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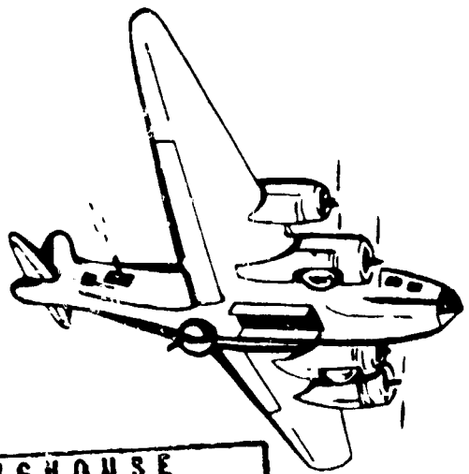
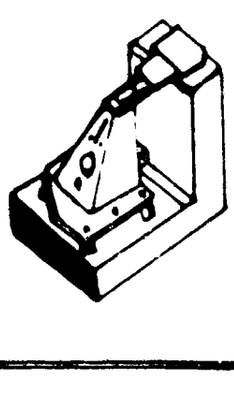


MEMORANDUM
REPORT No. 598

Note on the Determination of the Form of Air Shocks from the Decay Curve



JANE DEWEY

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B A L L I S T I C R E S E A R C H L A B O R I T I E S

MEMORANDUM REPORT NO. 598

February 1952

NOTE ON THE DETERMINATION OF THE FORM OF AIR SHOCKS FROM THE DECAY CURVE

Jane Dewey

Project No. TB3-0112J of the Research and
Development Division, Ordnance Corps

A B E R D E E N P R O V I N G G R O U N D, M A R Y L A N D

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BALLISTIC RESEARCH LABORATORIES MEMORANDUM REPORT NO. 598

JDewey/so
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February 1952

NOTE ON THE DETERMINATION OF THE FORM OF AIR SHOCKS FROM THE DECAY CURVE

ABSTRACT

Expressions connecting the first derivative of the shock parameters just behind the front with the decay curves are given in the form convenient for comparison with experimental results, for plane, cylindrical and shperical shocks. Comparison of pressure-time gage measurements on "spherical" shocks with the decay curve for shocks from spherical Pentolite charges is included. The discussion is preliminary to the determination of the form of the shock from these charges now being undertaken by integration of the hydrodynamic equations along the Kirkwood-Brinkley decay curve.

The determination of the pressure-time relation at a point and the pressure-distance relation at a given time from the decay of the peak pressure of an air shock has been hampered by the small range of peak pressures over which reliable measurements were available. Recent work in this Laboratory on spherical Pentolite charges has confirmed the general form of the decay curve obtained by compilation of data from a number of sources.[1] In the following note the slopes of the pressure-time and pressure-distance curves just behind the front are computed as a preliminary to a more complete study.

Following Kirkwood and Brinkley,[2] we calculate the pressure-time or pressure-distance variation at the shock front by combining the Hugoniot conditions with the hydrodynamic equations.* For comparison with experiment the Eulerian form is more convenient than the Lagrangian form used by Stanton and Cassen[3] and will therefore be used from the start, although the relations derived here can be obtained from theirs. To distinguish the variation of a quantity at the front from the usual total derivative, we write

$$\frac{D}{Dt} = U \frac{D}{DR} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial r} \tag{1}$$

where U is the shock velocity, r the space variable and R its value at the front.

$$\frac{\partial}{\partial t} + u \frac{\partial}{\partial r} = \frac{D}{Dt} - (U - u) \frac{\partial}{\partial r} = U \left(\frac{D}{DR} - \frac{\rho_o}{\rho} \frac{\partial}{\partial r}\right) \tag{1a}$$

where u is material velocity in the shock front, ρ and ρ_o the densities in and without the shock front.

The hydrodynamic equations can then be specialized at the front in the form:

Continuity equation and energy equation,

$$\frac{U}{\rho a^2} \left(\frac{DP}{DR} - \frac{\rho_o}{\rho} \frac{\partial p}{\partial r}\right) + \frac{\partial v}{\partial r} + \frac{\alpha u}{R} = 0 \tag{2}$$

where p is the pressure, P is the pressure at the front, and v is the material velocity, a^2 is the variation of pressure with density at constant entropy in the front, $\alpha = 0$ for a plane, 1 for a cylindrical, and 2 for a spherical shock front.

