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A TECHNIQUE FOR IMAGE FREQUENCY REJECTION

by

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A TECHNIQUE FOR IMAGE FREQUENCY REJECTION

One of the sources of interference in a radar is the reception of extraneous signals which are within the image pass band of the I-F amplifier. In this report, a method to reject the image pass band is presented.

A circuit diagram involving the basic principles of the image-rejection method is shown in Figure 1. With reference to this figure, the operation of the circuit may be described as follows:

The incoming signal is fed equally and in-phase to each of two mixers while the local oscillator signal is fed equally but with a phase quadrature difference to the two mixers. Thus, if we let the incoming signal be $\sqrt{2}\, E_s \cos \omega_s t$ and the local oscillator signal be $\sqrt{2}\, E_L \cos \omega_L t$, the input to mixer-A will be $E_s \cos \omega_s t$ and $E_L \cos \omega_L t$ while the input to mixer-B will be $E_s \cos \omega_s t$ and $E_L \sin \omega_L t$. The I-F output of each mixer will be the difference-frequency component of the product of the two inputs. The I-F output of the mixers, E_a and E_b, are thus:

(1) $$E_a = g_a E_s \cos(\omega_L - \omega_s)t = g_a E_s \cos \omega_d t$$

(2) $$E_b = g_b E_s \sin(\omega_L - \omega_s)t = g_b E_s \sin \omega_d t$$

where g is the conversion transconductance of the mixer, and $\omega_d = \omega_L - \omega_s$. We note that since E_a is an even function of ω_d, it will be given by equation (1) for $\omega_L \gtrless \omega_s$ However, E_b is an odd function of ω_d.

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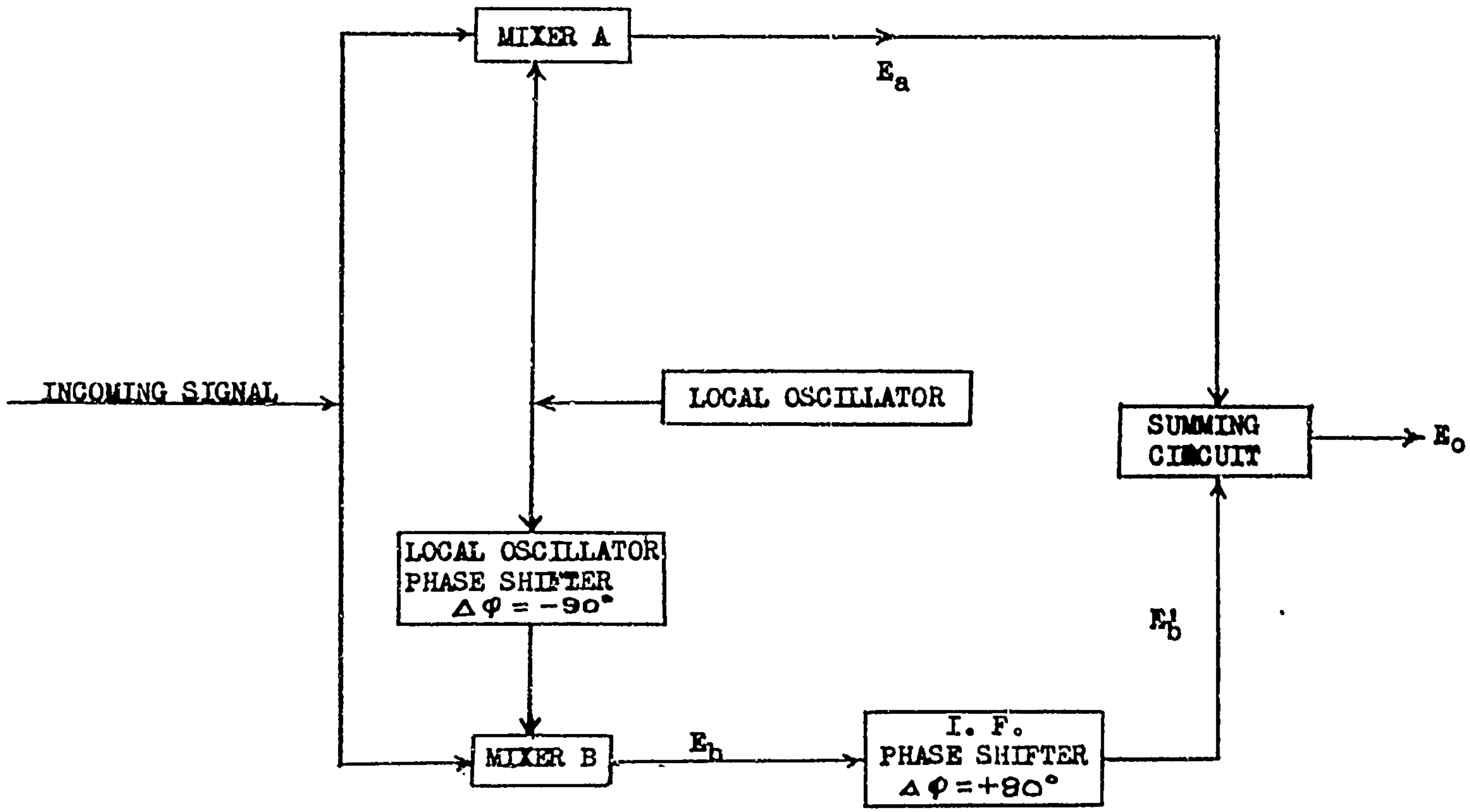

FIGURE 1

BLOCK DIAGRAM OF THE IMAGE REJECTION CIRCUIT

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Therefore:

(3) $$E_b = g_b E_s \sin \omega_d t \qquad \omega_s < \omega_L$$

(4) $$E_b = -g_b E_s \sin \omega_d t \qquad \omega_s > \omega_L$$

The signal, E_b, is then passed through a 90-degree I-F phase shifter. Thus from equations (3) and (4), the output of the phase shifter, E_b', is:

$$E_b' = g_b E_s \cos \omega_d t \qquad \omega_s < \omega_L$$

$$E_b' = -g_b E_s \cos \omega_d t \qquad \omega_s > \omega_L$$

The two signals, E_a and E_b' are then summed. Thus the output, E_o, is:

$$E_o = E_s (g_a + g_b) \cos \omega_d t \qquad \omega_s < \omega_L$$

$$E_o = E_s (g_a - g_b) \cos \omega_d t \qquad \omega_s > \omega_L$$

If $g_a = g_b$, then E_o will be zero for $\omega_s > \omega_L$. If $g_a \neq g_b$, E_o may still be made zero by placing an attenuator on one of the signal lines and adjusting the attenuator so that $E_{sa} g_a = E_{sb} g_b$. It should be noted that both mixers A and B may be balanced mixers, thus all the advantages of balanced-mixer operation may be obtained. Further, it should be noted that if the difference of E_a and E_b' were taken instead of the sum, the pass and reject bands would be reversed so that $E_o = 0$ for $\omega_S < \omega_L$.

Circuit Tolerances

Theoretically, it is possible to obtain infinite rejection of the image. However, due to amplitude and phase error, the rejection will be somewhat less and the sensitivity in the pass band will be slightly reduced.

Let α be the ratio of E_a and E'_b and θ the phase error between E_a and E'_b. It is seen that the output signal in the pass band, E_p, is:

$$E_p = g E_s \left| 1 + \alpha e^{j\theta} \right|$$

With no amplitude or phase error ($\alpha = 1$ and $\theta = 0$), $E_p = 2gE_s$. Thus the loss in sensitivity is:

$$S = \frac{g E_s \left| 1 + \alpha e^{j\theta} \right|}{2 g E_s}$$

or

$$S^2 = \frac{1 + \alpha^2 + 2\alpha \cos\theta}{4}$$

A graph of the sensitivity loss in db for various amplitude and phase errors is shown in Figure 2.

