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CIRCULAR ORBIT PATTERNS PROVIDING CONTINUOUS WHOLE EARTH COVERAGE

by

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SUMMARY

A study has been made of circular orbit patterns which ensure that every point on the earth's surface can always see at least one satellite (or two satellites for double coverage) above some minimum elevation angle. It is shown that five or six high altitude satellites can provide single coverage, and seven or nine satellites double coverage, at elevation angles which represent a significant improvement over previously published values; and a generalised approach to such coverage assessments is presented.

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CONTENTS

1 INTRODUCTION

The coverage of the earth's surface provided by various patterns of satellites has been the subject of many studies in the past, particularly in relation to the use of satellites as communications relays. Much of this work has been collated on an international basis by CCIR study groups; reference may be made particularly to Report 206-1 [1] regarding the choice of orbit for communications satellites serving fixed earth stations, and to Doc. IV/408 [2] regarding orbits for systems providing communication and radio-determination (position-fixing) for stations in the mobile service.

For commercial point-to-point communications it has been found operationally convenient to divide the inhabited portion of the earth's surface into large zones, within each of which all points are simultaneously within line of sight of a single satellite, so that all fixed earth stations within the zone may always communicate with one another via this satellite. However, there may be other satellite applications for which it is satisfactory to provide a simpler form of coverage, in which it is merely ensured that every point on earth can always see at least one satellite above some minimum elevation angle; such an arrangement seems likely to be acceptable for systems of a data-gathering nature, and possibly even for mobile communications.

Most previous studies of the provision of such simple coverage by high altitude satellites appear to have been of an *ad hoc* nature, the authors investigating particular orbital patterns which appeared to them as promising in relation to specific system studies on which they were engaged. This Report presents a more systematic approach to the analysis of coverage by means of circular orbit systems, and shows that better coverage is possible with smaller numbers of satellites than has often been assumed in the past. Only circular orbits have been considered since, while elliptical orbits have some advantages in the provision of coverage to limited areas mainly confined to either the northern or the southern hemisphere, circular orbits appear to have the advantage for zones extending equally into both hemispheres, and even more as regards whole earth coverage.

Many of the results presented in this Report were originally obtained in 1965 as part of an initial examination of possible requirements for aeronautical satellite systems, but as it appeared that such requirements would in fact be met by geostationary satellites the subject of whole earth coverage was not then pursued further. A more recent revival of interest, indicated by the preparation of Ref.2, led to the decision to prepare the original results for publication in this Report; while doing so, some additional examples have been calculated and the analysis has been generalised. A brief account of some of the original results was supplied at a late stage to the UK delegation at the CCIR study group meeting at which Ref.2 was prepared, and a few comments in Ref.2 reflect this; however, it would appear that it might usefully be expanded in some other aspects.

A shortened version of this Report, presenting the principal results without details of the methods of analysis, has been prepared[3].

2 COMPARISON WITH PREVIOUS RESULTS

It has long been appreciated that three satellites spaced 120° apart in high circular equatorial orbits can provide continuous coverage of most of the earth's surface, excluding the polar regions, the exact extent of this coverage depending upon the altitude of the satellites. Fig.1 shows the elevation angle ε at which a satellite is visible, as a function of the distance (expressed as the angle d subtended at the centre of the earth) between the observation point and the sub-satellite point, for satellites in circular synchronous or sub-synchronous orbits of periods from 24 down to 2 sidereal hours, as calculated from the formulae

$$R_e \cos \varepsilon = R_s \cos (d + \varepsilon)$$

and

$$R_s/R_e = 0.795\, T_{sh}^{2/3}$$

where R_e is the radius of the earth, R_s the radius of the orbit, and T_{sh} the orbital period in sidereal hours. This shows, for instance, that an elevation angle exceeding 5° is obtained out to a sub-satellite distance of 76.3° with 24 hour satellites, but only out to 48° with 3 hour satellites; and that an elevation angle exceeding 15° is obtained out to $d = 66.6^{\circ}$ with 24 hour satellites. Interpolation will give values for other periods.

Each satellite in 24 hour orbit is visible above 5° elevation from about 38% of the earth's surface. Nevertheless, even if the number of satellites in equatorial orbit is increased indefinitely, they cannot provide coverage above 5° elevation anywhere at a latitude greater than 76.3° (or, for 3 hour satellites, greater than 48°); the regions around the Poles will remain without coverage. If the satellites are in an inclined orbital plane, this lack of coverage will apply around the poles of the orbit. To secure whole earth coverage it is necessary that satellites should be disposed in at least two different orbital planes having a substantial relative inclination, so that the areas left uncovered by those in one plane may be covered by those in another plane.

Consider a number of satellites following equal radius circular orbits around the earth. If a line is drawn connecting each sub-satellite point to adjacent sub-satellite points, such that the earth's surface is divided into a number of spherical triangles with a sub-satellite point at each vertex, then the point on the earth's surface most remote from any of the sub-satellite points is the centre of the largest of the circumcircles of these triangles which does not enclose any other sub-satellite point. In seeking to provide whole earth coverage by means of a minimum number of satellites in high circular orbits, it is therefore natural to consider first the possibility of disposing them at the vertices of a regular solid figure, e.g. a regular tetrahedron, the circumcircles of whose sides would all be of equal radius.

The inverse problem of covering all possible satellite positions by a minimum number of tracking stations can indeed be solved in this manner; if four stations are disposed on the earth's surface at the vertices of a regular tetrahedron, then a sub-satellite point can never be more than 70.5^{o} from one or other of these stations (this being the radius of the circumcircle of one face of a regular tetrahedron). so that (from Fig.1) a satellite in a 24 hour circular orbit of any inclination can always be observed at an elevation angle not less than 11^{o}. It is interesting to note that, of INTELSAT's four TT & C stations, at Fucino (Italy), Carnarvon (W. Australia), Paumalu (Hawaii) and Andover (USA), the three first named are located very close to three of the vertices of a regular tetrahedron; however, to complete the figure the fourth would have to be located in South America instead of North America. INTELSAT was, of course, only concerned with low inclination satellites when choosing its TT & C station sites, so that the possibility of global coverage was irrelevant.

Similarly, if four synchronous satellites were located at the vertices of a regular tetrahedron, at least one could be seen at an elevation angle greater than 11^{o} from every point on the earth's surface. However, such a configuration could be established only momentarily, since the relative positions of satellites in different orbits are constantly changing as they progress around their orbits. Indeed, if one considers the plane which could be drawn at any instant through three of any four satellites in equal radius circular orbits (whatever their orbital planes), the fourth satellite must pass through that plane twice in each orbital period; this may be illustrated with the satellite pattern of, for example, Fig.2. In condition 1, when satellite A is at its ascending node, it is on the further side of the plane CDE (which passes through the points $C_1D_1E_1$). In condition 2, A is passing through the plane CDE (which passes through the points $C_2D_2E_2A_2$), and it continues on the nearer side of the plane CDE for the next half orbit, after which it passes through to the further side of the plane CDE for a further half orbit, until the satellites return to condition 2. Returning to our hypothetical four satellite system, it is evident that when all the satellites lie in one plane, whether or not it passes through the centre of the earth, whole earth coverage is impossible; hence whole earth coverage cannot be maintained at all times with less than five satellites.

However, it is of interest to consider how the coverage given by a tetrahedral pattern might be approximated by the use of five satellites. One such regular tetrahedron would consist of two satellites over the equator, at longitudes 54.7°E and 54.7°W, and two satellites over the 180° meridian, at latitudes 54.7°N and 54.7°S. Geostationary satellites could indeed be positioned at or near 54.7°E and 54.7°W, while three synchronous satellites in inclined orbits could follow at equal intervals a figure 8 ground track centred on the 180° meridian. A system rather similar to this was proposed in Ref.4 (based on an unpublished NASA contractor's report), which suggested that whole earth coverage (to an undefined standard) could be maintained by five satellites, two geostationary at 60°E and 60°W of a reference meridian and three in synchronous polar (or, in another version, 60° inclination) orbits following a figure 8 ground track centred on the meridian 180° from the reference. An examination of this configuration (which was, in fact, the first step in the present study) forms a convenient introduction to the main body of this Report.

The point on the earth's surface at which the minimum satellite elevation angle occurs is the circumcentre of the sub-satellite points of the nearest three satellites, under those conditions which result in the circumcircle being at its largest for any such group of three adjacent satellites; we shall describe this as the critical configuration, giving critical conditions at the circumcentre, the critical point. We define d_{max} as the maximum sub-satellite distance, i.e. the radius of the circumcircle in the critical configuration. It often happens that critical conditions occur when one satellite passes through the plane of another three satellites, i.e. its sub-satellite point lies on the circumcircle of their sub-satellite points.

For the configuration of Ref.4 we consider two geostationary satellites A and B symmetrically disposed either side of the reference meridian at angular distances s between 40° and 60°, with three synchronous satellites CDE in orbits of inclination i, for which we consider values between 60° and 105°. One critical configuration of the satellites occurs when one inclined satellite is at maximum excursion from the equator, over the 180° meridian at latitude i, i.e. if the orbit is polar one satellite (say E) is over (say) the south pole while the other two (C and D) are on opposite sides of the 180° meridian at latitude 30°N (or corresponding positions for other inclinations). A point in the northern hemisphere equidistant from the sub-satellite points of the two geostationary satellites (A and B) and the sub-satellite points of the two at 30°N (C_1 and D_1) is then at maximum distance from the sub-satellite points (Fig.3a); calculated values of this distance d_{max}, for different values of s and i, are plotted in full lines in Fig.3c.

Another critical configuration of the satellites appears to occur when the sub-satellite points ($C_2D_2E_2$) of the inclined satellites and that of one of the geostationary satellites (say B) all lie in one plane; one such condition (for polar orbits) is with the satellites CDE approximately at latitudes 70°N, 50°S and 10°S (Fig.3b). Values of d_{max} at the circumcentre of these four points could be calculated by the methods described later in this Report, but this would be a rather lengthy process; instead, they have been estimated with the aid of a globe and geometer, as described in Appendix A, and plotted as broken lines in Fig.3c. For any pair of values of i and s it is the higher of the two values of d_{max} which is significant, as indicated by thicker lines; the locus of points at which the two values of d_{max} are equal has been drawn on Fig.3c.

For the configuration suggested in the study described in Ref.4, it is seen from Fig.3c that with $s = 60^\circ$ and $i = 60^\circ$ the value of d_{max} is 81.4°, so that continuous whole earth coverage cannot quite be provided even at zero elevation angle. With $s = 60^\circ$ and $i = 90^\circ$ the value of d_{max} is 79.3°, giving a minimum elevation angle of only 2°; while nominally giving whole earth coverage, this is inadequate for practical purposes.

The optimum value of i with $s = 60^\circ$ is seen from Fig.3c to be about 100°; however, it appears that the best combination (as shown by the minimum of the equal values of d_{max} for the two critical configurations) would be in the region of $i = 78^\circ$ with $s = 42^\circ$, giving d_{max} about 77.0° and hence a minimum elevation angle of about 4.3°. These particular values are only approximate, due to the use of figures estimated from a globe, but they serve to illustrate the method adopted throughout this study of varying the available parameters in search of a minimum overall value of d_{max}; it should not be assumed that optimum conditions will be given by convenient round number values of parameters such as i.

However, we have so far retained the constraint of Ref.4 that three of the five satellites should be in inclined orbits (with nodes separated by 120^o to produce the single figure 8 ground track) while the other two are both in equatorial orbits. Such an arrangement cannot be expected to produce a uniform distribution of coverage over the earth's surface, which would help to avoid larger values of d occurring in one part of the pattern than another. It therefore seems desirable to investigate a more uniform arrangement in which all five satellites conform to similar rules, e.g. one having all five satellites in orbits of equal inclination to some reference plane (not necessarily the equator), with equally spaced nodes, and with a regular pattern of nodal passages as between the different orbits. Fig.2 shows one such arrangement; five satellites in orbits of equal inclination are so arranged that, when A is at its ascending node, B is 72^o past its ascending node, C is 144^o past its ascending node, and so on. In Fig.2 the ascending nodes are spaced with successive eastward longitude increments of 72^o in the order ABCDEA.... . Other possible regular patterns are ADBECA...., ACEBDA...., and AEDCBA...., but of these the last results in all the satellites following the same figure 8 ground track, while the other two are found on examination to give less satisfactory whole earth coverage than the arrangement of Fig.2.

With each of these regular patterns there is now only one variable parameter involved (apart from the phase angle) - the inclination δ of all five satellite orbits relative to the reference plane. In the pattern of Fig.2 (discussed further in section 3.4) critical conditions occur at the circumcentres of groups of sub-satellite points similar to $A_2D_2E_2$ (or $C_2D_2E_2$) when δ is small, and of groups similar to $B_2D_2E_2$ when δ is large; corresponding values of d_{max} are plotted against δ in Fig.4, from which it is seen that optimum coverage is obtained when $\delta = 43.7^o$, d_{max} then having the value 69.2^o at all critical points. This value corresponds to a minimum elevation angle in world-wide coverage by such a satellite system of 12.3^o with satellites in 24 hour orbits, 7.0^o in 12 hour orbits, or 2.5^o in 8 hour orbits - substantially better than with the configuration proposed in Ref.4.

However, this five satellite configuration gives inadequate coverage below the 12 hour orbit, so that for lower orbits it is necessary to consider larger numbers of satellites. Moreover, there are certain practical advantages in minimising the number of different orbit planes, and with six or eight satellites only two planes need be used. These configurations will be discussed in more detail in section 3.2, but the results obtained in terms of the variation of d_{max} with the inclination β of the two orbit planes to a reference plane are presented in Fig.5.

In a recent paper[5], Easton and Brescia concluded that a minimum of six satellites is required to obtain complete earth coverage, and that the maximum distance from any sub-satellite point with such a system is 69.3^o (marginally worse than our figure for five satellites). However, following a common preference for choosing symmetrical orbit patterns, they considered only even numbers of satellites distributed between two orbit planes having a relative inclination of 90^o. Fig.5 confirms that, for $2\beta = 90^o$, i.e. $\beta = 45^o$, the value of d_{max} for six satellites is indeed 69.3^o; however, the significantly lower value of 66.7^o may be obtained if β is reduced to 37.8^o. With eight satellites, four each in two planes, $d_{max} = 57.0^o$ at $\beta = 41.6^o$. Thus six satellites in 8 hour orbits can just provide continuous whole earth coverage with 5.0^o minimum elevation, and eight satellites can provide continuous whole earth coverage with 11.0^o minimum elevation in 6 hour or 3.1^o minimum elevation in 4 hour circular orbits.

Easton and Brescia[5] also concluded that 'a two-simultaneous-satellite-visibility constellation can be accomplished with twelve properly spaced satellites'. In section 3.5 we show that seven satellites in synchronous orbit, or nine satellites in 12 or 8 hour orbits, are sufficient to provide continuous duplicated whole earth coverage.

3 REGULAR CIRCULAR ORBIT PATTERNS

3.1 General

In this section we concentrate attention on systems of satellites in multiple equal-radius circular orbits, with an equal number of satellites in each orbital plane (n satellites in each of p planes). The criterion adopted for evaluation of coverage is the minimum elevation angle experienced at any point on earth at any time in viewing the nearest satellite; this is a function of the maximum sub-satellite distance (see Fig.1). We assume a near-perfect station-keeping capability in the satellites, i.e. they have control systems capable of establishing the desired orbits and maintaining them virtually unchanged despite perturbing forces, including maintaining satellite spacing of $2\pi/n$ within any one orbital plane.

Regularity of the satellite pattern is an obvious advantage in approaching uniformity of coverage, avoiding the isolated worst case (as evidenced by the comparison between Figs.3c and 4); this means rather more than symmetry of the orbits, which has often been suggested as desirable for optimum coverage. In particular, frequent recurrence of similar satellite patterns appears desirable. We consider as a starting-point the system indicated in Fig.6a, with several (e.g. three) polar orbit planes with ascending nodes equally spaced around the equator; is this an ideal arrangement from the point of view of whole earth coverage, and if not how could it be improved?

It is readily apparent that this arrangement could be improved upon, there being two particular drawbacks to it:

(a) Adjacent orbits, e.g. A and B, B and C, C and A, are everywhere contra-rotating, so that it is never possible to arrange station-keeping between satellites in adjacent orbits; they always pass one another in opposite directions.

(b) All orbits intersect at the poles, where coverage will therefore tend to be very good, whereas coverage will tend to be poor at the equator midway between the nodes.

The first drawback (a) may be minimised by changing the arrangement of Fig.6a to that of Fig.6b. Here the orbits are arranged so that all the ascending nodes are in one hemisphere and all the descending nodes in the other, with the result that only orbits of one adjacent pair (C and A) are contra-rotating, while the others (A and B, B and C) are co-rotating. This makes it possible to arrange station-keeping between satellites in adjacent co-rotating orbits, so improving coverage; and if part of this improvement is foregone by increasing the spacing between co-rotating orbits, this reduces the spacing between the contra-rotating orbits, providing a corresponding improvement of coverage there.

The second drawback (b) may be tackled by the alternative approach of changing from the polar orbits of Fig.6a to orbits of moderate inclination as in Fig.6c. In this case only two orbits intersect at any one point, so that there is a more even distribution of coverage over the surface of the earth; an optimum inclination from this point of view may be determined for each set of values of n and p.

If our ideal is completely uniform coverage of the whole earth's surface, with no preference given to any particular geographical area or areas, then we need not attach any particular significance to poles or equator, or give any consideration to the earth's rotation. Instead it is convenient to relate the various orbital patterns to an arbitrary reference plane, passing through the earth's centre but having no pre-determined inclination to the equator. The orbital patterns are independent of the common altitude and period of the satellites; this would not be the case if we worked instead with ground track patterns, with which it would not be permissible to treat the earth as non-rotating. It is also convenient to refer to the orbital patterns of Fig.6b and 6c by names; by an obvious analogy, we choose to refer to them as 'star patterns' and 'delta patterns' respectively. The manner in which it is found convenient to relate the reference plane to the orbital pattern (shown in this case for four orbit planes) is indicated for star patterns in Fig.7a and for delta patterns in Fig.7b.

The star pattern is typified by multiple orbits, sharing a common pair of nodes in the reference plane, and with equal (or approximately equal) relative inclinations of adjacent co-rotating orbits; the delta pattern is typified by orbits of equal inclination to, and with nodes equally spaced around, the reference plane. The five satellite pattern of Fig.2 is an example of the delta pattern, with $p = 5$ and $n = 1$. With only two orbit planes it is not possible to distinguish between star and delta patterns, the angle δ for the latter corresponding to the angle α for the former, but for analysis it is convenient to treat such systems as examples of the star pattern.

3.2 Star patterns

From Fig.7a it can be seen that while the pairs of adjacent orbits A and B, B and C, C and D are everywhere co-rotating, D and A are everywhere contra-rotating. We assume an equal relative inclination of 2α between adjacent pairs of co-rotating orbits, and of 2β between the contra-rotating orbits, so that

$$(p - 1)\,\alpha + \beta = \frac{\pi}{2} \quad . \tag{1}$$

Evidently the relative positions of satellites in co-rotating orbits will remain relatively little changed during an orbital period, apart from a lateral contraction as the nodes are approached and expansion as they are left behind; the distances between satellites will tend to be greatest in the vicinity of the great circle of which the nodes are the poles, which we shall refer to as the antinodal circle. As between contra-rotating orbits, on the other hand, the relative positions of satellites in the two orbits are constantly changing, with satellites meeting and passing one another in opposite directions; though here again, the distances between satellites will tend to be greatest in the vicinity of the antinodal circle.

Consider the projections on the earth's surface of the instantaneous positions of two successive satellites B and C (Fig.8a) following the same orbit at an interval of $2\pi/n$, and straddling the antinodal circle RQ'Q with B closer to it than C; let their respective distances from the intersection Q of their orbit with the antinodal circle be b and c, so that $c = \frac{2\pi}{n} - b$. Since each orbit contains n satellites there must always be in the adjacent orbit, whether co-rotating or contra-rotating, a satellite A lying adjacent to the arc BC, so that its distance a from the intersection Q' of its orbit with the antinodal circle does not exceed c. Considering the spherical triangle ABC, and the value of d at the circumcentre P of this triangle, it may be deduced using the equations developed in Appendix B that:

(i) For a given relative phasing of the satellites in adjacent orbits, such as would be maintained between co-rotating orbits (i.e. b-a = constant, if A and B are on the same side of QQ' as in Fig.8a), the value of d is greatest when P lies on the antinodal circle, i.e. when $b = c = \pi/n$.

(ii) For given positions of B and C, d is least when A is adjacent to (i.e. at the same distance from the nodes as) the mid-point of BC, i.e. when $a = \frac{c - b}{2} = \frac{\pi}{n} - \frac{b}{2}$, and d is greatest when A is adjacent to B, i.e. when $a = b$. Thus when $b = c = \frac{\pi}{n}$, d is least when $a = 0$ and greatest when $a = \frac{\pi}{n}$.

(iii) Starting from the situation $b = c = \frac{\pi}{n}$, $a = 0$, and moving A to increase a by Δa, the value of d is increased relatively rapidly, say by Δd. On the other hand, starting from the situation $a = b = c = \frac{\pi}{n}$ and moving all the satellites together so that a and b are both decreased (and c increased) by Δa, the value of d is increased relatively slowly, by an amount less than Δd.

Hence we may deduce that, if it is desired to minimise the maximum value of d experienced for a given pair of values of α and β, the best strategy is to choose the phasing of satellites in co-rotating orbits so that, when a satellite in one orbit is at a node, the nearest satellites in adjacent co-rotating orbits are each π/n away from the node (from (ii)); and to accept whatever condition results as between the single pair of contra-rotating orbits (since (iii) shows this to be of less significance). The resulting conditions actually vary depending on whether the values of p and n are odd or even; critical values of d are found to occur under the following conditions:

$$p \text{ even}, \; n \text{ even}: \quad a = b = \frac{\pi}{2n}$$

$$p \text{ even}, \; n \text{ odd}: \quad a = b = \frac{\pi}{n}$$

$$p \text{ odd}, \; n \text{ even}: \quad a = b = \frac{\pi}{n}$$

$$p \text{ odd}, \; n \text{ odd}: \quad a = b = \frac{\pi}{2n} \; .$$

No more favourable value than $a = b = \frac{\pi}{2n}$ could have been obtained under any circumstances.

