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JANUARY 1966
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Technical Memorandum

SCALING THE EFFECTS OF AIR BLAST ON TYPICAL TARGETS

by Col. H. S. MORTON

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THE JOHNS HOPKINS UNIVERSITY • APPLIED PHYSICS LABORATORY

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Operating under Contract NOw 62-0604-c, Bureau of Naval Weapons, Department of the Navy

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ABSTRACT

The interactions between shock waves, produced in air by detonation of explosives, and specific targets which they can destroy by air blast are described. A mathematical analysis is used which relates weights of explosive (or yields of nuclear devices) to the distances at which they can cause lethal damage over the entire range of blasts from a few pounds of conventional high explosive to kilotons or megatons of nuclear blast. Effects at sea level and higher altitudes are examined. In the analysis, typical targets are defined by two parameters for which specific numerical values can be established. Shock waves produced by detonation of specific explosives are similarly defined in mathematical terms which relate characteristics of the explosive to the ambient atmosphere. A dimensionless scaling parameter relating a shock wave parameter to a target parameter is the key to the scaling relationships derived.

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PROLOGUE

The General Scaling Equations presented in this report are based on interactions between shock waves generated by explosions in air and targets which can be fully defined by two parameters in a simple mathematical model. Simplicity is achieved by ignoring many factors which complicate rather than improve the study of blast phenomena. These scaling equations, and the analytic techniques based on their use, are not intended to be precision tools for computing specific effects with maximum accuracy, even though they are often as accurate as far more complicated computational techniques. Their unique value lies in their use of a dimensionless, universal scaling parameter whose values span the entire spectrum of blast damage phenomena, from the effects of a few pounds of conventional high explosive to those of megatons of equivalent TNT. The reader is cautioned, however, not to expect real targets to behave precisely like the simplified models from which the scaling equations were derived, but he is encouraged to use the equations freely as a means of correlating isolated pieces of data with the basic analytic structure of blast damage relationships.

These General Scaling Equations are a valuable aid in simplifying the complex effects of an air blast on a target. They will be invaluable to the novice searching for a basic understanding of air-blast phenomena. The equations will also serve those who are making detailed studies of particular phases of blast damage phenomena, by showing them just how their area of interest relates to the overall realm of air-blast theory.

I. PURPOSE AND SCOPE

This analysis deals with targets and the interactions between them and the shock waves generated by the detonation of explosives. It limits its attention to that fraction of the total range of scaled distances within which a variety of actual targets, varying widely in toughness, have been destroyed by air blast. Within this limited range, an abundance of excellent experimental data shows that shock waves behave in an orderly fashion allowing the parametric relationships which define their characteristics to be expressed mathematically by two simple, tractable equations.

Since the purpose of this report is to derive scaling equations for interactions between shock waves and targets which result in target "kills," these equations will be used only to compute the effects of interactions within the range of scaled distances at which such kills are realistic. For this reason, equations which validly and accurately define the characteristics of shock waves, over this range of scaled distances, can be used as the basis for deriving General Scaling Equations for blast kills. These kills will be equally valid and accurate because the kills occur within the range to which the basic equations apply.

These facts make it possible to derive scaling equations of classic simplicity in which specific target characteristic data (inherent in the "universal scaling parameter" used in their derivation) ensure that the equations will be valid for the range in which shock waves can and do kill such targets. This universal scaling parameter allows the equations defining the blast-shock wave characteristics to be converted successfully into equations which scale the interactions between shock waves and targets.

II. DEFINITION OF SHOCK WAVE CHARACTERISTICS

Appendix A* shows that experimental data, over the range of scaled distances with which this report is concerned, can be represented by consistent relationships between the parameters of the shock wave and its distance from the detonation of any given weight of explosives. Such data, for the well known military high explosive, 50/50 Pentolite (Ref. 1), which is frequently used as a standard for blast studies, yield the following equations:

$$P = \frac{13,300\ W}{R^3} \quad \text{(Eq. A-6)} \tag{1}$$

$$I = \frac{220\ W^{0.733}\ p^{0.267}}{\left(\frac{c}{c_o}\right) R^{1.20}} \quad \text{(Eq. A-7)} \tag{2}$$

where

W = weight of explosive charge (lb)

R = distance from explosive charge (ft)

P = "normally reflected" peak overpressure of shock wave (psi)

I = "normally reflected" positive impulse of shock wave (psi· msecs)

p = ambient air pressure (atmospheres), having the value of 1.00 at sea level

$\frac{c}{c_o}$ = ratio of the velocity of sound in air at any altitude to the velocity at sea level

These equations are valid and accurate analytical expressions for peak overpressure and impulse characteristics of shock waves produced by 50/50 Pentolite over a range of scaled distances from 2 to 10 and are

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*In the interest of brevity in the ensuing text, definitions of terminology, and the details of all derivations are given in the appendices.

very good approximations over a still wider range. Data for other high explosives can be analyzed to yield equations of similar form but with different numerical coefficients defining their characteristics.

Equations (1) and (2) may be solved simultaneously for W and R, yielding:

$$W = \frac{0.0083 \left(\frac{c}{c_o}\right)^3 I^3}{P^{1.20}\, p^{0.80}} \quad \text{(Eq. A-8)} \qquad (3)$$

$$R = \frac{4.8 \left(\frac{c}{c_o}\right) I}{P^{0.733}\, p^{0.267}} \quad \text{(Eq. A-9)} \qquad (4)$$

A third, and very important equation, derived either by combining Eqs. (3) and (4) or directly from Eq. (1) is given by

$$R = \frac{23.7\, W^{1/3}}{P^{1/3}} \qquad (5)$$

The significance of this equation is discussed in a later section of this report.

III. CHARACTERISTICS OF A TYPICAL TARGET

A typical target for air-blast destruction is an aircraft which has a relatively light skin covering the basic structural framework of ribs, stiffeners, and braces. Typical blast damage, assessed as a target kill, may consist of substantial crumpling and distortion of the surface. A characteristic of this damage is that the surface is forced inward a considerable distance toward the basic structural elements which support it.

In order to establish a basis for the numerical assessment of target characteristics, the following parameter is defined:

P_m = the minimum unit surface pressure which causes any permanent deformation of the target; it is further assumed that P_m is the unit pressure with which the target will resist deformation when pressures greater than P_m initiate and continue destructive deformation.

In the course of deriving functions for the total work done in deforming a target to the point of destruction (Appendix C), several other target parameters will be used temporarily and will ultimately be replaced by a second target characteristic parameter, defined as:

I_m = the lower limit for the value of an impulse which can deform the target enough to destroy it.

The minimum unit pressure, P_m, is the only parameter required in the first step of the derivation given in Appendix C.

IV. THE UNIVERSAL SCALING PARAMETER

This analysis is concerned with scaling the interactions between shock waves and targets. A scaling parameter for this purpose should be a dimensionless ratio between some distinctive characteristic parameter of the target and a similar distinctive characteristic parameter for the shock wave. The only characteristic common to both is a unit pressure.

A universal scaling parameter, which can uniquely correlate every possible combination of charge weight with a corresponding lethal radius, is the ratio ϵ, defined as

$$\epsilon = \frac{P_m}{P} \quad \text{(Eq. B-1)} \qquad (6)$$

where P_m is defined as before and P is the largest shock wave unit pressure which can just destroy the target without over killing it. The ratio, then, which exists when the shock wave can just kill the target is ϵ. It is impossible for ϵ to be zero since every real target offers some finite resistance to deformation. No target deformation can take place unless P is greater than P_m, which implies that ϵ must be less than one. Within these limits, ϵ spans the entire spectrum of charge weights (and corresponding lethal distances) from a few pounds of conventional high explosive to nuclear devices whose yield is measured in kilotons or megatons of equivalent TNT.

In the scaling equations derived in this report, the characteristics of the explosive, the target, and the ambient atmospheric conditions (altitude and ratio of sound velocities) are all defined by appropriate numerical values of the parameters. Every set of conditions represented by these parameters corresponds to a unique weight of explosive, and the unique lethal distance associated with it. Therefore, ϵ is a universal scaling parameter for the interactions between shock waves and the targets which they are capable of killing when the shock waves are spherical and the targets are correctly defined by the parameters, P_m and I_m.

V. A GENERAL RELATIONSHIP BETWEEN LETHAL DISTANCE AND CHARGE WEIGHT

The efficacy of the universal scaling parameter becomes readily apparent when it is introduced into Eq. 5; the shock wave parameter, P, is replaced by the more convenient target parameter, P_m, to give

$$R = \frac{23.7\, W^{1/3} \epsilon^{1/3}}{P_m^{1/3}} \quad \text{(Eq. G-9)} \tag{7}$$

The generality of this equation becomes obvious when ϵ approaches one, as it does for large nuclear charges. In this case the basic $R/W^{1/3}$ relationship is modified only by a term defining the threshold of damage for the target. However, when ϵ is less than one, its values modify this basic relationship and introduce the variable distance-weight relationships which characterize moderate-sized charges of conventional high explosives.

Although Eq. 7 is an enormously useful scaling equation, further modification is necessary before it can be used effectively. The problem of scaling blast damage would be considerably over-simplified if it were to be expressed in terms of a single parameter and the equation for shock wave peak overpressure. The cube root of P_m appears in the denominator of Eq. 7, but such factors as the amount of work per unit area which constitutes a complete kill and the altitude at which the blast occurs do not appear explicitly. A closer examination shows that these factors are implicit in the value of ϵ. It is then necessary to determine the proper value of ϵ to be used in Eq. 7.

This determination is not a trivial task; it is treated at length in Appendices C and G. It requires the derivation of functions for the total work per unit area which constitutes a kill, and the derivation of general parametric equations for R and W which are repeating functions of the universal scaling parameter, ϵ. R and W also contain terms dependent on

the explosive, target, and altitude. The derivation of the following equations is the major accomplishment of this analysis:

$$W = 0.0083 \left[\frac{I_m^{\,3}}{P_m^{\,1.20}}\right] \left[\frac{\left(\frac{c}{c_o}\right)^3}{p^{0.80}}\right] F_W(\epsilon) \quad \text{(Eq. G-1)} \tag{8}$$

$$R = 4.8 \left[\frac{I_m}{P_m^{\,0.733}}\right] \left[\frac{\frac{c}{c_o}}{p^{0.267}}\right] F_R(\epsilon) \quad \text{(Eq. G-2)} \tag{9}$$

In each of these equations the first factor, the numerical coefficient, is a characteristic of the explosive (50/50 Pentolite) and would be different for different explosives. The second factor consists of target parameters, and the third consists of the parameters which define the effects of altitude. Analytical expressions are derived in Appendix G for the functions of ϵ which constitute the fourth factor of each equation, but the complexity of these functions makes it more practical to use the values in Table I (p. 10). The two functions of ϵ in Eqs. 8 and 9 are related in the following way:

$$F_R(\epsilon) = [F_W(\epsilon)]^{1/3} \epsilon^{1/3} \quad \text{(Eq. G-7)} \tag{10}$$

The third or altitude factor may be written as $F_W(\text{alt})$ or $F_R(\text{alt})$. Average values of these factors are given as a function of altitude in Table II (p. 12). The scaling equations may thus be simplified to give:

$$W = 0.0083 \left[\frac{I_m^{\,3}}{P_m^{\,1.20}}\right] F_W(\text{alt})\, F_W(\epsilon) \tag{11}$$

$$R = 4.9 \left[\frac{I_m}{P_m^{\,0.733}}\right] F_R(\text{alt})\, F_R(\epsilon) \tag{12}$$

Solving Eqs. 11 and 12 for their respective functions of ϵ :

$$F_W(\epsilon) = \frac{120.5\ W}{\frac{I_m}{P_m}^{3} F_W(alt)} \qquad (13)$$

$$F_R(\epsilon) = \frac{R}{4.8 \frac{I_m}{P_m}^{1.20} F_R(alt)} \qquad (14)$$

Table I

Functions of ϵ					
ϵ	$F_W(\epsilon)$	$F_R(\epsilon)$	ϵ	$F_W(\epsilon)$	$F_R(\epsilon)$
0.01	0.00407	0.0344	0.36	0.976	0.706
0.02	0.00958	0.0577	0.37	1.063	0.733
0.03	0.01597	0.0782	0.38	1.159	0.761
0.04	0.02310	0.0974	0.39	1.264	0.790
0.05	0.03095	0.1157	0.40	1.380	0.820
0.06	0.0395	0.1333	0.41	1.509	0.852
0.07	0.0488	0.1506	0.42	1.651	0.885
0.08	0.0587	0.1675	0.43	1.809	0.920
0.09	0.0695	0.1842	0.44	1.985	0.956
0.10	0.0810	0.2008	0.45	2.181	0.994
0.11	0.0933	0.2173	0.46	2.400	1.033
0.12	0.1065	0.2338	0.47	2.642	1.075
0.13	0.1206	0.2503	0.48	2.915	1.118
0.14	0.1357	0.2668	0.49	3.22	1.164
0.15	0.1518	0.2835	0.50	3.56	1.212
0.16	0.1691	0.300	0.51	3.95	1.263
0.17	0.1875	0.317	0.52	4.39	1.316
0.18	0.2072	0.334	0.53	4.88	1.373
0.19	0.2283	0.351	0.54	5.44	1.432
0.20	0.2509	0.369	0.55	6.08	1.495
0.21	0.2751	0.387	0.56	6.80	1.562
0.22	0.3010	0.405	0.57	7.63	1.632
0.23	0.329	0.423	0.58	8.58	1.707
0.24	0.359	0.442	0.59	9.67	1.787
0.25	0.391	0.461	0.60	10.93	1.872
0.26	0.425	0.480	0.61	12.38	1.962
0.27	0.463	0.500	0.62	14.07	2.058
0.28	0.503	0.520	0.63	16.03	2.162
0.29	0.546	0.541	0.64	18.33	2.272
0.30	0.593	0.563	0.65	21.03	2.391
0.31	0.644	0.585	0.66	24.21	2.519
0.32	0.700	0.607	0.67	27.99	2.657
0.33	0.760	0.631	0.68	32.5	2.806
0.34	0.826	0.655	0.69	37.9	2.967
0.35	0.898	0.680	0.70	44.3	3.14

Table I (cont'd)

ϵ	$F_W(\epsilon)$	$F_R(\epsilon)$	ϵ	$F_W(\epsilon)$	$F_R(\epsilon)$
0.71	52.2	3.33	0.86	1595	11.11
0.72	61.7	3.54	0.87	2246	12.50
0.73	73.4	3.77	0.88	3249	14.19
0.74	87.9	4.02	0.89	4850	16.28
0.75	105.9	4.30	0.90	7513	18.91
0.76	128.4	4.60	0.91	12,180	22.29
0.77	157.1	4.95	0.92	20,870	26.78
0.78	193.7	5.33	0.93	38,400	32.9
0.79	241.1	5.75	0.94	77,500	41.8
0.80	303	6.24	0.95	117,500	55.2
0.81	385	6.78	0.96	488,700	77.7
0.82	496	7.41	0.97	1,800,000	120.4
0.83	648	8.13	0.98	11,240,000	222.5
0.84	859	8.97	0.99	256,600,000	633.3
0.85	1159	9.95			

Table II

Functions of Altitude					
Alt. (1000's of feet)	F_W(alt)	F_R(alt)	Alt. (1000's of feet)	F_W(alt)	F_R(alt)
1	1.015	1.005	31	1.79	1.21
2	1.03	1.01	32	1.83	1.22
3	1.05	1.02	33	1.87	1.23
4	1.07	1.02	34	1.92	1.24
5	1.09	1.03	35	1.96	1.25
6	1.11	1.03	36	2.03	1.27
7	1.13	1.04	37	2.11	1.28
8	1.14	1.05	38	2.19	1.30
9	1.16	1.05	39	2.28	1.32
10	1.18	1.06	40	2.37	1.33
11	1.20	1.06	41	2.46	1.35
12	1.22	1.07	42	2.55	1.37
13	1.25	1.08	43	2.65	1.38
14	1.27	1.08	44	2.76	1.40
15	1.29	1.09	45	2.86	1.42
16	1.32	1.10	46	2.97	1.44
17	1.34	1.10	47	3.09	1.46
18	1.37	1.11	48	3.21	1.48
19	1.39	1.12	49	3.34	1.50
20	1.42	1.13	50	3.47	1.51
21	1.45	1.13	51	3.61	1.53
22	1.48	1.14	52	3.75	1.55
23	1.51	1.15	53	3.90	1.58
24	1.54	1.16	54	4.07	1.60
25	1.57	1.16	55	4.23	1.62
26	1.61	1.17	56	4.40	1.64
27	1.64	1.18	57	4.57	1.66
28	1.68	1.19	58	4.74	1.68
29	1.72	1.20	59	4.93	1.70
30	1.75	1.21	60	5.12	1.72

VI. SCALING BLAST DAMAGE

Equation 7 was derived in Section V by inserting ϵ in an equation dependent solely on the characteristics of the explosive. It can also be derived from Eqs. 8 and 9.

Scaling computations start with a target which is defined by two parameters, P_m and I_m. These values must be known before the scaling equations can be used. The methods of determining these values are discussed at some length in Appendix J when P_m and I_m are known for any target and the altitude is specified. Equation 13 gives a value of $F_W(\epsilon)$ for any weight, W, and Table I gives the corresponding ϵ value.

The W and the corresponding ϵ inserted in Eq. 7 give the lethal radius at which the charge, W, can kill the specified target at the specified altitude. This process may be repeated indefinitely with many different values of W, and accurate lethal distances can be computed by simply finding the ϵ value that goes with each W for the specified target and altitude.

The use of Eq. 7 is a short cut in some cases; normally the known parameters are substituted in Eqs. 8 or 11 to find the values of $F_W(\epsilon)$ and ϵ. These, in turn, give $F_R(\epsilon)$ from Eq. 10 and allow R to be determined from Eqs. 9 or 12 for the specific target situation.

VII. A GENERAL R-W CURVE AND A SCALING NOMOGRAM

When the values of the functions of ϵ listed in Table I are plotted in terms of log $F_R(\epsilon)$ vs log $F_W(\epsilon)$, they present a general parametric equation. The value of such a graph becomes apparent when it is realized, from the scaling equations, that for any specific explosive, target, and altitude, R and W are simple linear functions of $F_R(\epsilon)$ and $F_W(\epsilon)$ respectively. If $F_R(\epsilon)$ vs $F_W(\epsilon)$ is plotted on a transparent overlay, it can then be oriented with respect to a logarithmic diagram whose ordinate is distance (in feet) and whose abscissa is charge weight (in pounds). The constants which relate R to $F_R(\epsilon)$ and W to $F_W(\epsilon)$ determine the vertical and horizontal displacements, respectively, of the overlay with respect to the distance-weight diagram.

A nomogram based on this principle was constructed at the Applied Physics Laboratory in 1954 (Ref. 2).. At that time it was classified confidential because data on specific targets were displayed on it as a guide for positioning the graph with respect to the scales. The principle has subsequently been applied to a somewhat more elaborate "Nomograph Calculator" covered by U.S. Patent No. 2,991,934, July 11, 1961 (unclassified), assigned by the author to the United States of America as represented by the Secretary of the Navy.

When both the graph and the scales are extended over many cycles, any fixed orientation of the component parts permits the reading of concurrent distances and weights over the entire available range of the scales. These concurrent values apply to the kind of explosive, the nature of the target, and the altitude for which the "setting" is made. The nomogram is merely a quick and convenient way of graphically performing the same analytic processes which are used in computing these relationships by means of the equations. The vital role which the dimensionless parameter, ϵ, plays is clearly evident in both the analytic and graphic solutions of the scaling relationships.

VIII. COMMENTS AND CONCLUSIONS

This analysis has avoided the necessity of treating the full range of shock wave phenomena and particularly the regions in which parameter relationships are far too complicated to fall into any simple pattern. By limiting its scope to the region in which parameter relationships can be expressed in simple analytic terms it fills a long-felt need for a direct and easy method of scaling blast effects on specific targets.

Fortunately a majority of targets of interest will fall well within the applicable range of the equations derived in this report. As soon as the parameters which define a target are known, it is a simple matter to determine the corresponding scaled distances and ascertain that they are within the applicable range of the equations.

