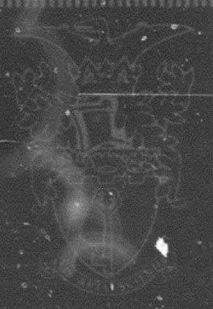


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THE ORBIT OF ARIEL 3

(1967-42A)

by

R. H. Gooding



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SUMMARY

The definitive orbit for Ariel 3 has been computed, from Minitrack observations, for a period of $27\frac{1}{2}$ months from the launch of the satellite. The orbit was represented by a model with seven independent orbital parameters and the values of these parameters were determined, and are listed, at three-day intervals. Typical accuracies are 10^{-5} in eccentricity and $4''$ in angular parameters, that is, about $\frac{1}{2}$ km in position.

A curious feature of the secular variation of orbital inclination, viz. that the expected decrease of about 0.02° appeared to occur over a three-month period instead of the full $27\frac{1}{2}$ -month period, is discussed but has not been explained.

Departmental Reference: Space 327

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1 INTRODUCTION

Ariel 3, the third of the series of satellites being launched in the scientific programme of Anglo-American co-operation, was the first spacecraft to be built entirely in Britain. The five experiments in the payload were concerned¹ with electron density and temperature (University of Birmingham), VLF radio waves (University of Sheffield), cosmic radio noise (Nuffield Radio Astronomy Laboratory, Jodrell Bank), molecular oxygen (Meteorological Office) and terrestrial noise sources (Radio and Space Research Station).

Known as S53 or UK3 before launch, and as Ariel 3 or 1967-42A afterwards, the satellite was placed in a near-polar, near-circular orbit at 18^h UT on 5 May 1967 by a Scout rocket launched from the Western Test Range, California.

The definitive orbit of Ariel 3, as for Ariel 2², has been derived at R.A.E. from Minitrack (interferometer) data provided by NASA. Orbital parameters have been obtained at three-day intervals by the use of the new computer program PROP³. They are tabulated, for the first 27½ months of the satellite's lifetime, in this Report. For epochs up to 1968 JAN 16 the work was done on London University's Atlas computer and for subsequent epochs it was done on the ICL 1907 computer at R.A.E.

2 OBSERVATIONS

The STADAN Minitrack network now consists of ten stations - five of the original twelve stations² have disappeared and there are three new ones. These are listed in Table 1, with their assumed positions in standard geocentric co-ordinates (x-axis towards the Greenwich meridian).

Observations consist of pairs of direction cosines. Their a priori accuracy (s.d.) has been taken, as usual^{3,4}, at 0.00029, equivalent to 1' in angular measure, though their true accuracy is believed to be worse than this.

Times are given in the UTC system, i.e. the system defined by WWV time transmissions from America, and have not been corrected during the orbital determination, except that the times of observations made on 1 February 1968 and used at epoch 1968 JAN 31 had to be reduced by 0.1^s to allow for the step advance of UTC at Feb 1.0. Times should be accurate to about 1 ms; no allowance for timing error was made by PROP.

In total, about 10000 Minitrack observations were used, covering the period from launch until about a week before the satellite's transmitter was switched off (1 September 1969); i.e. these were about 12 per day. They were received from NASA on punched cards, suitable for direct input to PROP. The epochs for orbit determination were taken at three-day intervals, and always at midnights (unlike the Ariel 2 epochs which were at ascending nodes). The orbit determination at each epoch used observations over a four-day period, allowing one-day overlaps in the periods of validity of the resulting orbital parameters, but observations were not (in general) used twice; observations on an 'overlap day' were divided into two sets, by alternate allocation, for use with the epochs before and after the overlap day. About 650 of the observations were rejected during analysis, but this includes (a) a rather high rejection rate during the first 3½ months - all the observations from Orroval were being rejected at one stage - and (b) nearly 100 observations, over a period of a month (Jan-Feb, 1968), which all had a one wavelength error in the north-south direction cosine due to a temporary error in the NASA program⁵ for processing the raw data; the normal rejection rate was about 4½%.

The number of Minitrack observations per day varied, of course, but there was at least one on every day of the period covered, apart from the week 28 November-4 December 1967, for which there was a complete absence of data.

A few observations of Ariel 3 were made by the Hewitt camera⁶ at Malvern. Among these, 8 observations came from a pass on the evening of 10 April 1969 and 12 observations came from two passes close to midnight on 19 June 1969 and 21 June 1969. It was decided to incorporate these 20 observations into re-runs of the orbit determinations at the appropriate two epochs, to see how fit and accuracy were affected. The remaining Hewitt camera observations, and the many visual observations of Ariel 3, have not been used.

3 ORBITAL MODEL

The orbital model of the program PROP is not the same as that used in the analysis of the orbit of Ariel 2. Eccentricity, inclinations etc. are defined slightly differently in the two programs, and the connecting relations are given in Appendix C of Ref.3.

The model allows some choice as to the set of orbital parameters which represent the orbit and which are determined from fitting to observations. The set chosen for the Ariel 3 orbit contained seven parameters, viz, e_0 (eccentricity), i_0 (inclination), Ω_0 (right ascension of the node), ω_0 (argument of perigee), M_0 (mean anomaly), M_1 (mean motion) and M_2 (half the mean acceleration). The first four parameters are epoch values of mean elements, as defined in Refs. 7 and 8 and the last three are the coefficients in the polynomial representation of (mean) mean anomaly:

$$M = M_0 + M_1 t + M_2 t^2 ,$$

where t is measured from epoch.

Secular rates of change of e , i , Ω and ω (i.e. the polynomial coefficients e_1 , i_1 , Ω_1 and ω_1) were computed, inside PROP at the beginning of each iteration of the differential-correction process, as functions of the seven independent parameters^{3,8}. These quantities, together with the long-periodic and short-periodic terms computed at each observation time, represented orbit perturbations due to drag and to the earth's zonal harmonics up to J_9 . The along-track effect of the tesseral harmonic $J_{2,2}$ was represented as usual^{3,8} using the value 1.8×10^{-6} , but, apart from this, tesseral harmonics were neglected. Luni-solar perturbations were ignored; their effect on Ariel 3 over a period of two or three days from any epoch is very small.

The decision not to have an eighth parameter M_3 , which would have made the M polynomial a cubic, was justified by some test runs, early in the lifetime, which showed that no significant improvement in fit would result and that the value of M_3 itself would not be significant. For the two epochs during the week of missing data, however, M_3 was included in the model and a reasonable fit thereby obtained to data before and after the gap, covering a period of $10\frac{1}{2}$ days. In retrospect, the decision to omit M_3 is open to question because drag increased fairly steadily through the $27\frac{1}{2}$ months considered (as the values of M_2 in Table 2 show), the maximum effects being at the end of March 1969; repetition of two of the runs, with M_3 included, showed that at this stage significant improvement in fit would be obtained (see section 4).

Apart from the omission of M_3 during periods of high drag, the main limitation of the orbital model is in the neglect of important tesseral-harmonic perturbations and, in particular, of the perturbation in inclination due to $J_{2,2}$. This perturbation has a period of just under 12 hours and an amplitude of about $0^{\circ}.002$, equivalent to a maximum position error of about $\frac{1}{2}$ km.

4 RESULTS

4.1 Main results

The orbital parameters obtained from the orbit determinations, together with certain additional information, are listed in Table 2. Successive columns of the table provide the following quantities, zero suffixes being omitted from a_0 etc:-

Epoch date (0^h UTC understood).

Semi-major axis, a (km).

Eccentricity, e .

Perigee height, h_p (km).

Inclination, i (degrees).

Right ascension of the node, Ω (degrees).

Argument of perigee, ω (degrees).

Mean argument of latitude, $M_0 + \omega$ (degrees).

Mean motion, M_1 (degrees/day).

Half acceleration, M_2 (degrees/day²).

Number of observations used, N .

Number of observations rejected, K .

Extent of the observations, D (days).

Standard deviation of an observation of unit weight, σ .

Modified Julian Day number of epoch date, MJD.

The orbital parameters are the seven quantities $e, i, \Omega, \omega, M_0, M_1$ and M_2 , but $M_0 + \omega$ is given instead of M_0 because of the high correlation between M_0 and ω . This correlation arises directly from the fact that the orbit is so nearly circular, and Ref.3 may be consulted for further explanation. (The appropriate value of the control parameter JELTYP³ was used to give the variance of $M_0 + \omega$ directly.)

The semi-major axis, a , is the mean element, as used by Merson⁷, defined from M_1 by

$$a = (\mu/M_1)^{1/3} - \frac{1}{2} J_2 R^2 (\mu/M_1^2)^{-1/3} (2 - 3 \sin^2 i) (1 - e^2)^{-1/2},$$

where μ is the earth's gravitational constant, J_2 is its second zonal harmonic coefficient and R is its mean equatorial radius, the values from Ref.3 being used.

The perigee height is given by

$$h_p = r_p - R_p,$$

where*

$$r_p = a(1 - e) + \frac{J_2 R^2}{4a(1 - e^2)} \left\{ \sin^2 i \cos 2\omega - (2 - 3 \sin^2 i) \left(1 + \frac{e}{1 + \sqrt{1 - e^2}} - \frac{\sqrt{1 - e^2}}{1 + e} \right) \right\}$$

and

$$R_p = R - 21.379 \sin^2 i \sin^2 \omega.$$

The right ascension of the node is nominally referred to the standard PROP equinox³, but contains a small error due to the fact that the times are given in UTC and no correction to UT1 has been made. To correct Ω to the time PROP equinox (epoch date still understood to be O^h UTC) add $0^{\circ}.004 \times (UT1 - UTC)$, where the time difference is in seconds.

The 'number of observations used' includes the number rejected; i.e. the parameters have been determined, in the end, from $N - K$ observations.

After nine of the tabulated quantities - the seven orbital parameters plus semi-major axis and perigee height - are given their computed standard

*The difference between r_p and $a(1 - e)$ is important. Thus, though r_p (or rather h_p) is the right parameter to use when correlating drag behaviour with perigee height, $a(1 - e)$ is the right parameter to work with when studying the effects of the earth's odd harmonics. For Ariel 3 the difference is approximately $1.54 \cos 2\omega$ km, and it was the use of r_p instead of $a(1 - e)$ which led to the apparent discrepancy mentioned in section 3.20 of Ref.13.

deviations, to one or two significant figures, the unit in each case being that of the final figure quoted for the main quantity. Every standard deviation includes ϵ as a factor, where ϵ is given by

$$\epsilon = \{\sum (\text{Res}/0.00029)^2 / (2N - 2K - 7)\}^{1/2} ;$$

here the summation is over all residuals, Res, in the $N - K$ accepted east-west and north-south direction cosines, and 0.00029 is the a priori accuracy referred to in section 2. Since the actual accuracy is worse than this a priori figure⁴, the values of ϵ in Table 2 are expected to be - and are - larger than unity.

Table 2 was obtained as direct computer output from a program known as TOP (Tabulation of Orbital Parameters). This program takes, as part of its input data, the punched card output from PROP runs, so there should not be any errors in the table.

The secular rates of change e_1 , i_1 , Ω_1 and ω_1 are not given in Table 2, since they are computed internally by PROP as part of the model. It is remarked, however, that the computation of the J_2^2 component of ω_1 contained an error until the PROP3 version of the program was introduced at the end of January, 1969. PROP2, which had this error in ω_1 , was able to compensate for it almost exactly, by fitting a slightly wrong value of M_1 , and this was one reason why the error was for a long time undiscovered. To correct the results from the PROP2 runs it was only necessary to correct M_1 by an amount equal to the error in ω_1 , and a special program was written to do this. The only reason for mentioning this point is that the values of M_1 in Table 2 are the corrected values, and so are different from the values provided in the first four provisional lists of Ariel 3 parameters to be issued. (To avoid having to ask AWRE Foulness to make a correction to the Ariel 3 telemetry data analysis program after PROP3 had been introduced, it was decided to continue sending incorrect values to AWRE, by adding the appropriate deliberate error to M_1 .)

Fig.1 gives a plot of orbital inclination, each value being represented by a vertical line, two standard deviations in length, centred on the fitted value. Fig.2 gives a plot of eccentricity, but most of the time the scale is too small for standard deviations to be shown. Fig.3 shows a short section of the eccentricity curve (covering just over half a period of the perigee) with

the scale expanded sufficiently for the standard deviations to be indicated as on the inclination plot. Figs. 1 and 2 give, in addition to the definitive inclinations and eccentricities obtained at R.A.E., the SDC (NORAD) values published in Spacetrack bulletins.

4.2 Results involving the parameter M_3

For two of the runs covered by Table 2 the parameter M_3 was included in the orbital model, namely, for those of epochs 1967 NOV 29 and 1967 DEC 2. These were the epochs which occurred during the week when no Minitrack data were supplied. Without M_3 the fit was twice as bad (ϵ 5.7 for the first epoch instead of 2.8), due to the number of days spanned by the observations. The value of M_3 , omitted from Table 2 to retain a regular format, was -0.00069, with standard deviation 0.00003, for both epochs; the same observations were used in both runs, so the second set of elements is really just the first set advanced three days, with small variations in the residuals (and hence a small change in ϵ) due to limitations of the orbital model.

It was stated in section 3 that, for epochs early in the satellite's lifetime, general introduction of M_3 would not have helped. To illustrate, the complete set of parameters, when M_3 is included, for 1967 DEC 20 is as follows (with last-figure standard deviations in brackets):-

$e = 0.007329$ (15), $i = 80.1802$ (19), $\Omega = 239.0276$ (19), $\omega = 155.22$ (10), $M_0 + \omega = 91.7171$ (15), $M_1 = 5433.0021$ (22), $M_2 = 0.0701$ (9) and $M_3 = 0.0007$ (9); the value of ϵ , viz. 3.6, was actually larger (unrounded value, 3.555 as against 3.546) than for the run without M_3 , due to the loss of a degree of freedom. For certain epochs later in the lifetime, however, introduction of M_3 would have led to better fits. This may be illustrated by considering the two worst fits obtained, namely, for epochs 1969 MAR 20 (ϵ of 5.0 in Table 2) and 1969 MAR 23 (ϵ of 5.1); on repeating these runs, with M_3 included, the following results were obtained:- for 1969 MAR 20, $e = 0.006902$ (45), $i = 80.1665$ (20), $\Omega = 10.3404$ (19), $\omega = 124.14$ (17), $M_0 + \omega = 189.4465$ (46), $M_1 = 5475.7830$ (24), $M_2 = 0.1282$ (9) and $M_3 = 0.0077$ (9), with $\epsilon = 3.1$; for 1969 MAR 23, $e = 0.006936$ (49), $i = 80.1677$ (30), $\Omega = 6.4259$ (22), $\omega = 116.38$ (16), $M_0 + \omega = 48.2682$ (47), $M_1 = 5476.6144$ (32), $M_2 = 0.1444$ (12) and $M_3 = 0.0085$ (13), with $\epsilon = 3.6$.

4.3 Results involving Hewitt camera observations

The runs at epochs 1969 APR 10 and 1969 JUN 21 were repeated, with (respectively 8 and 12) Hewitt camera observations included. The following results were obtained:- for 1969 APR 10, $e = 0.006778$ (7), $i = 80.1671$ (12), $\Omega = 342.9333$ (13), $\omega = 67.28$ (12), $M_0 + \omega = 326.4188$ (10), $M_1 = 5480.4510$ (8) and $M_2 = 0.0850$ (7), with $\epsilon = 2.0$ and only the same Minitrack observation rejected as was originally rejected; for 1969 JUN 21, $e = 0.005403$ (8), $i = 80.1610$ (10), $\Omega = 248.6570$ (13), $\omega = 175.80$ (6), $M_0 + \omega = 120.8871$ (10), $M_1 = 5490.2793$ (7) and $M_2 = 0.0584$ (3), with $\epsilon = 2.4$ and two of the Minitrack observations rejected that had previously been accepted. On comparison with corresponding entries in Table 2 it may be seen that for the first run there is little change - the maximum change in a parameter is for $M_0 + \omega$, the change being about twice the original standard deviation, and no standard deviation has decreased by a factor of more than 1½; for the other run, however, there is a large change in eccentricity, nearly five times the original standard deviation, and the standard deviations for e , ω and $M_0 + \omega$ have all been reduced by factors of more than 2. (It is worth remarking that the change in e was caused entirely by the introduction of the Hewitt camera observations, and not at all by the subsequent rejection of two Minitrack observations.)

A reasonable conclusion is that Hewitt camera observations, of high accuracy, are compatible with Minitrack observations of poorer accuracy. For a high-inclination satellite like Ariel 3 the effect is not very significant if the Hewitt camera observations all come from a single pass, but there is a great improvement in accuracy when observations from two or more passes are available.

5 ACCURACY OF POSITION COMPUTATION

As with Ariel 2 it was required, for correlation with on-board experiments, that the definitive orbital parameters should be good enough for position to be computable from them to better than 1 km. In the paper on the Ariel 2 orbit² the accuracy of position computation was considered by reference to plots of $\{\sigma^2(x) + \sigma^2(y) + \sigma^2(z)\}^{\frac{1}{2}}$, where the variances $\sigma^2(x)$, $\sigma^2(y)$ and $\sigma^2(z)$ are functions of time and the covariance matrix of the orbital parameters, and by comparison of such plots with plots, during overlap periods, of $\{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\}^{\frac{1}{2}}$, where x_1, y_1, z_1 denote satellite

co-ordinates computed from orbital parameters at the epoch before the given overlap period and x_2, y_2, z_2 denote co-ordinates computed from parameters at the following epoch. This approach would have been equally possible for Ariel 3, using the program PREP⁸, but it was decided that it would be adequate to consider the question by looking directly at standard deviations of orbital parameters and interpreting these as maximum position errors after $1\frac{1}{2}$ days.

The justification for this approach is that, with $M_0 + \omega$ rather than M_0 taken as a parameter, large correlations between parameters did not occur. (Occasional correlations as large as ± 0.4 occurred, usually involving e and one other parameter.) For a satellite, like Ariel 3, in an orbit which is nearly circular and not too far from polar, the maximum effects, on position after $1\frac{1}{2}$ days, of one-sigma errors in the parameters are approximately as follows: $2 \sigma(e)$, $\sigma(i)$, $\sigma(\Omega)$, $2 \sigma(\omega)$, $\sigma(M_0 + \omega)$, $\frac{1}{2} \sigma(M_1)$ and $2 \sigma(M_2)$, where angle sigmas are now taken to be in radians. The main effects here of e , ω and $M_0 + \omega$ are the along-track errors which arise from the expression of argument of latitude in the form

$$u = (M + \omega) + 2e \sin \{(M + \omega) - \omega\}.$$

Let us consider 'maximum position effects' for two different sets of sigmas: first, the maximum value of each sigma that occurs anywhere in Table 2, and, second, maximum values during, roughly speaking, the best ninety per cent of the time. Denoting sigmas from the two sets by σ_1 and σ_2 respectively (with angles in degrees again), and the corresponding maximum position effects, in km, by MPE_1 and MPE_2 , we have the following table:-

	<u>σ_1</u>	<u>MPE_1</u>	<u>σ_2</u>	<u>MPE_2</u>
e	0.000072	1.1	0.000020	0.3
i	0.0043	0.5	0.0030	0.4
Ω	0.0048	0.6	0.0025	0.3
ω	0.51	0.8	0.17	0.3
$M_0 + \omega$	0.0074	0.9	0.0031	0.4
M_1	0.0043	0.8	0.0023	0.4
M_2	0.0019	0.5	0.0019	0.5

Since $(\Sigma \text{MPE}_2^2)^{1/2} = 1.0 \text{ km}$ it is reasonable to claim that the accuracy requirements are met most of the time. If we consider the accuracy of height only, i.e. of $r = a \{1 - e \cos (M + \omega - \omega)\}$, then only the maximum one-sigma effects $\sigma(e)$ and $a e \sigma(\omega)$ are significant; this gives a $\Sigma^{1/2}$ of 0.4 km corresponding to the MPE_2 column in the table.

Some comments may be useful on the reason for some of the larger sigmas in Table 2. The large $\sigma(M_1)$ (and hence $\sigma(a)$) and $\sigma(M_2)$ at the first epoch arose partly because this epoch was only 8 hours after launch and partly because of the complete absence of observations between $2^{\text{h}} 43^{\text{m}}$ on 6 May 1967 and $20^{\text{h}} 18^{\text{m}}$ on 7 May; there was a correlation of -0.988 between the computed values of M_1 and M_2 . Similarly, a large $\sigma(M_1)$ arose for epoch 1967 DEC 5 because of the missing data for 3-4 December, which has already been mentioned. High sigmas for epochs from 1968 DEC 20 to 1969 JAN 16, inclusive, arose because of the paucity of observations during this period; with only 14 observations accepted, the run of 1969 JAN 1 gave the largest sigmas, for the parameters Ω and ω , of all the runs in Table 2. High sigmas for epochs from 1969 FEB 27 to 1969 APR 4 arose partly from paucity of observations and partly from the large values of e during that period; the epochs 1969 MAR 20 and 1969 MAR 23, for which the largest values of e (and of sigmas for e , i and $M_0 + \omega$) of all the runs in Table 2 were obtained, have already been discussed in section 4.

6 DISCUSSION

Although the perigee height of Ariel 3, during the period considered, was around 500 km, as opposed to about 300 km for Ariel 2, air drag was still important enough to be the chief limitation in the computation of orbital parameters by PROP. The value of M_2 , equal to half the mean angular acceleration of the satellite, was only about $0.02^\circ/\text{d}^2$ immediately after launch; when this parameter approached or exceeded $0.1^\circ/\text{d}^2$, as it did for a few days in October-November 1968 and for longer periods in 1969, or when it changed by more than about $0.1^\circ/\text{d}^2$ from epoch to epoch, for example in late December 1967, the orbit does not fit the data so well, as indicated by higher values of e . The period of validity of a set of orbital parameters is the same as the period spanned by the observations used in determining the parameters, i.e. between 3 and 4 days. If a set of orbital parameters is used to predict beyond the period of validity, then, when M_2 is changing rapidly (and e is large), error increases rapidly. Now since each set of orbital

parameters (after the first) was obtained by iteration from an initial set equivalent to the parameters at the preceding epoch, an immediate guide to the accuracy of three-day prediction - i.e. up to five days from a given epoch - is provided by the largest absolute value for the residuals in the first iteration of the orbit determination at the next epoch. This largest absolute value can change violently. As an extreme example, the figures for a series of successive epochs, starting at 1969 OCT 12, are:- 29, 45, 13, 7, 12, 67, 154, 189, 628, 52, 30, 59, 46, 17, 12, 74, 10; the value 628, equivalent to an angular error of about 10° , occurred for epoch 1968 NOV 5, and was obviously due to the unusually high value (at 1968 NOV 2) of 0.1144 for M_2 , which immediately afterwards fell to 0.0530. (A very large magnetic storm occurred on 1 November and had a devastating effect on the upper atmosphere⁹.)

The behaviour of the orbital inclination, as evidenced by Fig.1, is worth discussing in detail. There are two distinct features. First, and very striking, is the secular behaviour: i remained essentially constant at 80.18° until the middle of December 1967, then dropped to about 80.163 in a period of about three months, and thereafter again remained essentially constant. Second, there are the superimposed oscillations, in which certain frequencies and amplitudes can fairly readily be seen. It is not entirely easy to explain either of these features.

Apart from resonances - and there should be no relevant resonance associated with the orbital parameters of Ariel 3 - the only known cause of secular variation in the orbital inclination of an earth satellite is the rotation of the atmosphere. Applying the formula of King-Hele and Scott¹⁰, if the atmosphere at a height of 500 km is taken to rotate at twice the angular velocity of the earth ($\Lambda = 2.0$ in Ref.10), then i in Fig.1 should show a secular drop of about 0.02° , i.e. just about what it does show. The rate of drop should be proportional to M_1 , however, whereas in Fig.1, as already remarked, the total drop is concentrated into a period of about three months, starting in December 1967.

The phenomenon is sufficiently remarkable for a sceptical reader to wonder whether the inclinations in Fig.1 really are right. Here the SDC values, though less accurate than the R.A.E. values, are useful; they are completely independent of the R.A.E. values, and confirm - not that there was a serious doubt - the secular behaviour indicated by the R.A.E. values.

Slightly more credible, though still unlikely, is the possibility that the sharp drop in i is not a purely secular effect but an oscillation superimposed on the change due to atmospheric rotation. Such an oscillation would require a period of 500 days or more, however, and even then the next cycle might have been expected to appear before the end of the graph. The amplitude of the oscillation would have to be nearly 0.01° . The author is unable to see whence such a term could arise.

There remain two possibilities: a single, complete discontinuity, due for example to meteoric impact, and a genuine (secular) perturbation over the (roughly) three-month period. The former seems very unlikely, though a discontinuity near the beginning of January 1968 cannot be completely ruled out; so we are left with the possibility of a real perturbation. Bearing in mind that a perturbing force, to produce an effect on i without affecting i , has to act in a direction perpendicular to the orbital plane, and that, to avoid cancellation, it has to act in opposite directions at the ascending and descending nodes of the orbit, it is difficult to see what the force can be, other than atmospheric rotation.

Attempts to produce an explanation must therefore degenerate into mere speculation. The three-month period of interest corresponded to a period of maximum solar activity (mean 107 mm solar radiation in excess of $150 \times 10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$), and during this period M_2 was greater than before and after, though not enough greater to explain Fig.1 at once. Could it be that, at heights above 400 km, where no accurate measurements have been made, atmospheric rotation is significantly faster during periods of high solar activity, i.e. that King-Hele and Scott's Λ parameter is considerably less than 2.0 for most of Fig.1, but very much larger during the short period of maximum activity?

Apart from the correlation with solar activity, two other interesting (and unexplained) correlations should be mentioned. First, the direction of the spin axis of Ariel 3 has been monitored by RSRS, Slough¹. At injection the spin axis pointed 69° south, i.e. to a point on the celestial sphere with declination -69° . During the first three months the axis looped towards the south, reaching declination -86° on 19 June 1967, but after this it moved north and the declination remained positive after August 1967, for as long as the satellite was still spinning. The axis looped towards the north, reaching a declination of almost 90° on 25 or 26 February 1968; from early

January to early April 1968, i.e. roughly the period of sharp fall in orbital inclination, was the period during which the declination of the spin axis exceeded 45° . (There was another period, starting about the middle of September 1968, when the declination again exceeded 45° , but this only lasted for about one month instead of three.) Second, the satellite's tape recorder, which contained two litres of air at a pressure of one atmosphere, was operating only intermittently during the period of interest. Until 28 November 1967 the tape recorder worked successfully. It then failed, but recovered and worked, apparently perfectly again, for three periods (of two, four and three weeks), until it failed for the last time on 14 April 1968. It is tempting to speculate that air was leaking, but, even if this was so, it is difficult to see how the right inclination-reducing force could result.

On turning to the oscillations, it is clear from Fig.1 that a number of components, of differing frequency, amplitude and phase, are present. Since $\cos i$ is small and e is very small, the oscillatory perturbations due to the earth's odd harmonics, the amplitude of which is proportional to $e \cos i$, is completely negligible. The effect of the earth's tesseral harmonics, as remarked in section 3, is not negligible, and the amplitude of the $J_{2,2}$ perturbation is more than twice as big as some of the values of $\sigma(i)$ in Table 2; however, such effects should not appear in Fig.1, since they are averaged out during orbit determination. Hence the oscillatory components in Fig.1 may be thought of as being due solely to luni-solar perturbations, for which the various terms in di/dt are given in Ref.11 (equation (31)). The main term in the integral of the equation for di/dt is, for Ariel 3, $0^\circ.0015 \cos 2(u_s - \Omega)$, where u_s is the argument of latitude of the sun; the period of this term is 80 days and a complete cycle may be seen, in Fig.1, for example between MJD 39673 and 39753 and between MJD 39993 and 40073. The next largest terms are combined terms for the sun and moon which, if we ignore the small non-zero value (Ω_m) for the right ascension of the node of the moon's orbit, are given by $0^\circ.0012 \cos \Omega$ and $0^\circ.0007 \cos 2\Omega$, of period 280 days and 140 days respectively. The fourth largest term is also the principal one in which u_m , the argument of latitude of the moon, appears; ignoring Ω_m again, it is $0^\circ.0005 \cos 2(u_m - \Omega)$, of period $12\frac{1}{2}$ days approximately. Other terms are of smaller argument, but a combination of such terms could produce a detectable contribution to the graph of i .

In the absence of a spectral analysis or a complete analysis of all terms from Ref.11 it is difficult to be sure whether the oscillatory component of Fig.1 can be fully explained by luni-solar perturbations. It does appear, however, that Fig.1 contains a sinusoidal term with period about 30 days. A term in $\cos(k\Omega \pm u_m)$, for small integral k , would be appropriate here, but the only terms which arise have the eccentricity of the moon, i.e. 0.055, as a factor, and their amplitudes are too small.

One other known source of sinusoidal contributions to inclination variation should be mentioned. This is the precession and nutation of the earth's axis, which provides the reference with respect to which (the complement of) an orbital inclination is measured. The main contribution is from precession and may be taken from Ref.12 (in which the nutation terms have the wrong sign but the precession term is correct). For Ariel 3 this gives $-0^{\circ}.0007 \cos \Omega$, and so reduces (to about half) the amplitude of the direct luni-solar perturbation term of argument Ω .

7 CONCLUSIONS

Orbital parameters for the satellite Ariel 3, as for Ariel 2, have been determined at R.A.E., at three-day intervals, from Minitrack observations supplied by NASA. The accuracy of the computed parameters is about the same, in general, as was obtained for Ariel 2, i.e. better than 1 km except on very rare occasions, and should be adequate for experimenters' requirements. (Values of eccentricity were accurate enough to be used in determining the odd harmonics in the geopotential¹³.) During periods of high drag, better accuracy could have been obtained by inclusion of an eighth parameter, M_3 , in the orbital model, as was done with Ariel 2.

Orbital inclination was determined rather less accurately than for Ariel 2, no doubt due to the fact that there are no Minitrack stations, in either hemisphere, at latitudes as high as 80° . The accuracy was good enough, however, for an anomalous secular behaviour in inclination to be clearly observable. This behaviour has been discussed but not explained.

Acknowledgement

The author wishes to thank Jennifer Davies for preparing the card decks and supervising the computer analysis.

Table 1

MINITRACK STATIONS OBSERVING ARIEL 3

Station name	Location	x (km)	y (km)	z (km)
Fort Myers	Fort Myers, Florida, U.S.A.	807.885	-5652.020	2833.549
Johannesburg	Hartebeshoek, South Africa	5084.798	2670.474	-2768.164
Lima	Lima, Peru	1388.818	-6088.429	-1293.207
Newfoundland	St. Johns, Newfoundland	2602.801	-3419.184	4697.694
Quito	Quito, Ecuador	1263.617	-6255.010	-68.856
Santiago	Santiago, Chile	1769.707	-5044.642	-3468.192
Winkfield	Winkfield, England	3983.130	-48.404	4964.711
Ulaska	Fairbanks, Alaska	-2282.332	-1452.667	5756.942
Madagascar	Tananarive, Malagasy	4091.903	4434.373	-2064.537
Orroral	Canberra, Australia	-4447.361	2677.215	-3695.209

DATE	G	e	h _p	i	Ω	ω	H ₀ + ω	H ₁	H ₂	N	K	D	ε	HJD
1967 MAY 6	6530.0593	36	501.18	8	169.4546	163.21	134.1277	5419.3235	0.0220	40	6	2.7	1.8	39616
MAY 9	6929.9346	3	501.70	4	165.4410	154.27	182.4350	5419.3464	0.0168	64	0	2.9	1.5	39619
MAY 12	6929.8707	4	503.00	4	161.6310	145.50	231.0999	5419.4449	0.0171	5	0	2.9	1.5	39622
MAY 15	6929.7787	4	504.58	4	158.0188	136.79	280.0288	5419.5528	0.0183	4	2	3.7	1.8	39625
MAY 18	6929.6831	4	506.08	5	154.2069	128.36	329.2954	5419.6651	0.0188	5	2	3.8	1.7	39628
MAY 21	6929.5814	5	507.36	8	150.3958	119.99	378.8953	5419.7843	0.0196	5	3	4.0	2.2	39631
MAY 24	6929.4573	8	508.73	5	146.5827	111.41	428.8946	5419.9299	0.0209	4	3	3.8	1.7	39634
MAY 27	6929.2057	8	509.07	9	142.7737	102.76	478.8946	5420.2252	0.0257	9	4	3.8	1.5	39637
MAY 30	6928.9673	4	509.92	3	138.9548	94.70	528.8946	5420.5052	0.0442	4	2	3.8	1.7	39640
JUN 2	6928.7651	5	510.01	4	135.1417	86.40	578.8946	5420.7427	0.0521	5	3	3.7	2.0	39643
JUN 5	6928.6101	4	509.50	4	131.3280	78.21	628.8946	5420.9246	0.0594	5	4	3.7	2.0	39646
JUN 8	6928.4620	7	508.42	5	127.5145	69.94	678.8946	5421.0985	0.0631	8	2	3.8	2.5	39649
JUN 11	6928.3730	5	506.93	4	123.6980	61.58	728.8946	5421.2693	0.0628	5	4	3.8	1.5	39652
JUN 14	6928.3165	6	505.17	6	119.8865	53.55	778.8946	5421.4303	0.0628	6	3	3.9	2.0	39655
JUN 17	6928.2475	6	503.26	4	116.0692	44.98	828.8946	5421.5903	0.0647	6	7	3.6	1.8	39658
JUN 20	6928.1621	5	501.50	4	112.2529	36.20	878.8946	5421.7506	0.0647	5	4	3.6	1.5	39661
JUN 23	6928.0563	5	500.02	5	108.4413	27.17	928.8946	5421.9106	0.0647	5	4	3.6	1.5	39664
JUN 26	6927.9258	5	499.30	7	104.6257	18.01	978.8946	5422.0706	0.0647	5	4	3.7	1.8	39667
JUN 29	6927.7744	5	499.23	7	100.8101	8.78	1028.8946	5422.2306	0.0647	5	4	3.7	1.8	39670
JUL 2	6927.6453	11	500.30	7	96.9950	359.41	1078.8946	5422.3906	0.0647	5	4	3.7	1.8	39673
JUL 5	6927.5565	5	502.15	6	93.1785	369.77	1128.8946	5422.5506	0.0647	5	4	3.7	1.4	39676
JUL 8	6927.4913	8	505.00	5	89.3650	339.99	1178.8946	5422.7106	0.0647	5	4	3.7	1.1	39679
JUL 11	6927.4330	12	508.48	5	85.5491	330.04	1228.8946	5422.8706	0.0647	5	4	3.7	1.1	39682
JUL 14	6927.3508	4	512.54	5	81.7288	319.59	1278.8946	5423.0306	0.0647	5	4	3.7	1.3	39685
JUL 17	6927.2797	6	516.69	5	77.9113	308.77	1328.8946	5423.1906	0.0647	5	4	3.7	1.3	39688
JUL 20	6927.2095	8	520.54	5	74.0948	297.59	1378.8946	5423.3506	0.0647	5	4	3.7	1.3	39691
JUL 23	6927.1367	6	523.18	4	70.2774	286.40	1428.8946	5423.5106	0.0647	5	4	3.7	1.3	39694
JUL 26	6927.1007	5	524.63	4	66.4588	274.96	1478.8946	5423.6706	0.0647	5	4	3.7	1.3	39697
JUL 29	6927.0095	8	524.59	5	62.6421	263.55	1528.8946	5423.8306	0.0647	5	4	3.7	2.3	39700
AUG 1	6926.9181	3	522.96	4	58.8224	252.33	1578.8946	5423.9906	0.0647	5	4	3.7	1.1	39703
AUG 4	6926.8528	7	520.11	5	55.0065	240.87	1628.8946	5424.1506	0.0647	5	4	3.7	1.0	39706
AUG 7	6926.7767	4	516.28	3	51.1844	230.03	1678.8946	5424.3106	0.0647	5	4	3.7	1.6	39709
AUG 10	6926.6964	5	512.10	3	47.3641	219.51	1728.8946	5424.4706	0.0647	5	4	3.7	1.6	39712
AUG 13	6926.6030	3	507.92	4	43.5456	209.12	1778.8946	5424.6306	0.0647	5	4	3.7	1.4	39715
AUG 16	6926.4962	4	504.40	6	39.7269	198.97	1828.8946	5424.7906	0.0647	5	4	3.7	1.4	39718
AUG 19	6926.3602	7	501.56	7	35.9091	189.04	1878.8946	5424.9506	0.0647	5	4	3.7	1.8	39721
AUG 22	6926.1988	4	499.62	8	32.0900	179.27	1928.8946	5425.1106	0.0647	5	4	3.7	1.9	39724
AUG 25	6926.0251	7	498.57	11	28.2697	169.59	1978.8946	5425.2706	0.0647	5	4	3.7	2.3	39727
AUG 28	6925.8251	4	498.71	5	24.4474	160.26	2028.8946	5425.4306	0.0647	5	4	3.7	1.1	39730
AUG 31	6925.6073	14	499.47	9	20.6299	150.91	2078.8946	5425.5906	0.0647	5	4	3.7	2.2	39733

