AD683016

TRANSLATION NO. 334

DATE: July 1968

DDC AVAILABILITY NOTICE

~~This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of Commanding Officer, Fort Detrick, ATTN: SMUFD-AE-T, Frederick, Md. 21701~~

DEPARTMENT OF THE ARMY
Fort Detrick
Frederick, Maryland

#334

Structure of the Temperature Field in Turbulent Flow

By A. M. Obukhov

Translation Izvestiia Akademii Nauk S.S.S.R., Ser. Geogr. i Geofiz., Vol. XIII, No. 1, 1949, pp 58-69

Abstract

The mean square of the temperature difference at two points is used as the characteristic parameter of the temperature field. The dependence of this quantity on the distance between the points of observation is determined experimentally. The order of magnitude of the characteristic parameters of the field of temperature fluctuations in the atmosphere is evaluated.

The micro-structure of the atmospheric temperature field is a problem of considerable interest for meteorology. Small temperature fluctuations are responsible for the turbulent transport of heat, the twinkling of the stars, and also influence the propagation of sound and other phenomena in the atmosphere.

Rough measurements of the temperature fluctuations in the air layer near the ground show that the temperature field in the atmosphere is extremely complex, like the field of wind velocities. This arises from the turbulent state of the atmosphere.

In 1941, A. N. Kolmogoroff proposed using the mean square of the velocity difference at two adjacent points considered as a function of the distance ℓ between the points, as a characteristic parameter of the micro-structure of the turbulent field of velocities. This function we shall call the structural function of the velocity field.

A completely analogous method may be applied to the statistical description of the structure of the field of temperature fluctuation in the atmosphere by considering the mean square of the temperature difference at two points. The dependence of this value on the distance between the points of observation (structural function of the temperature field) characterizes the intensity of the temperature fluctuations in the sense of a spectrum.

The problem of the local structure of the velocity field in a turbulent stream has already been discussed in a series of theoretical and experimental papers (2,3,4), but the problem of the structure of the temperature field has not previously been discussed. Existing observations of the temperature fluctuations in the atmosphere do not permit even an approximate estimation of the structural function of the temperature field since suitable measurements at small distances of separation with sufficiently sensitive apparatus, do not exist.

In the present paper a preliminary attempt is made to consider theoretically the question of the structure of the field of temperature in a turbulent stream. By using the theory of locally isotropic turbulence, we succeed in obtaining a series of results relating to the structural function of the temperature field. For example, according to the theory developed below it is shown that for not too small distances, the mean square of the difference of temperatures is proportional to the 2/3's power of the distance. It is assumed that the amplitudes of the temperature fluctuations are small (compared with the mean absolute temperature) and do not influence the turbulent fluctuations of the velocity field which arise solely from dynamical causes. In other words, in the present work we neglect the buoyancy forces arising from the temperature and treat the heat transported by the stream as a "passive substance".*

To be consistent with the first assumption, it is also assumed that the turbulent motion in the atmosphere may be considered as "incompressible" (according to the terminology of Friedman).

Radiative heat exchanges in the medium are neglected in the present investigation, although on a very small scale the latter may become appreciable, together with molecular heat transfer in the medium.

The mechanism of the equalization of temperature in a sufficiently large volume can apparently only be explained by the joint action of turbulence and molecular heat transfer. Owing to the irregular motions of particles of air possessing different temperatures, the latter are able to converge within such small distances that equalization of temperature between these particles is only possible through the operation of molecular heat transfer. In other words, the turbulent motion inside a thermally heterogeneous medium with gradients which are initially weak can contribute to the local gradients of temperature, which are subsequently smoothed out by the action of molecular heat conductivity.

To obtain from this physical stream some quantitative conclusions it is necessary to introduce a number of auxiliary concepts. Fundamental for further developments is "the measure of the heterogeneity" of the temperature field.

Section 1. "Measure of Heterogeneity" and "Free Energy" of the Temperature Field.

