Technical Note 1965-52

M. Adler

Noise and Low-Frequency Amplifiers

21 December 1965

Prepared for the Advanced Research Projects Agency
under Electronic Systems Division Contract AF 19(628)-5167 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

BEST
AVAILABLE COPY

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This research is a part of Project Vela Uniform, which is sponsored by the U.S. Advanced Research Projects Agency of the Department of Defense; it is supported by ARPA under Air Force Contract AF 19(628)-5167 (ARPA Order 512).

90

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

NOISE AND LOW-FREQUENCY AMPLIFIERS

M. ADLER

Group 65

TECHNICAL NOTE 1965-52

21 DECEMBER 1965

LEXINGTON MASSACHUSETTS

Noise and Low-Frequency Amplifiers

M. Adler

Lincoln Laboratory, Massachusetts Institute of Technology
Lexington, Massachusetts

ABSTRACT

This paper is a study of low-noise amplification in the near direct current frequency range. The paper is a survey of the problem and no "ultimate amplifier" is presented in conclusion. The purpose of this paper is to analyze the problem and to present conclusions as to best class of amplifiers to use in this frequency range.

Two classes of amplifiers are considered. The "chopper" amplifier and the semiconductor diode parametric amplifier. It is concluded that the semiconductor diode parametric amplifier is inherently the lower noise device of the two. There are no intrinsic noise sources in this amplifier and noise arises mainly from parasitic effects which theoretically can be made arbitrarily small. The use of this type of amplifier at low frequencies marks a new and successful application of a device usually associated with microwave frequencies.

The first section of the paper is a critique of the various figures of merit used in specifying noise performance. The significance and proper use of the noise figure is discussed.

The section on the "chopper" amplifier includes analysis involving both the junction transistor and the field effect transistor. The conclusions are that the best overall performance of the chopper amplifier results when the junction transistor is used with a low impedance source.

Accepted for the Air Force
Stanley J. Wisniewski
Lt Colonel, USAF
Chief, Lincoln Laboratory Office

A. INTRODUCTION

The purpose of this paper is to present an analysis of low noise amplification of near DC signals. There are two main approaches to the problem; direct or DC amplification and amplification following or associated with up-conversion. Direct coupled amplifiers are the most straightforward, but unfortunately suffer from large amounts of unwanted thermal and 1/f noise associated with semiconductor devices. This paper will consider the more complex "chopper" amplifier and the parametric converter as solutions to these difficulties.

During the couse of the initial analysis it became convenient to consider several extensions and side topics. In conjunction with the discussion on chopper amplifiers, a comparison will be made between the junction transistor and the field effect transistor with relevance to noise performance. In addition, an attempt will be made to present the analysis of the parametric converter with sufficient detail to enable someone with little or no knowledge of the device to gain some understanding of it. Also, as an introduction to the subject, the paper will be prefixed by a general discussion on low-noise amplifiers.

1. The Noise Factor

Noise in a two port can be represented by a series noise voltage generator and a parallel noise current generator at the input of a noiseless equivalent two port.

3-65-4146

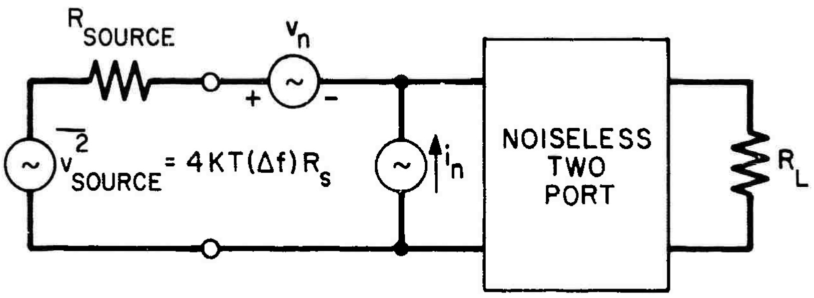


Fig. 1. Equivalent model of a noisy two port.

This noise representation is appealing because the voltage (v_n) and the current (i_n) are independent of the source resistor. As will be seen, other methods of representing noise performance can be conveniently expressed in terms of these noise generators.

The noise performance of an amplifier is commonly specified by means of the noise factor (F).

$$F = \frac{S_i/N_i}{S_o/N_o} \qquad \begin{array}{l} S_i, S_o = \text{signal power in, out} \\ N_i, N_o = \text{noise power in, out} \end{array} \quad . \tag{1}$$

The noise factor gives a measure of the deterioration of the input signal-to-noise ratio caused by the amplifier. It is thus an indicator of the amplifier's ability to act as a low-noise transducer between a source and a load.

$$F = \frac{N_o}{N_i} \times \frac{S_i}{S_o} = \frac{N_o}{N_i \cdot G} \quad ; \quad G = \text{gain of the amplifier},$$

$$F = \frac{\text{output noise to referred to input}}{\text{input noise}}$$

$$= \frac{4KT(\Delta f)R_s + \overline{(v_n + R_s i_n)^2}}{4KT(\Delta f)R_s} \; ,$$

$$F = 1 + \frac{\overline{v_n^2} + R_s^2 \; \overline{i_n^2} + 2\,\gamma\, R_s \sqrt{\overline{v_n^2}\;\overline{i_n^2}}}{4KT(\Delta f)R_s} \; ,$$

$$\gamma = \text{correlation coefficient } 0 \le \gamma \le 1 \quad . \qquad (2)$$

There is nothing sacred or terribly fundamental about the noise factor. It is simply one of the several ways of specifying noise performance. It can be a particularly useful noise criterion when used with understanding. However, like most figures of merit it does not tell the whole story.

The most important consideration in assessing noise performance is the output signal-to-noise ratio. The noise factor, however, normalizes the output signal-to-noise ratio by the input signal-to-noise ratio. When the input signal-to-noise ratio is fixed, (the source resistance is specified) the noise factor is a reasonable measure of noise performance. That is, minimizing the noise factor, maximizes the output signal-to-noise ratio. However, if the source resistor is considered a variable, the noise factor ceases to be meaningful noise criterion.

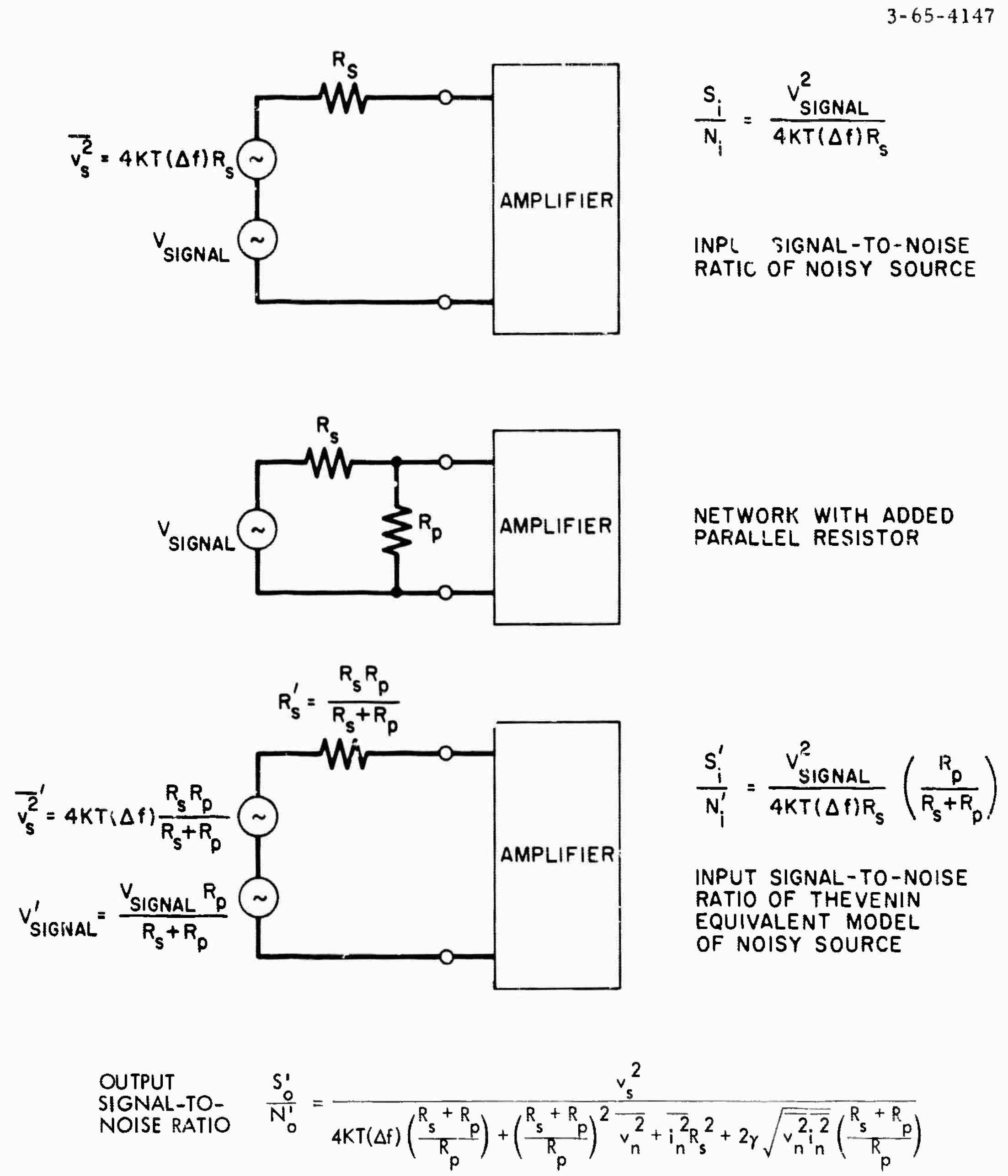


Fig. 2. Illustrating the deterioration of noise performance accompanied by adding a parallel resistor following the source.

$$\frac{S_o}{N_o} = \frac{V^2_{signal}}{4KT(\Delta f)R_s + \overline{v_n^2} + \overline{i_n^2}\,R_s^2 + 2\,\gamma\,R_s\sqrt{\overline{v_n^2}\;\overline{i_n^2}}} \; . \qquad (3)$$

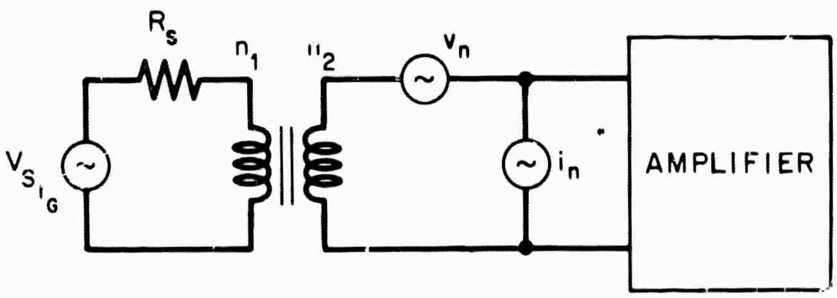
It is not hard to see that the optimum value of the output signal-to-noise ratio occurs for $R_s = 0$. However, a little checking will reveal that the noise factor is infinite. The apparent paradox results from the fact that the noise factor is only a measure of an amplifier's success as a low-noise transducer. It is not a measure of overall noise performance (amplifier and source). However, the noise factor can be very useful when matching an amplifier to a source as noted above.

The same argument applies to the problem of transferring power from a source to a load. The measure of this is the transducer gain (power out to available source power). In the case of zero source resistance, there is a maximum amount of power transferred to the load but the transducer gain is zero.

2. Lossy and Lossless Transformations

Consider adding a parallel resistor R_p between the source and amplifier as shown in Fig. 2. This transformation on the source has the undesirable affect of lowering the signal to noise ratio. Lossless transformations, however, can sometimes increase the signal-to-noise ratio.

3-65-4148



$$F = 1 + \frac{\overline{v_n^2} + R_s^2\,\overline{i_n^2} + 2\sqrt{\overline{v_n^2}\;\overline{i_n^2}}\;\gamma R_s}{4KT(\Delta f)\,R_s}$$

prior to adding the transformer. Now choose

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}\left(\frac{n_1}{n_2}\right)^2 = R_s$$

to minimize the noise factor

after adding the transformer

$$\left(\frac{S_o}{N_o}\right) = \frac{V_s^2\left(\frac{n_1}{n_2}\right)^2}{4KT(\Delta f)\,R_s\left(\frac{n_2}{n_1}\right)^2 + 2v_n^2(1+\gamma)}$$

$$F = 1 + 2\,\frac{\overline{v_n^2}(1+\gamma)\left(\frac{n_1}{n_2}\right)^2}{4KT(\Delta f)\,R_s} = 1 + 2\,\frac{\sqrt{\overline{v_n^2}\;\overline{i_n^2}}\,(1+\gamma)}{4KT(\Delta f)}$$

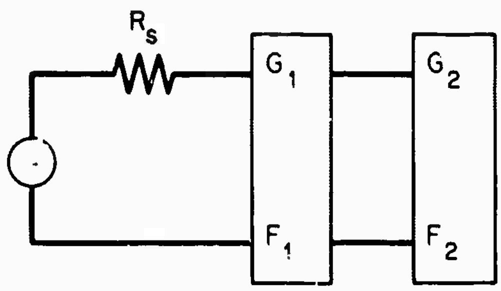
Fig. 3. Illustrating the improvement in noise performance accompanied by matching the source to the amplifier with a lossless transformer.

When given the problem of matching an amplifier to a source the best amplifier will have $\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = R_s$ (as is easily derivable from the noise factor expression in Eq. (2)). Consider an amplifier where $\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \neq R_s$. A transformer can theoretically be used to match the amplifier to the source. Since the transformer is lossless (ideally), the input signal-to-noise ratio stays constant throughout the transformation.

The process of adding the compensating transformer is illustrated in Fig. 3. The noise factor obtained after adding the transformer is, in fact, the lowest possible and corresponds to a perfect match of the amplifier and source. This optimum noise factor could also be obtained by adding resistors between the source and amplifier to "match the source to the amplifier." As was seen before, however, this will cause the output signal-to-noise ratio to increase rather than decrease.

In certain instances (core loss and poor frequency response) transformers may be impractical and means other than direct source transformation may have to be invoked to the achieve minimum noise figure. Possible alternatives are varying (v_n) and (i_n) through

3-65-4149



$G_{1,2}$ = available gain of amplifier 1, 2

$F_{1,2}$ = noise figure of amplifier 1, 2

$$N_2 = G_1 G_2 F_1 KT(\Delta f) + G_2(F_2 - 1)\, KT(\Delta f)$$

= available noise output power

$$N_{eq} = F_1 KT(\Delta f) + \frac{F_2 - 1}{G_1}\, KT(\Delta f) = \frac{N_2}{G_1 G_2}$$

= available noise output power referred to input of amplifier 1

$$F_{12} = \frac{N_{eq}}{KT(\Delta f)} = F_1 + \frac{F_2 - 1}{G_1}$$

= noise factor of cascade

Fig. 4. Noise factor of two cascaded amplifiers.

‡Available power gain is defined as the ratio of the power output if the output of the amplifier were matched to the available power of the source.

design changes in the amplifier and/or actually changing the type of amplifier to suit the source (using field effect transistors rather than junction devices for example).

The size of the signals the amplifier handles is another consideration in noise performance. The output signal-to-noise ratio will obviously be affected by the output signal range and the closeness the average signal is kept near the limits of this range. This is the issue that sets the upper limits on the amplifier's dynamic range.

3. Cascaded Amplifiers and Noise Measure

The problem of noise in a multistage amplifier is usually simplified by the fact that if the first stage has sufficient gain, the total noise figure of the amplifier will be essentially that of the first stage. (See Figure 4.)

An interesting question arises as to which amplifier to put first in a cascade assuming the gain of the circuit to be the same regardless of the order of placement. A reasonable criteria might be that the amplifiers be arranged such that the cascade noise factor F_{ij} is < F_{ji}.

For the example

$$F_1 + \frac{F_2-1}{G_1} < F_2 + \frac{F_1-1}{G_2} ,$$

$$F_1 - F_2 < \frac{F_1-1}{G_2} - \frac{F_2-1}{G_1} ,$$

(continued)

$$(F_1-1) - (F_2-1) < \frac{F_1-1}{G_2} - \frac{F_2-1}{G_1} \quad ,$$

$$[F_1-1]\,[1 - \frac{1}{G_2}] < [F_2-1]\,[1 - \frac{1}{G_1}] \; ,$$

$$\frac{F_1-1}{1 - \frac{1}{G_1}} < \frac{F_2-1}{1 - \frac{1}{G_2}} \quad . \tag{4}$$

These expressions have been called the "noise measure" (M) of the associated amplifier[1]. The implication is that for a cascaded system of amplifiers, when the earliest stages are obviously most critical, the best amplifier is the one with the lowest noise measure rather than noise factor. Furthermore, when operating under the premise that amplifiers are supposed to provide gain building blocks without adding excessively to the noise of the system, the noise measure is a more satisfactory criteria. Since noise measure and noise factor are essentially the same for amplifiers with high gain, the final performance evaluation of a practical multistage amplifier always rests numerically upon the familiar noise factor criterion.

In the following analysis, the problem of designing low noise multistage amplifiers will be reduced to that of designing a low noise first stage in light of the above comments.

B. CHOPPER AMPLIFIER

As mentioned earlier, direct current amplifiers are generally unsatisfactory due to large amounts of thermal and semiconductor 1/f noise in the pass band of the low frequency signal. In other words, the noise and the signal look the same. The choppper amplifier is a practical solution to this problem. By multiplying the signal by a high frequency* square wave, the signal is up-converted or modulated out of the region of the thermal and 1/f noise effects. The chopped signal is then amplified and demodulated with the overall result of a low-noise amplified version of the input signal.

It is relatively simple to build low-noise transistor amplifiers above 1 kc since the power density spectrum of transistor noise is essentially flat. I will take the view that no noise is contributed by the chopping device and that all of the noise is essentially due to the first stage.†

---

*The square wave must be at least twice as high in frequency as the highest signal frequency.

†This approximation can be a very bad one especially if semiconductor devices are used for chopping. I expect to show, however, that the parametric converter has a lower noise figure than even the idealized chopper amplifier. The approximation serves to limit the scope of the work to the primary issue of amplifiers.

1. A Junction Transistor First Stage

3-65-4150

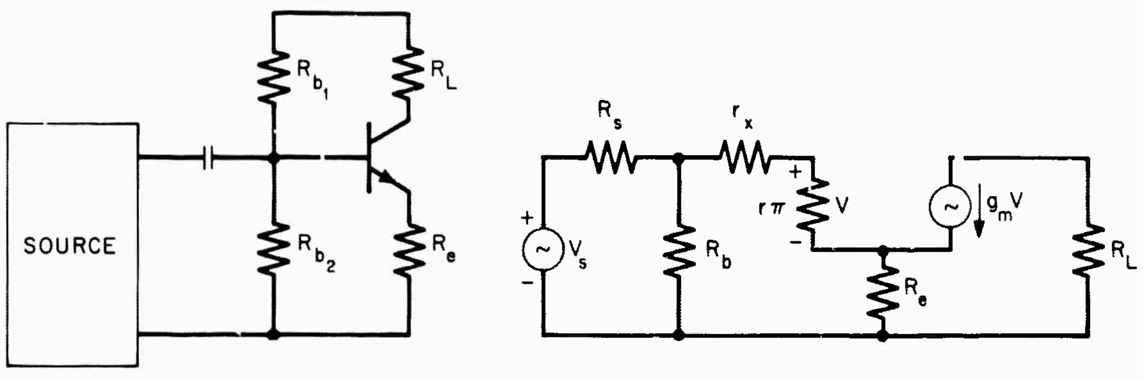


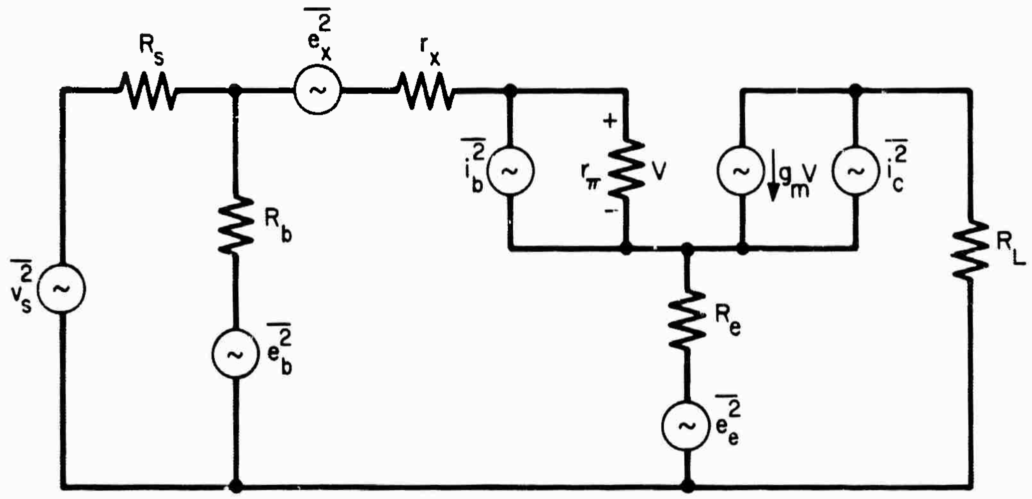
Fig. 5.

The small signal model for the junction transistor is the low frequency* hybrid π model. The emitter resistor is included for completeness sake and will, in fact, be removed in the final analysis.

Noise in the junction transistor comes from that noise caused by the base and collector currents and thermal noise due to the base resistance. The biasing resistors (R_b) and the emitter resistor also contribute thermal noise. The noise due to the load is not considered for purposes of calculating the noise performance of an amplifier. In a multistage amplifier, this noise will be reflected in the noise figure of the second stage.

