

Report No. 4090266

ASW SONAR
TECHNOLOGY REPORT

INTERNAL WAVES: THEIR INFLUENCE
UPON NAVAL OPERATIONS

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CAMBRIDGE, MASSACHUSETTS

DEPARTMENT OF THE NAVY
BUREAU OF SHIPS

NObsr - 93055

Project Serial Number SF - 101 - 03 - 21,

Task 11353

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TABLE OF CONTENTS

	<u>Page</u>
List of Figures and Tables	iii
I. SUMMARY	1
II. INTRODUCTION	2
III. THE NATURE OF INTERNAL WAVES	3
A. GENERAL CHARACTERISTICS	3
B. THE TWO-LAYER MODEL	3
C. THE VARIABLE-DENSITY MODEL	6
D. OBSERVATIONS OF INTERNAL WAVES	7
E. THE GENERATION OF INTERNAL WAVES	9
IV. THE INFLUENCE OF INTERNAL WAVES UPON DEPTH- AND PRESSURE-SENSITIVE DEVICES	13
V. THE ACOUSTIC SIGNIFICANCE OF INTERNAL WAVES	16
A. SHORT-PERIOD WAVES	16
B. LONG-PERIOD WAVES	19
VI. CONCLUSIONS AND RECOMMENDATIONS	21
A. NATURE OF INTERNAL WAVES	21
B. NONACOUSTIC EFFECTS	21
C. ACOUSTIC EFFECTS	23
D. AREAS FOR FUTURE WORK	24
E. RECOMMENDED PROGRAMS	25
APPENDIX A - CURRENT INVESTIGATIONS	27

LIST OF FIGURES AND TABLES

<u>Figure No.</u>		<u>Page</u>
1	Sound Level in the Medium With No Internal Waves On the Thermocline	17
2	Sound Level in the Medium With an Internal Wave On the Thermocline	17

<u>Table No.</u>		
1	Effect of Range On Magnitude of Focusing Effect	18
2	Characteristics of Internal Waves Observed in Various Ocean Areas	22

I. SUMMARY

Internal waves, traveling beneath the ocean surface with maximum amplitude at or near the pycnocline, have been observed in various areas of the oceans. This report reviews what is known concerning the generation, prevalence, and characteristics of internal waves, and summarizes available knowledge concerning the effects of internal waves on naval operations.

Internal waves with wavelengths of tens of miles and heights up to 300 feet have been observed in both the Atlantic and Pacific Oceans. Tidal-period waves and short-period waves as high as 50 feet are observed at some points on the continental shelves and in coastal waters.

Internal waves affect naval operations in two principal areas--depth control and detection capability. The passage of large-amplitude internal waves could make submarine depth control difficult, particularly when the submarine is running quietly at low speed. Perturbations caused by the internal wave could initiate uncontrollable sinking of a submarine without power if the submarine were more compressible than sea water.

The detection capabilities of ASW systems are influenced by internal waves because the latter affect transmission of acoustic signals through the ocean. The principal effect in deep water is a significant change in sonar range during the few hours required for the passage of high-amplitude internal wave. In coastal waters, the fluctuations in sonar signal due to short-period waves are of an appropriate frequency to interfere with target holding. It appears that the magnitude of the fluctuations in one-way transmission between source and target may be as high as 10 db.

Utilization of the effects of internal waves on naval operations depends on the degree to which the pressure and characteristics of internal waves can be predicted at any given time and place. Programs aimed at providing this prediction capability are outlined in Section VI. Computer studies are also recommended to delineate more definitely the effects of internal waves on sonar detection capabilities.

II. INTRODUCTION

Oceanographers have been interested in internal waves for more than 100 years, but their study was not given great emphasis before World War II because the data was difficult to obtain and had no practical application. Following the war, electronic methods of monitoring the temperature in the ocean became available, and many investigations of internal waves were undertaken. Recently, the United States Navy, largely through the Bureau of Ships, has become concerned with the effect of internal waves upon naval operations. The extent of this concern is reflected in the fact that in the fiscal years 1964, 1965, and 1966, a total of some two million dollars was earmarked by BuShips alone for studies of internal waves, tides, and similar variations in the thermal structure of the oceans (BuShips Program Summaries: Oceanography, SR 004 03, May 1964 (Conf.)). During these years our knowledge of the phenomena has significantly increased, not only as a result of Navy-sponsored work, but also due to independent work undertaken in the oceanographic community.

The following sections discuss internal waves and their effect on naval operations. Section III describes the nature and characteristics of internal waves and discusses some simple mathematical models of the waves. An explanation of the geographic distribution of internal waves is also proposed.

Nonacoustic effects of internal waves are considered in Section IV, including the possible effects of such waves upon depth control in submarines and torpedoes. Some attention is also given to mines; internal waves may not only affect their mooring and depth control, but the pressure fluctuations associated with the passage of a wave may be detected by a sensitive pressure-activated mine.

Section V considers the effect of representative internal waves on acoustic transmission in the various sonar modes. The effect on long-range sonar transmissions and the possibility of hot spots and shadow zones at short ranges resulting from internal waves are discussed, and comments are made concerning the manner in which a sonar operator might observe internal waves.

The final section of this report considers naval applications of the internal wave phenomena and recommends programs necessary to obtain the required knowledge.

Although most of the existing literature has been consulted during the preparation of this report, only a few papers are cited. An up-to-date internal wave bibliography has been published under a separate cover.

III. THE NATURE OF INTERNAL WAVES

A. GENERAL CHARACTERISTICS

Internal waves are one of the many phenomena which cause variations in the structure of the water column below the sea surface. They are like surface waves in that (1) small elements of water travel in elliptical orbits, and (2) energy travels at the group velocity. The maximum amplitude of the internal wave disturbance occurs at or near the pycnocline, and only small amplitude variations occur at the ocean surface. Internal waves differ from turbulence in that the elements of water in a turbulent eddy undergo translation, and the energy is transported as kinetic energy of the elements at a speed set by the speed of the turbulent flow.

Internal waves vary over a wide spectrum; they range from short-period stability oscillations with periods of 1 to 2 minutes and amplitudes of centimeters to long-period waves with periods of days or weeks and amplitudes of hundreds of meters. The temporal and spatial occurrence of internal waves is uncertain, but diurnal and semi-diurnal waves are commonly observed along the continental shelves, and short-period waves (1 minute to 1 hour) have frequently been observed close to the coast.