Dynamical equation,

$$U \frac{Du}{DR} - \frac{\rho_o}{\rho} U \frac{\partial v}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \tag{3}$$

or combining (2) and (3) and utilizing $\rho_o u U = P - p_o$, where p_o is the pressure of the undisturbed air,

$$U \frac{Du}{DR} + \frac{\rho_o U^2}{\rho a^2} \frac{DP}{DR} + \frac{\alpha (P - p_o)}{R} + \left(1 - \frac{\rho_o^2 U^2}{\rho^2 a^2}\right) \frac{\partial p}{\partial r} = 0 \quad . \tag{3a}$$

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* In an appendix the notation, equations, and Hugoniot conditions are summarized.

Since at very high peak pressures $D\rho/DP$ is very small, it is convenient to express u in terms of P and ρ at the front. The following expressions have been found useful:

$$-\frac{\partial p}{\partial r} = \frac{\left[\frac{1}{2}\frac{D\ln(\rho-\rho_o)}{D\ln(P-p_o)} + \frac{\rho}{2\rho_o} + \frac{P-p_o}{(\rho-\rho_o)a^2}\right]\frac{DP}{DR} + \frac{\alpha(P-p_o)}{R}}{1-\frac{\rho_o(P-p_o)}{\rho(\rho-\rho_o)a^2}} \tag{4}$$

$$\frac{1}{U}\frac{\partial p}{\partial t} = \frac{\left[1+\frac{1}{2}\frac{\rho}{\rho_o} + \frac{(P-p_o)}{\rho a^2} + \frac{D\ln(\rho-\rho_o)}{2D\ln(P-p_o)}\right]\frac{DP}{DR} + \frac{\alpha(P-p_o)}{R}}{1-\frac{\rho_o}{\rho}\frac{P-p_o}{(\rho-\rho_o)a^2}} \tag{5}$$

or

$$\frac{1}{U}\frac{dp}{dt} = \frac{1}{U}\frac{\partial p}{\partial t} + \left(1-\frac{\rho_o}{\rho}\right)\frac{\partial p}{\partial r} = \frac{\left[3+\frac{\rho_o}{\rho}\frac{D\ln(\rho-\rho_o)}{D\ln(P-p_o)}\right]\frac{DP}{DR} + \frac{2\alpha\rho_o(P-p_o)}{\rho R}}{2\left(1-\frac{\rho_o(P-p_o)}{\rho(\rho-\rho_o)a^2}\right)} \tag{6}$$

and

$$\frac{1}{u}\frac{\partial v}{\partial t} = \frac{\left[\frac{(3\rho-\rho_o)(P-p_o)}{\rho(\rho-\rho_o)a^2} + 1 + \left(1+\frac{(P-p_o)}{\rho a^2}\right)\frac{\rho_o}{\rho}\frac{D\ln(\rho-\rho_o)}{D\ln(P-p_o)}\right]\frac{DP}{DR} + \frac{2\alpha(P-p_o)}{R}}{2\rho_o\left(1-\frac{\rho_o(P-p_o)}{\rho(\rho-\rho_o)a^2}\right)}$$

$$= -\frac{1}{P-p_o}\frac{\partial p}{\partial t} - \tag{7}$$

$$\frac{\left(\frac{\rho+\rho_o}{\rho_o}+\frac{\rho-\rho_o}{\rho}\frac{D\ln(\rho-\rho_o)}{D\ln(P-p_o)}\right)\left\{\left(1-\frac{\rho_o(P-p_o)}{\rho)\rho-\rho_o)a^2}\right)\frac{1}{(P-p_o)}\frac{\partial p}{\partial t} - \frac{2\alpha(P-p_o{}^{1/2}\rho^{1/2}}{\rho_o{}^{1/2}(\rho-\rho_o)^{1/2}R}\right\}}{2+\rho/\rho_o+2(P-p_o)/\rho a^2+\frac{D\ln(\rho-\rho_o)}{D\ln(P-p_o)}}$$

Experimental work on spherical charges shows that decay is linear both at very low and at very high peak pressures. The equation

$$(P - p_o)Z = (2.26 + \frac{71.5}{1 + 0.23\ lb^{2/3} ft^{-2} Z^2})\ \text{atm ft lb}^{-1/3}, \quad (8)$$

where $Z = R/(\text{charge weight})^{1/3}$, has been found to fit data of this Laboratory on spherical Pentolite over the range from the charge surface to $Z = 100$, within the dispersion of measurements. Calculations of peak pressure from equation (8) can be relied upon at large distances from a detonation, probably to better than 5%, as is shown by the agreement between independent workers.[4] At small distances the non-spherical form of the shock produced by initiating a spherical charge with any existing detonator and other so-far unidentified factors introduce an uncertainty of the order of 10%. If the scaled distance is expressed in charge diameters, r_o, instead of in ft $lb^{-1/3}$, $x = r/r_o = 7.52\ Z$

$$(P - p_o)\ x = (17.0 + \frac{537}{1 + 4.1 \times 10^{-3} x^2})\ \text{atm charge radii} \quad (8a)$$

Figure 1 is a plot of the relevant parameters in these expressions taken from the tables of Kirkwood, Brinkley and Richardson.[5] These tables are also used in obtaining the peak pressure close to a charge from the measured shock velocity. In neither case can the possible errors in the equation of state used in the Hugoniot conditions introduce an error in the results at high pressures comparable to the uncertainty of measurement, as the relations between peak pressure and shock velocity and between peak pressure and particle velocity are very insensitive to variations in density behind the front, while the terms in equations 4-7 containing other parameters are small. We discuss pressures above and below 100 atmospheres separately.

Within a few charge radii from the surface we have the rough approximation

$$\frac{\partial \rho}{\partial r} - \frac{\rho}{2\rho_o}\frac{DP}{DR} = \frac{2(P - p_o)}{R} \sim 4\ P/R$$

Considering a better approximation, $\frac{D \ln(\rho - \rho_o)}{D \ln(P - p_o)}$ varies from 0 at the charge surface to 1/3 at 100 atmospheres pressure, and can be neglected in comparison with ρ/ρ_o over this range, while p_o is negligible compared to P. Results of computations from the expression

$$\frac{1}{P}\frac{\partial \rho}{\partial r} = -\frac{\rho/2\rho_o + P/(\rho - \rho_o)a^2\ \frac{1}{P}\frac{DP}{DR} + 2/R}{1 - \frac{\rho_o P}{\rho(\rho - \rho_o)a^2}} \quad (4a)$$

