output noise levels of about $1.9 \text{ m}\mu$ RMS. This is a difference in noise suppression of 4.8 dB. Comparison of these data with the data of Table III-1 indicates that the signal-to-noise ratios provided by the beamsteer and MCF processors are almost identical for this example.

The third sample was formed by scaling an $m_b = 5.2$ Sinkiang event with an 0.003 scale factor and adding it to the noise. After scaling the RMS noise to signal zero-to-peak ratio was 2.1. In this case six channels were available for processing. Figure III-3 gives the MCF results, again bandpassed with an 0.025 to 0.055 Hz filter. Here the original composite and a second in which the noise was shifted 600 seconds with respect to the signal were processed. In the original composite distortion of the signal is seen to be less severe than in the second composite. Table III-2 gives the ratio of signal amplitude in each of the composite outputs to that of the true signal, again measured from the chirp filtered outputs. Suppression of the noise in both composites was about 4.6 dB better for the MCF than for the



Epicenter 77.8 E 39.9 N

Figure III-3

Array Processing of Sinking Rayleigh Wave

TABLE III-2
CHINA SIGNAL AMPLITUDE IN PROCESSED COMPOSITE/TRUE
SIGNAL AMPLITUDE (dB)

	Relative Alignment of Signal and Noise in Composite (seconds)	
	0	600
Beamsteer	0.8	0.4
MCF	-1.2	-6.0

beamsteer. Thus in the case of the first composite the MCF yielded a signal to noise ratio about two dB better than the beamsteer, but in the second case just the reverse was true.

In summary the following observations can be made. When the MCF is designed from noise matrices estimated on a data gate containing the signal to be extracted, the response to plane waves coming from the epicentral direction appears to be good, but the MCF is able to work the noise against the signal so as to produce an attenuated and distorted version of the signal, at least for high design S/N ratios. When a beamsteer processor is applied to small signals the tendency is to produce a distorted and amplified version of the signals. Suppression of the noise by the MCF is about four dB better than that provided by the beamsteer. The difference in output signal levels, however, tends to cancel this difference in noise suppression. On the average the signal-to-noise ratios provided by the two processors is about equal. These results suggest that whether the noise matrices are estimated from a fitting interval preceding the signal as discussed in Section II, or are estimated from the signal gate itself has little influence on the MCF performance, and the simpler beamsteer performs about as well.

SECTION IV

WEIGHTED BEAMSTEER PROCESSOR

The foregoing results suggest that MCF processing is not a useful concept for the extraction of signals from noise at ALPA. It has been noted, however, that on occasion the ALPA noise levels at periods greater than twenty seconds are highly variable from site to site. The signal levels tend to be reasonably well equalized across the array. There exists then the possibility that weighted beamsteer processing may have some potential superiority to simple beamsteer processing. The weighted beamsteer as used here is actually an extremely simple type of MCF.

The data is first time shifted to align the desired signal across channels. A one point MCF is then designed with the objective of minimizing the mean square value of the MCF time-domain output, but subject to the constraint that the sum of the individual channel filter weights must equal one. To the extent that the signal is truly a plane wave coming from the designated source direction, this constraint ensures preservation of the signal. The resultant weighted beamsteer filter weight for channel n is

$$W_n = \frac{1}{\overline{N_n^2(t)} \sum_{i=1}^{NCH} \frac{1}{\overline{N_i^2(t)}}}$$

where: $\overline{N_i^2(t)}$ is the mean square value of the noise on channel i .

Both beamsteer and weighted beamsteer processors were applied to six different noise samples. The weighted beamsteer weights were estimated from 1000 seconds of bandpassed noise just preceding the segment to be treated in each case. The bandpass, 0.025 to 0.055 Hz, is the same as that over which the noise suppression of the processor was computed. In these six cases the RMS value of the weighted beamsteer output was less than the beamsteer output by 0.5 dB, 0.6 dB, 0.5 dB, 0.2 dB, 1.2 dB, and 0.7 dB. In four of the cases where a large signal existed in time proximity to the treated noise, the processors were also applied to the signal. In each of these cases the signal

outputs of the two processors were essentially equivalent. While the weighted beamsteer is always superior to the beamsteer, the difference in noise suppression in these six cases was not great.

It has been subsequently observed that on occasion the weighted beamsteer does suppress ALPA noise by as much as four dB more than the beamsteer (personal communication, George Bulin, VELA Seismological Center, Alexandria Virginia). It would appear that in a routine monitoring context, one would at least like to have the capability to do weighted beamsteer so as to achieve the best possible signal extraction in marginal cases.

SECTION V

TIME DOMAIN MCF DESIGN

Time-domain multi-channel filters were evaluated on the two five-hour noise samples discussed on page II-6. As was done for the frequency-domain design, the first 12288 seconds of each sample was designated as design noise, and the next 4096 seconds as off-design noise. In each case two maximum likelihood MCFs were designed, the first using the last 4096 seconds of the design noise gate for estimating the required noise statistics, and the second using the full 12288 seconds for this purpose. Both MCF processors were applied to the last 4096 seconds of design noise and to the off-design noise. The RMS values in the 0.02 to 0.06 Hz band of the processed outputs were measured. The figure of merit used was the ratio of MCF RMS noise to that of a beamsteer processor operating on the same data gate. Table V-1 gives the improvement over the beamsteer in noise suppression for the frequency-domain filters discussed in Section II, for the time-domain filters, and for time-domain adaptive filters.

When the optimum filters were designed from 4096 seconds of noise and applied back to the design noise, the frequency-domain design performed 2.7 dB better than the time-domain design in one case and 1.6 dB better in the other case. When these same filters were applied to the off-design noise the advantage switched to the time-domain design. In each of the designs discussed here the time-domain filters have 33 points per channel. Each frequency-domain filter was designed at 64 independent frequencies and consequently had 64 complex degrees of freedom per channel. The frequency-domain filter with its greater number of degrees of freedom was able to tune more sharply to the design noise, but consequently performed more poorly on the off-design noise. When the filters were designed from 12288 seconds of design noise the results changed to some extent. The frequency-domain filter was still superior when applied to the last 4096 seconds of design noise, but its superiority was less than in the former case. The frequency-domain and time-domain filters performed almost identically when applied to the off-design noise.

TABLE V-1
RATIO OF MCF TO BEAMSTEER RMS NOISE SUPPRESSION
IN 0.02 TO 0.06 Hz BAND (dB)

Noise Sample	Method of Design	MCF Designed From 4096 Seconds of Noise		MCF Designed From 12288 Seconds of Noise	
		Design Noise	Off-Design Noise	Design Noise	Off-Design Noise
NS5	Frequency Domain	5.2	0.9	3.7	1.5
	Time Domain	2.5	2.5	2.2	1.8
	Adaptive	2.6	6.0	2.8	4.9
NS6	Frequency Domain	5.2	-0.5	3.1	0.8
	Time Domain	3.6	0.4	2.6	0.8
	Adaptive	4.4	2.5	3.9	2.3

Time-domain adaptive multi-channel filters were also evaluated on these two noise samples using the technique of Booker et al, 1967. In each case two adaptive runs were made, each starting with the beamsteer filter. In the first run the filters adapted through the last 4096 seconds of design noise. The final state of the filters after this pass was used as the initial state for an adaptive pass through the last 4096 seconds of design noise and the immediately succeeding off-design noise. The RMS values of the filter output for design noise and off-design noise were computed during this pass and the corresponding increases over beamsteer performance are given in Table V-1. In the second run the initial beamsteer filters were allowed to adapt through the full 12288 seconds of design noise. As above the final state of the filters was used as the initial state for an adaptive pass through the last 4096 seconds of design noise and the off-design noise. The resultant performance is also given in Table V-1.

On the average the adaptive filter operating on the design noise performed slightly better than the time-domain optimal filters, but slightly poorer than the frequency-domain optimal filters. When operating on off-design noise, however, the adaptive filter is superior to either of the optimal filters.

In summary neither the time-domain or the frequency-domain optimal filter appears to significantly outperform the beamsteer for these two noise samples. The adaptive filter shows greater improvement over the beamsteer results, particularly in the case of NS5.

SECTION VI

CONCLUSIONS

Two approaches to the design of MCF's in the frequency domain have been considered. It was found that the use of noise crosspower spectral matrices estimated by smoothing conjugate products over adjacent frequencies of a long transform was superior to the use of matrices estimated by smoothing conjugate products over many short transforms at the same frequency. The results summarized below are based on the use of the former technique.

