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PROCESSOR:

THE EFFECT OF RELIABILITY OF
"SUPERVISORY" EQUIPMENT ON
THE ACCURACY OF A "SUPERVISED" SYSTEM

Alexander W. Boldyreff

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THE EFFECT OF RELIABILITY OF "SUPERVISORY" EQUIPMENT ON THE ACCURACY OF A "SUPERVISED" SYSTEM*

Alexander W. Boldyreff**

Summary: This study deals with a basic system, the performance of which degenerates through random errors increasing with the square root of operating time. The probable circular error of this system can be made independent of the time by the addition to the system of supervisory equipment capable of periodic correction of the errors of the basic system. Assuming the supervisory equipment "fail safe" and subject to constant hazard type of failure the performance of the supervised system is determined for various values of reliability of the supervisory equipment. It is shown that a great improvement in performance is possible even with unreliable supervision.

In the design of complex high performance automatic systems it is sometimes important to know how much additional complexity (with a corresponding loss in reliability) can be accepted to improve performance. This study is directed towards such a problem.

The basic system under consideration is designed to operate automatically for a maximum operating time T. Its performance is characterized (for operating time $t \geq 0.05T$) by a normal distribution of error with circular symmetry and zero mean, and the probable circular error c_a, such that

$$c_a^2 = a^2 + (b\sqrt{t})^2 \quad . \tag{1}$$

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By the addition of supervisory equipment the probable error of the supervised system is assumed to become independent of the time, provided the supervisory equipment does not fail, and the probable circular error of the supervised system c_1 is then assumed to be

$$c_1 = a. \tag{2}$$

The supervisory equipment is assumed to be "fail safe" and subject to the constant hazard type of failure, independent of the time it has been operating. Thus, the probability density of failure is

$$\frac{1}{L} \exp\,[-t/L] \;,$$

where L is the mean time to failure which depends on the design reliability. The probability of no failure during the operating time (0,T) is then the reliability of the supervisory equipment:

$$R = \exp\,[-T/L] \;. \tag{3}$$

It is assumed that a failure of the supervisory equipment results in the absence of corrective signals to the basic system, and that for failures of the supervisory equipment at time t the probable circular error of the basic system $c(t\ )$ is given at the end of the operating time T by

$$c^2(t\ ) = a^2 + (b\sqrt{T-t}\ )^2 \;. \tag{4}$$

With the above assumptions the question arises: what improvement in the performance of the basic system can be expected at the expense of added complexity by the addition to the basic system of supervisory equipment of variable reliability.

It is shown that a considerable gain in accuracy can be attained even with a relatively unreliable supervisory equipment.

The probability that a supervised system will have a radial error ρ not exceeding r at the end of operating time T, when the reliability of the supervisory equipment is R, is given by

$$P(r|R) = \int_0^T \Pr\left\{\text{failure at } t\right\} . \Pr\left\{\rho < r \text{ at } T| \text{ failure at } t\right\} dt +$$

$$+ \Pr\left\{\text{no failure over } (0,T)\right\} . \Pr\left\{\rho < r \text{ at } T| \text{ no failure}\right\} =$$

$$= \int_0^T \frac{1}{L} \exp(-t/L) \left\{ 1 - \exp[-\tfrac{1}{2}(1.1774r)^2/a^2 + b^2(T-t)]\right\} dt +$$

$$+ \exp\ (-T/L)\ .\ [\ 1 - \exp(-\tfrac{1}{2}(1.1774r)^2/a^2\ ] =$$

$$= 1 - \frac{1}{L}\int_0^T \exp(-t/L)\ .\ \exp[\ -\tfrac{1}{2}(1.1774r)^2/a^2 + b^2(T-t)\ ]\ dt$$

$$- \exp[\ -T/L\ ]\ .\ \exp[\ -\tfrac{1}{2}(1.1774r)^2/a^2\ ]\ , \tag{5}$$

where the constant $1.1774 = \sqrt{2 \ln 2}$ = the ratio of the probable circular error to the standard deviation.

For any given value of R the probable circular error of the system is given by that value of r for which $P(r|R)$ in (5) is equal to a half.

Since the use of equation (5) would involve a formidable computational job, the solution of the problem was obtained in a form avoiding the use of high speed computation.

This was done by dividing the interval $(0,T)$ into ten equal sub-intervals Δt_i with mid-points t_i, and replacing the exact expression for $P(r|R)$ by an approximation:

$$P(r|R) = 1 - \sum_{i=1}^{10} Pr\left\{\text{failure in } \Delta t_i\right\} \cdot \exp[-\tfrac{1}{2}(1.1774r)^2/a^2+b^2(T-t_i)]$$

$$- Pr\left\{\text{no failure over } (0,T)\right\} \cdot \exp[-\tfrac{1}{2}(1.1774r)^2/a^2] \qquad (6)$$

The expressions $P(r|R)$ were then computed for various R and r. This was done for b equal to $0.267a$ and $T = 210$ for each of the values of R: 0, 0.2, 0.4, 0.6, 0.8, 1, and for r varying from zero to $8a$ in steps of $a/3$.

The results are shown graphically by plotting $P(r|R)$ against r for each of the above values of R. The values of r corresponding to $P(r|R) = 1/2$ are then the probable circular errors at the end of total operating time T of the supervised system for supervisory equipment of varying reliability.

Comments

The significant result of this study is the great gain in accuracy attained even when fairly unreliable supervision is used. Thus, the use of supervision of reliability of only 0.6 decreased the probable circular error to one third of that of the unsupervised system.

The three basic assumptions, namely the circular normal distribution of error, the error propagation equation (1), and the constant hazard type of failure of the supervisory equipment, were supported by ample empirical data in the particular situation to which this analysis was applied. The real problem was the technical attainability of desired performance level, and the question of equipment cost was unimportant.

However, the above assumptions are satisfied in many other problems of control of automatic equipment (as in some types of long range automatic flight), and with obvious modifications the analysis is capable of still wider application. Specifically, by assuming (1) a different law for the propagation of error in the basic system, and (2) a different mechanism of failure for the supervisory equipment, it would be easy to adapt the analysis of this paper to the most general types of feedback systems, where the output of the basic system is measured, compared with a standard or with a supplementary measurement, and used to regulate the basic system. In industrial applications it would also be necessary to determine through cost analysis whether the improvement in performance justifies the added cost of the supervisory equipment.

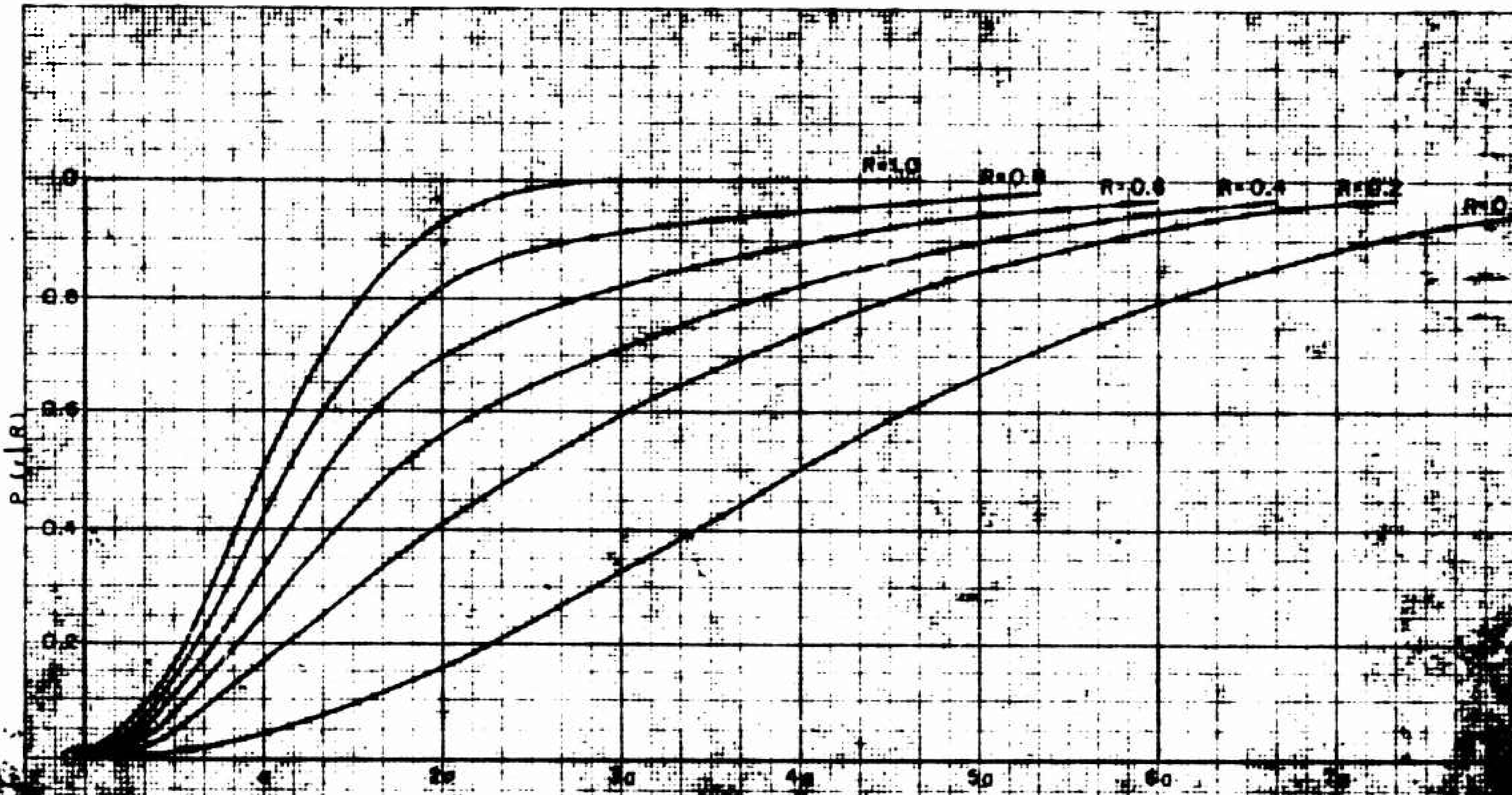

Fig. 1 — Effect of supervis[illegible] on [illegible] of supervised [illegible]