B. THE TWO-LAYER MODEL

Simple models of the ocean can provide some insight into the mechanisms of internal-wave propagation. Internal waves cannot exist in a homogeneous fluid; therefore a two-layer model is the simplest in which internal waves may exist. Consider two homogeneous layers of nonviscous, incompressible liquids of different densities on a nonrotating earth bounded on the bottom by a rigid boundary and on the top by a free surface. This is a good approximation to the ocean, where the thermocline separates two nearly homogeneous layers. The internal waves in this model have their maximum amplitude at the interface and a phase velocity given by

$$c^2 = \frac{g}{k} \frac{\rho - \rho'}{\rho \coth kh + \rho' \coth kh'}$$

The amplitude at the free surface is related to the amplitude at the interface by

$$\eta = \frac{-\eta_o}{\cosh kh' - (g/c^2 k) \sinh kh'}$$

where

c = phase velocity

g = acceleration of gravity

k = wavenumber (reciprocal of the wavelength)

ρ = density of the lower layer

ρ' = density of the upper layer

h = thickness of the lower layer

h' = thickness of the upper layer

η = amplitude at the free surface

η_o = amplitude at the boundary

These expressions are derived in Lamb (1932), p. 370.

Introducing the short-wave approximation (wavelength small compared to the thickness of the lower layer; that is, $kh \gg 1$) we arrive at

and

$$c^2 = \left[\frac{\rho - \rho'}{\rho \coth kh' + \rho'} \right] \frac{g}{k}$$

$$\eta = -\eta_o \left[\frac{\rho - \rho'}{\rho'} \right] e^{-kh'}$$

Short-wave Approximation
($kh \gg 1$)

If the wavelength is also small with respect to the thickness of the upper layer,

and

$$c^2 = \frac{g}{k} \left[\frac{\rho - \rho'}{\rho + \rho'} \right]$$

$$\eta = 0$$

} Infinitely Thick Layers Approximation
($kh \gg 1$,
 $kh' \gg 1$)

If, on the other hand, the thickness of the upper layer is small with respect to the wavelength

and

$$c^2 = \left[\frac{\rho - \rho'}{\rho} \right] gh'$$

$$\eta = -\eta_0 \left[\frac{\rho - \rho'}{\rho'} \right]$$

} Thin Upper Layer Approximation
($kh \gg 1$,
 $kh' \ll 1$)

In the long-wave approximation (wavelength large with respect to total depth; that is, $k(h + h') \ll 1$) we obtain

and

$$c^2 = \frac{\rho - \rho'}{\rho} \frac{ghh'}{h + h'}$$

$$\eta = -\eta_0 \frac{\rho - \rho'}{\rho'}$$

} Long-wave Approximation
($k(h + h') \ll 1$)

In both the long- and short-wave approximations we find that the phase velocity of internal waves is less than the phase velocity of surface waves of the same wavenumber in the same location. The phase velocity of the internal waves decreases as the difference in densities decreases, because the potential energy associated with the wave decreases. Long waves are affected by the presence of both surface and bottom boundaries; short waves are affected only by the surface boundaries.

The calculated disturbance of the free surface is 180° out of phase with the internal wave at the interface. This surface disturbance has not been observed in the ocean, because its amplitude is of the order of 0.1% of the amplitude of the internal wave at the interface.

The two-layer model can explain many features of internal wave motions, but it does not explain dispersion or distortion.

C. THE VARIABLE-DENSITY MODEL

Love (1891), Fjeldstad (1933), Groen (1948), and Eckart (1960) have considered models in which the density varies continuously as a function of depth and in which the influences of the rotation of the earth are introduced. The upper boundary is taken as a free surface and the lower boundary as a rigid surface. This model yields many interesting phenomena not present in the two-layer model.

A high cutoff frequency known as the Brunt-Väisälä or stability frequency exists for a free progressive wave. This high cutoff frequency, N , is given by

$$N = \left[\frac{g}{\rho} \frac{d\rho}{dz} - \frac{g^2}{c^2} \right]^{1/2}$$

and may be thought of as the frequency of oscillation of a volume of fluid given a vertical displacement when the density varies with depth and the motion of the displaced volume does not alter the density distribution. In the upper 4000 meters of the ocean, the period associated with this high cutoff or stability frequency varies from 1 minute to 3 or 4 hours (Eckart, 1961). Therefore, the short-period internal waves present depend on the stability of the water column.

The inertial frequency (twice the angular frequency of the earth times the sine of the latitude) is the low cutoff frequency for free progressive waves. The inertial period is equal to one-half of a pendulum-day, which increases from 12 hours at the poles to 24 hours at 30° latitude and becomes infinite on the equator. Yasui (1960) has suggested that periods longer than the inertial period are possible if the density distribution is not horizontally homogeneous.

The variable-density model yields a normal mode solution. For each frequency there are an infinity of modes with discrete wavelengths, each having a distinct amplitude-depth behavior. The longest wavelength (zero mode) corresponds to the surface wave. The next longest wavelength (first mode) corresponds to an internal wave that has a single amplitude maximum in the water column. The second mode has two amplitude maxima, the third mode has three, and so on. Theoretically, an infinite number of modes may occur, but in practice most observations may be explained by using only the lower modes (Cox and Sandstrom, 1962; Fjeldstad, 1936; Defant, 1961; O. S. Lee, 1965). Higher order modes may not exist, because of the higher shear and frictional forces which would be present. However, complete modal analysis requires accurate measurement throughout the entire water column, and such data are extremely rare. Phase velocity depends upon the wavelength, so waves in different modes diverge even though they have the same periods. Thus the waveforms, as indicated by isotherms, change in time and space.

D. OBSERVATIONS OF INTERNAL WAVES

It is difficult to identify internal waves experimentally, because many phenomena occurring in the ocean have characteristics similar to those of internal waves. The rise and fall of an isotherm may be caused by an internal wave, but it may equally well result from horizontal motion of a sloping thermocline, upwelling, meanders of a current system, variations in the strength of a current, convection, or still other causes. More than one parameter must be monitored to establish the existence of an internal wave.

The determination of the characteristics of internal waves is also complicated by the many modes and frequencies which may be present and by other phenomena that cause variations in the water column. Wave height, length, and period are not easily identifiable and are meaningful only as statistical parameters. To specify internal waves in the ocean, one must obtain the directional energy spectrum related only to the internal waves or identify that portion of the energy spectrum due exclusively to internal waves. In determining the spectrum from discrete observations, the problems of aliasing and resolution are encountered. The sampling interval must be short enough to represent high-frequency components, and the record length must be long enough to resolve low-frequency components. In addition, the spacing of the sensors and the geometry and size of the array must be chosen to detect all wavelengths present and the directional resolving power required. If an omnidirectional array is not used, the array must be oriented so that the interpretation of the data is not confused.

