SUBMARINE MAIN BALLAST TANKS - THEORY AND METHODS FOR REFINED STRUCTURAL DESIGN

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THEORY AND METHODS FOR REFINED STRUCTURAL DESIGN

by

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Opinions expressed are those of the authors, and do not necessarily represent the official view of the Naval Ship Engineering Center (NAVSEC), Bureau of Ships, or the Naval Service at Large.

U. S. S. PICKEREL (SS-524)

Surfacing with a 48° Up Angle from a Depth of 150 Feet

"...We want to bypass that reducer. But we are not quite ready to do it until we are sure we can make the ballast tank structure itself strong enough to take that immediate application...of pressure."*

*Brockett, Rear Adm. W. A., Chief, Bureau of Ships, Ref. (10), p. 36.

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SYNOPSIS

In a surfacing or emergency recovery maneuver, the safety of a submarine is dependent upon the structural integrity of its main ballast tanks. In addition to absorbing the cyclic effects of routine blowing over the life of the ship, in a casualty condition these tanks must sustain high and sudden air-blow pressures without rupturing. However, because of the weight criticality, the structure of these tanks must be efficiently designed.

Included is a discussion of the nature and history underlying the evolution of submarine ballast tanks, with reference to the Submarine Safety Program stemming from the loss of USS THRESHER. Theory and methods are developed for predicting the pressures applied to a ballast tank, and for the behavior of the tank structure under these pressures. Considered are variables of ballast tank and blow system configuration, initial transient loading and quasi-static pressures. Equations are developed to relate these variables and to determine the level of stress in any part of the tank structure, with theoretical development appended. General applications of these equations to analysis of a ballast tank are presented, with simplified curves and tables appended, and applicability to strutted sandwich shells other than ballast tanks indicated. Finally a complementary test program, established to confirm these equations and including large scale structural models and instrumented structural recovery trials, is briefly described.

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INTRODUCTION

The ability of a submarine to operate both under and on the surface of the ocean requires precise control over its displaced volume. Providing this control is the function of the ballast tanks. As shown on Figure 1, these tanks basically operate as follows: To submerge, vents in the top of the tanks are opened, thereby allowing air in the tanks to escape under the pressure of the sea water entering flood holes located in the tank bottom. To resurface, vents are closed and high pressure air is admitted to the tank to blow the ballast water back through the flood holes and out of the tank. For the water to flow out, the pressure inside of the tank must exceed the surrounding sea pressure.

Although differential blow pressures are relatively low in comparison with the tremendous hydrostatic pressure imposed on the pressure hull structure at operating depth, the internal tank pressure in an emergency ascent can be sufficiently high to stress the light weight ballast tank structure considerably beyond the threshold of yielding. Hence, because of its essential nature to ship operation and, in a casualty condition, to ship safety, the ability of the ballast tanks to sustain the imposed blowing loads without excessive straining or rupturing is of paramount importance.

It is the purpose of this paper to (1) investigate the order of magnitude of the ballast tank pressure and develop a method of predicting it, (2) derive a theory for accurately determining the accompanying response and level of stress throughout each component of the ballast tank structure, and (3) within the limits of security, to describe a comprehensive test program designed to confirm these equations. Although the effect of pressure hull deflections on the ballast tank have been included, solutions are proffered only for the ballast tank structure, as equations for the pressure hull have previously been published [(1)(2)].

HISTORICAL BACKGROUND

A brief insight into the historical background underlying the development of the submarine main ballast system is useful in establishing the importance of the main ballast tanks to submarine operation and safety.

Early Submarines

Any history of submarine development is incomplete without mention of the pioneering efforts of Lucullus, Leonardo da Vinci, William Bourne, and Cornelius van Drebbel [3]. Common to the submarine design of all of these individuals was the principle of changing the displaced volume of the craft by contracting or expanding a flexible leather hull.

The forerunner of today's submarine is often attributed to an Englishman named Symons who, in 1747, successfully built the first submarine which used water as ballast[3]. This ingenious craft employed large goat-skin flasks built inside the boat, and connected to openings in the bottom of the hull. The boat was submerged by untying the necks of the flasks, thus permitting them to fill with water. To resurface the craft, the water was squeezed out by twisting the flasks with a rod and retying the necks. Propulsion was achieved by oars.

To the principle of a water ballast system, David Bushnell of Connecticut, in 1775, added in his submarine, TURTLE, a sea chest which was provided with a manual pump for ejecting water ballast[4]. Lead ballast was also carried on the underside of the craft for use as an anchor, and this could be detached in an emergency.

Many of the submarines constructed subsequent to Bushnell's craft, including the NAUTILUS designed by Robert Fulton in 1801[5], and the submarine designed and built by an American named Phillips in 1851[6], carried compressed air tanks for replenishment of breathing air. Phillips' boat, as shown in Figure 2, contained many novel features and was streamlined for underwater performance. Of particular significance was the provision for an on-board air compressor. The compressor was manually operated to recharge the high pressure air banks, and to force the ship's air through the ballast water for purification. In addition, high pressure air was reportedly used[6] to blow bow and stern trim tanks automatically to keep the craft on an even keel. The reserve buoyancy for this boat was in excess of 100 percent, i.e., the ballast tank volume was sufficient to surface the craft with the interior completely flooded. Propulsion was effected by a hand driven crank shaft attached to the propeller.

The first use of compressed air for blowing ballast tanks is usually credited to the French submarine, LE PLONGEUR, launched in 1863 (Figure 3)[6]. This ship, designed by Brun and Bourgeois, carried large quantities of air stored at 180 psi, and used compressed air for propulsion. Unfortunately, air leakage and erratic submerged performance eventually condemned this design[4].

The Holland

The standard for the modern submarine was the familiar HOLLAND boat, designed and privately built by the J. P. Holland Company, and accepted by the U. S. Navy in 1900. As shown in Figure 4, the hull of this craft was shaped for underwater speed, as was Phillips' submarine. Displacing 75 tons, the HOLLAND employed compressed air for blowing water ballast from tanks carried inside the cigar shaped hull, similar to the location in Phillips' submarine. Air was stored at 2000 psi in flasks, also contained within the hull, and was reduced to 50 psi for services and to 10 psi differential pressure for ballast tank blowing. For emergency purposes this 10 psi reducer could be by-passed[7]. An air compressor was provided for recharging while underway on the surface. The reserve buoyancy of this boat was approximately 13 percent, and consequently in a surfaced condition the hull was almost awash. Power for propelling the craft was supplied by a gasoline engine on the surface, and by electric motor and rechargeable storage battery when submerged.

Subsequent U. S. Navy submarines of the A through K classes were essentially progressive developments from the HOLLAND with little change to the main ballast system. Beginning with the H class in 1913, gasoline engines were replaced with Diesels for surface propulsion.

With the longer range made possible by the more efficient Diesel engines, submarines of the L through S classes gradually evolved towards a submersible torpedo/gunboat. To provide the increased freeboard necessary for prolonged surfaced cruising and occasional deck gun action, the size of the main ballast tank system was gradually increased to produce a reserve buoyancy of about 20 percent. Although displacements were approaching 1000 tons for the S class, only slightly increased blowing pressures were provided.

WW II Submarines

The concept of the long range, high speed submersible torpedo/gunboat was fully realized by 1939 in the SARGO Class (SS 188), which had a displacement approaching 2000 tons and a test depth of 200 feet. The reserve buoyancy was approximately 25 percent and the ballast tanks were "wrapped" around the pressure hull in a saddle-like arrangement (Figure 5), with compressed air flasks stowed in the tanks. This tank arrangement was retained in the famed World War II "Fleet Boats" of the GATO (SS 212) and BALAO (SS 285) Classes which were also approximately 2000 tons displacement. However, test depths were increased to 400 feet during this period, while the reserve buoyancy was raised to approximately 35 percent.

The air system in these three classes provided for storage at 3000 psi, with reduction to a maximum of 15 psi differential pressure for the ballast blowing system[7]. No reducer was provided, as in the HOLLAND, but the system provided for throttling the air to the lower pressure, for blowing of the main ballast tanks.

Rapid advances in anti-submarine tactics, made possible by radar towards the end of World War II, required submarines to spend longer periods submerged to avoid detection. Hence, with submerged speed once again an important consideration, new submarine classes saw the beginning of a return to the HOLLAND concept of submarine operation.

The transition period, led by the snorkel breathing GUPPY and TANG (SS 563) classes, included the NAUTILUS (SSN 571) and SKATE (SS 578) classes and terminated with the USS TRITON (SSRN 586) in 1959. Although each of these classes was novel in some particular aspect, such as nuclear power for NAUTILUS or size for TRITON, the majority of the submarines built during this period were basically streamlined "fleet boats". High surfaced as well as submerged speeds were still desired, with the result that main ballast systems and tanks were much the same as on the WW II submarines.

Modern Submarines

With the successful application of nuclear power on USS NAUTILUS, submarines were no longer required to operate on or near the surface and could spend indefinite periods submerged. The marriage of this propulsion system to the hull form developed on the experimental submarine USS ALBACORE (AGSS 569) produced the prototype for the modern submarine in the SKIPJACK (SSN 585) class in 1959, from which followed the THRESHER (SSN 593) and STURGEON (SSN 637) classes.

With surface speed no longer a controlling operational requirement, the hull form once again could be designed for maximum performance submerged, as on the HOLLAND, with surface performance a secondary consideration. Accordingly, some significant changes were effected, as far as the main ballast system is concerned:

(1) Since these submarines would seldom operate on the surface, there was no need for either large freeboard or rapid submergence. Hence, the capacity of the main ballast tanks was reduced to a level between 12 and 15 percent.

(2) Higher submerged speeds necessitated use of much smaller flood holes with baffles, to eliminate undesirable resonance effects in the tanks.

(3) Smaller ballast tanks restricted the volume available for compressed air flasks, while increased demands on the air systems called for larger quantities of air which was still stowed in the tanks due to internal space limitations (Figure 1). This caused adoption of a 4500 psi air system.

(4) Operating depths were significantly increased below the previous 400 foot depth of the Fleet Boats[8][9].

The combination of smaller tanks, reduced flood hole sizes, increased hydrostatic pressures at depth, and adoption of 4500 psi air systems[7] generated new problems in MBT design. However, to maintain light weight ballast tank structure, air bank pressures were still reduced to obtain approximately 15 psi differential pressure in the tanks.

The Submarine Safety Program

As a result of the THRESHER disaster, the Bureau of Ships established a comprehensive program designed to review and, where necessary, improve all aspects of submarine design, construction, and operation with empha[illegible]n the safety of the ship[10].

An important result of this program has been a precise determination of the optimum recovery maneuver for each ship. Although a few controlled "emergency" surfacings had been run[10], such as conducted on USS PICKEREL (Figure 6), procedures to be followed in an actual emergency were left to the ship's discretion. Current procedures generally include planing to the surface at maximum achievable speed, and prompt and continuous blowing. Surfacing operations during a recent recovery maneuver are shown for USS NATHAN HALE in Figure 7. System modifications, permitting higher airflow rates, include bypassing all reducers, thereby blowing directly into the ballast tanks. The resulting rapid pressure increase, and the structural response thereto, will be examined herein, in detail.

PRESSURE LOADING

In determining the air pressure in a main ballast tank, there are two separate phenomena to be considered: the initial transient pressure resulting from the sudden release of high air bank pressures, and the volume expansion of the air bubble in the tank in transiting from the hydrostatic pressures at depth to atmospheric pressure at the surface. Calculation of the pressures incident to both of these phenomena can be simplified by making the following assumptions:

a. Air is a perfect gas.

b. The volume cross-section of the tank is uniform throughout. Thus, the volume of water in the tank is a linear function of the depth of ballast water.

c. The tank cross-section is considerably greater than the area of the flood holes. Thus, the velocity and acceleration of the water in the tank can be neglected, except in the vicinity of the flood holes[11].

Transient Pressure

Actuation of the emergency blow valve admits air at high pressure and flow rate directly into the ballast tank, and the maximum flow rate is achieved very quickly, varying principally with valve opening time. The mass of water in the flood hole accelerates more slowly and, until steady-state conditions are achieved, a transient overpressure exists in the tank which is dependent mainly on the rate of increase of the air flow, the stiffness of the tank boundaries, and the relative size of the mass to be accelerated.

As shown schematically in Figure 8, an analytical development for this transient pressure can be obtained from a simplified analog which is modeled as a single degree of freedom system by establishing the following analogies: the mass is the slug of water in the vicinity of the flood hole, and approximates the flood hole velocity; the spring is the flexibility of the tank boundary in the breathing mode; the damping is that provided by the flood hole as a sharp-edged orifice; and the exciting force is supplied by air pressure acting on the air-water interface.

For this system, the differential equation of motion is

$$\overline{m}\ddot{\overline{x}} + C\dot{\overline{x}}_2 \left| \dot{\overline{x}}_2 \right| + K^* (\overline{x}_2 - \overline{x}_1) = 0, \qquad \text{[B-5]*}$$

in which: $\overline{m}$ = mass of water slug in flood hole,

C = damping coefficient,

K^* = spring constant,

$\overline{x}_1$ = displacement of exciting force (ft), and

$\overline{x}_2$ = displacement of water slug (ft).**

(Dots denote successive differentiation with respect to time)

From the development in Appendix B, expressions are obtained for the above individual terms. Inserting these expressions into the above equation produces a second-order equation which is both non-linear and non-homogeneous (Equation [B-21]), and consequently is most readily solved by either digital or analog computer methods. By using this equation, it is possible to establish a sufficiently long valve opening time to keep transient pressures below those obtained from bubble expansion.

Expansion Pressure

A small bubble of air, once blown into a ballast tank at test depth, expands many times in volume as the submarine rises, since this bubble must reach atmospheric pressure at the surface. In this process of expansion, ballast water is forced out of

*Numbers in brackets refer to equations, and correspond to location of the equation in the Appendices.

**Complete Nomenclature appears after References.

the tank, through the flood holes. If this water could leave freely, the internal tank pressure would remain at ambient sea pressure. However, the flood holes normally present a considerable restriction to flow, such that flow velocity is achieved by establishing a pressure differential across the flood holes. This pressure differential is further increased by the addition of energy developed from air being continuously blown into the tank. Hence, the differential pressure is a function of water velocity, and thus of flood hole area, ship depth, air flow rate, and the air expansion rate, which in turn is a function of air volume and the rise rate of the tank.

With these assumptions and the development of Appendix B the pressure in the ballast tank can be expressed in terms of an effective rise rate, $\dot{z}_T$, which includes the effects of the actual ship rise rate, pitch rate, and the transfer of energy to the tank by the air blowing system, such that

$$\dot{Z}_T = \frac{-k^* W^* R^* T_s'}{\gamma_{sw} \bar{V}_A} + \left\{ \dot{Z}_{\mathfrak{L}} - [(Z_1 - \bar{h}_B - \bar{h}_W) \sin\theta^* + \bar{x}_A \cos\theta^*] \dot{\theta}^* \right\} , \quad \text{[B-27]}$$

in which, referring to Figure 10,

k = Specific heat ratio, $\frac{C_P}{C_V} = 1.4$ for air,

where: C_P = specific heat at constant pressure,

C_V = specific heat at constant volume,

W^* = Flow of air (lb/sec),

R^* = Universal gas constant (53.3 ft-lb/lb-^{o}R),

T_s' = Stagnation temperature of air (^{o}R),

γ_{sw} = Density of sea water (64 lb/ft^3),

$\bar{V}_A$ = Volume of air in tank (ft^3),

$Z_{\mathfrak{L}}$ = Depth to ship C.G. (ft),

Z_1 = Height of ship C.G. above baseline (ft),

$\bar{h}_B$ = Distance from baseline to top of flood hole (ft),

$\bar{h}_W$ = Height of blowable water in tank (ft),

θ^* = Pitch angle (radians), and

$\bar{x}_A$ = Axial distance from MBT to ship C.G. (ft).

The pressure inside the tank, p^*, can then be related to the equivalent rise rate by the following cubic equation:

$$(k^*)^2(p^*)^3 + \left[(2-k^*)k^* \gamma_{sw}\bar{h}_A - (k^*)^2 P_T^*\right](p^*)^2$$
$$+ \left[(1-2k^*)\gamma_{sw}\bar{h}_A - 2k^*P_T^*\right]\gamma_{sw}\bar{h}_A p^*$$
$$- \left[\gamma_{sw}\bar{h}_A + P_T^*\right]\gamma_{sw}^2 \bar{h}_A^2$$
$$= \frac{\gamma_{sw}^3}{2g} \cdot \left(\frac{\bar{V}_A}{C_D\bar{A}}\right)^2 \cdot \dot{z}_T^2, \qquad \text{[B-32]}$$

in which terms not previously defined are, referring to Figure 10:

p^* = Pressure in tank (lb/ft^2),

$\bar{h}_A$ = Vertical distance from tank top to water level (ft),

P_T^* = Ambient pressure outside top of tank (lb/ft^2 - abs.),

g = Acceleration of gravity (ft/sec^2),

C_D = Discharge coefficient, and

$\bar{A}$ = Area of flood hole opening (ft^2).

Hence, the differential pressure across the tank structure is merely the difference between internal and external pressure, or

$$\Delta p = \frac{(p^* - P_T^*)\ \text{lb/ft}^3}{144\ \text{in}^2/\text{ft}^2} \qquad \text{[B-33]}$$

Inspection of the equations for $\dot{z}_T$, p^* and Δp suggests that the differential pressure will peak at the instant the tank blows dry, and with a combination of maximum rise rate and air still entering the tank. Since the ballast expulsion rate increases exponentially with decreasing submergence, the maximum rise rate in a continuous blow would then occur at the surface, i.e., during broaching.

With the differential pressure, Δp, now determined, expressions for the response of the main ballast tank structure can be obtained.

STRUCTURAL RESPONSE

As shown on Figure 11, there are three structural systems which must be solved simultaneously due to their common interaction. These are the outer hull plating and frames, the inner hull plating and frames, and the connecting radial struts. At first glance, the presence of the outer hull frames and struts might appear redundant, since the function of the ballast tank plating is to contain an internal blowing pressure, for which stiffening is unnecessary. However, these struts and frames are necessary to maintain the concentricity of the outer hull with the inner hull when the ship is underway, and to resist concentrated external loadings, such as those produced by mooring, docking, wave slap, and breaking through an ice field. Because of the struts, very high bending stresses are periodically introduced into the outer hull framing, and thence into the outer hull plating. Although the subject of uniformly loaded and uniformly stiffened shell structures has been extensively treated both for single shells[2] and for sandwich shells[12], no known treatment exists for a sandwich shell structure having the inner and outer shells either periodically point connected or subjected to different types and magnitude of loading.

Deflection Sequence

An understanding of the deflected shape of the structure under load is essential to prediction of the distribution of stresses. As illustrated schematically in Figure 12, the sequence in which the ballast tank structure deflects can be described as follows: The inner (pressure) hull structure contracts uniformly due to the external hydrostatic pressure, p, occurring at a submergence depth, $D_{\mathbb{L}}$, and the attached radial struts are pulled inward (Figure 12b). This inward motion of the struts is resisted by the outer hull, such that the struts are elongated, thereby periodically loading the outer hull structure (Figure 12c). The outer hull then flexes, removing some of the strut load, and an equilibrium position is reached. Blowing of the ballast tanks then produces an additional internal pressure, Δp, inside the ballast tank (Figure 12d), which causes (1) the struts to elongate further until the axial strut load, W, is reached, and (2) the outer hull structure to expand under the hoop force, $T_{(x)}$, and to bulge outward under the moment, $M_{f(x)}$, applied to the outer hull frames. A similar deflection sequence occurs for the inner hull, due to the strut load, but since the inner hull is normally very stiff, with respect to the outer hull, secondary bending of the inner hull structure may usually be neglected in determining the inner hull strength.

Plastic Strength

Under an emergency recovery, account may be taken of the "plastic strength" of the main ballast tank structure, i.e., the reserve in strength that exists after the structure has begun to yield and permanently deform. Due to the complexity of this structure, expressions for the plastic strength must necessarily be semi-empirical in nature. A large scale structural model simulating a typical ballast tank has recently been subjected to internal pressure and tested well into the plastic range to determine an upper limit to the reserve in strength after yielding. Expressions for the plastic strength of individual elements in the tank are being developed from the results obtained from this test. This model, shown in Figure 22, will be discussed further in a later section.

Elastic Strength

For ballast blowing tests and training exercises, it is desirable to limit rise rates to prevent stresses in the tank structure from exceeding the yield strength, to minimize any possible low cycle fatigue cracking and to preclude formation of permanent bulges in the streamlined hull shape. Consequently, it is necessary to be able to predict, with a high degree of accuracy, stresses in each element of the structure.

Forces and Moments

The most difficult force to obtain, mathematically, is the axial force, W, in the radial strut (Figure 12d), due to the effects of the varying flexibility of the inner and outer hull structure. Once this strut load is determined, the membrane and bending loads on the outer hull structure can be determined, and expressions developed for the level of stress in each structural element. The strut load may be obtained by equating the deflections of the outer and inner hull structure with the strut elongation (Figure 13a), and expressed in descriptive terms as

$$W \approx \frac{\left[\begin{array}{l}\text{Effect of}\\ \text{radial expansion}\\ \text{of outer hull}\end{array}\right] + \left[\begin{array}{l}\text{Effect of}\\ \text{radial contraction}\\ \text{of inner hull}\end{array}\right] + \left[\begin{array}{l}\text{Effect of axial}\\ \text{contraction of}\\ \text{inner hull}\end{array}\right]}{\left[\begin{array}{l}\text{Effect of}\\ \text{elongation}\\ \text{of strut}\end{array}\right] + \left[\begin{array}{l}\text{Effect of strut}\\ \text{on stiffness of}\\ \text{both hulls}\end{array}\right] + \left[\begin{array}{l}\text{Effect of strut load}\\ \text{on hoop load in both}\\ \text{inner \& outer hulls}\end{array}\right]}$$

From the development in Appendix C, the load in the strut can be quantitatively written as

$$\begin{aligned} W = \Bigg\{ & 2P\left[1-\frac{\nu}{6}p\right] + \frac{p' R_H^2}{R_o^2}\left(\frac{E_o}{E_H}\right)\frac{A_{To}}{t_H}\left[2-\nu-\left(\frac{2A_{fH}-\nu(A_{FH})}{A_{TH}}\right)\right] \\ & + \frac{p' R_H A_{To}}{R_o t_o}\left[(1-2\nu)+\frac{\nu N}{\theta A_{TH}}\left(2\left(A_{fH}-b_H t_H\right)-\nu A_{fH}\right)\right]\Bigg\} \\ \div \Bigg\{ & \frac{2L_s A_{To} E_o}{R_o^2 A_s E_s} + 2\left[\delta\right]_{\frac{x}{\alpha}=1.0} \cdot \left[\frac{R_H^3}{R_o^2}\left(\frac{A_{To}}{I_H}\right)\frac{E_o}{E_H}+\frac{R_o A_{To}}{I_o}\right] \\ & + \Gamma\left[\frac{1}{R_o}+\frac{R_H E_o A_{To}}{R_o^2 E_H A_{TH}}\right]\Bigg\} \end{aligned} \qquad [C\text{-}27]$$

Examination of the above equation discloses that the strut load is proportional to the circumferential blowing load on the outer hull frame, P, and to the total pressure applied to the structure, p', where the latter includes both blow pressure, Δp, and hydrostatic pressure, p, incurred at the operating depth, $D_{\mathbb{L}}$.

Referring to Figure 14, the location of the structural element is denoted by the subscript "H" for the inner hull, "o" for the outer hull, and "s" for the strut, with the following general notation for the geometry of the element:

R = radius to midthickness of plating (in),

t = thickness of hull plating (in),

A_f = area of hull frame (frame only) (in^2),

A_F = area of hull frame plus area of plating in contact with the shell (usually frame web) (in^2),

A_T = total area of frame-shell combination, using the effective length of shell plating, L_e, (in^2),

where: L_e is defined by equations [C-5] and [C-21],

I = Inertia of frame-shell combination, using L_e (in^4), and

E = Modulus of Elasticity (lb/in^2).

Other symbols used denote non-dimensional coefficients, for which:

N = frame stiffness, relating to the effective length of plating acting with the frame, defined by equation [C-19],

θ = shell stiffness, defined by equation [C-20],

ν = Poisson's ratio,

ρ = Radius factor, defined by equation [C-2],

Γ = average normal (hoop) coefficient, defined by equation [C-16], and

$\left[\delta\right]_{\frac{x}{\alpha} = 1.0}$ = deflection coefficient, defined by equation [C-15], where the ratio x/α denotes the position along the frame relative to the strut (Figure 14b).

With the axial load in the strut determined, the circumferential hoop load in the outer hull frame at any position x along the frame may be expressed as

$$T_{(x)} = [PR_o - W\gamma_x] \left[\frac{A_{fo}}{A_{To}} \left(\frac{\alpha - x}{\alpha}\right) + \frac{x}{\alpha}\right] \qquad [C-41]$$

in which γ_x is a non-dimensional coefficient defined by equation [C-34]. The second term in the expression for hoop load represents a linear approximation of the effectiveness of the shell plating in absorbing the hoop load, for positions away from the strut.*

*The linear correction to the hoop load is based on effective width principles, rather than on the concept of effective breadth[(13)], since in many instances the compressive load induced by the inner hull contraction will be greater than the tensile load produced by ballast blowing.

The moment in the outer hull frame at any point x along the frame is

$$M_{f(x)} = WR_o [\xi_x] , \qquad [C-42]$$

in which ξ_x is a non-dimensional coefficient defined by equation [C-43].

Stress in Frames and Struts

With the hoop load and moment in the outer hull frame known, the stress in the frame may be readily calculated from the following equations: Taking Z_f and Z_p to represent the section modulus of the free flange and plate flange, respectively, for the outstanding flange of the frame

$$\sigma_{\phi f(x)} = \frac{T_{(x)}}{A_{To}} + \frac{M_{f(x)}}{Z_f} , \qquad [C-39]$$

and for the flange comprised of shell plating,

$$\sigma_{\phi p(x)} = \frac{T_{(x)}}{A_{To}} + \frac{M_{f(x)}}{Z_p} + \nu\sigma_{L(x)} . \qquad [C-40]$$

The last term in the expression for the stress in the plate flange represents the circumferential component of the longitudinal stress, $\sigma_{L(x)}$, in the outer hull plating adjacent to the fr· ne.

The axial stress in the strut is simply the strut load divided by the strut area, i.e.,

$$\sigma_s = \frac{W}{A_s} .$$

Stresses in Outer Hull Plating

By reversing equation [C-40] , the total longitudinal stress in the outer hull plating at any position along the edge of the frame can be written as

$$\sigma_{Lp(x)} = \sigma_{L(x)} + \nu \left[\frac{T_{(x)}}{A_{To}} + \frac{M_{f(x)}}{Z_p} \right] , \qquad [C-44]$$

where, from Appendix C,

$$\sigma_{L(x)} = \pm \frac{1.734\,K'}{(2V'N' - K'^2)(1-\nu^2)^{\frac{1}{2}}} \left\{ \frac{WR_o^2}{I_o} \left[\delta_x\right] + \frac{W}{A_{To}} \left[\gamma_x\right] \right.$$

$$+ \frac{(\Delta p) R_o}{A_{To}} \left[1 - \frac{\nu}{6} \, b\right] \left[\frac{A_{To}}{t_o} + H'_M \frac{A_{fo}}{t_o} - L_{eo} \right] \qquad [C-35]$$

$$\left. + \frac{p' R_H^2 E_o}{A_{TH} R_o E_H} \left[1 - \frac{\nu}{2}\right] \left[\frac{A_{TH}}{t_H} + \frac{H_M A_{fH}}{t_H} - L_{eH} \right] \right\}$$

$$+ \frac{(\Delta p) R_o b}{6 t_o}$$

and for which:

K', H'_M, N' and V' are stiffness coefficients for the outer hull, defined by equations [C-28] and [C-31] ,

H_M = stiffness coefficient for the inner hull, defined similar to equation [C-31] using θ in place of θ', and

δ_x = non-dimensional deflection coefficient for any position along the frame, and and is defined by equation [C-36].

For the case of the intersection of a wing bulkhead with the outer hull, the longitudinal stress in the outer hull plating reduces to

$$\sigma_{LB} = \pm \frac{1.734\,K'(1-\nu^2)^{-\frac{1}{2}}}{2V'N' - (K')^2} \left\{ \left[\frac{(\Delta p) R_o}{t_o} \right] \left[(1 + H'_M) + \left(\frac{L_{eo}\, t_o}{A_{To}} \right) C_3 \right] \right. . \qquad [C-38]$$

Continued on page 16

(Equation [C-38] continued)

$$\cdot \left[1 - \frac{\nu}{6} \beta\right] + \left[\frac{p' R_H^2 E_o}{A_{TH} R_o E_H}\right] \left[1 - \frac{\nu}{2}\right] \left[\frac{A_{TH}}{t_H} + \frac{A_{FH} H_M}{t_H} - L_{eH}\right] \Bigg\}$$

$$+ \frac{(\Delta p) R_o \beta}{6 t_o} \,, \qquad [C-38]$$

in which C_3 is a distribution factor relating the stiffness of the wing bulkhead and outer hull plating and is defined by equation [C-37].

Since the effective length of the outer hull plating is normally much smaller than the spacing between frames, the effect of bending of the shell at the mid-length position is negligible. Hence, the stresses at this position can be written simply as follows:

For the circumferential stress midway between frames

$$\sigma_{\phi} = \frac{(\Delta p) R_o}{t_o} + \nu \frac{(\Delta p) R_o \beta}{6 t_o} \,,$$

and for the longitudinal stress midway between frames

$$\sigma_{\frac{L}{2}} = \frac{(\Delta p) R_o}{6 t_o} \beta + \nu \frac{(\Delta p) R_o}{t_o} \,.$$

DESIGN APPLICATIONS

It is beyond the scope of this paper to proffer a detailed or sequential design procedure for the structure of submarine main ballast tanks. However, certain additional remarks may aid in application of equations contained herein to analysis of such structures and, in addition, to the general problem of strutted sandwich shells in which the two shells are subjected to different combinations of lateral and axial loading.

Pressure Loading

In equation [B-21] of Appendix B, which describes the forcing function in the equation of motion, [B-5], the denominator contains a term for absolute tank pressure, p^*. Since, in the equation, p^* is affected by depth, there is a suggestion that the transient differential pressure decreases with depth. This is an important consideration since the contraction of the pressure hull at depth can induce relatively high loads in the struts and frames of the ballast tank structure, which can thereby withstand only lower levels of differential pressure. The most significant parameter in determining the level of transient pressure is the valve opening time, since this controls the rate of air flow build-up. Further, modification of this opening time can be accomplished with fewer adverse effects on ship cost and performance than can modification of other parameters. It is preferable, therefore, to attack a transient pressure problem by increasing the valve opening time.

Although the expression for expansion pressure in a ballast tank, equation [B-32], is amenable to hand solution, a computer solution is preferable. Since the extreme condition for this pressure occurs when the tank blows dry while broaching, a solution is normally sought for this condition. In solving this equation, it has been found convenient to assume a reasonable discharge coefficient, $C_D = 0.6$[16], and a "standard" ratio of 100 ft^3/ft^2 for $\overline{V}_T/\overline{A}$ (relationship of ballast tank volume to flood hole area), and solve for the differential pressure, Δp, in terms of the tank depth. A series of solutions for this standardized condition can be obtained for a range of effective rise rates, $\dot{Z}_T$, spanning those which might occur in an emergency operation, and plotted as shown in Figure 18.

The Δp to be expected in a specific tank under a predetermined ship maneuver can be obtained by determining the effective rise rate, $\dot{Z}_T$, from equation [B-27], and then adjusting the solution obtained from Figure 21 for the actual flood hole area and volume of the tank in question. This adjustment is obtained by solving for the actual differential pressure

$$\Delta p = \frac{\overline{V}_T}{100\,\overline{A}} (\Delta p)_{STD} \;.$$

In comparison with the contribution of ship rise rate, the contribution of the pitch rate of the ship at broach is obviously quite small, and hence can generally be neglected. Referring to the effective rise rate, $\dot{Z}_T$, of the ballast tank [B-27], this means that the pitch-dependent terms (i.e., those containing θ^*) can be eliminated for most calculations. The term $\dot{Z}_{\mathbb{L}}$ in this equation represents the total rise rate of the submarine's center of gravity, and has components due to buoyancy and to the effect of axial velocity at an upward pitch angle. If neither the measured rise rate nor a solution of the equations of ship motion are available, it is possible to calculate the contribution of each of the components separately and combine them directly.

Referring to Figure 10, the component of the rise rate due to the pitch angle, θ^*, and the axial velocity, $\dot{u}$, is simply

$$\dot{Z}_u = \dot{u} \sin \theta^* .$$

The component of rise rate due to buoyancy is naturally dependent on the amount of ballast water blown, but it cannot exceed the velocity at which the drag force of the ship hull equals the buoyant force of fully-blown tanks. (The drag force is a maximum with the submarine moving perpendicular to its axis, and decreases as pitch angle increases.) For design purposes, the component of rise rate due to buoyancy normally can be assumed to be the rise rate of an unpowered submarine in a horizontal attitude, with fully-blown tanks.* Equating the buoyant force from fully blown tanks to the transverse drag of an appendageless submarine produces an expression for rise rate due to buoyancy,

$$\dot{Z}_B = 8.42 \sqrt{r \bar{B}}$$

in which r = reserve buoyancy factor (equivalent to weight of blowable ballast divided by weight of ship in Condition N, Surfaced) and

$\bar{B}$ = maximum beam (ft).

