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THE PROBLEM OF STRUCTURAL SAFETY WITH PARTICULAR REFERENCE TO SAFETY REQUIREMENTS

by

H. EBNER

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This Report was presented at the Sixth Meeting of the Structures and Materials Panel,
held from 4th to 8th November, 1957, in Paris

SUMMARY

The main topic of this Report is the historical development of the safety concept in aircraft design. The methods by which the prescribed degrees of safety in various national regulations have been arrived at are discussed and comparisons are made between the safety factors laid down in American, British, French and German airworthiness regulations. Other subjects dealt with are the relatively new statistical concept of safety, gust loads, fatigue, and cumulative damage in fatigue.

SOMMAIRE

Ce rapport a pour but principal de présenter l'historique du développement de la notion de la sécurité dans la construction des avions. Les principes à base de l'établissement des coefficients de sécurité prescrits par divers règlements nationaux sont exposés, avec une comparaison entre les facteurs de sécurité visés dans les règlements américain, britannique, français et allemand concernant la navigabilité. D'autres questions traitées portent sur la notion relativement nouvelle de la sécurité, considérée du point de vue statistique, sur les charges de rafale, la fatigue et les dommages cumulatifs dûs à la fatigue.

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THE PROBLEM OF STRUCTURAL SAFETY WITH PARTICULAR REFERENCE TO SAFETY REQUIREMENTS

H. Ebner*

1. INTRODUCTION

At the 4th meeting of the Structures and Materials Panel of AGARD, held in Brussels at the end of August 1956, Professor N.J. Hoff gave a paper in which he dealt with the 'Philosophy of Safety in the Supersonic Age'[1]. The main emphasis in Professor Hoff's paper was on the present-day conception of safety as regards magnitude and frequency of loads and on the fatigue and high-temperature properties of materials, the proposal for the criterion of safety being the probability of fracture. In the present paper the main emphasis is on the historical development of the safety concept (which was mentioned by Professor Hoff only in his Introduction) and the way in which a prescribed degree of safety is determined in various national regulations. (The *present-day* problem of safety is dealt with by Messrs Prot, Freudenthal and Mangurian in other AGARD Reports). It was not until 1956 that we in Germany were able to concern ourselves with these problems, so that I shall deal only briefly with the *fundamental* conception of safety.

2. HISTORICAL REVIEW OF THE DEVELOPMENT OF THE SAFETY CONCEPT IN AIRCRAFT CONSTRUCTION

Ever since the beginning of aeronautics, countries engaged in aviation have concerned themselves with problems of safety in aircraft design.

I would like to begin by saying something about the *purpose* of a safety factor. The safety factor is intended to cover certain inaccuracies which, in the present stage of technology, are unavoidable. For example:

(a) Inaccuracies in the assumed external loads;

(b) Inaccuracies in the theoretical or experimental determination of stresses due to the external loads and due to incalculable internal stresses;

(c) Inaccuracies and scatter in material properties (for example, ultimate strength), which may already be present before assembly or which may be caused by ageing;

(d) Inaccuracies in the specified dimensions of members owing to imperfections in manufacture or due to wear and tear (e.g., by friction or corrosion).

Because of the inaccuracies in the assumptions regarding external loads, there is a close relationship between the safety factor and the load assumptions. Until about 1930 no distinction was made between the *safety factor* and the *maximum load factor which could, with some specified degree of probability, be expected in the course of*

***Deutsche Versuchsanstalt für Luftfahrt E.V., Institut für Festigkeit, Mulheim - Ruhr, Germany* *operation* (i.e. the *limit load factor* or in German, so-called 'safe' load factor). Instead, the product of these two was used and called the *ultimate load factor*. Definite values were laid down for these factors in the regulations, depending on the loading conditions, application, and weight groups. However, the division of the ultimate load factor into a *probable 'safe' load factor and a safety factor of 1.8 to 2.0* had already been suggested in a French publication by P. James[2] as early as 1911.

The first scientific treatment of aircraft stress analysis and its associated problem of safety was given by H. Reissner in Germany in 1912. In a lecture before the 'Wissenschaftliche Gesellschaft fur Luftfahrt' (The Scientific Society for Aviation)[3] Reissner made various proposals regarding the loads effective during flight and the safety limits which should be adopted. To take account of all 'conceivably possible' loads he suggested (i) an *ultimate safety factor* of $\gamma = 3$ for tension and compression and (ii) a *permissible stress* to prevent the material suffering permanent distortion of 2/3 of the stress at the limit of elasticity. The second requirement represents a 1.5 - fold safety against the material yielding. In addition, Reissner emphasised the great importance of toughness, for the case of repeated loads, requiring the greatest possible difference between the ultimate strength and limit of elasticity of a material. At the same time he recommended the highest possible limit of elasticity for members subjected to buckling loads. This corresponds to present-day thought in these matters, viz., the use of high-strength material for the compression zone only.

Reissner's statements at that time were used as a guide in further scientific research in this field and have some significance even today.

On the basis of suggestions made by Reissner in a lecture which he gave on the subject of experimental research on aircraft loads, test flights were carried out by Wilhelm Hoff at D.V.L. in the years 1913-14. In these tests the loads were determined by means of specially developed devices; various flying operations were carried out, using two different aircraft. Measurements were based on the stresses in the wing cables; these were obtained mechanically and the load factors derived from them. Details of the measuring devices and a description of the experiments, together with the results, are contained in a Report by W. Hoff entitled 'The Strength of German Aircraft'[4]; because of the war, however, this report was not published until 1922. Based on the results obtained in these experiments, regulations relating to the design and acceptance of aircraft were issued by the German Flugzeugmeisterei (Military Technical Department) during the first World War - in the year 1916; these were extended and improved, and re-issued by the B.L.V. in 1918.

In these regulations, the ultimate load factors were now specified in terms of four different loading cases A, B, C and D, representing pull-out from a dive, gliding, diving, and inverted flight, respectively, and aeroplanes were divided into three groups, according to their weight. Later, they were divided into five groups. The ultimate load factors assumed for static calculations ranged from 3.5 to 5.0 for case A, the lower values applying to the higher weights. The safety factor included in these load factors was about 2. The ultimate load factors for the remaining cases were lower: for case B from 2.5 to 3.5, from 1.2 to 2.0 for case C, and from 2.5 to 3.0 for case D. For purposes of strength testing, ultimate load factors of from 10 to 30% greater were required, to allow for the fact that the beneficial effect of certain members were not to be included in the calculations. Further details about the load factors and load cases prescribed up to 1926, including specific conditions for fuselage, tail unit and landing gear, may be found in a paper published in 1932 by H.G. Kussner and K. Thalau entitled 'Entwicklung der Festigkeits vorschriften für Flugzeuge' (Development of Strength Requirements for Aircraft). Test flights were also carried out in France, by P. James[2] and Delaunay[6] amongst others - earlier even than those carried out in Germany - in order to determine the loads on an aircraft when pulling out of a dive and, also, for flight in gusty weather.

Later, tests similar to the French and German ones were carried out in U.S.A. with a fighter plane (the Fokker P-W 7), by F.H. Norton[7] in 1921 and by Doolittle[8] in 1923/24. In these tests accelerations were measured with an NACA accelerometer. The highest values were obtained when pulling out of a dive, and were around 8g. As a result of these tests it was stipulated that in the design of fighter planes ultimate load factors of up to 12g were to be adopted, giving a safety factor of approximately 1.5. Tests in gusty weather were also carried out, during which accelerations of up to 2.5g were measured.

After proposals had been submitted by van Gries (of the D.V.L.), as early as 1918, for a more realistic load factor and also for a safety factor, a clear subdivision into *'load factors definitely required' and* a *'safety factor'* was made for the first time in the provisional load assumptions of the D.V.L. issued in October, 1926. To achieve a desired state of safety, proof of a safety factor of 2.0 was required, and it was also required that no permanent set should remain after removal of the load. The Dutch regulations of 1924 had also included similar stipulations regarding permanent set.

In addition to the new method for determining the safety factor, a *subdivision of aircraft into five load categories* was introduced, and load factors independent of weight were adopted for case A. The safe load factors and air speed pressures were derived from the corresponding values of case A for the remaining load cases. However, in a further draft which appeared in 1927, dependance of the load factors on the weight was re-introduced for the first three load categories, whereby the values for airliners (categories 2 and 3) were reduced. To prevent safety being dependent solely on the ultimate strength, a safety factor of 1.4 on the 0.2% proof (or yield) was introduced. The ultimate factor was lowered to *1.8 for tension and compression.* As the ratio of the yield to the ultimate strength for materials then in use was 2.3, this reduction only affected the compression members in a structure. An ultimate factor of $j = 1.5 \times 1.4 = 2.1$ (which is greater than 2) results, for the tension members, by using a yield safety factor of 1.4 with a ratio of $\sigma b/\sigma s = 1.5$, assuming linear increase of stress with load.

The lower safety factor for compression members was considered justified by the argument that in compression and buckling the diminution of strength arising from local defects is less than it is for tension. It was, however, omitted to be taken into consideration that, due to unintentional, unavoidable eccentricities and initial deformations, reduction in strength does, in fact, occur in compression members also.

In the third draft of 1928 *a safety factor of 1.25* was laid down for the first time for fatigue. Also, a factor of 1.5 was prescribed for load tests of small aeroplanes, and the maximum permanent set under this load was limited to 5%. Based on the three drafts just described, the 'Vorlaufige Belästungsannahmen für die

Festigkeitsberechnung von Flugzeugen' (Provisional load assumptions for computation of strength of aircraft) appeared in December 1930, and this change was incorporated in an amendment which came into force in February 1931. This amendment provided for the following changes:

(a) With a factor of 1.8 on the assumed safe loads, the structure must not develop instability;

(b) An ultimate factor of 2.0 must be applied to the structure in tension, compression, and bending;

(c) For structures made in metal, a factor of 1.35 shall be applied to the 0.2% proof stress.

It was an additional requirement of the 'Festigkeitsvorschriften' (Strength Requirements) of April 1933 that the stresses corresponding to a factor of 1.0 on the assumed safe load must not exceed half the ultimate strength of the material; this was to take account of the non-linear increase of stress with load. This requirement only applied to structure in tension.

In the requirements for the strength of aircraft which appeared in December 1936, and which were valid until 1945, the higher safety factor of 2.0 for members in tension was reduced, and it was stipulated that with a factor of 1.35 the stress should not exceed the lower of the following:

(a) 75% of the ultimate strength

(b) The yield stress (or the 0.2% proof stress).

The ultimate factor was standardised again at 1.8 for tension and compression. The adoption of the same factor of safety for both stability and stress had the important advantage that the static calculation corresponding to double the assumed safe load was abolished, so that the strength in individual cases could be demonstrated with only one test; compression members with a factor of 1.8 had, earlier, to be re-inforced to give a factor of 2.0 to provide the necessary tension strength. Further, this version of the requirements implied that it had to be demonstrated, either by calculation or test, that these factors would not fall below the prescribed values by more than 10%, when allowance was made for tolerances in the material dimensions. Where such tolerances occur, the real ultimate safety factor is, of course, only 1.62.

It was, however, stipulated in the special strength requirements for military aircraft, of January 1935, that the prescribed factors must not be reduced at all because of tolerances.

Towards the end of the 1920's the C.I.N.A. issued regulations in which an ultimate safety factor of 2.0 and a proof factor of 1.25 were required. These were included in the regulations of many foreign countries, Italy and Holland, for instance, being two of them. In England, an ultimate safety factor of 2.0 was valid until 1936, but there was no proof requirement, and it was not until later that the ultimate safety factor was reduced to 1.5. The regulations of the U.S. Department of Commerce, of

October 1934, already contained an ultimate safety factor of 1.5, and a proof factor of 1.0.

The safety factor j prescribed in the German, American and CINA regulations since 1926 for ultimate and yield strength are shown in Figure 1. At the top of the figure it will be seen that the ultimate factor j_B, in Germany, has alternated between 2.0 and 1.8 in the course of the years, the lower value being 20% higher than the American one, which is only 1.5. However, if reduction of the ultimate factor due to low tolerances is taken into account, there is only a difference of 8%. The three lines at the bottom of the figure represent the proof or yield safety factors j_S. The ratios of the two factors, j_B/j_S, are then as follows:

German regulations:	1.35:2.0 to 1.35:1.8	= 0.675 to 0.750
American regulations:	1:1.5	= 0.667
CINA regulations:	1.25:2.0	= 0.625

Assuming linear increase of stress with load up to ultimate load, the proof or yield factors are only decisive if the preceding ratios exceed the ratio of yield stress to ultimate stress. This ratio varied from 0.6 to 0.7 until about the end of the 1920's. Since then it has steadily increased as a result of the greater use of high-strength materials having a comparatively high yield stress, and today it varies between 0.8 and 0.9. The proof or yield requirements are, therefore, no longer decisive for such materials.

It would appear, based on these values of the ultimate safety factors, that aircraft built to American and British regulations would be less safe than aircraft designed in accordance with the German and French (CINA) regulations. In order to clarify this point, the ultimate load factors for case A (i.e. pulling out of a dive) for the various regulations are compared in Figure 2. This comparison is based on the German regulations valid up to the end of the 1920's, which are identical with the last German structural requirements laid down (in 1936) as regards safety factors and load categories, and on the French regulations issued in 1929 by the Bureau Veritas. The regulations contained in 'Air Publication 1208' published in 1929 for civil aircraft, were taken for the British, and finally 'Airworthiness Requirements of Air Commerce Regulations for Aircraft' of 1931 were taken for the American requirements. Where the load factors were stated as safe values they have been multiplied by the prescribed safety factors 1.8 and 2.0, respectively.

Plotting of the ultimate load factors n_A in case A against the full flying weight demonstrates that the American load factors for normal aeroplanes, staggered according to power load (lines 10 and 11), are not below, but above, the German (lines 2 and 3) and the British (line 8) load factors in the case of low flying weights. Only the French ones are above the American ones in the case of larger flying weights. As regards acrobatic aeroplanes, the German load factors (lines 4 and 5) exceed for almost all flying weights (in fact, with the exception of very small flying weights) those of the other countries (lines 7, 9, 12). The Figure shows that there are differences, not only between the safety factors but also between the ultimate load factors for the individual countries.

At the end of this section I shall compare the safe load factors n_{Asi} laid down in the last German regulations (BVF 1936) - in force until recently - with the values contained in the American Civil Airworthiness Regulations (CAR, Parts 3 and 4b respectively), whereby the load groups 3, 4 and 5 of the German regulations are comparable with the N, U, and A (Normal, Utility, and Acrobatic) groups of the American regulations. In Figure 3 are plotted the American safe load factors (solid and lower chain-dotted lines) and the German load factors for load group 3 (lower broken line) for a weight range of 0 to 6 tons. It will be seen that the values of n for the American load group N are above those for the German load group 3. On the other hand, German safe factors for groups 4 and 5 (indicated by hatching) are above the American ones (groups U and A).

If the safe load values are multiplied by the prescribed safety factors n = 1.8 and 1.5 respectively, the American ultimate load factors (upper chain-dotted line) are found to be above the German ones (broken line). The largest difference, with W = 2 tons, is approximately 25%. For small and large values of W, however, the American and German values approach one another. For aircraft of the carrier category (W > 6 tons) the American regulations (CAR, Part 4b) lay down a constant, safe load factor of $n_{si} = 2.5$, whereas in the German regulations the relationship $n_{si} = 2 + 2/(W+2)$ (with W in tons) continues to be valid. Accordingly, the ultimate load factor for very large all-up weights according to CAR and BVF regulations are as follows:

CAR: $n_B = 1.5 \times 2.5 = 3.75$

BVF: $n_B = 1.8 \times 2.0 = 3.6$

i.e. the American value exceeds the German one by 4%.

The fluctuations of the safety factors contained in the various regulations in the course of the years 1926 to 1936 (Fig.1) and the differences in the ultimate load factors in the various countries (Fig.2) clearly reflect the uncertainty inherent in these values. The ultimate load factors and the safe load factors are both, first and foremost, empirical quantities which have hardly any physical significance. On the whole, they were staggered according to weight and general load grouping. The physically important effect of the air speed is merely indirectly contained in the load factors laid down in the American regulations - by dependence on the power load. The German regulations for the strength of aircraft in their version of December 1936 - valid without change up to the year 1945 - took the air speed into consideration only by making the dynamic pressure in cases B, C and D dependent on the dynamic pressure in the fastest unaccelerated horizontal flight condition.

3. THE NEW STATISTICAL CONCEPT OF SAFETY

This unsatisfactory situation for determining the load factors and safety factors suggested the advisability of putting these values on a new basis. It had already been pointed out in 1932 in the publication by Kussner and Thalau mentioned previously[5] that the required strength of an aircraft for a single load application should be based on the expectance of failure determined statistically from load measurements during flight. According to this, safety may be defined as a certain, if very low, probability of failure of the aircraft structure. These trains of thought were developed further by Kussner[9] in a paper published in 1935, in which frequency considerations were made for ascertaining the required strength of aircraft; these were limited to begin with to investigations of the static strength for the case of single high load applications. Similar proposals for the statistical investigations of the strength of aeroplanes were made in England by A.G. Pugsley[23, 24] in 1939.

A further argument in favour of considering aircraft strength from a statistical point of view came increasingly to the fore during the 1930's in connection with gust load cases. A gust load case was incorporated in the German strength requirements as a new requirement (Case G) in the year 1930, in the form of an amendment, prescribing a vertical gust velocity, both upward and downward, of $v = 10$ m/sec. The significance of this case is that the aircraft pilot can exert little influence on the magnitude of the load, especially if he wishes to adhere to the flying schedule and maintain normal cruising speed. Consequently, the aircraft is subjected to loads which are essentially of an accidental nature, and for which the magnitude and frequency can only be determined by statistical methods. For this reason, test flights with different aircraft types operating on various routes were commenced at the D.V.L. in the middle of the 1930's, in order to determine statistically the service loads on aircraft wing units and to derive from this the required fatigue strength. During these tests, extensive measurements of the acceleration of the centre of gravity of the aeroplane were made and their frequencies grouped into various classes. The results of these tests were published in two papers by H.W. Kaul[10, 11] in 1938, in the form of frequency distributions. These so-called gust statistics gave the frequency of load factors for the various aircraft in terms of flying distance.

Test flights to ascertain aircraft loads due to gusts had already been carried out at the N.A.C.A. by R.V. Rhode and E. Lundquist[12] in 1930/31. In these tests, also, measurements were made of the acceleration of the centre of gravity of the aircraft. Contrary to the investigations carried out by Kaul, however, measured accelerations in these tests were converted into gust velocities, without a reduction coefficient, using a simple gust formula given in a publication by Breguet and Devillers[13] as early as 1923, in order to make the results independent of the individual aircraft. By this method it was possible on the one hand to apply the values measured in a certain aircraft to other aircraft types, and on the other hand to compare the gust velocities calculated from the measured accelerations with available meteorological values. Substantial improvements have been made since then in the formula for gust load factor.

As the investigations carried out in this connection are described elsewhere and only concern the problem of safety indirectly, I will refrain from dealing with them in further detail in this paper. A report by R.V. Rhode and P.M. Donely[14] published during the war, as well as three further NACA reports published in 1950, 1952 and 1953 (TR 997, TN 2625, TN 3041) by P.M. Donely, W.G. Walker, and R. Steiner give a survey of the gust research carried out by the N.A.C.A. Mention may also be made of a lecture given last May at the D.V.L. by Mr. Donely, which provided an excellent survey of the latest achievements in the field of statistical gust research in the U.S.A., especially that carried out by the N.A.C.A.

The measurement of gust loads was continued by J. Taylor[15] in the beginning of the 1950's. The results of Taylor's investigations are illustrated in frequency diagrams in which the miles flown by an aircraft are plotted against the number of gusts encountered up to a certain given velocity. It is interesting to note that if Taylor's results are plotted on a log-log scale the result is a straight line. These tests were carried out with various aircraft and at various altitudes. Frequency investigations similar to those made with land-planes were carried out during the war with hydroplanes at the Institute for Marine Aviation, directed by myself at that time, in order to determine the stresses on the float undercarriage during take-off and landing in heavy seas. The results were published in a report by my assistant, H. Ehring[16], in the 1942 Yearbook of the Deutsche Luftfahrtforschung (German Aeronautical Research).

Mention may also be made of a contact extensometer designed by O. Svenson, and described by him at an earlier AGARD meeting; with this extensometer measurements of stresses during flight are to be made in the near future on certain aircraft of the German Lufthansa.

As the safety of an aircraft depends to a very large extent on the safe design load factors used, the safe gust load factors in current American regulations (to which the English ones approximately correspond) will be compared with those of the last German regulations (BVF 1936). According to the British and American regulations (BCAR Section D and CAR Parts 3 and 4b respectively) proof of the gust load is required for three different speeds V: (a) the stalling speed corresponding to maximum lift coefficient; (b) maximum cruising speed; and (c) the speed in a dive. Three different gust velocities U (both up and down) are associated with these speeds, of approximately 66, 50 and 25 ft/sec respectively. In the German regulations, however, a single gust of only 10 m/sec ($\approx$ 33 ft/sec) is required. This is assumed to be applied suddenly at maximum speed in unaccelerated horizontal flight. As the additional gust load factor Δn is proportional to the product UV, the German additional gust load factor would, accordingly, amount to only two-thirds of the comparable British or American values, corresponding to the most unfavourable case, which is the 50 ft/sec gust in horizontal flight.

A gust alleviation factor K has now been introduced, both in the German and in the British and American regulations. This is to take into consideration various favourable influences such as, for example, that the change of lift is not sudden, and that the gust velocity does not fluctuate violently, etc. In the German regulations this alleviation factor depends on the mass ratio of the aeroplane, μ (= $2WS/S\rho$ gca). The alleviation factor in the British and American regulations was, until last year, a function of the weight per square foot, namely, W/S. Now, quite recently, the aeroplane mass ratio has been introduced into the American regulations as well, so that the reduced gust velocity KU determined from American and German regulations can now be compared. The variation of these two values is entered above the aeroplane mass ratio in Figure 4. The difference between the two values decreases slightly because the safe gust velocity is allowed to be reduced to a lesser degree by the German regulations than by the American ones. However, by comparing the values multiplied by the different safety coefficients j = 1.5 and j = 1.8, i.e. the reduced ultimate gust velocities, it is found that the value given by the German regulations is only some 10% lower than the American one.

This comparison raises the question broached some time ago in a paper by Mangurian[17] as to whether the safety coefficient j is to be assumed for the entire value n or only for the additional value Δn in the case of the compound safe gust load factor $n = 1 \pm \Delta n$. When the safety coefficient is considered, first and foremost, as safety against exceeding the ultimate gust load, only the additional value should, of course, be taken into account. This is especially true in the case of the negative gust (directed downwards) because then the negative ultimate load factor becomes larger than if the safety factor were applied to the entire value. In the case of the positive gust, on the other hand, the ultimate load factor would become smaller, as the formula is $(1 + j.\Delta n) < j(1 + \Delta n)$.

As a result, the view is held in the regulations of 'Stahlbau' ('Steel Structure') that in the case of combined dead weight and live load the safety coefficient should be applied to both.

The slightly larger values required by the American regulations for the gust factor appear to be justified, as recent American investigations have established that not only the magnitude and frequency of individual gusts, but also the frequencies in the gust load spectrum, are of significance, and may, in the case of resonance with the natural frequency of the wing, lead to larger stresses. A further unfavourable aeroelastic effect is a possibility of a shift of the wing load to the wing tips, arising from the elastic deformation of the wing due to the aerodynamic forces, thereby causing increased bending moments at the wing root.

4. SAFETY AGAINST FATIGUE

When the maximum gust load expected in service is specified and gust statistics are known from flight tests, the *required* ultimate strength and fatigue strength for single and repeated loads may be obtained. In the case of a *single* load, the theoretical or experimental stresses can be related without undue difficulty to the ultimate strength, yield stress, limit of elasticity or the limit of stability. These are generally within a relatively small range of scatter. Any shock effects can be taken into account by intorducing a shock factor on the load. The safety factors to be applied can, therefore, be found with some degree of accuracy. In the case of a *repeated* load, however, it is much more difficult to determine the fatigue strength *existing* in the various structural members, the more so when stresses of variable amplitude and frequency are involved, as is the case with the gust load. In the present stage of research it is not at all possible to determine the fatigue strength theoretically. Consequently, this can only be achieved by experiment. Yet, for various reasons, even test results for repeated loads are subject to considerably more inaccuracy than is the case with a single load. In the first place, there is a much higher scatter, and secondly concentrations of stress have an important effect on fatigue strength. Such concentrations of stress arise, on the one hand, from irregularities or flaws in the material; on the other hand, they may be the result of details of the design, e.g., notches, holes, changes in sectional area, etc. A further reason is that the determination of the number of cycles to failure, and, therefore, the safe life, is possible only - because of the hardening and damaging effect of previous stress - if the test load is made to simulate the service load as nearly as possible. This, however, is expensive to do, and efforts have for some time past been made to simplify test conditions. In view of the effect of the testing procedure on safety, I should now like to discuss the different testing methods.

First attempts were directed towards replacing tests with various stress amplitude by tests with constant amplitude - the so-called 'Wohler' tests. The estimation of the fatigue strength under conditions of variable amplitude from such tests is based on a hypothesis which had already been applied to tests with ball bearings carried out by Palmgren[18] in 1924 and later adopted for tests with other structural members by Miner[19] in 1938. According to this hypothesis - known as the *'cumulative damage theory'* - the total damage is expressed as the sum of the part damages which are calculated from the ratios of the numbers of load cycles n_i at any given amplitude to the total number of cycles to failure N_i. Fatigue failure is assumed to occur when the sum of these ratios equals unity, i.e. when

$$\sum \frac{n_i}{N_i} = 1$$

On the basis of this hypothesis P.B. Walker[20,21] proposed in 1953 various design criteria for the determination of the service life of aircraft. Walker suggested that a structural member to be tested for fatigue should be subjected to an alternating load of 7½% of the ultimate load or, according to a later proposal by Walker - 7½% of the 50 ft/sec gust load multiplied by a factor of 1.5, superimposed upon the constant load in unaccelerated horizontal flight. Walker shows that this corresponds to the maximum damage, which is due to gusts of about 10 ft/sec. This value for the ultimate load is found by plotting, approximately, the gust velocity - which gives rise to the alternating stress - as ordinate of the S-N diagram, and then adding the load probability distribution (i.e., the straight line derived by Taylor). This is shown in Figure 5. The curve on the left side gives the ratios of the actual cycles n_i to the total cycles to failure N_i. The maximum value of this curve gives the point of greatest damage and corresponds to a gust velocity of about 3.5 m/sec (approximately 11 ft/sec), which is about the value suggested by Walker. The service life is now estimated for the number of cycles N taken from an S-N curve obtained from tests using the amplitude corresponding to the greatest damage.

According to Walker, a load cycle number of $N = 2 \times 10^6$ corresponds to about 30,000 flying hours at medium speed. The formula devised by Walker for the safe life thus reads:

$$L = KN/V$$

where L denotes the service life, V the flying speed and K is a factor. If L is in hours and V in knots, the value of K, estimated from all available evidence, is 2.5. This value of K is based on the logarithmic mean of the total number of cycles to failure estimated from 6 tests. The value N is then taken as 2/3 of the logarithmic mean, so that a service life safety factor of 1.5 is included. It is interesting to compare this value with proposals worked out by the C.A.A. in July, 1957. According to these, a safety factor for service life, based on the average of the test values, is suggested as follows:

For 1 test piece :	6
For 3 test pieces:	4.5
For 6 test pieces:	3.5

By comparison, the safety factor introduced by Walker would appear to be somewhat low. It should, however, be taken into consideration that in Walker's rule a safety factor of 1.25 is already actually embodied in the alternating load assumed, which represents a total safety factor of about 2 for the service life; also, the logarithmic mean is below the average figure given by the CAA rule.

Walker's investigations were extended later by Chilver[22], who evolved a mathematical relationship between the total number of cycles to failure, N, as obtained in the ordinary S-N test, and the required cycle number corresponding to the straight line gust statistics due to Taylor, using Miner's theory. On the basis of his investigations Chilver comes to the conclusion that Walker's criteria are valid only for ultimate load factors of up to 4.2 and that the service life is about double the value given by Walker's formula.

The problem of repeated loads in aircraft design was investigated in Germany at the D.V.L. by Gassner as early as the end of the 1930's. From a series of tests that he carried out at that time Gassner showed that the results obtained from ordinary S-N curves, calculated on the basis of the Miner damage theory, did not give satisfactory results in the case of randomly repeated variable loads.

Comparative tests of a similar nature with two and more stress levels were done by Heyer[25] in 1942. In recent years, very extensive tests at two and more levels have been carried out in the Netherlands by Hartman and Plantema[26], and also be Schijve and Jacobs[27], which showed major deviations from the values calculated by the Miner rule. Further suggestions were put forward by Lundberg[28] and Freudenthal[29] for the improvement of the damage theory by applying theories of statistical probability.

The main weakness of the damage theory is that it does not take into account the effects of stresses below the fatigue limit. It has been shown, from many tests, that these are unquestionably of importance when a succession of loads above the fatigue limit precede it.

In order to eliminate the uncertainties which exist when carrying out ordinary S-N tests and using the damage theory, Gassner proposed testing structural members in the laboratory using a succession of loads similar to those occurring in service. In such a test, the complete load spectrum is made up of separate spectra in which the loads have the same frequency as in the complete spectrum, thus giving a mixture corresponding to actual service conditions. A detailed description of a service endurance test is given in Reference 30.

Figure 6(a) shows the simplified loading proposed by Walker for the one-level type of test proposed by him and Figure 6(b) that due to Gassner for the service endurance test. Figure 6(c) shows a typical loading frequency distribution as measured during service. An accurate comparison of the service lives derived from the tests in Figure 6(a) and (b), and procedures for real service life in the case of a statistically distribued succession of random loads, is not yet possible. It may, however, be assumed that the life of a structure as obtained from a service endurance test is very closely representative of the actual service life. Tests similar to those of Gassner's[31] have been carried out recently by Freudenthal[32] as well, but with a more irregular distribution of loads.

Tests with a large number of 'Mustang' wings have been carried out by Payne[33] in Australia. These tests are especially valuable, as they reveal the behaviour of the complete wing during repeated loads. This investigation included also the beneficial effect on the fatigue strength of pre-loading the wing. It would seem that further tests in this direction are urgently needed.

5. CONCLUDING REMARKS

It is not possible within the scope of this paper to deal more fully with the results of fatigue research. By describing the various test methods I have shown, I hope, that a uniform assessment of safety, based on the expected service life, is hardly possible at present. On the question of whether safety should be based on load or on service life, I hold the view that the latter is more realistic, and therefore preferable; in the former case, multiplying the load by a safety factor alters the stress level, thereby changing the actual conditions.

The determination of safety in fatigue is particularly difficult because, in the main, there are major factors involved, namely, the total frequency of the service loads, the load factor or the mean stress level, the material used, and the stress concentration factor K_T.

The material used is of decisive significance in fatigue. It is well known that with aluminium alloys sensitivity to fatigue increases with the ultimate and yield strengths of the material. With high yield stress ratio, re-distribution of stress at points of stress concentration does not occur to the same extent as with aluminium alloys having a low yield stress ratio. Consequently, the fatigue strength of high-strength aluminium alloys is, generally, no higher than that of the lower-strength alloys. The introduction of the high-strength aluminium alloys has, in fact, made the problem of fatigue strength particularly acute.

Figure 7 clearly illustrates these points. The tensile strength has been plotted on the abscissa, where the values for the American alloys 24S-T4 and 75S-T5 have been marked. If the fatigue strength were of no consequence, the static limit design stress σ_D would be represented with the safety factor of 1.5 by the rising line. Service endurance tests carried out by Gassner show that with a required service life of 10^7 miles (or the equivalent number of load cycles $N = 40 \times 10^6$), the fatigue strength does not rise with the tensile strength, but remains constant. This occurs at a stress $\sigma_{SE} = 34{,}000$ lb/in.^{2} for a structural member with a stress concentration factor $K_T = 2.4$ and an ultimate load factor $n = 3.6$. Thus, the fatigue strength is lower than the static design stress. This means, however, as may be seen from Figure 7, that a quasi-static safety factor above 1.5, rising with the tensile strength, is required, which, for example, in the case of the high-strength alloy 75S-T5, reaches the value 2.3.

The preceding Figure showed the effect of the material on the value of the quasi-static safety factor; Figure 8 shows the effect of structural design on this factor. As the characteristic of structural design, the stress concentration factor was taken. It can be seen that the safety factor increases from 1.5 to 2.3 when the stress concentration factor increases from 2 to 4. The service life then becomes one order of magnitude less, that is, it attains a value required 20 years ago (1.8×10^6 instead of 18×10^6 miles).

The significance of these considerations is that with the required service life of modern aeroplanes and the use of high-strength materials, aeroplane structures can no longer be designed on the basis of static strength requirements. Additional weight is necessary for fatigue strength, and this weight increases if stress concentration is not avoided by careful design. These facts are responsible for the position today in the U.S.A., in which aeroplane structures are no longer designed for *safe life,* but on the basis of fatigue failure of a *primary single member of the structure.* After such a failure the remaining structure has to be able to withstand static loads with an ultimate load factor of 2 at cruising speed, and gust loads corresponding to gust velocities of 2/3 of the values for the undamaged structure.

For dynamic effects of failure under static loads, these must be multiplied by a factor of 1.15. Such a 'fail-safe' structure must be proved to posses the required ultimate strength with the omission of any member which it is suspected may fail in fatigue. This does not have to be done by test, although aeroplane factories do, generally, conduct fatigue tests and strain measurements in order to discover weak points and high stress levels. Space does not allow further consideration of the problem of whether a structure should be a 'safe-life' one or a 'fail-safe' one. My own opinion is that the right way would be a reasonable compromise between the two.

REFERENCES

1.	Hoff, N.J.	*Philosophy of Safety.* AGARD Report 87, 1956.
2.	James, P.	*Aérophile.* 19 Jahrgang., 1911, p.428.
3.	Reissner, H.	*Beanspruchung und Sicherheit von Flugzeugen.* Jahrbuch der WGL, 1912/13, pp.85-122.
4.	Hoff, W.	*Die Festigkeit deutscher Flugzeuge.* ZFM, Bd.13, 1922, p.139 und Ber. u. Abh. WGL, 1922, Heft 8, p.150. English translation: NACA Rep.143.
5.	Küssner, H.G. Thalau, K.	*Die Entwicklung der Festigkeitsvorschriften für Flugzeuge von den Anfängen der Flugtechnik bis zur Gegenwart.* Luftfahrtforschung, Bd. 10, 1932, pp.1-54.
6.	Delaunay	*L'Aérophile.* 20 Jahrg., 1912, p.317.
7.	Norton-Allen	NACA Report No.99, 1921.
8.	Doolittle, J.H.	*Accelerations in Flight.* NACA TR No.203, 1925.
9.	Küssner, H.G.	*Häufigkeitsbetrachtungen zur Ermittlung der erforderlichen Festigkeit von Flugzeugen.* Luftfahrtforschung, Bd.12, 1935, pp.57-61.
10.	Kaul, H.W.	*Die erforderliche Zeit- und Dauerfestigkeit von Flugzeugtragwerken.* Jahrb. 1938 der deutschen Luftfahrtforschung, pp.1274-1288.
11.	Kaul, H.W.	*Statistische Erhebungen über Betriebsbeanspruchungen von Flugzeugflügeln.* Jahrbuch 1938 der deutschen Luftfahrtforschung, Ergänzungsband, pp.307-313.
12.	Rhode, R.V. Lundquist, E.E.	*Preliminary Study of Applied Load Factors in Bumpy Air.* NACA TN 374, April 1931.
13.	Breguet Devillers	*L'Aérophile.* 31 Jahrg, 1932, p.102.
14.	Rhode, R.V. Donely, P.	*Frequency of occurrence of related loads on airplane structures.* NACA Wartime Report, Advance Restricted L 4121, Nov. 1944.
15.	Taylor, J.	*Measurement of gust loads in aircraft.* J. Roy. Aero. Soc., Vol.57, No.506, Feb. 1953, pp.78-88.
16.	Ehring, H.	*Anwendung der Häufigkeitsauswertung zur Beurteilung der Festigkeit von Seeflugzeugen.* Jahrb. 1942 der deutschen Luftfahrt, I, pp.484-491.

17. Mangurian, G.H. *Is the Present Aircraft Structural Factor of Safety Realistic?* Aeron. Eng. Rev., Sept., 1954, pp.63-75.

18. Palmgren, A. *Die Lebensdauer von Kugellagern.* Z. VDI 68, 1924, pp.339-341.

19. Miner, M.A. *Cumulative damage in fatigue.* Proc. Am. Soc. Mech. Eng. Metals 67, 1945, pp.159-164.

20. Walker, P.B. *Design Criterion for Fatigue of Wings.* Jour. Roy. Aeron. Soc. Vol.57, Jan., 1953, pp.12-18.

21. Walker, P.B. *Estimation of the Fatigue Life of a Transport Aircraft.* Jour. Roy. Aeron. Soc., Vol.57, Oct., 1953, pp.613-617.

22. Chilver, A.H. *The Estimation of Fatigue Damage in Aircraft Wing Structures.* Jour. Roy. Aeron. Soc., Vol.58, June 1954, pp.396-402.

23. Pugsley, A.G. *Note on Airworthiness Statistics.* R&M. No.2224, May, 1939.

24. Pugsley, A.G. *A Philosophy of Aeroplane Strength Factors.* R&M. No. 1906, September, 1942.

25. Heyer, K. *Mehrstufenversuche an Konstruktions-Elementen.* Bericht 152 der Lilienthal-Gesellschaft für Luftfahrtforschung, 1942, pp.29-54.

26. Plantema, F.J. *Some Investigations on cumulative damage.* IUTAM Colloquium on Fatigue, Stockholm, 1955.

27. Schijve, J.
Jacobs, F.A. *Research on Cumulative Damage in Fatigue of Riveted Aluminium Alloy Joints.* NLL Report M. 1999, Jan. 1956.

28. Lundberg, Bo K.O. *Fatigue Life of Airplane Structures.* FFA-Meddelande 60, Stockholm, 1955.

29. Freudenthal, A.M.
Heller, R.A. *Accumulation of Fatigue Damage. Fatigue in Aircraft Structures.* New York, 1956, pp.146-177.

30. Gassner, E. *Betriebsfestigkeit. Eine Bemessungsgrundlage für Konstruktionsteile mit statistisch wechselnden Betriebsbeanspruchungen.* Konstruktion 6, 1954, pp.97-104.

31. Gassner, E. *Performance Fatigue Testing with Respect to Aircraft Design.* Fatigue in Aircraft Structures, New York, 1956, pp.178-206.

32. Freudenthal, A.M. *Cumulative Damage under Random Loading.* International Conference on Fatigue of Metals, Session 3, Paper 4, 1956.

33. Payne, A.O. *An Investigation into the Fatigue Characteristics of a Typical 24S-T Aluminium Alloy Wing.* International Conference on Fatigue of Metals, Session 8, Paper 3, 1956.

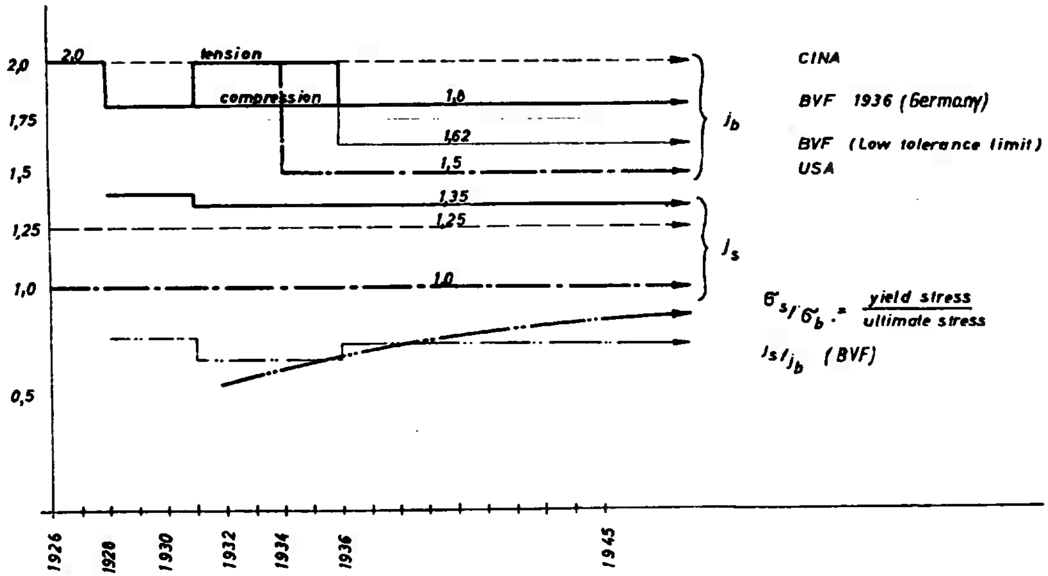
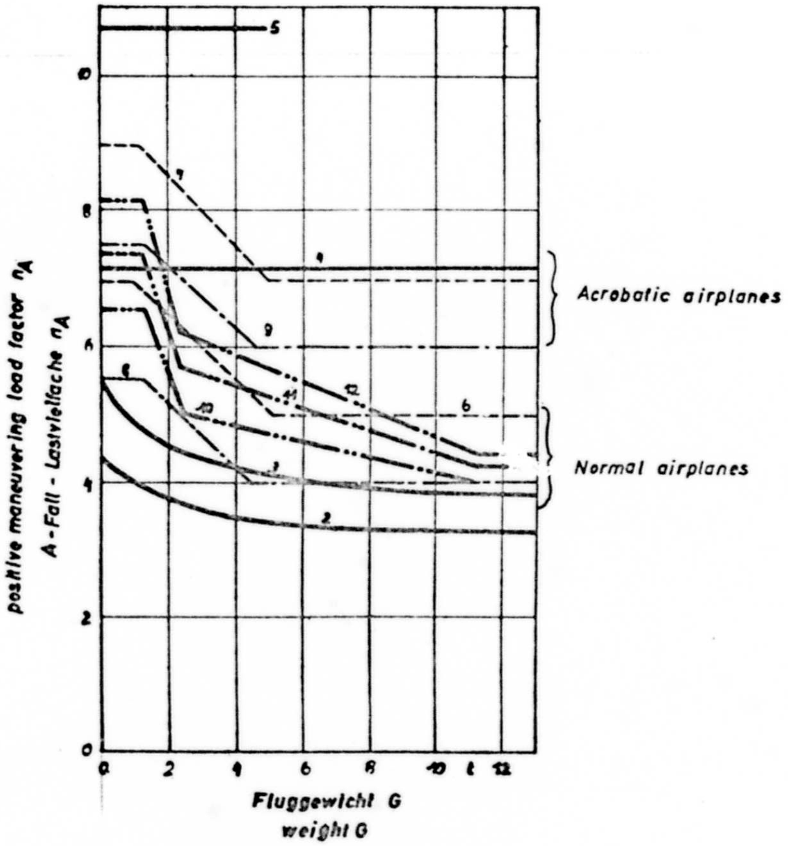