Some variability in noise suppression was observed when the optimal MCF's were applied to the design noise (the noise gate on which the required noise crosspower densities were estimated). One of the frequency-domain designs suppressed this noise by about five dB more than the beamsteer processor. In the practical case, however, when the filters were applied to off-design noise (a later data gate containing the presumed signal), the performance dropped and none of the optimal MCF's was materially superior to the beamsteer.

To overcome this performance loss in processing off-design data, several MCF's were designed from crosspower densities estimated on the data gate which contained the signal. When this was done the resultant MCF tended to degrade the signal, and the output signal-to-noise ratio was not any better than that obtainable with beamsteer processing.

There was some evidence that the adaptive time-domain filter can achieve somewhat better results. In one case the adaptive filter suppressed the noise by 6 dB more than the beamsteer.

The relatively simple weighted beamsteer processor was also considered. In most cases this type of processor achieves results very similar to the beamsteer, but on occasion it provides an additional four dB of noise suppression

SECTION VII
REFERENCES

- Booker, A.H., J.P. Burg, R.J. Holyer, 1967, Adaptive Filtering of Seismic Array Data; Advanced Array Research Special Report No. 1, AFTAC Contract No. F33657-67-C-0708-P001, Texas Instruments, Incorporated, Dallas, Texas, September.
- Cochran, W.T., J.W. Cooley, D.L. Favin, H.D. Helms, R.A. Kaenel, W.W. Lang, G.C. Maling, D.E. Nelson, C.M. Rader, P.D. Welch, 1967, What is the Fast Fourier Transform; IEEE Transactions on Audio and Electroacoustics, Vol. AU-15, No. 2, June.

APPENDIX

THEORETICAL MCF RESULTS

It is desired to design a least-mean-square-error MCF to extract the signal from multichannel time series data $x_m(n)$ (m is the channel subscript and n is the sample index). The time series data contains both signal and noise.

$$\begin{aligned} x_m(n) &= s_m(n) + n_m(n) & m &= 1, 2, \dots, NCH \\ & & n &= 0, 1, \dots, L-1 \end{aligned} \quad (1)$$

The design is to be accomplished in the frequency domain. The discrete Fourier transform (DFT) of the data is

$$\begin{aligned} X_m(k) &= \sum_{n=0}^{L-1} x_m(n) e^{-i2\pi \frac{nk}{L}} & K &= 0, 1, \dots, L-1 \\ &= S_m(k) + N_m(k). \end{aligned} \quad (2)$$

Let the frequency domain filter weight for channel m and frequency index k be denoted by $F_m(k)$. Then the frequency domain output of the MCF at frequency index k is given by

$$\begin{aligned} O(k) &= \sum_{m=1}^{NCH} F_m(k) X_m(k) \\ &= \sum_{m=1}^{NCH} F_m(k) [S_m(k) + N_m(k)] \end{aligned} \quad (3)$$

The desired output of the filter is the signal as it appears on one of the channels. Let this signal be denoted by $S_r(k)$. Then the error at frequency index k is given by

$$E(k) = S_r(k) - \sum_{m=1}^{NCH} F_m(k) [S_m(k) + N_m(k)] . \quad (4)$$

The error power density is expressed as $E(k)E^*(k)$ where $*$ indicates complex conjugation. The error power density at frequency index k (MSE(k)) is:

$$\begin{aligned} \text{MSE}(k) = & S_1(k)S_r^*(k) - \sum_{m=1}^{NCH} \left[F_m(k)S_r^*(k)S_m(k) + F_m^*(k)S_r(k)S_m^*(k) \right] \\ & + \sum_{m=1}^{NCH} \sum_{p=1}^{NCH} F_m^*(k)F_p(k) \left[S_m^*(k)S_p(k) + N_m^*(k)N_p(k) \right]. \end{aligned} \quad (5)$$

Implicit in (5) is the assumption that products involving signal and noise transforms are zero.