*It is assumed that the reduced drag resulting from the customary up-angle compensates for the fact that full buoyancy is presumed to be achieved only at the moment of surfacing, and that maximum rise rate thus cannot be fully achieved.

Of the ship parameters affecting differential pressure due to bubble expansion, the most important, and usually the most easily modified, is flood hole area.

Analysis of Structure

As in the case of pressure loading, a computer solution is preferable for structural equations for forces, moments and stresses developed in a main ballast tank. In the absence of such a facility, curves and tables are appended for the more complex of the non-dimensional coefficients. Figures 19 through 21 are plots of the stiffness coefficients H_M, H'_M, K', N, N' and V'. Tables 1 through 4, in turn, list calculated values for the coefficients applying to the outer hull frame (Γ, δ_x, ξ_x and γ_x), for strut spacing varying between 1 and 90 degrees, and for 20 equally spaced positions along the frame between adjacent struts.

By use of these intermediate frame coefficients for deflection (δ_x), moment (ξ_x) and hoop loading (γ_x), the variation in moment and axial load along the outer hull frame can be calculated using equations [C-41] and [C-42]. Stresses in both flanges of the frame can then be determined at intermediate positions from equations [C-39] and [C-40], and the corresponding longitudinal stress in the outer hull plating determined from equation [C-44].

Although equation [C-41] includes a linear correction for L_e in the hoop load, $T_{(x)}$, to be strictly correct a separate correction for L_e should be applied to the moment $M_{f(x)}$ (equation [C-42]).* Hence, it is anticipated that the frame flange stress, equation [C-39], would be in better agreement with actual values than would the plate flange stress, equation [C-40], since the latter depends upon the section modulus of the plate flange, Z_p. (Z_p will be more sensitive to L_e than will Z_f, the frame flange section modulus.)

In using the equations discussed under the preceding "Structural Response" section (and those appearing in Appendix C) for application to a particular submarine

*The length (breadth) of plating acting with a stiffener can be quite different for axial and uniform bending loading[15]. Hence, it may be expected that L_e would vary considerably more between struts due to the variation of $M_{f(x)}$.

ballast tank structure, expressions can be simplified readily by inserting the appropriate mechanical properties for the material used to fabricate the ballast tank.*

In their present form, however, structural expressions presented can be applied to structure other than submarine ballast tanks, and fabricated from materials other than steel. These expressions can be used to analyze the general problem of concentric ring-stiffened sandwich shells separated by rigid struts, with the shells subjected to various combinations of lateral and axial pressures. In addition, by the method of superposition, solutions can be obtained for deflections and stresses in strutted sandwich shells where the struts are not equally spaced.

The foregoing discussion has centered on the elastic behavior of a ballast tank structure under its intended loading. It is appropriate to mention that there are two other important aspects which must be considered in the design of such structures, and which are dependent upon the loading frequency and the environment in which the structure will be loaded. These are the fatigue strength of the structure, and its toughness under impact and dynamic loading. The ability of the steels (HTS and HY-80) normally used in submarine ballast tanks to resist brittle propagation of material defects, such as fabrication flaws or early fatigue cracks, has been previously discussed elsewhere[17]. For the general problem of fatigue strength of materials used in submarine ballast tanks, or of any other ductile structural material, expressions previously developed[18] may be used with an appropriate stress concentration factor representative of the weld notch.

CONFIRMING TEST PROGRAM

Since the high differential pressure loading developed in a main ballast tank during an emergency recovery is the type of loading which may be experienced only once during the lifetime of a ship, if ever, it is permissible to take account of the plastic strength of the tank structure. In other words, permanent bulging of the structure is not unacceptable, providing the tank boundary does not rupture or tear.

*For steel, $E \approx 30(10)^6$ lb/in^2 and $\nu \approx 0.3$.

Conversely, repeated loading that carries the structure into the plastic range is most undesirable due to a possibility of inducing low cycle fatigue propagation of flaws (cracks) in the material until a "critical" crack length is reached such that the crack length propagates rapidly.*

Hence, by comparison with the ship maneuvers anticipated in a recovery from a casualty condition, the maneuvers permissible during full-scale trials must remain quite mild. Consequently, full-scale testing can only provide experimental verification at the lower end of the scales of rise rate, differential blowing pressure, and structural response. As a result, it is necessary to maintain a considerable degree of conservatism in establishing criteria for loading and for structural design, until such time as large-scale land-based simulators can provide experimental information nearer the anticipated maximum values.

A relatively extensive test program designed to confirm the previously discussed pressure and stress equations has been established. Although the nature of results cannot be included herein, a brief description of the program follows.

Structural Testing

As mentioned previously, a large scale structural model simulating a typical submarine main ballast tank has been built and tested. The basic vehicle for this ballast tank model, shown in Figure 19, consisted of a discarded submarine fatigue model[14] previously tested under the Bureau of Ships' Submarine Structural Fatigue Program. This model was subjected to two separate tests, to simulate both the differential pressure and the hydrostatic pressure (operating depth) effects.

The first test, designed to simulate the depth effect, was conducted with the model in a 30-foot test tank[14], and with the ballast tank vented to the surrounding hydrostatic field. The test tank was gradually pressurized to produce an external

*Depending upon the individual viewpoint, an alternate concept is that of exhaustion of ductility.

load on the pressure hull, as would occur with a submarine operating at depth. The effect of the contraction of the pressure hull on strains in the ballast tank structure was then recorded by approximately 300 electric resistance strain gages placed at critical locations on the tank structure.

The second test of the ballast tank model was conducted outside the 30-foot test tank, with the ballast tank vents and flood holes covered. Hydrostatic pressure was then applied inside the ballast tank, to simulate the effect of differential pressure loading. The model was pressurized several times within the elastic range of the structure (to eliminate the effects of fabrication variables from the recorded strains), and then tested under gradually increasing and extreme pressures until the structure was loaded well into the plastic range. Strains were measured continuously during all pressure tests, to obtain a record of the elastic and plastic response of the ballast tank structure.

Full-Scale Trials

Supplementary instrumented blow tests are being conducted on several submarines, both at dockside and at sea.[10] Although conventional strain gages and pressure transducers are adequate for dockside blow tests, dynamic strain and pressure recorders are desirable for ascent trials conducted at sea to measure any transient differential pressures or non-linear response of the tank structure.

SUMMARY

Because of a substantial increase in the blow pressure now available for recovery from a casualty condition, the structure of submarine main ballast tanks may be taxed to perform well into the inelastic range, thus becoming one of the highest-performance structures in a modern submarine. Thus, ballast tanks must be designed accurately, with careful attention to uniformity of stress distribution and to avoidance of any detail which might initiate tearing.

By application of the equations contained herein, and appropriate design margins of safety, it is now possible to insure that the ballast tank structure is "strong enough to take that immediate application of pressure."*

*See FRONTISPIECE

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LISTING OF ABBREVIATIONS

SNAME - Society of Naval Architects and Marine Engineers

RINA - Royal Institution of Naval Architects

DTMB - David Taylor Model Basin

ASNE - American Society of Naval Engineers

NOMENCLATURE

The notation employed herein principally follows the standard used for submarine hull structure and, wherever possible, notation appearing in the literature. Use of the symbols is illustrated in Appendix A, Figures 8 through 17.

A_B	Area of wing bulkhead resisting blow pressure (in^2)
$\overline{A}$	Effective area of flood hole opening (ft^2)
A_{fH}	Area of inner (pressure) hull frame (frame only) (in^2)
A_{FH}	Area of inner hull frame plus area of shell in contact with frame ($A_{fH} + b_H t_H$) (in^2)
A_{fo}	Area of outer hull frame (frame only) (in^2)
A_s	Area of strut (in^2)
A_{TH}	Area of inner hull frame plus effective area of shell ($A_{fH} + L_{eH} t_H$), (in^2)
A_{TO}	Area of outer hull frame plus effective area of shell ($A_{fo} + L_{eo} t_o$), (in^2)
$\overline{B}$	Maximum beam of hull (ft)
b'_B	Radial width of wing bulkhead panel, measured perpendicular to outer hull plating (in)
b_H	Thickness of inner hull frame web or faying flange (in)
b_o	Thickness of outer hull frame web (in)
C	Damping coefficient ($lb\text{-}sec^2/ft$)
C_D	Flood hole discharge coefficient
C_P	Specific heat at constant pressure
C_V	Specific heat at constant volume
C_2	Function of flexural rigidity (non-dimensional)

C_3 — Distribution factor relating stiffness of wing bulkhead and outer hull plating (non-dimensional)

D_o' — Flexural rigidity of outer hull plating (in^4)

$D_{℄}$ — Depth to centerline of submarine hull when blowing ballast, measured at midlength of ballast tank (ft)

E_H — Modulus of elasticity of inner hull (lb/in^2)

E_o — Modulus of elasticity of outer hull (lb/in^2)

E_s — Modulus of elasticity of strut (lb/in^2)

F — Force causing flexure of tank boundary (lb)

F' — Force in outer hull due to end pressure on wing bulkhead (lb)

g — Acceleration of gravity (ft/sec^2)

H_M — Shell stiffness coefficient, inner hull (non-dimensional)

H_M' — Shell stiffness coefficient, outer hull (non-dimensional)

$\bar{h}_A$ — Vertical distance from tank top to water surface in tank (ft)

$\bar{h}_B$ — Depth of residual water, from base line to top of flood hole (ft)

$\bar{h}_T$ — Blowable depth of tank, from tank top to flood hole (ft)

$\bar{h}_W$ — Depth of blowable water in tank, from water surface to flood hole (ft)

I_H — Moment of inertia of inner hull frame-shell combination, including effective length of shell, L_{eH} (in^4)

I_o — Moment of inertia of outer hull frame-shell combination, including effective length of shell, L_{eo} (in^4)

J^* — Work (ft-lb)

K' — Shell stiffness coefficient for outer hull plating (non-dimensional)

K^*	Spring constant of tank (lb/ft)
k^*	Ratio of specific heats, $\frac{C_p}{C_v} \approx 1.4$ for air
L_{eH}	Effective length of inner hull plating interacting with frame (in)
L_{eo}	Effective length of outer hull plating interacting with frame (in)
L_H	Frame spacing of inner hull (in)
L_o	Frame spacing of outer hull (in)
L_s	Length of strut, denoted by distance between neutral axes for frame-shell combination of inner and outer hulls (in)
$M_{f(x)}$	Moment in outer hull frame developed from strut load, W (in-lb)
M_o	Longitudinal moment in outer plating due to frame stiffness (in-lb)
$\overline{m}$	Mass of water slug in flood hole (lb-sec^2/ft)
N	Frame stiffness coefficient, relating to effective width of shell interacting with frame, inner hull (non-dimensional)
N'	Frame stiffness coefficient, relating to effective width of shell interacting with frame, outer hull (non-dimensional)
N^*	Weight of air (lb)
P	Circumferential load on outer hull (lb/in)
P_e	Total end force in wing bulkhead due to blowing pressure (lb)
P^*_{ATM}	Atmospheric pressure at sea level (lb/ft^2)
P^*_T	Ambient sea pressure outside top of tank (lb/ft^2)
p	Hydrostatic pressure at hull axis (lb/in^2)
p'	Total pressure at hull axis (hydrostatic plus blow) (lb/in^2)

p^*	Pressure in tank (lb/ft^2)
Q_A	Flow of air (ft^3/sec)
Q_W	Flow of water (ft^3/sec)
Q^*	Heat energy (ft-lb)
R_H	Inner hull radius, to mid-thickness of plating (in)
R_o	Outer hull radius, to mid-thickness of plating (in)
R^*	Universal gas constant ($53.3 \frac{ft\text{-}lb}{lb\text{-}^oR}$)
r	Reserve buoyancy factor $\left(\frac{\text{ballast weight}}{\text{wt. in Cond. N, surf.}}\right)$
T_A	Average hoop load in frame, between struts (lb)
$T_{(x)}$	Hoop force in outer hull frame at any position x (lb)
T'_A	Air temperature (^oR)
T'_S	Stagnation temperature of air (^oR)
t_B	Thickness of wing bulkhead plating (in)
t_H	Thickness of inner hull plating (in)
t_o	Thickness of outer hull plating (in)
t^*	Time (differentiation with respect to time is indicated by dots, e.g., $\frac{dx}{dt} = \dot{x}$) (sec)
U	Energy (ft-lb)
$\dot{u}$	Axial velocity of submarine (ft/sec)
V'	Shell stiffness coefficient, outer hull (non-dimensional)

$\bar{V}_A$	Volume of air in tank (ft^3)
$\bar{V}_T$	Total blowable volume of tank (ft^3)
$\bar{v}$	Depth of flood hole baffles, parallel to flow (ft)
W	Load in strut (lb)
W*	Flow rate of air (lb/sec)
W^*_{MAX}	Maximum flow rate air (lb/sec)
$\bar{x}$	Distance along frame from midpoint between struts (radians or degrees)
$\bar{x}_1$	Displacement of exciting force, F (ft)
$\bar{x}_z$	Displacement of mass, $\bar{m}$ (ft)
$\bar{x}_A$	Axial distance from ship CG to ballast tank (negative aft) (ft)
y	Deflection of frame-shell combination (in)
y_s	Deflection of shell (unrestrained) (in)
y_2	Strut elongation (in)
y_3	Radial contraction of inner hull due to hydrostatic pressure (in)
$\bar{y}$	Center of pressure on segment of wing bulkhead, from hull axis (in)
$\bar{y}'$	Center of pressure on segment of wing bulkhead, from mid-thickness of inner hull plating (in)
Z	Net deflection of shell from frame ($y-y_s$) (in)
Z_1	High of ship CG above base line (ft)
Z_f	Section modulus of outer hull frame flange, for A_{To} (in^3)
Z_p	Section modulus of outer hull plate flange, for A_{To} (in^3)
Z_T	Depth from ocean surface to tank top (ft)

$Z_{\mathfrak{L}}$ — Depth from ocean surface to CG of ship (ft)

$\dot{Z}_B$ — Rise rate (vertical ship velocity) due to buoyancy (ft/sec)

$\dot{Z}_\mu$ — Rise rate due to axial ship velocity (ft/sec)

α — Half-angle between struts (radians or degrees)

α' — Hull parameter, $1.285\ (R_o t_o)^{-1/2}$

γ_{sw} — Density of sea water (lb/ft^3)

γ_x — Normal (hoop) force coefficient at position x (non-dimensional)

Γ — Average normal (hoop) force coefficient at position x (non-dimensional)

Δ — $y + y_3 - y_2$ (in)

Δ_H — Deflection due to normal (hoop) force caused by strut restraint (in)

Δ_{IH} — Total radial deflection of inner and outer hulls due to strut effect (in)

Δ_N — Deflection of frame at any position x from deflected position on unrestrained shell (in)

Δ_R — Radial deformation of outer hull caused by Δ_{ZO} (in)

Δ_{RS} — Radial deflection of frame centroid from original position, due to start restraint (in)

Δ_{ZO} — Average deflection of outer hull plating between frames (in)

Δ_1 — Total deflection of outer hull frame due to strut $(\Delta_{RS} + \Delta_H)$ (in)

Δp — Differential pressure across ballast tank structure, due to blowing or expansion (lb/in^2)

δp — Pressure drop across flood hole (lb/ft^2)

δ_x — Deflection coefficient at position x (non-dimensional)

ϵ_s — Axial strain in strut (in/in)

ϵ_ϕ — Circumferential strain (in/in)

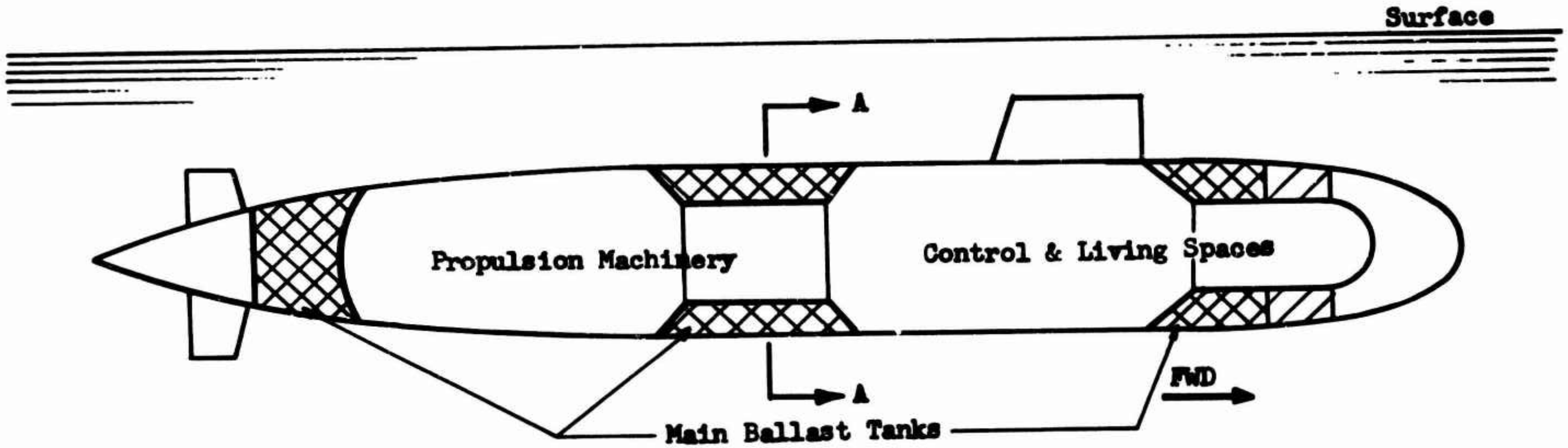
ϵ_Z	Longitudinal strain (in/in)
θ	Stiffness coefficient, inner hull (non-dimensional)
θ'	Stiffness coefficient, outer hull (non-dimensional)
θ^*	Pitch angle (degree of radians)
ν	Poisson's ratio
ξ_x	Moment coefficient at any position x (non-dimensional;
ρ	Radius factor (non-dimensional)
σ	Stress (lb/in^2)
σ_{LB}	Longitudinal stress in outer hull plating at edge of wing bulkhead (lb/in^2)
$\sigma_{Lp(x)}$	Longitudinal stress in outer hull plating, at edge of frame, for any position x (lb/in^2)
$\sigma_{L(x)}$	Longitudinal stress in plating at the frame of a uniformly ring-stiffened shell (lb/in^2)
$\sigma_{\frac{L}{2}}$	Longitudinal stress at midbay of outer hull (lb/in^2)
σ_s	Axial stress in strut (lb/in^2)
σ'_Z	Longitudinal membrane stress produced by axial contraction of inner hull under hydrostatic pressure, p (lb/in^2)
$\sigma_{\phi B}$	Circumferential stress in outer hull plating adjacent to wing bulkhead (lb/in2)
$\sigma_{\phi f(x)}$	Circumferential stress in outstanding flange of outer hull frame, for any circumferential position x (lb/in^2)
$\sigma_{\phi M}$	Mean circumferential stress in inner hull plating (lb/in^2)
$\sigma_{\phi o}$	Circumferential stress in outer hull plating, midlength between frames (lb/in^2)
$\sigma_{\phi oz}$	Stress in outer hull developed from longitudinal strain transferred from inner hull (lb/in^2)

$\sigma_{\phi p(x)}$ Circumferential stress in the outer hull shell flange, for any position x (lb/in^2)

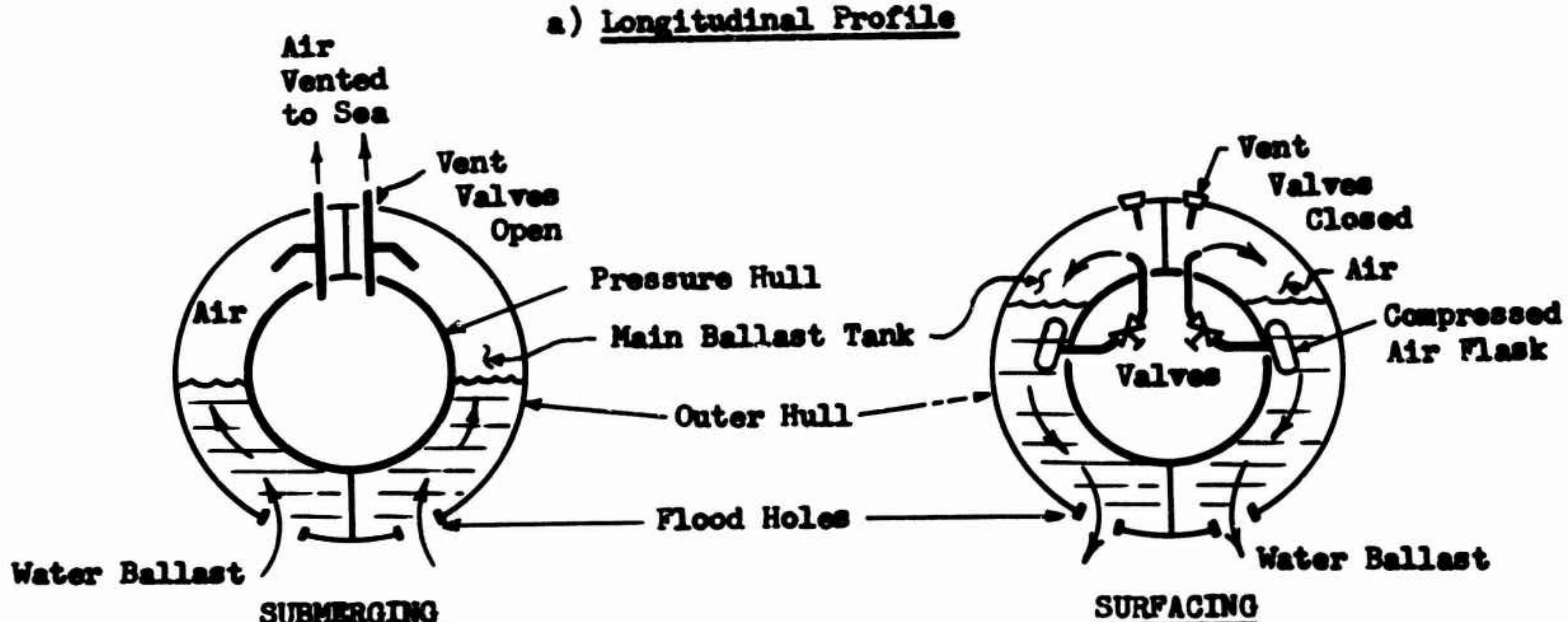
ϕ_N Parameters for adjusting shape of blow rate curve (non-dimensional)

APPENDIX A

FIGURES & TABLES



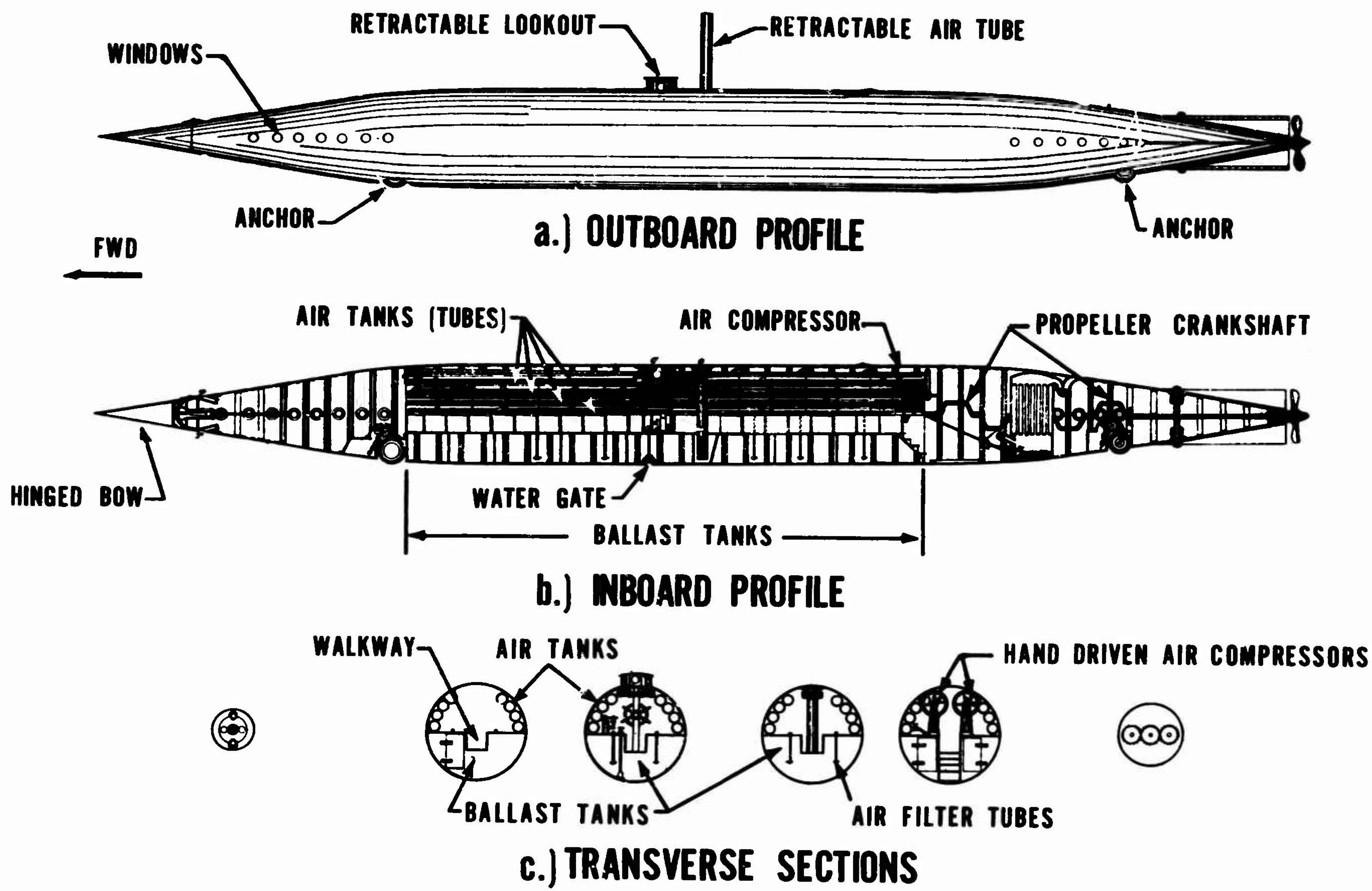
a) Longitudinal Profile



b) CROSS-SECTION THROUGH MAIN BALLAST TANKS (A-A)

OPERATION OF MAIN BALLAST TANKS IN MODERN SUBMARINES

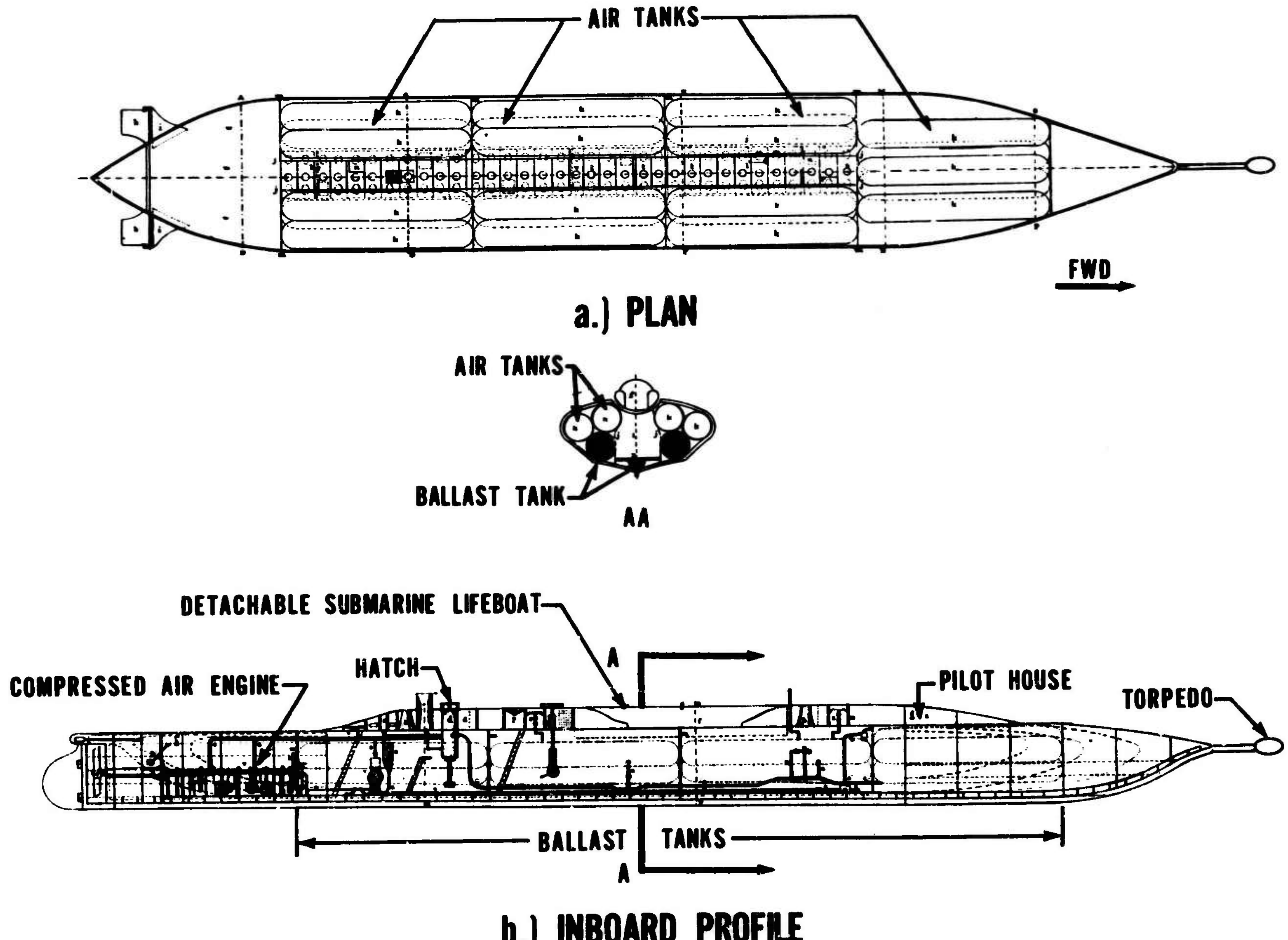
Figure 1



Phillips' Submarine (1851) *

Figure 2

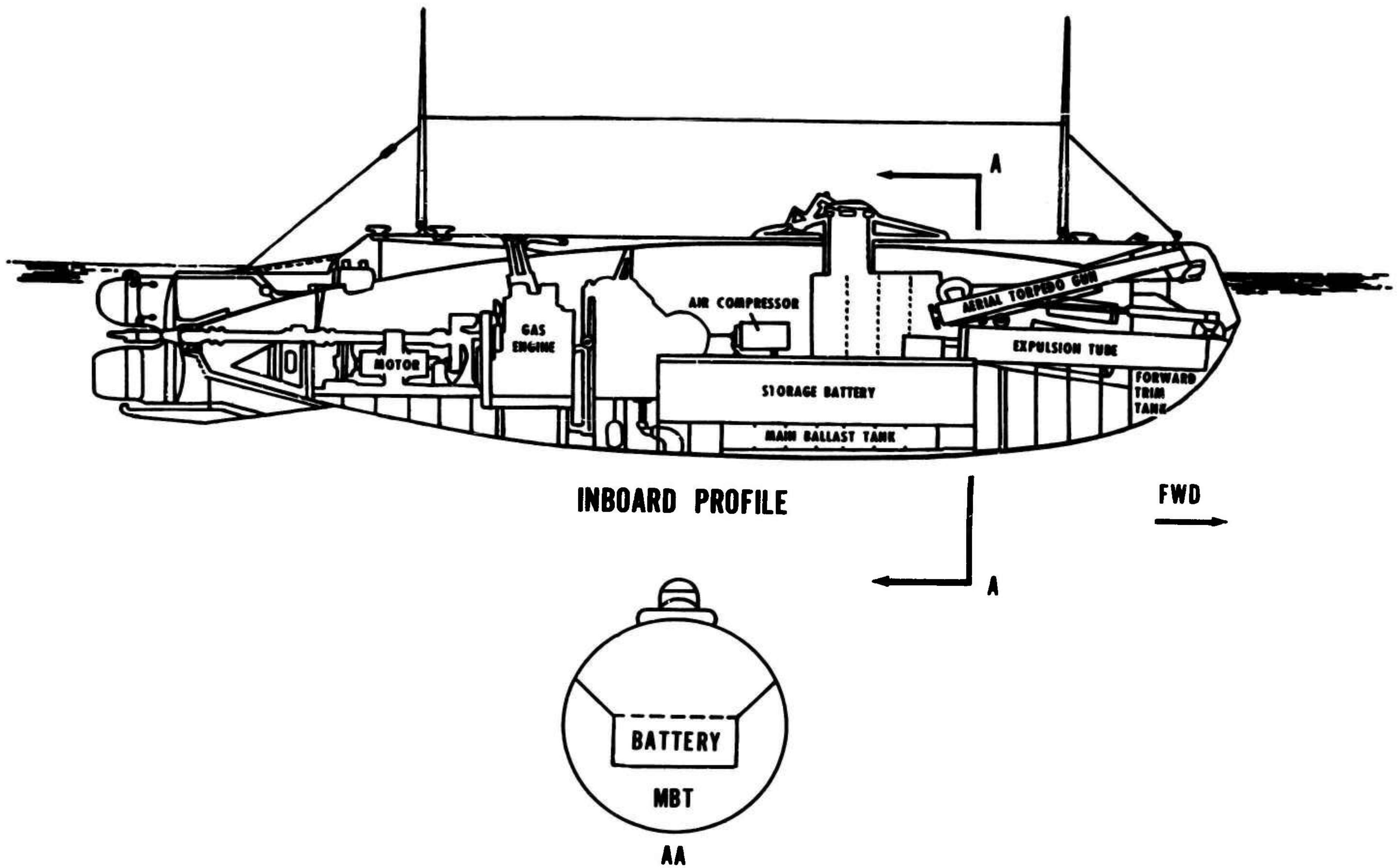
*From Reference (6)