The literature on internal waves contains very little information on the complete spectra of internal waves, due in part to the type of instrumentation used, in part to the duration of the observations, and in part to the fact that

individual investigators were interested in particular portions of the spectrum. Observations of internal waves fall into three groups--extratidal, tidal, and subtidal, depending on the periods which appear to dominate.

The few observations of internal waves with extratidal periods have all been of standing waves. O. Pettersson (1909) observed a standing internal wave with a period of 14 days in Gulmarfjord. Wedderburn (1909) calculated a period of 14.2 days for Gulmarfjord using a two-layer model with a 200-meter lower layer. Sverdrup (1940) interpreted the wave character of isobaric surfaces in the Gulf of California as a standing internal wave with three nodes inside the Gulf. His interpretation was supported by variation in the character of the sediment deposits (Revelle, 1939) and by a theoretical analysis (Munk, 1941) which revealed a 7-day period probably in resonance with the lunar-fortnightly period (13.6 days). Yasui (1961) reported an internal wave with an 80-day period at weather station "Tango" in the North Pacific south of Japan. This may not represent an internal wave but rather a nonpropagating perturbation associated with the Kuroshio current system. These exceptionally long periods may also suggest that the observations were of tidal motions of the second class, that is, planetary waves.

Observations of internal waves of tidal periods are numerous. Internal temperature and salinity fluctuations have been reported from anchor stations occupied during the early oceanographic expeditions such as the "Michael Sars," the "Goldseeker," the "Meteor," the "Dana," and the "Atlantis" in the Atlantic and the "Snellius" in the Indian Ocean. On all of these expeditions the water column was sampled every few hours for several days at discrete depths with water sampling bottles. Current measurements were also made in conjunction with the oceanographic stations on several of the expeditions, and the current data obtained established the observed fluctuations as internal waves. Tidal periods have been attributed to these fluctuations mostly by visual inspection.

Haurwitz (1954) has shown that the period of time over which these historic observations were made was much too short to establish statistically the existence of tidal periods. In most cases the probability that the observed variations could have occurred by chance in a randomly varying environment was greater than 1% and ranged as high as 70%; therefore, the possibility that no tidal period existed must be considered. However, the repeated observation of such phenomena increases their credibility.

Reid (1956) conducted a seven-day experiment off the coast of California which revealed the presence and dominance of internal waves of tidal periods at a station 40 miles offshore. Farther offshore Reid found that internal waves of tidal period no longer dominated; the spectrum contained no significant peaks, and periods ranged from 6 to 27 hours (including the inertial period). In three 24-hour multiple-ship surveys off the coast of southern

California, Summers and Emery (1963) observed internal waves of approximately semi-diurnal tidal period progressing shoreward. Cairns and LaFond (1965) have reported an internal tide with an amplitude five times as large as the surface tide in the area off Mission Beach near San Diego. Internal waves of tidal period have also been observed in the Gulf of Mexico (Boston, 1963 and 1961).

Observations of internal waves of subtidal periods are generally confined to waves with periods of less than one-half hour. Ufford (1947) reported a few occasions when internal waves with periods of from one to two hours were present. He also reported numerous observations of internal waves with 10- to 20-minute periods off Portsmouth, New Hampshire, and San Diego, California. Gaul (1961) reported internal waves with periods of several minutes at Texas Tower No. 4 off New York. Lee (1961) observed internal waves off Mission Beach with periods of 5 - 15 minutes, and LaFond (1959) reported internal waves with 2- to 20-minute periods at the same location.

E. THE GENERATION OF INTERNAL WAVES

To evaluate the significance of internal waves to the undersea warfare program, we will consider a typical spectrum of the oscillations in the water column. The spectra of thermal fluctuations and particle motion in the water column contain a background noise level of random or turbulent perturbations whose energy density decreases with increasing frequency.

The nature of the background noise spectra varies from place to place, with depth, and with time. Internal waves of many different frequencies (tidal, inertial, stability, local resonances in semi-enclosed basins, etc.) may be present all or most of the time in all parts of the ocean but may be indistinguishable from other irregular temperature fluctuations unless great care is taken in planning an experiment and collecting data. At some locations, internal waves of particular frequencies have sufficient energy to be distinguished above the background noise and determine the structure of the water column. The problem is one of determining where and when internal waves of given frequencies are likely to have this control over the water column.

The geographic distribution of internal waves can be explained in terms of the generating mechanisms, which include tidal currents, bottom topography interaction, hydrodynamic perturbations in current systems, meteorological disturbances, and mechanical disturbances. The relative importance of each of these mechanisms in the generation of internal waves would be expected to vary with time and space (Haurwitz, Stommel, and Munk, 1959; Cairns and LaFond, 1965; Car, 1965; Defant, 1961; Frassetto, 1960; Gaul, 1961).

Although various theories have been advanced to explain the mechanism of generation of internal waves, none have been definitely proved. It is the opinion of the authors of this report that most of the observations of internal waves may be explained by a model in which internal waves of tidal periods are generated at the seaward edge of the continental shelf by the action of tidal currents. Generation of internal waves of tidal periods through an interaction with the surface tide has been derived theoretically by Rattray (1960) and Cox and Sandstrom (1962). The internal tidal wave propagates both seaward and shoreward from the generating area and dissipates as it proceeds. Rattray (1957) has examined the dissipation of long internal waves theoretically and calculated that the amplitude of long internal waves decreases to 50% at a distance of 120 to 15,000 km from the point of generation, depending on wavenumber and damping coefficient.

The internal tidal wave that propagates seaward shows signs of dissipation in the open sea. The wave propagating shoreward slows down as it enters shallower water, and its wavelength decreases. This decrease in wavelength has been observed by Summers and Emery (1963) off southern California. The internal tidal wave causes significant disturbances in the water column in shallow water, in all likelihood dissipating part of its energy in the form of higher frequency waves, with the energy spectrum peaking at the frequencies of stability oscillations. These short-period stability waves generated by the internal tide would then be expected to propagate both seaward and shoreward and dissipate their energy rapidly.*

Rossby (1938 a & b), Cahn (1945), and Bolin (1953) have shown theoretically that perturbations in a current generate internal waves. The internal waves have stability and inertial periods and propagate away from the area of the disturbance. These hydrodynamically generated internal waves are important in areas of strong currents but do not affect the water column far from the current.