Fig.1 History of safety factors



Germany	Curve 2 to 5	———	Categories: 2 to 5
France (CINA)	" 6 " 7	- - - -	Categories: Normal and acrobatic
Great Britain	" 8 " 9	- - - -	Categories: Normal and acrobatic
USA	" 10 " 12	———	Powerloading 9.6 and 3 kg/HP

Fig.2 Comparison of ultimate positive manoeuvring load factor in 1929

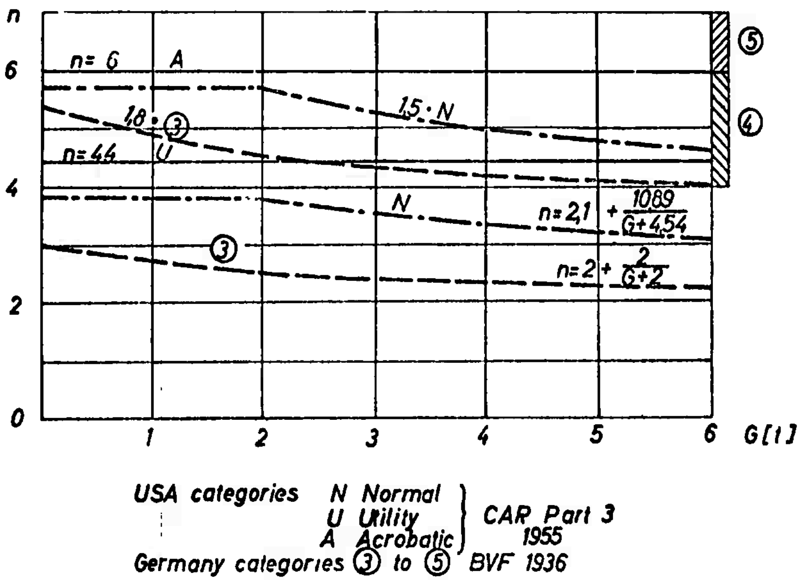


Fig.3 Further comparison of positive manoeuvring load factors

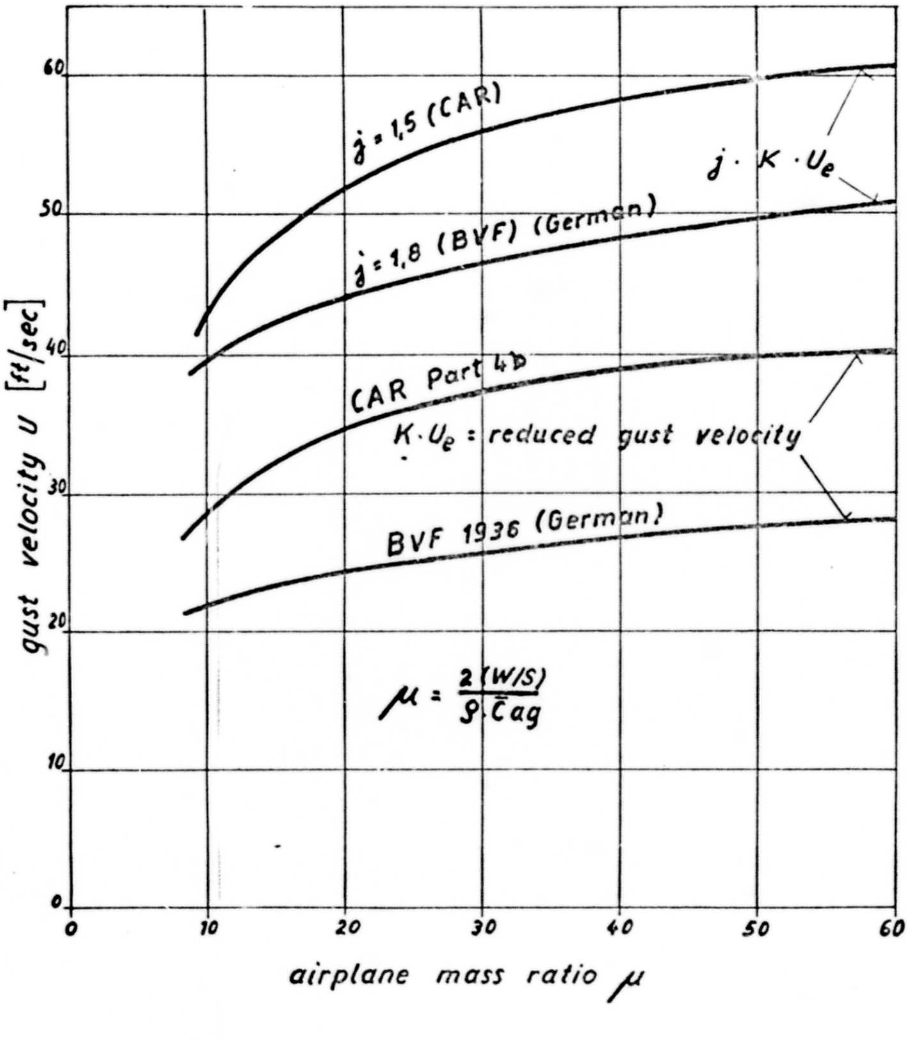


Fig. 4 Comparison of effective gust velocities according to German and USA regulations

