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THE LAMINAR BOUNDARY LAYER ON A CONE IN A SUPERSONIC AIR STREAM AT ZERO ANGLE OF ATTACK

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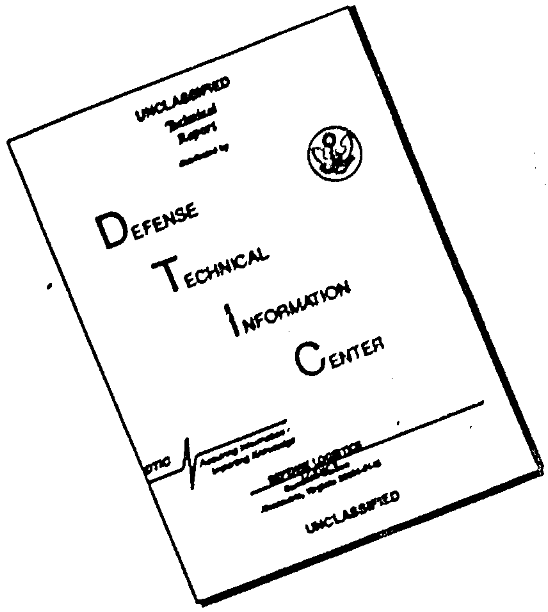

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THE LAMINAR BOUNDARY LAYER ON A CONE IN A SUPERSONIC AIR STREAM AT ZERO ANGLE OF ATTACK.

(Die laminare Grenzschicht bei einem mit Überschallgeschwindigkeit angeströmten nichtangestellten Kreiskegel)

This report demonstrates that the integration of the equations for the laminar boundary layer on a cone in a supersonic air stream can be reduced to the equations for the flat plate. The simple result is obtained, that on the cone the boundary layer is different by [illegible] as compared to the plate. The mean friction coefficient and the heat transfer coefficient are greater by the factor [illegible] times the squa

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If a circular-base cone is placed in a supersonic axial flow, then a conical shock front appears, behind which again potential flow is found, as long as friction is not taken into account. The flow behind the shock forms a conical field*; that is to say, constant pressure, density and velocity are found along all straight lines through the apex.

Now, let us discuss the laminar boundary layer on the cone. Let the axis of the cone be the positive x axis of a Cartesian system of co-ordinates, with the apex of the cone as its origin. Let us introduce the spherical co-ordinates r, ϑ, ϕ, by the relations:

$$x = r\cos\vartheta,\quad y = r\sin\vartheta\cos\phi,\quad z = r\sin\vartheta\sin\phi\ .$$

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* A. Busemann: Drücke auf Kegelförmige Spitzen bei Bewegung mit Überschallgeschwindigkeit. ZAMM, Volume 9 (1929), book 6, p. 496.

Let a meridian plane through the axis of the cone be described by a pair of straight lines, $\vartheta = \vartheta_0$ on the cone's surface. Let U and V be the velocity components in the meridian plane, U in the direction of r and V perpendicular to this direction (Figure 1). Due to the rotational symmetry, no component normal to this plane is found.

The differential equations for the laminar boundary layer are obtained in the usual manner from the equations of motion for a viscous fluid by a limiting process with respect to vanishing viscosity. If we assume here that the order of the magnitudes appearing in the boundary layer of the cone is maintained in the differentiation with respect to r, but that it is decreased by 1 in the differentiation with respect to ϑ, then, considering that the pressure on the surface of the cone is constant, the differential equations become:

(1)

where ρ denotes the density, i the enthalpy (heat content), λ the thermal conductivity, and μ the friction coefficient of the gas under consideration. Let the specific heat c_p and the Prandtl number $\sigma = \frac{c_p \mu}{\lambda}$ be constant. λ and μ are the well-known functions of the temperature T, or of i, with $i = c_p T$. The same holds for ρ since the pressure in the boundary layer is constant.

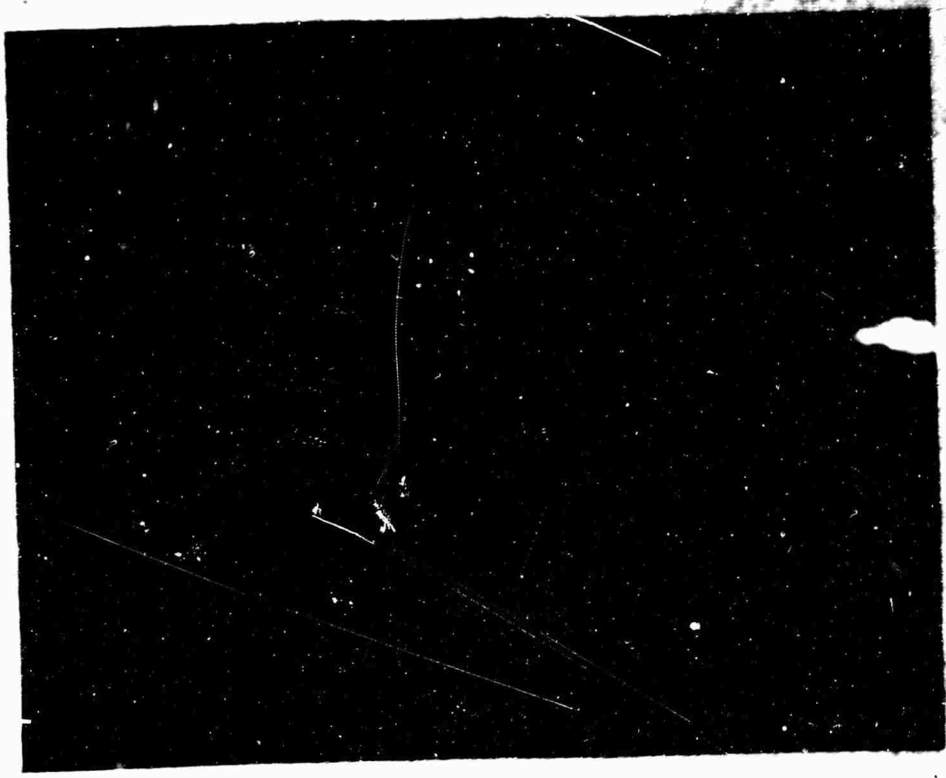

Fig. 1

Section through the axis of a cone in axial flow with compression shock. U_k = velocity along the surface of the cone in the potential flow behind the shock.

Let us now demonstrate that the integration of equations (1) for the cone in axial supersonic flow can be reduced to the integration of the equations for the laminar boundary layer on a flat plate. With the Cartesian system of co-ordinates, the positive x axis of which is in the plate, the y axis normal to the plate, and the origin of which lies at the foremost point of the plate, these equations are:

[illegible] (2)

Here, ρ , i, λ, μ , c_p denote the same magnitudes as above. u, v are the velocity components in the direction of the x and y axis. Let us demonstrate first that for the flat plate it is permissible to apply the assumption which is correct for incompressible flow, to compressible boundary layers also, i.e., u and i are functions of the single independent variable $\chi = \frac{y}{\sqrt{x}}$. In a previous treatise*, we integrated the system of equations (2) by assuming that i is a function of u only. However, with the result thus obtained, we could develop our calculations for $u = u(\chi)$, and then also for $i = i(\chi)$.

Now, if we use $u = u(\chi)$ and $i = i(\chi)$, then we obtain directly from the second equation of (2) that v has a form of $v = \frac{\bar{v}(\chi)}{\sqrt{x}}$. With that, the set of equations (2) is transformed into the system of common differential equations:

[illegible] (3)

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* W. Hantzsche and H. Wendt: Zum Kompressibilitätseinfluss bei der laminaren Grenzschicht der ebenen Platte. Jahrbuch 1940 der deutschen Luftfahrtforschung, p. 517.

In order to demonstrate that the integration of the equations (1) can also be reduced to a system of the form (3), let us first write in an analogous manner $U = U(\chi)$ and $i = i(\chi)$ where χ is correspondingly assumed to be

$$\chi = \frac{r(\vartheta - \vartheta_0)}{\sqrt{r}} = \sqrt{r}\,(\vartheta - \vartheta_0)\,.$$ Again, $v = \frac{\overline{V}(\chi)}{\sqrt{r}}$ and one obtains the equations:

[redacted] (4)

By a simple transformation of the dependent and independent variables:

$$(\overline{\chi}) = \sqrt{3\chi}\;,\quad \tilde{V}(\chi) = \sqrt{3}\left\{W(\overline{\chi}) - \frac{2}{3}\,\overline{\chi}\,U\right\}$$

we obtain a system of the form (3):

[redacted] (5)

The boundary conditions for this system of common differential equations on the surface of the cone ($\overline{\chi} = 0$) are $U = 0, W = 0$ (since $V = 0$) and a condition on i or $\frac{d\,i}{d\,\overline{\chi}}$, depending on the type of the special problem. If the temperature is predetermined, then i is given; if the heat transfer is predetermined, then $\left(\frac{d\,i}{d\,\bar{X}}\right)_0$ is given.

$\frac{d\,i}{d\,\bar{X}} = 0$ for $\bar{X} = 0$ corresponds to the thermometer problem (unheated or uncooled cone). Furthermore, U and i at the edge of the boundary layer must assume the values of the potential flow on the cone, U_K and i_K.

In order to obtain an integral of (5), we proceed in the following manner. At the corresponding boundary conditions:

[redacted]

we search for a solution of the differential equation (3) of the flat plate. If this has the form $u = u^*(X)$, $v = \frac{\tilde{v}^*(X)}{\sqrt{x}}$, and $i = i^*(X)$, then we obtain the solution for the cone in the form

[redacted]

The form of this solution shows that as a function of X or $\bar{X}$, respectively, the velocity profile (u-component) and the temperature profile in the boundary layer of the plate coincide with those in the boundary layer of the cone. However, for small $\vartheta - \vartheta_0$, $\bar{X}$ and X are different by $\sqrt{3}$ only, neglecting the chosen co-ordinates. Thus the boundary layer on the cone is distorted by the factor $\frac{1}{\sqrt{3}}$ as compared to the boundary layer on the plate. The dependency of the enthalpy $i = c_p T$ upon u in the boundary layer of the plate coincides exactly with that upon U in the case of the cone.

The shearing stress on the cone

[redacted]

is apparently greater by the factor $\sqrt{3}$ than that on the plate,

[redacted]

at the same x and r. However, the mean friction coefficient becomes different by the factor $\frac{2}{3}\sqrt{3}$ only, as compared to the value for the plate, because of the different rule of definition.

[redacted]

L = length of the plate, or length of the generatrix of the surface of the cone.

Finally, let us state the effect on the heat transfer coefficient, which is analogous to that on the friction coefficient. The local heat transfer coefficient on the cone is greater by the factor $\sqrt{3}$ than that on the plate, the mean heat transfer coefficient, by $\frac{2}{3}\sqrt{3}$.

Thus, the result for the laminar boundary layer on the flat plate may in this manner be applied directly to the cone in a supersonic air stream. Here the constant velocity on the surface of the cone in the potential flow behind the compression shock is to be employed as the reference velocity, U_k (Figure 1).