Brun And Bourgeois' "LE PLONGEUR" (1863)*

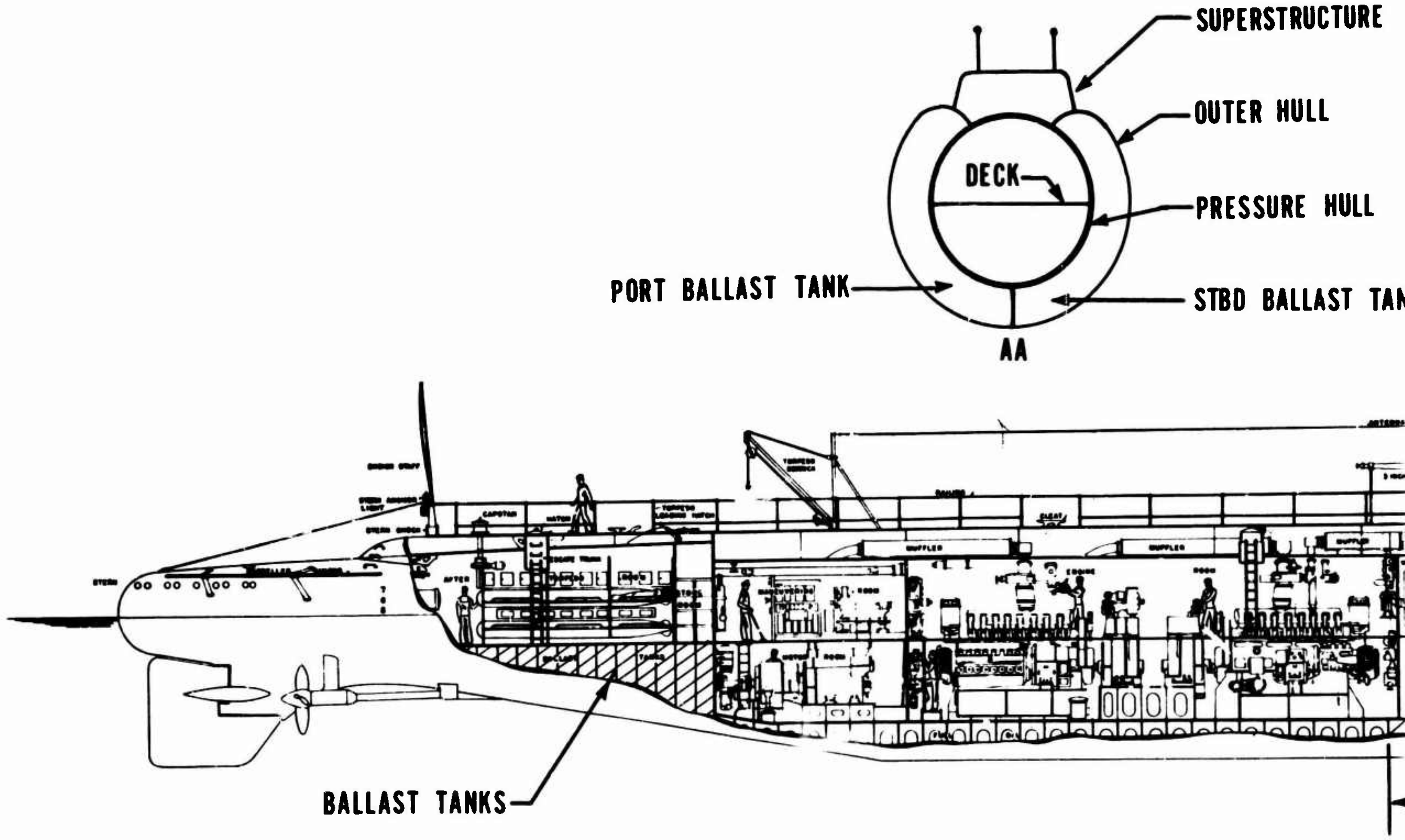
Figure 3

*From Reference (6)

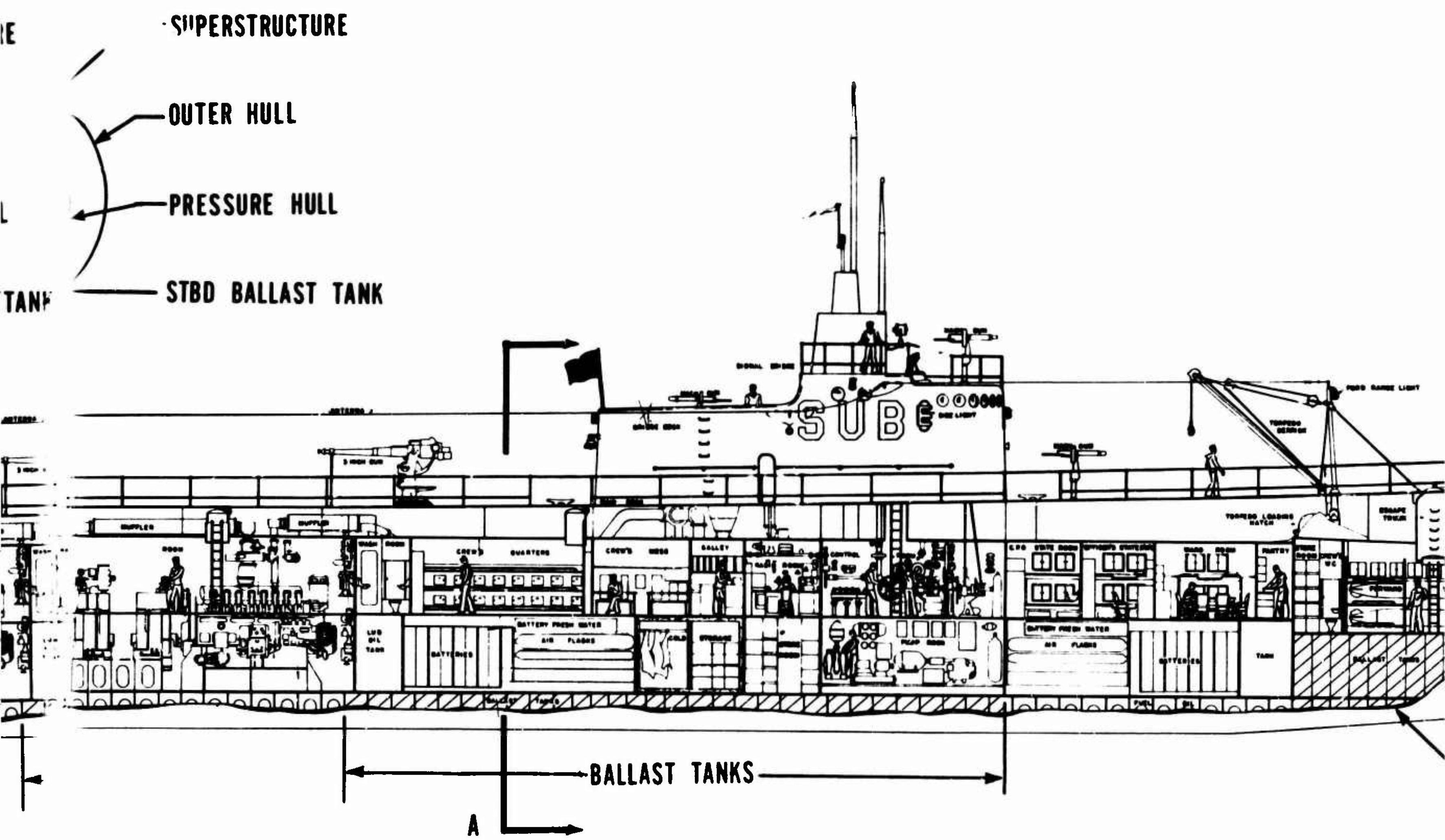


U.S.S. HOLLAND (SS-1)

Location of Water Ballast System
Figure 4

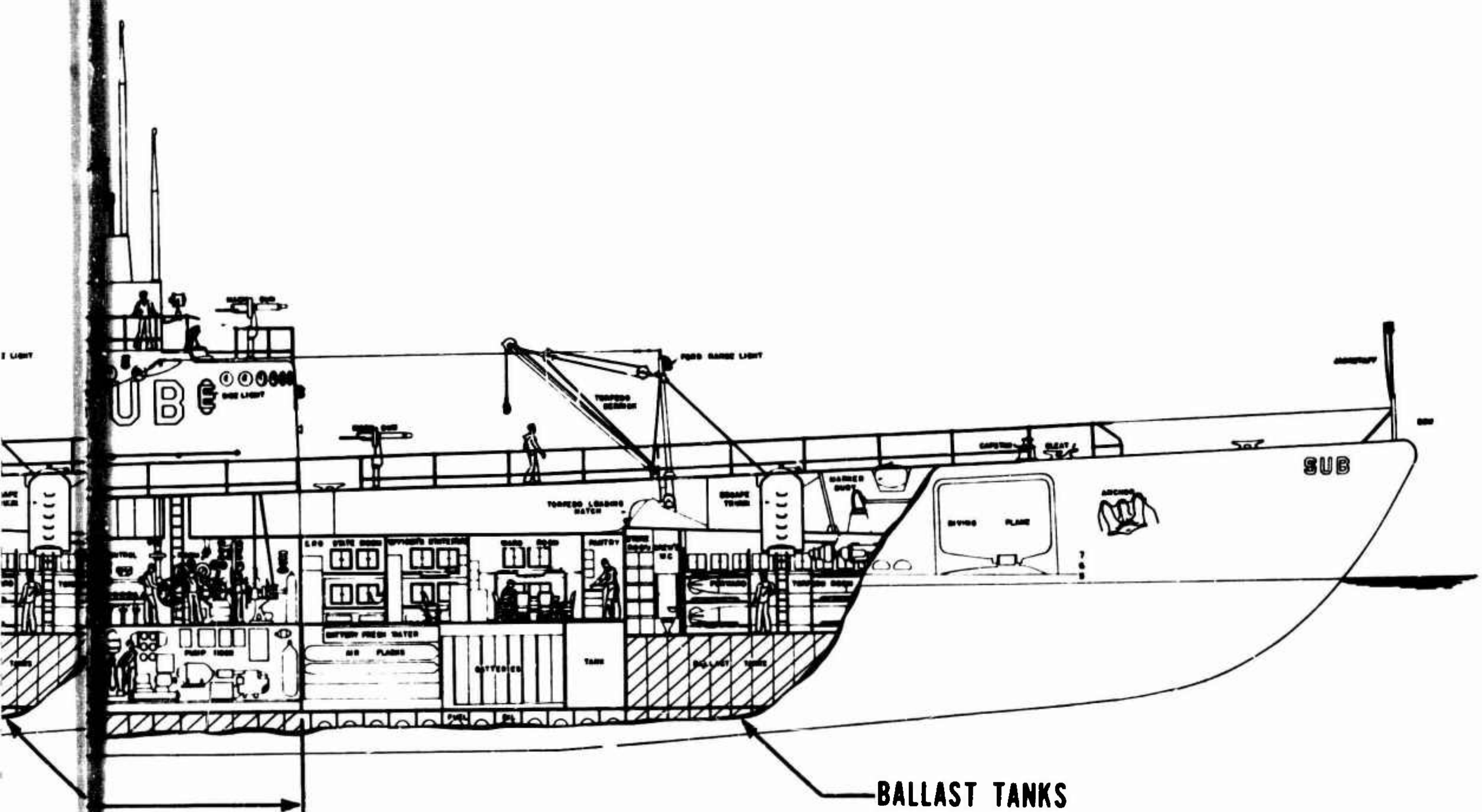

SUPERSTRUCTURE
OUTER HULL
DECK
PRESSURE HULL
PORT BALLAST TANK
STBD BALLAST TANK
AA
BALLAST TANKS

A



INBOARD PROFILE

B



Typical Submarine Of W W II Period

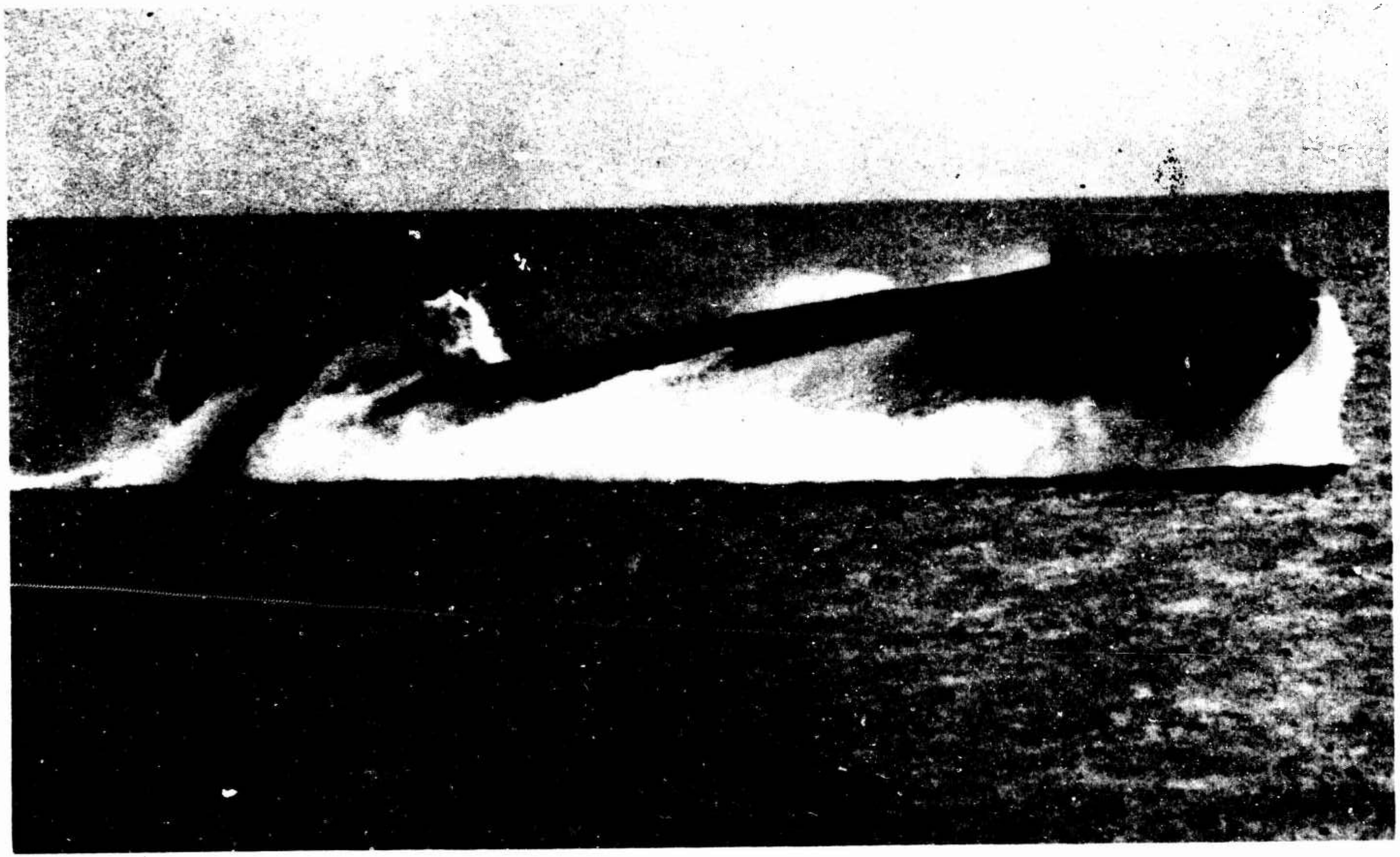
Figure 5

C

U.S.S. PICKEREL (SS-524)

Surfacing with a 48° Up Angle from a Depth of 150 Feet

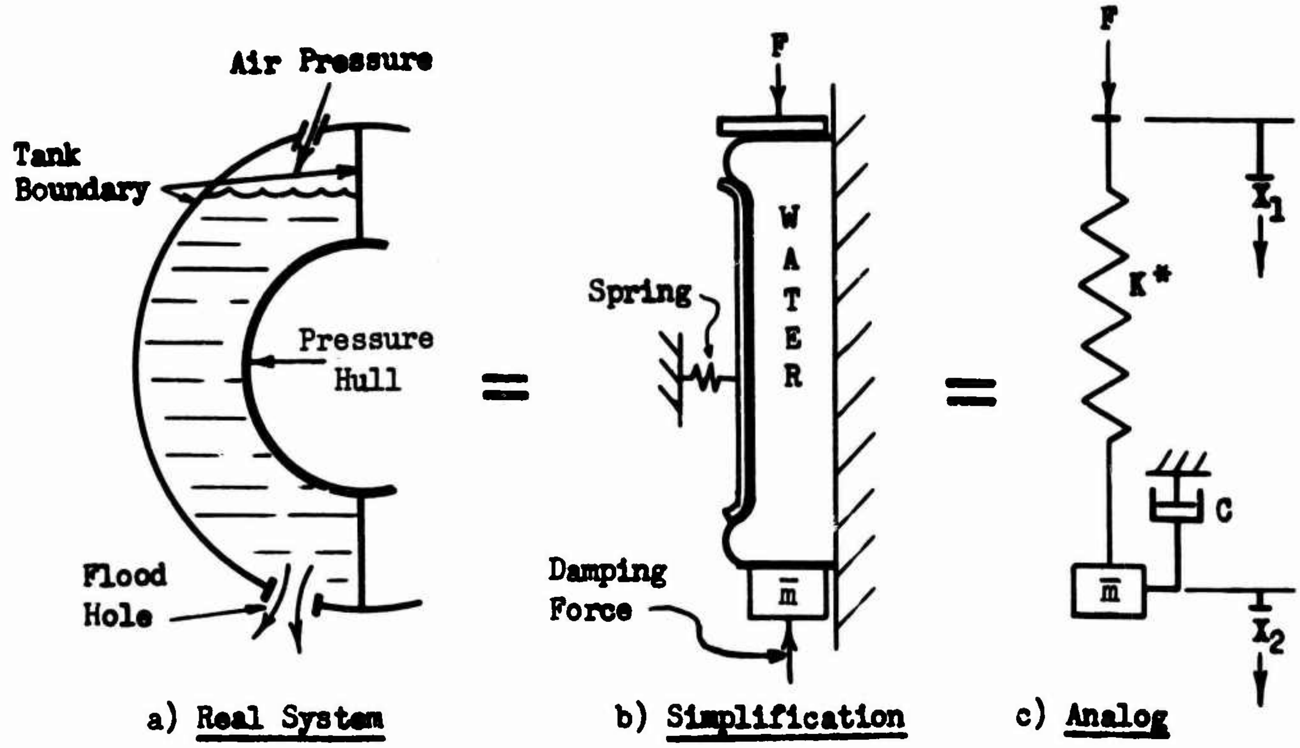
Figure 6



U.S.S. NATHAN HALE (SSBN 623)

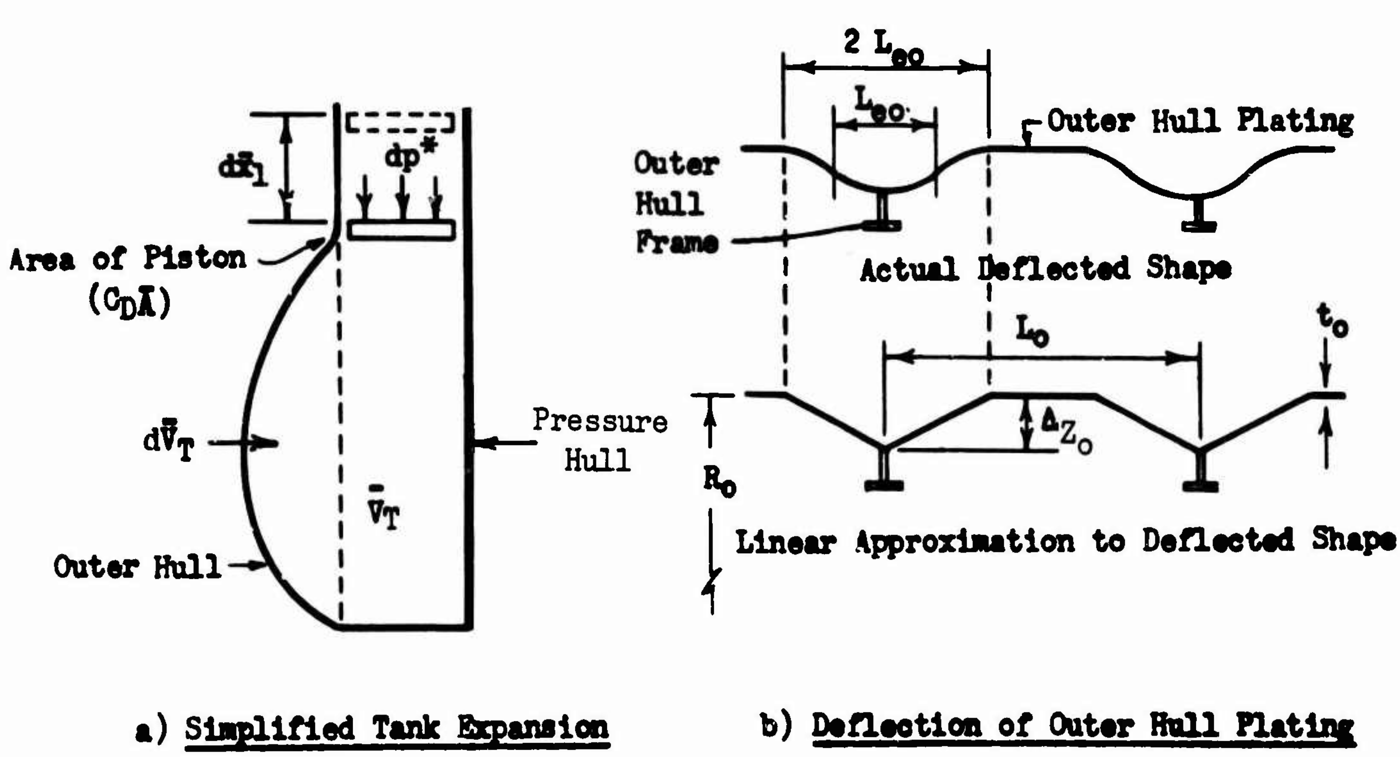
Surfacing with a 20° Up-Angle

Figure 7



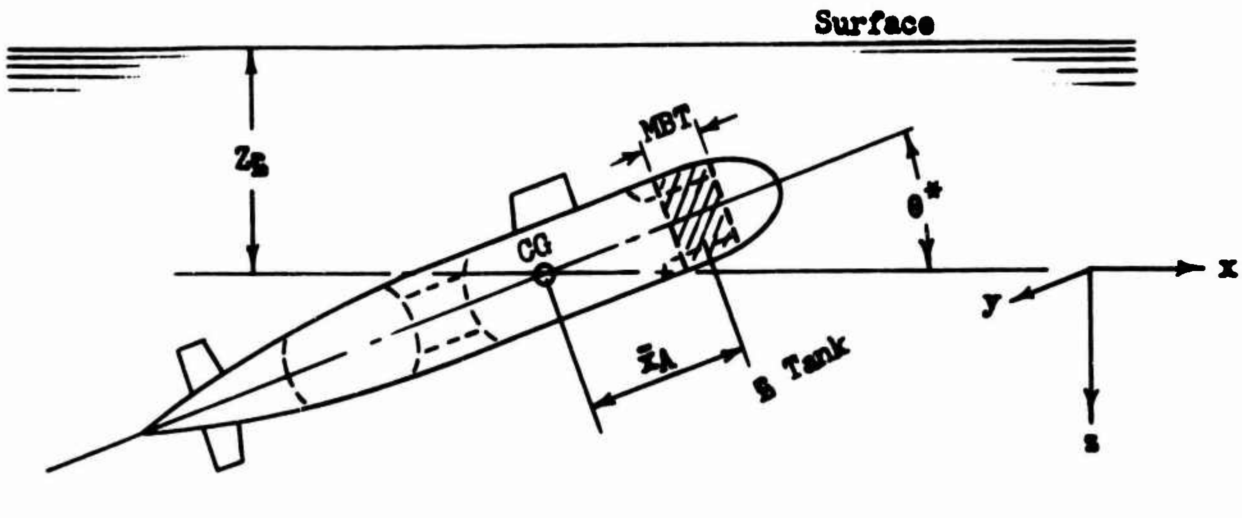
DEVELOPMENT OF ANALOG FOR TRANSIENT LOADING