The preceding text gives a brief indication of how the scaling parameter, ϵ, can be used to convert a function describing the characteristics of a shock wave produced by detonation of an explosive charge into a general equation for scaling blast interactions with targets. This use is but one of a multiplicity of uses to which this parameter may be put in simplifying the analysis of blast effects. In addition to defining shock wave and target characteristics, and giving details of derivations, the appendices which follow exemplify how the scaling parameter, ϵ, and equations derived therefrom can be used to illuminate the many obscure facets of the air-blast problem. The appendices also discuss the determination of the target parameters and attempt to answer various questions which may arise in the minds of readers.

As a final word, since the phenomena to be scaled are interactions between shock waves and targets, the parameter, ϵ, which relates a common characteristic of the two, is the most useful analytic tool that can be found for scaling these effects. Everyone who has to deal with blast damage will benefit by becoming acquainted with the scaling parameter, ϵ, and by using it freely in computations of blast effects.

APPENDIX A

Definition of Shock Wave and Target Parameters

This report deals with the interactions between shock waves produced by detonation of explosives and targets which are damaged or destroyed by their action.

In order to study these interactions, it is necessary to describe both the shock waves and the targets by parameters whose specific numerical values will uniquely identify their essential characteristics in terms which can be manipulated mathematically.

The definitions and symbols in this appendix constitute the foundation for all the derivations and discussions which follow.

1. The Nature of Shock Waves

A shock wave is a rapidly moving disturbance in air characterized by an abrupt rise in air pressure (at some fixed point in its path; after this critical point the air pressure gradually declines until it reaches the original ambient pressure. The behavior of a typical shock wave is shown in Fig. A-1.

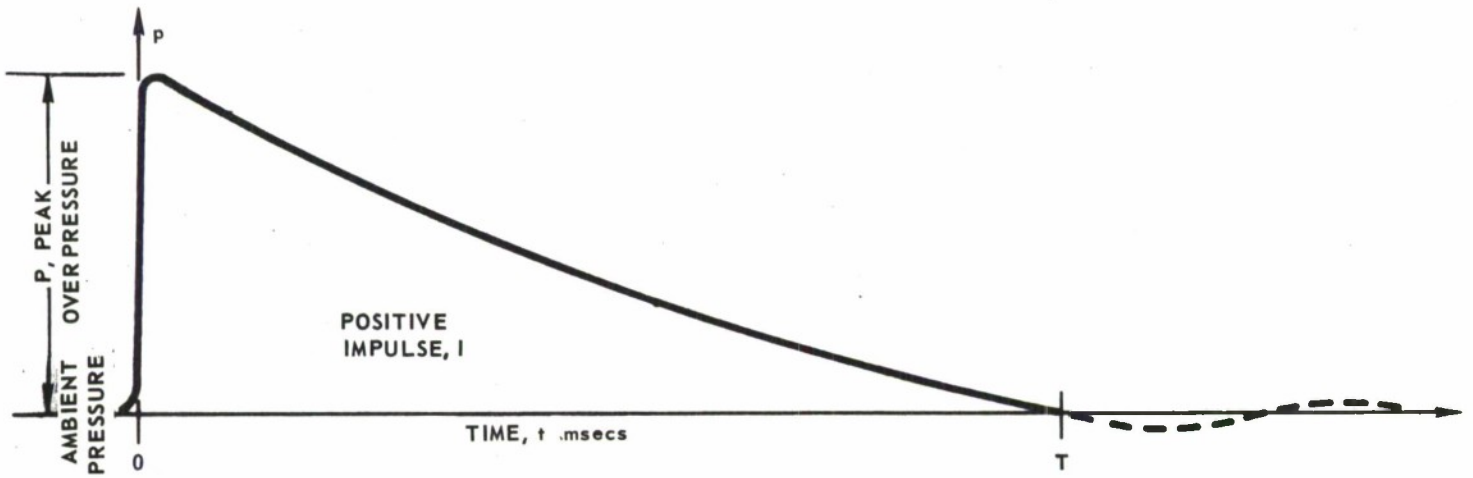


Fig. A-1 PRESSURE-TIME CURVE

The shock wave characteristics, at any point in its path, may be defined by the following parameters:

P = peak overpressure (psi)
= the difference between the absolute peak pressure and the original ambient pressure (see Fig. A-1).

T = time (msecs) during which the pressure of the shock wave is continuously greater than the ambient pressure. T is called the duration of the positive impulse.

t = time (msecs) measured from the instant at which the initial pressure rise begins.

p = overpressure of the shock wave (above ambient pressure) at an time, t.

I = total positive impulse (psi·msecs) up to time, T, i.e.,

$$I = \int_0^T p\,dt \qquad (A-1)$$

This is sometimes termed the first positive impulse, since there often is a second positive impulse beyond T. The pressure fluctuations beyond T are not important enough to warrant consideration in this analysis.

Values of P are usually given in terms of the "side-on" or "free-air" overpressures and the impulse taken assumes that nothing obstructs the free motion of the shock wave. Because this report deals with the interactions between shock waves and targets, we are not concerned with free-air conditions but will deal with the overpressures and impulses which act on a solid surface normal to the shock wave motion. For this reason the pressures and impulses and all other shock wave parameters which are defined above and used in subsequent derivations are face-on or normally reflected peak overpressures, impulses, etc. The reasons for using these values will be discussed more fully in Subsec. 2 of this appendix (Characteristics of a Target).

Shock Wave Characteristics--General.--Shock wave characteristics are functions of such factors as the distance from the detonation, the nature and size of the charge, and the pressure and temperature of the ambient atmosphere through which they propagate. These factors are defined as follows:

W = weight (lb) of the explosive charge.

R = the distance (ft) from the center of detonation to the point at which the shock wave properties are being evaluated.

Z = scaled distance, defined by the equations

$$Z = \frac{R}{W^{1/3}} \qquad \text{(A-2)}$$

p = ambient air pressure (atm), i.e., p = 1 at sea level (14.7 psi). At higher altitudes, $p < 1$.

c_o = speed of sound, sea level, 300°K.

c = speed of sound at any altitude and ambient temperature.

The absolute values of c and c_o are not important since only their ratio, c/c_o, occurs in the equations.

Shock Wave Characteristics--50/50 Pentolite.--The relationships between peak overpressures, positive impulses, scaled distances, and atmospheric environments differ from one explosive to another. Since 50/50 Pentolite is a military high explosive which gives consistently reproducible results, it is often used as a standard explosive for evaluating the effects of air blasts. Furthermore it is an explosive for which abundant and accurate data are available. For these reasons this analysis has been built around the characteristics of 50/50 Pentolite given by the data of Ref. 3.

Fortunately the interactions with which this report is concerned occur within a range of scaled distances in which parameter relationships behave consistently and the Sachs Scaling Laws appear to be valid. In accordance with these scaling laws, the data for normally reflected peak

overpressures for 50/50 Pentolite are shown as a function of the scaled distance in Fig. A-2. The straight line fitted to these data points is given by

$$\frac{P}{p} = \frac{13,300}{[Zp^{1/3}]^3} \qquad \text{(A-3)}$$

In similar fashion the data for normally reflected positive impulses for 50/50 Pentolite are shown as a function of the scaled distance in Fig. A-3.

The straight line fitted to these data points is given by

$$\frac{I}{W^{1/3}\, p^{2/3}} = \frac{220}{[Z_p^{1/3}]^{1.20}} \qquad \text{A-4)}$$

These two analytically tractable equations, Eqs. A-3 and A-4, define the characteristics of 50/50 Pentolite over the range of scaled distances with which this analysis is concerned. The anomalies which occur at scaled distances less than two or greater than ten or twelve have no bearing on the accuracy and validity of these equations since they lie outside the range of parameters under consideration.

Equation A-4 which is fitted to empirical data obtained at sea level ($p = 1.00$) indicates the qualitative effect of p on the impulse. At higher altitudes the temperature changes will affect the speed of sound and require a quantitative adjustment. These changes, which are independent of p, are accounted for by the independent parameter, c/c_o, which is included in Eq. A-4 in the following manner:

$$\frac{I\left(\frac{c}{c_o}\right)}{W^{1/3}\, p^{2/3}} = \frac{220}{[Zp^{1/3}]^{1.20}} \qquad \text{(A-5)}$$

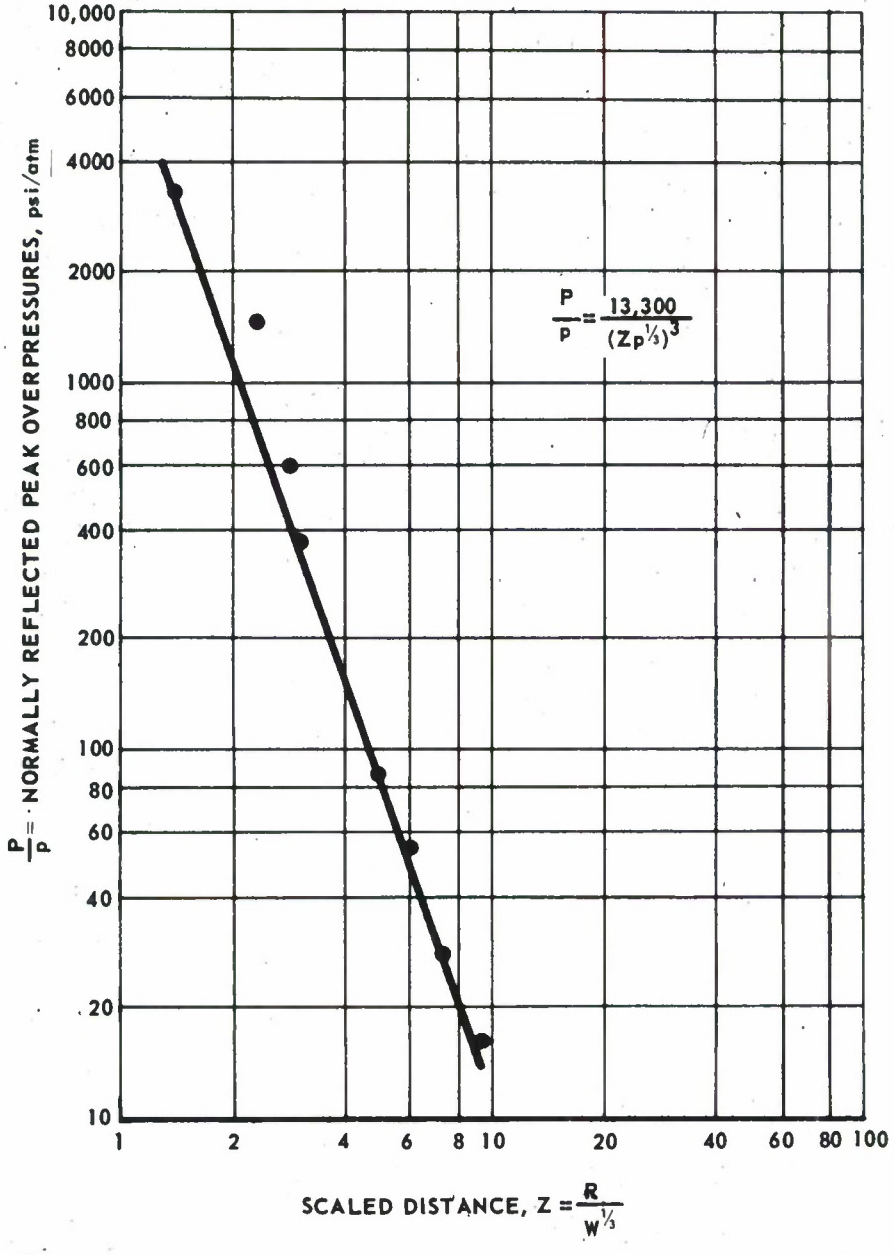


Fig. A-2 NORMALLY REFLECTED PEAK OVERPRESSURES FOR 50/50 PENTOLITE SPHERES

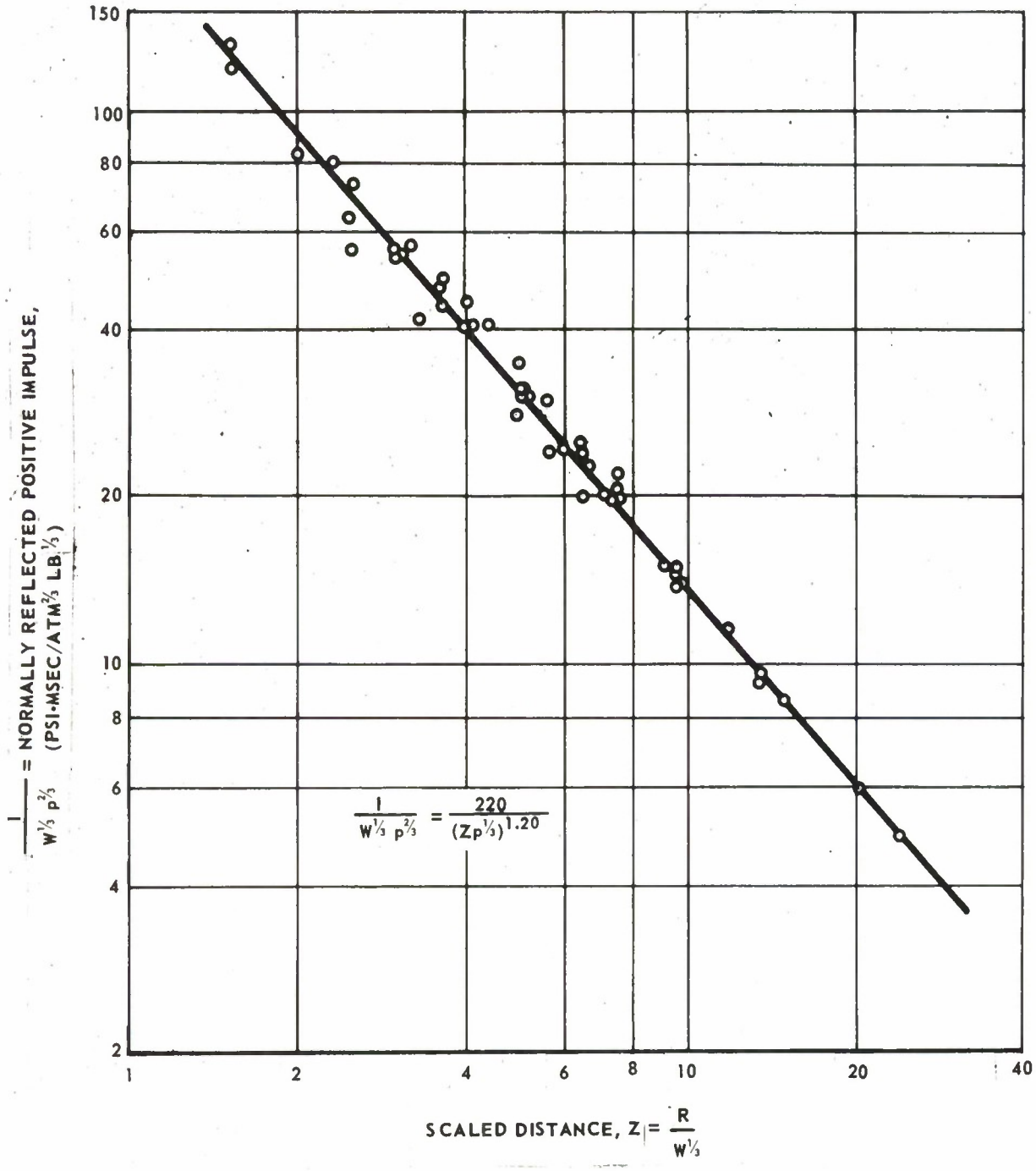


Fig. A-3 NORMALLY REFLECTED POSITIVE IMPULSE DATA FOR 50/50 PENTOLITE SPHERES

Equation A-2 may be used to eliminate Z from Eqs. A-3 and A-5, giving

$$P = \frac{13,300\ W}{R^3} \tag{A-6}$$

and

$$I = \frac{220\ W^{0.733}\ p^{0.267}}{\left(\frac{c}{c_o}\right) R^{1.20}} \tag{A-7}$$

Further analytical treatment converts these equations into:

$$W = \frac{0.0083 \left(\frac{c}{c_o}\right)^3 I^3}{P^{1.20}\ p^{0.80}} \tag{A-8}$$

and

$$R = \frac{4.8 \left(\frac{c}{c_o}\right) I}{P^{0.733}\ p^{0.267}} \tag{A-9}$$

These two equations are the first step in the derivation of a set of completely general parametric equations for W and R. The second step will be taken when more general expressions for P and I, derived in Appendix C, are used to eliminate P and I from Eqs. A-8 and A-9 in Appendix G.

The characteristics of shock waves produced by 50/50 Pentolite are fully defined by Eqs. A-6 and A-7; weights and distances have been related to these characteristics by Eqs. A-8 and A-9. Similar equations for other explosives can be derived in the same manner.

2. Characteristics of a Target

Aircraft structures, which are typical targets for air blast, consist of superficial coverings of comparatively thin material supported by rather complex assemblies of ribs, stiffening webs, and braces of various sorts which are attached to the basic framework. Target damage severe enough

to constitute a kill consists of substantial crumpling and distortion of the surface in the course of which it moves a considerable distance against the resisting forces offered by the structural elements which yield without breaking. The behavior of a target cannot be fully simulated, but a remarkably useful mathematical model can be defined in simple terms. Unit pressures below a certain minimum value will have no effect whatever on the target regardless of how long they are maintained. However, at some critical unit pressure the supporting structure will begin to yield, and if this pressure is maintained, the distortion will increase until the target is totally destroyed.

Therefore, it is possible to define a fundamental parameter representing target toughness as follows:

P_m = the minimum unit pressure (psi) on the target surface which will initiate destructive distortion and which, if continued long enough, will cause target destruction.

The mathematical model used in this analysis assumes a constant value of P_m throughout the period of distortion up to the point of destruction. Many real targets approximate this behavior closely enough to consider P_m a statistically significant measure of target toughness.

Other necessary parameters are:

m = mass per unit area (lb/g) of target material which is moved relative to its basic supporting structure during the course of destructive deformation.

S_o = total displacement (ft) of the target skin relative to the basic structure which will constitute a target kill.

E_o = total work done in producing this displacement which kills the target.

These are not independent parameters since

$$E_o = S_o P_m \tag{A-10}$$

These parameters will be eliminated from the final equations.

Although the behavior of real targets is not quite as simple as this model indicates, it is necessary to describe target behavior in terms which can be treated mathematically in the ensuing study of shock wave target interactions.

APPENDIX B

The Universal Scaling Parameter

A scaling parameter used in describing or evaluating the interactions between a shock wave and a target to which it imparts energy should be a dimensionless ratio of some physical characteristic common to both. The only such characteristic is a unit pressure. The significant pressure for the target is the minimum unit pressure, P_m, i.e., the pressure which causes target damage; for the shock wave it is the maximum unit pressure, P, which it can exert on the target. Hence, a universal scaling parameter, is defined by the equation

$$\epsilon = \frac{P_m}{P} \qquad \text{(B-1)}$$

No target damage can occur if ϵ is greater than 1.00 and no real target, in which there is any finite resistance to deformation, can have an ϵ value of zero. The entire gamut of shock wave target combinations, which produce target kills, is covered by the open interval $0 < \epsilon < 1$.

It will be shown that small values of ϵ are associated with small charges of explosives which kill the target at relatively short distances from the point of detonation, and that large values of ϵ can exist only with very large charges such as those of nuclear weapons whose yield is measured in kilotons, or even megatons, of equivalent TNT.

For a specific target in a specific atmospheric environment, each value of ϵ is associated with a unique combination of charge weight and lethal distance. All possible combinations of charge weight and corresponding lethal distance, which result in a kill in this environment, are associated with specific values of ϵ between the limits of zero and one.

APPENDIX C

Interaction Between a Shock Wave and a Target

This appendix treats the effects of shock waves on a target model. The target is defined by two parameters to which numerical values can be assigned. The target model accurately simulates the behavior of many real targets whose destruction by blast has been observed under test conditions. This target is assumed to have a relatively light superficial covering (similar to the skin of an airplane) supported by ribs, stiffeners, etc., anchoring the skin to the more massive components of the basic structure. Associated with a target is a minimum pressure exerted on its surface below which threshold there will be no damage. It is assumed that when this critical minimum pressure is reached, the supporting elements will begin to yield and permanent deformation will be initiated. It is further assumed that as long as deformation (consisting of displacement of the surface relative to the more massive elements of the basic structure) continues, it will be resisted by a constant force per unit area of the same magnitude as the unit pressure which initiated destructive deformation. Every target will have to be deformed by some definite minimum displacement of the surface relative to the basic structure before its usefulness is destroyed, i.e., a certain minimum target distortion, or a certain minimum work per unit, must be expended on the target to destroy it.

It is necessary then to derive functions for the total work per unit area which any given shock wave can impart to the target, and to specify the characteristics of the shock wave which can impart just enough work to achieve target destruction.

The normally reflected peak overpressures and the normally reflected positive impulses which are used in this study were derived in Appendix A for 50/50 Pentolite. Values for other explosives may be found in a similar manner. The justification for the use of these pressures and impulses to specify the required shock wave will be found in Appendix I.

It is now necessary to have an equation for the pressure-time profile of a shock wave acting against a target surface. This equation will give values of the overpressure as a function of time ($t < T$).

The linear diagram in Fig. C-1a shows

$$p = P\left(1 - \frac{t}{T}\right) \tag{C-1}$$

where

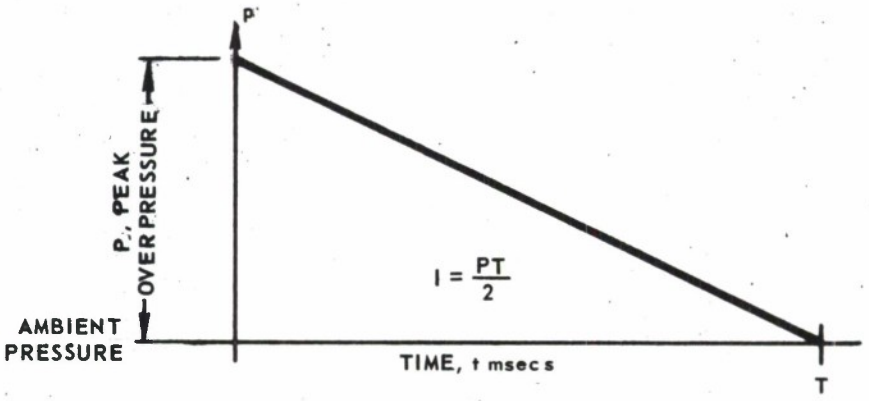
t = time from the first rise in pressure

p = the overpressure at time, t $(0 < t < T)$.

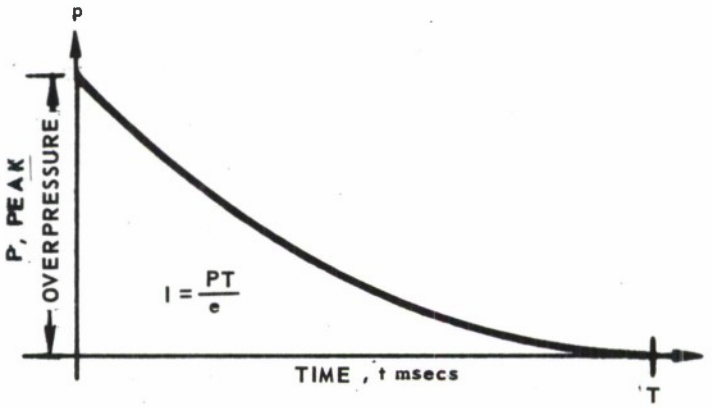
Figure C-1b gives

$$p = P\left(1 - \frac{t}{T}\right)e^{-\frac{t}{T}} \tag{C-2}$$

It can be shown that a completely self-consistent set of scaling equations can be derived from either of these pressure-time profiles and that the same relationships between charge weights and corresponding lethal distances are derivable from either equation. The linear relationship was used in Refs. 2, 4, 5, 6, and 7 with satisfactorily consistent results. However, self-consistency within a mathematical model is not sufficient to assure that values of ϵ, as well as the values of such target parameters as P_m, faithfully portray realistic interaction characteristics. The model will be comparable to a real target only if the pressure-time function used to derive scaling equations accurately reflects actual shock wave characteristics.



(a)



(b)

Fig. C–1 PRESSURE-TIME CURVES

The pressure-time relationships shown in Fig. C-1 differ from one another in the relationship of I to PT. For the linear curve (Fig. C-1a), the ratio is 1/2. For the exponential curve of Fig. C-1b, it is 1/e or 0.368. The data for 50/50 Pentolite (Ref. 3) show that the ratio of I to PT is not constant but varies at a moderate rate with scaled distance. Furthermore, it was found that the ratio 1/e of the exponential function (Eq. C-2) agrees with the 50/50 Pentolite data in the midrange of scaled distances used in this analysis. Since an accurate mathematical formulation of the actual pressure-time profile of a shock wave produced by 50/50 Pentolite over the full range of scaled distances has not yet been found, the exponential function (see Eq. C-2) is the best representation within the range of scaled distances covered by this analysis.

The mathematical parameters used in this analysis to define the shock wave and target characteristics are as follows:

P = normally reflected peak overpressure of the shock wave, also
= the maximum unit pressure (psi) acting on the target.

t = time (msecs) measured from the instant at which the shock wave first reaches the target.

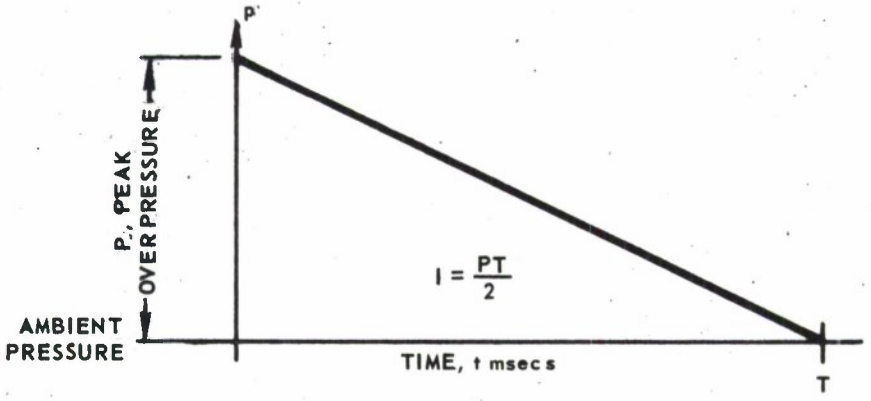
T = total duration (msecs) of the positive impulse.

p = unit pressure (psi) of shock wave, also
= unit pressure (psi) acting on target at time, t, where $0 < t < T$.

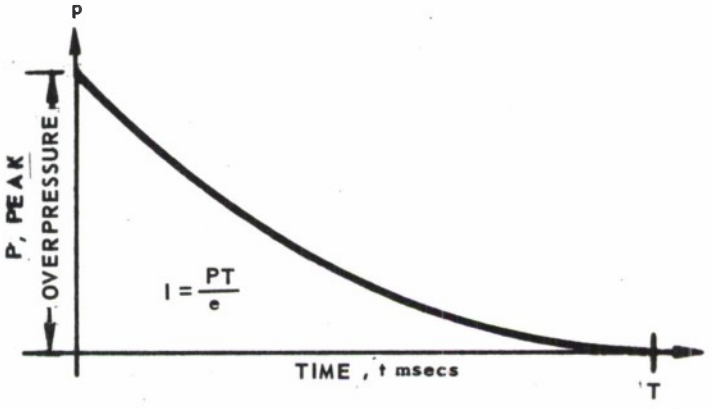
i = total positive impulse (psi·msecs) up to time, t (for $t \leq T$).

I = normally reflected positive impulse (psi·msecs) of the shock wave up to time, T.

P_m = the minimum unit pressure (psi) which can cause any target damage, also
= the unit pressure by which the target resists deformation



(a)



(b)

Fig. C–1 PRESSURE-TIME CURVES

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The mathematical parameters used in this analysis to define the shock wave and target characteristics are as follows:

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t = time (msecs) measured from the instant at which the shock wave first reaches the target.

T = total duration (msecs) of the positive impulse.

p = unit pressure (psi) of shock wave, also
= unit pressure (psi) acting on target at time, t, where $0 < t < T$.

i = total positive impulse (psi·msecs) up to time, t (for $t \leq T$).

I = normally reflected positive impulse (psi·msecs) of the shock wave up to time, T.

P_m = the minimum unit pressure (psi) which can cause any target damage, also
= the unit pressure by which the target resists deformation

m = mass per unit area (lb/g) of target material which is moved relative to the basic target structure when destructive deformation takes place. (This includes unit area of skin plus some additional fraction of the supporting elements which are distorted and moved.)

F = net accelerating force (lb) per unit area at time, t, acting on mass, m. May be positive or negative.

S = distance (ft) at time, t, that the mass, m, has moved from its original position.

S_o = minimum value of S which will cause target destruction.

E_o = total energy per unit area espended on target in displacing the surface a distance, S_o, against a resistance, P_m.

All of the target parameters except P_m will be replaced in the final scaling equation by a parameter, I_m, which will be defined at the conclusion of the derivation.

On a pressure-time diagram, impulses are represented by areas. Figures C-2 and C-3 will show that areas above the target resistance line, P_m, (designated by 1) are positive impulses which accelerate the mass, m, to a maximum velocity at time, t_m, and areas below P_m (designated by 2) are negative impulses which bring m to rest at some later time, t_o. Obviously the two impulses must be equal.

In setting up equations for the acceleration of the mass, m, one notes that between zero and T there is a constantly varying acceleration for which the pressure-time equation (Eq. C-2) gives an analytical value, but beyond the time, T, the negative acceleration has a constant value.

Interactions which are completed within the period (T) of the positive impulse can be treated by a single set of equations and will be designated Case I (see Fig. C-2). Interactions which continue beyond the end of the

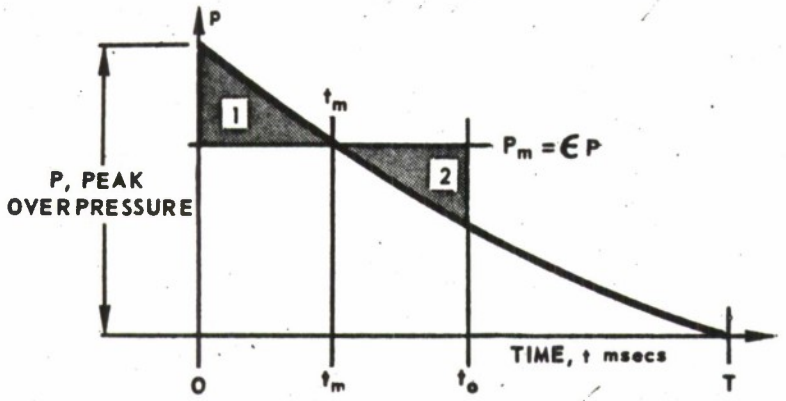


Fig. C-2 P VS t FOR CASE I ($1/e \leq \epsilon < 1$)

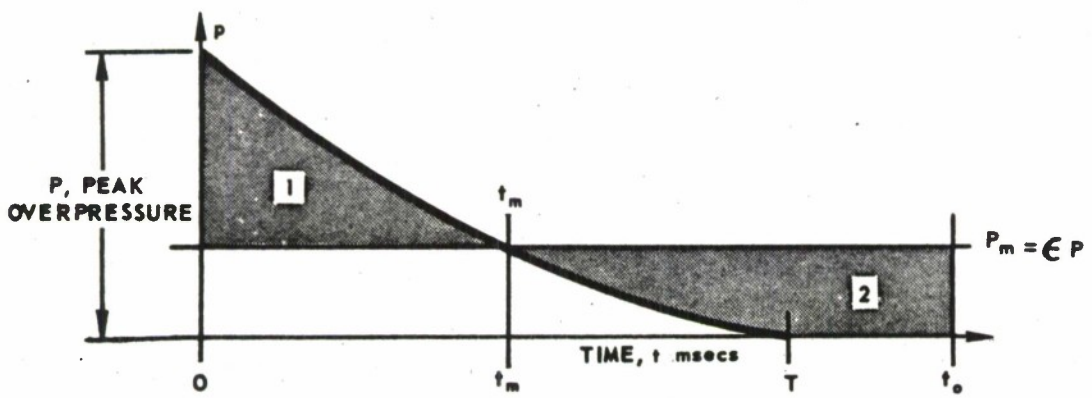


Fig. C-3 P VS t FOR CASE II ($0 < \epsilon \leq 1/e$)

positive impulse will require two sets of equations, one with a constantly varying acceleration (up to time, T), and one with a constant acceleration (beyond time, T). This will be called Case II (see Fig. C-3).

Equations will be set up which will apply over the entire range from zero to T. It will then be necessary to determine where Case I ends and Case II begins.

The accelerating force is given by

$$F = p - P_m \tag{C-3}$$

and hence

$$\frac{d^2S}{dt^2} = \ddot{S} = \frac{p - P_m}{m} \tag{C-4}$$

Using Eq. C-2 and the definition of ϵ, Eq. C-4 becomes

$$\ddot{S} = \frac{Pe^{-\frac{t}{T}}}{m} - \frac{Pte^{-\frac{t}{T}}}{Tm} - \frac{\epsilon P}{m} \tag{C-5}$$

Integrating Eq. C-5 with respect to t gives:

$$\dot{S} = -\frac{PTe^{-\frac{t}{T}}}{m} + \frac{Pte^{-\frac{t}{T}}}{m} + \frac{PTe^{-\frac{t}{T}}}{m} - \frac{\epsilon Pt}{m} \tag{C-6}$$

which reduces to:

$$\dot{S} = \frac{Pte^{-\frac{t}{T}}}{m} - \frac{\epsilon Pt}{m} \tag{C-7}$$

An integration of Eq. C-7 with respect to t yields:

$$S = -\frac{PT\,te^{-\frac{t}{T}}}{m} - \frac{PT^2e^{-\frac{t}{T}}}{m} - \frac{\epsilon Pt^2}{2m} + \frac{PT^2}{m} \tag{C-8}$$

By definition

$$i = \int_0^t p\,dt = Pte^{-\frac{t}{T}} \qquad (C-9)$$

and

$$I = \frac{PT}{e} \qquad (C-10)$$

Equations C-5 through C-10 are valid within the time range from zero to T and will be used in the subsequent analysis.

Case I. --It may be seen in Fig. C-2 that

$$t_o \leq T$$

Since areas 1 and 2 of this figure are equal and the velocity of mass, m, is zero at t_o, the rectangular area

$$\epsilon P t_o = i_{t_o} \qquad (C-11)$$

Substituting Eq. C-9 in Eq. C-11 gives

$$\epsilon P t_o = P t_o e^{-\frac{t_o}{T}} \qquad (C-12)$$

which reduces to

$$\epsilon = e^{-\frac{t_o}{T}} \qquad (C-13)$$

or

$$t_o = -T \ln \epsilon \qquad (C-14)$$

When

$$t_o = T, \quad \text{then} \quad \ln \epsilon = -1 \quad \text{and} \quad \epsilon = \frac{1}{e} \qquad (C-15)$$

This value of ϵ represents the dividing point between Case I and Case II. It may be seen from Figs. C-2 and C-3 that the applicable ranges of values of ϵ for these two cases are:

Case I: $$\frac{1}{e} \leq \epsilon < 1 \qquad \text{(C-16)}$$

and

Case II: $$0 < \epsilon \leq \frac{1}{e} \qquad \text{(C-17)}$$

In Case I at time, t_o, , Eq. C-8 becomes

$$S_{t_o} = -\frac{PT\, t_o\, e^{-\frac{t_o}{T}}}{m} - \frac{PT^2\, e^{-\frac{t_o}{T}}}{m} - \frac{\epsilon P\, t_o^2}{2m} + \frac{PT^2}{m} \qquad \text{(C-18)}$$

When the value of t_o, given by Eq. C-14, is substituted in Eq. C-18

$$S_{t_o} = \frac{PT^2}{2m}\left[2\epsilon \ln \epsilon - 2\epsilon - \epsilon \ln^2 \epsilon + 2\right] \qquad \text{(C-19)}$$

Equation C-19 gives a general relationship between the distance the target surface has moved, S_{t_o}, and the parameters characteristic of a shock wave, PT and T. This relationship can be made specific by replacing S_{t_o} with the minimum distance, S_o, which the surface must move to ensure target destruction. The term PT^2 in Eq. C-19 then will describe the shock wave which can just destroy the target characterized by $S_{t_o} = S_o$. Since

$$E_o = P_m S_o = \epsilon P S_o \qquad \text{(C-20)}$$

then

$$E_o = \frac{P^2 T^2 \epsilon}{2m}\left[2\epsilon \ln \epsilon - 2\epsilon - \epsilon \ln^2 \epsilon + 2\right] \qquad \text{(C-21)}$$

Using Eq. C-10, Eq. C-21 becomes

$$E_o = \frac{I^2 e^2 \epsilon}{2m} [2 \epsilon \ln \epsilon - 2\epsilon - \epsilon \ln^2 \epsilon + 2] \qquad (C-22)$$

which may be rewritten as

$$2E_o m = I^2 e^2 \epsilon [2 - \epsilon - \epsilon (1 - \ln \epsilon)^2] \qquad (C-23)$$

Thus for Case I ($\epsilon \geq \frac{1}{e}$):

$$I = \frac{\sqrt{2E_o m}}{e\epsilon^{1/2} [2 - \epsilon - \epsilon (1 - \ln \epsilon)^2]^{1/2}} \qquad (C-24)$$

This expression for the impulse which can destroy a target will be left in this form until a similar expression has been derived for Case II.

Case II. --Up to time, T, the distance which the mass, m, has moved is found by calculating S_T rather than S from Eq. C-8 by putting t = T. This equation reduces to:

$$S_T = \frac{PT^2}{2m} \left[2 - \frac{4}{e} \epsilon\right] \qquad C-25)$$

At time, T, the mass, m, has a velocity found by substituting T for t in Eq. C-7, i.e.,

$$\dot{S}_T = \frac{PT}{me} - \frac{\epsilon PT}{m} \qquad (C-26)$$

or

$$\dot{S}_T = \frac{PT}{m} \left[\frac{1 - e\epsilon}{e}\right] \qquad (C-27)$$

After time, T, the mass, m, is subjected to a constant negative acceleration which reduces its velocity from $\dot{S}_T$ at time, T, to zero at time, t_o. The distance moved is then:

$$S_{(t_o-T)} = \frac{1}{2} \dot{S}_T (t_o - T) \tag{C-28}$$

which, with Eq. C-27, becomes

$$S_{(t_o-T)} = \frac{PT}{2m} \left[\frac{1 - e\epsilon}{e} \right] (t_o - T) \tag{C-29}$$

In Fig. C-3, it may be seen that

$$\text{rect. } \epsilon P t_o = I + \text{area 2} - \text{area 1} \tag{C-30}$$

But since the velocity of mass, m, is zero at t_o, areas 1 and 2 must be equal. Equation C-30 then becomes

$$P t_o = I \tag{C-31}$$

Combining Eqs. C-31 and C-10 gives

$$\epsilon P t_o = \frac{PT}{e} \tag{C-32}$$

or

$$t_o = \frac{T}{e\epsilon} \tag{C-33}$$

and, hence

$$t_o - T = T \left[\frac{1 - e\epsilon}{e\epsilon} \right] \tag{C-34}$$

Substituting this value for $(t_o - T)$ in Eq. C-29 gives:

$$S_{(t_o-T)} = \frac{PT^2}{2m} \left[\frac{(1 - e\epsilon)^2}{e^2 \epsilon} \right] \tag{C-35}$$

The total distance moved is found from Eqs. C-25 and C-35 to be:

$$S_{t_o} = \frac{PT^2}{2m}\left[2 - \frac{4}{e} - \epsilon + \frac{(1 - e\epsilon)^2}{e^2\epsilon}\right] \qquad \text{(C-36)}$$

This general relationship between a distance (S_{t_o}) moved by the target surface and the characteristics (P and T^2) of the shock wave which produces the displacement may be made specific by substituting S_o for S_{t_o}. This equation will then relate S_o to the PT^2 of the specific shock wave causing this displacement and consequent destruction of the target. The energy equation for Case II which is equivalent to Eq. C-22 in Case I is given by

$$E_o = \frac{P^2T^2\epsilon}{2m}\left[2 - \frac{4}{e} - \epsilon + \frac{(1 - e\epsilon)^2}{e^2\epsilon}\right] \qquad \text{(C-37)}$$

which with Eq. C-10 gives

$$E_o = \frac{I^2e^2\epsilon}{2m}\left[2 - \frac{4}{e} - \epsilon + \frac{(1 - e\epsilon)^2}{e^2\epsilon}\right] \qquad \text{(C-38)}$$

which reduces to

$$E_o = \frac{I^2}{2m}\left[1 + (2e^2 - 6e)\epsilon\right] \qquad \text{(C-39)}$$

or

$$E_o = \frac{I^2}{2m}\left[1 - 1.5315\,\epsilon\right] \qquad \text{(C-40)}$$

Thus for Case II ($\epsilon \leq \frac{1}{e}$):

$$I = \frac{\sqrt{2E_o m}}{\sqrt{1 - 1.5315\epsilon}} \qquad \text{(C-41)}$$

For finite values of ϵ the denominator of Eq. C-41 is less than one and I is greater than $\sqrt{2E_o m}$, but as $\epsilon \to 0$, $I \to \sqrt{2E_o m}$.

It may be noted that $\sqrt{2E_o m}$ has the dimension of impulse per unit area. Since I approaches $\sqrt{2E_o m}$ as a lower limit, we may now define a more general target parameter:

I_m = the minimum impulse (i.e., the lower limit of the impulse) which can destroy a target of unit mass, m, requiring E_o to complete its destruction, i.e.,

$$I_m = \sqrt{2E_o m} \tag{C-42}$$

Equation C-42 may be substituted in Eqs. C-24 and C-41 to give:

For Case I ($\epsilon \geq \frac{1}{e}$):

$$I = \frac{I_m}{e\epsilon^{1/2}\left[2 - \epsilon - \epsilon(1 - \ln \epsilon)^2\right]^{1/2}} \tag{C-43}$$

For Case II ($\epsilon \leq \frac{1}{e}$):

$$I = \frac{I_m}{\sqrt{1 - 1.5315\epsilon}} \tag{C-44}$$

The use of these equations will be discussed in succeeding appendices.

APPENDIX D

Parametric Equations for a General P-I Curve

Equations C-43 and C-44 each cover a different part of the complete spectrum of values of ϵ between zero and one. These equations may be used to compute completely general pressure-impulse curves along which every point relates a peak overpressure to a positive impulse. These are the overpressures and impulses related to a shock wave which is capable of destroying a given target. The scaling parameter, ϵ, represents the peak overpressure, P. In order to make the curve completely general the coordinates are expressed as dimensionless ratios, i.e.,

$$\frac{P}{P_m} = \frac{1}{\epsilon} \qquad \text{(Eq. B-1)}$$

For Case I ($\epsilon \geq \frac{1}{e}$):

$$\frac{I}{I_m} = \frac{1}{e\epsilon^{1/2}\left[2 - \epsilon - \epsilon(1 - \ln \epsilon)^2\right]^{1/2}} \qquad \text{(Eq. C-43)}$$

For Case II ($\epsilon \leq \frac{1}{e}$):

$$\frac{I}{I_m} = \frac{1}{\sqrt{1 - 1.5315\epsilon}} \qquad \text{(Eq. C-44)}$$

Values of these functions in terms of ϵ are given in the table on the following page.

Table III

Coordinates of a General Pressure-Impulse Curve					
ϵ	$\frac{P}{P_m}$	$\frac{I}{I_m}$	ϵ	$\frac{P}{P_m}$	$\frac{I}{I_m}$
0.01	100.00	1.008	0.31	3.23	1.380
0.02	50.00	1.016	0.32	3.13	1.400
0.03	33.33	1.024	0.33	3.03	1.422
0.04	25.00	1.032	0.34	2.94	1.444
0.05	20.00	1.041	0.35	2.86	1.468
0.06	16.67	1.049	0.36	2.78	1.493
0.07	14.29	1.058	0.37	2.70	1.519
0.08	12.50	1.067	0.38	2.63	1.547
0.09	11.11	1.077	0.39	2.56	1.576
0.10	10.00	1.087	0.40	2.50	1.606
0.11	9.09	1.097	0.41	2.44	1.638
0.12	8.33	1.107	0.42	2.38	1.672
0.13	7.69	1.117	0.43	2.33	1.708
0.14	7.14	1.128	0.44	2.27	1.745
0.15	6.67	1.139	0.45	2.22	1.785
0.16	6.25	1.151	0.46	2.17	1.826
0.17	5.88	1.163	0.47	2.13	1.870
0.18	5.56	1.175	0.48	2.08	1.916
0.19	5.26	1.188	0.49	2.04	1.964
0.20	5.00	1.201	0.50	2.00	2.016
0.21	4.76	1.214	0.51	1.961	2.070
0.22	4.55	1.228	0.52	1.923	2.127
0.23	4.35	1.242	0.53	1.887	2.187
0.24	4.17	1.257	0.54	1.852	2.250
0.25	4.00	1.273	0.55	1.818	2.318
0.26	3.85	1.289	0.56	1.786	2.389
0.27	3.70	1.306	0.57	1.754	2.465
0.28	3.57	1.323	0.58	1.724	2.545
0.29	3.45	1.341	0.59	1.695	2.631
0.30	3.33	1.360	0.60	1.667	2.722

Table II (cont'd)

ϵ	$\frac{P}{P_m}$	$\frac{I}{I_m}$	ϵ	$\frac{P}{P_m}$	$\frac{I}{I_m}$
0.61	1.639	2.819	0.81	1.235	7.918
0.62	1.613	2.923	0.82	1.220	8.571
0.63	1.587	3.033	0.83	1.205	9.322
0.64	1.563	3.152	0.84	1.190	10.19
0.65	1.538	3.279	0.85	1.176	11.21
0.66	1.515	3.416	0.86	1.163	12.41
0.67	1.493	3.564	0.87	1.149	13.85
0.68	1.471	3.723	0.88	1.136	15.59
0.69	1.449	3.895	0.89	1.124	17.73
0.70	1.429	4.082	0.90	1.111	20.43
0.71	1.408	4.285	0/91	1.099	23.89
0.72	1.389	4.507	0.92	1.087	28.47
0.73	1.370	4.749	0.93	1.075	34.73
0.74	1.351	5.015	0.94	1.064	43.70
0.75	1.333	5.307	0.95	1.053	57.37
0.76	1.316	5.631	0.96	1.042	80.06
0.77	1.299	5.990	0.97	1.031	123.10
0.78	1.282	6.391	0.98	1.020	225.85
0.79	1.266	6.839	0.99	1.010	637.99
0.80	1.250	7.345			

Figure D-1 is a plot of the data given in Table III. It relates the shock wave pressures and impulses to the minimum pressures and impulses which can damage a given target. Thus, a single curve is the locus of all possible P-I combinations which can just destroy (without overkilling) any given target characterized by P_m and I_m.

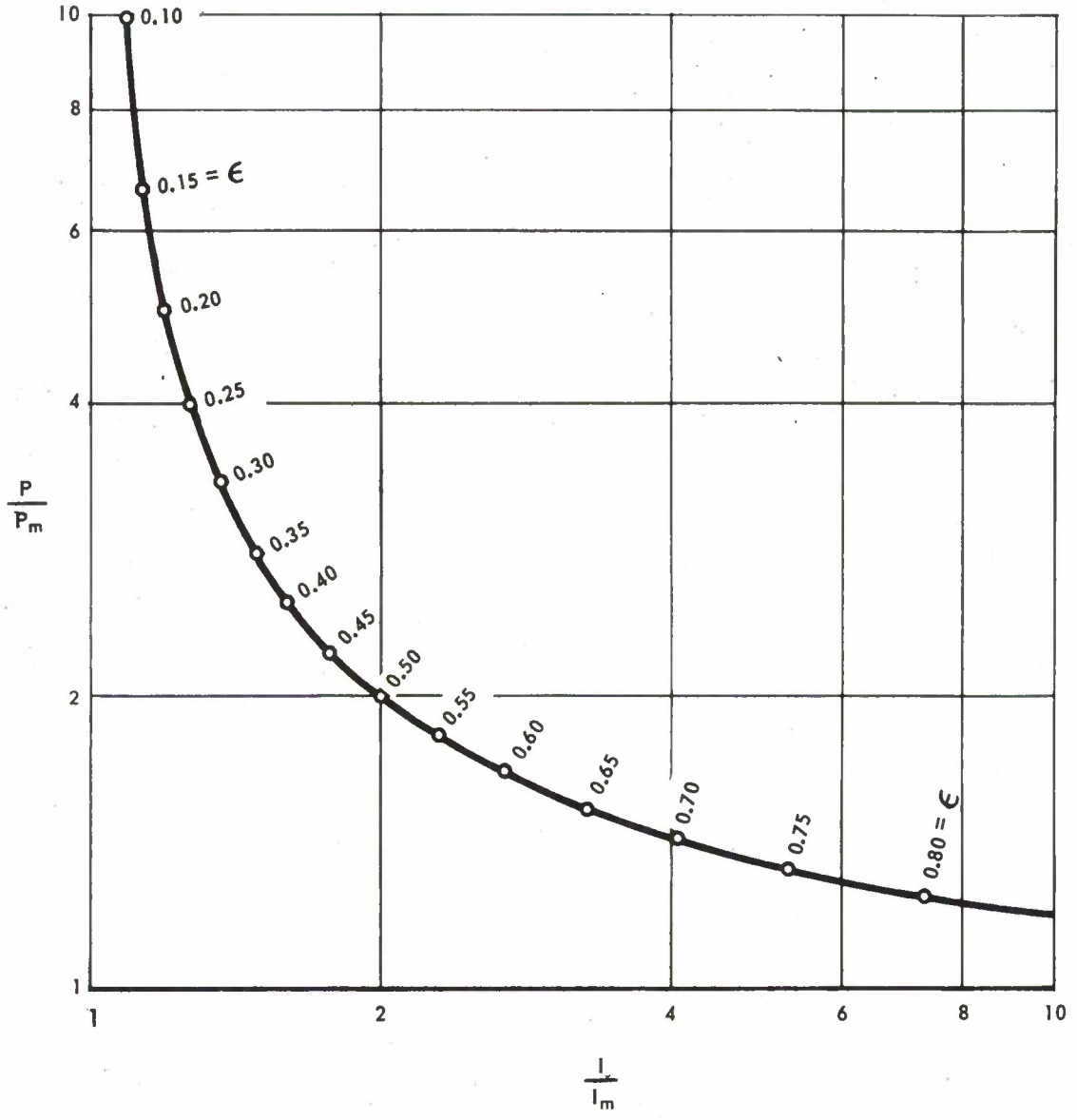


Fig. D–1 GENERAL PRESSURE-IMPULSE RELATIONSHIP

APPENDIX E

Efficiency of Energy Transfer

Whenever the surface of a target on which a shock wave acts is permanently displaced, some energy is imparted to the target. The amount of energy per unit area transferred from the shock wave to the target depends on two parameters: the mass per unit area of target which is moved, and the scaling parameter, ϵ .

Target with $\epsilon = 0$. --The maximum energy per unit area is imparted to a target when it offers no resistance to the shock wave (i. e,, when $\epsilon = 0$). This does not represent any real target but is introduced solely for the purpose of deriving an expression for the maximum energy transfer per unit area. The target, for the purposes of this first derivation, consists of a plate of mass, m, per unit area, originally stationary with respect to the ambient atmosphere but free to accelerate under the action of the shock wave which is not opposed by any other external force.

All of the energy imparted to the mass, m, becomes kinetic energy and imparts a velocity which is assumed to be constant. One further assumption is necessary for the following derivation: the mass, m, is large enough, and its final velocity small enough to make a relatively unimportant reduction in the shock wave pressure on the target. This criterion substantiates one of the basic assumptions of this study: the shock wave exerts its pressure on a non-moving target.

Figure E-1 shows the pressure-time relationship for a shock wave of unspecified shape whose equation is:

$$p = F(t) \tag{E-1}$$

and hence

$$I = \int_0^T F(t)\, dt \tag{E-2}$$

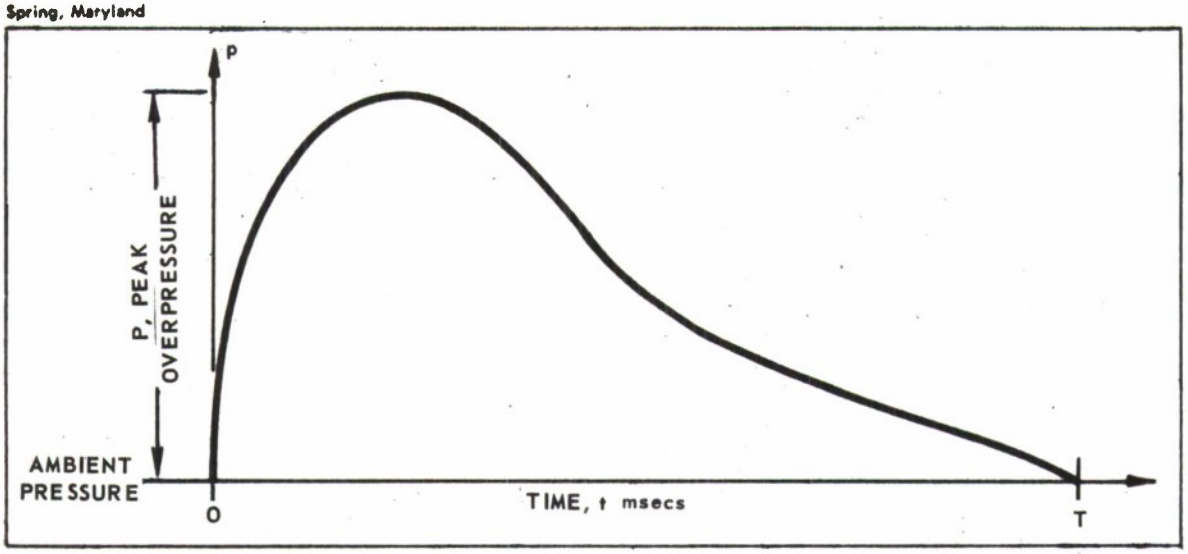


Fig. E-1 PRESSURE-TIME RELATIONSHIP FOR $\epsilon = 0$

Since there is no resistance to the target motion other than the inertia of its mass, m, it follows that

$$\ddot{S} = \frac{F(t)}{m} \tag{E-3}$$

The acceleration continues for the duration of the positive impulse and hence the velocity at time T, is

$$\dot{S}_T = \frac{1}{m}\int_0^T F(t)\,dt \tag{E-4}$$

which may by Eq. E-2 be reduced to

$$\dot{S}_T = \frac{I}{m} \tag{E-5}$$

If E_m is defined as the kinetic energy imparted to mass, m, then

$$E_m = \frac{1}{2} m \dot{S}_T^2 \tag{E-6}$$

$$= \frac{I^2}{2m}$$

Two conclusions can be drawn from this derivation:

1. The effectiveness of the shock wave is dependent only on the value of its positive impulse which is not affected by the shape of the pressure-time curve.
2. The effective energy of the shock wave varies inversely as the mass per unit area, m, of the target.

These conclusions are valid for any target which is free to move without opposition from any external force. All energy imparted to the target is kinetic energy. It will be shown that $I^2/2m$ is the maximum energy which the shock wave can impart to a unit area of any target, and that when ϵ has a finite value (between zero and one) the actual energy transferred is always less than $I^2/2m$.

Target with $\epsilon \geq 1/e$ (Case I). --In the case illustrated by Fig. E-2 where

$$\epsilon \geq \frac{1}{e}$$

it was shown in Appendix C that

$$t_o = -T \ln \epsilon \qquad \text{(Eq. C-14)}$$

and hence

$$\epsilon P t_o = - \epsilon PT \ln \epsilon \qquad \text{(E-7)}$$

but since

$$I = PTe^{-1} \qquad \text{(E-8)}$$

Eq. E-7 may be written as

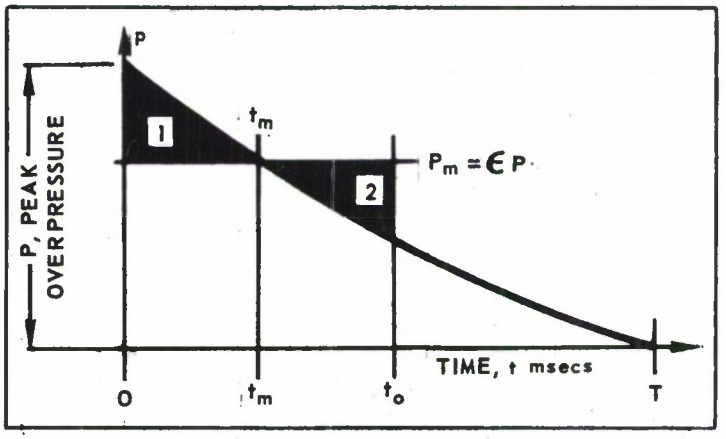


Fig. E-2 PRESSURE-TIME RELATIONSHIP FOR $\epsilon \geq 1/e$

$$\frac{\epsilon P t_o}{I} = - \frac{\epsilon PT \ln \epsilon}{PTe^{-1}} \qquad \text{(E-9)}$$

or

$$\frac{\epsilon P t_o}{I} = - e \epsilon \ln \epsilon \qquad \text{(E-10)}$$

This ratio of the impulse acting on the target to the positive impulse of the shock wave has a value of one when $\epsilon = 1/e$ and $t_o = T$, but is less than one for all higher values of ϵ, indicating that only a fraction of the positive impulse of the shock wave is able to act on the target in this range of ϵ values.

Target with $\epsilon \leq 1/e$ (Case II). --In the course of the derivation in Appendix C, it was shown that for Case II (see Fig. E-3) where

$$\epsilon \leq \frac{1}{e}$$

then

$$\epsilon P t_o = I \qquad \text{(Eq. C-31)}$$

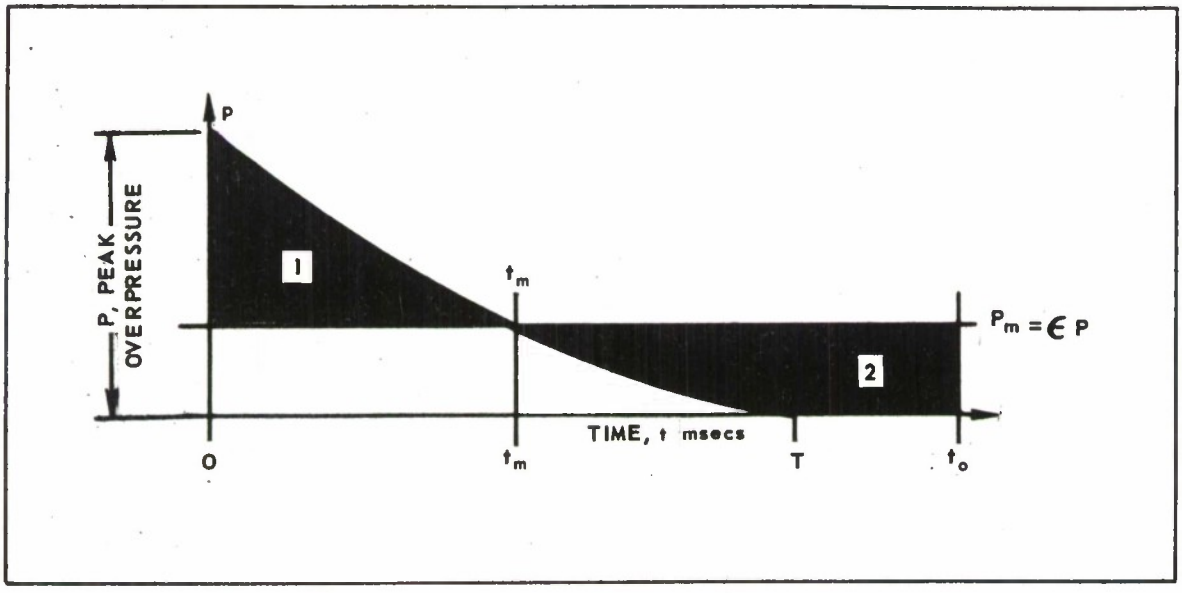


Fig. E-3 PRESSURE-TIME RELATIONSHIP FOR $\epsilon \leq 1/e$

Here, $\epsilon P t_o$ is the impulse acting on the target (a unit pressure, ϵ P, acting during a time, t_o) and is exactly equal to the positive impulse of the shock wave.

Figure E-4 shows the fraction of the positive impulse which acts on the target as a function of ϵ. It shows that the impulse is 100 percent effective for $\epsilon \leq 1/e$, but the percentage drops rapidly for higher values of ϵ.

The efficiency of energy transfer, as a function of ϵ, will be defined as the ratio of the energy imparted to a target having a finite value of ϵ to the energy which would be imparted if ϵ were zero.

In Appendix C, expressions were derived for energy per unit area in terms of I, m, and ϵ, as follows:

For $\epsilon \leq \frac{1}{e}$ $\qquad E = \frac{I^2}{2m} [1 - 1.5315 \epsilon]$ (Eq. C-40)

For $\epsilon \geq \frac{1}{e}$ $\qquad E = \frac{I^2}{2m} \left\{ e^2 \epsilon [e - \epsilon - \epsilon (1 - \ln \epsilon)^2] \right\}$ (Eq. C-23)

Let $\psi = \frac{E 2m}{I^2}$ = efficiency of energy transfer

The above equations simplify to:

For $\epsilon \leq \frac{1}{e}$ $\qquad \psi = [1 - 1.5315 \epsilon]$ (E-11)

For $\epsilon \geq \frac{1}{e}$ $\qquad \psi = e^2 \epsilon [e - \epsilon - \epsilon (1 - \ln \epsilon)^2]$ (E-12)

Figure E-5 shows the efficiency of the energy transfer as a function of ϵ. The efficiency is 100 percent at $\epsilon = 0$ and drops to 50 percent by the time ϵ reaches 0.32; at $\epsilon = 0.64$, it is reduced to 10 percent of its initial value.

Figure E-6 shows the efficiency of energy transfer in terms of $F_w(\epsilon)$, i.e., of the charge weight. It may be seen that where the latter is of the order of kilotons, the efficiency is very low.

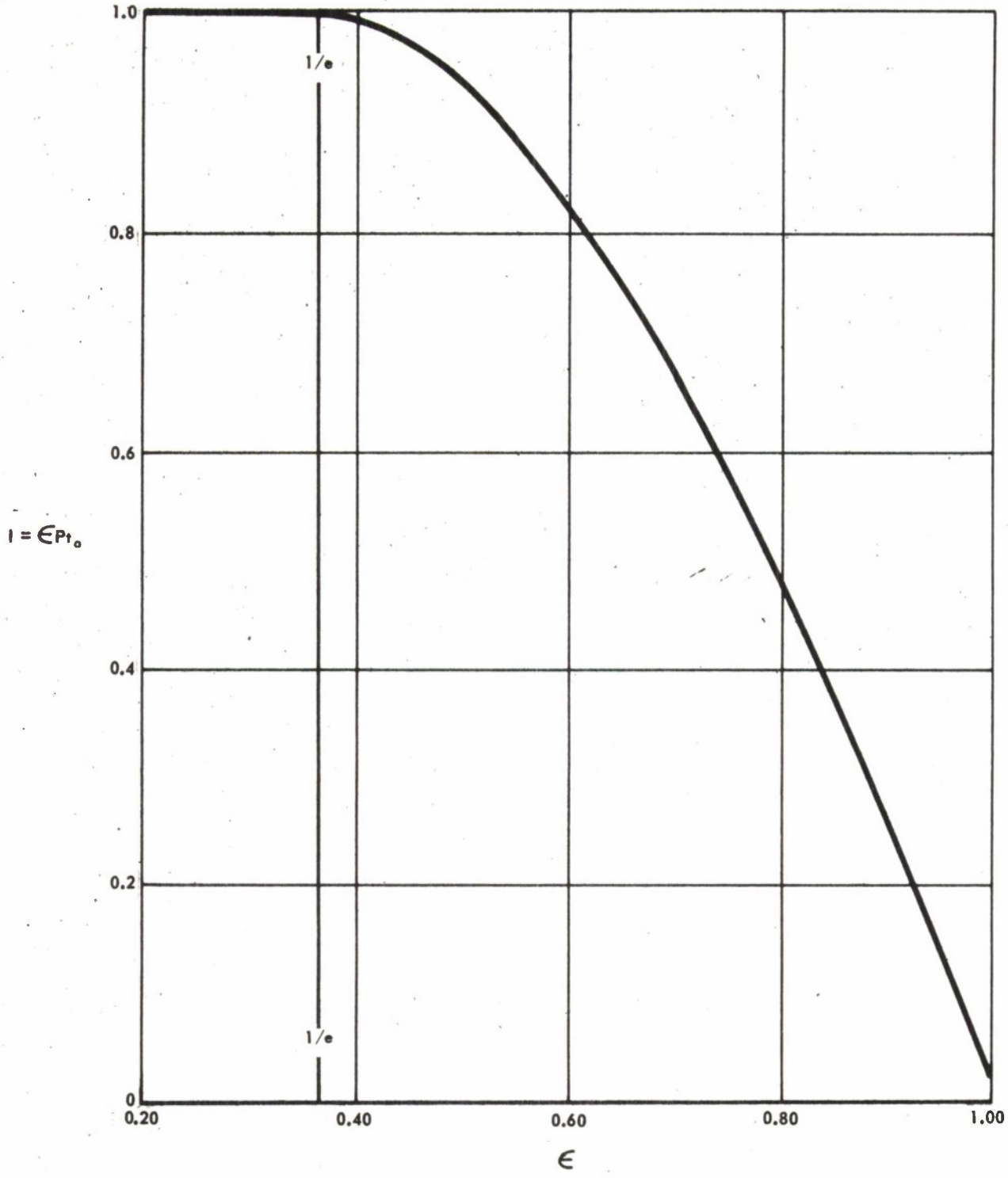


Fig. E-4 FRACTION OF IMPULSE WHICH ACTS ON TARGET AS A FUNCTION OF ϵ

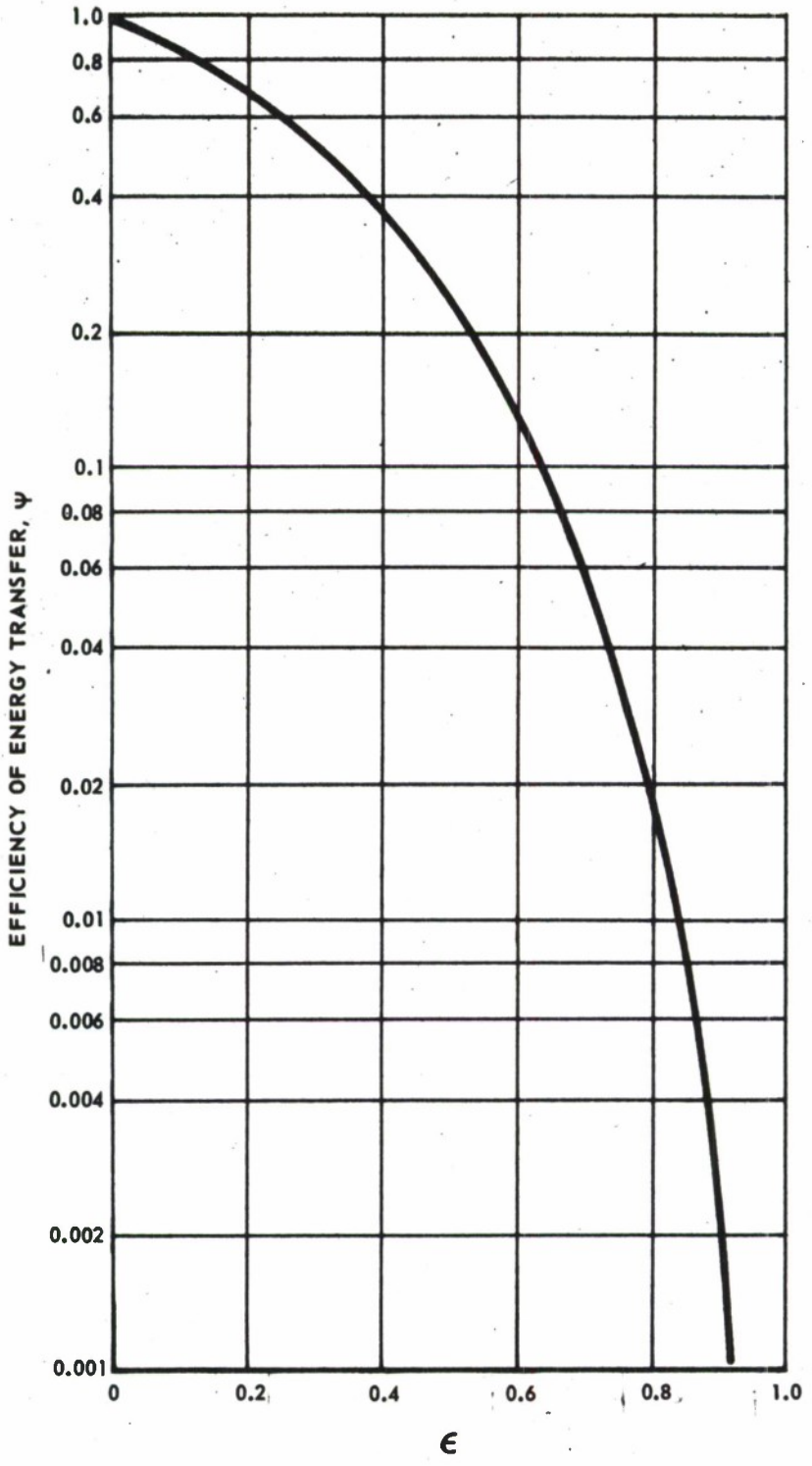


Fig. E-5 EFFICIENCY OF THE ENERGY TRANSFER, ψ, AS A FUNCTION OF ϵ

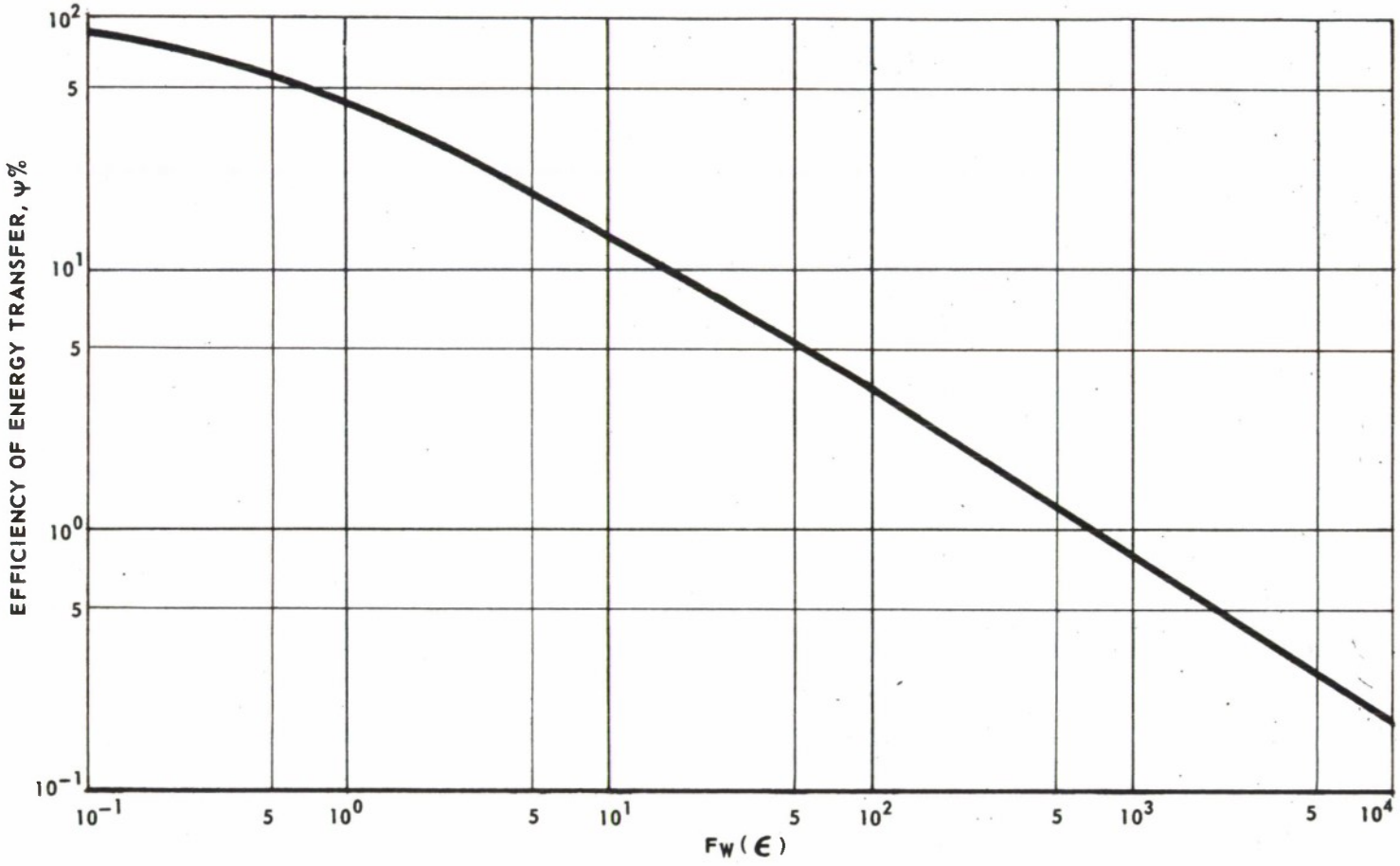


Fig. E-6 EFFICIENCY OF ENERGY TRANSFER, ψ, AS A FUNCTION OF THE CHARGE WEIGHT, $F_w(\epsilon)$.

APPENDIX F

Determination of the Equivalent Bare Charge

The remainder of this report deals with the effects of bare charges of explosive, i.e., those unencumbered by casings or enclosing structures which could modify their effectiveness. However, the report would not be complete if it failed to consider what happens when a substantial mass of non-explosive material surrounds the charge. In most anti-aircraft warheads the primary lethal agent is not the explosive blast but rather an additional effect incidental to the use of high explosives whose prime function is to impart kinetic energy to fragments, rods, or other inert solid materials surrounding the charge. Nevertheless the blast effects from such warheads are far from negligible; in fact they substantially augment the overall effectiveness of the warheads. An explosive charge encased in metal or other inert solid material must, of necessity, impart some energy and momentum to these materials, leaving less energy to be transferred by the shock wave to the target at some distance from the source. The study of the nature and magnitude of the effects of such inert casings can employ the same general relationships that have been derived in preceding appendices.

The usual way of expressing the effects of encasement is to determine the weight of an equivalent bare charge, i.e., that weight which can just destroy a given target at the same distance from the detonation as the weight of a standard encased explosive. In other words, the equivalent bare charge must do exactly the same amount of work per unit target area as does the standard encased charge.

Let C = actual weight (lb) of encased charge

KC = weight (lb) of equivalent bare charge

i.e., K = the proportionality factor relating equivalent bare charge to actual encased charge.

R = distance from detonation to target (same for both cases).

μ = weight (lb) inert casing material.

It is assumed that when completely inert casing materials (as opposed to those which may react either with the explosive or the atmosphere) are used, both the peak overpressure and the positive impulse produced by the charge, C, will be lower than those associated with a bare charge.

Let P_o = peak overpressure (psi) produced by bare charge at distance, R,

λP_o = peak overpressure (psi) produced at distance, R, by an encased charge.

The following equation, for work done on a unit area of target by a shock wave, was derived in Appendix C:

$$\text{for } \epsilon \leq \frac{1}{e} \qquad E = \frac{I^2}{2m}\left[1 - 1.5315\epsilon\right] \qquad \text{(Eq. C-40)}$$

Charge weights up to the order of about a half ton of explosive usually lie in this range of ϵ values, and thus include charge weights pertinent to this study.

The above equation shows that the target parameter, m, affects the total energy, E; this is a fixed quantity for both the encased charge and the equivalent bare charge. Total energy is affected by two other factors: the impulse, I, which is exclusively a shock wave parameter, and ϵ, which is the ratio of a target parameter, P_m, to the peak overpressure of the shock wave. In this analysis of the interactions between shock waves and targets, the equations which have been derived afford the basis for a new approach to the problem of determining equivalent bare charges. It is only necessary

to find appropriate analytical expressions of I and ϵ for the encased charge and the equivalent bare charge. These expressions are used in Eq. C-40, and the resulting energies equated.

This analytical approach is based on the postulation that the momentum imparted to the inert casing material is subtracted from the total positive impulse which the shock wave from a bare charge could exert against the entire inner surface of a hollow sphere of radius, R. This postulation is made merely to simplify the analytical study; however, it leads to some very interesting interpretations.

Equation A-7, which gives the positive impulse (psi·msecs) per unit area, may be rewritten as:

$$I = \frac{220\, W^{0.733}}{AR^{1.20}} \qquad (F-1)$$

where

I = normally reflected positive impulse (psi·secs/in^2)

W = weight (lb) of explosive

R = distance (ft) from detonation to target

A = value of a function of altitude above sea level (F_R(alt), Table II, p. 12).

The total normally reflected positive impulse over the entire inner surface of a hollow sphere of radius R is given by:

$$I_{total} = \frac{398 W^{0.733} R^{0.80}}{A} \qquad (F-2)$$

Using the following definitions of M and V

M = the momentum imparted to the weight, μ, of the inert casing material, and

V = the mean velocity imparted to the casing material

it may be seen that

$$M = \frac{\mu V}{32.2} \tag{F-3}$$

Then Eq. F-2 may be written:

$$I_C = \frac{398C^{0.733}R^{0.80}}{A} - \frac{\mu V}{32.2} \tag{F-4}$$

$$I_{CK} = \frac{398K^{0.733}C^{0.733}R^{0.80}}{A} \tag{F-5}$$

where

I_C = the total impulse produced by the encased charge, C

I_{CK} = the total impulse produced by the equivalent bare charge, KC

Care must be taken to use the correct value of ϵ for both the encased charge, C, and the equivalent bare charge, KC. Both of these will be expressed in terms of a third value of ϵ which would exist at distance, R, for a bare charge of weight, C.

These three values of ϵ are given by

$$\epsilon = \frac{P_m}{P_o} \tag{F-6}$$

$$\epsilon_{CK} = \frac{P_m}{KP_o} \tag{F-7}$$

and

$$\epsilon_C = \frac{P_m}{\lambda P_o} \tag{F-8}$$

where subscripts C and CK refer to an encased charge and its equivalent bare charge and λP_o = peak overpressure produced by the encased charge.

Both ϵ_C and ϵ_{CK} may be expressed as functions of ϵ as follows:

$$\epsilon_{CK} = \frac{\epsilon}{K} \tag{F-9}$$

and

$$\epsilon_C = \frac{\epsilon}{\lambda} \tag{F-10}$$

Equation C-40 may now be used to give two expressions for energy per unit area: one in terms of I_{CK} and ϵ_{CK}, and the other in terms of I_C and ϵ_C. These two can be equated to one another, i.e.,

$$\frac{I_{CK}^2}{2m}\left[1 - 1.5315\epsilon_{CK}\right] = \frac{I_C^2}{2m}\left[1 - 1.5315\epsilon_C\right] \tag{F-11}$$

Using Eqs. F-4, F-5, F-9, and F-10, Eq. F-11 may be re-written:

$$\frac{\left[398A^{-1}K^{0.733}C^{0.733}R^{0.80}\right]^2}{2m}\left[1 - \frac{1.5315\,\epsilon}{K}\right] = \frac{\left[398A^{-1}C^{0.733}R^{0.80} - \frac{\mu V}{32.2}\right]^2}{2m}\left[1 - \frac{1.5315\epsilon}{\lambda}\right] \tag{F-12}$$

or

$$K^{0.733} = \frac{\sqrt{1 - \frac{1.5315\epsilon}{\lambda}}}{\sqrt{1 - \frac{1.5315\epsilon}{K}}}\left[1 - \frac{\mu VA}{12,800C^{0.733}R^{0.80}}\right] \tag{F-13}$$

The scaled distance relationship:

$$Z = \frac{R}{W^{1/3}} = \frac{R}{C^{1/3}} \quad \text{(Eq. A-2)}$$

can be used to eliminate R from Eq. F-13 to yield:

$$K^{0.733} = \left[\frac{1 - \frac{1.5315\epsilon}{\lambda}}{1 - \frac{1.5315\epsilon}{K}}\right]^{1/2} \left[1 - \frac{\left(\frac{\mu}{C}\right)VA}{12,800Z^{0.80}}\right] \qquad \text{(F-14)}$$

i.e.,

$$K = \left[\frac{1 - \frac{1.5315\epsilon}{\lambda}}{1 - \frac{1.5315\epsilon}{K}}\right]^{0.682} \left[1 - \frac{\left(\frac{\mu}{C}\right)VA}{12,800Z^{0.80}}\right]^{1.364} \qquad \text{(F-15)}$$

This expression for K, the ratio of the equivalent bare charge weight to the actual weight of the encased charge, is complicated by the presence of a K in the denominator of the first term, which precludes derivation of a simple explicit function for K. The entire first term, however, may be considered as a correction factor which equals one when λ and K are equal and which varies from unity by only a small quantity even when λ and K are quite different. Therefore, its effect on K is relatively small, and Eq. F-15 may be further simplified to:

$$K \approx \left[1 - \frac{\left(\frac{\mu}{C}\right)VA}{12,800Z^{0.80}}\right]^{1.364} \qquad \text{(F-16)}$$

Implications--Equation F-16, which is based on the postulation that the total impulse of the shock wave at a scaled distance, Z, is reduced by the amount of momentum imparted to the inert casing material, presents some challenging implications. The negative term is the fraction by which the weight of the encased charge, C, is reduced to give the weight of the equivalent bare charge, CK. Unlike some other expressions for the equivalent bare charge, this derivation implies that not only the ratio μ/C but both the altitude and the scaled distance have an effect on the degradation in effective weight caused by the casing.

The parameters μ/C and V, are directly related to one another. For a spherical charge of pentolite the Gurney equation for V is:

$$V = 8400\sqrt{\frac{C/\mu}{1 + 0.6(C/\mu)}} \qquad \text{(F-17)}$$

which in conjunction with Eq. F-16 gives:

$$K \approx \left[1 - \frac{A}{1.58Z^{0.8}\sqrt{\frac{C}{\mu} + 0.6\left(\frac{C}{\mu}\right)^2}}\right]^{1.364} \qquad \text{(F-18)}$$

The altitude function, A, is identical with F_R(alt.). Values are listed as a function of altitude in Table II (p. 12).

The postulation upon which Eqs. F-16 and F-18 are based is still only a postulation and not an established fact. However, it has focused attention on the effect of altitude on the equivalent bare charge computations and suggests that the scaled distance also has some effect.

APPENDIX G

General Parametric Scaling Equations for Weight and Distance

The equations for W and R derived in Appendix A are:

$$W = \frac{0.0083\left(\frac{c}{c_o}\right)^3 I^3}{P^{1.20}\, p^{0.80}} \quad \text{(Eq. A-8)}$$

and

$$R = \frac{4.8\left(\frac{c}{c_o}\right) I}{P^{0.733}\, p^{0.267}} \quad \text{(Eq. A-9)}$$

These are combined with Eqs. B-1, C-43, and C-44 to give

$$W = 0.0083\left[\frac{I_m^{\,3}}{P_m^{\,1.20}}\right]\left[\frac{\left(\frac{c}{c_o}\right)^3}{p^{0.80}}\right] F_W(\epsilon) \qquad \text{(G-1)}$$

and

$$R = 4.8\left[\frac{I_m}{P_m^{\,0.733}}\right]\left[\frac{\left(\frac{c}{c_o}\right)}{p^{0.267}}\right] F_R(\epsilon) \qquad \text{(G-2)}$$

In this pair of equations, only the final terms, $F_W(\epsilon)$ and $F_R(\epsilon)$ are restricted to ϵ values above or below 1/e.

For $\epsilon \leq \frac{1}{e}$

$$F_W(\epsilon) = \frac{\epsilon^{1.20}}{(1 - 1.5315\,\epsilon)^{3/2}} \qquad \text{(G-3)}$$

and

$$F_R(\epsilon) = \frac{\epsilon^{0.733}}{(1 - 1.5315\,\epsilon)^{1/2}} \qquad \text{(G-4)}$$

For $\epsilon \geq \frac{1}{e}$ $$F_W(\epsilon) = \frac{1}{e^3 \epsilon^{0.30} \left[2 - \epsilon - \epsilon(1 - \ln \epsilon)^2\right]^{3/2}} \quad \text{(G-5)}$$

and

$$F_R(\epsilon) = \frac{\epsilon^{0.233}}{e \left[2 - \epsilon - \epsilon(1 - \ln \epsilon)^2\right]^{1/2}} \quad \text{(G-6)}$$

These two pairs of functions of ϵ are related as follows:

$$F_R(\epsilon) = \left[F_W(\epsilon)\right]^{1/3} \epsilon^{1/3} \quad \text{(G-7)}$$

This equation is valid for all values of ϵ. The functions $F_W(\epsilon)$ and $F_R(\epsilon)$ are listed in Table I (p. 10) for values of ϵ from 0.01 to 0.99.

The general parametric equations, G-1 and G-2, consist of four terms: the first is a numerical coefficient derived from the characteristics of the explosive; the second is a function of the target; the third is a function of the altitude; and the fourth is a function of the universal scaling parameter, ϵ. For any particular explosive, target and altitude, the first three terms are constants, i.e., only the functions of ϵ will vary. It will be found that for each value of ϵ, W and R have unique values and furthermore, ϵ in the open interval $0 < \epsilon < 1$ spans the entire spectrum of charge weights from a few pounds to kilotons or even megatons of equivalent TNT.

From Eqs. G-1 and G-2, one may obtain:

$$\frac{R}{W^{1/3}} = \frac{23.7\, \epsilon^{1/3}}{P_m^{1/3}} \quad \text{(G-8)}$$

or

$$R = \frac{23.7\, W^{1/3} \epsilon^{1/3}}{P_m^{1/3}} \quad \text{(G-9)}$$

The altitude functions to be used in Eqs. G-1 and G-2 are given in Table II (p. 12) where

$$F_W(\text{alt}) = \frac{\left(\frac{c}{c_o}\right)^3}{p^{0.80}}$$

and

$$F_R(\text{alt}) = \frac{\frac{c}{c_o}}{p^{0.267}}$$

It is often desirable to use Eqs. G-1 and G-2 rather than the short-cut afforded by Eq. G-9. $F_W(\epsilon)$ and $F_R(\epsilon)$ are easily obtained from these equations for any values of W and R.

All the analytical tools for finding lethal distances in terms of charge weight (or charge weight in terms of kill distance) are available in these general equations; the process can be further simplified by the use of the nomogram described in Appendix H.

APPENDIX H

Construction of a General R-W Nomogram

The two general parametric equations G-1 and G-2 for weight, W, and lethal distance, R, give all the necessary information with respect to interactions between spherical shock waves produced by explosive detonations and targets whose characteristics can be defined by P_m and I_m. The range of scaled distances must be sufficient to encompass the region in which targets of practical concern can be destroyed. These two equations epitomize the fundamental analytic structure of shock wave-target interactions, and are the most significant product of this entire study. In order to appreciate their usefulness and to apply them to some practical problems of blast scaling, it is well to review the statements on page 64, where it was made clear that the numerical constants in Eqs. G-1 and G-2 are functions of the explosive, the next factor in each equation is a function of the target characteristics, the next of the altitude and the last of ϵ. The first three are constant when the target, explosive and altitude are specified.

Eqs. G-1 and G-2 could be written:

$$W = C_W \cdot F_W(\epsilon) \tag{H-1}$$

and

$$R = C_R \cdot F_R(\epsilon) \tag{H-2}$$

They may be expressed as

$$\log W = \log C_W + \log F_W(\epsilon) \tag{H-3}$$

and

$$\log R = \log C_R + \log F_R(\epsilon) \tag{H-4}$$

A graph of log $F_R(\epsilon)$ versus log $F_W(\epsilon)$ constitutes a general R-W relationship. It can be applied to a specific combination of explosive, target, and altitude, by adding log C_R to the ordinate and log C_W to the abscissa. Such a graph of the $F_R(\epsilon)$ vs $F_W(\epsilon)$ values taken from Table I (p. 10) is shown on Fig. H-1, with corresponding values of ϵ given for various points along the curve.

Specific charge weight values are related to $F_W(\epsilon)$ by C_W (in Eq. H-1) which is the product of the first three factors in Eq. G-1. A rough indication of the order of magnitude of the charge weight is shown on Fig. H-1 from which it may be seen that over this range of ϵ the charge weights vary considerably--from 25 lb to 10 megatons.

Two other lines are shown on Fig. H-1. The first is a line of slope 1/3 which becomes tangent to the general curve as ϵ approaches 1.00 and $F_W(\epsilon)$ approaches infinity. This is consistent with the relationship $R \propto W^{1/3}$ which has been shown to be true for nuclear charges. The second is a line of slope 1/2 which is seen to be parallel to the general curve at values of ϵ around 0.20 to 0.25 and is consistent with the $R \propto W^{1/2}$ relationship known to apply to charges of a few hundred pounds. The change in the slope of the curve of log $F_R(\epsilon)$ vs log $F_W(\epsilon)$ is the most significant <u>scale effect</u> in the relationships between charge weights and lethal distances.

Let φ = the slope of the log $F_R(\epsilon)$ vs log $F_W(\epsilon)$ curve, i.e.,

$$\varphi = \frac{\frac{d}{d\epsilon} \log F_R(\epsilon)}{\frac{d}{d\epsilon} \log F_W(\epsilon)} \qquad \text{(H-5)}$$

and

$$R = \frac{\text{constant}}{W^{\varphi}} \qquad \text{(H-6)}$$

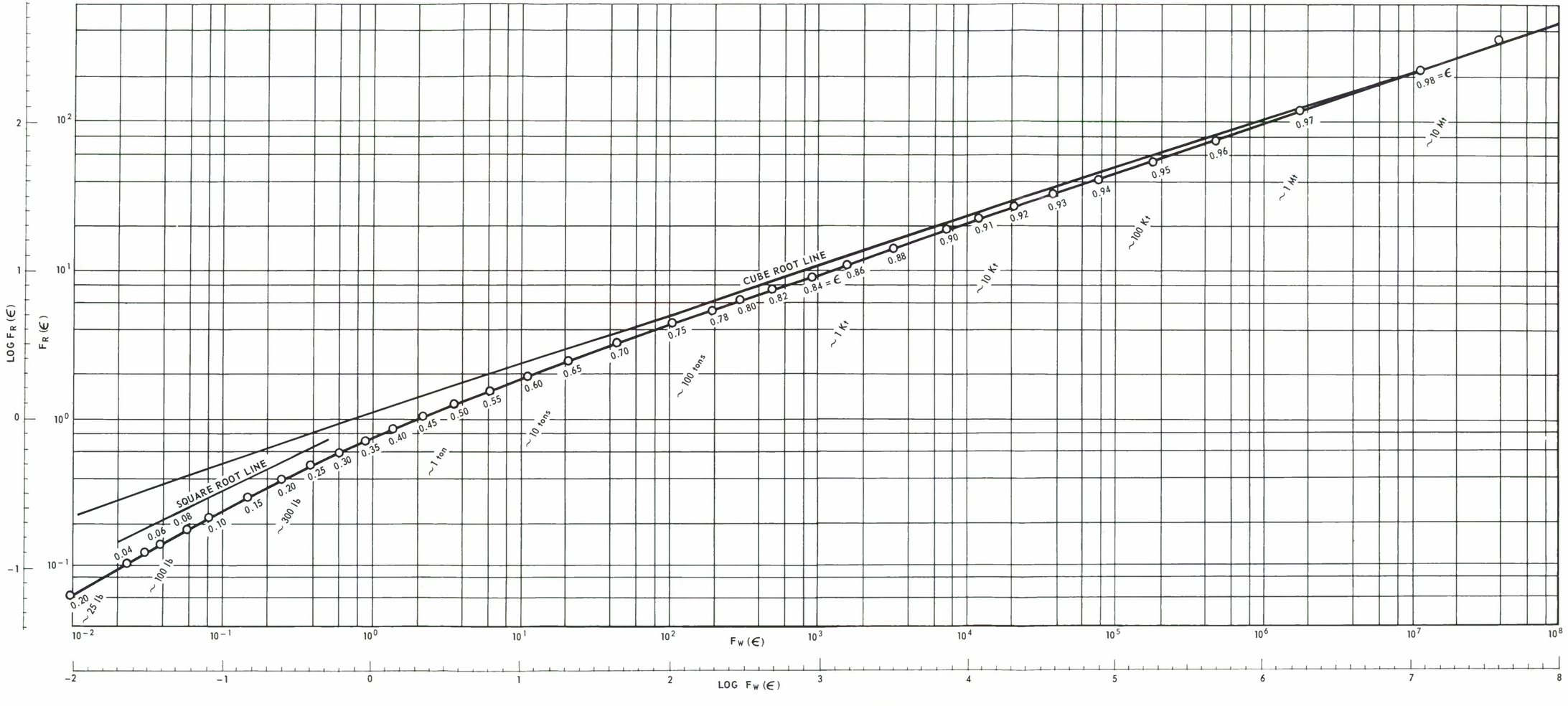


Fig. H–1 GENERAL DISTANCE–WEIGHT RELATIONSHIP

Differentiating the expressions for these functions of ϵ given in Eqs. G-3 to G-6, one obtains:

$$\text{for} \quad \epsilon \leq \frac{1}{e} \qquad \varphi = \frac{0.7333 - 0.3574\epsilon}{1.20 + 0.4590\epsilon} \tag{H-7}$$

and

$$\text{for} \quad \epsilon \geq \frac{1}{e} \qquad \varphi = \frac{\epsilon \ln^2 \epsilon + 0.4667 \left[2 - \epsilon - \epsilon (1 - \ln \epsilon)^2 \right]}{3 \epsilon \ln^2 \epsilon - 0.60 \left[2 - \epsilon - \epsilon (1 - \ln \epsilon)^2 \right]} \tag{H-8}$$

These equations have been evaluated for ϵ from 0.01 to 0.99. The results are given in Table IV and in Fig. H-2.

Table IV

φ vs ϵ

ϵ	φ	ϵ	φ	ϵ	φ	ϵ	φ	ϵ	φ
0.01	0.606	0.21	0.508	0.41	0.424	0.61	0.377	0.81	0.350
0.02	0.601	0.22	0.503	0.42	0.421	0.62	0.375	0.82	0.349
0.03	0.595	0.23	0.499	0.43	0.418	0.63	0.373	0.83	0.348
0.04	0.590	0.24	0.494	0.44	0.415	0.64	0.372	0.84	0.347
0.05	0.585	0.25	0.490	0.45	0.412	0.65	0.370	0.85	0.346
0.06	0.580	0.26	0.485	0.46	0.409	0.66	0.369	0.86	0.345
0.07	0.575	0.27	0.481	0.47	0.406	0.67	0.367	0.87	0.344
0.08	0.570	0.28	0.477	0.48	0.404	0.68	0.366	0.88	0.343
0.09	0.565	0.29	0.472	0.49	0.401	0.69	0.364	0.89	0.342
0.10	0.560	0.30	0.468	0.50	0.399	0.70	0.363	0.90	0.342
0.11	0.555	0.31	0.464	0.51	0.396	0.71	0.362	0.91	0.341
0.12	0.550	0.32	0.460	0.52	0.394	0.72	0.360	0.92	0.340
0.13	0.545	0.33	0.455	0.53	0.392	0.73	0.359	0.93	0.339
0.14	0.541	0.34	0.451	0.54	0.390	0.74	0.358	0.94	0.338
0.15	0.536	0.35	0.447	0.55	0.388	0.75	0.357	0.95	0.337
0.16	0.531	0.36	0.443	0.56	0.386	0.76	0.356	0.96	0.336
0.17	0.526	0.37	0.439	0.57	0.384	0.77	0.354	0.97	0.336
0.18	0.522	0.38	0.435	0.58	0.382	0.78	0.353	0.98	0.335
0.19	0.517	0.39	0.431	0.59	0.380	0.79	0.352	0.99	0.334
0.20	0.512	0.40	0.427	0.60	0.378	0.80	0.351	(1.00)	(0.333)

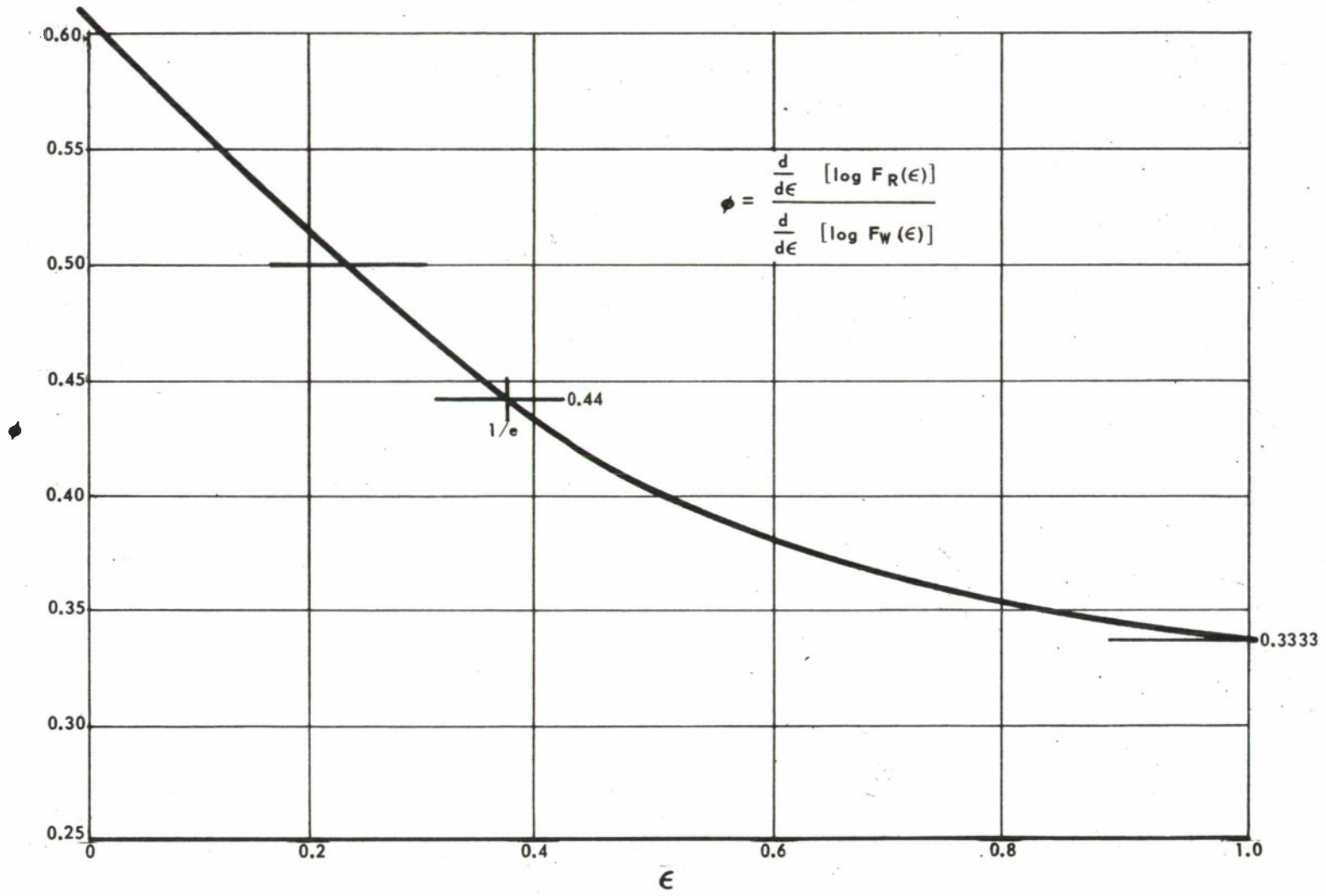


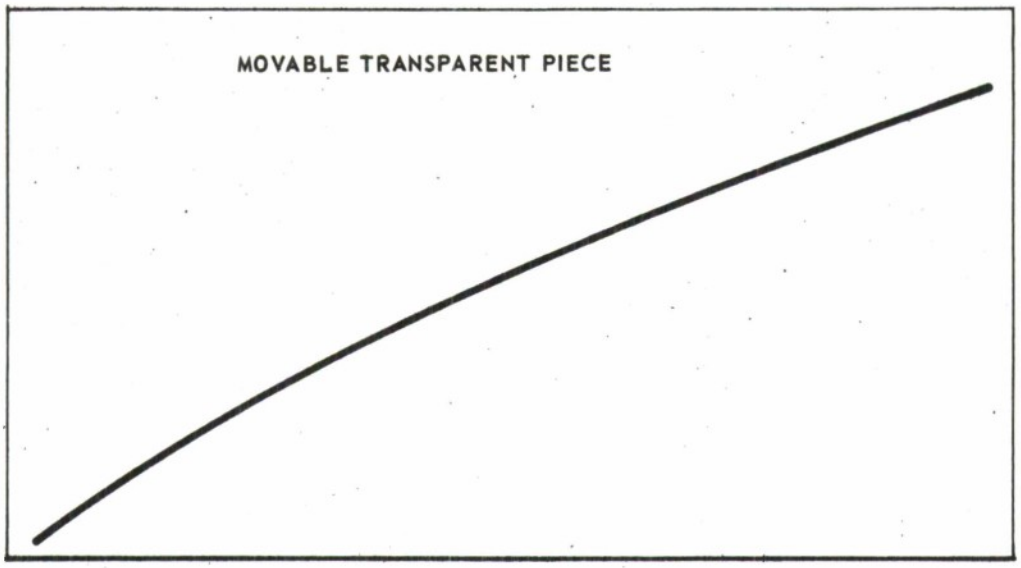
Fig. H-2 SLOPE OF LOG $F_R(\epsilon)$ VS LOG $F_W(\epsilon)$ AS A FUNCTION OF ϵ

Making the Nomogram. --It has been stated that the general graph of log $F_R(\epsilon)$ versus log $F_W(\epsilon)$ could be made specific by adding an appropriate constant, log C_R, to the ordinate, and a corresponding constant, log C_W, to the abscissa. These additions can be performed mechanically by a nomogram consisting of two separate pieces, one of which is a sheet of transparent plastic on which the general curve is drawn. This piece is movable both horizontally and vertically, with respect to the second piece, which is simply an ordinary log-log diagram of several cycles with its ordinate scale marked with distances in feet, and its abscissa in weights of explosive in pounds.

Figure H-3 shows the two parts of such a nomogram. The movable part made on transparent plastic uses the appropriate section or sections of Fig. H-1. The second or stationary part is merely a log-log grid with R as the ordinate and W as the abscissa. It may extend both ways for as many cycles as are required for the specific problems as will be explained in the next paragraph.

It is quite possible to construct a complete nomogram having 10 or 12 cycles on the abscissa, covering the entire spectrum of charge weights. However, it leads to more accurate reading of the scales of the nomograph if it covers only two or three cycles in the range of charge weights of immediate interest. Three or four nomographs with overlapping scales, to cover the entire range of possible charge weights are to be desired.

Positioning the Curve on the Scaled Diagram. --Referring to Eqs. G-1, G-2, H-1, and H-2, one notes that C_R and C_W are each the product of three terms, and their logarithms are the sum of three logarithms. Thus, the positioning of the movable curve involves adding three logarithms to log $F_R(\epsilon)$ and three others to log $F_W(\epsilon)$. Therefore, each of the three pairs



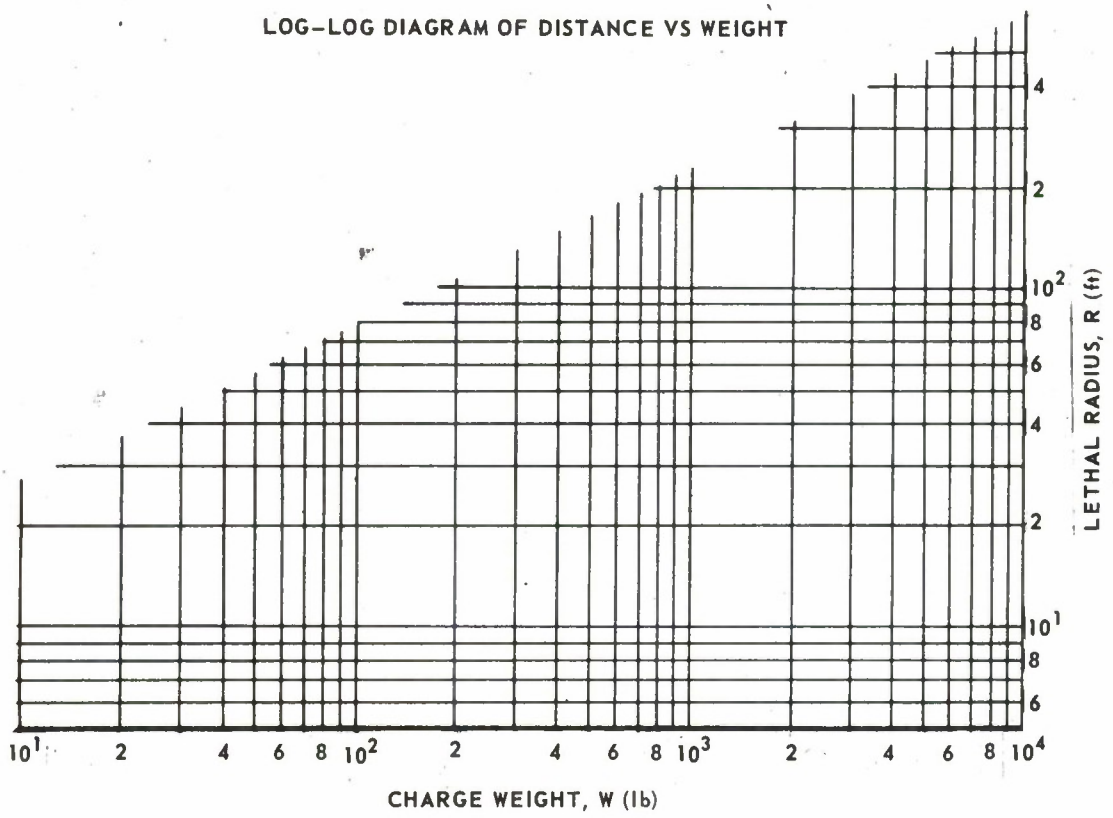


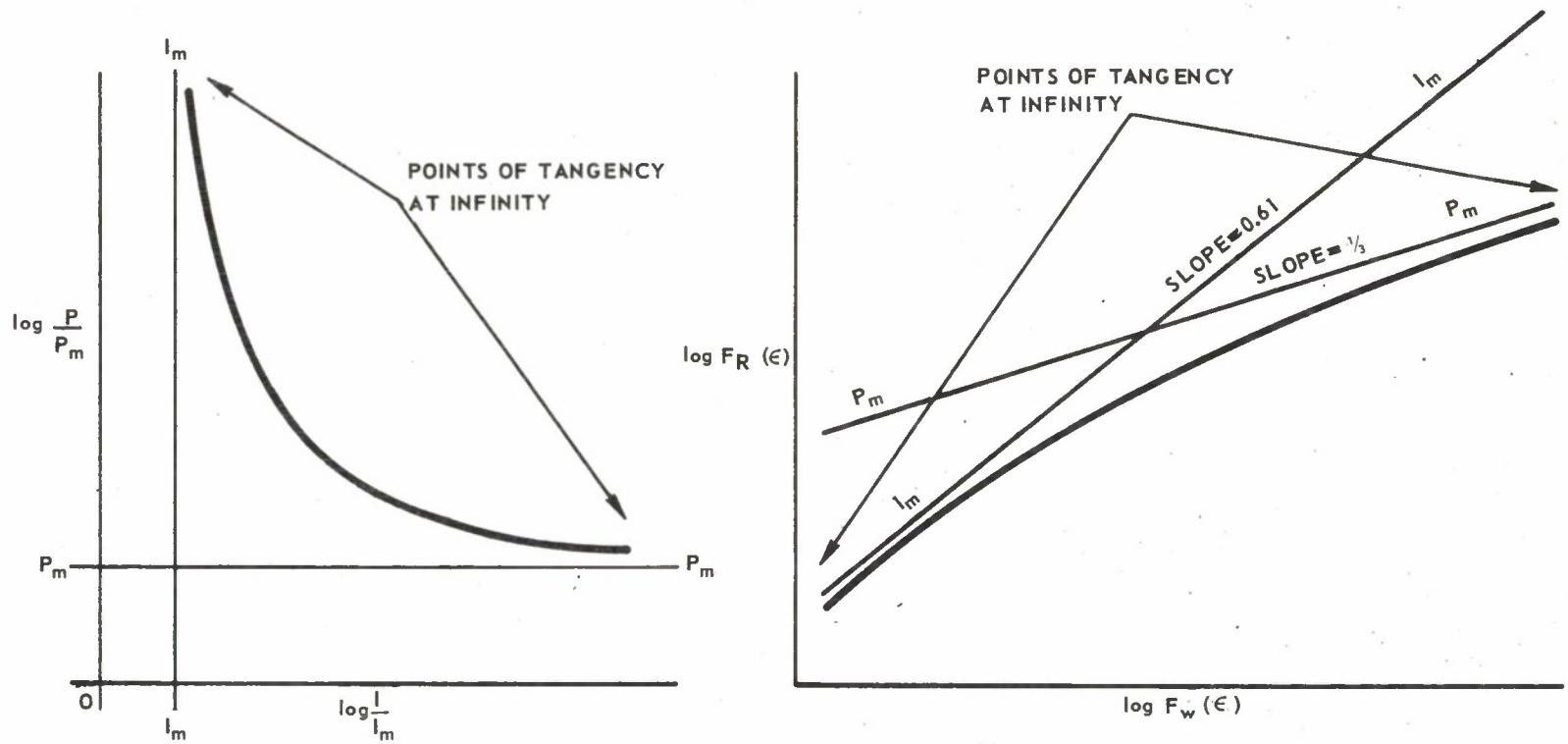
Fig. H–3 BASIC PLOTS FOR NOMOGRAPH

of terms, which define explosives, targets, and altitudes, produce displacements of the curve both vertically and horizontally. The numerical constants in Eq. G-1 and G-2 are only applicable to 50/50 Pentolite. The log of 0.0083 is negative and the log of 4.8 is positive, and therefore the curve displacement due to the two explosive characteristic terms in Eqs. G-1 and G-2 is upward and to the left, along a line with slope -1/3. The pressure-impulse curve on Fig. D-1 (p. 46) and Fig. H-1 (p. 69) are compared in Fig. H-4.

This figure shows that the two curves have a similar shape and could be superimposed. It is reasonable then to assume that the asymptotes of Fig. H-4a, (lines of constant I_m and constant P_m), would become asymptotes to the curve of Fig. H-4b. From Fig. H-2 it may be seen that the slope, ω, of the curve of Fig. H-4b, has the values 0.61 at $\epsilon = 0$ and 1/3 at $\epsilon = 1$. This implies that the lines of constant I_m and P_m will have these respective slopes when referred to the axes of Fig. H-4b.

The second terms of Eqs. G-1 and G-2 may thus be accounted for because: (1) movement on the log-log diagram due to I_m changes takes place along constant P_m lines of slope 1/3, and (2) the movement due to P_m changes takes place along constant I_m lines of slope 0.61.

This permits one to use a grid composed of constant P_m lines intersecting constant I_m lines as shown on Fig. H-5. This grid may conveniently be positioned on the same sheet as the log-log grid of R-W (Fig. H-3). Hence, a uniquely positioned point on the transparent movable part of the nomogram can be placed over the intersection of any pair of P_m and I_m lines (or interpolated points between lines) to make the appropriate adjustments for both characteristics of the target.



(a) Fig. D-1

(b) Fig. H-1

Fig. H–4 COMPARISON OF FIGS. D–1 AND H–1

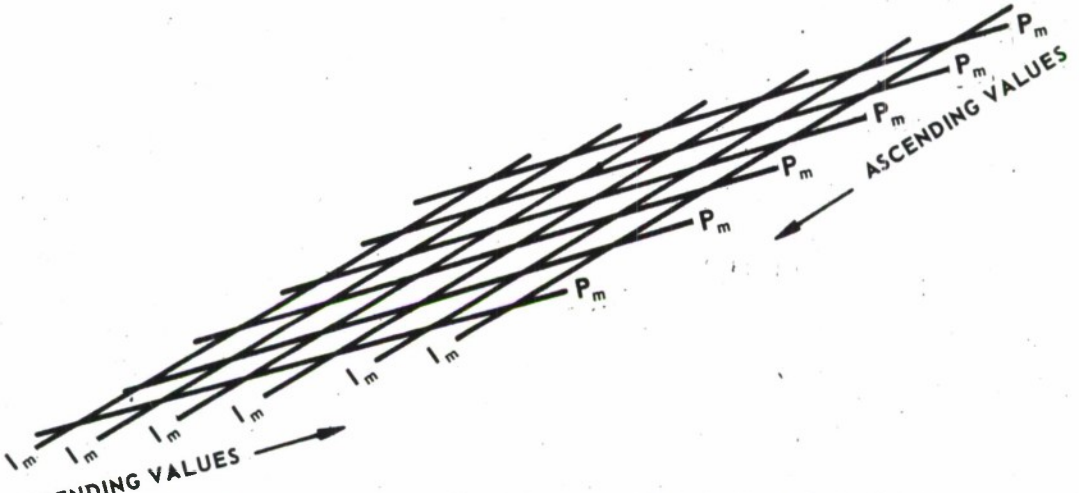


Fig. H–5 CONSTANT P_m AND I_m LINES ON SCALED DIAGRAM

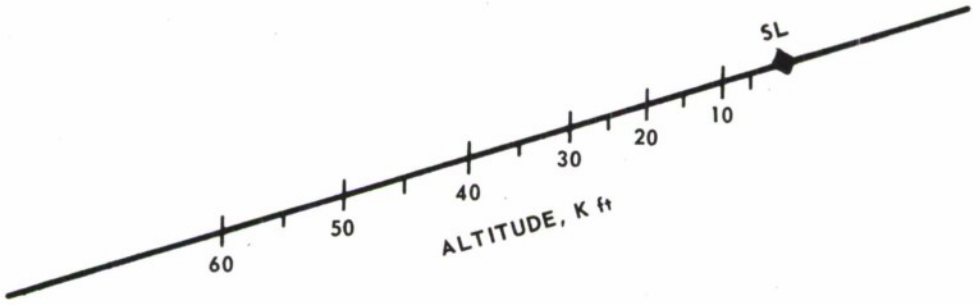


Fig. H–6 ALTITUDE SCALE (KILOFEET) ON MOVABLE PIECE BEARING GENERAL DISTANCE - WEIGHT CURVE

The third terms in Eqs. G-1 and G-2 have values of one at sea level and greater than one at all altitudes above sea level (see Table II, p. 12). Increasing the altitude corresponds to moving the curve upward and to the right along a line of slope 1/3. The displacement of the curve due to increasing altitude is in the same direction and along a line of the same slope (viz. 1/3) as the displacement due to increasing the value of I_m. This is to be expected, since the effect of increasing the altitude is to decrease the positive impulse of the shock wave, without materially affecting the normally reflected peak overpressure. Since the altitude displacement takes place along a line of slope 1/3, such a line is drawn on the transparent movable piece of the nomogram, as shown on Fig. H-6. In using the nomogram, the altitude point is superimposed on the point of P_m-I_m intersection and the altitude scale is aligned with the selected constant P_m line.

This nomogram is a simple mechanical device for expediting the solution of the scaling equations. It gives the same R-W relationship that could be computed by use of these equations and tables which relate functions of ϵ to ϵ . The nomogram is much more rapid and inherently has less chance of error than the equations.

APPENDIX I

Validity and Accuracy of Scaling Equations

The destruction of actual targets by blast is a far more complicated physical phenomenon than is indicated by the scaling equations derived in this report for an idealized target model. It has been shown that these scaling equations and the various parametric relationships derived therefrom are a valuable analytic tool. It remains to be shown that the results obtained from them are both valid and reasonably accurate over the range of scaled distances within which they are applicable. It is exceedingly important to know how much confidence can be placed in these equations and how closely a mathematical model of such simplicity can be expected to approximate actual phenomena.

In Appendix E it is shown that the energy per unit area which a shock wave can impart to a target is a function of both the mass per unit area and the total positive impulse of the target but is not dependent on the shape of the pressure-time curve. Further corroboration of this fact was obtained by repeating the analytical processes of Appendix C (which were based on an exponential pressure-time function) for two other pressure-time functions of the form

$$p = P\left[1 - \left(\frac{t}{T}\right)^n\right]$$

where $n = 1$ for the first function and $n < 1$ for the second. Using these functions, new and different expressions for I in terms of ϵ were derived, and new sets of parametric equations for R and W were then obtained by the techniques of Appendix G. Each set of parametric equations constituted a valid and self-consistent mathematical model as was shown when the results were plotted in the form of a universal graph of distance-weight relationships by the techniques of Appendix G. All three different pressure-time functions produced the same curve of $\log F_R(\epsilon)$ vs $\log F_W(\epsilon)$, although the specific ϵ points fell at different places along the curve.

The fact that there is only one general R versus W curve could have been anticipated from the nature of the derivation in Appendix G. This further corroboration shows that any self-consistent set of R-W equations will lead to identical R-W relationships when the target parameters used are consistent with the pressure-time function from which they are derived. It is desirable to use a pressure-time function which closely approximates physical reality even though R-W relationships could be computed with equal accuracy by any self-consistent set of equations.

Each pressure-time function produces a different set of functions for I in terms of ϵ and the values of ϵ fall at different points along the universal R-W curve for each pressure-time function. Therefore, the general P-I curve described in Appendix D is not the same for all pressure-time functions, but depends strongly on the shape of the pressure-time curve. The essential difference between the results of using different pressure-time functions lies in the relationship between I and PT. For the exponential pressure-time curve:

$$I = \frac{PT}{e} = 0.3679\ PT \qquad (I\text{-}1)$$

and for the linear pressure-time curve:

$$I = \frac{PT}{2} = 0.50\ PT \qquad (I\text{-}2)$$

For the third pressure-time curve:

$$I = PT\left(\frac{n}{n+1}\right) \qquad (I\text{-}3)$$

which for various values of n becomes:

n =	1.0	0.9	0.8	0.7	0.6	0.5
I/PT =	0.5000	0.4737	0.4444	0.4118	0.3750	0.3333

A certain value of I must be achieved for the destruction of a particular target, but the associated value of P obviously is dependent on the ratio of I to PT which, in turn, is dependent on the shape of the pressure-time curve. Specific values of ϵ fall at different points on the universal R-W curve, and hence the pressure-time function determines which value of ϵ will correspond to a specific set of R-W values. Since ϵ is the ratio of the target parameter, P_m, to the peak overpressure, P, of the shock wave, the value of P_m, which is consistent with a given set of R-W equations, is dependent on the ratio of I to PT for the pressure-time function from which they are derived. If the chosen pressure-time function accurately represents the ratio I/PT, the corresponding P_m values will approximate the actual minimum unit pressures which they nominally purport to represent. Otherwise they will merely be numbers which are consistent with a specific set of R-W equations.