The optimum filters are obtained by equating to zero the partial derivatives of $\text{MSE}(k)$ with respect to the real and imaginary parts of each filter weight. The resultant design equation in matrix form is (neglecting smoothing)

$$\begin{bmatrix} S_1^*S_1 + N_1^*N_1 & S_1^*S_2 + N_1^*N_2 & \dots & S_1^*S_{NCH} + N_1^*N_{NCH} \\ S_2^*S_1 + N_2^*N_1 & S_2^*S_2 + N_2^*N_2 & \dots & S_2^*S_{NCH} + N_2^*N_{NCH} \\ \vdots & \vdots & \ddots & \vdots \\ S_{NCH}^*S_1 + N_{NCH}^*N_1 & \dots & S_{NCH}^*S_{NCH} + N_{NCH}^*N_{NCH} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{NCH} \end{Bmatrix} = \begin{Bmatrix} S_1^*S_r \\ S_2^*S_r \\ \vdots \\ S_{NCH}^*S_r \end{Bmatrix} \quad (6)$$

The frequency index k has been suppressed in (6). This equation is solved at each index k to obtain the desired frequency-domain filter weights.

This design equation (6) results from the frequency-domain application of the filters given in (3). Thus for each channel at each frequency in the DFT the filter weight and the data transform are multiplied. This procedure corresponds to a circular convolution of the filter and data in the time domain, Cochran et al, 1967. This is illustrated by inverse transforming the output transform (3).

$$o(g) = \frac{1}{L} \sum_{k=0}^{L-1} \sum_{m=1}^{NCH} F_m(k) X_m(k) e^{i2\pi \frac{gk}{L}} \quad (7)$$

Substitution of (2) for X_m and the similar representation of $F_m(k)$ followed by change in the order of summation yields

$$o(g) = \sum_{m=1}^{NCH} \frac{1}{L} \sum_{n=0}^{L-1} x_m(n) \sum_{h=0}^{L-1} f_m(h) \sum_{k=0}^{L-1} e^{i2\pi \frac{k[g-(n+h)]}{L}} \quad g=0, 1, \dots, L-1 \quad (8)$$

Using the orthogonality relation

$$\sum_{r=0}^{N-1} e^{i2\pi \frac{r(n-m)}{N}} = \begin{cases} N, & \text{if } n \equiv m \pmod{N} \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

in (8) results in

$$o(g) = \sum_{m=1}^{NCH} \left\{ \sum_{h=0}^g f_m(h) X_m(g-h) + \sum_{h=g+1}^{L-1} f_m(h) X_m(g-h+1) \right\} \quad (10)$$

Here it is seen that frequency-domain application of the filters corresponds to time-domain convolution of the filter on each channel with the periodic extension of the data.

The discussion above is slightly incorrect since the noise crosspower terms in (6) are considered to be obtained from one segment of data. If this were the case the resultant filters would be highly tuned to that segment of data and would be very poor for any other data. In fact the crosspower spectral matrix in (6) would be at most of rank two and could not be inverted. In actual practice noise crosspower spectra are obtained by transforming many segments of data. At each DFT frequency the crosspower spectra are formed in each segment and then averaged over segments. Under these circumstances the filter design equation (6) implies time-domain circular convolution of the filters with each of the segments of data. This is in contrast with the type of application actually used to get the time-domain output. The frequency-domain filters are inverse transformed to get their impulse responses. These are then convolved with the data yielding the MCF output. Thus the MCF is optimal for the circular convolution implicit in the design algorithm but not for the type of convolution actually employed.

There is an alternate procedure for design and application, which is not subject to this difficulty. In this case the data gate used to estimate the noise crosspower spectral matrices is not divided into segments. Rather, a long DFT is taken over the full data gate for each channel. Crosspower matrices are formed at each of the many frequencies in the DFT. These are then smoothed

by averaging over blocks of adjacent frequencies. The resultant smoothed crosspower terms are used in the design equation (6). The filter weights are then applied to the data in the frequency domain. This however necessitates interpolation of the filter weights, if the data gate for application has the same length as the data gate used to estimate the noise statistics. If the crosspower spectral matrices are averaged over sixteen adjacent frequencies for example, the frequency-domain filters are designed only at every sixteenth frequency of the data transform. The correct interpolation scheme is simple. At each of the sixteen data transform frequencies over which the matrices are averaged to yield a smoothed matrix, we apply the filter weights designed from that matrix. In this way the actual application of the filters to the data transforms is consistent with the application implied by the design equation (6).