When the relative inclination (2α) of co-rotating orbits is large and that of the contra-rotating pair of orbits (2β) relatively small, the critical value of d would be expected to occur between co-rotating orbits; with α small and β large it would be expected to occur between the contra-rotating orbits; while at some intermediate pair of values of α and β the values of d_{max} for the two cases will be equal, this condition giving the best overall coverage for the particular values of n and p under consideration.

Considering the general case of adjacent orbits, whether co-rotating or contra-rotating, having a relative inclination 2γ, it is shown in Appendix B (equation (B-3)) that the value of d for a trio of satellites such as ABC is given by

$$d = \cos^{-1}\left\{\cos\frac{\pi}{n}\sin\tan^{-1}\left[\frac{\cos a \sin 2\gamma}{\cos\frac{\pi}{n} - \sin\cos^{-1}(\cos a \sin 2\gamma) \, . \, \cos\left[\tan^{-1}(\tan a \sec 2\gamma) + \frac{\pi}{n} - b\right]}\right]\right\} .$$

Inserting the values $a = 0$, $b = \frac{\pi}{n}$, $\gamma = \alpha$, this reduces to

$$d_{max} = \cos^{-1}\left\{\cos\frac{\pi}{n}\cos\tan^{-1}\left(\cos\frac{\pi}{n}\operatorname{cosec} 2\alpha - \cot 2\alpha\right)\right\} . \qquad (2)$$

When $a = b$, it is shown in Appendix B (equation (B-4)) that in the general case

$$d = \cos^{-1}\left\{\cos\frac{\pi}{n}\ \cos\tan^{-1}\tan\gamma\cos\left(\frac{\pi}{n} - b\right)\right\} .$$

Inserting the values $a = b = \frac{\pi}{2n}$, $\gamma = \beta$, this reduces to

$$d_{max} = \cos^{-1}\left\{\cos\frac{\pi}{n}\ \cos\tan^{-1}\tan\beta\cos\frac{\pi}{2n}\right\} , \qquad (3)$$

while with the values $a = b = \frac{\pi}{n}$, $\gamma = \beta$, it reduces to

$$d_{max} = \cos^{-1}\left\{\cos\frac{\pi}{n}\cos\beta\right\} . \qquad (4)$$

Equations (1) to (4) were used in the preparation of Fig.5, with $p = 2$ and $n = 3$ for the six satellite pattern and $p = 2$ with $n = 4$ for the eight satellite pattern.

It may be noted that, when n is large, these equations reduce (as would be expected) to $d_{max} = \alpha$ (equation (2)) and $d_{max} = \beta$ (equations (3) and (4)). The minimum value of d_{max} will then be obtained when $\alpha = \beta$; from equation (1), this will occur when each is equal to $\pi/2p$. Hence when n is large the relative inclination of adjacent orbital planes in a star pattern, whether co-rotating or contra-rotating, should preferably be 90^{o} for two planes, 60^{o} for three planes, etc. However, when n is small the preferred relative inclination may differ substantially from these values, as is apparent from Fig.5, a significant point which does not appear to have been generally appreciated in other studies.

The inclination at which the minimum value of d_{max} is obtained may be calculated directly by equating the formulae for d_{max} in terms of α and β respectively and substituting for β from equation (1) according to the number of planes under consideration. Thus when $(n + p)$ is an even number, from equations (2) and (3),

$$d_{max} = \cos^{-1}\left\{\cos\frac{\pi}{n}\ \cos\tan^{-1}\left(\cos\frac{\pi}{n}\operatorname{cosec}2\alpha - \cot 2\alpha\right)\right\}$$

$$= \cos^{-1}\left\{\cos\frac{\pi}{n}\ \cos\tan^{-1}\tan\beta\cos\frac{\pi}{2n}\right\} ,$$

i.e.

$$\cos\frac{\pi}{n}\operatorname{cosec}2\alpha - \cot 2\alpha = \cos\frac{\pi}{2n}\tan\beta$$

$$= \cos\frac{\pi}{2n}\cot(p-1)\alpha ,$$

and similarly when $(n + p)$ is an odd number, from equations (2) and (4),

$$\cos\frac{\pi}{n}\operatorname{cosec}2\alpha - \cot 2\alpha = \cot(p-1)\alpha .$$

When $p = 2$ and n is an odd number this reduces to

$$\cos\frac{\pi}{n}\operatorname{cosec}2\alpha - \cot 2\alpha = \cot\alpha$$

$$= \frac{1 + \cos 2\alpha}{\sin 2\alpha} ,$$

i.e.

$$\cos 2\alpha = \frac{1}{2}\left(\cos\frac{\pi}{n} - 1\right) ,$$

or when $p = 2$ and n is an even number

$$\cos 2\alpha = \frac{\cos\frac{\pi}{n} - \cos\frac{\pi}{2n}}{1 + \cos\frac{\pi}{2n}} .$$

Similarly when $p = 3$ and n is an even number, we have

$$\cos\frac{\pi}{n}\operatorname{cosec}2\alpha - \cot 2\alpha = \cot 2\alpha ,$$

giving
$$\cos 2\alpha = \frac{1}{2}\cos\frac{\pi}{n} ,$$

or when $p = 3$ and n is an odd number

$$\cos 2\alpha = \frac{\cos\frac{\pi}{n}}{1 + \cos\frac{\pi}{2n}} .$$

Hence the values of α and β for optimum coverage may be obtained directly and substituted in equation (3) or (4) as appropriate to obtain the minimum value of the maximum sub-satellite distance d_{max}.

This has been calculated for a number of star patterns incorporating up to 25 satellites, distributed between up to six planes, and the resultant values are plotted in Fig.9. Full lines have been drawn though points representing similar values of p, as an aid to identification. For $p = 2$ separate lines have been drawn for $(n + p)$ even and $(n + p)$ odd; for larger values of p the two cases do not differ sufficiently to justify this, though points may be seen displaced on alternate sides of the single line drawn. It can be seen that, in order to minimise the total number of satellites required in the system, if the minimum acceptable elevation angle corresponds to a sub-satellite distance (from Fig.1) greater than 53° then two orbital planes should be used, containing a total of 6, 8 or 10 satellites; if the maximum sub-satellite distance is to be between 53° and 36° then three orbital planes should be used, containing 12, 15, 18 or 21 satellites; if it is less than 36° then four orbital planes should be used, containing 24 or more satellites. Details of these cases are collected in Table 1.

The envelope of these curves has been indicated on Fig.9 by a broken line. If fractional values of n and p were possible, this envelope would indicate optimum conditions; in practice, of course, only the individual points plotted on Fig.9 have any real validity. However, it may be noted that elementary consideration would suggest that the distribution of satellites over the earth's surface would be most uniform, and hence coverage obtained most economically, when $n = 2p$, i.e. 2 satellites in one plane, 4 each in two planes (total 8), 6 each in three planes (total 18), and so on; the broken line indicates that this simple estimate is not far from the truth.

Caution is necessary regarding one aspect of such calculations. The value of d_{max}, calculated from equation (2) as the radius of the circumcircle, falls as 2α is reduced until they both reach the value π/n and then increases again as 2α falls further. This is because, when $2\alpha = \frac{\pi}{n}$, the centre p of the circumcircle lies on BC, and as 2α is further reduced it passes outside the triangle ABC. However, when this happens it is approaching a satellite in a third orbit, and the point most remote from any satellite becomes instead the point on BC equidistant from A and C. The corresponding value of d_{max} is given by

$$d_{max} = \tan^{-1}\left(\operatorname{cosec}\frac{\pi}{n}\sec 2\alpha - \cot\frac{\pi}{n}\right) .$$

The effect of this may be seen on Fig.5 when β exceeds 60° (n = 3) or 67.5° (n = 4).

When p is large and n relatively small then 2α may be less than π/n for all values of β, even when $\beta = 0^{\circ}$. A calculation based on equation (2) will then show $\beta = 0^{\circ}$ as the optimum inclination, but will give a value for d_{max} in terms of the corresponding value of α which is greater than π/n. As before, this is not the true value of d_{max}, which should be π/n (given by equation (3) or (4)); examples appearing in Fig.9 are for 3 satellites each in four or more planes, and for 4 satellites each in five or more planes. In such cases the strategy for choosing the phasing of satellites, described earlier in this section as being the best, is not strictly valid; however, as all such cases are far from optimum and hence of no real practical interest, we shall not pursue this aspect further.

3.3 Star patterns with staggered nodal crossings

It was concluded in section 3.2 that, for a star pattern to give optimum coverage, then when a satellite in one orbit is at a node the nearest satellites in adjacent co-rotating orbits should each be π/n away from the node. If this is the case then, when $p > 2$, two or more satellites in different orbits will arrive at the common node simultaneously.

In practice this need not necessarily cause any problem; the risk of a physical collision would be extremely small, and could in any case by obviated by a small displacement from the optimum phasing which would have an insignificant effect on the coverage. However there may be cases in which practical problems might arise, e.g. if there is a risk of radio interference between satellites, and in such cases it might be wished to keep the minimum distance between any two satellites in the pattern as large as possible. For this purpose nodal crossings should be so staggered that, when any one satellite is at one node, the two nearest satellites should be $\frac{2\pi}{np}$ distant from it on their own orbits. When p is an odd number, this means that satellites in one orbit must be displaced by $\frac{\pi}{np}$ from the optimum phasing relative to those in the adjacent co-rotating orbit as established in section 3.2. When p is an even number, the corresponding displacement required is either zero or $\frac{2\pi}{np}$, for alternate orbits; for example, in Fig.7a, satellites in orbit B need not be displaced from their optimum phasing relative to those in orbit A, those in orbit C must be displaced $\frac{2\pi}{np}$ from their optimum phasing relative to those in orbit B, and those in orbit D need not be displaced from their optimum phasing relative to those in orbit C.

In consequence the passing points for satellites in contra-rotating orbits are displaced by $\frac{(p-1)\pi}{2np}$ when p is an odd number, or by $\frac{(p-2)\pi}{2np}$ when p is an even number, relative to those determined in section 3.2. The critical position is always the one lying between $\frac{\pi}{n}$ and $\frac{\pi}{2n}$; the values of $a = b$ at the passing points thus become

$$p \text{ even}, \; n \text{ even}: \quad a = b = \frac{(p-1)\pi}{np}$$

$$p \text{ even}, \; n \text{ odd}: \quad a = b = \frac{(p+2)\pi}{2np}$$

$$p \text{ odd}, \; n \text{ even}: \quad a = b = \frac{(p+1)\pi}{2np}$$

$$p \text{ odd}, \; n \text{ odd}: \quad a = b = \frac{(2p-1)\pi}{2np} \; .$$

When p is an even number, we now have two different conditions as between adjacent pairs of co-rotating orbits - those for which optimum conditions are maintained, and those for which there is a relative displacement of $\frac{2\pi}{np}$. For optimum coverage, this should ideally be reflected by a difference in the orbit spacing. If, for those adjacent orbit pairs whose relative satellite phasing is displaced by $\frac{2\pi}{np}$ from the optimum, the relative inclination is reduced from 2α to $2\alpha'$, we then have

$$\beta + \frac{p}{2}\alpha + \left(\frac{p}{2} - 1\right)\alpha' = \frac{\pi}{2} \; .$$

In practice, however, the difference between α and α' is likely to be small enough to be ignored.

A complete list of the equations for d_{max} in terms of α, α' and β, as required to determine the coverage obtainable with star patterns either with or without staggered nodal crossings, is presented in Appendix C. The practical effect of thus staggering the nodal crossings is a small increase in the optimum value of β, and a small increase in the value of d_{max}, i.e. a small decrease in the minimum elevation angle. This is only noticeable for small values of n and p; when p is large the equations reduce to those of section 3.2, and when n is large they still reduce to $d_{max} = \alpha = \beta = \frac{\pi}{2p}$.

As an example, for $n = p = 3$ the value of d_{max} for the optimum coverage configuration is 60.9^o, which is only increased to 61.2^o with staggered nodal crossings.

3.4 Delta patterns

A typical delta pattern (for four orbit planes) is illustrated in Fig.7b. The orbits have equal inclinations δ to a reference plane round which their ascending nodes are equally spaced at intervals of $\frac{2\pi}{p}$. The number of satellites in each orbit plane is n, so that there is a phase interval of $\frac{2\pi}{n}$ between successive passages of satellites in the same orbit plane through the ascending node; and there is some regular pattern of phase intervals as between nodal passages of satellites in different orbits, this pattern being selected to give optimum coverage in each particular case.

It has already been noted that, with only two orbit planes, the star and delta patterns are in practice identical, with the angle δ for the delta pattern corresponding to the angle α for the star pattern. When $p > 2$, however, the patterns differ considerably. The delta patterns have considerable practical interest, appearing in general to give better coverage than the star patterns, with other convenient features; however, they have been found much less amenable to any general analysis, due to the complex manner in which the inter-relationship of the different orbital planes varies with inclination, particularly when n and p are not small. So far no rules have been found for determining preferred configurations (except for two plane systems, to which the star pattern analysis may be applied) without going through the somewhat tedious process, for each pair of values of n and p, of

(i) selecting suitable phase relationships between satellites, in the same plane and as between planes (several alternatives probably requiring examination);

(ii) determining in each case (with the aid of globe and geometer, as in Appendix A) which spherical triangles of satellites provide the critical values of d;

(a) when δ is small (and at what phase),

(b) when δ is large (and at what phase),

and whether any others become critical at intermediate values of δ; and hence

(iii) calculating the values of d_{max} for these cases, for different values of δ, leading finally to

(iv) finding the optimum value of δ to give the minimum overall value of d_{max}.

In the general case it would be necessary, at step (iii), first to find the sides abc and opposite angles ABC of each spherical triangle, from standard formulae of spherical trigonometry, and hence to calculate d, equal to the radius of the circumcircle, from the standard formula for the radius R of the circumcircle of a spherical triangle

$$\tan R = \tan \tfrac{1}{2} a \cdot \sec \tfrac{1}{2} (B + C - A) \quad .$$

Plotting d for several satellite phase angles would then enable d_{max} to be determined. However, in all cases so far examined it has been found that critical conditions fall into one of two categories, in each of which a considerably simpler approach to the calculation of d_{max} is possible:

(a) four satellites lying in the same plane, symmetrically disposed about a central meridian, as illustrated in Fig.10a (and as $A_2C_2D_2E_2$ of Fig.2); or

(b) one satellite at maximum excursion from the reference plane, at latitude δ, with two others symmetrically disposed either side of its meridian (as $B_2D_2E_2$ of Fig.2).

Such simplified calculations of d_{max} are considered in Appendix D; they have made feasible the examination of delta patterns involving relatively small numbers of satellites using hand methods. For patterns involving larger numbers of satellites it will be necessary to adopt a computerised approach based on the standard formulae; work on this has begun.

For the limited number of cases which it has been possible to examine, the minimum critical values of d_{max} have been plotted against np in Fig.11 to provide a direct comparison with the similar plot for star patterns in Fig.9. Points relating to $p = 1$ and $p = 2$ are identical on the two figures; notes on the other cases considered follow.

<u>p = 5, n = 1</u>. As has been noted in section 2 this pattern, illustrated in Fig.2, is the most satisfactory of four possible arrangements of the five orbits. Similar patterns to each of the two conditions shown in Fig.2 recur ten times in

each orbital period; for example, we may follow the development of the spherical triangle formed by three of the satellites as follows:

Phase of $A = 0^o$	$A_1C_1D_1$		
18^o	$A_2C_2D_2$		
36^o	$A_3C_3D_3$	$\equiv$	$D_1A_1B_1$
54^o	$A_4C_4D_4$	$\equiv$	$D_2A_2B_2$
72^o	$A_5C_5D_5$	$\equiv$	$B_1D_1E_1$
90^o	$A_6C_6D_6$	$\equiv$	$B_2D_2E_2$
108^o	$A_7C_7D_7$	$\equiv$	$E_1B_1C_1$
126^o	$A_8C_8D_8$	$\equiv$	$E_2B_2C_2$
144^o	$A_9C_9D_9$	$\equiv$	$C_1E_1A_1$
162^o	$A_{10}C_{10}D_{10}$	$\equiv$	$C_2E_2A_2$
180^o	$A_{11}C_{11}D_{11}$	$\equiv$	$A_1C_1D_1$

with the sequence repeating during the following half period. Values of d_{max}, obtained as in section D.4 of Appendix D, are plotted against δ in Fig.4; when $\delta = 43.7^o$, giving $d_{max} = 69.2^o$, this is thought to represent the optimum condition for continuous whole earth coverage with a minimum number of satellites. d_{max} is less than 76.3^o, corresponding to a minimum elevation angle exceeding 5^o with 24 hour satellites, for values of δ between about 28^o and 57^o.

$p = 7, n = 1$. As with the previous example, there are several possible regular arrangements of the satellites in a seven plane pattern. Taking B in each case as the satellite whose phase angle is $2\pi/7$ in advance of A, C as that $2\pi/7$ in advance of B, and so on, the eastward sequence of ascending nodes might be:

(a) ABCDEFGA....
(b) AEBFCGDA....
(c) AFDBGECA....
(d) ACEGBDFA....
(e) ADGCFBEA....
(f) AGFEDCBA....

Pattern (d) is illustrated in Fig.12; as noted in section 3.5, this proves to be the best for double coverage. Pattern (f) places all the satellites on a single figure 8 ground track, so is unsuitable for whole earth coverage. Patterns (a) and (c) turn out to be the best for single coverage, giving almost identical minimum values of d_{max}; for these cases d_{max} is plotted against δ in Fig.13, showing minimum values of 60.5^o at $\delta = 48.0^o$ for pattern (a), and of 60.2^o at $\delta = 55.7^o$ for pattern (c). Similar patterns recur 14 times in each orbital period.

p = 8, n = 1. The only suitable pattern found has satellite B in advance of A by $\pi/4$ in both phase angle and east longitude of ascending node, C in advance of B by $\pi/4$ in both respects, and so on. The minimum value of d_{max} is 61.9^o at $\delta = 43.6^o$; this is inferior to the result for the same total number of satellites with p = 2, n = 4. Similar patterns recur 8 times in each orbital period.

p = 4, n = 2. The only suitable pattern found has successive orbits with ascending nodes advanced by $\pi/2$ in east longitude and satellite phase angles advanced by $\pi/4$. This case also gives a less satisfactory result than does p = 2, n = 4; the minimum value of d_{max} is 57.6^o at $\delta = 43.7^o$. Similar patterns recur 8 times in each orbital period.

p = 3, n = 3. The preferred pattern, which recurs 18 times in each orbital period, has a phase difference of 40^o between satellites in adjacent orbits. It is illustrated in Fig.14, and values of d_{max} are plotted against δ in Fig.15 (curves marked 'Fig.14 single coverage'); the minimum value of d_{max}, 60.0^o at $\delta = 61.5^o$, is a little better than that for the corresponding star pattern. Fig.15 also shows (curves marked 'Fig.16 single coverage') the somewhat less satisfactory results obtained with an alternative pattern, illustrated in Fig.16, in which the satellite phasing is similar in the three orbit planes so that similar patterns recur only six times in each orbital period.

p = 5, n = 2. This pattern corresponds to that of Fig.2 (p = 5, n = 1) with a second satellite (A',B', etc.) added in each plane on the opposite side of the orbit. Resulting values of d_{max} are plotted in Fig.17, marked 'single coverage'; the minimum value of 52.2^o at $\delta = 57.1^o$ is somewhat better than the value obtained for p = 2, n = 5.

p = 3, n = 4. The pattern selected, which recurs 12 times in each orbital period, is illustrated in Fig.18. With this relatively large number of satellites the inter-relationships between satellites in different planes are more complex, and several different parts of the pattern are involved in critical conditions at different inclinations, as shown in Fig.19. The minimum value of d_{max}, 47.2^o at $\delta = 60.3^o$, is an improvement over that for the corresponding star pattern. Consideration was given to an alternative configuration, with satellite phasing similar in the three orbit planes, but this proved to give substantially less satisfactory coverage than the pattern chosen.

Details of the more useful of the foregoing cases are collected in Table 2, for ease of reference.

It was not considered practicable to investigate further delta patterns involving even larger numbers of satellites by the hand methods so far used.

3.5 Double coverage

We have so far considered orbital patterns ensuring whole earth coverage above some minimum elevation angle by at least one satellite. We now consider patterns providing double coverage, i.e. such that, at all points on (or above) the earth's surface and at all times, at least two satellites are visible above some minimum elevation angle.

For single coverage the critical conditions giving minimum elevation angles occurred at the circumcentres of groups of three satellites, with no other satellite enclosed within the circumcircle. For double coverage we must consider spherical triangles formed by three satellites which have one other satellite enclosed within their circumcircle. Similarly for triple coverage the circumcircle would have to enclose two other satellites, and so on.

For star patterns, the same basic approach may be used for double coverage as for single coverage, though the formulae need appropriate modification in detail. If the number of satellites per orbit is relatively large compared to the number of orbits (as, for example, with $p = 2$ and $n = 5$) then the three satellites to be considered are a satellite in one orbit, the next but one satellite in the same orbit, and the satellite in an adjacent orbit most nearly opposite the intermediate satellite in the first orbit; the circumcircle of these three encloses the intermediate satellite in the first orbit. For optimum coverage, in the sense of maximising the minimum elevation angle, satellites in co-rotating orbits should pass the node simultaneously. If the reason for providing double coverage were simply to ensure redundancy, such convergence at the nodes might well be acceptable; in practice, however, there may be other reasons for providing double coverage, such as using two widely spaced satellites for position-fixing purposes (navigation or surveillance). Even in this case it would always be possible to avoid use of more than one of the nominally coincident satellites in forming a pair, but steps would have to be taken to avoid interference, and it might be considered preferable to stagger the nodal crossings at the expense of some reduction in the minimum elevation angle. The particular example mentioned ($p = 2$, $n = 5$), even with simultaneous nodal crossings, gives a minimum value of d_{max} of 79.5^{o}, which is not really satisfactory for practical purposes; this occurs with $\alpha = 66.7^{o}$ and $\beta = 23.3^{o}$.

On the other hand, with star patterns in which the number of satellites per orbit is not large compared to the number of orbits (for example, with $p = 3$ and $n = 3$) then the three satellites to be considered are two adjacent satellites in one orbit and a satellite in the next but one orbit, their circumcircle always enclosing a satellite in the intermediate orbit. For optimum coverage, in the sense of maximising the minimum elevation angle, satellites in one orbit should pass a node midway between the nodal passages of satellites in the next but one co-rotating orbit; the relative phasing of satellites in adjacent co-rotating orbits may then be adjusted to improve as far as possible the coverage between one orbit and the next but one contra-rotating orbit. Where γ appeared in equation (B-3), 2α should now be substituted instead of α; and $(\alpha + \beta)$ instead of β should be substituted for γ in equation (B-4). The minimum satellite separation D_{min}, which may be of interest if pairs of satellites are to be used for position-fixing, occurs when two satellites are equidistant from a node, one approaching and the other receding; as shown in Fig.10b

$$D = AB = \cos^{-1} (\cos OA \cos OB + \sin OA \sin OB \cos AOB)$$

where $OA = OB$ and AOB is equal to either 2β or $2\alpha + 2\beta$. For the case of $p = 3$ with $n = 3$, optimum coverage is obtained with $\alpha = 32.6^{o}$ and $\beta = 24.8^{o}$, giving $d_{max} = 74.0^{o}$ and $D_{min} = 24.2^{o}$; this is clearly a more satisfactory result than that obtained with only two planes, even though the total number of satellites is now smaller.

For delta patterns the analysis of double coverage is treated in the same way as for single coverage, by individual examination of particular cases, though using spherical triangles formed by three satellites whose circumcircle encloses a fourth satellite. Only three cases have so far been examined in detail. For each of these, critical conditions for small values of δ were found to occur in one of the two types of symmetrical situations which were noted in section 3.4 as commonly associated with critical conditions for single coverage, making it possible to use the simplified approach to calculation of d_{max} described in Appendix D. For large values of δ, on the other hand, critical conditions did not correspond to such symmetrical situations in any of the three cases examined, so that it was necessary to follow the full procedure of solving the spherical triangle formed by three satellites (whose circumcircle enclosed a fourth satellite) for several satellite phase angles, calculating $d \equiv R$ from the standard formula in each case, and hence determining the value of d_{max}. The minimum satellite separation did, however, occur in each case in a symmetrical situation; considering the pair of satellites Y and Z in Fig.10a, with M the mid-point of YZ, then in the right spherical triangle MRY

$$\sin YM = \sin YR \sin YRM$$

and hence $$D_{min} = 2 \sin^{-1} (\sin YR \sin YRM) \quad .$$

For the three individual cases of double coverage by delta patterns which have so far been considered the minimum values of d_{max} appear on Fig.11, and further notes on the results obtained follow:

<u>p = 7, n = 1</u>. As mentioned previously, the most satisfactory of six possible patterns was found to be that illustrated in Fig.12; values of d_{max} for this case (pattern d) are plotted on Fig.13, showing a minimum value of 76.0^{o} at $\delta = 61.8^{o}$. Values of D_{min} are plotted on Fig.20; they exceed 30^{o} for all values of δ below 66^{o}, being 37.3^{o} when d_{max} is a minimum at $\delta = 61.8^{o}$, and reaching a maximum of 58.9^{o} at $\delta = 48.0^{o}$. Thus it is theoretically possible to provide continuous duplicated whole earth coverage with a minimum elevation angle of 5.3^{o} using only seven 24 hour satellites; however, ϵ only exceeds 5^{o} for values of δ between about 60^{o} and 63^{o}, and in practice this must be considered somewhat marginal. Nevertheless, it does not appear possible to do as well as this with any regular pattern comprising eight satellites.

<u>p = 3, n = 3</u>. The configuration of Fig.16 (but not the more complex pattern of Fig.14) has been re-examined in terms of double coverage; values of d_{max} are plotted on Fig.15 (marked 'Fig.16 double coverage') and values of

D_{min} on Fig.20. The minimum value of d_{max}, equal to 65.1°, occurs at δ = 57.5°, when D_{min} = 23.2°. d_{max} is less than 76.3° (i.e. ε exceeds 5° with 24 hour satellites) for values of δ between about 28° and 90°; D_{min} exceeds 30° for values of δ between about 66° and 72°, and reaches 33.5° at δ = 70.5°. These are substantially more satisfactory results than those obtained with the corresponding star pattern. The minimum value of d_{max} corresponds to double coverage with minimum elevation angles of 16.6° with 24 hour satellites, 11.3° with 12 hour satellites, or 6.6° with 8 hour satellites.

p = 5, n = 2. This is again the pattern of Fig.2 with a second satellite (A',B', etc.) added in each orbit. d_{max} is plotted on Fig.17 (marked 'double coverage'), with the single coverage results from Fig.4 replotted (broken line) for comparison; it is seen that for small values of δ the results are identical, but for large values of δ double coverage with double the number of satellites gives more favourable elevation angles. The minimum value of d_{max} is 65.1°, identical with the result for nine satellites, but D_{min} (plotted on Fig.20) is generally larger than for nine satellites.

It was not considered practicable to investigate double coverage by larger numbers of satellites in delta patterns by the hand methods so far used.

4 PRACTICAL CONSIDERATIONS

In section 3 we have considered theoretical methods of determining the coverage obtainable with two families of circular orbit satellite configurations, star patterns and delta patterns. We now turn to some of the practical considerations affecting the choice of an orbital configuration for a particular system requiring whole earth coverage.

It should be noted first that we have assumed a near-perfect station-keeping capability in the satellites, i.e. their control systems must be capable of effectively cancelling out all natural orbital perturbations. Even given this capability, some margin of error should be allowed for in practice. On the other hand, the minimum elevation angles (as given by the maximum sub-satellite distances) which we have determined have been the least favourable values experienced at any point on the earth's surface at any time; most places will experience better conditions than this all the time and all places will experience better conditions for most of the time. Hence the minimum elevation angles quoted may be considered reasonably realistic. Moreover, even when whole earth coverage is required with all places nominally of equal interest, there are likely in practice to be particular circumstances in certain areas which make them 'more equal than others', and the alignment of the pattern may be chosen to assure these areas of more favoured treatment.

It must be emphasized that the patterns considered in this Report are not, in general, suitable for a system to be organised on a zonal basis such that all points within the zone may always use the same satellite as the rest. However, except in the special case of the geostationary satellite, many more satellites are needed to provide such zonal systems (using synchronous or sub-synchronous orbits) than are needed for the simple form of coverage considered here; for example, a configuration[6] which would provide essentially whole earth coverage on a zonal basis using 8 hour circular orbits would require 14 or more satellites, whereas it has been shown in section 2 that six satellites in 8 hour circular orbits can provide whole earth coverage on the simple basis that every point on earth can always see at least one satellite at not less than 5° elevation. It is therefore most uneconomical to use a zonal system when its special characteristics are not essential; simple coverage should be adequate for systems generally of a data-gathering nature, and this might even include systems for mobile communications. Geostationary satellites, on the other hand, can provide a zonal system at no extra cost, and have outstanding advantages of convenience in many respects; we shall consider later in this section how they may be incorporated in a star or delta pattern.

However, it should be noted that it is possible, using the simple coverage approach, to define conditions so as to ensure that a limited form of zonal coverage may be established anywhere on earth. As an example, suppose the requirement were that any point on earth should always be able to communicate with any other point within 1500 km (13.5°) via a satellite of 12 hour period, the elevation angle in all cases being not less than 10°. This requires (from Fig.1) a value of d_{max} not exceeding 66.3° at each point, for the same satellite; this is ensured by a system with which d_{max} cannot anywhere exceed $66.3 - 13.5 = 52.8^{\circ}$. This approach applies at any altitude, not only at synchronous and sub-synchronous altitudes.

Coverage is only one of many factors to be taken into account in system design; launching considerations and overall system reliability are two others of particular importance. Launching considerations may lead, for instance, to a strong preference for orbital inclinations corresponding as closely as possible to the latitude of the launching site, to secure maximum payload in orbit from a given launching vehicle; the examination of coverage for a range of inclinations, as demonstrated in this study, shows how much coverage is being sacrificed by adopting an inclination other than the optimum. For example, Fig.5 shows that, at an inclination α of 32^o (i.e. $\beta = 58^o$), eight satellites are necessary to provide the same standard of coverage as six satellites at an inclination α of 52^o (i.e. $\beta = 38^o$).

Another consideration which may sometimes inhibit the selection of an inclination to give optimum coverage is some special inclination requirement, e.g. a requirement for sun-synchronism, demanding near-polar orbits at moderate altitude. While such cases have not received specific examination in this study, it seems likely that for near-polar orbits a modification of a star pattern, which would give optimum coverage with multiple polar orbits, would give better coverage for a given number of satellites than a true delta pattern, giving its optimum coverage at medium inclinations; in the special case of two-plane patterns, near-polar inclinations may be obtained by appropriate orientation of the optimum pattern. With more than two planes $(p > 2)$ then, as the inclination is increased or decreased from 90^o, the value of $(p - 1)\alpha + \beta$ increases above $\pi/2$, resulting in an increase in the minimum value of d_{max} over that obtained for a star pattern; however, with the change in inclination limited to a few degrees, the increase in the minimum value of d_{max} is typically only a fraction of a degree.

Considerations of overall system reliability, if not adequately covered by redundancy within individual satellites, may sometimes be deemed to require that spare satellites be maintained in orbit. Since it is not feasible (at least with current technology) to make substantial changes of orbit plane within a short time, though fairly rapid in-plane repositioning is possible, this might be taken to imply that one spare satellite should be maintained in each orbit plane. Hence, while the delta pattern of Figs.2 and 4, with $p = 5$ and $n = 1$, is the most economical for single coverage in terms of the number of operational satellites required, it becomes less so than the two-plane patterns of Fig.5 if in-orbit spares are necessary. Four satellites in each of two planes, repositioned after a single failure in each plane to leave a six satellite system, would provide a high standard of system reliability. On the other hand, a system might be preferred which ensured continuous duplication of coverage without any repositioning of satellites; in this case one of the double coverage patterns of section 3.5, using either seven satellites or, more probably, nine satellites in three planes, might be selected.

The star and delta patterns have been defined in terms of an arbitrary reference plane passing through the centre of the earth; in any practical case, it is necessary to select a preferred orientation for this reference plane. One consideration is that, to simplify launching arrangements, it is desirable that all satellites should have the same inclination. This implies that for delta patterns of more than two planes the reference plane should coincide with the equator; for star patterns of more than two planes, on the other hand, the reference plane should be at 90° inclination with the orbits intersecting at the poles, all orbits thus being polar but with their nodes unequally spaced around the equator, as in Fig.6b.

An alternative consideration if satellites of 24 hour period are involved is that, because of the great advantages offered by geostationary satellites, one of the orbital planes should be equatorial. Since it would be undesirable to have more than one other orbit inclination in the system in addition to equatorial, interest in such cases is likely to be confined to two plane or three plane systems. For example, in Figs.6b and 6c, plane B might be equatorial while planes A and C are of equal inclination to it, which means that the reference plane has an inclination of $2\alpha + \beta$ in the former (star) case and of δ in the latter (delta) case. As the results of section 3 have shown, for optimum coverage in combination with geostationary satellites the preferred inclination of the other orbits is not 90° for the two orbit cases, nor is it 60° for the three orbit cases, as has often been assumed.

Applying these arguments to some particular cases, it is unlikely that the five and seven plane delta patterns considered would be used in any other manner than with the reference plane coinciding with the equator, so that all the orbits were of equal inclination. Over a non-rotating sphere the orbital patterns of Figs.2 and 12 would also represent satellite ground tracks; but the earth's rotation results in the same orbital pattern producing different ground track patterns for different orbital periods, each satellite in 24 hour circular orbit crossing the equator at a single longitude only, in 12 hour orbit at two longitudes 180° apart, in 8 hour orbit at three longitudes 120° apart, and so on. The ground track of a satellite in 24 hour circular orbit is a figure 8; if the inclination exceeds 90°, the two loops encircle the poles. Figs.21a and 21b represent the ground track patterns of satellites in synchronous orbits following the orbital patterns of Figs.2 and 12 respectively, with all orbits in each pattern of equal inclination to the equator; satellite positions marked correspond to condition 2 in each case.

With the two plane six satellite system, which may be considered as either a star or a delta pattern, more possibilities present themselves.

Considering it first as a delta pattern, the two planes might be made of equal inclination to the equator; or one plane might be made equatorial, the other being near-polar. For 24 hour satellites, the corresponding ground track patterns are shown in Figs.22a and 22b, the former showing six figure 8 tracks, the latter three geostationary satellites and three whose figure 8 tracks encircle the poles. Considering it as a star pattern, both orbits might be made polar; for 24 hour satellites, the corresponding ground track patterns are shown in Fig.22c. While the three ground track patterns of Fig.22 appear very different, they represent identical orbital patterns, differing only in the orientation of the reference plane to the equator; the minimum elevation angle is almost 15^o in each case, but the points on the earth's surface at which this minimum is experienced are different for the three cases. With such two plane patterns there is, further, the possibility of selecting the orientation of the pattern to obtain pairs of orbits of similar inclination, with the inclination having any value between α and $\pi - \alpha$, and appropriate nodal spacing. Thus in terms of whole earth coverage all these patterns achieve identical results, but particular individual areas of the earth's surface may be better served by one ground track pattern than by another. Moreover, the perturbations which the control systems of the satellites must combat to maintain the orbital pattern will be different for different orientations of the reference plane.

Fig.23 shows two ground track patterns for nine 24 hour satellites following the delta pattern of Fig.16 (condition 2); again, one case is with all orbits of equal inclination and the other with one of the three orbits made equatorial so that the system includes three geostationary satellites.

Section 3 showed that it was a relatively simple matter, with star patterns, to determine optimum coverage arrangements for any number of satellites; for delta patterns, on the other hand, no simple general rules for rapid selection and analysis have been discovered, so that it has been necessary to study each example on a case-by-case basis. In each case which has been examined, a delta pattern has given somewhat better, but (for single coverage) not substantially better, coverage than a star pattern involving a similar number of satellites. It therefore seems appropriate to suggest that, when any particular requirement for whole earth coverage arises, it should be considered first in terms of a star pattern solution; when this has shown the approximate number of satellites required (from Fig.9, for instance), corresponding delta patterns should be examined. There will usually be several alternative possibilities to be considered as regards the relative phasing of satellites in different orbits; the superiority of the patterns of Figs.14 and 18 over corresponding patterns (such as Fig.16) with similar phasing in each orbit suggests that it is desirable to choose patterns which are repeated as frequently as possible.

It should be noted, however, that Fig.16 (with reference plane at the equator) represents a pattern which has been suggested in the past as suitable for providing a zonal coverage system using 24 hour satellites. This is because the relative phasing of the satellites in the different orbits is such that, when their ground tracks over the rotating earth are plotted (Fig.23a), only three separate figure 8's are formed, each of which is traced, at equal intervals, by one satellite from each of the three orbit planes. Zones may thus be formed, centred on the two loops of each figure 8, which use only the satellites following that particular figure 8. The analysis made in section 3 of the system of Fig.16, for single and for double coverage, was not based on such a zonal concept; on a simple coverage basis, a station at a particular location might at different times use satellites following different figure 8 tracks, thereby achieving better minimum elevation angles. Synchronous satellites in the alternative pattern of Fig.14, found better than that of Fig.16 for simple single coverage, follow nine separate figure 8 tracks spaced at equal intervals round the equator. The condition for a satellite to follow another one or the same figure 8 track is that its phase angle increment should equal the westward longitude increment of its ascending node relative to that of the first satellite; thus the pattern of Fig.18 would produce only four separate figure 8 tracks followed by three satellites each.

While only single and double coverage have received specific examination, the conclusion that single coverage can be ensured by a minimum of five satellites and double coverage by a minimum of seven makes it evident that triple coverage need not require more than twelve satellites or quadruple coverage more than fourteen; indeed, it seems probable that detailed examination of these cases would show smaller numbers to be adequate.

5 POSSIBLE APPLICATION TO MOBILE SERVICES

As noted in section 1, this study began as an initial investigation of the number of satellites which might be necessary to provide continuous whole earth coverage, on the assumption that such coverage might prove to be one of the requirements for a system of satellites to provide aeronautical communication and/or surveillance services. Subsequent discussion with experts in civil aircraft operations suggested that there was unlikely, in practice, to be a demand for whole earth coverage; geostationary satellites could cover all except a very limited number of routes, principally those between Europe and the Far East via Alaska, and the prospect that the opening of a trans-Siberian route would reduce usage of these polar routes seems, in early 1970, to be approaching reality. However, possibilities of whole earth coverage have been considered in the recent CCIR report[2] on orbits suitable for mobile services, and so the subject is considered briefly here.

Use of a zonal system of coverage, as for commercial point-to-point communication satellite services, is obviously convenient, but for whole earth coverage it would involve the use of many more satellites than would simple coverage, as noted in section 4. It would be for experts in aircraft and ship operations to determine whether the greater convenience outweighed the greater cost, or whether a simple coverage system (including the possibility of a limited form of zonal coverage) would be acceptable; questions of acceptable ground station locations would enter into such an assessment. Many of the patterns considered in this Report would allow part of the system to consist of geostationary satellites, with their great advantages even on a simple coverage basis; indeed, it might be expected that any such system would be established initially using geostationary satellites only, these being supplemented by other satellites in high inclination orbits only if the extension to provide whole earth coverage were considered justifiable at a later stage.

However, to examine the conditions which would prevail in a simple coverage system not including any geostationary satellites, consideration was given to use of the system of Fig.2 with reference plane coinciding with the equator, i.e. a system of five synchronous satellites at about 44° inclination. If the pattern of Fig.21a is located geographically so that the ascending node of A is at 108°W, of D at 36°W and of B at 36°E, then conditions along the principal North Atlantic air traffic lane from London (0°W) via Shannon (9°W) and Gander (54°W) to New York (74°W) during a 24 hour period are indicated in Fig.24. This shows, on a basis of time (from ascending nodal passage of D) and longitude along the track, the coverage available from satellites A, B and D at an elevation angle exceeding 12° as seen from sea level.

Fig.24 shows that satellite D is visible above 12° elevation over the whole route from London to New York except for the period from about 13 h to 23 h. Satellite B provides coverage between London and Gander from about 12 h to 18 h, and satellite A between Shannon and New York from about 19 h to 24 h and between London and New York from about 20 h to 23 h. There are short periods when (unless the London and New York ground stations operate below 12^{o} elevation, as might well be feasible) an aircraft cannot communicate direct with London (though it can with Shannon) unless it is east of about 40^{o}W, and somewhat longer periods when (subject to the same qualification) an aircraft cannot communicate direct with New York (though it can with Gander) while it is over the ocean. However, while the North Atlantic coverage thus provided by three satellites out of a five satellite pattern is of quite a high standard, it is inferior to that provided by a single geostationary satellite at (say) 36^{o}W; this would provide continuous coverage of the whole track, and might form part of a whole earth coverage pattern of six satellites, three in geostationary orbit at 36^{o}W, 156^{o}W and 84^{o}E, and three in orbits of about 104^{o} inclination crossing the equator at 96^{o}W, 144^{o}E and 24^{o}E, this representing one orientation (as in Fig.22b) of the optimum six satellite system discussed previously. With the addition of the 104^{o} inclination satellites, one or other of these could back up the coverage provided by the 36^{o}W geostationary satellite over most of the principal North Atlantic air traffic lane, as well as providing coverage to the polar regions.

An eight satellite system, using four geostationary and four 97^{o} inclination satellites, would provide duplicated continuous coverage of the North Atlantic route and of a large part of the rest of the world. Nine satellites in three planes could provide fully duplicated whole earth coverage on a simple coverage basis, given acceptable ground station locations. The pattern of Fig.16, as considered in section 3.5, might represent (for 17^{o} minimum elevation and 24^{o} minimum satellite spacing) either nine satellites all at about 58^{o} inclination (Fig.23a) or three in geostationary orbit with three each in two orbits of about 94^{o} inclination with ascending nodes about 94^{o} apart (Fig.23b). Alternatively, for 15^{o} minimum elevation and 30^{o} minimum satellite spacing, one might have either nine satellites all at about 66^{o} inclination or three geostationary with three each in two orbits of about 105^{o} inclination with ascending nodes about 110^{o} apart.

6 CONCLUSIONS

Consideration has been given to systems of satellites in multiple equal-radius circular orbits, with an equal number of satellites in each orbit, capable of providing continuous coverage of the whole earth's surface in orbits from synchronous altitude down to relatively low altitude. Two types of orbit pattern have been considered, identified as 'star patterns' and 'delta patterns' respectively; the latter appear most suitable for orbits of moderate inclination, or for combinations of equatorial and high inclination orbits, the former for polar (or near-polar) orbits. Coverage has been treated, not on the zonal basis adopted for commercial point-to-point communications systems, but on the simple basis of ensuring that every point on the earth's surface can always see at least one satellite (or two satellites for double coverage) above some minimum elevation angle; this requires substantially fewer satellites than a corresponding zonal system (other than one based on geostationary satellites, which cannot provide whole earth coverage), and while unsuitable for commercial point-to-point communication systems it is likely to be acceptable for data-gathering type systems and perhaps even for mobile communications.

Analysis of the coverage provided by the star patterns is considerably simpler than for the delta patterns; however, the delta patterns appear in general to provide somewhat better coverage than star patterns using similar numbers of satellites, while for double coverage their advantage may be even greater. The methods of analysis presented allow the determination of coverage for any orbital inclination, and the determination of an optimum inclination in each case, which does not usually correspond to any of the simple round number values of this parameter often selected for study in the past; for double coverage, satellite separation is also considered. The results obtained suggest that symmetry of the orbital pattern, often treated in previous studies as an obvious requirement, is of less significance than regularity of the satellite pattern, leading to frequent recurrence of similar satellite patterns during each orbital period. Systems involving only two orbital planes, which may be treated as either star or delta patterns, are particularly versatile.

Several particular cases have been examined. The most significant new results appear to be:

(a) Five satellites, in five circular orbits of about 44° inclination with ascending nodes 72° apart, can provide continuous whole earth coverage with a minimum elevation angle exceeding 12° for 24 hour orbits or 7° for 12 hour orbits. The minimum elevation angle will exceed 5°, for 24 hour orbits, for inclinations between about 28° and 57°.

(b) Six satellites, in two circular orbits of about 52° inclination with ascending nodes 180° apart, can provide continuous whole earth coverage with a minimum elevation angle exceeding 14° for 24 hour orbits, 9° for 12 hour orbits, or 5° for 8 hour orbits. The minimum elevation angle will exceed 5°, for 24 hour orbits, for inclinations between about 28° and 69°. Alternative configurations, similarly giving a minimum elevation angle exceeding 14° for 24 hour orbits, would be three geostationary satellites with three satellites at about 104° inclination, or three satellites in each of two polar orbits with ascending nodes about 104° apart; or, indeed, three each in two orbits of similar inclination, for any inclination between 52° and 128°, with appropriate nodal spacing.

(c) Seven satellites, in seven circular orbits of about 62° inclination with ascending nodes 51.4° apart, can provide continuous duplicated whole earth coverage with a minimum elevation angle exceeding 5° for 24 hour orbits, and with a minimum satellite separation exceeding 36°.

(d) Nine satellites, in three circular orbits of about 58° inclination with ascending nodes 120° apart, can provide continuous duplicated whole earth coverage with a minimum elevation angle exceeding 16° for 24 hour orbits, 11° for 12 hour orbits or 6° for 8 hour orbits, and a minimum satellite separation exceeding 23°. The minimum elevation angle will exceed 5°, for 24 hour orbits, for inclinations between about 28° and 90°, and the minimum satellite separation will exceed 30° for inclinations between about 66° and 72°. An alternative configuration, similarly giving a minimum elevation angle exceeding 16° for 24 hour orbits, would be three geostationary satellites with three satellites in each of two orbits of about 94° inclination.

These conclusions would be relevant to any revision of the CCIR report[2] on 'Satellite orbits for systems providing communication and radiodetermination for stations in the mobile service'.

A computerised approach, rather than the hand methods so far used, will be necessary to apply this analysis to systems using larger numbers of satellites, suitable for lower altitude orbits or for greater redundancy of coverage; work on this has begun.

Appendix A

NOTE ON THE USE OF THE NATIONAL GEOGRAPHIC SOCIETY GLOBE

(see sections 2 and 3.4)

The National Geographic Society 12 inch (or 16 inch) globe comprises an unmounted sphere; a transparent plastic stand, of which the upper portion is a great circle ring with degree, time and distance scales; and a transparent plastic geometer, shaped like a skull cap, with a radial degree scale extending for 70^{o} from its pole.

For initial approximate solution of coverage problems involving circular orbits, over an earth which may be considered as non-rotating, the geographical features on the globe may be ignored. The globe may be positioned on the stand so that the great circle ring indicates a desired orbit, and markers (of tacky coloured paper, for example) then placed on the globe to indicate desired positions of satellites on the orbit as shown by the degree scale on the great circle ring. After repeating this process for satellites in other orbits, the globe may be removed from the stand, and the geometer used to measure distances between satellites and radii of circumcircles of groups of satellites. The markers may then be removed and the process repeated for other satellite configurations.

Having eliminated non-critical conditions by this approximate screening process, critical conditions may be analysed accurately by means of spherical trigonometry, as described in Appendices B-D; the approximate values obtained with globe and geometer, which have usually been found to be accurate to about $\pm 1^{o}$, then provide a check on the correctness of the calculations.

Appendix B

CALCULATION OF THE MAXIMUM VALUE OF THE SUB-SATELLITE DISTANCE d BETWEEN SATELLITES IN ADJACENT ORBITS OF A STAR PATTERN

(see section 3.2)

B.1 General case

Fig.8a illustrates a typical situation. Here O and O' are the projections on the earth's surface (which may be treated as non-rotating) of the nodes of adjacent orbits, OAQ'DO' and OBQCO', having a relative inclination 2γ; R is the projection of the pole of the latter orbit, and RQ'Q is the antinodal circle. A, B, C and D are the instantaneous positions of the sub-satellite points of four satellites, the two in each orbit nearest to QQ', with A being the nearest and B next nearest.

P is the instantaneous position of the circumcentre of the spherical triangle ABC; in this analysis we need consider only cases in which P lies inside ABC, so that, with these satellites situated astride the antinodal circle, P is in fact the point in this hemisphere most distant at this instant from the sub-satellite points of any of the satellites in these two orbits.

From the spherical triangle ARP

$$\cos PA = \cos AR \cos RP + \sin AR \sin RP \cos ARP$$

and similarly from the spherical triangle BRP

$$\cos PB = \cos BR \cos RP + \sin BR \sin RP \cos BRP \quad . \qquad \text{(B-1)}$$

But PA = PB = d; hence

$$\cos AR \cos RP + \sin AR \sin RP \cos ARP = \cos BR \cos RP + \sin BR \sin RP \cos BRP \ ,$$

which reduces to

$$\tan RP = \frac{\cos AR - \cos BR}{\sin BR \cos BRP - \sin AR \cos ARP} \quad . \qquad \text{(B-2)}$$

From the right spherical triangle ARQ'

$$\cos AR = \cos AQ' \cos RQ'$$

and

$$\tan ARQ' = \tan AQ' \operatorname{cosec} RQ' \quad .$$

Putting $AQ' = a$, $BQ = b$, and noting that $RQ' = \frac{\pi}{2} - 2\gamma$ and $PRB = \frac{\pi}{n}$, so that $PRQ = \frac{\pi}{n} - b$, we have

$$ARP = ARQ' + PRQ$$

$$= \tan^{-1}(\tan a \sec 2\gamma) + \frac{\pi}{n} - b \quad .$$

Also $BR = \frac{\pi}{2}$. Substituting these values in equation (B-2) gives

$$\tan RP = \frac{\cos a \sin 2\gamma}{\cos\frac{\pi}{n} - \sin\cos^{-1}(\cos a \sin 2\gamma)\,.\cos\left[\tan^{-1}(\tan a \sec 2\gamma) + \frac{\pi}{n} - b\right]}$$

and substituting this also in equation (B-1) gives

$$d = \cos^{-1}\left\{\cos\frac{\pi}{n}\sin\tan^{-1}\left[\frac{\cos a \sin 2\gamma}{\cos\frac{\pi}{n} - \sin\cos^{-1}(\cos a \sin 2\gamma)\,.\cos\left[\tan^{-1}(\tan a \sec 2\gamma) + \frac{\pi}{n} - b\right]}\right]\right\}$$

.... (B-3)

B.2 <u>At the passing points</u>

When $a = b$, a simple expression for d is most readily obtained from consideration of Fig.8b, in which PM bisects BC at right angles. Then from the right spherical triangle PMO'

$$\tan PM = \tan PO'M \sin O'M$$

and from the right spherical triangle BMP

$$\cos PB = \cos BM \cos PM$$

$$= \cos BM \cos\tan^{-1}(\tan PO'M \sin O'M) \quad .$$

Substituting $BM = \frac{\pi}{n}$, $PO'M = \gamma$, $O'M = \frac{\pi}{2} - QM = \frac{\pi}{2} - \left(\frac{\pi}{n} - b\right)$ and $PB = d$, we have

$$d = \cos^{-1}\left\{\cos\frac{\pi}{n}\cos\tan^{-1}\tan\gamma\cos\left(\frac{\pi}{n} - b\right)\right\} . \qquad \text{(B-4)}$$

Appendix C

EQUATIONS FOR d_{max} FOR STAR PATTERNS

Provided the centre of the circumcircle of the spherical triangle formed by three satellites, two adjacent satellites in one orbital plane and one in the adjacent orbital plane, lies within the spherical triangle, then the value of the maximum sub-satellite distance d_{max} at the centre of the circumcircle, expressed as a function of the relative inclination of the orbital planes (2α, $2\alpha'$ or 2β), is obtained from the equation

$$d_{max} = \cos^{-1}\left\{\cos\frac{\pi}{n}\cos\tan^{-1} Q\right\} .$$

Here Q may take one of the nine values listed below, derived from either equations (B-3) or (B-4), as indicated in sections 3.2 and 3.3. The particular equation to be used for Q, depending upon whether the values of p and n are even or odd numbers, and whether d_{max} is required in terms of α, α' or β, is indicated in the table following.

$$Q = \tan\beta \tag{C-1}$$

$$Q = \tan\beta\cos\frac{\pi}{n} \tag{C-2}$$

$$Q = \tan\beta\cos\frac{\pi}{np} \tag{C-3}$$

$$Q = \tan\beta\cos\frac{\pi}{2np} \tag{C-4}$$

$$Q = \tan\beta\cos\frac{(p-1)\pi}{2np} \tag{C-5}$$

$$Q = \tan\beta\cos\frac{(p-2)\pi}{2np} \tag{C-6}$$

$$Q = \cos\frac{\pi}{n}\operatorname{cosec}2\alpha - \cot 2\alpha \tag{C-7}$$

$$Q = \frac{\cos\frac{\pi}{n} - \sin\cos^{-1}\left(\cos\frac{\pi}{np}\sin 2\alpha\right) . \cos\tan^{-1}\left(\tan\frac{\pi}{np}\sec 2\alpha\right)}{\cos\frac{\pi}{np}\sin 2\alpha} \tag{C-8}$$

$$Q = \frac{\cos\frac{\pi}{n} - \sin\cos^{-1}\left(\cos\frac{2\pi}{np}\sin 2\alpha'\right) . \cos\tan^{-1}\left(\tan\frac{2\pi}{np}\sec 2\alpha'\right)}{\cos\frac{2\pi}{np}\sin 2\alpha'} , \tag{C-9}$$

	p even, n even	p even, n odd	p odd, n even	p odd, n odd
Optimised coverage				
α	(C-7)	(C-7)	(C-7)	(C-7)
β	(C-2)	(C-1)	(C-1)	(C-2)
Staggered nodal crossings				
α	(C-7)	(C-7)	(C-8)	(C-8)
α'	(C-9)	(C-9)		
β	(C-3)	(C-6)	(C-5)	(C-4)

α, α' and β are connected by the relationships:

For optimised coverage: $$(p - 1)\,\alpha + \beta = \frac{\pi}{2}$$

For staggered nodal crossings: $$\frac{p}{2}\,\alpha + \left(\frac{p}{2} - 1\right)\alpha' + \beta = \frac{\pi}{2} \quad .$$

Appendix D

DETERMINATION OF d_{max} FOR DELTA PATTERNS

(see section 3.4)

D.1 General case

Where it has been necessary to use the full-length approach to calculation of d for a given trio of satellites, the following procedure has been found appropriate:

(i) For two of the three satellites, for a chosen value of δ,

(a) solve the spherical triangle formed by their orbits and the equator, giving the position and angle of intersection of these two orbits; and then

(b) solve the spherical triangle formed by the two satellites and the point of intersection of their orbits, giving the distance between the two satellites.

(ii) Repeat for the other two pairs formed by the three satellites, so finding the lengths of the sides of the spherical triangle formed by the three satellites, and solve this triangle to find the angles.

(iii) Find the radius R of the circumcircle (which is the same as d) from the standard formula

$$\tan R = \tan \tfrac{1}{2}a \sec \tfrac{1}{2} (B + C - A)$$

where abc are the sides opposite the angles ABC respectively.

Steps (i)(b), (ii) and (iii) are then repeated for several satellite phase angles in the neighbourhood of that condition thought to be critical, to give the corresponding values of d, and these values plotted against phase angle to determine the maximum value d_{max} for this particular value of δ. The entire procedure must then be repeated for other values of δ in order to find the optimum value of δ giving the minimum value of d_{max}.

However, it is frequently possible to short-circuit this approach, when the critical condition can be identified as corresponding to one of two symmetrical arrangements of satellites, considered in sections D.2 and D.3 below.

D.2 Four satellites in the same plane, symmetrically disposed about a central meridian

In Fig.10a the sub-satellite points W and X, Y and Z are symmetrically disposed about the meridian QPR, where Q and R are the poles of the reference plane, and P is the centre of the circumcircle of X, Y and Z (and also W).

From the spherical triangle XRP

$$\cos XP = \cos XR \cos PR + \sin XR \sin PR \cos XRP$$

and similarly from the spherical triangle YRP

$$\cos YP = \cos YR \cos PR + \sin YR \sin PR \cos YRP \quad .$$

But XP = YP = d; hence, as for equation (B-2), we obtain

$$\tan PR = \frac{\cos YR - \cos XR}{\sin XR \cos XRP - \sin YR \cos YRP} \quad .$$

Here XR and YR are functions of the latitudes of the sub-satellite points, and XRP and YRP of their longitudes, relative to the reference plane, and may be expressed as functions of the inclination δ of the satellite orbits to the reference plane in any specific case. Hence general solutions for d_{max} in terms of δ for such critical conditions may be obtained from the equations

$$d_{max} = \cos^{-1} (\cos XR \cos PR + \sin XR \sin PR \cos XRP) \qquad (D-1)$$

where

$$PR = \tan^{-1} \frac{\cos YR - \cos XR}{\sin XR \cos XRP - \sin YR \cos YRP} \quad . \qquad (D-2)$$

D.3 One satellite at maximum excursion from the reference plane, with two others symmetrically disposed about its meridian

For analysis, this may be regarded as a special case of Fig.10a, with W coinciding with X on the meridian QPR, so that $XR = 90^o + \delta$ and $XRP = 0^o$, i.e. $\sin XR = \cos \delta$, $\cos XR = - \sin \delta$, and $\cos XRP = 1$. Inserting these values in equations (D-1) and (D-2) we obtain

$$d_{max} = \cos^{-1}(\sin PR \cos\delta - \cos PR \sin\delta) \tag{D-3}$$

where

$$PR = \tan^{-1}\frac{\cos YR + \sin\delta}{\cos\delta - \sin YR \cos YRP} \tag{D-4}$$

D.4 Example: p = 5, n = 1 (Fig.2)

When δ is small, critical conditions occur in spherical triangles similar to $A_2D_2E_2$. Here $A_2 \equiv X$, $D_2 \equiv Y$, $C_2 \equiv Z$, so we have for insertion in equations (D-1) and (D-2):

$$XR = 90^\circ + \sin^{-1}\sin 18^\circ \sin\delta$$

$$YR = 90^\circ - \sin^{-1}\sin 54^\circ \sin\delta = \cos^{-1}\sin 54^\circ \sin\delta$$

$$XRP = 18^\circ + \tan^{-1}\tan 18^\circ \cos\delta$$

$$YRP = 54^\circ + \tan^{-1}\tan 54^\circ \cos\delta \quad .$$

When δ is large, critical conditions occur in spherical triangles similar to $B_2D_2E_2$. Here $B_2 \equiv X$, $E_2 \equiv Y$, $D_2 \equiv Z$, so we have for insertion in equations (D-3) and (D-4):

$$YR = 90^\circ - \sin^{-1}\sin 54^\circ \sin\delta = \cos^{-1}\sin 54^\circ \sin\delta$$

$$YRP = 126^\circ - \tan^{-1}\tan 54^\circ \cos\delta \quad .$$

Results are plotted on Fig.4 for values of δ from 0° to 90°.

Table 1

SINGLE COVERAGE: STAR PATTERNS

Total number of satellites np	Best combination of: number of planes p	satellites per plane n	Minimum value of maximum sub-satellite distance d_{max}	Corresponding semi-separation of: co-rotating planes α	contra-rotating planes β
6	2	3	66.7°	52.2°	37.8°
8	2	4	57.0°	48.4°	41.6°
10	2	5	53.2°	47.7°	42.3°
12	3	4	48.6°	34.7°	20.7°
15	3	5	42.1°	32.8°	24.5°
18	3	6	38.7°	32.2°	25.7°
21	3	7	36.3°	31.4°	27.2°
24	4	6	33.5°	24.7°	16.0°

Table 2

SINGLE COVERAGE: DELTA PATTERNS

Total number of satellites np	Best combination of: number of planes p	satellites per plane n	Minimum value of maximum sub-satellite distance d_{max}	Corresponding inclination to reference plane δ
5	5	1	69.2°	43.7°
6	2	3	66.7°	52.2°
7	7	1	60.2°	55.7°
8	2	4	57.0°	48.4°
10	5	2	52.2°	57.1°
12	3	4	47.2°	60.3°

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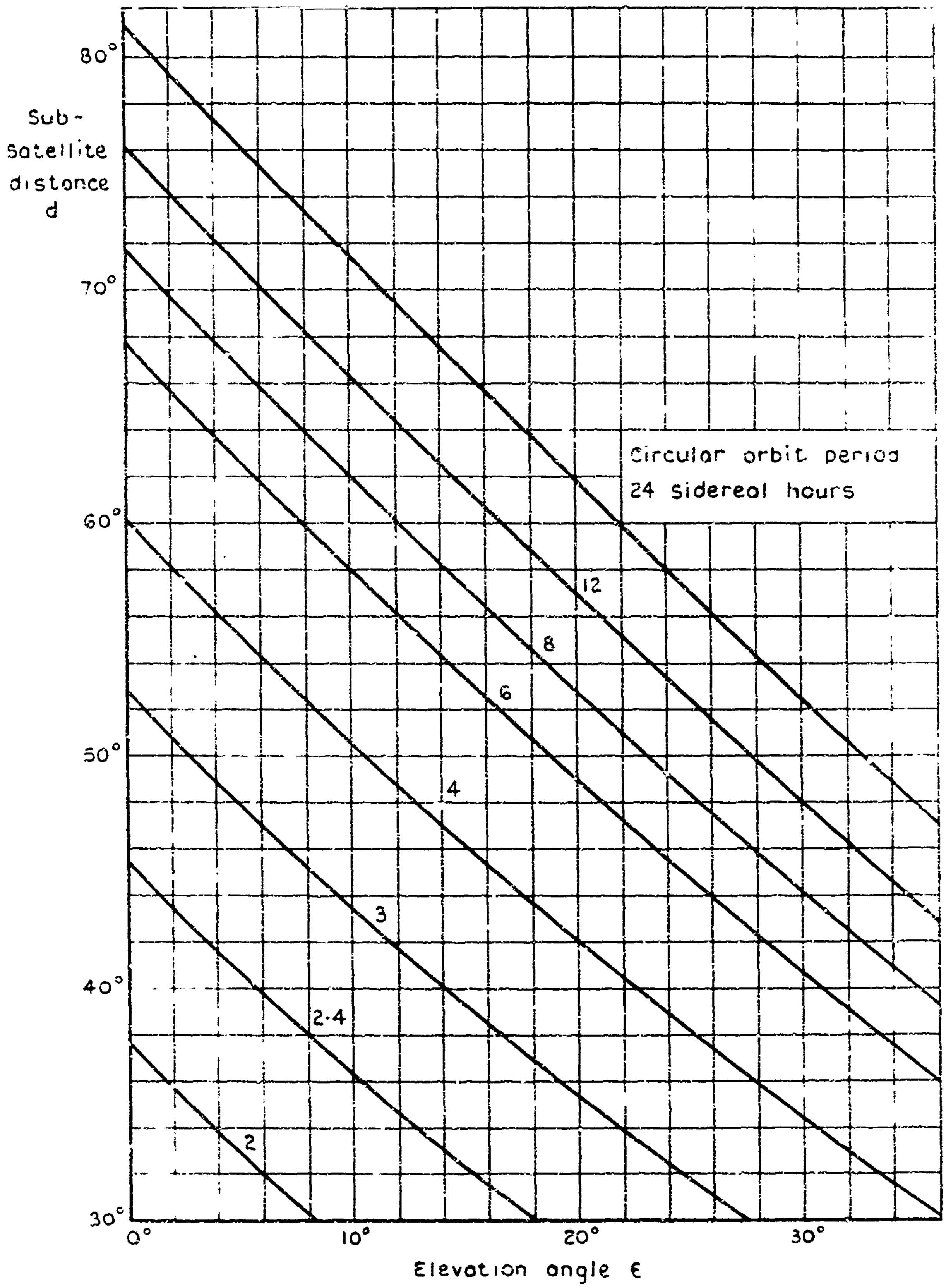

Fig.1 Dependence of elevation angle on sub-satellite distance and orbit period

Fig. 2

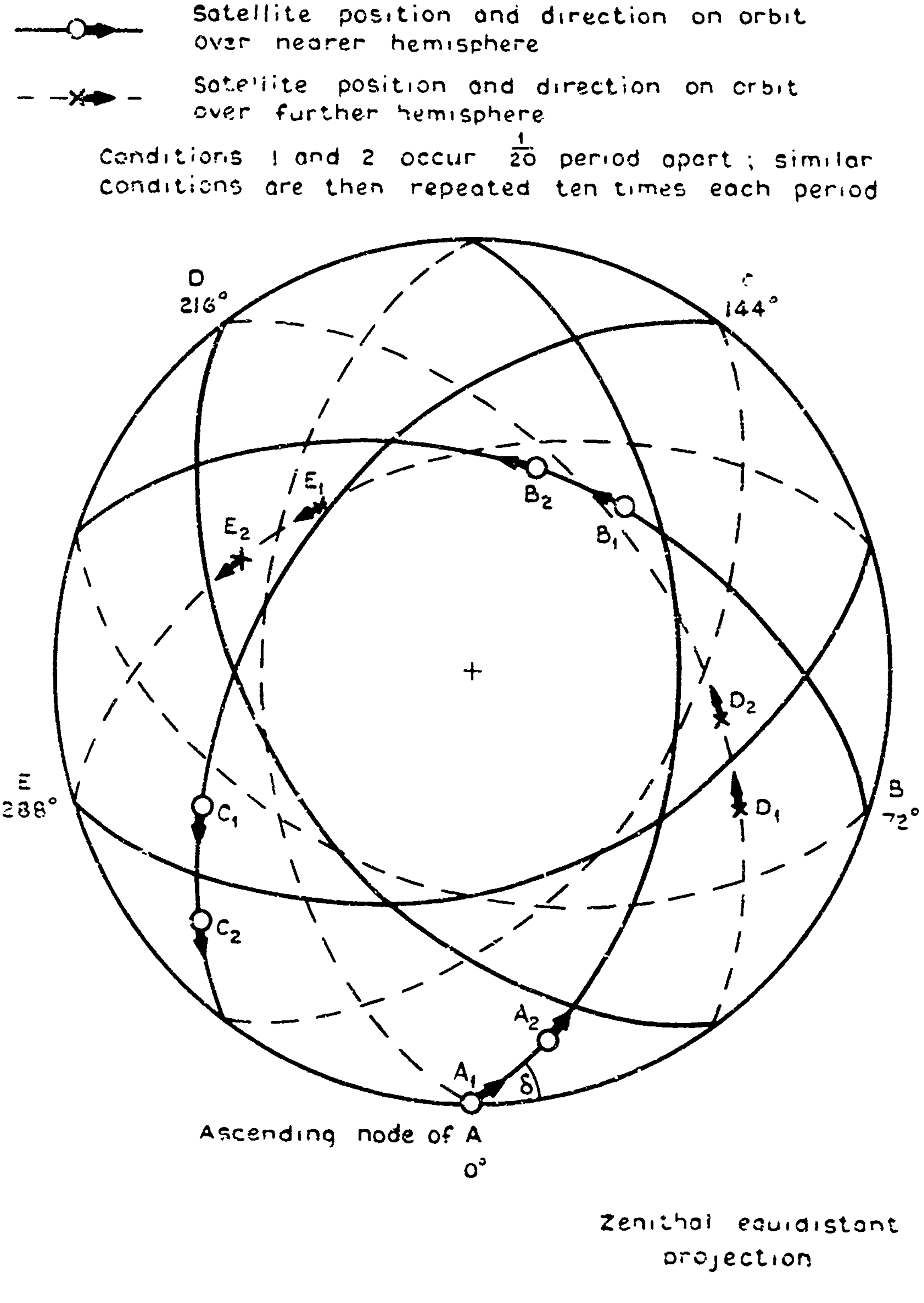

Fig. 2 5 satellite circular orbit pattern

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Fig.3 a-c

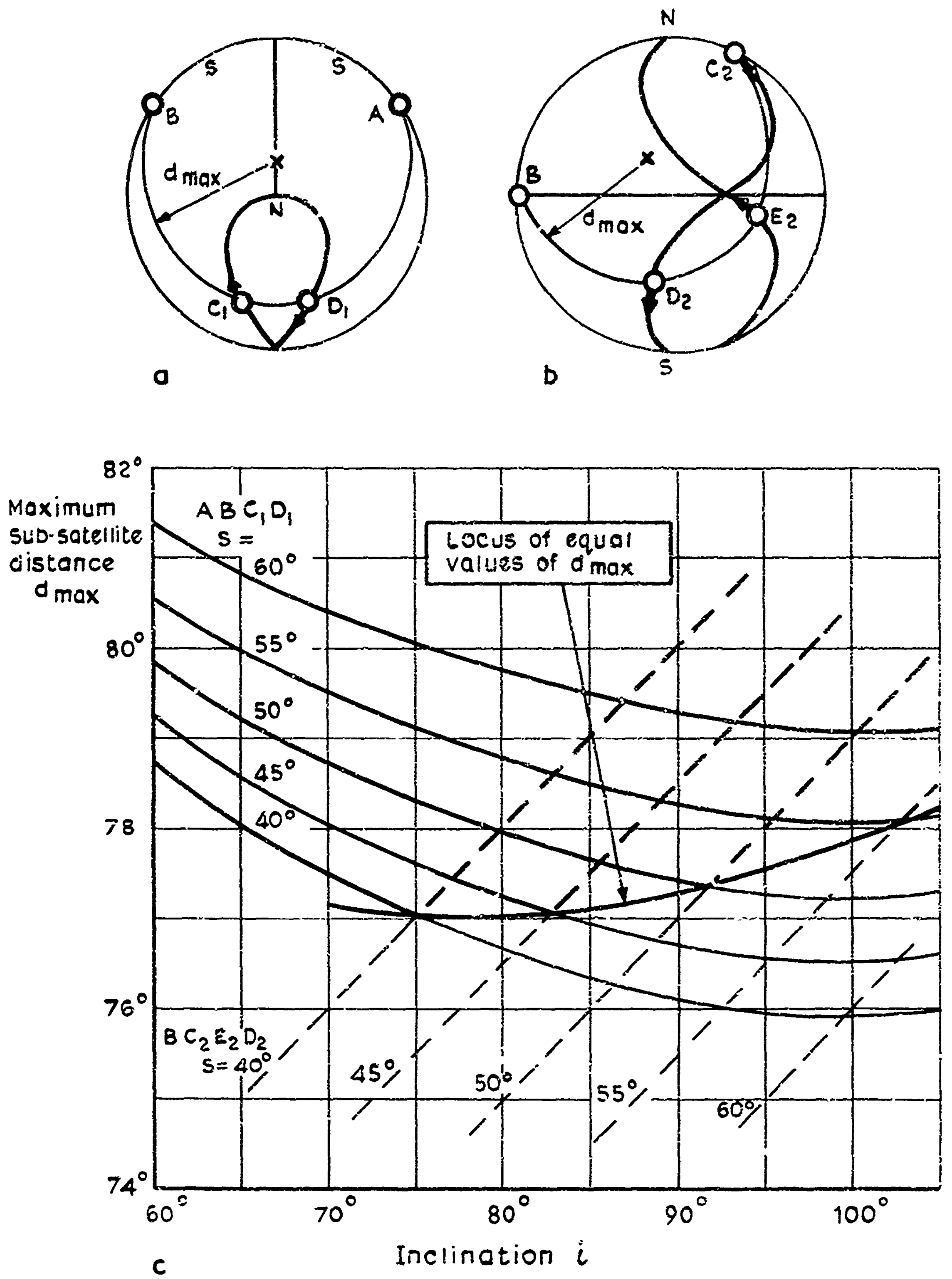


Fig. 3 a-c Coverage with two geostationary and three near-polar synchronous satellites

TR 70211

004 902393

Fig. 4

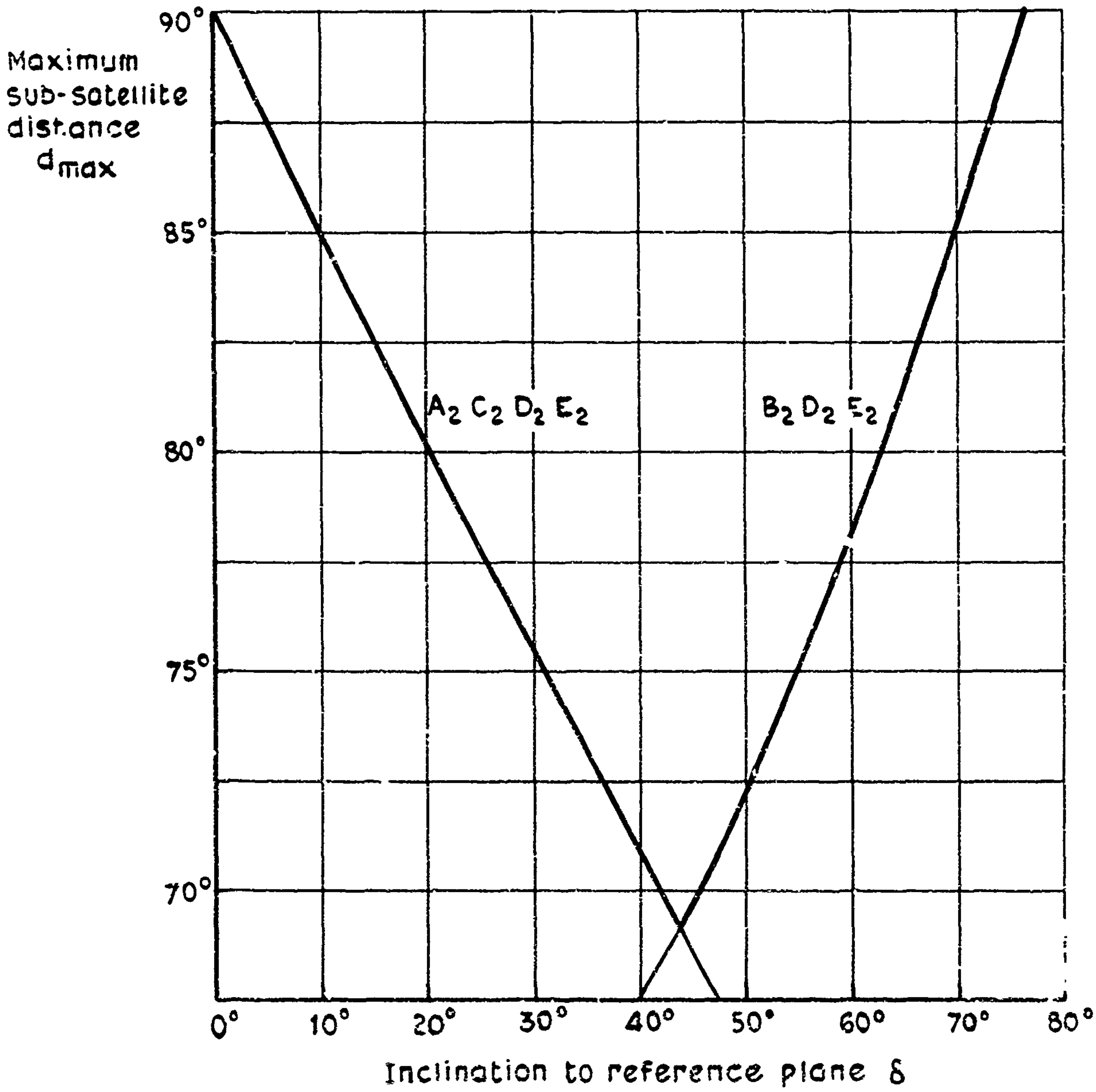


Fig. 4 Single coverage: 5 satellites in five planes

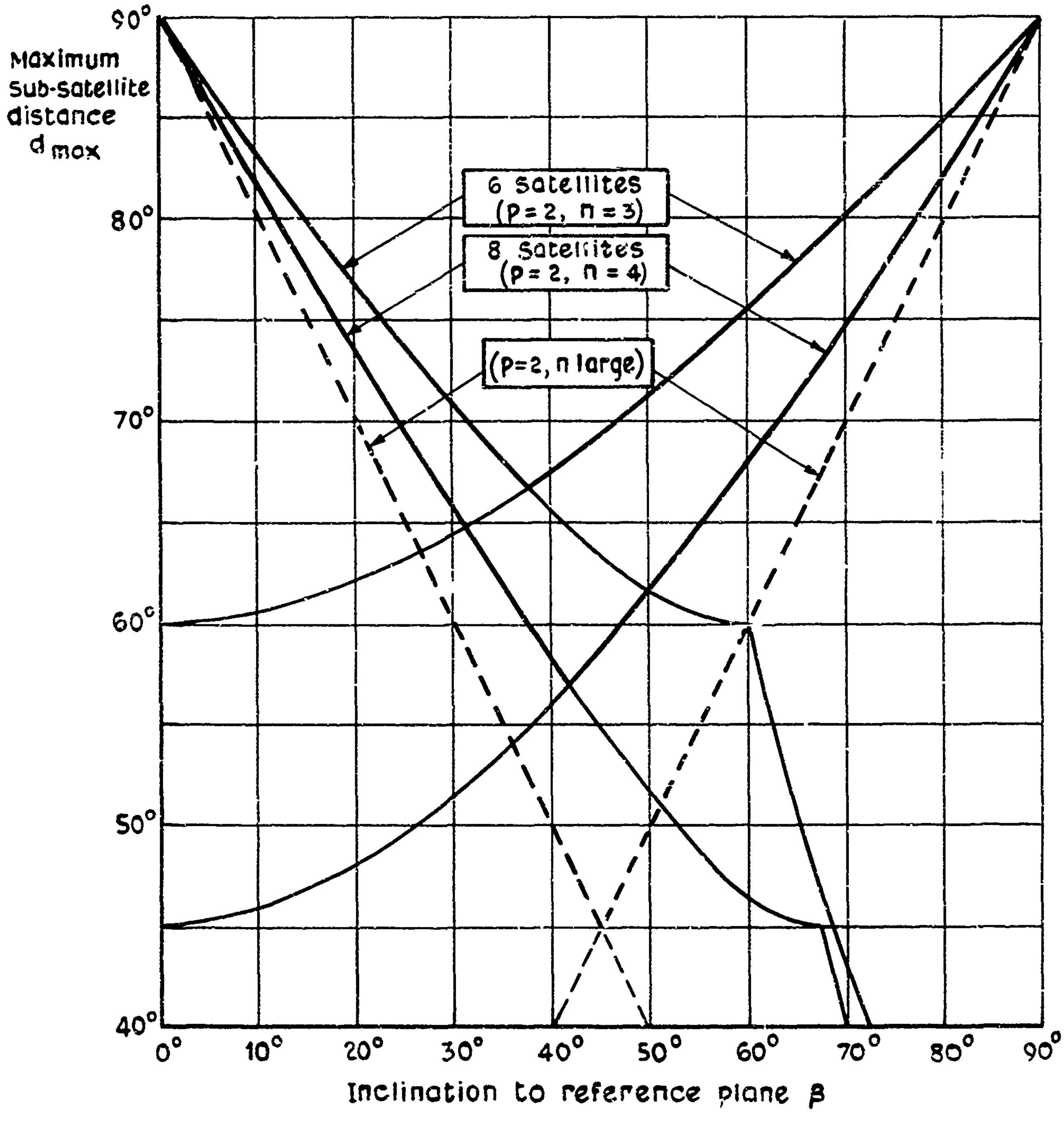


Fig.5 Single coverage: 6 or 8 satellites in two planes

Fig. 6 a-c

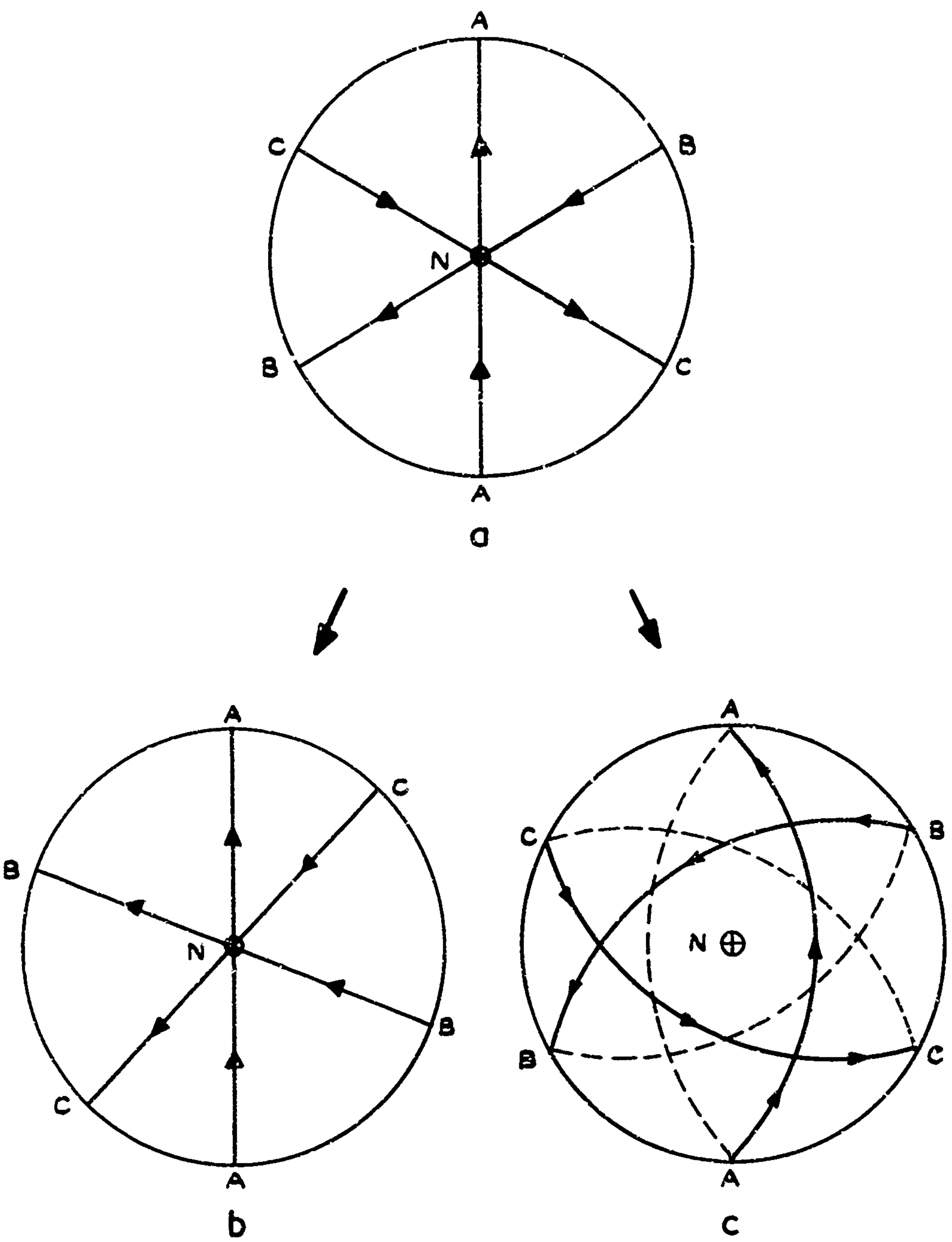
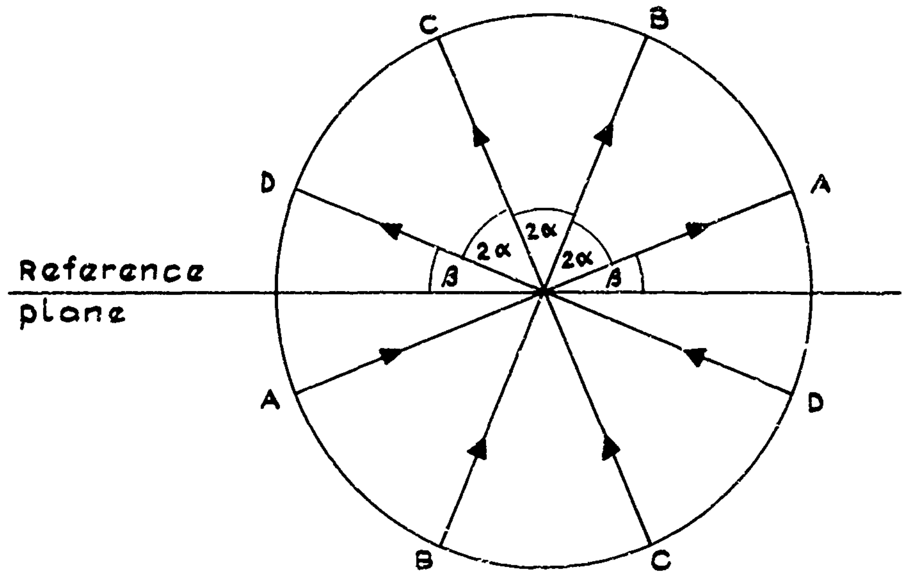
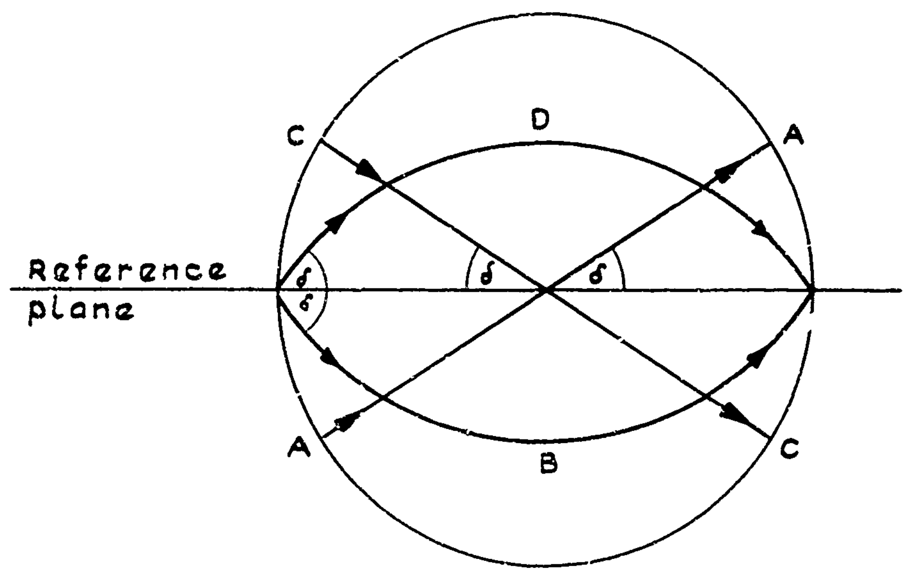


Fig. 6 a-c Multiple circular orbit patterns
(Three orbit planes)