*f ≤ 25 kc defines "low frequency" in this context.

3-65-4151



Noise Model of Transistor Amplifier

$$\left.\begin{aligned}\overline{i_b^2} &= 2q(\Delta f)\, I_b \\ \overline{i_c^2} &= 2q(\Delta f)\, I_c\end{aligned}\right\}\ \text{Schottky's theorem for shot noise}$$

$$\left.\begin{aligned}\overline{e_e^2} &= 4KT(\Delta f)\, R_e \\ \overline{e_b^2} &= 4KT(\Delta f)\, R_b \\ \overline{e_x^2} &= 4KT(\Delta f)\, r_x\end{aligned}\right\}\ \text{Nyquist's theorem for thermol noise}$$

Fig. 6. Noise model for junction transistor amplifier.

Each of the noise sources in the model can be regarded as independent. Hence, we can quadratically add their effects at the output and refer them to the input as an equivalent noise voltage generator in series with the source resistance.

Voltage gain of amplifier

$$\frac{V_o}{V_{in}} = \frac{R_b}{R_s+R_b} \times \left[\frac{g_m r_\pi R_L}{(R_s//R_b)+r_x+r_\pi+g_m r_\pi R_e}\right]$$

output noise voltage

$$\overline{v_o^2} = \overline{i_c^2}\, R_L^2 + (\overline{e_e^2} + \overline{e_x^2})\left[\frac{g_m r_\pi R_L}{(R_s//R_b+r_x+r_\pi+g_m r_\pi R_e}\right]^2$$

$$+ \overline{i_b^2}\left[\frac{(R_s//R_b)+r_x+g_m r_\pi R_e \;\; g_m r_\pi R_L}{(R_b//R_s)+r_\pi+r_x+g_m r_\pi R_e}\right]^2$$

$$+ \overline{e_b^2}\left[\left(\frac{R_s}{R_s+R_b}\right)\frac{g_m r_\pi R_L}{(R_b//R_s)+r_\pi+r_x+g_m r_\pi R_e}\right]^2 \quad . \qquad (5)$$

Output noise voltage referred to the input

$$\overline{v_{eq}^2} = \overline{i_c^2}\left(\frac{(R_s//R_b)+r_x+r_\pi+g_m r_\pi R_e}{g_m r_\pi}\right)^2 \left(\frac{R_s+R_b}{R_b}\right)^2$$

$$+ \overline{e_b^2}\left(\frac{R_s}{R_b}\right)^2 + (\overline{e_e^2} + \overline{e_x^2})\left(\frac{R_s+R_s}{R_b}\right)^2$$

$$+ \overline{i_b^2}\,[R_s//R_b+r_x+g_m r_\pi R_e]^2 \left(\frac{R_s+R_b}{R_b}\right)^2$$

$$g_m r_\pi = B = \text{current gain.} \qquad (6)$$

It is quite apparent that the emitter resistor not only lowers gain, but greatly reduces noise performance. For R_e sufficiently large $(g_m r_\pi R_e >> (R_s//R_b)+r_x+r_\pi)$ it will be the limiting factor in both the noise voltage and the voltage gain.

Setting $R_e = 0$ by removing or bypassing it and recognizing that $r_\pi >> r_x$ greatly simplifies the noise voltage expression.

$$\overline{v_{eq}^2} = 4KT(\Delta f)\left[\frac{R_s^2}{R_b} + r_x + \frac{R_s}{B}\right]$$

$$+ 2q(\Delta f)\frac{I_c}{B}\left[R_s^2 + 2R_s r_x + r_x^2\right]$$

$$+ 2\frac{(KT)^2}{qI_c}\left[\frac{R_s^2}{R_b} + 2\frac{R_s}{R_b} + 1\right] \quad . \tag{7}$$

This equation can be separated into terms dependent on R_s^2, R_s and R_s^0. The coefficients of the R_s^2 and R_s^0 terms are the parallel noise current source and the series noise voltage source. The coefficient of the R_s term is the average of the product of the noise voltage and noise current.

$$\overline{v_n^2} = 4KT(\Delta f)r_x + 2q(\Delta f)\frac{I_c}{B}r_x^2 + 2\frac{(KT)^2}{qI_c}$$

$$\overline{i_n^2} = 4KT\frac{(\Delta f)}{R_b} + 2q(\Delta f)\frac{I_c}{B} + 2\frac{(KT)^2}{qI_c}\frac{1}{R_b^2}$$

$$\overline{v_n i_n} = 2KT\frac{(\Delta f)}{B} + 2q(\Delta f)\frac{I_c r_x}{B} + 2\frac{(KT)^2}{qI_c R_b}$$

$$\overline{v_{eq}^2} = \overline{v_n^2} + R_s^2\,\overline{i_n^2} + 2\,\overline{v_n i_n}\,R_s \quad . \tag{8}$$

The previous equations and the voltage gain provide criteria sufficient to design a low noise junction transistor amplifier. It is immediately apparent that a high current gain (B) and low base resistance (r_x) are necessary for a low noise junction transistor. Also, the base biasing resistor R_b should be as large as bias stability allows. There is not much else that can be said without knowledge of the source resistor and values for the constants and transistors parameters appearing in the noise voltage equation.

Taking values from the 2N930 for r_x and B and typical values for the circuit resistors:

$$r_x = 25\Omega \qquad R_e = 10^4$$
$$R_b = 100K \qquad g_m = \frac{qI_c}{KT}$$
$$B = g_m r_\pi = 400 \qquad r_\pi = \frac{10}{I_c} \tag{9}$$

<u>For $I_c = 10$ ma</u>

$$\frac{\overline{v_{eq}^2}}{(\Delta f)} = 8.2 \times 10^{-24} R_s^2 + 4 \times 10^{-22} R_s + 2.58 \times 10^{-20}$$

$$F = 1 + 4.9 \times 10^{-4} R_s + \frac{1.55}{R_s}$$

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = 56 \ \Omega$$

$$\frac{V_o}{V_s} = \left(\frac{990}{990+R_s}\right) 4000 \tag{10}$$

For $I_c = 1$ ma

$$\frac{\overline{v_{eq}^2}}{(\Delta f)} = 9.66 \times 10^{-25} R_s^2 + 8.65 \times 10^{-23} R_s + 6.25 \times 10^{-19}$$

$$F = 1 + 5.8 \times 10^{-5} R_s + \frac{3.8}{R_s}$$

$$\frac{V_o}{V_s} = \left(\frac{9.1K}{9.1K+R_s}\right) 400$$

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = 8000\ \Omega \qquad (11)$$

For $I_c = 0.1$ ma

$$\overline{v_{eq}^2} = 2.46 \times 10^{-25} R_s^2 + 8.55 \times 10^{-23} R_s + 2.5 \times 10^{-18}$$

$$F = 1 + 1.48 \times 10^{-3} R_s + \frac{250}{R_s}$$

$$\frac{V_o}{V_s} = \left(\frac{50K}{50KTR_s}\right) 40$$

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = 3.3\ K \qquad (12)$$

As can be seen, the collector current can be varied to make the amplifier compatible with a range of source resistances. However, the constraints of the device together with the voltage gain limit the extent that the junction transistor can be used effectively with high source resistance.

