

EDF STATISTICS FOR GOODNESS-OF-FIT: PART I

by

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## 1. Introduction.

The goodness-of-fit problem is as follows: given a random sample  $x_1, x_2, \dots, x_n$ , to test  $H_0$ : the sample comes from a population with distribution function  $F(x)$ . The classical test for this problem is the  $\chi^2$ -test, which has several advantages: (a) it is well-adapted for the case when  $F(x)$  is discontinuous, i.e., represents a discrete distribution, and (b) it is known (at least to a good approximation) how to adapt the statistic for the case when parameters of  $F(x)$  must themselves be estimated from the sample.

This paper deals with another class of goodness-of-fit statistic—EDF statistics, so-called because they are based on a comparison of  $F(x)$  with the empirical distribution function  $F_n(x)$ . For the case when  $F(x)$  is continuous and completely specified (Case 0 below) it has been long known that, in general, EDF statistics give more powerful tests of  $H_0$  than  $\chi^2$ : the disadvantage is that they are not well-adapted for discrete distributions, nor for the case when parameters must be estimated from the sample. This last drawback has undoubtedly prevented their wider application in practice, together with the fact that they are relatively difficult to compute. Recent work has now made it possible to use these statistics very easily in Case 0, and also for two very important practical situations—when the distribution tested is normal, or exponential, with parameters to be estimated, and power studies suggest they should be brought into wider use.

In this report we concentrate on a practical guide to the use of EDF statistics; specifically, those usually called  $D^+$ ,  $D^-$ ,  $D$ ,  $W^2$ ,  $V$ ,  $U^2$ ,  $A$ . A suffix is often added to represent sample size, but this will be omitted.

Once a test statistic has been calculated, a table is entered to make the test. The choice of table depends on what is known of  $F(x)$ , so this is classified first, in section 2. The formulas and procedures are in sections 3 and 4. Comments on the tables and computational details are in sections 5 and 6, and Part 1 ends with some general observations on power and choice of statistic.

## 2. Knowledge of $F(x)$ .

The tables to be used with the statistics depend on knowledge of  $F(x)$ , classified as follows.

- (a) Case 0;  $F(x)$  continuous, completely specified. This is the classical case, and tables of significance points for all the statistics exist in the literature. For references see Stephens (1970b). The use of Table 0 as described below permits us to dispense with these tables.
- (b) Case 1:  $F(x)$  is the normal distribution,  $\sigma^2$  known,  $\mu$  estimated by  $\bar{x}$ .
- (c) Case 2:  $F(x)$  is the normal distribution,  $\mu$  known,  $\sigma^2$  estimated by  $[(x_i - \bar{x})^2 / (n-1)]$ .
- (d) Case 3:  $F(x)$  is the normal distribution, both  $\mu$  and  $\sigma^2$  unknown, estimated as above.

(e) Case 4:  $F(x)$  is the exponential distribution, i.e.,

$$F(x) = 1 - \exp(-\theta x), \quad \theta \text{ estimated by } 1/\bar{x}.$$

In the case of normality, Case 3 is the important practical situation, though Case 2 sometimes arises, e.g. in regression analysis, when  $\mu$  is known to be zero.

### 3. Test procedures.

The goodness-of-fit test takes the following steps:

(a) When necessary, parameters are estimated from the sample, as described above.

(b) The values of  $x_1, x_2, \dots, x_n$  are assumed to be in ascending order; then calculate  $z_i = F(x_i)$ , for  $i=1, 2, \dots, n$ , where  $F(x)$  may contain estimated parameters for Cases 1 to 4. Then

$$z_1 \leq z_2 \leq \dots \leq z_n.$$

(c) The desired statistic is calculated as described below:

suppose we call it  $T$ . The appropriate Table 1 is entered, (corresponding to Case  $i$ ) and  $T^*$ , the modified  $T$ , is found from the expression given; then  $T^*$  is referred to the adjoining set of significance points to make the test.

The test given is the usual upper tail test; on occasion the lower tail may have to be used (see section 8.3, and Seshadri, Csorgo and Stephens (1969)).

4. Calculation of statistics.

(a) The Kolmogorov statistics  $D^+$ ,  $D^-$ ,  $D$ .

$$D^+ = \max_{1 \leq i \leq n} \left( \frac{i}{n} - z_i \right); \quad D^- = \max_{1 \leq i \leq n} \left( z_i - \frac{(i-1)}{n} \right).$$

$$D = \max(D^+, D^-).$$

(b) The Cramer-von Mises statistic  $W^2$ .

$$W^2 = \sum_{i=1}^n \left( z_i - \frac{2i-1}{2n} \right)^2 + \frac{1}{12n}.$$

(c) The Kuiper statistic  $V$

$$V = D^+ + D^-$$

(d) The Watson statistic  $U^2$

$$U^2 = W^2 - n \left( \bar{z} - \frac{1}{2} \right)^2 \quad \text{where} \quad \bar{z} = \sum_{i=1}^n z_i / n.$$

(e) The Anderson-Darling statistic  $A$

$$A = - \left( \sum_{i=1}^n (2i-1) (\ln z_i + \ln(1-z_{n+1-i})) \right) / n$$

When the statistic is calculated, use Table 1 for Case 1:  $H_0$  is rejected if the statistic exceeds the point given at the chosen level of significance.