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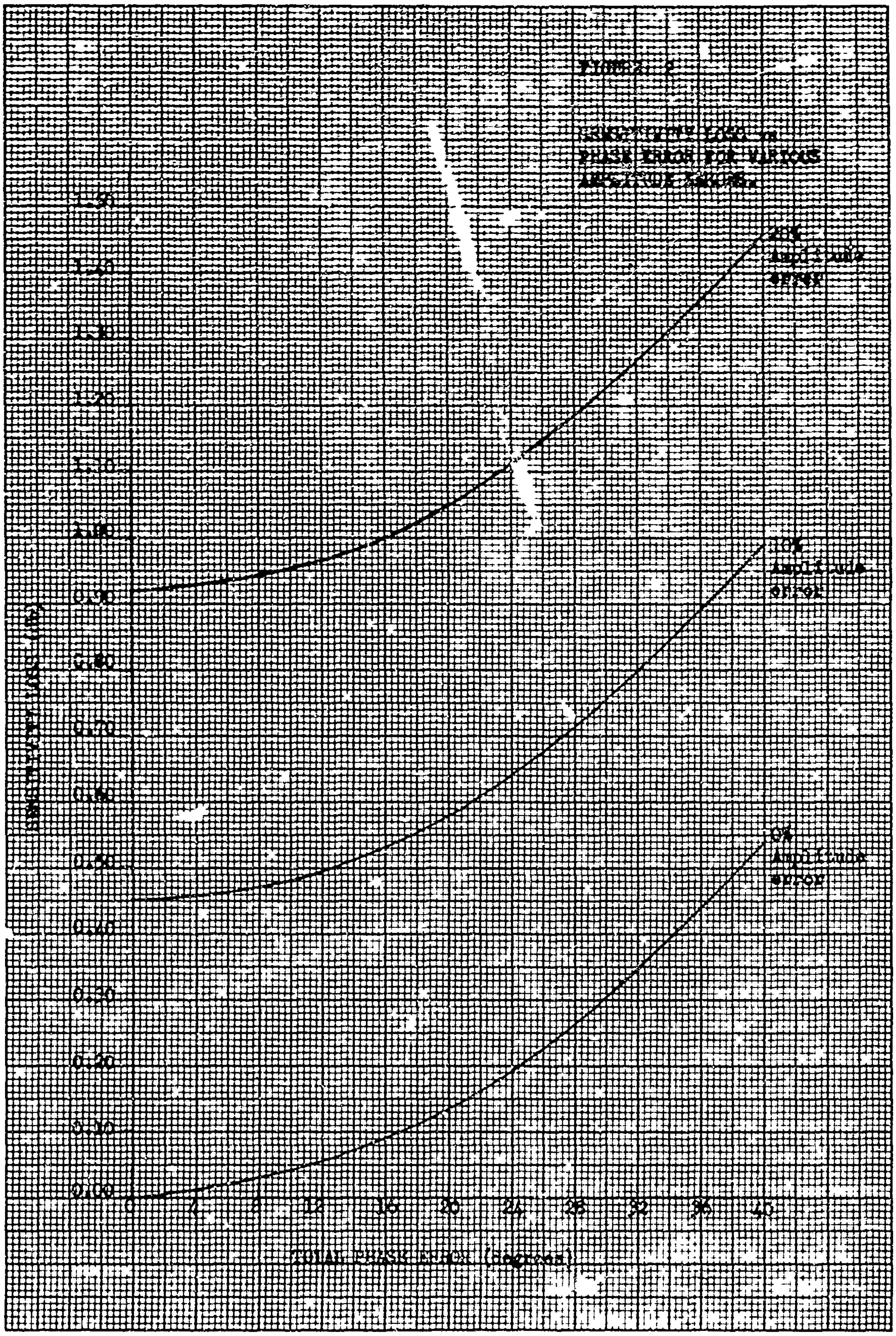

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In the reject band, the output signal, E_R, is:

$$E_R = g E_s \left| 1 - \alpha e^{j\theta} \right|$$

Thus the rejection ratio, R, is:

$$R = \frac{E_R}{E_P} = \frac{\left| 1 - \alpha e^{j\theta} \right|}{\left| 1 + \alpha e^{j\theta} \right|}$$

or

$$R^2 = \frac{1 + \alpha^2 - 2\alpha \cos\theta}{1 + \alpha^2 + 2\alpha \cos\theta}$$

A graph of the rejection ratio in db for various amplitude and phase errors is shown in Figure 3. From this graph it is observed that a minimum rejection of 20 db may be obtained if the amplitude error is held to 10% and the phase error is held to $\pm 10^\circ$. With these tolerances, it is shown in Figure 2 that the maximum loss in sensitivity is less than 0.5 db.

Circuit Considerations

It is seen from the preceding discussion that if a 20-db rejection ratio is to be obtained, the I-F phase shifter (re: Figure 1) must maintain a phase shift of 90 ± 10 degrees over the I-F amplifier bandwidth. A simple I-F phase shifter that could be used is a quarter wavelength transmission line. For such a phase shifter, the bandwidth, Δf, over which the phase shift would be 90 ± 10 degrees is:

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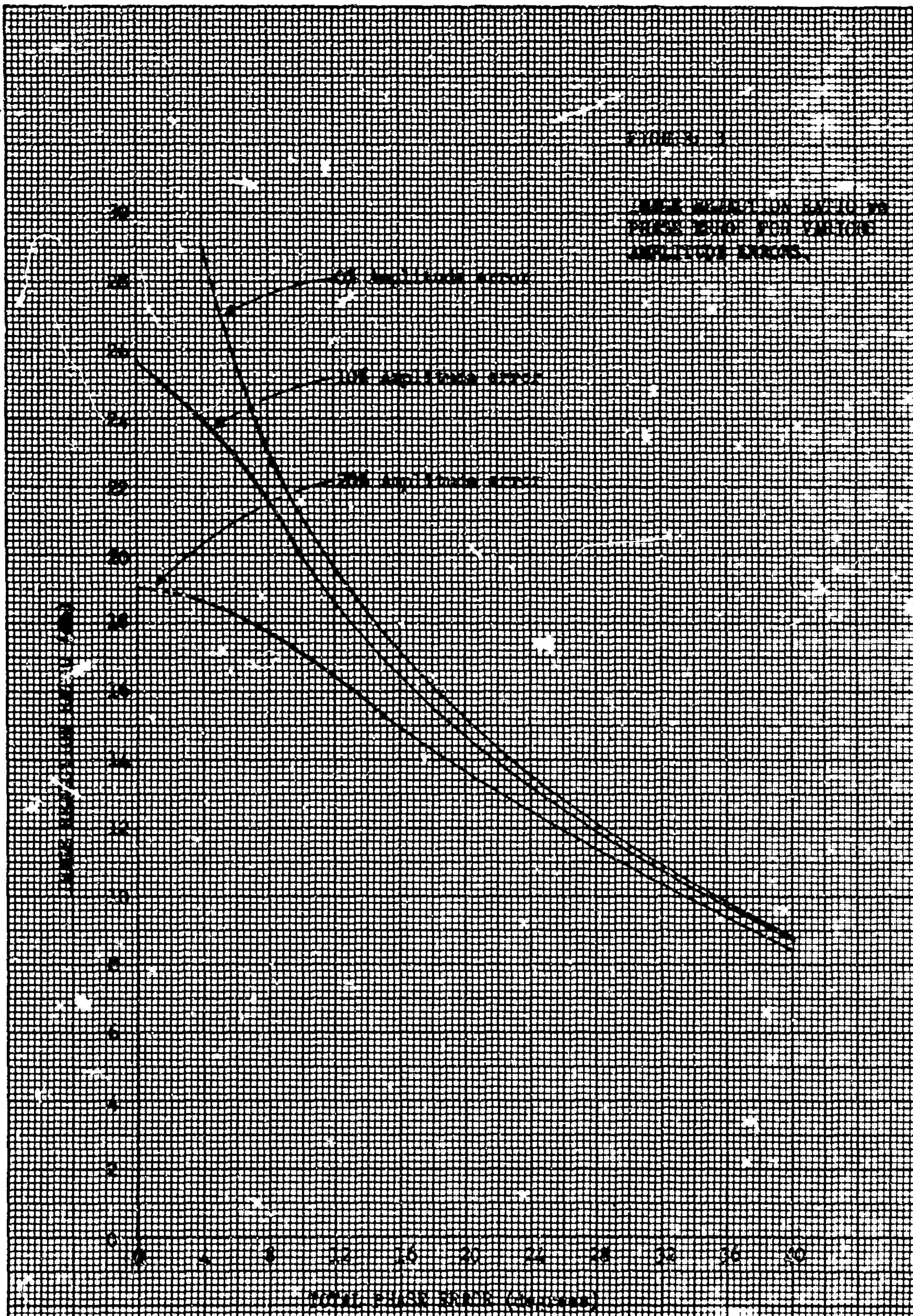

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$$\frac{\Delta f}{f_0} = 2\,\frac{\Delta\varphi}{\varphi_0} = 2\,\frac{10}{90} = 0.22$$

Thus for a 30-mc/s I-F amplifier, a 20-db rejection ratio could be obtained over a maximum band of 6-2/3 mc/s. The I-F amplifier of many radars (such as search and CW radars) have a percentage bandwidth much less than 22 percent. For such radars, the quarter wavelength transmission line offers a simple and stable I-F phase shift network. For those radars which utilize I-F bandwidths in excess of 22 percent (such as tracking radars), this technique of I-F phase shift would not be satisfactory. Any other passive network which could be designed to provide a 90-degree phase shift over a broad frequency band would necessarily result in increased attenuation through the network and thus a decrease in radar sensitivity.* To circumvent this difficulty, a double conversion system as shown in Figure 4 may be used. It is seen that the signal input to mixers C and D is E_a and E_b, respectively, as previously discussed. The second local oscillator input to mixers C and D is $E_{L2} \cos \omega_{L2t}$ and $E_{L2} \sin \omega_{L2t}$, respectively. Thus the difference frequency output of mixer C, E_c, is:

$$E_c = g_a\, g_c\, E_s \cos(\omega_s - \omega_{L_2})t \qquad \omega_s \lessgtr \omega_{L_1}$$

and the difference frequency output of mixer D, E_d, is:

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*This may be seen from the Hilbert transformation. Let the network transfer function be $h(\omega) = e^{-(\alpha + j\beta)}$. Then from the Hilbert transformation

$$\alpha(\omega) = -\frac{2\omega^2}{\pi}\int_0^\infty \frac{\beta(\eta)\,d\eta}{\eta(\eta^2-\omega^2)}$$

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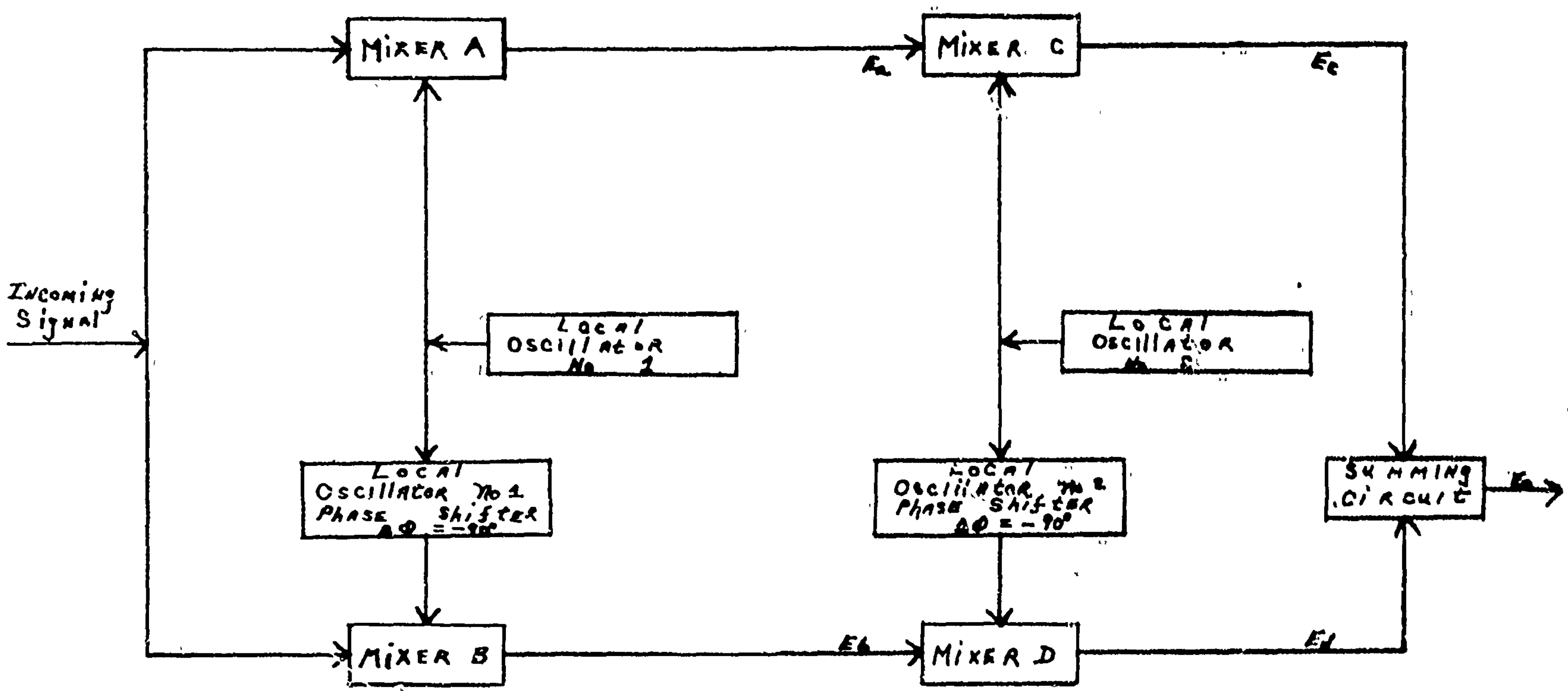

Figure 4

BLOCK DIAGRAM OF THE DOUBLE CONVERSION IMAGE REJECTION CIRCUIT

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$$E_d = g_b g_d E_s \cos(\omega_d - \omega_{L2})t \qquad \omega_s < \omega_{L1}$$

$$E_d = -g_b g_d E_s \cos(\omega_d - \omega_{L2})t \qquad \omega_s > \omega_{L1}$$

E_c and E_d are then summed. The output of the summing network, E_o, is thus the same as that of Figure 1. It is seen that the double conversion method of image rejection does not require a broadband I-F phase shifter and thus it is capable of providing an image rejection over a broad I-F bandwidth.