2. A Field Effect Transistor First Stage

3-65-4152

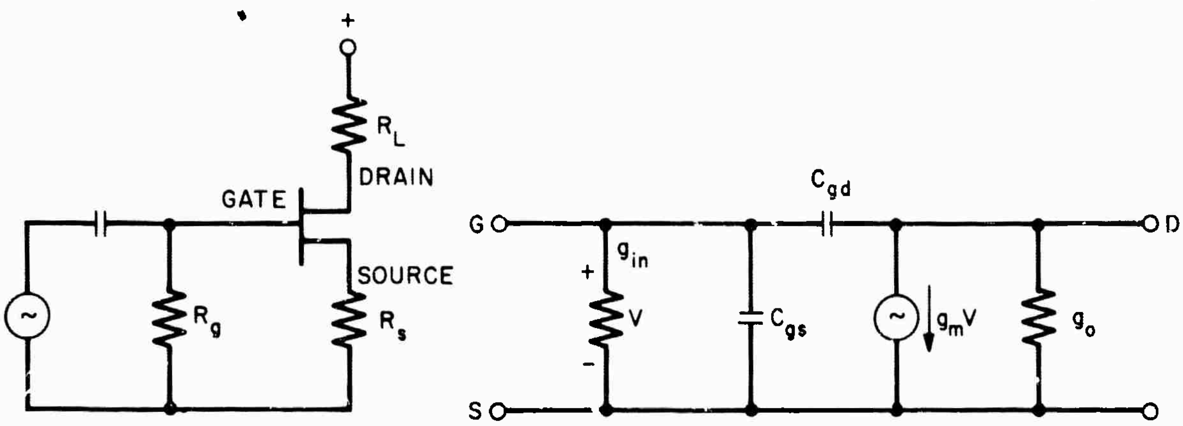


Fig. 7.

The junction field effect transistor is characterized by such a high input resistance that at frequencies above 1/10 cps the input impedance is essentially capacitive.

The small signal model can be simplified using the Miller approximation for handling the drain-to-gate capacitance. The result is an unilaterial model with an effective input capacitance $C_t = C_{gs} + C_{gd} (g_m(R_e+R_L) + 1)$.

3-65-4153

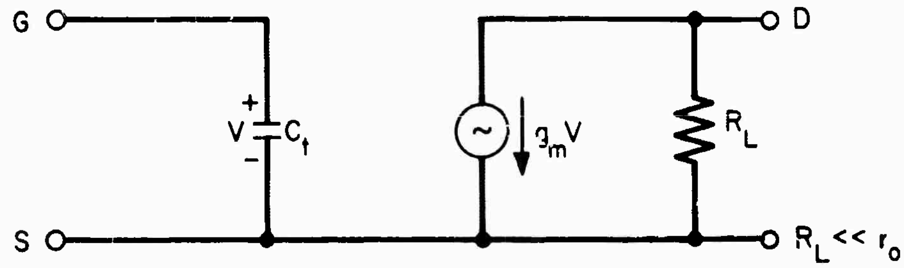


Fig. 8. Simplification of F.E.T. transistor small signal model using Miller approximation.

Since the field effect transistor does not depend on carriers injected across a potential barrier for its operation (as in a junction transistor), there is no shot noise to speak of. The only shot noise is a parasitic effect due to the very small amount of gate leakage current. The principle noise source is the thermal noise in the drain-to-source channel. This can be expressed in the current by a current source $\overline{i_d^2} = 4KT(\Delta f)g$ (Ref. 2) in parallel with the output current generator.

3-65-4154

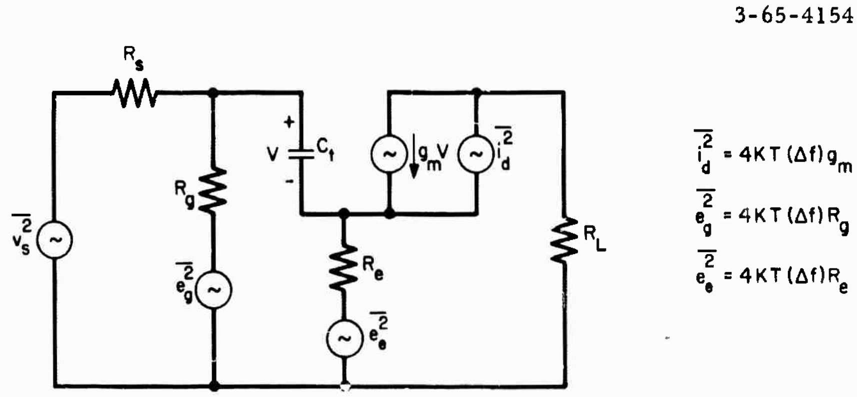


Fig. 9. Noise model for F.E.T. amplifier.

Again the "emitter" resistor is included in the analysis for completeness. The noise model is probably a bit simplified but it will serve to get a ballpark estimate of the noise performance of the field effect transistor.

Each of the noise sources is statistically independent and hence their effects can be added separately at the output.

Input Impedance of Amplifier

$$\frac{V_{in}}{I_{in}} = \frac{R_s + R_g + \dfrac{R_eR_gC_ts}{g_mR_e+1}}{1 + \dfrac{(R_e+R_g)C_ts}{g_mR_e+1}}$$

$$= R_s+R_g \quad \text{if} \quad R_e \ll \frac{1}{C_TW}$$

and

$$R_e+R_g \ll \frac{1}{C_TW} \quad . \qquad (13)$$

Voltage Gain of Amplifier

$$\frac{V_o}{V_{in}} = \frac{R_d}{R_s+R_g}\left(\frac{g_mR_L}{g_mR_e+1}\right)$$

Output Noise Voltage

$$\overline{v_o^2} = \overline{i_d^2}\, R_L^2 + \left(\frac{g_mR_L}{g_mR_e+1}\right)^2 \overline{e_e^2} + \overline{e_g^2}\left(\frac{R_s}{R_g+R_s}\right)^2\left(\frac{g_mR_L}{g_mR_e+1}\right)^2$$

Equivalent Input Noise Voltage

$$\overline{v_{eq}^2} = \overline{i_d^2}\left(\frac{g_mR_e+1}{g_m}\right)^2\left(\frac{R_s+R_g}{R_s}\right)^2 + \overline{e_e^2}\left(\frac{R_s+R_g}{R_g}\right) + \overline{e_g^2}\left(\frac{R_s}{R_g}\right)^2 \quad . \qquad (14)$$

Again the "emitter" resistor has a pronounced effect on the noise performance of the amplifier. It is slightly less important in the F.E.T. than in the junction transistor since the transconductance (g_m) for the field effect transistor is typically much less than that for the junction transistor. However, it is still undesirable and should be eliminated either by removing or bypassing it with a capacitor. With $R_e = 0$ the noise voltage expression simplifies to:

$$\overline{v_{eq}^{2}} = 4KT(\Delta f)g_m \left(\frac{R_s+R_g}{g_m R_g}\right)^2 + 4KT(\Delta f)R_g\left(\frac{R_s}{R_g}\right)^2 \quad . \qquad (15)$$

$$g_m = g_{m_o}\left[1 - \left(\frac{V_g}{V_p}\right)^{2/3}\right] \quad V_p = \text{pinch off voltage}$$

$$V_g = \text{gate voltage} \quad . \qquad (16)$$

From these expressions, it is evident that low-noise high-gain operation results when g_m is large or equivalently the gate voltage (V_g) is biased near zero volts. The gate biasing resistor R_g should be as large as possible.

Let

$$g_m = 10^{-3} \text{ mhos}$$

$$R_g = 10^6 \ \Omega$$

$$R_L = 10^4 \ \Omega \qquad (17)$$

$$\overline{v_{eq}^2} = 1.66 \times 10^{-26} R_s^2 + 3.32 \times 10^{-23} R_s + 1.66 \times 10^{-17}$$

$$\frac{\overline{i_n^2}}{(\Delta f)} = 1.66 \times 10^{-26} \; ; \; \frac{\overline{v_n^2}}{(\Delta f)} = 1.66 \times 10^{-17}$$

$$\overline{v_n i_n} = 1.66 \times 16^{-23}$$

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = 3.3 \times 10^4 \; \Omega \quad . \tag{18}$$

$$F = 1 + 10^{-6} R_s + \frac{10^3}{R_s}$$

$$\frac{V_o}{V_1} = \left(\frac{10^6}{10^6 + R_s}\right)^{10} \quad . \tag{19}$$

<u>For $R_g = 10^7 \; \Omega$</u>

$$F = 1 + 10^{-7} R_s + \frac{10^3}{R_s}$$

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = 10^5 \; \Omega \quad . \tag{20}$$

The conclusions are inescapable. The field effect transistor has very little current noise and hence is ideal for applications where the source resistance is large. The junction device is inherently a higher gain device than the F.E.T. but after the first 20 db of gain, the issue is essentially that of noise figure. Therefore, at high source impedances, the field effect transistor will make a better first stage than the junction transistor while at low source resistance, the junction transistor is better. The best overall noise performance in terms of the highest output signal-to-noise ratio results when a junction transistor is used at low source resistance.

C. PARAMETRIC CONVERTER

Any amplifier can be considered to be a modulator in that the input signal causes variations in the energy flowing from the amplifier's energy source. If the amplifier's source is direct, as in the case of a transistor amplifier, the output is ideally a replica of the input. Depending on the nature of the amplifier, it may exhibit gain or loss. On the other hand, if the energy source is not direct, but is an alternator, referred to as a local oscillator, the output is not a replica of the input, but occurs at a different frequency than the input, and consequently, the device is called an up-converter, down-converter or mixer. This shift in frequency occurs because signals of two different frequencies, other than zero, are applied to a non-linear element. These frequencies are sum and difference combinations of the applied frequencies. Again some of these devices exhibit a gain or a loss depending on their nature.

A non-linear reactor can be used to effect gain producing up-conversion. Consider a non-linear capacitor that is pumped from an alternating source. The non-linear capacitor can be imagined as replaced by a linear time varying capacitance. This in turn can be replaced, at least conceptually, by a linear capacitor with movable electrodes.

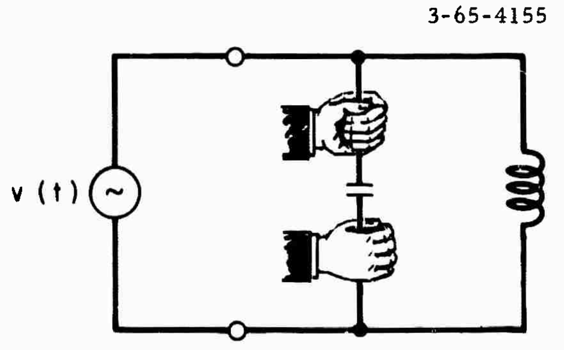


Fig. 10. Pertaining to the discussion of a reactance converter.

Imagine that it is possible to pull the capacitor plates apart and push them together at will. Suppose each time the applied voltage goes through a positive or negative movement, the capacitor plates are suddently pulled apart. Work is done in separating the charge on the two plates. The energy goes into the electric fields existing across the plates. The capacitance is reduced and since $V = \frac{Q}{C}$ the voltage is amplified. Each time the voltage goes through zero, the plates are suddently pushed back together again. When the plates are pushed back together, there is no charge on the capacitor and no work is done. The net result is amplification of the voltage across the capacitor, and the flow of energy being from whatever pumps the plates into the fields of the resonant tank.[3]

A reactor circuit usually contains several tuned resonant tanks.

3-65-4156

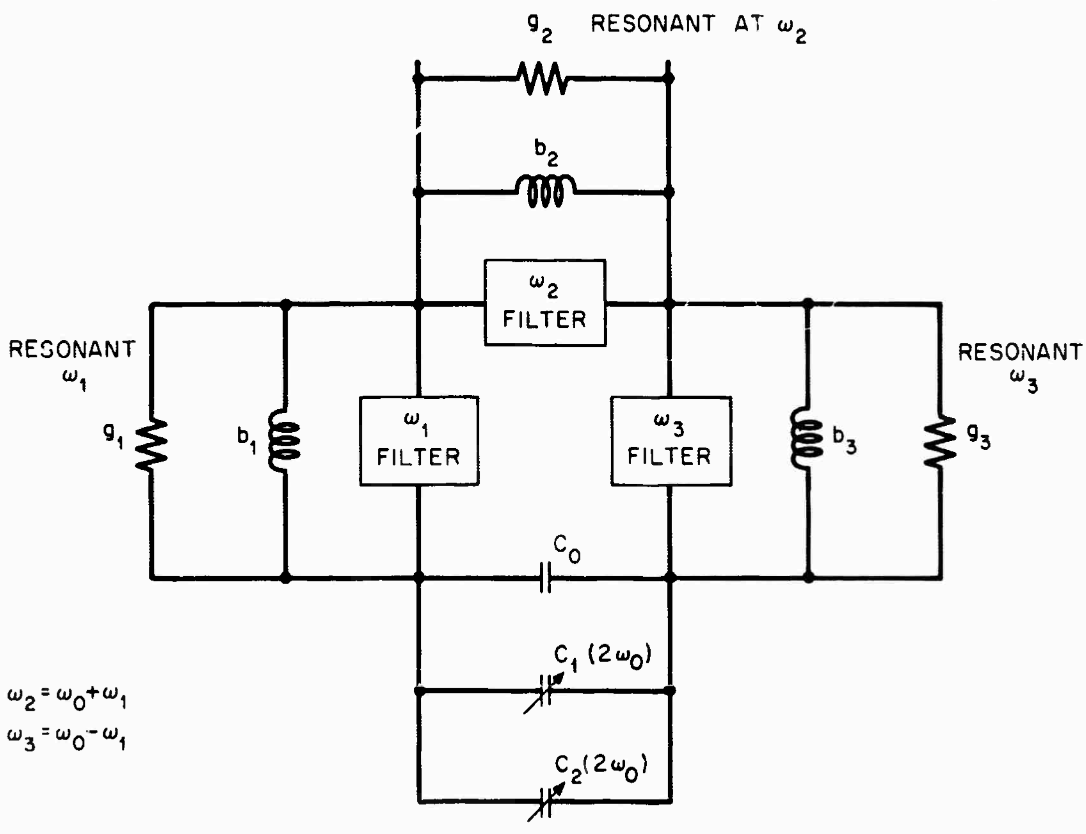


Fig. 11. Four frequency reactance converters.

There is no phase restriction on the tank circuit that existed in the one tank circuit. The variable capacitor serves to couple the three resonant tanks together. If a signal voltage exists across one of the tanks at its resonant frequency, a voltage will develop in the other tanks at their resonant frequency due to the mixing action of the variable capacitor.