Figure 8

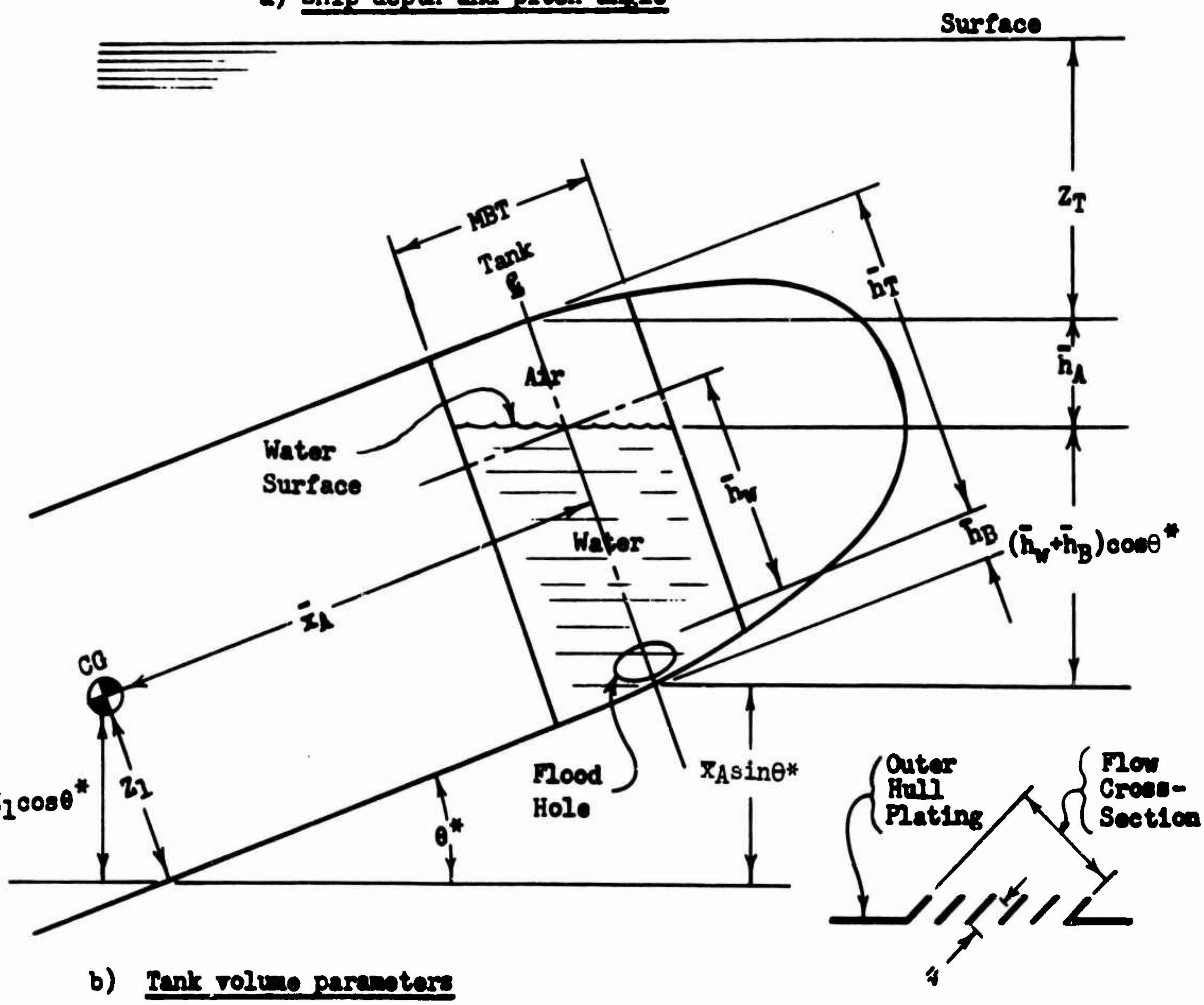


DETERMINATION OF TANK SPRING CONSTANT

Figure 9



a) Ship depth and pitch angle

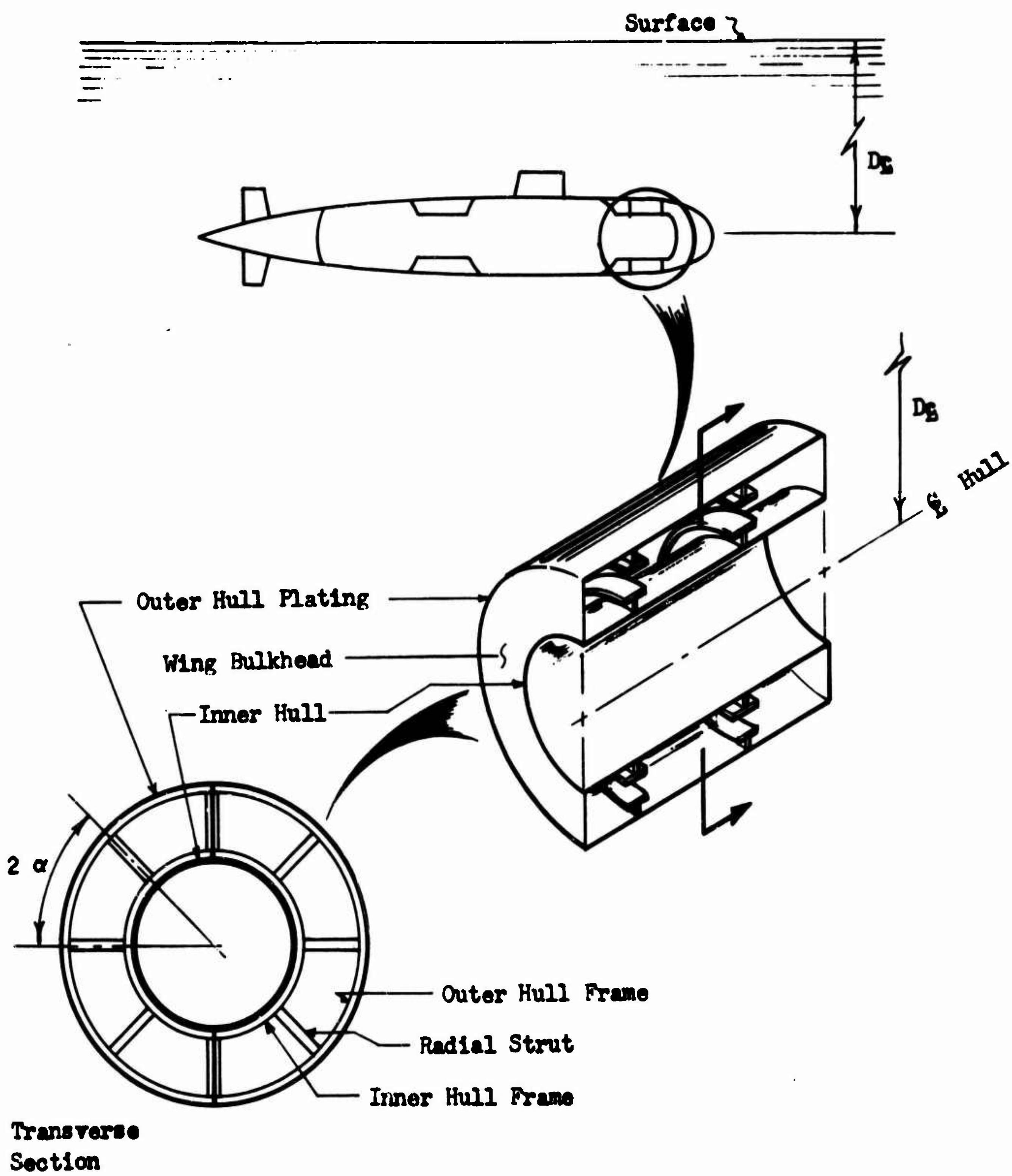


b) Tank volume parameters

c) Flood Hole Parameters

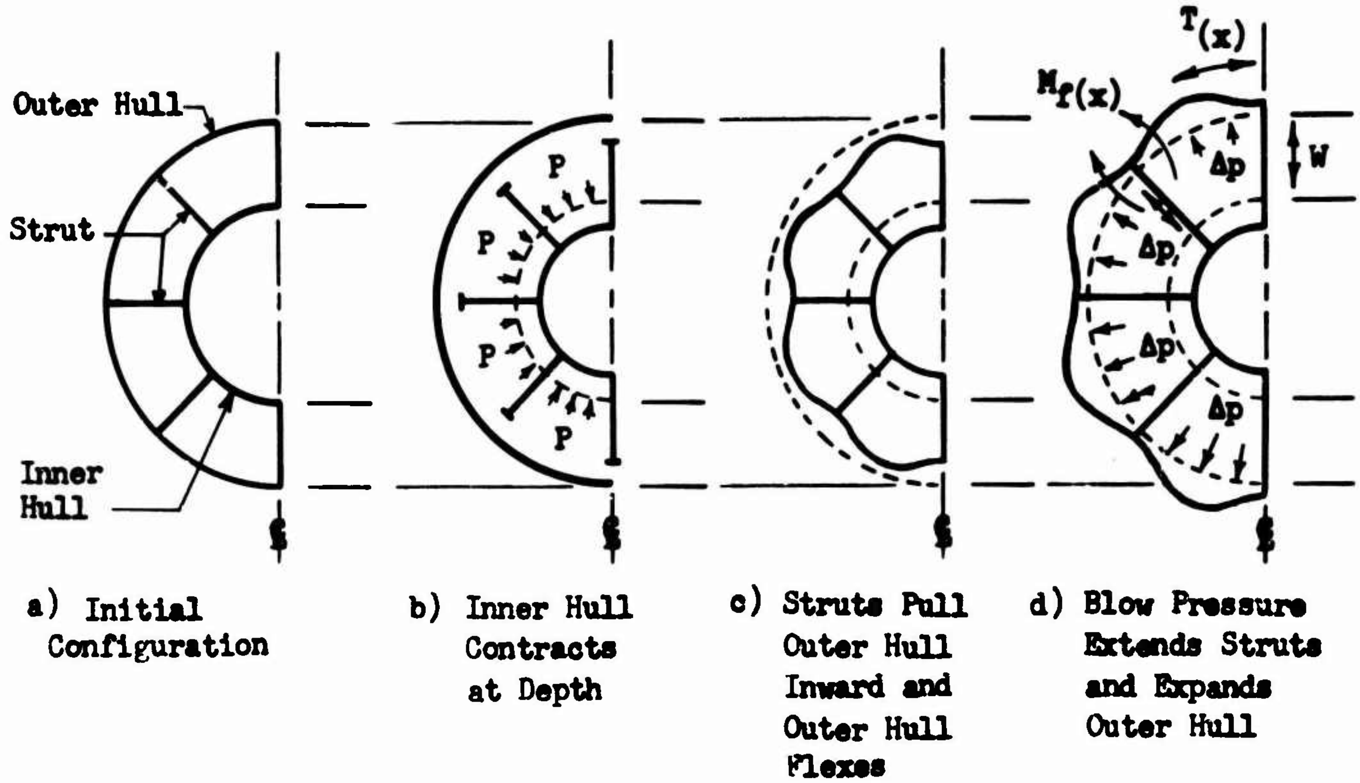
DIMENSIONS FOR TANK VOLUME AND LOCATION

Figure 10



CONFIGURATION OF MAIN BALLAST TANK STRUCTURE

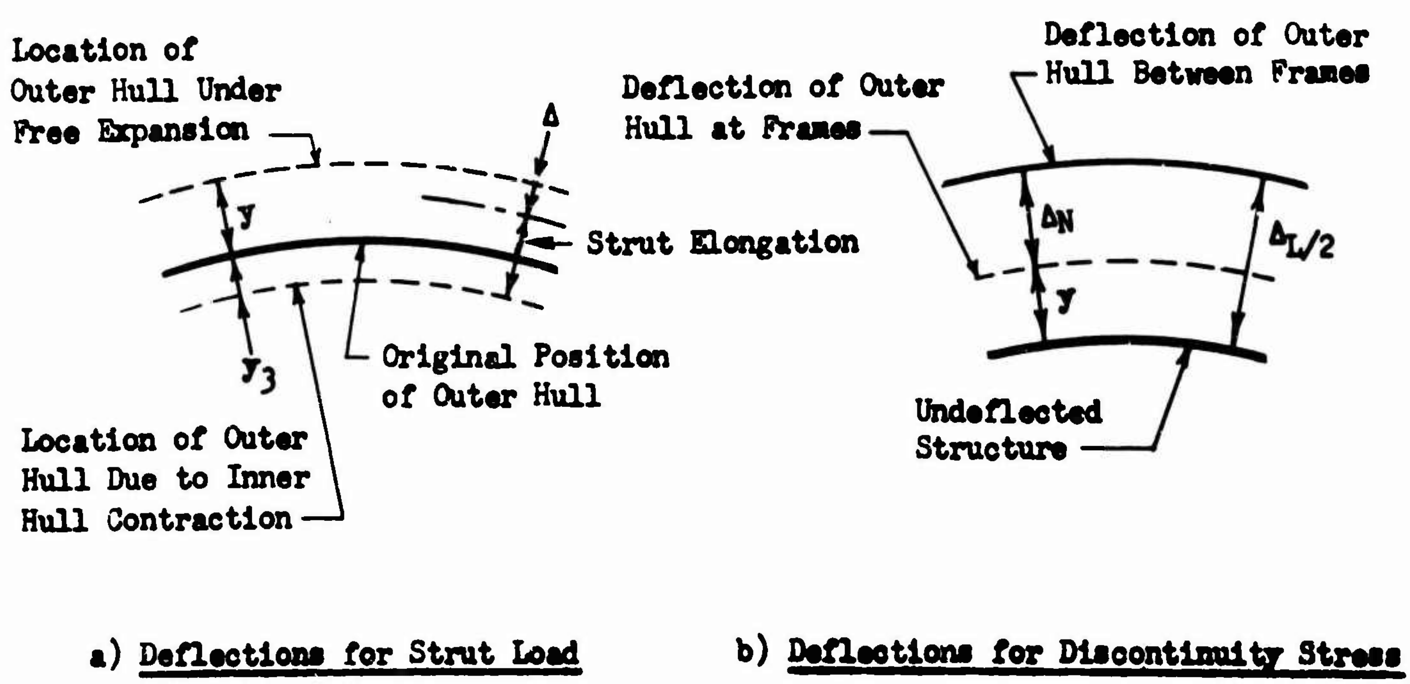
Figure 11



SEQUENCE OF LOAD DEFLECTIONS

(Transverse Section Through Main Ballast Tank)

Figure 12

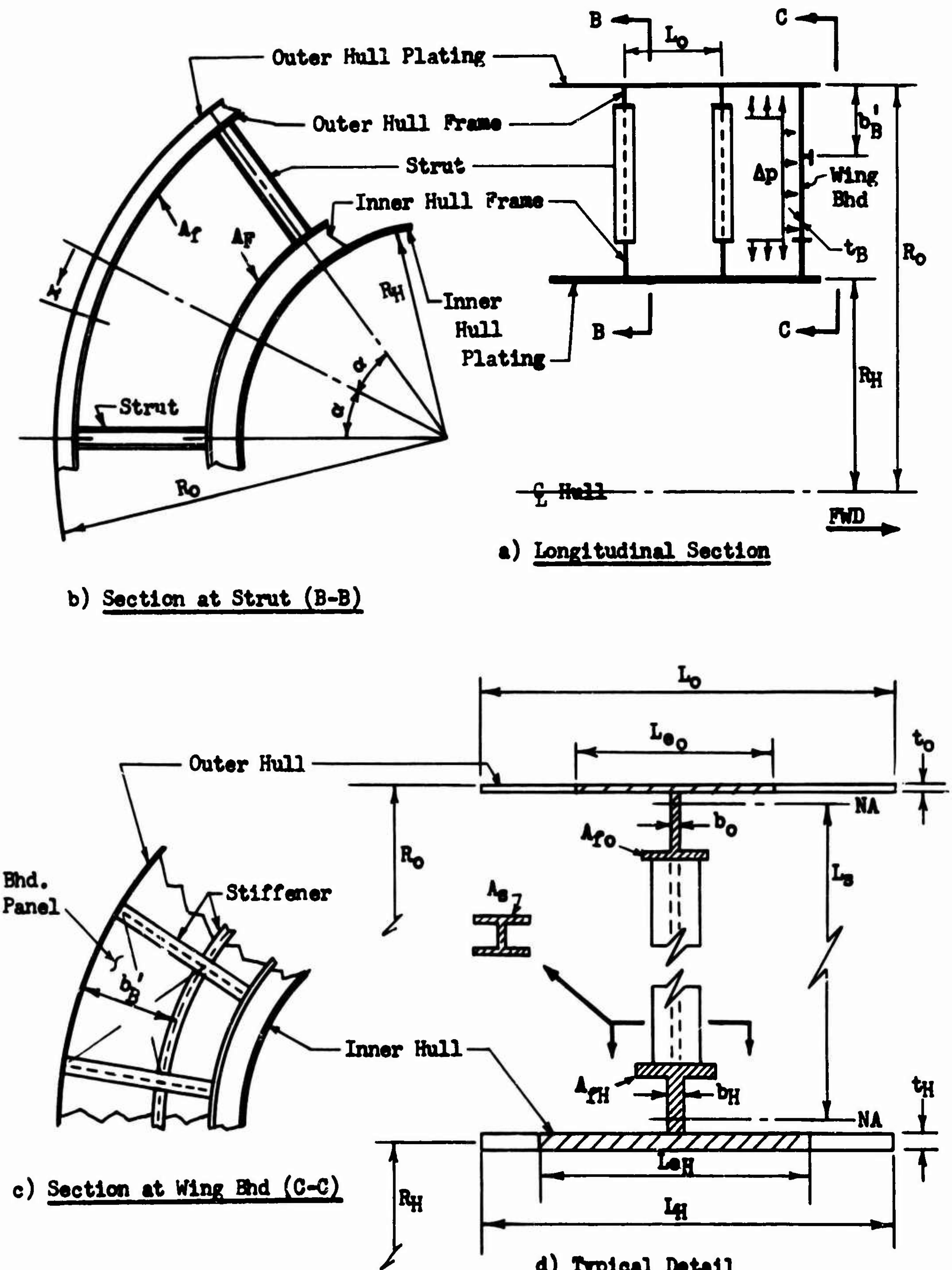


a) Deflections for Strut Load

b) Deflections for Discontinuity Stress

DEFLECTION NOMENCLATURE

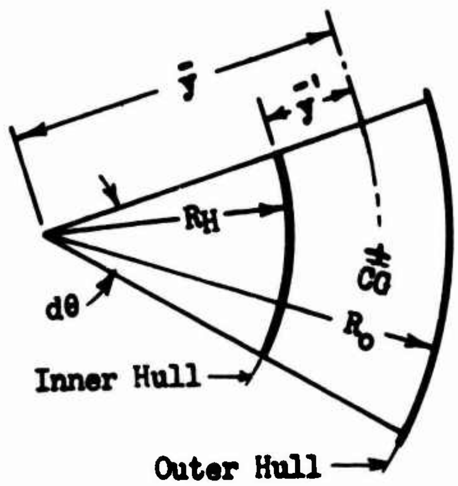
Figure 13



DIMENSIONS FOR TANK STRUCTURE

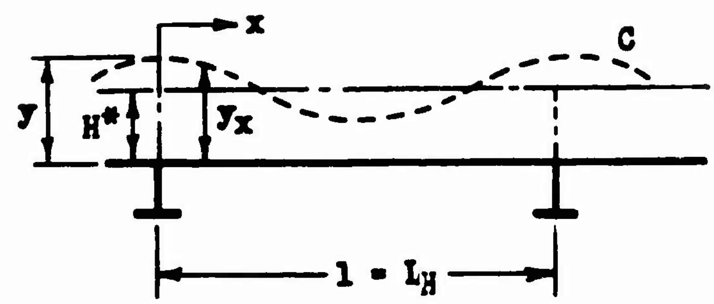
(Inner and Outer Hulls, and Wing Bulkhead)

Figure 14



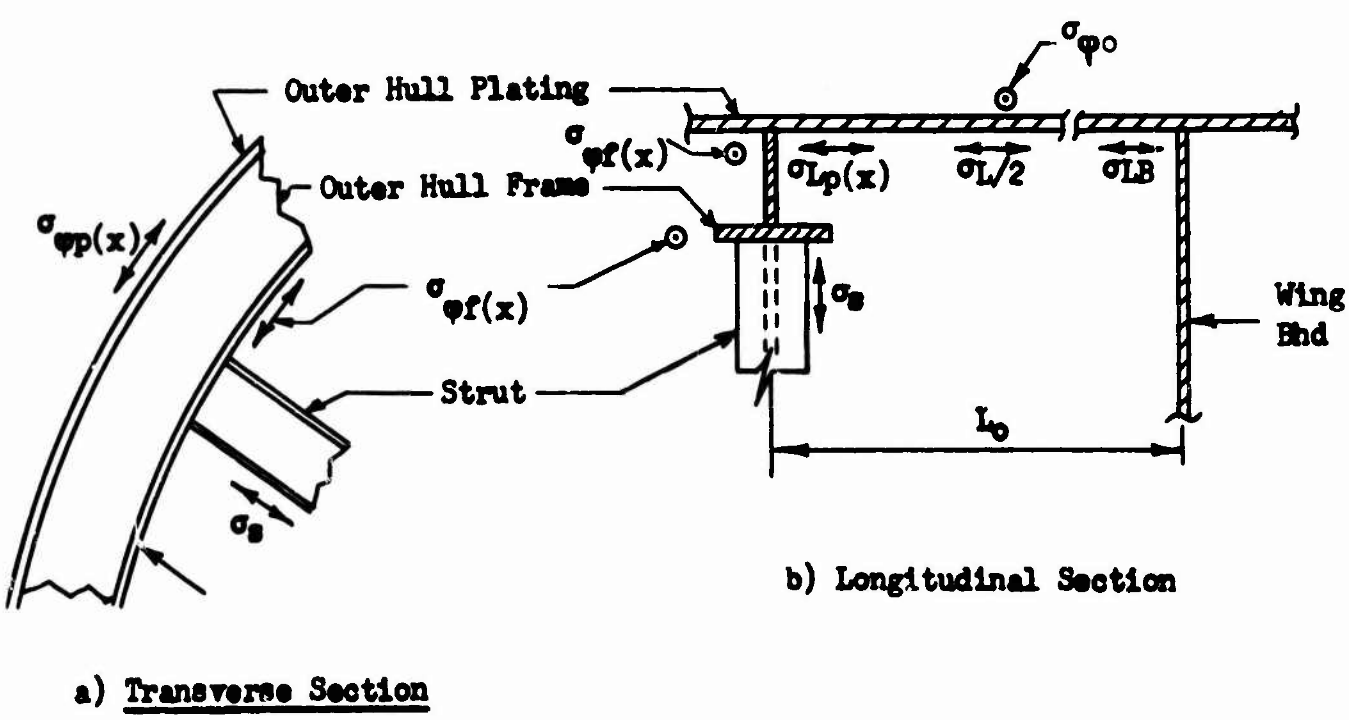
LOADING ON WING BULKHEAD

Figure 15



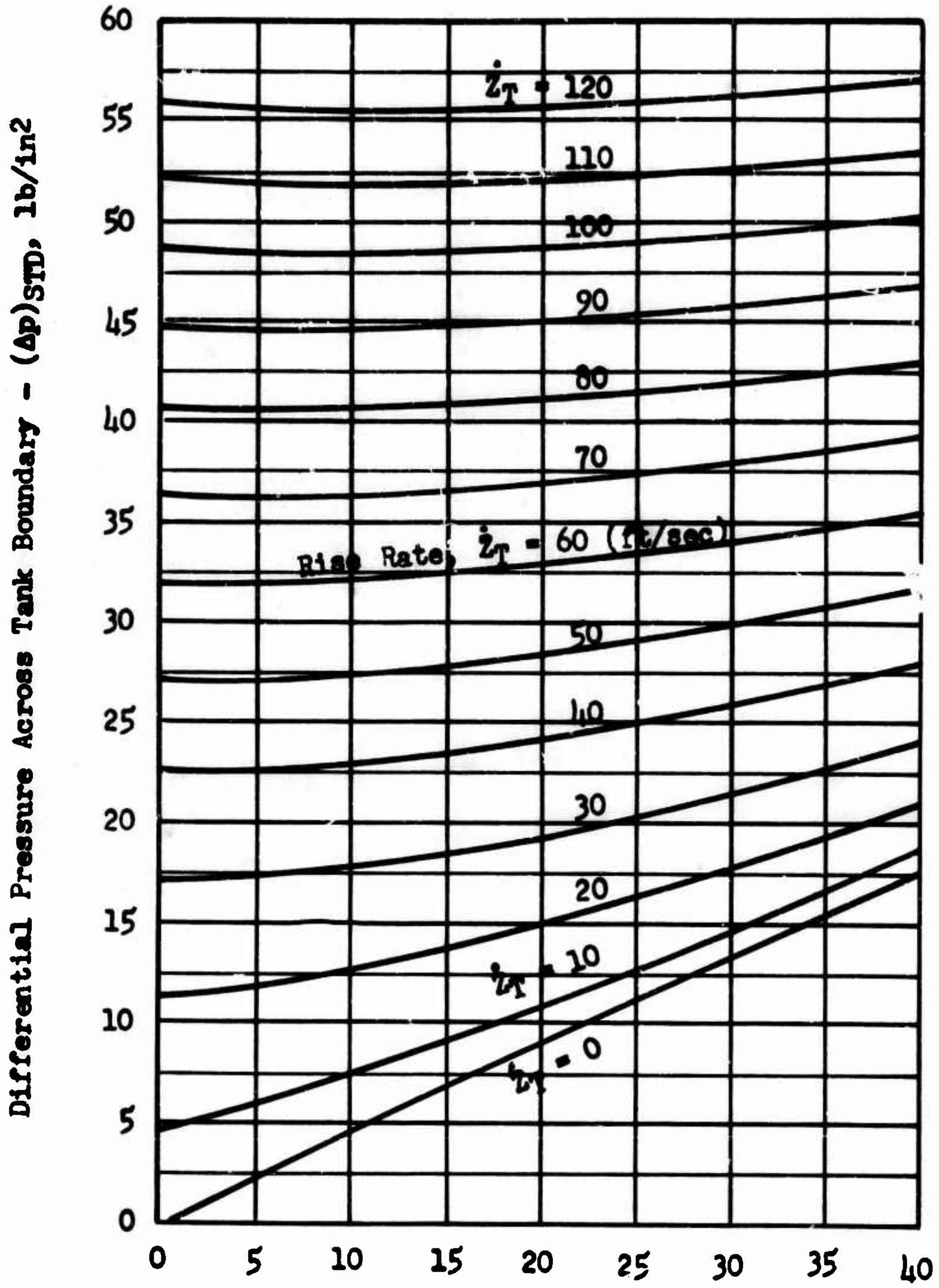
PRESSURE HULL DEFLECTION

Figure 16



LOCATION AND DIRECTION OF CONTROLLING STRESSES

Figure 17



Distance - Tank Top to Water Surface - $\bar{h}_A$, feet

(For maximum condition, use distance from tank top to flood hole, $\bar{h}_T$)

PRESSURE DIFFERENTIAL AT SURFACE OF BALLAST WATER*

(For "Standard" Tank with $\bar{V}_T$ = 100 $\bar{A}$)

Figure 18

*Reference 19

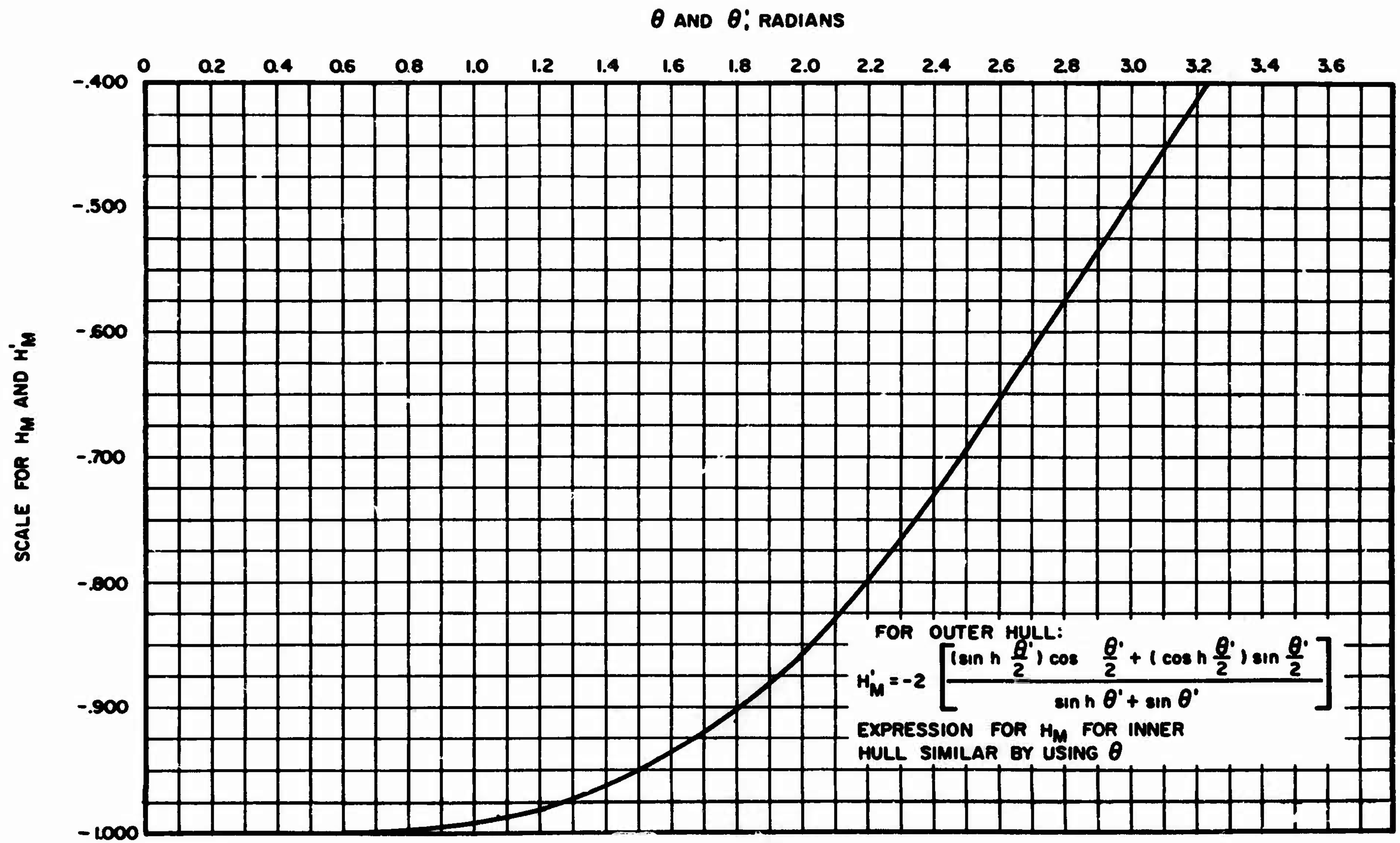


FIGURE 19A- SHELL STIFFNESS COEFFICIENTS, H_M & H'_M

$(0 < \theta \leqslant 3.2)$

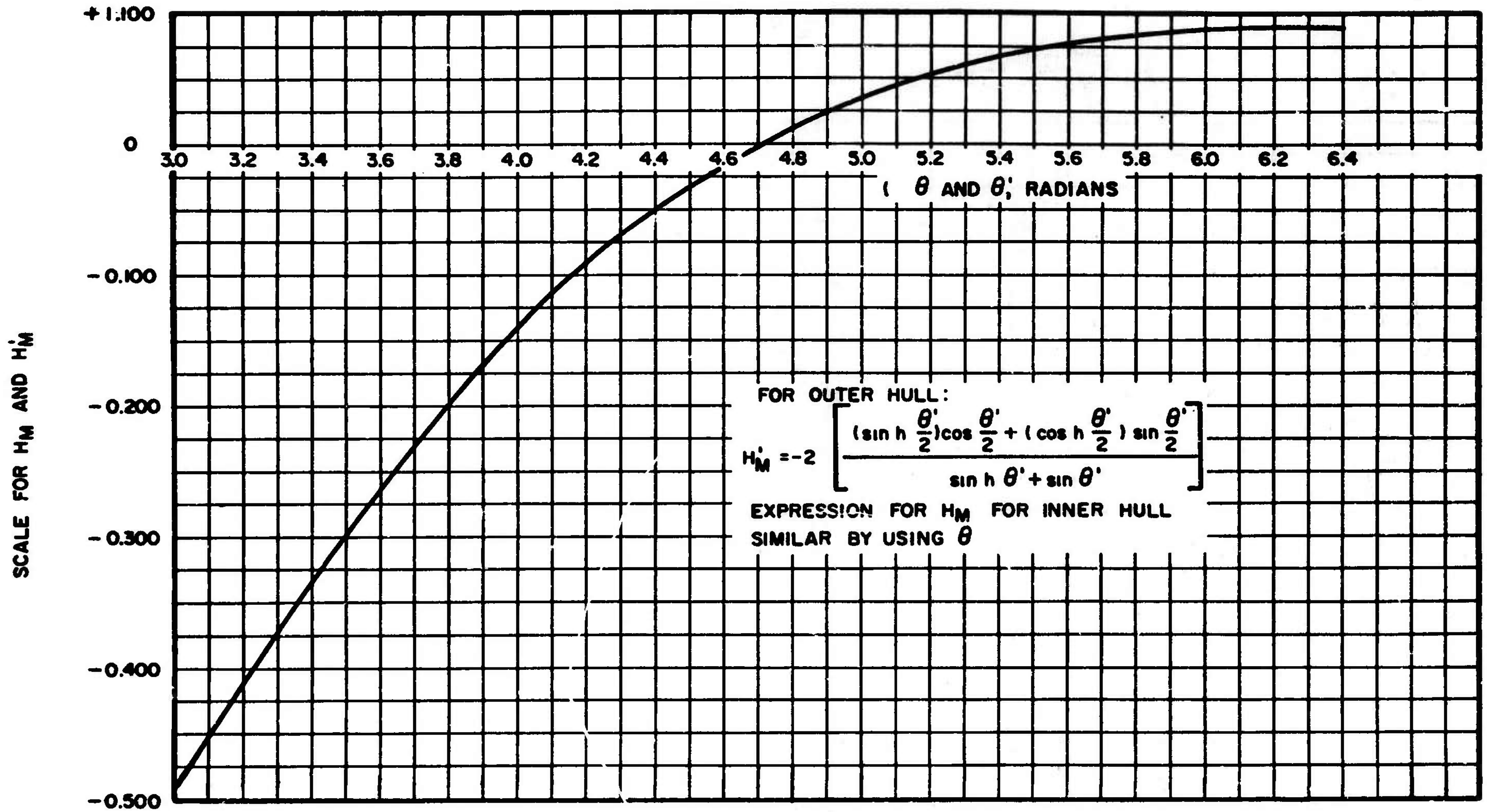


FIGURE 19B - SHELL STIFFNESS COEFFICIENTS, H_M AND H'_M
($\theta > 3.0$)

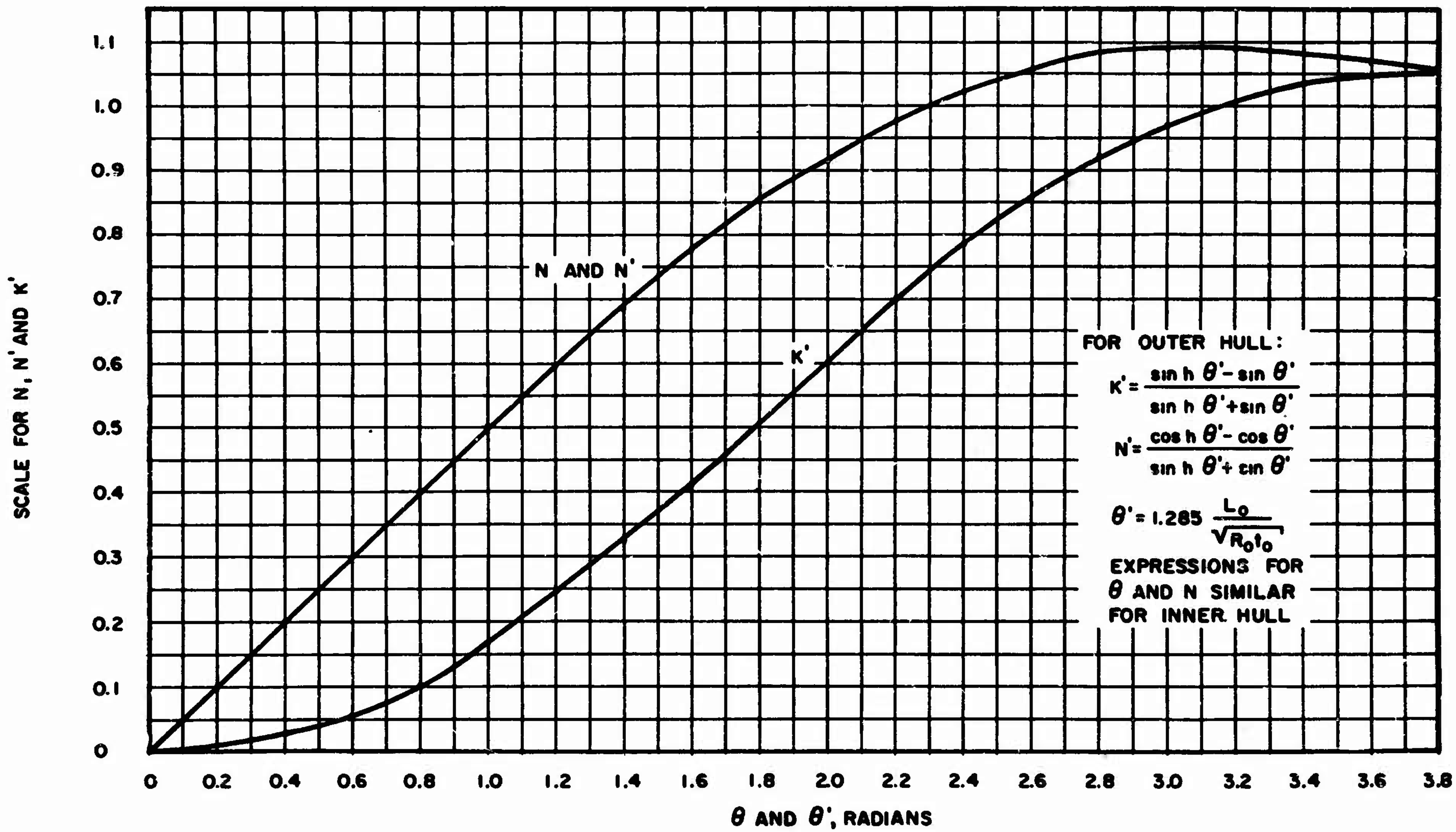


FIGURE 20A-SHELL STIFFNESS COEFFICIENTS, N, N' AND K'

$(0 < \theta, \theta' \leqslant 3.0)$

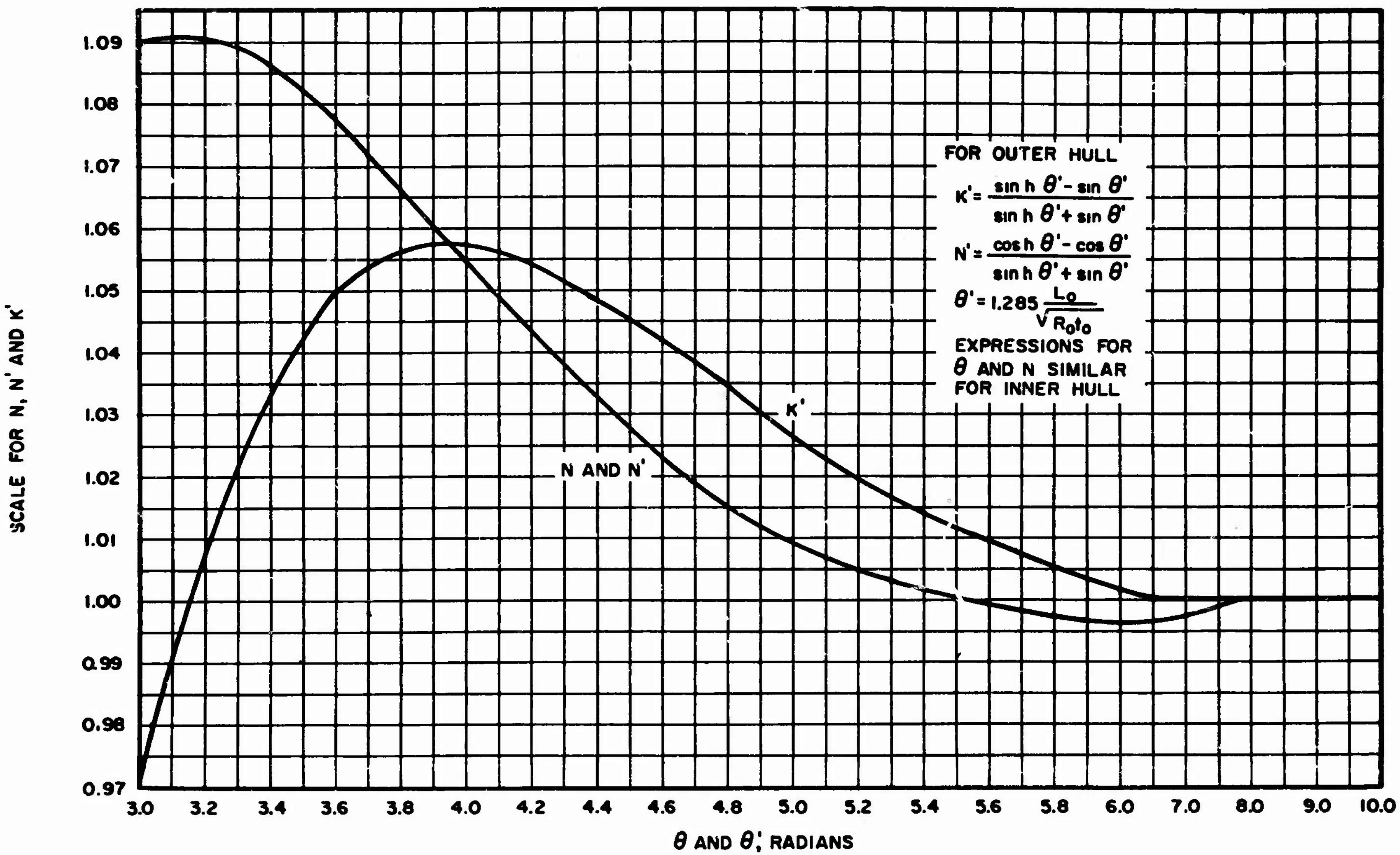


FIGURE 20B-SHELL STIFFNESS COEFFICIENTS, N, N' AND K'