Points on a circle. Although they can be used also, like the other statistics, for points on a line, the statistics  $U^2$  and  $V$  were introduced for points on a circle. Only these two statistics should be calculated for such points, and any suitable origin may be used; the other statistics will take different values according to choice of origin.

Illustration 1. Suppose  $F(x)$  is completely specified, and  $D$  is .27 for 25 observations. Then, in Table O, the modified  $D$  is

$$\begin{aligned} D^* &= .27(5 + 0.12 + 0.11/5) \\ &= 1.388 . \end{aligned}$$

Reference to the table of significance points for  $D^*$  in Table O shows  $D^*$  to be significant at the 5% level.

Illustration 2. A test is made that 20 observations are from a normal population with mean and variance unknown. The sample gives  $\bar{x}$  and  $s^2 = \sum (x_i - \bar{x})^2 / (n-1)$ . For each  $x_i$ , it is convenient first to find  $w_i = (x_i - \bar{x})/s$  and then  $z_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{w_i} \exp(-t^2/2) dt$ . Using the  $z_i$  as above, suppose  $W^2$  is found to be .054. In Table 3,  $W^*$  is  $W(1 + 0.5/n) = .054(41/40) = .055$ . This is not greater than 0.091, i.e. not significant at the 15% level.

##### 5. Tables.

Table A contains Tables O, 3, 4 for the three most practical cases. Table B contains tables for Cases 1 and 2.

Table O, with A added, comes from Stephens (1970b); note the different meaning for A in that paper. The Anderson-Darling statistic, in Case 0, converges so rapidly that no modification is needed in any realistic situation ( $n \geq 5$ ): see Marshall (1958) and Table B, (Table 6) for Monte Carlo studies by the author. In Tables 1-4, the asymptotic points for  $W^2$ ,  $U^2$  and A have been calculated theoretically (Stephens, 1971). For finite  $n$ , significance points from Monte Carlo studies, mostly based on 10,000 samples for each of many values of  $n$ , then smoothed, have been used to calculate the modifications. (Stephens 1969, 1970a contain original 5% and 1% points for all except A; for completeness these points, added later, are now given in Table B). Other workers have found points for some statistics as indicated:

Lilliefors (1967, 1969), D; van Soest (1967), D,  $W^2$ ; Koerts and Abrahamse (1969), V. The points agree well with those given by use of Table A, except for some differences in estimates of asymptotic points for D and V. Those given here are based on larger samples ( $n$  up to 100; other authors have  $n \leq 40$ ) but in any event the practical difference is negligible.

For Case 2, some Monte Carlo points and asymptotic points are given in Table B, Table 2. but no modifications have been calculated. For Case 1, the most unlikely situation, only theoretically calculated asymptotic points are known. (Table 1).

## 6. Computing details.

(a) The modifications to the well-known statistics were made in order to dispense with extensive tables; then computer subroutines can easily

be written to calculate the modified statistics in a given Case, and, for that Case, to print out the appropriate set of significance points so that the user can make his test. Such a routine is available (FORTRAN) from the author.

(b) At one time it seemed desirable to approximate the set of significance points by distributions of the form  $a + b\chi_p^2$  so that a modified statistic  $T^*$  could be used to calculate a further modification  $T^{**} = (T^* - a)/b$ , and the program would print out  $T^{**}$  and  $p$  with an instruction to compare with the  $\chi_p^2$  distribution. For practical use,  $p$  would need to be an integer. Even with this limitation, excellent approximations were found, (Stephens 1969, 1970a) and the values of  $a$ ,  $b$  and  $p$ , for Cases 0, 3 and 4, are in Table C.

#### 7. Power comparisons - general conclusions.

We end Part 1 with a resume of the power situation, based on the comparisons given in detail in Part 2. For all three practical cases, EDF statistics compare excellently with other goodness-of-fit statistics, the only serious rival being  $W$  (see below) for Case 3. On the whole,  $A$ ,  $W^2$  and  $U^2$  are recommended. Each case is now considered in turn.