The exponential function upon which these derivations have been based closely approximates the physical reality within the range of scaled distances of major interest. Experimental data for 50/50 Pentolite (Ref. 3) indicate that the ratio of I/PT is weakly dependent on the scaled distance, Z. Furthermore, the average of the ratio I/PT for scaled distances of the order of 5 to 8 is about 1/e (the ratio associated with the exponential function), i.e., the exponential function provides a good fit to the experimental data. Scaling equations based on the exponential function are as close to actual physical reality as any simple analytical function can be. They not only establish valid R-W relationships, but P_m values consistent with these equations are closest to the actual values of the minimum unit pressures which can destroy the targets.

The Effect of Non-Stationary Target Mass.--In Appendix C, the derivation of functions for total energy per unit area imparted to the target is based on the pressure-time function for a shock wave acting on a stationary surface. Actually the pressure on the target will become somewhat less than this function indicates, as soon as the target begins to move under the action of the shock wave. The decrement of pressure depends on how rapidly the target is moving away from the impinging shock wave. The mass, m, is initially accelerated by the difference between P and P_m, and the rate of acceleration decreases continually with time until it eventually becomes negative, i.e., the mass, m, finally returns to rest. Figure I-1 shows the nature of the velocity profile as a function of time.

Figure I-2 shows the effect of this velocity on the pressures actually exerted on the target surface. The dotted line shows the pressure-time curve for a stationary target. This is the value assumed by the preceding derivation which shows motion terminating at t_1 and a total work per unit area of $\epsilon\ Pt_1$.

In reality, the pressure profile is more correctly shown by the solid line and the motion terminates at t_2, where the total work per unit area is $\epsilon\ Pt_2$. This is considerably less than the work indicated in the preceding paragraph and raises a legitimate question regarding the validity, or at least the accuracy, of the derivation in Appendix C.

Re-examination of Appendix C and of the way in which E_o is defined discloses that the function for total work per unit area (whatever its analytical form may be) is simply defined as the work per unit area which can just destroy the target by moving the mass, m, a distance S_o. This work per unit area, E_o, which can just destroy the target is implicit in the value of I_m, as given by Eq. C-42, i.e.,

$$I_m = \sqrt{2E_o m}$$

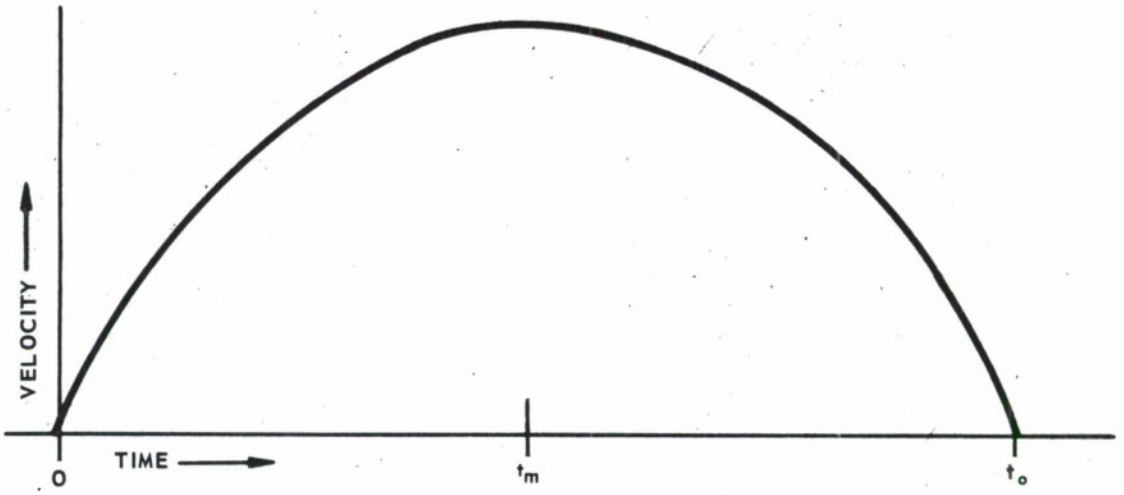


Fig. I–1 VELOCITY PROFILE OF TARGET MASS

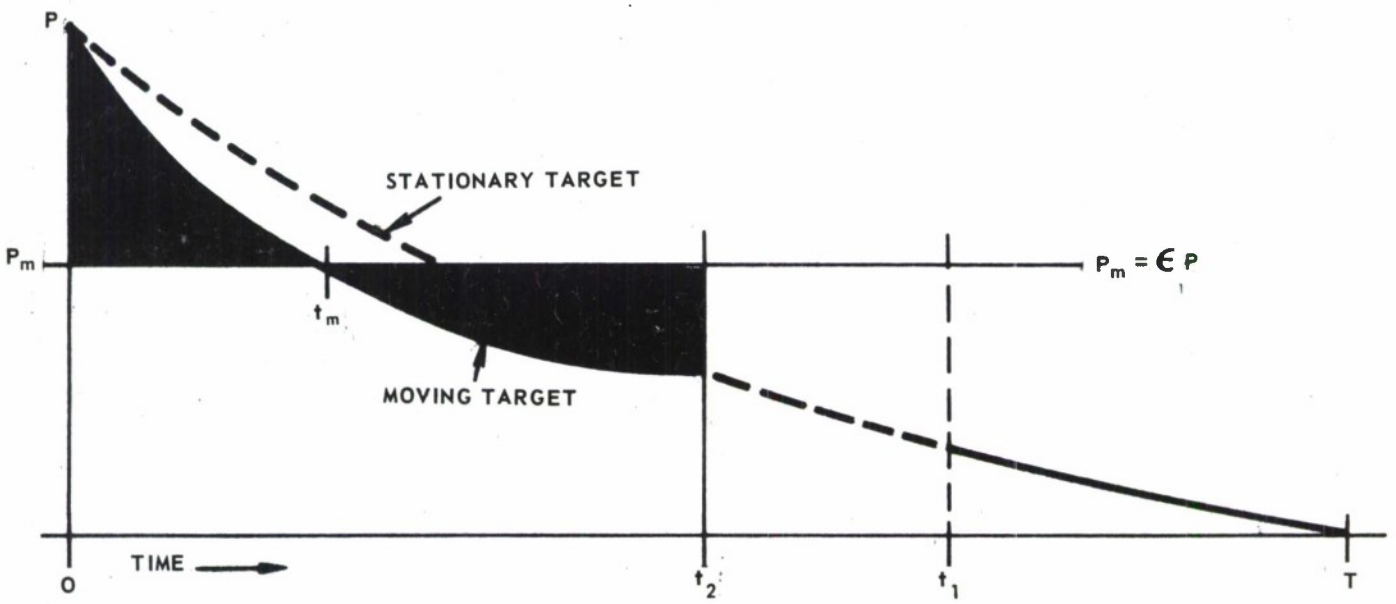


Fig. I–2 PRESSURE PROFILES FOR A STATIONARY AND A MOVING TARGET

I_m is not found by computing E_o from S_o (which obviously depends on t_2 in Fig. I-2), but is determined empirically from specific combinations of R and W which just destroy the given target. Thus, neither the value of t_2 nor its relationship to t_1 enter into the calculations.

The derivation in Appendix C merely establishes the analytical form of the function for I in terms of ϵ and I_m but is not used in evaluating I_m, since neither S_o nor t_2 are known, and furthermore nothing is known about the specific value of E_o beyond the fact that it is sufficient to destroy the target.

As long as authentic data for actual distances at which given charge weights can destroy a target are used to establish the values for both P_m and I_m, these values will be completely consistent with the scaling equations which relate them to R and W values. The methods for establishing these empirical values for P_m and I_m will be discussed in Appendix J.

The Nature of Target Response. --The general scaling equations are both valid and accurate for computing R-W relationships for targets which behave approximately like the mathematical model defined in Appendix C. Fortunately many actual targets behave sufficiently like this model that the scaling equations derived in this report can be used confidently. However, it would be a great mistake to assume that these equations may be applied to all targets. They can always be of great value in at least two ways:

1. They provide the analytical framework for correlating vast amounts of data from numerous diverse sources for targets whose response to shock waves conforms in general to the target model defined in Appendix C.

2. They can identify and isolate targets whose response characteristics are quite different from the model. In this way attention can be focused on analyzing the differences between them.

If substantial amounts of data disclose a consistent pattern of nonconformity to the scaling equations derived in this report, it is altogether possible that new target models (with different characteristics) could be defined and used to derive new scaling equations.

APPENDIX J

Determination of the Target Parameters, P_m and I_m

The numerical values for the two target characteristics, P_m and I_m, which are consistent with the scaling equations, are a necessary prerequisite to every application of these equations. This appendix will show how to determine these parameters for any specific target.

It is not easy, or even feasible to compute P_m and I_m directly in terms of their physical significance, since the pressure at which the structure will yield continuously and the energy which will produce destructive damage are not readily obtained from drawings and specifications or even from an examination of the target itself. These engineering data give rough estimates and relative orders of magnitude when comparing one target with another, but they are not the best or most accurate means of determining values which will be consistent with the scaling equations.

For any charge weight, W, the values of P_m and I_m which are truly consistent with the scaling equations must be those which give the exact lethal distance, R, in an accurately conducted set of experimental firings. Values for P_m and I_m are not ends in themselves, but are merely the analytic tools for scaling shock wave effects on a specific target. They are derived from conditions under which values of R for various charge weights can be accurately determined by experimental firings. The data can be scaled to simulate other ambient conditions and charge weights which are not amenable to experimentation.

Numerical values for P_m and I_m are thus derived analytically from experimental data which determine the lethal distances for at least two charge weights. The charge weights should differ from one another by

factors of three, four, or even five. Experiments for this purpose will doubtless be made at or near sea level and with charge weights in the range within which $\epsilon < 1/e$. In this region

$$I = \frac{I_m}{\sqrt{1 - 1.5315\epsilon}} \qquad \text{(Eq. C-44)}$$

or

$$I = \frac{I_m}{\sqrt{1 - 1.5315 \frac{P_m}{P}}}$$

from which equation

$$P_m = 0.653P\left[1 - \frac{I_m^2}{I^2}\right] \qquad \text{(J-1)}$$

and

$$I_m = I\left[1 - 1.5315 \frac{P_m}{P}\right]^{1/2} \qquad \text{(J-2)}$$

In an actual experiment the resultant distances are R_1 and R_2 for destruction by charge weights of W_1 and W_2. Since P_m and I_m in Eqs. J-1 and J-2 are the same in both tests with the same target, then it may be shown that

$$P_m = P_{m_1} = P_{m_2} = 0.653 \left[\frac{I_2^2 - I_1^2}{\frac{I_2^2}{P_2} - \frac{I_1^2}{P_1}}\right] \qquad \text{(J-3)}$$

and

$$I_m = I_{m_1} = I_{m_2} = \left[\frac{P_2 - P_1}{\frac{P_2}{I_2^2} - \frac{P_1}{I_1^2}}\right]^{1/2} \qquad \text{(J-4)}$$

For the sea level case ($p = 1$ and $c/c_o = 1$) Eqs. A-6 and A-7 become

$$P = 13,300\ W/R^3 \quad \text{and} \quad I = 220W^{0.733}/R^{1.2}.$$

These values, substituted in Eqs. J-3 and J-4, yield

$$P_m = 8685\left[\frac{\frac{W_2^{1.467}}{R_2^{2.40}} - \frac{W_1^{1.467}}{R_1^{2.40}}}{W_2^{0.467}R_2^{0.60} - W_1^{0.467}R_1^{0.60}}\right] \tag{J-5}$$

$$I_m = 220\left[\frac{\frac{W_2}{R_2^3} - \frac{W_1}{R_1^3}}{\frac{1}{W_2^{0.467}R_2^{0.60}} - \frac{1}{W_1^{0.467}R_1^{0.60}}}\right]^{1/2} \tag{J-6}$$

These equations are based on data obtained on Pentolite firings in free air where no ground reflections affect the shock wave at the target.

It should be reiterated that the prime purpose of this study was to derive a method of predicting the lethal distance, R, at which an explosive charge, W, will just destroy a target. An actual test firing is obviously the acid test of whether the scaling equations have fulfilled this purpose. Such tests must be carefully and accurately conducted to obtain two sets of R-W values either for computing P_m and I_m or for checking the results calculated for some other charge weight. Inaccurate tests will produce data which are not only meaningless but which may be actually misleading.

When W_1 is inserted into the scaling equations (with the P_m and I_m values computed from the preceding equations, and the same ambient atmospheric conditions as the test) the correct value of R_1 will emerge from the computation. Similarly, W_2 inserted in the equations with P_m, I_m and correct ambient condition terms will yield exactly R_2.

Furthermore, the determination of P_1, P_2, and P_m establishes ϵ values for the two test weights (W_1 and W_2):

$$\epsilon_1 = \frac{P_m}{P_1} \tag{J-7}$$

and

$$\epsilon_2 = \frac{P_m}{P_2} \tag{J-8}$$

For 50/50 Pentolite tested under normal sea level conditions:

$$\epsilon_1 = \frac{P_m R_1^3}{13,300 W_1} \tag{J-9}$$

and

$$\epsilon_2 = \frac{P_m R_2^3}{13,300 W_2} \tag{J-10}$$

Comments and Cautions on the Use of P_m and I_m.--The reader who is about to use the equations of these appendices to determine P_m and I_m values for targets, has reached the crucial point in his efforts to make the scaling equations and other techniques of this report into effective analytic tools for his own use.

Reference to the discussion of the scaling nomogram in Appendix H will shed further light on the vital role which P_m and I_m values play in correctly orienting the relationships between the totally general R-W curve (Fig. H-1) and specific R-W values (lower part of Fig. H-3).

It may be seen from Fig. H-2 that the slope of the general curve has a maximum value of 0.61 as ϵ approaches zero and a minimum value of exactly 1/3 as ϵ approaches 1.00. In Fig. H-4, the constant I_m lines have a slope of 0.61, and the constant P_m lines have a slope of 1/3. Figure H-1

shows that each value of ϵ has a specific place on the general curve and from Fig. H-2 (and Table IV) it may be seen that the slope of the general curve is different for each value of ϵ, decreasing as ϵ increases.

When the general curve is laid over the scaled portion of the nomogram (Fig. H-3) it is positioned by the numerical values of three parameters: altitude (see Fig. H-6), P_m and I_m (see Fig. H-5).

1. Increasing the altitude displaces the general curve, upward and to the right along a line of slope 1/3, thus <u>decreasing</u> R for a given W at all points except for large values of ϵ where the general curve approaches the slope of 1/3 and W is in the megaton range.
2. In the same way, increasing I_m without changing P_m, displaces the general curve upward and to the right along a line of slope 1/3, with the same effects noted above.
3. Increasing P_m, without changing I_m, displaces the general curve downward and to the left, and decreases the value of R for a given W, at all points where ϵ is greater than zero (i.e., when the charge weight is significantly greater than zero).

The change in slope of the general curve is small compared to changes in W. For example, when W is rather small ($\epsilon = 0.15$), $\varphi = 0.536$; when W is increased to five times the original value ($\epsilon = 0.33$), $\varphi = 0.455$; i.e., φ has changed -15 percent for a 500 percent change in W. The slope of the general curve differs from 1/3 (the slope of the line along which changes in I_m displace the general curve) by 0.203 at $\epsilon = 0.15$ for the smaller charge and by 0.122 at $\epsilon = 0.33$ for the larger charge. Two independent sets of R-W values, for the same target, establish a value of ϵ for each set (see (Eqs. J-7 to J-10), and also a value of φ for each R-W combination, and thus position the general curve with respect to the R and W scales on the fixed part of the nomograph.

It may be noted that experimental determinations of R_1 and R_2 determine P_m with much greater accuracy than I_m. Changes in R values change P_m values along constant I_m lines of slope 0.61 and the changes in P_m produced by a given change in R are 1.64 times greater than those in R. But the same changes in R cause I_m changes along constant P_m lines of slope 1/3, and cause I_m changes 3.0 times as great as those in R. If any small inconsistency in the measurements of R_1 and R_2 are such that the errors happen to be in opposite directions, they will cause an apparent change in the mean slope of the line between W_1 and W_2. This in turn will lead to quite erroneous I_m values and values for ϵ_1 and ϵ_2. For this reason it is imperative that careful and accurate measurements be made if one is to compute accurate values for P_m and especially for I_m. Unfortunately, there are no handbooks or tables to which one may go for P_m and I_m values.* This deficiency may be remedied if any individuals or agencies elect to apply the techniques of this report to the analysis of their own test data. Meanwhile it is suggested that the reader who wants to familiarize himself with these techniques of calculating P_m and I_m may use existing data on obsolete aircraft. The data are presented in a series of blast damage contours for military aircraft published by the Ballistic Research Laboratory, Aberdeen Proving Ground, Md. some fifteen years ago. The author is convinced that a discussion of how P_m and I_m values were computed from these contours more than a dozen years ago, is of far more value than a tabulation of such values at this point. The Aberdeen data consisted of experimentally determined

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* The reader who may have access to Ref. 8, which listed a few P_m and I_m values computed in the early fifties, is cautioned not to use them with the equations in this report, since they are intended for use with scaling equations having different numerical coefficients. The equations of Ref. 8 are on a linear pressure-time function for shock waves, rather than the more accurate exponential pressure-time curve used in the present study.

contours, about three mutually perpendicular axes, within which 90 lb (W_1) and 450 lb (W_2) charges of 50/50 Pentolite, in free air, could destroy the respective targets.

The six contours plotted from the data (three for 90 lb and three for 450 lb) in three different planes were measured with a planimeter, and were replaced for computational purposes by circles of the same areas. The projected areas of the three aspects of the target itself were similarly replaced by circles of the same areas. The mean radius at which the 90 lb charge destroyed the target (in each of the three aspects) was the difference between the radius of the equivalent 90 lb circle and the radius of the corresponding projected area circle; R_1 was simply the average of these three differences for the three aspects. R_2 was computed in similar fashion by averaging the three differences between the radius of the 450 lb circle and the radii of the corresponding projected area circles.

Experience has proven that the data presented in these contours were excellent and that these analytical techniques gave values of P_m and I_m with satisfactory accuracy.

After P_m and I_m values have been established for a variety of targets it will be noted that they are not only consistent with the scaling equations, but begin to show some degree of self-consistency. Really rugged targets will have higher P_m values than the more fragile aircraft. It will be particularly noticeable that I_m values agree well with Eq. C-42, i.e.,

$$I_m = \sqrt{2E_o m}$$

where m is the mass of material per unit area which is moved when the superficial covering of the target is displaced relative to the main structural framework.

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1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
The Johns Hopkins Univ., Applied Physics Lab. 8621 Georgia Avenue Silver Spring, Maryland 20910	Unclassified
	2b. GROUP N/A

3. REPORT TITLE

SCALING THE EFFECTS OF AIR BLAST ON TYPICAL TARGETS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report

5. AUTHOR(S) (Last name, first name, initial)

Morton, Harold S., Col.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
January 1966	95	11

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
NOw 62-0604-c b. PROJECT NO. X11 c. d.	TG 733
	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) N/A

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
N/A	Naval Weapons Support Activity, Department of the Navy

13. ABSTRACT

The interactions between shock waves, produced in air by detonation of explosives, and specific targets which they can destroy by air blast are described. A mathematical analysis is used which relates weights of explosive (or yields of nuclear devices) to the distances at which they can cause lethal damage over the entire range of blasts from a few pounds of conventional high explosive to kilotons or megatons of nuclear blast. Effects at sea level and higher altitudes are examined. In the analysis, typical targets are defined by two parameters for which specific numerical values can be established. Shock waves produced by detonation of specific explosives are similarly defined in mathematical terms which relate characteristics of the explosive to the ambient atmosphere. A dimensionless scaling parameter relating a shock wave parameter to a target parameter is the key to the scaling relationships derived.

DD FORM 1 JAN 64 1473

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