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THE RAND COLLECTION OF ILLUSTRATIVE APPROXIMATIONS

Cecil Hastings, Jr.

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## The RAND Collection of Illustrative Approximations

In recent years, the Numerical Analysis Department at RAND has been preparing loose leaf sheets that contain interesting and useful approximations to a number of the higher transcendental functions. <sup>are discussed</sup> A sample sheet is displayed, on the following page, somewhat reduced photographically from its original notebook size. Some sixty odd sheets in this series have been prepared to date, and up to date collections of these sheets have been distributed to some three hundred people working in the field of numerical analysis thruout the United States and in a few foreign countries.

The approximations given ~~in this series of loose leaf sheets~~ are of both a practical and of an illustrative nature. In a high speed digital machine, these approximations may take the place of bulky tables or the place of awkward series developments. For the hand computer, an easily evaluated expression will often be of use when a required table is unavailable for consultation. Beyond this, however, the sheets will be useful for the insight they give the practical computer in the approximation of functions. Starting with approximations concerning the common logarithm, sheets concerning an ever increasing number of the useful transcendental functions have been prepared and distributed. Sheets of approximations in this series are already available concerning the logarithmic function, the exponential function, the inverse tangent, the sine function, the inverse sine, the Gamma function, the Gaussian error integral, the inverse Gaussian error integral, the complete elliptic integrals of first and second kind, the exponential integral and a number of other ~~and~~ special functions.

As the approximations in this series are intended to be illustrative as well as practical, great care has been taken in the accurate leveling of

The RAND Corporation  
 Approximations in Numerical Analysis

Function:

$$-Ei(-X) = \int_X^{\infty} \frac{e^{-t}}{t} dt$$

Range:

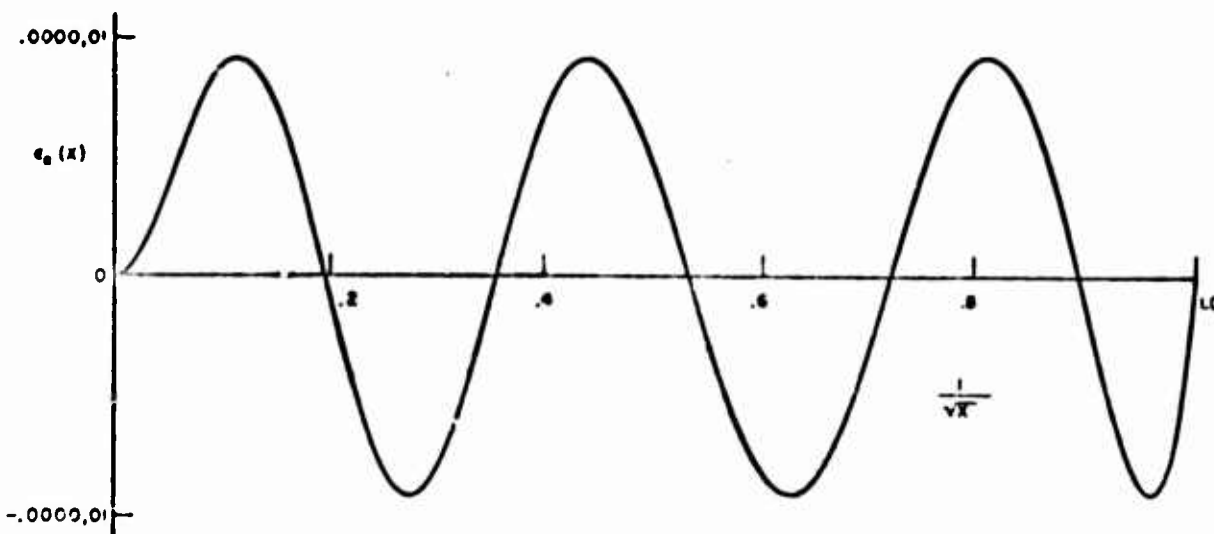
$$1 \leq X < \infty$$

Approximation:

$$-Ei^{\circ}(-X) = \frac{e^{-X}}{X} \left[ \frac{a_0 + a_1 X + a_2 X^2 + X^3}{b_0 + b_1 X + b_2 X^2 + X^3} \right]$$