We consider the temperature field in a medium with a specific heat C occupying some region V .**

---

* The influence of systematic temperature heterogeneities on turbulence is considered from another point of view (by the method of the semi-empirical theory of turbulence) in the work "Turbulence in a Heterogeneous Field"(4).

**In the present investigation of the temperature field of the atmosphere, the specific heat C will be identified with the specific heat C_p at constant pressure so that the total energy of the system replaces the internal energy.

Let $\overline{T}$ be the mean temperature of the field (averaged over the volume V)

$$\overline{T} = \frac{1}{M} \iiint_V \rho T(x,y,z)\, dv \tag{1}$$

where

$$dv = dx\,dy\,dz$$

$$M = \iiint_V \rho\, dv \qquad \text{(mass of the fluid)}$$

Denote the deviation of temperature from the mean by

$$T'(x,y,z) = T(x,y,z) - \overline{T} \tag{2}$$

As a measure of the "temperature heterogeneity" of the field in the region V, it is natural to introduce the quantity G defined as the integral over the region V of one-half of the square of the temperature deviation.

$$G = \frac{1}{2} \iiint_V \rho (T')^2\, dv \tag{3}$$

It is obvious that $G = 0$ if and only if the temperature is constant over the whole region. The factor $\frac{1}{2}$ is to make an analogy with the kinetic energy of the relative motion in the fluid. The latter is obtained if, in the above formula, the temperature is replaced by the vector of velocity. It is convenient to introduce a special symbol for the measure of the heterogeneity, referred to the unit mass of the medium.

$$g = \frac{G}{M} = \frac{1}{2M} \iiint_V \rho (T')^2\, dv$$

It may be remarked that the physical significance of G is that it is proportional to the maximum work W, which it is possible to obtain from the non-uniformly heated substance, considering it as an isolated system (in regard to heat). We shall call this maximum work W the "free energy" of the non-uniformly heated substance.*

---

* The term "free energy" as used above is not to be confused with the analogous conception in thermodynamics, which is significant only for isothermal processes. Inasmuch as, in the following, we shall not use the classical expression for free energy, this terminology should not lead to confusion.

The value of W for a uniformly heated substance is obviously zero since in this case the substance is in a state of thermodynamic equilibrium.

We shall now estimate W for the region V with a given distribution of temperature $T(x, y, z)$.

In order to extract the maximum quantity of heat from a system, we must bring the system into a state of thermodynamic equilibrium with the aid of some reversible process. The change in the total energy of the system in such a conversion from the given initial state to the final equilibrium state determines the value of W . We denote the temperature of the substance in the final state $\tilde{T}$. $\tilde{T}$ is obviously constant, inasmuch as it refers to a state of thermodynamic equilibrium. Thus, measuring the maximum work in heat units, we have

$$W = \iiint_V c\rho T(x,y,z)\,dv - \iiint_V c\rho \tilde{T}\,dv = cM(\bar{T} - \tilde{T}) \qquad (4)$$

where $\bar{T}$ is the mean temperature of the substance.

To determine $\tilde{T}$ we shall use a reversible process to connect the initial and final states of the system. The total entropy of the system in such a process remains constant. Equating the entropy of the heat substance in the initial and final states, we obtain an expression for the determination of $\tilde{T}$.

$$S = \iiint_V c\rho \log T(x,y,z)\,dv = \iiint_V c\rho \log \tilde{T}\,dv \qquad (5)$$

from which

$$\log \tilde{T} = \frac{1}{M} \iiint_V \rho \log T(x,y,z)\,dv \qquad (6)$$

$\tilde{T}$ may be termed the "geometric mean" temperature of the substance.