(θ, θ' > 3.0)

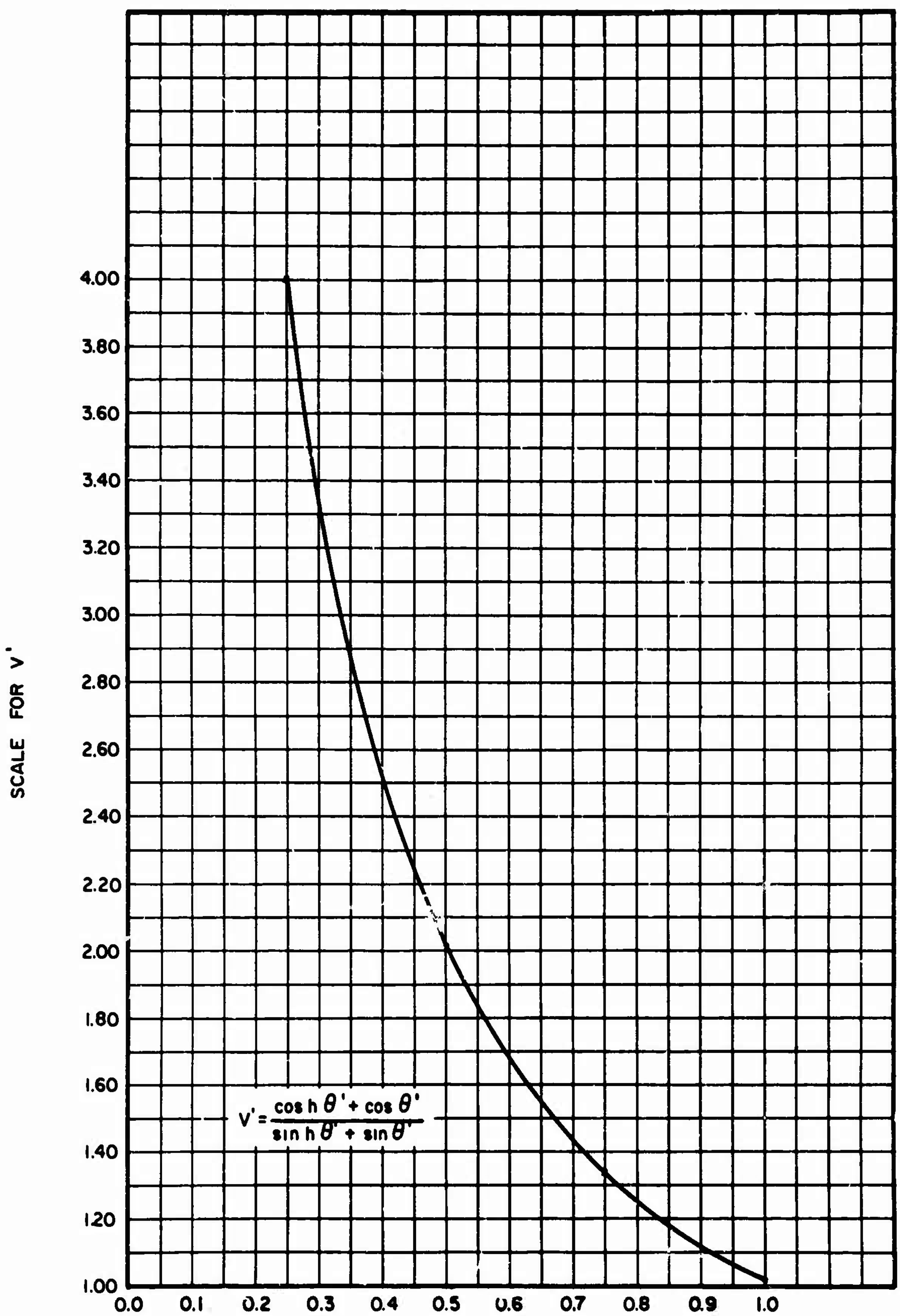


FIGURE 21A- SHELL STIFFNESS COEFFICIENT, V'

$(0 < \theta' \leq 0.8)$

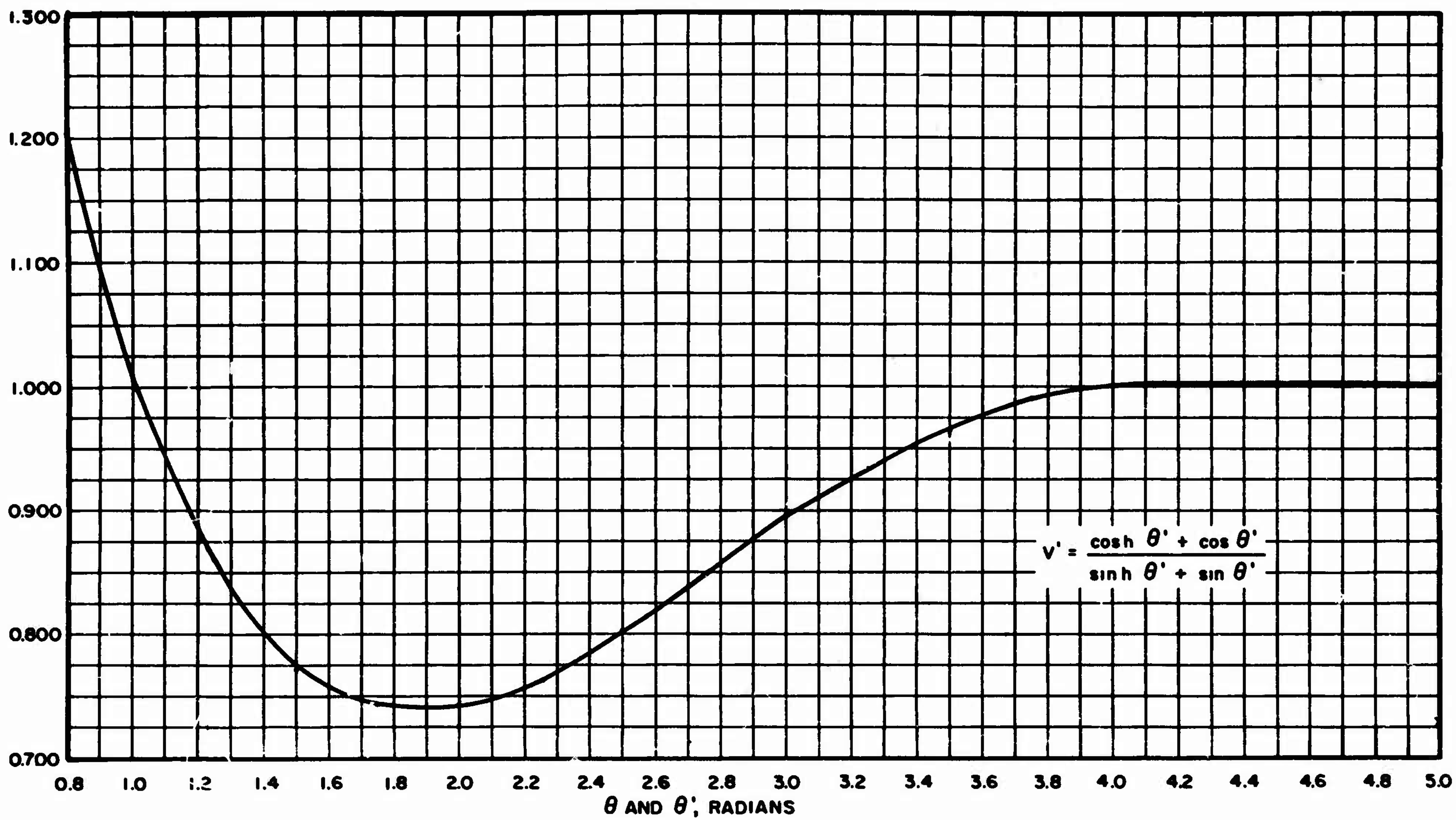


FIGURE 21B - SHELL STIFFNESS COEFFICIENT, V'

($\theta' > 0.8$)

α	Γ	α	Γ
1	57.29434500	46	1.17792630
2	28.64498500	47	1.14992140
3	19.09423500	48	1.12301850
4	14.31812900	49	1.09715000
5	11.45188300	50	1.07225360
6	9.54057000	51	1.04827190
7	8.17492900	52	1.02515200
8	7.15033400	53	1.00284490
9	6.35310300	54	.98130540
10	5.71502700	55	.96049120
11	5.19269900	56	.94036335
12	4.75718290	57	.92088555
13	4.38844430	58	.90202400
14	4.07217380	59	.88374715
15	3.79787760	60	.86602550
16	3.55768540	61	.84883175
17	3.34557860	62	.83213990
18	3.15687620	63	.81592600
19	2.98788270	64	.80016740
20	2.83564130	65	.78484295
21	2.69775890	66	.76993260
22	2.57227730	67	.75541770
23	2.45757880	68	.74128060
24	2.35231540	69	.72750460
25	2.25535450	70	.71407410
26	2.16573820	71	.70097430
27	2.08265020	72	.68819110
28	2.00539070	73	.67571135
29	1.93335680	74	.66352255
30	1.86602560	75	.65161260
31	1.80294200	76	.63997090
32	1.74370740	77	.62858625
33	1.68797190	78	.61744865
34	1.63542650	79	.60654860
35	1.58579760	80	.59587685
36	1.53884200	81	.58542490
37	1.49434260	82	.57518435
38	1.45210560	83	.56514730
39	1.41195660	84	.55530640
40	1.37373890	85	.54565435
41	1.33731090	86	.53618445
42	1.30254470	87	.52689015
43	1.26932410	88	.51776525
44	1.23754360	89	.50880375
45	1.20710690	90	.50000010

TABLE 1 -- AVERAGE NORMAL (HOOP) COEFFICIENT, Γ

α	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS						
	0*	0.1	0.2	0.3	0.4	0.5	0.6	
1	+.00000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.
2	+.00000025	.0000010	.0000005	.0000002	.0000005	.0000005	.0000005	.
3	+.00000100	.0000010	.0000007	.0000007	.0000002	0.0000000	0.0000000	0.
4	-.00000300	-.0000030	-.0000032	-.0000040	-.0000045	-.0000050	-.0000050	-.
5	-.00000675	-.0000067	-.0000077	-.0000087	-.0000100	-.0000110	-.0000112	-.
6	-.00001042	-.0000107	-.0000122	-.0000140	-.0000160	-.0000177	-.0000182	-.
7	-.00001762	-.0000182	-.0000205	-.0000235	-.0000267	-.0000295	-.0000302	-.
8	-.00002652	-.0000275	-.0000307	-.0000352	-.0000402	-.0000442	-.0000455	-.
9	-.00003790	-.0000395	-.0000440	-.0000505	-.0000577	-.0000635	-.0000650	-.
10	-.00005190	-.0000542	-.0000602	-.0000692	-.0000792	-.0000870	-.0000892	-.
11	-.00006897	-.0000720	-.0000802	-.0000922	-.0001052	-.0001157	-.0001187	-.
12	-.00009017	-.0000939	-.0001047	-.0001203	-.0001373	-.0001510	-.0001549	-.
13	-.00011480	-.0001196	-.0001333	-.0001532	-.0001749	-.0001923	-.0001974	-.
14	-.00014365	-.0001497	-.0001668	-.0001918	-.0002190	-.0002408	-.0002473	-.
15	-.00017692	-.0001844	-.0002055	-.0002362	-.0002699	-.0002969	-.0003050	-.
16	-.00021520	-.0002243	-.0002500	-.0002874	-.0003283	-.0003613	-.0003714	-.
17	-.00025872	-.0002696	-.0003005	-.0003456	-.0003949	-.0004347	-.0004470	-.
18	-.00030785	-.0003208	-.0003576	-.0004112	-.0004701	-.0005176	-.0005325	-.
19	-.00036277	-.0003781	-.0004215	-.0004847	-.0005542	-.0006106	-.0006285	-.
20	-.00042432	-.0004422	-.0004930	-.0005671	-.0006485	-.0007147	-.0007361	-.
21	-.00049255	-.0005133	-.0005723	-.0006584	-.0007532	-.0008303	-.0008557	-.
22	-.00056785	-.0005919	-.0006599	-.0007593	-.0008688	-.0009583	-.0009882	-.
23	-.00065077	-.0006783	-.0007563	-.0008704	-.0009962	-.0010994	-.0011344	-.
24	-.00074165	-.0007731	-.0008621	-.0009922	-.0011360	-.0012542	-.0012952	-.
25	-.00084102	-.0008767	-.0009777	-.0011254	-.0012890	-.0014238	-.0014714	-.
26	-.00094920	-.0009894	-.0011035	-.0012705	-.0014557	-.0016088	-.0016638	-.
27	-.00106670	-.0011120	-.0012402	-.0014283	-.0016369	-.0018101	-.0018735	-.
28	-.00119397	-.0012447	-.0013884	-.0015992	-.0018335	-.0020286	-.0021014	-.
29	-.00133150	-.0013881	-.0015484	-.0017840	-.0020461	-.0022652	-.0023485	-.
30	-.00147982	-.0015427	-.0017211	-.0019833	-.0022756	-.0025209	-.0026160	-.
31	-.00163937	-.0017091	-.0019069	-.0021979	-.0025229	-.0027966	-.0029048	-.
32	-.00181072	-.0018878	-.0021065	-.0024285	-.0027888	-.0030933	-.0032162	-.
33	-.00199440	-.0020793	-.0023204	-.0026758	-.0030741	-.0034122	-.0035513	-.
34	-.00219102	.0022884	-.0025495	-.0029407	-.0033800	-.0037544	-.0039115	-.
35	-.00240110	-.0025035	-.0027943	-.0032239	-.0037072	-.0041210	-.0042980	-.
36	-.00262522	-.0027372	-.0030556	-.0035262	-.0040569	-.0045130	-.0047123	-.
37	-.00286410	-.0029864	-.0033340	-.0038486	-.0044300	-.0049320	-.0051557	-.
38	-.00311827	-.0032514	-.0036304	-.0041919	-.0048277	-.0053791	-.0056298	-.
39	-.00338847	-.0035333	-.0039456	-.0045572	-.0052511	-.0058558	-.0061363	-.
40	-.00367535	-.0038326	-.0042802	-.0049452	-.0057013	-.0063634	-.0066767	-.
41	-.00397965	-.0041500	-.0046353	-.0053570	-.0061796	-.0069034	-.0072529	-.
42	-.00430210	-.0044864	-.0050117	-.0057938	-.0066873	-.0074774	-.0078667	-.
43	-.00464347	-.0048425	-.0054102	-.0062566	-.0072257	-.0080870	-.0085200	-.
44	-.00500455	-.0052192	-.0058319	-.0067464	-.0077962	-.0087340	-.0092149	-.
45	-.00538620	-.0056174	-.0062777	-.0072645	-.0084003	-.0094201	-.0099536	-.

TABLE 2A - DEFLECTION COEFFICIENT, δx (1 ⟨

* At x/α = 0, coefficient is at mid-length
** At x/α = 1.0, coefficient is at strut

A

FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α									
0.[illegible]	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0**
.0000[illegible]00	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000	.0000000
.0000[illegible]10	.0000005	.0000002	.0000005	.0000005	.0000005	.0000002	.0000005	.0000005	.0000015
0.0000[illegible]10	.0000007	.0000007	.0000002	0.0000000	0.0000000	0.0000000	.0000010	.0000020	.0000040
-.0000[illegible]30	-.0000032	-.0000040	-.0000045	-.0000050	-.0000050	-.0000045	-.0000030	-.0000002	.0000045
-.0000[illegible]67	-.0000077	-.0000087	-.0000100	-.0000110	-.0000112	-.0000102	-.0000075	-.0000017	.0000070
-.0000[illegible]107	-.0000122	-.0000140	-.0000160	-.0000177	-.0000182	-.0000167	-.0000117	-.0000020	.0000135
-.0000[illegible]182	-.0000205	-.0000235	-.0000267	-.0000295	-.0000302	-.0000275	-.0000195	-.0000042	.0000205
-.0000[illegible]275	-.0000307	-.0000352	-.0000402	-.0000442	-.0000455	-.0000412	-.0000295	-.0000067	.0000305
-.0000[illegible]395	-.0000440	-.0000505	-.0000577	-.0000635	-.0000650	-.0000592	-.0000425	-.0000100	.0000432
-.0000[illegible]542	-.0000602	-.0000692	-.0000792	-.0000870	-.0000892	-.0000812	-.0000582	-.0000135	.0000595
-.0001[illegible]720	-.0000802	-.0000922	-.0001052	-.0001157	-.0001187	-.0001085	-.0000775	-.0000180	.0000795
-.0001[illegible]39	-.0001047	-.0001203	-.0001373	-.0001510	-.0001549	-.0001414	-.0001014	-.0000240	.0001029
-.0001[illegible]196	-.0001333	-.0001532	-.0001749	-.0001923	-.0001974	-.0001804	-.0001294	-.0000308	.0001311
-.0002[illegible]497	-.0001668	-.0001918	-.0002190	-.0002408	-.0002473	-.0002261	-.0001624	-.0000390	.0001639
-.0002[illegible]344	-.0002055	-.0002362	-.0002699	-.0002969	-.0003050	-.0002790	-.0002006	-.0000485	.0002020
-.0003[illegible]243	-.0002500	-.0002874	-.0003283	-.0003613	-.0003714	-.0003399	-.0002447	-.0000596	.0002456
-.0004[illegible]696	-.0003005	-.0003456	-.0003949	-.0004347	-.0004470	-.0004094	-.0002952	-.0000726	.0002951
-.0004[illegible]208	-.0003576	-.0004112	-.0004701	-.0005176	-.0005325	-.0004882	-.0003524	-.0000874	.0003510
-.0005[illegible]781	-.0004215	-.0004847	-.0005542	-.0006106	-.0006285	-.0005766	-.0004168	-.0001042	.0004139
-.0006[illegible]422	-.0004930	-.0005671	-.0006485	-.0007147	-.0007361	-.0006759	-.0004894	-.0001235	.0004837
-.0007[illegible]133	-.0005723	-.0006584	-.0007532	-.0008303	-.0008557	-.0007864	-.0005703	-.0001454	.0005614
-.0009[illegible]919	-.0006599	-.0007593	-.0008688	-.0009583	-.0009882	-.0009090	-.0006604	-.0001701	.0006471
-.0010[illegible]783	-.0007563	-.0008704	-.0009962	-.0010994	-.0011344	-.0010446	-.0007602	-.0001979	.0007414
-.0011[illegible]731	-.0008621	-.0009922	-.0011360	-.0012542	-.0012952	-.0011938	-.0008704	-.0002291	.0008447
-.0013[illegible]767	-.0009777	-.0011254	-.0012890	-.0014238	-.0014714	-.0013577	-.0009919	-.0002640	.0009575
-.0015[illegible]894	-.0011035	-.0012705	-.0014557	-.0016088	-.0016638	-.0015370	-.0011252	-.0003029	.0010803
-.0017[illegible]120	-.0012402	-.0014283	-.0016369	-.0018101	-.0018735	-.0017328	-.0012712	-.0003464	.0012136
-.0019[illegible]447	-.0013884	-.0015992	-.0018335	-.0020286	-.0021014	-.0019460	-.0014308	-.0003947	.0013579
-.0021[illegible]881	-.0015484	-.0017840	-.0020461	-.0022652	-.0023485	-.0021776	-.0016048	-.0004483	.0015138
-.0024[illegible]427	-.0017211	-.0019833	-.0022756	-.0025209	-.0026160	-.0024289	-.0017942	-.0005077	.0016818
-.0027[illegible]091	-.0019069	-.0021979	-.0025229	-.0027966	-.0029048	-.0027008	-.0020001	-.0005734	.0016825
-.0029[illegible]878	-.0021065	-.0024285	-.0027888	-.0030933	-.0032162	-.0029946	-.0022234	-.0006461	.0020563
-.0033[illegible]793	-.0023204	-.0026758	-.0030741	-.0034122	-.0035513	-.0033116	-.0024654	-.0007263	.0022640
-.0036[illegible]884	-.0025495	-.0029407	-.0033800	-.0037544	-.0039115	-.0036533	-.0027272	-.0008147	.0024861
-.0040[illegible]035	-.0027943	-.0032239	-.0037072	-.0041210	-.0042980	-.0040207	-.0030102	-.0009121	.0027233
-.0044[illegible]372	-.0030556	-.0035262	-.0040569	-.0045130	-.0047123	-.0044156	-.0033156	-.0010193	.0029763
-.0048[illegible]864	-.0033340	-.0038486	-.0044300	-.0049320	-.0051557	-.0048395	-.0036451	-.0011372	.0032456
-.0052[illegible]514	-.0036304	-.0041919	-.0048277	-.0053791	-.0056298	-.0052940	-.0040001	-.0012667	.0035321
-.0057[illegible]333	-.0039456	-.0045572	-.0052511	-.0058558	-.0061363	-.0057810	-.0043823	-.0014090	.0038363
-.0063[illegible]326	-.0042802	-.0049452	-.0057013	-.0063634	-.0066767	-.0063021	-.0047936	-.0015651	.0041591
-.0068[illegible]500	-.0046353	-.0053570	-.0061796	-.0069034	-.0072529	-.0068596	-.0052358	-.0017363	.0045012
-.0074[illegible]864	-.0050117	-.0057938	-.0066873	-.0074774	-.0078667	-.0074553	-.0057110	-.0019240	.0048634
-.0080[illegible]425	-.0054102	-.0062556	-.0072257	-.0080870	-.0085200	-.0080915	-.0062214	-.0021298	.0052466
-.0087[illegible]192	-.0058319	-.0067464	-.0077962	-.0087340	-.0092149	-.0087705	-.0067694	-.0023551	.0056516
-.0094[illegible]174	-.0062777	-.0072645	-.0084003	-.0094201	-.0099536	-.0094949	-.0073574	-.0026020	.0060793

TABLE 2A - DEFLECTION COEFFICIENT, δ_x ($1 < \alpha \leq 45$)

* At x/α = 0, coefficient is at mid-length
** At x/α = 1.0, coefficient is at strut

α	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α								
	0*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
46	-.00578925	-.0060380	-.0067486	-.0078122	-.0090393	-.0101472	-.0107383	-.0102672	-.0
47	-.00621460	-.0064818	-.0072457	-.0083906	-.0097150	-.0109173	-.0115715	-.0110904	-.0
48	-.00666320	-.0069499	-.0077701	-.0090011	-.0104290	-.0117323	-.0124556	-.0119672	-.0
49	-.00713597	-.0074433	-.0083229	-.0096451	-.0111829	-.0125945	-.0133934	-.0129010	-.0
50	-.00763401	-.0079630	-.0089055	-.0103241	-.0119787	-.0135062	-.0143878	-.0138951	-.0
51	-.00815829	-.0085102	-.0095189	-.0110396	-.0128181	-.0144696	-.0154415	-.0149531	-.0
52	-.00870996	-.0090860	-.0101645	-.0117930	-.0137032	-.0154875	-.0165578	-.0160788	-.0
53	-.00929014	-.0096915	-.0108437	-.0125861	-.0146360	-.0165622	-.0177401	-.0172764	-.0
54	-.00990002	-.0103282	-.0115580	-.0134207	-.0156187	-.0176967	-.0189919	-.0185503	-.0
55	-.01054083	-.0109971	-.0123086	-.0142984	-.0166535	-.0188937	-.0203169	-.0199050	-.0
56	-.01121394	-.0116998	-.0130973	-.0152212	-.0177429	-.0201565	-.0217190	-.0213456	-.0
57	-.01192062	-.0124375	-.0139257	-.0161910	-.0188892	-.0214882	-.0232025	-.0228775	-.0
58	-.01266231	-.0132119	-.0147953	-.0172099	-.0200951	-.0228922	-.0247718	-.0245063	-.0
59	-.01344052	-.0140245	-.0157081	-.0182800	-.0213634	-.0243720	-.0264317	-.0262384	-.0
60	-.01425674	-.0148767	-.0166658	-.0194036	-.0226968	-.0259315	-.0281871	-.0280801	-.0
61	-.01511265	-.0157705	-.0176704	-.0205831	-.0240986	-.0275746	-.0300434	-.0300387	-.0
62	-.01600994	-.0167075	-.0187239	-.0218209	-.0255717	-.0293056	-.0320062	-.0321216	-.0
63	-.01695033	-.0176897	-.0198284	-.0231196	-.0271196	-.0311288	-.0340815	-.0343372	-.0
64	-.01793569	-.0187188	-.0209861	-.0244819	-.0287457	-.0330490	-.0362758	-.0366941	-.0
65	-.01896797	-.0197970	-.0221994	-.0259107	-.0304537	-.0350710	-.0385958	-.0392019	-.0
66	-.02004925	-.0209265	-.0234708	-.0274090	-.0322475	-.0372002	-.0410488	-.0418708	-.0
67	-.02118160	-.0221094	-.0248026	-.0289798	-.0341312	-.0394419	-.0436426	-.0447117	-.0
68	-.02236728	-.0233481	-.0261977	-.0306266	-.0361091	-.0418022	-.0463854	-.0477367	-.0
69	-.02360866	-.0246450	-.0276589	-.0323528	-.0381858	-.0442872	-.0492859	-.0509584	-.0
70	-.02490820	-.0260028	-.0291891	-.0341621	-.0403659	-.0469035	-.0523537	-.0543911	-.0
71	-.02626844	-.0274241	-.0307915	-.0360581	-.0426545	-.0496579	-.0555986	-.0580495	-.0
72	-.02769215	-.0289118	-.0324692	-.0380451	-.0450569	-.0525579	-.0590316	-.0619502	-.0
73	-.02918218	-.0304690	-.0342258	-.0401273	-.0475788	-.0556112	-.0626641	-.0661109	-.0
74	-.03074153	-.0320987	-.0360648	-.0423091	-.0502260	-.0588263	-.0665084	-.0705509	-.0
75	-.03237340	-.0338043	-.0379901	-.0445953	-.0530049	-.0622119	-.0705776	-.0752912	-.0
76	-.03408106	-.0355892	-.0400056	-.0469907	-.0559220	-.0657773	-.0748860	-.0803548	-.0
77	-.03586802	-.0374572	-.0421156	-.0495008	-.0589842	-.0695325	-.0794487	-.0857665	-.0
78	-.03773797	-.0394120	-.0443244	-.0521309	-.0621990	-.0734881	-.0842819	-.0915537	-.0
79	-.03969484	-.0414579	-.0466369	-.0548869	-.0655742	-.0776553	-.0894033	-.0977463	-.0
80	-.04174270	-.0435990	-.0490580	-.0577750	-.0691181	-.0820461	-.0948316	-.1043768	-.1
81	-.04388587	-.0458399	-.0515927	-.0608017	-.0728395	-.0866732	-.1005872	-.1114812	-.1
82	-.04612895	-.0481855	-.0542468	-.0639739	-.0767476	-.0915504	-.1066918	-.1190989	-.1
83	-.04847672	-.0506407	-.0570259	-.0672989	-.0808523	-.0966920	-.1131691	-.1272733	-.1
84	-.05093427	-.0532109	-.0599362	-.0707843	-.0851640	-.1021136	-.1200445	-.1360521	-.1
85	-.05350705	-.0559018	-.0629843	-.0744383	-.0896939	-.1078318	-.1273454	-.1454882	-.1
86	-.05620066	-.0587193	-.0661769	-.0782695	-.0944538	-.1138641	-.1351016	-.1556400	-.
87	-.05902118	-.0616697	-.0695214	-.0822871	-.0994561	-.1202294	-.1433451	-.1665721	-.
88	-.06197499	-.0647598	-.0730255	-.0865008	-.1047143	-.1269480	-.1521109	-.1783567	-.
89	-.06506882	-.0679967	-.0766974	-.0909208	-.1102424	-.1340415	-.1614367	-.1910735	-.
90	-.06830984	-.0713877	-.0805457	-.0955582	-.1160555	-.1415331	-.1713634	-.2048120	-.

TABLE 2B-DEFLECTION COEFFICIENT, δ_x (45 < α ≤ 90)

* At x/α = 0, coefficient is at mid-length

** At x/α = 1.0, coefficient is at strut

A

FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α									
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0**
-.0[illegible]80	-.0067486	-.0078122	-.0090393	-.0101472	-.0107383	-.0102672	-.0079883	-.0029723	.0065306
-.0[illegible]18	-.0072457	-.0083906	-.0097150	-.0109173	-.0115715	-.0110904	-.0086650	-.0031683	.0070064
-.0[illegible]99	-.0077701	-.0090011	-.0104290	-.0117323	-.0124556	-.0119672	-.0093905	-.0034922	.0075078
-.0[illegible]33	-.0083229	-.0096451	-.0111829	-.0125945	-.0133934	-.0129010	-.0101682	-.0038467	.0080358
-.0[illegible]30	-.0089055	-.0103241	-.0119787	-.0135062	-.0143878	-.0138951	-.0110020	-.0042348	.0085914
-.0[illegible]02	-.0095189	-.0110396	-.0128161	-.0144696	-.0154415	-.0149531	-.0118957	-.0046595	.0091759
-.0[illegible]60	-.0101645	-.0117930	-.0137032	-.0154875	-.0165578	-.0160788	-.0128535	-.0051244	.0097902
-.0[illegible]15	-.0108437	-.0125861	-.0146360	-.0165622	-.0177401	-.0172764	-.0138800	-.0056334	.0104357
-.0[illegible]82	-.0115580	-.0134207	-.0156187	-.0176967	-.0189919	-.0185503	-.0149804	-.0061906	.0111136
-.0[illegible]71	-.0123086	-.0142984	-.0166535	-.0188937	-.0203169	-.0199050	-.0161598	-.0068009	.0118252
-.0[illegible]98	-.0130973	-.0152212	-.0177429	-.0201565	-.0217190	-.0213456	-.0174243	-.0074694	.0125718
-.0[illegible]75	-.0139257	-.0161910	-.0188892	-.0214882	-.0232025	-.0228775	-.0187802	-.0082021	.0133550
-.0[illegible]19	-.0147953	-.0172099	-.0200951	-.0228922	-.0247718	-.0245063	-.0202343	-.0090052	.0141761
-.0[illegible]45	-.0157081	-.0182800	-.0213634	-.0243720	-.0264317	-.0262384	-.0217942	-.0098861	.0150367
-.0[illegible]67	-.0166658	-.0194036	-.0226968	-.0259315	-.0281871	-.0280801	-.0234680	-.0108526	.0159385
-.0[illegible]705	-.0176704	-.0205831	-.0240986	-.0275746	-.0300434	-.0300387	-.0252648	-.0119140	.0168832
-.0[illegible]75	-.0187239	-.0218209	-.0255717	-.0293056	-.0320062	-.0321216	-.0271941	-.0130800	.0178723
-.0[illegible]97	-.0198284	-.0231196	-.0271196	-.0311288	-.0340815	-.0343372	-.0292668	-.0143622	.0189080
-.0[illegible]88	-.0209861	-.0244819	-.0287457	-.0330490	-.0362758	-.0366941	-.0314943	-.0157729	.0199920
-.0[illegible]70	-.0221994	-.0259107	-.0304537	-.0350710	-.0385958	-.0392019	-.0338895	-.0173267	.0211264
-.0[illegible]265	-.0234708	-.0274090	-.0322475	-.0372002	-.0410488	-.0418708	-.0364665	-.0190396	.0223132
-.0[illegible]094	-.0248026	-.0289798	-.0341312	-.0394419	-.0436426	-.0447117	-.0392407	-.0209300	.0235548
-.0[illegible]481	-.0261977	-.0306266	-.0361091	-.0418022	-.0463854	-.0477367	-.0422290	-.0230186	.0248534
-.0[illegible]450	-.0276589	-.0323528	-.0381858	-.0442872	-.0492859	-.0509584	-.0454503	-.0253293	.0262115
-.0[illegible]028	-.0291891	-.0341621	-.0403559	-.0469035	-.0523537	-.0543911	-.0489252	-.0278893	.0276315
-.0[illegible]241	-.0307915	-.0360581	-.0426545	-.0496579	-.0555986	-.0580495	-.0526767	-.0307297	.0291163
-.0[illegible]118	-.0324692	-.0380451	-.0450569	-.0525579	-.0590316	-.0619502	-.0567303	-.0338865	.0306684
-.0[illegible]690	-.0342258	-.0401273	-.0475788	-.0556112	-.0626641	-.0661109	-.0611143	-.0374015	.0322911
-.0[illegible]987	-.0360648	-.0423091	-.0502260	-.0588263	-.0665084	-.0705509	-.0658603	-.0413231	.0339872
-.0[illegible]043	-.0379901	-.0445953	-.0530049	-.0622119	-.0705776	-.0752912	-.0710034	-.0457080	.0357600
-.0[illegible]892	-.0400056	-.0469907	-.0559220	-.0657773	-.0748860	-.0803548	-.0765833	-.0506228	.0376131
-.0[illegible]572	-.0421156	-.0495008	-.0589842	-.0695325	-.0794487	-.0857665	-.0826441	-.0561462	.0395499
-.0[illegible]120	-.0443244	-.0521309	-.0621990	-.0734881	-.0842819	-.0915537	-.0892356	-.0623717	.0415743
-.0[illegible]579	-.0466369	-.0548869	-.0655742	-.0776553	-.0894033	-.0977463	-.0964142	-.0694115	.0436902
-.1[illegible]990	-.0490580	-.0577750	-.0691181	-.0820461	-.0948316	-.1043768	-.1042436	-.0774007	.0459018
-.1[illegible]399	-.0515927	-.0608017	-.0728395	-.0866732	-.1005872	-.1114812	-.1127959	-.0865039	.0482136
-.1[illegible]855	-.0542468	-.0639739	-.0767476	-.0915504	-.1066918	-.1190989	-.1221537	-.0969230	.0506301
-.1[illegible]407	-.0570259	-.0672989	-.0808523	-.0966920	-.1131691	-.1272733	-.1324111	-.1089087	.0531565
-.1[illegible]109	-.0599362	-.0707843	-.0851640	-.1021136	-.1200445	-.1360521	-.1436765	-.1227753	.0557977
-.1[illegible]018	-.0629843	-.0744383	-.0896939	-.1078318	-.1273454	-.1454882	-.1560746	-.1389223	.0585594
-.1[illegible]193	-.0661769	-.0782695	-.0944538	-.1138641	-.1351016	-.1556400	-.1697499	-.1578652	.0614472
-.1[illegible]697	-.0695214	-.0822871	-.0994561	-.1202294	-.1433451	-.1665721	-.1848707	-.1802796	.0644675
-.2[illegible]598	-.0730255	-.0865008	-.1047143	-.1269480	-.1521109	-.1783567	-.2016337	-.2070684	.0676266
-.2[illegible]967	-.0766974	-.0909208	-.1102424	-.1340415	-.1614367	-.1910735	-.2202706	-.2394654	.0709313
-.2[illegible]877	-.0805457	-.0955582	-.1160555	-.1415331	-.1713634	-.2048120	-.2410553	-.2792007	.0743891

TABLE 2B-DEFLECTION COEFFICIENT, δx (45 < α ≤ 90)

* At x/α = 0, coefficient is at mid-length

** At x/α = 1.0, coefficient is at strut

B

α	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α								
	0*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
1	.0014560	.0014170	.0012865	.0010685	.0007630	.0003700	-.0001100	-.0006775	-.001
2	.0029100	.0028230	.0025610	.0021245	.0015135	.0007280	-.0002320	-.0013665	-.002
3	.0043650	.0042355	.0038425	.0031880	.0022710	.0010925	-.0003475	-.0020500	-.004
4	.0058215	.0056470	.0051230	.0042495	.0030270	.0014550	-.0004655	-.0027355	-.005
5	.0072790	.0070605	.0064050	.0053130	.0037840	.0018185	-.0005835	-.0034215	-.006
6	.0087382	.0084759	.0076891	.0063779	.0045423	.0021827	-.0007007	-.0041077	-.008
7	.0101990	.0098928	.0089743	.0074436	.0053009	.0025465	-.0008190	-.0047953	-.009
8	.0116623	.0113118	.0102613	.0085107	.0060603	.0029105	-.0009378	-.0054842	-.010
9	.0131278	.0127335	.0115506	.0095795	.0068207	.0032749	-.0010572	-.0061744	-.012
10	.0145963	.0141577	.0128422	.0106502	.0075823	.0036394	-.0011772	-.0068662	-.013
11	.0160680	.0155851	.0141365	.0117229	.0083450	.0040041	-.0012981	-.0075598	-.014
12	.0175431	.0170156	.0154336	.0127976	.0091088	.0043689	-.0014200	-.0082555	-.016
13	.0190220	.0184498	.0167338	.0138748	.0098742	.0047340	-.0015429	-.0089534	-.017
14	.0205050	.0198879	.0180374	.0149545	.0106410	.0050994	-.0016668	-.0096538	-.018
15	.0219924	.0213304	.0193448	.0160371	.0114095	.0054651	-.0017919	-.0103566	-.020
16	.0234845	.0227773	.0206561	.0171227	.0121797	.0058310	-.0019183	-.0110623	-.021
17	.0249818	.0242291	.0219716	.0182114	.0129518	.0061973	-.0020460	-.0117710	-.022
18	.0264846	.0256861	.0232917	.0193036	.0137258	.0065638	-.0021753	-.0124829	-.024
19	.0279931	.0271487	.0246166	.0203996	.0145021	.0069308	-.0023059	-.0131981	-.025
20	.0295077	.0286172	.0259466	.0214993	.0152806	.0072981	-.0024383	-.0139170	-.027
21	.0310288	.0300918	.0272819	.0226030	.0160614	.0076658	-.0025724	-.0146396	-.028
22	.0325568	.0315730	.0286230	.0237112	.0168448	.0080339	-.0027083	-.0153662	-.029
23	.0340919	.0330610	.0299700	.0248238	.0176308	.0084025	-.0028462	-.0160971	-.031
24	.0356346	.0345563	.0313233	.0259412	.0184196	.0087715	-.0029860	-.0168323	-.032
25	.0371852	.0360591	.0326831	.0270636	.0192112	.0091410	-.0031280	-.0175723	-.034
26	.0387441	.0375699	.0340499	.0281912	.0200059	.0095109	-.0032721	-.0183170	-.035
27	.0403117	.0390890	.0354238	.0293243	.0208038	.0098814	-.0034186	-.0190669	-.037
28	.0418883	.0406168	.0368053	.0304630	.0216050	.0102524	-.0035676	-.0198221	-.038
29	.0434744	.0421536	.0381947	.0316077	.0224097	.0106240	-.0037190	-.0205827	-.039
30	.0450703	.0436999	.0395922	.0327587	.0232179	.0109962	-.0038731	-.0213492	-.041
31	.0466765	.0452559	.0409982	.0339160	.0240299	.0113689	-.0040300	-.0221217	-.042
32	.0482934	.0468222	.0424132	.0350801	.0248458	.0117423	-.0041897	-.0229004	-.044
33	.0499214	.0483991	.0438373	.0362511	.0256658	.0121163	-.0043524	-.0236856	-.045
34	.0515608	.0499869	.0452709	.0374294	.0264898	.0124909	-.0045182	-.0244776	-.047
35	.0532122	.0515863	.0467145	.0386151	.0273183	.0128662	-.0046872	-.0252766	-.048
36	.0548761	.0531975	.0481684	.0398087	.0281513	.0132423	-.0048596	-.0260828	-.050
37	.0565527	.0548210	.0496330	.0410103	.0289889	.0136194	-.0050354	-.0268966	-.051
38	.0582428	.0564573	.0511086	.0422203	.0298315	.0139965	-.0052148	-.0277182	-.053
39	.0599466	.0581067	.0525957	.0434389	.0306789	.0143748	-.0053980	-.0285479	-.054
40	.0616647	.0597698	.0540946	.0446665	.0315316	.0147538	-.0055850	-.0293859	-.056
41	.0633975	.0614470	.0556057	.0459033	.0323896	.0151337	-.0057760	-.0302326	-.058
42	.0651456	.0631389	.0571295	.0471497	.0332531	.0155144	-.0059712	-.0310883	-.059
43	.0669096	.0648459	.0586664	.0484060	.0341224	.0158960	-.0061706	-.0319532	-.061
44	.0686899	.0665685	.0602169	.0496725	.0349975	.0162784	-.0063745	-.0328277	-.062
45	.0704870	.0683072	.0617813	.0509496	.0358787	.0166617	-.0065830	-.0337121	-.064

TABLE 3A-MOMENT COEFFICIENT, ξx ($1 \leq \alpha \leq 45$)

* At x/α = 0, coefficient is at mid-length
** At x/α = 1.0, coefficient is at strut

A

	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α								
0.	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0**
.0013	.0012865	.0010685	.0007630	.0003700	-.0001100	-.0006775	-.0013315	-.0020735	-.0029025
.0026	.0025610	.0021245	.0015135	.0007280	-.0002320	-.0013665	-.0026755	-.0041595	-.0058175
.0040	.0038425	.0031880	.0022710	.0010925	-.0003475	-.0020500	-.0040135	-.0062395	-.0087265
.0053	.0051230	.0042495	.0030270	.0014550	-.0004655	-.0027355	-.0053545	-.0083225	-.0116385
.0066	.0064050	.0053130	.0037840	.0018185	-.0005835	-.0034215	-.0066960	-.0104060	-.0145520
.0080	.0076891	.0063779	.0045423	.0021827	-.0007007	-.0041077	-.0080378	-.0124906	-.0174657
.0093	.0089743	.0074436	.0053009	.0025465	-.0008190	-.0047953	-.0093817	-.0145777	-.0203823
.0107	.0102613	.0085107	.0060603	.0029105	-.0009378	-.0054842	-.0107276	-.0166670	-.0233013
.0120	.0115506	.0095795	.0068207	.0032749	-.0010572	-.0061744	-.0120754	-.0187588	-.0262230
.0134	.0128422	.0106502	.0075823	.0036394	-.0011772	-.0068662	-.0134256	-.0208537	-.0291480
.0147	.0141365	.0117229	.0083450	.0040041	-.0012981	-.0075598	-.0147785	-.0229518	-.0320765
.0161	.0154336	.0127976	.0091088	.0043689	-.0014200	-.0082555	-.0161346	-.0250537	-.0350090
.0174	.0167338	.0138748	.0098742	.0047340	-.0015429	-.0089534	-.0174938	-.0271596	-.0379458
.0188	.0180374	.0149545	.0106410	.0050994	-.0016668	-.0096538	-.0188566	-.0292699	-.0408873
.0202	.0193448	.0160371	.0114095	.0054651	-.0017919	-.0103566	-.0202232	-.0313848	-.0438338
.0215	.0206561	.0171227	.0121797	.0058310	-.0019183	-.0110623	-.0215939	-.0335048	-.0467858
.0229	.0219716	.0182114	.0129518	.0061973	-.0020460	-.0117710	-.0229691	-.0356303	-.0497436
.0243	.0232917	.0193036	.0137258	.0065638	-.0021753	-.0124829	-.0243489	-.0377616	-.0527076
.0257	.0246166	.0203996	.0145021	.0069308	-.0023059	-.0131981	-.0257337	-.0398989	-.0556781
.0271	.0259466	.0214993	.0152806	.0072981	-.0024383	-.0139170	-.0271238	-.0420428	-.0586557
.0285	.0272819	.0226030	.0160614	.0076658	-.0025724	-.0146396	-.0285196	-.0441936	-.0616407
.0299	.0286230	.0237112	.0168448	.0080339	-.0027083	-.0153662	-.0299212	-.0463516	-.0646333
.0313	.0299700	.0248238	.0176308	.0084025	-.0028462	-.0160971	-.0313290	-.0485173	-.0676342
.0327	.0313233	.0259412	.0184196	.0087715	-.0029860	-.0168323	-.0327433	-.0506909	-.0706436
.0341	.0326831	.0270636	.0192112	.0091410	-.0031280	-.0175723	-.0341645	-.0528730	-.0736621
.0355	.0340499	.0281912	.0200059	.0095109	-.0032721	-.0183170	-.0355928	-.0550638	-.0766899
.0370	.0354238	.0293243	.0208038	.0098814	-.0034186	-.0190669	-.0370286	-.0572638	-.0797277
.0384	.0368053	.0304630	.0216050	.0102524	-.0035676	-.0198221	-.0384722	-.0594734	-.0827757
.0399	.0381947	.0316077	.0224097	.0106240	-.0037190	-.0205827	-.0399239	-.0616931	-.0858344
.0413	.0395922	.0327587	.0232179	.0109962	-.0038731	-.0213492	-.0413841	-.0639231	-.0889042
.0428	.0409982	.0339160	.0240299	.0113689	-.0040300	-.0221217	-.0428532	-.0661640	-.0919857
.0443	.0424132	.0350801	.0248458	.0117423	-.0041897	-.0229004	-.0443315	-.0684161	-.0950792
.0458	.0438373	.0362511	.0256658	.0121163	-.0043524	-.0236856	-.0458193	-.0706800	-.0981853
.0473	.0452709	.0374294	.0264898	.0124909	-.0045182	-.0244776	-.0473170	-.0729561	-.1013045
.0488	.0467145	.0386151	.0273183	.0128662	-.0046872	-.0252766	-.0488250	-.0752447	-.1044371
.0503	.0481684	.0398087	.0281513	.0132423	-.0048596	-.0260828	-.0503437	-.0775464	-.1075837
.0518	.0496330	.0410103	.0289889	.0136190	-.0050354	-.0268966	-.0518734	-.0798617	-.1107449
.0534	.0511086	.0422203	.0298315	.0139965	-.0052148	-.0277182	-.0534146	-.0821910	-.1139209
.0549	.0525957	.0434389	.0306789	.0143748	-.0053980	-.0285479	-.0549676	-.0845348	-.1171126
.0565	.0540946	.0446665	.0315316	.0147538	-.0055850	-.0293859	-.0565329	-.0868937	-.1203204
.0581	.0556057	.0459033	.0323896	.0151337	-.0057760	-.0302326	-.0581109	-.0892681	-.1235448
.0597	.0571295	.0471497	.0332531	.0155144	-.0059712	-.0310883	-.0597020	-.0916585	-.1267863
.0613	.0586664	.0484060	.0341224	.0158960	-.0061706	-.0319532	-.0613066	-.0940655	-.1300456
.0629	.0602169	.0496725	.0349975	.0162784	-.0063745	-.0328277	-.0629252	-.0964896	-.1333232
.0645	.0617813	.0509496	.0358787	.0166617	-.0065830	-.0337121	-.0645583	-.0989315	-.1366197

5)

TABLE 3A-MOMENT COEFFICIENT, ξx $(1 < \alpha \leq 45)$

* At $x/\alpha = 0$, coefficient is at mid-length

** At $x/\alpha = 1.0$, coefficient is at strut

B

α	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α								
	0*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0
46	.0723015	.0700626	.0633602	.0522375	.0367662	.0170459	-.0067962	-.0346067	-.06
47	.0741341	.0718352	.0649540	.0535367	.0376601	.0174311	-.0070144	-.0355119	-.06
48	.0759853	.0736256	.0665632	.0548475	.0385608	.0178173	-.0072375	-.0364280	-.06
49	.0778557	.0754344	.0681883	.0561704	.0394684	.0182044	-.0074659	-.0373553	-.07
50	.0797458	.0772621	.0698298	.0575055	.0503830	.0185926	-.0076998	-.0382942	-.07
51	.0816564	.0791093	.0714882	.0588534	.0413049	.0189818	-.0079393	-.0392452	-.07
52	.0835881	.0809767	.0731641	.0602144	.0422344	.0193720	-.0081845	-.0402085	-.07
53	.0855416	.0828650	.0748580	.0615891	.0431717	.0197634	-.0084357	-.0411845	-.07
54	.0875175	.0847746	.0765704	.0629776	.0441169	.0201558	-.0086931	-.0421738	-.07
55	.0895166	.0867065	.0783020	.0643806	.0450705	.0205494	-.0089568	-.0431766	-.08
56	.0915395	.0886611	.0800534	.0657985	.0460325	.0209441	-.0092271	-.0441934	-.08
57	.0935871	.0906393	.0818251	.0672317	.0470033	.0213401	-.0095043	-.0452247	-.08
58	.0956600	.0926417	.0836178	.0686806	.0479831	.0217372	-.0097884	-.0462709	-.08
59	.0977592	.0946693	.0854322	.0701459	.0489722	.0221355	-.0100797	-.0473325	-.08
60	.0998854	.0967226	.0872689	.0716279	.0499709	.0225352	-.0103786	-.0434099	-.09
61	.1020395	.0988026	.0891287	.0731272	.0509794	.0229360	-.0106852	-.0495037	-.09
62	.1042223	.1009100	.0910121	.0746443	.0519980	.0233382	-.0109[illegible]	-.0506143	-.09
63	.1064346	.1030457	.0929200	.0761797	.0530271	.0237418	-.01132[illegible]	-.0517424	-.09
64	.1086776	.1052107	.0948532	.0777342	.0540670	.0241466	-.0116538	-.0528883	-.09
65	.1109522	.1074058	.0968124	.0793081	.0551179	.0245529	-.0119939	-.0540528	-.10
66	.1132591	.1096319	.0987984	.0809021	.0561803	.0249606	-.0123430	-.0552364	-.10
67	.1155997	.1118901	.1008121	.0825169	.0572544	.0253698	-.0127015	-.0564396	-.10
68	.1179748	.1141813	.1028543	.0841530	.0583407	.0257804	-.0130697	-.0576632	-.10
69	.1203856	.1165067	.1049259	.0858112	.0594394	.0261925	-.0134479	-.0589077	-.10
70	.1228333	.1188672	.1070280	.0874921	.0605510	.0266061	-.0138364	-.0601738	-.11
71	.1253189	.1212639	.1091613	.0891965	.0616758	.0270212	-.0142356	-.0614622	-.11
72	.1278437	.1236982	.1113269	.0909250	.0628143	.0274380	-.0146458	-.0627737	-.11
73	.1304090	.1261710	.1135258	.0926784	.0639668	.0278563	-.0150674	-.0641089	-.11
74	.1330160	.1286838	.1157592	.0944576	.0651338	.0282763	-.0155008	-.0654686	-.12
75	.1356662	.1312377	.1180281	.0962633	.0663158	.0286980	-.0159464	-.0668537	-.12
76	.1383608	.1338342	.1203336	.0980964	.0675132	.0291213	-.0164046	-.0682649	-.12
77	.1411015	.1364745	.1226769	.0999577	.0687265	.0295464	-.0168758	-.0697031	-.12
78	.1438896	.1391602	.1250594	.1018482	.0699561	.0299732	-.0173604	-.0711692	-.13
79	.1467268	.1418927	.1274822	.1037688	.0712026	.0304019	-.0178590	-.0726642	-.13
80	.1496146	.1446736	.1299467	.1057205	.0724666	.0308323	-.0183721	-.0741889	-.13
81	.1525548	.1475045	.1324542	.1077043	.0737486	.0312646	-.0189000	-.0757443	-.13
82	.1555492	.1503871	.1350063	.1097213	.0750491	.0316987	-.0194434	-.0773316	-.14
83	.1585995	.1533231	.1376043	.1117725	.0763688	.0321348	-.0200028	-.0789518	-.14
84	.1617078	.1563144	.1402499	.1138592	.0777082	.0325728	-.0205787	-.0806060	-.14
85	.1648758	.1593628	.1429448	.1159824	.0790681	.0330128	-.0211718	-.0822954	-.14
86	.1681059	.1624703	.1456904	.1181436	.0804491	.0334547	-.0217827	-.0840211	-.15
87	.1714000	.1656390	.1484888	.1203438	.0818519	.0338988	-.0224119	-.0857846	-.15
88	.1747605	.1688711	.1513416	.1225846	.0832772	.0343449	-.0230603	-.0875870	-.15
89	.1781897	.1721687	.1542507	.1248673	.0847258	.0347931	-.0237285	-.0894299	-.16
90	.1816901	.1755342	.1572183	.1271933	.0861985	.0352435	-.0244172	-.0913145	-.16

TABLE 3B-MOMENT COEFFICIENT, ξx (45 < α ≤ 90)

* At x/α =0, coefficient is at mid-length

** At x/α =1.0, coefficient is at strut

A

		FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α								
0.8		0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0**
.06620	626	.0633602	.0522375	.0367662	.0170459	-.0067962	-.0346067	-.0662064	-.1013916	-.1399358
.0678	352	.0649540	.0535367	.0376601	.0174311	-.0070144	-.0355119	-.0678699	-.1038706	-.1432720
.06954	256	.0665632	.0548475	.0385608	.0178173	-.0072375	-.0364280	-.0695492	-.1063690	-.1466290
.07124	344	.0681883	.0561704	.0394684	.0182044	-.0074659	-.0373553	-.0712450	-.1088874	-.1500074
.07295	621	.0698298	.0575055	.0503830	.0185926	-.0076998	-.0382942	-.0729578	-.1114266	-.1534079
.0746	093	.0714882	.0588534	.0413049	.0189818	-.0079393	-.0392452	-.0746880	-.1139871	-.1568313
.0764	767	.0731641	.0602144	.0422344	.0193720	-.0081845	-.0402085	-.0764362	-.1165695	-.1602781
.0782	650	.0748580	.0615891	.0431717	.0197634	-.0084357	-.0411845	-.0782030	-.1191747	-.1637491
.0799	746	.0765704	.0629776	.0441169	.0201558	-.0086931	-.0421738	-.0799891	-.1218032	-.1672452
.0817	065	.0783020	.0643806	.0450705	.0205494	-.0089568	-.0431766	-.0817948	-.1244558	-.1707669
.0836	611	.0800534	.0657985	.0460325	.0209441	-.0092271	-.0441934	-.0836209	-.1271333	-.1743151
.0854	393	.0818251	.0672317	.0470033	.0213401	-.0095043	-.0452247	-.0854681	-.1298363	-.1778907
.0873	417	.0836178	.0686806	.0479831	.0217372	-.0097884	-.0462709	-.0873368	-.1325657	-.1814944
.0892	693	.0854322	.0701459	.0489722	.0221355	-.0100797	-.0473325	-.0892279	-.1353222	-.1851271
.0911	226	.0872689	.0716279	.0499709	.0225352	-.0103786	-.0484099	-.0911420	-.1381068	-.1887896
.0930	026	.0891287	.0731272	.0509794	.0229360	-.0106852	-.0495037	-.0930798	-.1409201	-.1924829
.0950	100	.0910121	.0743443	.0519980	.0233382	-.0109997	-.0506143	-.0950421	-.1437633	-.1962079
.0970	457	.0929200	.0761797	.0530271	.0237418	-.0113225	-.0517424	-.0970295	-.1466370	-.1999656
.0990	107	.0948532	.0777342	.0540670	.0241466	-.0116538	-.0528883	-.0990429	-.1495422	-.2037569
.1010	058	.0968124	.0793081	.0551179	.0245529	-.0119939	-.0540528	-.1010831	-.1524800	-.2075828
.1031	319	.0987984	.0809021	.0561803	.0249606	-.0123430	-.0552364	-.1031508	-.1554512	-.2114445
.1052	901	.1008121	.0825169	.0572544	.0253698	-.0127015	-.0564396	-.1052470	-.1584570	-.2153429
.1073	813	.1028543	.0841530	.0583407	.0257804	-.0130697	-.0576632	-.1073725	-.1614984	-.2192793
.1095	067	.1049259	.0858112	.0594394	.0261925	-.0134479	-.0589077	-.1095282	-.1645764	-.2232547
.1117	672	.1070280	.0874921	.0605510	.0266061	-.0138364	-.0601738	-.1117152	-.1676922	-.2272704
.1139	639	.1091613	.0891965	.0616758	.0270212	-.0142356	-.0614622	-.1139342	-.1708469	-.2313275
.1161	982	.1113269	.0909250	.0628143	.0274380	-.0146458	-.0627737	-.1161864	-.1740418	-.2354274
.1184	710	.1135258	.0926784	.0639668	.0278563	-.0150674	-.0641089	-.1184729	-.1772781	-.2395714
.1207	838	.1157592	.0944576	.0651338	.0282763	-.0155008	-.0654686	-.1207946	-.1805572	-.2437608
.1231	377	.1180281	.0962633	.0663158	.0286980	-.0159464	-.0668537	-.1231527	-.1838802	-.2479972
.1255	342	.1203336	.0980964	.0675132	.0291213	-.0164046	-.0682649	-.1255484	-.1872487	-.2522818
.1279	745	.1226769	.0999577	.0687265	.0295464	-.0168758	-.0697031	-.1279829	-.1906640	-.2566163
.1304	602	.1250594	.1018482	.0699561	.0299732	-.0173604	-.0711692	-.1304574	-.1941278	-.2610023
.1329	927	.1274822	.1037688	.0712026	.0304019	-.0178590	-.0726642	-.1329732	-.1976414	-.2654413
.1355	736	.1299467	.1057205	.0724666	.0308323	-.0183721	-.0741889	-.1355317	-.2012065	-.2699350
.1381	045	.1324542	.1077043	.0737486	.0312646	-.0189000	-.0757443	-.1381342	-.2048247	-.2744853
.1407	871	.1350063	.1097213	.0750491	.0316987	-.0194434	-.0773316	-.1407823	-.2084979	-.2790940
.1434	231	.1376043	.1117725	.0763688	.0321348	-.0200028	-.0789518	-.1434774	-.2122278	-.2837629
.1462	144	.1402499	.1138592	.0777082	.0325728	-.0205787	-.0806060	-.1462211	-.2160163	-.2884941
.1490	628	.1429448	.1159824	.0790681	.0330128	-.0211718	-.0822954	-.1490151	-.2198653	-.2932895
.1518	703	.1456904	.1181436	.0804491	.0334547	-.0217827	-.0840211	-.1518611	-.2237769	-.2981515
.1547	390	.1484888	.1203438	.0818519	.0338988	-.0224119	-.0857846	-.1547608	-.2277532	-.3030821
.1577	711	.1513416	.1225846	.0832772	.0343449	-.0230603	-.0875870	-.1577161	-.2317963	-.3080837
.1607	687	.1542507	.1248673	.0847258	.0347931	-.0237285	-.0894299	-.1607289	-.2359086	-.3131588
.16380	342	.1572183	.1271933	.0861985	.0352435	-.0244172	-.0913145	-.1638013	-.2400925	-.3183098

TABLE 3B-MOMENT COEFFICIENT, ξ_x ($45 < \alpha \leq 90$)

* At $x/\alpha = 0$, coefficient is at mid-length

** At $x/\alpha = 1.0$, coefficient is at strut

B

α	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS						
	0*	0.1	0.2	0.3	0.4	0.5	0.6
1	28.6493490	28.649310	28.649180	28.648962	28.648656	28.648263	28.647783
2	14.3268560	14.326769	14.326507	14.326071	14.325460	14.324674	14.323714
3	9.5536633	9.5535332	9.5531403	9.5524856	9.5515689	9.5503904	9.5489501
4	7.1677949	7.1676202	7.1670962	7.1662228	7.1650002	7.1634284	7.1615075
5	5.7368577	5.7366390	5.7359836	5.7348915	5.7333626	5.7313972	5.7289952
6	4.7833871	4.7831248	4.7823380	4.7810268	4.7791913	4.7768316	4.7739482
7	4.1027553	4.1024490	4.1015305	4.0999998	4.0978571	4.0951028	4.0917372
8	3.5926490	3.5922985	3.5912480	3.5894974	3.5870470	3.5838972	3.5800488
9	3.1962271	3.1958328	3.1946499	3.1926788	3.1899200	3.1863742	3.1820421
10	2.8793857	2.8789471	2.8776316	2.8754395	2.8723716	2.8684287	2.8636121
11	2.6204220	2.6199391	2.6184905	2.6160768	2.6126989	2.6083581	2.6030558
12	2.4048678	2.4043400	2.4027580	2.4001220	2.3964333	2.3916933	2.3859044
13	2.2227061	2.2221339	2.2204179	2.2175589	2.2135583	2.2084181	2.2021412
14	2.0667831	2.0661660	2.0643155	2.0612326	2.0569191	2.0513775	2.0446112
15	1.9318519	1.9311899	1.9292044	1.9258967	1.9212690	1.9153247	1.9080676
16	1.8139779	1.8132706	1.8111495	1.8076160	1.8026730	1.7963244	1.7885750
17	1.7101520	1.7093993	1.7071418	1.7033816	1.6981220	1.6913675	1.6831241
18	1.6180342	1.6172358	1.6148413	1.6108533	1.6052755	1.5981135	1.5893743
19	1.5357770	1.5349326	1.5324005	1.5281834	1.5222860	1.5147147	1.5054779
20	1.4619024	1.4610118	1.4583413	1.4538939	1.4476752	1.4396928	1.4299563
21	1.3952142	1.3942772	1.3914673	1.3867884	1.3802468	1.3718512	1.3616129
22	1.3347338	1.3337499	1.3307999	1.3258881	1.3190217	1.3102109	1.2994686
23	1.2796525	1.2786216	1.2755306	1.2703844	1.2631914	1.2539631	1.2427144
24	1.2292968	1.2282185	1.2249855	1.2196034	1.2120818	1.2024337	1.1906762
25	1.1831009	1.1819749	1.1785989	1.1729793	1.1651269	1.1550567	1.1427877
26	1.1405861	1.1394120	1.1358919	1.1300333	1.1218480	1.1113530	1.0985699
27	1.1013448	1.1001221	1.0964569	1.0903574	1.0818369	1.0709145	1.0576144
28	1.0650273	1.0637558	1.0599444	1.0536021	1.0447441	1.0333915	1.0195714
29	1.0313328	1.0300120	1.0260531	1.0194661	1.0102681	.9984824	.9841393
30	1.0000001	.9986296	.9945220	.9876884	.9781477	.9659259	.9510566
31	.9708021	.9693815	.9651238	.9580416	.9481555	.9354945	.9200956
32	.9435401	.9420689	.9376599	.9303268	.9200925	.9069890	.8910570
33	.9180393	.9165170	.9119553	.9043691	.8937837	.8802342	.8637656
34	.8941459	.8925721	.8878561	.8800145	.8690750	.8550760	.8380669
35	.8717235	.8700975	.8652258	.8571264	.8458296	.8313775	.8138240
36	.8506509	.8489723	.8439433	.8355835	.8239261	.8090171	.7909152
37	.8308202	.8290884	.8239004	.8152778	.8032564	.7878864	.7692320
38	.8121347	.8103492	.8050006	.7961123	.7837234	.7678885	.7486771
39	.7945079	.7926681	.7871570	.7780003	.7652403	.7489362	.7291633
40	.7778620	.7759672	.7702919	.7608638	.7477289	.7309512	.7106123
41	.7621266	.7601761	.7543348	.7446324	.7311187	.7138628	.6929530
42	.7472383	.7452316	.7392222	.7292424	.7153458	.6976071	.6761215
43	.7331397	.7310760	.7248965	.7146361	.7003525	.6821260	.6600594
44	.7197783	.7176570	.7113054	.7007610	.6860860	.6673669	.6447139
45	.7071069	.7049270	.6984012	.6875694	.6724985	.6532815	.6300368

TABLE 4A-NORMAL (HOOP) COEFFICIENT, γ

* At x/α = 0, coefficient is at mid-length

** At x/α = 1.0, coefficient is at strut

A

UTS,		FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α								
		0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0**
83	28.	28.649180	28.648962	28.648656	28.648263	28.647783	28.647216	28.6465620	28.6458200	28.6449910
14	14	14.326507	14.326071	14.325460	14.324674	14.323714	14.322508	14.3212710	14.3197870	14.3181290
501	9	9.5531403	9.5524856	9.5515689	9.5503904	9.5489501	9.5472479	9.5452840	9.5430584	9.5405712
075	7.	7.1670962	7.1662228	7.1650002	7.1634284	7.1615075	7.1592375	7.1566186	7.1536508	7.1503345
952	5.	5.7359836	5.7348915	5.7333626	5.7313972	5.7289952	5.7261570	5.7228827	5.7191726	5.7150269
482	4.	4.7823380	4.7810268	4.7791913	4.7768316	4.7739482	4.7705412	4.7666111	4.7621583	4.7571832
372	4.	4.1015305	4.0999998	4.0978571	4.0951028	4.0917372	4.0877609	4.0831745	4.0779785	4.0721739
488	3.	3.5912480	3.5894974	3.5870470	3.5838972	3.5800488	3.5755025	3.5702591	3.5643196	3.5576854
421	3.	3.1946499	3.1926788	3.1899200	3.1863742	3.1820421	3.1769249	3.1710239	3.1643404	3.1568762
121	2.	2.8776316	2.8754395	2.8723716	2.8684287	2.8636121	2.8579231	2.8513637	2.8439356	2.8356413
558	2.	2.6184905	2.6160768	2.6126989	2.6083581	2.6030558	2.5967941	2.5895754	2.5814021	2.5722774
044	2.	2.4027580	2.4001220	2.3964333	2.3916933	2.3859044	2.3790689	2.3711898	2.3622707	2.3523154
412	2.	2.2204179	2.2175589	2.2135583	2.2084181	2.2021412	2.1947306	2.1861902	2.1765244	2.1657382
112	2.	2.0643155	2.0612326	2.0569191	2.0513775	2.0446112	2.0366243	2.0274214	2.0170082	2.0053907
676	1.	1.9292044	1.9258967	1.9212690	1.9153247	1.9080676	1.8995029	1.8896363	1.8784747	1.8660257
750	1.	1.8111495	1.8076160	1.8026730	1.7963244	1.7885750	1.7794310	1.7688994	1.7569885	1.7437075
241	1.	1.7071418	1.7033816	1.6981220	1.6913675	1.6831241	1.6733991	1.6622011	1.6495398	1.6354265
743	1.	1.6148413	1.6108533	1.6052755	1.5981135	1.5893743	1.5790667	1.5672007	1.5537880	1.5388420
779	1.	1.5324005	1.5281834	1.5222860	1.5147147	1.5054779	1.4945857	1.4820501	1.4678849	1.4521057
563	1.	1.4583413	1.4538939	1.4476752	1.4396928	1.4299563	1.4184776	1.4052708	1.3903518	1.3737389
129	1.	1.3914673	1.3867884	1.3802468	1.3718512	1.3616129	1.3495457	1.3356658	1.3199917	1.3025447
686	1.	1.3307999	1.3258881	1.3190217	1.3102109	1.2994686	1.2868107	1.2722557	1.2558253	1.2375436
144	1.	1.2755306	1.2703844	1.2631914	1.2539631	1.2427144	1.2294634	1.2142315	1.1970433	1.1779263
762	1.	1.2249855	1.2196034	1.2120818	1.2024337	1.1906762	1.1768298	1.1609189	1.1429713	1.1230185
877	1.	1.1785989	1.1729793	1.1651269	1.1550567	1.1427877	1.1283434	1.1117512	1.0930427	1.0722536
699	1.	1.1358919	1.1300333	1.1218480	1.1113530	1.0985699	1.0835250	1.0662492	1.0467782	1.0251521
144	1.	1.0964569	1.0903574	1.0818369	1.0709145	1.0576144	1.0419662	1.0240045	1.0037692	.9813054
714	1.	1.0599444	1.0536021	1.0447441	1.0333915	1.0195714	1.0033169	.9846668	.9636656	.9403633
393	.	1.0260531	1.0194661	1.0102681	.9984824	.9841393	.9672756	.9479344	.9261653	.9020240
566	.	.9945220	.9876884	.9781477	.9659259	.9510566	.9335805	.9135456	.8910066	.8660255
956	.	.9651238	.9580416	.9481555	.9354945	.9200956	.9020039	.8812723	.8579616	.8321399
570	.	.9376599	.9303268	.9200925	.9069890	.8910570	.8723463	.8509152	.8268305	.8001674
656	.	.9119553	.9043691	.8937837	.8802342	.8637656	.8444323	.8222986	.7974379	.7699326
669	.	.8878561	.8800145	.8690750	.8550760	.8380669	.8181075	.7952681	.7696290	.7412806
240	.	.8652258	.8571264	.8458296	.8313775	.8138240	.7932346	.7696862	.7432665	.7140741
152	.	.8439433	.8355835	.8239261	.8090171	.7909152	.7696920	.7454311	.7182283	.6881911
320	.7	.8239004	.8152778	.8032564	.7878864	.7692320	.7473708	.7223940	.6944057	.6635225
771	.7	.8050006	.7961123	.7837234	.7678885	.7486771	.7261737	.7004773	.6717009	.6399709
633	.7	.7871570	.7780003	.7652403	.7489362	.7291633	.7060135	.6795937	.6500265	.6174487
123	.6	.7702919	.7608638	.7477289	.7309512	.7106123	.6868114	.6596644	.6293036	.5958769
530	.6	.7543348	.7446324	.7311187	.7138628	.6929530	.6684964	.6406182	.6094610	.5751843
215	.6	.7392222	.7292424	.7153458	.6976071	.6761215	.6510044	.6223907	.5904342	.5553063
594	.6	.7248965	.7146361	.7003525	.6821260	.6600594	.6342768	.6049235	.5721645	.5361844
139	.6	.7113054	.7007610	.6860860	.6673669	.6447139	.6182607	.5881632	.5545988	.5177652
368	.6	.6984012	.6875694	.6724985	.6532815	.6300368	.6029077	.5720615	.5376883	.5000001