Case 0. If  $F(x)$  is completely specified, the  $z$  should be uniformly distributed between 0 and 1, written  $U(0,1)$ . Power studies have therefore been confined to a test of this hypothesis concerning  $z$ , when the  $z$  are in fact drawn from alternative distributions. If the variance of the hypothesised  $F(x)$  is correct, but the mean is wrong, the points  $z$  will tend to move toward 0 or 1: if the mean is correct, but the variance wrong, the points will move to each end, or



will move towards 0.5.  $D$  and  $W^2$  tend to judge the same samples significant, and  $V$  and  $U^2$ ; the first pair will detect the change in mean better, and the second pair will detect the change in variance.  $W^2$  tends to be better than  $D$ , and  $U^2$  slightly better than  $V$ ; the best pair is always better than the  $\chi^2$  test. Thus in practice it would seem always worth while looking at  $W^2$  and  $U^2$ . Historically,  $D$  has been the most used EDF statistic, but it tends to be the least powerful, overall, for the four. Unfortunately, for this Case, few results exist for  $A$ . For references to earlier work on Case 0, see Kendall and Stuart, Vol. 2 (1961).

Case 3. For this case, many test statistics have been proposed in the past. The EDF statistics, with  $A$  in the lead, generally behave much better than all of them, including  $\chi^2$ . Another recently introduced statistic,  $W$ , (Shapiro and Wilk, 1965) has power comparable to that of  $A$ , possibly slightly greater, but not overwhelmingly so, as earlier reported. It has some disadvantages in the case with which the test can be made (see section 8.2). The results for EDF statistics, particularly  $A$ , and  $W$  are very highly correlated, and it would be interesting to see this connection explored further.

Case 4. For this case also many tests have been proposed. We have investigated the Case 4 procedure and three other transformations, each of which produces values  $z$  which must then be tested for uniformity. On the whole,  $W^2$  or  $A$ , with Case 4, seem to be best omnibus statistics, though further work needs to be done.

Other considerations. With the existence of modern computers, there is a temptation to investigate existing statistics, or invent new ones and investigate them, by Monte Carlo methods. Part 2 is full of such studies. Nevertheless, this can be a risky procedure, since it is easy to make mistakes, and yet not know it. Most checks can only be made by someone else repeating the experiment; most of the results in this paper have been so checked, except for some of the power studies. Apart from aesthetic reasons, the more mathematical results that can be produced to support Monte Carlo work the better. In connection with producing significance points, mathematical work can be and has been done on  $W^2$ ,  $U^2$  and  $A$  in Cases 1 to 4, to get reliable asymptotic percentage points, and the statistics all converge rapidly to their asymptotic distributions; similar work has not yet been done for  $D$ ,  $V$ . If we add the good overall power properties of  $W^2$ ,  $A$ , and  $U^2$ , and their ease of computation, it would seem that they should be brought into greater use.

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TABLE A

Modifications to D, V,  $W^2$ ,  $U^2$ , ATable O. Modifications for the test when  $F(x)$  is completely known

Statistic T	Modified form T*	Percentage points for T*				
		% 15.0	10.0	5.0	2.5	1.0
$D^+$ ( $D^-$ )	$D^+(\sqrt{n} + 0.12 + 0.11/\sqrt{n})$	0.973	1.073	1.224	1.358	1.518
D	$D(\sqrt{n} + 0.12 + 0.11/\sqrt{n})$	1.138	1.224	1.358	1.480	1.628
V	$V(\sqrt{n} + 0.155 + 0.24/\sqrt{n})$	1.537	1.620	1.747	1.862	2.001
$W^2$	$(W^2 - 0.4/n + 0.6/n^2)(1.0 + 1.0/n)$	0.284	0.347	0.461	0.581	0.743
$U^2$	$(U^2 - 0.1/n + 0.1/n^2)(1.0 + 0.8/n)$	0.131	0.152	0.187	0.221	0.267
A	For all $n \geq 5$ :	1.61	1.933	2.492	3.070	3.857

Table 3. Modifications for a test for normality,  $\mu$  and  $\sigma^2$  unknown

Statistic T	Modified form T*	Percentage points for T*				
		% 15.0	10.0	5.0	2.5	1.0
D	$D(\sqrt{n} - 0.01 + 0.85/\sqrt{n})$	0.775	0.819	0.895	0.955	1.035
V	$V(\sqrt{n} + 0.05 + 0.82/\sqrt{n})$	1.320	1.386	1.489	1.585	1.693
$W^2$	$W^2(1 + 0.5/n)$	0.091	0.104	0.126	0.148	0.178
$U^2$	$U^2(1 + 0.5/n)$	0.035	0.096	0.116	0.136	0.163
A	$A(1 + 4/n - 25/n^2)$	0.576	0.656	0.787	0.918	1.092