Thus the "free energy" of a non-uniformly heated substance is equal to the heat derived in a change between the "arithmetic mean" and "geometric mean" values of the temperature. Substituting for $\tilde{T}$ from (6) in (4), we obtain the final expression for W :

$$W = cM\bar{T}\left\{1 - \exp\left[\frac{1}{M} \iiint_V \rho \frac{\log T(x,y,z)}{\bar{T}}\right]\right\} \qquad (7)$$

Expression (7) may be simplified by assuming that the variation of the temperature T' is very small compared with $\bar{T}$. In this case, since

$$\iiint_V \rho T'(x,y,z)\,dv = 0$$

and neglecting the integration of powers of $T'/\bar{T}$ greater than the second:

$$W \approx \frac{c}{\overline{T}} \iiint_V \frac{1}{2} \rho (T')^2 dv = \frac{c}{\overline{T}} G \qquad (8)$$

Thus, the approximate expression (8) for the "free energy" W differs only by the factor $c/\overline{T}$ from the measure of the heterogeneity G of the temperature field introduced above.

We may also estimate in this case the approximate increase of entropy ΔS in the equalization of temperature due to non-reversible processes (heat conduction). Such a process brings the substance to a constant temperature $\overline{T}$, so that the variation of entropy can easily be estimated:

$$\Delta S = cM \log \overline{T} - c \iiint_V \rho \log T(x,y,z)\, dv = -c \iiint_V \rho \log \left[1 + \frac{T'(x,y,z)}{\overline{T}}\right] dv$$

from which, neglecting terms of higher order, we obtain a very simple expression for the increase of entropy:

$$\Delta S \approx \frac{c}{\overline{T}^2} G \qquad (9)$$

From (8) and (9) it follows that to the approximation considered

$$W = \overline{T} \Delta S \qquad (10)$$

which also follows from general thermodynamic reasoning.

Thus, based on expression (8) obtained for W and (9) for S, the value of G may be treated in the same manner as the "free energy" of the field, or as the measure of the deficiency of entropy of the temperature field ("negative entropy").

Section 2. Variation of the Measure of Heterogeneity of the Temperature Field with Time.

Consider the motion of an incompressible fluid of density ρ (assumed to be constant for simplicity) possessing a variable temperature $T(x,y,z,t)$. Let λ be the thermal conductivity and, correspondingly, K the thermal diffusivity of the fluid. The boundaries of the volume under consideration will be assumed to be fixed and non-conducting. We shall calculate the change in the value of G for the whole volume.

The temperature in a moving medium satisfies the following two equations:

$$c\rho \frac{dT}{dt} = \lambda \Delta T$$

or

$$\frac{\partial T}{\partial t} + \operatorname{div}(\vec{v}, \operatorname{grad} T) = K \Delta T \qquad (11)$$

Since $$\operatorname{div} \vec{v} = 0$$

we obtain on substituting $T(x,y,z,t) = \bar{T} + T'(x,y,z,t)$

in equation (11), the following equation for the variation of $T'(x,y,z,t)$:

$$\frac{\partial T'}{\partial t} + \operatorname{div}(\vec{v}, \operatorname{grad} T') = K \Delta T' \qquad (11a)$$

where K is the thermal diffusivity.

On multiplying this relation by $\rho T'(x,y,z,t)$, integrating for the whole volume V , and applying Gauss' theorem, observing that for the boundaries of the volume

$$v_n = 0 \quad , \quad (\operatorname{grad} T)_n = 0$$

we obtain
$$\frac{dG}{dt} = -K \iiint_V \rho (\operatorname{grad} T)^2 \, dv \qquad (12)$$

Using the measure of the temperature heterogeneity referred to a unit of mass $g = G/M$, equation (12) may be written in the form

$$\frac{dg}{dt} = -K \overline{(\operatorname{grad} T)^2} \qquad (12a)$$

where the mean refers to the volume V .

This equation is completely analogous to that for the dissipation of energy. If g is assumed by formal analogy to be the kinetic energy, then the dissipation function of Stokes referred to unit mass for a temperature field will be given by the expression:

$$N = K (\operatorname{grad} T)^2$$

which determines the rate of equalization of the temperature heterogeneities.

We recall that the thermal diffusivity K and the kinematic viscosity ν have the same dimensions and, for air, approximately the same numerical values ($\nu = 0.14, K = 0.19 \text{ cm}^2/\text{sec}$).

Equation (12) shows that in a hypothetical medium for which $K = 0$ inside a closed boundary, the measure of heterogeneity remains constant. On the other hand formula (12) also shows that in real medium with a small thermal diffusivity (air or water), the actual equalization of temperature heterogeneities (decrease of G) will in practice occur only when local gradients are sufficiently great.

We may now form a qualitative picture of the processes occurring in a temperature field in a turbulent medium possessing only a small thermal diffusivity. If the initial distribution of temperature is sufficiently uniform, then for a very small K, N may be considered practically equal to zero, not only initially but also during some period of time subsequently. As is demonstrated in practice, the irregular turbulent motion of a fluid affects the temperature field in such a way that the temperature tends to uniformity. If we divide the initial volume V into small cells of volume $\omega = V/k$, then through mixing the mean temperature in the cells will tend to approach a constant $\overline{T}$.

Nevertheless, if the total measure of heterogeneity G (or g) remains finite, then inside every cell the field must be extremely heterogeneous since the mean amplitude of the temperature fluctuations inside the small cell will approach in the mean the amplitude of the temperature fluctuations observed initially for the entire volume V. Owing to the fact that during the mixing process the size of the region ω, for which a higher average temperature prevails, must be decreasing, the actual temperature gradient must be increased in such a process, and from a certain stage the mechanism of molecular heat conduction must become important. At this stage, inside sufficiently small elements of volume (the size being smaller, the smaller the thermal diffusivity of the medium) owing to the influence of molecular heat conduction, equalization of the temperature heterogeneities will take place, that is, a decrease in the value of G will occur (increase of entropy). It is easy to see that during some period inside sufficiently small cells, a quasi-stationary (statistically) regime must be set up, through which increase of the measure of heterogeneity inside the volume ω owing to mixing is compensated by the actual equalization of the temperature field inside the volume ω as a result of molecular heat conduction.

Thus the influence of turbulence leads to a redistribution of the measure of the temperature heterogeneity into a "spectrum" of temperature inhomogeneities. It is possible to establish the concept of the spectrum of a temperature field more exactly by considering the resolution into a Fourier series (integral) and observing that the value of g will be the sum of contributions from different spectral components. This method of characterizing the temperature field may be carried out in a completely analogous way to that which has been applied to the field of velocities by A. M. Obukhov in 1941 (3) and somewhat later by Onsager (5).

In the present work we shall not consider in detail the application of the method of spectral resolution to the problem of the micro-structure of the temperature field. We shall attempt, on the basis of the above qualitative notions, to proceed directly to the investigation of the structural function of the temperature field, using for this purpose dimensional considerations. This method is analogous to that employed by A. N. Kolmogoroff (2) in the investigation of the micro-structure of a turbulent velocity field.

Section 3. Structural Function of the Temperature Field.

In the introduction we spoke of the structural function of a temperature field. This function describes in a statistical sense the mean value of the square of the difference of the temperatures at two points M and M'

$$H(M,M') = \overline{[T(M') - T(M)]^2} \tag{13}$$

We shall assume that the temperature field is locally isotropic. This means that the function $H(M,M')$ depends only on the distance ℓ between the points M and M', which are chosen so that the distance between them is small in comparison with the scale of turbulence ℓ_0. The scale ℓ_0 for given conditions of flow is determined by the geometry of the system. For example, ℓ_0 may be determined according to the theory of Prandtl. This method is completely analogous to that which was proposed by Kolmogoroff in 1941, for local isotropy of a turbulent velocity field. It is natural to assume local isotropy for the temperature field in a turbulent flow pattern.

Using the condition of the local isotropy in the temperature field, we may write

$$H(M,M') = H(\ell) \tag{14}$$

for $\ell < \ell_0$, where

$$\ell = \sqrt{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}$$

It is obvious that for $\ell = 0$, H equals zero. From symmetry it also follows that $H'(0) = 0$.

The second derivative of the structural function for zero value of its argument, as is readily shown, is directly expressible in terms of the mean value of the square of the temperature gradient. Considering the coordinate points M and M' as independent variables $(x_1, x_2, x_3, x_1', x_2', x_3')$, we have from equation (13)

$$\left(\left(\nabla_0 \nabla' H(M,M')\, \delta M . \delta M'\right)\right) = -2\left(\left(\overline{\operatorname{grad} T(M) . \operatorname{grad} T(M')}\, \delta M . \delta M'\right)\right)$$

Here the dot denotes the tensor product of vectors and the right and left hand sides represent bi-scalar products. Dividing by the product of the vectors δM and $\delta M'$, it follows that

$$\overline{\operatorname{grad}_\alpha T(M) . \operatorname{grad}_\beta T(M')} = -\frac{1}{2} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} H(\ell) \tag{15}$$

The right hand side of (15) may be evaluated, using the condition of local isotropy (14):

$$\overline{\operatorname{grad}_\alpha T(M) \operatorname{grad}_\beta T(M')} = \frac{1}{2\ell} H'(\ell) \delta_{\alpha\beta} + \frac{1}{2}\left[H''(\ell) - \frac{1}{\ell} H'(\ell)\right] n_\alpha n_\beta \tag{16}$$

$$\alpha, \beta = 1,2,3 \qquad \delta_{\alpha\beta} = \begin{cases} 1 & \alpha = \beta \\ 0 & \alpha \neq \beta \end{cases}$$

$$n_\alpha = x'_\alpha - x_\alpha / l$$

being the components of the vector determining the direction from the point M to M'.

Proceeding to the limit $M' \to M \; (l \to 0)$ and summing for the indices, we obtain the desired formula for the mean square of the temperature gradient.

$$\overline{[\operatorname{grad} T]^2} = \frac{3}{2} H''(0) \tag{17}$$

Multiplying both sides of equation (17) by the thermal diffusivity K, we obtain an expression for the parameter N (given above Section 2) determining the rate of equalization of the temperature heterogeneities.

$$\overline{N} = \frac{3}{2} K \, H''(0) \tag{18}$$

Hence for very small l (later we shall define more precisely the meaning of "small" l),

$$H(l) \approx \frac{1}{2} H''(0) l^2 = \frac{1}{3} \frac{\overline{N}}{K} l^2 \tag{19}$$

The structural function for the temperature field $H(l)$ may, roughly speaking, be treated as a measure of the intensity of the temperature heterogeneities q (calculated for unit mass) for values not exceeding l. The heterogeneities considerably larger than l will not influence to any extent the difference of temperatures at the distance l. The relation between the spectrum of temperature heterogeneities and the structural function may be established more exactly by the method of the Fourier integral.

Using the preceeding picture of the equalization process for the temperature heterogeneities in a turbulent stream, we may now obtain an expression for the structural function $H(l)$ for (not very small) values of l, where the direct influence of thermal diffusivity of the medium is negligibly small. It is natural to suppose that in this region in a quasi-stationary regime of temperature fluctuations, the size of $H(l)$ must be determined only by the value of (the analog of the dissipation energy in Kolmogoroff's theory) and by the characteristics of the turbulence.

In this region the coefficient of thermal diffusivity does not enter directly. According to Kolmogoroff's theory, in this range the structure of the field of turbulent fluctuations of velocities is completely determined by the mean dissipation energy $\overline{\varphi}$, calculated for unit mass of the medium. Thus we may write

$$H(l) = F(\overline{N}, \overline{\varphi}, l) \tag{20}$$

Before considering the form of the function $H(l)$ as derived from dimensional analysis, it is necessary to make one remark regarding the dimensions of temperature. Since we have assumed a "passive" character for the heat transported in the flow, i.e. that the heterogeneity of the temperature field does not influence the field of turbulence (this will be the case for small fluctuations of temperature from the mean and considerable turbulence of dynamic origin), the mechanical equivalent of heat does not appear among the determining parameters. Thus, in the dimensional analysis, we may use an arbitrary scale, not depending on the scales for the dynamic quantities.*

For the dimensions of the quantities appearing in formula (20) we write

$$[H] = \theta^2 , [\Phi] = L^2 T^{-3} , [N] = \theta^2 T^{-1} , [\ell] = L$$

(In this formula T is the dimension of time.)

From these values it is possible to set up only one dimensionless combination:

$$\frac{H \, \overline{\Phi}^{\frac{1}{3}}}{N \ell^{2/3}} = k^2 \text{ (number)}$$

from which it follows that the structural function $H(\ell)$ has the form

$$H(\ell) = k^2 \frac{\overline{N}}{\overline{\phi}^{\frac{1}{3}}} \ell^{2/3} \qquad (21)$$

or

$$\sigma_{\Delta T}(\ell) = \sqrt{\overline{(T(M') - T(M))^2}} = B \ell^{2/3} \qquad (22)$$

where k is a numerical constant obviously of the order unity.

$B = k \frac{\sqrt{\overline{N}}}{\sqrt[6]{\overline{\phi}}}$ appears as the fundamental characteristic of the local structure of the temperature field.

The fundamental form of the mean square of the difference of temperature in a turbulent stream as a function of the distance between points of observation is completely analogous to the "two thirds law" for a field of velocities as obtained by Kolmogoroff and Obukhov in 1941 (2,3).

$$\overline{[\vec{v}(M') - \vec{v}(M)]^2} = c \, \phi^{2/3} \ell^{2/3} \qquad (23)$$

---

* We may note that the transformation of mechanical energy into heat (due to dissipation) in turbulent flow causes negligible changes of temperature so that we may disregard this process.

where C is a numerical constant of the order unity.

Thus a unique similarity exists between the structure of a temperature field and a field of velocities in conditions of local isotropy. It follows that the behavior of the mean square amplitude of the difference of temperatures and the difference of velocities does not depend upon the distance between points of observation and has the form $\sqrt{\overline{N}/\overline{\Phi}}$.

It is now possible to estimate the value of the temperature heterogeneities for which the field approach as linearity owing to the absence of thermal diffusivity. This region corresponds to asymptotic expansion of the structural function $H(\ell)$ for small ℓ. We determine a suitable dimension ℓ_1 by the intersection of two asymptotic representations (19) and (21) for $H(\ell)$, corresponding to "small" and "large" values of ℓ (Fig. 1.--Here the broken line represents the hypothetical form of $H(\ell)$ in the transitional zone.)

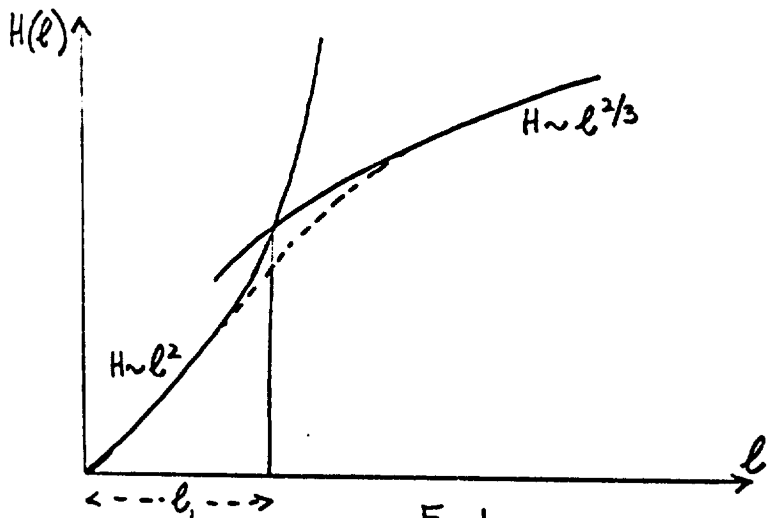

Fig 1

Thus ℓ_1 must satisfy the equation:

$$\frac{1}{3}\frac{\overline{N}}{K}\ell_1^2 = k^2 \frac{\overline{N}}{\overline{\Phi}^{1/3}} \ell^{2/3}$$

from where

$$\ell_1 = \sqrt{\frac{3k^6K^3}{\overline{\Phi}}} \tag{24}$$

The magnitude of ℓ_1 does not depend upon the intensity of the temperature fluctuations. Owing to the fact that the Prandtl number for air is of order of unity, the magnitude of ℓ_1, is of the same order as the characteristic parameter of turbulence η, given in the above cited theorem of Kolmogoroff:

$$\eta = \sqrt[4]{\frac{\nu^3}{\Phi}} \tag{25}$$

The above discussion concerning the micro-structure of the temperature field in a turbulent flow pattern might be applied in meteorology to the study of the fluctuations of temperature in the near-ground layer, provided the velocities of the field be sufficiently great and turbulence of dynamic origin.

It is possible to estimate roughly the order of magnitude of the basic parameters of the structural function of the temperature field in the atmosphere. Using the similarity conditions formulated above for the temperature and velocity fields, we may calculate the conversion factor from the amplitude of the velocity to amplitude of temperature fluctuations by comparing wind and temperature observations obtained under similar conditions. Using the data given in Lettau's book, it is possible to estimate this conversion factor to be of the order of magnitude of 0.5° for 1m/sec or $5\ 10^{-3} cm^{-1}$ sec.

Then using measurements of the fluctuation of wind velocity, Godecke (7) and the author (8), the characteristic of the micro-structure of the temperature field, B (the coefficient of proportionality for $\ell^{2/3}$ in the expression of the mean square difference of temperature) may be estimated at a few hundredths of a degree for $cm^{2/3}$. This corresponds for a base of one meter to a mean amplitude of the difference of temperatures of the order of a tenth of a degree.

According to data given by Godecke, the inner scale of turbulence may be estimated as being of the order of 1 cm. The value of ℓ_1 specifying the temperature heterogeneity of the atmosphere may be deduced from such a value.

This coarse evaluation must naturally be made more precise by means of special measurements of the rapid oscillations of temperature differences in the atmosphere over small distances (from several centimeters to a meter).

Investigations of the same type are of interest not only in connection with the theory developed above, but for the explanation of certain questions relating to accoustics and optics in the atmosphere.

Academy of Science USSR Geophysical Institute - Submitted 28 March 1948

Translated by V. Bartlett, Camp Detrick, September 1950

Bibliography

1. Holtzmann, Zur Frage des Genauigkeitsgrades der Lufftemperatur und Feuchtigkeitsmessungen in naturlichen Verhaltnissen, Meteorologische Zeitschrift, B. 53, S. 327, 1936.

2. Kolmogoroff, A. N., Local structure of turbulence in an incompressible viscous fluid, Doklady Akademiia Nauk, S.S.S.R., Vol. XXX, No. 4, 1941.

3. Obukhov, M. A., Concerning the distribution of energy in the spectrum of a turbulent stream, Izvestiia Akademiia Nauk, S.S.S.R., Ser. Geogr. i Geofiz No. 4-5, 1941.

4. Batchelor, G. K., Kolmogoroff's Theory of locally isotropic turbulence, Proc. of the Cambr. Phil. Soc., Vol. 43, Part 4, p 533, 1947.

5. Onsager, L., Phys. Review, v. 68, p 286, 1945.

6. Lettau, H., Atmospheric turbulence, Sec. 50, Leipzig, 1939.

7. Godecke, K., Measurements of atmospheric turbulence, Annalen d. Hydrographie, H. X., S 400, 1936.

8. Obukhov, A. M., On the theory of atmospheric turbulence, Izvestiia Nauk, S.S.S.R., Ser. Geofiz., Vol. VI, No. 1-2, 1942.