γx (

TABLE 4A-NORMAL (HOOP) COEFFICIENT, γx (1 < α ≤ 45)

th

* At x/α = 0, coefficient is at mid-length
** At x/α = 1.0, coefficient is at strut

B

α	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α							
	0*	0.1	0.2	0.3	0.4	0.5	0.6	0.7
46	.6950818	.6928429	.6861405	.6750178	.6595465	.6398262	.6159840	.5881735
47	.6836638	.6813649	.6744836	.6630664	.6471898	.6269608	.6025153	.5740177
48	.6728164	.6704568	.6633943	.6515787	.6353919	.6146484	.5895935	.5604031
49	.6625066	.6600853	.6528392	.6408212	.6241193	.6028553	.5771849	.5472956
50	.6527037	.6502199	.6427876	.6304633	.6133408	.5915504	.5652580	.5346636
51	.6433798	.6408327	.6332116	.6205768	.6030283	.5807052	.5537841	.5224782
52	.6345091	.6318978	.6240851	.6111355	.5931554	.5702930	.5427365	.5107125
53	.6260679	.6233912	.6153842	.6021153	.5836980	.5602897	.5320905	.4993417
54	.6180340	.6152912	.6070869	.5934942	.5746335	.5506723	.5218234	.4883427
55	.6103873	.6075772	.5991728	.5852514	.5659413	.5414202	.5119139	.4776941
56	.6031090	.6002306	.5916229	.5773680	.5576021	.5325137	.5023423	.4673760
57	.5961817	.5932339	.5844197	.5698263	.5495979	.5239347	.4930903	.4573698
58	.5895893	.5865709	.5775470	.5626098	.5419123	.5156664	.4841407	.4476582
59	.5833167	.5802268	.5709897	.5557034	.5345297	.5076930	.4754777	.4382250
60	.5773503	.5741875	.5647338	.5490928	.5274357	.5000000	.4670862	.4290549
61	.5716771	.5684402	.5587663	.5427648	.5206170	.4925737	.4589524	.4201339
62	.5662851	.5629728	.5530749	.5367071	.5140608	.4854011	.4510630	.4114484
63	.5611631	.5577743	.5476485	.5309082	.5077556	.4784703	.4434059	.4029861
64	.5563010	.5528341	.5424766	.5253575	.5016904	.4717700	.4359695	.3947350
65	.5516890	.5481427	.5375492	.5200449	.4958548	.4652898	.4287429	.3866840
66	.5473182	.5436909	.5328574	.5149611	.4902393	.4590196	.4217159	.3788226
67	.5431802	.5394706	.5283926	.5100974	.4848350	.4529503	.4148789	.3711408
68	.5392674	.5354739	.5241469	.5054456	.4796332	.4470729	.4082228	.3636293
69	.5355725	.5316935	.5201128	.5009981	.4746263	.4413793	.4017389	.3562791
70	.5320889	.5281228	.5162836	.4967478	.4698066	.4358617	.3954191	.3490817
71	.5288104	.5247554	.5126527	.4926880	.4651673	.4305127	.3892558	.3420292
72	.5257311	.5215856	.5092143	.4888124	.4607017	.4253254	.3832415	.3351137
73	.5228459	.5186079	.5059628	.4851154	.4564037	.4202933	.3773694	.3283280
74	.5201497	.5158175	.5028929	.4815913	.4522676	.4154101	.3716328	.3216650
75	.5176381	.5132096	.5000000	.4782352	.4482877	.4106699	.3660254	.3151181
76	.5153068	.5107801	.4972796	.4750424	.4444592	.4060673	.3605413	.3086810
77	.5131520	.5085250	.4947275	.4720083	.4407770	.4015970	.3551747	.3023474
78	.5111703	.5064408	.4923401	.4691289	.4372368	.3972539	.3499202	.2961114
79	.5093583	.5045243	.4901138	.4664004	.4338342	.3930334	.3447725	.2899673
80	.5077133	.5027723	.4880453	.4638192	.4305653	.3889310	.3397265	.2839097
81	.5062326	.5011822	.4861319	.4613820	.4274263	.3849423	.3347776	.2779333
82	.5049138	.4997516	.4843708	.4590858	.4244137	.3810633	.3299211	.2720329
83	.5037549	.4984785	.4827597	.4569279	.4215241	.3772901	.3251525	.2662035
84	.5027541	.4973607	.4812963	.4549055	.4187546	.3736191	.3204676	.2604403
85	.5019099	.4963968	.4799788	.4530165	.4161022	.3700468	.3158622	.2547386
86	.5012209	.4955854	.4788055	.4512586	.4135642	.3665698	.3113323	.2490938
87	.5006861	.4949252	.4777749	.4496300	.4111380	.3631849	.3068741	.2435015
88	.5003047	.4944153	.4768858	.4481288	.4088215	.3598891	.3024838	.2379571
89	.5000761	.4940552	.4761372	.4467537	.4066123	.3566796	.2981579	.2324565
90	.5000000	.4938441	.4755282	.4455032	.4045085	.3535534	.2938926	.2269953

TABLE 4B-NORMAL (HOOP) COEFFICIENT, γx (45 < α

A

* At x/α= 0, coefficient is at mid-length
** At x/α= 1.0, coefficient is at strut

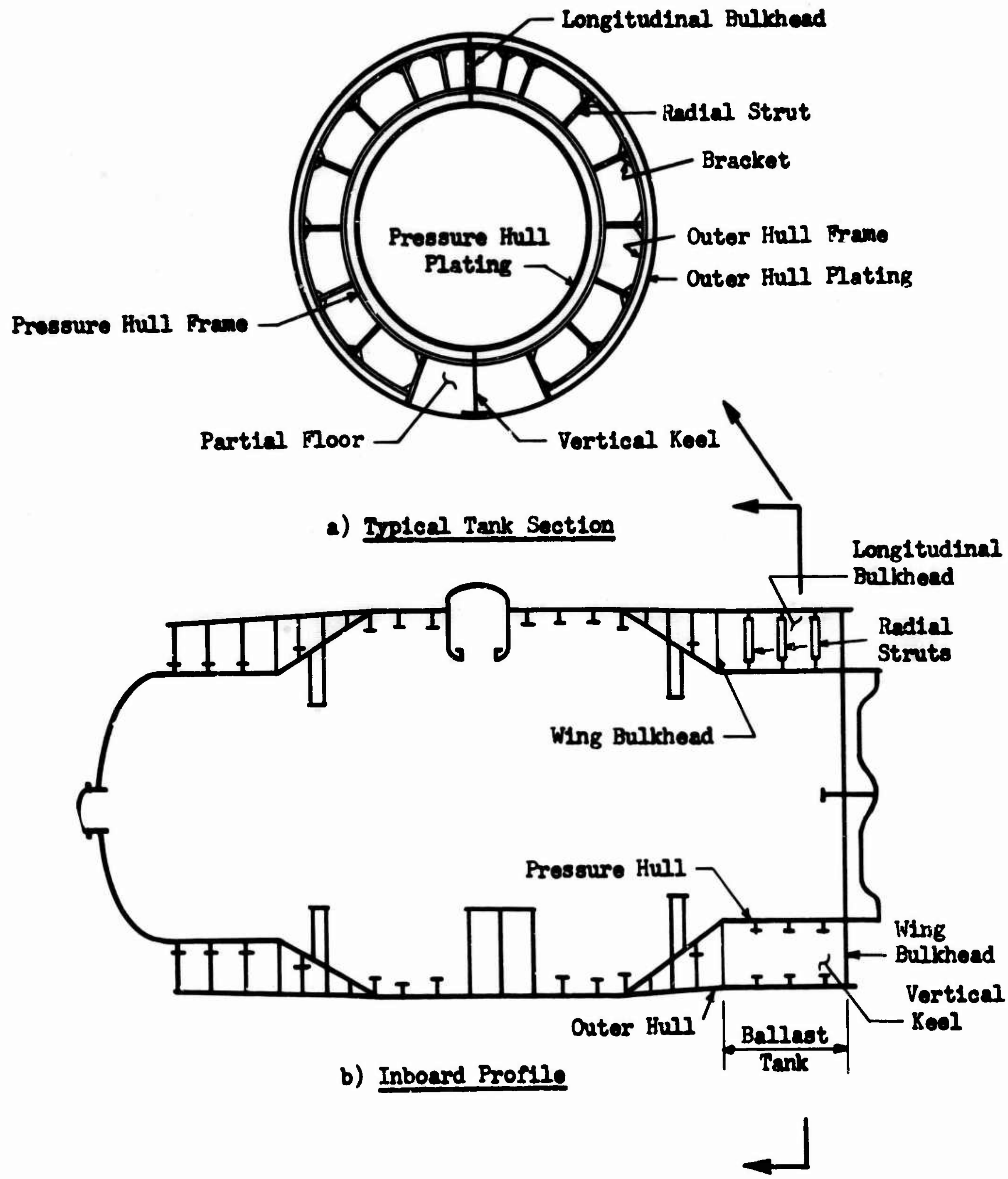
	FRACTION OF HALF-ANGLE (α) BETWEEN STRUTS, x/α									
	[illegible].1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0**
35	.5[illegible]3429	.6861405	.6750178	.6595465	.6398262	.6159840	.5881735	.5565739	.5213886	.4828445
77	.5[illegible]3649	.6744836	.6630664	.6471898	.6269608	.6025153	.5740177	.5416598	.5056590	.4662576
31	.5[illegible]568	.6633943	.6515787	.6353919	.6146484	.5895935	.5604031	.5272819	.4904621	.4502021
56	.51[illegible]0853	.6528392	.6408212	.6241193	.6028553	.5771849	.5472956	.5134058	.4757634	.4346435
36	.5[illegible]2199	.6427876	.6304633	.6133408	.5915504	.5652580	.5346636	.5000001	.4615312	.4195499
82	.4[illegible]327	.6332116	.6205768	.6030283	.5807052	.5537841	.5224782	.4870354	.4477363	.4048921
25	.4[illegible]978	.6240851	.6111355	.5931554	.5702930	.5427365	.5107125	.4744848	.4343514	.3906429
17	.4[illegible]912	.6153842	.6021153	.5836980	.5602897	.5320905	.4993417	.4623232	.4213515	.3767771
27	.4[illegible]912	.6070869	.5934942	.5746335	.5506723	.5218234	.4883427	.4505274	.4087133	.3632713
41	.4[illegible]772	.5991728	.5852514	.5659413	.5414202	.5119139	.4776941	.4390759	.3964149	.3501038
60	.4[illegible]306	.5916229	.5773680	.5576021	.5325137	.5023423	.4673760	.4279485	.3844362	.3372543
98	.41[illegible]339	.5844197	.5698263	.5495979	.5239347	.4930903	.4573698	.4171265	.3727583	.3247039
82	.40[illegible]709	.5775470	.5626098	.5419123	.5156664	.4841407	.4476582	.4065923	.3613634	.3124347
50	.3[illegible]268	.5709897	.5557034	.5345297	.5076930	.4754777	.4382250	.3963295	.3502352	.3004304
49	.3[illegible]875	.5647338	.5490928	.5274357	.5000000	.4670862	.4290549	.3863228	.3393580	.2886752
39	.3[illegible]402	.5587663	.5427648	.5206170	.4925737	.4589524	.4201339	.3765577	.3287174	.2771546
84	.3[illegible]728	.5530749	.5367071	.5140608	.4854011	.4510630	.4114484	.3670207	.3182995	.2658548
61	.3[illegible]743	.5476485	.5309082	.5077556	.4784703	.4434059	.4029861	.3576989	.3080914	.2547628
50	.34[illegible]341	.5424766	.5253575	.5016904	.4717700	.4359695	.3947350	.3485804	.2980811	.2438664
40	.3[illegible]427	.5375492	.5200449	.4958548	.4652898	.4287429	.3866840	.3396537	.2882568	.2331539
26	.3[illegible]909	.5328574	.5149611	.4902393	.4590196	.4217159	.3788226	.3309081	.2786077	.2226144
08	.3[illegible]706	.5283926	.5100974	.4848350	.4529503	.4148789	.3711408	.3223334	.2691234	.2122375
93	.31[illegible]739	.5241469	.5054456	.4796332	.4470729	.4082228	.3636293	.3139200	.2597941	.2020132
91	.30[illegible]935	.5201128	.5009981	.4746263	.4413793	.4017389	.3562791	.3056585	.2506104	.1919321
17	.2[illegible]228	.5162836	.4967478	.4698066	.4358617	.3954191	.3490817	.2975404	.2415634	.1819852
92	.2[illegible]554	.5126527	.4926880	.4651673	.4305127	.3892558	.3420292	.2895572	.2326445	.1721639
37	.28[illegible]856	.5092143	.4888124	.4607017	.4253254	.3832415	.3351137	.2817009	.2238455	.1624599
80	.27[illegible]079	.5059628	.4851154	.4564037	.4202933	.3773694	.3283280	.2739640	.2151587	.1528654
50	.2[illegible]175	.5028929	.4815913	.4522676	.4154101	.3716328	.3216650	.2663390	.2065765	.1433728
81	.25[illegible]096	.5000000	.4782352	.4482877	.4106699	.3660254	.3151181	.2588191	.1980916	.1339747
10	.25[illegible]801	.4972796	.4750424	.4444592	.4060673	.3605413	.3086810	.2513975	.1896972	.1246641
74	.24[illegible]250	.4947275	.4720083	.4407770	.4015970	.3551747	.3023474	.2440676	.1813864	.1154342
14	.23[illegible]408	.4923401	.4691289	.4372368	.3972539	.3499202	.2961114	.2368232	.1731528	.1062783
73	.22[illegible]243	.4901138	.4664004	.4338342	.3930334	.3447725	.2899673	.2296583	.1649901	.0971902
97	.22[illegible]723	.4880453	.4638192	.4305653	.3889310	.3397265	.2839097	.2225669	.1568921	.0881635
33	.21[illegible]822	.4861319	.4613820	.4274263	.3849423	.3347776	.2779333	.2155434	.1488529	.0791923
29	.20[illegible]516	.4843708	.4590858	.4244137	.3810633	.3299211	.2720329	.2085822	.1408666	.0702705
35	.20[illegible]785	.4827597	.4569279	.4215241	.3772901	.3251525	.2662035	.2016779	.1329274	.0613924
03	.19[illegible]3607	.4812963	.4549055	.4187546	.3736191	.3204676	.2604403	.1948251	.1250300	.0525522
86	.18[illegible]3968	.4799788	.4530165	.4161022	.3700468	.3158622	.2547386	.1880189	.1171686	.0437444
38	.18[illegible]5854	.4788055	.4512586	.4135642	.3665698	.3113323	.2490938	.1812539	[illegible]093380	.0349635
15	.17[illegible]9252	.4777749	.4496300	.4111380	.3631849	.3068741	.2435015	.1745253	.1015329	.0262040
71	.16[illegible]4153	.4768858	.4481288	.4088215	.3598891	.3024838	.2379571	.1678281	.0937479	.0174605
65	.16[illegible]0552	.4761372	.4467537	.4066123	.3566796	.2981579	.2324565	.1611575	.0859777	.0087276
53	.154[illegible]8441	.4755282	.4455032	.4045085	.3535534	.2938926	.2269953	.1545086	.0782173	.0000001

< α < 9[illegible]

TABLE 4B-NORMAL (HOOP) COEFFICIENT, γx (45 < α < 90)

* At x/α= 0, coefficient is at mid-length
** At x/α= 1.0, coefficient is at strut

B



LARGE-SCALE MODEL SIMULATION OF MAIN BALLAST TANK STRUCTURE

Figure 22

APPENDIX B

DEVELOPMENT OF EQUATIONS

FOR

PRESSURE LOADING

I. TRANSIENT PRESSURE EQUATIONS

The following development will employ a basic method similar to that used in reference (19)*. After determining the equation of state for air, expressions will be developed for the mass, spring and damping terms, and for the forcing function in the single-degree-of-freedom system of equation [B-5] and shown in Figure 8.

Equation of State

The energy change in a thermodynamic process may be expressed [20] as

$$dU = \delta Q^* - \delta J^* \qquad \text{[B-1]**}$$

in which: δQ^* = Heat added to a system (ft-lb), and

δJ^* = Work done by the system (ft-lb).

Assuming air to be a perfect gas, the terms δQ^* and δJ^* can be treated as perfect differentials, dQ^* and dJ^*.

The energy added by the incoming air is then

$$dQ^* = W^* C_p T_s' dt^*$$

in which: W^* = Flow rate of air (lb/sec),

C_p = Specific heat at constant pressure,

T_s' = Stagnation temperature (^oR), and

dt^* = Time interval (sec).

Work is performed by removing the water ballast, thereby changing the volume, or

$$dJ^* = p^* d\overline{V}_A$$

in which: p^* = Pressure in the tank (lb/ft^2), and

$d\overline{V}_A$ = Change in volume of air in the tank (ft^3)

---

* Numbers in parenthesis indicate similar entries in the Bibliography.

**Numbers in brackets refer to equations.

Substituting in [B-1] the rate of energy change is

$$\frac{dU}{dt^*} = W^*C_pT_s' - p^*\frac{d\overline{V}_A}{dt^*} .$$

However, for the air entering the tank,

$$U = N^*C_VT_A'$$

and

$$\frac{dU}{dt^*} = C_VN^*\frac{dT_A'}{dt^*} + C_VT_A'\frac{dN^*}{dt^*},$$

in which: N^* = Weight of air (lb),

C_V = Specific heat at constant volume, and

T_A' = Temperature of air (°R).

Equating, the energy equation becomes

$$C_VN^*\frac{dT_A'}{dt^*} + C_VT_A'\frac{dN^*}{dt^*} + p^*\frac{d\overline{V}_A}{dt^*} = W^*C_pT_s'. \qquad [B-2]$$

Differentiating the universal gas law

$$p^*\overline{V}_A = N^*R^*T_A' ,$$

yields

$$p^*\frac{d\overline{V}_A}{dt^*} + \overline{V}_A\frac{dp^*}{dt^*} = R^*N^*\frac{dT_A'}{dt^*} + R^*T_A'\frac{dN^*}{dt^*} \qquad [B-3]$$

Since the universal gas constant, $R^* = C_p - C_V$, and using $k^* = C_p/C_V$, equations [B-2] and [B-3] combine to produce the air equations of state,

$$p^*k^*\frac{d\overline{V}_A}{dt^*} + \overline{V}_A\frac{dp^*}{dt^*} = R^*k^*T_s'W^*. \qquad [B-4]$$

<u>Transient Differential Pressure</u>

As noted in the "Pressure Loading" section of the paper, determination of the transient differential pressure requires solution of the equation,

$$\overline{m}\,\ddot{\overline{x}}_2 + \overline{C}\dot{\overline{x}}_2|\dot{\overline{x}}_2| + K^*(\overline{x}_2 - \overline{x}_1) = 0, \qquad [B-5]$$

for which terms are defined in the paper and indicated on Figure 8.

In the following development mathematical expressions will be generated for each of the coefficients in this equation.

Effective Mass ($\overline{m}$)

The expression for effective mass of the water slug in the flood hole is[21]

$$\overline{m} = \frac{\gamma_{sw}}{g} \overline{A} \left(\overline{v} + 0.85 \sqrt{\overline{A}} \right), \qquad [B-6]$$

in which: $\overline{A}$ = net cross-section area of the flood hole, perpendicular to direction of flow (Figure 10c) (ft^2),

$\overline{v}$ = depth of flood hole vanes parallel to direction of flow (Figure 10c) (ft),

γ_{sw} = density of sea water (lb/ft^3), and

g = acceleration of gravity (ft/sec^2).

For a flood hole without baffles, the term $\overline{v}$ disappears, and the expression becomes

$$\overline{m} = \frac{0.85 \gamma_{sw} \left(\overline{A} \right)^{\frac{3}{2}}}{g}.$$

For multiple flood holes, there is generally no interaction between effective masses, and they can be combined directly.

Spring Constant (K*)

In the classical vibrating system, the spring constant is defined as the force required to produce a unit displacement, or

$$K^* = \frac{dF}{d\overline{x}_1},$$

in which: F = force applied to the system (lb), and

$\overline{x}_1$ = displacement in the direction of F (ft).

Since the inner (pressure) hull structure is normally very stiff in comparison with the outer hull structure, the flexibility of the ballast tank may be assumed as being concentrated in the outer hull structure. From the geometry of Figure 9a,

$$dF = C_D \overline{A} dp^*$$

and

$$d\bar{x}_1 = \frac{d\bar{V}_T}{C_D\bar{A}},$$

in which: C_D = discharge coefficient, and

$\bar{V}_T$ = tank volume (ft^3)

Substituting,

$$K^* = \frac{(C_D\bar{A})^2}{d\bar{V}_T/dp^*} . \qquad [B-8]$$

For instances in which the tank has more than one flood hole,

$$K^* = \frac{(C_D\Sigma\bar{A})^2}{d\bar{V}_T/dp^*} \qquad [B-9]$$

In the above expression the ratio $d\bar{V}_T/dp^*$ can be obtained experimentally, or approximated by the following procedure:

As described in the "Structural Response" section of the paper, the outer hull plating is periodically stiffened by outer hull frames which, in turn, are attached to the inner hull structure by radial struts (Figure 11). The stiffness of this outer frame-strut array is normally quite large in comparison with that of the outer hull plating. Consequently, for purposes of determining the change in tank volume, it may be assumed that the flexibility of the ballast tank is concentrated in the outer hull plating.

Approximating the deflected shape of the outer hull plating as shown in Figure 9b, the trapezoidal cross-section of the increase in tank volume is

$$\Delta A = \Delta_{Zo}\left(L_o - L_{eo}\right),$$

in which, Δ_{Zo} = radial (hoop) deflection of outer hull plating unaffected by frames (in),

where: $\Delta_{Zo} = \frac{dp^* R_o^2}{12^2 E_o t_o}$

R_o = radius of outer hull plating (in),

E_o = modulus of outer hull plating (lb/in^2), and

t_o = thickness of outer hull plating (in),

L_o = Length between outer hull frames (in),

L_{eo} = Effective length of outer hull plating acting with outer hull frame (in),

where: $L_{eo} = 1.56N'\sqrt{R_o t_o}$, and

N' = Frame stiffness coefficient for outer hull (Defined in Appendix C and plotted in Appendix A).

The increase in the tank volume will take the form of a torus over each frame space, for which

$$d\bar{V}_T = \frac{\Delta A}{12^3}(2\pi R_o).$$

Substituting,

$$\frac{d\bar{V}_T}{dp^*} = \frac{2\pi R_o^3 n_o}{12^5 E_o t_o}\left(L_o - 1.56N'\sqrt{R_o t_o}\right), \qquad [B\text{-}10]$$

where: n_o = number of frame spaces over which tank extends.

The solution to this expression is then substituted into equation [B-8] to obtain the spring constant, K*.

Damping Coefficient

With the system of equation [B-5], velocity-squared damping may be assumed to be opposing the motion of the mass. The damping coefficient is [19]

$$C = \frac{\gamma_{sw} C_D \bar{A}}{2g}. \qquad [B\text{-}11]$$

Displacement Terms

Referring to Figure 8c, if $\bar{x}_2$ is the displacement of the water slug in the flood hole, then $\dot{\bar{x}}_2$ is its velocity and $\ddot{\bar{x}}_2$ its acceleration.

Since $\dot{\bar{x}}_2 = \frac{Qw}{C_D \bar{A}}$, [B-12]

where: Q_w = flow rate of water (ft^3/sec),

then $\bar{x}_2 = \int^{t^*} \frac{Qw}{C_D \bar{A}} \, dt^*$ [B-13]

and $\ddot{\bar{x}}_2 = \frac{\dot{Q}w}{C_D \bar{A}}$ [B-14]

Under steady-flow conditions, air flow is equal to water flow, i.e.,

$$Q_A = Qw$$

The spring-mass model of Figure 8c requires that, under steady state conditinns,

$$\dot{\bar{x}}_1 = \dot{\bar{x}}_2 \; .$$

Thus, $\dot{\bar{x}}_1 = \frac{Qw}{C_D \bar{A}} = \frac{QA}{C_D \bar{A}}$ [B-15]

and, $\bar{x}_1 = \int^{t^*} \frac{QA}{C_D \bar{A}} \, dt^*$. [B-16]

Forcing Function

An expression must be obtained for the forcing function in terms of the motion of the mass of water in the flood hole.

Designating the pressure drop across the flood hole as δp^*, the air pressure in the tank, from Figure 10, becomes

$$p^* = P^*_{ATM} + \gamma_{sw} Z_T + \gamma_{sw} \bar{h}_A + \delta p^*, \quad [B-17]$$

in which: P^*_{ATM} = atmospheric pressure (lb/ft^2),

Z_T = depth to tank top (ft), and

$\bar{h}_A$ = depth from tank top to ballast water surface (ft).

Assuming that volume varies linearly with depth, as indicated in Figure 8b,

$$h_A = \frac{\bar{V}_A}{\bar{V}_T} \bar{h}_T \; ,$$

in which: $\bar{h}_T$ = blowable depth of tank (ft).

Substituting,

$$p^* = P^*_{ATM} + \gamma_{sw} Z_T + \gamma_{sw} \frac{\overline{V}_A}{\overline{V}_T} \overline{h}_T + \delta p^*. \qquad \text{[B-18]}$$

Differentiating with respect to time,

$$\frac{dp^*}{dt^*} = \gamma_{sw} \frac{dZ_T}{dt^*} + \gamma_{sw} \frac{\overline{h}_T}{\overline{V}_T} \frac{d\overline{V}_A}{dt^*} + \frac{d\,(\delta p^*)}{dt^*}. \qquad \text{[B-19]}$$

For the condition in which the ship has no vertical velocity when blowing is initiated, the first term in the above equation is zero.
Also, since

$$\frac{d\overline{V}_A}{dt^*} = Q_A,$$

and, from equation [B-15]

$$Q_A = C_D \overline{A}\, \dot{\overline{x}}_1,$$

then, considering that

$$\delta p^* = \frac{\gamma_{sw} (\dot{\overline{x}}_2)^2}{2g},$$

equation [B-19] becomes

$$\frac{dp^*}{dt^*} = \gamma_{sw} \frac{\overline{h}_T}{\overline{V}_T} (C_D \overline{A} \dot{\overline{x}}_1) + \frac{\gamma_{sw}}{g} \dot{\overline{x}}_2 \ddot{\overline{x}}_2 . \qquad \text{[B-20]}$$

Combining equations [B-20] and [B-15] with [B-4] and rearranging gives

$$\dot{\overline{x}}_1 = \frac{R^* k^* T_s^{\prime} W^* - \dfrac{\overline{V}_A \gamma_{sw}}{g} \dot{\overline{x}}_2 \ddot{\overline{x}}_2}{C_D \overline{A} \left(p^* k^* + \dfrac{\overline{V}_A \overline{h}_T}{\overline{V}_T} \gamma_{sw} \right)} .$$

Finally, substituting the above and equations [B-6], [B-8], [B-10], and [B-11] into [B-5] gives

$$\Sigma\left[\frac{\gamma_{sw}}{g}\,\bar{A}\left(\bar{v} + 0.85\sqrt{\bar{A}}\right)\right]\ddot{x}_2 + \frac{\gamma_{sw}\,C_D\Sigma\bar{A}}{2g}\,\dot{x}_2|\dot{x}_2|$$

$$+\ \frac{12^5 E_o t_o (C_D\Sigma\bar{A})^2}{2\pi R_o n_o\,(L_o - 1.56N'\sqrt{R_o t_o})}\left[\bar{x}_2 - \int^{t^*}\frac{R^*k^*T_s{}'W^* - \frac{V_A\gamma_{sw}}{g}\,\dot{x}_2\ddot{x}_2}{C_D\bar{A}\left(p^*k^* + \frac{V_A h_T}{V_T}\,\gamma_{sw}\right)}\right] = 0$$

Where summations are made over all flood holes in the tank. The above equation is a second-order, non-linear, non-homogeneous equation, requiring computer solution. The air flow, W^*, varies with time, from zero to a full-flow value, W^*_{MAX}, which can be calculated by the procedure of Reference (22), or obtained by measurement. The history of increase in the air flow rate may also be obtained by experiment, or may be approximated by a mathematical expression. One such expression, developed to investigage the effect of curve shape, is

$$W^* = W^*_{MAX}\left[1 - \left(\frac{1}{1-\varphi_2/\varphi_1}\right)e^{-\frac{t^*}{\varphi_1}} - \left(\frac{1}{1-\varphi_1/\varphi_2}\right)e^{-\frac{t^*}{\varphi_2}}\right]. \qquad \text{[B-22]}$$

The coefficients, φ_1, and φ_2, can be varied to produce different curve forms depicting the rate of increase in the air flow rate, to approximate experimental results.