Sandstrom (1908) demonstrated that meteorological disturbances on the sea surface could generate internal waves. Defant (1961) observed an internal wave on Meteor Station 254 which he attributed to a meteorological disturbance. Veronis and Stommel (1956) calculated the effect of moving meteorological systems on a two-layer ocean; they calculated that the internal wave period would be the inertial period. In the ocean, where the density varies continuously with depth, the internal waves would have energy peaks at the inertial and stability frequencies. Meteorologically generated internal waves are important only

*The mechanism of generation of stability oscillations is unknown but may be in some manner similar to the mechanism of conversion of surface to internal waves as treated by the following authors: Cox, 1959; Cox and Sandstrom, 1962; Eckart, 1961; Rattray, 1959, 1960.

in areas of strong meteorological disturbances, such as along storm tracks. Storm tracks and meteorological disturbances vary seasonally, as do the area and importance of meteorologically generated internal waves.

Ships and seismic disturbances can generate internal waves mechanically. Ekman (1906) showed both theoretically and experimentally that slow-moving ships generated internal waves in areas where a strong pycnocline is located below a thin surface layer. Ships are not a major source of internal waves, however, because areas with the proper conditions are quite localized and the speed of the ship must be less than a critical value, which is itself quite small.

Wong (1965) describes the mechanical generation of internal waves by underwater seismic disturbances. In areas of high seismic activity, such as the Gulf of Alaska (Robinson and Northrup, 1965), major shocks, earthquakes, or slumping may generate many small-amplitude internal waves. The periods of these waves are likely to be the stability periods (Brunt-Väisälä frequency) and the inertial period, as the system is most easily excited at these frequencies. Wave patterns and direction of propagation are confused within the generating area but become more regular away from the generating area as the waves progress outward. However, except in areas of high seismic activity, seismically generated internal waves do not constitute a major source of internal waves.

If we neglect areas of strong currents and high seismic activity, where the internal waves are likely to be hydrodynamically or mechanically generated, it is convenient to divide the oceans into three areas (open sea, continental shelf, and coastal) on the basis of the generation of internal waves by tidal currents. It should be noted that the generation of internal waves by surface tides has not been proven and is not unanimously accepted by the oceanographic community, but it is the most widely accepted theory.

The area seaward of the point on the continental shelf where the internal tidal waves are generated will be called the open sea. From this point shoreward to the region where the stability waves are generated will be called the continental shelf, and from this region to the shore will be called the coastal area. In the open sea, internal waves of tidal and inertial periods are generated either on the edge of the continental shelf or by impulsive disturbances. The amplitude of the internal tidal waves depends on their original amplitude, the damping coefficient, and interference. In the Pacific Ocean remote from the continental shelf, Rudnick and Cochrane (1951) and Reid (1956) were unable to distinguish internal waves of tidal periods from the background noise. In the Atlantic Ocean, on the other hand, although the measurements are not of sufficient duration to establish the definite existence of tidal periods, the evidence suggests that internal waves of tidal periods strongly influence the structure of the water column.

On the continental shelves (as defined on an earlier page) internal tidal waves progress shoreward, and short-period internal waves progress seaward. No measurements of the direction of propagation of short-period internal waves have been reported; however, short-period internal waves have been observed to progress shoreward in the coastal region (LaFond, 1959; Lee, 1960; and Gaul, 1961). The short-period waves are not coherent over long distances because of their diverse origins and interference with each other and are damped quickly; thus fluctuations in the structure of the water column on the continental shelf are dominated by tidal-period internal waves.

Much of the energy of the internal tidal waves that enter coastal regions is believed to be converted to short-period internal waves which dominate the fluctuations of the water column in this region much of the time. The generation of short-period internal waves depends on local conditions; their periods depend on the local stratification at the time of generation, and their propagation depends on local topography. Some band slicks on the sea surface are caused by internal waves. From these slicks it has been inferred (Ewing, 1950; LaFond, 1959) that the internal waves are refracted much like surface waves, but are not reflected.

In areas of strong current systems, hydrodynamically generated internal waves of stability and inertial periods dominate the variations in the water column. The direction of propagation and spatial coherence of these waves vary because of the impulsive nature of their generation. Turbulent eddies are also common in areas of strong current systems.

Internal waves of inertial and stability periods occur along storm tracks following the passage of storms (Haurwitz, Stommel, and Munk, 1959). Their direction of propagation would vary with the storm and they would probably be coherent only over short distances, because they would be generated over a long irregular front or line source and thus would be expected to be out of phase.

Seismically generated internal waves of stability and inertial periods dominate the variation in areas of high seismic activity. The variations at any point are confused by the presence of waves of different origins. Spatial coherence is limited, and the direction of propagation varies.

IV. THE INFLUENCE OF INTERNAL WAVES UPON DEPTH- AND PRESSURE-SENSITIVE DEVICES

The Navy uses a number of submerged systems that are designed to sense pressure and density and to use these parameters to position themselves or to activate certain components. The principal devices in this category are submarines, torpedoes, mines, and moored arrays or buoys.

The effect of internal waves on the control of submarine depth has been informally discussed, but little has appeared in print on the subject. There are two ways in which the wave could affect a submarine's depth. One is by changing the buoyancy of the vessel if the water temperature and density should suddenly change. A sudden temperature change of 5°C would be considered large under normal circumstances, for it causes the density of sea water to change by about 0.1%.

As a submarine is nearly neutrally buoyant, its total average density is about 1 gm/cm³. The buoyant force (positive or negative) exerted on a vessel as a result of a 5°C temperature change would thus be about 1 dyne/cm³, giving the vessel an acceleration of approximately 1 cm/sec², neglecting drag. A submarine under way could easily correct for the effect; however, this slight acceleration could cause serious problems for a submarine without power, hovering, or moving at very slow speeds. Under these conditions it would not be possible to trim the vessel by the diving planes alone. It would be necessary to blow ballast, a difficult process during power failure and undesirable during quiet running. If the vessel were more compressible than sea water, the acceleration would be accentuated as the vessel's density responded to changes in hydrostatic pressure. Vertical displacement could thus reach serious proportions unless promptly controlled. As the vessel reached thermal equilibrium with its new environment, its buoyancy would change in the direction tending to restore the original position and would thus have a damping effect on the influence of the internal wave.

The other possibility is that the submarine might be caught in a strong vertical current. If this current were caused by an underwater disturbance such as an explosion, landslide, or volcanic activity, it might be expected to reach serious proportions. It is also known that unusual internal wave phenomena occur at the edges of strong currents such as the Gulf Stream and Kuroshio and also in certain straits such as Gibraltar and Malacca. However, an ordinary internal wave moves at such a slow speed and has such a large ratio of wavelength to amplitude that the vertical motion of any region of water within the wave is of extremely low velocity; it would be well within the capability of a submarine commander to correct for any depth change caused by this slight motion. Again, it would be necessary to compensate for any buoyancy change brought about by changes in position and pressure.