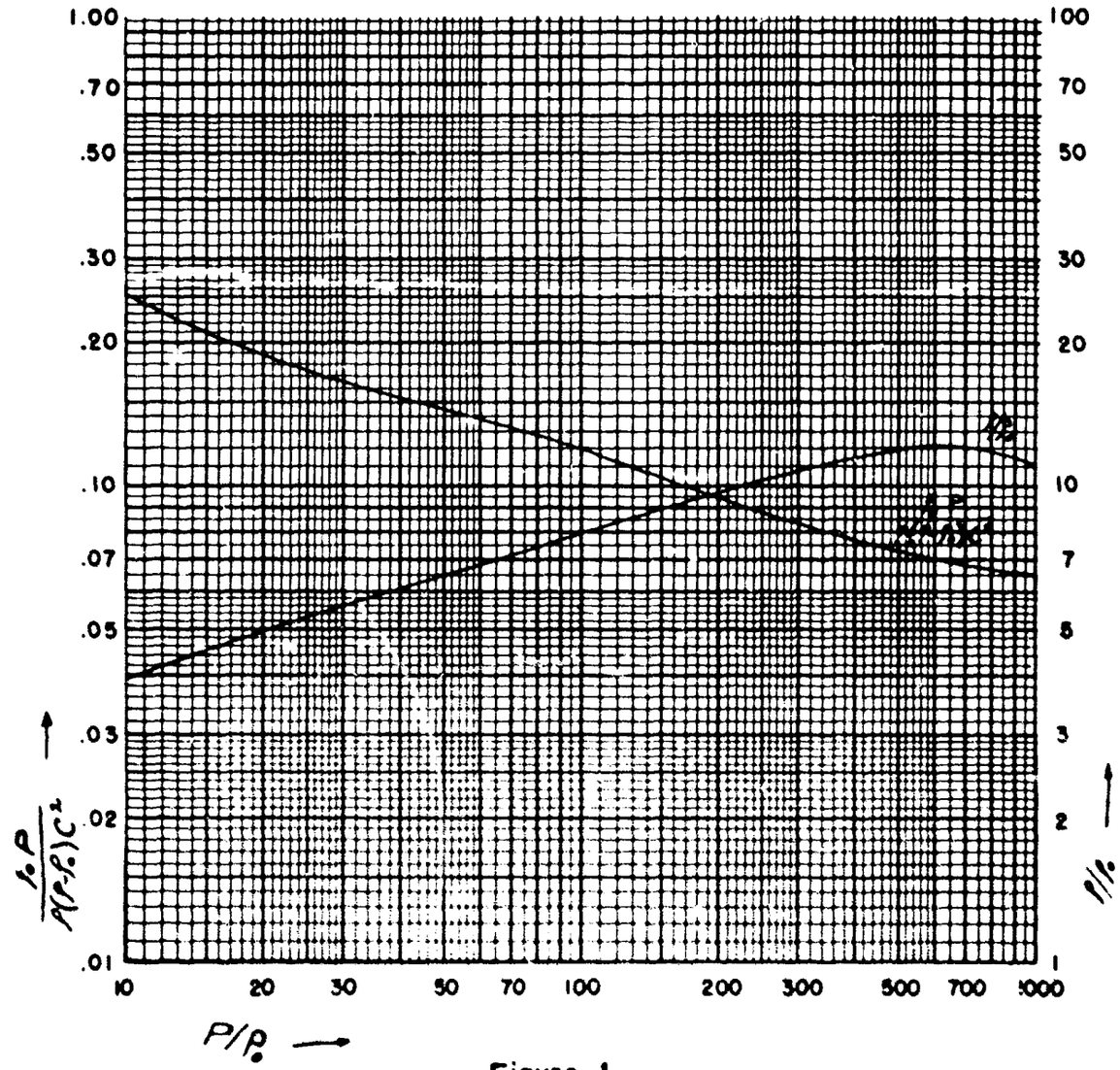

Figure 1.

Properties of air at the shock front, from Kirkwood - Brinkley tables.