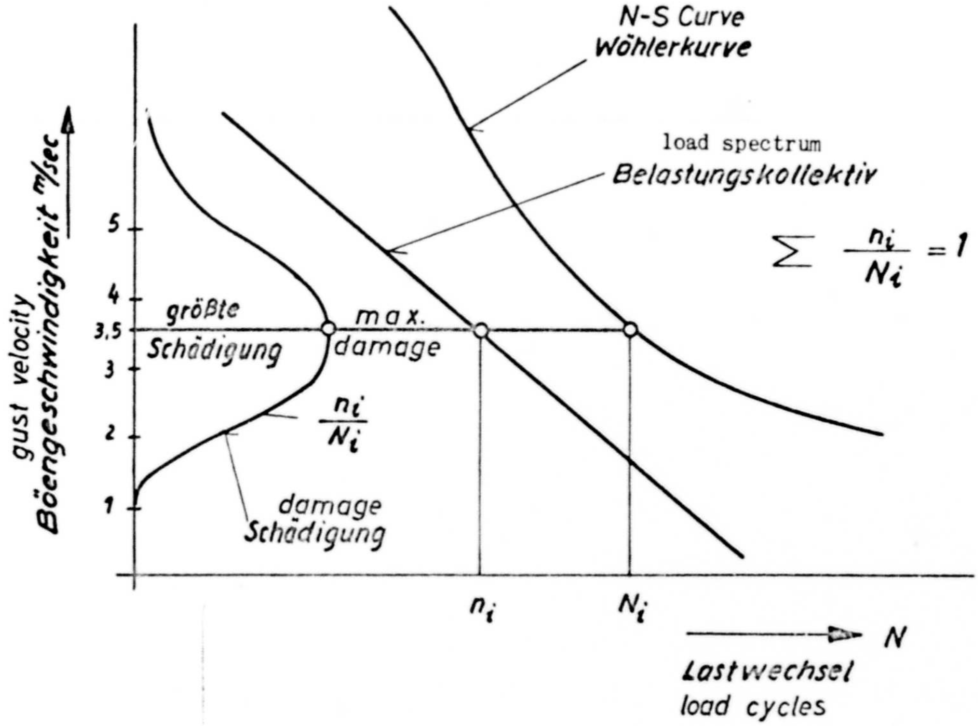
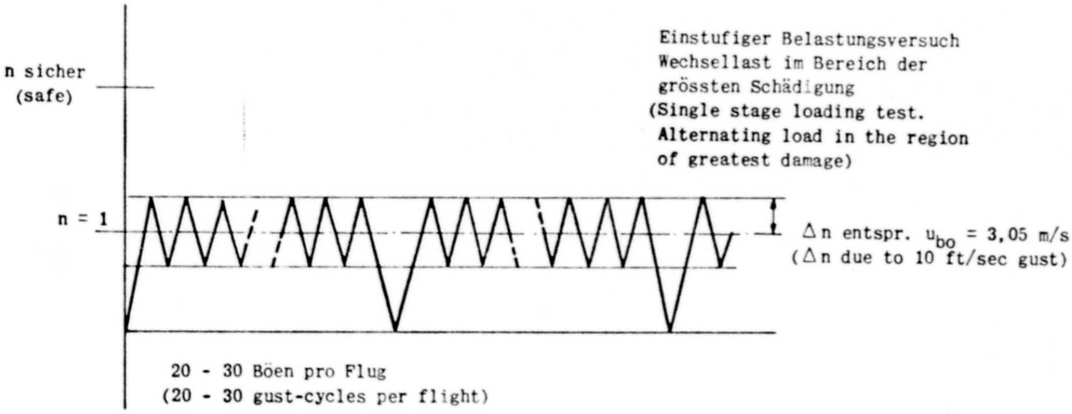


Fig.5 Cumulative damage theory



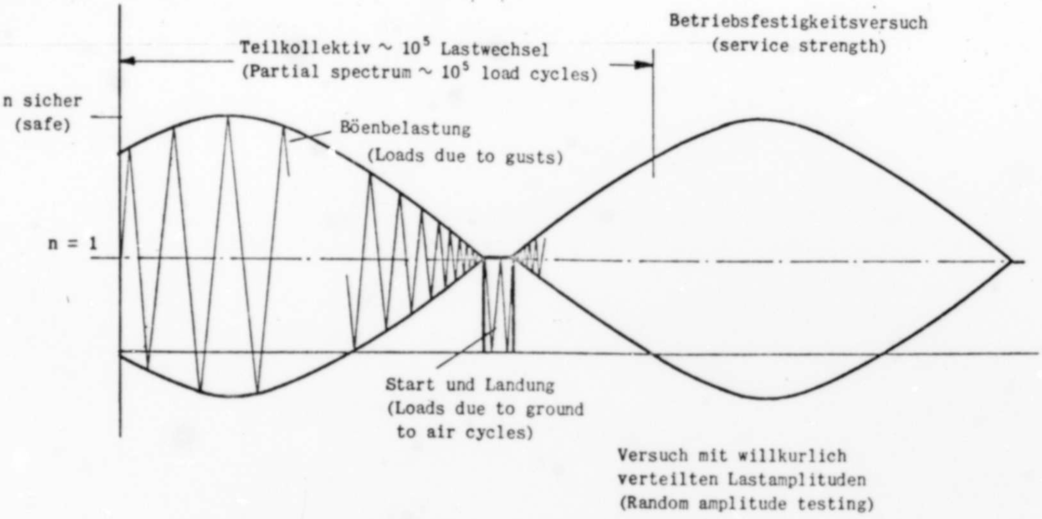
Einstufiger Belastungsversuch
Wechsellast im Bereich der
grössten Schädigung
(Single stage loading test.
Alternating load in the region
of greatest damage)

Δn entspr. u_{bo} = 3,05 m/s
(Δn due to 10 ft/sec gust)

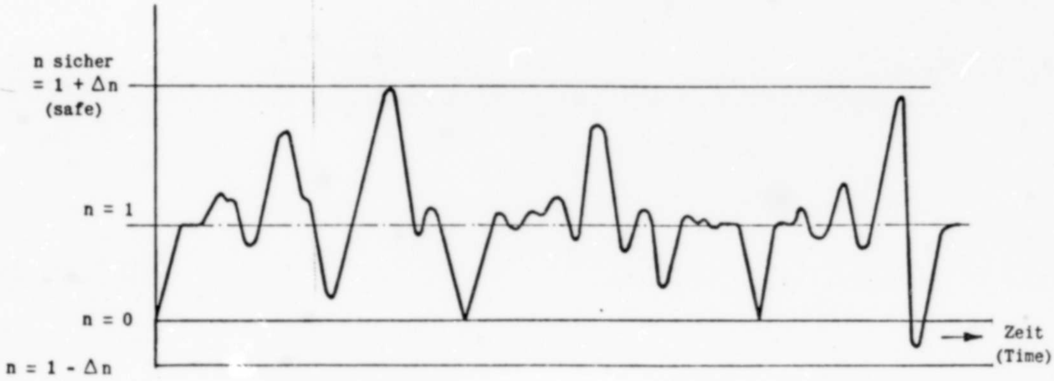
20 - 30 Böen pro Flug
(20 - 30 gust-cycles per flight)

(a) Idealized loading sequence composed of most damaging gust cycles and ground to air cycles

Fig.6 Methods for fatigue testing of wings



(b) Programme testing (Gassner)



(c) Random amplitude testing

Fig. 6 Methods for fatigue testing of wings - Cont'd.