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# THE STEREOGRAPHIC PROJECTION

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## ABSTRACT

The theory of the stereographic projection is reviewed and some applications in meteor astronomy are discussed briefly. It is shown that the grid lines for a hemispherical projection may be plotted with an accuracy sufficient for most purposes by drawing only arcs of circles.

## SOMMAIRE

La théorie de la projection stéréographique est rappelée et quelques unes de ses applications à l'astronomie des météores sont brièvement discutées. Il est démontré que dans la plupart des cas les lignes de coordonnées d'une projection hémisphérique peuvent être tracées avec une précision suffisante comme de simples arcs de cercles.

*Introduction.* When a meteor trail, normally part of a great circle, is extrapolated beyond its beginning and end points it appears to an observer as a straight line against the star background. If this trajectory is plotted on a star map, the form it takes depends on the particular map projection. The stereographic projection, which is used in the visual meteor observing programme of this laboratory, has several advantages (Millman 1960). There is no distortion of star groups and the entire hemisphere may be mapped on a single sheet. On the other hand, a complete meteor trajectory maps into an arc of a circle. A meteor, being only a short segment of an arc, may be validly plotted as a straight line, but because of the overall curvature distortion of the map the trail orientation must be determined with respect to near by stars. A set of stereographic grids was produced which is useful both in the reduction of data on visual meteor trail plots and for instruction of visual observers.

The more general uses of the stereographic projection in mapping the earth's surface may be found in most books on map making. However, since much of the detailed theory of this projection is to be found in publications now relatively inaccessible (Penfield 1901, Adams 1919), it is useful to review here the elementary mathematics and to illustrate methods of calculating and plotting grids.

*Theory of the projection.* In the stereographic projection, points on the sphere are projected on to a plane from the end of a diameter opposite to the point of tangency of the plane (figure 1). An arc of length  $S$  along

the sphere from the point of tangency  $Z$  projects into a radial distance  $R$  on the plane given by

$$S \rightarrow R = D \tan \frac{\theta}{2} \quad (1)$$

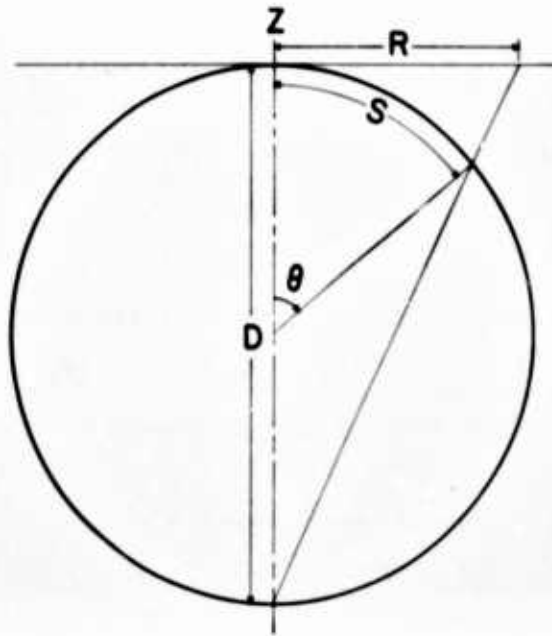


FIG. 1—The geometry of the projection of a sphere on to a plane which is tangent to the sphere at  $Z$ .

where  $D$  is the diameter of the sphere and  $\theta$  the angle at the centre of the sphere subtended by the arc  $S$ . For celestial co-ordinates  $D$  may be set to 1 unit; this unit is the arbitrarily chosen map radius of the full circle of a projected hemisphere ( $\theta = 90^\circ$ ).

The co-ordinates on the sphere may be one of several systems such as latitude and longitude or declination and hour angle. We will deal with the latter system and define the following symbols:

$t$ , hour angle;  $\delta$ , declination;  $h$ , elevation;  $A$ , azimuth;  $R$ , radial map distance;  $\phi$ , latitude.

It is desired to project the observer's hemisphere from zenith to horizon. The tangent point of the projection plane is the observer's zenith; hence the angle  $\theta$  is the complement of the elevation angle  $h$ , that is

$$R = \tan\left(\frac{90^\circ - h}{2}\right) \quad (2)$$

The position angle of the radius vector is determined by the azimuth  $A$ . The elevation and azimuth are calculated from the spherical trigonometric equations

$$\sin h = \sin \delta \sin \phi + \cos \delta \cos \phi \cos t \quad (3)$$

$$\sin A = \cos \delta \sin t / \cos h \quad (4)$$

In some cases  $R$  may be more easily calculated from the equation

$$R = \left( \frac{1 - \sin h}{1 + \sin h} \right)^{\frac{1}{2}} \quad (5)$$

The calculations are straightforward but tedious to carry out by hand. For example if a grid at  $5^\circ$  intervals of hour angle and declination is required there are some 600 intersection points to be calculated.\* A medium-sized digital computer will perform the calculations for a grid in about four minutes.

*Simplified methods.* The most accurate method of producing a grid is to calculate and plot intersection points and then draw smooth arcs through the points. With care in drafting, a grid that is suitable for most purposes may be produced by a shorter method.