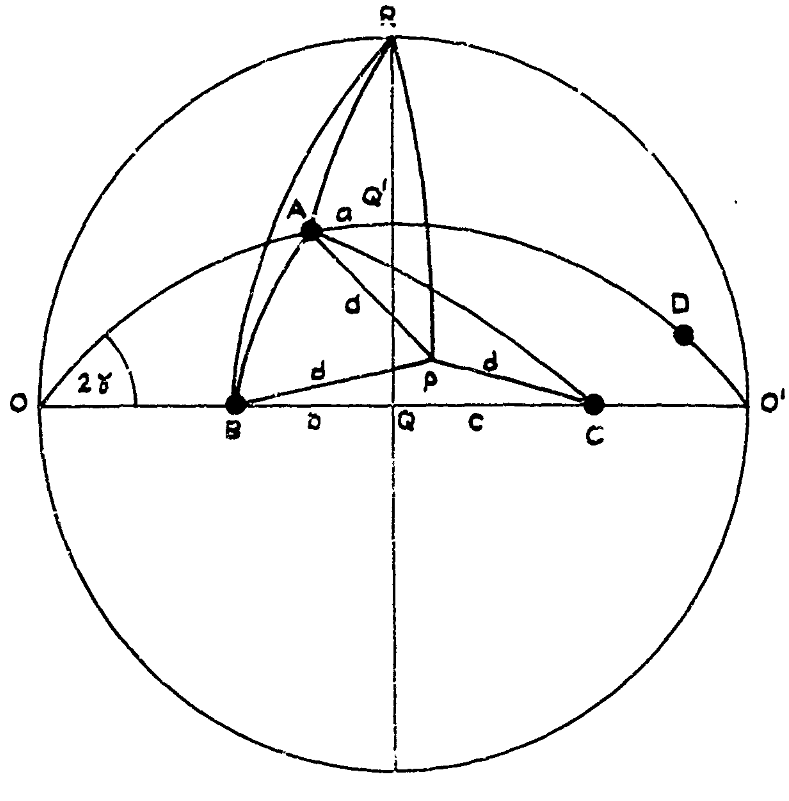
a Star pattern



b Delta pattern

Fig. 7 a&b Multiple circular orbit patterns
(Four orbit planes)

Fig. 8 a&b



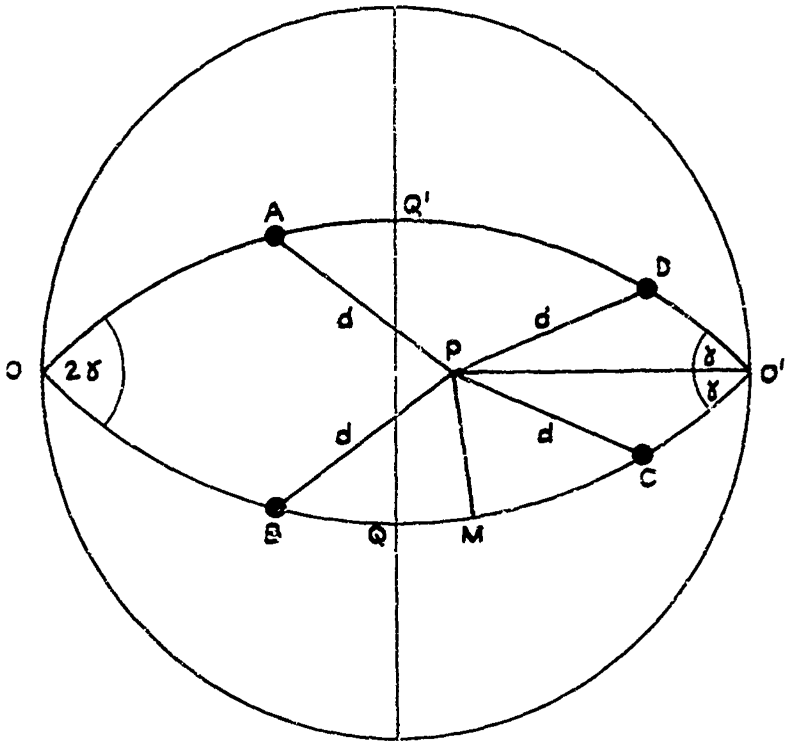


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Fig. 8 a&b Derivation of formulae for star pattern

004 902403

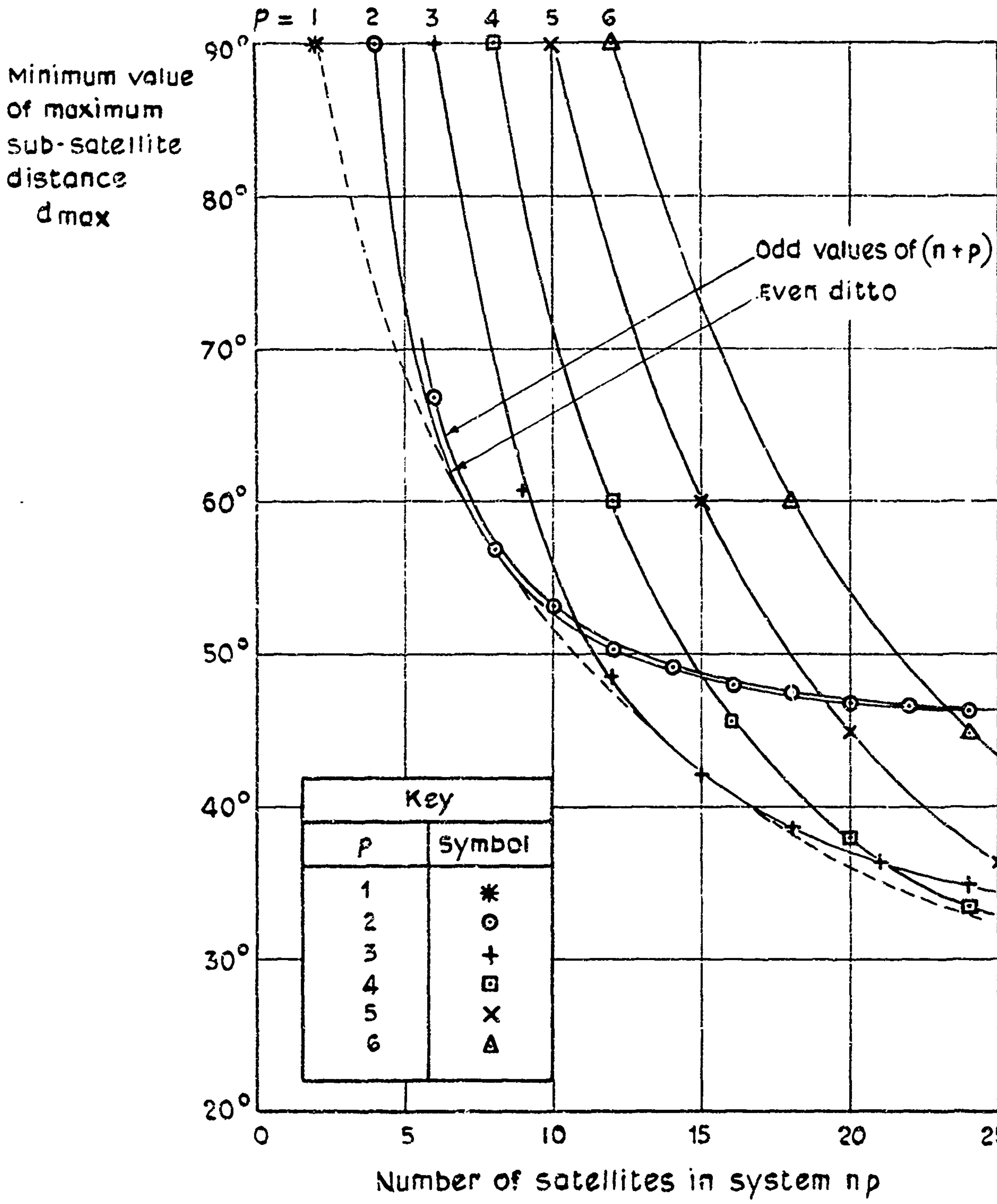


Fig. 9 Star pattern: effect of number of orbital planes

Fig. 10 a&b

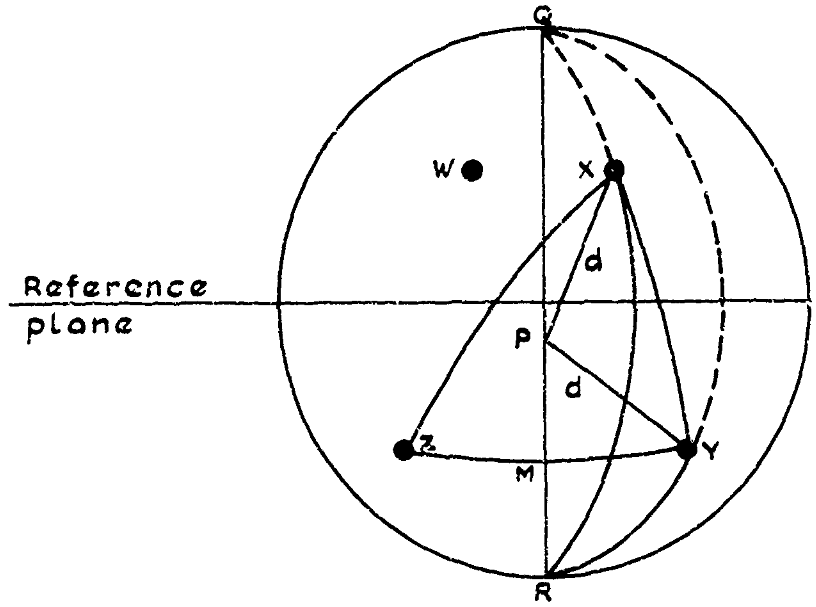


Fig. 10 a Delta pattern: critical conditions

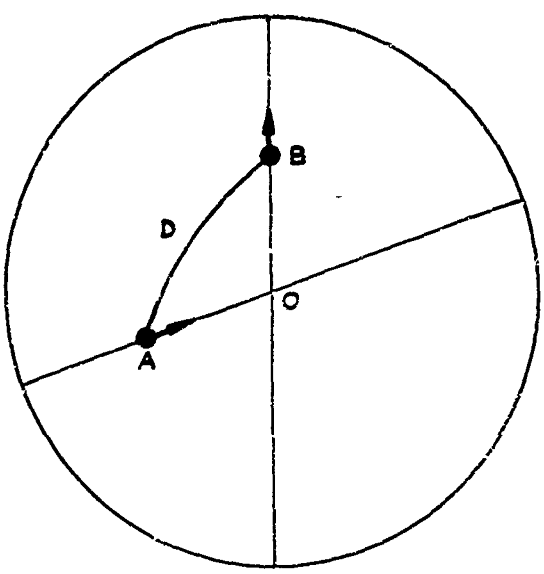


Fig. 10 b Star pattern: minimum satellite separation

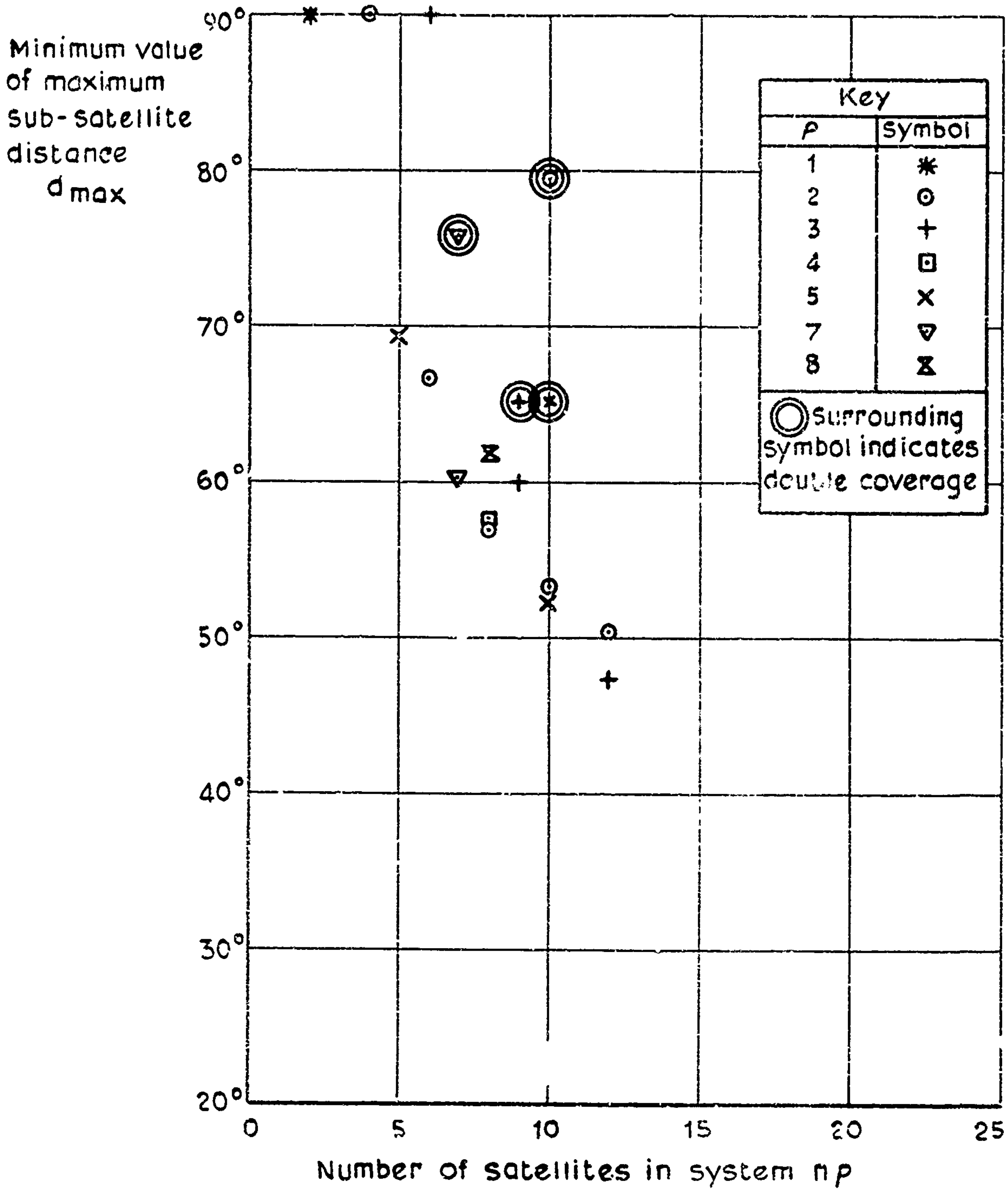


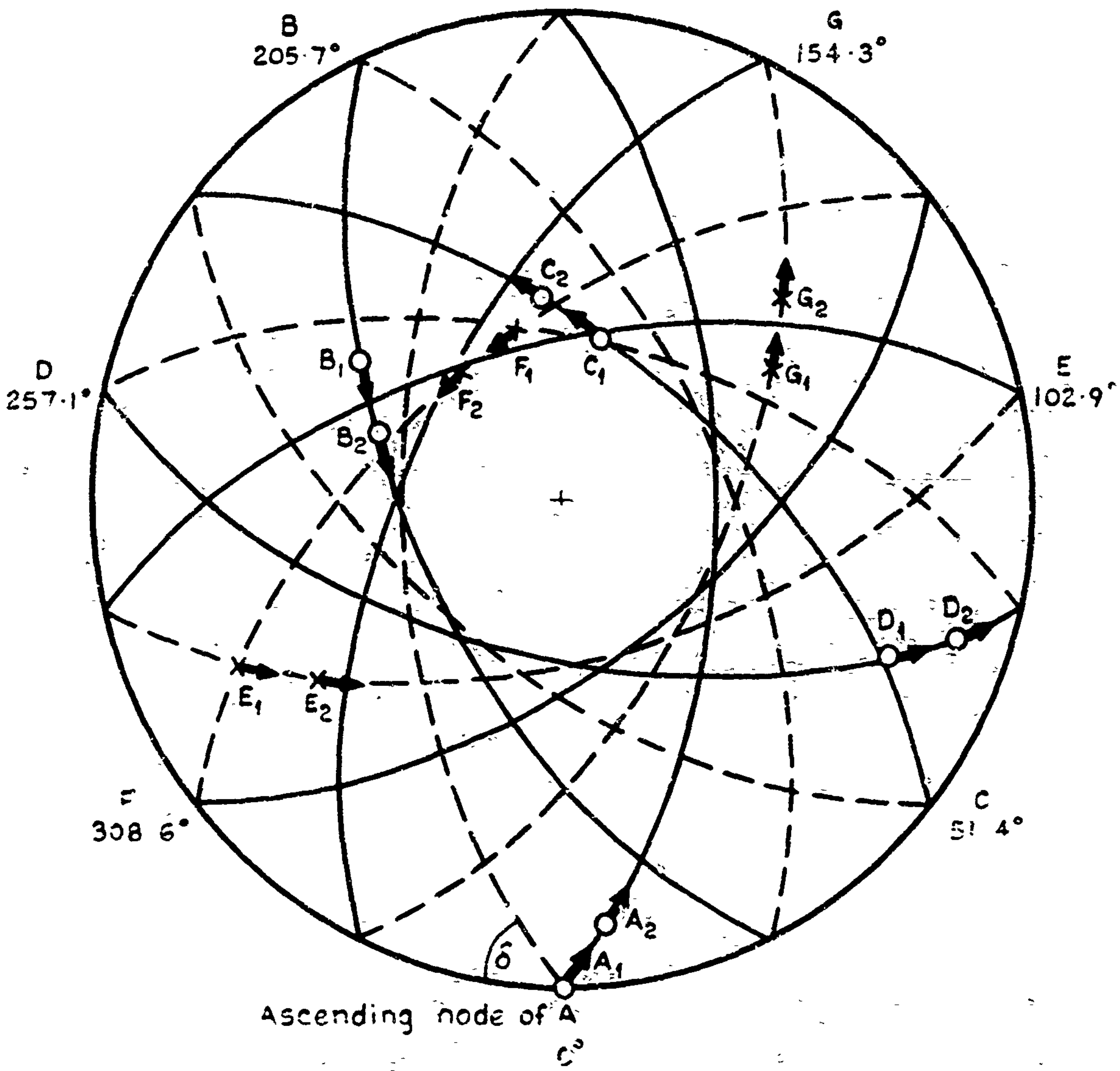
Fig.11 Delta pattern: examples

Fig.12

Satellite position and direction on orbit over nearer hemisphere

Satellite position and direction on orbit over further hemisphere

Conditions 1 and 2 occur $\frac{1}{28}$ period apart: similar conditions are then repeated 14 times each period



Zenithal equidistant projection

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Fig.12 7 satellite delta pattern (pattern d)

004 902418

Fig.13

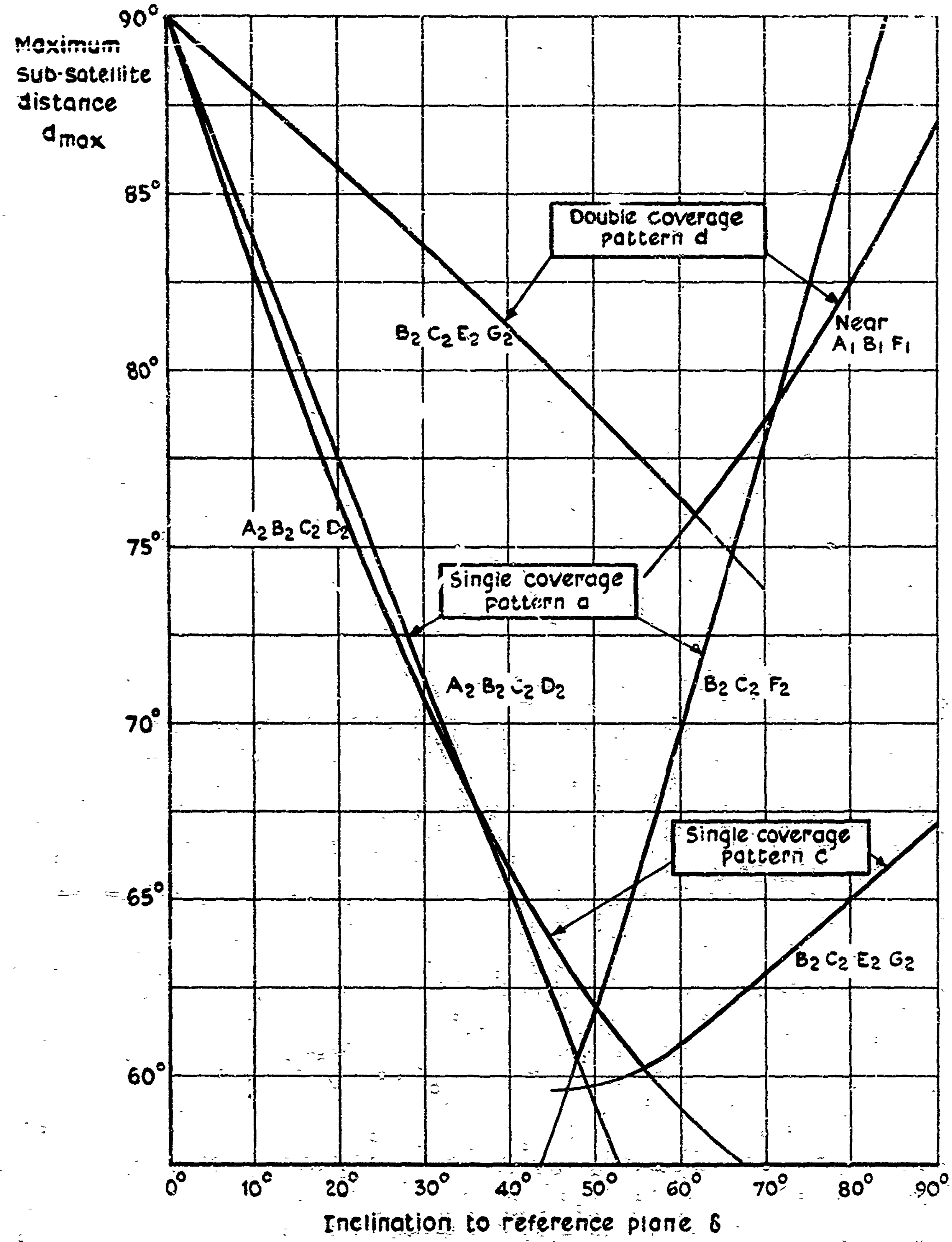


Fig.13 Single and double coverage: 7 satellite delta patterns

Fig.14

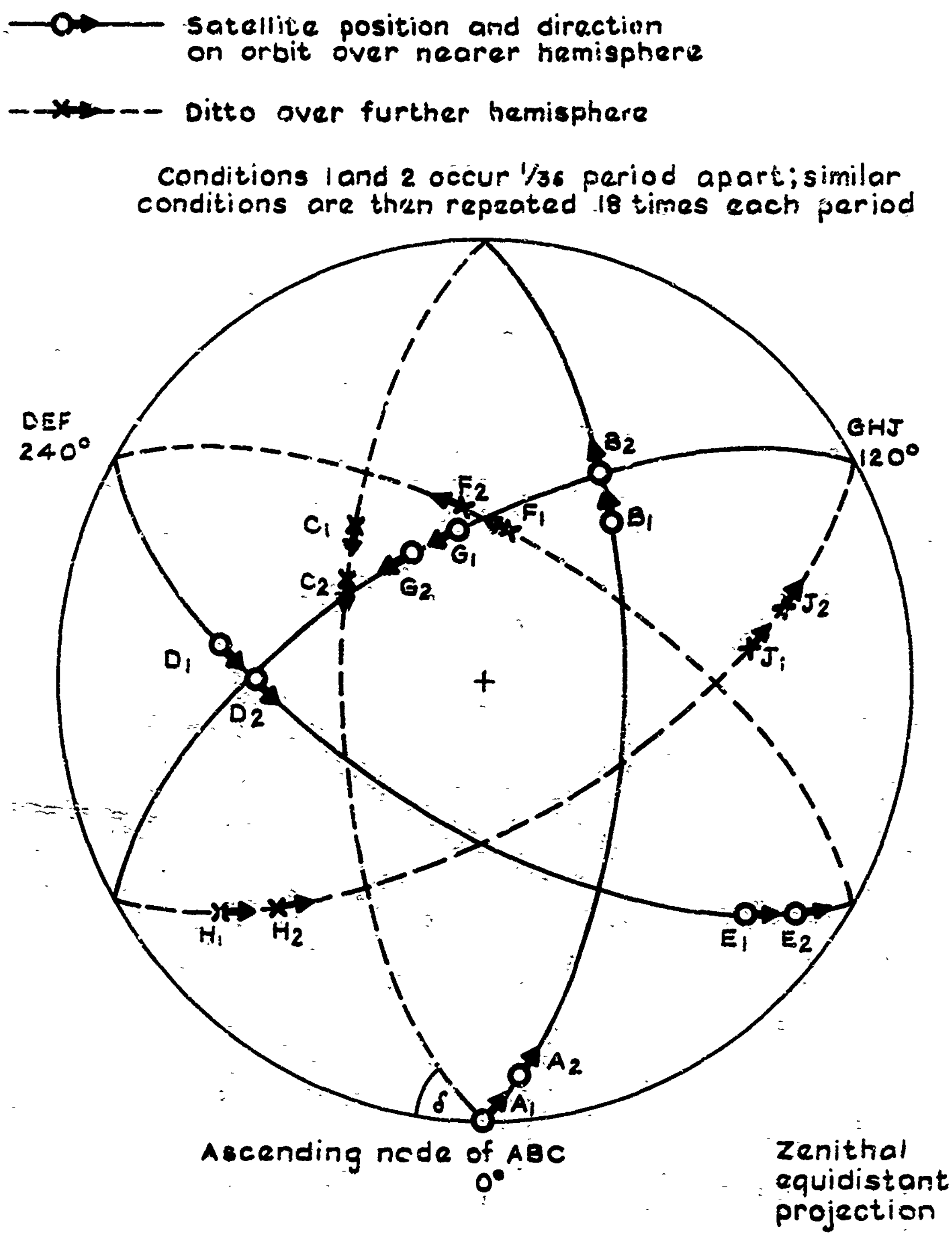


Fig. 14 9 satellite delta pattern
(3 planes, staggered phasing)

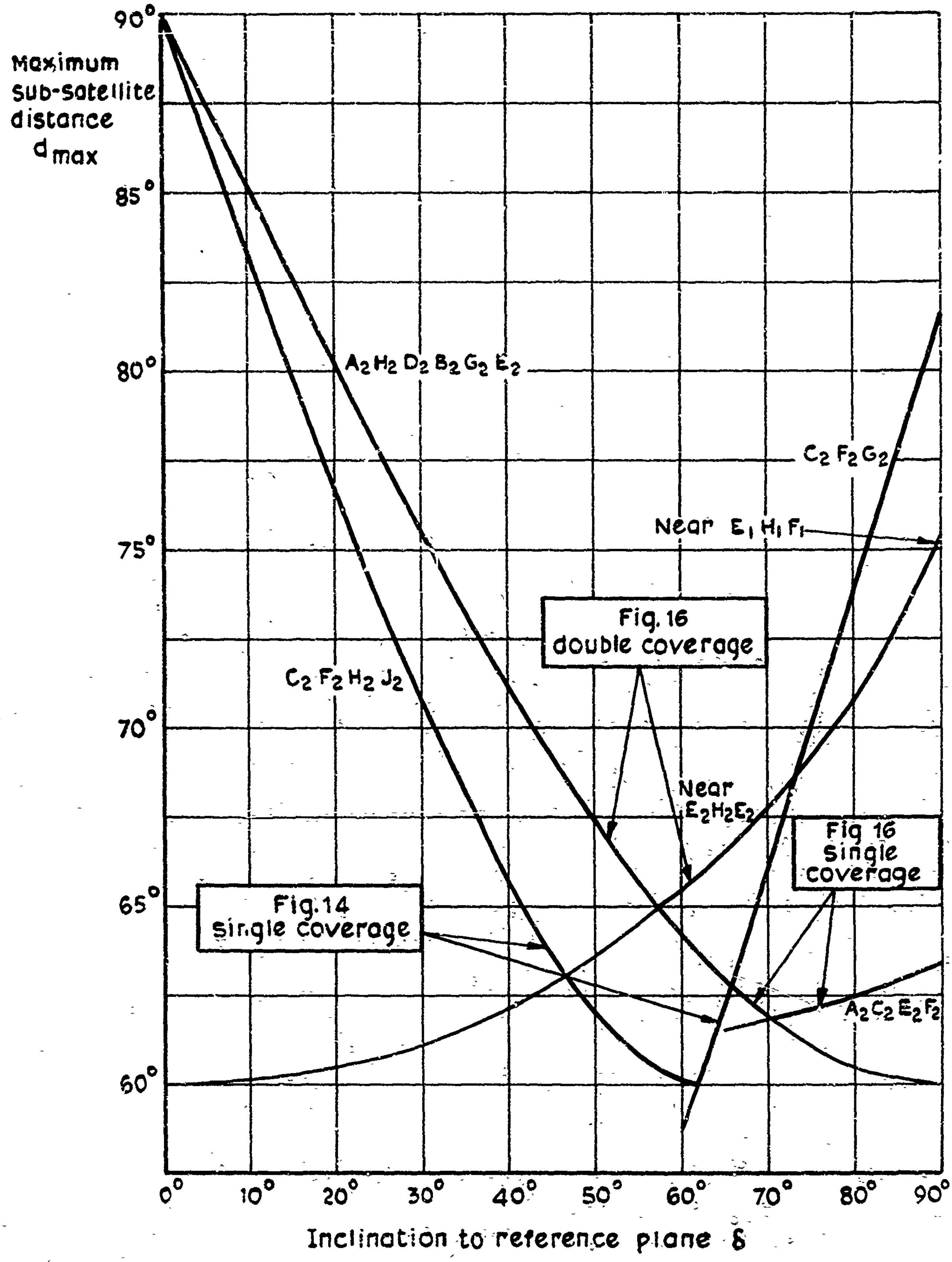


Fig.15 Single and double coverage: 9 satellite (3 plane) delta patterns

Fig.16

Satellite position and direction on orbit over nearer hemisphere

Ditto over further hemisphere

Conditions 1 and 2 occur 1/12 period apart; similar conditions are then repeated six times each period

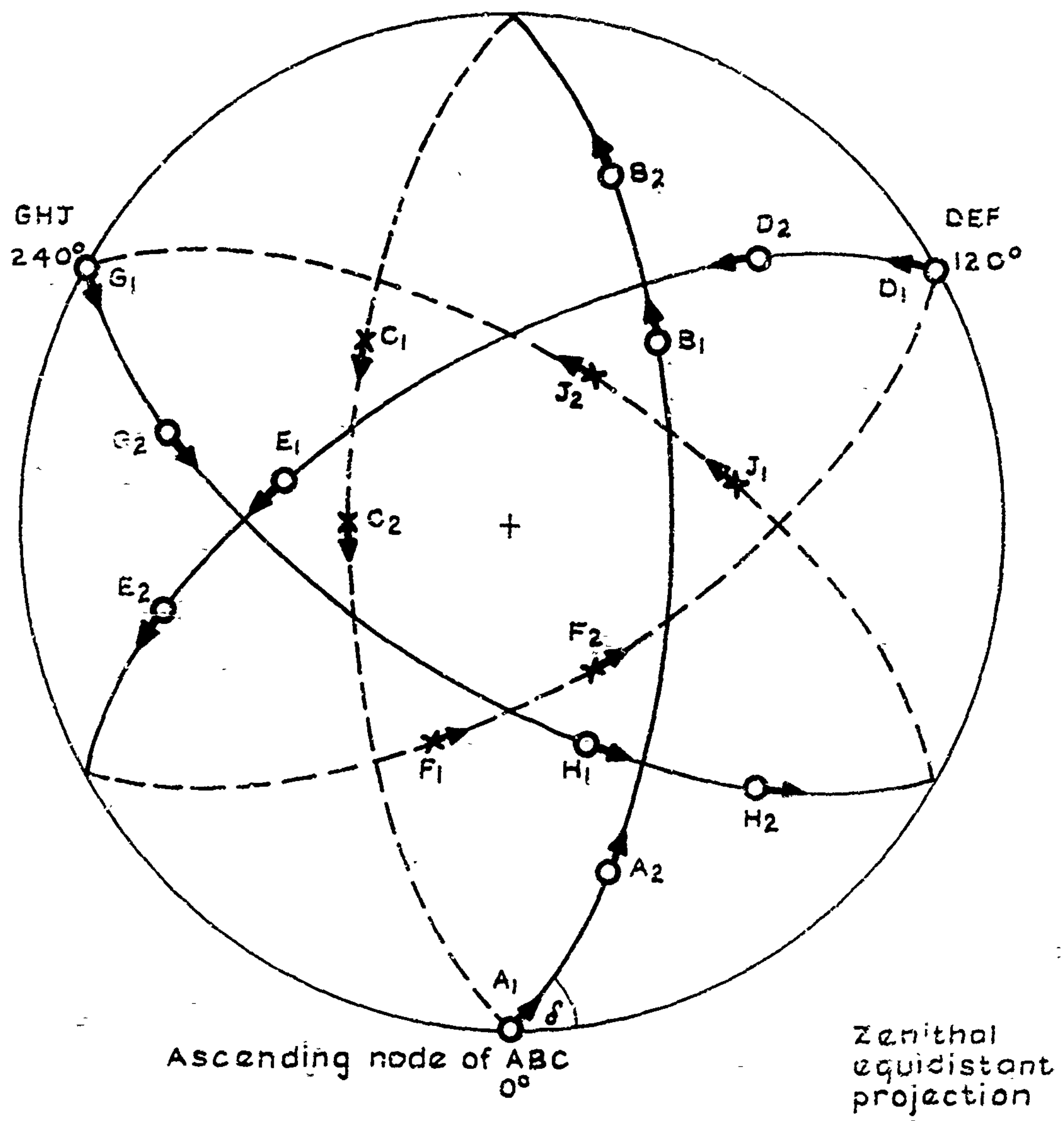


Fig. 16 9 satellite delta pattern (3 planes, similar phasing)

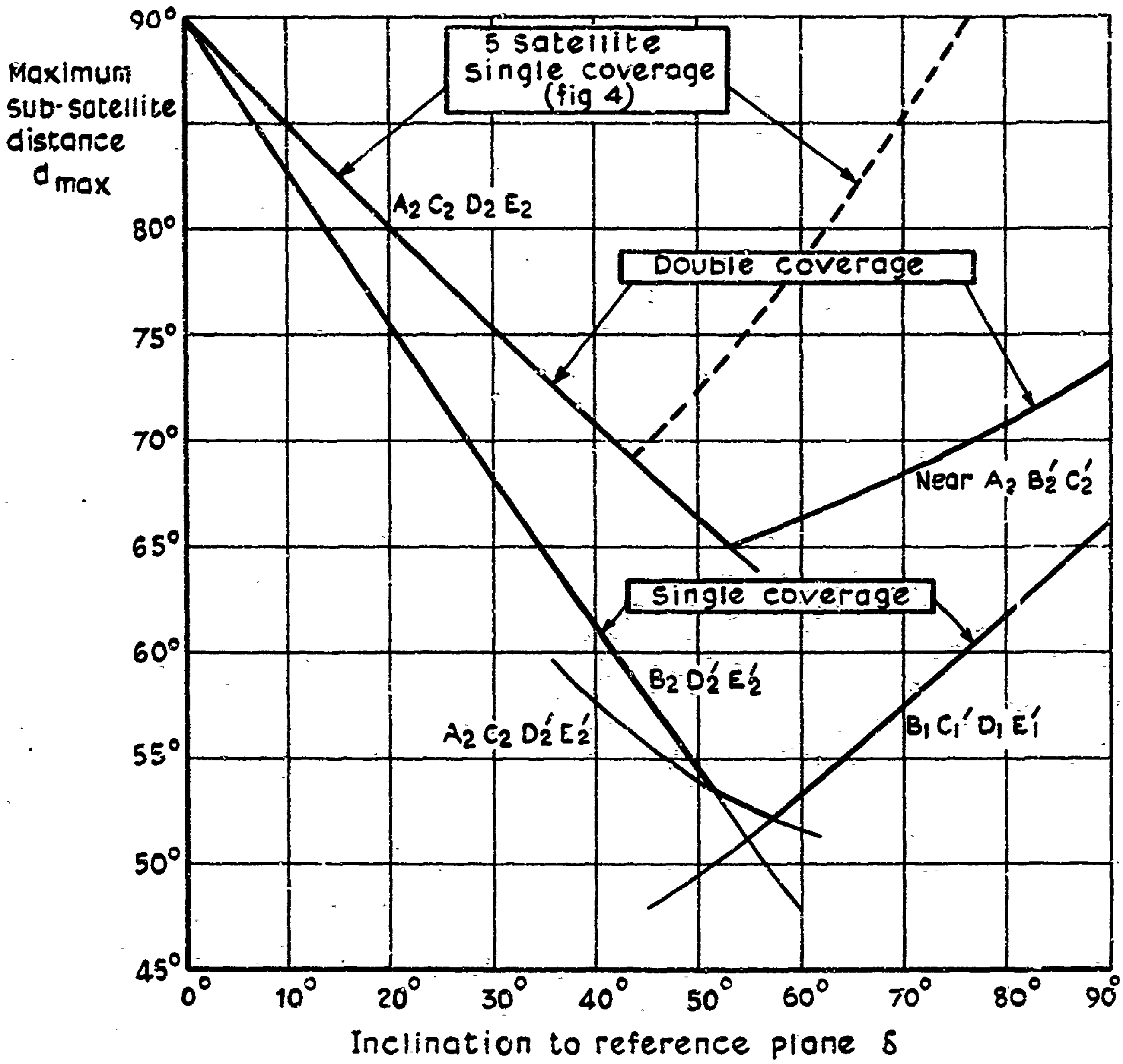


Fig.17 Single and double coverage: 10 satellite (5 plane) delta pattern

Fig.18

Satellite position and direction on orbit over nearer hemisphere

Ditto over further hemisphere

Conditions 1 and 2 occur $\frac{1}{24}$ period apart; similar conditions are then repeated twelve times each period

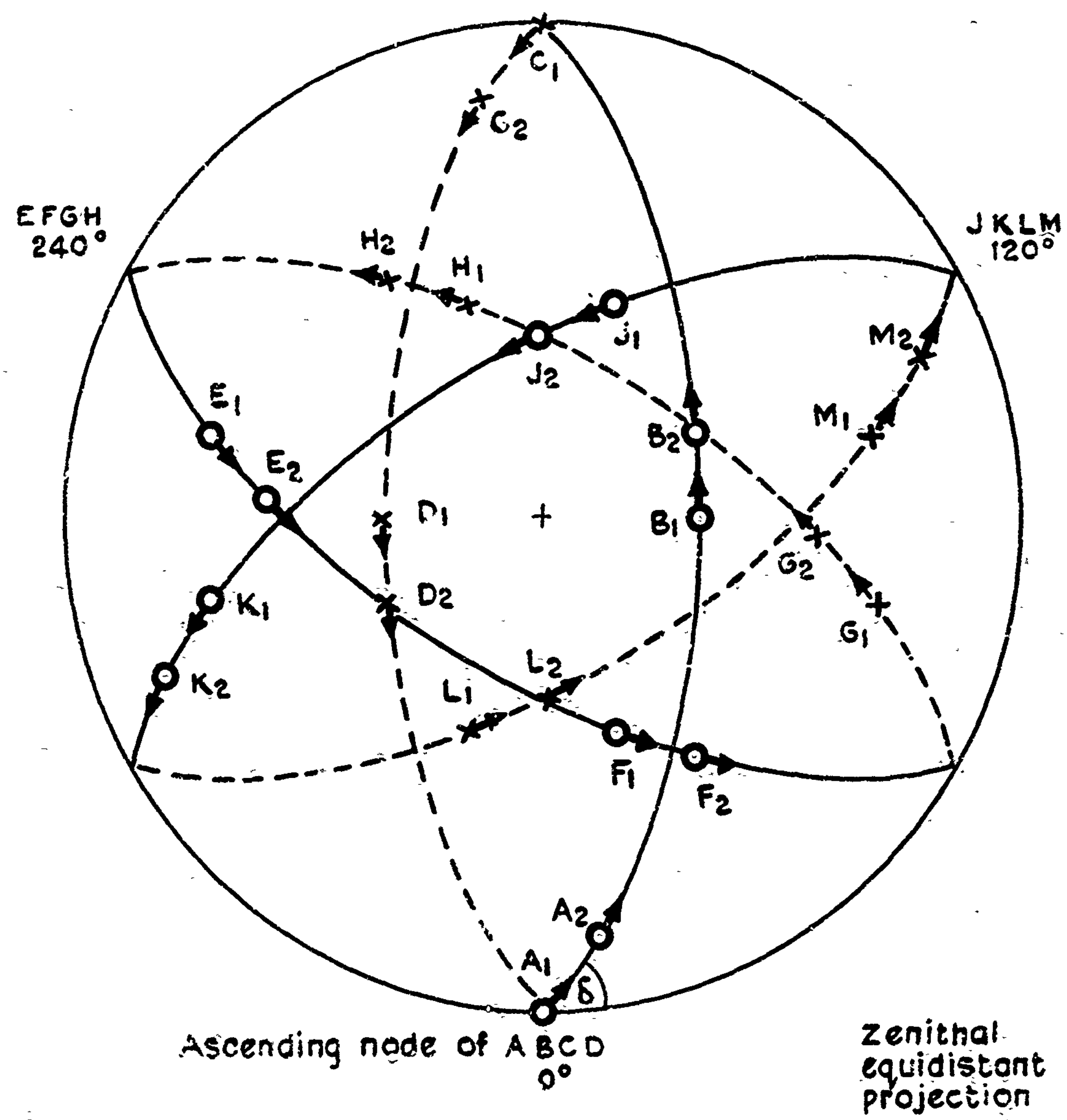


Fig.18 12 satellite delta pattern (3 planes)