At this point it is well to set up a model of an internal wave situation which may be considered typical of a rather pronounced wave. We will treat a reasonable exaggeration of the phenomena so that any effects will be amplified. As implied in a preceding section, real waves may be approximated by two superimposed sinusoidal waves, one of low frequency and high amplitude, and the other of high frequency and low amplitude. We will assume the first wave component to be of tidal frequency (12-hour period) and, in a two-layer ocean of $\Delta T = 10^{\circ}\text{C}$, to have an amplitude of 200 feet peak-to-peak. The second component will have a period of 10 minutes and an amplitude of 10 feet peak-to-peak. A 10°C temperature difference corresponds to roughly a 0.2% change in sea water density.

One should keep in mind that a second-order effect occurs here: the pressure change due to passage of an internal wave changes the density of the water beneath it. However, the compressibility of sea water is so slight that under these circumstances it may safely be neglected.

The short-period wave of 10-foot amplitude will have only slight influence on a moving torpedo. The degree of vertical motion is slight, and the changes in pressure caused by passage of a wave are negligible. The larger waves might, however, be troublesome. If a torpedo were keyed onto a particular density level rather than onto a pressure level, it might travel up or down a sloping isotherm as it proceeded across an internal wave of tidal period. Assuming a phase velocity for the wave of 1 knot, the wavelength would be 12 miles; in that distance a torpedo could change its depth by as much as 200 feet if guided by density alone.

The positions of mines which have positive flotation and are moored to the bottom will not be appreciably affected by the presence of internal waves. Neither the pressure nor density changes will be of sufficient magnitude to alter the vertical position of the mines. The velocity of these waves is so slight, at most about one knot, that horizontal displacement should also be slight.

Mines which are designed to be neutrally buoyant at a given density, however, will be affected. As the wave progresses, regions of constant density will rise and fall with the frequency of the wave. A mine under such circumstances will experience a pressure drop when the internal wave rises and a pressure increase as the wave descends. If the mine were to be pressure-activated, it would also have to be programmed to ignore pressure changes of the magnitude and frequency caused by the action of such waves.

Mines designed to remain at a constant pressure level would find their positions scarcely changed by the passage of an internal wave. Pressure fluctuation would also be minor. However, in designing the mechanism for maintaining the mine at constant pressure, two things must be taken into consideration. First,

the mine must be capable of changing its density rapidly enough to keep pace with the changes brought about by the passing wave; second, if such waves were to continue for some time, or for the life of the mission, it would be necessary for the mine to constantly readjust its density. If this were done, for example by automatic valving and venting from a cylinder of compressed gas, such constant adjustment of density might soon expend the gas supply unless allowances were made for such a condition.

An internal wave could affect a mine in still another way. If the mine were moored or otherwise fixed in position, passage of a wave could cause a pressure drop due to the Bernoulli effect alone. The formula for the pressure drop caused by the passage of an internal wave in a liquid over a smooth surface has been calculated by Eckart (1961) to be $\Delta P = \rho V u$ in which ρ is liquid density, V is phase velocity, and u is particle velocity. For an internal wave in sea water moving at a phase velocity of 2 knots, the particle velocity might approach ± 1 knot; from the above equation we would expect a pressure change of about ± 2 inches of water. This drop would occur only at the edges or sides of the mine which were parallel with the direction of wave motion. Pressure at other parts of the mine would vary with position, turbulence, etc. If the mine were restrained from rotating, the pressure fluctuations would occur at the same frequency as the internal wave.

The nonacoustic effects of internal waves on the operations of moored arrays and buoys is much the same as their effect upon fixed mines. It is doubtful if troublesome conditions of turbulence would occur about transducers or hydrophones, but the pressure changes should be considered.

V. THE ACOUSTIC SIGNIFICANCE OF INTERNAL WAVES

In practice, it is difficult to separate the acoustic effects of internal waves from similar effects which could be caused by other sources such as the thermal and density microstructures. Although unequivocal and quantitative data are scarce, there is good reason to believe that some observed types of internal waves have a significant effect on the propagation of acoustic signals in the ocean and, therefore, on the detection capabilities of ASW systems. Signal propagation is affected by the local rising and falling of the thermocline and by the wave-induced curvature of the thermocline. Also, breaking of internal waves and the movement of the scattering layer affect the reverberation level.

The effect of an internal wave on the detection capability of a sonar system depends mainly on the period of the internal wave and the sonar mode used. Direct short-range transmission between surface transducers is nearly independent of internal waves of any period, since little of the acoustic energy being transmitted passes through the region of the ocean which is strongly affected by internal waves. When both transducers are in or below the thermocline, the effect would be somewhat greater; here, especially in the thermocline, the temperature gradient is greater than above it, and the effect of the internal wave is accentuated. For other modes of transmission in which energy passes through the thermocline, the period is important. Two types of internal waves will be considered: 1) short-period waves with periods of a few minutes to a few hours and 2) long-period waves with tidal or extratidal periods.

A. SHORT-PERIOD WAVES

Most of the research effort on the acoustic effects of internal waves has been concerned with short-period waves, which appear to occur mostly on the continental shelves and in the very shallow water of coastal areas. Some recent studies indicate the magnitude of the expected effects.

Barkhatov and Cherkashin (1962, 1963) conducted laboratory-scale experiments with a two-layer fluid system. The interface between the fluids was excited to produce a quasi-periodic traveling wave. The intensity of an acoustic signal transmitted through the interface fluctuated by as much as 14 db within one internal wavelength. They reported that backscatter could be measured, the ratio of the acoustic pressure of the scattered field to that incident on the scattering surface being of the order of 10^{-3} to 10^{-4} .

Lee and Baltzer (1964) measured fluctuations in acoustic intensity of up to 10 db off the coast of southern California. Their hydrophone was placed near the bottom in 60 feet of water and was ensonified by a 92-kc source located

near the surface 500 feet away. Concurrent measurements of the vertical positions of isotherms at three points were used to characterize the internal wave structure. Fluctuations in acoustic intensity were directly related to the passage of the internal waves. In isothermal water, fluctuations of 4 db were observed. In this case, no measure of internal wave structure, if in fact internal waves were present, could be inferred.