The projection is such that circles on the sphere map into circles or arcs of circles on the plane. Consider first the declination circles as in figure 2(a), where  $P$  is the pole. On the map, the radius of a declination circle is given by†

$$R_d = \frac{\cos \delta}{\sin \delta + \sin \phi} \quad (6)$$

The distance along the meridian from the centre of the plot to the centre of the declination circle is

$$C_d = \frac{\cos \phi}{\sin \delta + \sin \phi} \quad (7)$$

As a further check when plotting, it is useful to calculate the radial distance  $E$  to the intersection of the circle with central meridian

$$E = \tan \frac{\delta - \phi}{2} \quad (8)$$

Note that the radius  $R_d$  becomes infinite when  $\delta = -\phi$ ; that is, this circle maps into a straight line. For  $\delta < -\phi$  both  $R_d$  and  $C_d$  become

\*There are actually 1200 points but since the grid is symmetrical about the central meridian only half need be calculated.

†Equations (6) to (10) are derived by Adams (1919) by complex number analysis. They may also be derived by simple algebra and trigonometry beginning with equation (1).

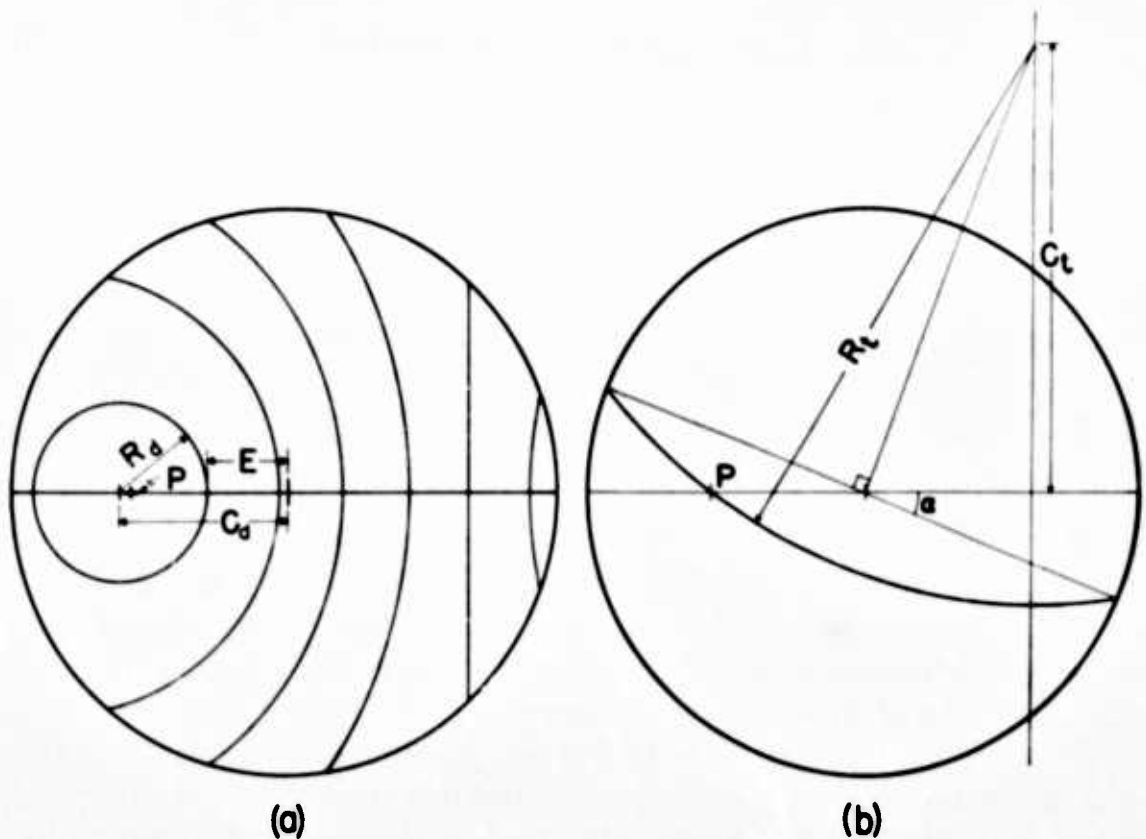


FIG. 2—(a) Declination (or latitude) circles. (b) Hour angle (or longitude) circles.

negative indicating that the arcs have reversed curvature and the circle centres are located on the other side of the  $\delta = -\phi$  line.

The hour angle arcs all pass through the pole  $P$  (figure 2(b)) and terminate on the ends of a diameter. The radius of the arc is

$$R_t = \csc t \sec \phi \quad (9)$$

The centres all lie on the  $\delta = -\phi$  line at a distance  $C_t$  from the meridian

$$C_t = \cot t \sec \phi = R_t \cos t \quad (10)$$

As a further check the angle  $\alpha$  of the diameter is given by

$$\sin \alpha = \frac{\sin t \sin \phi}{\cos z} \quad (11)$$

where

$$\sin z = \sin t \cos \phi.$$

It requires only a short time to compute these values by hand for a single grid.

The only problems involved in drawing grids by the radius-centre method arise when the radii become extremely long. For example at

latitude  $45^\circ$  the  $5^\circ$  hour circle has a radius of 97 inches for a 12-inch diameter map. In these cases recourse may be had to a drafting device in which a beam is bent to the arc of a circle of very large radius. The circular arc of the proper radius is fitted to the pole and to the end points of the diameter determined from equation (11). In figure 3 are shown grids at  $15^\circ$  intervals of latitude produced by the above method.

The original need for these grids involved only hour angle circles. If the pole is taken as the radiant of a meteor shower, the hour angle circles represent a family of possible shower meteor trajectories. Radiants can

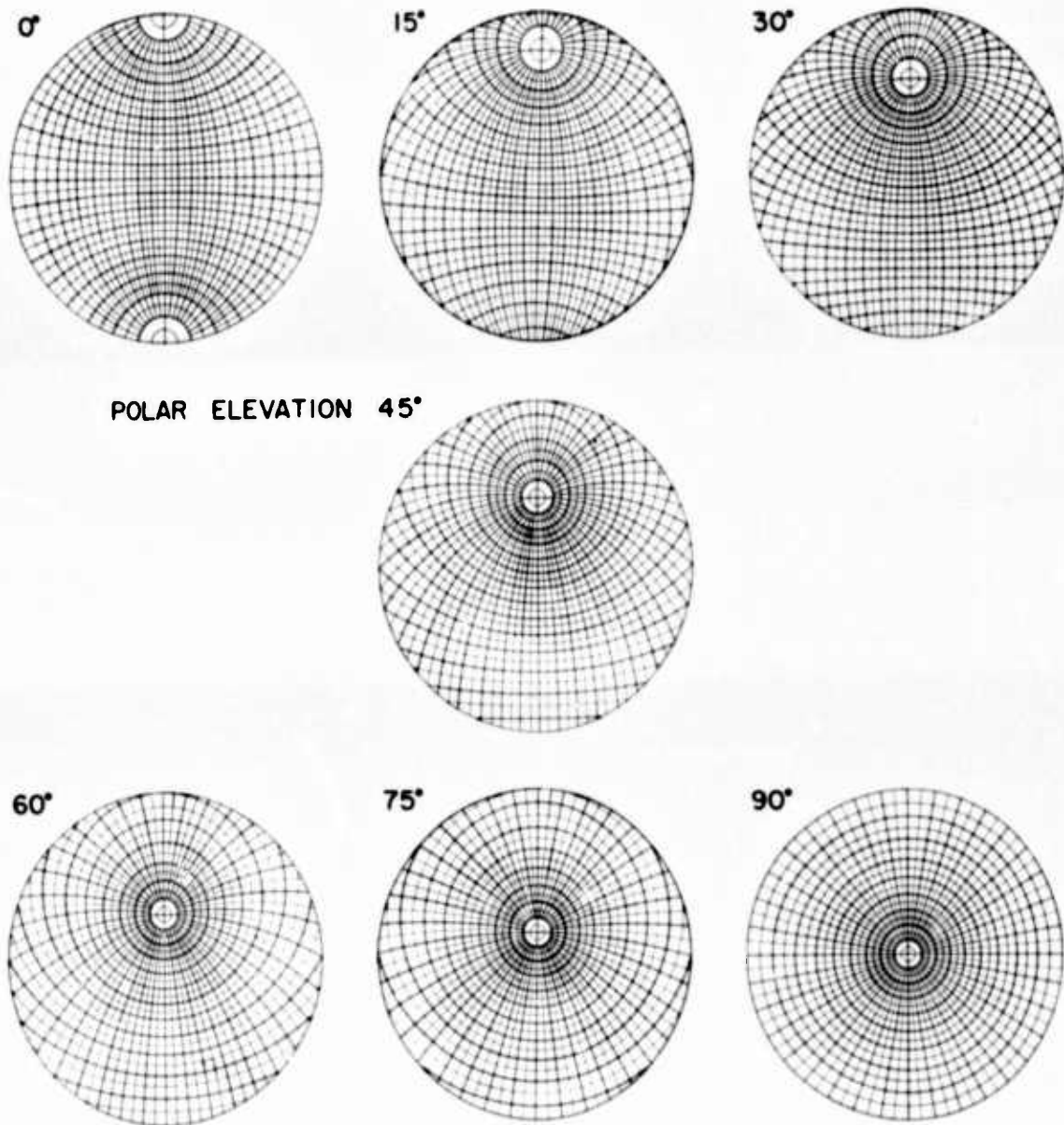


FIG. 3—A set of grids at  $15^\circ$  intervals of polar elevation produced by plotting only circles or arcs of circles. The 12-in. diameter grids are printed on stable plastic and are used as aids in reducing data on visual meteors plotted on star maps.

be determined from meteor trail plots using a series of such curves at intervals of polar elevation.

On the star maps employed here, the radiants of the major meteor showers group fairly well at  $15^\circ$  intervals of radiant elevation from  $0^\circ$  to  $75^\circ$ . Hence six grids of hour angle circles were required. The declination circles were added and the seventh grid at  $90^\circ$  polar elevation plotted also to give a complete set of grids of more general usefulness.

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