$a_0 = .2372,9050$	$b_0 = 2.4766,3307$
$a_1 = 4.5307,9235$	$b_1 = 8.6660,1262$
$a_2 = 5.1266,9020$	$b_2 = 6.1265,2717$

Error Curve (Approximation - Function)/(Function):



Comments:

from P1

the error curves. The sheets of approximations thus give interesting information concerning the location of roots - points at which function and approximation agree - and the location of extremals - points at which the deviation between function and approximation is locally a maximum in an absolute or relative sense. The ~~accurate and carefully drawn~~ error curves that appear on each sheet will do much to give the reader a feel for the ~~nature of analytical approximation~~ <sup>gives an</sup> in a wide variety of typical cases of practical importance. In order to provide this important information, the coefficients in the approximations given have been recorded to perhaps two, three or four more decimals than would be required for practical considerations of utility. Should the coefficients given be of awkward size for use in a given computing machine, it is permissible to round the numbers quite severely. Of course the beauty and character of the error curve will be lost as a result of such mistreatment, but this is not a matter of practical concern in the use of the approximation.

Cecil Hastings, Jr.

The RAND Corporation, 1500 Fourth Street, Santa Monica, California

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 Approximations in Numerical Analysis

Function:

$$-Ei(-X) = \int_X^{\infty} \frac{e^{-t} dt}{t}$$

Range:

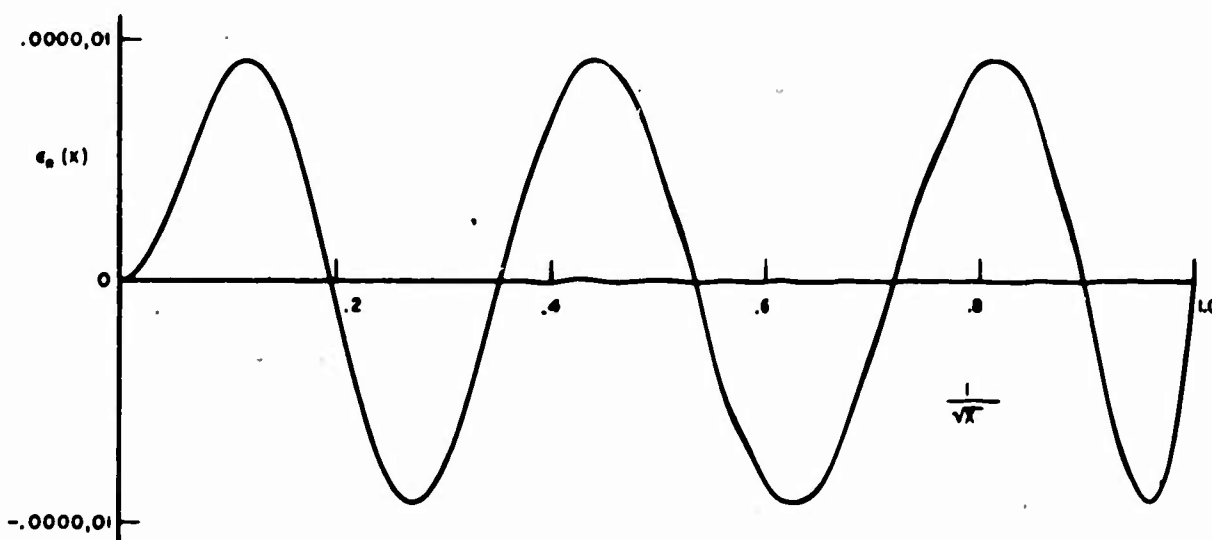
$$1 \leq X < \infty$$

Approximation:

$$-Ei^{\circ}(-X) = \frac{e^{-X}}{X} \left\{ \frac{a_0 + a_1 X + a_2 X^2 + X^3}{b_0 + b_1 X + b_2 X^2 + X^3} \right\}$$

$$\begin{aligned} a_0 &= .2372,9050 & b_0 &= 2.4766,3307 \\ a_1 &= 4.5307,9235 & b_1 &= 8.6660,1262 \\ a_2 &= 5.1266,9020 & b_2 &= 6.1265,2717 \end{aligned}$$

Error Curve (Approximation - Function) / (Function):



Comments: