FSTC-HT-23-395-68

U.S. ARMY FOREIGN SCIENCE AND TECHNOLOGY CENTER

MEASUREMENT OF THERMAL PROPERTIES OF SOLID AND LIQUID METALS AT HIGH TEMPERATURES

COUNTRY: USSR

TECHNICAL TRANSLATION

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MEASUREMENT OF THERMAL PROPERTIES OF SOLID AND LIQUID METALS AT HIGH TEMPERATURES

(Izmereniye teplovykn svoystu tverdykh i zhidkikh metallov pri vysokikh temperaturakh)

by

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SOURCE: MOSCOW UNIVERSITY
1967, pp 325
USSR

Translated for FSTC by ACSI

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Abbreviation	Meaning	Page
grad	gradient	6
div	divergence	9
max	maximum	41
min	minimum	41
const	constant	79
cos	cosine	132
exp	exponential	132
sin	sine	162
arctg	arc tangent	162

PREFACE

The development of the study of solid and liquid states of matter and, in particular, the study of the most important aspects of this problem--questions of the nature of thermal motion and the mechanism of heat transfer--is related not only with the development and close study of the corresponding theoretical concepts, but also with the broadening of the range of states encompassed by this study. Experiments having as their aim not only a check of the conclusions of theoretical considerations, but also a direct study of the existing correlations, form an inseparable part of such studies. Experimental studies of the properties of solid and liquid metals at high temperatures are not only for scientific interests; these studies are of an extremely important significance for practice. A knowledge of the thermal properties of metals at high temperatures is necessary for the development of many branches of new technology. These have vital interest for atomic energy, rocket building and cosmic flights, heat engineering, metallurgy, electronics, and many other branches of engineering. Studies of thermal properties at high temperatures are of special significance in the development of magneto-hydrodyamic generators. It is not accidental that a study of the thermal properties of solid and liquid metals comprises a significant part of the problem of "thermophysics of high temperatures," the development of which is under the jurisdiction of the committee for the coordination of scientific research work, which is attached to the Council of Ministers USSR/1/.

The presently existing experimental data on the thermal properties of metals at high temperatures are very limited with respect to the number of objects studied as well as the range of temperatures studied. The quality of existing experimental data also leaves much to be desired. Even for comparatively well studied solid refractory metals, the results of studies by different authors differ strongly from each other; as regards the heat capacity, these differences frequently amount to 15% (see ss20), for heat conductivity--20-30% (see ss23). As regards liquid metals, the situation is particularly unsatisfactory. The number of experimental studies dealing with the systematic measurements of the heat conductivity of liquid metals is very small; the studies are limited by the comparatively low temperatures, while the results by different authors frequently differ by scores of percents.

Such a situation is explained, in the first place, by the existing difficulties of experimental studies at high temperatures. These difficulties are essentially caused by the great magnitude of heat exchange by means of radiation, the consideration or exclusion of which is a very complex matter. A weak spot in the experimental technique at temperatures of 2,000-3,000°K is that of measuring temperatures.

The existing situation makes it entirely necessary to carry out work on the development and perfection of metals from mercuring thermal parameters of solid and liquid metals at high temperatures. In such work, a special place should be allocated to searches for finding ways and conditions for carrying out such a study, particularly such which are empirical precisely at high temperatures and do not encounter limitations during their use at temperatures of 2,000-3,000° K

Certain achievements have been attained recently in this direction. Matters of measurement have appeared and are being improved, which go hand in hand with the development of modern means of experimental techniques. Vacuum electronics, radio technical means, systems for automatic registration and control are coming into wide use. One of the basic tests of this monograph is to generalize the accumulated experience in this field.

In the first place, we are speaking about the experience gained by the author in the course of his creative work with these colleagues in the Department of Molecular Physics of the Physical Faculty at Moscow State University. This group included the graduate students

L. A Pigal'skaya, Yu. N. Simonova, R. P. Yurchak, I. N. Makarenko, I. P. Mardykin, L. N. Trukhsnova, Senior Instructor A. N. Nurumbetov and a group of graduates from this department. General guidance was by Professor A. S. Predvoditelev. The results of the work by this group have been reported in approximately 30 scientific communications and have been discussed at 10 scientific conferences and meetings. This monograph systematizes the existing experimental data (including also the data not published before), and in connection with the comparison of the work by other authors, proper generalizations are made wherever this is possible. In accordance with this nature of the monograph, considerable place therein is taken up by original data; this amounts to more than one-half of the scope. At the same time, the author, however, while realizing the danger of an excessively subjective and, for this reason, unilateral presentation of the problems under consideration, strived as much as possible for a fuller inclusion and comparison of literature data.

The study of the nature of the properties of metals comprises an important field in modern physics.

The specific features of the class metals--the existence of collective electrons, comparatively free electrons--leads to the problem which is specific primarily for metals--the description of the behavior of the totality of electrons in metal. A fundamental aspect of this problem is the study of the role of electrons in processes of the transfer of heat and electricity, that is, in those processes for which this role is, if not the determining, then at least very important. The electron theory of metals is so far not in a position to give a satisfactory quantitative description of these processes. In connection with this, great importance should be assigned to the experimental study of heat conductivity and electrical conductivity of metals, including refractory metals at high temperatures.

In addition to heat conductivity in eletrical conductivity, the study of the heat capacity of metals is of great interest, the behavior of which is determined by the nature of the thermal motion, in particular, the study of the problems of the participation of electrons in the thermal motion and the explanation of the specific features of high-temperature behavior of heat capacity, which is noticeably different from its behavior at moderate temperatures.

As regards liquid metals, the study is an important component of the study of one of the most fundamental problems of molecular physics--the problem of the liquid state. Work in this direction is all the more important since little work has been done here. The number of experimental studies of the thermal properties of liquid metals is insignificant, and there are no firmly established correlations. There is no clear information regarding the nature of the temperature function of the heat capacity, heat conductivity and the Lorentz number. Theoretical studies are in the initial stage. A broad field of activity is opening up for researchers in this field.

In accordance with the general experimental directivity of the monograph, the author did not consider it plausible to pay great attention to the presentation of the theory of the properties under consideration, all the more so since a number of special monographs deal with this. Only the most fundamental conclusions of the theory are presented conspectively, emphasis being made to define the basic most characteristic experimental correlations.

In presenting the methods of the experimental study, the author was governed by the following considerations.

1. Of the methods for measuring the thermal parameters, special preference is merited by the methods which make it possible to obtain data for any one of the parameters, but for their combination, that is, methods for determining the complex--heat conductivity, temperature diffusivity, and heat capacity. A study of this combination of properties with the same object and under the same conditions make the process of measurement comparatively brief and make it possible to obtain mutually related and sometimes mutually controllable data. Particularly convenient in this respect are the methods which make it possible to obtain a totality of the characteristics under study from data of a single experiment.

2. Of the diverse thermal conditions used for measuring the thermal properties, great preference is merited by the periodic processes (regular thermal conditions of the third type). The utilization of periodic processes makes it possible to insure the natural multiple repetition of an experiment under the same conditions and, what is most important, it yields an amount of information on the thermal properties of the medium under study, which is considerably greater in comparison with other processes. Actually, the source of information in the case of periodic processes can be: constant component of the temperature, amplitude of the variable component, its

phase, and, finally, its frequency composition (or the results of a variation in the amplitude and phase as a function of frequency for the case of a harmonic signal). This makes it possible to carry out diverse methods of measurement of the thermal properties which control one another within the limits of the same experiment. For the same reason, the utilization of periodic processes makes it possible to obtain the entire complex of thermal parameters from the results of a single experiment.

The monograph devotes considerable space to the description of the methods that are based on the utilization of periodic processes. Many of the methods described are original.

3. The methods of measurements, which are intended for use with small objects such as wire, foil, drops of liquid metal, deserve great attention. The utilization of these specimens makes it possible to get by without cumbersome installations that require much power, reduce the experimental time, and makes it possible to carry out measurements with comparatively simple means at temperatures up to 3,000°. Besides that, these methods, in principle, make it possible to carry out also complex measurements on the same object. Considerable space has also been allocated to work involving these methods.

In describing the methods of the experimental study, attention is, in the first place, given to the methods which are specially intended for high temperatures. The monograph does not include methods of classical calorimetry, which are used at high temperatures but are not specific for such high temperatures, all the more so since these problems have been presented sufficiently and fully in proper monographs.

The author considers it his duty to express gratitude to the corresponding member of the Academy of Sciences USSR, Professor A. S. Predvoditelev, for his constant attention to this work.

Chapter I

General Problems of the Procedure and the Experimental Technique

§1. Analysis of General Problems of the Experiment

The basis of all methods for measuring thermal properties is the Fourier law

$$\vec{q} = -\lambda \operatorname{grad} T \tag{1.1}$$

($\vec{q}$ is the specific heat flow, λ is the heat conductivity coefficient which will henceforth be called the heat conductivity) and the determination of the specific heat capacity

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p, \tag{1.2}$$

where h is the specific enthalpy.

The magnitudes that are directly measured in the experiment are the spacial and time changes of the temperature in the test objects and the amount of energy liberated by the sources of heat in the volume are on the surface of these objects.

The relationship of these magnitudes is expressed by the heat conductivity equation

$$c_p \rho \frac{\partial T}{\partial t} = \operatorname{div}(\lambda \operatorname{grad} T) + w, \tag{1.3}$$

where w is the specific power of the volume source of energy.

In using this equation for solving specific problems, it is necessary to set definite boundary conditions. Those usually taken as such are:

1) Boundary conditions for the first type when a known (experimentally determined) temperature on the surface of the body is proposed as a function of the coordinates and of the time:

2) Boundary conditions of the second type: the assignment of a specific heat flow q on the surface

$$\lambda \frac{\partial T}{\partial n} = q, \tag{1.4}$$

where $\frac{\partial}{\partial n}$ is the derivative with respect to the normals;

3) Boundary conditions of the fourth type in case of contact of two solid bodies

$$\lambda_1 \frac{\partial T_1}{\partial n} = \lambda_2 \frac{\partial T_2}{\partial n}; \; T_1 = T_2; \tag{1.5}$$

4) Generalized boundary conditions of the third type

$$\lambda \frac{\partial T}{\partial n} + \alpha (T - T_0) = q, \tag{1.6}$$

where α is the heat exchange coefficient, q is the density of the surface source of heat. (Equation (1.6) is the equation of the energy balance on the surface, the sum of the heat flow in the body and of the heat loss to the surrounding medium is equal to the power of the extraneous source.)

The heat conductivity equation (1.3), and the absence of heat liberation and when the temperature function of the heat conductivity coefficient can be disregarded, takes on a simpler usual form

$$\frac{1}{a} \cdot \frac{\partial T}{\partial t} = \nabla^2 T, \tag{1.7}$$

where

$$a = \frac{\lambda}{c_p \rho} \tag{1.8}$$

The temperature diffusivity (temperature diffusivity coefficient), while

$$\nabla^2 \equiv \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} -$$ is the Laplace operator.

In examining the type of the basic correlations (1.3)--(1.8), one can make the following general conclusions.

1. Experiments in which only the temperatures are being measured can yield information only about the temperature diffusivity of the test body. In this case, it is necessary to study the time function of the temperatures (non-steady experiments) because only in this case will equation (1.7) have solutions that contain as a parameter the magnitude of the temperature diffusivity α.

2. In experiments dealing with the measurement of temperature diffusivity in the absence of modern sources of heat, there is no need to know the absolute values of the measured temperatures; the temperature can be measured with accuracy up to an arbitrary factor. (This follows from the type of the equation (1.7) which remains invariant relative to the linear transformation T' = cT.) In this lies the important advantage of experiments for the determination of the temperature diffusivity.

The observed circumstance, however, does not mean that in such an experiment there is completely no need for measurements of the absolute values of the temperature. Absolute measurements are, of course, necessary in order to determine the temperature to which the results of the measurement of the temperature diffusivity should be referred. At the same time, the temperature field and the changes with time do not require absolute measurements.

3. In order to determine the heat conductivity in using boundary conditions of the first and second type, it is necessary to measure the power of the volume or surface sources of energy.

In using boundary conditions of the fourth type, the heat conductivity of the test medium λ should be determined on the basis of the heat conductivity λ_2 of another, standard body--relative method of measurements.

In using boundary conditions of the third type, the heat conductivity can, in principle, be determined if the heat exchange coefficient α is known. In some rather rare cases (for heat exchange through emission provided the coefficient of the emitting capacity of the body is known, see below), the heat exchange coefficient can be determined by calculation, and it is not mandatory to measure the power. However, if α is determined from an additional experiment, then in this experiment measurements of the power are mandatory and the heat conductivity, in the final analysis, is always determined by measuring the power; the heat exchange coefficient plays the role of an intermediate link in this case.

4. In purely steady experiments, determinations can be made only of the heat conductivity of the heat exchange coefficient and (the case when $\partial/\partial t \equiv 0$, equation (1.3) in conjunctivity with the boundary conditions does not contain other parameters).

5. The heat capacity can be determined from experiments of a quasi-steady type when the rate of the process which takes place with time is not essential.

Actually, it follows from equation (1.3) when $\operatorname{div} \lambda \operatorname{grad} T \ll w$ that

$$c_p\rho = \frac{\int_{t_1}^{t_2} w\,dt}{T_1 - T_2}.$$

This relationship forms the basis of the calorimetric experiment in which the values $Q = \int_{t_1}^{t_2} w\,dt$ (the amount of heat introduced into the system) are vital and $T_1 - T_2$ (rise in temperature) are also essential, while the nature of the function w(t) does not play a direct role.

6. A study of the unsteady processes makes it possible to determine, generally speaking, all three thermophysical magnitudes, heat conductivity, heat capacity and temperature diffusivity. This is an important quality of nonsteady methods.

For an analysis of the general problems of the experiment, it is expedient to utilize the methods of the theory of similitude.

The theory of similitude represents a device which makes it possible to disseminate information obtained for a definite number of objects (systems, processes) over a much broader diversity of these. This is achieved by reducing the number of independent magnitudes (variables and parameters) which participated in the formulation and the solution of the problems under consideration by means of grouping these magnitudes into dimensionless combinations--complexes. The corresponding functions are expressed, in this manner, as a relationship between complexes, while the equations are expressed as a relationship of dimensionless operators. In this instance, the form of the dimensionless functions is identical for multiplicity of processes which satisfy the so-called conditions of similitude. The number of required conditions of similitude include the requirement of geometric similitude of the systems under consideration and the condition of equality of the so-called determining criteria--complexes which are composed of parameters that figure in the equations that describe the process and in the boundary conditions. A detailed presentation of the fundamentals of the theory of similitude can be found in monographs [2-5].

Let us examine the basic determining criteria of the heat conductivity problems. In this case we assume, as this is usually done, that $c_p \rho$ and λ are constants. It follows from the type of basic heat conductivity equation (1.3) that there should exist two independent criteria

$$K_1 = \frac{\lambda t^*}{c_p \rho l^2} = \frac{a t^*}{l^2} = Fo \tag{1.9}$$

and

$$K_2 = \frac{w t^*}{c_p \rho T^*} = Po. \tag{1.10}$$

In these formulas, l, t*, and T* indicate a characteristic plant, time and temperature (difference of temperatures) which figure in the boundary conditions. The criterion K_1 is called the Fourier criteria. K_2 has been designated by A. V. Lykov as the Pemrantsev criterion [6].

The dimensionless form of equation (1.3) has the form

$$\frac{\partial \theta}{\partial \tau} = Fo\left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial^2 \theta}{\partial \xi^2}\right) + Po, \tag{1.11}$$

where $\theta = T/T^*$ is the dimensionless temperature,

$\tau = t/t^*$ is the dimensionless time, and

$\xi = \frac{x}{l};\ \eta = \frac{y}{l};\ \zeta = \frac{z}{l}$ are the dimensionless coordinates.

Two other determining criteria figure in the dimensionless form of the boundary condition (1.6):

$$\frac{\partial \theta}{\partial \nu} + Bi(\theta - \theta_0) = Ki. \tag{1.12}$$

here, $\frac{\partial}{\partial \nu}$ indicates the derivative with respect to the dimensionless normal,

$$Bi = \frac{\alpha l}{\lambda}, \tag{1.13}$$

$$Ki = \frac{ql}{\lambda T_*}. \tag{1.14}$$

The ratio (simplex) $\theta_0 = \frac{T_0}{T_*}$, , generally speaking, is a function of ξ, η, ζ and τ, because T_0 in the general case can depend on the coordinates and on the time. The first of these criteria is called the Biot criterion and the second the Kirpichev criterion.

The solution of equation (1.11) and for the boundary conditions (1.12) should have the form

$$\theta = \Phi(\xi, \eta, \zeta, \tau, Fo, Po, Bi, Ki). \tag{1.15}$$

In order to determine the thermophysical coefficients, use is made of the specific functions of the type (1.15) which contain 1-2 criteria, the magnitudes of which should be determined from the experimental values of the function Φ.

Let us examine the problem of the optimum values of the criteria to be determined. Let, for example,

$$\theta = \Phi(\xi, \eta, \zeta, \tau, K), \tag{1.16}$$

where K is the criteria that is to be determined. Let us assume at first that $K \ll 1$. In this case, Φ can be expanded into a series with respect to K and we can limit ourselves to the first member of the expansion:

$$\theta = \Phi_0(\xi, \eta, \zeta, \tau) + K\Phi_1(\xi, \eta, \zeta, \tau) + \dots \qquad (1.17)$$

The functions Φ_0 and Φ_1 do not contain dimensional coefficients and, for this reason, it is natural to expect that with respect to the order of magnitudes they lie between 0 and 1 (the opposite would indicate the existence of special points in the solution of the corresponding problem; we exclude such a case). If in formula (1.17) $\Phi_0 \sim \theta$, for the small K and $\theta \sim 0$, that is, the measured changes of the temperatures fall and the case is not suitable for practical purposes. However, if $\Phi_0 \sim 1$, then, when $K \to 0$ there is a trivial solution and K should be determined from the small difference between the trivial solution and the temperature test field θ. Such a case is also unsuitable for practical purposes because of the large errors. As a result, we come to the conclusion that the case of $K \ll 1$ is unfavorable for the measurement of the thermal properties. Exactly the same reasoning can be carried out for the case of $K \gg 1$; for this purpose, it is necessary to examine the expansion of Φ into a series with respect to K^{-1}. Thus, it follows from general considerations that, in order to carry out the experiment on the determination of the thermal properties, it is necessary to utilize such processes for which the values of the corresponding criteria have a magnitude of the order of unity. Thus, for measurements of heat conductivity, it is necessary that

$$Ki \sim 1, \qquad (1.18)$$

for measurements of temperature diffusivity

$$Fo \sim 1, \qquad (1.19)$$

for heat capacity, respectively,

$$Po \sim 1 \qquad (1.20)$$

or

$$KiFo = \frac{t^{*}q}{c_{p}\rho l T^{*}} \sim 1. \qquad (1.21)$$

The conditions (1.18)-(1.21) determine only the order of magnitude of the criteria. The specific boundaries of the region of optimum conditions of the experiment should be established in each specific case.

One more conclusion can be made on the basis of the given reasoning. It is precisely when the influence of any one of the criteria in the expression (1.15) is undesirable that it is necessary to accomplish conditions for which this criterion will be much less than unity unity and, conversely, much more than it. Thus, an undesirable criterion is usually the Biot criterion which contains the heat exchange process on the of the body. In order to decrease the role of the heat exchange, it is necessary thus to make it a relative magnitude of very small (heat insulation) or, conversely, very large (isothermal conditions).

All the considerations presented earlier are general and valid for high as well as for moderate and low temperatures. The high temperature reading which is of interest to us has its specificity. It consists, first of all, of the fact that the basic mechanism of heat exchange of free boundaries of a body is the heat exchange by emission; besides, the magnitude of this heat exchange is, as a rule, considerable and depends strongly on the temperature. This leads to the situation wherein, in many practically important cases, the Biot criterion is commeasurable with unity, as a result of which it becomes necessary to make a special calculation or to exclude a role of heat exchange. Either one is, however, made very difficult by the steep nonlinear dependence of the heat exchange coefficient on the temperature.

Let us examine this program in greater detail. In order to describe the heat exchange by emission between an element of the surface of the body under consideration and its surrounding isothermal shell at the temperature of T_0, use is usually made of the approximate formula

$$q' = \sigma\varepsilon\varphi (T^4 - T_0^4), \tag{1.22}$$

where σ is a constant of the Stephen Baltzmann law, ε is the reduced degree of blackness, ϕ is the coefficient of irradiance [7,8]. The heat exchange coefficient in this case can be formally recorded as

$$\alpha = \sigma\varepsilon\varphi (T^3 + T^2 T_0 + T_0^2 T + T_0^3). \tag{1.23}$$

The vital dependence of the coefficient α on the temperature makes extremely difficult the analytic solution of the corresponding problems even in the simplest cases. A way out from the situation is the utilization of the experiment of small spacial-time changes of the temperature, small temperature perturbations. Such small perturbations can be the result of small changes in the power of the surface and volume sources of energy or of the magnitude T_0--the temperature of the surrounding medium. Under these conditions, the temperature T of the body can be represented as the sum of the basic member T and the small increment ϑ :

$$T = \overline{T} + \vartheta, \tag{1.24}$$

$$\frac{\vartheta}{\overline{T}} \ll 1. \tag{1.25}$$

In the general case $\overline{T}$ is a function of the coordinates and of the time.

A specific case of the notation of (1.24) can be considered as the result of the expansion of T(x,y,z,t) - into a series close to the values of x_0, y_0, z_0, t_0 (the examination of the small local changes of temperature). In this case, $T = T(x_0, y_0, z_0, t_0)$

For the condition (1.25), the nonlinear dependence of the emission heat exchange on the temperature can be approximately linear due to the possibility of disregarding the members of the higher order of smallness in comparison with $\vartheta/\bar{T}$. Actually, in this case the condition (1.6) can be recorded approximately (with an accuracy up to the members of the higher order of smallness relative to ϑ) as

$$\lambda \frac{\partial\bar{T}}{\partial n} + \alpha(\bar{T})(\bar{T} - T_0) - q_0 = -\lambda \frac{\partial\vartheta}{\partial n} + \vartheta \left\{ \frac{\partial}{\partial T} [\alpha(T)(T - T_0)] \right\}_{T=\bar{T}} + \delta q. \tag{1.26}$$

Here, δq is the change in density of the surface source of energy, which leads to the origin of the test perturbation and the temperature ϑ. (If the change in temperature ϑ is due in its origin to the change in T_0, then instead of δq in this, the magnitude $\left[\frac{\partial\alpha(T - T_0)}{\partial T_0} \right]_{\bar{T}} \delta T.$ should figure obviously in this formula. This member will be present also when T_0 changes simultaneously with the change in q; in this instance, the relation δT_0 and δq should be known.)

The last part of the equation (1.26), which does not contain a magnitude of the temperature deviation ϑ, is equal to zero because the magnitude $\bar{T}$ is in a sense the solution of the problem under consideration and should satisfy the corresponding boundary conditions. Hence, it follows that ϑ should satisfy the boundary condition

$$\lambda \frac{\partial\vartheta}{\partial n} + \vartheta \left(\frac{\partial\alpha(T - T_0)}{\partial T} \right)_{T=\bar{T}} = \delta q. \tag{1.27}$$

If α does not depend on T, then we get a trivial result

$$\lambda \frac{\partial \vartheta}{\partial n} + \alpha\vartheta = \delta q.$$

The condition (1.27) has the same linear form also in the general case because the magnitude

$$\alpha' = \left(\frac{\partial \alpha (T - T_0)}{\partial T} \right)_{T=\bar{T}} \tag{1.28}$$

does not depend on ϑ

$$\lambda \frac{\partial \vartheta}{\partial n} + \alpha'\vartheta = \delta q. \tag{1.29}$$

Thereby, the condition (1.6), which is nonlinear in the general case for small changes in temperature (1.25), reduces itself to a linear condition. In this case, however, it is necessary to bear in mind that $\alpha' = \alpha'(\bar{T})$, generally speaking, is a function of the coordinates of the point on the surface of the body because $\bar{T}$ can depend on the coordinates. This circumstance makes it difficult to solve the corresponding problems even in such a linear approximation, particularly if one takes into account That $\bar{T}$ is practically assigned not in the form of a convenient analytic function, but as the result of an experiment for the straight multiplicity of points. Hence, it follows that the practically convenient cases of the perturbation method under consideration are only those for which $\bar{T}$ = const or the function $\bar{T}(x,y,z)$ has a symmetry for which $\bar{T}$ = const on the surface of the test body.

For emission heat exchange, the magnitude of the effective coefficient of heat exchange α' is, according to formulas (1.23) and (1.28), equal to

$$\alpha' = \sigma\varepsilon\varphi\left(4 + \frac{\partial \ln \varepsilon}{\partial \ln T}\right)\bar{T}^3. \tag{1.30}$$

A condition, sufficient for the validity of this formula, is, as it is easy to show, the inequality

$$\frac{\vartheta}{\bar{T}} \cdot \frac{1}{2} \cdot \frac{12 + 8\dfrac{\partial \ln \varepsilon}{\partial \ln T} + \dfrac{\bar{T}^2}{\varepsilon} \cdot \dfrac{\partial^2 \varepsilon}{\partial T^2}}{4 + \dfrac{\partial \ln \varepsilon}{\partial \ln T}} \ll 1,$$

or approximately, with an accuracy up to the temperature dependence or the degree of blackness,

$$\frac{3}{2} \cdot \frac{\vartheta}{\bar{T}} \ll 1,$$

which is practically equivalent to the inequality (1.25).

We obtain the basic form of the heat conductivity equation for small temperature perturbation ϑ by substituting formula (1.24) into equation (1.3) with consideration of the fact that $\bar{T}$ is the solution of the equation (1.3):

$$\left[(c_p\rho)_{\bar{T}} + \left(\frac{\partial c_p\rho}{\partial T}\right)_{\bar{T}}\vartheta\right]\frac{\partial \vartheta}{\partial t} + \left(\frac{\partial c_p\rho}{\partial T}\right)_{\bar{T}}\vartheta\frac{\partial T}{\partial t} =$$
$$= \operatorname{div}\left[\left(\frac{\partial \lambda}{\partial T}\right)_{\bar{T}} \operatorname{grad} \bar{T}\right] + \operatorname{div}\left[\left(\lambda_{\bar{T}} + \left(\frac{\partial \lambda}{\partial T}\right)_{\bar{T}}\right)\operatorname{grad}\vartheta\right] + \delta\omega = 0. \tag{1.31}$$

Here δw is the perturbation part of the density of the volume heat liberation. (In deriving this equation, the members of the expansion $c_p \rho$ and λ into the series with respect to ϑ above the first order of ϑ have been discarded.) For simplification of equation (1.31), we assume that

$$\left.\begin{aligned} &\frac{\partial \ln c_p \rho}{\partial T} \vartheta \ll 1, \\ &\frac{\partial \ln \lambda}{\partial T} \vartheta \ll 1, \end{aligned}\right\} \qquad (1.32')$$

$$\frac{\partial \vartheta}{\partial t} \sim \frac{\partial \overline{T}}{\partial t} \text{ and } \operatorname{grad} \overline{T} \sim \operatorname{grad} \vartheta. \qquad (1.32'')$$

The conditions (1.32), which are superposed on ϑ, are, as a rule (with the exception of special cases of an anomalously large change in $c_p \rho$ and λ near the points of phase transformations), considerably less rigid than the condition (1.25), in accordance with the lesser temperature dependence of the magnitude $c_p \rho$ and λ, in comparison with q' in the formula (1.22). The conditions (1.32") require that the spacial-time changes $\Delta \overline{T}$ of the basic temperature field should not exceed in the order of magnitude the temperature perturbation ϑ ($\Delta \overline{T} \sim \vartheta$). under consideration.

In satisfying the conditions (1.32), the equation (1.31) assumes a form of

$$c_p \rho \frac{\partial \vartheta}{\partial t} = \lambda \nabla^2 \vartheta + \delta \omega, \qquad (1.33)$$

with $c_p \rho$ and λ refer to a certain fixed point of the temperature field $\overline{T}$. We wish to emphasize once more that the limitations of the spacial-time changes in temperature (inequality (1.32)), which are necessary for the linearization of the heat conductivity equation, are weaker than the limitations (1.25) which are required for the linearization of the boundary conditions. In this lies the specificity of the high temperature region. (For moderate temperatures, only the first limitations are essential.)

In the linearization of the expression for emission heat exchange, the formula for the Biot criterion can, in accordance with formula (1.30), be written as

$$Bi = \frac{4\sigma\varepsilon\varphi T^3 l}{\lambda} \tag{1.34}$$

(in the case when $\frac{\partial \ln \varepsilon}{\partial \ln T} \ll 4$) , or, if $T_0 << T$, as

$$Bi = \frac{4q' l}{\lambda T}. \tag{1.35}$$

In such a form, the Biot criterion is similar with the Kirpichev criterion (1.14).

Table 1 is given in order to evaluate the order of magnitude of the Bi criterion and the experiments with metals at high temperatures; this table contains the Biot numbers for a number of values of l as a function of the temperature for three different values of λ. If magnitude of $\varepsilon\phi$ in this case is set equal to its maximum value, unity.

As can be seen from this table the Biot criterion is sufficiently small for characteristic dimensions of the order of 1-10 mm (for the latter up to 2,000°K). For larger dimensions there are no grounds for hoping for a smallness of heat exchange through emission. Hence, it follows that the rational tendency in accomplishing experiments at high temperatures is to strive to reduce the dimensions of the test specimens. Small scale methods are preferred also in another respect-

Table 1. Values of the Biot Criterion for Emission Heat Exchange.

T°K	$l = 0{,}1$ мм			$l = 1$ мм			$l = 1$ см			$l = 10$ см		
	λ wt/cm. degree			λ' wt/cm. degree			λ wt/cm. deg.			λ' wt/cm.deg.		
	4	1	0,2	4	1	0,2	4	1	0,2	4	1	0,2
1000	$5{,}8\cdot10^{-5}$	$2{,}3\cdot10^{-4}$	$1{,}1\cdot10^{-3}$	$5{,}8\cdot10^{-4}$	$2{,}3\cdot10^{-3}$	0,011	$5{,}8\cdot10^{-3}$	0,023	0,11	0,058	0,23	1,1
1200	$1\cdot10^{-4}$	$4\cdot10^{-4}$	$2\cdot10^{-3}$	$1\cdot10^{-3}$	$4\cdot10^{-3}$	0,02	0,01	0,04	0,2	0,1	0,4	2
1500	$1{,}9\cdot10^{-4}$	$7{,}7\cdot10^{-4}$	$3{,}7\cdot10^{-3}$	$1{,}9\cdot10^{-3}$	$7{,}7\cdot10^{-3}$	0,037	0,019	0,077	0,37	0,19	0,77	3,7
1700	$2{,}8\cdot10^{-4}$	$1{,}1\cdot10^{-3}$	$5{,}5\cdot10^{-3}$	$2{,}8\cdot10^{-3}$	0,011	0,055	0,028	0,11	0,55	0,28	1,1	5,5
2000	$4{,}5\cdot10^{-4}$	$1{,}8\cdot10^{-3}$	$9{,}0\cdot10^{-3}$	$4{,}5\cdot10^{-3}$	0,018	0,09	0,015	0,18	0,9	0,45	1,8	9
2200	$6\cdot10^{-4}$	$2{,}4\cdot10^{-3}$	0,012	$6\cdot10^{-3}$	0,024	0,12	0,06	0,24	1,2	0,6	2,4	12
2500	$9\cdot10^{-4}$	$3{,}6\cdot10^{-3}$	0,018	$9\cdot10^{-3}$	0,036	0,18	0,09	0,36	1,8	0,9	3,6	18
2700	$1{,}1\cdot10^{-3}$	$4{,}6\cdot10^{-3}$	0,022	0,011	0,046	0,22	0,11	0,46	2,2	1,1	4,6	22
3000	$1{,}5\cdot10^{-3}$	$6{,}2\cdot10^{-3}$	0,031	0,015	0,062	0,31	0,15	0,62	3,1	1,5	6,2	31

--they require a small expenditure of energy for heating, whereas for relatively large specimens, the energy consumed can be very considerable, which can be elucidated from Table 2.

Table 2. Energy Emitted From the Surface of the Specimen (ε = 1), in wt.

T °K	Surface		
	0,1 см²	1 см²	10 см²
1000	0,57	5,7	57
1200	1,2	12	120
1500	2,9	29	290
2000	9	90	900
2200	13	130	1300
2500	22	220	2200
2700	30	300	3000
3000	46	460	4600

§2. Methods of Measuring Small Differences of High Temperatures

It should be clear from §1 that a necessary element in thermophysical studies at high temperatures is the measurement of small differences of high temperatures and small time changes in temperature (pulsations). In this paragraph we will dwell on the first of these problems--measurement of steady temperature differences.

First of all, we shall clarify why we are speaking precisely about temperature differences and not simply about temperature fields. From a fundamental point of view, it is indifferent whether the absolute temperature in the function of the coordinates T = T(x,y,z) is measured or the difference of temperatures relative to a fixed point $\vartheta = T(x,y,z) - T(x_0,y_0,z_0)$. In practice, however, it is preferred to measure precisely differences. This is related with the fact that in absolute measurements of the temperature the allowable relative error of measurements should be much smaller than in measurements of the differences. Actually, if the required error of measurement of the magnitude ϑ should be equal to $\delta\vartheta/\vartheta$, then the relative error of the measurements of the absolute temperatures should in this case amount to

$$\frac{\delta\vartheta}{T+\vartheta} \approx \frac{\delta\vartheta}{T} = \frac{\delta\vartheta}{\vartheta} \cdot \frac{\vartheta}{T},$$

that is it should be ϑ/T times less. Thus, if in the experiment at the moderate temperature of 1,000°K it is required to determine a temperature difference of 10°, (which corresponds to the condition (1.25)) with an accuracy of 1% (three significant figures), then in order to measure the absolute temperatures, the allowable relative error should amount to 0.1°/1,000° = 0.01%, that is, the results of the measurements should have five significant figures. Further, measurements of absolute temperatures are usually classified with respect to time and, for this reason, a possible instability of the average temperature can lead to an additional error; for measurements of temperature difference, this error will be considerably less. Moreover, measurements of the temperature difference are possible also under quasisteady conditions when the temperature at each point of the system depends on the time, while the difference of the temperatures between any points remains constant (the so-called regular thermal condition of the second type--see §7).

In measuring differences of high temperatures, use is made essentially of two methods: thermocouple and optical (photoelectric). Let us examine each of these separately.

The problem of the utilization of thermocouples for measuring temperatures, including also high temperatures, is dealt with in many review papers (see for example [9, 10, 11, 12]). In this monograph, we shall dwell only on problems characteristic of measurements of temperature differences and on certain problems specific for high temperatures.

In measuring steady differences of temperature in metals, two different problems are encountered: the measurement of temperature differences in the body of metal specimens and the measurement of the temperature field of metal surfaces. Let us dwell at first on the first problem. In measuring temperatures in the body of specimens, it is necessary to have a lead of thermocouple wires to drill channels. On the basis of general considerations, it is clear that these channels should be as much as possible narrow and long and fall on isotherms.

In this case, the junctions of the thermocouples (or at least of one of these) should not have an electrical contact with the body of the metal; for otherwise, the metal will short circuit the thermocouple and measurements will require special devices (see below). One of the basic questions in this instance is that as to which temperature the thermo couple joint will register in this case and the extent to which this temperature differs from the temperature in a given point to the metal in the absence of a grill channel. It is difficult to give a general answer to this question because the result depends on the specific configuration resystem and temperature field. However, certain considerations can be advanced.

Let us examine the case when the temperature, in the absence of a thermocouple, depends linearly on one coordinate z. Let a channel be drilled in the point z = 0 and let the thermocouple be placed in this channel. Let us assume that a channel is infinitely long so that the temperature along it along the element is the same (thereby we shall disregard the loss of heat along the thermocouple and consider only the distortion of the temperature field caused by the introduction of the thermocouple).

Assuming that the radius r_0 is much less than its distance to the boundaries of the body, it can be assumed that "at infinity" (that is, at a sufficient distance from the drilled channel) there is an unperturbed temperature field

$$T = Uz. \tag{2.1}$$

The problem of determining the temperature field within the outside and within the channel reduces itself to the solution of steady state equations of heat conductivity $\Delta^2 T_1 = 0$ and $\Delta^2 T_2 = 0$ with the boundary condition of the fourth type on the surface of the drilled channel $\lambda_1 \frac{\partial T_1}{\partial r} = \lambda_2 \frac{\partial T_2}{\partial r}$; $T_1 = T_2$ (it is assumed that the channel was filled with a uniform medium with a conductivity of λ_2) and with the condition (2.1) at infinity. The solution of this problem can be found without much labor. However, the basic result can be obtained also from considerations of a general order. For this purpose, we wish to observe that from the formulation of the problem stems the oddness of the functions T_1 and T_2 with respect to the coordinate z. Actually, the substitution of z by -z with the simultaneous substitution of T_1 by $-T_1$ and T_2 by $-T_2$ leads to the identical problem. Hence, it follows that from z = 0, $T_1 = 0$, that is, the temperature on the channel axis is an accuracy equal to the unperturbed temperature--such a schematic thermocouple measures the temperature without distortions. The same considerations are valid also in a more complex case (for more detailed model of the thermocouple) when there are in the channel several coaxial sheets with any thermal characteristics (thermocouple wire, its insulation), and for a temperature field of more complex nature but odd with respect to the coordinate z.

In measuring the temperature differences on the surface of metals specimens, the electrical contact between the thermocouple junction and the metal is needed because, otherwise, it would be difficult to assure a sufficiently good thermal contact. In this case, the direct utilization of differential thermocouples is excluded. Nevertheless, here too different measurements are possible. One of such methods--the use of fast electromechanical switches in conjunction with "memorizing" condensers. The condensers are periodically connected either to the thermocouples or serially in opposition to each other because a different signal flows into the measurement system [13]. Another method is the connection of thermocouples parallel to the measurement instrument provided the resistances of the thermocouple wires are the same in accuracy [14]. Such a method is simpler; a disadvantage, however, is that it is necessary to equalize very thoroughly the resistances of the wires.

Another method of measuring temperature differences on metal surfaces is the utilization of the test metal as a thermalelectrode. In this method, thin wires are welded (soldered) at the required points of the metal; these are probes of a metal which gives a sufficiently large thermoelectromotive force paired with the test metal. The thermal electromotive force from each pair of probes, just as from an ordinary differential thermocouple is determined by the temperature difference of the corresponding points. In using probes, only one wire and not two runs to each point just as in the case of thermocouples. In accordance with this, there is a decrease in their indeterminacy of the coordinates of the points, the temperatures of which are being measured; the position of each point can be considered as known with an accuracy up to the radius of the probe. Advantages of the probe methods are particularly acceptable when measurements of the temperature field are required in many points of a small region of the object. A disadvantage of the measurements of the temperature difference by means of probes is that it is necessary to know the thermoelectromotive force of the pair of metals used and of which one is the test metal. This requires the performance of a special calibration experiment. The indicated disadvantage, however, drops out when it is necessary to know not the absolute values of the temperature differences but only their relative magnitudes.

The principle which forms a basis of the probe method of measuring temperature differences, namely the utilization of a test metal as an intermediate, can be used also for other purposes--in measuring absolute temperatures. In this case, when the surface of the test metal is isothermal (or an isothermal line is known on the surface), instead of a triple junction of thermocouple wires with the surface of the metal, one can make two separate junctions of each of the wires with the surface which are apart from each other at a distance that is convenient for their installation. The thermocouple readings should not in this case depend on the test metal.

The above said with respect to the probe differential thermocouples is valid not only for surface thermocouples. Such thermocouples can equally be utilized also for measuring temperatures in the volume of a metal.

In measuring temperatures of a metal surface by means of thermocouples, one of the basic questions is that of errors introduced by the heat loss along the thermocouple wire. Two cases should be differentiated: 1) heat exchange with the surface of the thermocouple in accordance with the Newtonian principle (the coefficient of heat exchanges is constant) and 2) the coefficient of heat exchange cannot be considered as constant.

The first case is encountered under conditions when the temperature readings along the thermocouple are comparatively small, that is, when the test specimen is not strongly overheated with respect to the surrounding space (although its temperature can be high). The second case corresponds to a strong overheating of the specimen (for example, a calcine specimen in a cold vacuum chamber). Under these conditions, very large temperature gradients can exist on the thermocouple wires, and the nonlinearity of the emission heat exchange makes impossible the utilization of Newton's law with a heat exchange coefficient that is independent of the temperature.

The solution of the problem of errors in the readings of the thermocouples is classified into two parts: 1) the calculation of the thermal flow at the end of the thermocouple on the basis of a solution of the heat conductivity equation for a thermocouple with consideration of the heat exchange on its lateral surface and 2) determination of the distortion of the temperature field on the test body for which the flow of heat through a transverse cross-section of the thermocouple is equal to the magnitude that has been found. Such a partition of the problem is possible as a first approximation of a more accurate solution in which the solution of the first part (boundary conditions of this solution) already assumes the solution of the second part as known, that is, the accurate distribution of the temperature. It is clear, however, that for small distortions of the temperature field such a simplification is fully justified.

In studying the problem of the temperature distribution and the flow of heat along thermocouple wires, we shall examine each wire separately, disregarding the heat exchange between wires. Besides that, we shall disregard the temperature gradient along the radius of the wire, assuming as valid the unidimensional equations of heat conductivity

$$\lambda \frac{d^2T}{dx^2} - \frac{2\alpha}{r_0}(T - T_0) = 0 \tag{2.2}$$

and

$$\frac{d}{dx}\lambda\frac{dT}{dx} - \frac{2\sigma\varepsilon T^4}{r_0} = 0, \tag{2.3}$$

where r_0 is the radius of the wire, T_0 is the temperature of the surrounding medium (see §4). The first of these equations has been reckoned for the Newtonian heat exchange and the second for the emission heat exchange when $T \gg T_0$. (In both cases, only the heat exchange of the wire with the external medium is taken into consideration but not that with the body, which is permissible only for approximate evaluative calculations.) The boundary conditions of the equations (2.2) and (2.3) are the condition on the left-hand column $T = T(0)$

when x = 0 and the condition on the right-hand $T = T_0$ when $x \to \infty$ (2.2) and $T = 0$ for $x \to \infty$ for (2.3). The linear equation (2.2) is solved elementarily and leads to a formula for the flow of heat when x = 0:

$$Q = \sqrt{2}\pi\lambda^{1/2}\alpha^{1/2}r_0^{3/2}[T(0) - T_0] = \sqrt{2}\pi\lambda r_0 Bi^{1/2}[T(0) - T_0]. \qquad (2.4)$$

The equation (2.3) is not solved in elementary functions.

An analysis of the quadratures that are possible for the case when ε and λ can be considered constant, makes it possible, however, to obtain the formula [15]

$$Q = \frac{2}{\sqrt{5}}\pi\lambda^{1/2}\sigma^{1/2}\varepsilon^{1/2}r_0^{3/2}T(0)^{5/2}. \qquad (2.5)$$

By using the expression (1.34) for the Biot criterion in the case of emission heat exchange, we can write this formula in a form similar to (2.4):

$$Q = \frac{\pi}{\sqrt{5}}\lambda r_0 Bi^{1/2}T(0). \qquad (2.6)$$

Formulas (2.4) and (2.6) solve the first half of the problem under consideration.

In solving the second half of the problem, we make a simplifying assumption. Namely, let us assume that the place of contact of the wire with the surface of the specimen represents a semisphere with the radius r_0; besides, the distortion of the temperature on the surface of this semisphere is the same. Absolute error introduced by such an approximation should, with respect to the order of magnitude, be close to the temperature gradient in the metal at the distance of the order or r_0, which can change the result of the calculation by a factor of the order of unity. Distribution of the isotherms in the metal is shown schematically in Figure 1. This assumption gives grounds for representing the temperature distribution in the body of the metal in the form of

$$T = \theta(x, y, z) + \frac{A}{r}, \tag{2.7}$$

where θ is the temperature distribution in the absence of a junction (speaking about the isothermal condition of the contact, we thus have in mind T - θ).

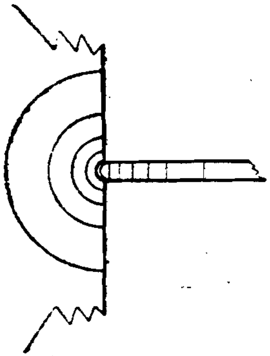


Figure 1. Schematic representation of the isotherms of the temperature distortions in the specimen--thermocouple system.

The second member of this formula, which describes the distorting influence of the thermocouple, satisfies the equation of the heat conductivity in the boundary conditions for the semi-space which is heat-insulated on the surface. An examination of the semi-limited medium practically means that influence of the transverse dimensions of the body in these evaluations is considered negligibly small, and this is natural for objects with dimensions that greatly exceed r_0 $\left(\frac{r_0}{R} \ll 1\right)$

The problem of the possibility of disregarding the heat exchange on the surface of the body requires consideration. A specific evaluation of the influence of the heat exchange on the result can be obtained from the comparison of the magnitudes of the specific thermal flows along the surface of the body and perpendicular to it (more accurately not the flows themselves but their changes due to the presence of the thermocouple). Measurement of the flow along the surface is equal to

$$q_l = \Lambda \frac{A}{r^2}, \tag{2.8}$$

where Λ is the heat conductivity of the specimen. A change in the flow due to the heat exchange is equal to

$$q_2 = \alpha' \delta T = \alpha' \frac{A}{r} \tag{2.9}$$

where α' is the heat exchange coefficient on the surface of the specimen. The ratio of these flows is equal to the Biot number

$$\frac{q_2}{q_1} = \frac{\alpha' r}{\Lambda} = Bi \tag{2.10}$$

For the most widely occurring case--emission heat exchange--one can utilize the values of the Biot criterion from Table 1. Besides, it is easy to become convinced that the values of the distance r, for which the Bi criterion is considerably less than unity, lies within the limits of 1 mm - 1 cm at temperatures of 1,000 - 2,000°, which is 1-2 orders greater than the radius of the thermocouple wire. Thus, formula (2.7) approximates real distribution of temperature and the specimen up to regions where the very temperature distortion becomes overly small.

In order to determine the constant A in the solution of (2.7), we shall utilize the above-determined expressions for heat flow (2.4) and (2.6). If

$$-\Lambda \frac{d(T-\theta)}{dr}\bigg|_{r=r_0} 2\pi r_0^2 = Q \tag{2.11}$$

we get

$$A = -\frac{Q}{2\pi\Lambda}. \tag{2.12}$$

from this follows the formula for the temperature distortion in the location of the junction

$$\delta T = \theta - T = \frac{Q}{2\pi\Lambda r_0}. \tag{2.13}$$

In particular, for the first case (small gradients on the wire)

$$\frac{\delta T}{T(0)} = \frac{1}{\sqrt{2}} \cdot \frac{\lambda^{1/2}}{\Lambda} \alpha^{1/2} r_0^{1/2} \frac{T(0)-T_0}{T(0)} = \frac{1}{\sqrt{2}} \cdot \frac{\lambda}{\Lambda} Bi^{1/2} \frac{T(0)-T_0}{T(0)}, \tag{2.14}$$

For the second case (large gradients)

$$\frac{\delta T}{T(0)} = \frac{1}{\sqrt{5}} \frac{(\varphi e r_0 \lambda T^3)^{1/2}}{\Lambda} = \frac{1}{2\sqrt{5}} \cdot \frac{\lambda}{\Lambda} Bi^{1/2}. \tag{2.15}$$

In both cases, the temperature distortion is comparatively slight, as the square root, and depends on the wire diameter and to a great degree on the heat conductivity of the material of the body then on the heat conductivity of the wire. For the first case, the magnitude of the heat exchange coefficient and the temperature difference

$T(0) - T_0$, are of fundamental significance, while for the second case it is the absolute temperature. It is pertinent to observe that in the attempt to obtain the formula (2.15) as the specific case of the formula (2.14) when $T(0) \gg T_0$ leads to results that differ approximately threefold. In order to estimate the specific values of δT druing evaluations with formula (2.15), Table 3 lists practical examples. It can be seen that for wires of a relatively large diameter and at high temperatures, the distorting influence of the heat loss can be considerable.

The examined system for evaluating the role of heat loss along the thermocouple wires can be utilized also for a case when the test object is a thin body--metal foil. Such a case can be encountered in practice during a study of heat conductivity and the electrical conductivity of metals (see § 5).

Assuming that the thickness of the foil is less than the radius of the wire or comparable with it (in the latter case, the resulting formulas will be valid with an accuracy up to the factor of the order of unity), one can disregard the temperature distribution across the foil and solve the unidimensional equation of heat conductivity in which the heat exchange on the surface of the foil plays the role of a source

$$\frac{d^2T}{dr^2} + \frac{1}{r}\cdot\frac{dT}{dr} = \frac{\alpha'}{h\Lambda}(T - T_0), \tag{2.16}$$

where α' is the heat exchange coefficient equal to the sum of the heat exchange coefficients on both sides of the foil, is the foil thickness. In this problem, it is necessary to take into account the heat exchange on the surface because in its absence the steady-state solution of the heat conductivity equation $T = \Theta + A \ln r$ does not satisfy the condition of infinity. Thus in this two-dimensional problem, unlike the problem for a three-dimensional body, the heat exchange on the surface of the body should play a substantial role.

Table 3. Examples of Evaluations of the Errors Due to the Heat Loss Along the Thermocouple Wires During Measurements of the Temperature of the Surface of a Massive Specimen.

Wire Diameter mm	Iron 1,000°K, Chromel Wires;	Wolfram Platinum Wires		
		1000°K	1500°K	2000 K
0,05	1,6°	0,6°	1,9°	5°
0,1	2,3	0,8	2,7	7
0,2	3	1,2	3,8	10
0,3	4	1,5	4,7	12
0,4	4,7	1,7	5,4	14
0,5	5,2	1,9	6	16

Equation (2.16) is nonlinear when there is emission heat exchange ($\alpha' = \alpha'(T)$), it is linearized for that small portion of the solution of ϑ, which describes the temperature change due to the presence of a junction with a wire

$$T = \theta + \vartheta, \tag{2.17}$$

$$\frac{d^2\vartheta}{dr^2} + \frac{1}{r} \cdot \frac{d\vartheta}{dr} = k\vartheta, \tag{2.18}$$

where

$$k = \frac{\alpha'}{h\lambda}. \tag{2.19}$$

For emission heat exchange

$$k = \frac{4\sigma(\varepsilon' + \varepsilon'')\vartheta^3}{h\lambda}. \tag{2.20}$$

The solution of equation (2.18) for r → ∞ can be written as

$$\vartheta = AH_0^{(1)}(i\sqrt{k}r), \tag{2.21}$$

where $H_0^{(1)}$ is the first Hankel function of zero order.

In order to determine the constant A, it is necessary to utilize the boundary condition similar to the conditions (2.11):

$$2\pi r_0 h\lambda \left.\frac{d\vartheta}{dr}\right|_{r=r_0} = Q. \tag{2.22}$$

(Here we disregard the small heat losses on the surface of the foil which lies opposite the wire end.) It follows from the solution of (2.21) and the condition (2.22) that

$$A = \frac{Qi}{2\pi r_0 h\lambda \sqrt{k} H_1^{(1)}(i\sqrt{k}r_0)}, \tag{2.23}$$

where $H_I^{(1)}$ is the first Hankel function of the first order. For the temperature distortion in the location of the junction, we get the formula

$$-\vartheta = \frac{Q}{2\pi r_0 h\lambda \sqrt{k}} \cdot \frac{iH_0^{(1)}(i\sqrt{k}r_0)}{-H_1^{(1)}(i\sqrt{k}r_0)}. \tag{2.24}$$

Before analyzing this expression, we shall evaluate the magnitude of the argument of the Hankel function. In accordance with formula (2.19)

$$r_0\sqrt{k} = \left(Bi_h^{1/2}\,\frac{r_0}{h}\right), \tag{2.25}$$

where

$$Bi_h = \frac{\alpha' h}{\Lambda}. \tag{2.26}$$

For conditions in which the Biot number is relatively small $Bi_h \ll 1$, the value of the argument $r_0\sqrt{k}$ is also small, because $\frac{r_0}{h} \leqslant 1$. In this case, instead of $iH_0^{(1)}/H_1^{(1)}$ one can utilize its asymptotic expression for small $r_0\sqrt{k}$, which leads to the formula

$$\delta T = \frac{Q}{2\pi h\Lambda}\ln\frac{2}{\gamma\sqrt{k}\,r_0}, \tag{2.27}$$

where $\gamma = 1.78107$ [16].

For heat exchange under conditions of small gradients, we get in this instance

$$\delta T = \frac{1}{\sqrt{2}}\,Bi^{1/2}\,\frac{\lambda}{\Lambda}\cdot\frac{r_0}{h}\ln\frac{2}{\gamma Bi_h^{1/2}\frac{r_0}{h}}\,[T(0) - T_0]. \tag{2.28}$$

(A more detailed derivation of this formula can be found in the monographs [17].)

For the case of large temperature gradients along the thermocouple, we get

$$\delta T = \frac{1}{2\sqrt{5}} Bi^{1/2} \frac{\lambda}{\Lambda} \cdot \frac{r_0}{h} \ln \frac{2}{\gamma Bi_h^{1/2} \frac{r_0}{h}} T. \qquad (2.29)$$

Formulas (2.28) and (2.29) are similar with (2.14) and (2.15). The difference is in the factor

$$\frac{r_0}{h} \ln \frac{2}{\gamma Bi_h^{1/2} \frac{r_0}{h}},$$

which increases the error δT in comparison with the case of a massive body. This factor depends substantially on the heat exchange conditions on the side surface of the bodies; besides, with increasing heat exchange, the entire factor becomes small. This is explained by the fact that in the case under consideration, the heat loss on the surface of the test metal plays a role of a factor which decreases the temperature gradients (in its absence, they would be infinitely large).

Examples of calculations with formula (2.29) are given in Table 4 (wolfram foil, platinum wire). One can see that relatively small magnitudes of the temperature distortion take place only when the wire diameter is approximately equal (or less than) the foil thickness, that is, when $\frac{h}{d} \sim 1-2.$. The utilization of formulas (2.28) and (2.29) for large values of this ratio is not valid because in this case the fundamental prerequisite of the calculation, the two-dimensional state of the temperature field in the foil--will be disturbed. At the same time, it is clear that for sufficiently large h/d, the magnitude δT will approach the values given by the formulas (2.14) and (2.15) which will thus become the maximum values in tables of type 4 tables.

Let us examine the photoelectric method of measuring small differences and high temperatures. This method is based on a comparison of the signals of photoelectric pickups which register the emission from two different sections of the test object.

Table 4. Examples of Evaluations of Errors Due to the Heat Loss Along the Thermocouple Wires With Measurements of the Temperature of Thin Specimens (Foils).

1000°K

d, мм	$h = 0{,}025$ мм	$h = 0{,}05$ мм	$h = 0{,}1$ мм	$h = 0{,}2$ мм
0,05	3,6°	1,9°	1,0°	—
0,1	8,8°	4,8°	2,5°	1,3°
0,2	21°	11°	6,2°	3,3°
		2000°K		
0,05	20°	11°	5,8°	—
0,1	47°	26°	14°	7.6°
0,2	108°	73°	34°	18°

The emission intensity of the surface of a body in the visible section of the spectrum at temperatures below 3,000°C can, as is known, be expressed by the formula [18]

$$I(\nu, T) = \varepsilon(\nu, T)\frac{2\nu^3 h}{c^2} e^{-\frac{h\nu}{kT}}, \qquad (2.30)$$

where ν is the emission frequency, h is the Planck constant, c is the speed of light, k is the Boltzmann constant. The factor $\varepsilon(\nu,T)$ is the spectral degree of blackness of the surface and it characterizes the difference in emission of the surface of the object from the emission of an absolutely black body. It is a comparatively weak function of the frequency and of the temperature.

The signal at the exit of the photoelectric pickup, which has a linear characteristic with respect to the emission intensity, will be equal to

$$Y=\int_0^{\infty} I(\nu,\ T)f(\nu)\,d\nu, \tag{2.31}$$

where f(ν) is the spectral characteristic of the system, which is the product of the spectral characteristic of the light filters employed by the magnitude of the spectral sensitivity of the pickup and the temperature factor.

The expression for the different signals of two identical pickups from the emission of two sections of one surface (or two identical surfaces) with temperatures of T and T + δT when

$$\frac{\delta T}{T}\ll 1, \tag{2.32}$$

can be represented as a series with respect to the powers of the parameter δT

$$\begin{aligned} Y(T+\delta T)-Y(T)&=\frac{\partial Y}{\partial T}\delta T+\frac{\partial^2 Y}{\partial T^2}\cdot\frac{(\partial T)^2}{2}+\ldots=\\ &=\delta T\int_0^{\infty}\left(\frac{\partial I}{\partial T}+\frac{\partial^2 I}{\partial T^2}\cdot\frac{\delta T}{2}+\ldots\right)f(\nu)\,d\nu. \end{aligned} \tag{2.33}$$

By taking into account the expression (2.30), this formula can be written as

$$Y(T+\delta T)-Y(T)\approx\frac{\delta T}{T}\int\limits_0^\infty If\left(\frac{h\nu}{kT}+T\frac{\partial\ln\varepsilon}{\partial T}\right)d\nu+$$

$$+\left(\frac{\partial T}{T}\right)^2\int\limits_0^\infty If\left(-\frac{h\nu}{kT}+\left(\frac{h\nu}{kT}\right)^2\cdot\frac{1}{2}+\frac{T^2}{2}\cdot\frac{d^2\ln\varepsilon}{\partial T^2}\right)d\nu+$$

$$+\left(\frac{\delta T}{T}\right)^2\frac{1}{2}\int\limits_0^\infty If\left(\frac{h\nu}{kT}+\frac{\partial\ln\varepsilon}{\partial T}\right)T\frac{\partial\ln\varepsilon}{\partial T}d\nu. \qquad (2.34)$$

In this expression, the members with the derivatives $T\frac{\partial\ln\varepsilon}{\partial T}$ and $\frac{T^2}{2}\cdot\frac{d^2\ln\varepsilon}{\partial T^2}$ are small in comparison with the dimensionless magnitude $h\nu/kT$. Actually, the values of $h\nu/kT$, shown in Table 5, are at least an order larger than the magnitudes $T\frac{\partial\ln\varepsilon}{\partial T}$, which lie usually within the interval of 0.1 - 0.5. (Thus, for Wolfram in the worst case, at the temperature of 2,000°K, $T\frac{\partial\ln\varepsilon}{\partial T}\sim 0{,}1$ [19].) This gives grounds for examining the member with $T\frac{\partial\ln\varepsilon}{\partial T}$ in the first interval of formula (2.34) as a small correction and not to take into account similar members in the subsequent integrals which are small in themselves. In this approximation, one can write

$$Y(T+\delta T)-Y(T)=\frac{\delta T}{T}\int\limits_0^\infty If\left(\frac{h\nu}{kT}+T\frac{\partial\ln\varepsilon}{\partial T}\right)\delta\nu+$$

$$+\left(\frac{\delta T}{T}\right)^2\int\limits_0^\infty If\frac{h\nu}{kT}\left(\frac{h\nu}{2kT}-1\right)d\nu. \qquad (2.35)$$

For the magnitude $\frac{1}{Y}\cdot\frac{dY}{dT}$ we get from this

$$\frac{1}{Y}\cdot\frac{dY}{dT}=\frac{1}{T}\cdot\frac{\int_0^\infty If\frac{h\nu}{kT}d\nu}{\int_0^\infty Ifd\nu} \qquad (2.36)$$

(Without taking into account the corrections for the temperature dependence of ε on T).

Table 5. Values of the Parameter $\frac{h\nu}{kT}$ as a Function of the Temperature and Lense of the Emission Wave.

	$T°K$				
Å	1000°	1500°	2000°	2500°	3000°
4000	36	24	18	14	12
6000	24	16	12	10	8

For a specific case, when monochromatic emission is used,

$$\frac{T}{Y}\cdot\frac{dY}{dT}=\frac{h\nu}{kT}. \qquad (2.37)$$

For another specific case of the p-shaped spectral characteristic of the measurement system, we get

$$\frac{T}{Y}\cdot\frac{dY}{dT}=\frac{\left(\frac{h\nu_{\max}}{kT}\right)^4 e^{-\frac{h(\nu_{\max}-\nu_{\min})}{kT}}-\left(\frac{h\nu_{\min}}{kT}\right)^4}{\left(\frac{h\nu_{\max}}{kT}\right)^3 e^{-\frac{h(\nu_{\max}-\nu_{\min})}{kT}}-\left(\frac{h\nu_{\min}}{kT}\right)^3}. \tag{2.38}$$

If the difference $\nu_{\max} -- \nu_{\min}$ is sufficiently large so that $h/kT\ (\nu_{\max} - \nu_{\min})$ is of the order of several units (for this, $\nu_{\max} -- \nu_{\min}$ should in the worst case amount to 1,000 - 2,000 Å), then the formula (2.38) will assume a form similar to formula (2.37)

$$\frac{T}{Y}\cdot\frac{dY}{dT}=\frac{h\nu_{\min}}{kT}. \tag{2.39}$$

The reason for the identity of formulas (2.29) and (2.37) is the steep nature or the frequency function of the emission intensity (2.30), as a result of which the role of the high-frequency boundary of the spectral characteristics turns out to be significant, when it is not too close to the low-frequency spectral characteristic. It follows from this that the relationship of the type (2.37) should be approximately valid also for the case of a more complex frequency function $f(\nu)$, if $f(\nu)$ has more or less a sharp low-frequency (long-wave) boundary. In the general case, we will, for this reason, write

$$\frac{T}{Y}\cdot\frac{dY}{dT}=\frac{h\nu^*}{kT}, \tag{2.40}$$

where ν^* is the effective frequency which lies near the long-wave boundary of the frequency characteristic of the system. If magnitude

ν^* can, in principle, be a function of the temperature. Although this function should not be as strong.

The magnitude of the second derivative of Y in the expansion (2.33), which characterizes the deviation from the linear type of relationship δT and δY on the basis of the formula (2.40), can be written as

$$\frac{d^2Y}{dT^2} = \frac{Y}{T^2}\left[\left(\frac{h\nu^*}{kT}\right)^2 - 2\frac{h\nu^*}{kT} + \frac{h\nu}{kT}\cdot\frac{d\ln\nu^*}{d\ln T}\right]. \qquad (2.41)$$

It follows from this that the correction for the non-linearity is determined essentially by the factor

$$\frac{h\nu^*}{2kT}\cdot\frac{\delta T}{T}. \qquad (2.42)$$

The magnitude of the maximum values of the temperature gradient δT, for which the linear dependence of δY on δT for a wave-length of 6,000 Å takes place with an accuracy of up to 1%, is shown in Table 6. As can be seen, this is a rather narrow range of temperature differences, which is a disadvantage of the optical method or more accurately of the modification of this method under consideration--the case when use is made of a system that is linear with respect to the intensity. This disadvantage, however, is compensated by the high sensitivity of the optical method of measurements. Actually, it follows from formula (2.40) that a relative magnitude of the signal measured by the photoelectric pickup exceeds the relative temperature difference 10-40 times (see Table 5).

In measurements of temperature differences, which are greater than those shown in Table 6, it was necessary to take into account the correction for the nonlinearity. For this purpose, one can utilize the formula

$$\frac{\delta Y}{Y}=\frac{h\nu^*}{kT}\cdot\frac{\delta T}{T}\left(1+\frac{h\nu^*}{kT}\cdot\frac{\delta T}{2}\right) \qquad (2.43)$$

or which is more convenient, the Inbliss formula

$$\frac{\delta T}{T}=\frac{\delta Y}{Y}\cdot\frac{1}{\left(\frac{h\nu^*}{kT}\right)}\left(1-\frac{1}{2}\cdot\frac{\delta Y}{Y}\right). \qquad (2.44)$$

A more radical method of broadening the working range of the method is the utilization of the logarithmic scale of readings on the pickup. In this case (without taking into account the role of the degree of blackness),

$$\delta\ln Y=\frac{\delta T}{T}\left(\frac{h\nu^*}{kT}\right)\left[1-\frac{\delta T}{T}\left(1-2\frac{d\ln\nu^*}{d\ln T}\right)\right]. \qquad (2.45)$$

The magnitude of the correction for nonlinearity is here determined already essentially by the ratio δT/T and the range of nonlinearity in the measurements increases 10-40 times in comparison with the nonlogarithmic scale.

Formulas (2.44) and (2.45) make it possible to reveal one fundamental specificity of the photoelectric method of measuring temperature differences, namely, the fact that the temperature difference being measured for the same (or one and the same) objects does not depend on the emission properties of these objects (with an accuracy up to small corrections of the type $\frac{d\ln\varepsilon}{d\ln T}\cdot\frac{1}{\left(\frac{h\nu}{kT}\right)}$). The measured temperature difference is a true difference and not the difference of the brightness temperatures. In this instance, however, the absolute temperature T in the formulas (2.44) and (2.45) should be known.

The examined method of determining the temperature difference by comparing two readings of identical pickups is most convenient in the zero-method. When this comparison is accomplished by means of a controlled attenuation of one of the signals to a magnitude at which the quality (compensation) of both signals takes place. The temperature difference in this method is related unambiguously with the scale of the attenuator.

Table 6. Temperature Differences for Which a Linear Relationship in the Readings of the Photoelectric Device (Linear With Respect to the Emission Intensity) With the Test Temperature Difference Takes Place

T°K	1000	1500	2000	2500	3000
δT	0,8°	2°	2,3°	5°	7,5°

The controlled attenuation of the signal in differential optical pyrometers can be accomplished in the optical as well as the electrical parts of the system. In the first case, there should be in the path of one of the rays a variable optical attenuator--photometric wedge, calibrated diaphragm, polarization attenuator, etc. (on the path of the second ray in this case, a constant attentuator is positioned). In the second case, the attenuator can be any electrical attenuator. In this and in the other case, during compensation the equality below should be valid

$$\int_0^\infty \varphi_0(\nu) I(\nu, T_0) f(\nu)\, d\nu = \int_0^\infty \varphi(\nu, x) I(\nu, T) f(\nu)\, d\nu, \qquad (2.46)$$

where $\phi(\nu, x)$ is the attenuation factor, x is the scale divisions of the attenuator, $\phi_0(\nu)$ is the magnitude of the constant attenuation in the second channel. (During attenuation of the electrical signals, the frequency function ϕ_0 and ϕ is absent.)

The most interesting, from the practical point of view, is the case when a photometric wedge is utilized as the attenuator. The function $\phi(x)$ for the wedge has the form

$$\varphi(x) = \varphi_0 e^{-\alpha x}, \tag{2.47}$$

besides, the relatively weak frequency function of the wedge constant α can be disregarded in comparison with strong function $I(\nu)$. For this case,

$$\varphi_0 \int_0^\infty I(\nu,\ T_0) f(\nu) d\nu = \varphi_0 e^{-\alpha x} \int_0^\infty \varphi(\nu,\ T)\, I(\nu T) f(\nu) d\nu$$

or

$$\ln Y = -\alpha x + \ln(Y + \delta Y). \tag{2.48}$$

Thus, by utilizing as an attenuator a photometric wedge, it is possible to carry out the compensation method of measurement with the logarithmic scale with respect to the signal and, consequently, linear with respect to the temperature difference (within a comparatively large range). The relationship between the shift in the wedge and the temperature difference is in this case given by the formula

$$\frac{\delta T}{\delta x} = \frac{T\alpha}{\left(\frac{h\nu^*}{kT}\right)}, \tag{2.49}$$

while the correction for the nonlinearity is determined by the magnitude $\delta T/T$.

It is necessary to observe that in deriving the formula (2.48) just as formula (2.36), there is no need to assume that both channels of the system are completely identical. It is important only that the signal from the comparison channels is constant when T_0 = const. However, when the channels are not the same, one of the above-indicated advantages of the compensation method--stability of the measurements of the temperature differences with changes in the average temperature--is lost. (A change in T_0 does not have the same effect on both channels). For this reason, it is highly desirable to utilize identical measurement channels.

In the practical implementation of the photoelectric methods of the measurement of small temperature differences, it is much more convenient to utilize no two identical pickups, but one pickup to which is fed the emission from one or from another of the comparable objects. Such a method makes it possible to discard concern about the identity of the photoelectric part of the measurement system and the compensation method becomes simple: in the absence of compensation, there is a variable tension at the exit of the system, which is due to the inequality of the signals from the variably connected optical channels; during compensation, this tension is close to zero.

In optical instrument building, use is made of different methods of switching optical channels. Those most widely used are modulators with an oscillating mirror, vibrators which overlap one or another diaphragm, rotating obturator disks with sectors which open one or another optical channel. Devices which compare the emission intensities of two objects by means of one photoelectric pickup are used in photometers (see, for example, [20]) spectropyrometers intended for thermometric work ([21, 22]), and differential optical pyrometers.

Below is described the construction of a differential optical pyrometer which is intended for studying the temperature fields of small objects and the experience of using these is given in [23, 24, 25].

The basic optical system of the described instrument is shown in Figure 2. The comparable sections of the object 1 and 2 are in the focus of the lenses of the objectives 3 and 4. In the path of one of the parallel beams is placed a neutral light filter 5 and in the path of the other beam--a photometric wedge 6. The lenses 7 and 8 give an image of the object on the plane of the diaphragms 9 and 10, which cut out narrow sections of the image. The diaphragms are in turn overlapped by the disk obturator 11 so that the image from one or from another section of the object strikes the photomultiplier 12. The measurements consist of finding the position of the photometric wedge at which the variable signal which is taken off the exit of the photomultiplier becomes equal to zero (more accurately minimum), which will take place when there is equality in the intensities of the beams that strike it. The change in temperature of one of the sections (or the shift of one of the objectives to another section) leads to a disturbance in this compensation, as a result of which, a variable signal appears at the exit of the photomultiplier. Its compensation will require the shifting of the wedge to another position. The relationship between the shifting of the wedge δx and the measured temperature difference δT is in this case given by the formula (2.49).

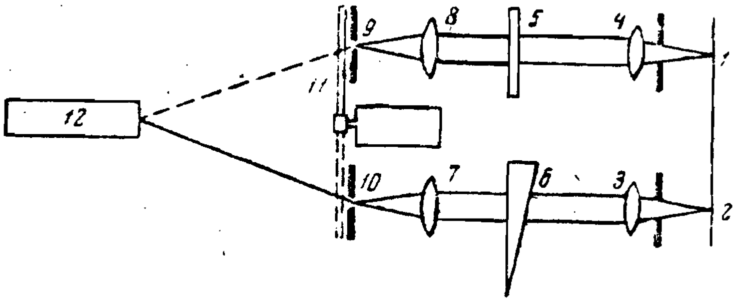


Figure 2. Basic Optical System of a Differential Pyrometer.

A general view of the pyrometer is shown in Figure 3. The test object is located in vacuum chamber 1 in a horizontal position under two like objectives 2 and 3 with a focal distance of 90 mm. Each of the objectives, together with its rotating prism 4 and 5, has two degrees of freedom: it can turn around the horizontal axis and it can shift along it. During measurements of the temperature distribution along the line on the object, both objectives should rotate at a small angle towards the object (Fig. 4a), in comparing the temperatures of two objects, one of the objectives (or both) can be turned 90° from the vertical (Fig. 4b). For translatory displacement, the micrometric screws 6 and 7 (Fig. 3) are used; these insure motion distances up to 25 mm and the accuracy of the displacement reading up 0.01 mm. The micrometric screw 8 shifts the photometric wedge in a vertical direction, each screw division having a value also of 0.01 mm. In the second optical channel, instead of the photometric wedge, there is a holder with a neutral light filter (not shown in the figure). The lenses in the sleeves 9 and 10 (focal distance 90 mm) serve for focussing the image of the object on diaphragms 11 and 13 (when the instrument is installed, the diaphragms are replaced by matte glasses). The width of the diaphragms, which determines the regions of the object, the temperatures of which are being compared, can be taken as differing from 0.5 - 0.1 mm. The disk obturator 12 is positioned flush against the diaphragms. During its rotation, one or the other diaphragm is overlapped (Fig. 5). The frequency of obturation is 30 cycles per second. The lenses 14 and 15 (Fig. 3) serve for focussing the images of the diaphragms 11 and 13 on the same place of the cathode and photomultiplier FEU -27. In the casine -16 of the photomultiplier is installed a cathode repeater, the signal from its exit is fed to the cathode voltmeter LV-9M. In the operating condition, the instrument is closed by a light-impermeable cover.

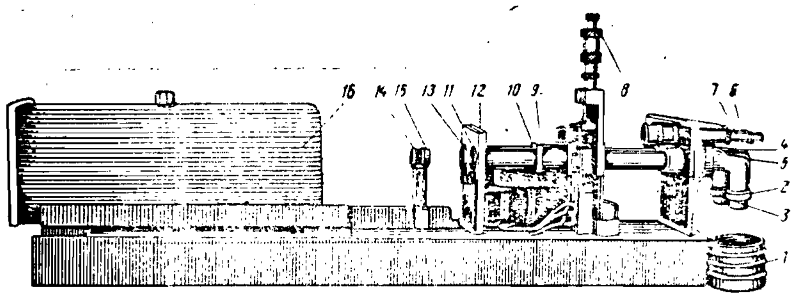


Figure 3. General Appearance of an Optical Differential Pyrometer.

The sensitivity of the described system is limited by the minimum signal which can be registered on the background of the systems noises. In this instrument, the ratio of the noise magnitude to the maximum signal, which is obtained with complete overlapping of one of the optical channels, amounts to approximately 5 $\times 10^{-4}$ for a temperature region above 1,000°K. Thus, the minimum relative change in signal, which can be registered by the instrument, is $\frac{\delta Y}{Y} \simeq 5 \cdot 10^{-4}$. From formula (2.48) we see that from $\delta T/T$ lies within the limits of 0.02 at temperatures of approximately 1,000°K to 0.2 at temperatures of approximately 3,000°K. These same figures are obtained in another method of evaluating the sensitivity, which is based on observing the minimum displacement of the wedge, which causes a noticeable change in the output signal. Thus, the minimum displacement for the utilized wedge $(\alpha \simeq 0{,}01\ 1/\text{см})$ is the displacement of approximately 0.02 mm. In accordance with formula (2.49), approximately the same values of 0,02 - 0.2° are obtained for δT in this case.

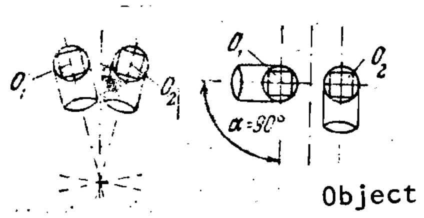
The actual reproducibility of the compensation position, that is, of the separate measurement of the temperature difference of fixed sections of the object, corresponds to these evaluations. Table 7 shows for illustrative purposes a series which consists of five successive readings of the wedge position (T ~1,900°K).

Table 7. Reproducibility of Rings of Differential Optical Pyrometer.

No. of Readings	Position of Wedge δx mm
1	10,23
2	10,26
3	10,28
4	10,30
5	10,25
Average	10,26
Average Deviation	0,02

The stability of the compensation of the emission flows is characterized by a magnitude of the change in position of the photometric wedge with a change in average temperature provided the objectives are focussed on the same specimen. At 1,500-2,000°K, the maximum change in compensation with a change in temperature of 100° corresponds to 0.2°. For 20°, that is, the working range of the instrument at these temperatures, the change thus amounts to 0.04°, which lies on the boundary of the sensitivity. The reproducibility of the measurement of the temperature distribution along the specimen during repeat recording of the same curve with the same objective of the parameter as characterized by an error which is close to the magnitude of the instrument sensitivity. The maximum deviations of individual points amount to 0.1°, the average to 0.05°. For illustration, Figure 6 shows the temperature distribution on a wolfram foil that is 50 mm thick at 1,630°K. The reproducible nonregularities of the demonstrated curve are explained by local heterogeneities in the test foil.

The results of the temperature distribution, which have been obtained by the displacement of one and the other objective, are also in good agreement with each other. Deviations from the average curve usually do not exceed 0.1°. An example of the temperature distribution curve recorded by two objectives is the curve shown in Figure 7.



a b

Figure 4. Position of Differential Pyrometer lenses.

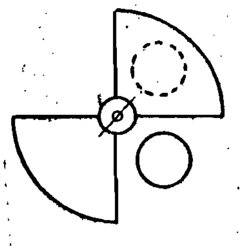


Figure 5. Pyrometer Obturator

The agreement in the readings with the use of different objectives is confirmed also by experiments in which both objectives were shifted at the same time along the object, that is, they were directed to the same sections. The maximum change in compensation in this case did not exceed the magnitude corresponding to 0.2°.

It should be observed that during the study of the temperature distribution and the specimens, the emission absorption in the window of the chamber in which these specimens are placed is not essential. Thus, when a neutral light filter with an optical density of 0.15 is placed on this window, the curve of the temperature change remains completely identical; only a systematic shift in the readings by magnitude corresponding to 0.1° takes place.

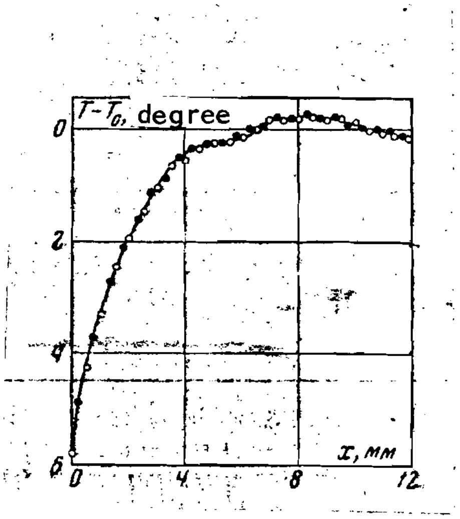


Figure 6. An example of a temperature distribution curve recorded by means of a differential pyrometer.

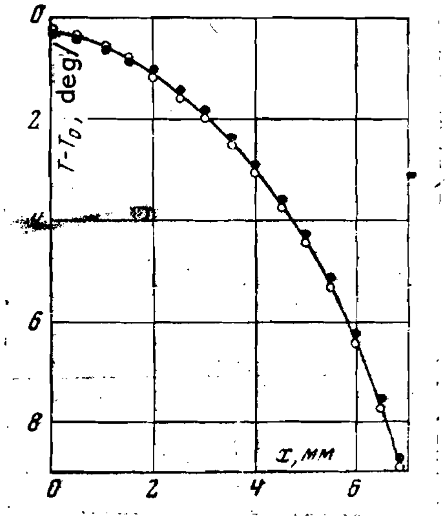


Figure 7. Results of temperature distribution measurements obtained by shifting different objectives of the parameter.

The focussing of the objectives by 1 mm along its optical axis leads only to a shift in the zero position of the wedge (by a magnitude corresponding to 0.4°); the wedge displacement values which are being measured and the temperature differences remain practically the same.

Let us dwell on the problem of the calibration of the instruments which are similar to that described here. This problem is solved more simply for a case when the emission which strikes the pickup is monochromatized. Under such conditions, there is no need for calibration as such, the instrument constant can be dteremined by means of formula (2.49) from the known frequency and the wedge graduation. Nevertheless, in practice this method is not the best. The use of a monochromator complicates substantially and raises the cost of the construction of the pyrometer and decreases its sensitivity because of a decrease in the light flux which strikes the photomultiplier. Design-wise it would be simpler to utilize an interference light filter instead of a monochromator. However, this path does not always lead to a desired result because the effective frequency of the system ν^* in this case does not coincide with the maximum of the filter transmission band [26].

The reason for this noncoincidence is the steep frequency dependence of the emission intensity, as a result of which the emission transmission in the long wave "tail" of the spectral characteristics of interference filter becomes essential.

The direct graduation of the differential parameter requires the recording of the dependence of the readings on the photometric work scale δx on the temperature T at a fixed temperature T_0. The value of the parameter divisions in this case can be determined from the angle of slope of the curve of δx as a function of T. The difficulty of accomplishing such a method of graduation is connected with the fact that it is necessary to know sufficiently accurately the temperature changes of a standard source; this accuracy should amount to 0.1 - 0.3° (1% of the instrument scale, which amounts to 10-30°). The systematic error on the measurement of the temperature can be in this instance 1-2 orders greater. It is possible, however, to accomplish the graduation of the parameter also when the temperature change can be known with a greater error than indicated above, but accordingly within a greater range than the range of the instrument scale. In this case, it is necessary to measure the signal I at the exit of the instrument as a function of the temperature. The magnitude of the effective frequency in this instance can be determined by means of formula (2.40) rewritten as

$$\frac{h\nu^*}{k} = -\frac{d\ln Y}{d\frac{1}{T}}. \tag{2.50}$$

The magnitude ν^* is, thus, determined from the angle of the slope of the curve $\ln Y$ as a function of 1/T. The dependence of $\ln Y$ on 1/T is, as a rule, linear within the temperature interval which amounts to several hundred degrees and this indicates the practical constancy of the effective frequency ν^* in such an interval. The determination of ν^* by means of the described procedure with the utilization of a pyrometer with the disappearing filament for reading the temperatures of the model of an absolutely black body can be carried out with an accuracy of the order of 1%. As an example, Figure 8 shows the dependence of $\ln Y$ on 1/T, which was recorded for the instrument described above; for temperature measurements, the micropyrometer OMP-19 was used.

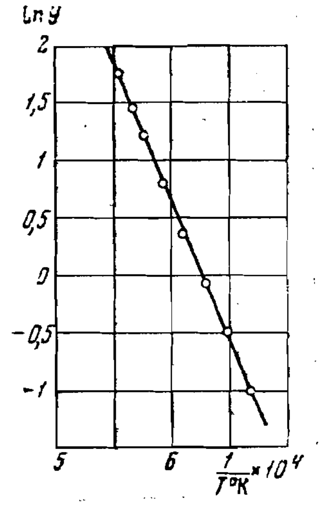


Figure 8. Determination of the effect of frequency of a pyrometer from the dependence of a signal logarithm at the exit on the reciprocal of the temperature.

The method of measuring temperature differences by means of a photoelectric differential pyrometer is not the only possibility of utilizing optical pyrometry. There is also another method--photographic pyrometry in which the test field of the temperatures is determined by photometering a photographic image of the object. Such a method is convenient in that the measurement process itself, the photographing, lasts a very brief period of time as a result of which this method can be utilized also for nonsteady, comparatively rapid processes (use in movie work). However, the method has also a substantial disadvantage. In order to convert the degree of blackening of the photographic plate (film) and the temperatures of the temperature differences, it is necessary to make an additional experimental determination of the function of the blackening--brightness temperature,(or blackening--emission intensity) functions. This circumstance complicates considerably the processing of the data and makes the processing prolonged, while the results are less accurate in comparison with measurements by means of differential optical pyrometer (although, apparently, the accuracy of the photographic method can be no less than the accuracy of the determination of the temperature field by means of absolute pyrometers). Nevertheless, in a number of experiments (and in the first place, nonsteady experiments), the utilization of photographic methods can be quite expedient, and in particular for measurements of an applied nature. Details with respect to the photographic method can be found in references [18, 27].

§3. Methods of Measuring Small Pulsations of High Temperatures

For measurements of small pulsations of high temperatures, just as in the case of measurements of small differences of high temperatures, use is made essentially of two methods--thermocouple and photoelectric (optical). It is true that these methods do not exhaust the entire arsenal of means for measurements of the temperature pulsations. For the number of objects, such as wires, thin rods, strips of foil, the pulsation of the temperature can be determined from the resistance of the very object under study or from the thermalelectron emission from its surface. These methods, however, are less universal and, for this reason, are examined in §18 which deals with the utilization of temperature pulsation measurements for determining the heat capacities.

In the practical utilization of thermocouples for temperature pulsation measurements, just as in temperature difference measurements, one of the essential problems is that of the distortion of the thermocouple readings because of heat loss. Here, we shall examine this problem for a case of surface temperature measurements, utilizing approximately the same system of considerations as in §2, but with consideration of the periodic nature of the temperature changes.

The nonsteady equation of heat conductivity for wire is written as

$$\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}=c_p\rho\frac{\partial T}{\partial t}+\frac{2\alpha}{r_0}(T-T_0) \qquad (3.1)$$

(see § 4).

For the case when the constant as well as the variable temperature components vary comparatively weakly along the wire (this will take place for the first of the examined cases, when $\frac{T-T_0}{T}\ll 1$), the solution of this equation has the form of

$$T=\overline{T}(x)+\theta_0 e^{-kx}e^{i\omega t}, \qquad (3.2)$$

where

$$k^2 = \frac{\omega}{a} i + \frac{2\alpha^*}{\lambda r_0}, \qquad (3.3)$$

θ_0 is the amplitude of the temperature variations of the frequency ω at the point $x = 0$, equal (with an accuracy down to the small distortion) to the pulsation of the temperature in the body of the object under study. (The use of the complex form for recording periodic functions is explained in §8.) The value of the effective coefficient of heat exchange α^* can in this case be referred to any of the temperatures $\overline{T}$ (0) or to T_0. (The form of the function (3.2) assumes that the variable perturbation of the temperature does not reach the second end of the wire.) In accordance with formula (3.2), the distorting influence of the thermocouple wire is equivalent to the action of a periodic source of heat, which creates a local flow equal to

$$Q = k\lambda\pi r_0^2 \theta_0 e^{i\omega t}. \qquad (3.4)$$

Just as in the case of the steady problem, we shall assume that this heat source in the body of the specimen acts within the isothermal (relatively variable temperature distortions) of the hemisphere with a radius of r_0. The inaccuracy introduced by this approximation should be of the order of unity.

The temperature distribution within the massive objective described by the equation

$$\frac{1}{A} \cdot \frac{\partial T}{\partial t} = \nabla^2 T, \qquad (3.5)$$

where A is the heat conductivity of the specimen. The solution of this equation is written as

$$T = \theta + \vartheta e^{i\omega t}, \qquad (3.6)$$

where θ is the temperature field in the absence of a thermocouple, while ϑ is the temperature distortion. For ϑ we use the formula

$$\vartheta = \frac{C}{r} e^{-r\sqrt{\frac{i\omega}{A}}}, \tag{3.7}$$

which is a spherically symmetric solution of equation (3.5), valid when the body is limited by only one plane on which there is no heat exchange. The assumption that a body can be regarded as semilimited, actually means that the influence of other boundaries of the body is not important. A sufficient condition for the fulfillment of this prerequisite is the smallness of the pulsation amplitude ϑ on other boundaries in comparison with the amplitude near the thermocouple wire, that is, the condition

$$\frac{r_0}{R} e^{-R\sqrt{\frac{\omega}{2A}}} \ll 1, \tag{3.8}$$

where R is the characteristic dimension of the body. This condition is less rigid than the condition

$$\frac{r_0}{R} \ll 1, \tag{3.9}$$

valid for the steady case, which is explained by the more rapid decrease in the amplitude of the spherical temperature wave (3.7) in comparison with 1/r. Since the condition (3.9) is practically always satisfied, the problem of the validity of disregarding the influence of the boundaries of the body is considered exhausted.

The role of heat exchange on the surface, just as in the steady case, is determined by the ratio of the normal intangential heat flows at the surface, which are governed by the temperature perturbation θ. The tangential flow is equal to

$$q_1 = -\Lambda \frac{\partial \vartheta}{\partial r} e^{i\omega t} = \Lambda \vartheta \left[\frac{1}{r} + \sqrt{\frac{i\omega}{A}} \right] e^{i\omega t}. \tag{3.10}$$

The absolute value of the amplitude of this flow is thus greater than the magnitude $\Lambda \frac{|\vartheta|}{r}$:

$$|q_1| > \frac{\Lambda |\vartheta|}{r}. \tag{3.11}$$

A change in the flow in the direction of the normal is equal to

$$q_2 = \alpha' \vartheta e^{i\omega t}, \tag{3.12}$$

where α' is the effective heat exchange coefficient on the surface of the specimen.

For the ratio of the absolute values q_2 to q_1, we get in accordance with formulas (3.11) and (3.12):

$$\frac{|q_2|}{|q_1|} < \frac{\alpha' r}{\Lambda} = Bi, \tag{3.13}$$

which coincides with a similar evaluation for the steady case.

Thus, the evaluations make it possible to consider the use of the solution (3.7) as substantiated.

In order to determine the constants C in the solution (3.7), it was necessary to equate the heat flow, found as a disturbing action of the thermocouple (3.4), to the heat flow found from the temperature distribution of this perturbation in the body of the specimen $Q=2\pi r_0^2 q_1$. In this instance, we get for the temperature distortion ϑ:

$$\frac{\vartheta}{\theta}=\frac{\lambda}{\Lambda}\cdot\frac{kr_0}{2\left[1+r_0\sqrt{\frac{i\omega}{A}}\right]}. \qquad (3.14)$$

The ratio ϑ/θ is complex, which indicates the existence of amplitude as well as phase distortions in the temperature variations.

The correction to the amplitude of the temperature pulsations has the form of

$$\frac{\Delta\theta}{|\theta|}=\frac{\lambda}{\Lambda}\cdot\frac{1}{2}\cdot\sqrt{\frac{\omega}{2a}}\,r_0\sqrt{\varkappa+\sqrt{\varkappa^2+1}}\,, \qquad (3.15)$$

where

$$\varkappa=\frac{2\alpha^* a}{r_0\lambda\omega}. \qquad (3.16)$$

In this instance, it has been assumed that the magnitude $r_0\sqrt{\frac{\omega}{2A}}$ is small in comparison with unity because the characteristic parameter which has an order of unity is $R\sqrt{\frac{\omega}{A}}$ (see §8 and 9). The correction to the phase is determined by the formula

$$\Delta\varphi=\frac{\lambda}{\Lambda}\cdot\frac{1}{2}\cdot\sqrt{\frac{\omega}{2a}}\,r_0\sqrt{-\varkappa+\sqrt{\varkappa^2+1}}\,. \qquad (3.17)$$

The resulting formulas can be utilized for specific evaluations of distortions and the measurements of temperature pulsations under conditions when the radiants of the average temperature along the thermocouple are relatively small. For large temperature gradients, these formulas can be utilized only for approximate evaluations of the order of magnitude of the distortions.

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The direct utilization of a mirror galvanometer--that is the simplest method for recording relatively the values of the amplitudes of temperature pulsations--is suitable for periods several times smaller than the natural period of vibrations; in practice, in the case of galvanometers of the type M-21, M-25, and similar ones, the registration of temperature variations with periods of 2 seconds and more is possible, although the sensitivity of the galvanometers, of course, drops noticeably for small periods. We shall allow reading the amplitude on the scale of the mirror galvanometer only in the case, of course, when the variations under study are close in form to sinusoidal, that is, when the fraction of the highest harmonics is small. Otherwise, a substantial error can take place through the distortion of the signal form because, in view of the non-uniform frequency characteristics of the galvanometer for periods that are co-measurable with its natural period, the relationship between the amplitudes and the phase of the harmonics will be changed. This non-uniformity in the frequency characteristics of the galvanometer can play, on the other hand, a positive role in measuring signals that are close to sinusoidal when the amplitude of the first harmonic should be the measured magnitude, while the higher harmonics play the role of distortions; the fraction of these distortions in changes by means of a galvanometer can be reduced.

The mirror galvanometer can be used also in circuits with an automatic registration of the signals under study. One of such circuits described in [28] is a system which tracks the position of a light spark of the galvanometer on the scale of the automatic potentiometer of the type EPP-09. The pickup of such a system is the differential photoelectric cell which is located on the carriage. The outlet of the photoelectric cell is connected to the inlet of an electronic circuit of a potentiometer. A shift in the light spot with respect to the photoelectric cell leads to the appearance of a voltage which shifts the carriage to the proper side until the signal from the exit of the differential for the electric cell is equal to zero. Similar circuits with two photocathodes and a differential photoresistance of the type FSK-7 as pickups in combination with a potentiometer EPS-152 (the run time of the carriage is 2 seconds) were used and tested in [29, 30]. In this instance, it was established that such circuits are convenient for use for periods on the order of 12-15 seconds. For smaller periods there is observed a certain lag of the carriage from the light spot of the galvanometer. An example of the recording of temperature variations by means of a tracking system is shown in Figure 27.

The method of reporting temperature variations on the scale of a self-recording instrument, with the use of a previously amplified signal is less inertial. As an amplifier, one can use in this case an electric amplifier of direct current or (for more sensitive circuits) an amplifier with the conversion of a constant voltage to a variable voltage. One can utilize also for the electric amplifiers with the low-inertial galvanometers as working elements, for example, the amplifier FEOU-18. One of such amplifiers, made in accordance with the circuit shown in the monograph [20], has been used successfully in studying the thermal properties of solid and liquid metal [31, 32, 33] (see §15). The electrical amplification circuit on the low-inertial mirror galvanometer with two photoresistances of the type FSK-1, similar with the amplifier of the type F-17 utilized in a standard photogalvanometer (see [34])has proved itself satisfactorily. Even less inertial are systems with the loop oscillograph as a recording instrument. The utilization of a loop oscillograph requires, however, considerable amplification of the signal with respect to current.

The photoelectric method of measuring small pulsations of high temperatures is based on the registration of the variable components of the signal from the photoelectric pickup. The theory of the method under consideration repeats almost completely the theory of the photoelectric method of measuring small temperature differences.

A change in the signal with a change in the temperature can be represented as

$$\delta Y = Y_{\sim} = Y(T + \delta T) - Y(T) = \frac{dY}{dT}\,\delta T + \frac{(\delta T)^2}{2}\cdot\frac{d^2Y}{dT^2} + \ldots = \frac{\delta T}{T}\int_0^\infty If\left(\frac{h\nu}{kT} + T\,\frac{\partial \ln \varepsilon}{\partial T}\right)d\nu + \left(\frac{\delta T}{T}\right)^2\int_0^\infty If\,\frac{h\nu}{kT}\left(\frac{h\nu}{2kT} - 1\right)d\nu + \ldots$$

(See (2,35]).

The first member of this expression can be expressed in the form of (2.39), with an accuracy of up to the small correction for the temperature dependence of the degree of blackness, as

$$\frac{T}{Y}\cdot\frac{dY}{dT}=\frac{h\nu^*}{kT},$$

where ν^* is the effective frequency. In this approximation, the ratio of the variable component of the signal to its constant component is equal to

$$\frac{Y_\sim}{Y_=}=\frac{d\ln Y_=}{d\ln T_=}\cdot\frac{T_\sim}{T_=}=\frac{h\nu^*}{kT_=}\cdot\frac{T_\sim}{T_=}. \tag{3.19}$$

This is the basic formula of the method. The boundaries, where this formula linear with respect to $T_\sim$ is accurate, are determined by the magnitude of the second, non-linear member of the formula (2.35). In particular, the maximum values of $\delta T=T_\sim$ for measurements with an accuracy of approximately 1% are given in Table 6. Just as for the method of measurements of small temperature differences, the relatively small maximum magnitude of the amplitudes of the temperature pulsations is a disadvantage of such a direct method. The exclusion of the role of non-linear members can be achieved when the test signal is harmonic (simple sinusoid) or when only the first harmonic of the complex signal is of interest. In such a case, the influence of the non-linear members in formula (2.35) shows up only on the second and higher harmonics, while the signal of the first harmonic remains undistorted.

For temperature changes which take place with the sound frequency, the signal of the first harmonic can be separated by radio technical means--selective amplifiers. For relatively slow changes in temperature, this can be accomplished by means of a harmonic analysis of the oscillograms of the output signal.

For the practical utilization of formula (3.19) it is necessary to know the constant of the measuring instrument $h\nu^*/k$. Just as in the method of measuring temperature differences, the simplest is the determination of this magnitude when monochromatic emission is used. The value of $h\nu^*/k$ in this case can be found by simple calculation. In the experimental determination of the magnitude $h\nu^*/k$ for non-monochromatic emission, it is most expedient to find it from the tangent of the angle of the slope of the curve $\ln Y = f(1/T)$:

$$\frac{h\nu^*}{k} = -\frac{d\ln Y_=}{d\frac{1}{T}}.$$

(An example of such a curve is shown in Fig. 8 as well as in [26].)

We shall cite practical examples of circuits which are used for measuring small pulsations of high temperatures by the photoelectric method.

One of the simplest systems of this type, which is suitable for measuring the pulsations of sound frequencies is shown in Figure 9.

As an emission receiver in this circuit, use is made of the photomultiplier 1 (FEU-27, FEU-31A, etc.) which is fed by a stabilized source of voltage (VFS or similar ones). The variable and constant components of the signal, which is taken off from the load of resistance 2 with a magnitude of 0.1-1 megohm, are amplified by the single-cascade balance amplifier on the tube 6N7S--3. In the absence of a signal on the photomultiplier, the amplifier is balanced by changing the anode load 4 (approximately 20 KΩ); in this way, the dark current is also compensated. The variable component is measured by a decathode voltmeter 5 (VZ-2A or a similar one), the constant component is measured by the needle instrument 6 (for example, M-193). Such a circuit, despite its simplicity, has a sensitivity of 0.03-0.05° within the temperature range of 2,000-3,000°. The temperature pulsations of approximately 3-5° can thus be measured with an accuracy of approximately 1%. Details regarding the operation of this circuit can be found in [26].

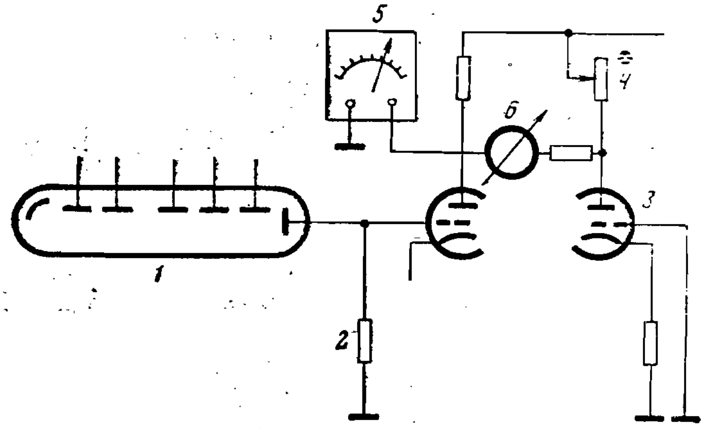

Figure 9. Circuit for the photoelectric registration of rapid pulsations of high temperatures.

During measurements of large pulsation amplitudes, in order to avoid errors due to the non-linearity in the relationship between the emission intensity and the temperature, it is expedient to utilize the circuit examined in combination with the selective amplification of the variable components. It is expedient to use selective amplification also in another case when the measured pulsations are small. The selectivity increases the sensitivity of the circuit by reducing the ratio of noise/signal. The negative aspect of selective amplification is that it is necessary to know the amplification coefficient of the selective cascade, without which one cannot utilize formula (3.19).

The circuit similar to that described can be utilized also with a photoelectric cell as the receiver [26, 35, 36]. The minimum pulsations that can be measured are in this case practically the same; however, large light fluxes are required for the measurements. For this reason, when the dimensions of the test objects are small or when use is made of monochromatized emission, it is expedient to use photoamplifiers. For sufficiently extended objects, when much light strikes the receiver, one can utilize also other receivers: photodiodes, photoresistances (for the latter it is necessary to pay attention to the linearity of their characteristics). Finally, one can use photoamplifiers also without an amplification cascade. The variable component in this instance should be measured by a high ohmic cathode voltmeter directly on the load of the photoamplifier while the constant component of the current should be measured through the load by a microameter [37, 38].

For measurements of temperature pulsations that are slower than sound pulsations, it is necessary to resort to oscillographic registration of the variations. The most convenient for this purpose are loop oscillographs. The relatively low sensitivity of loop oscillographs requires, however, the use of more complex amplifying circuits. Some of such circuits are shown in [39, 40, 41]. These circuits, when used with photoamplifiers of the type FEU-19M and oscillographs of the type MPO-2 (with vibrators MOV-2 with the sensitivity of 3.7 mm/ma) and in N=700, show registration of temperature variations of 0.03-0.1° for frequencies of the order of fractions of cycle per second in the region of moderate temperatures of 1,000-2,000°K.

Measurements of the thermal properties with a utilization of photoelectric methods of registering the temperature variations are described in §9, §16, §18.

Chapter II

Steady Methods for Measuring the Heat Conductivity of Metals

§4. Review of Steady Methods Utilized for Measuring High Temperatures

We shall start a review of the measurement methods with the problem of their classification. It is expedient to divide the entire diversity of steady methods for measuring heat conductivity into two large groups depending on the type of energy source utilized in the experiment: external (with respect to the test specimen) or internal (volume). The first group includes all methods which utilize external electrical heaters and methods in which use is made of heating by means of an electron bombardment; the second group includes methods which utilize heat with a current passing directly through the specimen. The proposed method of dividing all the methods into two groups makes it possible to carry out their distinct, although not completely absolute,separation. Thus, it is possible to indicate the measurement method which occupies to a certain degree an intermediate place between these two groups--the method in which use is made of heating with high frequency currents, when the heat liberation is not strictly of the surface type but is close to it. Such a method, however, can, without stretching,be classified in the first group because the distribution of the heat liberation through the volume of the specimen can be taken into account in the form of a small correction.

A second step in the classification of the steady methods under consideration are the delineations which are performed within each of the two indicated groups. The proposed classification utilizes as a basic index the form of isotherms or the temperature field in the specimen. Under conditions of the most simple types of symmetry, the isotherms can be flat, cylindrical or spherical. The latter, the spherical type of isotherms is not used in experiments with metals and thus two basic sub-groups of methods remain: with the flat and cylindrical isotherms. In the first sub-group the heat flow is directed along the axis of symmetry of the specimens, as a result of which this conductivity can be called methods with axial or longitucinal flow of heat. In the second sub-group, the heat flow is directed along the radius of the cylindrical speciment. Methods of this sub-group can be called methods with radial flow of heat.

The two indicated sub-groups do not completely exhaust the diversity of methods with an external heating. There is, of course, possible also intermediate cases when the temperature field is more complex as, for example, along the radius of the cylindrical specimen and along its axis. It is expediant to classify all intermediate cases of this type into a separate third sub-group of methods.

Besides the indicated classification, each of the indicated sub-groups of methods can, at least in principle, be subdivided depending on the nature of the measurements--absolute of relative. The utilization of relative variants of the measurement methods leads in many cases to a substantial simplification of the procedure, in the first place,due to possibility of avoiding direct determination of the magnitude of the heat flow. The practical accomplishment of relative methods is based on the problem of standardization, that is, the selection of specimens of metals with thermal properties that are known with a sufficient degree of accuracy. At the present time, however, one can speak of outstanding metals only up to temperatures of 1,000°C; as standards, one can use Armce iron copper and platinum [42, 43, 44]. The problem of standards for higher temperatures is to a considerable degree debatable and requires the organization of broad and high-quality experimental studies.

Let us examine the sub-group of metals with axial, longitudinal flow of heat. The simplest from a fundamental point of view, is the experiment under conditions where the temperature distribution in the specimen is uniform, that is, when the heat flow in each point of the specimen is directed strictly along the axis of symmetry. Such conditions will be created when the electrical surface of the specimen which is assumed there are parallelopipid, is heat-insulalated. The heat conductivity in this case is determined from the formula

$$\lambda = \frac{Ql}{\Delta T \cdot s}, \tag{4.1}$$

where Q is the amount of heat passing through cross-sections per unit time, ΔT is the temperature difference in the points of the rod, which are a distance of l from each other.

It is not difficult to assure the heat insulation of the lateral surface of the specimens at low temperatures when the emission heat exchange is very small--in this case, it is sufficient to place the specimen in vacuum. In the region of moderate, and all the more high, temperatures, it is necessary to take special measures to insure heat insulation and in order to create around the specimen a field of temperatures $T_0(z)$, which is as much as possible close to the temperature distribution along the length of the specimen. For this purpose, the specimen is surrounded by one of several "protective cylinders" which are equipped with special heaters in which the currents are so selected that the required temperature distribution along the length of the protective cylinders is obtained in this case. The space between the specimen and the protective devices is filled with a heated insulating filler or the entire system is vacuum. The procedure for adjusting the temperature field of the specimen--protective cylinder system is not simple; prolonged manipulation with many heating elements is required. (In [45], for example, eight heaters were used.)

Another aspect of the examined experiments with an axial flow of heat is the determination of the magnitude of Q. The ordinary method of determining this magnitude consists of finding the capacity of the heater that creates the heat flow. Such a procedure requires conditions under which the entire heat liberated by the heater enters completely the test specimen and is not dissipated. For this, it is necessary that all the surfaces of the heater, except that which adjoins directly the face of the specimen, should be heat-insulated, which can be achieved again by using a system of additional protective heaters. In experiments of such a type, it is desirable to have the best heat contact between the heater and the specimen, because the better this contact, the less the temperature difference between the heater and the specimen and the easier it is to insure heat insulation of the heater--specimen system. One of the commonly used methods for improving heat transfer from the heater to the specimen consists in placing the small-sized heaters in the hollow of the specimen itself (see, for example [46]), which makes it possible to dispense with one protective cylinder for heat insulation of the lateral surface and one additional heater for heat-insulation of the end face. (The need of using the latter can be avoided, if use is made of a symmetrical system consisting of two identical specimens, just as this was done in [46].) The difficulties related with the determination of the magnitude of the heat flow disappear when use is made of a relative diversity of the examined methods of axial heat flow with heat insulation. In the relative variant, use is made of two rods--the test rod and a standard rod; the heat conductivity of the latter should be sufficently well-known. The rods are positioned end to end within the system with a protective heater. The heater, which develops overheating, is installed on the free end of one of the rods; the heat insulation of the heater is already not mandatory here. If the lateral surface of the rod is heat-insulated, it follows from formula (4.1), that

$$\lambda = \lambda_э \frac{\Delta T_э}{\Delta T} \cdot \frac{l}{l_э} \cdot \frac{s_э}{s} \qquad (4.2)$$

where the subscript e belongs to the standard rod. In order to determine the heat conductivity λ in this manner, it is sufficient to measure the temperature differences in two points of the test and standard rods [47].

Let us cite an example of one of the better installations where use is made of the method of axial flow with heat insulation in its absolute variant. A description of this installation, besides the original paper by the author [48], is in the reference [49]. The heat insulation of the specimen--heater system in the installation under study is accomplished by means of a protective cylinder which has two heaters and a protective cup with a heater that adjoins from above the protective cylinder and encompasses the entire flat heater which is located on the end face of the specimen--a cylinder 25 mm in diameter and 80 mm long. The specimen is placed on the heat receiver which also contains a heater. The protective devices and the heat receiver are made of gold. The entire system is placed in vacuum within a tubular furnace made of stainless steel with multisectional heater. A detailed analysis of the operating conditions of the described installation included calculation of the heat loss due to emission, heat loss along the wires of the heater and the thermocouples, non-uniformity in the temperature field of the specimen, evaluations of the errors in the measurement of the power, temperatures, and emissions, as well as evaluations of the errors due to non-strict steady-state condition of the process and due to the change in the heat conductivity with the temperature. As a result of this analysis, the authors came to the conclusion that the maximum error of the results in the region of temperatures up to 800°C does not exceed 2%; besides, the systematic errors amount to approximately 1%. The maximum deviation of the results of heat conductivity measurements of iron from the average values amounts to 0.6%, the average scatter is - 0.35%.

The examined method of actual flow in the heat-insulated specimen has been used repeatedly to measure the heat conductivity of liquid metals. In working with liquid metals, it is necessary, of course, to take into account the heat conductivity of the walls of the vessel in which the liquid metal is contained. For the given case, when the isotherms are perpendicular to the axis of the specimen, this calculation is very simple: formula (4.1) should be multiplied by a correction factor

$$\delta = 1 + \frac{\lambda'}{\lambda} \cdot \frac{s'}{s}, \qquad (4.3)$$

where λ' and s' are the heat conductivity and the area of the cross-section of the wall. In the rest, the system of measurement as well as the corresponding unit do not differ fundamentally from such for solid metals (see, for example, [50, 51, 52].

Summarizing the results of the description of the method with heat insulation, we emphasize that, despite the fundamental simplicity of this method of measurement, its utilization at sufficiently high temperatures involves great difficulties. Experimental units are comparatively complex and the measurements are prolonged and painstaking. It is not accidental that all the experimental studies by means of this method are limited to a temperature of 1,000°C. Attempts to accomplish this method at higher temperatures are hardly expedient.

Let us examine the diversities of the method of axial flow, which do not require heat insulation of the lateral surface of the specimen. Before describing specific experiments, let us dwell on the fundamental question: in which cases and about which approximation can one speak about flat isotherms of the temperature and specimens which are not heat-insulated? This question is important for further presentation because we have included in the group of metals with axial flow feed precisely those in which the isotherms are flat, while in the presence of flows from the surface of the specimen, radial components of the temperature gradient must exist in the specimen. On the other hand, it is obvious that from metals which have a great heat conductivity, the presence of radial flows of heat in the specimens should not cause large temperature differences along the cross-section.

Let us examine the problem of the calculation of the lateral heat exchange analytically. In the case of smallness of the radial temperature gradients, the solution of the heat conductivity equation (here, we examine the general, non-steady case for a cylindrical specimen)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{2} \cdot \frac{\partial T}{\partial r} = \frac{c\rho}{\lambda} \cdot \frac{\partial T}{\partial t} - \frac{\omega}{\lambda} \tag{4.4}$$

can be given in the form of

$$T = a(x, t) + r^2 b(x, t) + r^3 c(x, t) + \dots \tag{4.5}$$

(the member with the first power of r is lacking in this expansion

because $\frac{\partial T}{\partial r} \to 0$ when $r \to 0$.)

For sufficiently small radial gradients, the members with the power of r^3 and higher powers of flow can be disregarded as a first approximation. Then the combination of the radial derivatives, which figures in equation (4.4), is simply expressed by the coefficient b:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{2} \cdot \frac{\partial T}{\partial r} = 4b. \tag{4.6}$$

The magnitude b can, in its turn, be expressed by the coefficient of heat exchange by means of the boundary condition

$$-\lambda \frac{\partial T}{\partial r} = \alpha (T - T_0) \tag{4.7}$$

(for r = R), which gives

$$2b = -\frac{\alpha}{\lambda R} (T - T_0). \tag{4.8}$$

substituting formula (4.8) in equations (4.6) and (4.4), we get an equation which does not contain derivatives with respect to the radius:

$$\lambda \frac{\partial^2 T}{\partial x^2} = c_p \rho \frac{\partial T}{\partial t} + \frac{2\alpha}{R} (T - T_0) - w, \tag{4.9}$$

or

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial T}{\partial t} + \nu (T - T_0) - \frac{w}{\lambda}, \tag{4.10}$$

where

$$\nu = \frac{2\alpha}{\lambda R}. \tag{4.11}$$

Equation (4.10) can be obtained also directly by following the heat balance on the assumption that the radial gradients in the specimen are absent, but at the same time there is a heat exchange on the lateral surface. Such a course of thinking is somewhat less consistent, but it is convenient for generalizing equation (4.10) for a specimen with a cross-section of random form (for example, rectangular). In this case, the equation obtained is of the same type as in (4.10), but

$$\nu = \frac{\alpha p}{\lambda s}, \tag{4.12}$$

where p is the perimeter and s is the cross-section (see [53]).

An evaluation of the limits of the applicability of the unidimensional equation (4.10) can be obtained from the following considerations. The main conditions utilized in the derivation of these equations is the assumption of the smallness of the third and successive members of expansion (4.5). An evaluation of the magnitude of these members, generally speaking, requires an examination of the boundary conditions not only in the lateral surfaces of the rod but also in its ends, which is related with the concrete definition of the corresponding problems. However, if we limit ourselves not to finding the corrections to the equations, but only to the evaluation of the order of magnitude, then one can take, as a criterion of the sufficiently rapid convergence of the series (4.5), the ratio of the second member to the first:

$$\xi = R^2 \frac{b}{a} = \frac{\alpha (T - T_0) R}{2\lambda T}.$$

The magnitude T--the temperature in the cross-section under consideration--is in its turn related to the boundary conditions on the ends with the characteristic temperature which is deducted from the temperature of the medium $T^* - T_0$ thus, with an accuracy up to the factor of the order of unity, the magnitude ξ is equal to

$$\xi \approx \frac{1}{2} Bi, \tag{4.13}$$

and the radial temperature gradients can be disregarded under conditions of the smallness of the Biot criterion.

For the examined steady methods with axial flow without inside heating, equation (4.10) assumes the form of

$$\frac{d^2T}{dx^3} = \nu (T - T_0). \tag{4.14}$$

This equation can be generalized also for the case of considerable axial gradient temperatures when it is necessary to take into account the temperature dependence of the coefficient of heat conductivity:

$$\frac{1}{\lambda} \cdot \frac{d}{dx} \lambda \frac{dT}{dx} = \nu (T - T_0). \tag{4.15}$$

For high temperatures, in the presence of heat exchange only through emission, in particular,

$$\frac{1}{\lambda} \cdot \frac{d}{dx} \lambda \frac{dT}{dx} = \frac{2\sigma\varepsilon}{R} (T^4 - T_0^4). \tag{4.16}$$

The utilization of equations (4.15) and (4.16) depend substantially on the magnitude of the temperature of the surrounding medium T_0 and how it depends on the coordinate x. We shall differentiate the following three types of conditions:

(1) T_0 = const are the isothermal conditions; besides $\theta = T - T_0$

is small in comparison with T_0:

$$\frac{\theta}{T_0} \ll 1; \tag{4.17}$$

(2) Non-isothermal conditions, $T_0 = T_0(x)$ with the preservation of the inequality (4.17);

(3) $T_0 << T$ - is the specimen which is strongly overheated with respect to the walls of the chamber in which it is located.

Let us examine the conditions of the first type. By using the variable θ, equation (4.14) is non-linear in the general case because $\nu = \nu(T)$ can be written as

$$\frac{d^2\theta}{\partial x^3} = \nu_0\theta, \tag{4.18}$$

where

$$\nu_0 = \frac{2\alpha'}{\lambda R} = \frac{2}{\lambda R}\left(\frac{\partial\alpha(T - T_0)}{\partial T}\right)_{T=T_0} \tag{4.19}$$

(see §1), and, in particular for emission heat exchange, as

$$\nu_0 = \frac{8\sigma\varepsilon T_0^3}{\lambda R}\left(1 + \frac{d\ln\varepsilon}{d\ln T}\right). \tag{4.20}$$

(Later on, the subscript 0 at a magnitude ν is dropped.)

The general form of the solution of equation (4.18) is such:

$$\theta = Ae^{\sqrt{\nu}x} + Be^{-\sqrt{\nu}x}. \tag{4.21}$$

The coefficients A and B are determined by the boundary conditions on the ends of the rod. The specific definition is so far not necessary.

Expression (4.21) gives the linear temperature distribution in a specific case when the indices of the exponent are sufficiently small so as to make possible the use of the expansion

$$e^{\sqrt{\nu}x} = 1 + \sqrt{\nu}x + \ldots \tag{4.22}$$

and not to take into account the members with higher powers of x.

Thus, the role of the right-hand part of equation (4.18), which leads to the non-linear temperature distribution along the specimen, is determined by the criterion

$$\eta = \sqrt{\nu}l, \tag{4.23}$$

where l is the length of the test rod (or its half for a symmetric problem). The magnitude η can be represented as

$$\eta = \sqrt{2}Bi^{1/2}\frac{l}{R}, \tag{4.24}$$

if in the formula for Bi the characteristic dimension is R (for emission heat exchange $Bi \approx \frac{4\sigma\varepsilon T^3 R}{\lambda}$). In order to evaluate the magnitudes η under conditions of emission heat exchange, Table 9 is given which lists the values of l for which $\eta = 1$ (λ - 1 wt/cm·degree $\varepsilon = 1$).

The dashes in the table correspond to the case when $\xi > 0, \Gamma$ and thus the unidimensional state of the temperature distribution is disturbed. It is seen from the table that the magnitude is sufficiently large so that the right-hand part of the equation (4.15) is real, starting already from 1,000°K (in any case, for specimen that are not very thick). It is not permissible to disregard the emission from the lateral surface in these cases; the temperature distribution along the axis is linear.

Let us examine further a similar evaluation also for the case of heat insulation with a filler. As the coefficient of heat exchange in this case, use is made of the formula

$$\alpha = \frac{\lambda_3}{R \ln R_3/R}, \qquad (4.25)$$

where λ_3 is the heat conductivity of the filler material, R is the radius of the rod, R_e is the radius of the protective screen. This formula is strict for the case when the heat flow through the heat insulation is radial in accuracy, which can take place only for a constant temperature of the rod along its slant. In the case when the temperature is not constant, formula (4.25) should be regarded as approximate. For the criterion η, we get

$$\eta = \left[\frac{2}{\ln R_3/R}\right]^{1/2} \left(\frac{\lambda_3}{\lambda}\right)^{1/2} \frac{l}{R}. \qquad (4.26)$$

For evaluations we assume that $R_3 \approx 3R, \frac{\lambda}{\lambda_3} < 10^2 - 10^3$. Then for l, for which $\eta = 1$, we get the values of $< 0.8 - 2$ for R = 1 mm;

$< 2\text{-}6$ for $R \sim 3$ mm and $< 8\text{-}20$ for $R \sim 1$ cm. Thus, in the case of the use of heat insulation, the magnitude η under real conditions is far from being negligibly small and it is necessary to take into account the heat loss from the lateral surface.

Let us examine some specific variants by accomplishing the methods with the axial flow of heat under isothermal external conditions.

It follows from the above obtained evaluations that for rods with a diameter of 2-6 mm and a length of 5-15 cm, even at temperatures of the order of 1,000°K, the magnitude η becomes so large ($\sqrt{\nu L} > 3$), that the influence of the second member in formula (4.21) will be insignificant in comparison with the first, and the temperature at the end of the rod will practically not differ from the temperature of the surrounding medium (the furnace) T_0. In this case, the temperature distribution along the rod should be described by the formula

$$\theta = \theta_0 e^{\sqrt{\nu} x}, \tag{4.27}$$

where

$$\theta_0 = T(0) - T_0.$$

The magnitude of the coefficient ν can then be found from the value of the temperature difference θ_l for $x = l$:

$$\sqrt{\nu} = \frac{1}{l} \ln \frac{\theta_l}{\theta_0}. \tag{4.28}$$

If the flow of heat to the face of the specimen Q is known, then the heat conductivity of the metal can be determined from the formula

$$\lambda = \frac{Q}{\sqrt{\nu}\theta_0 s} = \frac{Ql}{\theta_0 s \ln \frac{\theta_l}{\theta_0}}. \tag{4.29}$$

Such a method of measurements was first performed by Barat and Vinter [54]. The Barat and Vinter method is in principle somewhat more convenient than the method of axial flow with heat insulation because there is no need of adjusting the temperature distribution of the protective cylinder under the temperature distribution in the specimen. It is only necessary to have temperature uniformity of the screen T_0. In this instance, however, there is a difficulty which is related with the assurance of conditions required for determining the magnitudes of the heat flow Q. Another system of experiment by Hogan and Sawyer [55] is free of this difficulty. In their method, the heat exchange coefficient is not excluded, as is the case in the Barat and Vinter method, but, on the other hand, it is determined by means of an auxiliary experiment. For this purpose, a heating current is passed through the specimen which in the Hogan and Sawyer method is thin and long. The heat exchange coefficient is found from the heating power of a unit length of the specimen and from the temperature difference between the specimen and the screen. It is necessary to mention also the Forbes method in which the heat exchange coefficient is determined from an additional non-steady experiment (with discretion of the rate of chilling of the specimen) [56].

Although the utilization of systems with axial flow of heat under conditions of heat exchange with a shell of constant temperature, as already pointed out, reduces some with the difficulties of the experiment in comparison with the conditions of heat insulation, it is still insufficiently convenient for high temperatures, and, in particular, for this reason that the practical fulfillment of the conditions T_0 = const is related with difficulties. In connection with this, it is natural to perform experiments that do not require constancy of temperature of the protective cylinder. The principle which forms the basis of such experiments is the specific case of the method for studying small temperature perturbations and is briefly presented in §1.

Let $T_1(x)$ be the temperature distribution which is a solution of equation (4.14) in the absence of heating by an external heater, that is, the temperature distribution through the heat exchange of the rod with a protective cylinder, the temperature of which is $T_0(x)$. Let further, $T_2(x)$ be the temperature distribution of specimen when the end here of power Q is connected. Let us examine the difference $\vartheta = T_2 - T_1$. Assuming that ν is approximately constant (heat exchange through insulating filler), we obtain for ϑ the equation

$$\frac{\partial^2\vartheta}{dx^2} = \nu\vartheta$$

the same as for (4.18) for the case of

$$T - T_0 = \text{const.}$$

A weakness of these given considerations lies in the assumption that the temperature field of the screen before and after the connection of the hand heater remains identical. The Laubitz method [57, 58] is free of this disadvantage; it requires only that the walls of the shell which surrounds the specimen--screen system be at a constant temperature.

Another path of utilizing measurements with random temperature distribution of the screens for two different values of the heater power is related with the possibility of excluding the magnitudes ν from two solutions of the corresponding problem. Such a method of measurements was developed and performed by N. A. Nikol'skiy [59, 60] and was designated by him as the "method of successive steady states". The solutions of the equations are presented by N. A. Nikol'skiy in an integral form, which requires measurement of the temperature distribution in many points of the specimen--screen system. In the calculation of formulas of the method, only the ratios of the members with lateral heat exchange play a part, and the magnitude of the heat exchange coefficient is thereby excluded provided it is independent of the temperature. The method was used by N. A. Nikol'skiy for measuring the heat conductivity of liquid metals. Evaluations of the accuracy of the results obtained by this method are difficult in view of the complexity of calculating the error introduced by the approximation of the calculations of integrals that figure in the theory of the method from the results of temperature measurements in a discrete multiplicity of points.

Let us examine experiments in which use is made of the third type of boundary conditions of equation (4.14)--the case of strong overheating of the specimen with respect to the temperature of the surrounding medium. Under such conditions, difficulties of creating a shell around the specimen disappear and the system becomes low inertial, and the heat exchange is more stable. Overheating of the specimen with respect to the medium should be accomplished by adding great power to the face of the specimen. This becomes possible by using electron bombardment heating, by means of which one can not only obtain large heating power but also measure these in a convenient manner. The main complexity in accomplishing experiments with great overheating with respect to the medium is that under these conditions the temperature gradients along the specimen become very large, which makes it impossible to carry out the procedure for linearizing the equation (4.15) and requires the development of special methods for utilizing this equation. These methods, just as the practical accomplishment of such a method of measurements, will belong to D. L. Timrot and V. E. Peletskiy [61]. The first of these methods is based on the direct integration of equation (4.16) written as

$$\frac{d}{dx}\lambda\frac{dT}{dx}=\frac{2q}{R}, \qquad (4.30)$$

where q is the heat loss per unit lateral surface of specimen through emission. This integration gives

$$\lambda\frac{dT}{dx}=\frac{2}{R}\int_{x}^{L} q\,dx+q_2=Q, \qquad (4.31)$$

where q_2 are the heat losses from the unit surface of the cold end (x = L). The magnitudes q and q_2 as well as the temperature gradient dT/dx along the specimen are considered as known.

Another method for determining the magnitude of heat conductivity in such temperatures is related with the approximation of the function q(T) of the power relationship

$$q = AT^n. \tag{4.32}$$

Equation (4.27) can in this case be reduced to the form

$$dQ^2 = d\lambda^2\left(\frac{dT}{dx}\right)^2 = \frac{2\lambda}{R(n+1)}\, dTq. \tag{4.33}$$

From this, we get for λ the formula

$$\lambda = \frac{R(n+1)}{2} \cdot \frac{dQ^2}{dTq}. \tag{4.34}$$

Table 9. Estimate of Role of Transverse heat Exchange by radiation for rods. Values of length l for which η (4.24) is 1.

$T°K$	$l = \frac{1}{\sqrt{\nu}}$ см		
	$R = 1$ мм	$R = 3$ мм	$R = 1$ см
1000	1,6	2,8	5
1500	0,8	1,4	2,5
2000	0,5	0,9	1,7
2500	0,4	0,7	—
3000	0,3	0,5	—

The magnitude of the derivative dQ^2/dTq in this formula can be found from the angle of the dependence $Q(T_q)$, while Q can be determined from the formula (4.31) from the well-known function of q(T).

In order to determine the emission heat flow q(T) in the first paper by D. L. Timrot and V. E. Peletskiy, use was made of short ($L/D \approx 0.2$-0.5) specimens of the test material. Afterwards, D. L. Timrot, V. Yu. Voskresenskiy and V. E. Peletskiy developed a method for measuring the degree of blackness, which is also suitable for long specimens. This was achieved as a result of the utilization of electron heating of the lateral surface of the specimen by means of cathodes parallel to the specimen [62, 63].

In order to measure the temperature distribution along the specimens, D. L. Timrot, V. E. Peletskiy and V. Yu. Voskresenskiy used in their experiment an optical pyrometer which is cited on a drilled hole in the specimen. The maximum error on the measurement of temperatures of 2,500-2,600°K amounts to 15% ; at higher temperatures, the error increases considerably. A method was used for measuring the heat conductivity of wolfram up to temperatures of approximately 2,800°K [62], molybdenum (approximately 2,400°K), niobium (approximately 2,300°K), zirconium (approximately 1,900°K) [62, 63], and tantalum (approximately 2,900°K) [64].

Let us summarize briefly the examination of the group of methods with axial heat flow. The utilization of ordinary variants of method with retro-heat insulation or with an isothermal shell can lead to good results for temperatures up to approximately 1,000-1,500°K although the accomplishment of these methods involves serious experimental difficulties. For higher temperatures, it is apparent that only those methods are promising which are similar to the method of D. L. Timrot and V. E. Peletskiy in which a considerable overheating of the specimen with respect to the surrounding medium is utilized. The need for additional experiments in order to determine the emission heat flows in this method is completely compensated by the possibility of measurements at temperatures above 2,000°K.

Let us examine the second sub-group of methods with steady external heating--methods of radial heat flow in which temperature gradients develop that are directed strictly along the radius. Corresponding equation of heat conductivity has the form of

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} = 0, \tag{4.35}$$

and its solution leads to the formula

$$\lambda = \frac{Q \ln R_1/R_2}{2\pi l (T_1 - T_2)}, \tag{4.36}$$

where Q is the amount of heat liberated by the heating source, which is within the cylindrical specimen, R_1 and R_2 are the radii of the cylinder, $(T_1 - T_2)$ is the temperature difference on the surfaces, l is the length of the specimen. Measurements of the heat conductivity by the method of radial flow reduce themselves thus to the measurement of the temperature difference $T_1 - T_2$, the power of the heater Q, and the geometry of the system. Unlike the method of axial flow, the practical determination of the magnitude Q involves somewhat fewer difficulties because the heat loss is dangerous only from the end surfaces; for a relatively long cylinder, the fraction of the end heat loss can be made comparatively small even when use is made of elementary protective devices. Assurance of conditions of radially of heat flow in a specimen presupposes that the heat flows from the ends of the rod are negligibly small. In connection with this, it is desirable to utilize the longest possible specimens, and for heat protection of the ends to use protective cylinders. The danger of the formation of axial gradients can be reduced considerably when the specimen is made as a composite consisting of a stack of disks; poor heat contact between the disks in this instance promote a decrease in the axial heat flows ([66, 74, 75]).

One of the most high-temperature variants of the method of radial heat flow was encountered by Rasor and McClelland [66]. Unlike the ordinary system of measurements, they placed within the specimen not a heater but a heat drain (a pipe with running water). Heating on the other hand was accomplished with a cylindrical external furnace.

In characterizing the methods of radial flow as a whole, one can indeed observe that with respect to design and the range of temperatures, which is accessible for utilization, these methods are not inferior to those of the axial heat flow, although they do require a somewhat more complex form of the specimens.

As already observed, the methods of axial and radial flows far from exhaust the possible diversity of methods with external heating--other methods exist in which use is made of a more complex symmetry of the temperature field.

Let us examine one of such combination methods [68].

The test specimen, which is a cylinder with a diameter of 0.8-2.5 cm and a length of 0.16-4 cm,is placed in vacuum within a sufficiently long inductor of a high frequency furnace so that access of the cylinder is parallel to the direction of the magnetic field. In the case of induction heating of the specimen, due to the existence of a skin effect, the heat is liberated only in a thin layer near the lateral surface of the cylinder so that the heating of the specimen can be considered actually external. Temperature distribution along the end of the specimen under steady conditions is determined in this sensor by two factors: the coefficient of radiant heat exchange from the surface and the conductivity of the material. It follows from this that the study of the temperature distribution along the end permits in principle to obtain the value of the heat conductivity of the material.

The solution of the corresponding mathematical problem has been conducted by the authors using the approximation method on the assumption that the temperature distribution along the end can be approximated by parabola and that the temperature of the lateral surface does not depend on the height (deviation from these approximate conditions is taken into account by a correction member). The calculation formula includes a magnitude of the degree of blackness of the specimen,which is assumed to be known.

The described method was used by the authors for determining the heat conductivity of molybdenum and vanadium at temperatures of about 2,000°K. The temperature distribution was determined by pyrometer with a disappearing filament. The error of the results is evaluated as 15-20%. In a subsequent study [70], this method was generalized for a case of material with an isotropic heat conductivity and it was utilized for measuring the heat conductivity of different specimens of graphite within the temperature range of 1,260-2,200°K. The last study [261] dealt with measurements of the heat conductivity of different specimens of molybdenum within the temperature range of 1,500-2,300°K.

A disadvantage of the examined method is in need of utilizing values of the degree of blackness obtained from other experiments; in this respect the method is not closed. Nevertheless, the fundamental simplicity of the method and the possibility of using it at high temperatures impel us to pay attention to it.

In the group of methods based on heating with a current that passes through the specimen, as well as in methods with external heating, one can separate two sub-groups which differ in the symmetry of the isotherms: in the first sub-group the temperature varies along the axis of the object and in the second it varies along the radius of the cylindrical specimen. Let us examine the first most numerous group.

As already mentioned at the beginning of this section, for sufficiently small values of the Bi criterion, the temperature distribution along the specimen is described by the unidimensional equation

$$\frac{d}{dx}\lambda\frac{dT}{dx}=\frac{p\alpha}{s}(T-T_0)-w. \tag{4.37}$$

The magnitude of the specific power of the volume source w is equal in this instance to

$$w=\frac{\rho I^2}{s^2}, \tag{4.38}$$

where the ρ is the specific resistance($\rho = \rho(T)$!) I is the current strength, s is the transverse cross-section of the specimen. The magnitude of the temperature of the surrounding medium T_0 is usually selected or is comparatively close to the temperature of the specimen (first type of conditions) or, on the other hand, is small in comparison with its temperature (second type of conditions).

Let us examine at first the conditions of the first type--the specimen is surrounded by a medium with a temperature close to its temperature (practically, the specimen is placed within the furnace while the current which passes through the specimen overheats it moderately with respect to the temperature of the furnace). Let T_0 = const. We introduce the variable $\theta = T - T_0$ provided $\theta/T << 1$, which makes it possible to linearize equation (4.37) and to write it as

$$\frac{d^2\theta}{dx^2} = \nu\theta - \frac{w}{\lambda}, \tag{4.39}$$

where

$$\nu = \frac{\alpha p}{\lambda s} - \frac{d\rho}{dT}\cdot\frac{I^2}{s^2}, \tag{4.40}$$

and λ, α, ρ and $d\rho/dT$ refer to the temperature T_0. The solution of this equation for boundary conditions $\theta = \theta_0 = T_{00} - T_0$ when $x = \pm L$ (symmetric conditions on the ends of the specimen) has the form of

$$\theta = \frac{w}{\lambda\nu} + \left(\theta_0 - \frac{w}{\lambda\nu}\right)\frac{e^{\sqrt{\nu}x} + e^{-\sqrt{\nu}x}}{e^{\sqrt{\nu}L} + e^{-\sqrt{\nu}L}} = \frac{w}{\lambda\nu} + \left(\theta_0 - \frac{w}{\lambda\nu}\right)\frac{\operatorname{ch}\sqrt{\nu}x}{\operatorname{ch}\sqrt{\nu}L}. \tag{4.41}$$

This solution assumes the simplest form under conditions of

$$|\sqrt{\nu}|L \ll 1 \tag{4.42}$$

(we arrive here $|\sqrt{\nu}|$, because in principle the $\sqrt{\nu}$ can be also imaginary). In this case, the expansion of equation (4.41) is a series with respect to the index of the exponent gives there a simple parabola

$$T - T_{00} = \theta - \theta_0 = \frac{w}{2\lambda}(L^2 - x^2). \tag{4.43}$$

From equation (4.43) we get the formula for determining the heat conductivity

$$\lambda = \frac{w}{2(T_M - T_{00})} L^2, \tag{4.44}$$

where T_M is the temperature in the point $x = 0$ (that is, the maximum temperature of the specimen). Thus, under conditions of smallness of the heat exchange from the surface of the specimen, the heat conductivity can be determined from the values of the temperature in two points (in the middle and at the end) of the specimen or from the value of the heating power. This is the essence of the well-known Kohnrausch method.

Examining the subsequent members of the expansion (4.41) and the series, we obtain a more accurate formula which contains a correction for the lateral heat exchange:

$$\lambda = \frac{wL^2}{2} \cdot \frac{1}{T_M - T_{00} - \varepsilon\left(T_M - T_0 - \frac{T_M - T_{00}}{6}\right)}, \tag{4.45}$$

where

$$\varepsilon = \frac{\nu L^2}{2}.$$

Such a formula was obtained and utilized by Jaeger and Disselhorst [69].

It follows from formula (4.45) in particular that the correction for the heat exchange as a first approximation will be equal to zero, if the temperature of the wall differs from the temperature of the end of the rod by 1/6 the magnitude of the temperature gradient on the specimen.

The refinement of the heat exchange corrections in the case of a parabolic temperature distribution of the surrounding medium is given by D. L. Timrot [71]. The problem of the calculation of the heat exchange corrections is dealt with also in [72, 73].

From formula (4.45), as a condition of the smallness of the heat exchange corrections, follows the inequality

$$\frac{|v| L^2}{2} \ll 1 \qquad (4.46)$$

or provided

$$\frac{\alpha p}{\lambda s} \gg \frac{dp}{dT} \cdot \frac{l^2}{s},$$

$$\frac{\eta^2}{2} \ll 1, \qquad (4.47)$$

where

$$\eta = \sqrt{2} Bi^{1/2} \frac{L}{R} \qquad (4.48)$$

for a rod of circular cross-section in accordance with formula (4.24).

Evaluations of the values of L for which η is approximately 1, which were made at the beginning of this section, indicate that the conditions of (4.47) can be fulfilled only within the temperature region below 1,000°K. At these low temperatures, the Kohlrausch method justifies itself completely and is one of the most popular methods.

Let us dwell somewhat in greater detail on problems related with the practical utilization of the Kohlrausch method. Formula (4.44) presupposes that the temperature pickup--the thermocouple--is directly in the point of the maximum temperature T_M. In practice, this is of course difficult to achieve, all the more so since the temperature distribution along the specimen can be also non-symmetric with respect to its ends, if the ends are not under identical conditions. In such a case of measurements, the temperatures in both points of the specimen will be insufficient and it will be necessary to know the temperature in three points. If the coordinates of the points in which the temperatures are measured are x_1, x_2 and x_3, while the corresponding temperatures are equal to T_1, T_2 and T_3, then the formula which replaces (4.44) will have the form of

$$\lambda = \frac{w}{2} \cdot \frac{x_3 - x_1}{\frac{T_1 - T_2}{x_1 - x_2} - \frac{T_2 - T_3}{x_2 - x_3}}. \qquad (4.49)$$

In particular, for points that are equal distance from each other ($x_1 - x_2 = x_2 - x_3 = L$):

$$\lambda = \frac{wL^2}{2\Delta T}, \qquad (4.50)$$

where

$$\Delta T = T_2 - \frac{T_1 + T_3}{2}. \qquad (4.51)$$

Formula (4,49) with consideration of (4.38) can be written also as

$$\lambda = \frac{V^2}{2\rho \Delta T}, \tag{4.52}$$

where V is the difference of the potentials on a section having a length of L. From formula (4.52) is was possible to directly determine the ratio of the heat conductivity to the electrical conductivity:

$$\frac{\lambda}{\sigma} = \frac{V^2}{2\Delta T} \tag{4.53}$$

Formula (4.53) can be written in a more general form which takes into account the temperature function of the ratio λ/σ:

$$\frac{\lambda}{\sigma}(T_M) = \frac{1}{2} \cdot \frac{\partial V^2}{\partial T_M}. \tag{4.54}$$

It is necessary to pay attention to the fact that in the formulas (4.53) and (4.54) the geometric dimensions of the specimen are not included. Moreover, a similar relationship is valid also for a more complex configuration of the system when the specimen is a thin and short connecter between two massive metallic blocks and the distribution of the electrical potential in temperature is essentially not only in the specimen but also in the blocks. The utilization of such a system is advantageous in that the correction for the heat exchange from the lateral surface is negligibly small up to the highest temperatures. On the basis of this, Hopkins [76] undertook the measurement of the heat conductivity of platinum up to the melting point and even in the molten state. The temperature T_M in the middle of the connecter was measured by him with the aid of an optical pyrometer. A disadvantage of metal is the difficulty of converting from the measured [illegible - Tr.] temperature to the true temperature, which is aggravated by the presence of emission reflected from the blocks.

The accuracy of determining the heat conductivity by the Kohlrausch method in its usual variant, in accordance with [77], amounts to 3%; at relatively high temperatures, the error increases considerably, however, due to the role of retro-heat exchange. In the temperature range up to 1,000-1,200°K, the Kohlrausch method is one of the most convenient ones, has found wide application in research and applied measurement (see, for example [77-83]).

Another method of conducting experiments to determine the heat conductivity under conditions of the same type as in the Kohlrausch method is related with another integral method for determining the heating of a specimen by a current. In this method, the magnitude of the average temperature of the specimen is determined from its resistance--the specimen thereby plays the role of a resistance thermometer. The difference in resistances of the specimen during its heating and during the measurements by means of a small current which does not practically heat the specimen is determined by the overheating of the specimen with respect to the medium, which, in its turn, is directly related with the heat conductivity of the material (see formula (4.93)). Such a method of measurement, was used in different variants by Diesselhirst and Knudsen, Weber, Cannulic and Meissner (see [77]); it is convenient, of course, in the first place for fine wire specimens, the resistance of which is measured simply and with great accuracy. Also, it is necessary, however, to take into account that the thinner the specimen, the greater the role of heat losses from the lateral surface and, for this reason, the smaller the temperatures at which this method can give satisfactory results. Measurement by means of the examined variant of the Kohlrausch method are limited by lower temperatures. An exception is the work by Cutler and others in which use was made of a configuration of the system that was used by Hopkins--a narrow connecter between massive blocks [84]. Thus, the heat conductivity of monocrystals of molybdenum and wolfram was measured up to temperatures of 1,700°K [85], as well as the heat conductivity of liquid semiconductors and metals [86]. In the latter case, the role of the connecter was played by a hole in the partition that separated the liquid. The Cutler method combines conveniently all the advantages of the Hopkins method--smallness of heat loss correction with convenience of registration of overheating with respect to the resistance. There are all grounds for assuming that such a method of measurement, even taking into account the complexity of the preparation of the specimens, can find wide application, particularly if one considers the prospects for the development of this method also for measurements of a non-steady nature, which give along with the heat conductivity also the temperature diffusivity. Further refinement of this procedure of measurements for liquid metals is also expedient. The first experiments in this direction have lead to the conclusion regarding the existence of distortions due to convective mixing, which is explained apparently by the large magnitude of the temperature gradients which take place within the short connector even for comparatively small temperature differences [86]. This difficulty can, apparently, still be overcome.

Another type of heating conditions of the test specimens is the considerable overheating of the specimens with respect to the surrounding medium. All experiments of such a type are conducted in vacuum, the specimen is heated by the current, the chamber walls are not heated specially. Equation (4.37) has, under emission heat exchange conditions, the form of

$$\frac{d}{\partial x}\lambda\frac{dT}{dx}=\frac{p\sigma\varepsilon}{s}T^4-\frac{\rho I^2}{s^2} \tag{4.55}$$

(in this case, we disregard the magnitude T_0^4 in comparison with T^4).

In this equation, the first member of the right-hand part, which describes the emission heat exchange from the lateral surface of the specimen, is real and cannot be considered as small.

In utilizing equation (4.55) for determining the heat conductivity, there are two basic ways: numerical solution of the equation and the creation of conditions under which the temperature changes on a certain section of the specimen are small, which makes it possible to linearize this equation. Let us examine the first of these possibilities.

Assuming that the heat conductivity depends comparatively weakly on the temperature and that the gradients along the specimen are not excessively large, equation (4.55) can be utilized directly:

$$\lambda = \frac{\frac{P\sigma\varepsilon T^4}{s} - \frac{\rho I^2}{s^2}}{\frac{d^2T}{dx^2}}. \tag{4.56}$$

The magnitude of the second derivative d^2T/dx^2 can in this case be calculated approximately from the values of the temperature and three points which are at a distance of l from one another:

$$\frac{d^2T}{dx^2} \sim \frac{T_1 + T_3 - 2T_2}{l^2}. \tag{4.57}$$

Such a method of measurement was utilized by Ye. S. Platunov and V. B. Fedorov [87] for measurements of the heat conductivity of wolfram at temperatures up to 2,900°C. It should be observed that the errors of such a method of determining the heat conductivity can be considerable in view of the inaccuracy of the determination of the second derivative of the temperature.

In the other method of determining the heat conductivity, use is made not of the equation (4.55) itself but the result of its integration:

$$\lambda = \frac{1}{\frac{dT}{dx}} \int_{x}^{x_0} \left(\frac{p\sigma\varepsilon T^4}{s} - \frac{\rho I^2}{s^2} \right) dx. \tag{4.58}$$

The point x_0 pertains to the section of the object with a constant temperature, where $dT/dx = 0$. The resulting formula can be directly utilized for determining the heat conductivity from the results of measurements of the temperature distribution along the object $T = T(x)$, if determinations are first made of the dependence of the degree of blackness and of the specific resistance on the temperature $\varepsilon = \varepsilon(T)$ and $\rho = \rho(T)$. Formula (4.58) can be written in a somewhat different form, if it is taken into account that on the section with a constant temperature

$$\frac{p\sigma\varepsilon T^4}{s} = \rho\frac{I^2}{s^2}, \tag{4.59}$$

For this reason,

$$\bar{\lambda} = \frac{I^2}{\frac{dT}{dx}s^2}\int_x^{x_0}\left(\frac{I_T^2}{I^2} - 1\right)dx, \tag{4.60}$$

where I_T is the current strength which assures heating of the section of the object with a constant temperature up to a temperature of T which is equal the temperature of the given point of the specimen.

A procedure similar to that described above was utilized by Worthingom for determining the heat conductivity of wolfram and tantalum at temperatures up to approximately 2,500°K [88], and then by Osborn for determining the heat conductivity of wolfram and molybdenum at the range of 1,100-2,000°C [89], by R. L. Rudkin and others for wolfram up to 2,800°K, molybdenum up to 2,100°K, and rhenium up to 2,700°K [90]. The objects of the measurement were thin wires. The temperature distribution along the wire was measured by an optical pyrometer with a disappearing filament. A disadvantage of the method under consideration for determining the heat conductivity is the need for utilizing graphical integration and differentiation (determination of dT/dx) of the results of the experiment and (to a smaller degree) the need of preliminary experiments for determining $\rho(T)$ and $I_T(T)$ (or, which is the same, $\varepsilon(T)$).

The second method of utilizing equation (4.55) is to examine small temperature differences $\theta = T_m - T$, where T_m is the temperature of a certain point of the specimen, for which it is most convenient to take the maximum temperature; for a symmetric specimen, this will be the temperature of the mid-point. In order to observe the conditions

$$\frac{\theta}{T_m} \ll 1 \tag{4.61}$$

it is necessary that the length of the specimen be sufficiently long so that the temperature changes near the maximum would be small on the section of the given extent.

For the condition of (4.61), equation (4.55) can be written as

$$\frac{\rho I^2}{s^2} - \frac{p\sigma\varepsilon T_m^4}{s} = \lambda\frac{d^2\theta}{dx^2} - \theta\left(\frac{4\sigma T_m^3 p}{s} + \sigma\frac{pT_m^4\frac{d\varepsilon}{dT}}{s} - \frac{d\rho}{dT}\cdot\frac{I^2}{s^2}\right) \tag{4.62}$$

with an accuracy up to the members of a higher order of smallness with respect to Θ. All the coefficients and their derivatives in equation (4.62) should be referred to the temperature T_m.

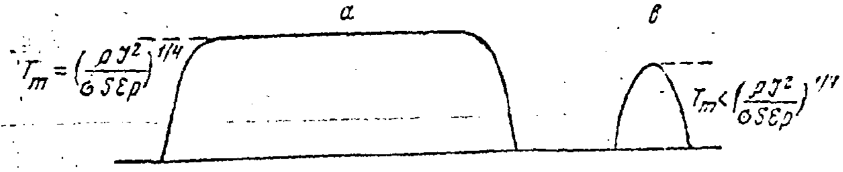

Figure 10. Temperature distribution along the wire which has been heated by a current: a, long wire; b, short wire.

Henceforth, it is important to delineate two physically different cases of temperature distribution near the mid-point of the linear specimen--a case of a "long" and "short" specimen. In the first case, there is near the mid-point of the specimen a sufficiently clearly expressed section with a practically constant temperature (Figure 10), after which there is a more or less steep drop in the temperature. In the second case, there is no such section; the temperature varies also in the immediate vicinity of the mid-point. For the first case, on the section with the constant temperature, the temperature gradients are practically absent and all the Joule heat liberated per unit plant is completely lost from the lateral surface through emission (the heat flow along the specimen is negligibly small). For this section

$$\frac{\rho I^2}{s^2} = \frac{\sigma \varepsilon p T_m^4}{s}, \tag{4.63}$$

whence

$$T_m = \left(\frac{\rho I^2}{sp\varepsilon\sigma}\right)^{1/4}. \tag{4.64}$$

(Formula (4.64) is obtained also formerly from equation (4.62) because $d^2\theta/dx^2$ is the curvature of the temperature distribution of approximately 0.) For a short specimen, the axial heat flow cannot be disregarded; the liberated heat is lost incompletely through emission, as a result of which the temperature in the mid-point of the specimen is smaller than the maximum value even by the formula (4.64).

Let us examine the possibilities opened up by the study of the temperature distribution and a "long" and "short" specimen for determining the heat conductivity. Let us start with a "long" specimen. The temperature distribution in the region adjoining the section with a constant temperature is, in accordance with equation (4.62), described by the equation

$$\frac{d^2\theta}{dx^2} = \nu\theta, \tag{4.65}$$

where

$$\nu = \frac{1}{\lambda}\left(\frac{4\sigma\varepsilon p T_m^3}{s} + \frac{\sigma p T_m^4 \frac{d\varepsilon}{dT}}{s} - \frac{d\rho}{dT}\cdot\frac{j^2}{s^2}\right). \tag{4.66}$$

By taking into account formula (4.64) and, the coefficient ν can be written as

$$\nu = \frac{1}{\lambda}\cdot\frac{\rho j^2}{s^2}\left(\frac{4}{T_m} + \frac{d\ln\varepsilon}{dT} - \frac{d\ln\rho}{dT}\right). \tag{4.67}$$

The solution of equation (4.65) for the left-half of the temperature distribution can be approximated by the formula

$$\theta = Ae^{-kx}, \tag{4.68}$$

where $k = \sqrt{\nu}$, while A is a constant (4.69)

(By means of a similar formula it is possible to describe also the second, right-hand drop of the curve on Figure 10, curve a.) The section with a constant temperature corresponds here to large (within the limit infinitely large) values of x.

Formula (4.68) makes it possible to determine the magnitude of k directly from the results of the measurement of the temperature distribution from the angle of the slope of the curve which expresses $\ln\theta$ as a function of x.

$$\frac{d\ln\theta}{dx} = -k, \tag{4.70}$$

By means of k, the value of the heat conductivity coefficient can be found from the formula

$$\lambda = \frac{\rho I^2}{k^2 s^2}\left(\frac{4}{T_m} + \frac{d \ln \varepsilon}{dT} - \frac{d \ln \rho}{dT}\right). \tag{4.71}$$

In order to determine λ besides k, it is necessary to know the current strength, the temperature T_m, the electrical conductivity $1/\rho$, as well as the magnitudes of the temperature coefficients $d\varepsilon/dT$ and $d\rho/dT$ (the latter however, play the role of corrections).

The method for determining the heat conductivity from the exponential temperature distribution was first used by Jain and Crishnan [91, 92]; afterwards, this method was developed in [24, 93, 25]. This procedure is described in detail in §5.

Let us examine the case of a short specimen (Figure 10, curve b). For a short specimen, the origin of the coordinates ($x = 0$) can be conveniently located in the mid-point, at a point of maximum temperature. The solution of equation (4.62) in this case has the form of

$$\theta = \theta_0 \left[\frac{e^{kx} + e^{-kx}}{2} - 1\right] = \theta_0 [\operatorname{ch} kx - 1], \tag{4.72}$$

where, as before, $k = \sqrt{\nu}$, while the constants θ_0 is determined by the formula

$$\theta_0 = \left(\rho \frac{I^2}{s^2} - \frac{p\sigma\varepsilon T_m^4}{s}\right)\frac{1}{\nu\lambda}, \tag{4.73}$$

that is, proportional to the difference of the quantities of heat liberated by the current and dissipated from the lateral surface of the specimen.

In the region close to the mid-point of the specimen (kx << 1), formula (4.72) can be written in an approximate form by expanding into a series with respect to kx:

$$\theta = \frac{\theta_0 x^2 \nu}{2} = \frac{1}{2\lambda}\left(\rho \frac{I^2}{s^2} - \frac{\rho\sigma\varepsilon T^4_n}{s}\right) x^2. \tag{4.74}$$

The temperature distribution can thus be approximated by a parabola. The boundaries of the validity of this approximation can be determined from the condition of the smallness of the roll of the subsequent members of the expansion (4.72), which gives

$$\frac{\nu x^2}{12} \ll 1. \tag{4.75}$$

Formula (4.74) can be directly utilized for determining the heat conductivity λ from the known temperature distribution (for this, it was necessary to find the angle of the slope of the curve which describes θ as a function of x^2.). And inconvenience of such a method is in the need of knowing the degree of blackness. However, even with consideration of this, it is of course, more convenient to utilize formula (4.74) than integral relationship (4.58).

The use of the parabolic formula (4.74) for determining the heat conductivity was first recommended and carried out by Crishnan and Jain [91, 92]. The performance of the examined procedure for measuring the heat conductivity is particularly convenient when it is possible to utilize at the same time a long and short specimen. In this case, the magnitude $\rho\ \sigma\varepsilon T^4{}_m / s$ can be determined by the means of the relationship (4.63) for a long rod and formula (4.74) acquires the simple form of

$$\frac{\theta}{x^2} = \frac{1}{2\lambda} \cdot \frac{\rho}{s^2}\left(I^2 - I_1^2\right), \tag{4.76}$$

where I_1 is the current strength at which the mid-point of the long specimen is heated to the temperature T_m. The method of determining the heat conductivity by means of formula (4.76) was proposed and carried out by V. V. Lebedev [94, 95].

This procedure was utilized by V. S. Gumenyuk and V. V. Lebedev for measurements of the heat conductivity of wolfram and graphite at temperatures up to 2,200°C [95]. The objects of the study were rods (wires) 1-2 mm in diameter. The temperature distribution was determined by means of OPPIR-09 pyrometer which was fastened on the carriage of a cathetometer. The error of the measurements was evaluated by the authors as 6%.

An inconvenience of such a method of measurement involves the need of having sufficiently long specimens. Another variant of the procedure, proposed by V. V. Lebedev [96,97], is free of this disadvantage. In this variant, two specimens of different plans are also utilized, but in this case it is not mandatory that a second specimen be so long that it have a section with constant temperature. The magnitude p/s $\sigma\varepsilon T_m^4$ in formula (4.74) in this variant is excluded simply by deducting two such formulas, which assumes that the experiments are carried out with two specimens of different length under conditions such that the mid-points of the specimens are heated to the same temperature:

$$\frac{\theta_1}{x_1^2} - \frac{\theta_2}{x_2^2} = \frac{\rho}{2\lambda s^2}(I_1^2 - I_2^2). \qquad (4.77)$$

This procedure was utilized by V. V. Lebedev [97] for the measurement of the heat conductivity of molybdenum up to 1,400°C and by V. S. Gumenyuk, V. Ye. Ivanov and V. V. Lebedev [98, 99] for measurement of the heat conductivity of wolfram and molybdenum up to 2,200°C and of tantalum up to 2,500°C.

The described methods of measuring the heat conductivity, which are based on a study of the temperature distribution along specimens that are strongly overheated with respect to the walls of the chamber in vacuum, do not require a knowledge of the specific conditions on the ends of the specimens. The temperature of the ends can be random and, in particular, close to the temperature of the cold walls--such a case is most convenient in practice. There is also possible, however, another type of experiments which are carried out under conditions when the temperature of the ends of the specimen is close to the temperature of its mid-point. In order to create such conditions, it is necessary to resort to artificial preheating of the ends of the specimen, which can be accomplished conveniently, of course, only in the case of thin specimens (wires, foil), when the leads and attachments are also not excessively massive.

Let us examine the theory of experiments of such a type. As a "base temperature", in this case, it is convenient to utilize not the temperature of the mid-point of the specimen T_m, as we did before, but the temperature of the ends T_l, determining the variable θ as the difference

$$T - T_l = \theta \ldots \tag{4.78}$$

As before, we shall assume that the magnitude θ is relatively small

$$\frac{\theta}{T_l} = \frac{T - T_l}{T_l} \ll 1. \tag{4.79}$$

The linearized equation of the heat conductivity will in this case be identical to equation (4.62), if, instead of T_m we substitute therein T_l and change the sign for θ. The solution of this equation will have the form ($x = 0$) corresponds to the mid-point of the specimen

$$\theta = \theta_0 \frac{e^{kx} + e^{-kx} - e^{kL} - e^{kL}}{2 - e^{kL} - e^{-kL}}, \tag{4.80}$$

while the temperature of the mid-point of the specimen θ_0 is equal to

$$\theta_0 = \theta(0) = \frac{1}{\nu\lambda}\left(\rho\frac{I^2}{s^2} - \frac{\varepsilon p\sigma T_l^4}{s}\right)\frac{e^{kL}+e^{-kL}-2}{e^{kL}+e^{-kL}}. \qquad (4.81)$$

It can be seen that for a definite current I_0, which is determined by the condition

$$\rho\frac{I_0^2}{s^2} = \frac{p\sigma\varepsilon T_l^4}{s}, \qquad (4.82)$$

$\theta_0 = 0$ and $\theta = 0$, that is, the specimen will be heated uniformly, its temperature at all the points will be the same and equal to the temperature of the ends. For large currents, the temperature of the mid-point of the specimen will be greater than the temperature of the ends; for small currents, on the other hand, the mid-point of the specimen will be colder than the ends. By means of formula (4.82), formula (4.81) can be written as

$$\theta_0 = \frac{1}{\nu\lambda}\cdot\frac{\rho}{s^2}(I^2 - I_0^2)\frac{e^{kL}+e^{-kL}-2}{e^{kL}+e^{-kL}}, \qquad (4.83)$$

and by taking into account formula (4.66), it can be written as

$$\frac{\theta_0}{T_l} = \left(\frac{I^2}{I_0^2} - 1\right)\frac{1-\frac{1}{\operatorname{ch} kL}}{4\left(1+\frac{1}{4}\cdot\frac{\partial \ln \varepsilon/\rho}{\partial \ln T}\right)}. \qquad (4.84)$$

This expression can in principle be utilized for determining the magnitude of k and, by means of it the heat conductivity

$$\lambda = \frac{4\rho I_0^2}{s^2 k^2 T_l}\left(1 + \frac{1}{4}\cdot\frac{d\ln\varepsilon/\rho}{d\ln T}\right). \tag{4.85}$$

The measured magnitudes should in this case be the following: overheating of the mid-point of the specimen with respect to the ends θ_0, current strength I, which causes this overheating, current strength I_0, for which the temperature of the specimen is uniform, and temperature of the ends T_l. Besides that, it is necessary to know the correction 1/4 ,d ln ε/ρ /d ln T to the temperature function of the degree of blackness and the specific resistance. However, another way of using formula (4.84) is possible which is not related with the need measuring θ_0--a method which utilizes the determination of the average temperature from the resistance of the specimen, just as this is done in the method by Knudsen and others under conditions of a small heat exchange.

For the average temperature of the specimen, it follows from expressions (4.80) and (4.84) that

$$\bar{\theta} = \frac{1}{2L}\int_{-L}^{+L}\theta\,dx = T_l\left(\frac{I^2}{I_0^2} - 1\right)\frac{1}{4\left(1 + \frac{1}{4}\cdot\frac{d\ln\varepsilon/\rho}{d\ln T}\right)} \times \\ \times\left[1 - \frac{e^{kL} - e^{-kL}}{e^{kL} + e^{-kL}}\cdot\frac{1}{kL}\right]. \tag{4.86}$$

If it is taken into account that resistance of the specimen is

$$R = R_0 + \frac{dR}{dT}\bar{\theta},$$

then we get

$$\frac{\left(\frac{R}{R_0}-1\right)}{\left(\frac{I^2}{I_0^2}-1\right)}=\frac{\beta T_l}{4\left(1+\frac{1}{4}\cdot\frac{d\ln \varepsilon/\rho}{d\ln T}\right)}\left[1-\frac{e^{kL}-e^{-kL}}{e^{kL}+e^{-kL}}\cdot\frac{1}{kL}\right], \tag{4.87}$$

where $\beta = 1/R_0 \cdot dR/dT$ is the temperature coefficient of the resistance of the specimen.

Formula (4.87) makes it possible to determine the coefficient k and, by means of it, the heat conductivity from the measured change in the resistance as a function of the strength of the current that heats the specimen. The procedure of utilizing this equation requires a solution of the transcendental equation

$$1-\frac{e^{kL}-e^{-kL}}{e^{kL}+e^{-kL}}\cdot\frac{1}{kL}=1-\frac{\operatorname{th} kL}{kL}=\varphi. \tag{4.88}$$

The form of the function $(\phi = \phi(kL)$ is given in the curves (Figure 11). The maximum cases for this formula are

$$\varphi=\frac{k^2L^2}{3} \qquad kL \ll 1 \qquad (4.89) \tag{4.89}$$

and

$$\varphi=1-\frac{1}{kL} \qquad kL \gg 1. \tag{4.90}$$

This same figure shows also the function $\psi(kL)$ which characterizes the sensitivity of the relationship $\phi(kL)$:

$$\frac{d\,kL}{kL}=\frac{1}{\psi}\cdot\frac{d\varphi}{\varphi}. \tag{4.91}$$

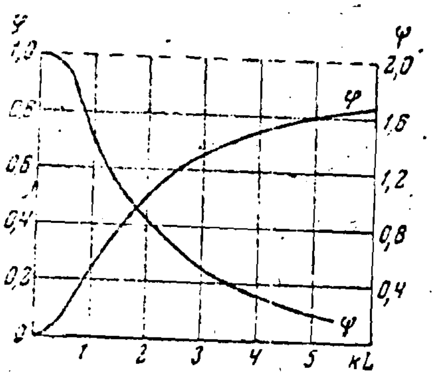

Figure 11.

The function ψ shows how many times the relative error of the magnitude kL, which is found from the relationship φ(kL), is less than the relative error φ. It can be seen that for values up to $kL \simeq 1$, the error in determining kL is approximately twice as great as the error of φ; with increasing kL, the magnitude ψ decreases, that is, the error in determining kL increases, which is explained by the decrease of the curvature of the function φ(kL). When kL is ~ 3:4, the error $\delta kL/kL$ is 2-3 times greater than $\delta\phi/\phi$. The values of kL ~ 3:4 are for this reason maximum magnitudes which can be recommended for similar experiments. For the magnitude 2L (the length of the specimen), which corresponds to such values of kL, we get at 1,000°K values of 70-120 mm for wires to 0.2-0.6 mm in diameter; for 2,000°K, we get 24-40 mm and for 3,000°K, we get 12-20 mm (for $\varepsilon \sim 0.3$, and $\lambda \sim 1$ wt/cm degree). These magnitudes are quite acceptable for the practical performance of the method.

The most advantageous case for performing the examined method of measurement is that of $kL \ll 1$, which corresponds to the maximum sensitivity in the determination of kL. For the condition of $kL \ll 1$ formula (4.87) acquires the form of

$$\Delta R = R - R_0 = \frac{\beta L^2 R_0 (I^2 - I_0^2) \rho}{3s^2\lambda}. \qquad (4.92)$$

Specific evaluations of the factor $\beta L^2 R_0\rho/3_s{}^2\lambda$ lead to the conclusion that noticeable differences in ΔR can be obtained only when $I \gg I_0$. The formula (4.92) can therefore be written in the form of

$$\Delta R = \frac{dR}{dT} \cdot \frac{L^2 I^2 \rho}{3s^2\lambda} = \frac{1}{12} \cdot \frac{dR}{dT} \cdot \frac{V^2}{\rho\lambda}. \qquad (4.93)$$

Since the case of $kL \ll 1$ corresponds to conditions under which one can disregard the heat loss through emission, then such an accurate formula is naturally obtained from the expression (4.43) for experiments with a variant of the Kohlrausch method (see above).

The set-up of experiments under conditions of $kL \ll 1$ for high temperatures becomes possible only for very short specimens (mm if not fractions of mm).

The examined procedure of measurement which is based on the observation of the overheating of the mid-point of the wire located between the heated leads, was carried out by Bode [100]. Unlike the presented variants of the method, which are based on formulas (4.84) and (4.87), Bode utilized a different, so-to-speak intermediate variant, in which information is simultaneously required about a change of R and θ_0 as a function of I. The formula for determining the heat conductivity by the Bode method can be obtained by excluding the exponential members from the formulas (4.84) and (4.87), which gives

$$\lambda = \frac{L}{s} V \frac{dI_0}{dT_l} \cdot \frac{\left[1 - \left(\frac{dR}{dI}\right)_{I_0}\right]^2}{1 - \left[1 - \frac{d(\theta/dI)_{I_0}}{dT_l/dI_0}\right]^2}. \tag{4.94}$$

By means of such a method, Bode measured the heat conductivity of molybdenum, platinum, nickel at a temperature of abour 900°C. In our opinion, the Bode procedure is not the most successful, not only because it requires the simultaneous measurement of θ_0 and R, but also because the functions R(I) and θ (I), in accordance with formulas (4.84) and (4.87), are not linear and the procedure of finding the corresponding derivatives is, for this reason, not convenient.

All the examined methods of measurement, which are based on heating by current, were classified in the sub-group of methods in which use is made of the temperature distribution along the axis of the specimen. In the second sub-group, which we are taking under examination, different conditions are accomplished: the temperature along the axis of the specimen does not vary--there is a section with a constant temperature, but due to the utilization of specimens of comparatively large radius, there is a temperature distribution along the radius (the temperature on the surface differs from the temperature in the depth of the specimen). The equation of heat conductivity for such conditions has the form of

$$\lambda\left(\frac{d^2T}{dr^2}+\frac{1}{r}\cdot\frac{dT}{dr}\right)+w=0 \tag{4.95}$$

(due to the proposed smallness of the change in temperature, we disregard the temperature function of the heat conductivity). The magnitude of the volume density of the source of heat is equal to

$$w=\frac{V^2}{\rho L^2}. \tag{4.96}$$

The solution of equation (4.95) with boundary conditions of $T = T_1$ and $dT/dR = 0$ for $r = R_1$ (two) has the form of

$$\lambda(T-T_1)=\frac{w}{4}(R^2-R_1^2)-\frac{wR_1^2}{2}\ln\frac{R}{R_1}. \tag{4.97}$$

Hence, if we substitute also the value of the temperature T_2 on the external surface of the tube ($R = R_2$), we get a formula for the determination of the heat conductivity

$$\lambda=\frac{w}{4}\cdot\frac{(R_2^2-R_1^2)}{T_2-T_1}-\frac{wR_1^2}{2(T_2-T_1)}\ln\frac{R_2}{R_1}. \tag{4.98}$$

The heat conductivity, in accordance with this formula, can be found from the results of measurements of the temperature differences on two surfaces of the tube for a known difference of potentials, the electrical conductivity and geometric dimensions. It is pertinent that the calculation formula does not include values for the degree of blackness of the surface. The examined method of measurement was carried out for the first time by Engel [101] and then by Powell and Schofield [102]. Strauss [103, 104] used this method for measurements of the heat conductivity of different specimens of graphite up to temperatures of 3,000°C. The heat conductivity of graphite, molybdenum, and tantalum within the temperature range of 2,500-3,000°C has been determined by Rasor and McClelland [67].

The utilization of the examined method for the measurement of the heat conductivity of metals is expedient chiefly for high temperatures of the order of 2,000-3,000°K. For small temperatures, the measured temperature difference $T_2 - T_1$ will be extremely small to make possible its accurate measurement. In order to become convinced of this, we shall make the following calculations. Formula (4.98), which has, for a solid rod, the form of

$$\lambda = \frac{wR^2}{4\Delta T}, \tag{4.99}$$

considering that w = 2Q/R, where Q is the heat flow from a unit surface of the rod, can be written also as

$$\lambda = \frac{\sigma\varepsilon T^4 R}{2\Delta T}. \tag{4.100}$$

Hence, it follows that

$$\frac{\Delta T}{T} = \frac{Bi}{8}. \tag{4.101}$$

Let us evaluate ΔT/T for one typical case, heat exchange in vacuum $\left(Bi = \frac{4\sigma\varepsilon T^3 R}{\lambda}\right)$, $\varepsilon=0{,}3$, $2R=1$ $c\mathit{м}$, $\lambda=1$ = 1 wt/cm·deg. In this case, we get $\Delta T = 7°$ for 2,000°K and $\Delta T = 35°$ for 3,000°K. Besides the small difference of the measured temperatures, a disadvantage of the method is the fact that the test rod should be sufficiently long in order to have a section with a constant temperature in its mid-point; in this case, very large currents and powers are required for heating it.

Let us make a brief summary of the examination of the methods which have been based on heating the specimens by a current.

For specimens which are in the form of rods, bars and thin wires, one of the most convenient methods of measuring is that based on the parabolic temperature distribution in the variant of the V. V. Lebedev method. This method is sufficiently simple, but is suitable for high temperatures and doesn't require material with a significant degree of wetness.

Its disadvantage is in need of conducting the measurements with two specimens of different length. This disadvantage is absent in the method which is based on the measurement of the exponential temperature distribution on specimens in the form of a wire or strip of foil. Such a method is also simple, it is suitable for high temperatures, and does not require a knowledge of the degree of blackness. The experiment using this procedure is presented in the next section. (This section also notes its additional advantages.)

§5. Experiment Based on the Method of the Exponential Temperature Distribution

The essence of the method under consideration, the basis of the theory of which are presented in the preceding section (see also [93]), consists of determining the index of the exponent of the distribution of the temperature T, which has been established near the section of the object with a constant temperature. In order to determine the heat conductivity, use is made of formula (4.71) which can be written also as

$$\lambda = \frac{4\rho I^2}{k^2 s^2 T_m}(1+\Delta), \tag{5.1}$$

where k is the coefficient in the index of the exponent of the formula that describes the dependence of the temperature on the coordinates:

$$\theta = T_m - T = Ae^{-kx} \tag{5.2}$$

(thus, $k = \frac{\partial \ln \vartheta}{\partial x}$), , T_m is the asymptotic temperature (the temperature of the average, "flat" section of the curve T(x)--see Figure 10, curve a), ρ is the specific resistance, I is the strength of current which heats the specimen, and s is the transverse cross-section.

The correction member Δ, which takes into account the temperature dependence of ρ and ε , has the form of

$$\Delta = \frac{1}{4} \cdot \frac{\partial \ln \varepsilon/\rho}{\partial \ln T_m}. \tag{5.3}$$

By taking into account the relationship (4.59), this expression can be written as

$$\Delta = \frac{1}{2} \cdot \frac{\partial \ln I/T_m^2}{\partial \ln T}, \tag{5.4}$$

which does not contain explicitly the dependence of ρ and ε on the temperature.

Thus, in order to determine the heat conductivity by the given method, it is necessary to know:

(1) the temperature distribution along the heated object;
(2) the current strength;
(3) the transverse cross-section of the specimen;
(4) the electrical conductivity.

It is necessary to emphasize at once the important characteristic of the method: in determining the distribution of the temperature $T(x)$, there is no need to know the true differences of the temperature θ, it is sufficient to obtain the dependence $\theta(x)$ on any random scale of magnitudes. The units in which the values of θ are determined affect only the pre-exponential factor A in the formula (5.2), the magnitude k in formula (5.1) does not depend on the unit of measurement of θ. This is a considerable advantage of the given method in comparison with other methods that are based on a study of the temperature distribution along the specimen because it makes it possible to utilize for measuring the temperature distribution the differential optical pyrometer without its graduation, which facilitates considerably the procedure of the measurements.

The above-said does not, of course, indicate that one can use this method without absolute temperature measurements in general. The absolute measurements are not required only for the temperature differences θ. The value of the temperature T_m in formula (5.1) should, of course, be known (it is also the reference temperature) just as well as the value of the temperatures in formula (5.4). Point 1 and the recalculation of the magnitudes necessary for determining the heat conductivity should for this reason be refined in the following manner: The value of the temperature distribution in random units and the value of the absolute temperature T_m. Returning to the remaining points of this recalculation, we observe that the determination of the electrical conductivity (or of the specific resistance) can be carried out in the same experiment if the difference of potentials V on the section of length L with a constant temperature is known. (For this purpose, use can be made of fine probed taps; for details see below.) In this case, formula (5.1) can be represented in one of the following forms:

$$\lambda = \frac{4IV}{sLk^2T_m}(1+\Delta), \qquad (5.5)$$

$$\lambda = \frac{4V^2}{\rho L^2 k^2 T_m}(1 + \Delta). \quad (5.6)$$

Formula (5.5) does not contain a magnitude of the specific resistance, it includes only magnitudes that are directly measurable. Formula (5.6) makes it possible to determine directly the ratio of the heat conductivity to the electrical conductivity. ($1/\rho$).

The condition under which formula (5.1) was derived in §4 is the requirement of the relative smallness of the temperature changes

$$\frac{\theta}{T_m} \ll 1. \quad (5.7)$$

This condition is sufficient for the fulfillment of the formula (5.1) and its derivative formulas (5.5) and (5.6). An analysis of the general non-linear equation (4.37) leads, however, to the conclusion that this condition is not a mandatory condition. As such, the weaker condition [105, 106, 92] should figure therein

$$\frac{\theta}{2T_m \ln\theta} \ll 1. \quad (5.8)$$

The values of θ, for which $\theta/T_m \ln\theta = 0.01$, are shown in Table 10. The comparatively large allowable temperature differences are a second advantage of the method which thought should be emphasized. (Here, also, however, it is worthwhile to observe that the utilization of relatively large temperature differences can be recommended only when the temperature dependence for the heat conductivity of the test material is small because an evaluation of (5.8) has been obtained without taking into account this dependence. In practice, apparently, the change in heat conductivity within the interval of Θ should not exceed the magnitude of the allowable experimental error).

Table 10. Values of Maximum Temperature Differences Allowed in Experiments with Exponential Temperature Distribution (Without Taking Into Account the Temperature Dependence of the Properties of the Substance).

T_m °K	θ
1000	90
1500	150
2000	220
2500	280
3000	350

The selection of specimen dimensions which would insure the existence of a section with a constant temperature is a fundamental problem in the procedure. In order to evaluate the minimum dimensions of the specimen, it is necessary to know the pre-exponential factor A in the formula for determining the temperature (5.2). According to Jain and Crishnan [106], in formula (5.2) (which is valid for a small temperature difference which is determined by the condition (5.8)) the magnitude A is equal to

$$A = T_m e^{\varkappa}, \tag{5.9}$$

where $\varkappa = \frac{1}{2} + \frac{1}{16} - \frac{1}{240} + \ldots$ for x, which are deducted from the value T = 0 on the end of the specimen of infinite length. The value of the magnitude A makes it possible to determine the distance l from the end at which the magnitude of the temperature difference θ will be equal to the small value of θ_0, for example, $\theta_0 = 1°$. For this distance of l, we obtained

$$kl = \ln T_m - \ln \theta_0 + \varkappa. \tag{5.10}$$

The length of the specimen for which the temperature at the midpoint differs from the asymptotic T_m by a magnitude of the order of θ_0 should be, apparently, at least twice as large as the value of l. Finally, we obtain

$$kL > 2\ln\frac{T_m}{\theta_0} + 2\varkappa. \qquad (5.11)$$

This same formula can be written as

$$Bi^{1/2}\frac{L}{r} > \sqrt{2}\left(\ln\frac{T_m}{\theta_0} + \varkappa\right), \qquad (5.12)$$

for wire, r is its radius, and for a strip of foil, r = h is the thickness.

The values of L (in cm), which correspond to the sign of equality in formula (5.12) and (5.13) for one of the typical cases (λ = 1 wt/cm•deg, ε = 0.3, $\theta_0 = 1°$), are shown in Table 11. It can be seen that for a wire with a diameter of <0.3 mm in the foil with a thickness of <150 mc, the length of the specimen ~ 10 cm is sufficient for measurements at temperatures above 1,000-1,500°K.

The convenient dimensions of the specimens and the small power required for heating these is a third advantage of the method. The determination of the index of the exponent k in the region where there are temperature gradients due to the flow of heat to the ends of the test wire is not the only possibility of determining k. Another method, which is carried out in [93], consists in utilizing temperature nonuniformities which are developed artificially on a section with a constant temperature by the hanging of a rider. Due to the heat dissipation from the surface of the rider near it, at temperature gradients develop on the specimen, a local temperature nonuniformity originates which is described by equation (5.2), if the dimensions of the section with a constant temperature are sufficiently large. The method of determining k on an artificially developed temperature nonuniformity is one important advantage in comparison with the use of the temperature distribution near the end. This advantage is related with the possibility of studying the temperature distribution on a section of the specimen before and after the hanging of the rider. Measurements conducted before the hanging of the rider make it possible in this case to reveal the existence of small local perturbations in the temperature constancy on a specimen, which are related with the nonuniformity of the specimen itself--temperature "background". (Practice shows that it is difficult to find a section of wire or foil without small nonuniformities.) The deduction of this "background" from the temperature distribution in the presence of a rider makes it possible to obtain a completely smooth exponential curve. However, the measurements of the temperature distribution are conducted without taking into account the temperature "background", then the resulting curves show local perturbations, the magnitude of which can reach a degree; in order to determine the index of the exponent in this case, smoothing of the curves is required.

Table 11. Length of Wire for Which the Temperature at the Mid-point Differs from the Maximum Temperature T_m By Not More than 1° (In cm).

	$2h = d = 2r$			
T° K	0,05	0,1	0,2	0,3
1000	6	9	12	15
1500	3,5	5	7	9
2000	2,5	3,5	5	6
2500	1,8	2,5	3,6	4.5
3000	1,4	2	2 8	3,5

The utilization of temperature nonuniformities developed by the writer in measuring k presupposes that there is on the specimen a sufficiently standard section with a temperature that is constant. For a priori evaluation of the real dimension of the section with a constant temperature, we shall utilize the following considerations. The temperature (reckoned from T_m) on the middle of the segment, i.e. at the distance of L/2 from the ends, is determined by the formula

$$\theta_0 = Ae^{-\frac{kL}{2}}.$$

At a distance of l from the mid-point, that is, in the point for the coordinate $x = \frac{L}{2} - l$, the temperature will be equal to

$$\theta_0 + \Delta\theta = Ae^{-k\left(\frac{L}{2} - l\right)} = \theta_0 e^{kl}.$$

Hence, for the dimensions of a section equal to $2l$, on which the temperature varies by not more than the given magnitude of $\Delta\theta$, we get the formula

$$2l = \frac{2}{k}\ln\left(1 + \frac{\Delta\theta}{\theta_0}\right). \qquad (5.13)$$

In order to determine the magnitude of θ_0, one can utilize the formula (5.12). Calculations show that the magnitude θ_0 for a specimen of given length decreases extremely rapidly with rising temperature, that is, the temperature of the mid-point of the specimen rapidly approaches an asymptotic value of T_m. Table 12 gives the values of θ_0 for one of the typical cases (L = 10 cm, λ = 1 wt/cm deg, ε = 0.3). The magnitudes $2l$ are the length of the section with the constant temperature (in cm), which is determined by the change in temperature $\Delta\theta$, equal to θ1° for the same conditions, are shown in Table 13. One can become convinced that for a wire with a diameter of up to 0.2 mm and a foil with a thickness of up to 100 microns at temperatures above 1,000-1,500°K, there are normally constant-temperature sections with an extent of the order of a cm and higher. (It is also necessary to take into account in this case that a condition of constancy $\Delta\theta = 0.1°$, for which this table has been compiled, is rather rigid; in practice, large differences are permitted in use.) Such conditions are quite sufficient for experiments with a rider (see below).

Table 12. Difference in Temperature of the Mid-Point From the Maximum Value of T_m for Wire 10 cm Long

$d = 2r$

T° K	0,05	0,1	0,2	0,3
1000	0,02	0,5	4	25
1500	$2\cdot10^{-6}$	10^{-3}	0,06	2
2000	10^{-11}	$3\cdot10^{-7}$	$3\cdot10^{-4}$	0,8
2500	10^{-17}	10^{-10}	$3\cdot10^{-7}$	10^{-2}
3000	10^{-23}	10^{-15}	$3\cdot10^{-10}$	$6\cdot10^{-3}$

The following problem in the procedure of measurements, which is subject to discussion, is related with the method of determining the function of $\theta(x)$. In practice, the temperature is never measured strictly in the point $x = x_0$; there is always a certain averaging of the temperature. In particular, in using optical pyrometers, including the differential pyrometer which is described in § 2, which was used in this work, it is not $\theta(x)$ that is measured, but the value of θ_{ef} which is average with respect to certain finite although small section Δ:

$$\theta_{ef} = \frac{1}{\Delta}\int_{x}^{x+\Delta} \theta\, dx. \qquad (5.14)$$

(The magnitude Δ is equal to the width of the slope's diaphragm of the optical parameter, which cuts out a test section of the object image; for a round diaphragm, Δ is in order of magnitude close to the diameter of the diaphragm).

Table 13. Length of Section With the Constant (Up to 0.1°) Temperature on a Wire Having a Total Length of 10 cm.

T° K	$d = 2r$			
	0, 05	0,1	0,2	0.3
1000	0,7	0,25	0,04	0,006
1500	5,1	3	1	0,04
2000	7,0	5,5	3,6	0,13
2500	7,8	6,4	5,6	1,7
3000	8,3	7,4	6,6	4,0

For the exponential temperature distribution (5.2), we get

$$\theta_{ef} = \frac{(1 - e^{-\Delta k})}{\Delta k} Ae^{-xk} = A'e^{-kx}. \tag{5.15}$$

The directly measurable dependence of $\theta_{ef}(x)$ is the same exponent with the somewhat (insignificantly) changed pre-exponential factor. The index of the exponent in this case remains unchanged and, for this reason, the final width of the diaphragm of the pyrometer does not introduce an error in the results of the determination of the magnitude k. This is one more advantage of the examined method of measurement.

Let us now pass on to a description of the practical performance of the method. The test object--wire or narrow (~ 2mm) strip of foil 1 having a length of 11 cm (Fig. 12) is placed horizontally in the cylinder chamber 2, the upper part of which 3 is taken off during the installation of the specimen. The left-hand of the specimen is firmly clamped by the screw 4 and the right is tightened by the porcelain roller with the screen spring 6. The relay 8 serves for the hanging of the rider 7. The probe caps 9 and 10, which are used for determining the electrical conductivity, are made of thin (0.05 mm) wolfram wire. The ends of the probes are drawn back by small logs. The chamber is exhausted through the connection 11 by means of the diffusion pump VA-05-1 and a fore vacuum pump PVN-20 to a vacuum of $\sim 10^{-5}$ mm mercury.

In order to cool the walls of the chamber, use is made of a brass tube 12 which is sorted on its outside surface and through which water is passed. The plain parallel glass 13 is ground down to the upper surface of the chamber under the window 5. For sealing, use is made of a vacuum case. The chamber is located above the objectives of the differential optical pyrometer which is described in §2 so that the direction of displacement of the objectives coincides with the access of the specimens. In measurements of absolute temperature of the specimens, a prism is placed on the window of the chamber; the micropyrometer OMP-19 is used for the measurements.

The specimens are heated from a 24-volt storage battery. The voltage on the probes and the current strength are determined by means of a PPTB potentiometer and a zero galvanometer (for measuring the current strength, normal resistances of 0.1 and 1 Ω are used).

For measurements, the specimen is annealed at a temperature above the maximum temperature of the measurements. Then, at the given temperature T, a record is made of the "background", that is, of the function $\theta(x)$. In the absence of a rider the values I and V are measured. The last stage in the measurements is the determination of the curve $\theta(x)$ in the presence of the rider. Typical curves obtained in the experiment are shown on Figure 13 (molybdenum, wire 0.1 mm in diameter). The curve a is the temperature distribution in the absence of a rider, b is the same with the rider, c is the results of the deduction of these two curves (one can become convinced that unlike the first two curves, the curve c is smooth; the small nonuniformities in temperature are excluded). Curve d represents curve c plotted on a semilogarithmic scale. Curve d is satisfactorily approximated by straight line in accordance with exponential nature of the function $\theta(x)$. From the angle of the slope of this straight line, it is possible to determine directly the magnitude k:

$$k = -\frac{d \ln \theta}{dx}.$$

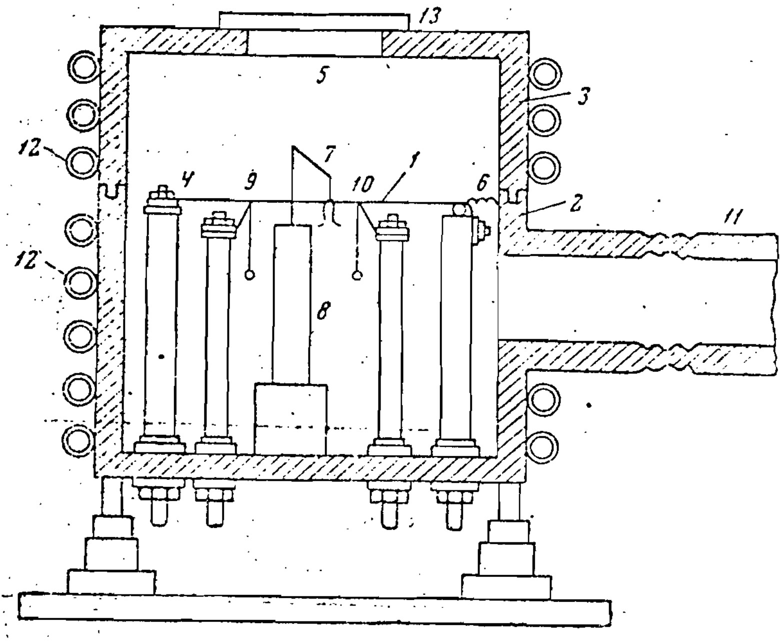

Figure 12. Chamber with the specimen.

The reproducibility of the values of the coefficient k is illustrated in Table 14 which gives the values of k taken from two sections of the same wolfram foil to the left and to the right of the rider; besides, each measurement was made twice by one and the other objectives. The deviation from the average value in this series of measurements does not exceed 2%. The scatter values as approximately the same order as in the series of other similar measurements.

In order to determine the heat conductivity, use is made of formula (5.5,. The magnitude of the transverse cross-section s which is in this formula is most simple determined by weighing the average part of the specimen of length l (weighing is accomplished after the measurements are completed; the absence of a noticeable atomization of the specimen is controlled by measurements of the electrical conductivity). The error in the measurement of s is in this case determined essentially by the inaccuracy in the measurement of l. When $l \sim 5$ cm and $\delta l \sim 0.5$ mm, the relative error $\delta s/s$ amounts to 1%.

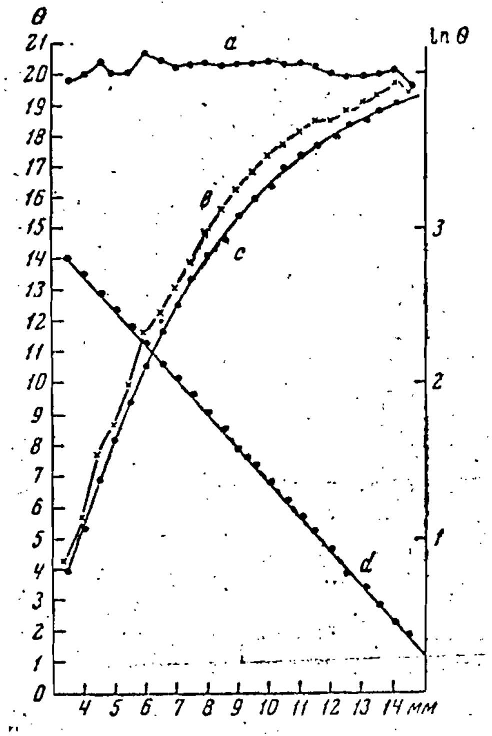

Figure 13. An example of the measurements of the temperature distribution along the wire: a, without a rider; b, with a rider; c, results of the deduction of curve a from curve b; d, curve c on a semilogarithmic scale.

In order to determine the absolute temperature T_m from the readings of the OMP-19 micropyrometer which is graduated for the brightness temperature, it is necessary to know the spectral degree of blackness of the test material. Within the framework of the given experiment, the spectral degree of blackness $\varepsilon\lambda$ can be determined only if the absolute temperature is in its turn known (for example, from the known electrical conductivity of the specimen). For a material with properties that are unknown beforehand, the experiment is not close. Additional determination of the magnitude $\varepsilon\lambda$ is required. This circumstance is, no doubt, a disadvantage of the method. However, here it is necessary to emphasize that all methods which are based on the heating of a wire by a current suffer from this disadvantage to an equal degree, even those in which a use is made of calculation of formulas that do not contain an explicitly absolute temperature (it is necessary to know to which temperature the result has to be referred). The additional experiment for determining the degree of blackness (spectral as well integral) of the same objects under the same conditions and on the same equipment will be described at the end of this section. For the present, we shall return to the determination of the heat conductivity.

Table 14. Reproducibility of the Values of the Coefficient $k(-d \ln \theta/dx)$ for Measurements on a Section of Wire to the Left or Right of the Rider With Displacement of Different Objects of the Differential Pyrometer.

Section	Objective	k, см$^{-1}$ ($T = 1700°$ K)
I	1-й	2,76
	2-й	2,70
II	1-й	2,80
	2-й	2,70
Average value	—	2,74
Maximum deviation	—	2%

The accuracy of the determination of the absolute temperature T_m is composed of the accuracy of the determination of the brightness temperature, which is ~0,5%, and the accuracy of the recalculation of the brightness temperature to the absolute temperature. The latter depends on the error of the determination of $\varepsilon\lambda$. Taking into account that $\varepsilon\lambda$ can be determined to an accuracy of 5-10% (see below), we find an error in the recalculation ~ 0.5%. The total error in the determination of T_m amounts, thus, to 1%.

The magnitude of the correction for the temperature dependence of the specific resistance in integral degree of blackness Δ can be found directly from formula (5.4) from the results of the measurements of T_m and I. The values of the correction Δ are in most cases small, amounting to several percent, (only in exceptional cases, as for example for molybdenum at temperatures > 2,200°K does this correction reach 20%). Accordingly, the errors become reduced through the inaccuracy in the determination of Δ usually to not exceed fractions of a percent. The maximum error in the determination of the heat conductivity, in accordance with the above presentation consists of the double error in the determination of k (4-6%), the error in the determination of T_m (1%) and of the error in the determination of s in L (each ~ 1%); The errors in the measurements of I and V are considerably smaller.

The total error, thus, does not exceed 7-9%. (We do not take into account the errors of reference, the magnitude of which depends on the temperature coefficient of the heat conductivity; this error is usually negligibly small.) The natural scatter of the results of measurements does not, as a rule, exceed this maximum magnitude. For illustrating the reproducibility of the results of the measurements under different conditions, we give Table 15 which was obtained in experiments with wolfram [93, 107] (see also §23).

Table 15. Reproducibility of Results of Measuring the Heat Conductivity of Wolfram.

Object, Conditions of Measurement	λ wt/cm·degree
Foil 60 microns thick, dia. of rider 0.2mm, round diaphragms in the instrument;	1.21
Foil 60 microns thick, dia. of rider 0.3 mm round diaphragms	1.17
Foil 60 microns thick, dia. of rider 0.2mm, diaphragms are in the form of slots	1.18
Wire 0.2 mm in dia., dia. of rider 0.2 mm, diaphragms are in the form of slots	1.10
Average value	1.17
Maximum deviation	6%

As observed above, the accomplishment of a close experiment in determining the heat conductivity by the method under consideration presupposes the possibility of determining the absolute temperature of the object. This possibility can be realized only in two ways: (1) by performing additional experiments on determining the electrical donductivity, when the object is placed within the model of an absolutely black body; in this case, the true temperature in the experiment for determining heat conductivity is found from the values of the resistance of the specimen, in comparison with the brightness temperature gives in this case values of the spectral degree of blackness, the integral degree of blackness is determined from the formula (4.59); (2) by determining the spectral degree of blackness under conditions when the very test object forms a similitude of an absolutely black body; the value of the absolute temperature makes it possible in this case to find also the electrical conductivity in the degree of blackness as functions of the temperature. A realization of the first of these ways is in principle simple. The object is placed within a long tube which is heated by current or by an outside heater. The true temperature is determined in a small opening of this tube. The negative aspect of this method is the need to have a special, comparatively cumbersome (it is necessary to have a sufficiently long section with a constant temperature) and powerful furnace which would make it possible to perform optical measurements of the temperature and which would have electrical leads for determining the electrical conductivity of the specimen. At temperatures above 2,000°, the performance of such experiments involves great difficulties. These difficulties are absent in the second test in which the role of an absolutely black body should be played by the specimen itself which is rolled into a tube or into a spiral. In this case, there is not need to utilize a separate furnace or chamber. Measurements can be carried out under the same conditions as in the experiments for determining the heat conductivity and within the same temperature range, that is, at temperatures above 2,000°. The second advantage of the method under consideration is the possibility of internal control of the experiment. Actually, a study of the specimen--marble of a black body--makes it possible to determine simultaneously the functions $\rho(T)$ and $\varepsilon\lambda(T_0)$. In this case, the first as well as the second functions can be utilized for the independent determination of the values of the true temperature of the object in experiments for determining the heat conductivity: information on the temperature of the object becomes all the more superfluous. Thus, the additional experiments of the examined type make it possible to accomplish by means of the same equipment a close circle of measurements of a complex of magnitudes: heat conductivity, electrical conductivity, integral spectral degrees of blackness. A description of the experiment in this direction is given below. Let us examine at first the objects in the form of a foil. The literature [108] describes three models of an absolutely black body, which can be made from a strip of foil. The first model represents a strip folded lengthwise in such a way that a wedge-like gap with an angle of approximately ~ 10° forms within the two halves. This model is one of the simplest, but at the same time it has a number of disadvantages: it requires the introduction of comparatively large corrections, there is danger of formation of temperature difference along the resulting wedge (across the strip of foil). The second model--a cylindrical spiral with narrow gaps between the turns--can in principle better assure the degree of blackness close to one; however, the practical accomplishment of this model with consideration of the expansion of the metal at high temperatures and the sagging of the spiral is sufficiently complex. The third model--a tube with a small opening--is the best as regards the approach to conditions of an absolutely black emission, however, it is not a simple matter to prepare an integral tube from foil. The disadvantages inherent to the three examined models of an absolutely black body have justified the work involved in the study of another fourth model which is a tube rolled from a strip of foil in such a way that the non-linking edges of the strip form a comparatively narrow (0.3-0.5 mm) slot. Specific evaluations of the magnitude of the effective degree of the blackness of emission of the slot lead to the conclusion that this magnitude is sufficiently close to one. Thus, for a tube of the diameter of ~ 1.5-2 mm, $\varepsilon \sim 0.5$ and for a wide slot of ~ 0.4mm, the effective degree of blackness is $e_{ef} = 0.96$, that is, the correction for the "nonabsoluteness" amounts to 4% (for details see [109]).

For specimens in the form of a wire, a model was tested which has the shape of a long cylindrical spiral with a relatively small distance between the turns. Unlike the model with a spiral made of foil emitted by a "window," here there were no gaps between the turns of the spiral but the end of the spiral. For oblique beams of emission which comes from within the spiral, the integrals between the turns play a lesser role and the effective degree of blackness of such emission is comparatively close to unity, although naturally also not so close as for the preceding model. Thus, for a spiral made of wire 0.2 mm in diameter with a distance between the turns ~0.1 mm for $\varepsilon = 0.5$, the effective degree of blackness ε_{ef} is equal to 0.93 (see [109]).

In performing measurements with the described models of the black body, the specimens were placed in the same vacuum chamber as during the measurements of the heat conductivity (Fig. 12). In experiments with a foil, the tube, which is made by rolling a foil, is placed horizontally. One end of it is secured to the current lead and the other is tightened by a spring and the current is fed to it through a flexible conductor. In two places of the tube, probes were connected to it (they were passed through the tube and tightened by loads). In experiments with wire, the spring was suspended vertically in the chamber, and the probe caps were fastened near the ends of the spiral. For sighting the pyrometer, a prism with complete internal reflection was placed on the lateral surface of the spring within the chamber (in processing the results, a correction was made for the attenuation of the emission by the prism).

The determination of the spectral degree of blackness of the material is related with the determination of relatively small differences of the reciprocal values of the absolute values of the brightness temperatures on the surface of the tube and the slot:

$$\ln \frac{\varepsilon_\lambda}{\varepsilon_{ef}} = \frac{h\nu}{k}\left(\frac{1}{T_{elk}} - \frac{1}{T_{pov}}\right). \tag{5.16}$$

For this reason, the error in determining $\varepsilon\lambda$ in these experiments is incidentally and always comparatively large. Thus, in using an optical pyrometer OMP-19, which assures a relative error in the determination of the temperature amounting to 0.5%, for a wavelength $\lambda = 0.66$ microns, the error in determining $\varepsilon\lambda$ amounts to ~ 20% of 1,000°K and ~ 10% at ~ 2,000°K. These errors pertain only to individual measurements and are essentially governed by the random error due to inaccuracy in determining the temperatures. The statistical averaging of the series of measurements can to a considerable degree exclude these random errors and decrease the total errors severalfold.

As an example of the determination of the spectral degree of blackness, we cite the results of measurement for molybdenum wire 0.2 mm in diameter (Fig. 14). The degree of blackness was determined for a spiral of ~ 1-1.2 mm in diameter, 6-8 centimeters long with a distance between the turns equal to 0.07-0.1 mm. Each point on the curve was obtained by averaging four separate measurements. The maximum deviation of the point from the averaging straight line amounts to 5%, the average deviation is 1.5%. The dashed curve in this figure shows the data by Worthing (see [108]). The maximum difference (at 1,200°K) amounts to ~10%. The existence of such divergence from the Worthing data, although small, can partly be determined by the difference in the surface of the specimens; in the Worthing experiment, polished specimens were used.

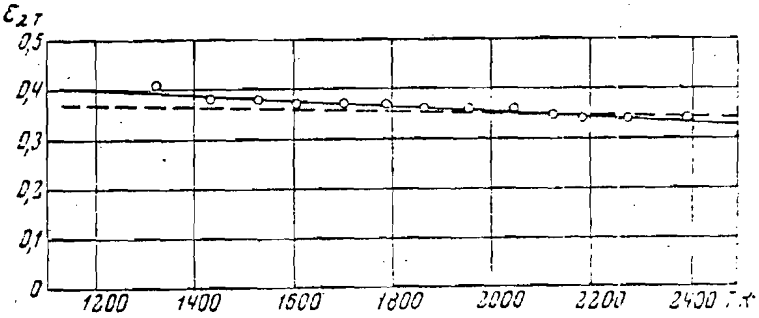

Figure 14. Temperature function of the spectral degree of blackness of molybdenum wire.

The error in determining the true temperature, obtained by recalculating the measurements of the blackness temperature by means of the values of the spectral degree of blackness, found by means of the described method, can be found by the aid of formula (5.16). If the maximum values of 20-10%, which are valid for individual measurements, are taken as the errors in the determination of $\varepsilon\lambda$, then the total error in determination of the true temperature would amount to 1.5%, if the error in determining the brightness temperature is 0.5%. For averaged and smoothened values, the errors of $\varepsilon\lambda$ will at least be half as large in the maximum error in determining temperature amounts to ~ 1%.

It was observed above that the experiments with models of an absolutely black body, which have been made from a specimen, make it possible to perform internal control of the measurements of the totality of the magnitudes $\varepsilon\lambda$--ρ. This control can be carried out by comparing the values of the true temperatures found from the functions $\varepsilon\lambda(T)$ and $\rho(T)$ or by comparing the values of $\rho(T)$ obtained in experiments with an absolutely black body and with a specimen under conditions of measurements of the heat conductivity (tightened specimens). A comparison of the functions $\rho(T)$, which has been described in [109], has shown that compatible values with an accuracy up to the errors of one or another experiment are obtained.

In conclusion, we wish to dwell on the determination of the integral degree of blackness. Values of ε can be found on the basis of the relationship (4.59), from which it follows that

$$\varepsilon = \frac{\rho I^2}{Sp\sigma T^4}. \tag{5.17}$$

This formula can be written in a more convenient form:

$$\varepsilon = \frac{IV}{Lp\sigma T^4}, \tag{5.18}$$

where V is the drop in voltage between the probe taps which are resistance of L from one another.

The maximum error in determining ε, in accordance with this formula, is equal to the quadrupled error in the measurement of the temperature (4%) and the error in the measurement of the geometric parameters L and p (1% + 1%), which amounts to a total sum of 6%. The results of the determination of the integral degree of blackness of the same specimen of molybdenum wire for which data were obtained on the spectral degree of blackness shown above is illustrated in Figure 15.

The maximum deviation of points from the averaging straight line on this diagram does not exceed 5%, the average deviation amounts to 2%. The dashed line in the figure shows the Worthing data (see [108]). The positive deviations from these data, just as in the case of the spectral degree of blackness, can be explained by the fact that the measurements by Worthing were carried out with polished specimens.

In confirmation of this explanation, we can refer to [110] where it is shown that the degree of blackness of molybdenum, just as this should be true for all metals, depends substantially on the degree of roughness of its surface. Data in [110] for a strongly rough specimen of molybdenum lie above those shown in Figure 15.

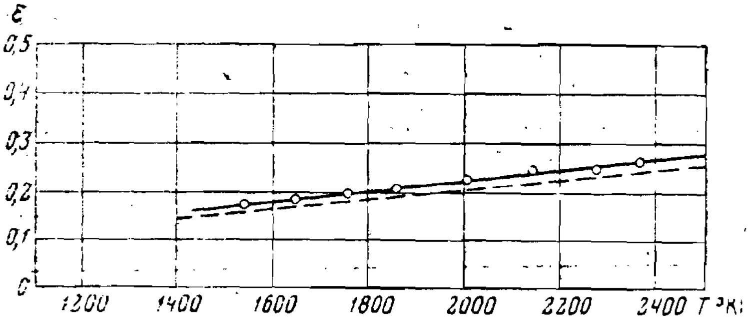

Figure 15. Results of the measurement of the temperature function of the integral degree of blackness of molybdenum wire.

In concluding the presentation of the procedure for determining the degree of blackness, we wish to emphasize once more that measurements of the complex of thermophysical measurements such as the heat conductivity, electrical conductivity, spectral and integral degree of blackness, can be accomplished in a single experiment with the same equipment within a temperature range from ~ 1,000°K to temperatures limited only by the intensive vaporization of the test object (in case of a noticeable vaporization, the range of maximum temperatures that can be measured can be raised if the measurements are carried out not in vacuum but in an atmosphere of a pure inert gas). We wish to observe here also that the same equipment (with small changes) and the same objects can be utilized also for determining the heat capacities within the same temperature range (see § 19). Such complex measurements of all thermophysical parameters of a material are, of course, particularly valuable.

Chapter III

Non-Steady Methods for Measuring Thermal Propertie

§6. False Methods. Utilization of a Method With a Standard for Liquid Metals

It was observed in Chapter 1 that a study of non-steady processes gives in principle considerably more information on the thermal properties of the system in comparison with the amount of information in the case of steady conditions. This circumstance is related, in the first place, with the fact that the temperature fields for non-steady processes depend not so much on the coordinates but also on one more variable--time; the number of independent variables increases and, in accordance with this, the volume of information also increases. On the other hand, the basic equation which describes the space-time distribution of the temperature--the heat conductivity equation--in the general, non-steady case includes a new, in comparison with a steady case, parameter--the volume heat capacity. This makes it possible to utilize the increased volume of information not only for expanding the diversity of methods for studying the heat conductivity of substances, but also for carrying out measurement of other thermophysical characteristics such as heat capacity and temperature diffusivity.

The great adversity of non-steady experimental methods, which differ not only in the space symmetry of the test temperature field and in the nature of measurements of the temperature with respect to time but also in the beams of the very experiment, that is, with respect to the magnitudes that are precisely being determined in the experiment, hinder greatly the classification in the systematization of the corresponding method. Nevertheless, it is believed that the classification of measurements method with respect to the indication of the magnitude that is precisely being measured cannot be the best, because in this case, measurement methods which have very little in common with one another get placed in one group, while the measurement methods of different magnitudes, which utilize the same measurement system, are dispersed; inconveniences arise in describing experiments, the aim of which is the measurement, not of any one magnitude, but of the entire totality of the thermal properties, (namely such methods which have been developed recently are of the greatest practical value).

It seems more judicious to use the classification of non-steady methods, the basis of which is, in the first place, the difference in the nature of the change of the temperature field with respect to time and in the second place--the difference in the nature of heating (external, internal) and then, just as in the case of steady methods--the symmetry of the isotherms of the temperature field and the more frequent characteristic signs. We shall adhere henceforth to precisely such a system.

With respect to the nature of the dependence of the temperature field on the time, the Soviet literature has adopted a classification of metals into two large groups. The first group includes metals for which the initial conditions are pertinent, that is, the temperature field at a time when the test process started. In the methods of the second group, on the other hand, use is made of a process in its stage when the initial conditions do not play a role--so-called regular conditions. In this section we examined only the first group of methods. As regards studies of the thermal properties of metals at high temperatures, these methods have not been developed substantially, which makes it necessary to limit the presentation to general information, making an exception only for the pulse method of heating small flat specimens, which is successfully being used at the present. In order to familiarize the readers with many highly diverse methods of non-regular conditions, which are used for the measurement of the thermal properties of four conductors of heat, the reviews in [111, 112, 113] are recommended. The examined group of non-steady methods can be divided into two sub-groups with respect to the method of creating temperature changes. In the first sub-group of methods, the temperature changes are achieved by connecting (most frequently for brief periods of time) surface sources of heat. These are the so-called pulse methods. In the second sub-group, the test objects are brought into thermal contact with bodies which have another temperature; the thermophysical characteristics of these bodies are assumed to be known and for this reason such methods are called methods with a standard.

Let us now pass on to an examination of pulse methods. The simplest is the accomplishment of the method under conditions with the space change of the temperature unidimensionally, in the first place for flat isotherms. As is known from §4, the general equation of heat conductivity for specimens in the shape of cylinders reduces itself to the unidimensional equation provided there is smallness of the

Biot criterion in which the transverse dimension of the cylinder figures. Another maximum case, when one can examine flat isotherms is a specimen in the shape of a plate with a thickness that is much less than the transverse dimensions. The distortion of the isotherms in this case will be observed only near the edges of the plate; in the central region of the isotherm, the isotherms will be flat. (The same conditions can obviously take place for a massive specimen which is limited on one side by a flat surface if the space-time changes of temperature are realized only on a small distance from this surface, in comparison with the transdiameter of the specimen.)

Let us examine one of the most developed theories of the post methods which had been worked out specifically for specimens in the shape of a plate [114]. The essence of this method is a study of the time dependence of the temperature on the end surface of a flat specimen after the opposite side had been subjected to pulse heating. (By pulse heating we mean the liberation of a finite amount of heat during a small, within the bounds of an infinitely small time.) The theory of the method assumes that as a first approximation, the surfaces of the specimen can be considered as heat-insulated. The solution of a unidimensional non-steady equation of heat conductivity

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial T}{\partial t}, \qquad (6.1)$$

provided that at the moment of action of the heat pulse the temperature of the specimen was the same and equal to 0

$$T(x, 0) = 0, \qquad (6.2)$$

has in this case the form

$$T(x, t) = \frac{q}{c_p \rho L} \left[1 + 2 \sum_{n=1}^{\infty} \cos \frac{n \pi x}{L} \exp \left(- \frac{n^2 \pi^2}{L^2} at \right) \right]. \qquad (6.3)$$

Hence, for the point x = L on the end of the specimen, we get

$$T(L,t)=\frac{q}{c_p\rho L}\left[1+2\sum_{n=1}^{\infty}(-1)^n\exp\left(-\frac{n^2\pi^2}{L^2}at\right)\right]. \tag{6.4}$$

For in dimensionless variables

$$\tau=\frac{T}{T_M} \tag{6.5}$$

and

$$\zeta=\frac{\pi^2 at}{L^2}=\pi^2 Fo, \tag{6.6}$$

where T_M is the asymptotic maximum temperature

$$\tau=1+2\sum_{n=1}^{\infty}(-1)^n\exp(-n^2\zeta). \tag{6.7}$$

The curve of the function $\tau(\zeta)$ is shown in Figure 16. This curve can be utilized for determining the temperature diffusivity a from the results of the change in T during a fixed moment of time t and from the magnitude T_M. For this it is sufficient to find from the curve the value of ζ corresponding to the given ratio T/T_M, and determine from it a. The optimum conditions for carrying out such a procedure correspond to the values of $T/T_M = 0.5$. If $T/T_M = 0.5$, then the temperature diffusivity can be determined directly from the formula

$$a=\frac{1{,}38L^2}{\pi^2 t_{1/2}}, \tag{6.8}$$

where $t_{1/2}$ is the time required for heating the surface, x = L to half of the maximum temperature. Just as in all experiments which deal with the measurement of the temperature diffusivity, in this case a knowledge of the absolute values of the temperature and of the power of the heating is not required (see § 1).

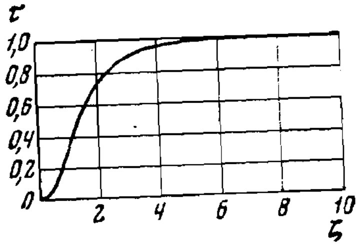

Figure 16. Curve of the dependence of the dimensionless temperature τ on the dimensionless time ζ in experiments with pulse heating of the plate.

In experiments by Parker *et alia* [114], the specimens were metal plates 1-3.5 mm thick and ~ 2 cm^2 in area. The pulse heating was accomplished by illuminating the specimen with emission of a flashbulb. The temperature measurements were recorded by a thermocouple. The measurements were limited to room temperatures. The error of the measurements amounted to 5%.

A weak spot in the examined method is a disregard of the heat loss from the end surface of the specimen x = L, which is particularly dangerous at high temperatures. The problem of errors introduced by this effect has been examined in [115] where a generalized solution has been obtained of the corresponding problem and values are given of the correction to the formula (6.8). The magnitude of the corrections is naturally an unambiguous function of the Biot criteria:

$$Bi = \frac{\alpha L}{\lambda}. \tag{6.9}$$

The correction are small for Bi of ~ 0.1 and smaller,which makes it possible to utilize this procedure of measurements under conditions of vacuum (heat exchange by emission) up to temperatures of ~ 3,000°K, if the thickness of the specimens is equal to ~ 1 mm (see Table 1).

Another pertinent problem of the procedure is the role of the finiteness of the heating time which is being subjected to analysis in [116]. In this same reference, brief descriptions are given of the measurements which have been carried out at temperatures up to 700°C. A laser was utilized as the source of pulse heating.

The method by Kennedy *et alia* [117] can be considered a variant of the method of pulse heating, in which the temperature diffusivity of the specimen is determined by comparing a series of solutions of the heat conductivity equation, which have been obtained on an electronic computer, with experimental curves of the temperature change with time in a fixed point of the specimen. In determining the calculated curves, information of the initial and boundary conditions of the specimen, which have been obtained in experiment, are fed into the machine. Thus, the authors used this method to determine the temperature diffusivity of iron up to 1,100°C.

The prospects for the development of pulse method of measuring temperature diffusivity, besides projecting it into the region of high temperatures, are related apparently with the possibility of accomplishing this method as a complex method which makes it possible to determine not only the temperature diffusivity but also the heat capacity and consequently also the heat conductivity. On the basis of the solution of (6.4), it is possible to obtain the heat capacity if T_M and Q are known. The determination of Q can in principle be accomplished by utilizing pulse heating by means of electron bombardment. Work in this direction seems expedient.

Let us now pass on to the procedure with a standard. These methods are based on the solution of the heat conductivity equation for the test and standard media with boundary conditions of the fourth type as conditions of the conjugation of these solutions. It follows from this that the value of the heat characteristics of the standard medium is necessary. The source of information in these methods is as a rule the dependence of the temperature in one or several points of the standard on the time. The methods with a standard, which have been worked out in the first place by the efforts of A. V. Lykov and his students, make it possible to determine in a convenient manner the complex of heat characteristics of poor heat conductors (see, for example, the review [113]). These methods have, however, not been developed for metals. But such a situation does not at all indicate that a utilization of the methods with a standard for metals is inexpedient. It seems to us that a number of variants of these methods could successfully be utilized specifically for liquid metals. Below is described work with some of the variants of the method with a standard--the method for determining the coefficient of heat activity of liquid metals.

The coefficient of heat activity of a substance is, according to A. V. Lykov [6], called the magnitude[1]

$$b = \frac{\lambda}{\sqrt{a}} = \sqrt{\lambda c_p \rho} \tag{6.10}$$

This combination of heat characteristics shows up in a description of the non-steady temperature field in systems of boundary conditions of the second, third and fourth types and during the calculation of the non-steady flows of heat in the medium. In its physical sense, the heat activity is similar to acoustic impedance; this analogy is, however, not complete [118]. By itself, heat activity, although it has a direct practical significance, still represents limited interests. However, its utilization in combination with other parameters makes it possible to determine other heat characteristics and, in the first place, the heat conductivity. Such a roundabout way of determining the heat conductivity is in principle and theory to the method of direct measurement. Nevertheless, in a number of cases one should not disregard even such indirect methods which could serve as a convenient control of the results of direct measurements and sometimes as their substitution. This pertains, in the first place, to liquid metals and semiconductors, since the means for their study are rather limited. It is particularly expedient to combine measurements of the coefficients of heat activity with measurements of the temperature diffusivity. Actually, in this case a maximum error in the determination of λ and $c_{p\rho}$ will be made up of the error of the determination of b and half of the error of the measurement of a:

$$\frac{\delta\lambda}{\lambda} = \frac{\delta(c_p\rho)}{c_p\rho} = \frac{1}{2}\cdot\frac{\delta a}{a} + \frac{\delta b}{b}. \tag{6.11}$$

---

[1] The terminology here has not been fully established. The literature has terms such as the coefficient of heat penetration, the coefficient of heat assimilability, or the coefficient of heat accumulation. Henceforth, we shall instead of the term coefficient of heat activity, write simply heat activity.

If it is taken into account that the errors $\delta a/a$ and $\delta b/b$ are approximately equal $\frac{\delta a}{a} \simeq \frac{\delta b}{b} = n,$ then it follows from this that

$$\frac{\delta\lambda}{\lambda} = \frac{\delta(c_p\rho)}{c_p\rho} \simeq 1{,}5n. \tag{6.12}$$

In the other case, however in utilizing the combination, the heat activity is the heat capacity

$$\frac{\delta\lambda}{\lambda} = 2\frac{\delta b}{b} + \frac{\delta(c_p\rho)}{c_p\rho} \tag{6.13}$$

and

$$\frac{\delta a}{a} = 2\frac{\delta b}{b} + 2\frac{\delta(c_p\rho)}{c_p\rho}. \tag{6.14}$$

Hence, even if it is taken into account that $\frac{\delta(c_p\rho)}{c_p\rho}$ can be several times less than $n = \frac{\delta b}{b}$, we get

$$\frac{\delta\lambda}{\lambda} > 2n, \tag{6.15}$$

$$\frac{\delta a}{a} > 2n. \tag{6.16}$$

The combination of measurements of the coefficient of heat activity and of the temperature diffusivity should lead, for this reason, to more accurate results.

Let us now pass on to the presentation of the essence of the described method for measuring the coefficient of heat activity. Its basis is the simple "everyday" fact: during cold weather, metal objects seem to feel considerably colder than stone and wooden objects. This circumstance is explained by the fact of the temperature, which becomes established upon contact of bodies with a different temperature, depends fundamentally on the thermophysical characteristics of these bodies. An analytic study of the problem of the temperature distribution in two semi-infinite media (each medium is a semispace) after their contact leads to the formulas [119]

$$T(x, t) = \begin{cases} T_1^0 + \frac{\gamma}{\gamma+1}(T_2^0 - T_1^0)\,Erfc\left(\frac{-x}{2\sqrt{a_1 t}}\right) x < 0, \\ T_2^0 + \frac{1}{v+1}(T_1^0 - T_2^0)\,Erfc\left(\frac{x}{2\sqrt{a_1 t}}\right) x > 0, \end{cases} \tag{6.17}$$

where

$$Erfc = 1 - \frac{2}{\sqrt{\pi}}\int_0^z e^{-\xi^2}d\xi, \tag{6.18}$$

T_1^0 and T_2^0 are the temperatures of the bodies prior to contact,

$$\gamma = \frac{b_2}{b_1}. \tag{6.19}$$

It follows from formula (6.17) that the temperature of the surface of contact (x = 0) does not depend on the time and is equal to

$$\theta = \frac{\gamma}{\gamma+1}(T_1^0 - T_2^0) + T_1^0 \equiv T_2^0 + \frac{1}{\gamma+1}(T_1^0 - T_2^0). \tag{6.20}$$

This formula can be written also as

$$\frac{\theta - T_1^0}{T_2^0 - \theta} = \gamma = \frac{b_2}{b_1}. \qquad (6.21)$$

Thus, the ratio of the heat activity of two media can be found by measuring the constant temperature which becomes established on the surface of two extending bodies. If one of the bodies is a standard, that is, its heat activity is known, then such an experiment can in principle serve as a relative method for determining the heat activity.

The method of determining the heat activity, which is based on the examined principle, was utilized by A. F. Chudnovskim for studying the heat activity of soils and grounds, that is for conditions under which at least one to two bodies participating in the experiment approaches semi-infinite state to the maximum degree [111, 112].

A detailed examination of the experimental conditions which apply specifically to metals, the details of which cannot be dealt with by us here, leads to the following conclusions.

1. The conditions of smallness of the influence of the lateral heat exchange of the contacing bodies is the inequality

$$\frac{t}{\tau} \ll 1, \qquad (6.22)$$

where τ is the characteristic cooling (heating)time. Hence, it follows in particular that one of the bodies participating in the experiment can be placed within the heater, if the heating time in this case is not so small as to disturb the condition (6.22).

2. Consideration of the difference in the transverse cross-sections of the contacting bodies leads to the requirement

$$Fo \ll 1. \qquad (6.23)$$

3. Consideration of the infinity of the length of the specimen makes it necessary to satisfy the inequality

$$\frac{at}{L^2} < 0,06. \qquad (6.24)$$

4. The existence of an initial temperature gradient in the specimens, equal on the order of magnitude to the initial temperature differences of the contacing surfaces, will little effect the final results if the condition (6.23) is satisfied.

5. Condition of smallness of the role of the non-ideality of the heat contact has the form

$$\frac{b}{b_1} \cdot \frac{\delta}{\sqrt{at}} \ll 1, \qquad (6.25)$$

where b and δ are the heat conductivity and the thickness of the intermediate layer between the contacting surfaces. A specific analysis of the resulting evaluations lead to the conclusion that the corresponding corrections are small for the time interval of 0.01 to 0.06-1.5 sec provided the magnitude of the surface of the liquid metal differs little from the surface of the standard. Measurements of the temperature during the time of the order of 1 sec is not complex and the examined procedure of measurements can be realized.

The main problem in the practical accomplishment of this method of measurement, specifically with respect to metals is in the measurement of the temperature of the standard--specimen interphase. There are two possible methods of measurement: 1) determination of the temperature from the magnitude of the thermal electromotive force which develops in the test metal--standard thermocouple, and 2) utilization of thermocouples or thermal probes located near the surface of contact. At first, let us examine the first method by dwelling not on all its possible variants but only on one of those that are most convenient ([118]). The basic system for accomplishing this method is shown on Figure 17. The liquid metal is placed in this case in a U-shaped tube; besides, the second elbow of the tube is kept at a temperature of T_1 which is equal to the temperature of the standard metal (for this purpose, the standard metal can be immersed in the elbow with the liquid metal or disequality in temperatures can be reached by artificial control). The measured difference of the potentials is V_{12}, which is proportional to the difference in temperatures Θ--T_2 and V_{13}, and proportional to T_1 -- Θ. The ratio of the heat activities is in this case determined from the formula

$$\frac{b_2}{b_1} = \frac{V_{13}}{V_{12}}. \tag{6.26}$$

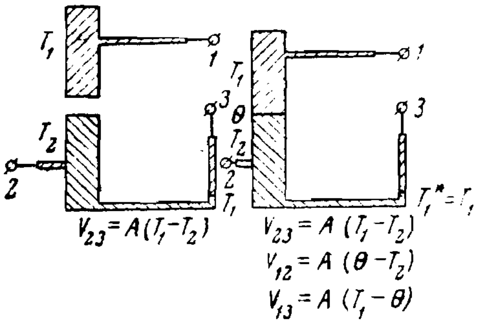

Figure 17. Schematic diagram for conducting the experiment (method of contact with a standard).

The accomplishment of the thermoelectric method of measuring presupposes that the emf of the junction of the metal with a thermoelectric probe (wire) of the same metal is equal to zero or, in any case, is very small in comparison with V_{12} and V_{23}. In practice, this takes place in a very small number of cases. As a rule, the wires and the corresponding sample, even if of sufficiently pure metal, have a noticeable thermal emf. An exception is copper as well as iron samples and pair with annealed shavings from the same samples. Apparently, for other metals one can use as a thermoelectrode also small rods cut from the same specimen; however, it is clear that a need for special care about the smallness of the parasitic thermal emf s is a minus factor in this method of measurements.

The second method of measurements--with the thermocouple in a standard sample near the surface which contacts the test metal--is free of this indicated disadvantage in principle.

The first of the variants of this method is shown in Fig. 18. The initial difference in temperatures in this variant is measured up to contact by means of thermocouples that are connected differentially as shown in Figure 18a. After this, the thermocouples are connected in Figure 18b; besides, the reading of the thermocouple is proportional to the temperature difference T_1--Θ. The ratio of the heat activities in this method of measurement should be determined from formula

$$\frac{b_1}{b_2} = \frac{T_1 - T_2}{T_1 - \theta} - 1 = \frac{V_{12}}{V_{13}} - 1. \tag{6.27}$$

This formula is somewhat less convenient that (6.26) because it error in determining the magnitude $\gamma = b_1/b_2$ in this case is greater. Actually, it follows from (6.27) that

$$\frac{\delta\gamma}{\gamma} = (1 + \gamma)\left[\frac{\delta V_{12}}{V_{12}} + \frac{\delta V_{13}}{V_{13}}\right], \tag{6.28}$$

whereas formula (6.26) gives

$$\frac{\delta\gamma}{\gamma} = \frac{\delta V_{12}}{V_{12}} + \frac{\delta V_{13}}{V_{13}}. \tag{6.29}$$

Thus, in the first case the error is $1 + \gamma$ times greater and the utilization of the examined method of measurements leads to better results when b_2/b_1 is comparatively small, that is, the heat activity of the standard is greater than that of the test sample.

Another variant of the same method of measurements is shown in Figure 19.

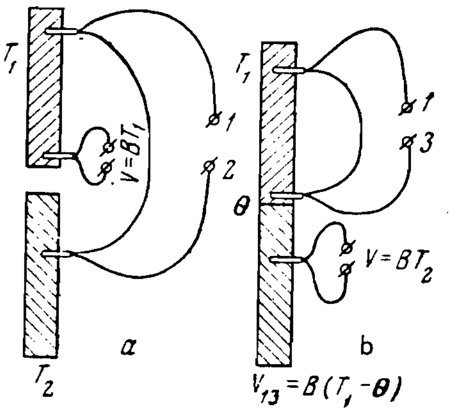

Figure 18. Schematic for conducting the experiment. Variant with thermocouples.

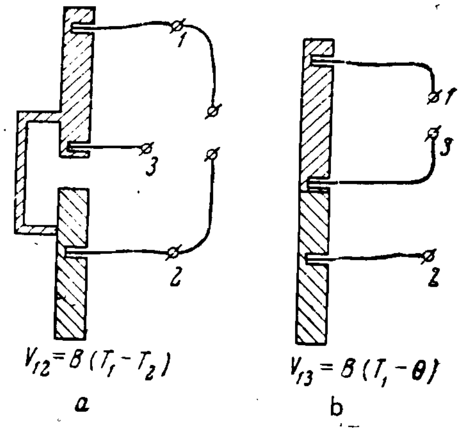

Figure 19. Schematic for conducting the experiment. Variant with thermoprobes.

It differs from the first variant only in the fact that instead of thermocouples it has thermoprobes, which simplify somewhat the installation of the system (chiefly the junction 3 which should be located as close as possible to the end). In order to determine the initial difference in temperatures in accordance with this system, it is, however, necessary to utilize the thermoelectrode of the standard material, which prior to the experiment should close the circuit as shown in Fig. 19a. The rest remain just as in the preceding variant. In accomplishing the second method of measurements, an important problem is that of correction for the thickness of the layer of the specimen that is between the surface of contact and the thermocouple. This correction can be written as

$$\frac{\delta}{\sqrt{\pi a t}} \qquad (6.30)$$

For $\delta \sim 0.05$ cm (which is practically realizable) and $a \sim 0.2$ cm^2/sec (iron), we find that the correction amounts to 6% for $t \sim 1$ sec.

Thus, this correction is not negligibly small, but it is still sufficiently small that it can be taken into account precisely as a correction. It is plain that in this instance a substantial role is played by the fact as to how small it is possible to make the distance and with what accuracy it can be determined.

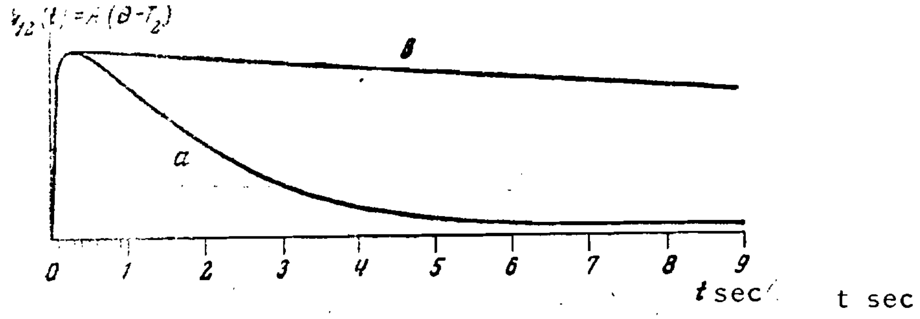

Figure 20. Change in temperature difference in the measured metal with respect to time: a, test specimen with a diameter considerably greater than that of the standard; b, diameters of the test standard specimens are alike.

Further we shall pass on to a presentation of the results of the practical study of different variants of the examined method for measuring heat activity.

Measurements of temperature gradients in the standard and in the sample, as shown above, should be carried out during the time that ranges from ~ 0.1 - ~1 sec after contact. The measurement device should for this reason be of flow inertia. In order to measure the thermoelectromotive forces in the described experiments, use was made of an UF-220 device with a mirrorgalvanometer, which has a stabilization time of ~0.1 sec. The dependence of the measured emf on the time was recorded on a photographic film. Typical curves for the differences of the temperature in the test metal are shown in Fig. 20. The curve a corresponds to the case when the rod with~ 1 cm diameter was immersed in the liquid metal that was in a comparatively wide crucible. The observed temperature drop at time has a rather steep character and the determination of V_{12} at the initial moment and involves a certain indeterminacy. The curve b pertains the case when the transverse dimensions of the standard and the specimen were comparatively close: the determination of V_{12} here involves no labor. From this one can make the conclusion that experiments should be carried out as much as possible under conditions for which the radial temperature gradients are not significant. This conclusion agrees in general with the results of the *a priori* evaluations carried out above.

The first of the methods for measuring heat conductivity, which is based on the registration of the temperature of the surface directly from the thermoelectromotive force of the corresponding pair of metals, was tested on pairs of solid metals such as tin-copper, tin-iron. The taps were made from this same samples (tin strip and iron shavings) and, in the case of copper, a copper wire was used. The temperature T_1 was room temperature (the standard sample and the junction of T_1 were brought into thermal contact). In order to carry on the registration of the thermal emf V_{12} and V_{13}, without repeating the experiment a second time (only one registration system was available), use was made of a somewhat different procedure of measurements from that described on page 104). It is precisely for this reason that, instead of the magnitude V_{13}, a determination was made of the magnitude V_{23}, prior to contact, which, as can be seen from Figure 17 is proportional to $T_1 - T_2$. In accordance with this, instead of formula (6.26), use was made of the formula

$$\frac{b_2}{b_1} = \frac{T_1 - T_2}{T_1 - \theta} - 1 = \frac{V_{12}}{V_{13}} - 1. \qquad (6.31)$$

The results obtained in a series of measurements with the iron-solid tin pair are shown in Table 16. (The values of V_{23} and V_{12} in the table are given in arbitrary units.) The table makes it possible to judge the reproducibility of results in this method of measurements. If it is taken into account that accuracy of the determination of V_{23} and V_{12} amounts to ~2%, then the maximum of the individual measurements of b_2/b_1 should be equal to approximately 10% without taking into account the errors of the fulfillment of the equality of the temperatures $(T_1^*-T_1)$. Thus, the deviations of the measurement results from the average value can be considered consistent with the *a priori* evaluation of the error for the given conditions. The average value of the ratio b_{sn}/b_{Fe} which was determined in this experiment, coincides within the limits of the accuracy of the measurements with the magnitude 0.62 which is obtained from the data by R. P. Yurchak for the same samples of iron and tin [31, 33]. Similar experiments were carried out also with the iron-liquid tin pair [118]. The results however, are of a preliminary nature so far and, for this reason, are not cited here.

Table 16. Results of the Measurements of the Ratio of the Coefficients of Heat Activity of Iron and Solid Tin.

V_{23}	V_{13}	b_{Sn}/b_{Fe}
9,8	6	0,63
9,5	5,9	0,61
9,1	5,3	0,59
9,6	5,8	0,65
9,5	5,7	0,67
9,7	5,8	0,68
48,(8)	28,(5)	0,71
45,(5)	29,(0)	0,68
49,(8)	31,(3)	0,59
52,(3)	33,(5)	0,56

Average Magnitude 0,64
Maximum Deviation 12%
Average Deviation 6%

Table 17. Results of the Measurements of the Ratio of the Coefficients of Heat Activity of Iron and Liquid Mercury.

T_1-T_2	$T_1-\Theta$	b_{Fe}/b_{Hg}
90	17	4,3
87	17	4,1
87	17	4,1
87	17	4,1
87	17	4,1
87	17	4,1
86	17	4,1
85	17	4,0
86	17	4,1
87	17	4,1
86	16	4,4

Average Value $4{,}1_3$
Maximum Deviation 7%
Average Deviation 2%

The second of the considered methods of measurements--with the utilization of thermocouples near the end of the standard rod--was tested specifically in determining the heat activity of liquid metal--mercury. In these experiments, the mercury was poured into an iron cup made by proper treatment of the end of a massive iron rod (Figure 21). The diameter of the cup exceeded seomwhat that of the standard iron rod. Constantan probes 1 and 3 were welded to the surface of the iron, probe 2 was welded within the hold at a distance of ~ 0.4 mm from the end. The piece of fine shaving 4 was, prior to the experiment, immersed in the mercury. The results of one series of measurements are shown in Table 17 (the temperature is given in arbitrary units).

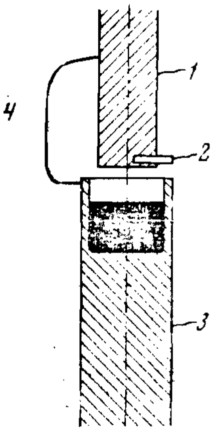

Figure 21. Diagram of an experiment with liquid mercury.

It can be seen that in the series of eleven measurements the results are reproduced rather well. The maximum deviation from the average value does not exceed the calculated error which under these conditions amounts to 8%. (The error of the determination is

$$T_1 - T_2 \sim 1\%, \quad T_1 - 0 \sim 5\%.)$$

The average value of the ratio of the activities agrees with literature data

$$\left(\frac{b_{Fe}}{b_{Hg}} = 4{,}1 \right)$$

The described studies, which are in the nature of "zeroing" exploratory experiments make it possible nevertheless, despite the limited nature, to conclude that further work in this direction is justified, inasmuch as there are ground for assuming that measurements of such a type could be sufficiently convenient for liquid metals and semiconductors.

§7. Methods of Controlled Conditions

As already observed, the regular thermal conditions are used to designate the stage of the process in which the space-time changes in temperature cease to depend on the initial conditions and thereby acquite a more general "regular" character. At the present time, it is customary to distinguish three types of regular conditions depending on the nature of the conditions which lead to changes in temperature. The regular conditions of the first type take place upon heating or cooling of the body of the medium with a constant temperature. In another case, when the temperature of the medium varies at a constant rate, regular conditions of the second type are realized. Finally, regular conditions of the third type correspond to a process of a periodic change in temperature. In this section, we present briefly problems pertinent to the utilization of regular conditions of the first and second types. The regular conditions of the third type, as more productive and promising, will be dealt with in succeeding sections.

Let us examine the regular thermal conditions of the first type. The basis of the theory of these conditions is the classifical method of solving non-steady problems of heat conductivity--the method of separating variables [6], the essence of which is in presenting the general solution of the heat conductivity equation as the sum of specific solutions each of which is the product of a function that depends only in the coordinates, on a function which depends only on the time (this latter function is the exponent):

$$T - T_0 = \sum_{i=0}^{\infty} A_i U_i(x, y, z)\, e^{-m_i t}, \qquad (7.1)$$

where T_0 is the temperature of the surrounding medium.

The functions U_i in this case satisfy the differential equation

$$\nabla^2 U + \frac{m}{a} U = 0 \qquad (7.2)$$

and the boundary condition which, for the problem under consideration, is the condition of Newtonian heat exchange

$$\left(\frac{dU}{dn}+\frac{\alpha U}{\lambda}\right)_s=0. \tag{7.3}$$

Solution of the equation (7.2), which satisfies the condition (7.3), is possible only for discrete values of the numbers m of the characteristic numbers. The corresponding discrete totality of the solution U_i is designated as the system of characteristic functions. The summation in formula (7.1) is carried out thus in accordance with the characteristic numbers and functions. The factors A_i in (7.1), which are the coefficients of the expansion of the function $T(x, y, z, 0)$ in a series by functions of U_i, are of initial conditions.

An important characteristic of the totality of the characteristic numbers m_i is that they are all positive numbers which form an increasing sequence, that is, each m_i is greater than m_{i-1} and less than m_{i+1}. Hence, it follows that with increasing t, each of the members of the sum (7.1) decreases more rapidly than the preceding member, so that the greater the time since the start of the process, the smaller the number of members of this series which is real; the space distribution of the temperatures (T (x, y, z, t) for sufficiently large t will without limit approach the asymptotic

$$T-T_0=AU(x, y, z)e^{-mt}. \tag{7.4}$$

The stage of the process where T(x, y, z, t) does not differ practically from the expression (7.4) comprises the regular thermal conditions of the first type [120].

Let us examine the basic characteristic specifities of the regular conditions of the first type. The magnitude m, which bears the designation of the rate of cooling (heating), is a parameter that does not depend on the coordinates, as a result of which the changes in temperature in any point of the body is the same function of time. The temperature fields thus vary similarly themselves; the ratio of the temperatures

$$\frac{T(x, y, z)-T_0}{T(x', y', z')-T_0}=\frac{U(x, y, z)}{U(x', y', z')}$$

does not depend on the time.

The magnitude of the rate of cooling, which is determined from equation (7.2), can, in totality with the condition (7.3), be a function only of the temperature diffusivity a, the ratio α/λ, and the characteristic dimension L: $m = m(a, \alpha/\lambda, L)$. The dimensionless expression of this dependence should have the form

$$p=f(Bi), \tag{7.5}$$

where

$$p=\sqrt{\frac{m}{a}}L \;\&\; Bi=\frac{\alpha L}{\lambda}. \tag{7.6}$$

From considerations of similitudes, the function f should be the same for geometrically similar bodies.

An important sequence of (7.5) is the assertion that for Bi → ∞ (intensive heat exchange on the surface of the body)

$$a=m_{\infty}L^2\,\mathrm{const}=m_{\infty}K, \tag{7.7}$$

where K is the so-called coefficient of form--in magnitude which is determined only by the form and the geometry of the body.

The existence of a simple dependence of temperature and time, which is the same for all points of the body and this characteristic of the stage of the controlled conditions of the first type and is a simple relationship of the magnitude of the rate of cooling m with the thermal characteristics of the medium, makes these conditions a convenient means for studying the thermal properties. Numerous variants of the corresponding methods of measurement--methods of regular conditions of the first type have been developed by G. M. Kondrat'yev and his students [120, 121]. These methods have found wide use in the study of the thermal properties of a solid body and partly of liquids at low temperatures. As an example, we shall examine one of the most convenient methods--the method of measuring the temperature diffusivity ("method of the α-calorimeter").

It follows from formula (7.7) that a temperature diffusivity of a body can be found directly from the magnitude of the rate of cooling of the body under conditions when the Biot number is sufficiently large (with intensive mixing of the cooling medium). The determination of the rate of cooling m does not present any problem because it follows from equation (7.4) that

$$m = \frac{\ln (T(t_1) - T_0) - \ln (T(t_2) - T_0)}{(t_2 - t_1)}. \tag{7.8}$$

One needs the measurements of the temperature difference of one point of the body T(t, r) for two moments of time and the temperature of the medium T_0 (instead of the differences $T - T_0$ in this formula, the differences of the temperature of two points of the body $T_1(t) - T_2(t)$). can figure in this formula. The measurements, consequently, are sufficiently simple. It is not, however, possible to apply such a procedure to metals because, in view of the large magnitude of the heat conductivity of the metals, it is impossible to reach sufficiently high values of the Biot criterion even with the utilization of liquid metal heat carriers. One of the possible roundabout ways is to conduct the experiments for infinite Biot values.

Experiments of such a type require a knowledge of the magnitude of α, which can be found by conducting experiments of cooling under the same conditions of specimens of standard metals the heat constants of which are known [122]. Such a type of experiment, which is relative in its nature, involves of course larger errors and is less convenient.

* * *

Let us now pass on to an examination of the regular conditions of the second type. As already mentioned, this type of regular conditions is realized during heating (or cooling), of a body of the medium with a temperature that varies with time in accordance with the linear law

$$T_0 = T_{00} + kt. \tag{7.9}$$

An analytic study of the general type of the solution of the heat conductivity equation with boundary conditions of the third type (Newtonian heat exchange) leads to the conclusion that an asymptotic expression which the solution of the problem approaches, should for sufficiently large values of t have the form [6, 121] when

$$T = T_{00} + kt + \theta(x, y, z), \tag{7.10}$$

besides, the function of Θ, which does not depend on the time, should satisfy the equation

$$\nabla^2\theta = \frac{k}{a} \tag{7.11}$$

and the boundary condition

$$\left(\frac{d\theta}{dn} + \frac{\alpha\theta}{\lambda}\right)_s = 0. \tag{7.12}$$

The solution of (7.10) characterizes the space-time variation in the temperature of a body during the stage of regular conditions of the second type. The basic distinguishing characteristic of this is the constancy of the derivative $\partial T/\partial t$ for all points of the body, the same rate of temperature change. It follows in particular from this that the difference in temperatures of any two points of the body under regular conditions of the second type does not depend on the time (the temperature field changes with time differently for all points of the body). Such a nature of change in the temperature fields with time is naturally very convenient in the practice of thermal measurements.

For specific evaluations of the methods for determining thermal constants in experiments that are based on the utilization of regular conditions of the second type, we shall refine the dependence of Θ on the parameters a and α/λ. An analysis of the dimension of the corresponding magnitudes leads to the conclusion that the only form of the dimensionless representation of the function of Θ is as follows:

$$\frac{\theta a}{kL^2} = \Phi\left(\frac{x}{L}, \frac{y}{L}, \frac{z}{L}, Bi\right), \qquad (7.13)$$

(The only pyrometric unit with the dimension of temperature is kL^2/a. where L is a characteristic dimension.)

The function (7.13) should be universal for geometrically similar bodies. Further, on the basis of the type of the boundary condition (7.12), one can define specifically also the dependence of Φ on the Biot parameter by writing

$$\frac{\theta a}{kL^2} = F\left(\frac{x}{L}, \frac{y}{L}, \frac{z}{L}\right) - \frac{1}{B_i}\left(\frac{dF}{d\frac{n}{L}}\right)_s, \qquad (7.14)$$

where F is the function that satisfies the equation

$$\left(\frac{\partial^2}{\partial\left(\frac{x}{L}\right)^2} + \frac{\partial^2}{\partial\left(\frac{y}{L}\right)^2} + \frac{\partial^2}{\partial\left(\frac{z}{L}\right)^2}\right)F = 1 \qquad (7.15)$$

and the boundary conditions

$$(F)_s = 0. \qquad (7.16)$$

An important result stemming from the expression (7.14) is that the difference in temperatures of two points of the body under regular conditions of the second type is inversely proportional to the temperature diffusivity

$$\theta_1 - \theta_2 = \frac{kL^2}{a}(F_1 - F_2). \qquad (7.17)$$

The simple method of measuring the temperature diffusivity, which reduces itself to the measurement of the constant difference of temperatures in two points of the body is based on the utilization of (7.17). This and other similar methods of measurement, which have been developed by A. V. Lykov and his students, found use for measurements of the thermal properties of nonmetallic materials [6, 121] (for the utilization of such a method for liquids see [123, 124]). The possibilities of using the method of regular conditions of the second type is not exhausted however by this. A rather large diversity of experiments can be realized in an aggregate study of the temperature fields as well as of the heat flows on an external surface of the bodies. Such a type of experiment requires naturally the realization of other boundary conditions than the conditions of heat exchange (7.12). Let us clarify first of all that the bases of the theory of regular conditions remain unchanged in this instance.

Let us start from the fact that the boundary conditions of the third type, written in the form (7.12), change in a natural manner to boundary conditions of the first type (the assignment of temperatures of the surface) when Bi → ∞ . This circumstance follows directly from (7.12) because when Bi → ∞ the first member of the relationship $\frac{\lambda}{\alpha}\left(\frac{\partial\theta}{\partial n}\right)_s + \theta_s = 0,$ approaches zero, which is equivalent to the setting of the temperature of the surface as $T = T_0$. Thus, we come to the conclusion that the basic formula of the theory of regular conditions (7.10) remains valid in setting the temperature of the surface of the body, which varies linearly with time.

On the other hand, it follows from formula (7.10) that the flow of heat on the surface is under regular conditions of the second type constant. It is easy to understand that the inverse assertion should be valid also.

In assuring constancy (with respect to time) of the heat flow to the surface of the body in the course of a certain period of time, regular conditions of the second type are established. Actually, the solution of the type (7.10) can satisfy the equation of the heat conductivity as well as the boundary condition

$$\left(\frac{\partial T}{\partial n}\right)_s = q, \qquad (7.18)$$

besides, in the general case (q = q(x, y, z) (the condition (7.12) can be regarded as a specific case of this, more general condition).

The method of heating with a constant heat flow makes it possible to utilize the regular thermal conditions of the second type as a means for determining the temperature diffusivity at considerably higher temperatures in comparison with the method of heating in the medium the temperature of which varies linearly with time. An example of such a high-temperature experiment is that in [125] in which the test cylindrical specimen is placed in a vacuum within a hollow cylinder that is heated with high-frequency currents. Under definite conditions, the heat flow to the surface of the specimen change little with time and regular conditions were realized in the system (the method provides at the same time the possibility for introducing a small correction for the inconstancy in the heat flow). The temperature diffusivity was determined from the difference between the time during which the temperature on the axis of the specimen becomes equal to a certain fixed value in the time during which this same temperature was achieved in the point close to the surface. Measurements of the temperature in this reference were conducted by means of a photoelectric pyrometer which was sighted from the end of the specimen on a channel (drilled hole) on the axis and on a channel parallel to it near the lateral surface (the experiment in this case should be repeated twice).

The method was utilized for measurements of the temperature diffusivity of tantalum in the temperature range of 1,300 - 1,600°C; the maximum error amounted to 7%.

As pointed out above, the heating with constant power is distinguished not only by convenience at high temperatures, but chiefly by the possibility of developing diverse methods of study, including also such which give information about the entire complex of thermal properties. (Speaking about this, we have in mind those experiments in which the heating power can be measured.) Let us turn to the fundamental aspect of the problem. Let us examine the simplest unidimensional problem: an infinite (with respect to the axis y and z) plate of L thickness, subjected to heating from one (x = L) side. The opposite side (x = 0) we shall assume to be heat-insulated.

The solution of equation (7.11), with consideration of the boundary condition for x = 0($\partial\theta/\partial x = 0$) gives

$$\theta = \frac{kx^2}{2a} + A, \tag{7.19}$$

For the difference in temperature in two points with the coordinates $x = x_1$ and $x = x_2$, we get

$$T_1 - T_2 = \frac{k}{2a}(x_1^2 - x_2^2). \tag{7.20}$$

The magnitude of k in this formula plays the role of a parameter not given directly. It can be expressed through boundary conditions on the surface of x = L, that is, by the magnitude of the power per unit surface of the plate q. In this case we get

$$k = \frac{aq}{L\lambda}. \tag{7.21}$$

On the other hand, the magnitude of k is nothing other than the derivative of the temperature with respect to time:

$$k = \frac{\partial T}{\partial t}. \tag{7.22}$$

In order to determine k in the experiment, one can in this manner utilize directly the results of the measurement of the speed of temperature rise. A combination of the expressions (7.20), (7.21), and (7.22) makes it possible to obtain at once formulas for determining the different thermal magnitudes:

$$a = \frac{\frac{\partial T}{\partial t}}{T_1 - T_2} \cdot \frac{x_1^2 - x_2^2}{2} \tag{7.23}$$

$$\lambda = \frac{q}{L(T_1 - T_2)} \cdot \frac{x_1^2 - x_2^2}{2}, \tag{7.24}$$

$$c_p\rho = \frac{q}{L\frac{\partial T}{\partial t}}. \tag{7.25}$$

Attention should be turned to the last of these formulas. It is easy to see that it can be written also as

$$c_p \frac{\partial T}{\partial t} = Q, \tag{7.26}$$

where c_p is the heat capacity of the entire plate, while Q is the complete heat flow. This formula is identical to the general thermodynamic expression which is used for quasisteady processes, that is, for cases when the heating takes place without temperature gradients in the body. The validity of this formula for regular conditions of the second type stems from the constancy $\partial T/\partial t$ in each point of the body.

The totality of the formulas (7.23)-(7.25) (or similar formulas for bodies of another symmetry) makes it possible to determine the entire complex of thermal properties of bodies by the measurement of: 1) the constant temperature difference of two points of the body; 2) speed of temperature rise of any point; 3) heating power of the last are the most difficult of these measurements.

There are two basic possibilities of ensuring control of the heat flow on the surface of the body: 1) utilization of electron bombardment in which the surface heat liberation can be determined from the strength of the electron current or from the voltage; 2) the use of a shell heat gauge which surrounds the specimen on which the temperature difference is measured and which is proportional to the flow (the heat gauge should first be calibrated).

Let us examine the first test. The utilization of electron heating for determining the complex of thermal properties of metals by the method of regular conditions of the second type has been accomplished by Ye. V. Kudryavtsev and K. N. Chakolev [126, 127]. A characteristic feature of their experiment is the utilization, for determining the thermal coefficients, or the readings of one temperature pickup--thermocouple, and note too as this was assumed in formulas (7.23) and (7.24). Let us dwell on this problem in greater detail. The utilization of two temperature pickups makes it possible to exclude from the solution of (7.19) the indeterminate constant A, the magnitude of which cannot be found from an examination of only one regular stage of the process. At the same time, if the entire process of temperature change, starting from the moment of t = 0, is examined, then it is clear that a constant should be a magnitude that is completely determined by boundary conditions. Thus, the utilization of the solution for a body which has been initially heated uniformly ($\theta(x, 0) \equiv 0$), gives [6, 126]

$$A = -\frac{qL}{6\lambda}$$

Thus, for Θ we get the solution

$$\theta = \frac{q}{\lambda L} \cdot \frac{(3x^2 - L^2)}{6} . \tag{7.27}$$

From expression (7.27) follows the formula for determining the heat conductivity

$$\lambda = \frac{q}{L} \cdot \frac{3x^2 - L^2}{6\theta} = \frac{q}{L} \cdot \frac{3x^2 - L^2}{6} \cdot \frac{1}{T - T_{00} - kt} . \tag{7.28}$$

In order to determine k, one can utilize in this case

$$k = \frac{T(x, t_1) - T(x, t_2)}{t_1 - t_2} . \tag{7.29}$$

the formula (7.21) makes it possible to find also the heat capacity:

$$c_p \rho = \frac{q}{Lk} . \tag{7.30}$$

Thus, in order to determine the complex of thermal properties in a given variant of the procedure, it is necessary to know the heating power, the initial temperature of the sample, and the values of the temperature in one point at two different times and that stage of the process in which regular conditions have already been established. (in [126] the corrections for nonregularity are discussed.)

The utilization of a single temperature pickup in such an experiment is of course an advantage. However, this advantage cannot apparently always be decisive because the measurement of a temperature difference which is constant with respect to time and the first variant of the method (formulas (7.23) and (7.24)) is practically more convenient than the measurement of the temperature as a function of time. (True, it is necessary to state the qualification here that a measurement of the temperature difference by thermocouples is difficult. in the case of metals when there is an electrical contact of the thermocouple junctions with the metal--see §2.)

Another disadvantage of the method with a single thermocouple is more fundamental. Formula (7.27) can be utilized only in the initial stage of regular conditions and provided the thermal properties of the test substance differ little with temperature, because in deriving this formula the magnitudes λ and $c_{p\rho}$ were soon to be constant. However, if the heating process starts with relatively low temperatures, while the study is carried out at high temperatures, then the utilization of formula (7.27) can generally speaking lead to considerable errors. On the basis of these considerations, the procedure of measurements, which is based on the utilization of the readings of several thermocouples, seems to us more promising.

The second test for determining the magnitude of the flow on the surface of the emissive body is the use of a shell heat gauge or as its author Yu. P. Barskiy calls this method--the diathermal shell. The diathermal shell represents a homogeneous layer of substance, on the opposite surfaces of which are layers of a differential thermal battery. (For flat objects, the shell comprises two plates which are placed from two sides on the sample.) The magnitude of the heat flow from the shell to the test body can be expressed by the formula

$$q = A\Delta t - hk, \qquad (7.31)$$

in which the coefficients A and h are determined only by the properties of the shell and do not depend on the thermal characteristics of the test body [128]. These coefficients are found by calibrating the shell--a procedure which is part of the experiment with the substance and the thermal properties of which (for example, the heat capacity) are sufficiently well-known. The method under consideration is thus relative.

The method of the diathermal shell, just as the method of the electron heating, makes it possible in principle to determine the entire totality of the parameters a, λ and $c_p\lambda$ by utilizing the formulas (7.23)-(7.25). Examples of the accomplishment of such measurements can be found in [129-133]. [129] in particular describes the high-temperature (up to 1,100°C) variant of the complex procedure for measuring the thermal properties of solid bodies. Specifically for metals, however, the method of the diathermal shell has so far not been developed.

Summarizing briefly the examination of the methods which are based on regular conditions of the second type, we emphasize that these methods possess a number of undoubted merits which are the result of the simplicity of general correlations of the type (7.10)-(7.14), as a result of which these methods can be utilized for measurements of a complex nature. Besides, one should in particular delineate the circumstance that a process of the measurements ensures by itself an overlap of a definite interval of reference temperatures; the results give in principle at the same time also the temperature function of the test magnitudes.. Here, however, it is necessary to state a serious qualification. The fact of the matter is that the theory of the method of regular conditions of the second type, which is built on the assumption of constancy of thermophysical characteristics of the object and on its unqualified application to the case in which these characteristics vary noticeably with the temperature, is not permissible because it is far from excluded that measurements made at the different rate of heating lead under such conditions to different results. In connection with this, it seems highly desirable to carry out the work dealing with a general analysis of the conditions for utilizing regular conditions of the second type in measuring the temperature function of thermal parameters.

§8. Regular Conditions of the Third Type

As already stated in the preceding section, regular thermal conditions of the third type are used to designate established periodic process, that is, a process in the stage in which the space-time temperature changes of the system do not depend on the initial conditions. In carrying out regular conditions of the third type, the temperature in each point of the body varies with the same period around the average temperature which is generally speaking, a function of the coordinate

$$T = \overline{T}(x, y, z) + \theta(x, y, z, t), \tag{8.1}$$

besides,

$$\theta(x, y, z, t + \tau) = \theta(x, y, z, t). \tag{8.2}$$

Just as for any periodic process, the temperature variations can be represented as a Fourier series

$$\theta = \sum_{n=1}^{\infty} [a_n \cos \omega t + b_n \sin \omega t], \tag{8.3}$$

where $a_n(x, y, z)$ and $b_n(x, y, z)$ are the harmonic components of the variations which determine the amplitude A_n and the phase ϕ_n of each of the sine harmonics

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad \varphi_n = \operatorname{arctg} \frac{a_n}{b_n}. \tag{8.4}$$

A study of the established temperature variations in the body thus gives at once several functions: $\overline{T}(x, y, z)$, $A_1(x, y, z)$, $A_2(x, y, z), \ldots$; $\phi_1(x, y, z)$, $\phi_2(x, y, z) \ldots$, (or, if the variations are harmonic, instead of the complex A_n and ϕ_n it gives $A(x, y, z, \omega)$ and $\phi(x, y, z, \omega)$). It follows directly from this that the method of regular conditions of the third type ensures a considerably larger amount of information in comparison with other conditions. This circumstance, which has been emphasized in the foreword, determines the great possibilities which are opened by the method of regular conditions of the third type.

The existence of a large amount of information makes it possible to carry out very diverse methods of measuring all the thermal characteristics of a substance; besides, also such methods which possess possibilities of internal control of the results of the measurements due to the existence of excess information. On the other hand, the same characteristic of the method of regular conditions of the third type makes it possible, in case of need, to limit oneself to the minimum number of temperature pickups; below are described many methods which require registration of the temperature variations in only one point of the object under study. Finally, and this is particularly important, a large amount of information makes the method of regular conditions of the third type convenient for performing methods of complex study of the thermal properties of solid and liquid metals. The enumerated basic merits of the method of regular conditions of the third type far from exhaust its positive aspects. We shall become acquainted with many of these during the subsequent presentation. Here, we wish to point out so far only one important aspect. The method of regular conditions of the third type ensures a natural repeat recurrence of the resulting data. The results of the experiments are repeats as many times as the periods of the variations that are registered (these periods, as a rule, are small). This repeat reproducibility at the same time can be conveniently combined also with the study of the temperature function, if the average temperature $\overline{T}$ in the given (slow) manner varies with time. The prospects of carrying out such quasiregular conditions are very great.

In the practical accomplishment of regular conditions of the third type, just as in a predominant number of other methods, the assumption of the relative smallness of the amplitude of the pulsation of the temperature Θ is fundamental. This assumption, which is formulated by the inequality (1.25). And totality with the inequalities (1.32) (or only by the letter when the boundary conditions are set in such a manner that the nonlinear emission heat exchange is not fundamental), makes it possible to linearize a problem, reduce it to the solution of linear equation with respect to Θ with linear boundary conditions. In a linear approximation for describing the periodic changes of a complex form, one can utilize the principle of superposition (the solution being sought can be represented in the form of a series of specific solutions for harmonics sinusoidal and cosinusoidal changes). This makes it possible to limit the theory of all variants of the method to an examination of the harmonic case, and for a description of specific signals of the non-harmonic type, to utilize

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their representation in the form of a Fourier series, by separating the members of this series from the experiment curves by means of a harmonic analysis.

The classification of different theories and systems and the utilization of the method of regular conditions of the third type can most conveniently be carried out as a function of the type of method that is used to create and maintain harmonic variations of the temperature. Besides, all the variants of the method (just as earlier for steady methods) will be divided into two large groups--the group with external (outside) heating and the group with internal heating (heating by current). The intermediate case--induction heating, when the volume of heat liberation is close to the surface heat liberation, is classified in the first group.

Each of the groups can have its own internal classification with respect to the nature of the symmetry of the isotherms: flat, cylindrical, spherical and, in principle, more complex.

Let us examine general problems of the theory of the methods of regular conditions. In the first group of methods, the variable temperature of the surrounding medium or the variable density of the surface source of heat (variable heat flow) can be set under boundary conditions. In the first case, these boundary conditions can be written as

$$\lambda \frac{\partial T}{\partial n} + \alpha [T - T_0(1 + me^{i\omega t})] = 0, \qquad (8.5)$$

and in the second case as

$$\lambda \frac{\partial T}{\partial n} + \alpha (T - T_0) = q + q_0 e^{i\omega t} = q(1 + me^{i\omega t}), \qquad (8.6)$$

where q and q_0 are the constant and variable components of the surface heating power and m is the modulation coefficient.

For the given periodic function, we utilize the complex form of notation, in accordance with which, the sought-for temperature is represented as

$$T = \overline{T}(x, y, z) + \theta(x, y, z)\, e^{i\omega t}, \tag{8.7}$$

where $\overline{T}$ is the constant component of the temperature (average temperature), and $\theta e^{i\omega t}$ is the variable component.

The utilization of a complex representation simplified considerably all the calculations. Let us clarify the sanctioned method of the utilization of the complex representation of the harmonic processes. By writing the given process in the form of $Ae^{i\omega t} = A\cos\omega t + iA\sin\omega t$ and taking the real or imaginary part of this expression, we get the representation of this function in the form of a cosine or sine. In the general case, the sought-for function will have in this instance a complex amplitude of $\theta = \theta_1 + i\theta_2$. For the real and imaginary part of the solution $\theta(\cos\omega t + i\sin\omega t)$ и, we then get the formulas: $\theta_1\cos\omega t - \theta_2\sin\omega t$ и $\theta_2\cos\omega t + \theta_1\sin\omega t$, which can be represented also in the form of $|\theta|\cos(\omega t + \varphi)$, where Θ is the modulus of the complex number equal to $\sqrt{\theta_1^2 + \theta_2^2}$, а фаза while the phase of the variations is ния $\varphi = \mathrm{arctg}\,\theta_2/\theta_1$.

Thus, the complex representation of the periodic processes makes it possible to determine with a convenient and single method the amplitude of the sought-for temperature variations as well as the phase with respect to the given periodic change.

In accordance with the representation of the temperature in the form of (8.7), the boundary conditions (8.5) and (8.6) can be each divided into two parts, for the constant and variable components of the temperature:

$$\lambda \frac{d\overline{T}}{dn} + \alpha(\overline{T} - T_0) = 0 \tag{8.8}$$

and

$$-\lambda \frac{d\theta}{dn} + \alpha'\vartheta = \alpha m T_0, \tag{8.9}$$

$$\lambda \frac{\overline{dT}}{dn} + \alpha(\bar{T} - T_0) = q \tag{8.10}$$

and

$$\lambda \frac{d\theta}{dn} + \alpha'\vartheta = qm. \tag{8.11}$$

The magnitude of the effective heat exchange coefficient α' is equal to

$$\alpha' = \frac{d[\alpha(T - T_0)]}{dT}\bigg|_{T=\bar{T}}; \tag{8.12}$$

for the important, specific case of emission heat exchange α' is determined by the formula (1.30).

In accordance with the general methods of linearization, which are represented in §1, and the derivation of teh formulas (8.8)-(8.11), it was assumed that the amplitude of the temperature variations $|\Theta|$ was sufficiently small so as not to take into account the temperature dependence (Λ) and not to take into account the members of the order of Θ^2.

A comparison of the type of conditions (8.9) and (8.11) leads to the conclusion that they are formerly alike, if one designates the constant member $\alpha(T)T_0$ by q. Hence, it follows that the solution of the problem of temperature variation for both cases of a periodic temperature change of the medium and a periodic change of the surface heat liberation is the same, although in practice, these cases are far from equivalent because in the first case the magnitude $q = \alpha T_0$ is usually indeterminant (due to the indeterminacy of α) and should, for this reason, be excluded from the calculation of formulas, whereas the power of the surface source qm can in many experiments be measured directly.

Just as the boundary conditions (8.8)-(8.11), the very heat-conductivity equation for the sought-for function of the type (8.7) and with respect to small $|\Theta|$ also decomposes into two equations: the equation for the constant component of the temperature

$$\nabla^2 \bar{T} = 0 \tag{8.13}$$

and the equation for the variable component

$$\nabla^2\theta - \frac{i\omega}{a}\theta = 0 \tag{8.14}$$

(It is assumed here as well that the changes $\bar{T}(x,y,z)$ are comparatively small).

Equation (8.13) in conjunction with the boundary conditions (8.10) or (8.8) assures completely the determination of $\bar{T}(x,y,z)$, while the equation (8.14) in conjunction with the condition (8.11) assures the determination of the complex amplitude $\Theta(x,y,z)$.

In the second group of methods of regular thermal conditions of the third type, methods with heating by current, it is possible to use the same boundary condition (8.5) and the conditions (8.8) and (8.9) which devolve therefrom by assuming that $m \equiv 0$. This pertains to the boundary condition on the lateral surface of the specimens. For the end surfaces, to which the current is fed, the boundary condition is given in the form of a condition of the first side $T = T_0$.)
(As regards the equation of heat conductivity, it will have the form of

$$c_p\rho\frac{\partial T}{\partial t} = \operatorname{div}(\lambda \operatorname{grad} T) + w(1 + me^{i\omega t}). \tag{8.15}$$

By writing the solution in the form of (8.7) for small $|\Theta|$ and comparatively small changes of $\overline{T}$ with the coordinates, we get again to equations. The equation for the constant component will have the form of

$$\nabla^2\overline{T} = \frac{w}{\lambda} \tag{8.16}$$

The equation for the variable component is written in the form of

$$\left(\frac{i\omega}{a} - \frac{1}{\lambda}\cdot\frac{dw}{dT}\right)\theta = \nabla^2\theta + \frac{wm}{\lambda}. \tag{8.17}$$

In this equation, the change in the volume power of the source w as a function of the temperature, which can take place as a result of the temperature function of the electrical conductivity, was taken into consideration. In the general case, $w = w(x, y, z)$ and

$$\frac{\partial w}{\partial T} = \left(\frac{\partial w}{\partial T}\right)_{T=\overline{T}(x,y,z)}$$

For samples of constant transverse cross-section s (in practice only such samples are used),

$$w = \frac{I^2\rho}{s^2}, \tag{8.18}$$

where ρ is the specific resistance.

The dependence of w on the coordinates can in this case be determined only by the function of $\rho(\overline{T})$ which can be disregarded under conditions of linearization of the equation.

Actually, for small changes in the temperature $\overline{T}$ one can utilize the decomposition of w into a series around a certain average temperature T_0:

$$w = w_0 + w\frac{\partial \ln \rho}{\partial T}(\overline{T} - T_0) + \ldots \tag{8.19}$$

Changes in the temperature of $\overline{T} - T_0$ should, in accordance with (8.16), be proportional to $(\frac{wl^2}{\lambda})$, where l is a characteristic dimension (with an accuracy of up to the factor of the order or unity). Thus,

$$w \simeq w_0 + \frac{w_0^2 l^2}{\lambda} \cdot \frac{\partial \ln \rho}{\partial T} \tag{8.20}$$

On the other hand, it follows from equation (8.17) that an amplitude of pulsation of the temperature $|\Theta|$ should be proportional to the factor $(\frac{wml^2}{\lambda})$. Then from formula (8.20) we get

$$w \simeq w_0\left(1 + \frac{|\theta|}{m} \cdot \frac{\partial \ln \rho}{\partial T}\right) \tag{8.21}$$

In view of the smallness of $|\Theta|$, the correction for the change in heating power with the coordinates is small (it gives in equation (8.17) members which are proportions of Θ^2). Similar considerations make it possible to become convinced of the smallness of the correction for the temperature function of w in the left-hand part of the equation (9.17) through a change in the variable component of the temperature. Equation (8.17) then assumes the form of

$$\frac{i\omega}{a}\theta = \nabla^2\theta + \frac{l^2\rho}{s^2\lambda} m. \tag{8.22}$$

Let us examine the form of solutions of the equations for the variable component. It follows from equations (8.14) and (8.22) that the determining criterial magnitude which contains a frequency is the combination (124a) $\frac{\omega l^2}{a}$, which can be expressed by the Fourier criterion

$$\frac{\omega l^2}{a} = \frac{2\pi}{\left(\frac{a\tau}{l^2}\right)} = \frac{2\pi}{Fo}.$$

In the Soviet literature, frequent use is made of the "Predvoditelev criterion":

$$\frac{\omega l^2}{a} = Pd. \tag{8.23}$$

for the magnitude (124a) $\frac{\omega l^2}{a}$ which was introduced by A. V. Lykov [6]. (In the more general determination by the Predvoditelev criterion, A. V. Lykov calls this magnitude .

$$Pd = \left(\frac{d\theta_c}{dFo}\right)_{\max} \tag{8.24}$$

"which characterizes the intensity of temperature rise of the surrounding medium Θ_c". For a periodic process, which is set by the temperature change of the outside medium, $\left[\frac{d\theta_c}{dt}\right]_{\max} = \omega T_0$, , from which, by taking into account $Fo = \frac{at}{l^2}$, it follows that $Pd = \frac{\omega l^2}{a}$.).

For most problems, it is more convenient to utilize not the Pd. criterion directly but the square root of this magnitude:

$$\varkappa = \sqrt{Pd} = \sqrt{\frac{\omega}{a}}\, l. \tag{8.25}$$

The second criterial magnitude, which figures in the solution for the first group of methods with external heating, should contain the magnitude qm. The dimensionless combination which includes q is known (§ 1). This is the Kirpichev criterion:

$$Ki = \frac{qlm}{\lambda\theta}. \tag{8.26}$$

In this criterion, instead of quasicharacteristic temperature T*, we have written the pulsation amplitude of Θ , because no other magnitudes with a temperature dimension are contained in the problem of Θ. Hence, it follows that the Kirpichev criterion in the given problem is not determining but is on the other hand the sought-for complex.

A similar sought-four complex for the second group of methods with heating by current is the modified Pomerantsev criterion:

$$Po = \frac{wm}{c_p \rho \omega \theta}. \tag{8.27}$$

Finally, the general criterion for the first and second groups of methods will be the determining Biot criterion:

$$Bi = \frac{\alpha' l}{\lambda}, \tag{8.28}$$

which is equal in the case of emission heat exchange to the magnitude

$$Bi = \frac{4\sigma\varepsilon\varphi T^3 l}{\lambda}, \tag{8.29}$$

(see (1.34)).

Thus, the general formula of the solution for the first group of methods should have the form of

$$\frac{\theta\lambda}{qlm} = \Phi\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \varkappa, Bi\right) \tag{8.30}$$

and for the second group

$$\frac{\theta c_p \rho \omega}{wm} = \Psi\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \varkappa, Bi\right). \tag{8.31}$$

Both types of solutions of (8.30) and (8.31) can be written in a similar uniform form, if, instead of q and w, we introduce in the first and second cases the total heat liberation Q (its variable component):

$$Q = qsm \tag{8.32}$$

(where s is the surface)
and

$$Q = mwv$$

(where v is the volume).

Then, the complex $\left(\frac{\theta c_p \omega}{w}\right)$ can be written in the form of

$$\frac{\theta c_p \omega}{mw} = \frac{\theta}{\theta_0}, \tag{8.33}$$

where

$$\theta_0 = \frac{Q}{Mc_p\omega} \tag{8.34}$$

is the amplitude of the temperature pulsation which would take place if the process were to proceed very slowly $(\omega \to 0)$, while the boundary conditions would ensure the absence of gradients along the sample. (Actually, in this case the equation of the heat balance could be written in the form of

$$Mc_p i\omega\theta = Mc_p\frac{d\theta}{dt} = Q. \tag{8.34'}$$

from which, for the pulsation amplitude of the temperature Θ_0, follows the formula (9.34).)

The complex $\left(\frac{\theta\lambda}{mql}\right)$ can be expressed by Θ/Θ_0:

$$\frac{\theta\lambda}{mql} = \frac{\theta}{\theta_0}\cdot\frac{\eta}{\varkappa^2}, \tag{8.35}$$

where η is the purely geometric factor equal to

$$\eta = \frac{sl}{v}. \tag{8.36}$$

It can be said equal to 1, if we assume that ends with l is the magnitude

$$l = \frac{v}{s}. \tag{8.37}$$

In accordance with this, the expression (8.30) and (8.31) can be written as one general formula

$$\frac{\theta}{\theta_0} = F\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \varkappa, Bi\right). \tag{8.38}$$

For the first group of methods, the equation for F is

$$\nabla^2 F - i\varkappa^2 F = 0 \tag{8.39}$$

(∇^2) indicates the dimensional or plus operator), while the boundary condition (9.11) in this case has the form of

$$F_s' + BiF_s = \varkappa^2. \tag{8.40}$$

For the second group, the equation has the form of

$$\nabla^2 F - i\varkappa^2 F + \varkappa^2 = 0, \tag{8.41}$$

while the boundary condition

$$F_s' + BiF_s = 0 \tag{8.42}$$

on the lateral surface and

$$F_s = F_0 \tag{8.43}$$

on the end surfaces.

In the equations (8.40) and (8.42) as well as below, the prime sign indicates the dimensionless derivative with respect to the normal while the subscript s indicates that the functions are taken on the surface.

In a predominant number of methods, which will be described below, the Biot criterion is small in comparison with the unit of magnitude. In such cases, the Bi role will reduce itself to a small correction in the functions (8.38) can be written approximately as

$$\frac{\theta}{\theta_0} = F\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \varkappa\right) + Bi f\left(\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \varkappa\right), \tag{8.44}$$

discarding the members of the higher order of smallness. For corrections to the amplitude and to the phase of the temperature variations, the following formulas will be valid in this instance

$$\frac{\Delta|\theta|}{|\theta|} = Bi \operatorname{Re} \frac{f}{F}, \tag{8.45}$$

$$\Delta\varphi = -Bi \operatorname{Im} \frac{f}{F}, \tag{8.46}$$

where Re and lm are the real and the imaginary part of the corresponding expressions. The minus sign has been placed in formula (8.46) because the phase delay which is measured in the experiment is equal to $-\phi$. The boundary conditions for F and f for the first group of methods (external heating) will have the form of

$$\begin{aligned} F'_s &= \varkappa^2, \\ f'_s &= -F_s. \end{aligned} \tag{8.47}$$

Besides, F as well as f should satisfy the same heat conductivity equation

$$\nabla^2 F - i\varkappa^2 F = 0,$$
$$\nabla^2 f - i\varkappa^2 f = 0. \qquad (8.48)$$

Hence, it follows that the function $1 - f\frac{\varkappa^3}{F_s}$ should be identical to the function of F (the same equation, the same boundary conditions). Because of this, formula (8.44) can be written in the form of

$$\frac{\theta}{\theta_0} = F\left[1 - Bi\frac{F_s}{\varkappa^2}\right] \qquad (8.49)$$

From equation (8.49) we get a simple formula for the amplitude and the phases of the temperature variations with consideration of the corrections for the Bi values:

$$\frac{\Delta|\theta|}{|\theta|} = -\frac{Bi}{\varkappa^2}F_{1s}, \qquad (8.50)$$

$$\Delta\varphi = \frac{Bi}{\varkappa^2}F_{2s}, \qquad (8.51)$$

where F_1 and F_2 are the real and imaginary front parts of the function F:

$$F = F_1 + iF_2. \qquad (8.52)$$

For the second group of methods, it is not possible to establish a direct relationship between F and f of the type (8.49); however, the boundary conditions for F and f in this case have the simple form of

$$
\begin{aligned}
F'_s &= 0,\\
f'_s &= -F_s
\end{aligned}
\tag{8.53}
$$

on the lateral surface and

$$
\begin{aligned}
F_{s'} &= F_0,\\
f_{s'} &= 0
\end{aligned}
\tag{8.54}
$$

on the end surfaces.

The function F in this case satisfies the equation (8.41), while f satisfies the equation (8.39):

$$\nabla^2 f - i\varkappa^2 f = 0. \tag{8.55}$$

The above-cited considerations pertain only to the case in which the boundary condition with heat exchange has been set on one surface. In practice, a more general setup of the problem is encountered in which the boundary conditions of such a type are set differently on two surfaces. This takes place, for example, in experiments with hollow cylinders of n-spheres and unilateral heating of plates.

By different conditions of heat exchange on two surfaces, we mean a difference in the effective heat exchange coefficients, in connection with which two Biot criteria instead of one should already figure in the problems under consideration.

For the case of small values of these criteria, one can as an approximation of (8.44) write

$$\frac{\theta}{\theta_0} = F + Bi_1 f_1 + Bi_2 f_2, \tag{8.56}$$

where f_1 and f_2 are correction functions and which the first takes into account the heat exchange on the heated surface (first side) and the second--on the opposite surface (second side). The derivatives of f_1 and f_2 are equal to zero on the opposite side: on the first and second--for f_1 and f_2, respectively.

For the group of methods with external heating, the function f_1, just as the earlier function f, can be represented in the form of

$$f_1 = -\frac{FF_{s_1}}{x^2}, \tag{8.57}$$

in the problem of heat exchange on the heated surface is solved just as simply as in the case of a body with one heated surface. As regards the function f_2, it satisfies the following conditions:

$$f_2' = C \text{ and } s_1, \tag{8.58}$$

$$f_2' = -F_{s_2} \text{ for } s_2. \tag{8.59}$$

(Utilizing these boundary conditions, it is necessary to take into account that f' here, just as before, indicates the derivative with respect to the dimensionless normal, while the direction of the normal on s_2 is opposite to that on s_1.

The conditions (8.58) and (8.59) do not make it possible to express f_2 directly through F as in the case of f_1. It is not difficult however, to realize that f_2 can be represented in a form similar to the expression (8.57):

$$f_2 = -\frac{F_{s_2}\widetilde{F}}{x^2}, \tag{8.60}$$

if we introduce the function $\tilde{F}$, which is the solution of the equation of heat conductivity for the case in which not the surface s_1, but the opposite surface s_2, is subjected to heating.

Let us examine the general type of solution of (8.44) from the point of view of the possibility of its utilization for determining the thermal characteristics of the substance. Assuming that Bi is the small correction, we can make the following assertions.

1. The magnitude of the difference in phases ϕ between the temperature variations in any point of the sample and the variations in the heating power is an unambiguous function of the number κ, that is, in the final analysis, of the temperature diffusivity.

Hence, it follows that by measuring the phase of the temperature variations, it is possible to determine directly the temperature diffusivity of a substance. This method of measuring the temperature diffusivity will henceforth be called the phase method, the phase variant of the method of regular conditions of the third type. Fundamentally, this method requires the utilization of only one temperature pickup.

Under some conditions, the calculation of the phase from the variations in the power can be inconvenient In this case, the temperature diffusivity can be determined from the difference in the phases of the temperature variations and two points of the sample. Such a variant of the phase method will henceforth be called the phase method of two points.

2. The ratio of the amplitudes of the temperature variations at two points is also determined only by the value of κ (for the given geometry and frequency). The temperature diffusivity can be thus determined from the magnitude of the ratio of the amplitude of the variations in two points of the sample. This method of measuring the temperature diffusivity will be called the amplitude method (the amplitude method of two points).

3. The ratio of the amplitudes of the temperature variations from one point of the sample for different frequencies and constant power of variable heat is also determined only by the temperature diffusivity. Unlike the amplitude method of two points, this method

makes it possible to limit ourselves to the readings of one temperature pickup. Such a method of determining the temperature diffusivity will be called the amplitude-frequency method.

4. By analogy with the amplitude-frequency method, one can separate also the phase-frequency method, although there is no particular need for utilizing such a method because, in accordance with point one, the magnitude ϕ for one frequency makes it possible to find the temperature diffusivity. The frequency dependence of ϕ can be utilized as additional information for controlling the results.

5. The determination of the absolute value of the amplitude of the temperature pulsation $|\Theta|$ makes it possible to find the magnitude of Θ_0 and, consequently, also the heat capacity of the samples, if the power Q fed into the sample is known and the temperature diffusivity has been determined (for values of κ).

A combination of these measurements with determinations of the temperature diffusivity by methods described in points 1-3 leads to different methods of carrying out the complex measurements, that is, methods which make it possible to determine the entire totality of the main thermal characteristics such as heat capacity, temperature diffusivity and heat conductivity from the data on the same experiment.

The simplest of the complex method is that in which the temperature diffusivity is determined by means of the phase variant. This method is convenient in that it requires the utilization of only one pickup--temperature gauge; the phase of the recorded temperature variation makes it possible to determine the temperature diffusivity of the specimen, amplitude-heat capacity.

In examining diverse variants of complex measurements, it is necessary to consider the following problem. Properly which parameters of the totality of $c_{p\rho}$, λ and a are determined directly in such type experiments, considering that these three magnitudes are related by the identity $\alpha = \lambda/c_{p\rho}$. It seems to follow from the above considerations that a and $c_{p\rho}$ are determined while the heat conductivity can be found already as a combination of these magnitudes. Such a solution is not quite valid because returning to another notation of the general formula of the solution, namely to formula (8.30), we can it would seem to assert with equal justification that in the experiment we determined a and λ, while $c_{p\rho}$ is already found as their combination. If this is so, then the problem as to what magnitudes are precisely determined directly in the experiment seems to lose sense and it is always necessary to speak about the determination of only the complex of thermal characteristics. Actually, this is not entirely true. First of all, let us turn attention to the fact that the magnitude of the temperature diffusivity in the experiments under consideration always plays a special role because it can be determined by many methods regardless of the measurements of $c_{p\rho}$ or λ. It is natural to approach with the same measure also the remaining parameters and to see which of the determinable magnitudes $c_{p\rho}$ or λ depends less on the determination of a, that is, which of the functions, Φ in the expression (8.30) or F in the expressions (8.38) and (8.44), depends less on κ. If in this instance the difference in the nature of the dependence on a is considerable (this is most frequently so in view of the fact that Φ and F differ by the factor κ^2), then it can be said which of the parameters in the experiment is determined "more directly" in the sense that the results of its determination are less affected by the results of the determination of the temperature diffusivity.

As an example, one can examine the case of external heating (first group of methods) under conditions in which the frequencies ω (and consequently, also κ) are comparatively small. It follows from the nature of the process in this instance that, as κ decreases, the amplitude of the temperature variations $|\theta|$ should differ comparatively little from the magnitude $|\theta_0|$ --the amplitude at infinitely slow variation (variations in absence of temperature gradients). This means that the function of F in its modulus should differ little from unity and, consequently, the determination of the heat capacity by means of formula (3.41) will depend little on the determination of the temperature diffusivity (the latter will be needed only for determining the correction for the deviation of $|F|$ from unity). In this case, one can speak quite definitely about independent measurements of the temperature diffusivity and the heat capacity; the heat conductivity will be a magnitude of the derivative.

Let us examine another maximum case--that of large values of κ. For sufficiently large values of κ, the amplitude of the temperature variations will decrease so strongly with increasing distance from the heated surface that the body will become practically semi-infinite. Only the properties and the configuration of the boundary layered material will be essential for the process. Under such conditions, the role of the transverse dimension of the body l should not be determining and the dimensionless form of the solution of the heat conductivity equation should, as a first approximation, have the form of

$$\frac{\theta\lambda}{qlm}\varkappa = \frac{\theta\lambda}{qm}\sqrt{\frac{\omega}{a}} = \Phi\left(x\sqrt{\frac{\omega}{a}}\right), \tag{8.61}$$

where x is the coordinate along the normal to the surface. In a specific case of x = 0, for the temperature variations of the heated surface

$$\vartheta = \frac{\text{const}\, qm}{b\sqrt{\omega}}, \tag{8.62}$$

where b is the heat activity. Measurements of the amplitude of the temperature variations on the surface in the case of large values of κ under consideration no longer yield the heat capacity for the heat activity, and the corresponding complex procedure for the measurements is a procedure for measuring the temperature diffusivity and the heat activity; the heat capacity and the heat conductivity are derivative magnitudes. In subsequent sections, we shall become acquainted with different combinations of parameters that are obtained in complex measurements.

In concluding this section, we wish to state a few words about the terminology. The variation heat process in the volume of the body, initiated by periodic heating of the surface, is customarily called the temperature wave. This designation reflects the fact that the amplitude and the phase of the temperature variations vary regularly from point to point just as in the case of the propagation of a wave in the medium with absorption. Nevertheless, one should take into account the fact that the periodic heat process is not a weight process because the propagating temperature wave does not have a front which delineates the region reached by the wave from the unperturbed region and is not related with the direction of the displacement of the energy. The heat flow in each point pulsates, changing periodically, so that, on the average, during the period is equal to 0. Actually,

$$\vec{q} = -\lambda \operatorname{grad} \theta e^{i\omega t} \qquad \bar{\vec{q}} = \int_0^\tau \vec{q}\, dt = 0).$$

§9. Flat Temperature Waves

In this section, we examine flat temperature waves, that is, the vibrational process which is determined by the periodic heating of one surface of an infinite (in a mathematical sense) plate of a finite thickness. Quasiflat waves--waves which propagate along the axos of cylindrical samples with finite transverse dimensions--are very similar with the flat temperature waves. In the former case, the existance of lateral heat exchange introduces its specificity, as a result of which such actual waves are examined by us in the next section where our examination is, however, to a considerable degree bases on results in this section.

In examining flat temperature waves, we shall limit ourselves to cases with small values of the Biot criterion, which will be equivalent to an examination of comparatively thin plates. It follows from table 1 that for temperatures of ~ 1,000°K, plates with the thickness of < 1 cm, can be considered thin plates, for temperatures of ~ 2,000°K--less than several millimeters, for temperatures of ~ 3,000°K--less than a millimeter.

The mathematical formulation of the problem in accordance with the preceding section will be as follows.

The sought-for functions are F(x/L, κ), which determines the dimensionless complex amplitude of the variations of the temperature θ, and the function f (x/L, κ), which determines the correction for the heat exchange:

$$\frac{\theta}{\theta_0} = F\left(\frac{x}{L}, \varkappa\right) + Bi f\left(\frac{x}{L}, \varkappa\right), \tag{9.1}$$

where L is the plate thickness, x is the coordinate which is obtained from the surface that is not subjected to heating (such a selection of coordinates leads to more convenient formulas);

$$\varkappa = \sqrt{\frac{\omega}{a}}L,$$

of Θ_0, in accordance with formula (8.34), is equal to

$$\theta_0 = \frac{Q}{Mc_p\omega} = \frac{mq}{c_p\rho\omega L} \tag{9.2}$$

for the case of heating with a source which insures variable heat liberation mq on a unit surface.

For conditions of heating by varying the temperature of the external medium, one can, instead of formula (9.2), write

$$\theta_0 = \frac{m\alpha T_0}{c_p\rho\omega L}, \tag{9.3}$$

where α is the heat coefficient of heat exchange on the boundary x = L, mT_0 is the amplitude of the temperature variations of the medium. The Biot criterion is equal to

$$Bi = \frac{\alpha' L}{\lambda}, \tag{9.4}$$

where α' is the effective coefficient of heat exchange on the boundary x = 0.

The functions of F and f are subject to the equations

$$\frac{d^2F}{d\left(\frac{x}{L}\right)^2} - i\varkappa^2 F = 0, \tag{9.5}$$

$$\frac{d^2f}{d\left(\frac{x}{L}\right)^2} - i\varkappa^2 f = 0 \tag{9.6}$$

and the boundary conditions

$$\left.\begin{aligned} &\frac{dF}{d\left(\frac{x}{L}\right)} = 0 \\ &\frac{d}{d\left(\frac{x}{L}\right)} f = F\zeta \end{aligned}\right\} \text{for } x = 0, \tag{9.7}$$

$\zeta = \alpha'_0/\alpha'_L$ (the ratio of the effective coefficients of heat on the boundaries x = 0 and x = L),

$$\left.\begin{aligned} &\frac{dF}{d\left(\frac{x}{L}\right)} = \varkappa^2 \\ &\frac{df}{d\left(\frac{x}{L}\right)} = -F \end{aligned}\right\} \text{for } x = L. \tag{9.8}$$

Let us examine at first the function F, relegating for the time being the problem of the correction for heat exchange to a secondary place. The solution of the equation (9.5) with the conditions (9.7) and (9.8) leads to the function

$$F = \frac{\varkappa}{\sqrt{i}}\,\frac{e^{\sqrt{i}\varkappa\frac{x}{L}} + e^{-\sqrt{i}\varkappa\frac{x}{L}}}{e^{\sqrt{i}\varkappa} - e^{-\sqrt{i}\varkappa}}. \tag{9.9}$$

On the boundaries of the plate, we get

$$F(0) = \frac{2\varkappa}{\sqrt{i}\,(e^{\sqrt{i}\varkappa} - e^{-\sqrt{i}\varkappa})} = \frac{\varkappa}{\sqrt{i}\,\mathrm{sh}\,(\sqrt{i}\varkappa)} \tag{9.10}$$

and

$$F(L) = \frac{\varkappa}{\sqrt{i}}\cdot\frac{e^{\sqrt{i}\varkappa} + e^{-\sqrt{i}\varkappa}}{e^{\sqrt{i}\varkappa} - e^{-\sqrt{i}\varkappa}} = \frac{\varkappa}{\sqrt{i}}\cdot\frac{1}{\mathrm{th}\,(\sqrt{i}\varkappa)}. \tag{9.11}$$

The hyperbolic functions sh and th from the argument $\sqrt{ik}$ can in the final analysis be expressed by the product of the hyperbolic and trigonometric functions from the actual argument. It is more convenient, however, to utilize the already tabulated functions

$$\left.\begin{aligned} \mathrm{sh}\,(\sqrt{i}\varkappa) &= A_s(\varkappa) + iB_s(\varkappa) \\ \mathrm{ch}\,(\sqrt{i}\varkappa) &= A_c(\varkappa) + iB_c(\varkappa) \end{aligned}\right\} \tag{9.12}$$

(see [6]).

For the amplitude and the phase of the temperature variations on the surface x = 0, which lies opposite that being heated, we get

$$\frac{|\theta|}{\theta_0} = \frac{\varkappa}{\sqrt{A_s^2 + B_s^2}}, \tag{9.13}$$

$$\varphi = \operatorname{arctg} \frac{A_s + B_s}{A_s - B_s} = \frac{\pi}{4} + \operatorname{arctg} \frac{B_s}{A_s}. \qquad (9.14)$$

Let us examine the phase relationships. $\phi = \phi(\kappa^2)$ is shown in Figure 22. For $\kappa \to 0$, the shift in phases between the temperature variation and the power approaches $\pi/2$ (90°). This is a general principle that is valid for all cases (when Bi = 0), and it is easy to become convinced of this with the aid of formula (8.34'). (For slower temperature changes, when the gradients in the body are small, the heat flow is proportional to the derivative of the temperature with respect to time and, consequently, the temperature is shifted in phase by $\pi/2$.) The dependence of ϕ on κ^2 is quite close to linear $\phi - 90° \approx 20.7\kappa^2$, which makes it possible to evaluate in a convenient manner the accuracy of the determination of κ^2 and, consequently, of a and the phase variant of the method for measuring the temperature diffusivity

$$\frac{\delta a}{a} = \frac{\delta \kappa^2}{\kappa^2} = \frac{\delta \varphi}{\varphi - \frac{\pi}{2}} \qquad (9.15)$$

When $\delta\phi \sim 1°$ and $\phi - -\pi/2 > 30°$ $(\kappa^3 > 4)$, the *a priori* error in determining the temperature diffusivity amounts to ~ 3%, that is, the phase method of determining the temperature diffusivity can in principle be made quite accurate.

It follows from formula (9.15) that the error in determining the temperature diffusivity for a given absolute error in the determination of the phase $\delta\phi$ becomes smaller, the larger the value of the measured phase difference ϕ, that is, the greater the value of κ. An increase in κ, however, only to known limits increases the accuracy in determination of the temperature diffusivity because, with an increase in the frequency the errors simultaneously decrease also in the amplitude of the test variations due to a decrease in Θ_0 and of the factor $|F|$.

In connection with this, in order to evaluate the optimum conditions for accomplishing the phase variant of the method, it was necessary to examine also the dependence $|F|$ on κ^2, a knowledge of which, on the other hand, is decisive for the problem of the conditions of accomplishing complex measurements. Dependence of $|F|$ on κ^2 shown on Figure 23. For small values of κ, the magnitude $|F|$ approaches unity in accordance with the general characteristic of the function F for all problems with external heating. In this region,

$$|0| \sim \theta_0 = \frac{Q}{Mc_p\omega} \tag{9.16}$$

and the measurement of the variation amplitudes of the temperature for known power Q makes it possible to determine the heat capacity of the test material. The relatively small variation and $|F|$ for a comparatively broad integral of values of κ makes the measurement of the heat capacity little dependent on the measurements of the temperature diffusivity. If we are speaking about complex measurements, the temperature diffusivity--the heat capacity, then the optimum values of κ should naturally be considered the greatest of this interval ($\kappa^2 \sim 4$) because in this instance, the maximum accuracy in the measurement of the temperature diffusivity will be achieved (of course, provided the amplitude of the temperature variations for these variations of κ is sufficiently high).

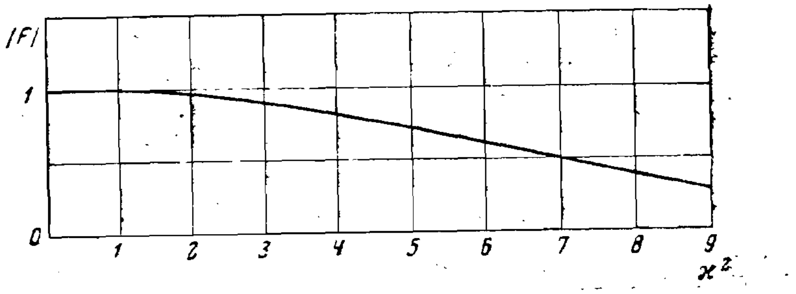


Figure 23. Dependence of the modules of the function F -- the magnitude which determines the amplitude of the temperature variations-- on the parameter κ^2.

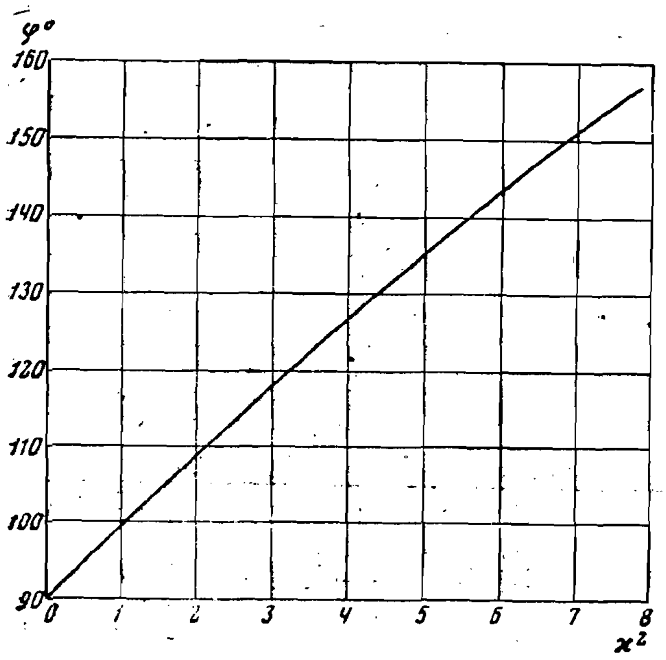


Figure 22. Dependence of phase difference between the variation in the power on the surface of the plate and the temperature variation on the opposite surface on the parameter κ^2.

As regards individual measurements of the temperature diffusivity by the phase method, it is seen from Figure 23 that within a sufficiently large integral of values of κ (up to $\kappa^2 \sim 10$ and greater) the function $|F|$ changes relatively little in comparison with unity and the frequency dependence with the amplitude of the temperature pulsations is essentially determined by the factor $\Theta_0 \sim 1/\omega$. The selection of maximum values of $|\Theta|$ is determined exclusively by the possibilities of the recording equipment. The smaller the pulsation amplitudes $|\Theta|$ it can be recorded without substantial interferences, the greater the values of κ that can be utilized in the phase variant of the method for measuring the temperature diffusivity than the greater will be the phase differences that will be measured in this case, and the greater will be the accuracy.

The possibility of conducting measurements for relatively large values of κ, greater than 2, is of interest not only for the phase variant of measuring the temperature diffusivity, but also for carrying out complex measurements. For $\kappa^2 > 4$, the function $|F|$ becomes noticeably dependent upon κ and the measurement of the heat capacity cannot already be considered independent of the measurements of the temperature diffusivity. However, from an analysis of the function $|F|$ one can make another positive conclusion, namely that the product $\kappa|F|/2$ in the region of values of κ^2 ranging from 4 to 16 differs from unity by not more than 10%. This fact makes it possible to assume that a complex method for measuring in this region gives the combination $\frac{c_p\rho}{\sqrt{a}}$ -- a unlike the combination $c_{p\rho}$ -- a for smaller values of κ.

In speaking about measurements of the temperature diffusivity, we have so far limited ourselves to the phase variant. Let us dwell on the amplitude variant. It follows from the relatively weak nature of the dependence $|F|$ on κ that the amplitude-frequency variant can hardly compete with the phase variant because the errors in the determination will here be comparatively large. The amplitude-frequency measurements can be regarded only as a means for additional control of phase measurements and only within the region of $\kappa^2 > 10$. As regards the amplitude variant in two points, we shall dwell on it somewhat later in connection with theory of temperature waves in a semi-infinite medium. The accomplishment of this variant in the case of a plate is also of little expedience.

Let us examine the following approximation of the theory of the method, the function f, which describes the correction for the heat exchange. On the basis of the equation (9.6) and the boundary conditions (9.7) and (9.8), we get for the function f

$$f = -\frac{e^{-\sqrt{i}\frac{x}{L}\varkappa}[\zeta F(0)e^{\sqrt{i}\varkappa} + F(L)] + e^{\sqrt{i}\frac{x}{L}\varkappa}[\zeta F(0)e^{-\sqrt{i}\varkappa} + F(L)]}{\varkappa\sqrt{i}\,(e^{\sqrt{i}\varkappa} - e^{-\sqrt{i}\varkappa})}. \qquad (9.17)$$

For x =0, in particular,

$$f(0) = -\frac{1}{\varkappa\sqrt{i}}\cdot\frac{\zeta + 1}{\operatorname{sh}(\sqrt{i}\varkappa)\operatorname{th}\sqrt{i}\varkappa}, \qquad (9.18)$$

or in a more convenient form

$$f(0) = -F(0)\frac{(\zeta + 1)F(L)}{\varkappa^2} \tag{9.19}$$

(the formula (9.19) could have been obtained also directly from the general relationships of (8.57) and (8.60), if one takes into account the relationship between the functions of F and $\tilde{F}$(138a) , which stems from the symmetry of the problem.) Hence, for corrections to the amplitude and the phase of the variations, we obtain the formulas

$$\begin{aligned} \frac{|\theta|}{\theta_0} &= |F|\,[1 - Bi(\zeta + 1)\psi_1], \\ \varphi &= \varphi_0 - Bi\psi_2(\zeta + 1), \end{aligned} \tag{9.20}$$

where

$$\begin{aligned} \psi_1 &= \frac{F_1(L)}{\varkappa^2}, \\ \psi_2 &= -\frac{F_2(L)}{\varkappa^2}. \end{aligned} \tag{9.21}$$

F_1 and F_2 are the real and imaginary parts of the function F which can be expressed through the functions of (9.12):

$$\begin{aligned} F_1(L) &= \varkappa\sqrt{2}\,\frac{A_c(A_s - B_s) + B_c(A_s + B_s)}{(A_s - B_s)^2 + (A_s + B_s)^2} = \\ &= \frac{\varkappa}{\sqrt{2}} \cdot \frac{(A_c + B_c)A_s + (B_c - A_c)B_s}{A_s^2 + B_s^2}, \\ F_2(L) &= \varkappa\sqrt{2}\,\frac{B_c(A_s - B_s) - A_c(A_s + B_s)}{(A_s - B_s)^2 + (A_s + B_s)^2} = \\ &= \frac{\varkappa}{\sqrt{2}} \cdot \frac{A_s(B_c - A_c) - B_s(A_c + B_c)}{A_s^2 + B_s^2}. \end{aligned} \tag{9.22}$$

(For $\zeta = 0$, the formulas (9.20)-(9.21) are identical to the formulas (8.50) and (8.51) in accordance with the deductions in the preceding section.

The functions $\psi_1(\kappa^2)$ and $\psi_2(\kappa^2)$ are shown in Figure 24. In accordance with the nature of $\psi_1(\kappa^2)$, the correction to the amplitude of the variations for small Bi values is small for all the values of κ. The strong increase if ψ_2 for small κ values limits the region of the optimum values of κ in the experiments for the determination of the heat capacity of the magnitude $\kappa \sim 1$.

The experimental accomplishment of the phase method for measuring the temperature diffusivity of metals at high temperatures has been described by O. A. Krayev and A. A. Stel'makh [134]. Their procedure was intended for materials in the form of thin (0.1 - 0.5 mm) plates, in connection with which the frequencies ω, which are optimum for such experiments, amount to hundreds of cycles per second. The utilization of such sonic frequencies makes it possible to use radio technical means for developing and recording the temperature variations, which makes the experiment very convenient and sophisticated.

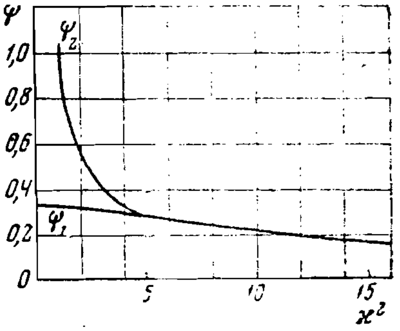


Figure 24. Function of Ψ_1 and Ψ_2 which determine the corrections to the amplitude and the phase of the temperature oscillations due to the heat exchange.

According to the method of O. A. Krayev and A. A. Stel'makh, the samples were heated by using electron bombardment on one of these plate surfaces. A wolfram spiral cathode, located parallel to the plate, served as the electron emitter. The variable power component was obtained by modulating the anode voltage by means of a sonic generator. For recording the temperature variations on the opposite side of the plate, use was made of a photoelectric method. The variable signal from the photoelectric cell was amplified by a selective amplifier and was fed to a vertical amplifier of the cathode oscillograph. The signal from the modulating sonic generator was fed to the horizontal amplifier of the oscillograph through a phase shifter. Measurements of the phase difference consisted in selecting the position of the phase shifter for which the ellipse on the oscillograph screen was converted to a straight line.

The described procedure was utilized for measuring the temperature diffusivity of wolfram, molybdenum, niobium and tantalum, within the temperature range of 1,800° - 2,000°K to 2,500 - 3,200°K [134,135]. The error in the measurements is evaluated as 5% by the authors.

The system of the experiment by O. A. Krayev and A. A. Stel'makh is convenient in principle not only for measurements of the temperature diffusivity, but also for measurement of the heat capacity, which we have already indicated in the reviews [136]. For this purpose, it is necessary to measure the current strength in the voltage (their variable components) in the cathode-sample circuit and to determine absolute values of the variation amplitude of the temperature from the signal at the exit of the recording system, which also does not represent much of a difficulty (see §3). The work for accomplishing the complex procedure of testing thin samples seems very promising. So far we have limited ourselves to an examination of flat temperature waves and plates of finite thickness.

Another specific case of flat waves are waves which propagate in the semilimited (semi-infinite) space. In practice, this means conditions under which the temperature wave is almost completely dampened (absorbed) at the distance comparable with the real object. In order to find an expression of Θ, one can utilize in this case the solution of (9.9) by changing x for L - x (by shifting the origin of the coordinates on the heated surface and changing the direction of axis x) and approaching L to infinity. In this case, we shall get

$$\theta = \frac{mq}{b\sqrt{\omega i}} e^{-\sqrt{\frac{i\omega}{a}} x}, \qquad (9.23)$$

where $b = \sqrt{c_p \rho \lambda}$ is the heat activity.

For the amplitude and the phase of the temperature variations at a distance x from the surface, we get the expressions

$$|\theta| = \frac{mq}{b\sqrt{\omega}} e^{-\sqrt{\frac{\omega}{2a}}x}, \tag{9.24}$$

$$\varphi = \frac{\pi}{4} + \sqrt{\frac{\omega}{2a}}x. \tag{9.25}$$

The formulas (9.23-(9.25) describe the simplest flat temperature wave which in the ordinary notation has the form of

$$T - \overline{T} = T_0 e^{-\sqrt{\frac{\omega}{2a}}x} \cos\left(\omega t - \sqrt{\frac{\omega}{2a}}x\right). \tag{9.26}$$

The amplitude of this wave, which is equal to T_0 on the surface decreases exponentially with increasing depth in the body, the leg with respect to the phase increasing linearly. The length of the temperature wave is equal to

$$L = 2\pi\sqrt{\frac{2a}{\omega}}. \tag{9.27}$$

At a distance equal to the wavelength, the temperature wave is dampened $e^{2\pi}$ times, that is, practically completely. (This is one more of the reasons which account for the concept of the temperature wave being conditional.)

On the surface x = 0,

$$|\theta| = \frac{mq}{b\sqrt{\omega}}, \tag{9.28}$$

$$\varphi = \frac{\pi}{4}. \tag{9.29}$$

In accordance with the general considerations in §8, the amplitude of the temperature variations on the surface is determined by the magnitude of the heat activity. A shift in phases between the variations in the power and temperature on the surface by $\pi/4$ is a general characteristic for conditions of semi-infinity. It is related with the fact that Φ in formula (8.30) is a function of the complex variable

$\sqrt{i\frac{\omega}{a}}x$, , which follows from the type of equation (8.39).

An analysis of the type of expressions (9.24)-(9.29) makes it possible to conclude regarding the possibility of accomplishing the following methodical variants for a "semi-infinite" body: 1) phase variants of the method of measuring the temperature diffusivity (method of one of two points); 2) temperature variant of measuring the temperature diffusivity (variant of two points; frequency-amplitude variant is possible, but it has no advantages); 3) independent method of measuring the heat activity. A combination of this method with any of the methods for measuring the temperature diffusivity yields a procedure for measuring the complex of the properties.

A description of specific experiments for a semi-infinite body will be given in the next section which deals with temperature waves and rods.

§10. Longitudinal Temperature Waves and Rods

This section deals with an examination of the problem of the utilization of temperature waves which propagate in rods along the axis--longitudinal or axial temperature waves. Another type of temperature waves and rods--radial or cylindrical--will be examined in §13 and 14.

In accordance with the data in§ 4, the equation of heat conductivity for rods can be written in the form of

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial T}{\partial t} + \nu(T - T_0), \tag{10.1}$$

where $\nu = \frac{\alpha p}{\lambda s}$, (α is the coefficient of heat exchange on the lateral surface of the rod, which generally speaking is a function of T and T_0, p is the perimeter and s is the cross-section). Equation (10.1) is valid under conditions when the radial temperature gradients are small, which takes place in the case of Bi << 1. (We wish to point out that equation (10.1) still implies the existence of radial gradients which are approximated by a parabolic function, that is, the examined temperature waves are not strictly flat.)

In accordance with the old procedure for linearizing the heat conductivity equation, which is already known from §1 and 8, we set

$$T = \overline{T}(x) + \theta(x) e^{i\omega t}. \tag{10.2}$$

for the complex amplitude of the variable component of the temperature θ, we obtain the equation

$$\frac{d^2\theta}{dx^2} = \left(\frac{i\omega}{a} + \nu'\right)\theta, \tag{10.3}$$

where $\nu' = \frac{\alpha' p}{\lambda s}$, while α' is the effective coefficient of heat exchange (see formula (8.12)). In accordance with the designations in §8 and 9, we set

$$\theta = \theta_0 F\left(\frac{x}{L}\right). \tag{10.4}$$

Then equation (10.3) can be written in the form of

$$\frac{d^2F}{d\left(\frac{x}{L}\right)^2} = L^2\left(\frac{i\omega}{a} + \nu'\right)F, \tag{10.5}$$

which is completely identical to the type of equation (9.5), if we introduce the designation

$$i\varkappa^2 = \frac{i\omega L^2}{a} + \nu' L^2. \tag{10.6}$$

Thus, the solutions of the equation for axial temperature waves in the complex form are identical to the solution for flat waves, if by κ we mean a magnitude which is determined by the expression (10.6).

For most experiments, the examination of one specific case of the function F, which corresponds to the conditions of "semi-infinity," is of interest. In practice, this condition means the selection of such periods of temperature variations for which the temperature wave is almost completely dampened on the length of the test rod. (In such a problem, there is no longitudinal characteristic (determining) dimension and the role of the parametric unity of length will be played by the magnitude $L = \sqrt{\frac{a}{\omega}}$, which is proportional to the length of the flat temperature wave.)

By presenting the solution of (9.23) for the flat temperature wave in the form of

$$\theta = \theta_0 e^{-\sqrt{i}\varkappa \frac{x}{L}}, \tag{10.7}$$

the sought-for solution for the actual wave is obtained therefrom by replacing κ by the expression (10.6):

$$\theta = \theta_0 e^{-\sqrt{\frac{i\omega}{a}+\nu'}\,x} = \theta_0 e^{-\frac{1}{\sqrt{2}}\sqrt{\sqrt{\nu'^2+\frac{\omega^2}{a^2}}+\nu'}\,x} \times$$
$$\times e^{-\frac{i}{\sqrt{2}}\sqrt{\sqrt{\nu'^2+\frac{\omega^2}{a^2}}-\nu'}\,x}. \qquad (10.8)$$

For the amplitude of the temperature variations, it follows therefrom that

$$|\theta| = |\theta_0|\,e^{-kx},$$
$$k = \sqrt{\frac{\omega}{2a}}\sqrt{\sqrt{\frac{\nu'^2a^2}{\omega^2}+1}+\frac{\nu'a}{\omega}}, \qquad (10.9)$$

which is also an exponential damping, just as in the case of a simple flat temperature wave, but with a greater damping decrement (the damping increases through heat losses from the lateral surface).

For the phase of the temperature variations, we get

$$\varphi = \varphi_0 + k'x,$$
$$k' = \sqrt{\frac{\omega}{2a}}\sqrt{\sqrt{\frac{\nu'^2a^2}{\omega^2}+1}-\frac{\nu'a}{\omega}}, \qquad (10.10)$$

which is also a linear law of the change in phase with the coordinate. (Here, the expression for the amplitude $|\Theta_0|$ is not decoded and the phases ϕ_0 of the temperature variations on the end of the rod $x = 0$ because there is no need for this so far.)

The utilization of the relationships (10.9) and (10.10) in order to determine the temperature diffusivity of the material amounts to the substance of the Angstrom method of temperature wave [53, 113, 137].

The main problem of this method is the exclusion of the coefficient ν'. The two basic ways for excluding ν': 1) the direct utilization of the relationships (10.9) and (10.10) and 2) the accomplishment of the experiment under conditions for which the role of ν' reduces itself to a small correction. Let us examine at first the first of these ways, and we shall return to the second one in the second part of this section.

In accordance with the sources' presentation §8, the functions $\overline{T}(x)$, $\phi(x,\omega)$, $|\Theta|$ (x,ω), serve as sources of information in the utilization of the regular thermal conditions of the third type. Different pairs of combinations of these functions give in principle six different methods for the simultaneous determination of the coefficients a and ν'. Here, we examine not all of these, but only those which are not related with the knowledge of the constant component of the temperature, although, generally speaking, the utilization of information on $\overline{T}(x)$ is far from excluded.

One of the simplest and most common methods for excluding ν' is the utilization of information of the amplitude and phases of the temperature variations for one definite frequency. It follows from the expressions (10.9) and (10.10) that

$$kk' = \frac{\omega}{a}\,. \tag{10.11}$$

In its turn,

$$k = \frac{1}{l}\ln\frac{\theta_1}{\theta_2}\,, \tag{10.12}$$

$$k' = \frac{\varphi_2 - \varphi_1}{l}\,, \tag{10.13}$$

where $\theta_1, \theta_2, \varphi_1, \varphi_2$ — are the amplitudes and phases of the temperature variations in two points that are at the distance of l from one another.

From this, we get the formula for determining the temperature diffusivity:

$$a = \frac{\omega l^2}{(\varphi_2 - \varphi_1) \ln \frac{\theta_1}{\theta_2}} . \tag{10.14}$$

In order to determine a, we need thus the recording of the temperature variations in two points of the sample.

The second method of excluding the magnitude ν' is the utilization of information on the frequency dependence of the amplitudes of the temperature variations in two points. From expression (10.9) we can get the formula

$$a = \frac{1}{2} \cdot \frac{\left[\frac{\omega_1^2}{k_1^2} - \frac{\omega_2^2}{k_2^2}\right]^{1/2}}{(k_1^2 - k_2^2)^{1/2}} , \tag{10.15}$$

where k_1 and k_2 are the values of k for frequencies of ω_1 and ω_2. The determination of the temperature diffusivity for this method requires thus a measurement of the amplitudes of the temperature variations in two points for two different frequencies.

The third method of excluding the magnitude ν' is completely similar to the preceding one but it pertains to the phases. In the case of measurements of the phases in two points for two different frequencies, the formula, which is identical to (10.15), is valid

$$a = \frac{1}{2} \cdot \frac{\left[\frac{\omega_1^2}{k_1'^2} - \frac{\omega_2^2}{k_2'^2}\right]^{1/2}}{k_1'^2 - k_2'^2} . \tag{10.16}$$

As already pointed out, the first of the examined variants of the Angstrom method is most common. It is precisely by means of this that measurements are accomplished in most studies (see, for example [138, 82, 137, 139-141]. The main advantage of this variant is the need not to repeat measurements for another frequency. The second variant of the method, which is based on formula (10.15), has been accomplished by Starr [142]. The third method has been realized by King [143] who determined the delay time in the temperature variations in two points for different frequencies.

Let us dwell on the problem of the practical accomplishment of the examined variants of the Angstrom method. In most studies, the length of the test rod is 15-50 cm, the temperature pickups are thermocouples which are distributed at distances of several centimeters from the end; the corresponding periods of the variations lie within the range of 10 to 30 minutes.

The simplest method for developing temperature variations is that of the periodic connection and disconnection of a heater (see, for example [137, 144]). A disadvantage of this method is a need to carry out a harmonic analysis of the curves of temperature changes, which complicates the processing of the measurement results. As a means of harmonic analysis, the approximate method of twelve coordinates is used most frequently. A method of twelve coordinates insures the registration of five harmonics of the test signal. The non-registered seventh harmonic of the temperature variations (even ones, including also the sixth harmonic, for equality of the periods of connection and disconnection of the heater should be insignificant) with a П-shaped change of the power on the end of the rod amounts to approximately 5%. If the first thermocouple is removed from the end of the rod by a distance which ensures a twofold attenuation of the main variations, then the seventh harmonic at this distance attenuates approximately 6 times and its fraction does not exceed 1%. Under these conditions, the utilization of the method of twelve coordinates does not lead to an error greater 1%, however, if the thermocouples are located closer to the end of the rod, it is expedient to utilize more accurate methods of harmonic analysis, for example, the method of twenty-four coordinates. A utilization of mechanical analyzers yields results in all cases that are not too bad. It is necessary to point out also the method of analysis of the curves of temperature change, which has been proposed by Bosanquet and Aris [145]. These authors prepared normagrams which make it possible to process the curves of the temperature variations without the harmonic analysis, directly from the maximum spread of the variations in the position of the mean arithmetic value of the temperature variations. The method assumes that a change in the power on the end is strictly П-shaped in nature, which practically takes place when the heater has a small thermal inertia while the period of the variations is large.

Another method of accomplishing periodic heating involves the utilization of special devices which ensure the sinusoidal change in the heater power [138, 140, 146-148] (the latter work pertains to the case in which a solar furnace was used for heating).

These devices are usually mechanical systems which by means of shaped cams control the rheostats or autotransformers in the circuit of the heater. The design and production of such a system complicates the set-up of the unit, but it simplifies the processing of the measurements, making it possible to dispense with the methods of harmonic analysis. It is worthwhile to observe that the utilization of mechanical sinusoidal modulators of the current doesn't always give a purely sinusoidal temperature change, which can be seen as an example in [138]. The curves of the temperature change shown in this reference clearly differ from sinusoidal curves. The reason for this is apparently that in heating with large currents (in order to obtain sufficient amplitudes of variations), the resistance of the heater wires changes noticeably in the course of the cycle due to a change with the temperature. This nonlinear fact leads to the appearance of a second harmonic of temperature variations.

Let us pass on to an examination of the second method for excluding the constant of the heat exchange ν'--the creation of conditions under which this magnitude becomes a part of the calculation formulas only as a small correction. It follows from the formulas (10.9) and (10.10) that the dimensionless complex which characterizes the role of heat exchange from the electrosurface is the magnitude

$$\xi = \frac{\nu' a}{\omega}. \qquad (10.17)$$

For small values of this complex, the formulas (10.9) and (10.10) can be written in the form of

$$k = \sqrt{\frac{\omega}{2a}}\left(1 + \frac{\xi}{2}\right),$$

(10.18)

$$k' = \sqrt{\frac{\omega}{2a}}\left(1 - \frac{\xi}{2}\right) \qquad (10.19)$$

from which follow the formulas for determining the temperature diffusivity

$$a = \frac{\omega^2}{2k^2}(1 + \xi) = \frac{\omega^2 l^2}{2\left(\ln \frac{\theta_1}{\theta_2}\right)^2}(1 + \xi), \qquad (10.20)$$

$$a = \frac{\omega^2}{2k'^2}(1 - \xi) = \frac{\omega^2 l^2}{2(\varphi_2 - \varphi_1)^2}(1 - \xi). \qquad (10.21)$$

It is seen from these formulas that a factor ξ for its smallness is directly a correction to the experimental values of the temperature diffusivity obtained from the formulas which are valid for strictly flat temperature waves.

Let us examine the conditions under which the factor ξ is small, limiting ourselves to the case of emission heat exchange. Let us represent the magnitude ξ for a round rod with radius R in the form of

$$\xi = \frac{2Bi}{\varkappa^2} \qquad (10.22)$$

where

$$Bi = \frac{4\sigma\varepsilon T^3 R}{\lambda},$$

and

$$\varkappa = \sqrt{\frac{\omega}{a}}\,R. \qquad (10.23)$$

It is obvious that for rods of the given radius of material, a magnitude ξ will be smaller, the larger the frequency of the temperature wave. For sufficiently large values of ω, the magnitude ξ can be in principle made as small as is desired. The frequency of the wave, however, cannot be increased too much because, with an increase in ω, there is an increase in the damping coefficient of the wave and its amplitude on the end drops. In order to determine the upper boundary of the frequencies (values of κ) for which the experiment can be carried out without a noticeable drop in its accuracy, we shall go through the following reasoning.

For the given frequency of variations, the selection of the distance l between the thermocouples and the realization of the amplitude method of measuring the temperature diffusivity is determined from the condition of the minimum magnitude of the relative error $\delta a/a$, which is determined by the inaccuracy in determining the ratio of the amplitudes of the temperature variations. In accordance with formulas (10.12) and (10.18),

$$\frac{\delta a}{a} = 2\frac{\delta k}{k} = \frac{2}{kl}\left(\frac{\delta\theta_1}{\theta_1} + \frac{\delta\theta_2}{\theta_2}\right). \qquad (10.24)$$

The greatest magnitude in this expression is possessed by the second member which contains the relative error in the determination of the second, smaller amplitude of the temperature variations. In order to evaluate this magnitude, the dependence of $\delta\Theta_2$ on the amplitude Θ_2 is essential. For small values of Θ_2, it can be assumed that $\delta\Theta_2 =$ const, that is, the error in the measurements is limited by the maximum error of the recording device (otherwise, it would be advantageous to utilize even still smaller values of Θ_2 and thereby reduce the error of $\delta a/a$ to an increase in k). Under this condition, the selection of the optimum position of the second thermocouple reduces itself to finding the value of $\theta_2=\theta_1 e^{-kl}$, which makes the magnitude $kl=\sqrt{\frac{\omega}{2a}}\,l=\ln\frac{\theta_1}{\theta_2}$ the minimum. In order to solve this problem, we take into account that $\frac{2}{kl\theta_2}\approx\frac{2\delta k}{k\delta\theta_2}$, while the magnitude Θ_1 is constant when ω = const. Determination of the extremum of the ratio $\left(\frac{e^{kl}}{kl}\right)$ gives kl = 1, that is, the optimum ratio of the amplitude is 2.5 - 3. It is pertinent that the values of klθ 1 ensure simultaneously also sufficiently high accuracy in determining the temperature diffusivity by the phase method because the magnitude of the phase difference, in accordance with formulas (10.13) and (10.19), is obtained in this case of an order of one radiant. By introducing into formulas (10.22) and (10.23) the value of the optimum distance l between the thermocouples,

$$l=\frac{1}{k}=\frac{1}{\sqrt{\frac{\omega}{2a}}}, \qquad (10.25)$$

it is possible to exclude the values of the frequencies from these formulas:

$$\xi=Bi\left(\frac{l}{R}\right)^2 \qquad (10.26)$$

Formula (10.26) makes it possible to evaluate the values of l for which the correction ξ does not exceed the given smallness of the magnitude and thereby determine the range of temperatures in geometric dimensions for which the effect of heat exchange from the electrical surface of the rod plays a role of a small correction. The results of these calculations for one of the typical cases ($\lambda\sim$ 1 wt/cm·degree, $\varepsilon\sim$0.5) are shown in Table 18. The magnitude l is shown in centimeters, ξ is set equal to 0.1 (10%).

Table 18. The Length of Samples for which the Correction for the Heat Exchange from the Lateral Surface Does Not Exceed 10%.

T K	$2\ r=1$ см	$2r=0,5$ см
1000	2,3	1,2
1500	1,2	0,6
2000	0,7	0,4

If it is assumed that values of $l > 1$ cm are allowed for the experiment (for thermocouples or thermal probes 0.2 mm in diameter, the error in determining the distant cell in this instance amounts to 0.2 mm, that is, 2%), then it follows from the table that the heat exchange can be considered as a correction for thin (~ 5 mm) rods up to temperatures of ~ 1,000°K if a thick (~ 1 cm) rods up to temperatures of about 1,500-2,000°K. The corresponding values of the periods of temperature variations, determined from formula (10.25), amount to 5--25 sec; which are at least an order less than those usually employed. Other evaluations from formula (10.22) lead to similar conclusions [30].

The accomplishment of experiments with short periods of temperature variations has a number of very substantial advantages. The basic advantage of these is due to the fact that as a result of the smallness of the correction ξ, a smaller amount of information, in comparison with methods which require the exclusion of relatively large values of ν', is required for the experiment. This advantage is the possibility of the independent determination of the temperature diffusivity in the phase and amplitude variants of the method by the utilization of the sample calculation formulas (10.20) and (10.21). The possibility of determining the temperature diffusivity from the amplitudes of the temperature variations alone or from their phases alone makes it possible to simplify substantially the experiment and the processing of its data, while the utilization of amplitude as well as phase data makes it possible to exert internal control on the experiment.

Another merit of the procedure under consideration is the small length of the samples employed. Because of the exponential nature of the drop in the amplitude of the temperature wave with the distance, a sample length of ~3l ensures completely no effect of the second end of the rod so that samples with a length of 3-5 cm are suitable for the experiment.

the experiment. The small of the samples and the corresponding dimensions of the unit--quality--is particularly important for high temperatures.

Further, the utilization of small periods of temperature variations reduces considerably the time for the measurements, makes it possible to record practically any number of repeated periodic curves, and thereby obtain repeatedly reproducible results.

Another circumstance related precisely with the smallness of the period of the variations is the low sensitivity of the measurement system to nonregular changes and the average temperature of the experiment, which is determined by the instability of the thermal process. Actually, at average temperature of the measurement varies with the speed c ($T = T_0 + ct$), then the ratio of the "drift" of the temperature during the period of $\tau \delta T = c\tau$ to the amplitude of the temperature variations near the end $\theta \sim C\sqrt{\tau}$ for a constant heating power (see formula (9.24) amounts to

$$\frac{\delta T}{\theta} = \frac{c}{C}\sqrt{\tau},$$

that is, it decreases substantially with a decrease in period. Besides, for still greater decrease in the effect of the temperature drift for periods of the order of seconds, it is possible to utilize electrical filters which decrease the signal with a frequency much less than that recorded (see [146]). These considerations make it possible in the case of small periods to carry out measurements under quasiregular conditions when the average temperature increases progressively.

A negative aspect of work with brief periods is a relatively large error which is introduced by the inaccuracy in the determination of the distance between the thermocouples. A method for decreasing this error will be described in §11. Another negative aspect is the small amplitude of the temperature variations obtained under conditions of small periods with the introduction of a definite power into the samples;

in accomplishing the experiment, it is required, for this reason, to pay special attention to the construction of the heaters and to utilize more sensitive recording systems than in operating with long temperature waves (see §11).

The indicated negative aspects of the examined variant of the procedure cannot, however, be compared with its positive aspects; the advantages of the utilization of small periods exceed considerably the disadvantages. The advantages of short temperature waves are particularly large for experiments of a complex nature, in which the minimum amount of information required for determining the thermal characteristics in the absence of a noticeable effect of lateral heat loss makes it possible to accomplish measurement methods which are very simple in nature. The description of these complex experiments is dealt with in §12. In §11 we presented first the experience gained from the utilization of short temperature waves for measuring thermal diffusivity and we dwell in particular on the specific features of the experiment with liquid metals.

§11. Practice of Measuring the Temperature Diffusivity by Means of Short Longitudinal Temperature Waves and Rods

In the preceding section, we presented a theory of the method of measuring the temperature diffusivity by means of temperature waves and rods (Angstrom method) and we cited arguments in favor of the expediency of the utilization of short temperature waves for which the calculation of the heat exchange from the lateral surface can be achieved by the introduction of a small correction. This section deals entirely with the practical utilization of precisely such short temperature waves.

One of the important practical problems which arise in the accomplishment of experiments with small periods of temperature variations is, as was observed in §10, that of the selection of the most efficient construction of the heater. The importance of this problem is explained by the need to introduce into the test sample a power that is as great as possible, because, in accordance with formula (9.24), the amplitude of the temperature variations on the end noticeably decreases with a rise in the frequency or a constant power, while on the other hand, the power transferred from the heater to the sample also decreases with rising frequency through an increase (sometimes significant) of the thermal resistance of the heater-sample.

The uniformity of heating over the end is of essential significance for short temperature waves because the relative smallness of the amplitude of the temperature variations makes desirable a distribution of a heat receiver as close to the end of the test rod as possible (for the evaluation of the role of nonuniformity in heating of the end, see below).

In the first study where short temperature waves were used [146], the heater was a thin ceramic plate covered on both sides by a layer of molybdenum. This layer was repeatedly cut on one side in such a manner that its total resistance amounted to ~ 2 ohms at room temperature; lead electrodes were soldered to this surface of the heater. Another surface of the heater was soldered with a solid solder to the end of the sample. An advantage of such a heater is the sufficiently good uniformity in heating along the end. The complexity of the technology in the preparation of the described heater has impelled us to study other types of heaters [30]. The most successful of these are shown in Figure 25.

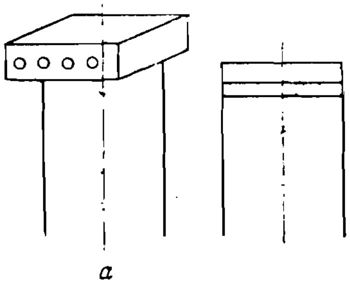


Figure 25. Construction of heaters.

The body of the heater, a, comprises a single entity with the rod (it is milled from one piece of the material). The wires of the heater are placed in horizontal rods having a diameter of ~ 1 mm, which are located a distance of 2 mm from one another and are insulated by a ceramic straw (beads). The heater ensures sufficiently uniform temperature field along the cross-section and makes it possible to introduce into the sample a comparatively large power. Its disadvantage is a need to have the test material in the form of a rod (bar) of large diameter (20 mm).

The heater, b, is a layer of conducting power-- a mixture of graphite with aluminum oxide--located between the end of the test straw and the contact molybdenum surface, squeezed from above by the spring [29]. The composition of the powder is selected in such a manner that the resistance of the heater would amount to 10-15 ohms. Such a construction of the heater combines good uniformity of temperature field on the end with simplicity of installation and manufacture. Another way for solving the problem regarding the heating of the end is the utilization of electron bombardment which is described in §12.

One of the methodical problems on which we dwell here is that of thermal pickups and the recording system.

In §2 attention was paid to the expediency of utilizing for the measurement of the temperature differences of thermal probes and thermal electrodes which form a differential thermocouple, an intermediate element of which is the test metal. Advantages of the use of thermal ohms are particularly great for the problem under consideration when the distances between the thermal pickups are small and should, for this reason, be measured particularly thoroughly. The utilization of thermal probes, that is, single electrodes instead of thermocouples, makes more definite the distance between points with the temperatures measured in these two smaller errors in the determination of the distance.

The circuit for including the thermoprobes is shown in Figure 26. The thermoprobes 1, 2, 3 (any number)are soldered to the surface of the test rod in locations where the temperature pulsations should be recorded. The general thermoprobe 4 is located near the end of the rod where the temperature variations are practically absent. Intermediate junctions with copper leads should be at the same temperature (for example, placed together in a Dewar flask) so that a change in the temperature would not give an additional shift in the readings of the thermoprobes with time.

The differential diagram shown in Figure 26 has also another advantage in comparison with thermocouples: slow systematic changes in average temperature of the sample show up on it considerably less as a results of the mutual compensation of these changes by the two thermal probes.

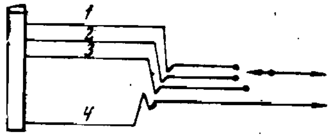


Figure 26. Diagram for including the thermal probes.

In working at temperature up to 1,000°C, it is expedient to use alumel leads as thermal probes, which give a rather considerable thermal emf in the pair with the most metal; up to 800°C one can use constantan probes. It is convenient to weld the probes by means of an electrical spark. Prior to welding, it is expedient to make on the surface of the sample the thin annular reference marks (by using a milling bench) at distances which correspond to the position of the probes and to weld the thermal probes at these reference lines; to determine on a comparator the distances between the reference lines with an accuracy of ~ 0.1 mm is no difficulty. The indeterminacy of the distance between the thermal probes, with consideration of their proper thickness (diameter of the thermal probes should amount to 0.1 - 0.2 mm) can in this case be taken as ~ 0.2 mm.

As a recording system for the experiment under consideration, use can be made of any of the systems with photoelectric amplification (for example, the system described in the second half of this section) or the system with a highly sensitive electronic amplifier using direct current. However, the specificity of the measurements of the temperature diffusivity is that the measurements do not require the knowledge of the absolute values of the temperature variations and the true phases of these variations (only the phase differences are important), it permits the utilization of amplification and recording elements which have, generally speaking, random amplitude-frequency and phase-frequency characteristics; the only requirement for these is that of linearity. In particular, it is possible to utilize elements which have a definite inertia and even resonance characteristics. These elements include precisely ordinary mirror galvanometers. For periods of variations rated in ten seconds, the system of objective registration of the readings of a mirror galvanometer, which is briefly described in §3, is convenient. The example of a recording of temperature variations (4 thermoprobes) shown in Figure 27 pertains to the case when the power modulation of the heater is accomplished by its periodic connection and disconnection, that is, when the change in power was Π-shaped (with the same interval of connection and disconnection). Despite such a nature of change, the power, which registered the temperature variations, differs very little from the purely sinusoidal ones even for thermal probes that are located in the immediate vicinity of the heated end. The reason for this lies essentially in the considerable damping of the higher harmonics of the temperature during the passage of the signal from the heater to the sample. The large thermal resistance of the heater-sample (including also the heater shown in

Figure 25b is the damping in the powder) is unfavorable for the transfer of variable heat flow to the sample and from this point of view it plays a positive role. Besides that, decrease in the share of the higher harmonics takes place also in the recording system itself due to the amplitude characteristics of the galvanometer (considering this, it was expedient to work with periods close to the natural frequency of the galvanometer and to utilize its purely vibrational conditions). The closeness of the recorded curve to the sinusoidal curves is an additional advantage in working with short temperature waves, which makes it possible to utilize the simplest type of modulation--"connected-disconnected" system with minimum corrections for the inharmonic state of the recorded curves. (In connection with this, we wish to observe that the given circumstance was apparently not properly clarified in the first study with short temperature waves [146] because its authors paid much attention to the creation of the mechanism for the sinusoidal modulation of the heater power.)

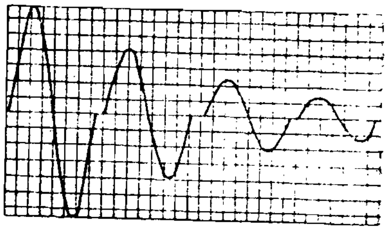


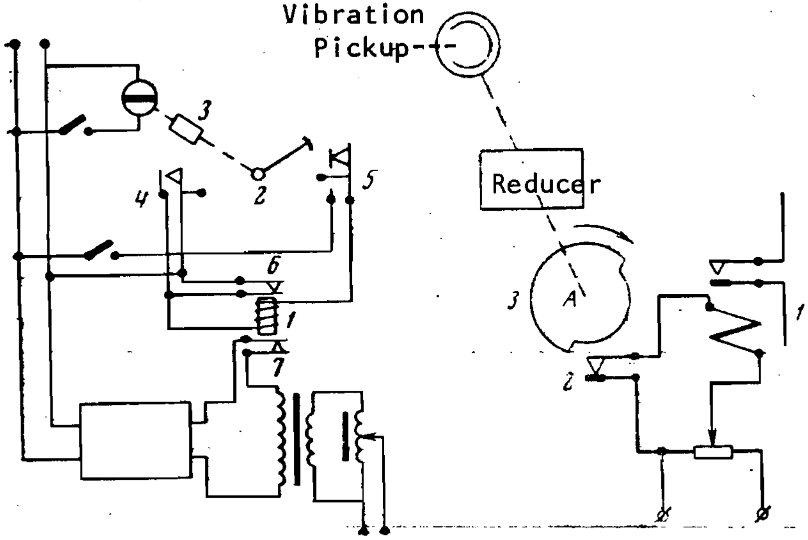
Figure 27. Example of the recording of temperature variations.

A system of automatic power modulation in accordance with the "connected-disconnected" system can be accomplished by means of a mechanical or an electronic device. An example of the first of these is described in [29, 30, 33]; the system of this device is shown in Figure 28.

The connection and disconnection of current in the heater circuit is accomplished by means of the magnetic starter 1. The starter is controlled by a lever 2 which is rotated by a motor through the reducer 3 which gives the necessary range of periods from 3-30 seconds. (For this purpose, the reducer from an automatic PS-383 device is convenient.) As it rotates, the lever closes the contact 4 which leads to the actuation of the starter (contact 5 is in this case closed) and the closing of the contact 6 and 7. The starter remains connected also after the disconnection of the contact 4 on the path of movement of the lever to the contact 5 (contact 4 and contact 6 of the electromagnet are connected parallel to one another). When the lever opens contact 5, the starter is disconnected and remains disconnected during the entire second half of the period until lever 2 again closes contact 4.

The described modulator ensures sufficiently clear and stable operation; the relationship between the connection and disconnection intervals are easily controlled by varying the positions of the contacts 4 and 5.

Another type of mechanical modulator is described in [39, 40, and 41]; its circuit is shown in Figure 29. The connection and disconnection of current is accomplished here by means of the relay 1; the current through the winding of this relay is interrupted by the contact 2 which is controlled by the rotating disk 3. The disk closes the contact and connects the relay in the course of a half-period of rotation. For stable operation of such a device, good quality of the contact 2 is very important.



Figures 28, 29. Circuits of the mechanical modulators (interrupters) of the circuit.

The electronic modulating devices are convenient in operation. The simplest of these is a multivibrator with a stepwise control of the frequency, which controls the work of the electromagnetic or electronic relay.

Somewhat more complicated is the electronic modulator which ensures continuous control of frequency within a broad range from scores of cycles per second to hundreds of cycles per second.

The master generator in this device is the sonic generator the signal from which flows to the frequency divider which ensures its 1,000-fall degrees (for this purpose, use can be made of the scaling device of the type PS-1000). The signal of the required frequency flows to the differentiating cascade, at the outlet of which sharp voltage peaks form. These peaks control the operation of the trigger device which feeds every half-period a regulating signal to the outlet of the electrical relay. The complexity of this device is completely compensated by the possibility of its use withing a range of frequencies where the utilization of mechanical modulators is practically excluded.

Let us examine the problem of the accuracy in determining the temperature diffusivity. In the amplitude variant of the method of short temperature waves, the error in determining the magnitude

$k = \frac{1}{l} \ln \frac{\theta_1}{\theta_2}$ is equal to

$$\frac{\delta k}{k} = \frac{1}{kl}\left(\frac{\delta\theta_1}{\theta_1} + \frac{\delta\theta_2}{\theta_2}\right) + \frac{\delta l}{l}. \qquad (11.1)$$

The random error in determining the greater amplitude of Θ_1 is comparatively small, of the order of 0.5%, of which one can become convinced from an examination of Table 19 which gives the results of the measurements of the amplitude of five successive periods of variations (one of a typical series).

If it is taken into account that the error introduced by the inaccurate calculation of the harmonics can also amount to ~ 0.5% then the maximum error $\delta\Theta_1/\Theta_1$ is equal approximately to 1%.

In order to evaluate the error in the determination of $\delta\Theta_2/\Theta_2$

we shall take into account that, in accordance with §10, the magnitude of Θ_2 should in the optimum case be approximately 2.5-3 times less than that of Θ_1. In recording the curves of the temperature variations on the tape of an automatic potentiometer of the EPP-09 type, without a change in the coefficient of amplification of the recording device in changing from one thermocouple (thermal probe) to another, the magnitude of the amplitude of Θ_2 will amount to ~ 4 cm, if the doubled amplitude of Θ_1 extends almost over the entire scale of the potentiometer (25 cm), Besides, the inaccuracy in the reading of the amplitude of $\delta\Theta_2$ in the worst case will amount to ~ 0,5 mm. The corresponding relative error is ~ 1,2%, while with consideration the error due to the harmonics is ~ 1,5% (on the second thermocouple, the fraction of the harmonics is considerably less than on the first).

Thus, the total maximum error is

$$\frac{\delta\theta_1}{\theta_1} + \frac{\delta\theta_2}{\theta_2} \simeq 2,5\%.$$

For kl $\not\approx$ 1, which is the optimum condition (see § 10),

$$\frac{\delta k}{k} = 2,5\% + \frac{\delta l}{l}$$

The magnitude $\delta l/l$ in accordance with evaluations in § 10, amounts to 2% in the worst case. Finally, we get $\frac{\delta k}{k} \simeq 4,5\%$. The error in determining the temperature diffusivity is, in accordance with formula (10.20), equal to

$$\frac{\delta a}{a} = 2\frac{\delta k}{k} + 2\frac{\delta\omega}{\omega} + \delta\xi. \tag{11.2}$$

The error of $\delta\omega/\omega$ is negligibly small because it is not difficult to measure the time of a sufficiently large number of periods. The inaccuracy in the correction for the heat exchange $\delta\xi$ depends fundamentally on the magnitude of this correction. For temperatures of hundreds of degrees, $\delta\xi$ is practically equal to zero; in the worst case, when ξ approaches 10% due to the inaccuracy in the knowledge of the degree of blackness (if the correction is introduced purely by calculation), this error can reach 2-3%. Thus, finally for the maximum error in the determination of the temperature diffusivity, we get 9-12%.

Let us examine the phase variant of the method. The error in determining the magnitude $k' = (1/l)\ \Delta\phi$ is equal to

$$\frac{\delta k'}{k'} = \frac{\delta\Delta\varphi}{\Delta\varphi} + \frac{\delta l}{l}. \tag{11.3}$$

The magnitude of the absolute error in determining the difference of the phases $\Delta\phi$ is made up of the errors in reading the phase of the variations of each of the thermocouples (thermal probes) with respect to the marks of the connection and disconnection of heating on the tape of the automatic potentiometer.

In order to evaluate these errors, we have assumed that the position of the nodal points of the periodic curves recorded on the tape of the automatic potentiometer can be determined with an accuracy of ~0.3 mm (for a line width of ~0.5 mm, the position of each midpoint is fixed with an accuracy of 0.2 mm for a vertical line and ~ 0.3 mm for a line with a slope of ~45°). If the period of the temperature variations extends over a length of 10-20 cm, then the error in determining the phase amounts in this case to 0.5-1°. The stability of the recorded curves actually ensures such accuracy in determining the phases, if not for individual readings, then in any case for the average. For illustration, we present Table 20 which contains the results of the reading of the phase of five successive variations with respect to the bench marks on the potentiometer tape.

For $\delta\Delta\phi = 2\delta\phi = 1\text{-}2°$, the summary error in the determination of k', is thus equal to 3-5%. The maximum error in determining the temperature diffusivity is in accordance with formula (10.21) equal to

$$\frac{\delta a}{a} \simeq 2\frac{\delta k'}{k'} + \delta\xi, \qquad (11.4)$$

which gives 6-13%, that is, approximately the same magnitude as in the amplitude method.

Table 19. Results of the Recording of the Amplitudes of Five Successive Temperature Variations.

Number	1	2	3.	4	5	Avg.	Deviation Max.	 Avg.
φ Degree	45,3	45,6	46,5	45,3	45,3	45,7	0,8	0,4

Table 20. Results of the Recording of the Phase of Five Successive Temperature Variations.

Number	1	2	3	4	5	Avg.	Deviation Max.	 Avg.
A, мм	117	117	116	117	116,5	116,7	0,6%	0,3%

The evaluations of the maximum errors in determining the temperature diffusivity in the amplitude and phase variants of the method are more typical than large for the range of high temperatures. Nevertheless, it is quite natural to strive to decrease these errors. The real possibility of accomplishing this is opened with the utilization of a large number of thermal probes. In particular, it is no problem to utilize in the determination of the temperature diffusivity the readings of 4-5 thermal probes which are located at the distance of 2-5 mm from one another. In this case, the magnitudes k and k' can be determined already as the most probable values for the entire totality of the thermal pickups by means of the formulas

$$k = -\frac{d \ln \theta}{dx}, \tag{11.5}$$

$$k' = \frac{d\varphi}{dx}, \tag{11.6}$$

k and k' should thus be regarded as angles of the slope of the straight lines ln Θ as a function of x and $\phi = \phi(x)$, which corresponds in the best manner to the experimental points. Typical curves of these functions are shown in Figure 30 a and b. Then the section of the curves in Figure 30 is explained by the fact that for different periods the coefficient of amplification was different. A comparison of the values of k found from formulas (11.5) and (11.6) during a repeat of the experiments under identical conditions leads to the conclusion that a maximum error of k and k' for 4-5 thermal probes does not exceed ~ 2-3%.

Table 21. Results of the Determination of the Temperature Diffusivity from the Amplitudes and Phases of the Variations for Different Periods.

<table>
<tr><th colspan="3">a, cm²/sec</th></tr>
<tr><th>Seconds</th><th>from amplitudes</th><th>from phases</th></tr>
<tr><td>10,2</td><td>0,093</td><td>0,097</td></tr>
<tr><td>15,2</td><td>0,095</td><td>0,098</td></tr>
<tr><td>20,0</td><td>0,093</td><td>0,097</td></tr>
</table>

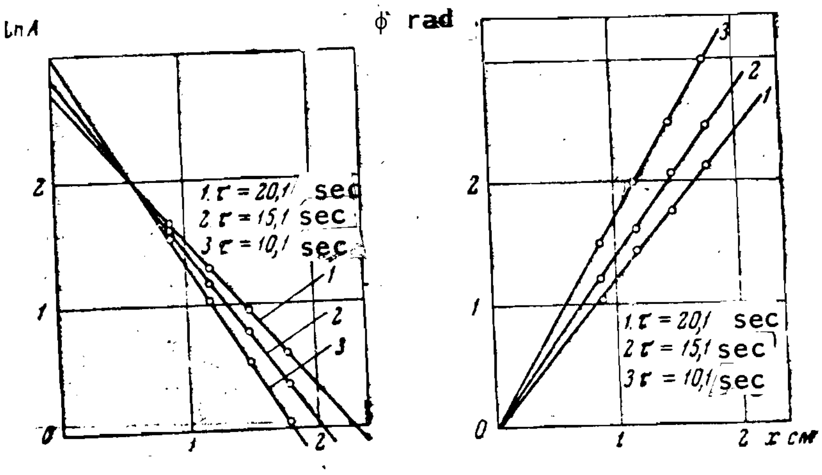


Figure 30. a, Dependence of the logarithm of the temperature variations on the coordinates of the thermal probe; b, The dependence of the phase of the temperature variations on the coordinates of the thermal probes.

The corresponding values of the maximum error in determining the temperature diffusivity amounts to 4-9% (with consideration of the possible systematic error due to the inaccuracy of the introduction of a correction for the heat exchange). The reproducibility of the individual experimental values, including also those for different periods, are completely in agreement with this evaluation. In results of the amplitude and phase measurements agree with one another with the same accuracy. For illustration, we show Table 21 (iron at the temperature of 433°C, four probes were used).

In conclusion of this section, we wish to dwell on a problem of the utilization of the method of longitudinal temperature waves for measuring the temperature diffusivity of liquid metals.

The main difficulty in this case is related with a need of keeping track of the role of rolls of that tube (crucible) in which the liquid metal is contained. It is obvious that the effect of the tube walls on the results of the measurement will be comparatively small when the thickness of the walls is small in comparison with the diameter of the tube. For such thin-walled tubes, the role of the wall can be reduced to a small correction. The specific expression for this correction is shown in [149]. (Corrections are given as the first approximation of the solution of the corresponding accurate problem; there no conclusion.)

For small values of the Bi criterion, for which, properly, it makes sense to introduce an accurate correction and small values of $= \frac{R_2^2 - R_1^2}{R_1^2}$, , that is, for small values of the correction itself, the expression shown there changes to the following:

$$\Delta = \frac{R_2^2 - R_1^2}{R_1^2} \cdot \frac{c_{\text{CT}} \rho_{\text{CT}}}{c\rho} \left(1 - \frac{a_{\text{CT}}}{a}\right). \tag{11.7}$$

This formula agrees in accuracy with the formula for the correction given in [150] where it was obtained from the expression for the effective temperature diffusivity which was determined as the combination of the average heat conductivity along the cross-section, average heat capacity with respect to the volume, and average weight density.

Measurements of the temperature diffusivity of liquid sodium and lithium within the temperature range of 350 to 876° and 1,000°C are respectively described in [150]. The test metals were placed in a stainless steel tube 230 mm long, 8.6 mm in diameter, and a wall thickness of 0.2 mm. The error of the results of measurements amounted to 10%. The results of similar measurement of the temperature diffusivity of liquid sodium and potassium and of the alloid containing 78% potassium and 22% sodium within the temperature range of 100-150° to 600--750°C are given in [149].

§12. Procedure for Measuring the Complex of Thermal Parameters of Metal by Means of Longitudinal Temperature Waves and Rods

The possibilities of the method of regular thermal conditions of the third type, as already pointed out in §8, make it possible to utilize it not only for measurements of the temperature diffusivity of material, but also for determining the entire complex of thermal characteristics such as heat conductivity, heat capacity, temperature diffusivity [151, 136]. Such a development of the method of temperature waves presupposes a knowledge of the variables of the heat flow on one of the surfaces of the sample during its periodic heating. The problem of the experimental determination of the flows is told by the utilization of heating by means of electron bombardment when a specific surface of the sample (in the case under consideration--one end of the rod) is heated by a stream of electrons which are emitted by a calcine cathode or a special gun; the heating power is in this case determined from the strength of the electron current and the difference of potential between the cathode and the sample-anode.

Let us examine the theory of the method by limiting ourselves to the case of relatively short temperature waves, the advantages of which have been explained in § 10 and 11. For such waves, that is, under conditions of smallness of the role of emission heat exchange, the equation for the complex amplitude of the temperature waves in a semi-infinite rod should have the form of

$$\theta \doteq \vartheta_0 e^{-\sqrt{\frac{\omega}{2a}} \cdot ix\left(i - \frac{\xi}{2} i\right)} \tag{12.1}$$

where $\xi = \nu' a/\omega$ (see formulas (10.17)-(10.19)).

The amplitude of the temperature variations on the heated end of the rod θ_0 should in this case be found from the boundary condition on the end:

$$-\lambda \frac{d\theta}{dx} = qm - \alpha_1 \vartheta \tag{12.2}$$

(this condition is similar to (8.11) with consideration of the erection of the axis x), where α_1 is the effective coefficient of emission heat exchange on the end, q is the specific power of the electron bombardment, m is the coefficient of modulation. Substitution of the expression (12.1) in equation (12.2) gives for the amplitude of the temperature variations and of the phase (with respect to the variations of the power) the formulas

$$|\vartheta_0| = \frac{qm}{\lambda\sqrt{\frac{\omega}{a}}}\left(1-\frac{\xi'}{2}\right), \tag{12.3}$$

$$\varphi = \frac{\pi}{4} - \frac{\xi'+\xi}{2}, \tag{12.4}$$

where

$$\xi' = \frac{\alpha_1}{\lambda\sqrt{\frac{\omega}{2a}}}. \tag{12.5}$$

In the derivation of these formulas, it was assumed that $\xi \ll 1$ и $\xi' \ll 1$.

Formula (12.3) makes it possible to determine the value of the heat activity $\left(\frac{\lambda}{\sqrt{a}}\right)$ from the results of the measurements of the power q and the amplitude of the temperature pulsation on the end θ_0. In practice, it is however, inexpedient to determine not θ_0 but the amplitude of the temperature variations at a certain distance l from the end. For the amplitude and phase we get in this case the formulas

$$|\theta| = \frac{qm}{\lambda\sqrt{\frac{\omega}{a}}}\, e^{-\sqrt{\frac{\omega}{2a}}\,l}\,(1-\delta_1), \tag{12.6}$$

$$\varphi = \frac{\pi}{4} + \sqrt{\frac{\omega}{2a}}\,l - \delta_2. \tag{12.7}$$

The members δ_1 and δ_2 are small corrections for the emission heat exchange and are equal to

$$\delta_1 = \frac{\xi'}{2} + \frac{\xi}{2}\sqrt{\frac{\omega}{2a}}\,l, \tag{12.8}$$

$$\delta_2 = \frac{\xi'}{2} + \frac{\xi}{2}\left(1 + \sqrt{\frac{\omega}{2a}}\,l\right). \tag{12.9}$$

The formulas (12.6) and (12.7) make it possible to determine both thermophysical coefficients λ and a, which figure therein, from the readings of one thermocouple:

$$a = \frac{\omega l^2}{2\left(\varphi - \frac{\pi}{4}\right)^2}\left(1 - \frac{2\delta_2}{\varphi - \frac{\pi}{4}}\right), \tag{12.10}$$

$$\lambda = \frac{qml}{\sqrt{2}\left(\varphi - \frac{\pi}{4}\right)|\theta|}\, e^{-\left(\varphi - \frac{\pi}{4}\right)}(1 - \delta_3), \tag{12.11}$$

while

$$\delta_3 = \delta_1 + \delta_2\left(1 + \frac{1}{\sqrt{\frac{\omega}{2a}}\,l}\right) = \xi\left(1 + \sqrt{\frac{\omega}{2al}} + \frac{1}{2\sqrt{\frac{\omega}{2al}}}\right) + \xi'\left(1 + \frac{1}{2\sqrt{\frac{\omega}{2a}}\,l}\right). \tag{12.12}$$

In utilizing the readings of the thermocouples (thermal probes) for determining the temperature diffusivity, these formulas can, in accordance with §11, be written in the form of

$$a = \frac{\omega}{2k'^2}(1 - \xi), \tag{12.13}$$

$$\lambda = \frac{qm}{\sqrt{2}k'|\vartheta|} e^{-k'}(1 - \delta_3'), \tag{12.14}$$

where

$$\delta_3' = \delta_1 + \frac{\xi}{2}\left(1 + \sqrt{\frac{\omega}{2a}}l\right) = \frac{\xi}{2}\left(1 + 2\sqrt{\frac{\omega}{2a}}l\right) + \frac{\xi'}{2}. \tag{12.15}$$

Besides the determination of temperature diffusivity from the value of the phase of the temperature variations under conditions of controlled power, there is also another possibility of determining this magnitude: from the amplitude of the temperature variations for two or several frequencies (amplitude-frequency method). The temperature diffusivity can in this case be found from the formula

$$a = \frac{\omega l^2}{2} \cdot \frac{(\sqrt{n} - 1)^2}{\left[\ln \frac{|\vartheta|_\omega}{|\vartheta|_{n\omega}} \cdot \frac{1}{\sqrt{n}}\right]^2}(1 - \delta_4),$$

$$\delta_4 = \frac{2(\delta_{1\omega} - \delta_{1n\omega})}{l(\sqrt{n} - 1)\sqrt{\frac{\omega}{2a}}}.$$

The utilization of the amplitude-frequency method makes it possible to accomplish additional control of the resulting data, which is one of the advantages of the described method of measurement.

In the examined complex method, an independent method is used to determine the magnitude of the temperature diffusivity, the second thermal characteristic--heat conductivity (or heat activity) is related with the determination of the temperature diffusivity (which figures in formula (12.11) as the amplitude as well as the phase of the temperature variations).

Let us now pass on to a presentation of the problems of the practical accomplishment of the procedure. The circuit of the experimental unit which is intended for measurements by the cathode under consideration is shown in Figure 31 (see [151]).

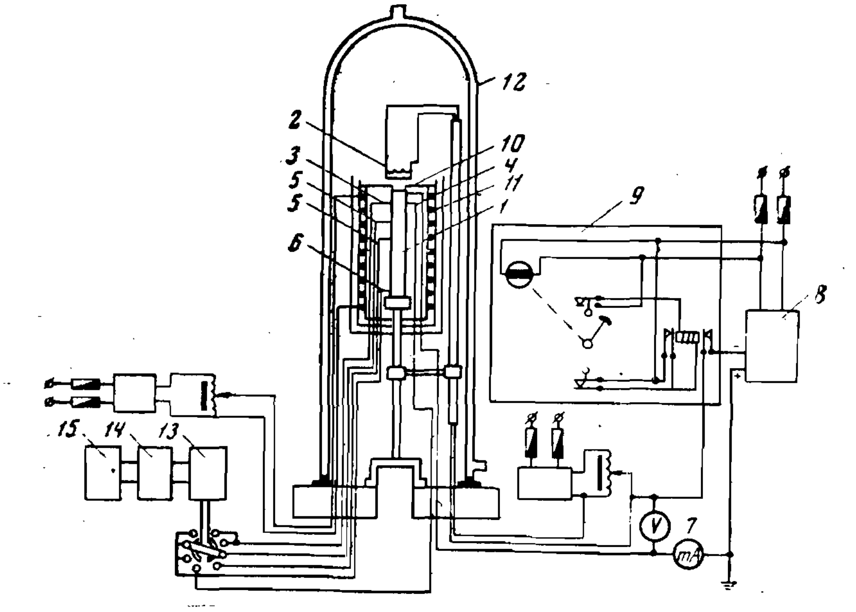


Figure 31. Circuit of unit for measuring the complex of thermal properties by the method of longitudinal temperature waves.

The test sample 1 is placed opposite the cathode 2 (oxidized cathode or wolfram spiral). Two wires 3 and 4 (chromel and alumel, 0.2 mm in diameter) are welded to the surface of the sample at a distance of several mm from the end, which forms the thermocouple that is used for recording the temperature variations as well as for measuring the average temperature of the sample (the wires are welded at an equal distance from the end). In order to obtain additional information regarding the space-time distribution of the temperature with respect to the rod, several thermoprobes--alumel wires 5 which are ~ 3 mm from one another, are welded to its surface. A common thermoelectric 6 made of the same wire, is welded to the lower end of the sample. Each of the thermal probes in pair with the electrode 6 forms a differential thermocouple, the intermediate metal of which is the sample itself. (Figure 26). The sample is grounded through the millimeter 7 which records the electron current. The cathode is connected with the negative pole of the UIP-I type rectifier 8. The voltage of the rectifier is periodically connected and disconnected by means of a mechanical modulator 9 (see §11). Screen 10 which is usually under zero potential, protects the thermocouples against the entry of an electron current. The electrical heater 11 is a cylinder of molybdenum sheet on which is wound a spiral. The spiral is insulated from the cylinder by aluminum rods. On top of the heater are mounted thermal screens which together with the heater are supported on the bottom round screens. The entire system is placed within a glass vacuum cap 12 with double walls between which water circulates. The base of the unit is mounted directly on the flange of a diffusion pump of the VA-05-4 type.

The problem of the methods for recording the temperature variations is given in §11. In this unit, the constant component of the thermal electromotive force of the thermocouples is compensated and measured by means of the potentiometer 13 of the PMS type, the variable component is fed to the amplifier 14 of the F-117 type, the signal from the outlet of which is recorded by the automatic potentiometer 15 of the EPP-09 type (an example of the recording of temperature variations is shown in Figure 32, the spikes correspond to the moment connection and this connection moments of the electron heating). It can be seen that the temperature variations in points distant from the end are close to sinusoidal as a result of the rapid damping of the higher harmonics.

The curves are processed by the method of approximate harmonic analysis: the variations in the points closest from the end by the method of twelve coordinates, while in points distant--by the method four ordinates.

The determination of the temperature diffusivity from the difference in phases between the variations in the power and the temperature in this method does not differ substantially from the experiment described in the preceding section. It is worthwhile to mention only that the dependence of ϕ $\pi/4$ on x for thermal probes in the examined case of electron heating should pass through the origin of the coordinates in accordance with formula (12.7) (with an accuracy up to the correction δ_2), whereas for inertia heaters described in §11, the initial difference in the phases was indeterminate. This circumstance makes it possible to determine the coefficient k' (see formula (11.6)) with a somewhat greater accuracy or with the same accuracy from a smaller number of probes.

Figure 32. Example of the recording of temperature variations.

The magnitude of the instantaneous power of the electron heating can be determined as the product of the strength of the electron current I, which passes from the cathode to the sample, by difference of potentials between them V. Accordingly, the specific power q is equal to

$$q = \frac{IV}{\pi R^2}. \qquad (12.16)$$

This formula can be considered quite accurate in the absence of a secondary electron emission from the end. In the described experiment where the values of V are small (100-200 volts), the influence of secondary emission should not be great. A decrease in the role of secondary emission is facilitated by the screen 10 (see Fig. 31) on which any potential can be fed. In this case, practice shows that the change in the potential of the screen from zero (with respect to the anode--sample) to a double negative does not show up at all on the results, which can be regarded as an argument in favor of the absence of a noticeable effect of secondary emission in the described experiment. (Identical results are received also when the upper flat part of the screen 10 is replaced by a plate made of mica.) Another proof of the absence of the role of secondary emission is in the coincidence of the results obtained from different values of the power (inseparately of the voltages) which differ severalfold. Some of these results will be cited below.

In order to pass from values of instantaneous electrical power to the magnitude of the amplitude of the first harmonic of the very power variations for a П-shaped type of modulation in the formula (12.16), it is necessary to introduce the factor equal to $2/\pi$:

$$q = \frac{2}{\pi^2 R^2} IV. \qquad (12.16')$$

In some cases, for example, for the close location of the cathode with respect to the sample and for high temperatures, the change in power with time will not be strictly П-shaped due to the possible change in the anode current in the course of a half-period. For small changes of such a type, it can be assumed that the power changes linearly, as shown in Figure 33.

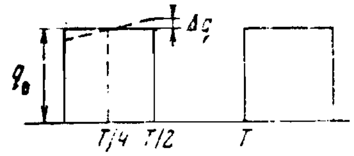


Figure 33.

In this case, for the magnitude of the amplitude of the first harmonic the following formula will be valid

$$q = \frac{4}{\pi} q_0 \left[1 + \frac{1}{2\pi}\left(\frac{\Delta q}{q_0}\right)^2\right].$$

A correction to formula (12.16) will in this case be negligibly small even when Δq amounts to $\sim 10\%$. A correction to the phase of the power variation is another matter. It is equal to

$$\delta\varphi = -\frac{2}{\pi} \cdot \frac{\Delta q}{q_0} = -\frac{2}{\pi} \cdot \frac{\Delta I}{I}.$$

The magnitude $-\delta\phi$, which is added to the difference in phases between the variations in the power and the temperature, can generally speaking be perceptible. In connection with this, in performing the experiment it is recommended to record the curve of the change in anode current with time by feeding to the potentiometer a voltage with a resistance of ~ 0.1 ohm connected in the gap of the anode circuit.

The accuracy in determining the power from formula (12.16) is determined by the class of accuracy of the instruments used and by the stability of the measured values of I and V. In the described experiment, use was made of class 0.5 instruments; no noticeable variations in I and V during the time of the experiment were observed for the stabilized process; the error in determining q was close to 1%.

The error in determining the absolute values of the amplitude of the temperature variations is made up of the errors in determining the first harmonic of the amplitude on the scale of the visions of the tape in the automatic instrument, the errors in determining the value of the divisions of the instruments in units of the electromotive force and the errors of recalculating the electromotive forces of the thermocouple to the temperature difference. In accordance with the presentation in §11, the first of these magnitudes is determined with an accuracy of ~ 1%. The value of the division of the potentiometer can be found with an error of ~ 0.5% (for control purposes, several readings are given), the graduation of the thermocouples makes it possible to determine their sensitivity with an accuracy also of 0.5-1%. The total error of $\delta|\Theta|/|\Theta|$ thus does not exceed ~ 2%.

Let us examine the problem of corrections for emission. In accordance with the evaluations of §10, the magnitude of the dimensionless parameter $\sqrt{\frac{\omega}{2a}}\,l$ under optimum conditions should be close to unity. In accordance with this, the corrections for emission δ_1, δ_2 and δ_3 are determined by the order of magnitude of ξ and ξ'. Evaluations of the first of these have been made in §10. For a sample diameter ranging from 5-10 mm the correction turns out to be small up to temperatures of 1,000-1,500°K. Thus, for an iron sample 8 mm in diameter, the temperature of 1,000°K and the period of 10 seconds, the magnitude ξ is ~ 0.3%.

Formula (12.5), which determines the magnitude ξ' can be represented also in the form of

$$\xi' = \frac{2a\alpha_1}{\omega\lambda R} R \sqrt{\frac{\omega}{2a}}. \qquad (12.17)$$

In case of equality of the coefficients of heat exchange on the end and the lateral surface of the sample,

$$\xi' = \xi R \sqrt{\frac{\omega}{2a}} \qquad (12.18)$$

The dimensionless factor $R\sqrt{\frac{\omega}{2a}}$ is with respect to the order of magnitude either equal to unity of less than $(R \leqslant l,\ l\sqrt{\frac{\omega}{2a}} \sim 1)$ Hence, it follows that

$$\xi' \leqslant \xi. \qquad (12.19)$$

This conclusion naturally remains valid also when the coefficient of heat exchange on the end α is, with respect to accuracy, not equal to α, but as close to it in order of magnitude.

Thus, each of the correction members δ_1, δ_2, and δ_3 is small, amounting to a magnitude of the order of several percent. For the temperature range of ~ 1000- 1,500°K, the accuracy introduced as a result of the determination of the heat conductivity and the temperature diffusivity due to the inaccuracy in the calculation of these corrections does not exceed ~ 0.5%.

Another possible source of errors is the nonuniformity of the temperature field along the radius of the sample, which is determined by the nonuniformity of the electron flow on the end. In order to evaluate the role of this factor, let us examine the solution of the three-dimensional problem on the assumption that the distribution of the heating power is symmetric to the axis of the sample, for the specific heat liberation depends on the radius (q = q(r)). The temperature distribution in the sample is in this case described by the equation

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T\right)+\frac{\partial^2 T}{\partial x^2}=\frac{1}{a}\cdot\frac{\partial T}{\partial t}. \tag{12.20}$$

The solution of this equation will be sought in a zero approximation, disregarding the corrections for the emission heat exchange. In this approximation, the boundary conditions reduce themselves to the form of

$$-\lambda\frac{\partial\theta}{\partial x}=qm \text{ for } x=0, \tag{12.21}$$

$$\frac{\partial\theta}{\partial r}=0 \text{ for } r=R \tag{12.22}$$

and

$$|\theta|\rightarrow 0 \text{ when } x\rightarrow\infty, \tag{12.23}$$

while the very equations for the complex amplitude of the temperature pulsation has the form of

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\theta+\frac{\partial^2\theta}{\partial x^2}-\frac{i\omega}{a}\theta=0. \tag{12.24}$$

In order to solve the problem, we shall utilize the method for the separation of the variables, assuming that

$$\theta=\vartheta(x)\psi(r). \tag{12.25}$$

Specific solutions of equation (12.24) in this case have the form of

$$\theta = \vartheta_0 e^{-\sqrt{\frac{i\omega}{a}+\mu}} I_0(\sqrt{\mu}\, r), \qquad (12.26)$$

where I_0 Bessel function of the first type of zero order, μ is a numerical coefficient the magnitude of which, in accordance with the boundary condition (12.22), should be determined from the equation

$$I_1(\sqrt{\mu}\, R) = 0 \qquad (12.27)$$

(I_1 is the Bessel function of the first type of the first order), which has an infinite discrete series of actual positive roots

$$\sqrt{\mu}\, R = \sqrt{\mu_1}\, R,\ \sqrt{\mu_2}\, R\ . \quad = 0;\ 3{,}8;\ 7, \qquad (12.28)$$

The totality of the values of the characteristic numbers $\mu_1 \ldots \mu_i$ determines the system of specific solutions of the equation $\theta_1 \ldots \theta_i$. The general solution of the equation can in this case be represented in the form of an infinite series of characteristic solutions:

$$\theta = \sum_n C_n e^{-\sqrt{\frac{i\omega}{a}+\mu_n x}} I_0(\sqrt{\mu_n}\, r). \qquad (12.29)$$

The coefficients C_n of this series can be found from the boundary condition (12.21) which leads to the expression

$$\sum_n C_n \sqrt{\frac{i\omega}{a}+\mu_n}\, I_0(\sqrt{\mu_n}\, r) = \frac{q(r)\, m}{\lambda}. \tag{12.30}$$

The expression (12.30) represents the expansion of the given function q(r) as a series with respect to the Bessel functions $I_0\sqrt{\mu}r$.

By utilizing the properties of the orthogonality of the Bessel functions, we get for the composition coefficients C_n the formula

$$C_n = \frac{2m\int\limits_0^R q(r)\, r I_0(\sqrt{\mu_n}\, r)\, dr}{R^2\lambda\sqrt{\frac{i\omega}{a}+\mu_n}\, I_0^2(\sqrt{\mu_n}\, R)}. \tag{12.31}$$

We get the final solution of the problem in the form of

$$\theta = \frac{2m}{R^2\lambda}\sum_n \frac{1}{\sqrt{\frac{i\omega}{a}+\mu_n}}\, e^{-\sqrt{\frac{i\omega}{a}+\mu_n}\, x} \times$$

$$\times \frac{I_0(\sqrt{\mu_n}\, r)}{I_0(\sqrt{\mu_n}\, R)}\int\limits_0^R qr\, \frac{I_0(\sqrt{\mu_n}\, r)}{I_0(\sqrt{\mu_n}\, R)}\, dr, \tag{12.32}$$

Let us turn attention to the fact that for q = const (uniform heat liberation on the end) all the members of the series (12.32), with the exception of the first ($\mu = 0$), are equal to zero and the solution corresponds to a flat temperature wave

$$\theta = \frac{mqe^{-\sqrt{\frac{i\omega}{a}}x}}{\lambda\sqrt{\frac{i\omega}{a}}}, \tag{12.33}$$

the amplitude and the phase of which are described by formulas (12.3) and (12.4) (without corrections for the emission).

For $q \neq$ const, the same flat wave is given by the first member of the series (12.32); besides, instead of q in the formula (12.33), the average heat liberation figures

$$\bar{q} = \frac{2\int\limits_0^R qrdr}{R^2}, \tag{12.34}$$

which is equal to the ratio of the full power liberated on the end to its area:

$$\bar{q} = \frac{Q}{\pi R^2} = \frac{\int\limits_0^R q2\pi rdr}{\pi R^2}. \tag{12.35}$$

The subsequent members of the series (12.32) described the difference of the real temperature wave from the flat wave. In order to evaluate the magnitudes of the corresponding distortions, we shall examine the first of these members in the points of the surface (r-R):

$$\Delta\theta = \frac{2m}{R^2\lambda}\cdot\frac{e^{-\sqrt{\frac{i\omega}{a}+\mu}\,x}}{\sqrt{\frac{i\omega}{a}+\mu}}\int_0^R qr\,\frac{I_0(\sqrt{\mu}r)}{I_0(\sqrt{\mu}R)}\,dr. \tag{12.36}$$

We shall assume that $\sqrt{\mu}R\sim 4$ and $\sqrt{\frac{\omega}{2a}}R<1$,

for which

$$\frac{\omega}{2a\mu}\ll 1. \tag{12.37}$$

As an approximation of (12.37), formula (12.36) can be written in the form of

$$\Delta\theta = \frac{2m}{R^2\lambda}\cdot\frac{e^{-\sqrt{\mu}x}\,e^{-i\frac{\omega}{2\mu a}\left[1+\frac{x}{R}\sqrt{\mu}R\right]}}{\sqrt{\mu}\left[1+\frac{i\omega}{2a\mu}\right]}\int_0^R qr\,\frac{I_0(\sqrt{\mu}r)}{I_0(\sqrt{\mu}R)}\,dr.$$

The amplitude and phase of this member given by the formulas

$$|\Delta\theta| = \frac{m}{2R\lambda}e^{-\frac{4x}{R}}\left|\int_0^R qr\,\frac{I_0\left(4\frac{r}{R}\right)}{I_0(4)}\,dr\right|, \tag{12.38}$$

and

$$\psi = \frac{\omega R^2}{32a}\left(1+\frac{4x}{R}\right). \tag{12.39}$$

The corrections to the main member (flat wave) can be found from the expressions

$$\frac{\Delta|\theta|}{|\theta|}=\frac{|\Delta\theta|}{|\theta|}\cos(\psi-\varphi), \tag{12.40}$$

$$\frac{\Delta\,\mathrm{tg}\,\varphi}{\mathrm{tg}\,\varphi}=\frac{|\Delta\theta|}{|\theta|}\sin(\psi-\varphi). \tag{12.41}$$

In order to find the magnitude $|\Delta\Theta|/|\Theta|$, which determines these corrections, we have the formula

$$\frac{|\Delta\theta|}{|\theta|}=\frac{1}{4}\sqrt{\frac{\bar{\omega}}{a}}\,Re^{-\frac{x}{R}\left(4-\sqrt{\frac{\omega}{2a}}R\right)}\frac{\left|\int\limits_0^R qr\frac{I_0\left(4\frac{r}{R}\right)}{I_0(4)}\,dr\right|}{\int\limits_0^R qr\,dr}. \tag{12.42}$$

For specific evaluations, it is necessary to set the type of function q(r). As an example, we shall examine one specific case when the electron flow strikes on the surface which is limited by half of the radius, that is, when

$$q(r)=\begin{cases} q_0 & r<\frac{R}{2} \\ 0 & r>\frac{R}{2} \end{cases} \tag{12.43}$$

(Only one-fourth of the surface of the end is in this case subjected to heating).

The substitution of the functions (12.43) and formula (12.42) gives for the ratio of the integrals of this formula a magnitude which is ~ 1.5. We shall further assume that under optimum conditions

$\sqrt{\frac{\omega}{2a}}x \sim 1.$. Then we get

$$\frac{|\Delta\theta|}{|\theta|} \simeq \frac{R}{x} e^{-\frac{4x}{R}} \tag{12.44}$$

The dependence of /R, expressed by this formula, is very steep. The ratio of ($\frac{x}{R}$) becomes very small even for ($\frac{|\Delta\theta|}{|\theta|}$; when x/R = 1.5, ($\frac{|\Delta\theta|}{|\theta|}$) is negligibly small (~ 0.1%). The result should be of a general nature because for the function q(r) of another type the results will differ from formula (12.44) only by a factor of the order of unity. Thus, even very significant variations from the uniformity of the electron flow near the end should not in any noticeable manner show up also on the distance from the end of the order of one radius.

Let us now pass on to an evaluation of the total error in determining the heat conductivity. In accordance with the above presentation, the error introduced by the inaccuracy in the measurement of the magnitudes q, $|\Theta|$ and the correction factors amounts to a total of 3.5%. In using the readings of one thermocouple for determining the temperature diffusivity, that is, in calculations with the use of formula (12.11), it is necessary to add to this magnitude the relative error in the determination of the distance l from the end to the thermocouple, which is equal to 1-2% (δl ~0.1 mm, l --5--10 mm), and the member

$$\frac{\delta\left(\varphi-\frac{\pi}{4}\right)}{\varphi-\frac{\pi}{4}}\left[\frac{1}{\varphi-\frac{\pi}{4}}e^{-\left(\varphi-\frac{\pi}{4}\right)}+1\right], \tag{12.45}$$

which takes into account the inaccuracy in the determination of the phase. For optimum conditions, when $\varphi-\frac{\pi}{4}\sim 1$ and $\delta\left(\varphi-\frac{\pi}{4}\right)=0{,}5\div 1$ (see §11), this leaves 1-2%. The total maximum error in determining the heat conductivity thus amounts to 5.5-7.5%.

In utilizing formula (12.14), the error in the determination of l does not disappear, and instead of (12.45), the magnitude

$$\frac{\delta k'}{k'}\left(\frac{1}{k'}e^{-k'}+1\right),$$

enters which is equal to approximately 2-3% $\left(k'\sim 1,\ \frac{\delta k'}{k'}\sim 2-3\%,\right.$ (see §11). The maximum error in this case lies within the limits of 4-7%. The results obtained during the refinement of the procedure and the experience gained from its use [151] are in agreement with the determined evaluations of the experimental accuracy. For illustration, Table 22 is given which contains the results of the measurements of the heat conductivity and the temperature diffusivity of Armco iron for two considerably differing heating powers and three different periods.

It can be seen that the resulting values are in good agreement with each other and with literature data.

The method described here for determining the complex of thermal properties can in principle be applied also to the case in which the test samples have a length that is comparable with the length of the temperature wave, that is, when the conditions of semi-infinity, which are used in a theory of this method, cease to be valid. The utilization of short samples with a length of ~ 0.5-1 cm is of practical interest in the study of costly, rare-earth elements, monocrystals, and semiconductors.

Formula (9.9) for a flat temperature wave can form the basis of the theory of the experiment with short samples; this gives the solution of the sought-for problem as a zero approximation (with corrections for heat exchange):

$$\frac{\theta}{\theta_0}=\frac{\varkappa}{\sqrt{i}}\cdot\frac{\operatorname{sh}\left(\sqrt{i}\,\varkappa\,\frac{x}{L}\right)}{\operatorname{ch}(\sqrt{i}\,\varkappa)}, \qquad (12.46)$$

where

$$\varkappa = \sqrt{\frac{\omega}{a}}\,L, \quad \theta_0 = \frac{mq}{c_p \rho \omega L}.$$

Table 22. Example of Simultaneous Measurements of the Temperature Diffusivity and Heat Conductivity (Armco iron).

t°C	q,wt	τ, sec.	a, cm²/sec	a [146]	λ, cal/cm·sec·deg	λ [42]
300	2,52	10,2 15,2 20,1	0,120 0,121 0,119	0,125	0,126 0,127 0,125	0,131
370	6,25	10,1 15,2 20,2	0,105 0,107 0,106	0,110	0,122 0,122 0,120	0,121

For the phase of the temperature variations (with respect to the power variations) at the distance of x from the nonheated end of the rod and from the expression (12.46) there follow the formulas

$$\varphi = \frac{\pi}{4} + \operatorname{arctg} \frac{A_s\left(\varkappa \frac{x}{L}\right) B_c(\varkappa) - A_c(\varkappa) B_s\left(\varkappa \frac{x}{L}\right)}{A_c(\varkappa) A_s\left(\varkappa \frac{x}{L}\right) + B_c(\varkappa) B_s\left(\varkappa \frac{x}{L}\right)}, \qquad (12.47)$$

or

$$\varphi = \frac{\pi}{4} + \operatorname{arctg} \frac{B_c(\varkappa)}{A_c(\varkappa)} - \operatorname{arctg} \frac{B_s\left(\varkappa \frac{x}{L}\right)}{A_s\left(\varkappa \frac{x}{L}\right)}, \qquad (12.48)$$

where A_s, B_s, A_c and B_c are functions which are determined by the relationships (9.12). Formula (9.14) for the phase of the variations in the point x = 0 (on the end) follows from formula (12.47) as a specific case. The form of the curves $\phi = \phi(\kappa)$ for the three fixed values of the ratio x/L, which are equal to 0, 1/2 and 1, is shown in Figure 34. For all the curves, for small values of κ, the difference of the phases differs little from $\pi/2$. This is the general characteristic of slow temperature changes, which has been observed in §9. For x/L = 0, the curve $\phi(\kappa)$, with an increase in κ (actually the same as $\phi(\kappa^2)$ in Figure 22) approaches an asymptotic straight line

$$\varphi = \frac{\pi}{4} + \frac{\varkappa}{\sqrt{2}}, \tag{12.49}$$

which corresponds to a flat temperature wave in semi-infinite rod (see formula (12.47)). This should be understood because, with increasing κ, there is a decrease in the ratio of the length of the temperature wave to the dimension of the sample.

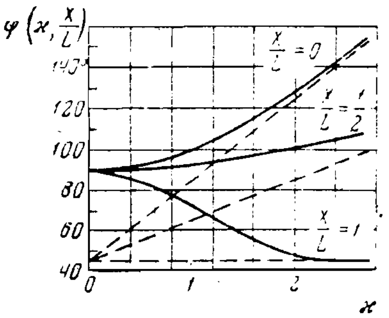


Figure 34. Dependence of the difference in the phases between the variation and the heating power of the end and the variation in the temperature on the parameter κ in different points of a short rod.

In the case when x/L = = 1, that is, for temperature variations on the heated end of the sample, the asymptote of the curve $\phi(\kappa)$ for relatively large values of κ is the horizontal line

$$\varphi = \frac{\pi}{4}.$$

As observed earlier (see §9), a shift in the phases by $\pi/4$ is a property which is characteristic of temperature variations of a heated surface during the propagation of the temperature wave in a semi-infinite medium.

For temperature variations in the middle point of the specimen when x/L = 1/2, the dependence $\phi(\kappa)$ has an intermediate nature in comparison with the cases when x/L = 0 and 1. Here, the asymptotic line is the straight line

$$\varphi = \frac{\pi}{4} + \frac{\varkappa}{2\sqrt{2}}. \tag{12.50}$$

An examination of the curves $\varphi\left(\varkappa, \frac{x}{L}\right)$ specifically for the experiment on the determination of the temperature diffusivity makes it possible to clarify that in measuring the temperature of the heated end, κ (and consequently also the temperature diffusivity) can be determined sufficiently accurately within the range of $\kappa \sim 1.5$; in recording the temperature on the opposite end, the region of $\kappa \geq 1.5$ is convenient. As regards intermediate values of x/L, they are all less favorable in comparison with the indicated extreme values (curves $\phi(\kappa)$ are more slanting). If it is taken into account besides that that during the recording of the temperature of the heated end it is necessary to take special measures to insure uniformity of heating on the surface, then the most rational position of the heat receiver should be the free end of the sample. Experiments with the recording of the temperature on the free end of the sample have also the advantage that at distance x = L, which figures in the calculation formulas, can be determined in this instance in a most convenient and accurate manner. (Besides that, the installation of the thermocouples is also facilitated, and the wires of these can be welded practically to any two points of the surface of the end, whereas in welding to the electrical surface, the wires should without fail be on a single line.)

The formula for the amplitude of the temperature variations in the general case has the form of

$$\frac{|\theta|}{|\theta_0|} = \varkappa \frac{\sqrt{A_c^2\left(\varkappa \frac{x}{L}\right) + B_c^2\left(\varkappa \frac{x}{L}\right)}}{\sqrt{A_s^2(\varkappa) + B_s^2(\varkappa)}}. \tag{12.51}$$

It can be written also in the form of

$$\frac{|\theta|}{|\theta_0|} = |F_0| \sqrt{A_c^2\left(\varkappa \frac{x}{L}\right) + B_c^2\left(\varkappa \frac{x}{L}\right)}, \tag{12.52}$$

where $|F_0|$ is the value of $|\Theta|/|\Theta_0|$ when x/L = 0, that is, the function (9.13), as shown in Figure 23. The curves $|\Theta|/|\Theta_0|$ of the dependence on κ for the values of x/L = 0, 1/2 and 1 are shown in Fig. 35.

In all cases for small values of κ, $|\Theta| \sim \Theta_0$ (the amplitude of slow temperature variations is determined only by the heat capacity of the sample). The curve for x/L = 0 represents the function $|F_0|$ which has been examined in §9. The dependence of $|\Theta|/|\Theta_0|$ on κ for x/L = 1 (the amplitude of the variations on the heated end) has a steeper nature. With increasing κ, the values of this function approach the asymptotic line

$$\frac{|\theta|}{|\theta_0|} = \varkappa, \tag{12.53}$$

that is, to the function which is characteristic of the temperature wave that penetrates into the semispace (see §9); the amplitude of the temperature variations $|\Theta|$ varies here with the frequency less strongly than in the case of slow variations which heat up the entire sample. The case of x/L = 1/2 is intermediate in comparison with the examined extreme cases. It is seen from an examination of the curves of Fig. 35 that when $\kappa \sim 2$, the amplitude of the temperature variations on the heated end exceeds the amplitude on the opposite end by 1.5-2 times. Nevertheless, the above presented considerations during the examination of the conditions for determining the temperature diffusivity compel us still to give preference to the method for recording temperature variations on the free end of the sample. Thus, the experiment with samples in the form of short rods should not in its fundamental basis differ from the method of determining the complex of thermal properties of flat samples, as presented in §9.

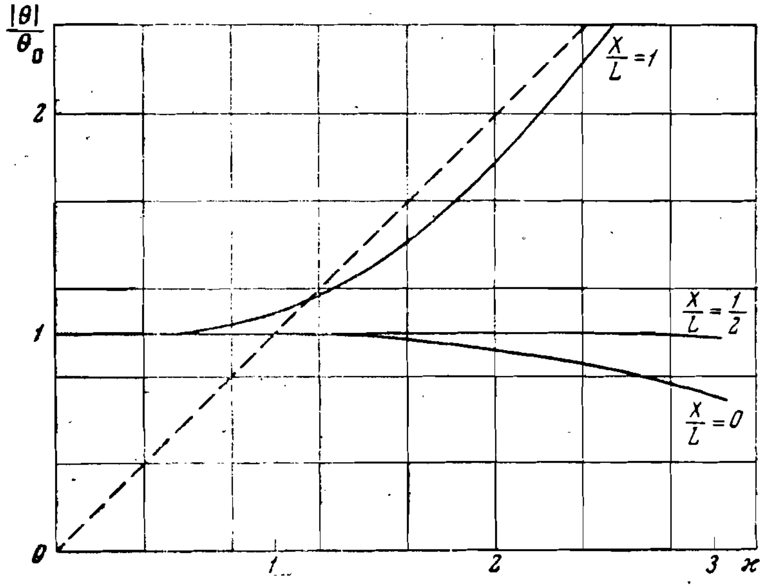


Figure 35. Dependence of the amplitude of the temperature variations on the parameter κ in different points of a short rod.

§13. Radial Temperature Waves. Measurement of the Temperature Diffusivity.

Radial or cylindrical temperature waves comprise a vibrational process in cylindrical bodies, under which the amplitude of the temperature variations is a function only of the distance of the point from the axis of the cylinder, that is, the isothermal surfaces are cylindrical.

The existence of radial temperature waves of two types--converging and diverging--is possible. In the first case, the amplitude of the temperature variations increases with the distance from the axis of the cylinder and in the second it decreases. Converging temperature waves take place during variable heating of the surface of a cylindrical sample and diverging temperature waves take place during the action of internal (volume) periodic heat sources or during periodic heating of the internal surface of a hollow cylinder. (The case of volume heating is not examined by us in this section). Below is given the theory of radial temperature waves, which encompasses both of these types of waves (see also [152, 153].

Let us examine a hollow round cylinder with the radius of the inside surface being R_1 and that of the external surface R_2. The length of the cylinder will be assumed to be so large that the temperature distribution along the axis of the cylinder can be disregarded. In accordance with the general theory of temperature waves, which has been presented in §8, the complex pulsation amplitude of the temperature Θ as a first approximation (when $Bi \to 0$), is determined by the function F:

$$F = \frac{\theta}{\theta_0} = \frac{\theta}{(Q/Mc_p\omega)} \tag{13.1}$$

(See formula (8.44)), which depends on the dimensionless variable r/l in the parameter

$$\varkappa = \sqrt{\frac{\omega}{a}}\, l, \tag{13.2}$$

where l is a characteristic dimension equal to the ratio of the volume of the cylinder to the heated surface

$$F = F\left(\frac{r}{l}, \varkappa\right). \tag{13.3}$$

The function F satisfies the equation

$$\nabla^2 F - i\varkappa^2 F = 0 \tag{13.4}$$

(∇^2 is here a dimensionless Laplace operator) and the boundary conditions

$$F' = \varkappa^2 \quad \text{on the heated surface} \tag{13.5}$$

and

$$F' = 0 \quad \text{on the opposite surface} \tag{13.6}$$

(F' is the derivative with respect to the dimensionless normal, $F' = \pm \frac{\partial}{\partial \frac{r}{l}}$; the plus sign corresponds to outside heating, the minus sign corresponds to internal heating).

The general solution of equation (13.4) for a case of cylindrical symmetry can be written in the form of

$$F = C_1 I_0(z) + C_2 H_0^{(1)}(z), \tag{13.7}$$

where I is a Bessel function of the first type of zero order, $H_0^{(1)}$ is the first Hankel function of zero order. The argument z is equal to

$$z = \sqrt{-i\varkappa}\,\frac{r}{l} = r\sqrt{\frac{-i\omega}{a}}. \tag{13.8}$$

The constants C_1 and C_2 should be found from the boundary conditions (13.5) and (13.6). Let us designate by z_a and z_b the values of z on the heated and the nonheated surfaces. (In case of internal heating

$$z_a = R_1 \sqrt{\frac{-i\omega}{a}} = z_1,$$
$$z_b = R_2 \sqrt{\frac{-i\omega}{a}} = z_2; \qquad (13.8)$$

in case of external heating

$$z_a = R_2 \sqrt{\frac{-i\omega}{a}} = z_2,$$
$$z_b = R_1 \sqrt{\frac{-i\omega}{a}} = z_1). \qquad (13.9)$$

For C_1 and C_2 we get

$$C_1 = \pm \frac{\varkappa}{\sqrt{-i}} \cdot \frac{H_1^{(1)}(z_b)}{I_1(z_b) H_1^{(1)}(z_a) - I_1(z_a) H_1^{(1)}(z_b)},$$
$$C_2 = \mp \frac{\varkappa}{\sqrt{-i}} \cdot \frac{I_1(z_b)}{I_1(z_b) H_1^{(1)}(z_a) - I_1(z_a) H_1^{(1)}(z_b)}. \qquad (13.10)$$

where I_1 is a Bessel function of the first type of the first order, $H_1^{(1)}$ is the first Hankel function of the first order. For the function F, we thus get the expression

$$F(z) = \pm \frac{\varkappa}{\sqrt{-i}} \cdot \frac{H_1^{(1)}(z_b) I_0(z) - H_0^{(1)}(z) I_1(z_b)}{I_1(z_b) H_1^{(1)}(z_a) - I_1(z_a) H_1^{(1)}(z_b)}. \qquad (13.11)$$

From expression (13.11) we can obtain directly formulas for the amplitude and phase of temperature pulsation at any point of the sample.

Let us examine at first the expression for F(z) when $z = z_b$, that is, on the side opposite that being heated. From expression (13.11) we get in this case

$$F = \frac{R_2^2 - R_1^2}{\pi R_1 R_2} \cdot \frac{1}{I_1(z_2) H_1^{(1)}(z_1) - I_1(z_1) H_1^{(1)}(z_2)}. \tag{13.12}$$

In the derivation of this formula, use was made of the identity

$$I_0 H_1^{(1)} - I_1 H_0^{(1)} = \frac{2}{\pi i z}. \tag{13.13}$$

The resulting expression for F is the same for inside and outside heating, that is, for converging and diverging cylindrical waves.

From expression (13.12) we get the following formulas for the difference in phases between the variation in the power and temperature and for the amplitude pulsation of the temperature:

$$\varphi = \operatorname{arctg} \frac{\Phi_2}{\Phi_1}, \tag{13.14}$$

$$\frac{|\theta|}{\theta_0} = \frac{R_2^2 - R_1^2}{\pi R_1 R_2} \cdot \frac{1}{\sqrt{\Phi_1^2 + \Phi_2^2}}, \tag{13.15}$$

where Φ_1 and Φ_2 are the real and imaginary parts of the denominator of the function F:

$$\Phi_1 + i\Phi_2 = I_1(z_1) H_1^{(1)}(z_2) - I_1(z_2) H_1^{(1)}(z_1). \tag{13.16}$$

Φ_1 and Φ_2 can be expressed through the derivatives of the Thomson (Kelvin) function ber, bei, her and hei [16]:

$$\left.\begin{aligned} \Phi_1 = \Phi_1(\varkappa_1, \varkappa_2) = \operatorname{bei}'(\varkappa_2)\operatorname{her}'(\varkappa_1) + \operatorname{hei}'(\varkappa_1)\operatorname{ber}'(\varkappa_2) - \\ - \operatorname{bei}'(\varkappa_1)\operatorname{her}'(\varkappa_2) - \operatorname{hei}'(\varkappa_2)\operatorname{ber}'(\varkappa_1), \\ \Phi_2 = \Phi_2(\varkappa_1, \varkappa_2) = \operatorname{ber}'(\varkappa_1)\operatorname{her}'(\varkappa_2) + \operatorname{bei}'(\varkappa_2)\operatorname{hei}'(\varkappa_1) - \\ - \operatorname{bei}'(\varkappa_1)\operatorname{hei}'(\varkappa_2) - \operatorname{ber}'(\varkappa_2)\operatorname{her}'(\varkappa_1), \end{aligned}\right\} \quad (13.17)$$

$$\varkappa_1 = \sqrt{\frac{\omega}{a}}R_1, \quad \varkappa_2 = \sqrt{\frac{\omega}{a}}R_2. \quad (13.18)$$

In the specific case when $R_1 = 0$ (external heating of a solid cylinder) we get from the expression (13.12) the formulas [155]:

$$F = -\frac{\varkappa}{\sqrt{-i}\,I_1(z)}, \quad (13.19)$$

$$\varphi = \operatorname{arctg}\frac{\operatorname{bei}'(\varkappa_2)}{\operatorname{ber}'(\varkappa_2)}, \quad (13.20)$$

$$\frac{|\theta|}{\theta_0} = \frac{\varkappa_2}{2\sqrt{\operatorname{bei}'(\varkappa_2)^2 + \operatorname{ber}'(\varkappa_2)^2}}. \quad (13.21)$$

For practical calculations, it was convenient to utilize the existing tables for the amplitude and phase of the function

$I_1(\sqrt{i}\varkappa) = be^{i\beta}$. Then,

$$\bar{\varphi} = \beta + \frac{1}{4}\pi, \quad (13.21')$$

$$\frac{|\theta|}{\theta_0} = \frac{\varkappa}{b}. \quad (13.21'')$$

The magnitudes ϕ and $|\Theta|/\Theta_0$ can conveniently be represented as the functions of the variables κ^2 ~~and~~ ζ:

$$\kappa = \sqrt{\frac{\omega}{a}} \cdot \frac{R_2^2 - R_1^2}{2R_2} \qquad (13.22)$$

and

$$\zeta = \frac{R_1}{R_2}. \qquad (13.23)$$

The first of these variables represents the magnitude of κ from the formula (13.2) for outside heating. The selection of precisely such a variable and not, shall we say, κ_1 or κ_2 from the formulas (13.18) is explained by the circumstance that when $R_1 \to R_2$ (thin-walled cylinder), the magnitude $\kappa \to \kappa \sqrt{\frac{\omega}{a}} L,$ where $L = R_2 - R_1$ is the thickness of the walls, that is, that variable which was used during the examination of the problem of flat temperature waves (see §9). However, the utilization of κ^2 instead of κ is more convenient for two reasons. In the first case, the curves $\phi(\kappa^2)$ are close to straight lines (in a definite region); in the second place, the parameter κ^2 contains the sought-for magnitude of the temperature diffusivity in the first power. The dependence of $\phi(\kappa^2, \zeta)$ and $|\Theta|/\Theta_0$ are represented in Figures 36 and 37.

In this section, we limit ourselves to an examination of only one aspect of the utilization of radial temperature waves--the procedure of measuring the temperature diffusivity. For this reason, we shall approach the analysis of the indicated results precisely from this point of view. As was observed in the case of the general examination of the problem of the use of temperature waves in §8, the first of these diverse methods for determining the temperature diffusivity--the phase method--is based on the study of the phase of the temperature variation. The first variant of the phase method (the method of a single point) is based on the utilization of information regarding the magnitude ϕ--the difference in phases between the variations in temperature and the power of variable heating.

The curve in Figure 36 makes it possible to carry out an analysis of the conditions for the fulfillment of such an experiment. From an examination of this curve it is seen that the dependence of ϕ of κ^2 is most steep, and consequently is most convenient for examining κ for the solid cylinder. ($\zeta = 0$). With increasing $\zeta = R_1/R_2$, the slope of the curves decreases, approaching in the case of thin-walled cylinder ($\zeta = 1$) the case of a flat layer (the dependence $\phi(\kappa^2)$ for $\zeta = 1$ is identical to that in Figure 22). Hence, it follows that in order to obtain results equivalent in accuracy for larger values of ζ (for thin-walled samples), it is necessary to utilize larger values of κ^2.

Actually, $\frac{\delta\varkappa^2}{\varkappa^2} = \frac{1}{\delta\varphi/\delta\varkappa^2}\cdot\frac{\delta\varphi}{\varkappa^2}$ For this reason, for a constant magnitude $\delta\phi$, in order to obtain a given magnitude of the relative error $\delta\kappa^2/\kappa^2$ it is necessary to have equality of the product $\partial\phi/\partial\kappa^2, \kappa^2$. It is seen from Figure 36 that $\partial\phi/\partial\kappa^2$ for $\zeta = 0$ is approximately three times larger than for $\zeta = 1$. The magnitude κ^2 for a solid cylinder can for this reason be selected three times smaller than for a flat layer. However, a decrease in κ^2 is convenient, first of all, in that the amplitude of the variations of the temperature ($\theta_0 \sim \frac{1}{\varkappa^2}$ increases in this case and the ratio $|\theta|/\theta_0$ also increases somewhat with decreasing κ^2, as this is seen from Figure 37).

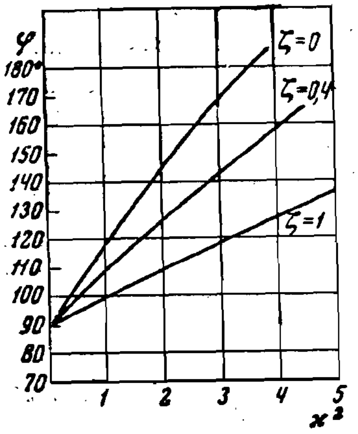


Figure 36. Dependence of the phase of the temperature variations (with respect to the phase of the power variations) on the parameter κ^2 for the case of radial temperature waves.

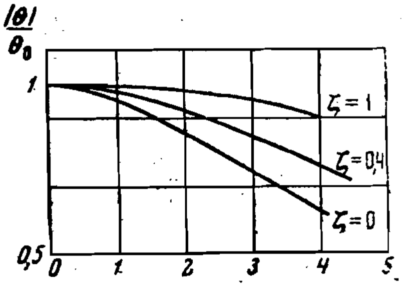


Figure 37. Dependence of the amplitude of temperature variations on the parameter κ^2 for radial temperature waves.

The accomplishment of the examined variant of the phase method for determining the temperature diffusivity requires the capacity for determining the phase of the power variation, which is related with the need of utilizing an inertia-free method of heating which is exemplified by heating with the aid of electron bombardment. The use of electron heating solves at the same time also another problem that of determining the power of the variable heating component, which makes it possible as already known from §8, to carry out complex measurements of the thermal properties. In connection with this, we postpone to § 14 the description of the experimental performance of the phase variant of one point as part of a study of the complex of the properties, and we pass on to an examination of other methods for measuring temperature diffusivity--the phase and amplitude variants of the method of two points (the determination of the temperature diffusivity from data on the temperature variations in two points of the sample.

Let us return to the general formula (13.11). With consideration of the expression (13.12), it can be written in the form of

$$F(z)=\frac{\pi}{2} iz_b\,[H_1^{(1)}(z_b)\,I_0(z)-H_0^{(1)}(z)\,I_1(z_b)]\,F(z_b). \qquad (13.24)$$

From this expression there follow directly the formulas for the ratio of the amplitudes and the difference in phases of the temperature variations in the points that correspond to the variables z_1 and z_b, that is, in the body of the test sample (including as a maximum case the heated surface) and on the surface that is opposite to the heated surface:

$$\frac{|\theta_1|}{|\theta_b|} = \frac{\pi}{2}\varkappa_b\sqrt{\Phi_3^2+\Phi_4^2}, \tag{13.25}$$

$$\Delta\varphi = \operatorname{arctg}\frac{\Phi_3}{\Phi_4}, \tag{13.26}$$

where

$$\begin{aligned}\Phi_3 = {} & \mathrm{ber}(\varkappa_1)\,\mathrm{her}'(\varkappa_b) + \mathrm{bei}'(\varkappa_b)\,\mathrm{hei}(\varkappa_1) - \\ & - \mathrm{bei}(\varkappa_1)\,\mathrm{hei}'(\varkappa_b) - \mathrm{her}(\varkappa_1)\,\mathrm{ber}'(\varkappa_b),\end{aligned} \tag{13.27}$$

$$\begin{aligned}\Phi_4 = {} & \mathrm{hei}(\varkappa_1)\,\mathrm{ber}'(\varkappa_b) + \mathrm{bei}'(\varkappa_b)\,\mathrm{her}(\varkappa_1) - \\ & - \mathrm{ber}(\varkappa_1)\,\mathrm{hei}'(\varkappa_b) - \mathrm{bei}(\varkappa_1)\,\mathrm{her}'(\varkappa_b),\end{aligned} \tag{13.28}$$

$$\varkappa_1 = \sqrt{\frac{\omega}{a}}\,r_1, \quad \varkappa_b = \sqrt{\frac{\omega}{a}}\,R_b. \tag{13.29}$$

Let us examine the most important cases; internal heating of a hollow cylinder and external heating of a solid cylinder.

In the first case, we shall utilize formulas (13.25)-(13.29), assuming that $R_b = R_2$, in the second case--simplified formulas obtained from previous indeterminacies by expanding for $z_b \to 0$:

$$\frac{|\theta_1|}{|\theta_0'|} = \sqrt{\mathrm{ber}^2(\varkappa_1) + \mathrm{bei}^3(\varkappa_1)}, \tag{13.30}$$

$$\Delta\varphi = \operatorname{arctg}\frac{\mathrm{bei}(\varkappa_1)}{\mathrm{ber}(\varkappa_1)}. \tag{13.31}$$

(These same formulas can be obtained also directly from the solution of (13.7), if it is taken into account that for a solid cylinder $C_2 = 0$.) The dependence of $\Delta\varphi$ and $\frac{|\theta_1|}{|\theta_0|}$ of on

$$\varkappa^2 = \frac{\omega}{a}\left(\frac{R_2^2 - r_1^2}{2R_2}\right)^2$$

for $\zeta = r_1/R_2 = 1$ and 0.4 are shown in Figures 38 and 39 (for κ the same magnitude (13.22) was taken as above). The curves for outside heating of a solid cylinder are dotted, those for inside heating of the hollow cylinder are solid.

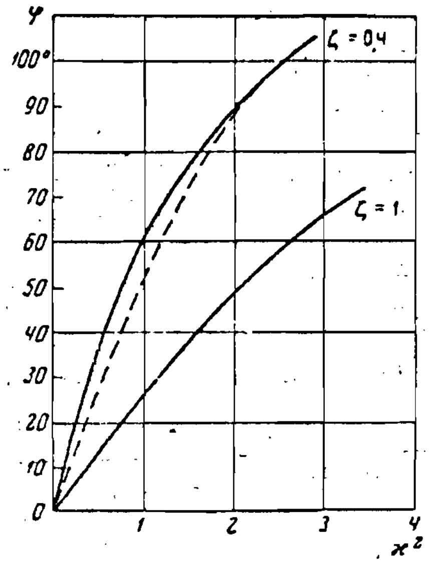


Figure 38. The dependence of difference in phases between temperature variations in two points of a cylinder on the parameter κ^2. The solid curves indicate internal heating of the hollow cylinder, the dotted curves indicates external heating of a solid cylinder.

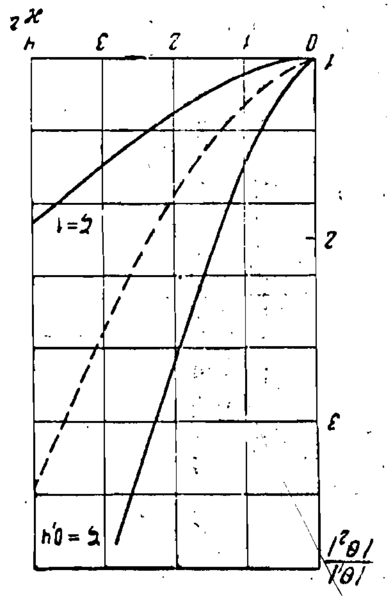


Figure 39. The same for the ratio of the amplitudes of the variations.

It can be seen that the dependence of $\Delta\phi$ and $|\Theta_1|/|\Theta_2|$ on κ^2 are quite steep and it follows from this that the amplitude as well as the phase methods for determining the temperature diffusivity could be sufficiently effective in external as well as in internal heating.

Let us examine the problem of the optimum values of the parameter κ. From the phase curves themselves, one can make only a single conclusion: for the same absolute error $\delta\phi$, the accuracy of the determination of κ^2 will be greater, the greater the magnitude ϕ, that is, the greater κ^2. On the other hand, however, an increase in κ above a certain limit can lead to a decrease in accuracy due to an increase in the error in the determination of the phase with a decrease in the amplitude. Specific evaluations of the upper boundary of κ require a knowledge of the specific relationship of the error in determining the phase and the magnitude of the amplitude. This relationship can be established only practically on the basis of an analysis of the operation of a quite fully definite recording system. As regards the lower boundary of the magnitude of κ, one can find directly from Figure 38 the values of κ^2, which ensure the given error $\frac{\delta a}{a} = \frac{\delta \varkappa^2}{\varkappa^2}$ when $\delta\phi = 1^\circ$. The corresponding magnitudes are shown in Table 23.

Table 23. Lower Boundary of the Values of the Parameter κ^2, for which the Error in Determining the Temperature Diffusivity, Determined by the Inaccuracy in the Measurement in the Difference in Phases of 1°, Amounts to the Given Magnitude.

$\frac{\delta\varkappa^2}{\varkappa^2}$, %	Outside Heating	Internal Heating	
		$\zeta=0.4$	$\zeta=1$
2	1	3	4
3	0,7	1	1,5
4	0,4	0,3	1

The error in determining κ^2 in the amplitude variant in accordance with the formula $\frac{\delta\varkappa^2}{\varkappa^2}=\frac{1}{\varkappa^2\frac{\partial\left|\frac{\theta_1}{\theta_2}\right|}{\partial\varkappa^2}}\,\delta\frac{|\theta_1|}{|\theta_2|}$, depends on two factors: the multiplier $\varkappa^2\frac{\partial\left|\frac{\theta_1}{\theta_2}\right|}{\partial\varkappa^2}$, which decreases with rising κ^2, and the magnitude

$$\delta\frac{|\theta_1|}{|\theta_2|}=\frac{|\theta_1|}{|\theta_2|}\left(\frac{\delta|\theta_1|}{|\theta_1|}+\frac{\delta|\theta_2|}{|\theta_2|}\right),$$

which, on the other hand, increases with rising κ^2. For a detailed study of the totality of the effect of these factors, it is necessary to know the dependence of the absolute errors $\delta|\theta_1|$ and $\delta|\theta_1|$ on the magnitudes of the amplitudes $|\theta_1|$ and $|\theta_2|$ and, besides that, the absolute values of the amplitude $|\theta_1|$. The magnitude $|\theta_1|$ depends not only on κ^2, but also on the other thermal characteristic c_{pp} (or λ) while the relationship of $\delta|\theta_1|$ with $/|\theta|$ depends on the specific characteristics of the system being registered. All this together makes difficult a *a priori* analysis of the error in determining the temperature diffusivity as a function of κ^2. Certain conclusions with respect to the region of optimum values of κ^2 can, however, be made in the following manner. Let us assume that absolute errors in determining the amplitudes for $\delta|\theta_1|$ and $\delta|\theta_2|$ are the same: $(\delta|\theta_1|=\delta|\theta_2|=\delta|\theta|)$. In this case also the relative error in determining $\varkappa^2\left(\frac{\delta\varkappa^2}{\varkappa^2}\right)$ is related with the relative error $|\theta|\left(\frac{\delta|\theta_2|}{|\theta_2|}\right)$ by the factor

$$\frac{\frac{\partial\varkappa^2}{\varkappa^2}}{\frac{\partial|\theta_2|}{|\theta_2|}}=\frac{1+\frac{|\theta_1|}{|\theta_2|}}{\varkappa^2\frac{\partial\frac{|\theta_1|}{|\theta_2|}}{\partial\varkappa^2}} \tag{13.32}$$

The dependence of this factor on κ^2, although it does not reflect the complete dependence of the experimental error on κ^2 because of the above-said, nevertheless it is a sufficiently convenient characteristic of the sensitivity [155, 156, 33]. The corresponding curves are shown in Figure 40.[1] The solid curves represent internal heating of a hollow cylinder while the dotted curve--external heating.

From the shape of the curves in this figure, it can be concluded that an increase in κ^2 above the value of 2-4 does not lead to a knowledgeable decrease in the error of determining $\delta\kappa^2/\kappa^2$, but will more likely increase this error if it is taken into account that $\delta|\theta_2|/|\theta_2|$ increases in this instance. Thus, the values of $\kappa^2 \sim 2 \div 4$ can be considered optimum for the amplitude variant of the method. As can be seen from Table 23, these same values of κ^2 are sufficient for a relatively accurate determination of the temperature diffusivity by the phase method.

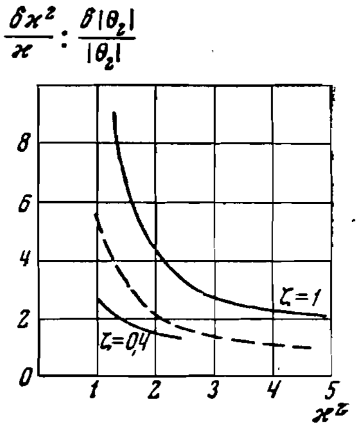


Figure 40. Curves which characterize the error in determining the temperature diffusivity in the amplitude variant of the method. Solid curves represent internal heating of hollow cylinder; dotted curve--external heating.

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[1] In order to avoid misconceptions, we wish to observe that in [155 and 156] similar curves are given but not for

$$\frac{\partial\varkappa^2}{\varkappa^2} : \frac{\delta|\theta_2|}{|\theta_2|}, \text{ a when } \frac{\partial\varkappa^2}{\varkappa^2} : \frac{\delta|\theta_1|}{|\theta_1|},$$

as a result of which the corresponding curves have a minimum.

Hence, it follows that the determination of the temperature diffusivity by the amplitude of phase method is possible from the results of the same experiment; registration of the curves of the temperature variation in two points makes it possible to obtain values of the temperature diffusivity by two methods that are independent of each other. This makes possible internal control of the results: data obtained from information regarding the phases of the variations can be controlled by the results of determining the amplitude of the same curves.

The phase variant of the method of two points was used for the first time by Van Zee and Babcock [157] for determining the temperature diffusivity of molten glasses within the temperature range of 700-1,400°C. The amplitude variant of the method (with external heating) was developed by Yu. A. Kirichenko [155, 156, 158, 159] specifically for measurement of the temperature diffusivity of non-metallic materials (plastics). The utilization of the amplitude and phase methods with external and internal heating for solid and liquid metals has been accomplished in [31, 160, 161]. Most of the experience gained in these studies is presented below.

A schematic representation of the main part of the unit (a distribution of the specimen, heater and thermocouples) is shown in Figure 41.

Figure 41a corresponds to a case of external heating for a solid specimen; Figure 41b corresponds to external heating for liquid metals; Figure 41c corresponds to internal heating for liquid metals.

The heater which is used for external heating is coiled of nichrome, of wolfram (wire) on top of a thin-walled tube made of molybdenum sheet, 25 mm in diameter and 200 mm long. In order to insulate the wires from the body, six thin ceramic straws were used which were located symmetrically. At the top and bottom, the ceramic straws were secured by molybdenum rings. The heater which is used in internal heating consists of a ceramic tube on top of which is wound a wire spiral.

In case of external heating, the specimen of solid metal is made in the form of two cylinders each ~ 20 mm in diameter and ~ 50 mm in length. The wires of the thermocouple (chromel and alumel 0.2 mm) are calked or welded with an electric spark from a charge condenser to the end of one of the cylinders on its axis and near the periphery. In order to insulate the wires, use is made of 2-channel porcelain straw 1 mm in diameter; the wires in the straw pass through vertical holes in the second half of the cylinder.

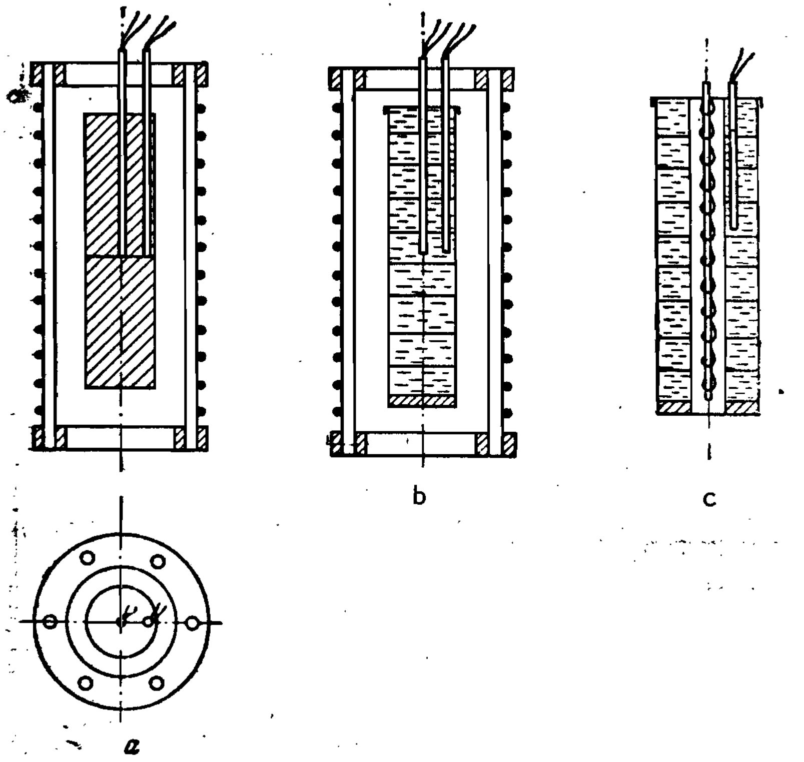

Figure 41. Schematic drawing of the main part of the unit for measuring the temperature diffusivity.

The crucible with the liquid metal, shown in Figure 41b, is made of a thin-wall ($a \sim 0.1$ mm) tantalum tube; the bottom of the crucible is made of molybdenum. In the crucible is placed a system of horizontal partitions made of tantalum or molybdenum (sheet) ~ 0.1 mm thick, located at a distance of 10-15 mm from one another. The partitions are secured by two wires made of tantalum, located along the sides (these are not shown in Fig. 41b). The partitions are intended for increasing the possibility of the development of convector circulation of the liquid metal. Besides this, the partitions play also another role--they fix the position of the thermocouples. The thermocouples, just as in the case of the solid metal, are housed in thin (1 mm) ceramic straws. In order to prevent penetration of the liquid metal into the channels of the straws, the ends of the straws, (but not the junction of the thermocouples) is coated with a ceramic paste. The crucible is closed on top with a cover made of tantalum (or molybdenum sheet). The crucible is filled with the metal in the following manner. A test metal is first melted in a vacuum so that after it solidifies, a result would be a cylindrical specimen with a diameter somewhat larger than the diameter of the crucible. This cylinder is used to mill washers which are installed together with the partitions and thermocouples as an integral unit. There is also possible another method: the test metal in the form of granules is poured into the crucible as the shell with the partitions and the thermocouples is lowered into it. After this, a funnel with the same granulated metal is inserted into the upper part of the crucible and the entire system is placed in a vacuum furnace. Prior to the start of the experiment, the crucible with the molten metal, regardless of the method of filling, was held for a definite time in a vacuum. Control cross-sections of the crucible with the solidified metal have convinced us that there are no noticeable voids, the metal filled the entire space between the partitions.

The crucible for the experiments with internal heating, shown in Figure 41c, is made of two thin-walled (0.1 mm) tantalum tubes (in the experiments, tubes 8 and 23.8 mm in diameter were used). The bottom of the crucible is a molybdenum washer while a cover is provided for the upper section. A system of horizontal washers, just as in the case of external heating, is placed in the crucible. One thermocouple is placed in the crucible near the inside wall (its position is fixed by the washers), while the second thermocouple is welded directly to the outside wall of the crucible.

The installation of the working section of the unit (for the variant with external heating) is shown in Figure 42. The test specimen 1 is secured on three ceramic rods within the heater 2 which creates temperature variations. The second heater 3, which is coaxial with the first heater, serves for varying the average temperature. This entire system is mounted on the support 4 which is within the aluminum oxide washer.5. Bolts along the periphery of this washer feed voltage to the heaters. Screens 6 made of molybdenum sheet are mounted thereon. The vacuum cylindrical cap with a diameter of 120 mm and a length of 650 mm with twin walls is held by means of clamp 7 through the rubber lining 8 against the base 9. The base current leads 10 to the heaters, the leads for the thermocouple wires (they pass through the common thick-rubber lining 11), conducts for the tubes of the vacuum meter, and a connection for the inlet of the inert gas. The vacuum in the chamber ($\sim 10^{-5}$mm mercury) is created by the forevacuum pump RVN-280 and the diffusion pump TsVL-100. The electrical circuit of the unit is shown in Figure 43. Periodic heating of the specimen is accomplished by automatic connection and disconnection of the current through the heater by means of a device which consists of a magnetic starter 1 which is controlled by contacts that close and open with the rotation of the lever 2. The operation of such a device is described in §11. The interval of the variation period used in the experiment is 3-30 seconds, which corresponds to the optimum conditions κ^2 ~ 2 :4) for the interval of values in the temperature diffusivity ranging from 0.03 to 0.3 cm/ 2sec (for specimens with R ~ 1cm).

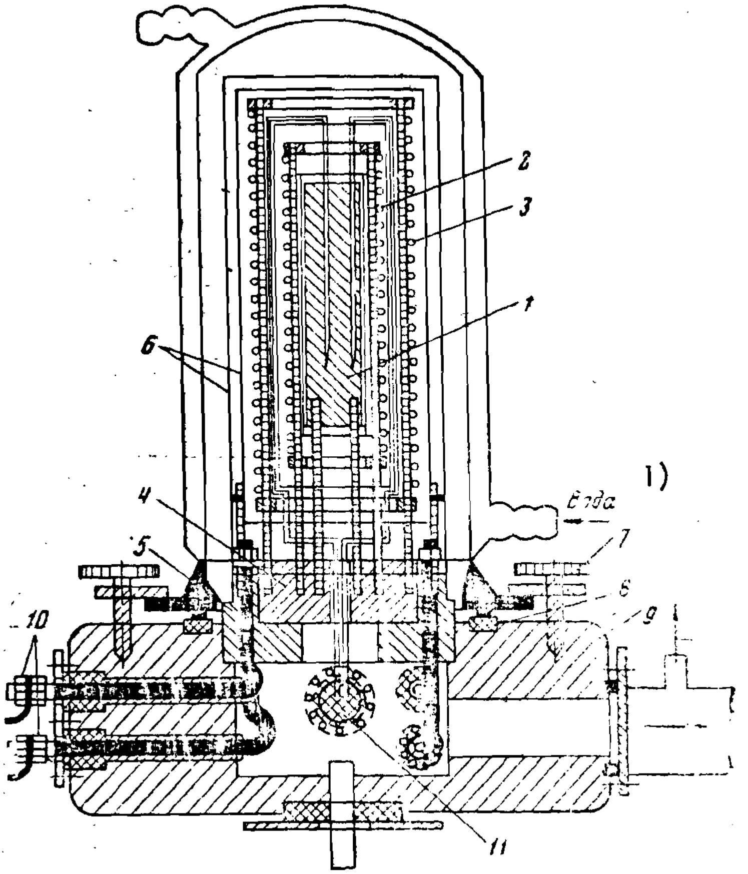

Figure 42. Installation of the working part of the unit.
1, water.

The thermoelectromotive force is fed through the thermocouples through the nonthermal precision switch 3 to the low-ohm potentiometer 4 PMS-49 by means of which the constant component of the thermoelectromotive force, which corresponds to the average temperature of the experiment, is compensated and measured. The variable component, the magnitude of which usually lies within the limits of 20 to 100 microvolts (which corresponds to temperature variations of 0.5 to 2.5°) is fed to the inlet of the single-tube photoelectric amplifier 5 which is assembled in accordance with the scheme described in [20]. The signal from the outlet of the amplifier is fed to the automatic recording potentiometer 6 EPP-09. By means of an additional relay, records are made on the potentiometer tape of the moments of connection and disconnection of the voltage to the heater, which are intended for calculating the phases. An example of the recording of the temperature variations is shown in Figure 44.

It is pertinent that a form of the temperature variations come even for the thermocouple that is located near the heated surface, is very close to a sinusoidal curve. This circumstance is additionally illustrated in Table 24 which lists data of the amplitude of the first three harmonics for the external thermocouples with external heating for different periods (in arbitrary units, with different amplification); the calculation was carried out by the method of twelve ordinates. The influence of the third harmonic was greater than that of the second, which can be understood because the П-shaped curve of the change in the power of the heater, with equality of the half-periods of connection and disconnection, does not contain a second harmonic. The relative smallness of the amplitudes of the harmonics simplifies the processing of the results, making it possible to utilize simpler approximation matters of harmonic analysis (for example, the method of four ordinates) or (for central thermocouples with small periods) dispense with these in general.

The stability of the registration of the temperature variations in the described experiment is not worse than in measurements with longitudinal temperature waves (§11). The error in determining the relative values of the amplitude of the temperature pulsation with consideration of the error of harmonic analysis, amounts to 1% from the difference in phases is determined with an error not exceeding 1-2° (see [31, 33]).

The experiment is conducted as follows. After the installation of the specimen, the chamber is closed and evacuated; in operating with liquid metals, the evacuation is conducted with the heaters connected. After evacuation, the chamber is filled with the inert gas helium up to a pressure of 100-200 mm mercury. The utilization of helium makes it possible to improve considerably the heat exchange between the heater and the specimen and thereby obtain a sufficiently large amplitude of temperature variations. Simultaneously with the evacuation and the filling, variable heating is started. The establishment of periodic conditions is controlled by means of a mirrorgalvanometer. The regular thermal conditions start usually after 15-20 minutes. (Theoretical evaluations of the time required for the establishment of regular conditions are given in [156].) With the establishment of regular conditions, the average temperature of the specimen is measured, after which the recording device is connected. For control, the measurements are usually repeated for another value of the period of the variations, which differs twofold. The results of the measurements are processed with curves of the type shown in Figure 39, which have been plotted for the utilized value of the relationship ($\zeta = R_2/R_1$).

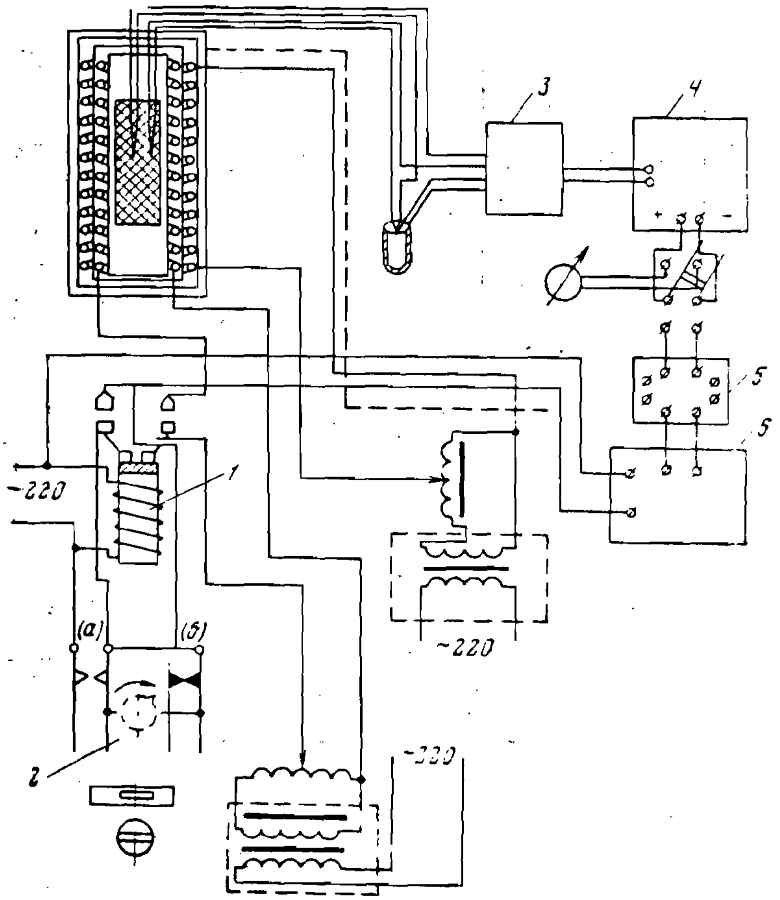

Figure 43. Electrical circuit of the unit.

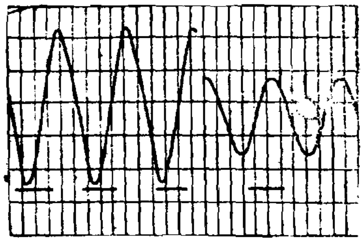

Figure 44. Example of the recording of temperature variations.

Let us examine the problem of the sources of errors in the experiment. One of the most significant errors is related with the inaccuracy in determining the magnitudes R_1 and R_2. By utilizing external heating, the inaccuracy in the installation of the central thermocouple (deviation from the axis) plays a secondary role because $\partial\theta/\partial R = 0$ when $R \sim 0$, in view of the symmetry of the problem, and the error due to the existence of $\delta R \neq 0$ is determined thus only by the square of the magnitude δR. As regards the error in determining the position of the second, external thermocouple, it leads to an error in determining the temperature diffusivity equal to

$$\frac{\delta a}{a} = 2\frac{\delta R}{R}. \qquad (13.33)$$

For $\delta R \sim 0{,}2$ mm (the error which is equal to the diameter of the thermocouple wires) and $R \sim 10$ mm, this gives $\frac{\delta a}{a} \approx 4\,\%$,

In case of internal heating, the position of the external thermocouple can be considered as determined with a greater accuracy if the wires of the thermocouple are welded directly to the external surface of the specimen. The magnitude δR_2 in this case will be equal to the indeterminancy in the measurement of the external radius; can be taken as sequel to 0.05 mm, with consideration of the possible deformation of crucible, which gives an error of $\frac{\delta a}{a} \sim$ [illegible] As regards the distance R_1, the error in δR_1 shows up on the inaccuracy in determining the parameter ζ: $\frac{\delta R_1}{R_1} = \frac{\delta\zeta}{\zeta}$ and on the inaccuracy in determining l. It follows from an analysis of the curves in Figures 38 and 39 that in the region of optimum values of κ^2, a change in $\delta\zeta/\zeta$, equal to 1%, leads to an error in the determination of κ^2, equal to 0.3% when $\zeta \sim 0,4$ for the phase variant and 0.15% for the amplitude variant. It it is taken into account that $\frac{\delta\zeta}{\zeta} \simeq \frac{0.2\ мм}{3 \div 4\ мм} \simeq 5 \div 7\%$, then for the values of ζ the error in $\delta\kappa^2/\kappa^2$ due to the factor ζ amounts to 0.8-2%.

A second source of error is related with the correction for the heat exchange. For the case of external heating of a solid cylinder, the problem of the nature of the heat exchange on the external surface does not play any role because the magnitude q--the variable power on the external surface--does not figure in the formulas for the difference of the phases and the ratio of the amplitude. This is an important characteristic of this method.

For the case of internal heating, the nature of the heat exchange on the internal surface of the specimen, on the basis of the same considerations, is incidental. As regards the heat exchange on the external side, its role should be taken into consideration.

Table 24. Harmonic Composition of Oscillations recorded in temperature

τ, sec	A_1	A_2	A_3
26,4	85,2	1,0	5,6
13,2	86,8	0,3	1,6
6,6	79,4	0,7	1,5

In deriving the formulas for the function F, it was assumed as a first approximation to the heat exchange on the external surface is absent (F' = 0). In order to take this into account, it was necessary to examine the following approximation, assuming that

$$\frac{\theta}{\theta_0} = F + Bif \tag{13.34}$$

(see §8). Function f should satisfy the same equation as F:

$$\nabla^2 f - i\varkappa^2 f = 0 \tag{13.35}$$

and the condition on the external surface

$$\frac{\partial f}{\partial \frac{r}{l}} = -F, \tag{13.36}$$

if Bi in the expression (13.34) contains a heat exchange coefficient on this external surface. As regards the boundary condition on the external surface, for this problem it can be selected entirely at random, inasmuch as the total magnitude of the heat flow on this surface will be excluded in an examination of the ratio of the amplitudes and the difference of the phases. In view of this, we shall utilize the simplest condition

$$f = 0 \tag{13.37}$$

for $r = r_1$.

The solution of the equation (13.35) with the conditions (13.36) and (13.37) leads to the function

$$f(z) = F(z_2)\frac{I_0(z) H_0^{(1)}(z_1) - I_0(z_1) H_0^{(1)}(z)}{I_1(z_2) H_0(z_1) - I_0(z_1) H_1(z_2)}. \tag{13.38}$$

For corrections to the magnitudes of the ratio of the amplitudes and the difference of the phases, we get

$$\frac{\delta \frac{|\theta_1|}{|\theta_2|}}{\frac{|\theta_1|}{|\theta_2|}} = Bi \operatorname{Re}\frac{f(z_2)}{F(z_2)} = Bi \frac{1}{\varkappa} \frac{\Phi_5 \Phi_3 - \Phi_4 \Phi_6}{\Phi_3^2 + \Phi_4^2} = -Bi\, \psi_1, \tag{13.39}$$

$$\delta(\Delta\varphi) = Bi \operatorname{Im}\frac{f(z_2)}{F(z_2)} = Bi \frac{1}{\varkappa} \frac{\Phi_3 \Phi_6 + \Phi_4 \Phi_5}{\Phi_3^2 + \Phi_4^2} = Bi\, \psi_2, \tag{13.40}$$

where Φ_3 and Φ_4 are determined by the formulas (13.27) and (13.28) for $\kappa_b = \kappa_2$,

$$\begin{aligned} \Phi_5 = {} & \operatorname{ber}(\varkappa_2) \operatorname{her}(\varkappa_1) + \operatorname{bei}(\varkappa_1) \operatorname{hei}(\varkappa_2) - \\ & - \operatorname{bei}(\varkappa_2) \operatorname{hei}(\varkappa_1) - \operatorname{ber}(\varkappa_1) \operatorname{her}(\varkappa_2), \end{aligned} \tag{13.41}$$

$$\begin{aligned} \Phi_6 = {} & \operatorname{ber}(\varkappa_2) \operatorname{hei}(\varkappa_1) + \operatorname{bei}(\varkappa_2) \operatorname{her}(\varkappa_1) - \\ & - \operatorname{ber}(\varkappa_1) \operatorname{hei}(\varkappa_2) - \operatorname{bei}(\varkappa_1) \operatorname{her}(\varkappa_2). \end{aligned} \tag{13.42}$$

The dependence of ψ_1 and ψ_2 on κ^2 for a fixed $\zeta = 0.4$ is shown in Fig. 45. In the region of values of κ^2, which are used in the experiment, the correction factors ψ_1 and ψ_2 are less than unity and the correction for the heat exchange thus does not exceed the magnitude Bi. For emission heat exchange within the temperature range up to 1,500°K, the correction for the heat exchange, even in extreme cases, does not exceed several percent and the error introduced by this in determining the temperature diffusivity is 1%.

The following possible source of error in the experiment--the role of the finiteness of the length of the test specimen--has been analyzed in [155] in which it was shown that in the region of optimum values the magnitude of the ratio of the diameter of the specimen to its length, equal to less than 1/3, insures an error in the measurements of the amplitudes which is less than 0.5%; a similar order of magnitude should be also in the error of the phases. It is necessary to underscore that the favorable nature of such evaluations distinguishes advantageously the methods of the regular conditions of the third type from the steady-state experiments of the same type. (Under conditions of steady radial flow of heat in cylinders, the problem of heat losses on the ends is one of the most significant ones.)

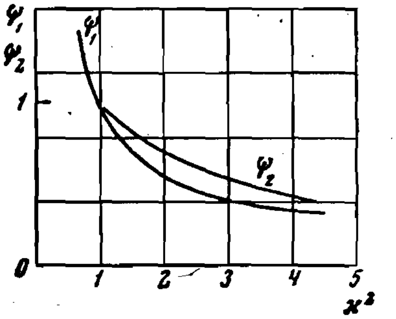

Figure 45. Functions ψ_1 and ψ_2 which determine the dependence of the corrections for the heat exchange in the amplitude and phase variants of the method on the parameter κ^2

In the variant with internal heating, for the case of liquid metals, when the thermocouple is located on the external side, it is necessary to take into account one more factor--the correction for the role of the external wall of the crucible. A detailed examination of this problem, which takes into consideration the distribution of the temperature in the wall and the distortion of the temperature field in the liquid metal, leads to the conclusion that for the case of a thin wall the correction member to the function F should have the form of

$$\Delta F = -\sqrt{i}\,\varkappa \frac{2hR_2}{R_2^2 - R_1^2} F(z_2) \sqrt{\frac{a_{\text{zh}}}{a_{\text{cт}}}} = -\sqrt{i}\sqrt{\frac{w}{a_{\text{cт}}}}\, hF(z_2), \tag{13.43}$$

where h is the thickness of the wall, that is, the correction gives the same change in temperature and phase as for a flat temperature wave in a thin layer.

In order to derive the formula (13.43), the temperature distribution of liquid metal is represented in the form of the sum of the functions $F_0(z) + f_1$ where $F_0(z)$ is the "unperturbed" function F(z).

The temperature distribution in the wall is characterized by the function $F_1(x/h)$ which satisfies the equation $\frac{d^2 F_1}{d\left(\frac{x}{h}\right)^2} = i\varkappa_h^2 F_1$, where $\varkappa_h = \sqrt{\frac{\omega}{a_{\text{ст}}}}\, h$. The utilization for F_1 of the equation for a flat layer assumes that $\frac{h}{R} \ll 1$. f_1 satisfies the equation (13.4): $\nabla^2 f_1 - i\varkappa^2 f_1 = 0$ and the boundary conditions $\lambda \frac{df_1}{dr} = \Lambda \frac{dF_1}{dx}$ and $f_1 = F_1$ for $r = R_2$ $(x = 0)$ and $f_1 = 0$ for $r = R_1$ (the condition on the internal surface can be selected at random for this problem because the temperature on the internal surface is excluded in the method of two points). The boundary conditions F_1 on the external surface is the condition of heat insulation: F' = 0. The solution of the problem under consideration leads to the formula

$$\Delta F(z_2) = F(z_2) - F_1(1) = F(z_2)\Bigg[1 - 2 \times$$

$$\times \frac{\frac{\lambda}{\Lambda} i \frac{\varkappa}{\varkappa_h} e^{\sqrt{i}\,\varkappa_h}}{(1 - e^{2\sqrt{i}\,\varkappa_h}) \frac{H_0^{(1)}(z_1) I_0(z_2) - H_0^{(1)}(z_2) I_0(z_1)}{I_1(z_2) H_0^{(1)}(z_1) - I_0(z_1) H_1^{(1)}(z_1)} + \frac{\lambda}{\Lambda} i \frac{\varkappa}{\varkappa_h} (1 + e^{2\sqrt{i}\,\varkappa_h})}\Bigg].$$

when $\varkappa_h \ll 1$, we get (13.43).

For conditions of the described experiment, the magnitude of the correction to the amplitude and phase lies within the limits of 3-6%;

the error due to the inaccuracy in the introduction of this correction can in the worst case amount to 0.5%.

The last of the secondary factors, the existence of which must be taken into account, is the convective mixing of the liquid metal. The *a priori* evaluation of the role of convection is difficult in view of the lack of a reliable criterial relationship which could be utilized for these purposes. The decisive factor here is the experiment. The most direct way for clarifying the effect of the role of convection is to conduct experiments with different temperature gradients. The difference in the radial gradient can be carried out by changing the power of the variable heating (with preservation of the average temperature through a change in the power of the additional heater) as well as by changing the frequency. These as well as other changes under conditions of the described experiment have always lead to reproducible results that are in agreement with each other [33, 160], which can be regarded as a significant argument in favor of the absence of a noticeable effect of convective mixing. Another argument is the agreement of the results of measurements with external and internal heating (see below). On the other hand, attempts to make measurements under conditions when the partitions which separate the liquid metal were removed did not give satisfactory stable results apparently due to the origin of convective flows. The utilization of partitions can, therefore, be considered necessary, (we wish to observe in connection with this that the partitions themselves do not introduce a perceptible distortion in the temperature field on the surface of the crucible; one can become convinced of this by comparing the readings of several thermocouples located at a different height [33]).

We shall summarize the information regarding the possible experimental errors. In the variant with external heating, the maximum error in determining the temperature diffusivity consists of the error in determining $\delta\kappa^2/\kappa^2$, equal to $(1 \div 1.5)\frac{\delta|\theta_2|}{|\theta_2|} = 1 \div 1{,}5\%$ in the amplitude method and $(1 \div 1{,}5)\,\Delta\varphi = 1 \div 1.5^\circ$ in the phase method (for κ^2 2:4). To this is added the error due to the inaccuracy in determining the distance to the external thermocouple, equal in both cases to ~ 4%.

Thus, the maximum errors in determining a are the same because for the amplitude as well as the phase methods and are equal to 5-5.5%.

In the variant with internal heating, for values of $\zeta = \frac{R_1}{R_2} \sim 0,4$ the error in determining $\delta\kappa^2/\kappa^2$ without taking into account the error in ζ, is somewhat less in the amplitude method than for external heating for the same values of $\varkappa \left(\frac{\delta\varkappa^2}{\varkappa^2} \simeq (0,8 \div 1,5) \frac{\delta|\theta_2|}{|\theta_2|} \simeq 1\ \%\right.$; for the phase method, it is approximately equal to that for the case for external heating $\left(\frac{\delta\varkappa^2}{\varkappa^2} \sim 1 \div 1,5\ \%\right)$. The error due to determination of R_2 amounts to 1% and this variant, and due to the determination of R_1 it amounts to 0.8:2%. To these errors one must add the possible error due to the introduction of the correction for the heat exchange (0:1%) and the corrections for the wall thickness in the case of liquid metal (0.5%) the sum of all these errors is 3.5 + 6% in the amplitude as well as in the phase method.

The results obtained in a refinement of the procedure confirm the above cited evaluations. Values of the temperature diffusivity, obtained in measurements under different conditions (different variants of the procedure, different frequencies, different installation of the thermocouples), agree with each other within the limits of several percent. Details with respect to the conditions of the experiment and the control experiments can be found in [33] as well as in [31, 160 and 161]. We shall cite several illustrations.

Table 25 lists the results of measurements of the temperature diffusivity of liquid tin, obtained in variants with external and internal heating by the phase as well as the amplitude methods.

All these results are in good agreement with each other; the maximum difference from the average value amounts to 2.5%.

Figure 46 shows the results of measurements of the temperature diffusivity of Armco-iron (position C, 0.015%; Si, 0.18%; Mn, 0.17%; S, 0.025%; P, 0.006%), obtained with external heating by the amplitude and phase methods at three different frequencies, each of which is two times greater than the preceding one. All these data are in good agreement with each other and with the results in [146].

The results of a study of the temperature diffusivity of solid and liquid tin, lead, cadmium, and bismuth, obtained by the described method, are given in §24.

Table 25. Results of Measurements of the Temperature Diffusivity of Liquid Tin, Obtained in Variants With External and Internal Heating by the Phase As Well as By the Amplitude Methods.

t°C	Temperature diffusivity cm^2/sec				
	External Heating		Internal Heating		
	with respect to amplitudes	with respect to phases	with respect to amplitudes	with respect to phases	Average Value
420 570	0,167 0,165	0,169 0,169	0,165 0,164	0,170 0,171	0,168 0,167

In concluding this paragraph, we shall compare the variants of the method with external and internal heating. Both methods give approximately the same accuracy. The method of external heating makes it possible to utilize instruments of a more simple configuration, does not require care in making corrections for heat exchange and for the wall of the crucible in working with liquid metals. The method with internal heating requires placing within the specimen (in particular in liquid metal) not two thermocouples, but one thermocouple, which simplifies somewhat the technique of the experiment; it makes it possible for the same average to introduce into the specimen a greater power and thereby obtain greater amplitudes of the temperature pulsation. Finally, the variant of the method with internal heating is more convenient in making measurements of a complex nature; the presentation of this problem is dealt with in the next section.

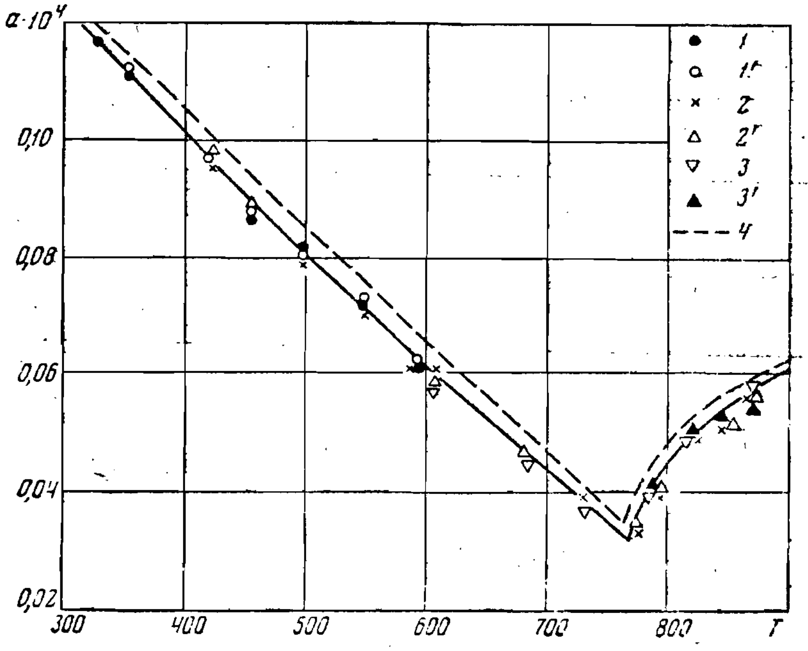

Figure 46. Dependence of $a(T)$ for Armco iron, and m^2/sec: 1, 2, 3, measurements by amplitude for 6.6 seconds of heating; 13.2; 26.4; 1', 2', 3', measurements by phases respectively; 4, from [146].

§14. Utilization of Radial Temperature Waves for Determining the Complex of Thermal Properties of Solid and Liquid Metal

Just as in the case of longitudinal temperature waves, information on the amplitudes and the phases of temperature variations make it possible to determine not only the temperature diffusivity of the test medium but also the heat capacity and the heat conductivity, if the variable component of the power on the surface of the specimen is known in this case. It was already observed in §9 and 12 that a problem of determining the power is effectively solved by utilizing heating by means of electron bombardment. Below is given an analysis of the theory and a description of such an experiment.

Let us return to formulas (13.4)-(13.18) of the previous section. The simplest way for determining the temperature diffusivity by utilizing low-inertia heating, as exemplified by heating by means of electron bombardment, is the measurement of the phase differences between the temperature variation on the side opposite to that being heated (or in the center of the specimen in case of external heating) and the power variation, that is, the phase method of one point. From the curves

$$\varphi = \varphi(\varkappa^3, \zeta)\ \left(\zeta = \frac{R_1}{R_2},\ \varkappa = \sqrt{\frac{\omega}{a} \cdot \frac{R_2^2 - R_1^2}{2r_2}}\right)$$

shown in Figure 36, one can obtain the dependence of the magnitude $\frac{\delta\varkappa^2}{\varkappa^2} / \delta\varphi$, (that is, the errors in determining κ^2 per degree in the determination of the phase) on κ^2. From the shape of this dependence, shown in Figure 47, it follows that an accuracy of ~ 3%, which is quite sufficient for the experiment, can be insured for values of κ^2 ranging from 1.5 to 3 depending on the magnitude of ζ.

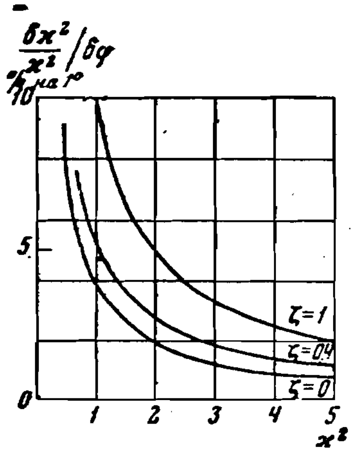

Figure 47. Error in determining κ^2 per degree in the determination of the phase and the phase method of a single point.

We shall clarify further what information can be obtained from data on the amplitude of the temperature variation in the same point.

In the general case, the magnitude $|\theta|$ depends on two thermal parameters: the heat capacity which is contained in Θ_0 and the temperature diffusivity on which the function F depends. This dependence on two parameters for small values of κ degenerates into the trivial relationship

$$|\theta| = \theta_0 = \frac{Q}{Mc_p\,\omega}, \qquad (14.1)$$

because $|F|$ in this case differs

little from 1 (for slow temperature variations, the amplitude of the variation depends only on the heat capacity; the temperature diffusivity, which characterizes the spacial nonuniformities of the nonsteady field of temperatures, is immaterial in this case). It follows from the shape of the curves $F(\kappa^2,\zeta)$ (Figure 37) that the magnitude $|F|$ differs comparatively little from 1 not only for $\kappa << 1$, but also for $\varkappa^2 \sim 1{,}5 \div 4$, that is, approximately for the same values of the parameter κ^2, for which, as already clarified, a sufficiently accurate determination of the temperature diffusivity.from the phases of the temperature variation is possible. It follows from this that such a performance of the complex experiment, in which the heat capacity is determined in a way which is practically independent on the temperature diffusivity (the value of κ^2 is necessary only in order to refine the correction which differentiates $|F|$ from 1) is possible, despite the fact that both magnitudes are found from results of the same single experiment. Such conditions for conducting the experiment are preferred, although they are not necessary. In principle, it is possible to carry out measurements for large values of κ^2, for which $|F|$ differs already significantly from 1. However, in such a case, the results of the determination of the heat capacity will be related with the data on the temperature diffusivity and the error due to this can increase. Besides that, a decrease in $|F|$ in comparison with unity causes a rather steep drop in absolute values of the amplitude of the temperature variations (due to Θ_0, a decrease because of $|\theta|$ is imposed on the change of $|F|$), which leads to an additional increase in the errors. On the basis of this, we shall in this section examine only the case in which the value of κ^2 is selected within the interval that can choose a deviation in $|F|$ from 1 within the limits of the order of 10%. (In §15 we shall examine the additional possibility of determining the temperature diffusivity by the amplitude-frequency method which is based on the change in $|F|$ with a frequency.) For such values of κ^2, the *a priori* error in determining $\left(\frac{\delta\varkappa^2}{\varkappa^2}\right)$ for $\delta\varphi = 1^\circ$, in accordance with Figure 47, amounts to $\sim 2\%$ for $\zeta = 0$ (solid cylinder), 2.5% for $\zeta = 0.4$ and $\zeta = 1$. The difference in these maximum errors is, as we see, not great; however, in this instance it is necessary to turn attention to the fact that in the last case ($\zeta = 1$) the magnitude κ^2, equal to 4, should exceed κ^2 for $\zeta = 0$ twofold, in view of which the magnitude of the amplitude in the temperature variations will be half as great in comparison with the case of $\zeta = 0$.

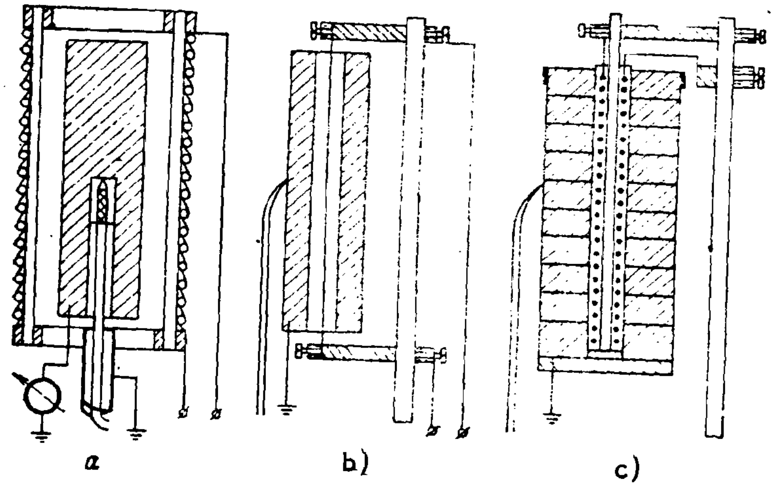

Figure 48. Diagram of the installation of specimens during measurements of the complex of thermal parameters.

And so, the determination of the temperature diffusivity--heat capacity complex is carried out almost in an independent manner from the results of a single experiment; in order to find these magnitudes, it is necessary to know the variable component of the power in the electron bombardment, the magnitude of the amplitude of the temperature variation one point of the specimen, and the difference in the phases between the variation in temperature and power. (In this respect, there is complete analogy with the utilization of temperature in place of the short rods (see §9 -12).

We shall now pass on to a description of the experiment. The installation of the specimens together with the cathode for electron heating is shown in Figure 48.

The diagram a represents α variant with external heating for a solid specimen--solid cylinder. The cathode in this case represents a wolfram spiral wound on six vertical ceramic rods. The thermocouple wires in a thin (1 mm) ceramic straw are introduced into the hole on the axis of the specimen and welded to the metal by means of a spark.

The specimen is grounded through the milliammeter. In operating with high temperatures, it is rather necessary to surround with insulation the thermocouple caps with a shield--metallic tube--for otherwise, in view of the noticeable electrical conductivity of the ceramic material, a portion of the electron current will strike the thermocouple and the tube should not have a direct electrical contact with the specimen, for otherwise, the anode current which strikes the tube will pass through the milliammeter and the tube should be grounded independently (the insulation of the thermocouple caps can apparently be solved in another way--by using a horizontal metal shield near the lower end of the specimen.

Diagram b represents the variant with internal heating for solid metals. The cathode is of wolfram wire drawn here along the axis within the specimen (in order to increase the heating power, it is possible to utilize also a cathode in the form of a spiral-small springs--wound on a ceramic tube just as in the diagram c, or simply drawn). In this variant, the thermocouple is welded directly to the external surface of the specimen, which, of course, is considerably more convenient than installation within a narrow hole in diagram a. Another advantage of diagram b is the effect that there is no need for special shielding of the thermocouple wires; the role of the shields is here played by the specimen itself.

The variant with internal heating for liquid metals is shown in diagram c. The specimen in this case represents a crucible, the cylindrical surfaces of which are made of thin-walled (0.12 mm) tantalum tubes (the diameters of the tubes were taken equal to 24 and 8 mm, respectively). The arrangement of the crucible and the method of filling with metal have been described in §13. The configuration of the cathode is seen directly in the illustration. The thermocouple wires are welded to the external surface of the crucible.

The diagram of the installation as a whole is shown in Figure 49. The working part of the device is installed directly on the flange of the diffusion pump VA-0,5-1, and is closed on the top with a quartz cap that is water cooled. The current and thermocouple wires are tapped through the side flange; four plastic washers are used for sealing.

The source of the cathode heat is the stabilized variable voltage which is varied by an autotransformer; the anode voltage is fed from the stabilized rectifier UIP-1, while a voltmeter M80/1 (accuracy class 0.5) is used for measuring the voltage. The current instrument--milliammeter 23 MA11/2 (accuracy class 0.5)--is connected between the specimen and the ground when diagram a is used (Figure 48) or between the rectifier and the ground when diagram b and c are used (the last connection is safer because when the instrument overburns, the thermocouples in the interexperimental setup will not be under an anode voltage.

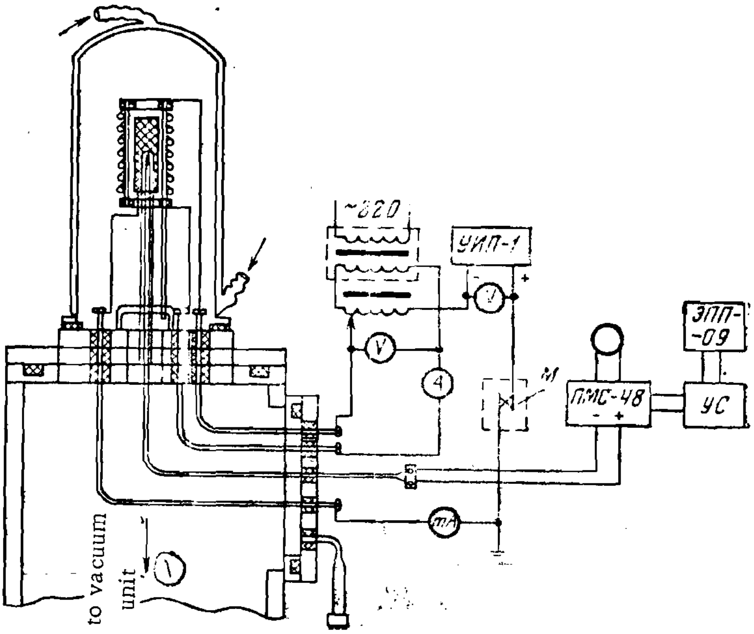

Figure 49. Diagram of the unit.

The modulating device of the unit has been described in §11-13. The recording system does not differ from that described in §13. The magnitude of the absolute values of the temperature variations, is just as in the preceding experiment, lies within the limits of 0.5-2.5°.

For illustration of the degree of stability of the recording of temperature variations, Table 26 lists the results of measurements of the amplitude and phase of five successive periods of change in the temperature (one of a typical series). The scatter of the individual values of the amplitudes in this and a similar series amounts to 0.3%, the absolute error in determining the phases is equal to approximately 0.5°. The maximum error in determining the absolute values with an amplitude of the temperature pulsation, with consideration of the errors in the harmonic analysis, a calibration of the thermocouples, and determination of the value of the potentiometer scale divisions, just as in the experiment described in §12, amounts to 2%.

The determination of the variable component of the power is made just as for the measurements with longitudinal temperature waves. The error of the determination of the magnitude q ~ 1%.

The errors introduced by the inaccuracy in determining the position of the thermocouples are comparatively small in all the variants of the method, which distinguishes advantageously the method used here for measuring the temperature diffusivity from the method of two points described in the preceding section. Assuming that $(\delta R_1 \simeq \delta R_2 \sim 0{,}05$ mm and $R_2 \sim 1$ cm, we get $\frac{\delta \varkappa^2}{\varkappa^2} \sim 1 \div 1{,}5$ %).

Let us examine the problem of corrections which are necessary to introduce into the complex method under consideration and which are related with these errors.

Table 26. Results of Measurements of the Amplitude and Phase of Five Successive Periods of Temperature Variation--Illustration of the Stability of the Recording.

Parameters	Period				
	1	2	3	4	5
Amplitude (in mm on the diagram tape)	185	184,5	185,0	184,0	184,5
Phase	120°	121°	121°	120°	120°

The error introduced into formula (14.1) in determining the correction factor |F| can be found from the formula

$$\delta|F| = \frac{\partial |F|}{\partial \varkappa^2} \varkappa^2 \frac{\delta \varkappa^2}{\varkappa^2}. \qquad (14.2)$$

In the selected range of values of κ^2, the maximum value of the magnitude $\left(\frac{\partial|F|}{\partial\varkappa^2}\varkappa^2\right)$ is equal approximately to 1/4 (Figure 37) and thus,

$$\delta|F| \approx \frac{1}{4}\cdot\frac{\delta\varkappa^2}{\varkappa^2}.$$

The problem of corrections for emission heat exchange should be examined separately for the variants with external and internal heating. In the case of external heating, the correction of function f in the expression

$$\frac{\theta}{\theta_0} = F + Bif \tag{14.3}$$

is found in the simplest manner in accordance with the deductions from the general theory of temperature waves (§8):

$$f = -\frac{F_{r=R}\,F}{\varkappa^2}, \tag{14.4}$$

where $R = R_2$ is the radius of the cylinder (see 8.49). For correction to the amplitude and the phase of the temperature variations, we get the formulas

$$\frac{\Delta|\theta|}{|\theta|} = -\frac{Bi}{\varkappa^2}\,ReF = -\frac{Bi}{\varkappa}\cdot\frac{\mathrm{ber}\,\mathrm{ber}' + \mathrm{bei}\,\mathrm{bei}'}{\mathrm{ber}'^2 + \mathrm{bei}'^2} = -Bi\psi_1, \tag{14.5}$$

$$\Delta\varphi = \frac{Bi}{\varkappa^2}\,\mathrm{Im}F = \frac{Bi}{\varkappa}\cdot\frac{\mathrm{bei}\,\mathrm{ber}' - \mathrm{ber}\,\mathrm{bei}'}{\mathrm{ber}'^2 + \mathrm{bei}'^2} = -Bi\psi_2, \tag{14.6}$$

besides, the functions ber, bei and their derivatives are referenced to the value of the argument equal to 2κ, because $\varkappa = \sqrt{\frac{\omega}{a}} \cdot \frac{R}{2}$ at the same time $\varkappa_R = \sqrt{\frac{\omega}{a}} R$ (see §13). The dependence of ψ_1 and ψ_2 on κ^2 is expressed in Figure 50. The curves $\psi_1(\kappa)^2$ and $\psi_2(\kappa^2)$ are qualitatively similar to such for the case of a flat layer (see Figure 24).

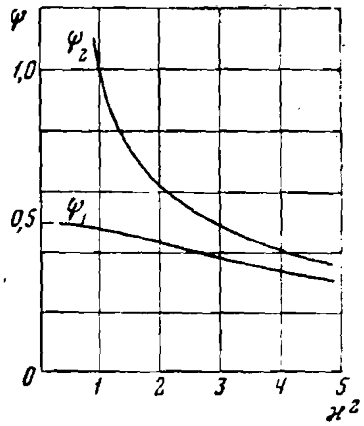

Figure 50. Functions of ψ_1 and ψ_2 which determine the corrections for the heat exchange to the amplitude and phase of temperature variations for the variant with external heating.

To introduce specific corrections for the magnitude of the heat exchange, it is necessary to possess at least approximate values of the effective coefficient of radiant heat exchange, which figures in Bi. In evaluating this magnitude, generally speaking, it is necessary to take into account that the temperature of the cathode can vary somewhat as a function of the temperature of the surface, as a result of which the heat exchange will have a comparatively complex nature.

Let us examine this problem analytically. The equation which describes the emission heat exchange between the cathode and the specimen has the form of

$$q(1 + me^{i\omega t}) = \lambda \frac{\partial T}{\partial r} + \varepsilon\varphi\sigma(T^4 - T_0^4) + \varepsilon'\sigma\varphi'(T^4 - T_k^4), \tag{14.7}$$

where ε and ε' are the effective degrees of blackness of the surface and of the cathode; ϕ and ϕ' are the coefficients of irradiation $(\varphi \sim 1)$; T, T_k and T_0 are the temperatures of the surface of the specimen, the cathode and the walls of the chamber. For small temperature variations, when

$$T = \bar{T} + \theta e^{i\omega t} \tag{14.8}$$

and

$$\frac{|\theta|}{\bar{T}} \ll 1, \tag{14.9}$$

we get the linearized equation

$$q_0 m = \lambda \frac{d\theta}{dr} + 4\sigma(\varepsilon\varphi + \varepsilon'\varphi')\bar{T}^3\theta - 4\sigma\varepsilon'\varphi'\bar{T}_k^3\theta_k, \tag{14.10}$$

where Θ_k is the small amplitude of the cathode temperature pulsation

$$T_k = \bar{T}_k + \theta_k e^{i\omega t}, \tag{14.11}$$

$$\frac{|\theta_k|}{\bar{T}_k} \ll 1. \tag{14.12}$$

Equation (14.10) contains a member with the temperature pulsation of the cathode, the magnitude of which so far is unknown. In order to determine this, it was necessary to employ additional equations--an equation of the heat balance of the cathode. In the nonlinearized form, it has the form of

$$c_p m_k \frac{dT_k}{dt} = \sigma\varepsilon'(T^4 - T_k^4)s + w - \sigma\varepsilon''(T_k^4 - T_0^4)s, \tag{14.13}$$

where c_p and m_k are the heat capacity and the mass of the cathode, s is its surface which faces the side of the specimen or the opposite side (the coefficients of irradiation, which are close to unity, are not taken into account here), w is the Joule heat liberated from heating with the current. In linearization of (14.13), we get

$$\frac{c_p m_k}{s}\vartheta_k i\omega = 4\sigma\varepsilon'(\bar{T}^3\vartheta - \bar{T}_k^3\vartheta_k) - 4\sigma\varepsilon''\bar{T}_k^3\vartheta_k. \tag{14.14}$$

For relatively slow temperature variations, the inertia member $\frac{c_p m_k}{s} i\omega\vartheta_k$ can be disregarded. Then we get

$$\vartheta_k \approx \frac{\vartheta}{2}\cdot\frac{\bar{T}^3}{\bar{T}_k^3} \tag{14.15}$$

(we disregarded also the difference between ε' and ε"). The substitution of (14.15) in (14.10) gives

$$q_0 n = \lambda\frac{d\vartheta}{dr} + 4\sigma\varepsilon\varphi\bar{T}^3\vartheta\left(1 + \frac{1}{2}\cdot\frac{\varepsilon'\varphi'}{\varepsilon\varphi}\right). \tag{14.16}$$

The Biot criterion thus has the form of

$$Bi = \frac{4\sigma\varepsilon\varphi\bar{T}^3\vartheta}{\lambda}l\left(1 + \frac{1}{2}\cdot\frac{\varepsilon'\varphi'}{\varepsilon\varphi}\right). \tag{14.17}$$

The correction member of this expression, which is within the records, and which takes into account the effect of the temperature pulsation of the cathode, is not large. Actually, the magnitude of the irradiation coefficient ϕ' for the cathode, which is a comparativelysparse wire spiral, is much less than unity. Thus, for a spiral with a distance between the turns 5 times greater than the diameter of the wire, $\frac{1}{2}\cdot\frac{\varepsilon\varphi}{\varepsilon'\varphi'}\approx\frac{1}{10}$. The temperature pulsation of the cathode thus changes the magnitude of Bi by approximately 10%.

Formula (14.15) for the temperature pulsation of the cathode is useful also in another respect: it can be used to evaluate the pulsation of the anode current, which is determined by the temperature variation of the cathode. Taking into account the exponential nature of the dependence of the anode current on 1/T, we get

$$\frac{\delta I}{I}=\frac{eu}{kT_k}\cdot\frac{1}{2}\cdot\frac{\theta}{\bar{T}}\left(\frac{\bar{T}}{T_k}\right)^3, \qquad (14.18)$$

where u is the work function of the electron, e is the electron charge. In the case of a cathode made of wolfram, $(\frac{u}{k}\sim 50\,000,\ T_k\sim 2500°\mathrm{K})$ for $T \sim 1{,}000°$K and $\iota \sim 1°$ the magnitude of a relative change in current $\delta I/I$ turns out to be equal to 0.06%, i.e. the current is practically constant. In utilizing thoriated and oxidized cathodes, this magnitude can however be noticeable.

In case of internal heating, the correction for the heat exchange, generally speaking, consists of two parts: corrections for the heat exchange on the heated internal surface and corrections for the heat exchange on the external surface.

However, the first of these corrections does not have to be taken into account for a low-inertia cathode because the total amount of energy fed to the cathode, including the power of the electron bombardment, strikes the specimen in one way or another.

For heat exchange on the external surface of the specimen, we have

$$\frac{\theta}{\theta_0} = F + Bif. \qquad (14.19)$$

For f, in accordance with the general conclusions in §8

$$f = \frac{-\widetilde{F}F(z_2)}{\varkappa^2}, \qquad (14.20)$$

where $\widetilde{F}$ is the function F for the case of external heating.

For corrections to the amplitudes and phases in the point which is located on the external side of the specimen, we get the formulas

$$\frac{\Delta|\theta|}{|\theta|} = -Bi\,\psi_1 = -\frac{Bi}{\varkappa^2}\operatorname{Re}\widetilde{F}(z_2) \qquad (14.21)$$

and

$$\Delta\varphi = -Bi\,\psi_2 = -\frac{Bi}{\varkappa^2}\operatorname{Im}\widetilde{F}(z_2). \qquad (14.22)$$

$$\psi_1 = \frac{\Phi_1\Phi_4' + \Phi_2\Phi_3'}{\Phi_1^2 + \Phi_2^2}\cdot\frac{1}{\varkappa} \qquad (14.23)$$

and

$$\psi_2 = \frac{\Phi_2\Phi_4' - \Phi_1\Phi_3'}{\Phi_1^2 + \Phi_2^2}\cdot\frac{1}{\varkappa}, \qquad (14.24)$$

where the function Φ^1_3 and Φ^1_4 are equal to the negative values of Φ_3 and Φ_4, with the indices 1 and 2, i.e.

$$\Phi_3'(\varkappa, \zeta) = -\Phi_3\left(\varkappa, \frac{1}{\zeta}\right), \quad (14.25)$$

$$\Phi_4'(\varkappa, \zeta) = -\Phi_4\left(\varkappa, \frac{1}{\zeta}\right). \quad (14.26)$$

Where the functions ψ_1 and ψ_2 are shown in Figure 51 (ζ = 0.4).

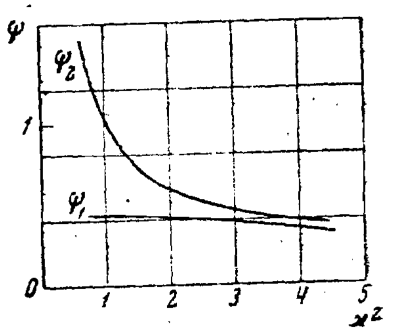

Figure 51. Functions of ψ_1 and ψ_2 which determine the corrections for the heat exchange in the amplitude and phase of the temperature variations for the variants with internal heating.

In introducing corrections for emission, besides the calculated evaluation of the effective values of the magnitudes Bi, their experimental determination is also possible. For this purpose, it was necessary to measure the change in average temperature of the sample ΔT when a constant, non-modulated electronic heating is introduced. In the case when the power of heating is relatively small in comparison with the heating by thermal emission of the cathode, the inequality

$$\frac{\Delta T}{\bar{T}} \ll 1. \quad (14.27)$$

will be valid.

In case of external heating, we get from equation (14.7) in this case

$$4\sigma\bar{T}^3\left(\varepsilon\varphi + \frac{\varepsilon'\varphi'}{2}\right) = \frac{\bar{q}}{\Delta T} \quad (14.28)$$

(with consideration of the change in the temperature of the cathode).

From this,

$$Bi = \frac{\bar{q}l}{\Delta T\lambda}. \qquad (14.29)$$

If it is taken into account further that $\bar{q} = q(\pi/2)$, where q is the first harmonic of the power variation with Π - shaped modulation of the power of the magnitude $\bar{q}$ (see formula 12.17), and the magnitude q is expressed by the amplitude of the temperature pulsation Θ_0, then (14.29) can be written in the form of

$$Bi = \frac{\pi}{2}\cdot\frac{\theta_0}{\Delta T}\varkappa^2 = \frac{\pi}{2}\cdot\frac{|\theta|}{\Delta T}\cdot\left(\frac{\theta_0}{|\theta|}\right)\varkappa^2. \qquad (14.30)$$

Formula (14.30) can be thus directly utilized for determining Bi. We wish to observe in this case that the accuracy of this formula is greater, the better the inequality (14.27) is fulfilled, that is, it is greater, the smaller ΔT or the greater the value of Bi itself. For ΔT/T of the order of several percent and Θ of the order of a degree, Bi amounts also to several percent, that is, good accuracy of the formula (14.30) is assured precisely under those conditions when the correction for the emission heat exchange becomes noticeable.

In returning to the curves in Figures 50 and 51, we wish to observe that the values of the functions of ψ, which determine the corrections for emission, and which within the optimum range of values of κ^2 do not exceed 1, are equal to 0.3-0.5 while the corrections themselves thus amount to a fraction of the Bi numbers. In accordance with this, the magnitude of the corrections in the worst case does not exceed several (~ 3%) percent and the error introduced into the results from the inaccuracy in determining Bi does not obviously exceed 1%. The remaining sources of error due to the finiteness of the length of the cylinder and to the wall of the crucible in experiments with liquid metal have been examined in §13.

The entire presentation with respect to the conditions for conducting the experiment makes it possible to evaluate the errors in the determination of the temperature diffusivity and the heat capacity. The maximum error in determining the difference in phases (with consideration of the correction for the phase of power variations due to the difference of modulation from Π-shaped) amounts to 1°, which gives for a maximum error $\delta a/a$ 2-2.5%. To this error is added the error due to the inaccuracy in determining the radii R_1 and R_2 ($\sim$ 1-1.5%), the maximum error in determining the corrections for emission 0-1% and (for liquid metals) the accuracy from the introduction of the correction for the wall of the crucible (0.5%). The total error thus amounts to 3-4.5% for the case of solid metals and 4-5.5% for liquid metals.

The maximum error in determining the heat capacity consists of the error in determining the amplitude of the temperature pulsation (2%), the errors in determining the power ($\sim$ 1%), and the error of the correction for the temperature diffusivity $\left(\sim \frac{1}{4} \cdot \frac{\delta \varkappa^2}{\varkappa^2} \sim 1\,\%\right)$, which add up to $\sim$4%.

The experimental study of the described procedure for making measurements included a broad program of studies which dealt with the study of the reproducibility of the results with the variation of all the basic parameters of the experiment; the frequency of modulation, the power and conditions (relationship of current and voltage) of the electron bombardment, the geometry of the specimens, the configuration of the cathode, the installation of the thermocouples, etc. The same materials were in this instance studied by means of internal and external variants of the method, which could serve as a means for additional control. We shall cite some of the data obtained as a result of testing the procedure.

Table 27 lists the results of measurements of the thermal properties of Armco iron at 870°C (C, 0.015%, Si, 0.18%, Mn, 0.17%, S,0.025%, P, 0.006%) by means of the variant of external heating. The table makes it possible to compare the results of measurement for different heating times and different powers. Similar data for the variant of internal heating are given in Table 28 (iron, 630°C). Table 29 shows the reproducibility of the results of measurements in working with liquid metals (lead, 635°C). In all cases, the deviations of the individual measurements from the average will not exceed the maximum errors found above.

The developed complex procedure for measurement was utilized in a study of the thermal properties of liquid tin and lead. The results, obtained for the heat capacity and at temperature diffusivity are given in §24.

Table 27. Example of the results of the Simultaneous Measurement of Temperature Diffusivity and Heat Capacity (Variant of External Heating).

Heating time t, sec	Periodically Varying capacity q, wt	Temperature Diffusivity a, cm^2/sec	Heat Capacity c_p, cal/gm·deg
13,2	24,4 51,7	0,0535 0,0528	0,185 0,188
26,4	24,4 51,7	0,0540 0,0537	0,192 0,194
Average value	—	0,0535	0,190
Max. deviation %	—	1,3	2,7

Table 28. Example of Results of Measurements with Internal Heating.

Heating time t, sec	Periodically Varying capacity q, wt	Temperature Diffusivity a, cm^2/sec	Heat Capacity c_p, cal/gm·deg
13,2	24,4 51,7	0,0538 0,0540	0.185 0.187
26,4	24,4 51,7	0,0549 0,0536	0.189 0.191
Average value		0,0540	0,188
Max. deviation %		2,5%	2%

Table 29. Example of the Results of Measurements of the Temperature Diffusivity and Heat Capacity of Liquid Lead (635°C).

Heating Time t, sec	Periodically Varying Capacity q, wt	Temperature Diffusivity a, cm^2/sec	Heat Capacity c_p, cal/gm deg
13,2	12,9	0,107	0.0338
	24,0	0.104	0,0336
26,4	12,9	0,105	0,0352
	24,0	0,109	0.0340
Average Value		0.106	0,0342
Max. Deviation %		3%	2,6%

In conclusion, we wish to dwell on a comparison of the variants of the method with internal and external heating. As we have seen, both variants insure approximately the same accuracy in the determination of the temperature diffusivity--heat capacity complex. The advantage of the variant with external heating is a more simple form of specimen. This advantage, however, is not particularly significant in the case of liquid metals. The fact of the matter is that for a crucible with liquid metals, the introduction of the thermocouple must be made from the top of the crucible and in this case it is comparatively difficult to protect the wires of the thermocouple, which pass by the cathode, against the anode current. The variant of the method with internal heating in this respect is more convenient because the wires of the thermocouple are shielded by the specimen itself. Further, the variant with internal heating makes it possible to use an additional external heater without complications which are related with its bombardment from the cathode as is the case with external heating. Finally, the advantages of the internal heating are rather considerable in carrying out the high-temperature modification of the measurement procedure under consideration, which is related with the utilization of a photoelectric recording of the temperature variations; the recording of the emission of the external surface of the cylinder is simpler than that of the emission of a narrow channel along its axis. (Such a high-temperature modification of the procedure has at present been carried out by the author and by I. P. Mardykin; the recording portion of the unit is similar to that described in §16.)

§15. Radial Temperature Waves and High-Frequency Induction Heating.

The case of periodic heating of the specimen in a high-frequency induction furnace, the field of which is modulated by a low-frequency, pertains, strictly speaking, to the case of volume, internal heating of the specimen. Nevertheless, the presentation of the theory of such temperature vats and of the experimental method based thereon should expediently be carried out at once in accord with the above presented material because the depth of penetration of induction heating is not great and the heat liberation is close to the surface heat liberation. The magnitude of the effective thickness of the skin-layer in which the heating properly takes place is determined by the formula

$$\sigma = \sqrt{\frac{\gamma}{\pi\mu f}}, \tag{15.1}$$

where μ is the magnetic permeability, f is the carrier frequency of the induction furnace, γ is the specific resistance [162]. (The formula is described in the system of NKS units). For frequencies f of the order of megahertz and $\mu \sim 1$, the magnitude σ for metals amounts, as a rule, to a fraction of a mm which is much less in diameter of the specimens which amount to mm. This circumstance makes it possible for us thereafter to consider the parameter

$$\eta = \frac{\sigma}{2R} \tag{15.2}$$

as a magnitude which is considerably less than unity and to utilize the expansion in series with respect to this parameter.

The specific heat liberation in a solid infinitely long cylinder with variable induction heating will be represented in the form of

$$\omega(r, t) = \omega(r)\,[1 + me^{i\omega t}], \tag{15.3}$$

where ω is the frequency of the modulation of the induction heating and m is the modulation coefficient.

The function ω(r) has the form [162] of:

$$\omega(r) = \frac{W}{\sqrt{2}\,\pi R\sigma}\,\frac{\left[\mathrm{ber}'\left(\frac{r\sqrt{2}}{\sigma}\right)\right]^2 + \left[\mathrm{bei}'\left(\frac{r\sqrt{2}}{\sigma}\right)\right]^2}{\mathrm{ber}\left(\frac{R\sqrt{2}}{\sigma}\right)\mathrm{ber}'\left(\frac{R\sqrt{2}}{\sigma}\right) + \mathrm{bei}'\left(\frac{R\sqrt{2}}{\sigma}\right)\mathrm{bei}\left(\frac{R\sqrt{2}}{\sigma}\right)}, \tag{15.4}$$

where

$$W = 2\pi\int_0^R \omega(r)\,r\,dr \tag{15.5}$$

is the complete power of heating a unit length of the specimen.[1]

The distribution of the temperatures in the body of the test cylindrical specimen is described by the equation of the heat conductivity with the volume source of heat

$$\nabla^2 T = \frac{1}{a}\cdot\frac{\partial T}{\partial t} - \frac{\omega(r,t)}{\lambda}. \tag{15.6}$$

For the variable components of the temperature, we get in this case the equation

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[1] In formula (5) of [163] a misprint has crept in: instead of the factor $\frac{1}{\sqrt{2}\,\pi R\sigma}$ $\frac{\gamma}{\sqrt{2}\,\pi R}$. is written.

$$\nabla^2\theta = \frac{i\omega}{a}\theta - \frac{m\omega(r)}{\lambda}. \tag{15.7}$$

As a first approximation, when the effect of the Biot parameter can be disregarded, the boundary condition on the surface of the specimen can be considered the condition

$$\frac{d\theta}{dr} = 0. \tag{15.8}$$

The general solution of the equation (15.7) for an infinitely long cylinder has the form [164] of

$$0 = C_1 I_0(z) + C_2 H_0^{(1)}(z) + H_0^{1} F_1(z) - I_0(z) F_2(z). \tag{15.9}$$

where

$$z = \sqrt{\frac{-i\omega}{a}}\, r; \tag{15.10}$$

I_0 is the Bessel function of the zero order of the first type;

($H_0^{(1)}$) is the first Henkel function of zero order;

$$F_1(z) = \frac{\pi}{2i}\int_0^z z I_0(z) f(z)\, dz, \tag{15.11}$$

$$F_2(z) = \frac{\pi}{2i}\int_0^z z H_0^{(1)}(z) f(z)\, dz, \tag{15.12}$$

$$f(z) = \frac{\omega(r) m}{i\omega c_p \rho}. \tag{15.13}$$

It follows from the condition of finiteness of the magnitude θ on the axis of the cylinder that $C_2 = 0$. In order to determine the constant C_1, we shall utilize the boundary condition (15.8), which gives

$$C_1 = \frac{I_1(z) F_2(z) - H_1^{(1)}(z) F_1(z)}{I_1(z)}, \tag{15.14}$$

$$Z = \sqrt{\frac{-i\omega}{a}}\, R. \tag{15.15}$$

For the temperature pulsation on the surface of the specimen, we get the formula

$$\theta(Z) = \frac{2i}{\pi Z} \cdot \frac{F_1(Z)}{I_1(Z)}, \tag{15.16}$$

or which is the same thing,

$$\theta(Z) = \int_0^Z \frac{z}{Z} \cdot \frac{I_0(z)}{I_1(Z)}\, f(z)\, dz. \tag{15.17}$$

(In the derivation of (15.16), use was made of the identity

$H_0^{(1)} I_1 - H_1^{(1)} I_0 = \frac{2i}{\pi z}$.) .)

In order to obtain the formula for the temperature pulsation on the axis of the cylinder ($z = 0$), it is necessary to develop the indeterminacy of the type 0∞, which originates in the members of

$(H_0^{(1)} F_1$. By utilizing the asymptotic expressions for the Thompson

function when $z \to 0$, we get

$$H_0^{(1)} F_1 \to 0 \tag{15.18}$$

when $z \to 0$.

The formula for $\Theta(0)$, has thus the form of

$$\theta(0) = C_1 = F_2(Z) - \frac{H_1^{(1)}(Z)}{I_1(Z)} F_1(Z), \tag{15.19}$$

or

$$\theta(0) = \frac{\pi}{2i} \int_0^Z z \frac{H_0^{(1)}(z) I_1(Z) - H_1^{(1)}(Z) I_0(z)}{I_1(Z)} f(z)\, dz. \tag{15.20}$$

The formulas (15.17) and (15.20) in principle describe completely the temperature variation on the axis and surface of the test specimen, giving the amplitude as well as the phase of the variations in a function of two variables: $\varkappa = \sqrt{\frac{\omega}{a}} \cdot \frac{R}{2}$ and $\eta = \frac{\sigma}{2R}$. For practical use, however, it is more convenient to rearrange these formulas to another form by conducting integration by parts:

$$\theta(Z) = -\frac{mW}{2\pi\lambda} \cdot \frac{1}{Z} \cdot \frac{I_0(Z)}{I_1(Z)} + \int_0^Z \frac{U(z) I_1(z)}{I_1(Z)} dz, \tag{15.21}$$

$$\theta(0) = -\frac{mW}{2\pi\lambda} \cdot \frac{1}{Z} \cdot \frac{1}{I_1(Z)} +$$

$$+ \frac{\pi}{2i} \int_0^Z U(z) \frac{H_1^{(1)}(z) I_1(Z) - H_1^{(1)}(Z) I_1(z)}{I_1(Z)} dz, \tag{15.22}$$

where

$$U(z) = \int_0^z z f(z)\, dz = -\frac{mW}{2\pi\lambda} \cdot \frac{z}{Z} \times \tag{15.23}$$

$$\frac{\operatorname{ber}\left(\frac{r\sqrt{2}}{\sigma}\right) \operatorname{ber}'\left(\frac{r\sqrt{2}}{\sigma}\right) + \operatorname{bei}\left(\frac{r\sqrt{2}}{\sigma}\right) \operatorname{bei}'\left(\frac{r\sqrt{2}}{\sigma}\right)}{\operatorname{ber}\left(\frac{R\sqrt{2}}{\sigma}\right) \operatorname{ber}'\left(\frac{R\sqrt{2}}{\sigma}\right) + \operatorname{bei}\left(\frac{r\sqrt{2}}{\sigma}\right) \operatorname{bei}'\left(\frac{R\sqrt{2}}{\sigma}\right)}, \tag{15.24}$$

$$\left(U(Z) = -\frac{mW}{2\pi\lambda}\right).$$

The first members of the formulas (15.21) and (15.22) represent the solution of the problem for the case in which the volume heat liberation is absent, (it degenerates into surface heat liberation), and the second integral member describes the addition due to the distribution of the heat sources to the volume. The smaller the relative thickness of the skin-layer, the smaller the role that is played by these members and the closer is the solution for Θ to the ideal case of a purely surface heating.

We shall leave for a while the problem of the quantitative calculation of the extent of the heat liberation and we shall analyze the possibilities of utilizing variable induction heating in order to determine the temperature diffusivity, on the basis of the nature of the functions described by the first members of (15.21) and (15.22).

The formula for the temperature pulsation on the axis of the cylindrical specimen in the approximation under consideration can be written in the form of

$$\frac{\theta(0)}{\theta_0} = \frac{-\varkappa}{\sqrt{-i}\, I_1(Z)}, \qquad (15.25)$$

where

$$\theta_0 = \frac{mQ}{Mc_p\omega}. \qquad (15.26)$$

The solution in the form of (15.25) was already obtained and studied earlier (see formula (13.19)). The solution for the temperature pulsation of the heated surface Θ(Z) in the direct form was not investigated.

The formula for Θ(Z) is written in a form similar to (15.25):

$$\frac{\theta(Z)}{\theta_0} = -\frac{\varkappa}{\sqrt{-i}} \cdot \frac{I_0(Z)}{I_1(Z)}. \qquad (15.27)$$

The expressions for the amplitude and phase (with respect to the power variation) have the form of

$$|\theta| = \theta_0 2\varkappa^2 \sqrt{F_1^2 + F_2^2}, \tag{15.28}$$

$$\mathrm{tg}\,\varphi = -\frac{F_2}{F_1}, \tag{15.29}$$

where

$$F_1 - iF_2 = -\frac{I_0}{ZI_1}, \tag{15.30}$$

$$F_1 = \frac{\mathrm{ber}\,\mathrm{ber}' + \mathrm{bei}\,\mathrm{bei}'}{(\mathrm{ber}'^2 + \mathrm{bei}'^2)\,2\varkappa}, \tag{15.31}$$

$$F_2 = \frac{\mathrm{bei}\,\mathrm{ber}' - \mathrm{ber}\,\mathrm{bei}'}{(\mathrm{ber}'^2 + \mathrm{bei}'^2)\,2\varkappa}. \tag{15.32}$$

The magnitudes ber and bei are in this sense in functions of the variable $\kappa_R = 2\kappa$. For practical calculations, these expressions can conveniently be represented in the form of

$$\frac{|\theta|}{\theta_0} = \varkappa b(2\varkappa), \tag{15.33}$$

$$\varphi = \beta(2\varkappa) + \frac{3}{4}\pi, \tag{15.34}$$

where b and β are the modulus and phase of the function $\frac{I_0(x\sqrt{i})}{I_1(x\sqrt{i})}$, tabulated in the monograph [16]. The form of the functions $\frac{|\theta|}{\theta_0}(\varkappa^2)$ and $(\varphi\,(\varkappa^2))$ is shown in Figure 52 and 53 (solid line).

Let us turn at first to the second of these illustrations. The phase delay lag in the variations of the surface temperature, in comparison with the power variation, is close to $\pi/2$ for small values of κ^2; this is a general characteristic of slow thermal processes, to which attention was already drawn in §8, 9, 12, and 14. For large values of κ^2, the phase displacement approaches asymptotically the maximum value of $\pi/4$, which is characteristic of temperature waves that propagate in semi-infinite medium (see §8 and 12). In the region of intermediate values of κ^2, there is a section with a comparatively steep dependence on κ^2, which can be utilized for determining the temperature diffusivity. In order to evaluate the sensitivity of such a method, Figure 54 shows the curve of the relative error in $\delta\kappa^2$ κ^2 per degree in change of ϕ.

It can be seen that near the value of κ^2 equal to 1, the *a priori* error in determining the temperature diffusivity amounts to ~ .5-6%. This magnitude is greater than the corresponding error in measurements on the axis of the specimen (Figure 47), but nevertheless it is still not so great as to make the corresponding experiment invalid, in particular if it is taken into consideration that a recording of the temperature on the external surface of the specimen is the simplest and most convenient. Thus, along with the already described type of experiment with radial temperature waves, when the temperature variations are studied on surfaces that are opposite to that being heated, the determination of the temperature diffusivity by the phase method is possible even with the recording of the temperature directly on the heated surface. Such a possibility can be practically realized only with induction heating and photoelectric method of recording the temperature variations. (Under conditions of heating by means of electron bombardment, the photoelectric recording of the temperature of the heated surface is difficult because of the presence of an incandescent cathode, while a recording by means of thermocouples, it is more expedient to distribute these on the opposite surface.)

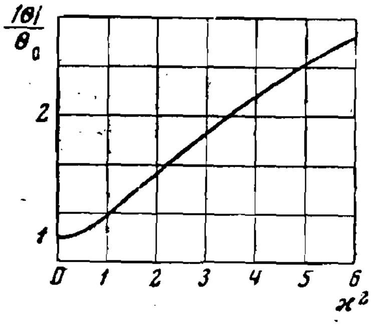

Figure 52. The amplitudes of the temperature variations on the surface of a cylinder with variable heating as a function of κ^2.

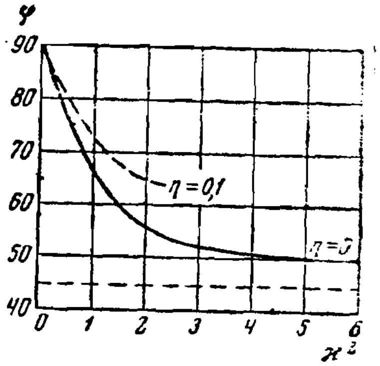

Figure 53. The difference in phases between the temperature variations on the surface of a cylinder and the variation of the heating power as a function of the variable κ^2

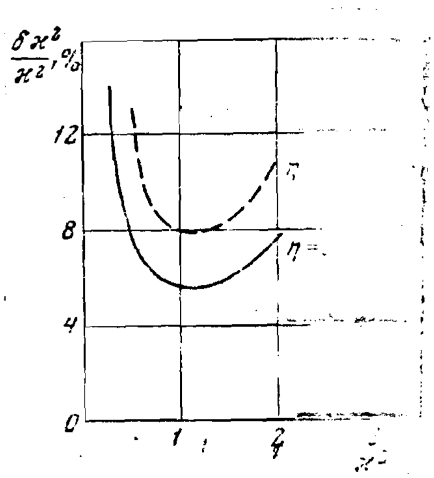

Figure 54. The error in determining the temperature diffusivity in the phase variant of the method with recording of the variations in the surface temperature of the specimen.

Along with the determination of the temperature diffusivity by the phase method, the recording of the temperature on the external heated surface of the specimen makes it possible to realize also another amplitude method for determining temperature diffusivity, namely, that mentioned in §8--the amplitude-frequency method which is based on information of the frequency dependence of the amplitude of the temperature variations with constant power of variable heating. In order to clarify the possibilities of the amplitude-frequency method, let us examine Figure 55 which gives a curve of the ratio of the amplitude of the variations for the given and doubled frequencies as a function of the parameter κ^2 (reference to the greater of the frequencies)--solid line.

The magnitude $|\Theta|_{\omega}/|\Theta|_{2\omega}$ equal to $2\omega/\omega$, for small values of $\varkappa^2\left(|0|\sim 0_0\sim\frac{1}{\omega}\right)$, , for large values of κ^2 approaches $\sqrt{2}$ because under conditions of a considerable attenuation of the temperature wave it penetrates into the body $|\theta| \sim \theta_0 \varkappa \sim \frac{1}{\sqrt{\omega}}$. In the region of values of κ^2 ranging from 1 to 2 there is a section of comparatively steep dependence of $|\Theta|_\omega / |\Theta|_{2\omega}$ on κ^2, which can be utilized for determining the temperature diffusivity. The curve of the dependence of the errors of such a determination on κ^2, when

$$\frac{\delta \frac{|\theta|_\omega}{|\theta|_{2\omega}}}{\frac{|\theta|_\omega}{|\theta|_{2\omega}}} = 1\,\%$$

(as shown in Figure 56).

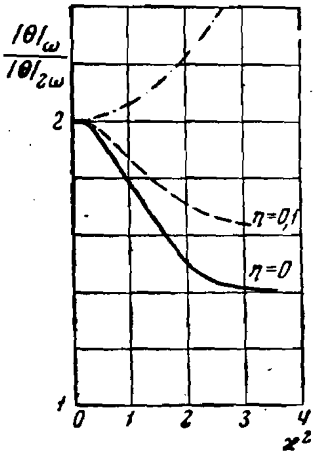

Figure 55. Ratio of the amplitudes of the temperature variations at frequencies which differ twofold as a function of the parameter κ^2.

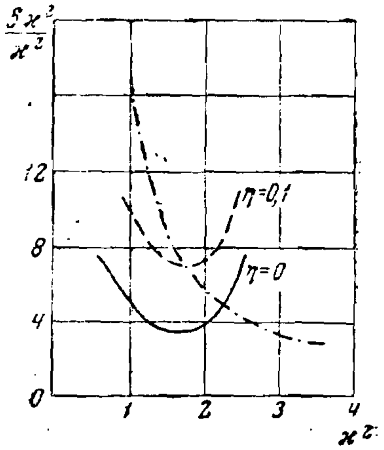

Figure 56. Error in determining the temperature diffusity by the amplitude method (with the ratio of the frequencies equal to two) as a function of the parameter κ^2.

The region of minimum errors (~3.5% lies here near the same values of $\varkappa^2 (\varkappa^2 \simeq 1{,}5)$, for which the errors in the phase method are also minimum. (If κ^2 is referred to the geometric mean of the frequencies of ω and 2ω, these regions coincide in accuracy); the absolute values of the errors are comparatively small. Thus, information on the variations in the surface temperature of the specimen under conditions of variable induction heating permit in principle to find the temperature diffusivity of the test material by two independent ways: by the phases of the temperature variations and the frequency function of the amplitudes. It is of interest to compare the possibilities of the amplitude-phase determination of the temperature diffusivity in the case under consideration when the temperature is recorded on the external heated surface with such during the recording of the temperature on the axis of the specimen. The curve of the function $|\Theta|_{\omega}/|\Theta|_{2\omega}$ in this last case is shown in Figure 55 by the dot and dash curves just as the curve of the errors in Figure 56. It can be seen that in order to obtain the same accuracy of measurements as in the first case, one needs here to utilize the values of κ^2 of the order of 3 and higher. A comparison of the amplitudes of the temperature variations leads to the conclusion that the amplitudes of the temperature variations on the axis, obtained in this case, are four times less than when the variations are recorded on the surface. Thus, the "eggshell" variant of the procedure is in this respect less favorable.

The above-made evaluation of the possibilities of the method pertain so far to an idealized state of a purely surface heating. We shall clarify now how the volume character of heat liberation in induction heating shows up; for this purpose we shall examine the second integral members of the formulas (15.21) and (15.22).

For the rearrangement of the expressions

$$\frac{\Delta\theta(Z)\,2\pi\lambda}{Wm} = \int_0^Z \frac{U(z)\,I_1(z)}{I_1(Z)}\,dz \tag{15.35}$$

and

$$\frac{\Delta\theta(0)\,2\pi\lambda}{Wm} = \frac{\pi}{2}\int_0^Z U(z)\,\frac{H_1^{(1)}(z)\,I_1(Z) - H_1^{(1)}(Z)\,I_1(z)}{I_1(Z)}\,dz \tag{15.36}$$

to a form convenient for calculations, we shall perform a series of successive integrations by prods with consideration of (15.23) and the recurrent relationships for the Thompson functions. As a result, we obtain

$$\frac{\Delta\theta(Z)\,2\pi\lambda}{mW} = \frac{\Delta\theta(Z)}{2\theta_0\varkappa^2} = \frac{\eta}{2}\cdot\frac{\mathrm{ber}^2+\mathrm{bei}^2}{\mathrm{ber}\,\mathrm{ber}'+\mathrm{bei}\,\mathrm{bei}'} - $$
$$- Z\eta^2\,\frac{\mathrm{ber}\,\mathrm{bei}'-\mathrm{bei}\,\mathrm{ber}'}{\mathrm{ber}\,\mathrm{ber}'+\mathrm{bei}\,\mathrm{bei}'}\cdot\frac{I_0}{I_1} - \frac{\eta^3}{\sqrt{2}}\,Z^2\,\frac{\mathrm{bei}'^2+\mathrm{ber}'^2}{\mathrm{ber}\,\mathrm{ber}'+\mathrm{bei}\,\mathrm{bei}'} +$$
$$+\eta^4 Z^2\left(\frac{I_0}{I_1}Z-2\right)+\ldots \qquad (15.37)$$

$$\frac{\Delta\theta(0)\cdot 2\pi\lambda}{mW} = \frac{\Delta\theta(0)}{2\theta_0\varkappa^2} = \frac{\eta}{\sqrt{2}}\;\frac{1}{\mathrm{ber}\,\mathrm{ber}'+\mathrm{bei}\,\mathrm{bei}'} -$$
$$- Z\,\frac{\eta^2}{I_1}\cdot\frac{\mathrm{ber}\,\mathrm{bei}'-\mathrm{bei}\,\mathrm{ber}'}{\mathrm{ber}\,\mathrm{ber}'+\mathrm{bei}\,\mathrm{bei}'} + Z^3\,\frac{\eta^4}{I_1}+\ldots \qquad (15.38)$$

A continuation of such an operation will lead to the appearance of members with higher powers of the small parameters η. The resulting expressions, however, are not an expansion of $\Delta\Theta$ in a power series with respect to η because the Thompson functions in the formulas (15.37) and (15.38) are functions of the argument $1/\eta\sqrt{2}$. The utilization of series for their asymtotic representation [165] makes it possible to rearrange (15.37) and (15.38) into real series with respect to η. After the real and imaginary parts of these series, we obtain expressions for the amplitude and phase of the temperature variations with consideration of the role of the skin-layer:

$$|\theta| = \theta_0\,2\varkappa^2\sqrt{\widetilde{F}_1^2+\widetilde{F}_2^2}, \qquad (15.39)$$

$$\mathrm{tg}\,\varphi = -\frac{\widetilde{F}_2}{\widetilde{F}_1} \qquad (15.40)$$

where for the points on the surface of the specimen

$$\widetilde{F}_1 = F_1+\eta+\eta^2(1+4\varkappa^2F^2)+\eta^3\left(\frac{5}{4}+4\varkappa^2F_2\right)+$$
$$+\eta^4\left(\frac{3}{2}+6\varkappa^2F_2-16\varkappa^2F_1\right)+\ldots \qquad (15.41)$$

$$\widetilde{F}_2 = F_2 - 4\eta^2\varkappa^2 F_1 - 4\eta^3(1 + F_1)\varkappa^2 - 4\varkappa^2\eta^4\left(\frac{3}{2}F_1 + 2 + 4\varkappa^2 F_2\right) + \dots \qquad (15.42)$$

while F_1 and F_2 are given by the formulas (15.31) and (15.32). Similarly, for temperature variations on the axis,

$$\widetilde{F}_1 = F_1^0 + \eta^2 4\varkappa^2 F_2^0 + 4\eta^3\varkappa^2 F_2^0 + \eta^4(6\varkappa^2 F_2^0 - 16\varkappa^4 F_1^0) + \dots \qquad (15.43)$$

$$\widetilde{F}_2 = F_2^0 - 4\eta^2\varkappa^2 F_1^0 - 4\eta^3\varkappa^2 F_1^0 - 4\eta^4\varkappa^2\left(\frac{3}{2}F_1^0 + 4\varkappa^2 F_2\right) + \dots \qquad (15.44)$$

where

$$F_1^0 + iF_2^0 = \frac{1}{Z I_1(Z)}.$$

Specific calculations by means of the resulting formulas make it possible to become convinced that for $\eta < 0.1$ the role of the last term members of these formulas (η^4) is negligibly small (<1%), which makes it possible to disregard the subsequent members of such expansions.

For the practical utilization of the formulas (15.39)-(15.44), it is convenient to represent the effect of η in the form of corrections to the previously determined functions $\phi(\kappa^2)$ and $|\Theta|_\omega/|\Theta|_{2\omega}$, more accurately to their reciprocal functions, that is, to the magnitudes of κ_0^2, determined for $\eta = 0$ from the original values of the angle ϕ or the ratios of the amplitudes $|\Theta|_\omega/|\Theta|_{2\omega}$. Such correction factors, which pertain to the case in which the temperatures of the heated surface are recorded, given in Tables 30 and 31.

It can be seen that the effect of the finite thickness of the skin-layer in the phase method is not so weak that it can be considered a correction when $\eta > 0.03$. Greater values of η lead to a noticeable decrease in the curvature of the function $\phi(\kappa^2)$, which show up on an increase in the error of the experiment. In order to evaluate the role of the relatively large values of η, Figures 53 and 54 show the corresponding curves for η = 0.1.

Table 30. Correction Factor Which Takes into Account the Finite Depth of Heat Liberation. Phase Variant of the Method; Recording of the Temperature Variations on the Surface.

$\varkappa_0^2$	$\varkappa^2/\varkappa_0^2$		
	$\eta=0,01$	$\eta=0,03$	$\eta=0,05$
0,3	1,04	1,12	1,21
0,4	$1,04_5$	1,13	$1,22_5$
0,5	1,05	1,13	$1,23_5$
0,6	1,05	1,14	1,24
0,7	$1,05_5$	1,15	1,25
0,8	$1,05_5$	1,15	$1,25_5$
0,9	$1,05_5$	$1,15_5$	1,26
1,0	$1,05_5$	1,16	1,27
1,1	1,06	1,17	1,29
1,2	1,07	1,18	1,33

Table 31. Correction Factor for the Amplitude Variant (Ratio of Frequencies Equal to Two), Recording of Temperature Variations on the Surface.

$\varkappa_\eta^2$ (for 2ω)	$\varkappa^2/\varkappa_0^2$		
	$\eta=0,01$	$\eta=0,03$	$\eta=0,05$
0,25	1,00	1,02	1,07
0,5	1,00	1,07	1,12
0,75	1,00	1,08	1,15
1,0	1,00	1,09	1,18
1,25	1,00	1,10	1,21
1,5	1,00	$1,11_5$	$1,21_5$

The effect of the magnitude η on the phase and amplitude of the temperature pulsation on the axis of the specimen is small, which is understandable because the point where the temperature is recorded is in this instance removed from the region of heat liberation. The correction factors for this case are shown in Tables 32 and 33.

The introduction of a correction is necessary for values greater 0.01.

In order to evaluate the values of the parameter η in specific conditions, we cite Table 34 which contains the values of η for wolfram at 2,000°C.

The table shows how important is the utilization of relatively high carrier frequencies in induction heating.

Table 32. Correction Factor for Phase Variant with Recording of the Temperature on the Axis.

$\varkappa_0^2$	$\varkappa'/\varkappa_0^2$		
	$\eta = 0,05$	$\eta = 0,07$	$\eta = 0,1$
1,0	1,025	1,05	1,11
1,5	1,025	1,05	1,11
2,0	1,025	1,05	1,11
2,5	1,025	1,055	1,12
3,0	1,025	1,05	1,12

Table 33. Correction Factor for the Amplitude Variant With Recording of the Temperature on the Axis.

$\varkappa_0^2$	$\varkappa^2/\varkappa_0^2$	
	$\eta = 0,07$	$\eta = 0,1$
2,0	1,02	1,05
2,5	1.03	1,07
2,0	1,035	1,08

Table 34. Values of the Relative Thickness of the Skin-Layer for Wolfram at 2,000°C.

R of the specimen, cm	f of the Furnace, Megahertz		
	0,1	1	10
0,5	0,13	0,040	0,013
1	0,06	0,020	0,006

Let us summarize the results of the examination of the theory of radio temperature waves as applied to experiments for determining the thermal properties. We have clarified that the method of induction heating can be utilized for measuring the temperature diffusivity by the phase method with recording of the temperature on the axis of the test specimen or on its surface. The latter method is more convenient, although it does insure a somewhat smaller accuracy. An additional advantage of recording the temperature variations on the lateral surface of the specimen is a possibility of realizing another independent method for determining the temperature diffusivity--the amplitude-frequency method.

Experience with the method of induction heating is presented in the next section.

§16. Measurements of the Temperature Diffusivity by the Method of Variable Induction Heating

The utilization of high-frequency induction heating in order to create radial temperature waves in the experiment for determining the temperature diffusivities has a number of important advantages. Induction heating makes it possible without difficulty to introduce into the test specimen comparatively large powers and thereby insure at the same time the maintenance of a high ($\sim 2{,}000^\circ$ and higher) average temperature as well as the creation of sufficiently large amplitudes of temperature variations. In this instance, it is easy to realize practically inertia-free control of the heating power and thereby make it possible to determine the temperature diffusivity from the phase shift between the variations in the temperature and power. An important merit of the method of induction heating is also the possibility of recording the temperature directly on the external heated surface of a cylindrical specimen, which makes it possible, on one hand, to utilize a very simple method of contact-free photoelectric recording the temperature variations and, on the other hand, it makes it possible to realize simultaneously too independent methods for determining the temperature diffusivity: the phase and amplitude-frequency methods. Finally, the utilization of the induction heating makes it possible to carry out experiments not only in deep vacuum, as is the case in electronic heating, but also in an atmosphere of inert gas, it is important when working in the range of temperatures within which there is a comparatively intensive atomization of the test materials.

A schematic representation of the main parts of the experimental unit is shown in Figure 57. The case a corresponds to the recording of the temperature variations on the external surface and the case b to the registration on the axis of the specimen. In this and the other cases, the test specimen with the diameter of 8-15 mm and a length of 5-8 cm is placed on the axis of the inductor in the high-frequency furnace, in its middle. In recording the temperature variations on the surface, emission from the section of the image of the lateral surface of the specimen, which is seen through the slots between the turns of the inductor, strikes the photomultiplier. In recording the variations on the axis, the emission from the image of the channel (perforation) 1 mm in diameter on the axis of the specimen is fed to the photomultiplier. Besides, a more thorough adjustment of the optical system is of course required than in the first case.

The block diagram entire unit is shown in Figure 58: 1, induction generator; 2, modulating device which controls the voltage of the rectifier 3; 4, photoelectric multiplier; 5, feedblock with a stabilization circuit; 6, amplifier; 7, loop oscillograph.

As a generator, use was made in these experiments of a standard induction generator GLP-15 from the MVP-5 unit. The generator power, equal to 15 kw, insured the attainment of average temperatures up to 2,000°K. In order to decrease the thickness of the skin-layer on the specimen, the working frequency of the generator, which lies within the limits of 0.25 to 0.4 megahertz, was raised to 0.6 megahertz by disconnecting a portion of the condensors in the circuit.

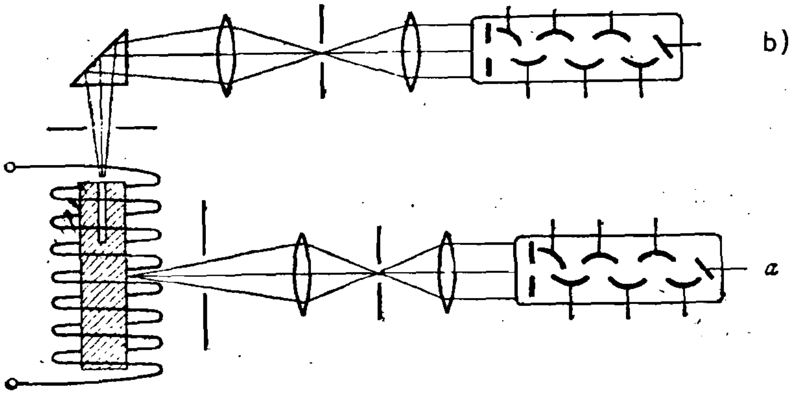

Figure 57. Schematic representation of two variants recording the temperature variations in the method of inducting heating.

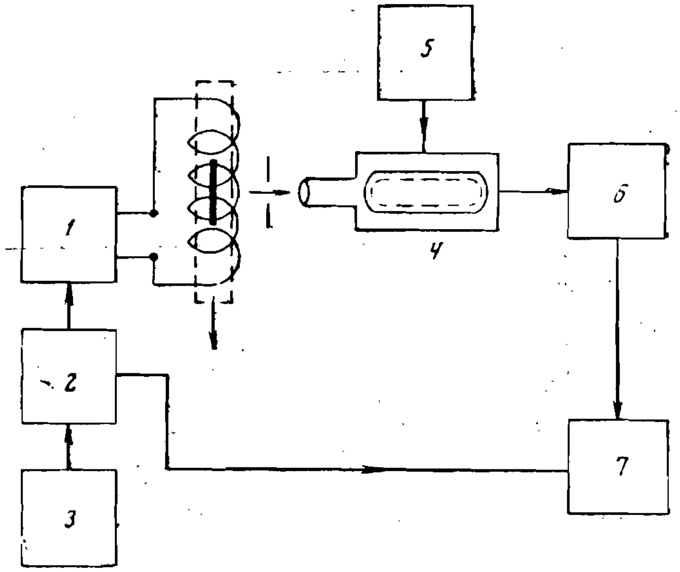

Figure 58. Block diagram of the unit.

As a modulating device, use was made of a mechanical system shown in Figure 25 b (see also [40, 41]). Recently, this system was replaced by an electronic circuit which insures greater stability. In one as well as in the other cases, the modulation had a П-shaped character: the generator tube was periodically wiped out by feeding a negative voltage to its grid. (For details see [41].) A portion of this control voltage was fed to one of the loops of the oscillograph for recording the moments of the generator connection. The interval of the frequency of modulation was 0.2-2 hertz (periods 0.5-5 seconds).

In the recording portion of the unit we utilized a photomultiplier of the FEU-19M type which was fed by a stabilized rectifier of the VSF type. The circuits of the utilized amplifiers are shown in [41, 39]. For recording the variations, use was made of the loop oscillographs of the MPO-2 and N-700 types. The diagram of the installation of the specimen (for the simplest case--recording of the temperature variations on the external part) is shown in Figure 59.

The test specimen 1 was usually secured on a ceramic straw 2 within the water-cooled quartz cap 3. In the upper part of the cap there is a flat-parallel quartz window 4 which is utilized in recording the temperature variations on the axis and in measurements of the average temperature. (For this purpose, use was made of an OMP-19 micropyrometer.)

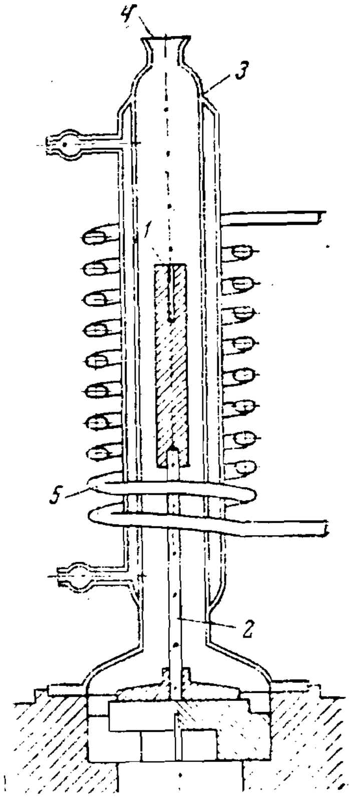

Figure 59. Installation of the specimen.

The inductor 5 was made of copper tubing; within the tube flowed water for cooling. The length of the inductor exceeds 2-3 times the length of the specimen (the scales are not shown properly in the illustration). The entire system was continuously evacuated by the MVP-5 vacuum unit of the furnace (diffusion pump TsVL-100, forevacuum pump VK-46-1M). The system also provides for the admission of an inert gas.

The experiment is carried out in the following order. After the achievement of vacuum, in the system and warming up of the generator and amplifier of the recording system, high-frequency heating in the modulating device are connected. A stable average temperature is reached comparatively rapidly--5-10 minutes after the heating is connected. Control of the amplification system reduces itself to the compensation of the constant component of a signal at the inlet of the amplifier and the establishment of an amplitude of the variable component for which the variable signal would not go beyond the limits of the linear section of its characteristics. The selected conditions for the operation of the amplifier should remain unchanged in changing from one frequency of modulation to another.

Typical oscillograms of the variations in the surface temperature, when recorded on the MPO-2 loop oscillograph, are shown in Figure 60.

The resulting curves are printed in magnification on photographic paper (by using the M-700 oscillograph, there is no need for magnification) and are processed by the method of twelve ordinates. (There is no need for using more accurate methods of approximate harmonic analysis--see [41].) The stability of the resulting values of the amplitude and phases of the first harmonic is shown in Table 35 which gives the results of the processing of five successive periods of variations in the surface temperature of the specimen (wolfram, 1,890°K, frequency 0.336 hertz; values of the amplitude are given in millimeters on the photo-paper). Similar results are obtained also in recording the temperature variations on the axis.

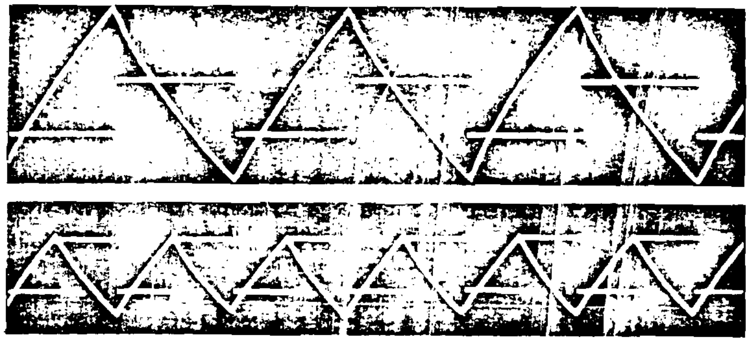

Figure 60. Typical oscillograms.

Table 35. Results of Processing Five Successive Periods of Temperature Variations.

No. of Periods	Amplitude mm	Phase, deg.
1	68.5	79.6
2	67.2	79.4
3	67.0	79.2
4	67.8	80.1
5	68.0	79.8
Average Value	67.7	79.6
Average Deviation	0.5 mm (0.7%)	0.3°
Maximum Deviation	0.8 mm (1.3%)	0.5°

The absolute values of the amplitude of the temperature variations lie within the limits of 0.2 to 10° depending on the material, frequency of modulation, and average temperature, so that the condition of smallness of the amplitude, in comparison with the average temperature, required for the theory of temperature waves (§8), is fulfilled in this experiment.

The values of the phase in the first harmonic of the temperature variations with respect to the first harmonic of the power variations are utilized directly for determining the temperature diffusivity. The calculations carried out by means of curves of functions as shown in Figures 36 and 53. The values of the magnitude η are determined from the formulas (15.1) and (15.2).

In determining the temperature diffusivity by the amplitude-frequency method, it is possible to utilize the curves in Figure 55. In order to increase the accuracy of the results, however, it is expedient to carry out the measurements not for two different frequencies, but for a considerably larger number of these. Thus, the usual series of measurements, which is carried out on the described unit, included seven frequencies (the recording of such a series of curves does not require too much time because the regularization of the thermal conditions in passing from one frequency to another takes place rapidly in the course of 1-2 minutes. Processing of the results of such a series by the paired combination of data for different frequencies which, in addition, are not multiples of one another, represents a little effective procedure. In connection with this, utilization was made of another system of processing, which consists of comparing the experimental dependence of $\ln \theta\tau^{-1}$ on $\ln\tau^{-1}$ (τ is the period of vibrations) with the calculated curve $\ln(|\theta|/\theta_0$ on $\ln\kappa^2$. In practice, this comparison was made by superimposing the experimental points large on a graph paper and the $(\ln\frac{\theta}{\tau} - \ln\tau^{-1}$. coordinates on the curve $\ln\frac{|\theta|}{\theta_0} - \ln\varkappa^2$ (for a specific value of η), plotted on millimeter paper. The purpose of this superposition, which was carried out with strict parallelism of the axis of the coordinate network, consisted in finding a position for which all the experimental points could be combined with the theoretical curve as well as possible. The difference between the abscissas of these two curves made it possible to determine the value of the temperature diffusivity in the best manner which describes the entire series of experimental values of the amplitudes:

$$\ln \varkappa^2 - \ln \tau = \ln \frac{2\pi R^2}{a}.$$

The described procedure, when repeated, leads, as a rule, to satisfactorily reproducible results. For illustration, we show Table 36 which contains data obtained during a repetition of the determination of the most reliable value of the temperature diffusivity (series of seven frequencies for wolfram at a temperature of 1,725°K).

Let us examine the problem of corrections for the heat exchange. As in all the other cases, which have been examined in preceding sections, the solution of the problem of the temperature variations, with consideration of the correction for heat exchange, is written in the form of

$$\frac{\theta}{\theta_0} = F + Bi f. \tag{16.1}$$

The correction function f when Bi is small should satisfy the equation and boundary condition

$$\nabla^2 f = i\varkappa^2 f, \tag{16.2}$$

$$f' = -F(Z). \tag{16.3}$$

In comparing (16.2) and (16.3) with the equation in the boundary condition for a case of external (purely surface) heating of a solid cylinder ($\Theta/\Theta_0 = \hat{F}$), we come to the conclusion

$$f = -\frac{\hat{F} F(Z)}{\varkappa^2}. \tag{16.4}$$

Table 36. Illustration of the Reproducibility of the Results in Determining the Temperature Diffusivity by the Amplitude-Frequency Method.

№	a, cm^2/sec
1	0,379
2	0,370
3	0,373
4	0,371
5	0,375
Average Values	0,374
Average Deviation	0,8%
Maximum Deviation	1,4%

Hence, for corrections to the temperature pulsation on the surface of a cylinder, we get

$$\frac{\Delta|\theta(Z)|}{\theta_0} = -Bi\,\mathrm{Re}\frac{\hat{F}(Z)}{\varkappa^2} = -Bi2F_1, \tag{16.5}$$

$$\Delta\varphi = Bi\,\mathrm{Im}\frac{\hat{F}(Z)}{\varkappa^2} = Bi2F_2, \tag{16.6}$$

because $\hat{F}(z) = 2\varkappa^2(F_1+iF_2)$, , in accordance with formulas (15.27)-(15.32). The corrections given by formulas (16.5) and (16.6) are identical to the earlier examined corrections -Bi$_{\psi 1}$ and -Bi$_{\psi 2}$ (see formulas (14.5) and (14.6)). From the nature of the functions $\psi_1(\kappa^2)$ and $\psi_2(\kappa^2)$, which are shown in the curves of Figure 50, it is seen that for the optimum regional values of $\varkappa^2 \sim 1-1,5$ the magnitude ψ_1 varies very insignificantly, by $\sim 3\%$. The effect of the parameter Biψ_2 on the ratio of the amplitudes of the temperature variations on the surface of the specimen does not exceed, for this reason, the magnitudes 0.03·0.5 Bi, i.e. ~ 1.5% of Bi. As regards the corrections to the phase, it lies in the range of 0.7 Bi to Bi within the same region of values of κ^2.

For corrections to the temperature variations on the axis of the specimen, we get from (16.4) the formulas

$$\frac{\Delta|\theta(0)|}{\theta_0} = -Bi\,\mathrm{Re}\,\frac{\hat{F}(0)\,F(Z)}{\varkappa^2 F(0)}, \tag{16.7}$$

$$\Delta\varphi = Bi\,\mathrm{Im}\,\frac{\hat{F}(0)\,F(Z)}{\varkappa^2 F(0)}. \tag{16.8}$$

For conditions for which the effect of the magnitude η has the nature of a small correction, it is possible to disregard the difference between $\hat{F}(0)$ and $F(0)$ (i.e. between the magnitudes $|\Theta|/\Theta_0$ in surface and volume heat liberation). In this case, also, for the temperature pulsation on the axis of the cylinder we get the same formulas (16.5) and (16.6) if the above made evaluations of the effect of the Biot criterion are valid.

Let us now pass on to the problem of evaluating the errors of the experiment under consideration.

Let us start with the phase method. As follows from the above presentation with respect to the reproducibility of the results in determining the phase differences, for the first harmonics the maximum deviation of the results from the average value usually amounts to ~ 0.5°. The maximum error in determining the true phase difference can be somewhat greater than this magnitude due to the inaccuracy which is introduced in calculating the corrections for the frequency characteristic of the amplifying circuit and (for mechanical modulator) due to the different durations of the connection and disconnection (for details we cannot stop here). These factors could increase the error in determining $\Delta\phi$ to 0.7-1°. The error reproduced by the inaccuracy in the calculation of the heat exchange amounts, as determined, to (0.7:1) δBi. For emission heat exchange the magnitude of the effective Bi criterion can be considered known with an accuracy of 10 to 30% depending on the accuracy of the available values of the degree of blackness; the indeterminacy in the values of the correction amounts thus to

(0.07-0.3) Bi. For values of heat conductivity $\lambda \sim 1$ wt/cm deg, the degree of blackness $\varepsilon \sim 0.5$, and the diameter of the specimen ~ 8-15 mm, the diameter Bi lies within the limits of 0.002 to 0.005 at 1,000°K, from 0.008 to 0.02 at 1,500°K and from 0.02 to 0.04 at 2,000°K. Hence, it follows that for the most unfavorable conditions the error introduced by the inaccuracy in the calculation of the emission heat exchange does not exceed ~0.5°. The total error in determining the phase difference thus lies within the limits of 0.7-1.5°. The error introduced into the magnitude of the temperature diffusivity due to the found error in determining the phase difference depends significantly on the magnitude η. For the case in which the correction for the finite thickness is negligibly small, the error of $\delta a/a$ for measurements of the temperature variations on the surface of the specimen for optimum values of κ^2 is, in accordance with Figure 54, equal to 5.5% χ (0.7 : 1.5) = 3.5 - 8%; for measurements on the axis of the specimen for the same values of κ^2 we get from Figure 47 $\frac{\delta a}{a} \simeq 3(0{,}7 \div 1{,}5) = 2 \div 4{,}5\%$, ; besides for larger values of κ^2, the accuracy can in principle be even greater if, of course, the amplitude of the variation is sufficiently large. For finite values of η, the error increases noticeably. For ~0.1 in the same cases $\frac{\delta a}{a} = 8(0{,}7-1{,}5) = 5{,}5 \div 12\%$ for measurements on the surface. To these figures must be added also the error due to the inaccuracy and the knowledge of the magnitude η. The latter amounts to ~ 3%, if the inaccuracy of the values of the specific resistance for the given temperatures is equal to 5%. When $\eta \sim 0.1$, a change in η by 3% can lead to an additional error of $\delta a/a$ equal to 1%.

An analysis of other sources of errors such as the finite length of the specimens, the possible nonuniformity in the temperature field with the height and parameter, inaccuracy in the knowledge of the frequencies of modulation and the carrier frequency of the generator, errors in the measurement of the dimensions, the role of nonlinearity in the recording system, lead to the conclusion that the contribution of these factors to this total error cannot be taken into account (see [41]).

The above-found errors in the determination of the temperature diffusivity in the phase variant of the method characterize the errors of individual measurements which include systematic as well as random errors. In averaging the series of measurements, the random errors are to a considerable degree compensated and the accuracy of the results increases. Thus, for the series of seven measurements, the error, which is equal to the sum of the average quadratic error and the maximum systematic error will be equal to (2.2-6.5)% and (1.2-3.5)% for the case of $\eta = 0$ for measurements on the surface and on the axis, respectively.

Let us examine further the errors in the amplitude-frequency method of measurement.

The error in determining the first harmonic of the amplitude of the temperature variations amounts to a magnitude of the order of 1%. For the error in determining a, with respect to the amplitudes for two frequencies, we get from the curve on Figure 56 for the optimum values of κ^2 $\delta a/a \sim 3.5 \times 2\% = 7\%$ where $\eta = 0$ and $\delta a/a \sim 14\%$ where $\eta = 0.1$. The latter magnitude should be increased by ~1% more due to the indeterminacy in the values of η. The use of the above-described method for processing the results of measurements for a series of frequencies makes it possible to decrease significantly the values of these errors. Random errors in the utilization of such a method for determining the temperature diffusivity are characterized by an error in the determination of the most probable value of the temperature diffusivity in each series (this is ~1.5-3% depending on the magnitude of η) and by the scatter of these probable values for equivalent series. As some of these errors amount to ~3-5% for $\eta = 0$ and 6-9% for $\eta \sim 0.1$.

Let us cite some results obtained in processing and in testing the procedure.

Table 37 compares the data obtained by the phase (recording on the surface) and the amplitude-frequency methods (wolfram, 1,720°K, $R = 4.97$ mm, $L = 80$ mm, f --0.5-0.6 megahertz). The values given in the table for the temperature diffusivity, which have been obtained by the amplitude method, have been found for each frequency after the entire totality of these data was superposed on the calculated curve by a method described above. The deviations of individual values of a_A and a_ϕ from the average values do not achieve the *a priori* maximum errors. Average values of a_A and a_ϕ are in good agreement with each other. Good agreement has been obtained also for the results of control experiments which were performed with specimens of different diameter and different length [166].

Table 37. Comparison of the Results of Measurements of the Temperature Diffusivity of Wolfram, Obtained by the Phase and Amplitude Methods.

ν, Hertz	A, мм	φ, Degree	a_A, $\frac{\text{см}^2}{\text{sec}}$	a_φ, $\frac{\text{см}^2}{\text{sec}}$
0,294	80,9	82,9	0,376	0,390
0,344	69,6	82,1	0,373	0,400
0,417	60,1	79,9	0,371	0,376
0,477	52,5	78,5	0,375	0,373
0,572	43,0	77,3	0,381	0,393
0,678	38,2	75,5	0,366	0,397
0,744	33,8	73,7	0,378	0,375
Average Value			0,374	0,386
Average Deviation.			1,0%	2,6%
Maximum Deviation			2,1%	3,6%

Table 38 lists data obtained by the amplitude-phase method under conditions when the connection intervals were three times greater than the disconnection intervals (wolfram, 1,890°K, the same specimen as in Table 37.)

The utilization of such conditions in the operation of the unit, besides additional possibilities for checking the reliability of the results, makes it possible to increase somewhat the maximum temperatures of the experiment (by an introduction of greater power) as well as to utilize in processing the results not only the first but also the second harmonics of the temperature variations and thereby obtain a greater volume of information. An analysis of the conditions under which the relationship of the magnitude of the first and second harmonics is optimum leads to the conclusion that the durations of the connection and disconnection intervals in this instance should be as 3/1 or 1/3 [40, 166].

It is seen from Table 38 that the values of the temperature diffusivity, obtained from data for the first and second harmonics, agree with each other, the deviations from the average value do not have a systematic nature. This circumstance makes it possible to utilize the second harmonics of the variations on the same footing with the first and thereby reduce the series of measurements for different frequencies twofold without a significant decrease in the accuracy of the results.

Table 38. Results of Measurements of Temperature Diffusivity Obtained from Data of the Amplitudes Under Conditions of Inequality of the Heating and Cooling Intervals.

ν_I, Hertz	ν_{II}, Hertz	A, мм	a, $\frac{см^2}{sec}$
0,289		76,5	0,373
0,336	—	67,8	0,373
0,399	—	59,0	0,375
0,448	—	53,8	0,378
0,529	—	46,2	0,375
0,542	—	41,9	0,371
—	0,578	39,8	0,367
0,667	—	35,6	0,365
—	0,672	35,4	0,375
—	0,798	31,1	0,375
—	0,896	28,3	0,381
—	1,058	24,7	0,366
Average Value			0,373
Average Deviation			1,0%
Maximum Deviation			2,1%

Table 39 (wolfram, 1,920°K) compares the data obtained from the amplitudes and phases for a ratio of the connection and disconnection duration equal to 3:1. The somewhat greater scatter of individual values of the temperature diffusivity, in comparison with the case of equal half-periods, is related apparently with the decrease in the specific weight of the first harmonic. The average values of the temperature diffusivity, obtained from the amplitudes and phases, are hereto in agreement with each other as well as with the values obtained for different half-periods. The measurements for a ratio of the connection disconnection intervals equal to 1:3 lead also to similar results [40, 166].

Some results obtained by means of the described unit are given in §23 and in [166, 167, 168].

Let us summarize the results. The described procedure in the unit makes it possible to determine the temperature diffusivity of metals within the temperature range of 1,000-2,000°K; besides, by utilizing a more powerful generator, the upper limit can be increased. Measurements are made on specimens of cylindrical form, the recording of the temperature variations is made by the contact-free photoelectric method.

Table 39. Comparison of the Result of the Determination of the Temperature Diffusivity of Wolfram from the Amplitudes and Phases Under Conditions of Inequality of the Heating and Cooling Intervals.

ν, Hertz	φ, Degrees	A, мм	a_φ, $см^2$/sec	a_A, $см^2$/sec
0,400	80,0	66,2	0,373	0,362
0,443	79,6	59,8	0,386	0,353
0,494	77,3	55,0	0,350	0,356
0,553	76,8	47,3	0,366	0,346
0,627	76,5	43,5	0,385	0,359
0,722	75,2	40,2	0,372	0,374
Average Value			0,372	0,358
Average Deviation %.			2,4	1,8
Maximum Deviation %			5,9	4,4

The simplest method is that of recording the variations on an external surface. The temperature diffusivity in this instance can be determined by two methods: the phase and amplitude methods. Both methods insure an approximately equal accuracy result, which varies from 3-13% depending on the conditions of the experiment, and in the first place on the ratio of the skin-layer to the diameter of the specimen. The phase method is more convenient because each measurement of the phase difference makes it possible to determine directly the temperature diffusivity. The amplitude method requires measurements of a series of values for several frequencies and has a somewhat more complex procedure for processing the results. Nevertheless, the amplitude method is a useful addition to the phase method as a method for the independent control of the resulting data within the framework of the same experiment.

The variant of the method for recording the temperature variations on the axis is more complex experimentally, but for equal conditions has a greater accuracy than the variant with recording of the surface temperature. It is expedient in this instance to determine the temperature diffusivity by the phase method alone.

The prospects for further development of the described procedure are related with the possibilities of its utilization as a complex procedure which yields not only values of the temperaute diffusivity but also of the heat conductivity (or heat capacity). This can be accomplished if it is possible to determine the power introduced into the specimen. One of the possible ways for determining the power is to measure the electromotive force which originates in an open annular turn which encompasses the specimen. Another indirect method of determining the power is to utilize information regarding the degree of blackness of the surface of the specimen. It can be utilized in principle also under conditions when the degree of blackness is not known sufficiently accurately, if the surface of the specimen is processed in such a manner that the degree of blackness approaches unity (this can be done, for example, by coating the surface with a thin layer of graphite).

§17. Nonstationary Variant of the Kohlrausch Method

The procedure of measurements described in this section belongs to those methods which utilize volume heating of the test specimen--heating by a current that passes through the specimen.

The scheme for accomplishing this experiment is in principle similar with that of the known Kohlrausch method (see 4). The test specimen represents a comparatively thin rod which is secured between two massive bodies that insure the feed of current and removal of heat. In the middle of the rod and in one (or two) points near the end, thermocouples are secured to the specimen. The lateral surface of the rod is heat insulated. In the usual steady-state variant, the rod is heated by direct current, the measured temperature difference makes it possible in this instance to determine the heat conductivity of the specimen from the formula

$$\lambda = \frac{wL^2}{2\Delta T}, \qquad (17.1)$$

where w is the power liberated per unit volume of the specimen, L is the distance between the central and peripheral thermocouples, ΔT is the corresponding temperature difference.

In the nonsteady variant which is examined here, the current which passes through the specimen and, consequently, also the power are modulated with a frequency of ω and not only the constant component of the temperature (average temperature) is measured, but also the variable component (temperature variations).

The values of the constant components of the temperature in the nonsteady experiment could be utilized exactly in the same manner as the constants of the temperature in the steady-state method; formula (17.1) makes it possible to obtain in this instance the magnitude of the heat conductivity. The variable components, their amplitude and phase make it possible to obtain information also on the heat capacity and the temperature diffusivity. The nonsteady measurements thus add new information and enrich significantly the possibilities of the experiment.

In accordance with the general theory of regular thermal conditions of the third type, which is described in §8, the dimensionless function F, which describes the spacial distribution of the complex amplitude of the temperature variations Θ

$$F = \frac{\theta}{\theta_0} = \frac{\theta \omega c_p \rho}{\varpi m} \qquad (17.2)$$

(see formulas (8.33), (8.34), (8.38)), satisfies the equation (8.41)

$$F'' - i\varkappa^2 F + \varkappa^2 = 0 \qquad (17.3)$$

in the boundary condition (8.43)

$$F = F_0 = \frac{\theta_L}{\theta_0} \text{ for } \frac{x}{L} = 1. \qquad (17.4)$$

(The temperature pulsation is set in the point to the distance of L from the middle of the rod, in particular, at the end.)

Solution of the equation (17.3), which is symmetric in the point x = 0 (middle of the rod),has the form of

$$F = -i + C\left(e^{\sqrt{i}\varkappa\frac{x}{L}} + e^{-\sqrt{i}\varkappa\frac{x}{L}}\right). \tag{17.5}$$

The value of the constant C is found from the condition (17.4):

$$C = \frac{F_0 + i}{e^{\sqrt{i}\varkappa} + e^{-\sqrt{i}\varkappa}}. \tag{17.6}$$

For the temperature pulsation in the point x = 0, we get

$$\frac{\theta}{\theta_0} = \left(\frac{\theta_L}{\theta_0} + i\right)\frac{1}{\operatorname{ch}\sqrt{i}\varkappa} - i. \tag{17.7}$$

Let us examine at first the case when $(\theta_L \sim 0)$ (the temperature pulsation on the ends of the specimens is small), which will take place if the relatively tense specimen is held in massive tongs.

For the amplitude and phase of the temperature variations, we get

$$\frac{|\theta|}{\theta_0} = \sqrt{\frac{(1 - A_c)^2 + B_c^2}{A_c^2 + B_c^2}}, \tag{17.8}$$

$$\operatorname{tg}\varphi = \frac{B_c^2 - A_c(1 - A_c)}{B_c}, \tag{17.9}$$

or

$$\varphi = \operatorname{arctg}\frac{B_c}{A_c} + \operatorname{arctg}\frac{A_c - 1}{B_c},$$

where A_c and B_c are the real and imaginary parts of $\operatorname{ch}\sqrt{i}\,\varkappa$, tabulated in the monograph [6].

For slow variations, when κ is small, A_c is close to 1, while $B_c \ll 1$.

$$\frac{|\theta|}{\theta_0} \approx B_c \approx \frac{\varkappa^2}{2} \tag{17.10}$$

and

$$\operatorname{tg}\varphi \approx B_c, \quad \varphi \approx 0. \tag{17.11}$$

The amplitude of the temperature variations can, in accordance with (17.10), be written in the form of

$$|\theta| = \frac{wmL^2}{2\lambda}. \tag{17.12}$$

Comparing (17.12) and (17.1), we come to the conclusion that the amplitude of the slow temperature variations, with an accuracy up to the modulation coefficient m is equal to the steady-state difference in temperatures or, which is the same thing, to the difference of the constant component of the temperature.

$$|\theta|_{\omega\to 0} \to \Delta T m. \tag{17.12'}$$

The phase of the temperature variations in this instance coincides with the theories of the power variations.

Thus, the low-frequency measurements of the amplitude of the temperature variations can be utilized for determining the heat conductivity. Formula (17.12') makes it possible in this instance to compare the information on the constant and variable components and thereby accomplish internal control of the experiment.

In another case, for relatively small values of κ

$$\frac{|\theta|}{\theta_0} \to 1 \quad \& \quad \varphi \to \frac{\pi}{2}.$$

The amplitude of the temperature variations is unambiguously related to the heat capacity

$$|\theta| = \frac{wm}{c_p\rho\omega}. \tag{17.12''}$$

For relatively high frequencies, the process in the experiment under consideration takes place just as for low frequencies in the group of methods with surface heating. The reason for this can be understood, if one takes into account that it is precisely at high frequencies, as a result of the significant denting of the temperature waves, that the effect of the ends of the test specimen becomes relatively small and the process in the middle of the specimen approaches the case of variable heating of an infinitely long rod, which should precisely be described by formula (17.12") in view of the absence of temperature gradients on the test section.

The general form of the dependence of $\varphi, \frac{|\theta|}{\theta_0}$ & $\frac{|\theta|}{m\Delta T}$ of $\varkappa$ on κ is shown in Figures 61 and 62. In order to evaluate the errors in the determination of the magnitude κ^2 from these curves, Figure 63 shows curves of the relative error $\delta\kappa^2/\kappa^2$ for $\delta\phi = 1°$ and for $\delta|\Theta| = 0.01\,|\Theta|$.

The following conclusions can be made from an examination of all the indicated curves.

1. It is expedient to carry out the determination of temperature diffusivity by the phase method within the interval of the parameters κ ranging from 1 to 3 (the optimum value is 1.5). The sensitivity of the method in this instance will be equal to 3-5% for $\delta\phi = 1°$. The

magnitude of the amplitude of the temperature pulsation under these conditions amounts to 0.9 to 0.2 values of the temperature gradient in the steady-state Kohlrausch method (for modulation coefficient m equal to unity).

2. Within the interval of values of the parameter ranging from 0 to 1, it is possible to determine the heat conductivity from formula (17.12), which is practically independent of the measurements of the temperature diffusivity. (A correction to formula (17.12) for the difference in the function $|\Theta|/\Delta T_m$ from unity, does not exceed 10%.)

3. The determination of the heat capacity by means of formula $|\Theta| = \Theta_0$, which is independent of the measurements of the heat conductivity, is possible for values of $\kappa > 4$. (Correction for the difference in the magnitude $|\Theta|/\Theta_0$ from unity does not in this instance exceed 10%.) Values of the amplitudes of the temperature pulsations in this instance amount to 1/10 of ΔT and less. The utilization of this method is thus feasible only under conditions when the volume heat liberation of the specimen can be made sufficiently large (thin tubes, wires at high temperatures).

4. Within the intermediate region of values of the parameter κ ranging from 1 to 4, it is possible to determine the temperature diffusivity from the experimentally determined ratio $|\Theta|/\Delta T$; in this instance, the error in the determination of κ amounts to 1-5% when

$\frac{\delta|\theta|}{|\theta|} = 1\%$.

5. In case it is desired to determine the complex of thermal properties from one experiment, that is, from one value of κ, it is expedient to combine the phase method of determining the temperature diffusivity with the determination of the heat conductivity from the formula

$$\lambda = \frac{\omega m L^2}{|\theta|}\left(\frac{|\theta|}{\Delta T m}\right). \qquad (17.13)$$

(The magnitude within the brackets should be found from the curve in Figure 62.) The optimum values of κ for such a complex method lie within the interval of 1:1.5.

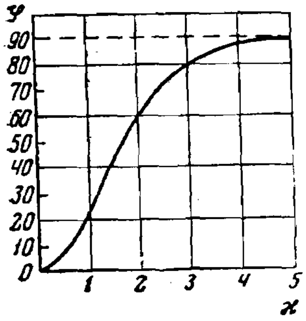

Figure 61. Dependence of the phase difference between the power and temperature variations on the parameter κ.

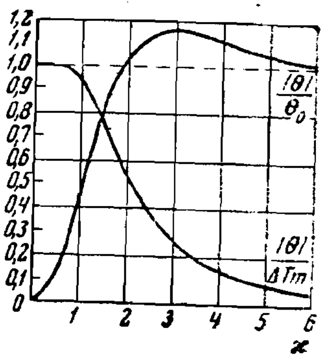

Figure 62. Dependence of the amplitude of the temperature variations (with respect to the constant component ΔT or with respect to Θ_0) on the parameter κ

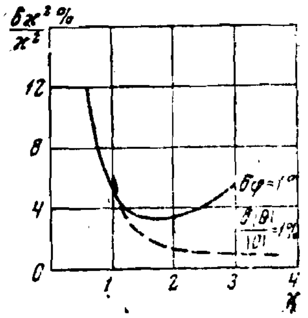

Figure 63. Errors in determining the temperature diffusivity from data of the amplitude and phases of the temperature variations.

The entire presentation pertains to the case in which the temperature pulsation and the point at a distance of L from the middle of the specimen is negligibly small. If these conditions are not observed completely accurately in practice, then formula (17.8) and (17.9), in accordance with (17.7), should be replaced by the more complex

$$\frac{|\theta|}{\theta_0} = \sqrt{\frac{(f\cos\varphi_0 + B_c)^2 + [f\sin\varphi_0 + (1 - A_c)]^2}{A_c^2 + B_c^2}}, \qquad (17.14)$$

$$\varphi = \operatorname{arctg}\frac{B_c}{A_c} + \operatorname{arctg}\frac{(A_c - 1) + f\sin\varphi_0}{f\cos\varphi_0 + B_c}, \qquad (17.15)$$

where ϕ_0 is the phase lag for the temperature pulsation in the points $x = \pm L$, f is the modulus of the function F_0, that is

$$f = \frac{|\theta|_L}{\theta_0} = \frac{|\theta|_L \varkappa^2}{2\Delta T}. \tag{17.16}$$

For small values of κ, when $B_c \sim \varkappa^2,\ A_c \sim 1\ \ \varepsilon\, \varphi_0 \sim 0,$, formula (17.14) degenerates to the analog of (17.12):

$$|\theta| - |\theta|_L = \frac{wmL^2}{2\lambda}. \tag{17.17}$$

This relationship can be utilized directly for determining the heat conductivity under conditions of slow variations.

In another maximum case, relative to the large values of κ, the expression (17.14) approaches the previous asymptotic formula (17.12"), which is understandable because the boundary conditions with an increase in κ, play a decreasing role as a result of an increase in the damping of the temperature waves and a decrease in its related "thermal relationships"--the middle of the specimen with its ends.

Within the intermediate range of values of κ, the formulas (17.14) and (17.15), when f << 1 , can be represented in the form of

$$\frac{|\theta|}{\theta_0} = \sqrt{\frac{(A_c - 1)^2 + B_c^2}{A_c^2 + B_c^2}}\,(1 + \delta_1), \tag{17.18}$$

$$\varphi = \operatorname{arctg}\frac{B_c}{A_c} + \operatorname{arctg}\frac{A_c - 1}{B_c} + \delta_2, \tag{17.19}$$

where

$$\delta_1 = f\,\frac{B_c \cos\varphi_0 + (A_c - 1)\sin\varphi_0}{B_c^2 + (A_c - 1)^2} \tag{17.20}$$

and

$$\delta_2 = f\,\frac{-B_c \sin\varphi_0 + (A_c - 1)\cos\varphi_0}{B_c^2 + (A_c - 1)^2} \tag{17.21}$$

Corrections of δ_1 and δ_2, if they are not large can be introduced after κ and λ have been found in their absence. The problem of the role of heat exchange in the experiment under consideration will be studied, just as earlier, on the assumption of the smallness of the Bi number.

As shown in §4, for small values of Bi the heat exchange, lateral surface of the rod can be calculated by introducing into the unidimensional equation of the heat conductivity a member with a negative source of heat, proportional to the temperature:

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{a} \cdot \frac{\partial T}{\partial t} + \frac{w}{\lambda} - \nu (T - T_0) = 0 \qquad (17.22)$$

(see equation (4.10)), where $\nu = \frac{2\alpha}{\lambda R}$.

For the function F, we get in this instance the equation

$$F'' - i\varkappa^2 F + \varkappa^2 - Bi^* F = 0, \qquad (17.23)$$

where

$$Bi^* = \frac{\alpha' 2}{\lambda} \cdot \frac{L^2}{R} = Bi \left(\frac{L}{R} \right)^2. \qquad (17.24)$$

The form of the resulting expression for the effective criterion Bi* makes it possible to clarify the role of the transverse cross-section of the test specimen, the magnitude of which did not figure in the problem without taking into account the heat exchange. It is seen that a utilization of thin specimens can lead to a serious increase in the role of the heat exchange.

In particular, for emission heat exchange in the region of temperatures of ~1,000°K, the values of Bi* can already be of the order of unity for specimens of the same type as used in the steady-state Kohlrausch method. The problem of taking into account the heat exchange in this case is just as urgent as in the steady-state method. However, in these conditions also it is possible to conduct the experiment for relatively large values (conditions for measuring the heat capacity) by introducing comparatively small corrections. For an analysis of this problem we shall examine the solution of equation (17.23), without limiting ourselves to the assumption regarding the smallness of Bi* in comparison with unity. This solution for x = 0 has the form of

$$F=\frac{\varkappa^2}{Bi^*+i\varkappa^2}\left[1-\frac{1}{\operatorname{ch}\sqrt{i\varkappa^2+Bi^*}}\right]. \tag{17.25}$$

The amplitude of the phase of the function F are expressed by the formulas

$$|F|=\frac{1}{\sqrt{1+\nu^2}}\sqrt{\frac{(A-1)^2+B^2}{A^2+B^2}}, \tag{17.26}$$

$$\varphi=\operatorname{arctg}\frac{1}{\nu}+\operatorname{arctg}\frac{B}{A}-\operatorname{arctg}\frac{B}{A-1}, \tag{17.27}$$

where

$$A=\operatorname{ch}\frac{\varkappa}{\sqrt{2}}\sqrt{\sqrt{\nu^2+1}+\nu}\cos\frac{\varkappa}{\sqrt{2}}\sqrt{\sqrt{\nu^2+1}-\nu},$$

$$B=\operatorname{sh}\frac{\varkappa}{\sqrt{2}}\sqrt{\sqrt{\nu^2+1}+\nu}\sin\frac{\varkappa}{\sqrt{2}}\sqrt{\sqrt{\nu^2+1}-\nu}, \tag{17.28}$$

$$\nu=\frac{Bi^*}{\varkappa^2}. \tag{17.29}$$

(17.30)

Specific evaluations of the changes in the modules and phase of the function F for values of $Bi^* \sim 1$ lead in the case of $x \sim 0 - 1$ to values that amount to ~30%; with an increase in κ, these changes, however, decrease comparatively rapidly, which is related with the circumstance that in the formulas (17.26) and (17.27) the Bi* criterion figures only in the combination Bi^*/κ^2. For relatively small values of the parameter ν, it is expedient to expand the expressions (17.26) and (17.27) into a series with respect to ν. In this instance, we get the formulas

$$\frac{\Delta F}{F} \frac{Bi^*}{\varkappa^2}\left\{i + \frac{\varkappa}{2\sqrt{i}} \quad \frac{\operatorname{th}\sqrt{i}\,\varkappa}{(\operatorname{ch}\sqrt{i}\,\varkappa - 1)}\right\}, \tag{17.31}$$

$$\frac{\Delta\theta}{|\theta|} = Bi^*\frac{1}{\sqrt{2\varkappa}} \cdot \frac{\widetilde{A}A_s + \widetilde{B}B_s}{\widetilde{A}^2 + \widetilde{B}^2}, \tag{17.32}$$

$$\Delta\varphi = \frac{Bi^*}{\varkappa^2}\left[1 + \frac{\varkappa}{\sqrt{2}} \cdot \frac{\widetilde{A}B_s - \widetilde{B}A_s}{\widetilde{A}^2 + \widetilde{B}^2}\right], \tag{17.33}$$

where

$$\widetilde{A} = (A_c - 1)(A_c - B_c) - B_c(A_c + B_c), \tag{17.34}$$

$$\widetilde{B} = (A_c - 1)(A_c + B_c) + B_c(A_c - B_c). \tag{17.35}$$

The dependence of $\frac{\Delta|\theta|}{|\theta|\,Bi^*}$ and $\frac{\Delta\varphi}{Bi^*}$ on κ^2 is shown in Figure 64. It can be seen that the corrections to the amplitudes amount to a small fraction of the Bi* criterion. For $\kappa > 1.2$, this correction is $< 1/10$ Bi* and decreases sharply with increasing Bi*. Corrections to the phases do not exceed 1/10 Bi*, starting with $\kappa > 1.5$, i.e. precisely in those places where phase measurements make sense.

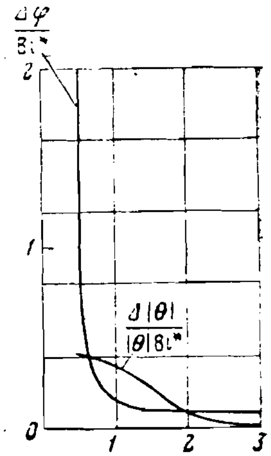

Figure 64. Corrections to the amplitudes and phases of the temperature variations due to lateral heat exchange.

And so, the problem of the role of heat exchange in the nonsteady variant of the Kohlrausch method is significantly more important than in other previously examined methods which are based on the utilization of the regular conditions of the third type. Nevertheless, the effect of the heat exchange on the conditions of the experiment for the determination of the heat capacity ($\kappa > 3$) is much less noticeable than in the steady-state experiment and in the measurements of the heat conductivity and temperature diffusivity for $\kappa \sim 0$-2. In one way or another, in operating within the temperature range starting with 700°K and above, it is desirable to achieve a decrease in the heat exchange from the lateral surface of the specimens by utilizing a heat insulating filler.

For the practical realization of the nonsteady variant of the Kohlrausch method, it is significant that in this instance use can be made, without any kind of modification, of the working part of the unit in the ordinary steady-state Kohlrausch method. All that is required is the introduction of a modulating device and the addition of an electron measuring part with a device for recording the temperature variations. The complete presentation makes it possible to consider the development of the method under consideration as very promising.

§18. Measurement of the Heat Capacity by the Method of Variable Heating with Current

The preceding section, during the examination of the problem of variable heating of rods by means of a current passing there through, it was observed that under conditions when

$$\varkappa^2 = \frac{\omega L^2}{a} > 20, \tag{18.1}$$

the temperature pulsation of the middle of the specimen is entirely determined by its heat capacity

$$|\theta| = \theta_0 = \frac{2m}{c_p \rho \omega}. \tag{18.2}$$

The conditions for carrying out such experiments are most favorable for comparatively thin specimens (rods, wires, strips of foil) because with all other conditions being equal the amplitude of the temperature pulsation in thin specimens is greater, the smaller the diameter of the specimen. Moreover, for specimens in the form of wires, a sufficiently large magnitude of the amplitude of the temperature pulsation can be obtained on frequencies that lie within the range of sonic frequencies, which makes it possible to utilize in the recording part of the unit a radial technical means for measurement. On the other hand, the performance of the experiment with the use of wires is advantageous in the study of critical and costly materials (for example, rare-earth elements). This procedure has particularly great advantages in the region of high temperatures when the methods of measurement, in particular calorimetric, require the use of a comparatively cumbersome and powerful equipment. However, in the case of wires, there is no need to use any kind of additional heating devices. The wire itself can be heated to a high temperature.

We shall dwell below on some other merits of the method of variable heating with current.

A significant characteristic of the experiment with thin and long specimens is the existence of more or less significant sections with a practically constant average temperature of the specimen and a magnitude of the temperature pulsation. The problem of the steady-state distribution of the temperature along the wires was examined in §5. In order to evaluate the length of the section of wire with a constant amplitude and phase of the temperature pulsation, we shall examine the asymptotic form of the solution of equation (17.22), which is valid

for $\frac{B_i^*}{\varkappa^2} \ll 1$, $\varkappa^2 \gg 1$ and x > y; and x > 0:

$$\frac{\theta}{\theta_0} = -i\left[1 - e^{-\sqrt{i}\,\varkappa\left(1-\frac{x}{L}\right)}\right]. \tag{18.3}$$

(This solution implies a constancy of thermal coefficients along the wire up to the "cold quartz" end; for evaluative calculations, this is however, not pertinent.) For the amplitude and phases, we get from the solution of (18.3) the formulas

$$\frac{\theta_0 - |\theta|}{\theta_0} = 2e^{-\frac{\varkappa}{\sqrt{2}}\left(1-\frac{x}{L}\right)} \cos\frac{\varkappa}{\sqrt{2}}\left(1-\frac{x}{L}\right), \tag{18.4}$$

$$\varphi - \frac{\pi}{2} = \operatorname{arctg} e^{-\frac{\varkappa}{\sqrt{2}}\left(1-\frac{x}{L}\right)} \sin \frac{\varkappa}{\sqrt{2}}\left(1-\frac{x}{L}\right). \tag{18.5}$$

For the values

$$\frac{\varkappa}{\sqrt{2}}\left(1-\frac{x}{L}\right) > 4 \tag{18.6}$$

the first formula gives 3.5%, the second < 1°. In order for such or smaller changes in the amplitude and phase of the temperature variations to take place on the length which amounts to one-half the length of the entire wire, the magnitude κ, in accordance with the inequality (18.6), should be graded in 10:

$$\varkappa > 10. \tag{18.7}$$

The fulfillment of the condition (18.7) insures that the existence on the wire of a considerable section where the temperature pulsation is constant. For such conditions, the summary equation of the heat balance will have the form of

$$c_p M \frac{dT}{dt} = W - \alpha(T - T_0)s, \tag{18.8}$$

where M is the mass of the section of the specimen with a constant temperature, W is the total power liberated on this section, s is the surface. Assuming that

$$W = \overline{W}(1 + me^{i\omega t}), \tag{18.9}$$

we get equations for the constant and variable components of the temperature provided the latter is small in comparison with the former:

$$\overline{W} = \alpha(\overline{T} - T_0)\, s \tag{18.10}$$

and

$$c_p M i\omega\theta = m\overline{W} - \alpha'\theta s, \tag{18.11}$$

where

$$\theta e^{i\omega t} = T - \overline{T} \tag{18.12}$$

$$\left(\frac{\theta}{\overline{T}} \ll 1\right), \tag{18.13}$$

$$\alpha' = \frac{\partial\alpha(\overline{T} - T_0)}{\partial\overline{T}}. \tag{18.14}$$

In a specific case, when the specimen is placed in the chamber with cold walls and vacuum,

$$\alpha \approx \sigma\varepsilon\overline{T}^3, \tag{18.15}$$

$$\alpha' = 4\sigma\varepsilon\overline{T}^3\left(1 + \frac{1}{4}\cdot\frac{d\ln\varepsilon}{d\ln\overline{T}}\right) \approx 4\sigma\varepsilon\overline{T}^3, \tag{18.16}$$

$$\overline{T} = \left(\frac{\overline{W}}{\sigma\varepsilon s}\right)^{1/4}. \tag{18.17}$$

The equation (18.11) gives for the complex amplitude of the temperature pulsation the formula:

$$\theta = \frac{m\overline{W}}{c_p M i\omega + \alpha' s}. \tag{18.18}$$

which can be written in the form of

$$\theta = \frac{-i\theta_0'}{1 + i\frac{Bi'}{\varkappa_l^2}}, \tag{18.19}$$

where

$$Bi' = \frac{\alpha' l}{\lambda}, \tag{18.20}$$

$$l = \frac{v}{s} \tag{18.21}$$

($l = \frac{R}{s}$ for specimen with a circular cross-section)

$$\varkappa_l = \sqrt{\frac{\omega}{a}}\, l. \tag{18.22}$$

It follows from formula (18.19) that for small values of Bi'/κ_l^2 the correction to the amplitude of the temperature variations is determined normally by the negligibly small magnitude $\left(\frac{Bi'}{\varkappa_l^2}\right)$ while the correction to the phase is equal to

$$\Delta\varphi = \frac{Bi'}{\varkappa_l^2}. \tag{18.23}$$

In order to evaluate the magnitude of this ratio, the formula for the amplitude of the temperature pulsation is represented in the form of

$$\frac{|\theta|}{\bar{T} - T_0} = \frac{Bi}{\varkappa_l^2}, \tag{18.24}$$

where

$$Bi = \frac{\alpha l}{\lambda}. \tag{18.25}$$

For emission heat exchange, when $T_0 << \overline{T}$

$$4Bi = Bi' \tag{18.26}$$

(with an accuracy up to the small correction for the temperature function for the degree of blackness). In this instance,

$$\frac{Bi'}{\varkappa_l^2} = 4\frac{|\theta|}{\overline{T}}. \tag{18.27}$$

Taking into consideration the inequality $\frac{|\theta|}{\overline{T}} \ll 1$, , we come to the conclusion that

$$\Delta\varphi = 4\frac{|\theta|}{\overline{T}} \tag{18.28}$$

is a small correction. The same conclusions are obviously valid also for another type of heat exchange because Bi and Bi' have the same order of magnitude.

In the measurement method described below, $\frac{|\theta|}{\overline{T}} \sim 10^{-3}$. ($\Theta$ comprises the degrees for average temperatures of ~ 1,000°K and higher). The correction $\Delta\phi$ is thus very small and, in accordance with formula (18.19), the temperature variations lag behind the power variations by exactly $\pi/2$.

The heat capacity can in this instance be determined from formula

$$c_p = \frac{m\overline{W}}{M\omega|\theta|}. \tag{18.29}$$

Let us examine the basic methods of modulation which are used for creating a variable heating. There are two of these: heating with an alternating current and heating with a current containing alternating and direct components.

Let us start with heating by an alternating current. Let the test object which has the resistance of R (strictly speaking, by R we should understand the resistance of the section with a constant temperature), connected in series with the resistance R_1 to the source of the variable voltage

$$V = V_{\sim} \cos \omega t. \tag{18.30}$$

The current strength to the specimen will in this instance be equal to

$$I = \frac{V.}{R + R_1 + \frac{dR}{dT} \vartheta}, \tag{18.31}$$

where θ is the variable component of the temperature. This formula takes into account the periodic change in the resistance of the object due to the temperature pulsation.

For the power W, which is liberated in the specimen, we get

$$W = I^2 R = \frac{V^2 \left(R + \frac{dR}{dT} \vartheta \right)}{\left(R + R_1 + \frac{dR}{dT} \vartheta \right)} \tag{18.32}$$

(just as in the preceding sections, the conversion factor here is omitted). Taking into account the relative smallness of the magnitude $\left(\frac{dR}{dT} \vartheta \right)$, this expression can be written, with an accuracy up to the members of the second order, in the form of

$$W = \frac{V^2 R}{(R+R_i)^2}\left[1 - \frac{R-R_i}{R+R_i}\vartheta\beta\right], \tag{18.33}$$

where

$$\beta = \frac{1}{R}\cdot\frac{dR}{dT} \tag{18.34}$$

is the temperatue coefficient of the resistance.

Taking into account that

$$V^2 = \frac{V_\sim^2}{2}(1 + \cos 2\omega f), \tag{18.35}$$

we get further

$$W = \frac{V_\sim^2 R}{2(R+R_i)^2}\left[1 - \frac{R-R_i}{R+R_i}\vartheta\beta + \cos 2\omega t\right] = \overline{W}(1 + m\cos 2\omega t), \tag{18.36}$$

besides

$$\overline{W} = \frac{1}{2}\overline{I}^2 R. \tag{18.37}$$

A change in the power thus takes place with a frequency equal to the double frequency of the change in voltage. After the changeover to the complex variable, we get for the effective modulation coefficient the expression

$$m = 1 - \theta\beta\frac{R-R_i}{R+R_i}, \tag{18.38}$$

where Θ is the complex amplitude of the temperature variations. The substitution of this expression in formula (18.18) (for Bi ~ 0) leads to the conclusion that a correction for the temperature function of the resistance of the specimen changes only the phase of the temperature pulsation by the magnitude

$$\Delta\varphi = |\theta| \beta \frac{R - R_1}{R + R_1}, \tag{18.39}$$

leaving practically unchanged its amplitude. In the worst case, when $R_1 \sim 0$ $\varepsilon |\theta| \sim 10^\circ$, the correction to the phase amounts to a total of only 3°.

We wish to underscore the basic characteristics of the method of heating with alternating current:

1) Heating takes place with a frequency equal to the double frequency of the change in voltage; besides, the power varies harmonically;

2) The modulation coefficient is constant and with an accuracy up to the small correction it is equal to unity.

Let us now pass on to an examination of the case of modulation in the presence of a constant component of voltage when

$$V = V_{=} + V_{\sim} \cos \omega t. \tag{18.40}$$

The current through the specimen, with consideration of the temperature pulsation, is equal to

$$I = \frac{V_{=} + V_{\sim} \cos \omega t}{R + R_1 + \frac{dR}{dT} \vartheta}. \tag{18.41}$$

The power liberated on the specimen is determined by the formula

$$W=\left(V_{=}^{2}+\frac{V_{\sim}^{2}}{2}+2V_{=}V_{\sim}\cos\omega t+\frac{V_{\sim}^{2}}{2}\cos 2\omega t\right)\frac{R}{(R+R_{1})^{2}}\times \\ \times\left(1-\frac{R-R_{1}}{R+R_{1}}\beta\vartheta\right). \qquad (18.42)$$

It is seen from this formula that, unlike heating with alternating current, the change in power is not strictly harmonic because the signal of the change in the power has a second harmonic (frequency 2ω). A constant component of the power is, in accordance with formula (18.42) equal to

$$\overline{W}=\left(V_{=}^{2}+\frac{V_{\sim}^{2}}{2}\right)\frac{R}{(R+R_{1})^{2}}. \qquad (18.43)$$

The modulation coefficient for the first component is determined by the formula

$$m=\frac{2V_{=}V_{\sim}}{V_{=}^{2}+\frac{V_{\sim}^{2}}{2}}\left[1-\theta\beta\frac{R-R_{1}}{R+R_{1}}\right]. \qquad (18.44)$$

Just as in the case of heating with alternating current, the correction factor within the brackets has practically no effect on the amplitude of the temperature variations and gives only a small correction to the phase. The absolute values of the modulation coefficient can vary from 0 to a maximum magnitude equal to

$\left[\sqrt{2}\ \text{(when}\ \frac{V_{\sim}}{V_{=}}=\sqrt{2}\right]$, depending on the relationship between the direct and alternating components of the voltage. For a maximum value of the modulation coefficient, heating with the direct current te component can insure thus a $\sqrt{2}$ greater magnitude of the amplitude of the temperature pulsation in comparison with heating with alternating current for the same frequency of modulation and an average temperature.

Formula (18.29) for determining the heat capacity, with consideration of the formulas (18.43) and (18.44), takes on the form of

$$c_p = \frac{2V_{\sim} V_{=} R}{(R + R_1)^2 M\omega \, |\theta|} \qquad (18.45)$$

or, if $V_{\sim}$ and $V_{=}$ are measured directly on the specimen:

$$c_p = \frac{2V_{\sim} V_{=}}{RM\omega \, |\theta|} \, . \qquad (18.46)$$

The basic characteristics of the method of heating with a direct current component are:

1) Heating takes place with the same frequency as the frequency of the change in voltage, but the second harmonic of the power is also present;

2) The modulation coefficient depends on the ratio $\frac{V_{\sim}}{V_{=}}$ and varies from 0 to $\sqrt{2}$.

A comparison of the two examined methods for modulating the heating does not make it possible to express the decisive preference for either one.

The method of heating with alternating current is simpler and convenient as a result of the harmonic nature of the power and temperature variations. In the method of heating with a direct current component, it was necessary to separate the second harmonic of the signal of the temperature change or to work with small modulation coefficients in order to make the amplitude of this harmonic relatively small. At the same time, this latter method has advantages when the recording device employs synchronous detection because the voltage itself, which is used for modulation, can here be used as the reference signal of the synchronous detector; in the method of heating with alternating current, an additional frequency multiplier is required for this purpose.

A second fundamental problem of the procedure of measurement is that of determining the magnitude of the temperature pulsation. In order to measure the amplitude of rapid temperature variations of wires (or other such objects) along with the photoelectric method already described in §3, it is possible to use also methods which are based on a change in the thermoelectron emission and the resistance of the specimen. Here, we combine the examination of the characteristics of realizing different methods for measuring the temperature pulsations with a description of specific experimental operations.

Measurements of the heat capacity by the photoelectric method of recording the temperature variations are described in [26, 37, 35, 36, 38, 169]. In all these references, heating with alternating current was employed.

In order to determine the amplitude of the temperature pulsation in [26, 35, 36], use was made of the relationship

$$\theta_1 = \frac{\bar{T}^2}{\alpha} \cdot \frac{Y_\sim}{Y}, \qquad (18.47)$$

where Y ~and Y ~are the alternating and direct current components of the signal at the exit (by Y is meant the current as well as the voltage). The coefficient α is equal to

$$\alpha = \frac{h\nu^*}{k}, \qquad (18.48)$$

where ν^* is the effective frequency of emission. The relationship (18.47) has been discussed in detail in §2 and 3. Measurements were carried out with a narrow spectral interval of emission (use was made of interference light filters) as well as with a broad interval (work was carried out without light filters). The coefficient α was found from the angle of the slope of the experimental curve of lnY as a function of 1/T. The magnitude α was in this instance practically constant within the temperature range of the order of hundreds of degrees. Photomultipliers and photoelectric cells were used to receive the emission. One of the systems used is shown in Figure 9. Such a method of measuring the magnitude of the temperature pulsation was subjected to a detailed check by a broad variation of the experimental conditions. In particular, a study was made of the reproducibility of the results obtained with different emission receivers, and work with light filters and without light filters, a study was made of the role of the operating conditions of the photomultipliers, the effect of noises was evaluated, etc. The object of the study in this work was the wolfram filament of the LM-2 manometric tube which was heated with an alternating current of a frequency of 50 Hertz. The average temperature of the filament was determined by a twin method--with an OMP-19 optical pyrometer and from the resistance of the filament itself; both methods gave results that agreed within the limits of $\sim$1%. The resistance of the filament was measured by means of a potentiometric circuit with the filament being heated with direct current. The average temperature of heating with a direct current was established as being equal to the sought-for average temperature of heating with an alternating current by selecting a direct current for which the signal at the exit of the recording device would be equal to such during measurements with alternating an current (see [26, 35]).

As a result of the completed work, it was established that the method of measurements under consideration insures reproducibility of the values of the amplitude of the temperature variations of the order of degrees with an error of 0.5-1%. The maximum error consists, essentially of random error and possible systematic errors due to the determination of ($Y \sim$ (0.5-2%); Y = (0.5-4.5-7%); α(0.5-1%) and f^2(1 + 1%): its sum can reach up to 4.5-7%. The difference in the values of the heat capacity of wolfram, obtained for the temperature range of 2,000-3,000°K by means of the described variant of the procedure from data by other authors, does not exceed this magnitude [26].

The relative variant of the measurements of the amplitude of the temperature variations by the described method was employed by I. A. Akhmatova for determining the heat capacity of liquid tin within the temperature range of 900-1,700°C [38]. Tin was included in the capillaries made of niobium, 60-80 mm long and having an internal and outside diameter (in mm) as follows: 0.155 and 0.28; 0.285 and 0.485, respectively. Heating was with alternating current of a frequency of 18 Hertz. A photomultiplier served as the emission receiver; the direct current component was determined by means of a current instrument which was connected in series with the load of the photomultiplier, the alternating current voltage on the load was determined by means of a cathode voltmeter (such a circuit of measurements was used earlier by Lowenthal [37].) The specific feature of the experiment, which makes the measurements relative, consisted in carrying out two measurements--with hollow and filled capillaries. These measurements were carried out for the same values of the direct current component of the current through a photomultiplier, that is, for equal average temperatures. Such a method measurements made it possible to exclude from the final formula the value of the constant α and to express the heat capacity of the emitting liquid metals through the heat capacity of the material of the capillary walls. The error in determining the volume heat capacity was evaluated by the author as 5-7%. (Data by I. A. Akhmatova are given in §21). A somewhat different method to determine the amplitude of the pulsation temperature was used in the work by Ya. A. Krafmakher and V. O. Shestopal [169]. Formula (18.47) is also the basis of this method; however, this formula is not used directly for determining the amplitude, but for a formula which contains in the right-hand part the difference of two similar expressions obtained from experiments with two light filters:

$$|\theta| = \frac{\bar{T}^2}{A}\left(\frac{Y_{1\sim}}{Y_1} - \frac{Y_{2\sim}}{Y_2}\right). \qquad (18.49)$$

In the opinion of the author, such a method should give values of the constant A that are more steady than the values of α, because in determining A, the effect of the temperature function of the degree of blackness is almost completely excluded. (The correction to the magnitude A due to the temperature function of the degree of blackness is determined not by the magnitude $(d\ \ln\varepsilon_\nu/d1/T)$, as a correction to α (see formula (2,35)), but by the magnitude $\frac{d \ln \varepsilon_{\nu_1}/\varepsilon_{\nu_2}}{d1/T}$, in which the ratio of the degree of blackness for the two wave lengths figures as well as for the correction to the color temperature.) It should be observed, however, that a utilization of such a procedure of measurements, which is twice as complicated, would be justified if the magnitude α varied with the temperature so strongly that this would make it inconvenient to process the results and it would show up on the accuracy of the measurements. In practice, however, the magnitude α varies weakly with the temperature. Besides, the existing dependence of α on the temperature is explained apparently not so much by the role of the temperature function of the degree of blackness as by the insufficient monochromatization of the emission (see Evaluations in §2).

This pertains all the more to the case of measurements of the heat capacity of graphite, which were carried out in the work under consideration because in the case of graphite, the magnitude ε_ν varies with the temperature more weakly than in the case of metals.

The recording device used by Ya. A. Krafmakher and V. O. Shestopal is similar with that described in the previous work. The test specimens were rods 0.6-0.8 mm in diameter; heating was with an alternating current of a frequency of 50 hertz. The measurement encompassed the temperature range of 1,750 to 2,850°. A study was made of five specimens of pure graphite. The error of the measurement was 5%.

Another method of recalculating the readings of the recording system to values of the amplitude of the temperature variations was used by Lowenthal [37]. He used for this purpose the power formula

$$Y = gT^{b(T)}, \tag{18.50}$$

from which as a first approximation

$$|\theta| = \frac{T}{b(T)} \cdot \frac{Y_\sim}{Y}. \tag{18.51}$$

Such a form of the dependence Y(T) was selected by Lowenthal apparently because of a desire to approach this dependence to the Stephen-Boltzmann law, and having in mind the relatively broad spectral interval which was used in the experiment without the light filter.

The utilization of formula (18.51) does not have any advantages in comparison with formula (18.47). On the other hand, the dependence of b on the temperature shows up comparatively strongly. Thus, with a change in the temperature from 1,200 to 2,400°K, the magnitude b varies from 12 to 7.3. The measurement system which was used by Lowenthal, and as already pointed out, consisted of a multiplier, current instrument, and cathode voltmeter. Measurements were made of the heat capacities of wolfram, (from 1,267 to 2,410°K), molybdenum (from 1,288 to 2,015°K), tantalum (from 1,256 to 2,300°K), and niobium(from 1,471 to 2,260°K).

The prospects for further perfection of the measurement of the heat capacity with photoelectric recording of the temperature pulsations are related with the utilization of heating by means of a sum of direct and alternating currents. For such heating, we get from formulas (18.46) and (18.47) the equation

$$c_p = \frac{2V_{=}\alpha Y}{RM\omega\bar{T}^2} \cdot \frac{V^2_{\sim}}{Y_{\sim}} . \qquad (18.52)$$

A fundamental characteristic of this formula is the fact that the magnitude V ~ and Y ~ , the error in the measurement of which introduces one of the most significant contributions into the error of the determination of the heat capacity, enter the formula as a ratio. This makes it possible to exclude the systematic error in the determination of V ~ as well as of Y ~, if the measurement of these magnitudes is carried out by the same instrument (cathode voltmeter) so that this instrument would always show the same voltage, which is a part of V ~ and Y ~, which is taken off from the calibrated voltage dividers. As a variable arm of the voltage dividers, use can be made of the nonreactive resistance boxes (for example, MSRB-48). Formula (18.52) will in this instance have the form of

$$c_p = \frac{2V_{=}Y_{\alpha}}{RM\omega\bar{T}^2} \cdot \frac{(r + r_1)\, r_2}{(r + r_2)\, r_1} , \qquad (18.53)$$

where r is the resistance of the constant arms of the dividers; r_1 and r_2 are the resistances of the variable arms of the voltage dividers in the circuit V ~and Y ~ , respectively, for which readings of the recording instruments are equal to e ~ --to the same value in all the measurements, selected on the edge of one of the instrument scales. The errors in the determination of r_1 and r_2 in such measurements will be determined only by the sensitivity of the instrument employed near the values of e ~ and by the stability of V ~and Y ~ . The maximum error in the determination of the heat capacity in these measurements will amount to 2.5-4%.

A second method of determining the magnitude of the temperature pulsation, as specifically for the method of heating with an alternating current, is based on the utilization of the effect of the thermal electron emission. This method is obviously suitable under conditions of comparatively high temperatures when the emission current from the test object is sufficently great.

In order to determine the amplitude of the temperature pulsations from the thermal electron emission, the test specimen should be a cathode of a double-electrode electron tube, that is, it should be placed in a vacuum of the order of 10^{-5} mm mercury along (or within) with the second electrode--anode. In heating the cathode-specimen with alternating current (with the direct current component or without it), the anode current through the tube will have, along with the direct current component (average current I), also an alternating current component $I\sim$ which is determined by the temperature pulsation of the specimen. The relationship between $I\sim$ and the amplitude of the pulsation $|\Theta|$ for small $|\Theta|/\overline{T}$ is obviously given by the formula

$$|\theta| = \frac{I_{\sim}}{\frac{dI}{dT}}, \tag{18.54}$$

which can be rewritten in the form similar to formula (18.47):

$$|\theta| = \frac{\overline{T}^2}{\alpha} \cdot \frac{I_{\sim}}{I}, \tag{18.55}$$

where

$$\alpha = \frac{d \ln I}{d\frac{1}{T}} \tag{18.56}$$

The temperature function of the anode current described by the Dashman formula (see, for example, [162])

$$I = AT^2 e^{-\frac{eu}{kT}} \tag{18.57}$$

where e is the electron charge, u is the work function of the electron, A is the coefficient which does not depend on the temperature. From this formula we get for α

$$-\alpha=\frac{eu}{k}+2T. \qquad (18.58)$$

Considering that eu/k for refractory metals is in order of magnitude equal to $5 \cdot 10^4$, we come to the conclusion that α should depend weakly on the temperature just as the similar coefficient in the photoelectric method of recording. Besides, in accordance with formula (18.58), the correction for the temperature function can be made in a simple manner:

$$\alpha=\frac{d\ln\frac{I}{T^2}}{d\frac{1}{T}}-2T. \qquad (18.59)$$

Thus, the coefficient α can be determined from the calibration experiment from the angle of the slope of the straight line $ln\ (1/T^2)$ as a function of 1/T.

The first work which deals with the measurements of the heat capacity with recording of the temperature variations from the thermal electron emission was carried out in 1922 by Smith and Bigler [170]. In order to record the temperature variations, they utilized a loop oscillograph which was connected in the anode circuit of the diode, the test wire, which was heated by alternating current to the frequency of 60 hertz, served as the cathode of this. Measurements were carried out within a narrow temperature range of 2,370-2,485°K. In 1925 similar measurements were made by Bocksteller [171]. A much wider range of temperatures was utilized by Zwikker [172] (2,200-2,500°K for untreated wolfram and 1,600-1,873°K for thoriated wolfram.) In order to record the alternating current component, Zwikker used a more improved system--rectification by means of a separate diode, a measurement with a mirror galvanometer.

The data obtained by Zwikker agrees with these measurements which were carried out by another method in observing the rates of temperature change of wires with a rapid change in heating. (The characteristic time of the process of temperature change was determined by means of a pendulum system. As soon as the tension on the test wire, which varies with the variation in the resistance with time, reached a definite magnitude, a high voltage was fed from the Ruhmkorff coil to the pendulum by means of a relay. As a result of this, a spark discharge developed at the end of the pendulum, the position of this spark was determined). Here, it is pertinent to mention also other work which deals with the measurement of the heat capacity of wires at high temperatures by means of the pulse variation and heating. The first of these was carried out in 1912 by Corbino [173]. The rate of temperature change in the wire with time was determined from the integral effect--the reading of a ballistic galvanometer to which a voltage was fed which was proportional to the deviation of the resistance of the test wire from the equilibrium, steady-state voltage. The Corbino method was utilized in 1918 by Worthing in order to measure the heat capacity of wolfram [174]. Along with this method, Worthing utilized also another method which was his own and in which the time-dependence rate of change in the current and voltage, which passed through the test wire, were determined by means of a pendulum system which included a recording galvanometer, after a definite, known time interval following change in heating. The last work with pulsed stepwise heating was carried out in 1964 by Taylor and Finch [175]. In order to record changes in current and voltage, use was made of a cathode two-beam oscillograph. (The utilization of stepwise heating of samples in a form of rods is described also in [67, 90]. The work by Ye. S. Platunov [176] deals with the utilization of so-called pulsed-dynamic conditions under which the average temperature of the surrounding cells varies monatomically, while the heat is introduced into the specimen in pulses.) A more complex form of the pulse, which consists of two half symmetric, lines with respect to a horizontal straight line, was utilized in 1954 by Pokhapskiy [177]. The measurements, which were of a relative nature (graduation by measurements with platinum) encompassed a temperature range from 0 to 650°C.

The results of a detailed study of the thermal electronic method for determining the temperature pulsations are presented in [35]. The circuit which was utilized for this purpose is shown in Figure 65.

The object of the measurements was the same manometric tube LM-2 (the same speciment as in the above-described experiments with the photoelectric method of determining pulsations). The two grids made contact with the anode. For measuring the constant component of the anode current, use was made of the M-193 milliammeter, the variable component of the kind was determined from readings of a cathode voltmeter (LV-9M) and from the known load resistance. The form of the voltage oscillations was controlled with the aid of a cathode oscillograph. The temperature was determined from the resistance of the incandescent filament by means of a potentiometric circuit with a constant current (potentiometer PPTN-1 with a mirror galvanometer M 21/4). The magnitude of the direct current was selected so that the constant component of the anode current would be equal to such during incandescence with an alternating current at the selected temperature. The evacuation system consisted of a forevacuum pump of the VN-461 type, a diffusion pump TsVL-100, and a titanium pump. The program of studies included measurements with different loads of the tube, with a different anode voltage, for different vacuums, depending on the different conditions of prior incandescence of the specimen. The basic results are as follows:

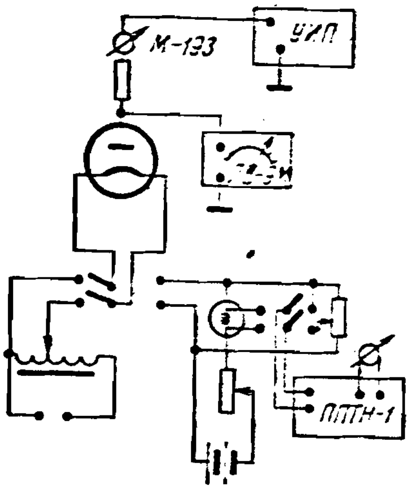

Figure 65. Circuit of unit for measuring the heat capacity (thermo-electric method for recording temperature variations).

The determination of the coefficient α by means of the relationship (18.59) requires that a reading of the values of the currents and of the average temperature should be conducted as rapidly as possible. If the series of measurements is recorded in the course of an hour, the dependence of I/T^2 on I/T differs from a rectilinear function, which is explained by the slow change of the anode current with time, which takes place despite the preliminary incandescence of the specimen for many hours at a much higher temperature. Experience shows, however, that this change is not related in any way with any noticeable change of the work functions because the values obtained for different times but sufficiently rapidly, are reproduced satisfactorily, within the limits of 0.5-1% (it is most convenient to determine α directly from the two values of I and T). The good reproducibility (approximately 1%) is observed also as a whole for values of the amplitude of temperature variations, obtained under the most diverse experimental conditions; besides, the values found agree with the same accuracy with the values obtained by the photoelectric method (maximum difference 1.6% and average difference 0.5%). This circumstance is illustrated by Figure 66 (the ordinate axis shows the ratio of the amplitude of the temperature pulsations $|\theta|$ to the heating power W).

As a result of the work, it can be concluded that the method which is based on the thermo-electron emissions assures measurements of temperature pulsations with approximately the same accuracy as the photoelectric method. A disadvantage of the method is the need to carry out measurements in vacuum and the above indicated instability of the current.

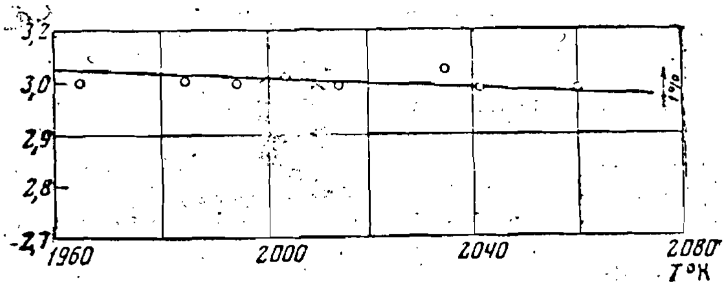

Figure 66. Comparison of the values of temperature pulsations, obtained by recording the thermo-electron emission (solid curve) and by the photo-electric method (points).

A third method for determining the amplitude of temperature pulsations--from the resistance pulsations--is accomplished in a different manner for heating with alternating current and for heating with direct components.

Let us examine the first of these cases. In heating with alternating current, the specimen resistance pulsates with a double frequency:

$$R_\sim = \frac{dR}{dT}|\theta| \sin 2\omega t. \tag{18.60}$$

It is easy to show that such a periodic change in the resistance of the circuit of variable voltage (V ~ cosωt) leads to the appearance of a component of alternating current of a combination, triple frequency. Actually,

$$\begin{aligned} I_\sim &= \frac{V_\sim \cos \omega t}{R+R_1+R_\sim} = \frac{V_\sim \cos \omega t}{R+R_1} - \frac{V_\sim R}{(R+R_1)^2}\beta|\theta| \cos \omega t \sin 2\omega t = \\ &= \frac{V_\sim}{R+R_1}\left[\cos \omega t - \frac{1}{2} \cdot \frac{R|\vartheta|}{R+R_1} \sin \omega t\right] - \\ &\quad - \frac{V_\sim R\beta|\theta|}{2(R+R_1)^2} \sin 3\omega t = I_\omega - I_{3\omega}. \end{aligned} \tag{18.61}$$

Measurement of the current component of triple frequency was carried out for the first time by Corbino in 1911 [178]. In order to observe $I_{3\omega}$, Corbino used an oscillographic device in which scanning along one of the axis was carried out with a current of basic frequency and along the other axis--by the difference of the currents ($I\sim - I_{\omega}$) (For this purpose, use was made of a coil with two windings). The relative magnitude of the current of tripled frequency was evaluated from the dimensions of the Lissajou figure.

Another method of separating the current component of triple frequency is to measure the voltage on the diagonal of the bridge circuit, in one of the arms of which the test specimen is connected, under conditions such that the circuit is balanced on the basic frequency. On the basis of (18.61), it is easy to show that the voltage of the triple frequency on the diagonals of the bridge in this case (with an accuracy up to a small correction proportional to $|\theta|^2\beta^2$) will be equal to

$$V_{3\omega} = \frac{V_{\omega}}{2} \cdot \frac{R \cdot R_1}{(R + R_1)^2} \beta |\theta|, \qquad (18.62)$$

where R_1 is the resistance of the adjoining arm.

Such a circuit for measurements was utilized in [179, 124, 180], in which the measurements of the temperature pulsations of a strip of foil placed in a liquid served as a means for determining the thermal activity of the liquid (see Figure 67). In order to separate the voltage of triple frequency, the signal from the diagonal of the bridge of variable current 1 (rheochord bridge R-38) was fed to the selective amplifier 2 assembled in accordance with the system with T-shaped RC-filters in a circuit of negative feedback. In order to control the absence of voltage of basic frequency at the output of the amplifier, the cathode oscillograph 3 was connected, the frequency of scanning of which was selected equal to the frequency of 3ω. In the presence of voltage of frequency ω, the screen shows three shifted curves, one with respect to the other, which merge into one single curve with complete equilibration of the bridge at the frequency of ω.

In order to compensate the reactive component of the resistance, which develops as a result of the appearance in the current of basic frequency of a component that is shifted in phase with respect to the feed voltage (under conditions when the foil is placed in a liquid), a magazine of capacities 4 was introduced into the bridge circuit. The measurements of the voltage $V_{3\omega}$ reduce themselves to the selection of the magnitude of resistance 5 (the magazine of nonreactive resistances of the type MSRB-48)--one of the arms of the voltage divider taken from the diagonal of the bridge--for which the signal at the output of the amplifier was equal to a previously selected magnitude, at the edge of the scale of the cathode voltmeter c of the LV-9-M type). Such a method of measuring the voltage decreased the errors related with the reading of the voltmeter scale and the errors which could be caused by the non-linearity of the amplifier characteristics. In order to see the bridge, use was made of a sonic generator of the 3G-2A type (the working frequencies were within the range of 20 to 200 Hz) or a voltage of 50 Hz from a network. The generator 7 was utilized for calibrating the amplifier. The adjustment of this generator for a frequency of 3ω was controlled by the oscillograph 8 (from daily Lissage figures). (Instead of a generator, one can use a frequency multiplier which insures the development of a third harmonic.)

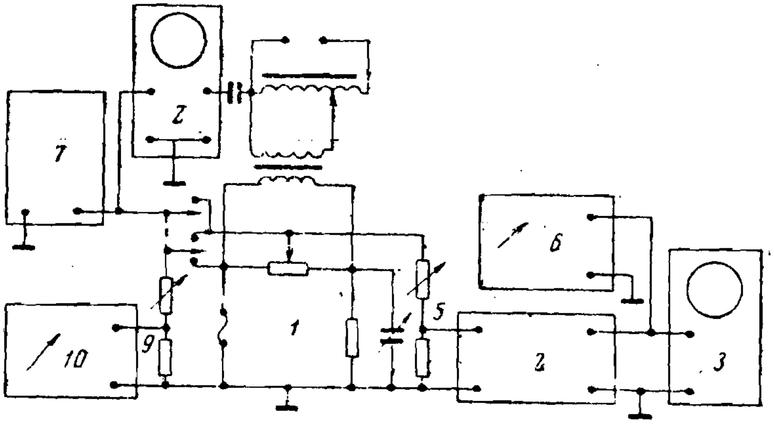

Figure 67. Circuit of unit for measuring resistance pulsations of a wire heated with alternating current.

The amplitude of the alternating voltage of frequency ω on the sample was measured also not directly but from the ratio of the arms of the voltage divider 9, similar to the divider 5. The voltage indicator was the cathode voltmeter 10 (the functions of the oscillographs 3 and 8 and of the voltmeters 6 and 10 could of course be combined). The scribe system insured measurement of temperature pulsations on the order of tenths of a degree with a random error of ~ 1%.

A similar method of determining temperature pulsations was utilized by Holland [181] in measurements of the heat capacity of titanium. As a bridge circuit, Holland used a Thompson double bridge. The signal from the diagonal of the bridge, after amplification, flowed to a synchronous detector. As a support signal for the synchronous detector, use was made of the signal from the frequency tripler, the input of which was connected to the source of the alternating current that fed the bridge circuit (frequency 8.6 and 14.95 Hz). The sample had a diameter of 0.25 mm and a length of 40 mm. The measurements encompass the temperature range from 600° to 1,345°K. The error in the results varied from 3-7%.

Another, somewhat more complex method for determining temperature pulsations from the temperature function of the resistance when heating with alternating current was used by Gerlich,Abeles,and Miller [182] for measuring the heat capacity of semiconductors. In this method, a low frequency (1 Hz) voltage for variable heating and a voltage of carrying frequency of 300 Hz were fed simultaneously to the test specimens. The temperature pulsation of the specimens resulted in the appearance of a low-frequency two-Hertz modulation of the signal of the carrying frequency, while the component of the signal of 1 Hz was compensated at the input. The modulated voltage was amplified and detected with the aid of a phase detector and was fed to the vertical plates of the cathode oscillograph. The voltage of the heating current of 1 Hz was fed to the horizontal plates of the oscillograph. The Lissajou figures on the oscillograph were photographed. By means of the described method, the authors measured the heat capacity of germanium, silicon and two of their alloys within the temperature range of 300-1,000°K. The reproducibility of the results in this work was approximately 2% and the systematic errors ranged from 3-5%.

Later, we shall present the method of determining temperature pulsations from the resistance under conditions of heating with a direct current component. The idea of this modification of the method of measurement, which belongs to Ya. A. Kraftmakher [183] is based on the fact of a shift in phases, which is close to $\pi/2$ between the modulating voltage and the temperature and thus between the voltage and the pulsation of the resistance, this shift in phases, which exists also between the voltage and the additional component of the alternating current, caused by the pulsation of the temperature, is equivalent to the apparent reactive component of the resistance of the specimen, which has a capacitance nature. This reactive component, which depends on the heat capacity of the specimen, can be measured from the electrical capacitance with which it is necessary to shunt the adjoining arm of the bridge in order to balance the bridge on the basic frequency.

In order to derive the relationship between the equivalent electrical capacitance C and the heat capacity c_p, we present formula (18.41) for current through the specimen in the form of

$$I = \frac{V_{=} + V_{\sim} e^{i\omega t}}{R_1 + R - R\beta \, | 0 | \, i e^{i\omega t}} \simeq I_{\omega} + I_{2\omega} + \frac{e^{i\omega t} V_{\sim}}{R_1 + R - R \frac{V_{=}}{V_{\sim}} \, | \theta | \, i\beta}. \quad (18.63)$$

(In this instance, it was assumed that $|R_{\sim}|/R \ll 1$.) The last member of this formula gives the expression for current of frequency ω. For impedance of the entire circuit we get from this

$$Z = R_1 + R - R \frac{V_{=}}{R_{\sim}} | 0 | \, i\beta. \quad (18.64)$$

This impedance can be represented as a sum of the active resistance R_1 and the impedance Z_R, formed by the active resistance R and its parallel capacitance resistance iωC:

$$Z_R = \frac{R}{1 + iC\omega R} \approx R - iC\omega R^2. \quad (18.65)$$

For the equivalent capacitance, we get from this the formula

$$C = \frac{V_{=} | \theta | \beta}{V_{\sim} \omega R} = \frac{2 V_{=}^2 \beta}{\omega^2 M c_p (R + R_1)^2}. \quad (18.66)$$

or, if $V_{=}$ is measured directly on the specimen,

$$C = \frac{2 V_{=}^2 \beta}{\omega^2 M c_p R^2}. \quad (18.67)$$

Formula (18.67) makes it possible to determine the heat capacity from the magnitude of the measured equivalent capacitance C.

In the unit developed by Ya. A. Kraftmakher [183], use was made of specimens in the form of a wire 0.03-0.05 mm in diameter and 30-60 mm long; the frequency of the modulation was equal to 120 Hz. The signal from the output of the bridge circuit, after amplification was fed to the synchronous detector. The sensitivity amounted to 0.1% for an amplitude of temperature pulsations of ~1°. The reproducibility of the results reached 1%. An evaluation of the maximum error was not made. The heat capacity of wolfram [184], molybdenum [185], tantalum [186], and niobium [187] was measured.

In comparing the presented modifications of the experiments, which are based on determining the temperature pulsations from the temperature function of the resistance, we wish to emphasize the following points.

The advantages of the Ya. A. Kraftmakher method are the simple, compensatory nature of the measurement and the circumstance that, in order to determine the heat capacity by this method, alternating voltages are not required. A disadvantage of the method is the comparatively strong, quadratic decrease in the measured capacitance with rising frequency, which hinders the utilization of this method for relatively large frequencies (for the desirability of studies on high-frequencies, see the next section). The method which was used by Gerlich, Abeles and Miller, is distinguished by the fact that it is possible to use it for measurements with small frequencies which are required when working with comparatively massive specimens, that is, under conditions when others of the methods examined here are not suitable. As regards the first of the described methods, which is based on the separation of the signal of triple frequencies when heating with alternating current, it is not inferior to the Ya. A. Kraftmakher method with respect to its sensitivity, but it can give a somewhat greater error because of the need of measuring the alternating voltages. In order to work with relatively large frequencies, this method should, however, be more suitable than the other two.

The above described methods for determining temperature pulsation, which are based on the temperature function of emission, thermal electron emission, and resistance, can be carried out in one more modification, the idea of which is, for all the methods that utilize heating with a totality of direct and alternating currents.

Let us examine the circuit in which the signal $Y_{\sim}$, which is taken from the outlet of the recording circuit, is fed to the block that contains the wide-band amplifier with a constant amplification coefficient k and a phase filter which shifts the signal phase by $\pi/2$. The signal from the exit of this block flows to the circuit for heating the specimen in succession with a source of direct current voltage, that is, it plays the role of the magnitude $V_{\sim}$..

Such a circuit, which contains the circuit of positive feedback, will represent an auto-oscillating system, the frequency of the oscillations of which can be found from formula (18.46) and the relationships:

$$V_{\sim} = kY_{\sim} \tag{18.68}$$

and

$$Y_{\sim} = \frac{\partial Y_{=}}{\partial T} |\theta|. \tag{18.69}$$

In this instance, we get the expression

$$\omega = \frac{2kV_{=}^{2}}{RMc_{p}\frac{\partial \ln V_{=}}{\partial T}}. \tag{18.70}$$

This formula can be utilized for determining the heat capacity from the magnitude of the constant component of the voltage, frequency, and amplification coefficient. The advantages of such a method are related with the absence of the need for measuring the values of the variable voltages. If the magnitude k is stable and the frequency range under consideration, the determination of c_p by means of this formula will give comparatively small error.

Let us summarize finally the results of all the methods for determining the temperature pulsations. With respect to the sensitivity, all the basic methods of measurement are approximately equivalent, they ensure a reproducibility of results with an accuracy of 0.5 to 2% for an amplitude of temperature pulsations from several tenths of a degree to degrees. All three methods in one or another form need calibration measurements: in the photoelectric method it is necessary to determine the effective length of the wave; in the thermal electron method, it is necessary to determine the work function of the electrons; in the resistance method, it is necessary to determine the temperature coefficient of the resistance. Each of these magnitudes can be found approximately with the same accuracy. The differences amount essentially to the convenience in determining these magnitudes. The effective wavelength and the temperature coefficient of the resistance are determined alike simply. However, measurements of the second magnitude is preferred because this very magnitude is of fundamental interest, being an important characteristic of the metal. On the other hand, in monochromatization of the emission in the photoelectric method, it is possible in general to avoid calibration measurements, whereas in the method which is based on temperature function of the resistance, measurement of the coefficient β is always desirable (it is not recommended to utilize tabular values). As regards the thermal electron method, as already pointed out, the calibration measurement in this case are less convenient. There is a difference also in the minimum average temperatures allowable for the study. The method based on the resistance makes it possible to study a broader temperature range, then follows the photo-electric method and, finally, the thermal electron method.

As regards the convenience of conducting measurements and the simplicity of the experimental units, the thermal electron method is in this respect, indeed, the simplest method (if one does not take into account the need for utilizing deep vacuum), while the remaining two methods are approximately alike. An additional advantage of the photoelectric method is that in the convenience of combining it with measurements of the heat conductivity by the wire method with the utilization of a differential optical pyrometer (see §5). In this case, the same recording circuit insures measurement of both thermal parameters [188].

It follows from all that has been presented above that it is hardly expedient to give a universal recommendation for the selection of a method for determining pulsations and the examined method for measuring the heat capacity; the problem can be solved under different conditions in a different way. At the same time, it can be emphasized that all the diverse methods for accomplishing such experiments possess advantages that are common for all of them: the working part of the instrument (specimen plus vacuum chamber) is simple to the maximum degree; the experiment is carried out rapidly; a temperature range up to the very melting point of the most refractory materials is assessible; the measurement circuit, which is based on the utilization of radio technical devices, ensures a convenient and reliable recording of the necessary parameter; the accuracy of the measurements is comparable with the accuracy of the best calorimetric experiments.

Chapter IV

Heat Capacity of Metals

§19. Basic Correlation in the Behavior of the Heat Capacity of Solid Metals at High Temperatures

We shall briefly present the basic physical considerations which pertain to the problem under consideration.

It is known that a basic theoretical correlation in the temperature range above the Debye point is the Dulong and Petit law in accordance with which the heat capacity for a constant volume of solid mono-atomic bodies, referred to as gram-atom should be equal to the constant magnitude of $3\,R \simeq 6$ cal/deg (see, for example, [189]). Experimental studies show, however, that such a simple correlation takes place only for comparatively low temperatures and in addition not completely accurately. Deviations from the Dulong and Petit law increase rather rapidly with rising temperature, reaching in a number of cases tens of percent. In order to become convinced of this, it is sufficient to examine one of the curves shown in §20.

The most elementary reason for the deviation of the experimental data from their magnitude predicted by the DeLong and Petit law is the difference in the measured values of C_p from the heat capacity at the constant volume of C_v. For the difference $C_p - C_v$, we get from thermo dynamics the general formula

$$C_p - C_v = \frac{vT\alpha^2}{\beta}, \tag{19.1}$$

where α is the coefficient of the volume expansion, β is the isothermal compressibility, v is the atomic volume. Specific calculations by means of this formula are difficult because of the absence of experimental data for the magnitudes of β at high temperatures. Nevertheless, if it is taken into consideration that the magnitude β varies with the temperature usually comparatively weak, evaluative calculations become quite real. Such calculations have been conducted in particular by Lowenthal [37] on the basis of the assumption that a ratio v/β remains constant and equal to such at 300°K. The results of the calculations lead to the conclusion that the difference of C_p from C_v is relatively small at low temperatures, increases comparatively rapidly with rising temperature, and reaches magnitudes which amount only to a noticeable, although far from basic, part of the difference $C_p - 3R$. Thus, for example, for wolfram $C_p - C_v/C_p$ at 300°K is equal to 0.01, at 1,500°K, it is equal to ~0.05, while above 2,200°K ~0.1.

Another method for the approximate calculation of the difference $C_p - C_v$ has been proposed by B. N. Oshcherin [190]. For the magnitude $\gamma = C_p/C_v$ he gave the formula

$$\gamma = 1 + \frac{\alpha T}{a}, \tag{19.2}$$

where a is the relative packing density of the atoms; a = 0.74 for a cubic phase centered lattice; 0.68 for a body centered lattice; and 0.34 for a lattice of the type of diamond. Calculations in accordance with this formula lead apparently to better agreement with the calculations in accordance with a strict thermodynamic formula (19.1) than the calculations in accordance with the Nernst and Lindemann empirical relationship

$$C_p - C_v = aC_p^2 T, \tag{19.3}$$

(a is the constant), which was utilized in most references ([174, 191, 192]).

The formula by B. N. Oshcherin gives a weaker dependence of $C_p - C_v$ on the temperature in comparison with the formula (19.1) for the ratio v/β which was independent of the temperature. In accordance with this, the magnitude $C_p - C_v/C_p$ at high temperatures is, in accordance with the evaluations by B. N. Oshcherin, somewhat smaller than in accordance with the evaluations by Lowenthal. Thus, for the same wolfram at a temperature of 2,300°K, B. N. Oshcherin gives $C_p - C_v/C_p \approx 0.065$.

Thus, the difference between the heat capacity at a constant pressure and constant volume is one of the factors which determine the deviations of the experimental values of the heat capacity from the Dulong and Petit's law at high temperatures, in particular near the melting point. The contribution of this factor explains, however, only a small share of these deviations.

A second possible reason for the deviations from the Dulong and Petit law at high temperatures is the role of the anharmonicity of the vibrations of the atoms in the lattice. The physical cause of the influence of this effect consists of a violation of the classical law of the uniform distribution of energy by degrees of freedom when the thermal vibrations of the atoms are not harmonic, that is, when the potential energy of the system contains not only quadratic members of the magnitude of the displacements of the atoms from the equilibrium positions, but also members of the third and higher powers. An analytic examination of this problem [194] leads to the conclusion that a magnitude of the anharmonic contribution to the heat capacity depends simultaneously on two anharmonic parameters: the coefficient for the third and fourth powers of decomposition of the potential energy with respect to the redisplacement of the atoms; besides, the first of these gives a positive contribution and the second a negative. Such a complex nature of this function makes any kind of accurate calculations of the anharmonicity difficult.

Evaluations of the magnitude of the anharmonic coefficients, obtained by using experimental data with respect to the coefficient of expansion and the temperature function of the module of elasticity, give only approximate values for the summary contribution of the anharmonic members in view of the small accuracy of the original data. Nevertheless, the results of such evaluations for sdium [194] and iron [195] give grounds for assuming that the role of anharmonicity is not negligibly small and should be taken into consideration. This, however, does not mean that one can use the anharmonicity to account for all deviations from the Dulong and Petit law, as this was done, for example, by Holland [181].

A third possible cause for the violation of the Dulong and Petit law is the role of the electron heat capacity. It is known that, as a result of the degeneration of the electron gas and metals, the magnitude of the electron heat capacity amounts usually to a very small share of the total heat capacity. It is possible to detect the existence of electron heat capacity only in the range of very low temperatures due to the fact that electron heat capacity varies with the temperature linearly:

$$C_{\text{electron}} = \gamma T \qquad (19.4)$$

whereas the heat capacity of the lattice at low temperatures depends on the temperature at T^3:

$$C_{\text{lattice}} = bT^3 \tag{19.5}$$

The electron heat capacity at low temperatures becomes noticeable on a background of low heat capacity of the lattice.

The region of high temperatures is the second region where the role of the electron heat capacity can be noticeable as the result of the constancy of the lattice heat capacity (in a harmonic approximation) all the more so since, in accordance with formula (19.4), the electron heat capacity for large T becomes greater.

A quantitative study of the problem of the magnitude of the electron heat capacity at high temperatures is of great interest because it makes it possible to expand substantially the information which is obtained from low temperature studies. Work in this direction is all the more desirable since a knowledge of the coefficient of electron heat capacity γ, in accordance with the zonal theory, makes it possible to obtain information on the fundamentally important physical magnitude--density of electron state near the Fermi surface $N(E_F)$ [196].

We wish to point out that the term Fermi surface is used to designate a surface in the k-space, the space with wave numbers of electrons, which are characterized by this fact that the energy of the electrons at this surface is equal to the Fermi level E_F, determined as the upper boundary of the energy of the electrons at absolute zero:

$$n = \int^{E_F} N(E)\,dE. \tag{19.6}$$

The Fermi level for a degenerated gas does not practically differ from the Fermi energy--chemical potential of electrons, which figures in the Fermi-Dirak distribution law:

$$f = \frac{1}{e^{\frac{E-E_F}{KT}} + 1} \tag{19.7}$$

For a degenerate electron gas in a metal, the function f differs little from the stepwise function

$$f = 1 \quad \text{for} \quad E < E_F, \tag{19.8}$$

$$f = 0 \quad \text{for} \quad E > E_F. \tag{19.9}$$

The values of the function f, which differ from 1 and 0, lie within a narrow range of values of E near E_F. The width of this interval has an order of magnitude of kT; besides $E_F \gg kT$ (it is precisely this condition that is the criterion of degeneracy of the electron gas). It is pertinent that only the electrons which are in this narrow layer near the Fermi surface have practically a finite probability of condition from one state into the other, that is, it is precisely these and only these that respond to an external effect, they are active electrons while the remaining electrons are "frozen". It is precisely this circumstance that determines this primary role which the Fermi surface plays in the electron theory of metals.

In the case when the Fermi surface differ comparatively little from a spherical surface $N(E_F)$ makes it possible to determine the so-called effective mass of the electron m*--the magnitude which characterizes directly the dynamics of the electrons in the external field. If the ratio of m* to the natural mass of the electron is in this case close to unity, the behavior of the electron differs little from such for a model of free electrons--the most simple model which does not take into account the direct interaction of the electrons with the ionic lattice. Considerable differences between the ratio of m*/m from unity indicates the strong influence of the ionic shell on the inertia properties of the electrons [197].

The problem of a possible explanation of the deviations from the Delong and Petit law by means of the role of the electron heat conductivity was discussed by Lowenthal [37] and Kohlhaas, Braun and Otmar [193]. Lowenthal obtained for the coefficient of electron heat capacity of molybdenum and wolfram at high temperatures values which were quite close to those found from low-temperature measurements; for niobium and tantalum, the high-temperature coefficients are however, 3-4 times smaller. Kohlhaas, Braun and Otmar obtained for the coefficient of electron heat capacity of titanium, vanadium and chromium, values which in order of magnitude agree with the theoretical calculations.

The conclusion which can so far be made from these references is that the electron heat capacity introduces without a doubt a potential contribution into the heat capacity of metals at high temperatures. As regards the problem of the possibility of making conclusions of a quantitative nature, it is necessary so far to show care in this case, considering the existence of errors caused by the inaccuracy of the calculation of the difference $C_p - C_v$ and by the indeterminancy of the correction for the anharmonicity. All the effects that have been examined up to now explain only a relatively weak cause of the dependence of the heat capacity on the temperature, it is close to linear. Not one of these is in a position to explain the steep rise of the heat capacity, which was observed in the range of temperatures above approximately 0.8 T_{melting}. This latter effect is ascribed by most authors to the role of the vacancies (Shottki effects)--unoccupied points of the lattice--which form in crystals near the melting point. The transition from one temperature to another is related with the change in the equilibrium concentration of the vacancies; formation of new vacancies with rising temperatures requires the expenditure of energy, as a result of which the heat capacity increases. In accordance with this, the additional heat capacity should be equal to

$$C_{\text{vacancies}} = E\frac{\partial n}{\partial T}, \tag{19.10}$$

where E is the energy of formation of the vacancies, n is the equilibrium concentration of the vacancies. For the magnitude n, the statistical theory gives

$$n = Ae^{-\frac{E}{kT}}, \tag{19.11}$$

from this,

$$C_{\text{vacancies}} = \frac{E^2}{kT^2}\, n = \frac{E^2 A}{kT^2}\, e^{-\frac{E}{kT}}. \tag{19.12}$$

The magnitude $C_{\text{vacancies}}$ should increase steeply with rise in temperature.

A comparison of the formula (19.12) with the results of the experiment (with a magnitude of the excess of heat capacity at high temperatures over the linear function at low temperatures) makes it possible in principle to determine the energy of the formation of vacancies as well as their equilibrium concentration. Evaluations of such a type have been performed in [184-187, 198]. For the energy of activation in this case, values have been obtained which amount to approximately 1/3 of the heat of vaporization, which is in agreement with the considerations of a theoretical nature. Reasonable values (from fractions of a percent to several percent) are obtained in this case also for maximum concentration of the vacancies (at the melting point). On the other hand, in accordance with the evaluations by Taylor and Finch [175], the magnitude of the energy of the vacancies, found from the measurements of the heat capacity, is considerably greater than that calculated theoretically, which leads these authors to conclude that there are other facts that are responsible for the high-temperature rise in the heat capacity. Rasor and McClelland [67], come to approximately the same conclusions. It is necessary to mention also the work by V. N. Kostrukova and P. G. Strelkov [199] who established a defect of the anamolous increase in heat capacity near the melting point of mercury can be caused by the introduction of a small amount (fractions of a percent) of impurities.

In considering the problem of the role of vacancies, one should apparently pay attention to the following situation.

Formula (19.12) was obtained on the assumption that a transition from one temperature to the other is accomplished so slowly that equilibrium concentration to vacancies has time to become established fully. At the same time, in measurements which were carried out by the method of intermittant heating with current, the changes in temperature take place comparatively rapidly; the change cycle takes place in fractions of a second. In this instance, the natural question is whether the equilibrium concentration of the vacancies can take place under such conditions. It is known that with sufficiently fast cooling of the body it is possible to "quench" vacancies, that is, to preserve in the body an excess number of vacancies in comparison with the equilibrium condition. The problem of the role of the relaxation time of vacancies in heat capacity measurements has to this day not been discussed.

For quantitative evaluations of the relaxation time of the equilibrium concentration of the vacancies, we site the following considerations.

The space-time distribution of the concentration of the vacancies in the body of the test object should be described in the diffusion equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n, \qquad (19.13)$$

while the coefficient of diffusion D is equal to

$$D=\frac{1}{6}\cdot\frac{\delta^2}{\tau}, \tag{19.14}$$

where δ is the width of the potential barrier, approximately equal to the dimension of the atom of the metal; τ is the average well time of a vacancy in one position (see [200]).

For a formulation of the boundary condition, we shall examine the flow of the vacancies on the boundary of the body. A change in the number of vacancies in the surface layer due to the "swallowing" of the "holes" by the surface and a decrease in the number of formed vacancies as the result of the emergence of the vacancies on the surface.

The first of these causes makes a contribution to the flow of the vacancies, which does not depend on the concentration (because this concentration is very small). The second cause results in a flow that is greater the larger the concentration of the vacancies. The summary flow of the vacancies at the surface is, thus, equal to

$$j=A-Bn.$$

In the equilibrium condition, when there is no directed flow of vacancies at the surface,

$$A=Bn^*,$$

where n* is the equilibrium concentration.

Thus,

$$j=B(n^*-n).$$

By utilizing Fick's first law, we get for the boundary condition

$$-D\frac{\partial n}{\partial \nu}=B(n^*-n),$$

where $\partial/\partial\nu$ is the derivative with respect to the normal.

The magnitude of the coefficient B can, on the other hand, be expressed by the average speed of movement of the vacancies

$$B = \frac{1}{6} \cdot \frac{\delta}{\tau}.$$

As a result, we get

$$\delta \frac{\partial n}{\partial \nu} = (n - n^*). \qquad (19.15)$$

The problem of the space-time distribution of vacancies, determined by the diffusion equation (19.13) and the boundary conditions (19.15), is fully analogous to the problem of the leveling of the temperature with the boundary condition of the third type.

The role of the Biot number in this case is played by the ratio L/δ, where L is the characteristic dimension of the body. The equivalent Biot number is thus very large, as a result of which we have on the surface of the body

$$n \simeq n^*. \qquad (19.16)$$

This conditions is equivalent to the isothermal conditions in the corresponding heat problems. Physically, the equality (19.16) is the result of the fact that a diffusion process of the vacancies in the volume is much slower than the establishment of the equilibrium concentration on the surface. The utilization of ready solutions for the establishment of the temperature under isothermal conditions on the surface (see [6]) gives for the characteristic times the expressions

$$\tau_L = \frac{L^2}{\pi^2 D} \qquad (19.17)$$

for a sample in the form of a plate and

$$\tau_R = \frac{R^2}{2,4D} \qquad (19.18)$$

for a sample in the form of a cylinder.

It is necessary to turn attention to the fact that solutions of the problem are obtained not in the form of simple exponential relationships and that the relaxation time is more or less determined clearly only for the regular stage of the process; the nature of the leveling of the nonequilibrium concentrations of the vacancies depends in the general case on the initial conditions.

The substitution of expression (19.14) for the coefficient of diffusion of the vacancies in the formulas (19.17) and (19.18) gives

$$\tau_L = \left(\frac{L}{\delta}\right)^2 \frac{6}{\pi^2} \tau \tag{19.19}$$

and

$$\tau_R = \left(\frac{R}{\tau}\right)^2 \frac{6}{2{,}4} \tau . \tag{19.20}$$

then we shall take into account that

$$\tau = \tau_0 e^{\frac{E}{kT}}$$

where E is the energy of activation of the jump of the vacancies (the energy of activation of self-diffusion), close in order of magnitude to the energy of the formation of the vacancies, while τ_0 is the time which is characteristic of the vibrational movement of the atom in the crystal: $\tau_0 \sim 10^{-13}$ sec. As a formula for evaluating the order of magnitude of the time required to establish the equilibrium concentration of the vacancies, we get the expression

$$\tau_L \approx \tau_R = \left(\frac{L}{\delta}\right)^2 \frac{6\tau_0}{10} e^{\frac{E}{kT}} = \left(\frac{2R}{\delta}\right)^2 \frac{6\tau_0}{10} e^{\frac{E}{kT}} . \tag{19.21}$$

Specific evaluations by means of this formula lead to the conclusion that at temperatures close to the melting point the magnitude τ_L becomes less than $\sim 10^{-2}$ sec. only when L is less than 10^{-4} cm. For greater distances between the sources (runoffs) of the vacancies, the values of the relaxation time will be greater, which should show up on the experiments that are conducted by the method of intermittent heating of wires. It follows from this that either under real conditions $L \sim 10^{-4}$ cm (which can take place if the sources--runoffs of the vacancies are dislocations) or that the results of the experiment conducted at high rates of heating are distorted by the role of the kinetics of the formation of the vacancy. In order to solve this problem, it is very desirable to conduct experiments by the method of intermittent heating within a wide range of frequencies. For periods of temperature change, which are comparable with the relaxation time, the excess heat capacity due to the vacancies should be less than for slow vibrations; this effect should increase with rising temperatures. The systematic performance of such status is one of urgent problems.

Summarizing the results of the examination of the problem regarding the behavior of the heat capacity of metals at high temperatures, we emphasize that a detailed experimental study of the correlations that exists here is important and gratifying work which makes it possible to obtain information on magnitudes which are interesting from a physical point of view--electron heat capacity, concentration in energy of the formation of lattice defects. For more effective performance of this work, it seems desirable to conduct a complex of studies of the heat capacity, thermal expansion, and ultra-acoustic characteristics which would make it possible to review clearly the behavior of the heat capacity at a constant volume and thereby facilitate a quantitative study of the electron heat capacity and of the contribution made by the defect.

Chapter 4 Heat Capacity of Metals

§20. Results of the Measurements of the Heat Capacity of Refractory Metals at High Temperatures

The content of this section deals with the systematization, comparison, and analysis of literature data on the measurements of the heat capacity of refractory metals. We shall limit ourselves here essentially to an examination of metals of the fifth and sixth groups of the periodic system. It is precisely for these metals (tantalum, niobium, wolfram, molybdenum) that studies are available which encompass a wide range of temperatures including the highest temperatures; besides, not only absolute temperatures but also those reduced to T/θ_{Debye}.

Studies have been performed recently with the utilization of the most diverse experimental methods, including also methods of intermittent heating, which are examined in this monograph. A comparative analysis had resulted in measurements of refractory metals and is for this reason, of interest from a methodical point of view. On the other hand, the metals under consideration crystallized in the same lattice--cubic body centers--and they do not have polymorphic transformations over the entire interval of state. This circumstance makes the refractory metals convenient objects for studying the behavior of the heat capacity within a wide range of temperatures, and, in particular, at the highest existing ratios of T/θ_{Debye}. → p. 391

One of the basic results of the examination should be tables (curves) of the values of the heat capacity which could be recommended as the most reliable at this time. The performance of such work involves a serious difficulty which develops almost always when a comparison is made of the data by different authors. The fact of the matter is that the divergencies among the data by different authors, as a rule, exceeds considerably the author's evaluations of the experimental accuracy, so that it turns out to be very difficult to judge the degree of reliability of the results. The situation is made more complex also by the fact that in many cases the authors describe the experimental conditions in a skimpy manner so that it is far from always possible to reveal the possible sources of the errors. This makes unavoidable the introduction into the work of definite elements of subjectivism, which always accompany the utilization of intuition.

The simplest way of comparing the results is by averaging in which the weight factors are determined in accordance with the evaluations of the measurement errors. Such a method is however far from ideal because the results, which differ considerably from the remaining due to the systematic errors not considered by the authors, can cause a considerable deviation of the average values from those that are most probable. A more consistent way in this respect is that utilized by us earlier in compiling tables of the heat conductivity of liquids [201]. This way, which is used when data are available for a number of substances from many authors and the results of the study by individual authors encompass a broad circle of substances, consist of the following.

For each of the well-studied substances, there are at first average values of zero approximation, obtained by simple averaging of all the available data. Then, for the data of each of the authors, we find the magnitude of deviations from these average values. In a predominant number of cases, these deviations have a nonsystematic nature. The magnitudes, which is the reciprocal of the average square of these deviations is utilized then as a weight factor in finding the average values of the first approximation. Then, the entire procedure is repeated. After this, the refined values of the squares of the deviations of the data by individual authors, which objectively characterize the errors of the corresponding experiments, utilize for finding the average values of the second, last approximation. The degree of reliability of the resulting data is characterized by the magnitude Δ which is determined by the formula

$$\Delta = \frac{1}{\sqrt{\sum_i \frac{1}{\Delta_i^2}}}, \qquad (20.1)$$

where Δ_i^2 are the quadratic deviations of the data by two different authors. The utilization of such a procedure for finding the most reliable values of the thermal characteristics of metals at high temperatures is difficult to a considerable degree because of the small amount of experimental data and the need of conducting the averaging within a wide range of temperatures, considering the possible dependence of the error of the results on the measurements on the the temperature. Nevertheless, we will strive to approach to a maximum degree such a system in order to acquire the possibility of revealing least subjectively the most probable data.

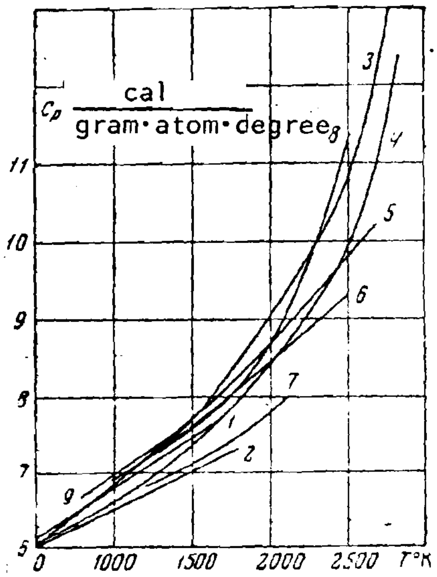

Figure 68. Literature data on the heat capacity of molybdenum; 1, Eger and Venster [202]; 2, Richert, Bennett and Johnston [37]; 3, Rasor and McClelland [66]; 4, Taylor and Finch [175]; 5, Kirillin, Sheyndlin and Chekhovskoy [203, 204]; 6, Lazareva, Kantor and Kandyba [205]; 7, Lowenthal [37]; 8, Kraftmakher [185]; 9, Yurchak and Filippov (see Figure 69).

Below is given a comparison of specific experimental data and a presentation of the averaging procedure. The fitted literature data for the refractory metals of molybdenum, wolfram, tantalum, niobium at high temperatures are shown in Figures 68, 70, 71 and 72. (This collection includes only the results pertaining to a comparatively broad range of high temperatures). Data for molybdenum have been supplemented by the results of measurements, obtained by the method of intermittent electronic heating [167] (for a description of the method, see §14). These results are shown in particular in Figure 69. The specimen contained 99.9% molybdenum, 0.01% nickel, 0.01% sesquioxides, 0.001% silicon monoxide.

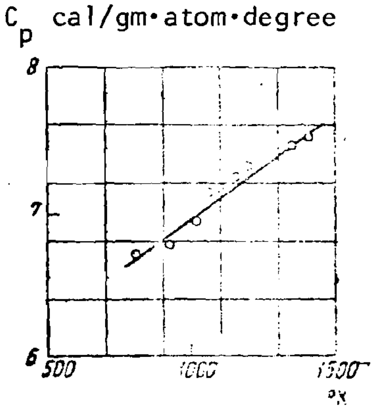

Figure 69. Results of measurement of the heat capacity of molybdenum, obtained by the method of intermittent electronic heating (see §14).

For my comparison of the cited literature data, the following conclusions can be made. In the range of temperature up to ~ 2,000°K, where there is still no steep rise in the heat capacity, the divergeencies between the data of different authors are relatively small; deviations from the average values, as a rule, do not exceed 5%. At higher temperatures, these deviations increase considerably.

The existence of comparatively numerous experimental data for refractory metals makes it possible not only to determine the most probable average values, but also to reveal the experiment, the results of which are closest to those most probable values for the entire totality of the substances, and to give to these a quantitative characteristic--the average quadratic deviation. This characteristic can then be utilized for finding the refined average values obtained by the averaging procedure in which the data by different authors are utilized with different weight factors, as was already stated about this above.

Let us examine specific results of such evaluations and of the averaging. The group of early calorimetric experiments, the results of which have been published by Jaeger and Veenstra[202] and Jaeger and Rosenbohm [206], belong to the best. The average quadratic deviations of these data from the most probable (average) curves amount for wolfram, molybdenum, and niobium to 0.8%. In finding the average weighted values the magnitude $1.5 \approx (1/0.8)^2$ was taken as the weight for these studies.

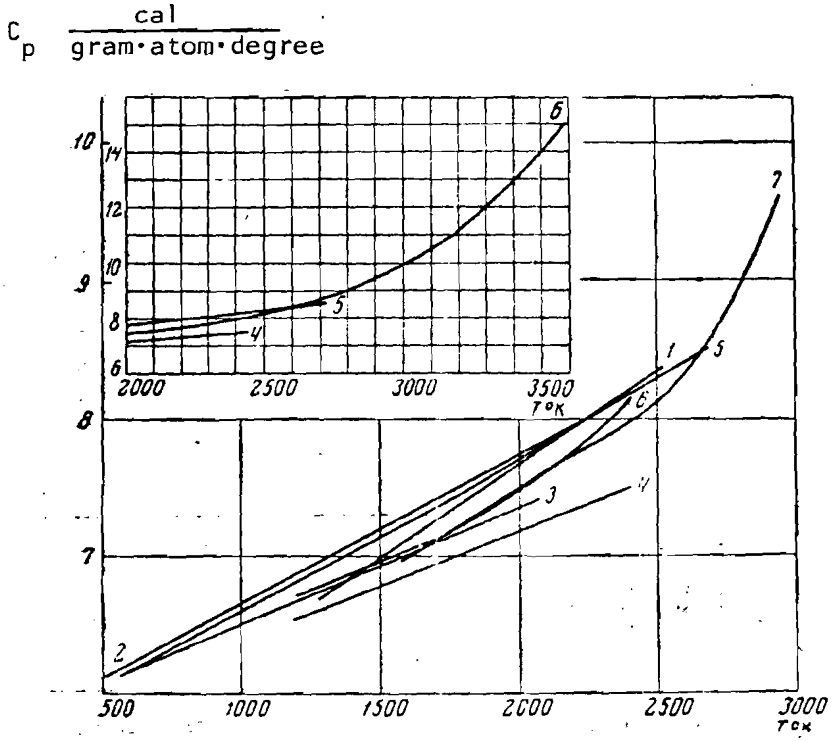

Figure 70. Literature data for the heat capacity of wolfram: 1, Worthing [174]; 2, Zwikker [172]; 3, Jaeger and Rosenbohm [206]; 4, Hoch and Johnston [207]; 5, Kirillin, Shenydlin, Chekhovskoy [208]; 6, Lowenthal [37]; Kraftmakher [184].

In good agreement with the average values are the latter calorimetric experiments which encompass a wide range of temperatures--the work by V. A. Kirillin, A. Ye.Shenydlin, V. Ya. Chekhovskoy and I. A. Zhukova [198, 203, 204, 208, 241]. The average quadratic deviations for these data amount in the case of molybdenum to 2.4%, wolfram 1.4%. (Here and henceforth, in examining the deviations from the average values, only the range of temperatures up to 2,000-2,400°K where there is a sufficient number of data by various authors, is taken into consideration.) As a weight of the results of these experiments, the magnitude $0{,}25 \approx \frac{2}{2{,}4^2 + 1{,}1^2}$ was taken. In finding the average weighted values at temperatures where the heat capacity depends more steeply on the temperature, a somewhat smaller weight should be ascribed to these measurements just as to all other calorimetric data because of the inaccuracies which arise in determining the heat capacity on the basis of data for the enthalpy when the changes in the enthalpy are small but take place over a relatively small range of temperatures. (Thus, according to the data from [198], a change in the enthalpy due to the formation of vacancies amounts to a total of 6%.) Considering this, we henceforth ascribed a weight to the high-temperature calorimetric data, which was equal to half of the weight at temperatures below 2,000°K. Approximately the same agreement with the average values takes place for the results of the calorimetric study by L. S. Lazareva, P. B. Kantor and V. V. Kandyba [205]. The deviation of their data from the average values amounts to 1.8%, which does not exceed the author's evaluation of the accuracy (5%); the weight up to a temperature of 2,000° was taken equal to 0.3, and above that--0.15.

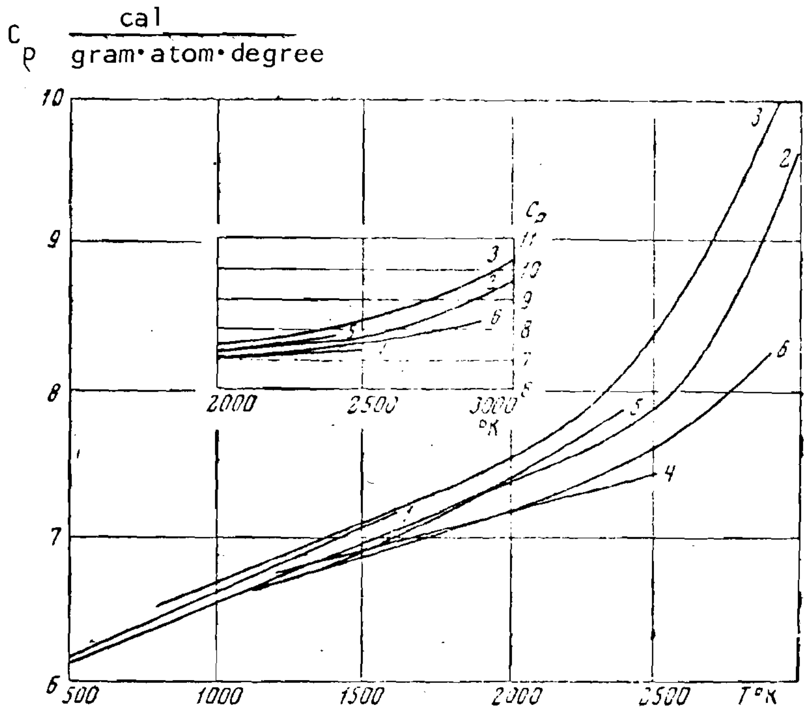

Figure 71. Literature data on the heat capacity of tantalum; 1, Jaeger and Veenstra[202]; 2, Taylor and Finch [175]; 3, Rasor and McClelland [66]; 4, Hoch and Johnston [207]; 5, Lowenthal [37]; 6, Kraftmakher [186].

The results of the calorimetric measurements of the heat capacity of niobium, obtained by P. V. Gol'd and F. G. Kusenko [209], close to the data obtained by V. A. Kirillin, A. Ye. Sheyndlin, V. Ya. Chekhovskoy and I. Ya. Zhukova [198] and Jaeger and Veenstra[202]. For these data, the accepted weight was 0.1 (with consideration of the additional error due to the correction for the content of impurities in the sample under study by these authors.).

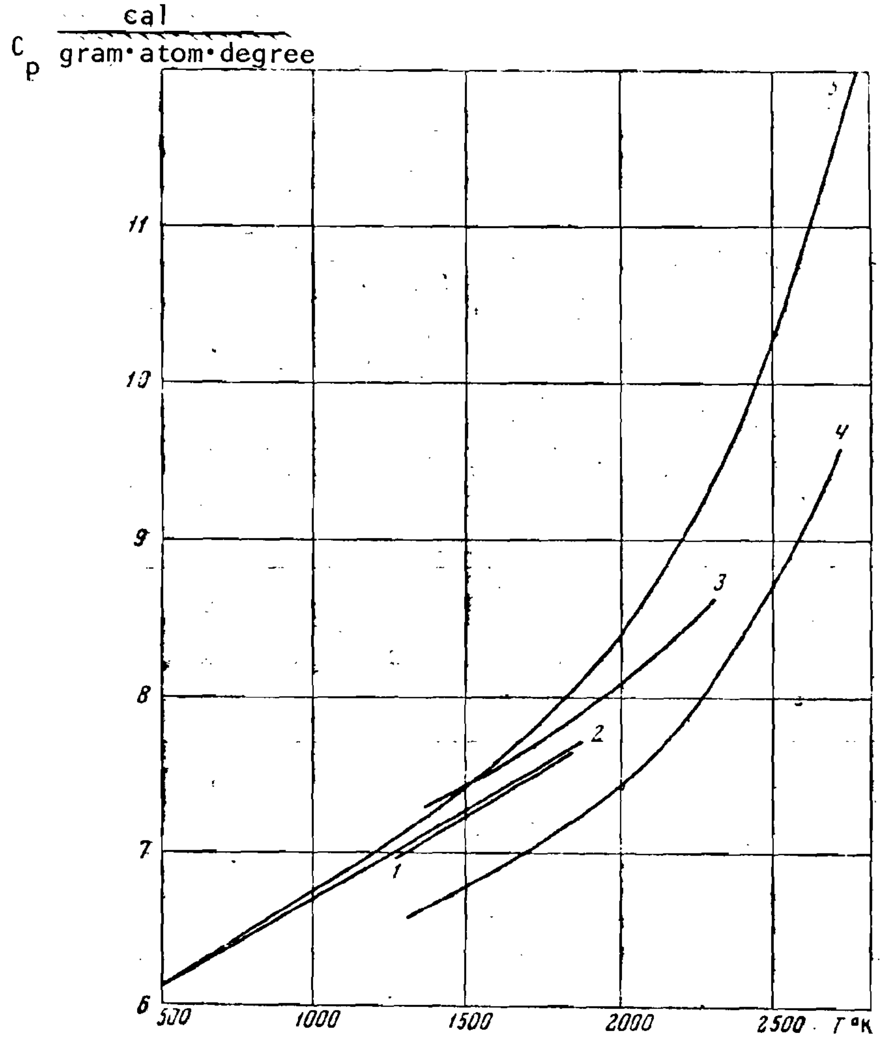

Figure 72. Literature data on the heat capacity of niobium: 1, Jaeger and Veenstra[202]; 2, Gol'd and Kusenko [209]; 3, Lowenthal [37]; 4, Kraftmakher [187]; 5, Kirillin, Sheyndlin, Chekhovskoy and Zhukova [198].

The situation was different with respect to the groups of calorimetric experiments by Hoch and Johnston [207] and Reichert, Bennett, and Johnston (the results of the latter work are taken from [37]). These data differ most strongly from the average values; besides, the deviation increases with rising temperature. For wolfram, the average quadratic deviation amounts to 4.5%, for molybdenum it is 4.2%, for tantalum it is 1.4%. The average weight of these data is 0.08.

A contradictory picture is obtained for data by Lowenthal [37] which had been obtained by the method of intermittent heating with a current with registration of the temperature variations by means of a photoelectric method (see §18). For wolfram and tantalum, the data by Lowenthal are in good agreement with the average values (the quadratic deviation is 0.7%), for molybdenum however the average quadratic deviation amounts to 5.9%, which is twice as great as the evaluation of the maximum error by the author. In an analysis of the evaluations of the error of the author, the very small magnitude of the error in determining the magnitude of the ratio of the anode current at the axis of the photomultiplier i_a to the voltage of the alternating-current component ΔV is, in accordance with Lowenthal, equal to 0.7%. This magnitude includes apparently only the random but not the systematic error. Assuming that $\delta i_a/i_a \sim 1\%$ and $\Delta V/V \sim 1.5\%$, we get for the maximum error a magnitude of 3 to 5%. We took a weight of 0.05 for the Lowenthal data.

The results of the series of studies by Ya. A. Kraftmakher [183-187], which have been obtained by the method of intermittent heating with a current with the registration of the temperature pulsations with respect to the resistance, the method of compensating the reactive components, see §8) for molybdenum, wolfram, and tantalum, in comparatively good agreement with the average values (the quadratic deviations amount to 1.9, 1.0 and 1.5%). For niobium, however, the data by Ya. A. Kraftmakher are 8-10% below those for the remaining. The source of error in the data by Ya. A. Kraftmakher could cause inaccuracies in the values of the temperature coefficient of the resistance, which the author did not measure directly but took from literature data. Apparently it is precisely this that is used to explain the indicated difference in the data for niobium. Evaluations of the maximum errors of the experiments in the work by Ya. A. Kraftmakher are lacking; there is only information on the reproducibility of the results (~ 1%, for wolfram above 3,300°K it is 4%). If it is taken into consideration that for wolfram, molybdenum, and tantalum the temperature coefficient of the resistance is known apparently with an accuracy of 1-2%, then values of 2-3% should be taken as the errors of the results for these metals (with exception of the highest temperatures). (The weight factors are 0.25-0.1).

The results of the work by Taylor and Finch [175], performed by the method of pulse heating of a wire specimen with a current, are in good agreement with the average values. The average quadratic deviation for molybdenum and tantalum is equal to 0.9%. The author gives for the error of the average values 4% up to 2,400°K and 7% at higher temperatures. One can completely agree with these evaluations of the author because the work contained a study of the reproducibility of the results for different specimens, while for the temperature coefficient of the resistance, their own results were utilized. The weight factors for the data by Taylor and Finch lie thus within the limits of 0.05-0.02.

The results of early experiments in which wires were heated are in comparatively good agreement with the average values also. Thus, the data by Worthing [174] for wolfram (see §18) have an average quadratic deviation of 1.2%. Considering, however, that the scatter of points for the work by the author is comparatively large (6%), a magnitude of 0.1 was taken for the weight factor in this work.

Data by Zwikker [172], obtained by two methods (pulse heating and heating with alternating current with registration of the temperature variations with respect to thermal electron emission, see §18), differ from the average values by 2.2%. The weight factor of the Zwikker data is equal to 0.2.

The results of measurements of the heat capacity for molybdenum by R. P. Yurchak and L. P. Filippov (Figure 69) differ from the average values by 3.5%, which lies within the limits of the maximum error of the results of the measurement (4%), see §14. The weight factor is 0.05.

The results of the work by Rasor and McClelland [67](method of monatomic heating of a rod and wire) differ from the average values for molybdenum by 4.4%, for tantalum by 2.0%. The scatter of points with respect to the average curves by these authors is comparatively large; the deviations frequently reach 10%. For this reason a value of 0.03 was taken as the weight factor for these data.

The curves of the temperature function for the most probable values of the heat capacity of refractory metals, which have been obtained by the above-described method, are shown in Figure 73, 74, 75 and 76. On each of these curves, the region of the probable error is indicated. In Figure 73, a fine dotted curve, which has been obtained as a first approximation, with simple averaging, has been drawn for molybdenum. It can be seen that the refinement introduced by adding the differences in the weights during averaging is small; hence, it follows that the elements of randomness which take place in the evaluation of the weight factors affect relatively little the final results. In the case of wolfram, tantalum and niobium curves have also been drawn for the heat capacity at a constant volume C_v; these have been obtained by utilizing the magnitudes C_p/C_v from the already mentioned work by B. N. Oshcherin [190].

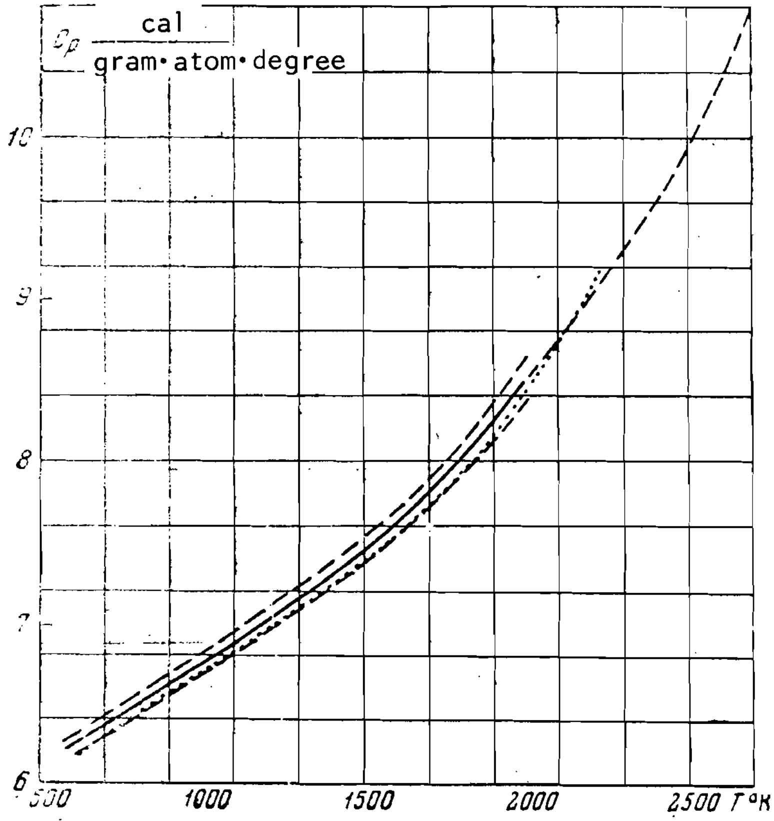

Figure 73. Heat capacity for molybdenum. The most probable values.

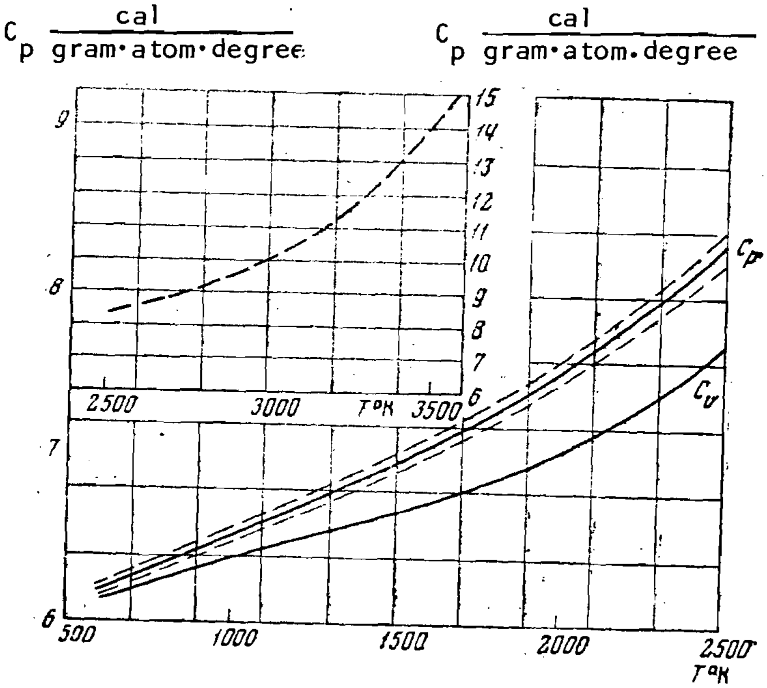

Figure 74. Heat capacity of wolfram. The most probable values.

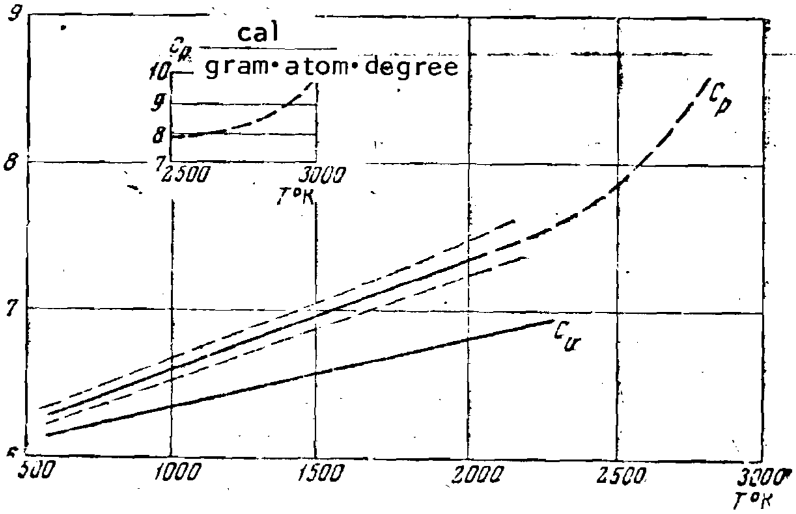

Figure 75. Heat capacity of tantalum. The most probable values.

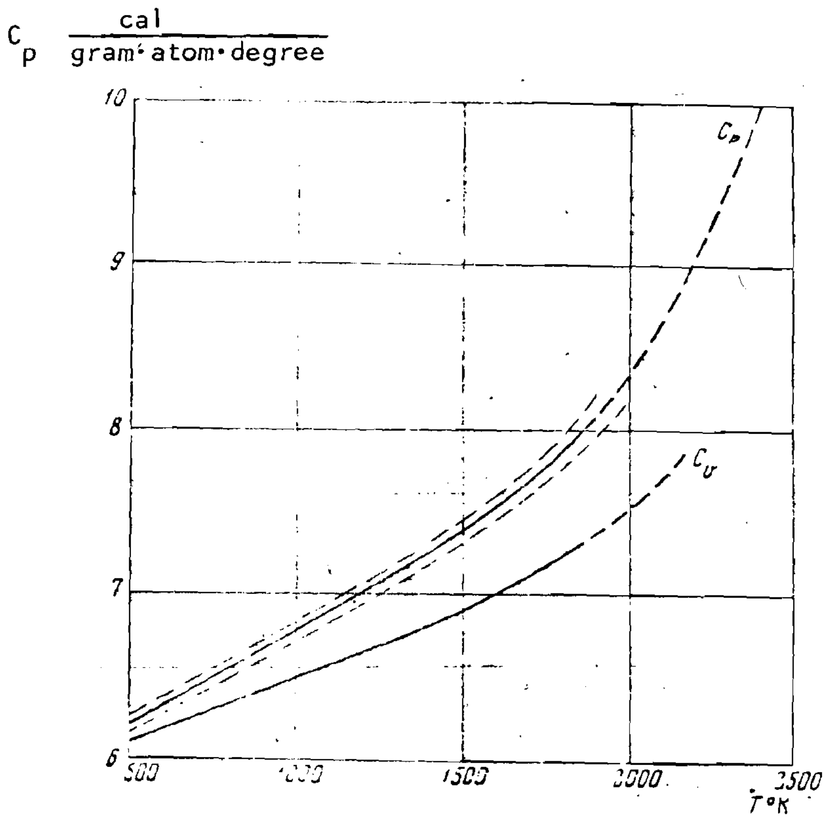

Figure 76. Heat capacity of niobium. The most probable values.

From an examination of the inside of curves the following conclusions can be made. In all cases up to temperatures of ~ 1,500-2,000° (i.e., approximately up to 0.5 melting point) the heat capacity varies with the temperature in a practically linear manner.

The magnitude of the angle of the slope (dC_p/dT) (cal/gm·atom·degree²) decreases in the following sequence: molybdenum $1.39 \cdot 10^{-3}$, niobium $1.15 \cdot 10^{-3}$, wolfram $0.88 \cdot 10^{-3}$, and tantalum $0.75 \cdot 10^{-3}$. Similar derivatives for the heat capacity at a constant volume dC_v/dT are equal to the following: for niobium $0.82 \cdot 10^{-3}$, for wolfram $0.65 \cdot 10^{-3}$, and for tantalum $0.43 \cdot 10^{-3}$. These magnitudes for niobium and tantalum are several times less than the values of the coefficient of electron heat capacity γ, found from low-temperature data [210]; for wolfram on the other hand the magnitude γ is approximately half as large as dC_v/dT.

As regards the values of the heat capacity in the temperature range above 2,000°, the values here, which have been given as the most probable, depend much more on the selection of the weight factors of averaging because the data by different authors in this range of temperatures differ strongly. The utilization of these values for quantitative evaluations of the concentration and the energy of the formation of defects is indeed premature. Here, further systematic studies are necessary.

In conclusion, we shall make an attempt at an empirical generalization of the results of measurements of the heat capacity of refractory metals. For this purpose, we shall examine the dependence of the atomic heat capacity on the dimensionless variable T/T_{melting}, where T_{melting} is the melting point in Kelvin degrees. The utilization of the dimensionless variable for the temperature makes it possible, from point of view of the theory of similitude, to exclude from the function under consideration one of the implicitly appearing molecular constants which characterize the individuality of the substance. In this case, one can expect the appearance of functions which are related to a smaller degree with the individuality of the separate substances, which are approximately valid for a relatively wide group of metals. In this instance, it is necessary to take into consideration that the magnitude of the atomic heat capacity is, from the point of view of the theory of similitude also dimensionless (with an accuracy up to the dimension of the constant of the gas constant or completely dimensionless, if the temperature is expressed in energy units). For this reason, there is no need of introducing any other dimensional factor in order to obtain dimensionless relationships.

The curves of the function

$$C_p = C_p\left(\frac{T}{T_{\text{melting}}}\right) \qquad (20.2)$$

are shown in Figure 77.

In plotting the curves, use was made of the above-found values of the heat capacity of molybdenum, wolfram, tantalum and niobium, supplemented by data for other representatives of the same groups of metals--chromium, vanadium--the results for which are taken from the work by Kohlhaas *et alia* [193]. Besides that, the same curve shows data for metals of the eighth group--osmium and iridium--the heat capacity of which has been measured by Jaeger and Rosenbohm [206], and the metal from the seventh group--rhenium--(data by Taylor and Finch [175]). All the examined metals have a cubic body centered (V, Nb, Ta, Cr, Mo, W), cubic face centered (Ir), or hexagonal densely packed (Os, Re) structures.

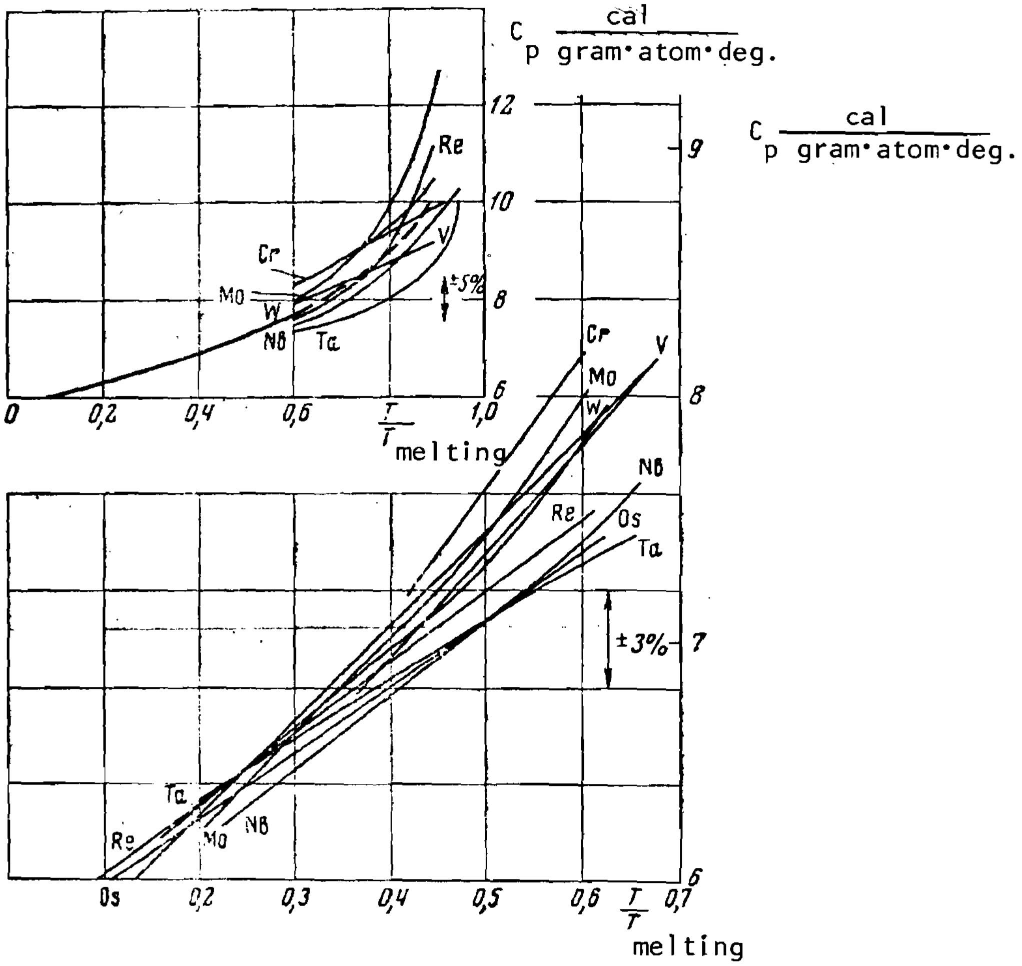

Figure 77. Dependence of the heat capacity C_p, cal/gm atom degree on the dimensionless temperature $T/T_{melting}$.

From the figure it can be concluded that within the interval of values of $-T/T_{melting}$ ranging from 0.1 to 0.5 the corresponding curves for all the examined metals are rather close. A maximum deviation of individual values from the average straight line is

$$C_p = 5{,}69 + 3{,}1_8 \frac{T}{T_{\text{melting}}} \tag{20.3}$$

only in one case (chromium) exceeds 2.5%. The accuracy with which the relationship (20.3) is valid is only a little less than the accuracy of the corresponding experimental data.

At temperatures in the range of T/T_{melting}, the deviations from the average curve (it is shown by a solid line in Figure 77) increased to 5%; at higher temperatures, the differences become even greater; however, they are not much more than those which take place between the data of different authors for the same substance. It is interesting that even in those cases when the very nature of the temperature function of the heat capacity is different (positive curvature), as this was found by Jaeger and Rosenbohm for paladium and rhodium [206], the values of C_p in the function T/T_{melting} do not all the same, differ very strongly from the average values in the totality under examination.

Thus, we come to the conclusion that the relationship of the variables C_p and T/T_{melting} leads to a function which gives a good correlation for a group of refractory metals. The formula (20.3) can be recommended for practical use.

We wish to point out that the function of the type (20.3) can be established also for the group of metals with relatively low melting points. Thus, L. I. Ivanova [211] has shown that the data for the heat capacity of monoatomic substances (predominantly metals) is satisfactorily described by the single formula

$$C_p = 5{,}283 + 1{,}987 \frac{T}{T_{\text{1st phase}}}, \tag{20.4}$$

where $T_{\text{1st phase}}$ is the temperature of the first phase transition (melting distillation or conversion). The average deviation of the individual values from this formula amounts to 2.5%. This same formula is valid also for a wide class of compounds, if the right-hand part of the formula (20.4) is multiplied by the number of atoms of the substance.

§21. Heat Capacity of Liquid Metals

The problem of the liquid state of matter is one of the most serious problems in molecular physics. The nature of liquid has at the present time been studied much less than the nature of solids and gases. The reason for this is the complex, dual nature of the structure of liquids and of the thermal motion of molecules therein. On one hand, liquids have much in common with gases: the molecules of the liquid are mobile (the liquid has fluidity), there is a continuous series of states of matter from that of an ideal gas to a liquid near the crystallization point. On the other hand, the liquid at relatively low temperatures has also characteristic features of a solid body: the liquid has a structure which indicates to a definite degree the structure of the corresponding phase. This structure is however, different from the structure of the crystal lattice and extends over a distance of only several molecular radii; it describes the statistical ordering of the nearest environment of the molecule under consideration, the so-called "close order," which differs from the "distant order" in the crystal lattice (ordered distribution of molecules in an ideal lattice is not limited in space). There are also other features which relate the liquid at temperatures close to the crystallization point with a solid body: the relative smallness of the latent heat of melting, the small change in density, closeness of the heat capacity [200].

A theoretical study of the liquid state of matter involves a number of very serious difficulties. Despite the considerable number of studies (see, for example, the monographs [212, 213]), the theory of liquids is still far from the possibility of giving a satisfactory quantitative description of even the basic thermodynamic properties. In connection with this, broad experimental studies of the properties of liquids are an absolutely necessary element in studying the nature of the liquid state.

Studies of the properties of liquid metals are of special interest. Liquid metals belong to the simplest mono-atomic liquids and from this point of view, a study of the structure and properties is particularly desirable. On the other hand, liquid metals have their specificity--the existence of an electron gas which shows up in particular on the existence of electrical conductivity and electron heat conductivity. A study of the electron properties of liquid metals is all the more important, because there are still very little clarity in understanding the specificity of these phenomena. At the same time, liquid metals are finding greater use in the new technology, particularly in atomic power engineering, which makes an exhaustive study of the properties and in particular of the heat capacity and heat conductivity, an absolute must.

In studying the heat capacity of liquid metals, the following two problems are, in the first place of interest: 1) change in the heat capacity during melting; 2) temperature function of the heat capacity of liquid metals. Information on the change in heat capacity of metals during melting can be found in [214-218, 199].

The basic correlation which takes place here is as follows: the change in heat capacity is small, amounting to several percent (thus for 10 it is $\Delta C_p/C_p \sim 3\%$ [217]; for lithium, it is 3.5% [216]; for sodium, it is 1.5% [215]; for lead it is $6 \pm 1\%$ [214]; for mercury it is close to zero [199], [218]). The smallness of the change in the heat capacity during melting is regarded usually as one of the arguments in favor of the closeness of the nature of the liquid and solid phases near the melting point.

However, one should not assume that the problem of the jump in the heat capacity during melting is exhausted by the existing studies. The fact of the matter is that for a sure quantitative analysis of this magnitude, it is necessary to have very detailed and thorough measurements of the heat capacity in the solid phase in view of the complex nonlinear nature of the change in heat capacity near the melting point, which is determined by the formation of lattice defects (see §19 and 20). This change, although it is comparatively large (tens of percents), takes place within a relatively narrow range of temperatures and can be revealed sufficiently clearly in measurements of a calorimetric nature because the very change in enthalpy in this case is small, amounting only to several percent. Systematic experimental studies in this direction are very desirable. During their accomplishment, it is expedient to pay attention to the problem of the behavior of the heat capacity at the constant volume, the magnitude of which reflects in a more direct manner the changes in the spectrum of the collective vibrations of the atoms of corresponding media.

The problem of the temperature function of the heat capacity of liquid metals also cannot be considered at present sufficiently studied. A number of experimental studies performed within a wide range of temperatures is very small. The results to which different authors come are contradictory to considerable degree. In a number of papers, a decrease in heat capacity with rising temperature has been found, while in others--increases;in some studies the heat capacity is almost constant while in others--the function is nonmonotonic.

Below we give a comparison and analysis of the results of experimental studies.

The values of the heat capacity of alkali metals--lithium, sodium, and potassium--are shown in Figures 78, 79, and 80.

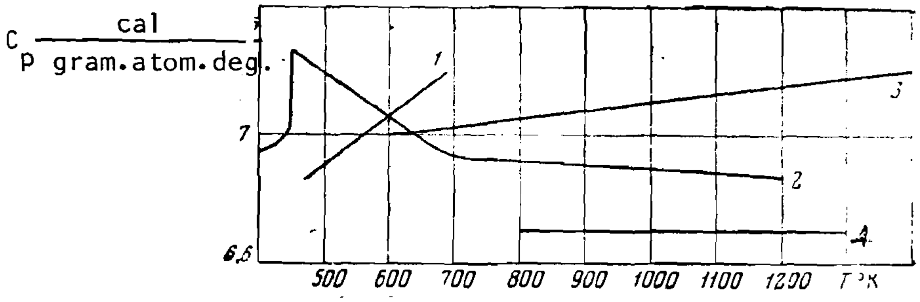

Figure 78. Heat capacity of liquid lithium: 1, Nikol'skiy et al [60]; 2, Douglas et al [217]; 3, Redmond and Lons (see [217]); 4, Bates and Smith (see [217]).

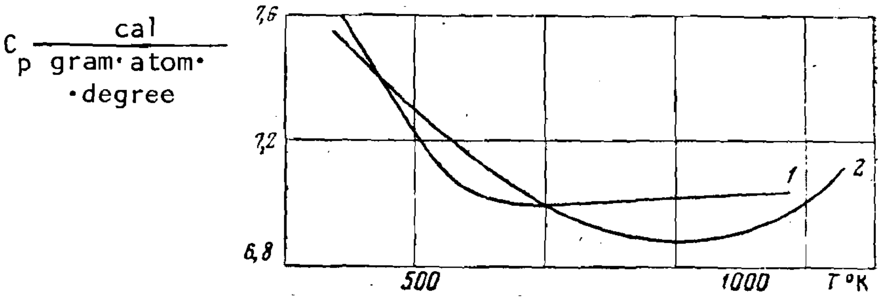

Figure 79. Heat capacity of liquid sodium: 1, Nikol'skiy et al [60]; 2, Ginnings et al [215].

It is seen from these figures that for sodium the results of two fundamental studies by Ginnings, Douglas and Ball [215] and N. A. Nikol'skiy, N. A. Kalakutskaya, I. M. Pchel'kina, T. V. Klassen and V. A. Vel'tishcheva [60] (see also [220]) are close to each other; the data for lithium are to a certain degree contradictory; for potassium, there is observed good agreement between two existing experiments [60] and [219].

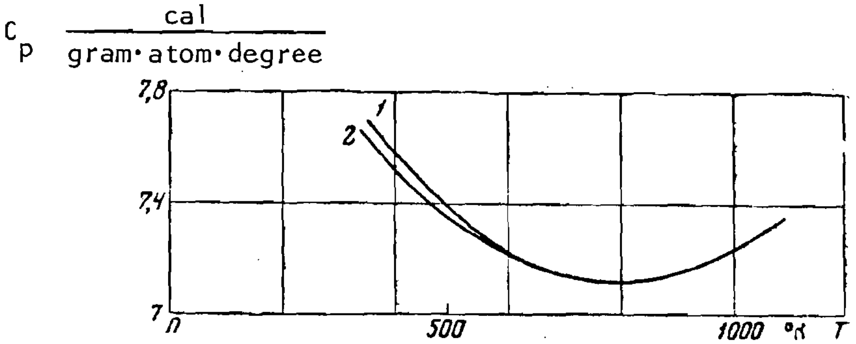

Figure 80. Heat capacity of liquid potassium: 1, Nikol'skiy et al [60]; 2, [219].

Yu. P. Os'minin [221] made attempts to generalize the experimental data on the heat capacity of alkali metals; he utilized for this purpose the relationship of the variables $C_p/C_{p_{mp}}$ and T/T_{mp}, as well as the variables $C_p/C_{p_{mp}}$ and T/T_k, where T_k is the critical temperature. U.P. Os'minin showed that in these variables, the experimental data for sodium and potassium fits satisfactorily on single curves and on the basis of this he attempted to evaluate the temperature function of the heat capacity of rubidium and cesium. Although the conclusions made on the basis of the comparison of the behavior of the properties of only two substances seemed doubtful, the utilization of the simplex T/T_{mp} as a dimensionless variable, as we have already seen where the example of the heat capacity of solid metals, can lead to practically useful relationships. In connection with this, it is pertinent to recall the references [222, 223 and 224] in which the variables T/T_{mp} was successfully utilized to describe the viscosity and the heat conductivity of liquid metals. As regards the variable $C_p/C_{p_{mp}}$, it is our view that the introduction of such a simplex is not necessary because the magnitude C_p is by itself dimensionless (which has been pointed out in the preceding section). The practical usefulness of the relationships written for the variable $C_p/C_{p_{mp}}$ is not great because by means of these it is possible to evaluate only the temperature function of the heat capacity and not the heat capacity itself.

The curves of the function

$$C_p = f\left(\frac{T}{T_{mp}}\right) \qquad (21.1)$$

for liquid alkali metals are shown on Figure 81 (for lithium, only the data by Douglas *et alia* have been used; for sodium and potassium, the values are the average ones of those shown in Figures 79 and 80). It is seen from the figure that the curves for sodium and potassium in these variables are in agreement with each other exactly as if the ratio $C_p/C_{p_{mp}}$ had been taken as the dependent variable. The curves for lithium lie somewhat below, but even for lithium the maximum deviation from the average (dotted) curve does not exceed 3%.

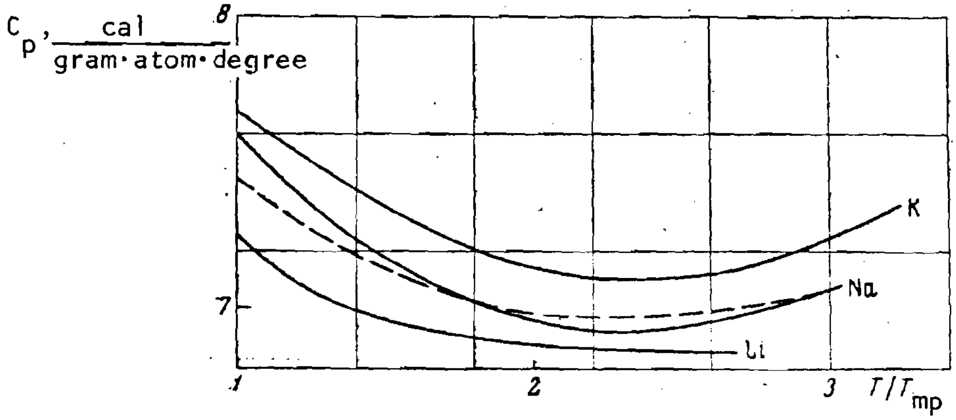

Figure 81. Heat capacity of liquid metals C_p cal/gm·atom·degree as a function of the variable T/T_{mp}.

The succeeding Figures 84 and 85 show the values and heat capacity of tin and lead which have been studied most in this respect (for the remaining metals only single data are available as a rule).

Figures 82 and 83 separately show the primary results of measurements of the heat capacity of tin and lead, obtained by means of the method of measurement of a complex of heat properties, described in §14 (the high temperature points of these data differ from earlier published data [227] by 1.5 for tin and 2.8% for lead because corrections were made for emission.) This figure shows in different ways the results obtained for different heating periods, which makes it possible to judge the reproducibility of these measurements. The maximum deviation of the results of individual measurements from the smoothing straight lines for tin does not exceed 1.5% and for lead 3%.

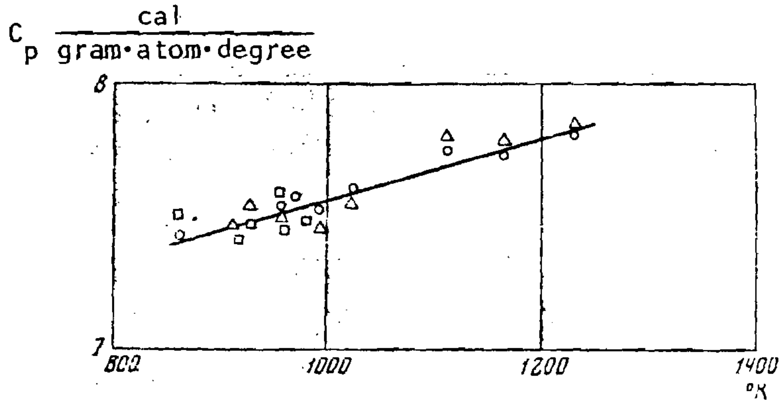

Figure 82. Results of measurements of the heat capacity of liquid tin, obtained by means of the method described in §14. Δ , period 6.6 sec; ○ ,period 13.2 sec; □, 26.4 seconds.

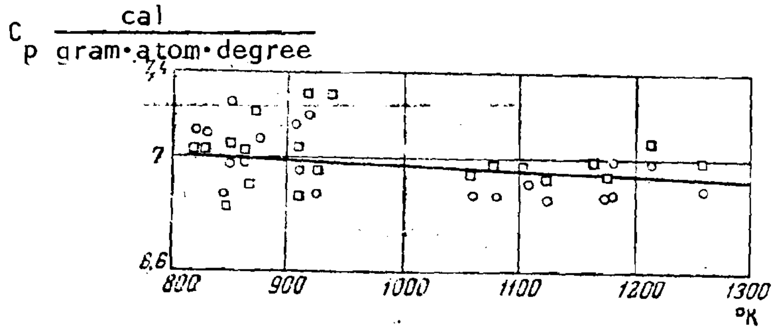

Figure 83. Results of measurements of the heat capacity of liquid lead, obtained by means of the method described in § 14. ○ , period 13.2 sec; □, 26.4 sec.

From an examination of Figures 84 and 85 one is astonished by the serious contradiction of the results obtained by different authors. The differences in the values of the heat capacity reach up to 10 and even more percent, without mentioning even the different temperature cores. The greatest deviations from the available totality are shown by the by the data of I. A. Akhmatova [38] and the temperature range of 1,100-1,200°K. From an examination of the primary, non-smoothed results of the measurements by I. A. Akhmatova, it is seen that a curve in the temperature range below 1,600°K has been drawn only from three experimental points. Apparently the data by I. A. Akhmatova should be considered as sufficiently reliable only at temperatures above 1,600°K where there are comparatively many measurements, besides it is necessary to take into account that the error in the method of measurements utilized by I. A. Akhmatova (see §18) increases with decreasing temperature.

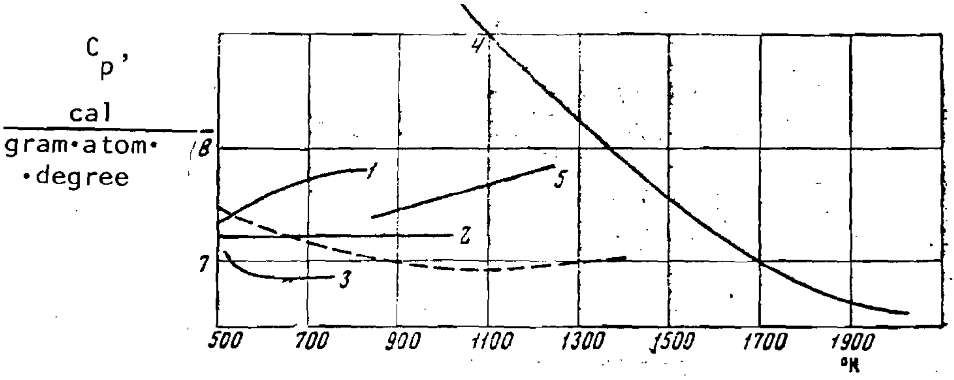

Figure 84. Literatire data on the heat capacity of liquid tin: 1, Klinkhardt [216]; 2, Nikol'skiy et al [60]; 3, Orr et alia [225]; 4, Akhmatova [38]; 5, Figure 82.

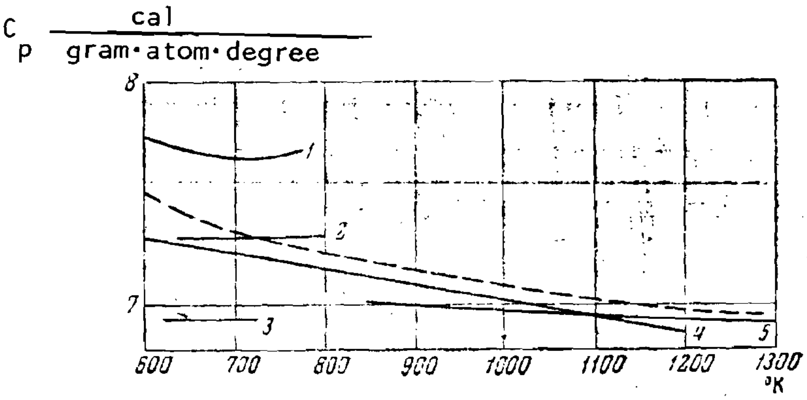

Figure 85. Literature data on the heat capacity of liquid lead: 1, Klinkhardt [216]; 2, Nikol'skiy et alia [60]; 3, Dixon and Rodebush [226]; 4, Douglas and Dever [214]; 5, Figure 83.

It is impossible to give preference to any one of these or to select a group of most reliable values; the scatter of the results is too great. It is interesting, however, to see how the curve of the most probable function $C_p(T/T_{mp})$ which has been found from the results of measurements of the heat capacity of alkali metals (see Figure 81), fits in among the cited data. Such curves are shown in Figures 84 and 85 as dotted curves. It can be asserted that the corresponding functions are far from contradictory with respect to the experimental data. The difference from the data by different authors is not greater than the difference between these very data. This gives grounds for assuming that a single function $C_p(T/T_{mp})$ encompasses a comparatively wide group of liquid metals. For more definite evaluations of the correlations which govern the heat capacity of liquid metals, further experimental studies are necessary.

Chapter V

HEAT CONDUCTIVITY OF METALS

§22. Heat Conductivity of Solid Metals at High Temperatures

A distinctive characteristic of the heat conductivity of metals, as is known, is the existence of electron conductivity of heat, which, as a rule, exceeds considerably the heat conductivity of the lattice and makes the heat conductivity of metals one to two orders greater than the heat conductivity of dielectrics.

In accordance with the existence of two mechanisms of heat transfer in metals, the observed heat conductivity of the metals can be represented as the sum of two members

$$\lambda = \lambda_{\text{electron}} + \lambda_{\text{lattice}} \qquad (22.1)$$

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electron and lattice heat conductivity. The summation of these members, however, does not mean that the mechanisms of electron and lattice heat conductivity are additive and do not affect each other. In the general case, the process of heat transfer by the lattice of the metal is not identical to the process which would take place in a similar lattice of a dielectric; the existence of "electron gas" can affect the nature of heat transfer by the vibrations of the lattice.

The basic fundamental correlation which pertains to the electron heat conductivity of metals at high temperatures is the Widemann-Franz law which establishes the relationship between the magnitudes of the electron heat conductivity and the electrical conductivity of metals σ:

$$\lambda_{\text{electron}}/\sigma T = L = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2, \qquad (22.2)$$

where k is the Boltzmann constant, e is the electron charge. The numerical values of the constant L, the so-called Lorentz number, is equal to $2.445 \cdot 10^{-8}$ wt·ohm/deg^2. The Widemann-Franz law is derived in the electron theory of metals with a minimum number of initial assumptions and is a law which is valid for all metals at temperatures above the Debye temperature.

The existence of the Widemann-Franz law does not yet solve completely the problem of the heat conductivity of metals even if it is assumed that a magnitude of electrical conductivity is known accurately. The Widemann-Franz law gives information only about one of the components of the heat conductivity. The problem of the relationship between the electron and lattice heat conductivity requires a separate knowledge of the magnitude of the lattice component. The process of heat transfer by the lattice is usually described by means of the concept of quasi-particles--quanta of elastic vibrations of the lattice, which are called phonons. For high temperatures, however, the introduction of this concept is not mandatory; it is possible to utilize also the "classical" terminology--to speak about the propagation and scattering of elastic waves in the continuum.

A theoretical examination of the problem of the magnitude of the lattice heat conductivity, based on simple considerations of the model type, leads to a formula which is similar to the corresponding formula of the molecular-kinetic theory of gases:

$$\lambda_{\text{lattice}} = \frac{1}{3} c_v \rho v l, \qquad (22.3)$$

where v is the average speed of propagation of the phonons (elastic waves), l is the average length of their run.

The main difficulty in calculating the heat conductivity of the lattice (just as the heat conductivity of dielectric) is in the magnitude of the average length of the free run. Several processes take place which limit this magnitude: scattering of the phonons on phonons (interaction of lattice vibrations, which is determined by the anharmonic effects), scattering of phonons on lattice defects (on point defects: atoms of impurities, vacancies, Frankel defects; on linear defects--dislocations, and planar defects--boundaries of crystal and grains or boundaries of a monocrystal) and a scattering of phonons on electrons. A theoretical examination of all these processes involves many complications. A theory gives usually only quantitative results--the order of magnitude of the corresponding effect, its dependence on the temperature. The case of high temperatures examined here is one of the most simple. At high temperatures, only one effect of scattering of phonons on phonons is pertinent; besides, the theory leads to a simple expression for the dependence on the temperature

$$l \sim \frac{1}{T}.$$

The dependence $l \sim 1/T$ leads also to similar temperature dependence of the lattice heat conductivity because the magnitude of c_p at high temperature is constant while ρ and v vary with the temperature slightly

$$\lambda_{\text{lattice}} \simeq \frac{C}{T}. \tag{22.4}$$

The relationship (22.4) for solid dielectric is called sometimes the Eucken law.

The existence of formula (22.4), even if it is regarded as absolutely correct, is far from solving the problem of lattice heat conductivity because one still needs to know the value of the constant C, the theoretical calculation of which does not lead to satisfactory quantitative data. An experimental study is necessary.

A somewhat different approach to the examination of the problem of lattice heat conductivity, even though it is related to the ideas of Debye, is given by A. S. Predvoditelev [228]. The starting point in the theory by A. S. Predvoditelev is the principle of the conformance between the acoustic vibrations which describe the heat movement in the body and the temperature waves in this medium (the length of the temperature wave is equal to the average length of the wave of transverse or longitudinal acoustic vibrations). The theory leads to the formula

$$\lambda_{\text{lattice}} = 2{,}41 c_p \rho \sqrt{\frac{E}{\rho}}\, d, \tag{22.5}$$

where E is Young's modulus, d is the average distance between the atoms. This formula is similar with (22.3) and is derived therefrom if it is assumed that the average length of the free run of elastic vibrations is proportional to the distance between the atoms. The temperature dependence λ_{lattice} should, in accordance with formula (22.5), possess a weaker character than (22.4).

Let us now return to the Widemann-Franz law. For the dependence of the electrical conductivity of metals on the temperature within the range of high temperatures, the theory predicts a relationship close to linear

$$\sigma \sim T. \tag{22.6}$$

In this instance, it follows from the Widemann-Franz law that the electron heat conductivity should be a constant magnitude. If it is taken into account that the lattice heat conductivity could exist with rising temperature, the summary heat conductivity should decrease weakly with rising temperature, approaching the constant value equal to the electron heat conductivity.

The conclusion regarding the constancy of the electron heat conductivity the high temperature can be made also directly without reference to the Widemann-Franz law, if the electron heat conductivity is written in a form similar to (22.3),

$$\lambda_{\text{electron}} = 1/3 C_{\text{electron}} n v_{\text{electron}} \cdot l_{\text{electron}}, \tag{22.7}$$

by utilizing the concept of the average length of the run of electrons in the metals l_{electron}. Let us assume that because of the statistical degeneracy of the electron gas in the metal

$$C_{\text{electron}} = \gamma T \tag{22.8}$$

the average thermal speed of the electrons v_{electron} does not depend on the temperature and that the volume concentration of electrons is also constant. As regards the magnitude l_{electron}, it depends on the temperature similar to (22.4):

$$l_{\text{electron}} \sim \frac{1}{T}. \tag{22.9}$$

It follows directly from formulas (22.7), (22.8) and (22.9) that $\lambda_{\text{electron}} \sim$ const.

The results of an experimental study of the heat conductivity indicates that as a rule the heat conductivity of metals at high temperatures is actually a weakly decreasing function of the temperature (Figures 104 and 106). At the same time, there are cases when the experiments lead to other results. Thus, for iron there is a comparatively strong decrease in the heat conductivity with rising temperature. For the heat conductivity of tantalum and niobium, a positive temperature coefficient has been obtained (see below). These examples indicate that the existing theory of the heat conductivity of metals cannot yet give an exhaustive qualitative description of the temperature function of the heat conductivity. A broad experimental study of the heat conductivity of metals is, for this reason, absolutely necessary.

Besides the study of the general correlations of the behavior of the heat conductivity of metals at high temperatures, one can point out also a number of specific problems which are of interest in such studies.

We have already indicated the importance of a study of the lattice component of heat conductivity. Here, it is of interest to clarify the correlations which govern the absolute values of the lattice heat conductivity for different metals and to clarify the nature of its temperature function. The latter problem is far from useless because the literature data do not always confirm formula (22.4) and, moreover, sometimes they give also a positive dependence of the heat conductivity of the lattice on the temperature (see, for example, [77]). The other problem is the study of the role of vacancies in the crystal lattice, which originate at high temperatures (see §20). From general considerations, it can be expected that the formation of vacancies should in principle show up on the processes of transfer of electricity in heat because these disturbances in the ordered distribution of the atoms are a source of additional scattering of electrons and phonons. The effect of the formation of vacancies on the electrical resistance of metals is considered an established fact--small, but an anomalous with respect to the nature of the temperature dependence increase in the electrical resistance of a number of metals, found in a number of references (see, for example, [229]), is ascribed precisely to the role of vacancies and is utilized for evaluating the energy of their formation. (The effect of vacancies on the electrical resistance is evaluated also from the change in resistance during rapid cooling--quenching of metals--see, for example [230, 231].) It is obvious that because of the existence of the Widemann-Franz law a similar effect should take place also in the case of electron heat conductivity of metals. On the other hand, the existence of defects in the lattice can change also the lattice heat conductivity and lead to its steeper decrease at high temperatures. For metals with a noticeable contribution of lattice heat conductivity, it is possible in principle not only to clarify this effect, but also to evaluate the energy of the formation of vacancies.

It is necessary to say a few words about the problem of the role of the grain dimensions in polycrystalline specimens on the heat conductivity. At high temperatures, the length of the free run of electrons and phonons are substantially less than the grain dimensions, and the scattering on the boundaries of the grain should not lead to a noticeable effect. Nevertheless, it is not excluded that specimens of the same composition subjected to different heat treatments could exhibit a different heat conductivity as the result of a different distribution of impurities in the volume. A rather large effect of such a type was discovered not too long ago by Jun and Hoch [261]. Although the results of the measurements by these authors contain, apparently, considerable systematic errors (see §24), the problem of the possible effect of heat treatment deserves attention.

all

§23. Results of Studies of the Temperature Diffusivity and the Heat Conductivity of Refractory Metals

This section as well as §20 and 21 compare and analyze the existing experimental data on the temperature diffusivity and the heat conductivity of refractory metals at high temperatures. The objects of the studies were the same metals as in §20.

The task of analyzing data on the heat conductivity and temperature diffusivity, and in particular the qualification of the most reliable values, is much more complex in comparison with such for the heat capacity. There are several reasons for this.

In the first place, differences in the data by different authors for the heat conductivity and the temperature diffusivity are, as a rule, much greater than for the heat capacity. In the second place, in discussing the reasons for these differences, there is always concern that these differences are determined not only by the systematic errors in the experiments but also by differences in the material itself, because it is known that the heat conductivity and the electrical conductivity are magnitudes which are rather sensitive to impurities, lattice defects, etc., although it must be said that in the range of high temperatures this effect is less than at low and medium temperatures.

In the third place, it is necessary to take care that the data proposed as the most reliable for the heat conductivity, temperature diffusivity, and heat capacity are mutually coordinated, although, on the other hand, the need for such coordination can be regarded also as a positive factor that facilitates additional control of these data.

Let us examine the material on the temperature diffusivity. The number of references dealing with measurement of the temperature diffusivity of refractory metals at high temperatures is not large and this makes the possibility of directly separating the most probable values of the temperature diffusivity debatable. A more thoughtout procedure is to find the most reliable values of the temperature diffusivity from those for the heat conductivity and heat capacity. On the other hand, the utilization of specific experimental data for the temperature diffusivity and the most probable values for the heat capacity makes it possible to obtain values of the heat conductivity, which could be utilized in determining the most reliable data for the heat conductivity. In determining the weight of these values, one can utilize deviations of the experimental data with respect to the temperature diffusivity from the above-indicated most probable values of the temperature diffusivity. Thereby the procedures for finding the most reliable data for the heat conductivity and the temperature diffusivity will be interlinked.

The results of measurements of the temperature diffusivity of molybdenum, wolfram, niobium, obtained by the method of variable heating in a high-frequency furnace, described in § 16, are shown in Figures 86, 87 and 88 [166, 167, 168, 41]. The test specimen of molybdenum contained 99.9% molybdenum, 0.001% nickel, 0.01% sesquioxides, 0.001% silicon oxide. The density of the specimen at room temperature (25°C) was equal to 10.20 gram/cm^3, the specific resistance of the same temperature amounted to $5.78 \cdot 10^{-6}$ ohm·cm. Besides this specimen, measurements were also made with a specimen of technical molybdenum which contained 0.18% Zr and 0.28% Ti (lower curve in Figure 86). The density of this specimen at room temperature was equal to 10.17 gm/cm^3, the specific resistance was $6.52 \cdot 10^{-6}$ ohm·cm--13% greater than for the first specimen. The specimen of wolfram contained 99.95% wolfram, 0.035% molybdenum. The density of the specimen at room temperature was equal to 19.17 gm/cm^3. Measurements of electrical conductivity, carried out within the temperature range of 500-900°K, yielded values which were 9% less than the electrical conductivity of pure wolfram [19]. The specimen of niobium had the position of 99.20% niobium, 0.3% tantalum, 0.08% titanium, 0.04% iron, and 0.04% silicon. The density at room temperature was 8.54% gm/cm^3. The specific resistance at room temperature was $14.8 \cdot 10^{-6}$ ohm·cm.

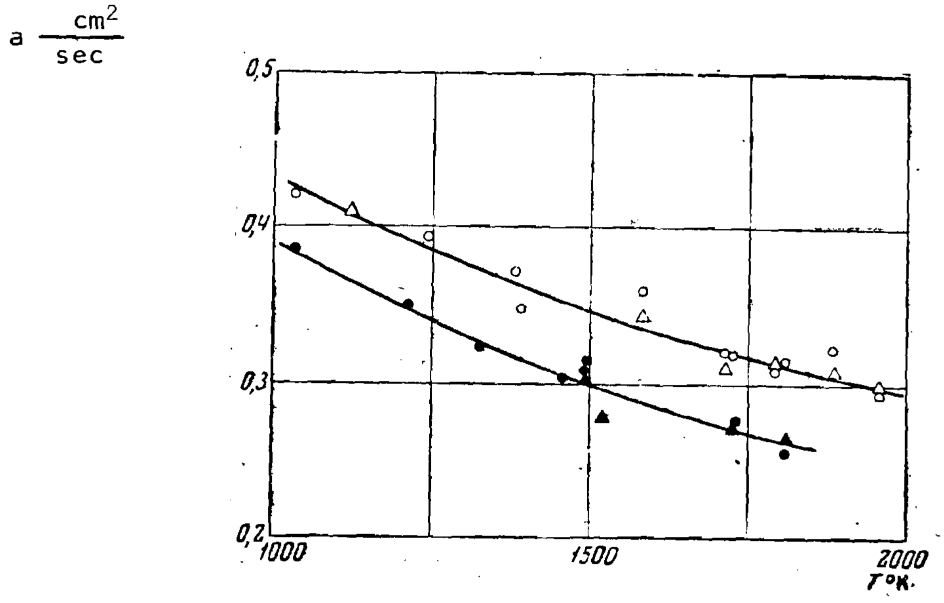

Figure 86. Results of measurements of the temperature diffusivity of two specimens of molybdenum, obtained by the method of intermittent heating in a high-frequency furnace (§16): ○ , data obtained from phases of temperature variations; △ , from the amplitudes.

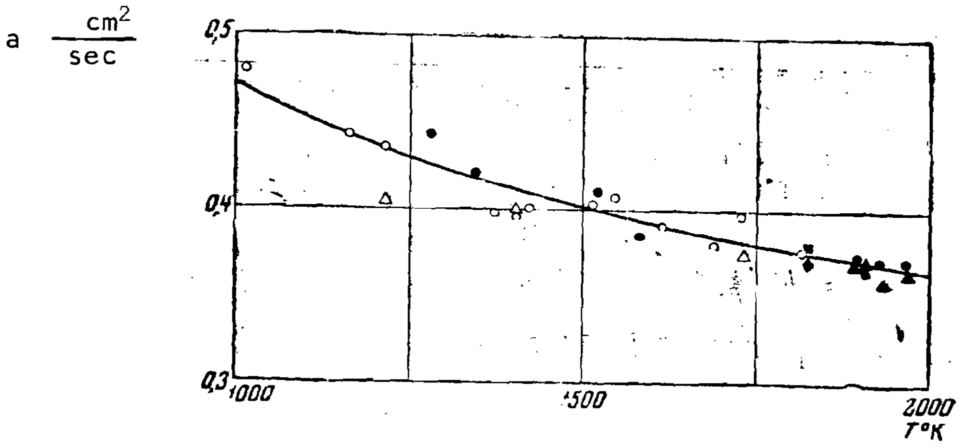

Figure 87. Results of measurements of the temperature diffusivity of wolfram, obtained by the method of variable heating in a high-frequency furnace: △ ○ , results of phase and amplitude measurements, respectively, for an equality of heating and cooling intervals; ● ▲ , the same for unequal intervals; ◆, ■ , the results of phase and amplitudes measurements of a specimen of smaller length.

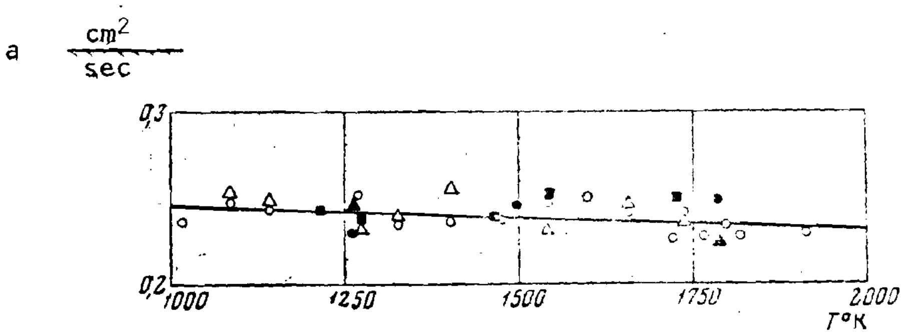

Figure 88. Results of measurements of the temperature diffusivity of niobium: O,Δ , results of phase and amplitude measurements, respectively, obtained by the method of variable heating in an induction furnace (§16); ●,Δ , the same for a specimen of smaller diameter; ■ results obtained on a unit with electronic heating (§14).

The data obtained by means of the amplitude and phase variants of the method are shown differently in the figures. For wolfram, besides that, the figures show the results of measurements with unequal intervals of connection and disconnection of the generator. For niobium, data are given which were obtained for two specimens of different diameter (12 and 15 mm). The same figures show also experimental data obtained for the same specimen of niobium by I. P. Mardykin and the author by using the method of radial temperature waves with electronic heating on a unit with a photoelectric registration of the temperature variations, mentioned at the end of §14, [232]).

The smoothed data by different authors for molybdenum, wolfram, tantalum, and niobium are shown in Figures 89-91.

For molybdenum, the results from the work by O. A. Krayev and A. A. Stel'makh [135] at temperatures of 2,000°K lie between the data for pure technical molybdenum, shown in Figure 86, closer to technical molybdenum. Unfortunately, the references by O. A. Krayev and A. A. Stel'makh have no information on the purity of the metals which they tested as well as data on the electrical conductivity from which one could indirectly evaluate the magnitude of the impurities. If it is assumed that these authors measured specimens of technically pure metals, the agreement in the data for molybdenum can be considered satisfactory. Data by Wheeler [233] for molybdenum lie considerably below the three preceding values. It is significant that these data give also values of the

Lorentz numbers obtained by utilizing tabular values for the electrical conductivity of pure molybdenum, 14% less than the theoretical Sommerfeld number $2.45 \cdot 10^{-8}$ wt•ohm/deg^2. The reason for this can be the presence of considerable impurities in the specimen tested by Wheeler, although according to data by the author the content of impurities (Fe) did not exceed 0.01%. The problem of a contradiction in the result for molybdenum will be taken up by us when we discuss the data on the heat conductivity.

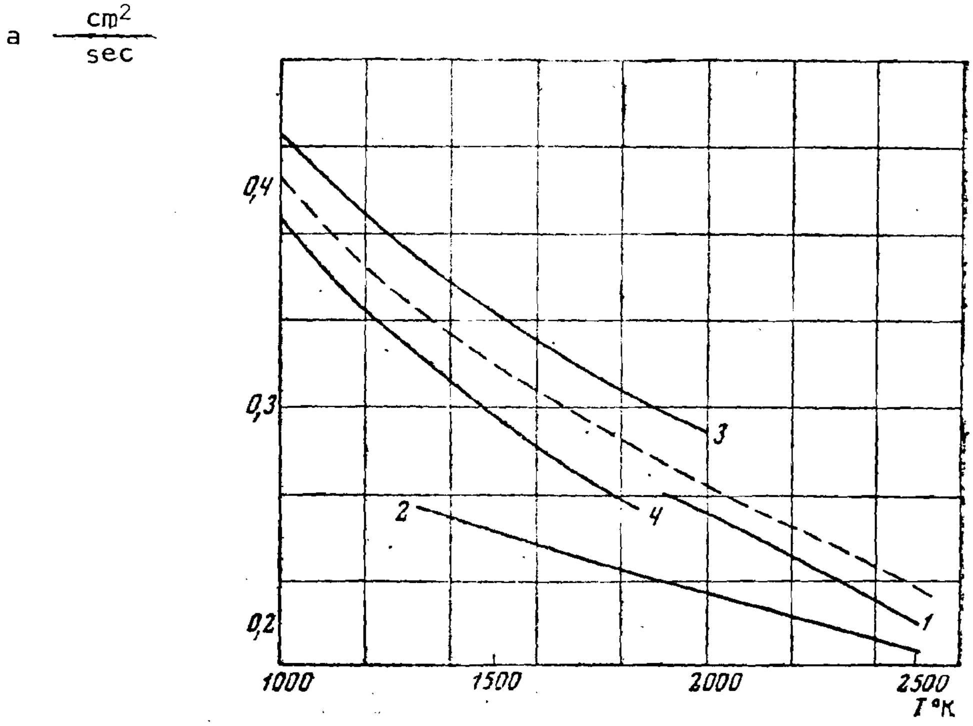

Figure 89. Literature data on the temperature diffusivity of molybdenum: 1, Krayev and Stel'makh [135]; 2, Wheeler [233]; 3, 4, Figure 86.

The differences between the data by various authors for wolfram also depend apparently to a considerable degree on the purity of the test materials. The greatest values are obtained for wolfram with a purity of 99.95%, then follow the data by Wheeler for wolfram of 99.5% (Impurities of fine molybdenum) and the results by O. A. Krayev and A. A. Stel'makh for wolfram of a known composition.

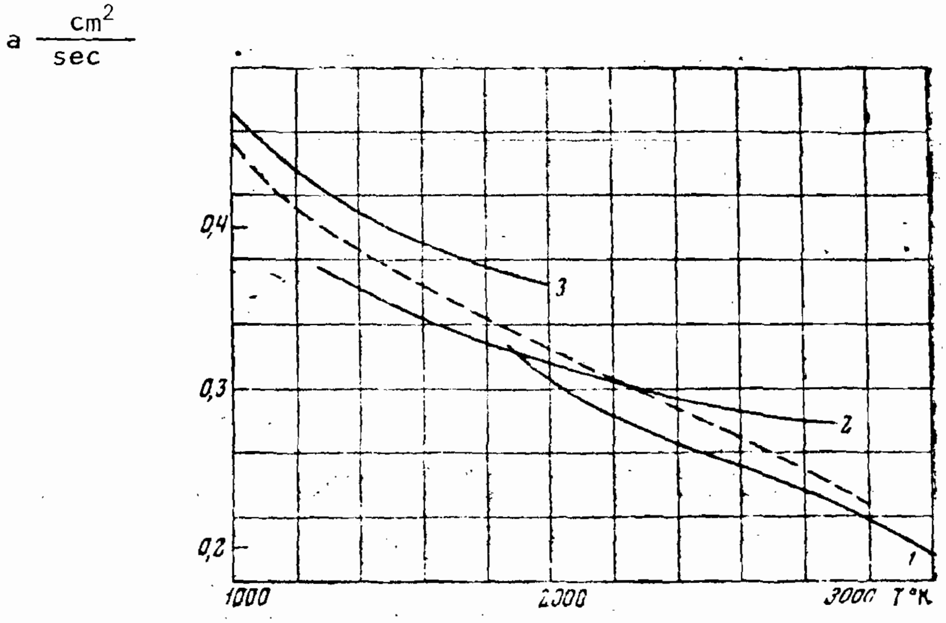

Figure 90. Literature data on the temperature diffusivity of wolfram: 1, Krayev and Stel'makh [135]; 2, Wheeler [233]; 3, See Figure 87.

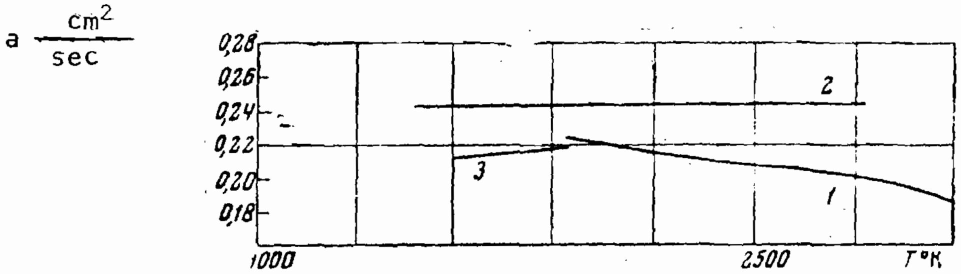

Figure 91. Literature data on the temperature diffusivity of tantalum: 1, Krayev and Stel'makh [135]; 2, Wheeler [233]; 3, Cape et alia [125].

The data by Wheeler and O. A. Krayev and A. A. Stel'makh for tantalum are in relatively good agreement at temperatures of 2,000°K and diverge at higher temperatures. Here it is necessary to point out that the less steep nature of the temperature dependence is characteristic for all the metals which they measured. In running ahead, we wish to observe that this leads to a constancy in the Lorentz numbers obtained by Wheeler, whereas the data by most other authors and considerations of a theoretical nature yield Lorentz numbers which decrease with rising temperature although weakly. The data by Cape *et alia* [125] near the same temperature of 2,000° are close to one in the other, however, the positive temperature coefficient found by these authors is of little reliability due to the considerable scatter of points in this experiment and the narrow temperature range.

The agreement between the data which exists for niobium can be considered satisfactory, which is apparently due to the closeness in the composition of the test specimens.

In comparing the data on the heat conductivity, the problem arises as to whether it is necessary to compare directly the values of the heat conductivity or to give preference to the Lorentz numbers or, finally, to concentrate attention only on the lattice heat conductivity. In principle, the comparison of the Lorentz numbers is a procedure that is more effective than a comparison of the values of the heat conductivity, because relatively large changes in the heat conductivity, determined by the presence of impurities, have a much smaller effect on the Lorentz numbers. At the same time, if we limit ourselves to a comparison of the Lorentz numbers only, we will lose the opportunity of utilizing the experimental material for which the data on the electrical conductivity are lacking. This pertains, for example, to the high-temperature data by O. A. Krayev and A. A. Stel'makh [134, 135]. Besides that, in order to pass from Lorentz numbers, selected as the most reliable, to values of heat conductivity, it is necessary to make a similar analysis for the data on the electrical conductivity. Considerations of this type lead to the conclusion that in an analysis of the experimental material on heat conductivity it is hardly feasible to limit ourselves to some single possibility of those under discussion. It is desirable to study the heat conductivity's values as well as the Lorentz numbers in the magnitude of the lattice components, all the more so since this creates an additional possibility for the mutual control of the results in averaging each of these magnitudes.

The smoothed values of the results of heat-conductivity measurements and of the Lorentz numbers for molybdenum, wolfram, tantalum, niobium are shown in Figure 95.and those that follow. Among those under comparison are also the values obtained from data of the temperature diffusivity and the heat capacity; besides, in the case of the heat capacity, use was made of values which have been recommended as the most reliable (Figures 91, 92, 93, 94). The experimental material for molybdenum and wolfram was supplemented by us with data obtained by the method described in §5 [234, 109, 107] (see Figures 93, 94). The test molybdenum (wire 0.5 mm in diameter) contains 0.1% iron. The measurements of the electrical conductivity of this specimen, obtained within the temperature range of 1,200-2,400°K by two methods (see §5), gave values which are in agreement with each other within the limits of 2-3%.

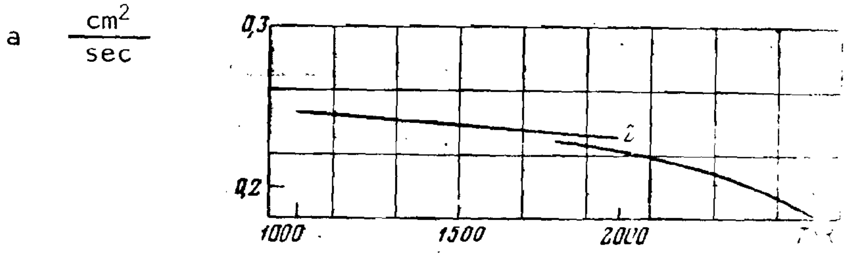

Figure 92. Literature data on the temperature diffusivity of niobium: 1, Krayev and Stel'makh [135]; 2, Figure 88.

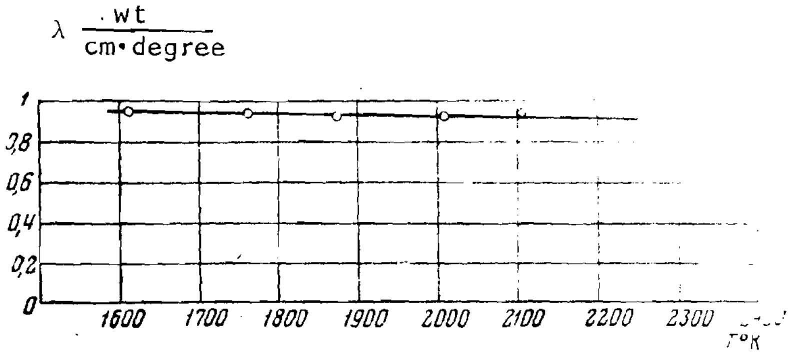

Figure 93. Results of the measurement of the heat conductivity of molybdenum (see §5).

Measurements of the heat conductivity of wolfram were carried out for different specimens: wires 0.3 and 0.2 mm in diameter (wolfram of grade VA-3-P) and foil 60 mk thick (wolfram of grade VCh).

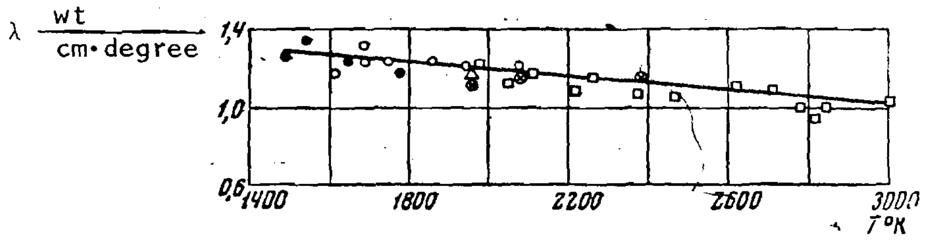

Figure 94. Results of the measurement of heat conductivity of wolfram: O, Δ, foil 60 mk thick, 2 mm wide (2 specimens); ●, foil 60 mk thick, 3 mm wide; □, ⊕, wires 0.3 and 0.2 mm.

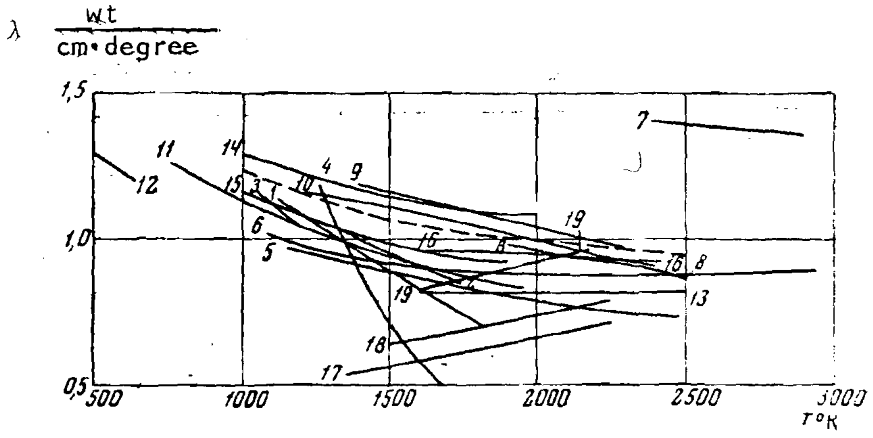

Figure 95. Summary of literature data on the heat conductivity of molybdenum: 1, Osborn [89]; 2, Cutler and Cheney [85]; 3, Bode [100]; 4, Lebedev [97]; 5, Gumenyk, Ivanov and Lebedev [98]; 6, Rason and McClelland [66]; 7, Allen et alia [235]; 8, Krayev and Stel'makh [135] (calculated from temperature diffusivity); 9, Timrot and Peletskiy [62]; 10, Timrot, Peletskiy and Voskresenskiy [63]; 11, Fielhouse (see [238]); 12, Tye [236]; 13, Wheeler [233]--calculated from data on temperature diffusivity; 14, 15, from data on the temperature diffusivity and Figure 86; 16, see Figure 93; 17, 18, 19, Jun and Hoch [261].

Let us examine the heat conductivity of molybdenum. It is seen from Figure 95 how large the difference is between the data by different authors with respect to the absolute values as well as with respect to the nature of the temperature function. In order to select an aggregate of more reliable data, it is of great help to examine the behavior of the Lorentz numbers shown in Figure 96 for data from papers in which the authors cite information on the electrical conductivity of the test specimens, even though for a limited temperature range (the data by Wheeler are an exception). As the result of an analysis of the cited data, we excluded the following from the most reliable and did not utilize these for finding the most probable values:

1. Data by Allen *et alia* [235] fall out sharply from the aggregate under consideration, and not only for molybdenum; a critique of this work is given by V. Yu. Voskresenskiy [237, 238].

2. Data by V. V. Lebedev [97] and V. S. Gumenyuk, V. Ye. Ivanov, and V. V. Lebedev [98] not only because these authors, in using the same method obtained two series of results which differ strongly from each other, but also because their data yield obviously reduced values of the Lorentz numbers.

3. The Wheeler data [233] which also leads to sharply reduced values of the Lorentz numbers.

4. The Osborn data [89], which also give anomalously low values of the Lorentz numbers at high temperatures.

5. The Jon and Hoch data [261] which have an anomalous temperature course. The reason for possible distortions of their results can lie in the approximation of the assumptions which are utilized in the theory of the method of measurement (see §4), in particular in not taking into account the corrections for the volume character of the heat liberation. A certain suspicion is aroused also by the data which have been found by these authors for the degree of blackness (for 2 specimens these were smaller than the tabular data for polished molybdenum). It is a pity that the authors do not cite information on the electrical conductivity of their test specimens; such data could have been of great help in clarifying the problem of the actual role of heat treatment.

Besides this, in averaging the data on the heat conductivity, we did not take into account the results of measurements for technical molybdenum, which are shown in Figure 86 (lower curve). The averaging of the remaining aggregate of the data which, in our opinion, are the most reliable was carried out in an independent manner from the heat conductivity from the Lorentz numbers; besides all the data were given equal weight (simple averaging). Recalculation of the average values of the Lorentz numbers with the utilization of tabular values for the electrical conductivity gave heat conductivity results which lie systematically 4% above the curve that was obtained by averaging the heat conductivity data. Assuming that the most reliable curve is the average between these two, the data for the average values of the heat conductivity will increase by 2%, while the data for the average values of the Lorentz numbers were reduced by 2%. The curves obtained in this case are shown in Figures 104 and 105 (dotted curves). The data by different authors from the separated aggregate differ, as a rule, by not more than 8% from the found most probable values of the heat conductivity, which is close to the error of the measurement.

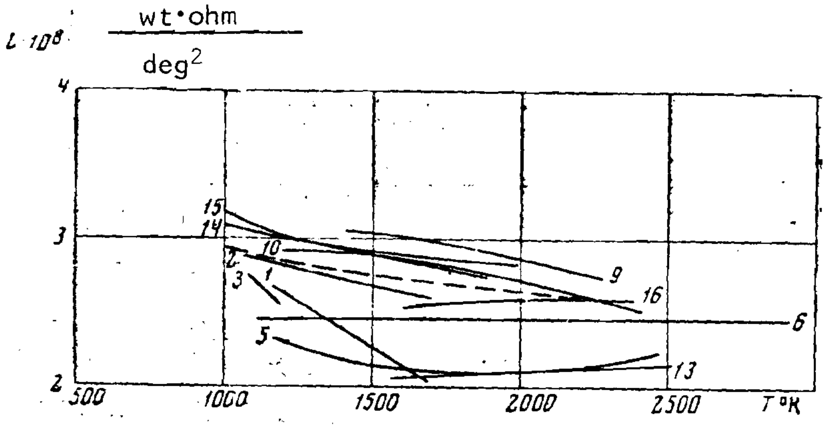

Figure 96. Values of the Lorentz number for molybdenum. The numbers correspond to Figure 95.

The greatest difference (16%) is observed for the Rason and Mc Clelland data [67]; the results by these authors differ also in their large scatter of individual measurements (up to 20%).

From the found probable values of the heat conductivity and heat capacity of molybdenum, the most probable values of the temperature diffusivity were determined. The results are shown by the dotted curve in Figure 89.

A summary of the results of the measurements of the heat conductivity of wolfram is shown in Figure 97. It can be seen that the sum of the results here are no less contradictory than for molybdenum. Just as for molybdenum, some data were excluded by us from the examination during the evaluation of the most reliable data. This pertains to the sharply falling out data by Allen *et alia* [235] and the data by Worthing [88], Zwickert [239], and Forsyte and Worthing [240], which have a positive temperature coefficient. The values of the Lorentz numbers are shown in Figure 98. Just as for molybdenum, the average data were obtained separately for the heat conductivity and the Lorentz numbers and were then compared. The maximum difference of the corresponding curves (at 1,000°K) amounted to 5%. The average, most reliable values of λ and L are shown in Figure 97 and 98 by a heavy dotted line. The same type of dotted line in Figure 90 gives the most reliable data for the temperature diffusivity.

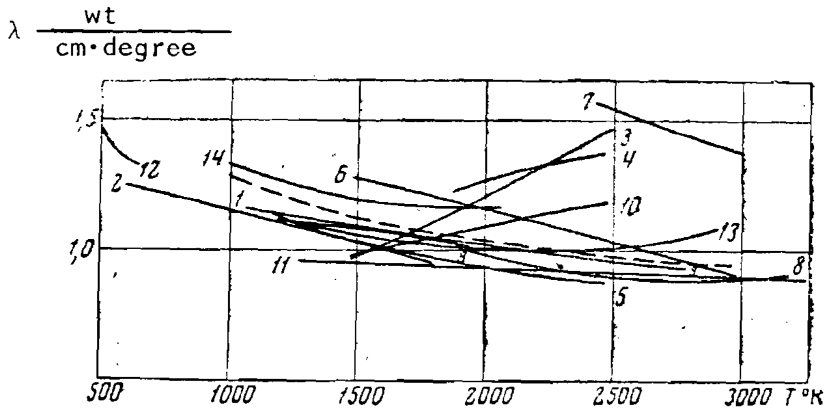

Figure 97. Summary of literature data on the heat conductivity of wolfram: 1, Osborn [89]; 2, Cutler and Cheney [85]; 3, Worthing [88]; 4, Zwikker [239]; 5, Gumenyuk, Ivanov and Lebedev [98]; 6, Figure 94; 7, Allen et alia [235]; 8, Krayev and Stel'makh [134] (calculated from the temperature diffusivity); 9, Timrot and Peletskiy [61]; 10, Forsyte and Worthing [240]; 11, Platunov and Fedorov [87]; 12, Tye [236]; 13, Wheeler [233] (calculated from temperature diffusivity data); 14, from temperature diffusivity data of Figure 87.

The results of measurements of the heat conductivity of tantalum are shown in Figure 99. With the exception of the data by V. S. Gumenyuk, V. Ye. Ivanov and V. V. Lebedev [98], Worthing [88], and Allen *et alia* [235], the results of high-temperature measurements are in general quite satisfactorily in agreement with each other and have a qualitatively like positive dependence on the temperature. The values of the heat conductivity, which are most probable, lie close to the data by Rasor and McClelland [66] and V. E. Peletskiy and V. U. Voskresenskiy [64]. As regards the Lorentz numbers for tantalum, it is seen from Figure 100 that the values lie close to the theoretical magnitude of $2.45\ 10^{-8}$ wt. ohm/deg and, if they exceed this, then it is insignificant.

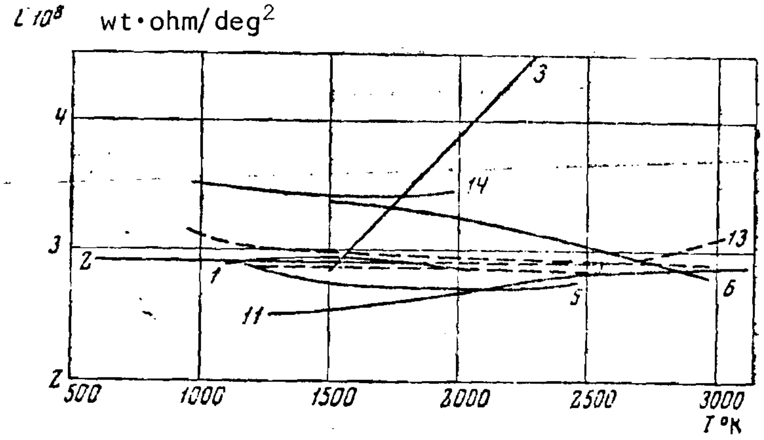

Figure 98. Values of the Lorentz number for wolfram. The numbers correspond to Figure 97.

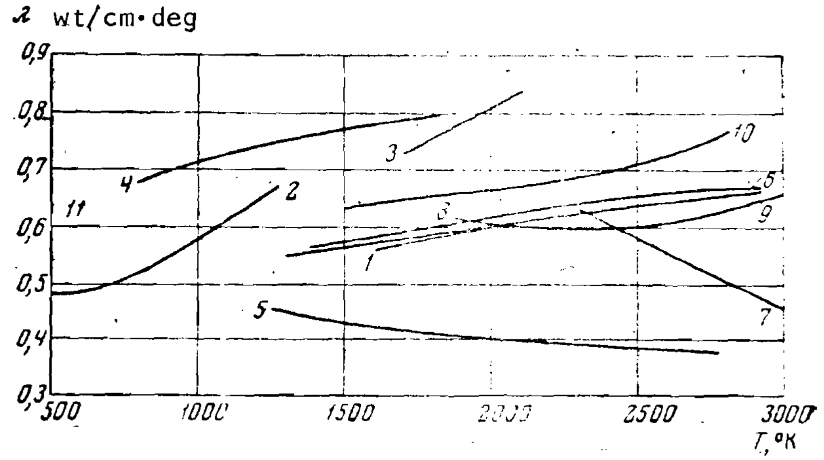

Figure 99. Summary of literature data on the heat conductivity of tantalum: 1, Cape et alia [125] (calculated from temperature diffusivity data); 2, Pozdnyak and Akhmetzyanov [242]; 3, Worthing [88]; 4, Fielhouse (see [238]); 5, Gumenyuk, Ivanov and Lebedev [98]; 6, Rasor and McClelland [66]; 7, Allen et alia [235]; 8, Krayev and Stel'makh [135] (recalculated from temperature diffusivity data); 9, Peletskiy and Voskresenskiy [64]; 10, Wheeler [233] (calculated from temperature diffusivity data); 11, Tye [236].

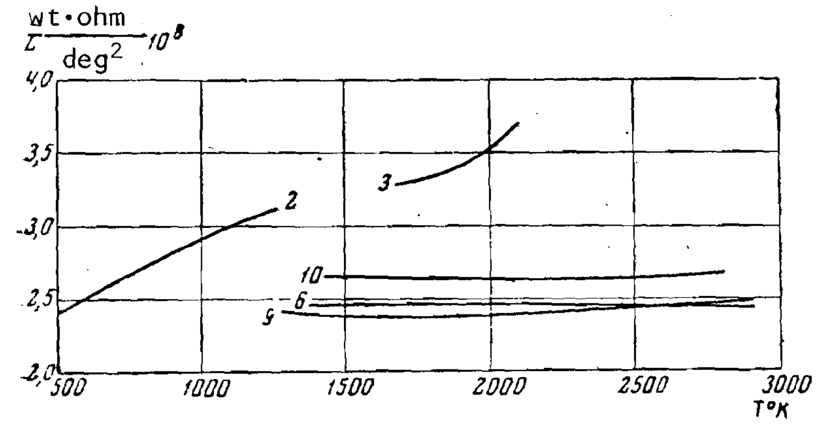

Figure 100. Values of the Lorentz number for tantalum. The numbers correspond to Figure 99.

The heat conductivity of niobium has a positive temperature core similar to tantalum (Figure 101). In the region of temperature above 1,000°K, the results of four studies are generally in good agreement with each other (the most probable values are given by the dotted curves), there is satisfactory agreement also in the range of temperature of 500-1,000°K. The data by N. Z. Pozdnyak and K. G. Akhmetzyanov [242], obtained by the Kohlrausch method, differ most strongly from the remaining; this difference is determined apparently by the role of the heat exchange [238]. The values of the Lorentz number for niobium (Figure 102) exceed the theoretical value by several percent.

Let us summarize some of the results of the study of the heat conductivity of refractory metals.

The main share of heat transfer in the metals under consideration is by the electron heat conductivity. The share of the lattice heat conductivity amounts to several percent in the case of tantalum and niobium and up to 17-20% in the case of molybdenum and wolfram (at a temperature of ~ 1,000°K). The absolute values of the lattice heat conductivity lie below 0.2-0.25 wt/cm. deg. The lattice heat conductivities of other metals [77] as well as of solid dielectrics of the oxide type [245] have approximately the same order of magnitude. With rising temperature, the lattice heat conductivity of the metals under consideration decreases. This decrease for molybdenum is close to 1/T. For wolfram, the temperature function is weaker. This fact, however, cannot at all be considered as firmly established because the reliability of the values obtained as the most probable is not so high: small changes in these values lead to a comparatively strong change in the nature of the temperature function of the lattice heat conductivity.

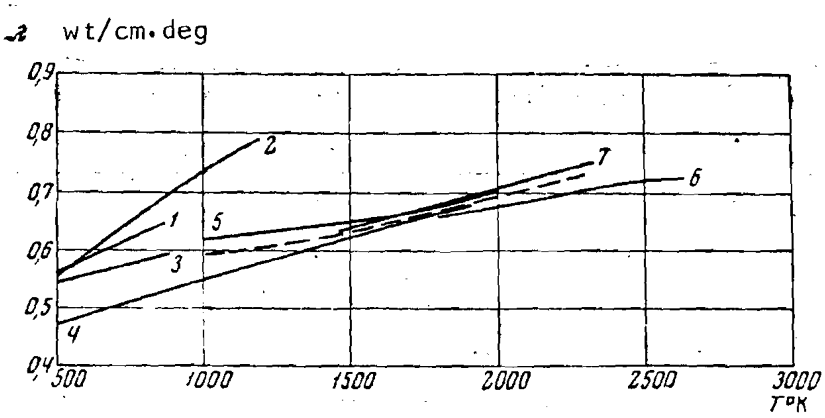

Figure 101. Summary of literature data on the heat conductivity of niobium: 1, Tottle [243]; 2,Pozdnyak and Akhmetzyanov [242]; 3, from [244]; 4, Fielhouse (see [238]); 5, from temperature diffusivity data of Figure 88; 6, Krayev and Stel'makh [134] (recalculated from temperature diffusivity data); 7, Timrot, Peletskiy and Voskresenskiy [63].

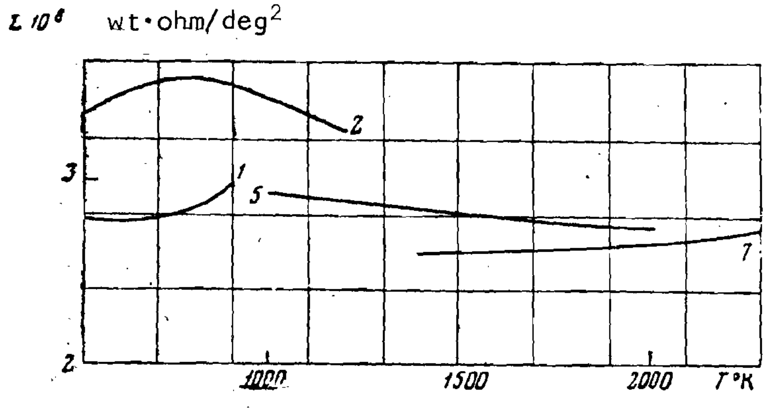

Figure 102. Values of the Lorentz number for niobium. The numbers correspond to Figure 101.

The temperature function of the summary heat conductivity is determined in the first place by the electron heat conductivity. It is precisely the behavior of the electron heat conductivity which determines the positive nature of the temperature function of the heat conductivity of tantalum and niobium, which distinguishes these from most metals. In order to determine the reasons for this, it is necessary to make a theoretical analysis of the specific characteristics of the mechanism of the transfer of heat and electricity to these metals. Since we do not find it possible to take upon ourselves such a task, we shall risk, however, in making the statement that these characteristics are related to the behavior of the electron heat capacity

§24. Heat Conductivity of Liquid Metal

The problem of the heat conductivity of liquid metals belongs to those areas that have been least studied theoretically as well as experimentally. The complexity of the theoretical study of the processes of the transfer of heat and electricity in liquid metals is understandable if one takes into account that in this instance difficulties which are inherent in the similar problem of solid bodies, multiplied by the complexity of the problem of the liquid state of matter (see§ 21). The paucity of the experimental materials is also understandable. It is explained by the serious experimental difficulties which stem from the difficulties that are usual for high-temperature studies and the additional difficulties such as concern about the lack of convective mixing, the need of taking into account the role of the walls of the vessel in which the liquid metal is contained, intensive evaporation of the metal, and others.

At the same time, the experimental study of heat conductivity is one of the fundamental means of studying the electron properties of liquid metals, which is all the more important because the data on the heat conductivity are basically necessary in a number of branches of the new technology such as atomic power.

We shall briefly dwell on the ways for developing the theory of electron properties of liquid metals. The attempt to apply to liquid metals the methods of research developed for solid metals belongs to A. I. Gubanov [246]. The starting position of A. I. Gubanov is the zonal theory of solid bodies, modified in such a way as to take into account the disturbances in the ideally ordered distribution of atoms in the lattice, which are inherent in the liquid state. These disturbances were assumed by A. I. Gubanov to be very small so that a significant disturbance in the ordered distribution of the atoms takes place only at relatively large distances. On the basis of such concepts,

A. I. Gubanov obtains a generalization of the basic results of the zonal theory, which pertain to the energy spectrum of electrons and processes of electron transfer. In particular, the theory leads to the conclusion regarding the existence in liquids of an additional, specifically liquid mechanism for scattering electrons, scattering which is governed by the structure of this order, by the disturbances in the ideal-crystalline packing. This additional scattering, which is added to the scattering due to thermal vibrations, leads to decrease in the effective length of the free run of the electrons and to an increase in the thermal and electrical resistance. At the same time, the electrical conductivity and the electron heat conductivity change parallel to each other in such a manner that the Widemann-Franz law for liquid metals should, in accordance with A. I. Gubanov, preserve its validity.

The fundamentals of the theory by A. I. Gubanov are not free of criticism. The concept of the structure of a liquid as a little-distorted crystal lattice does not apparently characterize sufficiently fully the disorder which exists in the liquid. In any case, such an approach is contradictory to the concept by D. Bernal in accordance with which the disorder distribution of the molecules of the liquid differs radically from the ordered structures of a solid body so that the continuous transition from the solid body to the liquid is basically impossible [247].

Another direction in the development of the theory of electron properties of liquid metals is to a certain degree in opposition to the first because it is based on the relationship of liquids, not with a solid body, but with a gas, more accurately with plasma. This direction has recently been under intensive development, but the results obtained so far pertain chiefly to the electrical conductivity, they do not make yet possible any kind of a reliable quantitative description of the aggregate of the electron properties of liquid metals.

The absence of more or less definite theoretical concepts regarding the basic correlations of the behavior of the heat conductivity of liquid metals makes it entirely necessary to perform a broad program of experimental studies. The basic problems in an experimental study of the heat conductivity of liquid metals are in this case as follows.

1. Clarification of the correlations of the change in the heat conductivity of metals during melting.

2. Study of the nature of the temperature function of the heat conductivity of liquid metals within a broad temperature range.

3. Comparison of the heat conductivity and electrical conductivity of liquid metals, determination of the temperature function of the Lorentz number, and clarification of the problem of the Widemann-Franz law.

A study of the first of these problems assumes the availability of information regarding the jump in the heat conductivity during melting, which would make it possible to evaluate the radical extent of the change in the nature of heat transfer during the transition from the solid state to the liquid state. A comparison of the jumps in the electrical conductivity and the heat conductivity reflects in this case the degree of preservation of parallelism in the mechanisms of the transfer of heat and electricity, even though in an ambiguous manner, because the measured heat conductivity is the sum of the electron and "lattice" (molecular) components. The establishment as much as possible of the general correlations of the change in the heat conductivity with temperature makes sense not only from the viewpoint of empirical systematization, but also directly for solving the problem of the heat transfer mechanism. A special role in this belongs to the study of the behavior of the Lorentz number, the absolute values and the temperature function of which reflects fundamentally the nature of the processes of electron transfer. In attempting a theoretical interpretation of the existing experimental correlations, it is expedient to turn back to the conclusions which can be made from analysis of the results that pertain to the electrical properties of liquid metals; the electrical conductivities and the Hall effect. In accordance with A. R. Regel's [249], it follows from the available data that during the melting of metals, as a rule, there is no fundamental change in the mobility of the current carriers. In order to explain the changes in electrical conductivity with an invariable mobility, it is necessary thus to make the conclusion that the concentration of the electrons during melting in most metals decreases approximately two times. On the other hand, Bush and Tieche [250] came to the conclusion that their experiments are in good agreement with the calculations on the assumption that a number of free electrons per atom in a liquid metal is equal to the number of valance electrons (lead and bismuth are an exception). The ambiguity of the resulting conclusions is related with the contradiction of the experimental material with respect to the Hall effect; values of the Hall constant in accordance with data by different authors differ not only in magnitude but sometimes even in sign (for example, for antimony and tellurium).

Before studying a systematization and discussion of the experimental data, we shall cite the original material which is at our disposal. Figure 103 and 104 show the results of measurements of the temperature diffusivity of tin, lead, bismuth, cadmium in the solid and liquid state

[160], [227]. The measurements were made by the method of radial temperature waves, described in §14. The high-temperature data for tin and lead were obtained by the method of measuring the complex of thermal properties, described in §15. (The primary material: results of measurements obtained by the phase as well as the amplitude variants of the method, for each--at 2-3 different periods are given in [160].) Table 40 gives the results of the measurement of the specific resistance of the same specimens of tin, lead, and bismuth [160].

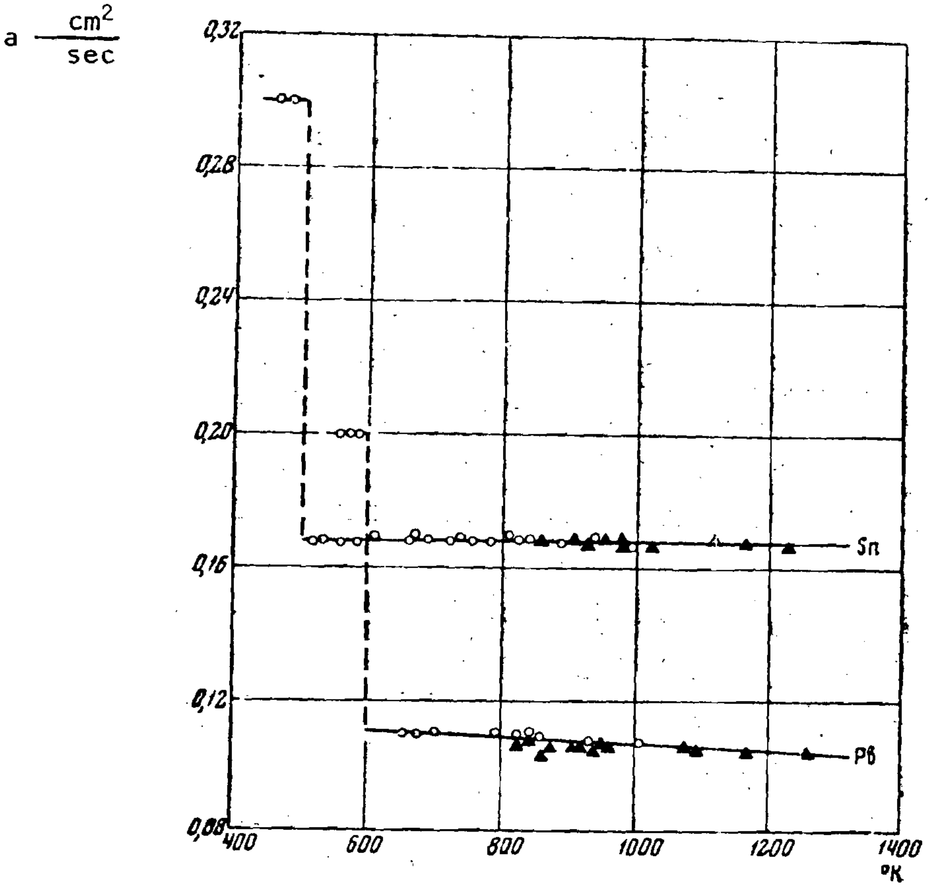

Figure 103. Results of measurement of the temperature diffusivity of tin and lead: ○, measurement by the method of radial temperature waves, described in §13; ▲, measurement by means of a complex procedure described in §14.

During measurements of electrical conductivity, the liquid metal was in a tantalum tube 24 cm long and 8 mm in diameter with a wall thickness of 0.1 mm. the tube was placed along the axis of the cylindrical heater. Current leads were fed to the end of the tube and near the middle, two pairs of thin potential wires which were at a distance of 5 and 10 cm from each other were welded thereto. Current of the order of ampheres was pressed through the tube, its magnitude being determined from the drop in voltage on a standard resistance of 0.001 ohm which was connected in series with the tube. The voltages on a standard resistance and in the working section of the tube, which amounted to hundreds of microvolts, were measured by a PMS-48 potentiometer. In order to exclude thermal electromotive forces, the measurements were carried out in two opposite directions of the current through the tube and the results were averaged. Before each series of measurements, a determination was made of the resistance of the empty tube. In experiments with liquid metals, this resistance was considered as connected in parallel. In order to have confidence in the observence of the conditions of equipotentiality of the transverse sections, the measurements were conducted on two sections of the tube (short and long). Agreement of the values of the electrical conductivity obtained in this case indicated the smallness of the distortion of the equipotentiality due to the influence of boundary conditions.

In measuring the electrical conductivity of solid metals, use was made of rods 6 mm in diameter. The maximum error in the resulting values of the electrical conductivity for solid as well as for liquid metals does not exceed 1-2%. The resulting data are in agreement with the literature (see [160]).

The values of the heat conductivity of liquid tin and lead, obtained from the temperature diffusivity data in Figure 103 and the values of the heat capacity found in the same complex experiment (see Figures 82 and 83), are shown in Figures 105 and 106 together with the results by the other authors. (For the solid phase the literature data of heat capacity was taken; values of the heat capacity below 850°K were extrapolated.)

Similar results for cadmium and bismuth are shown in Figures 107 and 108. In recalculating the data with respect to the temperature diffusivity, we made use of literature data [219]. The Figures 109-111 contain a summary of literature data on the heat conductivity of liquid alkali metals. The comparable data contain also results of determination of the heat conductivity, obtained from measurements of the temperature diffusivity by I. I. Novikov *et alia* [149] and I. I. Rudnev *et alia* [150].

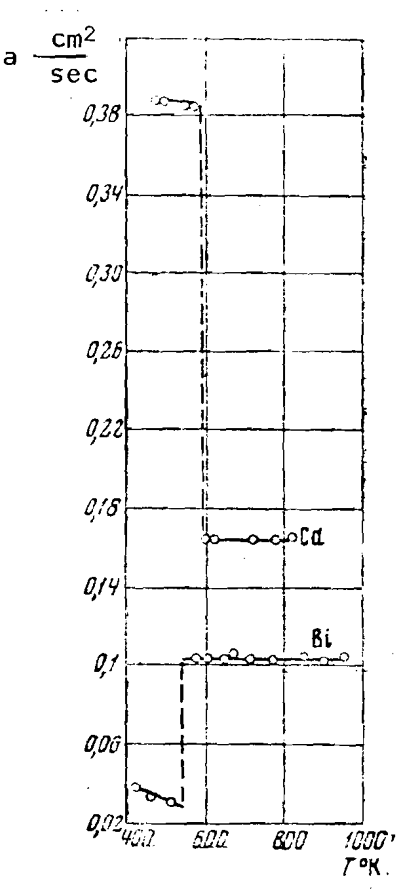

Figure 104. Results of measurements of the temperature diffusivity of cadmium and bismuth obtained by means of the method described in §13.

An examination of the entire aggregate of decided data convinces one of the considerable contradiction in most results. Deviations of several tens of percent are not rare. As a rule the differences increase with rising temperature; near the melting point agreement is better. The cited data do not permit any kind of general conclusions regarding the nature of the temperature function of the heat conductivity of liquid metals, not only because this function is frequently different depending on the data of different authors, but also because the general tendency of the change in the heat conductivity for different metals is apparently also different. Of the data pertaining to divalent metals, attention is drawn by the constancy of the heat conductivity over a large temperature (600-700°, which was clearly traced for tin and lead) and partly for cadmium and bismuth, which we consider sufficiently firmly established, considering the multiple control of the results of the corresponding experiment (see 13 and 14).

The summary of the results of the measurements of the jump in the heat conductivity during melting is given in Table 41. It can be seen that these data are, as a rule, in satisfactory agreement with each other.

The last column of this table shows the values of the jump in electrical conductivity during melting. The data which are given within the brackets pertain to the results from Table 40.

A comparison of the values of $\lambda_{solid}/\lambda_{liquid}$ and $\sigma_{solid}/\sigma_{liquid}$ makes it possible to observe the presence of parallelism between the jump in the heat conductivity and the electrical conductivity. This parallelism shows up clearly in the data for bismuth in the case of which the anomalous positive change in the heat conductivity during melting is accompanied also by a similar anomalous change in electrical conductivity. (The anomalous nature of these changes is related with the fundamental change in the structure of the short-range order of bismuth during melting). A detailed comparison of the magnitudes $\lambda_{solid}/\lambda_{liquid}$ and $\sigma_{solid}/\sigma_{liquid}$ makes it possible to establish that a jump in electrical conductivity is greater as a rule than the jump in the heat conductivity. If it is taken into account in this case that the Lorentz number before melting just as well as after melting most frequently exceeds the theoretical magnitude $2.45 \cdot 10^{-8}$ wt·ohm/deg^2, then this difference can be ascribed to the lesser change in the molecular (lattice) heat conductivity during melting in comparison with the change in the electron component. This conclusion is confirmed in directly by the results of the measurement of the change in heat conductivity during melting of nonmetallic materials, which were carried out by us [259].

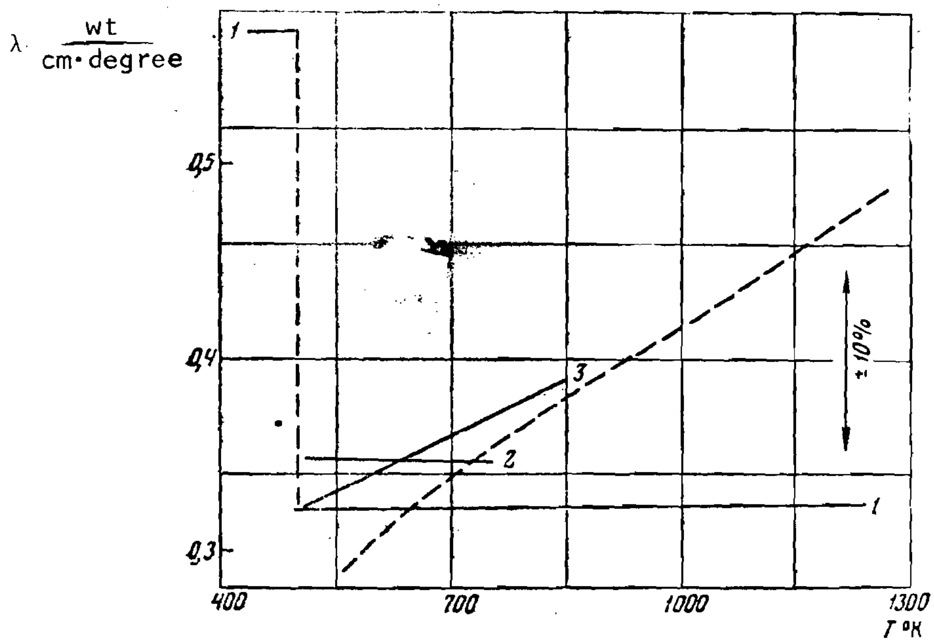

Figure 105. Results of measurements of the heat conductivity of solid and liquid tin: 1, data obtained by the method of measurements described in 14; 2, data by Konno [251]; 3, data by Nikol'skiy et al [60].

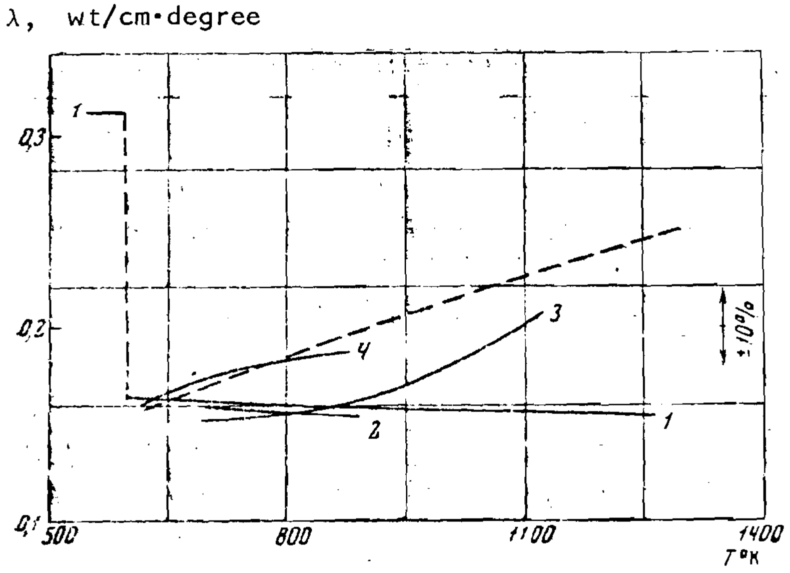

Figure 106. Results of measurement of the heat conductivity of solid and liquid lead: 1, data obtained by the method of measurements described in 14; 2, data by Konno [251]; 3, data by Nikol'skiy et al [60]; 4, data by Powell and Tye [252].

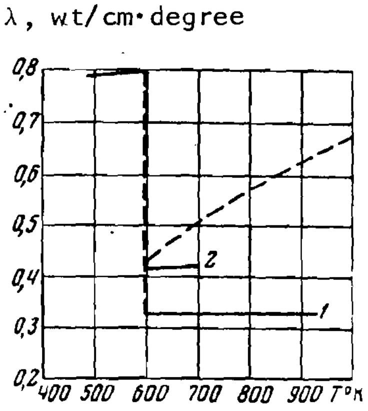

Figure 107. Results of measurements of the heat conductivity of solid and liquid cadmium: 1, data obtained from results of measurements of the temperature diffusivity in Figure 104; 2, data by Brown [253].

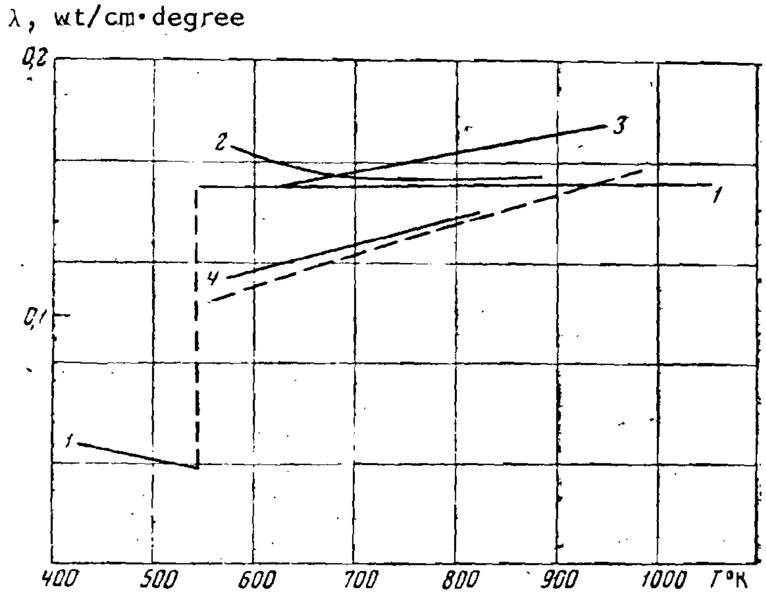

Figure 108. Results of measurements of the heat conductivity of solid and liquid bismuth: 1, data obtained from results of measurements of the temperature diffusivity of Figure 104; 2, data by Konno [251]; 3, data by Nikol'skiy et alia [60]; 4, data by Powell and Tye [252].

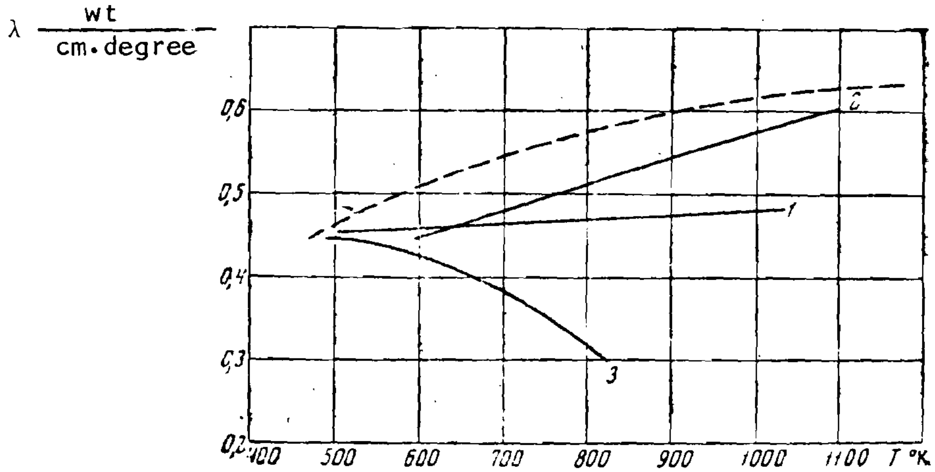

Figure 109. Summary of literature data on heat conductivity of liquid lithium: 1, Nikols'kiy et alia [60]; 2, Cooke [51]; 3, Webber et alia [254].

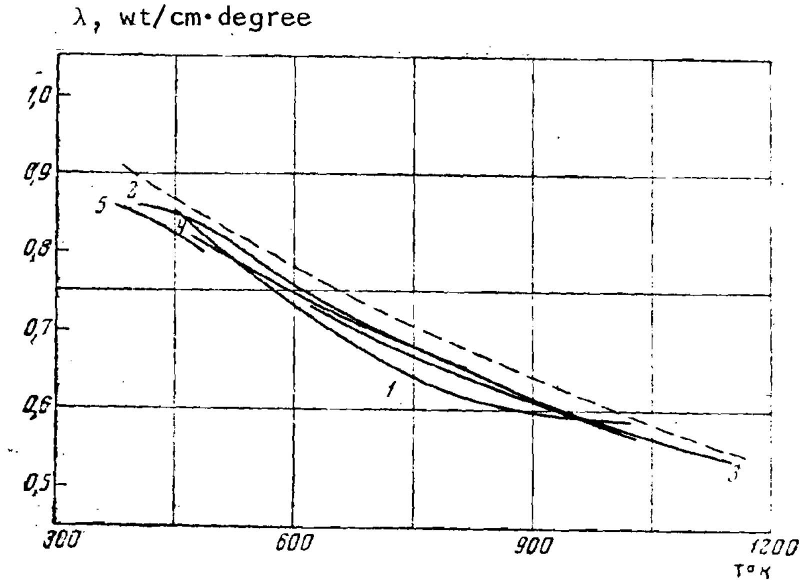

Figure 110. Summary of literature data on the heat conductivity of liquid sodium: 1, Nikol'skiy et alia [60]; 2, Novikov et alia [149] (recalculated from temperature diffusivity data); 3, Rudnev et alia [150] (recalculated from temperature diffusivity data); 4, Ewing et alia [50]; 5, Hall [52].

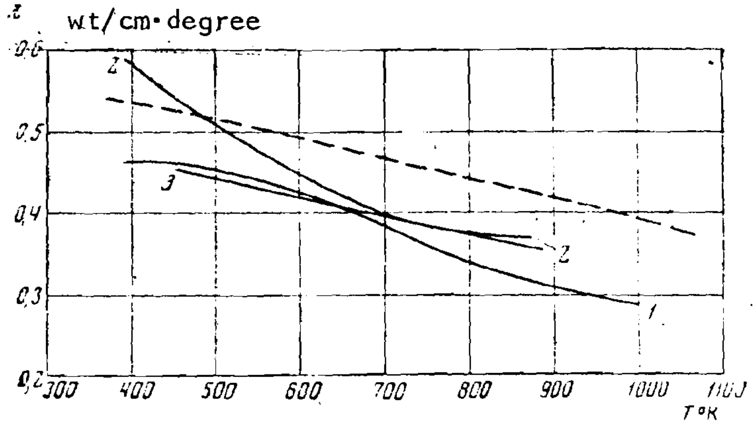

Figure 111. Summary of literature data of heat conductivity of liquid potassium: 1, Nikol'skiy et alia [60]; 2, Novikov et alia [149] (recalculated from data on temperature diffusivity); 3, Ewing et alia [51].

TABLE 40. RESULTS OF MEASUREMENT OF THE SPECIFIC ELECTRICAL RESISTANCE (MICRO OHM CM) OF TIN, LEAD, AND BISMUTH IN THE SOLID AND LIQUID STATES.

t° C	σ^{-1} Sn	t° C	σ^{-1} Pb	t° C	σ^{-1} Bi
20	11,65 solid	20	21,4 solid	20	294,2 solid
108	15,05	138	30,8	169	276,1
145	16,9	172	34,2	195	274,0
212	20,7	255	42,4	227	270,7
222	21,56	260	43,5	292	131,9 liquid
241	49,1 liquid	280	44,3	305	132,4
295	51,6	300	47,0	320	133,3
404	53,2	350	97,6 liquid	425	138,4
485	55,8	375	99,5	455	139,3
565	57,6	425	102,6	510	143,7
640	59,4	460	105,0	575	146,0
700	61,1	509	107,0	725	155,1
750	62,5	600	112,0	—	—
—	—	701	117,8	—	—

TABLE 41. SUMMARY OF RESULTS OF MEASUREMENTS OF THE JUMP IN HEAT CONDUCTIVITY DURING MELTING.

Metal	$\lambda_{solid}/\lambda_{liquid}$ Experiment	$\lambda_{solid}/\lambda_{liquid}$ Calculation	$\sigma_{solid}/\sigma_{liquid}$ [246]
Li	2,01 for [255])	1,90	1,68
Na	1,33 [51]	1,52	1,45
K	1,55 for [255])	1,53	1,56
Sn	1,75 [251], 1,82 [256], 1,82*	2,34	2,1 (2,28)
Pb	1,77 [251], 1,93*	1,67	1,94 (1,98)
Cd	2,6 [251], 2,4 [257], 2,5*	1,86	1,93
Zn	1,57 [251]	1,77	2,24
Al	1,61 [251]	2,00	2,2
Hg	1,90 for [255])	1,82	3,4
Bi	0,42 [251], 0,5 [258], 0,3*	—	0,45 (0,5)

An asterisk indicates values obtained from experiments described in §13 and 14.

The calculated values of $\lambda_{solid}/\lambda_{liquid}$ shown in Table 41 have been obtained from the formula

$$\frac{\lambda_{solid}}{\lambda_{liquid}} = e^{c\frac{r_{mp}}{T_{mp}}} \qquad (24.1)$$

($r_{melting}$ is the heat of melting), proposed by Mott for the jump in the electrical conductivity. For the jump in the heat conductivity, the coefficient c from this formula should in accordance with I. Z. Kopp [255], be taken equal to 1/4. It is seen from the table that the calculated values correspond only approximately to the results of experiments and correlate more readily with the values of the jump in the electrical conductivity (attention should be paid to the data for tin and cadmium). Considering the rather unilateral nature of the assumptions made by Mott in deriving these formulas (see [246]), the approximate nature of the relationship of the type (24.1) is not surprising.

In order to evaluate the dependence of the Lorentz number of liquid metals on the temperature, the dotted curves in Figures 114-120 show the calculated values of the electron components of the heat conductivity. It is characteristic that in most cases the experimental values of the heat conductivity at temperatures close to the melting point lie above the magnitude of electron heat conductivity, that is, the Lorentz number is somewhat greater than $2.45 \cdot 10^{-8}$ wt•ohm/deg^2. In this region one can apparently speak about the fulfillment of the Widemann-Franz law for the electron component. With rising temperature, however, in almost all cases there is observed a distinct tendency to a decrease in the Lorentz number beyond the limits of the value $2.45 \cdot 10^{-8}$ wt•ohm/deg^2, that is, the measured heat conductivity becomes less in a calculated electron heat conductivity. The nature of the temperature function of the Lorentz number differs with the data from different authors; however, it is natural to assume that the decrease in L with the temperature is monotonic. In any case, this fact arouses no doubt in the case of tin and lead.

The negative deviations from the Widemann-Franz law are apparently one of the characteristic features of the behavior of the heat conductivity of liquid metals.

Summarizing the result of the examination of the experimental material on the heat conductivity of liquid metals, it is necessary to assert that a study of this problem is in the very initial stage and in order to make a clear clarification of the definite correlations and the possibility of the theoretical interpretation, it is absolutely necessary to perform further studies on the heat conductivity and the complex with the electrical conductivity, the thermoelectromotive force, and the Hall effect.

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Foreign Science and Technology Center US Army Materiel Command Department of the Army	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

MEASUREMENT OF THERMAL PROPERTIES OF SOLID AND LIQUID METALS AT HIGH TEMPERATURES

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Translation

5. AUTHOR(S) *(First name, middle initial, last name)*

L. P. Filippov

6. REPORT DATE 17 September 69	7a. TOTAL NO. OF PAGES 437	7b. NO. OF REFS N/A
8a. CONTRACT OR GRANT NO. b. PROJECT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) FSTC-HT-23- 395-68	
c. 02RD500 d. SMUPA-VF	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* ACSI Control Number J-6000	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY US Army Foreign Science and Technology Center

13. ABSTRACT

The development of the study of solid and liquid states of matter and, in particular, the study of the most important aspects of this problem -- questions of the nature of thermal motion and the mechanism of heat transfer -- is related not only with the development and close study of the corresponding theoretical concepts, but also with the broadening of the range of states encompassed by this study. It is not accindental that a study of the thermal properties of solid and liquid metals comprises a significant part of the problem of "thermophysics of high temperatures," the development of which is under the jurisdiction of the committee for the coordination of scientific research work, which is attached to the Council of Ministers USSR.

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Experiment Metals Thermal properties Heat capacity Heat Conductivity						

☆ U. S. GOVERNMENT PRINTING OFFICE : 1969 391-032 (1674)