Table 2. Orbital parameters of Ariel 3

Table 2(cont'd.)

DATE	Q	e	h _p	i	Ω	ω	M ₀ +ω	M ₁	M ₂	N	K	D	ε	HJD							
1967 SEP 3	6925.3506	5	500.89	4	80.1831	9	16.8056	6	142.00	3	220.8084	5	5424.7339	6	0.0473	7	37	2	3.0	1.3	39736
SEP 6	6925.1570	7	502.76	4	80.1817	13	12.9885	7	133.20	3	285.8482	12	5424.9814	9	0.0337	3	24	0	4.9	1.4	39739
SEP 9	6924.9797	4	504.43	4	80.1808	14	9.1667	7	124.74	3	331.5117	6	5425.1899	5	0.0371	5	35	2	3.1	1.5	39742
SEP 12	6924.8036	6	505.94	4	80.1819	16	5.3460	8	116.41	7	37.8177	8	5425.3969	5	0.0318	5	32	2	3.7	1.8	39745
SEP 15	6924.6180	5	507.17	4	80.1831	12	1.5257	8	108.00	7	124.7375	9	5425.4322	6	0.0357	6	37	1	3.7	2.1	39748
SEP 18	6924.4613	5	507.92	4	80.1849	11	357.7073	11	99.67	7	192.2814	9	5425.7993	6	0.0296	6	34	0	3.7	2.0	39751
SEP 21	6924.2779	7	507.99	7	80.1839	12	353.8841	13	91.43	10	260.3797	14	5426.0191	8	0.0437	7	37	0	3.7	2.8	39754
SEP 24	6924.1083	6	507.57	6	80.1819	12	350.0528	12	83.31	9	329.1769	12	5426.2144	7	0.0246	6	32	0	3.7	2.4	39757
SEP 27	6923.9703	7	506.55	7	80.1823	14	346.2408	14	75.32	14	38.4612	20	5426.3768	8	0.0283	8	23	1	3.7	2.5	39760
SEP 30	6923.7676	8	505.12	7	80.1824	16	342.4170	15	66.89	13	108.3183	15	5426.6192	9	0.0449	8	28	2	3.4	2.3	39763
OCT 3	6923.6107	5	503.40	5	80.1817	10	338.5932	11	58.43	7	178.8867	9	5426.7997	6	0.0244	5	32	0	3.7	2.0	39766
OCT 6	6923.4813	4	501.53	4	80.1793	8	334.7685	9	49.74	5	249.9153	7	5426.9318	5	0.0256	5	39	3	3.7	1.7	39769
OCT 9	6923.3437	5	499.63	5	80.1786	14	330.9434	11	40.97	6	321.4037	8	5427.1112	6	0.0303	5	40	1	3.8	2.1	39772
OCT 12	6923.1719	4	498.10	5	80.1777	11	327.1172	8	32.01	4	33.4439	7	5427.3157	5	0.0320	5	41	3	3.5	1.6	39775
OCT 15	6923.0109	4	496.82	5	80.1782	13	323.2937	6	22.94	4	106.0775	6	5427.5051	4	0.0331	4	46	2	3.8	1.6	39778
OCT 18	6922.8430	4	496.33	5	80.1783	9	319.4679	6	13.77	3	179.2902	6	5427.7027	5	0.0345	5	42	2	3.4	1.6	39781
OCT 21	6922.6723	5	496.72	7	80.1781	12	315.6426	9	4.42	5	253.1044	8	5427.9035	6	0.0356	5	43	1	3.8	2.0	39784
OCT 24	6922.4766	5	498.19	7	80.1791	11	311.8163	10	354.89	5	327.5352	9	5428.1303	6	0.0428	5	41	3	3.8	2.1	39787
OCT 27	6922.2590	9	500.57	12	80.1774	19	307.9862	19	345.25	10	42.7060	15	5428.3898	10	0.0484	9	39	2	4.0	3.3	39790
OCT 30	6921.9522	9	503.64	12	80.1765	18	304.1576	17	335.27	10	118.7789	16	5428.7308	11	0.0624	10	32	3	3.6	3.0	39793
NOV 2	6921.6745	8	507.64	10	80.1753	16	300.3293	15	323.02	8	193.9471	12	5429.0776	9	0.0509	7	29	2	3.8	2.4	39796
NOV 5	6921.4201	5	511.80	6	80.1763	11	296.4997	10	314.44	6	274.0446	9	5429.3770	6	0.0449	5	33	0	3.7	2.1	39799
NOV 8	6921.2304	5	515.94	6	80.1766	12	292.6709	11	303.62	7	352.9327	9	5429.6804	6	0.0361	5	37	2	3.8	2.2	39802
NOV 11	6921.0603	5	519.37	5	80.1772	13	288.8397	10	292.52	8	72.4408	8	5429.8007	5	0.0303	5	43	3	3.8	2.1	39805
NOV 14	6920.8900	4	521.62	4	80.1769	9	285.0093	9	281.26	6	132.5321	6	5430.0011	5	0.0338	5	44	3	3.9	1.8	39808
NOV 17	6920.7058	4	522.26	4	80.1775	11	281.1777	8	269.70	8	233.2411	7	5430.2180	5	0.0398	5	50	3	3.7	2.0	39811
NOV 20	6920.4900	6	521.23	5	80.1781	14	277.3662	11	258.21	9	314.6614	8	5430.4722	7	0.0450	6	47	1	3.9	2.4	39814
NOV 23	6920.2475	7	518.51	8	80.1769	19	273.5144	14	248.94	11	36.8888	10	5430.7377	8	0.0486	4	43	2	3.7	3.0	39817
NOV 26	6919.9821	5	514.73	5	80.1761	15	269.6824	10	235.65	8	120.0260	8	5431.0702	6	0.0492	6	48	0	3.7	2.3	39820
NOV 29	6919.7398	7	510.16	6	80.1789	16	265.8547	10	224.52	7	204.0586	29	5431.3536	9	0.0460	4	58	0	10.5	2.8	39823
DEC 2	6919.5208	7	505.79	7	80.1789	17	262.0189	10	213.80	7	288.9116	31	5431.6136	8	0.0398	2	58	0	10.5	2.9	39826
DEC 5	6919.3423	19	501.49	6	80.1781	13	258.1892	8	203.60	6	14.4837	15	5431.8240	23	0.0373	8	37	0	2.8	1.8	39829
DEC 8	6919.1521	5	498.10	6	80.1790	13	254.3560	7	193.21	5	100.6832	8	5432.0481	5	0.0355	5	49	1	3.9	2.2	39832
DEC 11	6918.9895	5	495.92	6	80.1804	12	250.5240	7	183.47	5	187.5169	7	5432.2396	6	0.0280	5	45	3	3.7	1.9	39835
DEC 14	6918.8397	6	494.62	7	80.1814	14	246.6908	10	173.70	6	274.8754	9	5432.4161	7	0.0310	6	41	2	3.7	2.4	39838
DEC 17	6918.6310	6	494.01	8	80.1813	14	242.8597	13	164.46	7	2.8371	10	5432.6820	7	0.0303	6	49	0	3.8	2.8	39841
DEC 20	6918.3194	8	494.13	10	80.1802	19	239.0274	19	155.22	10	91.7171	13	5433.0292	9	0.0699	9	44	0	3.0	3.3	39844
DEC 23	6918.0284	7	494.63	10	80.1789	15	235.1930	18	146.48	9	181.7599	13	5433.3722	8	0.0471	7	46	2	3.8	3.3	39847
DEC 26	6917.8217	5	496.03	6	80.1791	10	231.3581	11	137.61	5	272.6506	8	5433.6188	6	0.0350	5	53	5	3.8	2.2	39850
DEC 29	6917.6294	7	497.58	9	80.1788	14	227.5230	15	128.99	8	4.2180	12	5433.8424	9	0.0354	8	43	6	3.8	2.8	39853

* FOR THESE TWO EPOCHS THE ADDITIONAL PARAMETER M_3 WAS INCLUDED IN THE ORBITAL MODEL. THE VALUE OBTAINED, IN BOTH CASES, WAS -0.00069 3.

Table 2(cont'd.) Orbital parameters of Ariel 3

DATE	G	e	hp	i	Ω	ω	$M_0 + \omega$	M_1	M_2	N	K	D	ϵ	MJD
1968 JAN 1	6917.4121	7	498.75	8	80.1787	14	223.6878	13	120.47	9	90.4704	16	5434.086	39836
JAN 4	6917.1810	5	499.94	8	80.1756	10	219.8503	10	112.14	6	189.3927	10	5434.3710	39859
JAN 7	6916.9643	5	500.68	8	80.1759	10	216.0118	11	103.74	6	283.4428	10	5434.4264	39862
JAN 10	6916.7298	7	500.98	8	80.1743	13	212.7071	13	95.40	8	18.1186	14	5434.9029	39865
JAN 13	6916.4463	8	500.70	7	80.1733	13	208.3353	13	87.04	7	113.6706	12	5435.2372	39868
JAN 16	6916.1630	8	499.95	10	80.1721	21	204.4944	14	78.90	10	210.2856	16	5435.5913	39871
JAN 19	6915.9404	5	498.57	5	80.1709	11	200.6511	10	70.47	6	307.7029	9	5435.8518	39874
JAN 22	6915.7573	5	496.92	6	80.1731	11	196.8099	10	62.13	7	46.0080	10	5436.0698	39877
JAN 25	6915.5907	5	495.00	6	80.1728	12	192.9679	13	53.68	8	144.8192	11	5436.2463	39880
JAN 28	6915.3837	7	492.92	6	80.1697	15	189.3249	15	45.05	9	244.2677	11	5436.4004	39883
JAN 31	6915.1114	5	490.85	5	80.1701	13	185.2848	12	36.33	8	344.5688	10	5436.8117	39886
FEB 3	6914.7760	7	487.01	7	80.1697	15	181.4604	15	27.54	10	85.9147	12	5437.2073	39889
FEB 6	6914.4047	8	487.84	8	80.1698	17	177.5922	16	18.50	10	188.8943	14	5437.6284	39892
FEB 9	6914.1237	12	487.50	14	80.1680	29	173.7501	25	9.32	13	292.3965	21	5437.9771	39895
FEB 12	6913.8176	12	488.09	13	80.1698	24	169.9029	24	359.97	17	37.2396	21	5438.3384	39898
FEB 15	6913.6033	10	489.37	13	80.1704	23	166.0532	22	350.83	14	143.0215	21	5438.5914	39901
FEB 18	6913.4091	6	491.89	8	80.1684	14	162.2163	13	340.99	6	249.4816	10	5438.8283	39904
FEB 21	6913.1948	5	495.41	8	80.1651	10	158.3698	10	330.99	6	356.6707	10	5439.0934	39907
FEB 24	6912.9728	5	499.24	8	80.1651	11	154.5200	11	320.67	6	104.6575	10	5439.3352	39910
FEB 27	6912.7332	6	503.38	8	80.1647	12	150.6704	12	310.07	8	213.4214	12	5439.6185	39913
MAR 1	6912.4088	7	507.11	8	80.1633	14	146.8197	13	299.21	10	323.1838	12	5440.0014	39916
MAR 4	6912.1074	6	510.13	7	80.1616	12	142.9374	12	287.93	9	74.0819	11	5440.3574	39919
MAR 7	6911.8210	7	511.82	8	80.1612	12	139.1139	13	274.41	12	186.0576	13	5440.6337	39922
MAR 10	6911.5263	5	511.90	5	80.1610	11	135.2021	11	265.01	10	298.8637	9	5440.9236	39925
MAR 13	6911.2428	5	510.30	5	80.1603	11	131.4953	11	251.48	9	52.3190	9	5441.1824	39928
MAR 16	6911.0661	4	507.15	5	80.1605	11	127.5324	12	241.88	9	166.4342	8	5441.3735	39931
MAR 19	6910.8834	4	503.05	5	80.1588	12	123.6933	11	230.53	7	281.2458	8	5441.5874	39934
MAR 22	6910.6834	4	498.42	5	80.1596	9	119.8392	8	219.67	5	36.6810	7	5441.8034	39937
MAR 25	6910.4420	4	494.24	5	80.1601	11	115.9846	6	209.02	4	132.8004	6	5442.0350	39940
MAR 28	6910.1900	4	490.49	5	80.1554	10	112.1243	6	198.69	4	269.8205	6	5442.3064	39943
MAR 31	6909.9864	3	487.38	5	80.1589	11	108.2677	6	188.70	4	27.8672	6	5442.7453	39946
APR 3	6909.7823	4	485.26	5	80.1606	10	104.6090	6	179.14	6	146.9998	6	5443.1038	39949
APR 6	6909.5307	4	484.21	6	80.1615	15	100.5380	10	169.88	6	267.1878	8	5443.4033	39952
APR 9	6909.2229	6	484.16	4	80.1630	11	96.6547	8	160.92	5	28.0768	6	5443.6480	39955
APR 12	6909.0514	5	484.87	6	80.1633	15	92.8358	12	152.18	7	149.6516	11	5443.8597	39958
APR 15	6908.8427	4	484.07	9	80.1641	9	88.9791	8	143.42	4	271.9061	6	5444.0974	39961
APR 18	6908.7402	4	487.66	5	80.1602	12	85.1282	9	134.77	5	34.9028	8	5444.3371	39964
APR 21	6908.5658	3	489.41	7	80.1661	7	81.2630	6	126.23	3	158.5776	5	5444.5434	39967
APR 24	6908.3844	4	490.79	4	80.1645	10	77.4039	6	117.73	5	282.8310	7	5444.7528	39970
APR 27	6908.1849	4	491.87	4	80.1643	10	73.5462	10	109.23	5	47.8043	7	5444.9938	39973

Table 2(cont'd.) Orbital parameters of Ariel 3

Table 2(cont'd.)

DATE	Q	e	h _p	i	Ω	ω	M ₀ + ω	M ₁	M ₂	N	K	D	ε	HJD
1968 APR 30	6907.9801	0.008078	492.61	80.1649	69.6840	100.93	173.4034	5445.2300	0.0395	26	0	3.9	1.5	39976
MAY 3	6907.7640	0.008131	492.62	80.1625	65.8286	92.48	299.9036	5445.4916	0.0436	32	1	3.7	2.1	39979
MAY 6	6907.5427	0.008132	492.27	80.1621	61.9656	84.48	47.1113	5445.7336	0.0435	32	1	4.0	2.0	39982
MAY 9	6907.2610	0.008115	491.27	80.1628	58.1033	76.24	195.1821	5446.0887	0.0534	37	1	3.8	2.0	39985
MAY 12	6906.9929	0.008034	490.10	80.1641	54.2407	68.07	326.2605	5446.4039	0.0500	40	0	3.6	2.0	39988
MAY 15	6906.7581	0.007925	488.58	80.1654	50.3793	59.68	94.2335	5446.6818	0.0432	40	1	3.9	1.5	39991
MAY 18	6906.5194	0.007787	486.88	80.1661	46.5160	51.25	228.0190	5446.9633	0.0507	40	2	3.8	1.6	39994
MAY 21	6906.2481	0.007613	485.15	80.1666	42.6530	42.56	356.7873	5447.2855	0.0576	50	2	3.8	1.5	39997
MAY 24	6905.9706	0.007419	483.55	80.1682	38.7904	33.63	129.3991	5447.6160	0.0512	42	0	3.6	1.5	40000
MAY 27	6905.7282	0.007219	482.31	80.1690	34.9287	28.48	263.0135	5447.9010	0.0432	51	1	3.8	1.5	40003
MAY 30	6905.5046	0.007004	481.75	80.1687	31.0642	15.14	39.4373	5448.1637	0.0366	44	0	3.6	1.5	40006
JUN 2	6905.3086	0.006816	481.82	80.1683	27.2011	5.72	172.240	5448.3976	0.0366	44	0	3.6	1.5	40009
JUN 5	6905.1391	0.006625	482.87	80.1649	23.3359	35.17	308.4616	5448.5935	0.0276	43	1	3.7	1.5	40012
JUN 8	6905.0039	0.006423	485.00	80.1631	19.4705	36.60	84.8194	5448.7835	0.0285	43	2	3.8	1.6	40015
JUN 11	6904.8492	0.006215	488.14	80.1626	15.6051	33.51	221.6810	5448.9450	0.0305	41	1	3.7	1.7	40018
JUN 14	6904.6736	0.006044	491.83	80.1628	11.7337	32.13	359.1641	5449.1495	0.0295	47	0	3.8	1.5	40021
JUN 17	6904.5323	0.005869	496.20	80.1642	7.8672	31.14	137.1768	5449.3088	0.0271	56	0	3.9	1.5	40024
JUN 20	6904.4037	0.005749	500.29	80.1641	3.9988	30.82	275.6699	5449.4691	0.0250	56	2	3.8	1.5	40027
JUN 23	6904.2859	0.005690	503.53	80.1648	0.1313	29.24	54.6118	5449.6037	0.0221	55	1	3.8	1.5	40030
JUN 26	6904.1790	0.005659	505.57	80.1649	356.2621	28.54	193.9522	5449.7353	0.0203	54	1	3.8	1.5	40033
JUN 29	6904.0883	0.005661	506.03	80.1673	357.3941	26.99	335.6736	5449.8695	0.0206	56	3	3.8	1.5	40036
JUL 2	6903.9553	0.005686	504.93	80.1669	348.5271	25.52	113.7938	5449.0004	0.0205	53	0	3.7	1.6	40039
JUL 5	6903.8567	0.005715	502.54	80.1633	344.6595	24.99	254.2951	5450.1171	0.0178	43	0	3.9	1.6	40042
JUL 8	6903.7691	0.005776	499.07	80.1637	340.7904	23.77	38.1229	5450.2208	0.0167	27	1	4.0	1.4	40045
JUL 11	6903.6649	0.005892	494.66	80.1637	336.9221	22.19	176.2630	5450.3163	0.0265	36	0	3.3	1.7	40048
JUL 14	6903.5221	0.006022	490.46	80.1642	333.0530	21.56	317.8500	5450.5134	0.0328	43	3	3.8	1.6	40051
JUL 17	6903.3568	0.006205	486.42	80.1641	329.1836	20.09	108.0018	5450.7093	0.0359	46	3	3.7	1.5	40054
JUL 20	6903.1800	0.006383	483.28	80.1638	325.3135	19.26	248.7520	5450.9188	0.0390	52	1	3.7	1.6	40057
JUL 23	6903.0030	0.006575	481.05	80.1670	321.4435	18.48	26.1320	5451.1237	0.0371	44	1	3.8	1.5	40060
JUL 26	6902.8309	0.006748	479.91	80.1693	317.5742	17.11	170.1459	5451.3327	0.0350	58	3	3.7	1.6	40063
JUL 29	6902.6669	0.006907	479.79	80.1678	313.7061	16.68	314.7534	5451.5270	0.0330	47	1	3.8	1.6	40066
AUG 1	6902.5368	0.007084	480.34	80.1657	309.8374	15.40	99.8892	5451.6832	0.0216	43	4	3.6	1.6	40069
AUG 4	6902.4189	0.007265	481.40	80.1635	305.9672	14.29	243.4409	5451.8208	0.0235	52	3	3.5	1.5	40072
AUG 7	6902.2820	0.007409	482.93	80.1643	302.0961	13.42	31.4460	5451.9831	0.0216	43	4	3.6	1.6	40075
AUG 10	6902.1500	0.007534	484.54	80.1627	298.2265	12.86	177.9709	5452.1632	0.0302	53	0	3.8	1.8	40078
AUG 13	6901.9771	0.007633	486.09	80.1617	294.3529	11.43	325.0386	5452.3444	0.0299	43	2	3.5	1.4	40081
AUG 16	6901.7807	0.007674	487.58	80.1631	290.4793	11.15	112.7018	5452.5773	0.0424	43	1	3.8	1.4	40084
AUG 19	6901.5850	0.007686	488.71	80.1669	286.6073	10.90	261.1070	5452.8095	0.0320	51	0	3.8	1.3	40087
AUG 22	6901.4502	0.007702	489.24	80.1643	282.7352	9.57	50.0993	5452.9853	0.0222	43	2	3.9	1.5	40090
AUG 25	6901.3299	0.007716	489.06	80.1630	278.8640	8.27	199.5206	5453.1118	0.0226	53	0	3.7	2.0	40093

Table 2(cont'd.) Orbital parameters of Ariel 3

Table 2(cont'd.)