In both of these studies, the range was of the order of one internal wavelength. For practical purposes, the effects at much larger ranges are important. While no data are available, an analytical study by Lee sheds some light on these effects. Lee (1961) mapped the acoustic intensity as a function of depth and range in a three-layered medium ensonified by a projector in the upper layer. The interfaces between layers were taken to be sinusoids with a wavelength of 300 feet and an amplitude of approximately 9 feet, these parameters being crudely representative of internal waves observed in shallow water off the California coast. Figures 1 and 2 are reproduced from Lee's work and show his results.

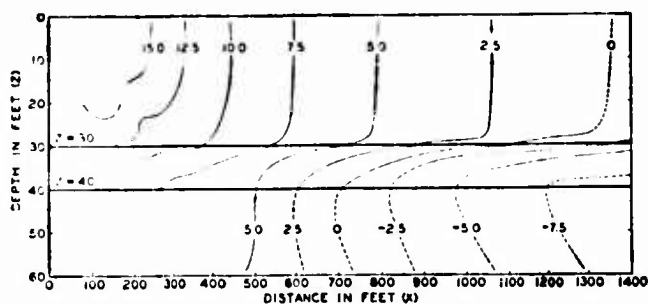


Fig. 1 Sound level in the medium with no internal waves on the thermocline. The db reference level is that corresponding to a sound level of 60 db at 1 ft from the directional source ($X = 0, Z = 10$) along the horizontal. The field is contoured at intervals of 2.5 db. Dashed lines indicate isolines that were smoothed by sight.

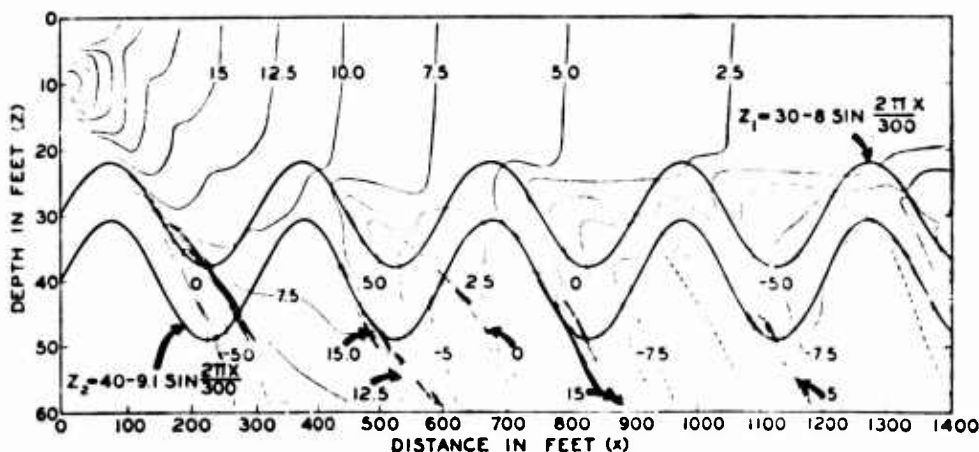


Fig. 2 Sound level in the medium with an internal wave on the thermocline. The db reference level is that corresponding to a sound level of 60 db at 1 ft from the directional source ($X = 0, Z = 10$) along the horizontal. The field is contoured at intervals of 2.5 db. Dashed lines indicate isolines that were smoothed by sight.

Consider the region below the thermocline. With no internal waves (Figure 1), the acoustic intensity decreases monotonically with range. In Figure 2, the focusing-defocusing effect of the internal wave is evident. Beneath the thermocline, the sound level varies periodically with range, with both the peaks and troughs of level appearing near the troughs of the internal wave. A rough picture of the effect of range on the magnitude of this focusing-defocusing effect can be obtained from the figure. Table 1 shows the differences between maximum and minimum sound levels occurring within the stated range intervals at a depth of 55 feet.

TABLE 1

EFFECT OF RANGE ON MAGNITUDE OF FOCUSING EFFECT

<u>Range Interval (ft)</u>	<u>Peak-to-Peak Fluctuation (db)</u>
200 to 500	20
500 to 800	20
800 to 1100	22.5
1100 to 1400	12.5

While the calculations cover a limited range, it seems that the peak-to-peak fluctuation decreases with increasing range.

The calculated peak-to-peak fluctuation at a range of 500 feet is about 20 db, while that measured by Lee and Baltzer (1964) under apparently similar conditions is 10 db. The real internal wave was certainly not a pure sinusoid, and the difference in values is probably due to the difference in waveforms. Superposition of high-frequency components in the internal wave would be expected to reduce the magnitude of the focusing-defocusing effect below that associated with a pure sinusoid of the principal frequency of the real wave.

The work cited above dealt only with acoustic transmission in the direction of wave propagation--that is, with acoustic paths which are normal to the ridge lines and trough lines of the traveling wave. The results show that appreciable fluctuation in acoustic intensity (or, equivalently, transmission loss) is associated with the presence of short-period internal waves. The magnitudes of fluctuation reported are for one-way transmission and would be doubled for active sonar systems. The frequency of the fluctuation of transmission loss between a fixed transmitter and receiver depends on the wavelength and velocity

of the internal wave. For a wavelength of 200 feet and a velocity of 0.3 knot, the elapsed time between a maximum and minimum is approximately five minutes. A sonar operator attempting to fix or track a target would be seriously hampered by large signal fluctuations on this time scale.

The effects of range and waveform on the magnitude of intensity fluctuations have not been determined in any detail. Present indications are that at ranges greater than several internal wavelengths, the fluctuations in one-way transmission will be less than 10 db and of an appropriate frequency to interfere with target holding.

No data have been published on acoustic transmission perpendicular to the direction of wave travel. However, if the internal wave is spatially coherent, no focusing-defocusing effect would be expected. In acoustic paths running parallel to the advancing wave fronts, the thermocline depth will be independent of range. The thermocline level will change with time in a quasi-periodic manner due to the passage of successive wave fronts, so that a sonar operator attempting to fix a target would experience signal fluctuation on the same time scale as in the previous case. The magnitude of the fluctuation depends on the mean level of the thermocline and the sensitivity of transmission loss to thermocline level, as well as on the amplitude of the internal waves. Another manner in which internal waves would affect sonar operation is by giving rise to inaccurate bearings on the target. In this manner the wave would have much the same effect as other horizontal "patchiness."

If the distance over which short-period internal waves are spatially coherent is short relative to the sonar range, signal fluctuations will not be quasi-periodic. Instead, the fluctuations will resemble noise.

The quasi-periodic or noisy fluctuations in acoustic transmission caused by internal waves might be used to advantage by a submarine commander entering coastal waters. He would need to know when and where internal waves are likely to occur, information which is not now available. Additional information on the magnitude of the fluctuations to be expected is also required.