In general there are an infinite number of frequencies that are excited in a pumped reactor circuit. As mentioned earlier, they correspond to the various combinations of the pump and signal frequency ($n\omega_0+n\omega_1$). Manley and Rowe[4] have derived two independent expressions that relate the time average power flow at different frequencies in a non-linear reactor.

$$\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\frac{m\,W_{mn}}{m\omega_1+n\omega_0}=0$$

$$\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\frac{n\,W_{mn}}{m\omega_1+n\omega_0}=0 \qquad (21)$$

W_{mn} is the time average power into the device at angular frequency $m\omega_1+n\omega_0$.

The relations reduce to an identity for a linear reactor since time average power can then exist at only one frequency. The relations are independent of the detailed slope of the non-linear characteristics and the external circuit to which the device is connected. The only restriction is that the non-linear characteristics be single valued. The relations are quite useful in considering the gain and stability of non-linear reactor circuits.

In practical circuits, power is allowed to exist at only a few frequencies by filtering out most of the frequency components. If the circuit is constrained to have only four frequencies as shown above, the Manley-Rowe relations become quite simple and open to interpretation.

$$\frac{W_{10}}{\omega_1} + \frac{W_{11}}{\omega_0+\omega_1} - \frac{W_{1-1}}{\omega_0-\omega_1} = 0$$

$$\text{or} \quad \frac{W_{10}}{\omega_1} = \left(\frac{-W_{11}}{\omega_0+\omega_1}\right) - \left(\frac{-W_{1-1}}{\omega_0-\omega_1}\right)$$

$$\frac{W_{11}}{\omega_0} + \frac{W_{11}}{\omega_0+\omega_1} + \frac{W_{1-1}}{\omega_0-\omega_1} = 0$$

$$\text{or} \quad \frac{W_{01}}{\omega_0} = \left(\frac{-W_{11}}{\omega_0+\omega_1}\right) + \left(\frac{-W_{1-1}}{\omega_0-\omega_1}\right) \quad . \tag{22}$$

$(-W_{mn})$ = power out of reactor.

W_{10} = time average signal power input.

W_{01} = time average pump power input.

The first equation states that the power delivered to the load at the sum frequency reflects a positive resistance or loading effect at the signal frequency while the power delivered to a load at the difference frequency reflects a negative resistance or regenerative action at signal frequency. The second equation states that power delivered to the load at both the sum and difference frequency reflects a load at the pump frequency.

$$G = \text{power gain} = \frac{(-W_{11})+(-(W_{1-1}))}{W_{10}}$$

$$= \frac{\omega_0+\omega_1}{\omega_1} \left(\frac{1 + \frac{W_{1-1}}{W_{11}}}{1 - \frac{\omega_0+\omega_1}{\omega_0-\omega_1} \frac{W_{1-1}}{W_{11}}} \right) \quad . \tag{23}$$

As can be seen, a possibility of infinite power gain occurs if $\frac{W_{11}}{\omega_0+\omega_1} = \frac{W_{1-1}}{\omega_0-\omega_1}$. This condition is actually one of potential instability. It is interesting to note that the previous conclusions were made without considering a specific circuit configuration. This illustrates the utility of the Manley-Rowe equations.

Non-linear capacitance amplifiers and converters have been used primarily at frequencies well into the microwave region. However, they have an important and largely overlooked application at frequencies near dc. With the development of high capacitance varactor diodes, it is feasible and practical to build low frequency reactance converters.

The advantages over conventional chopper schemes for low-noise dc amplication are numerous. Because the basic transducing element is a capacitance there is no inherent noise source. Noise comes primarily from parasitic sources such as inductor loss and series diode resistance and can be made quite low by careful choice of components. Furthermore, amplification is intimately associated with the up-conversion and consequently only one stage is needed for both conversion and amplification.

The low frequency reactance converter has to be at least a four frequency circuit. The sum and difference sidebands are essentially the same frequency for a high pump frequency (~455kc). I will consider a low frequency, four frequency reactance converter and develop expressions for voltage gain, power gain, input impedance and noise figure.

The circuit to be considered is essentially the Texas Instrument RA-5 reactance converter.

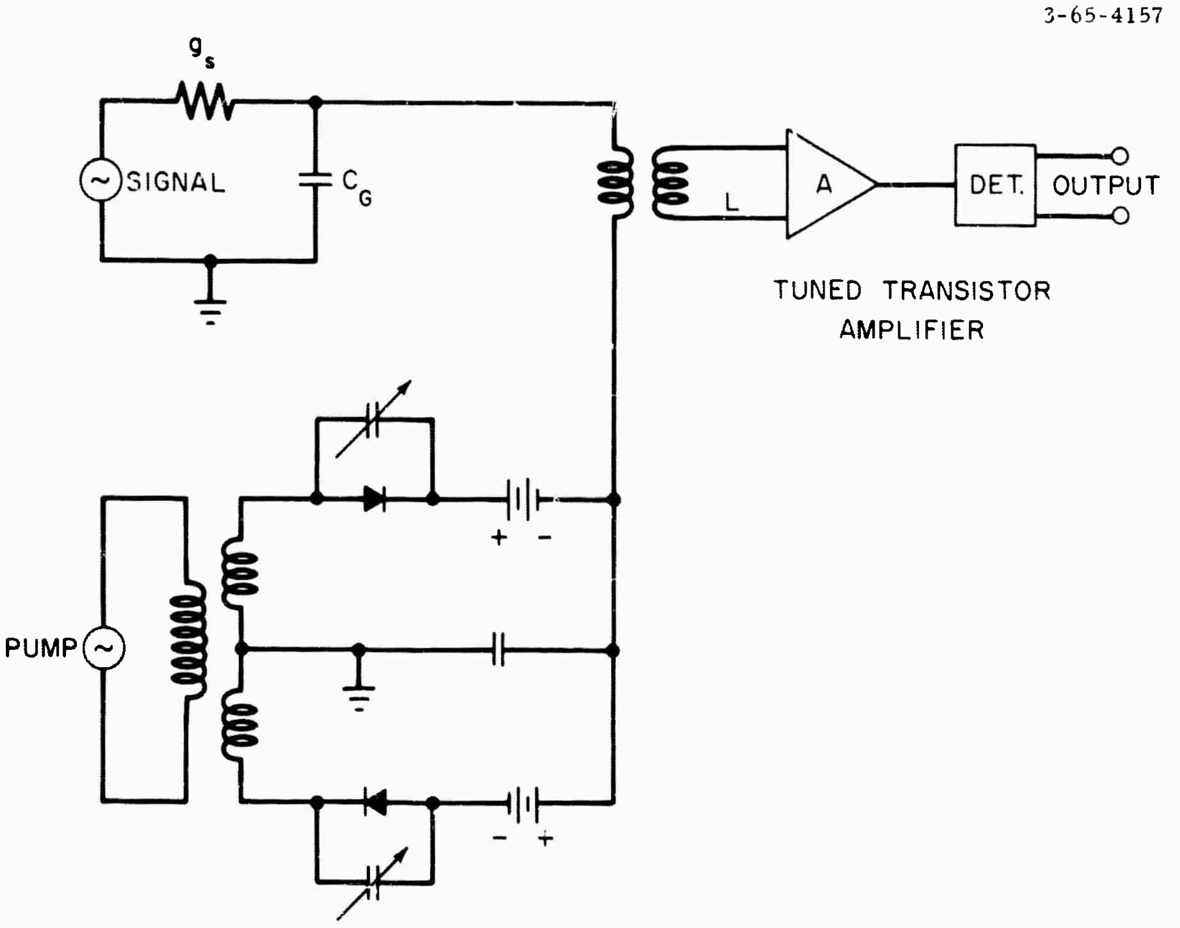


Fig. 12. Texas Instruments RA5 reactance converter.