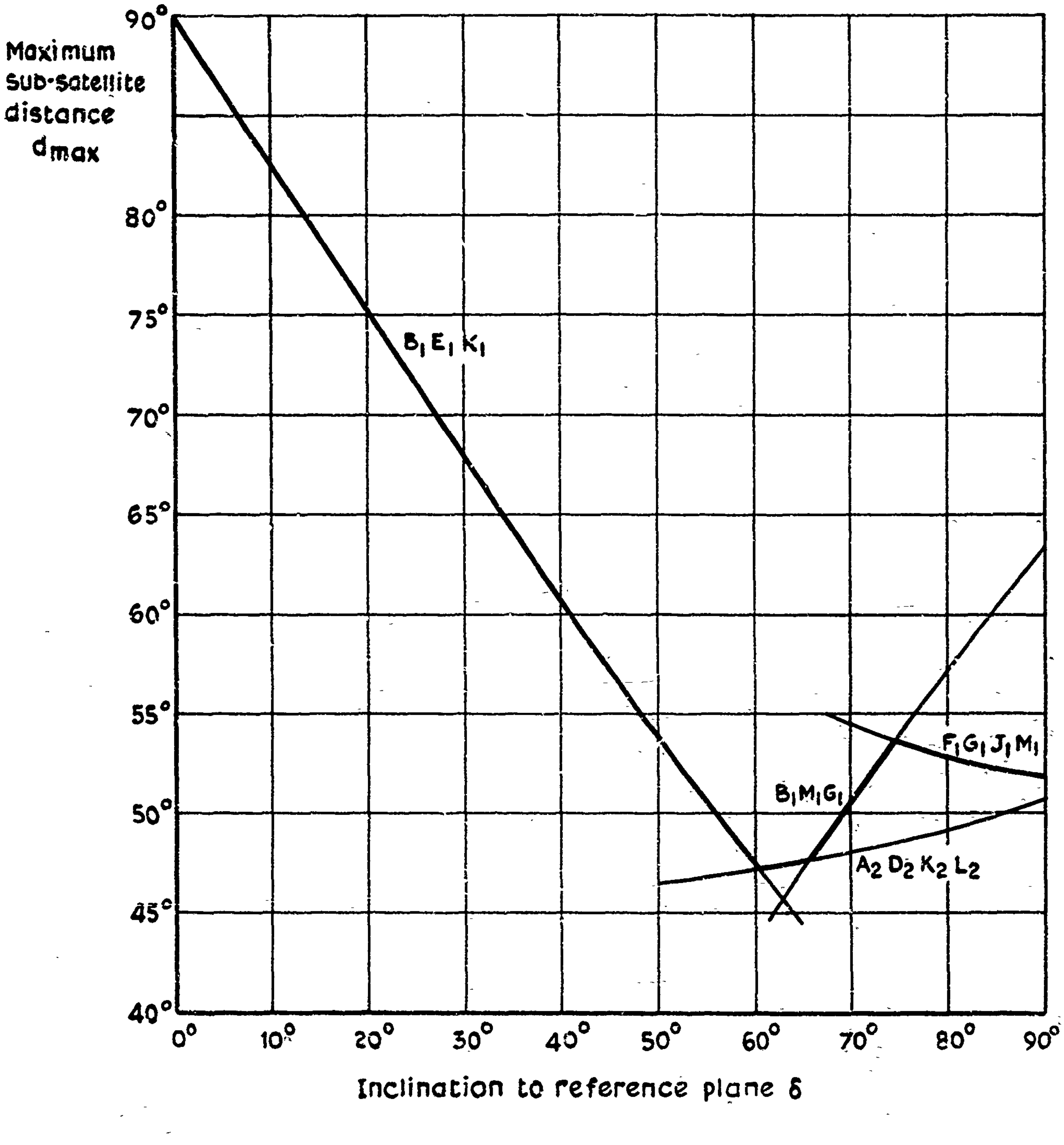


Fig.19 Single coverage: 12 satellite (3 plane) delta pattern

Fig.20

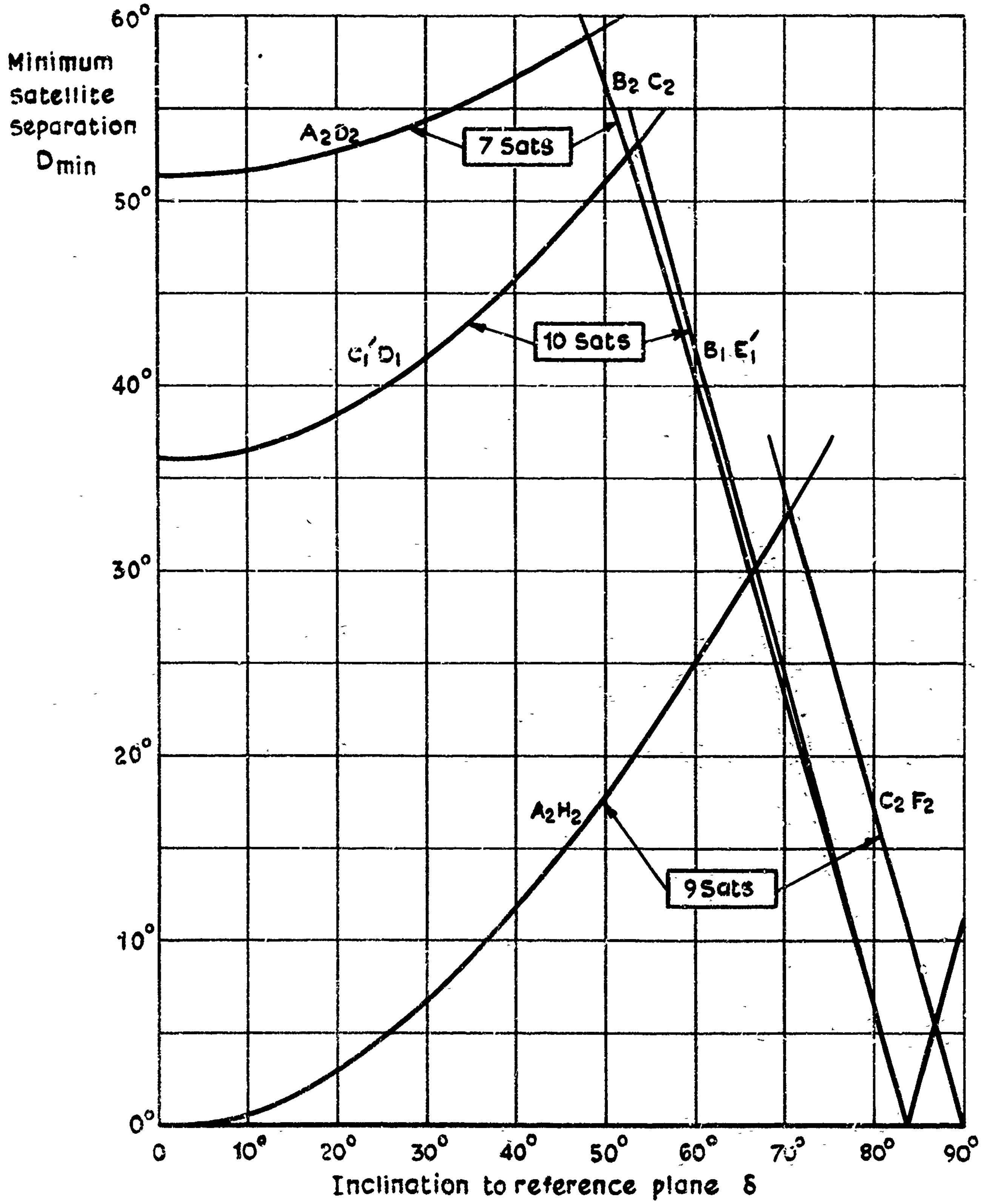
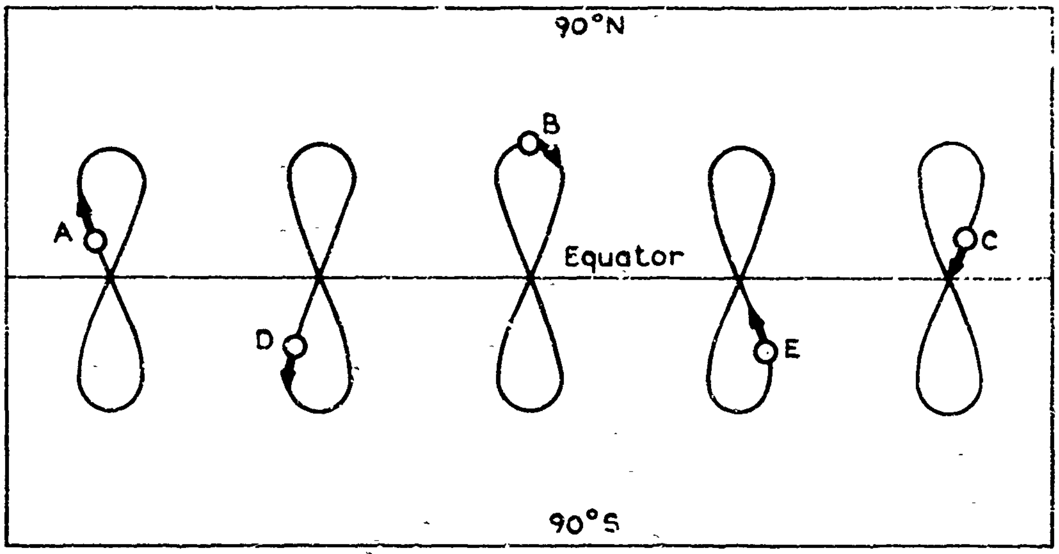
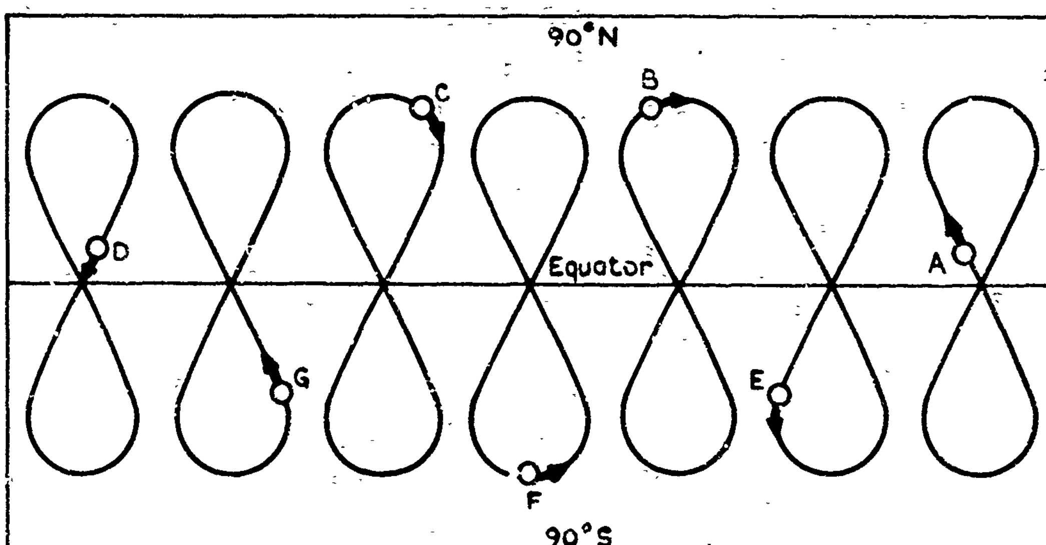


Fig. 20 Double coverage by delta patterns: minimum satellite separation



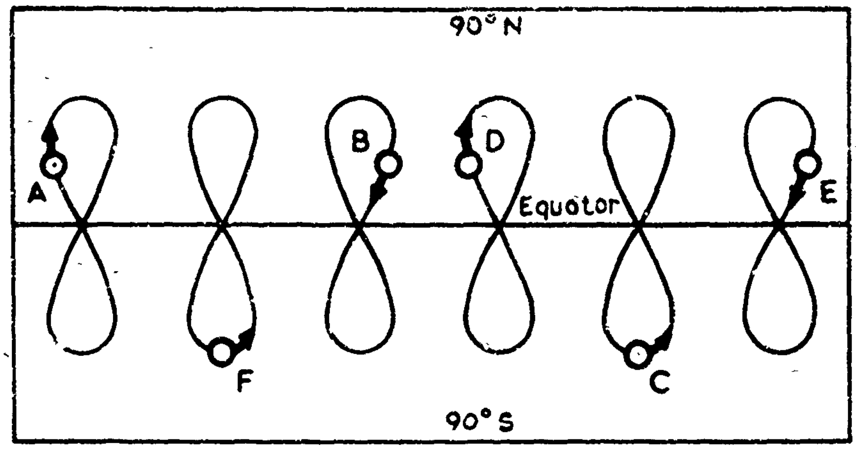
a 5 satellite synchronous system
(Five 24 hour orbits, all of 44° inclination)



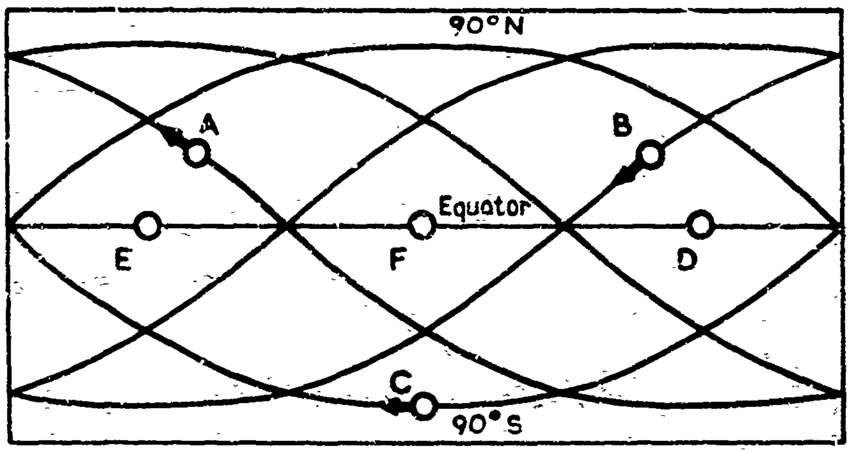
b 7 satellite synchronous system
(Seven 24 hour orbits, all of 62° inclination)

Fig.21 a&b Ground tracks: 5 and 7 satellite patterns
(Plate carrée projection)

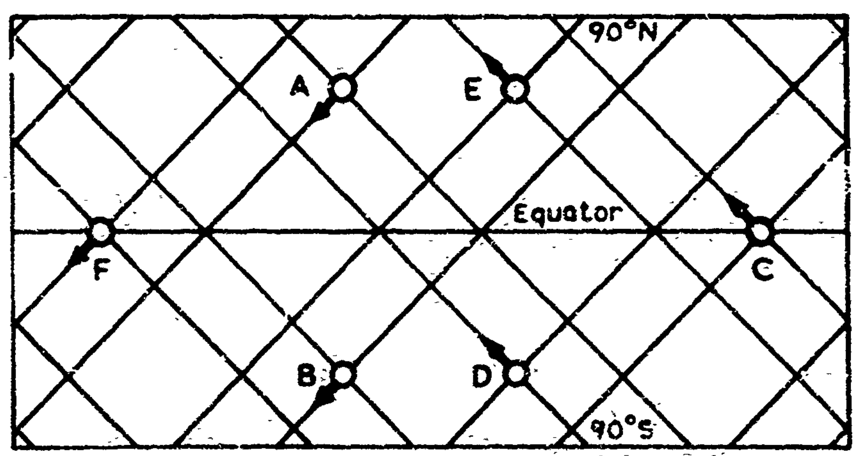
Fig. 22



a Two 24 hour orbits, both of 52° inclination



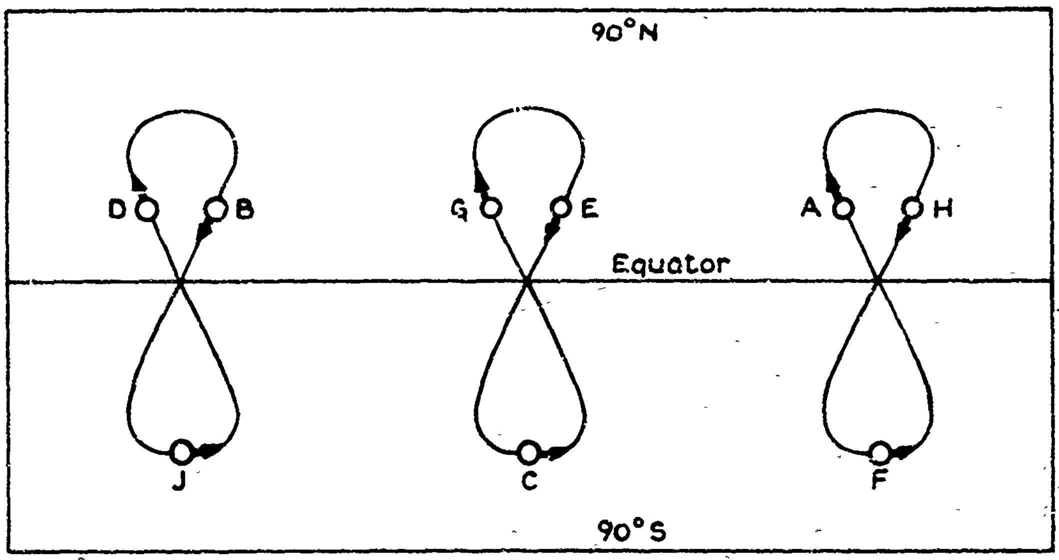
b Two 24 hour orbits, one equatorial and one of 104° inclination



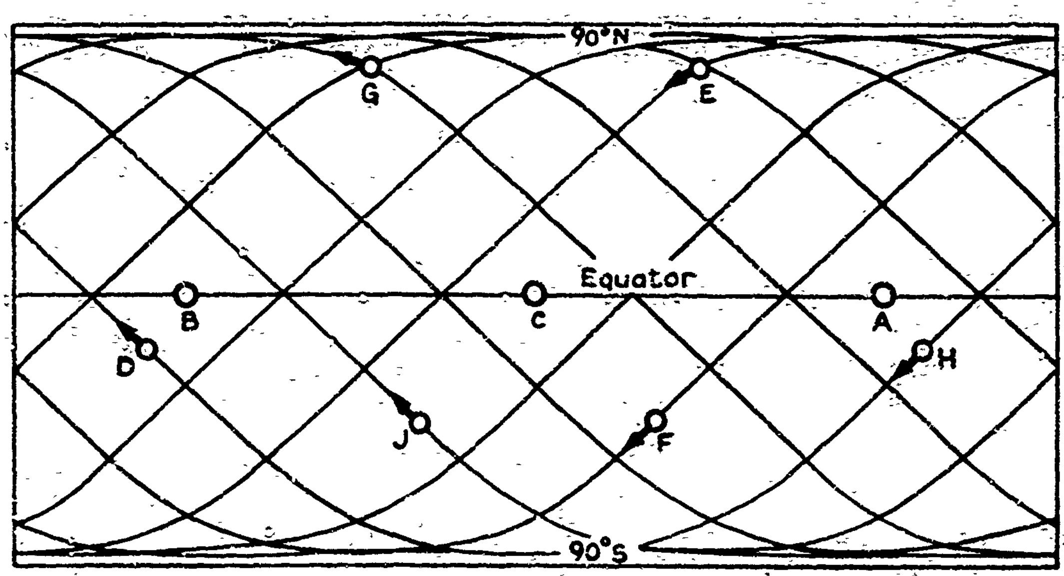
c Two 24 hour polar orbits with ascending nodes 104° apart

Fig 22 a-c Ground tracks: 6 satellite pattern (Plate carrée projection)

Fig. 23



a Three 24 hour orbits, all of 58° inclination



b Three 24 hour orbits, one equatorial and two of 94° inclination

Fig.23 a & b Ground tracks: 9 satellite pattern

(Plate carrée projection)

Fig. 24

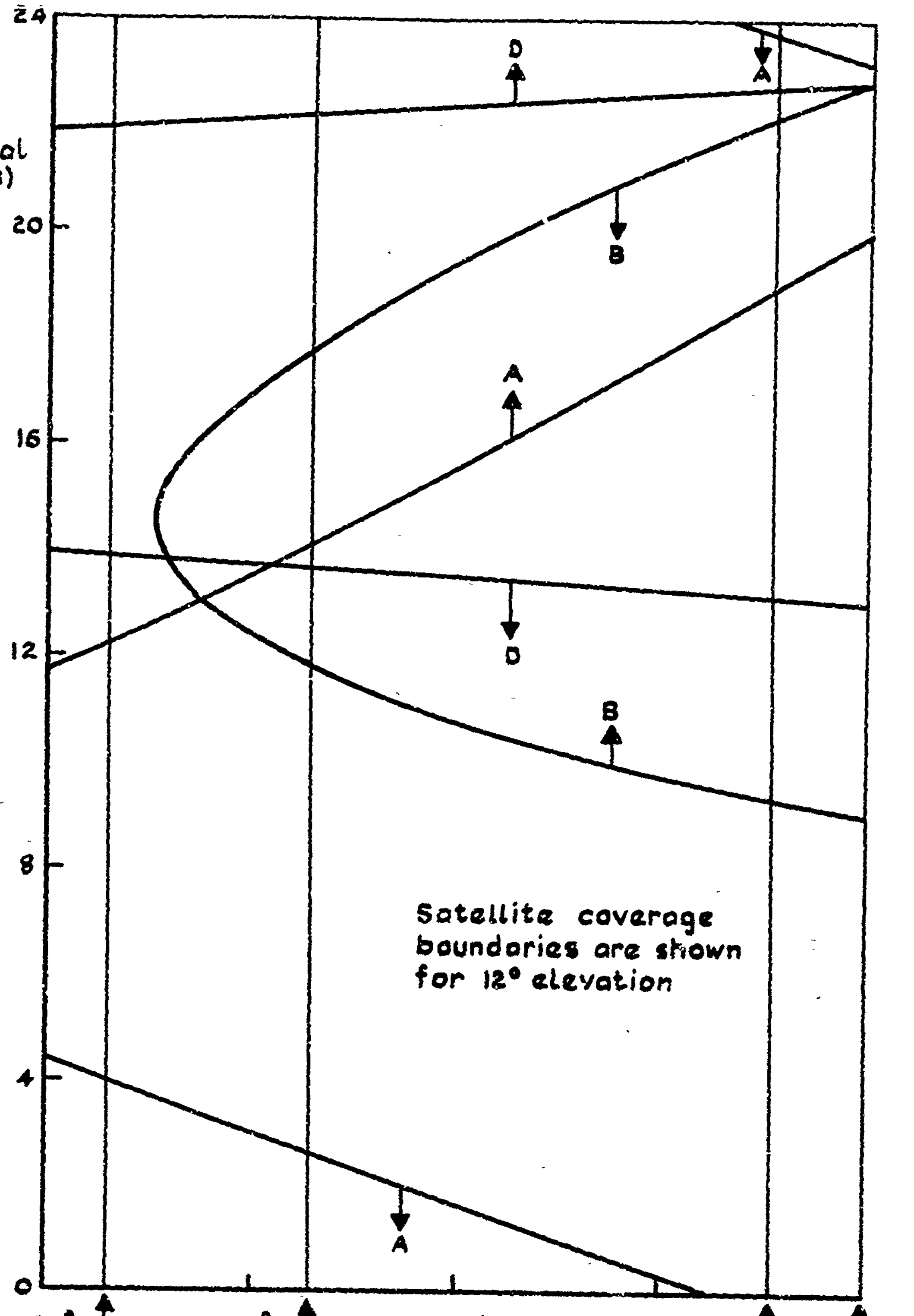


Fig. 24 Coverage of North Atlantic route by three out of five synchronous satellites