LONG-PERIOD WAVES

Long-period waves, which appear to be prevalent in the Atlantic Ocean, are characterized by wavelengths of the order of 25 n.m., velocities of 2 to 7 knots, and amplitudes of up to 320 feet. These values indicate a spatial rate of change of thermocline level of approximately 0.004 ft/ft, and a temporal rate of change of 0.01 to 0.03 ft/sec. No data appear to be available on the acoustic effects of long-period waves, but some reasonable conjectures can be made.

With direct-path, short-range, bottom-bounce and variable-depth sonar, the radius of coverage is less than one internal wavelength, and the periodic nature of the wave would not influence the acoustic transmission pattern. Because of the small rate of spatial variation of thermocline depth, the transmission loss experienced by an acoustic signal passing between a fixed source and receiver is expected to be little different than if the internal wave were not present.

Appreciable changes in transmission loss due to temporal variations in thermocline depth will take place over a period of hours. Although the frequency is too low to cause signal fading within the time required for a target detection, this temporal variation will cause long-term changes in sonar range. The rate of change and, therefore, the frequency with which the oceanographic parameters needed for range prediction should be measured depend on the characteristics of the internal wave. At a stationary ship, the depth of the thermocline could change by as much as 40 to 120 feet in one hour.

Both the temporal and spatial variations of thermocline depth contribute to the changing sonic conditions experienced by a ship under way. The most rapid change in conditions will occur when the ship moves countercurrent to the wave; if it is traveling at 15 knots, the ship will pass from maximum to minimum in approximately 40 minutes. Revised range prediction estimates should be made whenever the expected change in thermocline depth at the ship exceeds the resolution of the prediction method.

The performance of convergence-zone sonar might be noticeably affected by long-period internal waves. Although the effects of slope and curvature of the thermocline at the points where the beam passes through should be small, the difference in thermocline depth between points of passage could be as much as 320 feet. It is not known whether such differences could cause changes in zone width and separation which are significant in comparison with those caused by other nonperiodic horizontal variations in the velocity structure.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. NATURE OF INTERNAL WAVES

Internal waves in deep water may be caused by perturbations in current systems and by meteorological or seismic disturbances. Tidal currents also generate internal waves and are the probable cause of most of the internal wave action over continental shelves and in coastal waters.

The available information on the existence of internal waves is summarized in Table 2 according to the nature of the ocean topography. The entries made in Table 2 are subject to the following qualifications:

- (1) Internal waves on the continental shelves and in coastal areas have been observed intensively at only a few specific points. Whether the findings are applicable to all areas of similar topography is not known.
- (2) Relatively little data from the open ocean is available, and much of it does not meet stringent statistical requirements with respect to internal wave identification and classification.

B. NONACOUSTIC EFFECTS

Internal waves have a negligible effect on the operation of buoys, torpedoes, mines, and submarines, with the following exceptions:

- (1) A submarine, hovering during a power failure, may suddenly find itself in a region where it is appreciably negatively buoyant during the passage of a large-amplitude internal wave. In this situation, the submarine might sink rapidly for tens of meters.
- (2) Neutrally buoyant mines or buoys which might be designed to maintain position by valving and venting gas from a compressed gas cylinder could exhaust their gas supply during passage of successive peaks and troughs of an internal wave.

TABLE 2

CHARACTERISTICS OF INTERNAL WAVES OBSERVED IN VARIOUS OCEAN AREAS

<u>Area</u>	<u>Type</u>	<u>Period</u>	<u>Directionality</u>	<u>Wavelength (km)</u>	<u>Velocity (m/sec)</u>	<u>Height (m)</u>
<u>Open Ocean</u> Atlantic	Short Tidal	2 min-2 hr Diurnal Semi-diurnal	Random { Confused Seaward	1-2 10-500	0.5-5 0.5-5	1-10 10-100
	Inertial	12-30 hr	Random	25-600	0.5-5	10-100
	Short Tidal	2 min-2 hr Diurnal Semi-diurnal	Random { Confused Seaward	1-2 10-500	0.5-5 0.5-5	1-10 <1-25
<u>Pacific</u>	Inertial	12-30 hr	Random	25-600	0.5-5	10-100
	Tidal	Diurnal Semi-diurnal	Shoreward	25-300	0.5-3.5	5-50
	Short	2 min-2 hr	Seaward	0.05-20	0.5-3.5	5-50
<u>Continental Shelves</u>	Short	2 min-2 hr	Shoreward	0.01-5	0.1-0.5	1-10
	Tidal	Diurnal Semi-diurnal	Shoreward	10-50	0.1-0.5	1-15
	Resonant	Minutes to days	Standing waves			Depends on size and shape of basin
<u>Coastal</u> Open	Short Tidal	2 min-2 hr Diurnal Semi-diurnal	Shoreward	0.01-5	0.1-0.5	1-10
	Short	2 min-2 hr	Shoreward	0.01-5	0.1-0.5	1-10
	Tidal	Diurnal Semi-diurnal	Shoreward	10-50	0.1-0.5	1-10
<u>Semi-enclosed Bays, Inlets, etc.</u>	Short	2 min-2 hr	Shoreward	0.01-5	0.1-0.5	1-10
	Tidal	Diurnal Semi-diurnal	Shoreward	10-50	0.1-0.5	1-10
	Resonant	Minutes to days	Standing waves			Depends on size and shape of basin

Notes: 1. This table is based to some degree on the definitions and theories put forward on earlier pages and therefore should not be read out of context.

2. Directionality refers to general direction of propagation; in semi-enclosed areas, such as the Gulf of Mexico, long-period internal waves may rotate around the basin or assume a confused pattern. In some cases the directionality given in the table has been inferred from existing data and from assumptions of reasonable models.

3. Wavelength depends on velocity and period. Values shown indicate the order of magnitude.

4. Wave height refers to crest-to-trough dimension. Values shown are ranges observed.

5. Existing data for the Pacific Open Ocean Area are not sufficient to indicate the prominence of internal tidal waves.

C. ACOUSTIC EFFECTS

The detection capabilities of sonar systems are affected by the presence of internal waves. The rising and falling of the thermocline as well as the time-varying slope and curvature imparted to the thermocline by internal waves change the patterns of signal propagation and result in variations in transmission loss. Sonar modes most affected are those which depend on acoustic transmission through the thermocline. In addition, internal waves will affect the reverberation level where waves are breaking or where the scattering layer undergoes significant vertical translation.