DATE	G	e	h _p	i	Ω	ω	M ₀ + ω	M ₁	M ₂	N	K	D	ε	MJD							
1968 AUG 28	6901.2363	4	0.007710	3	80.1598	8	274.9904	8	78.05	3	349.3393	7	5455.2327	5	0.0172	4	41	1	3.9	1.7	40094
AUG 31	6901.1391	5	0.007673	4	80.1595	11	271.1167	10	69.93	4	350.4498	7	5455.3370	4	0.0241	4	45	4	3.4	1.8	40099
SEP 3	6900.9737	3	0.007581	3	80.1585	9	267.2414	6	61.48	4	290.0009	5	5455.5340	3	0.0359	3	51	4	3.0	1.5	40102
SEP 6	6900.7820	3	0.007547	3	80.1596	9	263.3647	6	53.05	4	81.2080	5	5455.7614	4	0.0388	3	56	3	4.0	1.6	40105
SEP 9	6900.5445	5	0.007264	5	80.1617	13	259.4891	9	44.39	5	233.2764	7	5456.0432	4	0.0509	5	50	3	3.8	2.0	40108
SEP 12	6900.2949	4	0.007087	4	80.1637	11	255.6141	11	35.62	4	26.7459	8	5456.3392	7	0.0538	4	44	3	3.7	2.2	40111
SEP 15	6899.9888	5	0.006916	5	80.1611	12	251.7396	12	26.84	7	179.9848	9	5456.7023	6	0.0614	6	35	4	3.6	2.1	40114
SEP 18	6899.7099	7	0.006712	8	80.1593	13	247.8628	14	18.17	10	334.9123	13	5458.0331	8	0.0692	9	35	5	3.4	2.3	40117
SEP 21	6899.4432	4	0.006527	8	80.1593	9	245.9842	9	8.91	5	130.7726	8	5455.3496	5	0.0543	5	46	3	3.7	2.0	40120
SEP 24	6899.1494	4	0.006293	7	80.1621	11	240.1058	7	359.46	4	287.6238	6	5455.6983	5	0.0598	4	49	2	3.8	1.7	40123
SEP 27	6898.8431	4	0.006070	7	80.1637	12	236.2269	7	349.20	5	85.5477	7	5456.0617	5	0.0649	4	52	3	3.7	1.7	40126
SEP 30	6898.5021	4	0.005836	6	80.1651	15	232.3490	12	338.68	8	244.6193	9	5456.4665	7	0.0705	6	48	3	3.8	2.0	40129
OCT 3	6898.1135	4	0.005656	10	80.1627	13	228.4721	12	327.94	8	44.9762	9	5456.9277	7	0.0802	7	42	2	3.7	2.0	40132
OCT 6	6897.7831	5	0.005460	9	80.1629	11	224.5910	11	317.13	8	206.0828	10	5457.3200	6	0.0576	7	37	1	3.7	2.3	40135
OCT 9	6897.4690	5	0.005303	9	80.1628	10	220.7095	11	306.12	9	9.4797	9	5457.6929	6	0.0624	6	41	3	3.9	2.2	40138
OCT 12	6897.1632	7	0.005183	8	80.1634	16	216.8296	14	294.34	13	173.3799	12	5458.0561	8	0.0678	7	52	2	3.9	3.0	40141
OCT 15	6896.8286	6	0.005114	9	80.1628	18	212.9478	13	282.18	13	338.4620	11	5458.4534	7	0.0539	6	44	3	3.9	2.8	40144
OCT 18	6896.5398	4	0.005125	6	80.1681	11	209.0681	11	270.07	8	144.5825	7	5458.7729	5	0.0561	5	49	4	3.8	1.9	40147
OCT 21	6896.2740	6	0.005127	8	80.1670	12	205.1853	13	258.19	10	311.4912	9	5459.1123	3	0.0598	6	48	2	3.8	2.3	40150
OCT 24	6896.0038	13	0.005139	18	80.1633	27	201.3062	29	246.21	26	119.7787	23	5459.4332	16	0.0420	15	30	0	3.8	4.7	40153
OCT 27	6895.6471	7	0.005219	10	80.1643	16	197.4232	15	234.52	13	288.9330	12	5459.8532	8	0.0651	7	41	1	3.9	2.7	40156
OCT 30	6895.2881	5	0.005353	7	80.1634	15	193.5377	9	222.93	9	99.3534	9	5460.2835	4	0.0896	5	43	3	3.8	2.3	40159
NOV 2	6894.7621	7	0.005512	7	80.1677	17	189.6515	12	212.03	9	271.3513	10	5460.9089	3	0.1144	8	44	4	3.7	2.8	40162
NOV 5	6894.3453	6	0.005676	10	80.1669	15	185.7645	13	201.63	9	88.1874	11	5461.3756	1	0.0530	7	41	0	3.7	2.8	40165
NOV 8	6894.1241	6	0.005829	9	80.1614	11	181.8826	13	191.12	8	260.0850	10	5461.6670	7	0.0520	7	51	0	3.7	2.1	40168
NOV 11	6893.8485	5	0.006028	9	80.1615	11	177.9947	11	180.89	6	73.7875	9	5461.9947	6	0.0533	6	42	1	3.7	2.1	40171
NOV 14	6893.5945	4	0.006217	8	80.1604	12	174.1077	8	171.37	4	282.5915	8	5462.2043	5	0.0483	5	45	0	3.9	2.1	40174
NOV 17	6893.3393	4	0.006397	10	80.1629	16	170.2159	12	162.42	8	70.0896	10	5462.6003	7	0.0587	7	45	2	3.8	2.0	40177
NOV 20	6893.0485	5	0.006560	7	80.1633	11	166.3274	11	153.25	7	248.7003	9	5462.9461	6	0.0501	5	33	2	3.9	2.0	40180
NOV 23	6892.8017	4	0.006700	9	80.1616	11	162.4388	12	144.09	5	68.2364	8	5463.2393	5	0.0432	5	29	0	3.7	1.7	40183
NOV 26	6892.5514	4	0.006848	6	80.1599	12	158.5575	11	135.12	8	248.6585	9	5463.5372	5	0.0561	5	34	1	4.0	2.2	40186
NOV 29	6892.2691	6	0.006962	4	80.1579	13	154.6575	8	126.64	7	70.0400	7	5463.8730	7	0.0514	7	42	1	3.6	2.1	40189
DEC 2	6892.0138	4	0.007028	4	80.1622	10	150.7646	9	118.34	5	282.3080	7	5464.1769	5	0.0541	5	42	2	3.8	1.8	40192
DEC 5	6891.7244	5	0.007098	7	80.1608	12	146.8746	11	109.89	6	75.6673	8	5464.5211	5	0.0571	5	32	0	3.8	1.9	40195
DEC 8	6891.4845	6	0.007163	9	80.1587	13	142.9844	12	101.53	7	299.9584	9	5464.8665	7	0.0603	6	36	0	3.8	2.2	40198
DEC 11	6891.2708	6	0.007182	8	80.1543	17	139.0901	9	93.28	9	85.0047	9	5465.0600	7	0.0480	6	25	0	3.8	1.7	40201
DEC 14	6891.0533	8	0.007135	11	80.1623	18	135.1948	12	84.79	8	270.8092	12	5465.3162	10	0.0351	9	32	0	3.5	2.2	40204
DEC 17	6890.8845	4	0.007096	9	80.1657	14	131.2986	14	76.53	8	97.7485	9	5465.5208	7	0.0345	6	24	0	3.7	2.1	40207
DEC 20	6890.6933	11	0.007070	45	80.1524	37	127.4235	46	68.23	18	284.4834	61	5465.7480	13	0.0433	11	18	0	3.2	2.6	40210
DEC 23	6890.4331	4	0.006964	37	80.1572	16	123.5193	11	59.90	12	118.4883	38	5466.0378	17	0.0585	7	22	1	3.4	1.6	40213

Table 2(cont'd.) Orbital parameters of Ariel 3

Table 2(cont'd.)

DATE	G	e	h _p	i	Ω	ω	M ₀ + ω	M ₁	M ₂	N	K	D	E	HJD
1968 DEC 26	6890.1374	0	477.68	80.1606	119.6199	50.77	301.3706	5466.4099	0.0363	23	0	3.3	2.4	40216
DEC 29	6889.8541	8	475.62	80.1584	115.7192	41.95	331.3975	5466.7475	0.0541	15	0	3.2	1.9	40219
1969 JAN 1	6889.3904	10	473.01	80.1554	111.8412	33.43	322.3101	5467.0811	0.0741	16	1	3.2	2.4	40222
JAN 4	6889.3649	7	471.89	80.1579	107.9390	25.39	324.1453	5467.3297	0.0458	21	0	3.4	1.6	40223
JAN 7	6889.7349	9	471.07	80.1633	104.0360	15.46	346.7988	5467.6309	0.0462	20	1	3.4	2.3	40228
JAN 10	6888.9233	7	471.11	80.1673	100.1342	5.43	185.2897	5467.8810	0.0383	18	0	3.2	1.8	40231
JAN 13	6888.7267	8	471.85	80.1661	97.3456	356.86	14.4846	5468.0901	0.0368	18	0	3.3	2.1	40234
JAN 16	6888.5214	7	474.06	80.1603	92.3456	344.76	220.3294	5468.3344	0.0503	21	1	3.0	1.4	40237
JAN 19	6888.2898	5	476.92	80.1619	88.4418	336.51	45.0316	5468.6342	0.0538	34	0	3.5	1.8	40240
JAN 22	6888.0800	4	480.73	80.1662	84.5396	325.62	241.5445	5468.8805	0.0508	30	0	3.5	1.4	40243
JAN 25	6887.9095	8	484.74	80.1601	80.6454	314.89	78.6350	5469.0334	0.0464	26	1	3.4	2.3	40246
JAN 28	6887.6762	4	489.13	80.1601	76.7421	303.42	274.4804	5469.3814	0.0423	31	1	3.6	1.3	40249
JAN 31	6887.4917	4	492.44	80.1630	72.8382	291.51	115.1033	5469.5614	0.0338	31	0	3.5	1.3	40252
FEB 3	6887.2796	9	494.68	80.1647	68.9338	279.26	314.3484	5469.8142	0.0387	27	1	3.9	3.1	40255
FEB 6	6887.0048	6	495.07	80.1545	65.0393	267.28	154.5403	5470.1113	0.0408	28	0	3.2	1.8	40258
FEB 9	6886.7448	3	493.58	80.1603	61.1309	255.14	559.4619	5470.4513	0.0311	34	1	3.9	1.4	40261
FEB 12	6886.4480	7	490.21	80.1623	57.2259	243.30	197.7323	5470.8033	0.0310	46	3	3.7	2.6	40264
FEB 15	6886.2096	4	486.14	80.1599	53.3220	231.34	48.8475	5471.0895	0.0374	42	2	3.8	1.4	40267
FEB 18	6886.0037	6	481.54	80.1606	49.4164	219.73	244.7237	5471.3350	0.0416	33	1	3.7	2.6	40270
FEB 21	6885.7394	5	476.90	80.1612	45.5080	208.79	89.3765	5471.6301	0.0450	39	4	3.8	1.6	40273
FEB 24	6885.5901	4	472.86	80.1639	41.6038	198.27	255.1513	5472.0468	0.0358	31	1	3.7	1.7	40276
FEB 27	6885.0175	10	465.78	80.1614	37.6972	188.09	142.2337	5472.5911	0.0397	26	1	3.8	3.5	40279
MAR 2	6884.5708	9	467.75	80.1661	33.7915	177.72	350.7906	5473.0441	0.0332	23	1	3.7	2.5	40282
MAR 5	6884.2249	8	466.54	80.1635	29.8816	168.13	200.8176	5473.4367	0.0616	23	1	3.7	2.7	40285
MAR 8	6883.8985	7	466.51	80.1620	25.9763	158.86	51.9716	5473.8461	0.0702	22	1	3.6	2.1	40288
MAR 11	6883.5562	8	467.13	80.1648	22.0488	149.94	264.3410	5474.2347	0.0490	29	0	3.7	3.1	40291
MAR 14	6883.2018	7	467.91	80.1645	18.1565	141.24	117.9736	5474.6777	0.0441	27	2	3.8	2.4	40294
MAR 17	6882.8018	10	469.04	80.1637	14.2503	132.54	338.8772	5475.1531	0.0937	23	0	3.6	3.1	40297
MAR 20	6882.2606	13	470.35	80.1664	10.3406	124.43	199.4465	5475.6013	0.1264	24	0	3.7	3.0	40300
MAR 23	6881.5632	15	472.25	80.1682	6.4266	116.00	48.2878	5476.0342	0.1478	26	1	3.9	3.1	40303
MAR 26	6880.8331	11	471.99	80.1629	2.5158	108.11	269.7521	5476.4342	0.1151	23	1	3.6	4.0	40306
MAR 29	6880.3043	9	471.91	80.1670	358.6041	100.11	133.6676	5476.8379	0.1083	23	0	3.4	2.8	40309
APR 1	6879.7749	8	472.55	80.1668	354.6855	92.21	339.0223	5477.2882	0.0984	23	0	3.2	2.3	40312
APR 4	6879.2599	9	472.26	80.1622	350.7747	83.80	226.4504	5477.7359	0.0983	24	0	3.6	3.4	40315
APR 7	6878.8026	6	471.21	80.1640	346.8569	75.96	95.6466	5479.0326	0.0896	26	0	3.6	2.4	40318
APR 10	6878.3687	7	470.10	80.1640	342.9326	67.49	324.4219	5480.4314	0.0848	29	1	3.6	2.6	40321
APR 13	6877.9545	10	468.48	80.1627	338.0175	59.40	198.7106	5480.9445	0.0843	26	1	3.6	3.0	40324
APR 16	6877.4873	5	466.49	80.1630	335.0973	51.05	72.5388	5481.5054	0.0997	29	1	3.7	1.9	40327
APR 19	6877.0112	7	464.46	80.1611	331.1758	42.28	308.1365	5482.0748	0.0803	30	2	3.4	3.0	40330
APR 22	6876.6464	5	462.80	80.1607	327.2526	33.40	185.2050	5482.5111	0.0749	21	2	3.7	2.4	40333