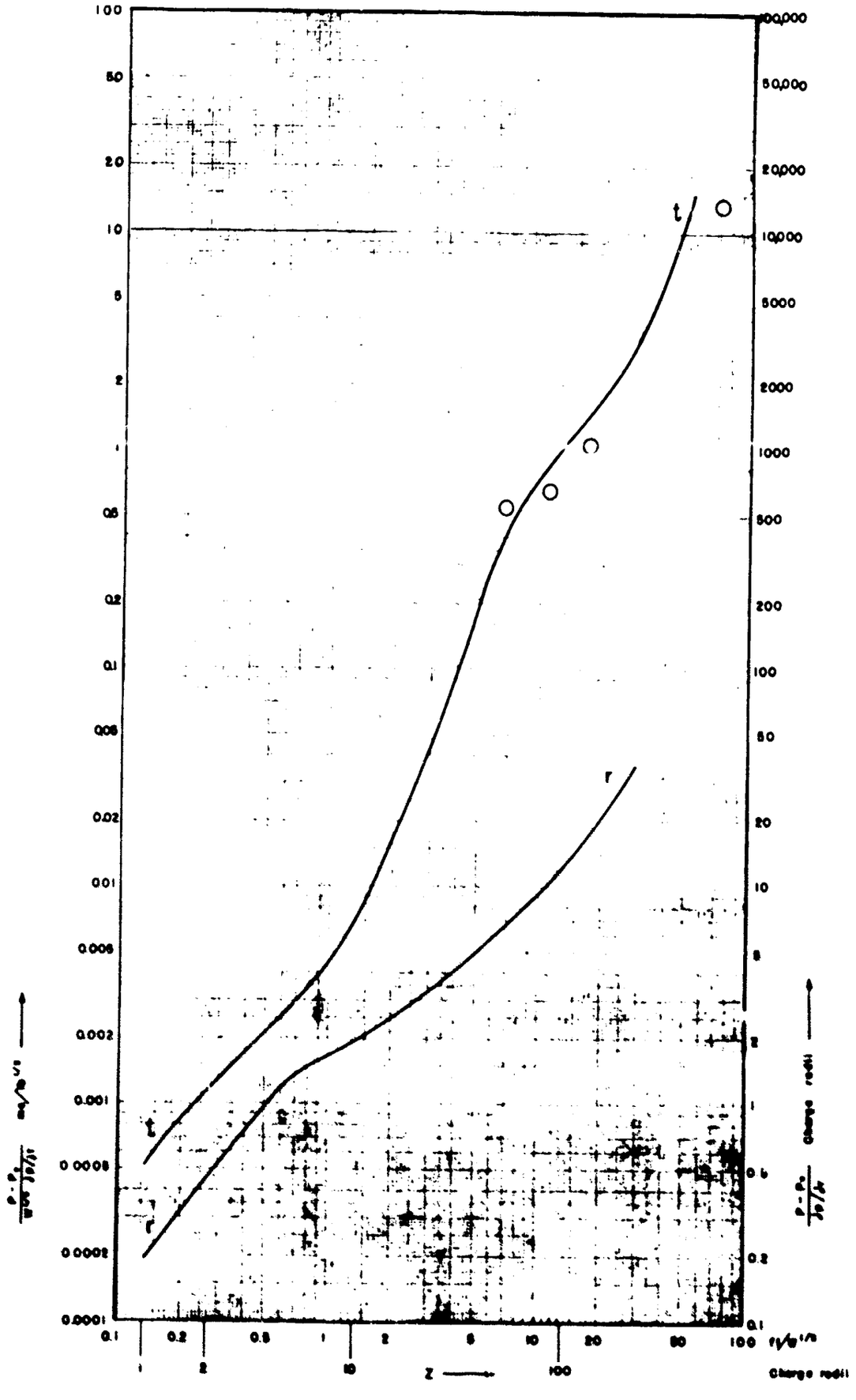

Figure 2

Rates of change of pressure at the shock front, computed from decay curve. Points experimental.

are shown in Figure 2. The slopes are given in charge radii for better visualization of the shock. $P/\frac{\partial p}{\partial r}$ is the distance in charge radii from the front to the intersection of the tangent to the pressure distance curve at the front and the line of $P = p_o$. It is interesting that this distance remains about 1/5 the distance between the front and the center of the charge throughout the range of these measurements. $-\frac{1}{P}\frac{\partial p}{\partial t}$ is computed from $1/P\ \partial p/\partial r$, using equation (1).

At pressures less than 100 atomospheres the perfect gas approximation can be used. For a gas of specific heat ratio γ, setting $(P-p_o)/p_o = y$

$$W^{1/3}\frac{1}{(p-p_o)}\frac{\partial p}{\partial r} = \frac{1}{\gamma+1}\frac{(\gamma+1)(2\gamma-1)y^2+\gamma(9\gamma+1)y+8\gamma^2}{(\gamma-1)y+2\gamma y^2}\frac{Dy}{DZ} + 2\frac{\alpha\gamma(y+1)}{yZ} \qquad (9)$$
$$- \frac{(\gamma+1)(2\gamma-1)y^2+\gamma(9\gamma+1)y+8\gamma^2}{(\gamma+1)\left[(\gamma-1)y+2\gamma\right]Zy^2}\frac{D(Z^{\alpha/2}y)}{DZ} - \frac{\alpha}{2}\frac{(2\gamma^2-5\gamma+1)y+\gamma(3\gamma-5)}{(\gamma+1)\left[(\gamma-1)y+2\gamma\right]Z}$$

$$\frac{1}{p-p_o}\frac{\partial p}{\partial t} = \frac{U}{(\gamma+1)\left[(\gamma-1)y+2\gamma\right]R}\left[\frac{(\gamma+1)(3\gamma-2)y^2+\gamma(11\gamma+3)y+8\gamma^2}{y^2}\frac{D(Z^{\alpha/2}y)}{DZ}\right. \qquad (10)$$
$$\left. + \frac{\alpha}{2}\left(\gamma^2-5\gamma+2\right)y+\gamma(\gamma-7)\right]$$