With our present state of knowledge of internal waves, only qualitative statements as to the magnitude of effects can be made:

- (1) Over continental shelves, short-period internal waves may cause a fairly high-amplitude (~10 db), quasi-periodic (~6 cph) variation in one-way transmission loss along acoustic paths in the direction of wave travel. Along acoustic paths perpendicular to the direction of wave travel, noise variations of uncertain amplitude are expected.
- (2) In deep water, the presence of high-amplitude, long-period internal waves may cause appreciable variations in sonar range over a period of a few hours due to large shifts in the local level of the thermocline.

It is interesting to consider what sort of useful information could be made available to the ASW commander if our knowledge of internal waves were increased. Detailed knowledge of the instantaneous wave patterns would be required to exploit the traveling shadow zones associated with the focusing and defocusing effect of internal waves. It is inconceivable that we could ever obtain this kind of knowledge of internal waves; therefore, the possibility of exploiting the focusing and defocusing effect is remote. If we had detailed statistical knowledge (for example, the energy, velocity, and directional spectra) of internal waves at any time and place in the ocean, we could refine the qualitative statements made in the preceding paragraph. We would be able to specify:

- (1) the spectrum of variations in sonar range to be expected at any given position, and
- (2) the change in the spectrum of variation in sonar range as a function of position.

Such information would undoubtedly shed light on the inexplicable departures of achieved sonar range from that predicted. Knowledge of areas in which acoustic propagation is likely to be confused by short-period internal waves would be useful to a submarine commander entering unfriendly coastal waters. The characteristics of long-period waves in deep water could be translated into charts showing how frequently an ASW commander should measure the local velocity structure to maintain up-to-date range predictions.

D. AREAS FOR FUTURE WORK

Two lines of approach seem warranted. It is and will be impossible to search any appreciable portion of the oceans for internal waves. Therefore, the degree to which the effects of internal waves on ASW operations can be included in strategic planning and tactical operations depends on the predictability of internal waves. This predictability is limited to the characteristics of quasi-periodic components and must rest on established cause and effect relationships or on verified empirical relationships between the characteristics of internal waves and easily observable ocean properties. Present knowledge in these areas is limited, although the literature hints at many effects which may be useful for prediction. Important questions which could be answered by careful analysis of internal wave data, already collected but mostly unreported, include the following:

- Do short-period internal waves at a given point occur in a fixed relationship to the tide?
- Does the relationship between the occurrence of short-period internal waves and the tide vary with location?
- Are the amplitudes of internal waves related to the amplitude of the tides?
- Do the periods of short-period internal waves vary with thermal stratification, either spatially or seasonally?

Whatever information is available concerning wave propagation should be combed from existing data and additional experiments performed as required to seek answers to these questions:

- How far do internal waves propagate?
- How do wave characteristics change during propagation?
- On what does the spatial coherence of internal waves depend? Does bottom topography play a major and predictable role in shallow water?

The investigation of existing data and acquisition of additional data should be conducted with the goal of constructing prediction models that can be used to forecast and describe the nature of internal waves in areas where it is not possible to make internal wave measurements. Priority should be given to experimental programs designed to test prediction models.

In addition, the analytical investigation of the acoustic effects of short-period waves should be continued. Computer simulations should be extended to media with more realistic velocity gradients and internal waveforms. The focusing-defocusing phenomenon, which affects direct transmission through the thermocline, should be studied to ascertain whether the fluctuations in transmission loss found at short ranges are also present with significant amplitude at or near the maximum detection range. The effect of internal waves on the short-range, bottom-bounce mode and transmission through the thermocline in the VDS mode should also be studied.

E. RECOMMENDED PROGRAMS

Programs aimed at the development of a prediction capability and further delineation of the acoustic effects of internal waves are recommended as follows:

I. Internal Wave Studies

(A) Basic studies to determine the mechanisms of internal wave generation.

- (1) An examination of all existing internal wave data in an effort to identify mechanisms.
- (2) Model testing to determine the mechanics of the implied mechanisms. This could be extended to cover nonlinear mechanisms if they proved feasible. More extensive tank studies would be valuable.
- (3) Field testing to evaluate the importance, or to establish the existence, of these mechanisms.

(B) Studies to determine empirical relationships for predicting internal waves.

- (1) Examination of existing data for empirical relationships and determination of the influence of local conditions on internal waves.

- (2) Collection of internal wave spectra from areas of strong current systems, high seismic activity, and along storm tracks. Appropriate data should be collected to permit complete model analysis of certain typical wave types or locations. An effort should also be made to observe short-period waves in deep water.
- (3) Development and testing of prediction models.

II. Acoustic Studies

- (A) Simulate acoustic transmissions through internal waves on computers using ray and normal theory and realistic waveforms for those sonar modes utilizing transmission through the thermocline.
- (B) Conduct model and field studies to verify the computer results. Field studies must include internal wave studies to be meaningful and, whenever possible, should be conducted in areas where instrumentation already exists. Test locations should be chosen to be representative of operating environments.
- (C) Investigate the effect of internal waves on other sonar parameters (directionality of ambient noise, reverberation level, etc.). Study, for example, the effect on sonar performance of conditions under which the deep scattering layer rises and falls with the passage of an internal wave.

APPENDIX A

CURRENT INVESTIGATIONS

Internal wave investigations have been and are currently being carried out at several laboratories by various investigators. These investigations include:

1. Navy Electronics Laboratory

E. C. LaFond and O. S. Lee - Studies of the spectrum of free internal waves and the general temperature structure of the Pacific.

2. Scripps Institution of Oceanography

J. L. Reid, Jr. - Investigations of tidal and inertial period internal waves on the continental shelf.

C. Cox - Theoretical investigations of the generation of internal waves.

M. Robinson - Investigations of the spectrum of variations using Flip.

3. University of Washington

M. Rattray, Jr. - Theoretical investigations of the generation and propagation of internal waves.

4. Pacific Naval Laboratory

Status of internal wave investigations

5. Fisheries Research Board of Canada

Regional studies along Canadian coast.

6. University of Kiel

W. Krauss - Spectrum of internal waves in the Baltic Sea.

7. Texas A & M

Roy Gaul working in conjunction with other laboratories around the Gulf of Mexico - Internal tide studies in the Gulf of Mexico.

8. National Institute of Oceanography
H. Charnock - Directional spectra of internal waves .
9. Japan Meteorological Agency
M. Yasui - Internal waves around Japan and theoretical studies of internal waves along oceanic fronts .
10. Southern California Industries
A. J. Carsola and others - Internal tides and spectra of short-period internal waves off California coast .
11. Massachusetts Institute of Technology
D. Halpern - Modes of internal waves in Cape Cod Bay .
12. USNUSL - Hasse, Barakos - Theory of acoustic interactions with internal waves .