Table 4. Modifications for a test for exponentiality,  $\theta$  unknown

Statistic T	Modified form T*	Percentage points for T*				
		% 15.0	10.0	5.0	2.5	1.0
D	$(D - 0.2/n)(\sqrt{n} + 0.26 + 0.5/\sqrt{n})$	0.926	0.990	1.094	1.190	1.308
V	$(V - 0.2/n)(\sqrt{n} + 0.24 + 0.35/\sqrt{n})$	1.445	1.527	1.655	1.774	1.910
$W^2$	$W^2(1 + 0.16/n)$	0.149	0.177	0.224	0.275	0.337
$U^2$	$U^2(1 + 0.16/n)$	0.112	0.130	0.161	0.191	0.230
A	$A(1 + 0.6/n)$	0.922	1.078	1.341	1.606	1.957

TABLE B

All asymptotic points in Tables 1, 2, 6 for  $W^2$ ,  $U^2$ , A are theoretically derived (Stephens, 1971)

TABLE 1

Asymptotic points for  $W^2$ ,  $U^2$ , A, Case 1

Significance level (%):		15	10	5	2.5	1
$W^2$			0.135	0.165	0.196	0.237
$U^2$			.128	.157	.187	.227
A			.908	1.105	1.304	1.573

TABLE 2

Significance points for Case 2. (Monte Carlo results for D, V).

Statistic:		Percentage level (%):				
		15	10	5	2.5	1
$\sqrt{n} D$	n					
	10	1.050	1.138	1.270	1.380	1.530
	20	1.070	1.160	1.290	1.415	1.570
	50	1.080	1.170	1.310	1.432	1.595
	100	1.100	1.180	1.320	1.440	1.610
	$\infty$	1.120	1.190	1.333	1.455	1.625
$\sqrt{n} V$	10	1.305	1.385	1.500	1.595	1.710
	20	1.345	1.410	1.535	1.642	1.770
	50	1.380	1.450	1.570	1.680	1.810
	100	1.390	1.470	1.590	1.697	1.825
	$\infty$	1.410	1.490	1.612	1.720	1.845
$W^2$	all n		.329	.443	.562	.723
$U^2$	all n		.123	.153	.182	.221
A	all n		1.760	2.323	2.904	3.690

TABLE B. (Cont.)

TABLE 6Monte Carlo points for A: Case 0, Case 3, Case 4

		Percentage level (%):				
n		15	10	5	2.5	1
Case 0	5	1.63	1.94	2.54	3.09	3.97
	$\infty$		1.933	2.492	3.020	3.857
Case 3	10	.514	.578	.683	.779	.926
	20	.528	.591	.704	.815	.969
	50	.546	.616	.735	.861	1.021
	100	.559	.631	.754	.884	1.047
	$\infty$	.576	.656	.787	.918	1.092
Case 4	10	.887	1.022	1.265	1.515	1.888
	20	.898	1.045	1.300	1.556	1.927
	50	.911	1.062	1.323	1.582	1.945
	100	.916	1.070	1.330	1.595	1.951
	$\infty$	.922	1.078	1.341	1.606	1.957

TABLE C

Values of  $a$ ,  $b$ ,  $p$  for an approximation of type  
 $a + bX_p^2$ , to significance points in Table A

Statistic	Case	$a$	$b$	$p$
$D^+, D^-$	0	$(2D^+)^2$	is $X_2^2$	distributed
$D$	0	0.1743	0.049	15
$V$	0	.178	.3358	30
$W^2$	0	.061	.105	1
$U^2$	0	.031	.026	2
<hr/>				
$D$	3	0.115	0.022	23
$V$	3	-0.251	.022	60
$W^2$	3	.0187	.0136	3
$U^2$	3	.0114	.0111	4
$A$	3	.212	.095	2
<hr/>				
$D$	4	0.017	0.0343	20
$V$	4	-.336	.0295	50
$W^2$	4	.046	.0466	1
$U^2$	4	.0265	.0266	2
$A$	4	.454	.231	1



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13. ABSTRACT.  <p style="text-align: right;">(F-114)</p> <p>This paper deals with EDF statistics, a class of goodness-of-fit statistics. They are based on a comparison of <math>F(x)</math> with the empirical distribution function <math>F_n(x)</math>. Recent work has now made it possible to use these statistics very easily when <math>F(x)</math> is continuous and completely specified and also for two very important practical situations - when the distribution tested is <u>normal</u>, or <u>exponential</u>, with parameters to be estimated. Power studies suggest they should be brought into wider use. In this report, we concentrate on a practical guide to the use of EDF statistics; specifically, those usually called <math>D</math>, <math>D^*</math>, <math>D</math>, <math>W</math>, <math>V</math>, <math>U</math>, <math>A</math>.</p>		

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**13. ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

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**14. KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.