II. EXPANSION PRESSURE EQUATIONS

Once steady flow has been established in the tank, the differential pressure acting on the tank structure becomes a function of the energy added to the tank and of the resistance of the tank to air expansion. As noted earlier, this differential pressure is the difference between internal and external pressure. The external pressure is simply a function of the depth at which the submarine is operating, while the internal pressure, referring to Figure 10, may be expressed as

$$p^* = \gamma_{sw}\left[Z_{\mathbb{L}} + \left(Z_1 - \bar{h}_B - \bar{h}_W\right)\cos\theta^* - x_A \sin\theta^*\right] + P^*_{ATM} + \frac{\gamma_{sw}}{2g}\left(\frac{1}{C_D\bar{A}}\ \frac{d\bar{V}_A}{dt^*}\right)^2 . \qquad [B-23]$$

The last term in the above equation represents the pressure drop across the flood holes. Differentiation of [B-23], neglecting the higher derivative, gives

$$\frac{dp^*}{dt^*} = \gamma_{sw}\left\{\dot{Z}_{\mathbb{L}} - \left[\left(Z_1 - \bar{h}_B - \bar{h}_W\right)\sin\theta^* + X_A\cos\theta^*\right]\dot{\theta}^* - \dot{\bar{h}}_W\cos\theta^* .\right\} \qquad [B-24]$$

The assumption that tank volume is linear with depth can be expressed as

$$\bar{h}_W = \left(\frac{\bar{V}_T - \bar{V}_A}{\bar{V}_T}\right)\bar{h}_T .$$

Its derivative is

$$\dot{\bar{h}}_W = -\frac{\bar{h}_T}{\bar{V}_T}\ \frac{d\bar{V}_A}{dt^*}$$

Substituting in [B-24] gives

$$\frac{dp^*}{dt^*} = \gamma_{sw}\left\{\dot{Z}_{\mathbb{L}} - \left[\left(Z_1 - \bar{h}_B - \bar{h}_W\right)\sin\theta^* + \bar{X}_A\cos\theta^*\right]\dot{\theta}^* + \frac{\bar{h}_T}{\bar{V}_T}\ \frac{d\bar{V}_A}{dt^*}\cos\theta^* .\right\} \qquad [B-25]$$

Substituting into [B-4], the air equation of state, and solving for the air expansion rate,

$$\frac{dV_A}{dt^*} = \frac{R^*k^*T_s^{\prime} W^* - \overline{V}_A \gamma_{sw} \left\{ \dot{Z}_{\mathfrak{L}} - \left[\left(Z_1 - \overline{h}_B - \overline{h}_W \right) \mathrm{SIN}\theta^* + x_A \mathrm{COS}\theta^* \right] \dot{\theta}^* \right\}}{p^*k^* + \frac{\overline{V}_A}{\overline{V}_T} \gamma_{sw} \overline{h}_T \mathrm{COS}\theta^*}.$$

The term in braces is a velocity - the rate of rise of the tank in question. Dividing the first term in the numerator by $\overline{V}_A \; \gamma_{sw}$ leaves it, too, in terms of a velocity. The resulting equation can be written,

$$\frac{d\overline{V}_A}{dt^*} = \frac{\overline{V}_A \gamma_{sw} \left(-\dot{Z}_T \right)}{p^*k^* + \frac{\overline{V}_A}{\overline{V}_T} \gamma_{sw} \overline{h}_T \mathrm{COS}\theta^*}, \qquad \text{[B-26]}$$

in which:

$$\dot{Z}_T = \frac{-k^*W^*R^*T_s^{\prime}}{\gamma_{sw} \overline{V}_A} + \left\{ \dot{Z}_{\mathfrak{L}} - \left[\left(Z_1 - \overline{h}_B - \overline{h}_W \right) \mathrm{SIN}\theta^* + \overline{x}_A \mathrm{COS}\theta^* \right] \dot{\theta}^* \right\}. \qquad \text{[B-27]}$$

A further simplification, based on the assumption of linearity of volume with depth, yields a new symbol,

$$\overline{h}_A = \frac{\overline{h}_T \overline{V}_A}{\overline{V}_T} \mathrm{COS}\theta^*,$$

which, when substituted into [B-26] gives

$$\frac{d\overline{V}_A}{dt^*} = \frac{\overline{V}_A \gamma_{sw} \left(-\dot{Z}_T \right)}{k^*p^* + \gamma_{sw} \overline{h}_A}. \qquad \text{[B-28]}$$

By letting

$$Z_T = Z_{\mathfrak{L}} + \left(Z_1 - \overline{h}_B - \overline{h}_W \right) \mathrm{COS}\theta^* - \overline{x}_A \mathrm{SIN}\theta^*, \qquad \text{[B-29]}$$

the ambient sea pressure at the level of the top of the tank can be written

$$P_T^* = \gamma_{sw} Z_T + P^*_{ATM}, \qquad \text{[B-30]}$$

and equation [B-23] can be written

$$p^* = P_T^* + \gamma_{sw} \bar{h}_A + \frac{\gamma_{sw}}{2g} \left(\frac{1}{C_D \bar{A}} \frac{d\bar{V}_A}{dt^*} \right)^2 . \qquad \text{[B-31]}$$

Substituting [B-28] in [B-31] gives a cubic equation in pressure,

$$(k^*)^2 (p^*)^3 + \left[(2-k^*) \; k^* \gamma_{sw} \bar{h}_A - (k^*)^2 P_T^* \right] (p^*)^2$$

$$+ \left[(1-2k^*) \; \gamma_{sw} \bar{h}_A - 2k^* P_T^* \right] \gamma_{sw} \bar{h}_A p^* - \left[\gamma_{sw} \bar{h}_A + P_T^* \right] \gamma_{sw}^2 \bar{h}_A^2$$

$$= \frac{\gamma_{sw}^3}{2g} \left(\frac{\bar{V}_A}{C_D \bar{A}} \right) \left(\dot{Z}_T \right)^2 . \qquad \text{[B-32]}$$

The differential pressure loading the tank structure is the difference between internal and external pressure, or

$$\Delta p = \frac{p^* - P_T^* \quad \text{lb/ft}^2}{144 \quad \text{in}^2/\text{ft}^2} \qquad \text{[B-33]}$$

APPENDIX C

DEVELOPMENT OF THEORY AND EQUATIONS FOR ELASTIC RESPONSE OF STRUCTURE

I. STRUT LOAD

Axial Load in Outer Hull from Blowing

Referring to Figures 11 and 14a, when the ballast tank is subjected to blowing pressures, there is an end load applied to the wing bulkhead. This load is distributed between the inner and outer hull structures based upon the center of pressure. From the geometry of Figure 15, the general equation for obtaining the center of pressure on a segment is

$$\bar{y} = \frac{\frac{1}{3}\left\{\int_0^{\frac{\pi}{2}} R_o^3 \, d\theta - \int_0^{\frac{\pi}{2}} R_H^3 \, d\theta\right\}}{\frac{1}{2}\int_0^{\frac{\pi}{2}} \left(R_o^2 - R_H^2\right) d\theta}$$

Integrating and substituting limits,

$$\bar{y} = \frac{2}{3}\,\frac{\left(R_o^3 - R_H^3\right)}{\left(R_o^2 - R_H^2\right)} \quad ,$$

but, since

$$\bar{y} = \bar{y}' + R_H \quad ,$$

the foregoing equation reduces to

$$\bar{y}' = \frac{2R_o^2 - R_o R_H - R_H^2}{3\left(R_o - R_H\right)}$$

Since the end force on the wing bulkhead

$$P_e = \pi (\Delta p)\left(R_o^2 - R_H^2\right) \quad ,$$

where: Δp = differential pressure across ballast tank (Appendix B). The force on the outer hull may be written as

$$F' = \frac{P_e \overline{y}'}{R_o - R_H} ,$$

and the longitudinal membrane stress in the outer hull is

$$\sigma_x = \frac{F'}{A_B} ,$$

where: $A_B = 2\pi R_o t_o$.

Substituting and rearranging for the longitudinal membrane stress,

$$\sigma_x = (\Delta p) \frac{R_o}{6t_o} (\wp) , \qquad [C-1]$$

$$\text{where: } \wp = 2 - \frac{R_H}{R_o} - \left(\frac{R_H}{R_o}\right)^2 . \qquad [C-2]$$

Unrestrained Deflection of Outer Hull

For a uniformly stiffened circular shell subjected to a uniform lateral pressure, the loading may be written as a second order differential equation

$$\frac{d^2 M}{dx^2} - \frac{\sigma_\phi t_o}{R_o} + (\Delta p) = 0 , \qquad [C-3]$$

for which: $\sigma_\phi = E_o \epsilon_\phi + \nu \sigma_x$,

$$\epsilon_\phi = \frac{y}{R_o} ,$$

$$M = -D_o' \frac{d^2 y}{dx^2} , \text{ and}$$

$$D_o' = \frac{E_o t_o^3}{12(1 - \nu^2)} .$$

Substituting the above and equation [C-1] into [C-3], and rearranging, the fourth order differential equation of a uniformly ring stiffened shell subjected to a variable end pressure may be written as

$$\frac{d^4 y}{dx^4} + \frac{12(1-\nu^2)}{R_o^2 t_o^2}\left\{ y - \left[\frac{(\Delta p) R_o^2}{E_o t_o} - \frac{\nu}{2}\,\frac{(\Delta p) R_o^2 p}{3 E_o t_o}\right]\right\} = 0 \ .$$

The net deflection of an unrestrained shell, y_s, at any longitudinal position, relative to the initial position, is represented by the last two terms in the brackets. By letting the net deflection of the shell from the frame

$$Z = y - y_s \quad ,$$

and defining

$$C_2^4 = \frac{3(1-\nu^2)}{(R_o t_o)^2} \quad ,$$

the fourth order differential equation may be written as

$$\frac{d^4 Z}{dx^4} + 4 C_2^4 Z = 0 \quad .$$

This is a standard fourth order equation, for which solutions of the form $Z = e^{\alpha x}$ may be assumed. The particular solution to this equation may be written as

$$y = \frac{(\Delta p) R_o^2}{E_o t_o}\left[1 - \frac{\nu}{6} p - \left(1 - \frac{\nu}{6} p - \frac{b_o t_o}{A_{fo} + b_o t_o}\right)\frac{k}{1+\beta}\right] \quad , \qquad [C-4]$$

in which: $k = \{\mathrm{SINH}\alpha x \cdot \mathrm{COS}\alpha(\ell - x) + \mathrm{COSH}\alpha x \cdot \mathrm{SIN}\alpha(\ell - x)$

$+ \mathrm{SIN}\alpha x \cdot \mathrm{COSH}\alpha(\ell - x) + \mathrm{COS}\alpha x \cdot \mathrm{SINH}\alpha(\ell - x)\}$

$\div \{\mathrm{SINH}\alpha\ell + \mathrm{SIN}\alpha\ell\}$, and for which k represents the deflection of the shell at any longitudinal location. This term degenerates to unity at the frame.

Hence, simplifying and letting

$$\beta = \frac{L_{eo} t_o}{A_{fo} + b_o t_o} ,$$

$$\text{where: } L_{eo} = 1.555 N' \sqrt{R_o t_o} , \qquad [C-5]$$

and, since the shell thickness is normally small, taking

$$A_{fo} \cong A_{fo} + b_o t_o ,$$

equation [C-4] simplifies, for the radial expansion of the outer hull at the frame, to

$$y = \frac{P R_o^2}{E_o A_{To}} \left(1 - \frac{\nu}{6} p\right) , \qquad [C-6]$$

where: $P = L_{eo} (\Delta p)$, and

$$A_{To} = A_{fo} + b_o t_o + L_{eo} t_o ,$$

Equation [C-6] applies to the case in which no shell inserts are used, or in which the web thickness of the frame is small, since the term $(b_o t_o)$ has been dropped. For other instances, equation [C-4] should be used to obtain the deflection at the frame.

Elongation of Strut

The elongation of the radial strut (Figure 13a) is simply

$$y_2 = \epsilon_s L_s = \frac{\sigma_s L_s}{E_s} = \frac{W L_s}{A_s E_s} , \qquad [C-7]$$

in which: ϵ_s = axial strain in the strut (in/in),

W = axial load on the strut (lb), and

L_s = length of strut (in) (Figure 14).

<u>Unrestrained Deflection of Inner Hull</u>

By substituting the appropriate nomenclature in equation [C-4] for the inner hull (Figure 14), and rearranging, the radial deflection (contraction) of the inner hull at the frame is

$$y_3 = \frac{p' R_H^2}{2 E_H t_H} \left[2 - \nu \left(\frac{2 A_{fH} - \nu (A_{FH})}{A_{TH}} \right) \right] \qquad [C-8]$$

in which: $p' = p + (\Delta p) = \frac{D_{\text{℄}}}{2.25} + (\Delta p)$, (Figure 11),

and $A_{FH} = A_{fH} + b_H t_H$.

Since pressure hull frames in some instances have a wide faying flange, b_H, the above equation will not be simplified similar to equation [C-6].

<u>Effect of Strut on Outer Hull Deflection</u>

To determine the load in the strut, deflections for the inner and outer hull will be added algebraically to the strut elongation. From Figure 13a,

$$\Delta = y + y_3 - y_2 \ .$$

Substituting expressions from [C-6], [C-7] and [C-8], the deflection at the strut for the case of the outer hull is

$$\Delta = \frac{P R_o^2}{E_o A_{To}} \left[1 - \frac{\nu}{6} p \right] + \frac{p' R_H^2}{2 E_H t_H} \left[2 - \nu - \left(\frac{2A_{fH} - \nu (A_{FH})}{A_{TH}} \right) \right] - \frac{W L_s}{A_s E_s} \qquad [C-9]$$

This deflection, Δ, must now be equated to the deflection of a ring under equally spaced concentrated radial loads of a magnitude equal to W.

By application of the principle of Least Work, the following expression may be obtained for the deflection of a ring subjected to equally spaced loads of equal magnitude. (This development may be found in many standard textbooks on structural analysis, for example reference (23).) Referring to Figure 14b,

$$\Delta_{RS} = \frac{W R_o^3}{4 E_o I_o}\left[\frac{\alpha \cos\alpha}{\sin^2\alpha\,(\cos x)} + \frac{\cos x}{\sin\alpha} - \frac{2}{\alpha}\right] . \qquad [C-10]$$

Similarly, an expression may be developed for the average normal force

$$T_A = \frac{W}{4}\left[\frac{1 - \cos\alpha}{\sin\alpha}\right] , \qquad [C-11]$$

where, in the above equations,

α = half angle between adjacent struts (degrees or radians), and

x = angular distance measured from the mid-position between adjacent struts (degrees or radians).

For large values of the angle α, the effect of hoop loading induced by W becomes negligible. However, as α becomes smaller, the effect of hoop loading increases until, as a limit, a condition similar to that of a wing bulkhead is reached. For this limiting case no circumferential bending occurs in the frame, and $W = p L_{eo}$.

In the case of a strut, the deflection due to hoop load may be written as

$$\Delta_H = \epsilon\phi_o R_o = \frac{T_A R_o}{A_{To} E_o} , \qquad [C-12]$$

in which: $\epsilon\phi_o$ = circumferential strain in outer hull (in/in).

Substituting for T_A, and adding equations [C-10] and [C-12], the deflection of the ring at the strut, for $x = \alpha$, is

$$\Delta_1 = \Delta_{RS} + \Delta_H = \frac{WR_o^3}{2E_oI_o}\left[\frac{\alpha}{2\,SIN^2\alpha} + \frac{COT\,\alpha}{2} - \frac{1}{\alpha}\right]$$

$$+ \frac{WR_o}{4E_oA_o}\left[\frac{1}{SIN\,\alpha} + COT\,\alpha\right] \quad . \qquad [C-13]$$

Effect of Strut on Inner Hull Deflection

The inner hull will be subjected to the same concentrated strut loads as is the outer hull. However, whereas the outer hull deflected inward, the inner hull will deflect outward at points around the hull space 2α degrees apart. Hence, equation [C-13] must have another term to represent this deflection, which will be of a form similar to [C-13] with the appropriate nomenclature for the inner hull. Referring to Figure 14, the deflection of the outer hull frame at the strut, including both the inner and outer hull components, may be written as

$$\Delta_{IH} = \frac{WR_H^3}{2E_HI_H}\left[\frac{\alpha}{2\,SIN^2\alpha} + \frac{COT\,\alpha}{2} - \frac{1}{\alpha}\right] + \frac{WR_H}{4E_HA_{TH}}\left[\frac{1}{SIN\,\alpha} + COT\,\alpha\right]$$

$$+ \frac{WR_o^3}{2E_oI_o}\left[\frac{\alpha}{2\,SIN^2\alpha} + \frac{COT\,\alpha}{2} - \frac{1}{\alpha}\right] \qquad [C-14]$$

$$+ \frac{WR_o}{4E_oA_o}\left[\frac{1}{SIN\,\alpha} + COT\,\alpha\right] \quad .$$

Defining the deflection coefficient at the strut as

$$\left[\delta_x\right]_{\frac{x}{\alpha} = 1.0} = \frac{1}{4}\left[\frac{\alpha}{SIN^2\alpha} + COT\,\alpha - \frac{2}{\alpha}\right] \quad , \qquad [C-15]$$

and the average normal (hoop) coefficient as

$$\Gamma = \frac{1}{2}\left[\frac{1}{SIN\,\alpha} + COT\,\alpha\right] \quad , \qquad [C-16]$$

equation [C-14] may be rewritten as

$$\Delta_{IH} = W\left[\delta_x\right]_{\frac{x}{\alpha} = 1.0} \cdot \left[\frac{R_H^3}{E_H I_H} + \frac{R_o^3}{E_o I_o}\right] + \frac{W}{2}\Gamma\left[\frac{R_H}{E_H A_{TH}} + \frac{R_o}{E_o A_{To}}\right] . \quad [C-17]$$

<u>Axial Contraction of Inner Hull</u>

For ballast tanks located in the "necked-down" portion of the hull near 'midships (Figure 1), an additional deflection term representing the effect of the axial contraction of the pressure hull (due to the hydrostatic pressure at depth) on the outer hull must be included. Using equation [C-4], simplifying and solving for the circumferential stress in the inner hull,

$$\sigma_{\phi H} = p\frac{R_H}{t_H}\left[1 - \left(1 - \frac{\nu}{2} - \frac{b_H t_H}{A_{fH} + b_H t_H}\right)\frac{1}{1+\beta} \cdot \frac{C''}{\text{SINH}\,\alpha' \ell + \text{SIN}\alpha' \ell}\right] \quad [C-18]$$

in which:

$$C'' = \text{SINH}\alpha' x \cdot \text{COS}\alpha' \ell - 2\,\text{SINH}\alpha' x + \text{COSH}\alpha' x + \text{COSH}\alpha' x \cdot \text{SIN}\alpha' \ell$$
$$-2\,\text{COSH}\alpha' x \cdot \text{SIN}\alpha' x + \text{SIN}\alpha' x \cdot \text{COSH}\alpha' \ell + \text{COS}\alpha' x \cdot \text{SINH}\alpha' \ell ,$$

$$\alpha' = \frac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{R_H t_H}} ,$$

and ℓ is defined in Figure 16.

Integrating the deflection coefficient C'' between limits of 0 and ℓ, and reducing,

$$C' = \frac{1}{\alpha'}\left[\text{COSH}\alpha' \ell - \text{COS}\alpha' \ell\right] .$$

This expression represents the area under the deflected curve "C" in Figure 16. Setting this area equal to that of the rectangle $H^* \cdot \ell$ to obtain the mean stress, and since

$\theta = \alpha' \ell,$

$$H^* = [\cosh\theta - \cos\theta]\,\frac{1}{\theta}$$

Substituting for C'' in equation [C-18], the mean circumferential stress in the inner hull expressed in terms of the total pressure, p', is

$$\sigma_{\phi M} = \frac{p'R_H}{t_H}\left[1 - \left(1 - \frac{\nu}{2} - \frac{b_H t_H}{A_{fH} + b_H t_H}\right)\frac{1}{1+\beta} \cdot \frac{1}{\theta} \cdot \frac{\cosh\theta - \cos\theta}{\sinh\theta + \sin\theta}\right] .$$

Letting $N = \dfrac{\cosh\theta - \cos\theta}{\sinh\theta + \sin\theta}$, [C-19]

where: $\theta = \sqrt[4]{3(1-\nu^2)}\left(\dfrac{L_H}{\sqrt{R_H t_H}}\right)$, [C-20]

defining $L_{eH} = 1.555\, N \sqrt{R_H t_H}$, [C-21]

and using appropriate notation for frame and shell areas, the mean circumferential stress may be rewritten as

$$\sigma_{\phi M} = \frac{p'R_H}{t_H}\left[1 - \frac{N}{2\theta A_{TH}}\left(2\left(A_{fH} - b_H t_H\right) - \nu A_{fH}\right)\right] . \qquad \text{[C-22]}$$

Since $\sigma_{\phi M} = E_H \epsilon_\phi$, the longitudinal membrane strain in the inner hull may be written as

$$\epsilon_Z = \frac{(1-\nu^2)}{E_H}\,\sigma_Z' - \nu\,\frac{\sigma_{\phi M}}{E_H} , \qquad \text{[C-23]}$$

in which: $\sigma'_Z = \dfrac{p' R_H}{2 t_H}$ [C-23]

Effect of Shortening of Inner Hull on Outer Hull

By substituting for σ'_Z and $\sigma_{\phi M}$ from [C-22] into [C-23], an expression may be obtained for the longitudinal membrane strain in the inner hull. Since the outer hull will be subjected to the same strain, the longitudinal membrane stress in the outer hull will be in proportion to the relative thickness of the two hulls, such that for the outer hull the average hoop stress is

$$\sigma_{\phi oZ} = E_H \epsilon_Z \frac{t_H}{t_o} = \frac{p' R_H}{2 t_o} (1 - \nu^2)$$

$$- \nu \frac{p' R_o}{t_o} \left[1 - \frac{N}{2 \theta A_{TH}} \left(2 (A_{fH} - b_H t_H) - \nu A_{fH} \right) \right] \qquad \text{[C-24]}$$

Taking $(1 - \nu^2) \cong 1$, and since the radial deflection

$$\Delta_R = \frac{R_o}{E_o} \sigma_{\phi oZ} ,$$

substituting,

$$\Delta_R = \frac{p' R_H R_o}{2 E_o t_o} \left[(1 - 2\nu) + \frac{\nu N}{\theta A_{TH}} \left(2 (A_{fH} - b_H t_H) - \nu A_{fH} \right) \right] . \qquad \text{[C-25]}$$

Adding to equation [C-9], for the case of a ballast tank located near midships and between two large pressure hull sections (Figure 1), the deflection of the strut becomes

$$\Delta = \frac{P R_o^2}{E_o A_{To}} \left[1 - \frac{\nu}{6} p \right] + \frac{p' R_H^2}{2 E_H t_H} \left[2 - \nu - \left(\frac{2 A_{fH} - \nu (A_{fH})}{A_{TH}} \right) \right] + \qquad \text{[C-26]}$$

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[Equation [C-26] Continued)

$$+ \frac{p'R_H R_o}{2E_o t_o}\left[(1 - 2\nu) + \frac{\nu N}{\theta A_{TH}}\left(2(A_{fH} - b_H t_H) - \nu A_{fH}\right)\right] \qquad [C-26]$$

$$- \frac{WL_s}{A_s E_s}$$

Load in the Strut

Equating deflections from equation [C-17] and [C-26], solving for W, and re-arranging, the load in the strut may be written as

$$W = \left\{2P\left[1 - \frac{\nu}{6}p\right] + \frac{p'R_H^2}{R_o^2} \cdot \frac{E_o}{E_H} \cdot \frac{A_{To}}{t_H}\left[2 - \nu - \left(\frac{2A_{fH} - \nu A_{FH}}{A_{TH}}\right)\right]\right.$$

$$\left. + \frac{p'R_H A_{To}}{R_o t_o}\left[(1 - 2\nu) + \frac{\nu N}{\theta A_{TH}}\left(2(A_{fH} - b_H t_H) - \nu A_{fH}\right)\right]\right\}$$

$$\div \left\{\frac{2L_s A_{To} E_o}{R_o^2 A_s E_s} + 2\left[\delta\right]_{\frac{x}{\alpha} = 1.0} \cdot \left[\frac{R_H^3 A_{To} E_o}{R_o^2 I_H E_H} + \frac{R_o A_{To}}{I_o}\right]\right.$$

$$\left. + \Gamma\left[\frac{1}{R_o} + \frac{R_H E_o A_{To}}{R_o^2 E_H A_{TH}}\right]\right\} \qquad . \qquad [C-27]$$

In the above equation, the third term drops out for ballast tanks located near the ends of the pressure hull, since the contraction of the inner hull under hydrostatic pressure does not affect the outer hull.

II. STRESS EQUATIONS

Net Deflection of Outer Hull Plating between Frames

For a uniformly ring-stiffened circular cylindrical shell under uniform lateral loading, equations are available[24] for the axial force and moment in the shell plating and, expressed in the usual notation, are of the form

$$P = \frac{2\Delta_N E_o t_o}{\beta R_o^2} \cdot \frac{\chi_3 (2\alpha)}{2\chi_1 (2\alpha) \chi_3 (2\alpha) - \chi_2^2 (2\alpha)}$$

and

$$M_o = \frac{P\chi_2 (2\alpha)}{4\beta\chi_3 (2\alpha)} ,$$

for which, in the notation used herein for the outer hull,

$$V' = \chi_1 (2\alpha) ,$$

$$K' = \chi_2 (2\alpha) ,$$

$$N' = \chi_3 (2\alpha), \text{ and}$$

$$\alpha' = \beta ,$$

where:

$$V' = \frac{\text{COSH } \theta' + \text{COS } \theta'}{\text{SINH } \theta' + \text{SIN } \theta'} ,$$

$$K' = \frac{\text{SINH } \theta' - \text{SIN } \theta'}{\text{SINH } \theta' + \text{SIN } \theta'} ,$$

$$N' = \frac{\text{COSH } \theta' - \text{COS } \theta'}{\text{SINH } \theta' + \text{SIN } \theta'} , \text{ and} \qquad [C-28]$$

$$\alpha' = \frac{\theta'}{L_o} = \frac{\sqrt[4]{3(1 - \nu^2)}}{\sqrt{R_o t_o}} .$$

Combining the above equations and rewriting in the notation of this paper, the longitudinal moment in the outer hull plating at any position is

$$M_o = \frac{0.289 K'}{(2 V'N' - K'^2)(1 - \nu^2)^{\frac{1}{2}}} \cdot \frac{E_o t_o}{R_o} \cdot \Delta_N , \qquad [C-29]$$

in which: Δ_N = net deflection of the outer hull plating, with respect to the frame (Figure 13b), (in).

From equation [C-4], the total deflection of the outer shell plating midlength between frames can be obtained by evaluating k for $\ell/2$ and, after simplification, is

$$\Delta_{\frac{L}{2}} = \frac{(\Delta p)\, R_o^2}{E_o A_{To}} \left[1 - \frac{\nu}{6} p\right] \left[\frac{A_{To}}{t_o} + H'_M \frac{A_{fo}}{t_o}\right] , \qquad [C-30]$$

where:

$$H'_M = \left[k\right]_{x = \frac{\ell}{2}} = -2 \left[\frac{\mathrm{SINH}\,\frac{\theta'}{2}\,\mathrm{COS}\,\frac{\theta'}{2} + \mathrm{COSH}\,\frac{\theta'}{2}\,\mathrm{SIN}\,\frac{\theta'}{2}}{\mathrm{SINH}\,\theta' + \mathrm{SIN}\,\theta'}\right] \qquad [C-31]$$

For ballast tanks located in the "necked down" portion of the inner hull (Figure 1), there will be an additional deflection component in the outer hull plating resulting from the longitudinal contraction of the inner hull under hydrostatic loading. To obtain this added deflection component, it may be assumed that the inner hull deflection between frames will produce a similar (but not equal) deflection of the outer hull in the opposite direction. In addition, the effect of the inner hull contraction on the deflection of the outer hull may be approximated by an equation having a form similar to equation [C-30] by insertion of the appropriate nomenclature. Adding both deflections, the total deflection at midlength between frames becomes

$$\Delta_{\frac{L}{2}} = \frac{(\Delta p)\, R_o^2}{E_o A_{To}} \left[1 - \frac{\nu}{6} p\right] \left[\frac{A_{To}}{t_o} + H'_M \frac{A_{fo}}{t_o}\right]$$

$$+ \frac{p' R_H^2}{E_H A_H} \left[1 - \frac{\nu}{2}\right] \left[\frac{A_{TH}}{t_H} + H_M \frac{A_{fH}}{t_H}\right] . \qquad [C-32]$$

To obtain the net midlength deflection of the shell at any circumferential position, the deflection at a uniform frame must be subtracted (equation [C-6]) and the restraint on the frame induced by the radial struts added. Substituting equation [C-13] and simplifying, the net deflection of the outer hull plating becomes

$$\Delta_N = \frac{WR_o^3}{E_o I_o}\left[\delta_x\right] + \frac{WR_o}{E_o A_o}\left[\gamma_x\right]$$

$$+ \frac{(\Delta p) R_o^2}{E_o A_{To}} \left[1 - \frac{\nu}{6} p\right] \left[\frac{A_{To}}{t_o} + H'_M \frac{A_{fo}}{t_o} - L_{eo}\right] \qquad [C-33]$$

$$+ \frac{p' R_H^2}{E_H A_{To}} \left[1 - \frac{\nu}{2}\right] \left[\frac{A_{TH}}{t_H} + H_M \frac{A_{fH}}{t_H} - L_{eH}\right] ,$$

where: $\gamma_x = \frac{1}{2}\left[\frac{COSx}{SIN\,\alpha}\right]$. [C-34]

Discontinuity Stress in Outer Hull Plating

The general equation for the longitudinal discontinuity stress in the outer hull plating may be written as

$$\sigma_{Lx} = \pm \frac{6 M_o}{t_o^2} + \frac{(\Delta p) R_o p}{6 t_o} .$$

Substituting equations [C-29] and [C-33] and simplifying, the longitudinal discontinuity stress in the outer hull plating at the frame, for any circumferential position with respect to the struts (Figure 17), is

$$\sigma_{Lx} = \pm \frac{1.734 K'}{(2 V' N' - K'^2)(1 - \nu^2)^{\frac{1}{2}}} \left\{ \frac{WR_o^2}{I_o}\left[\delta_x\right] + \frac{W}{A_{To}}\left[\gamma_x\right] \right.$$

$$+ \frac{(\Delta p) R_o}{A_{To}} \left[1 - \frac{\nu}{6} p\right] \left[\frac{A_{To}}{t_o} + H'_M \frac{A_{fo}}{t_o} - L_{eo}\right] + \qquad [C-35]$$

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(Equation [C-35] Continued)

$$+ \frac{p' R_H^2 E_o}{A_{TH} R_o E_H} \left[1 - \frac{\nu}{2}\right] \left[\frac{A_{TH}}{t_H} + \frac{H_M A_{fH}}{t_H} - L_{eH}\right]\Bigg\} + \frac{(\Delta p) R_o p}{6 t_o} \quad ,$$

where, for any position x between struts (Figure 14b), the deflection coefficient,

$$\delta_x = \frac{1}{4} \left[\frac{\alpha \, COS \alpha}{SIN^2 \alpha \; COSx} + \frac{COSx}{SIN \alpha} - \frac{2}{\alpha}\right] \quad . \qquad [C-36]$$

For ballast tanks located at the ends of the pressure hull, since there will be no deflection component due to inner hull contraction, the term containing the hydrostatic pressure, p', drops out.

To obtain the longitudinal discontinuity stress at a wing bulkhead, the term containing W becomes insignificant since the struts merge, and

$$\left[\frac{A_{To}}{t_o} + H'_M \frac{A_{fo}}{t_o} - L_{eo}\right] \longrightarrow \left[1 + H'_M\right] \quad .$$

In addition, since the wing bulkhead is very stiff, and consequently may be assumed as having a negligible deflection, the deflection of the adjacent outer hull frame will produce an additional longitudinal moment in the plating at the bulkhead. Hence, this discontinuity stress may be expressed as

$$\sigma'_{Lx} = K'' \left[\frac{E_o \Delta}{R_o}\right] C_3 \quad ,$$

where:

$$K'' = \frac{1.734 K'}{(2 V' N' - K'^2)(1 - \nu^2)^{\frac{1}{2}}} \quad , \quad C_3 = \frac{b'_B t_o^3}{b'_B t_o^3 + L_{eo} t_B^3} \qquad [C-37]$$

and C_3 is a distribution factor relating the relative stiffness of the wing bulkhead and the outer hull plating.

Substituting for Δ from equation [C-5],

$$\sigma'_{LS} = K'' \left[\frac{(\Delta p) R_o}{t_o} \left(1 - \frac{\nu}{6} p\right) \left(\frac{L_{eo} t_o}{A_{To}}\right)\right] \left[\frac{b'_B t_o^3}{b'_B t_o^3 + L_{eo} t_B^3}\right] .$$

Rearranging and substituting appropriate portions of equation [C-35], the longitudinal discontinuity stress at a wing bulkhead (Figure 17) is

$$\sigma_{LB} = \pm \frac{1.734 K' (1 - \nu^2)^{-\frac{1}{2}}}{2 V' N' - (K')^2} \left\{ \left[\frac{(\Delta p) R_o}{t_o}\right] \right.$$

$$\cdot \left[(1 + H'_M) + \left(\frac{L_{eo} t_o}{A_{To}}\right)\left(\frac{b'_B t_o^3}{b'_B t_o^3 + L_{eo} t_B^3}\right)\right] \left[1 - \frac{\nu}{6} p\right] \quad \text{[C-38]}$$

$$\left. + \left[\frac{p' R_H^2 E_o}{A_{TH} R_o E_H}\right] \left[1 - \frac{\nu}{2}\right] \left[\frac{A_{TH}}{t_H} + \frac{A_{FH} H_M}{t_H} - L_{eH}\right] \right\} + \frac{(\Delta p) R_o p}{6 t_o} .$$

Stresses in Outer Hull Frame

The stress in the outer hull frame at any circumferential position may be generally expressed as

$$\sigma_{\phi f(x)} = \frac{T_{(x)}}{A_{To}} + \frac{M_{f(x)}}{Z_f} \qquad \text{[C-39]}$$

and

$$\sigma_{\phi p (x)} = \frac{T_{(x)}}{A_{To}} + \frac{M_{f(x)}}{Z_p} + \nu \sigma_{L(x)} , \qquad \text{[C-40]}$$

where the last term in equation [C-40] represents the effect of axial contraction of the inner hull and, referring to Figure 17,

$\sigma_{\phi p(x)}$ = stress in outstanding frame flange (lb/in^2),

$\sigma_{\phi p(x)}$ = stress in plate flange (lb/in^2),

Z_f = Section modulus of outstanding frame flange (in^3), and

Z_p = Section modulus of plating flange (in^3).

The hoop force, $T_{(x)}$, and the moment, $M_{f(x)}$, in the outer hull frame at any position x may be written, respectively, as

$$T_{(x)} = \left[P R_o - W(\gamma_x)\right] \left[\frac{A_{fo}}{A_{to}} \left(\frac{\alpha - x}{\alpha}\right) + \frac{x}{\alpha}\right] , \quad \text{[C-41]}$$

where the second term represents a linear approximation to the effectiveness of the shell plating in absorbing the hoop load, and

$$M_{f(x)} = W R_o \left[\xi_x\right] , \quad \text{[C-42]}$$

where the moment coefficient at any position

$$\xi_x = \frac{1}{2} \left[\frac{\text{COS}x}{\text{SIN}\,\alpha} - \frac{1}{\alpha}\right] . \quad \text{[C-43]}$$