Table 2(cont'd.) Orbital parameters of Ariel 3

DATE	α	e	h_p	i	Ω	ω	$M_0 + \omega$	M_1	M_2	N	K	D	E	MJD		
1969 APR 25	6876.2819	5	461.36	6	80.1628 12	323.3265 10	24.47 9	5482.9473 9	6	0.0685	5	44	8	3.8	2.1	40336
APR 28	6875.9602	5	460.48	6	80.1614 12	319.4033 11	14.93 9	5483.3322 9	6	0.0660	5	39	6	3.9	1.9	40339
MAY 1	6875.6463	5	460.62	5	80.1590 9	315.4796 7	5.86 5	5483.9548 6	6	0.0544	4	44	7	3.8	1.6	40342
MAY 4	6875.3863	4	461.68	4	80.1623 10	311.5497 7	355.75 5	5484.0191 6	4	0.0467	4	45	4	3.6	1.6	40345
MAY 7	6875.1606	7	463.77	8	80.1603 15	307.6229 14	345.59 10	5484.2892 12	9	0.0452	6	35	5	3.8	2.6	40348
MAY 10	6874.9646	4	466.70	6	80.1612 11	303.6977 7	335.24 7	5484.5937 6	5	0.0574	4	43	7	3.6	1.6	40351
MAY 13	6874.7943	10	470.78	10	80.1636 22	299.7671 23	323.86 24	5484.9672 19	12	0.0728	11	26	3	3.7	3.1	40354
MAY 16	6874.6885	11	474.81	11	80.1597 21	295.8419 21	312.39 32	5485.3403 25	13	0.0837	11	32	1	3.7	4.2	40357
MAY 19	6873.8195	7	478.54	5	80.1605 14	291.9102 11	301.09 16	5485.7170 11	8	0.0618	8	31	0	3.7	1.8	40360
MAY 22	6873.5133	10	481.52	6	80.1615 11	287.9846 15	289.23 18	5486.0949 13	12	0.0597	10	24	1	3.9	1.9	40363
MAY 25	6873.2236	4	483.24	5	80.1597 9	284.0519 8	276.74 13	5486.4687 5	5	0.0560	5	32	0	3.6	1.6	40366
MAY 28	6872.9662	6	482.96	7	80.1627 10	280.1206 10	264.41 11	5486.8591 10	7	0.0485	6	27	0	3.6	1.6	40369
MAY 31	6872.7293	5	481.16	7	80.1617 9	276.1912 10	252.43 11	5487.2409 5	0.0471	5	28	1	3.7	1.6	40372	
JUN 3	6872.5101	5	477.70	8	80.1615 12	272.2571 9	240.47 8	5487.6236 10	5	0.0424	6	23	1	3.6	1.8	40375
JUN 6	6872.2487	6	473.52	11	80.1618 10	268.3288 13	223.93 13	5487.7769 7	0.0604	6	24	1	3.6	1.8	40378	
JUN 9	6871.8698	6	468.85	11	80.1593 14	264.3932 9	217.45 9	5488.3830 17	7	0.0889	6	28	2	3.6	1.8	40381
JUN 12	6871.4237	6	464.24	11	80.1635 11	260.4588 14	206.72 11	5488.7656 8	0.0900	8	27	2	3.3	2.2	40384	
JUN 15	6870.9359	5	460.51	10	80.1613 11	256.5276 9	196.06 9	5489.3503 7	0.1005	5	33	4	3.9	2.0	40387	
JUN 18	6870.4774	13	457.54	16	80.1632 26	252.5907 24	186.39 24	5489.9001 23	15	0.0770	13	21	1	3.5	3.4	40390
JUN 21	6870.1597	6	456.04	11	80.1597 12	248.6569 15	176.19 15	5490.2810 18	7	0.0588	7	22	0	3.7	2.3	40393
JUN 24	6869.9059	6	455.47	7	80.1595 13	246.7181 11	166.67 14	5490.5854 8	0.0477	9	33	3	3.5	2.1	40396	
JUN 27	6869.6849	6	455.58	8	80.1623 12	240.7828 14	157.56 14	5490.8505 15	7	0.0375	7	24	1	3.6	2.2	40399
JUN 30	6869.5072	6	456.63	6	80.1584 15	236.8446 10	148.51 14	5491.0636 8	0.0376	8	33	3	3.6	2.0	40402	
JUL 3	6869.2841	6	457.98	6	80.1593 15	232.9086 20	139.79 16	5491.3312 13	7	0.0450	6	28	1	3.6	2.0	40405
JUL 6	6869.0854	5	459.47	4	80.1572 10	228.9696 8	131.26 11	5491.6175 10	6	0.0522	6	36	1	3.5	1.9	40408
JUL 9	6868.7654	7	461.03	5	80.1595 11	225.0307 14	122.80 13	5491.9535 8	0.0509	8	25	1	3.3	1.9	40411	
JUL 12	6868.4837	5	462.27	4	80.1572 11	221.0918 8	114.44 8	5492.2914 6	0.0545	6	33	1	3.7	1.8	40414	
JUL 15	6868.2055	5	463.39	5	80.1594 12	217.1502 11	106.46 7	5492.6234 9	7	0.0485	7	35	1	3.8	2.3	40417
JUL 18	6868.0054	4	463.82	4	80.1574 8	213.2111 8	98.37 6	5492.9226 7	5492.8655	4	40	1	3.7	1.9	40420	
JUL 21	6867.8334	5	464.13	4	80.1588 10	209.2683 8	90.44 6	5493.0720 6	5	0.0326	5	48	6	3.2	1.8	40423
JUL 24	6867.6693	4	463.46	3	80.1578 10	205.3303 8	82.23 6	5493.2688 7	5493.2688	5	38	2	3.6	1.7	40426	
JUL 27	6867.4870	5	462.98	4	80.1560 12	201.3852 10	74.26 7	5493.4877 6	0.0438	5	42	2	3.6	2.0	40429	
JUL 30	6867.2846	7	461.73	6	80.1578 14	197.4469 13	66.10 12	5493.7282 11	8	0.0388	7	38	1	3.7	2.6	40432
AUG 2	6867.0288	6	460.22	5	80.1590 12	193.5018 10	58.14 12	5493.9802 11	5493.9802	7	30	1	3.7	1.9	40435	
AUG 5	6866.6660	6	458.40	6	80.1591 12	189.5630 13	49.86 17	5494.2434 14	7	0.0798	6	21	1	3.7	2.2	40438
AUG 8	6866.2557	5	456.60	5	80.1606 12	185.6171 10	41.28 14	5494.4961 13	6	0.0837	5	31	0	3.7	1.9	40441
AUG 11	6865.8483	6	454.97	5	80.1594 11	181.6753 11	32.73 15	5494.7555 12	8	0.0716	7	31	0	3.7	2.4	40444
AUG 14	6865.4953	7	453.71	9	80.1603 14	177.7275 16	24.16 20	5495.0193 17	5495.8995	8	27	1	3.7	2.5	40447	
AUG 17	6865.2333	5	453.29	5	80.1625 11	173.7831 9	14.83 18	5495.2842 14	6	0.0407	6	27	0	3.6	1.8	40450
AUG 20	6865.0070	10	453.57	19	80.1600 15	169.8373 16	5.43 26	5495.5460 30	0.0453	9	15	0	3.7	2.2	40453	
AUG 23	6864.7924	7	454.77	9	80.1580 14	165.8943 9	356.30 46	5495.8083 38	0.0403	8	21	0	3.2	1.9	40456	

Table 2(concl'd.) Orbital parameters of Ariel 3

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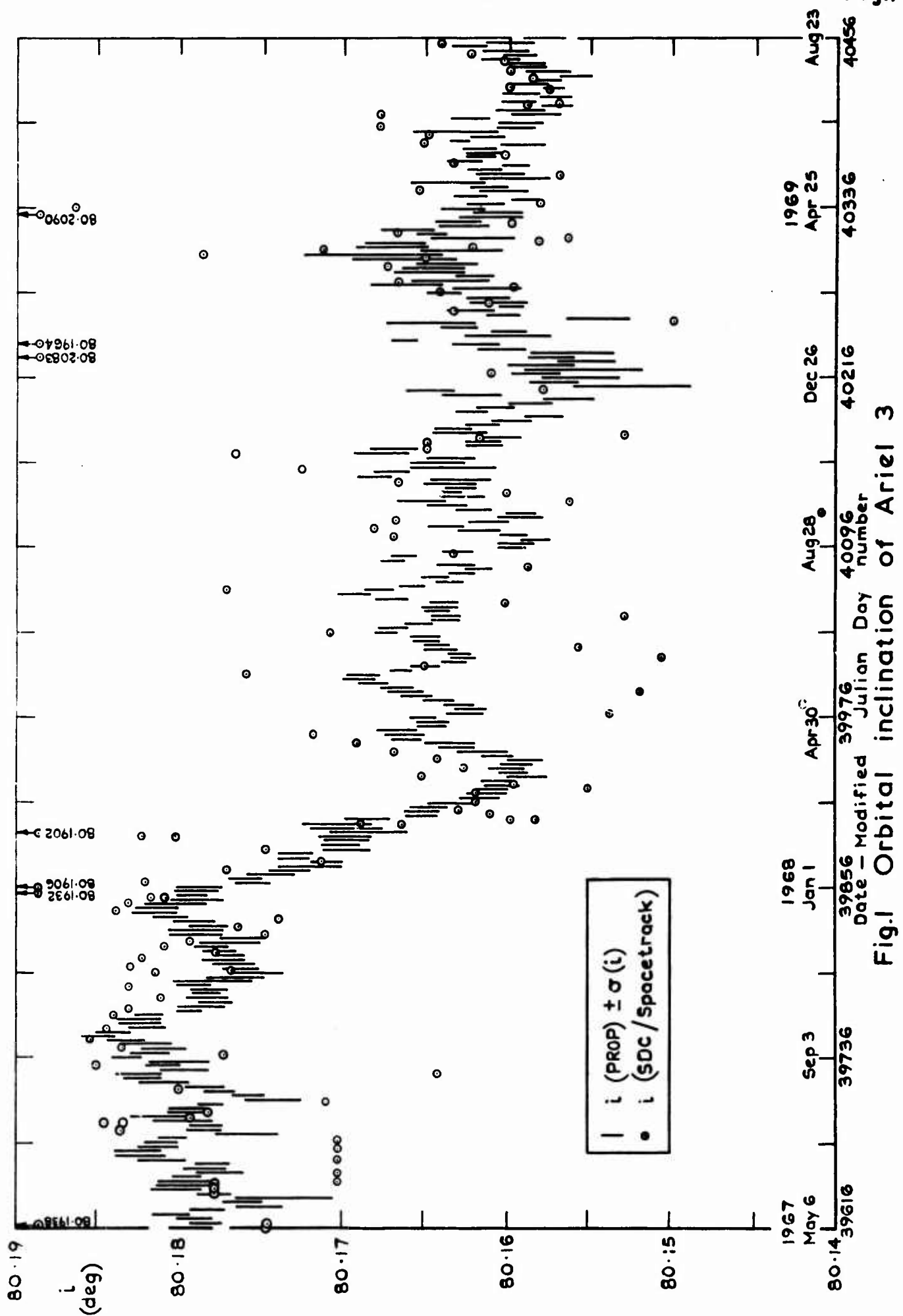


Fig. 1 Orbital inclination of Ariel 3

Fig. 2

004 902145

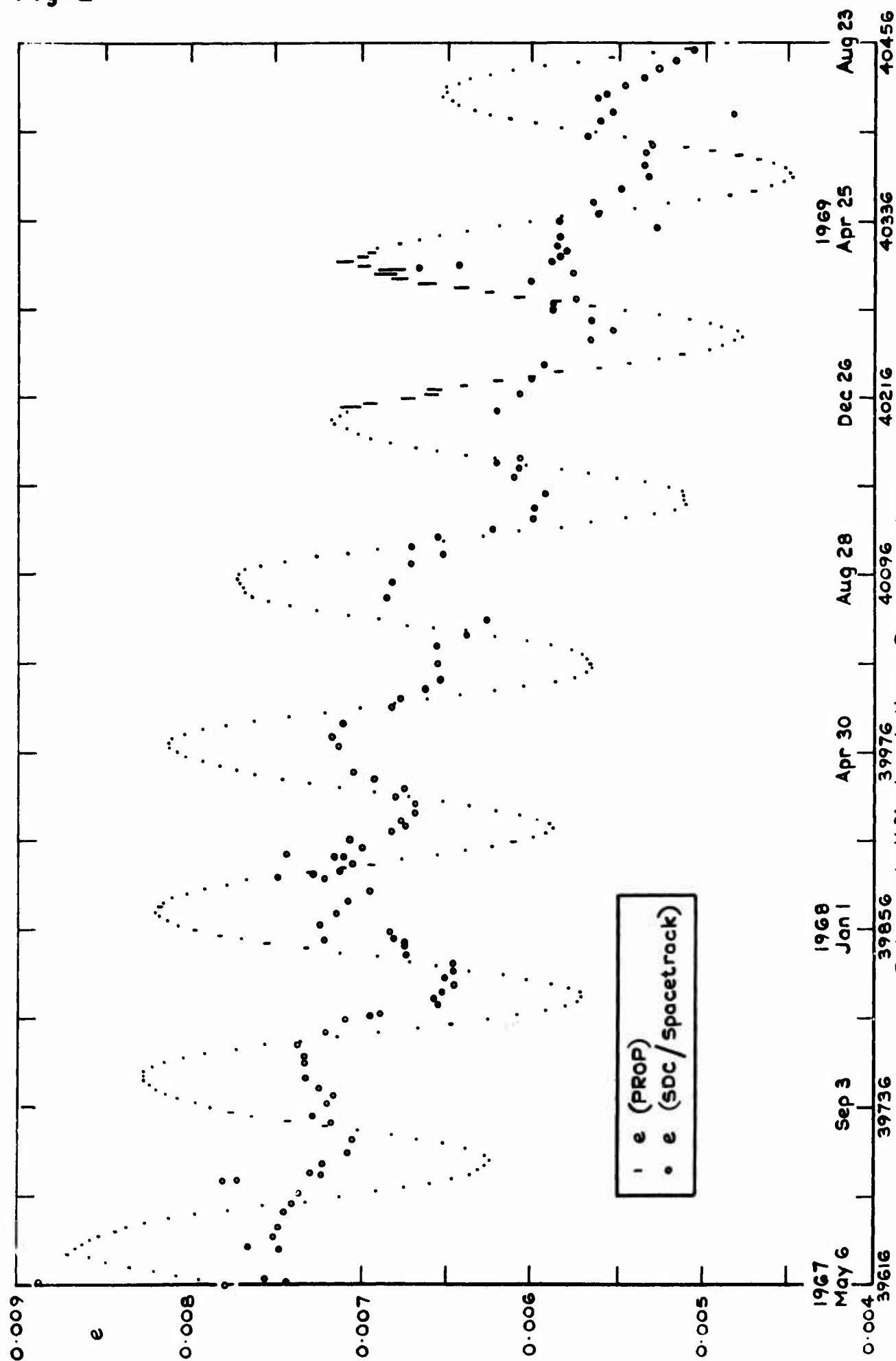
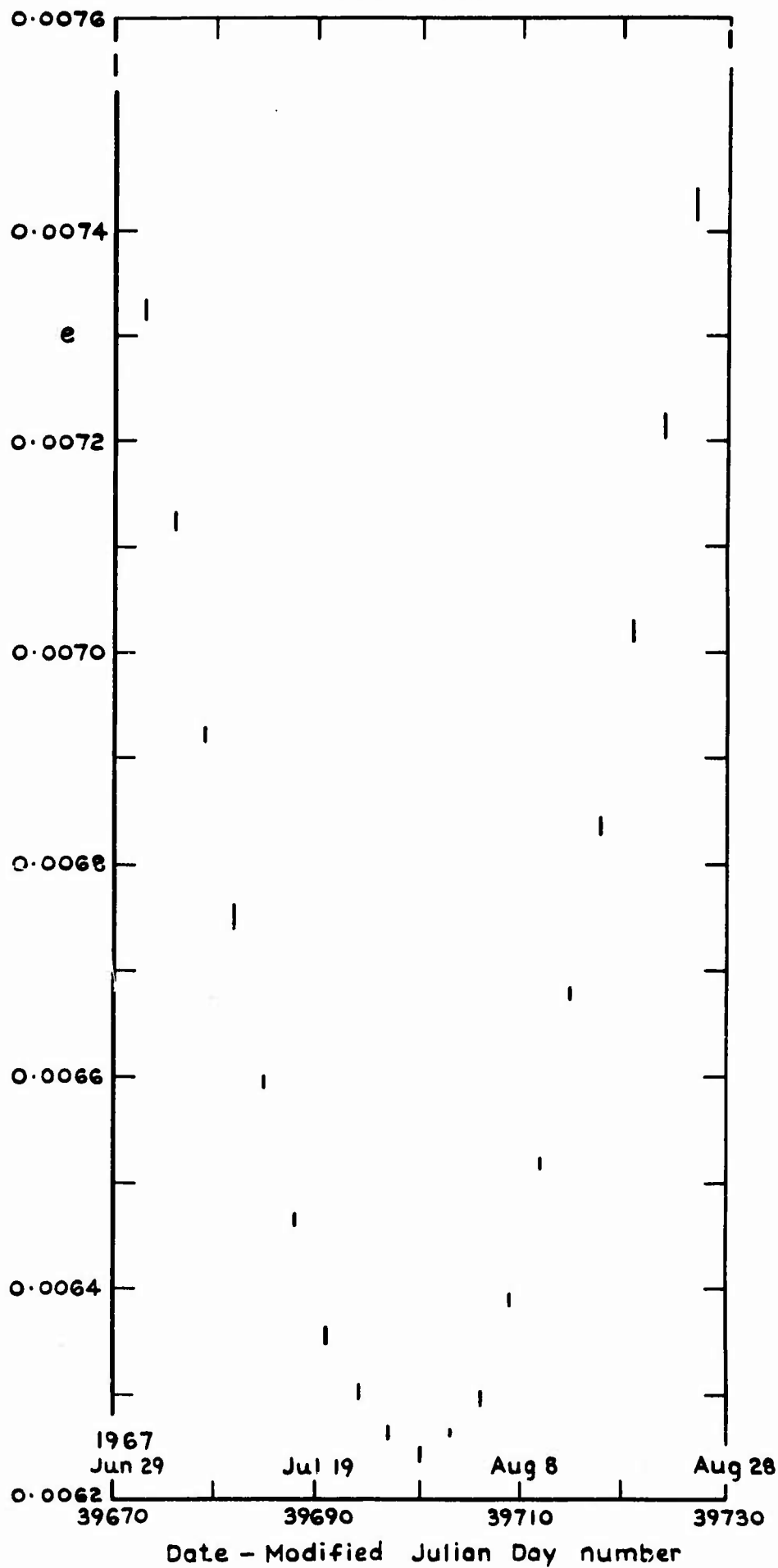


Fig.2 Orbital eccentricity of Ariel 3

004 902146

Fig.3



46 cgs.

ROYAL AIRCRAFT ESTABLISHMENT

Technical Report 69275

December 1969

THE ORBIT OF ARIEL 3 (1967-42A)

by

R. H. Gooding

ADDENDUM

The striking change of i (orbital inclination), from 80.18° in December 1967 to about 80.163° in March 1968, was remarked upon in section 6, but left unexplained. It is believed that the explanation is now known. During this period Ariel 3 was passing through a resonance associated¹⁴ with the earth's tesseral harmonics of order 15. In fact the mean motion of the satellite was exactly commensurate with the earth's rotation rate just before 0 hours on MJD 39889 (3 February 1968), and the variable $M + \omega + 15 (\Omega - n_0 t)$ of Ref. 14 varied by no more than 120° from its resonance value, during the three-month period. The author, in section 6, dismissed resonance, thinking of luni-solar resonance rather than tesseral-harmonic resonance, and discussed alternative explanations which must now be considered irrelevant.

ERRATA

Page 7, line 3: for the first ' M_1 ' read ' M_1^2 '.

Page 14, line 13: for the second ' i ' read ' M_1 '.

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