The second form is obviously convenient for the use of equation (8). Results of this computation for air ($\gamma = 1.4$) are plotted in Figure 2. Equation (7) has been used to compare the observed slope of the pressure-time curve recorded oscillographically with the slope to be expected from the decay curve (peak pressure-distance relation). Agreement has usually been obtained when the shock was very long compared to the gage size. Good agreement cannot be expected with data on shocks close to small charges when large gages are used (the BRL gage and fractional pound spheres, for instance) since the gage gives an average pressure over a considerable part of the shock wave. A few points obtained in this way with 1/4 pound Pentolite charges are included in Figure 2.

It is obvious that the method used to obtain equation (4) can be iterated, differentiating the hydrodynamic equations, to obtain successive derivatives at the front. These in turn can be inserted in a Taylor's series in the distance from the front and the form of the pressure time or pressure distance relation behind the front obtained to the discontinuity in any derivative. This discontinuity would be expected to be the boundary between the explosive gases and air, at which even the first derivative of the pressure is discontinuous, unless a second shock has formed. Although theoretical discussion of the form of a spherical shock is difficult,[6] it is clear that a second spherical shock will form much more slowly than a second plane shock. As experimental studies on small charges have not been carried out with initiation

resulting in a spherical shock, the appearance of multiple shocks in pressure-time records cannot be regarded as evidence as to their formation from a detonation having spherical symmetry. The question of the region of continuity thus remains unanswered until further theoretical work has been done. It is doubtful that the experimental results are sufficiently accurate so that reliance can be placed on higher derivatives of equation (8) than the first, in any case. Computations of the pressure behind the shock front as a function of time by the method of characteristics are being made, using the Kirkwood-Brinkley peak pressure vs distance curve[7]. Similar computations will be made using the experimental curve, if further experimental work shows significant differences between the curves. The estimation of positive impulse from decay may prove more accurate than its experimental determination[3] close to charges.

Jane Dewey

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APPENDIX

Symbols used:

r = space variable, for a spherical shock the distance from the charge center

$R(t)$ = the value of r at the shock front

t = time

$p(r,t)$ = pressure

$P(R)$ = pressure at the front

ρ = density

$$a^2 = \frac{\partial p(\rho,S)}{\partial \rho}$$

S = entropy

$U(R)$ = shock velocity relative to gas in front of shock

$v(r,t)$ = material velocity relative to gas in front of shock

$u(R)$ = value of v at the front

α is defined by $\nabla \cdot = \frac{\partial}{\partial r} + \frac{\alpha}{r}$

W = charge weight

$$Z = R/W^{1/3}$$

$$y = (P - P_o)/P_o$$

γ = ratio of specific heats

The subscript o refers to the gas in front of shock, which is assumed to be uniform.

No subscripts are used for quantities just behind the front.

The one dimensional hydrodynamic equations are: in Eulerian coordinates

Continuity, $\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial r} + \rho \nabla \cdot v = 0$

Dynamical, $\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} + \frac{\partial p}{\partial r} = 0$

Energy, $\frac{\partial S}{\partial t} + v \frac{\partial S}{\partial r} = 0$

Hugoniot Conditions

A. For any material

$$\rho_o U = \rho(U - u)$$

$$\rho_o u U = P - p_o$$

Change in internal energy of material passing through the front is

$$\frac{1}{2} (P + p_o) \left(\frac{1}{\rho_o} - \frac{1}{\rho_1}\right)$$

B. For a perfect gas

$$a_o^2 = \gamma p_o / \rho_o$$

$$u^2 = 2a_o^2 y^2 / \gamma \left[(\gamma + 1) y + 2\gamma\right]$$

$$U^2 = a_o^2 \left[(\gamma + 1) y + 2\gamma\right] / 2\gamma$$

$$\rho = \rho_o \left[(\gamma + 1) y + 2\gamma\right] / \left[(\gamma - 1) y + 2\gamma\right]$$

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