The diodes in the bridge are back biased by the series batteries. The trimmer capacitors are used to correct for differences in the diodes and to create a controlled imbalance in the bridge to provide a pump signal of proper phase to act as carrier for the sidebands created by the input signal. This allows the modulated wave to be peak detected. The analysis of the amplifier presented in the Appendixes A and B is more general than that presented in a paper by the designer, J. R. Biard.[5] Although a specific amplifier is being analyzed, the circuit constraints on the low frequency converter are stiff enough that the analysis can be said to apply to the class of low frequency parametric converters.

The small signal circuit is essentially a two tank circuit with the output tank tuned to the pump frequency and the input circuit designed to short out the pump frequency.

3-65-4158

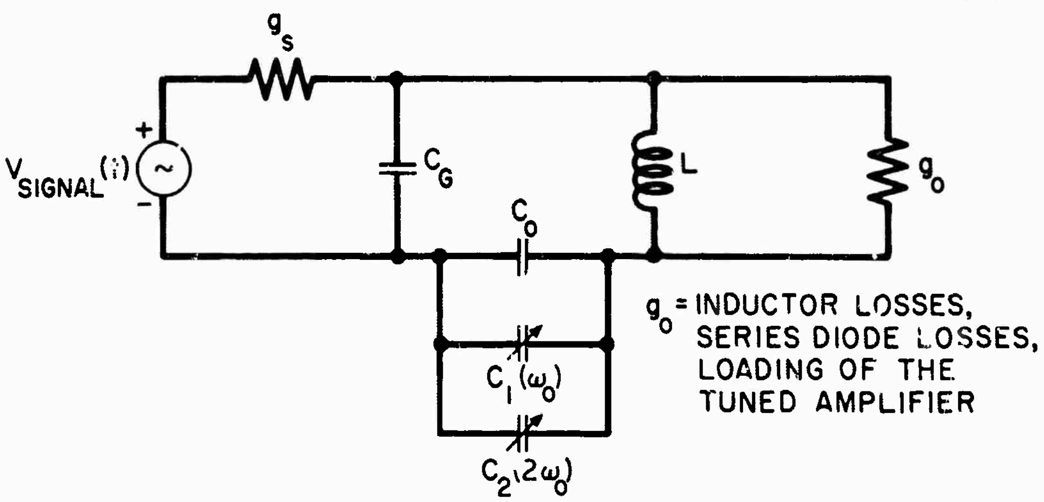


Fig. 13. Small signal model of reactance converter.

$$V_2^{\dagger} = V_1 \omega_0 C_1 \frac{g_o^2 + \omega_0^2 C_2^2}{g_o^2 - \omega_0^2 C_2^2} e^{j\varphi_2} ; \varphi_2 = \mathrm{TAN}^{-1}\left(\frac{g_o}{\omega_0 C_2}\right)$$

$$= \pi/2 + \mathrm{TAN}^{-1}\left(\frac{\omega_0 C_2}{g_o}\right)$$

$$V_3 = V_1^{*} \omega_0 C_1 \frac{g_o^2 + \omega_0^2 C_2^2}{g_o^2 - \omega_0^2 C_2^2} e^{j\varphi_2} ;$$ $V_{2,3}$ = Voltage across the output tank at the sum, difference frequency.

$$Y_{in} = j\omega_1 \left(C_0 + C_G + \frac{2(\omega_0 C_1)^2 C_2^2}{g_o^2 - \omega_0^2 C_2^2}\right) ; \omega_2 = \omega_0 + \omega_1 , \omega_3 = \omega_0 - \omega_1$$

Y_{in} = input impedance at signal frequency.

$$Y = g_s + Y_{in} \cong g_s \; ; \quad v(t) \cong v_1(t) \quad . \tag{24}$$

(See Appendix A for derivations.)

Excess noise in the reactance converter arises from two main sources. The first is the result of the presence of noise currents in the output tank and the mixing action of the second harmonic component of time varying capacitance. The noise current comes from thermal noise associated with the losses in the converter.

---

† Capital letters when used for voltages and currents refer to the complex amplitudes of complex time functions. Also, since the frequency of the input signal is not preserved in the output signal, there is no true voltage gain. The gain of the device will be the ratio of the magnitude of the complex amplitude of the output signal to the input.

3-65-4159

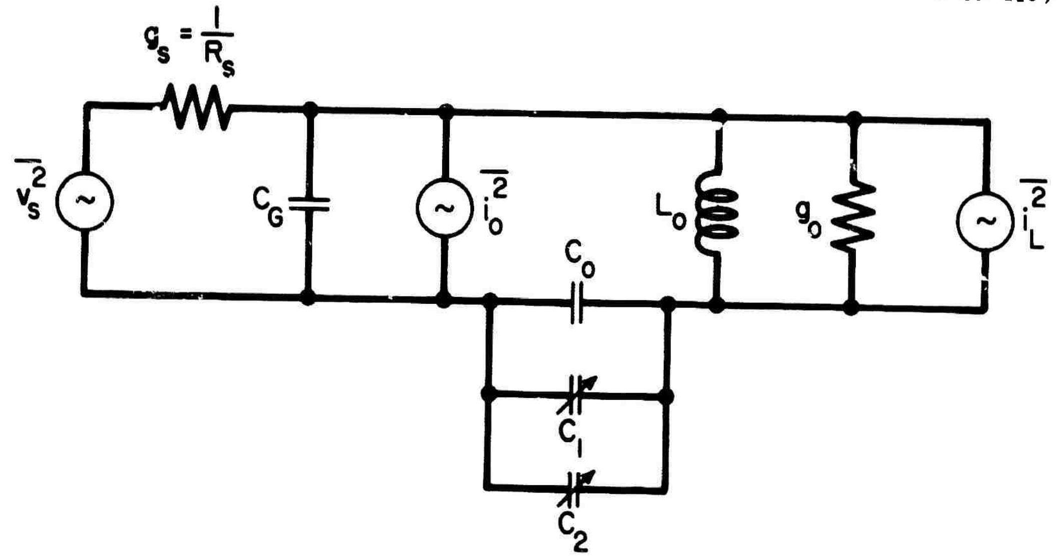


Fig. 14. Noise circuit of low frequency reactance converter.

$\overline{i_L^2} = 4KT(\Delta f)g_T$ $\qquad g_T = g_o - g_L$ (portion of g_o due to losses in the converter itself.)

$\overline{i_o^2} = 2q(\Delta f)I_{sat}$ $\qquad$ Diode leakage

$\overline{v_s^2} = 4KT(\Delta f)R_s$ $\qquad$ Source noise voltage

$$\overline{v_L^2} = \frac{g_o^2 + \omega_0^2 C_2^2}{(g_o^2 - \omega_0^2 C_2^2)} 2 \quad \overline{i_L^2}$$

Output noise voltage due to mixing action of second harmonic component of time varying capacitance with the thermal noise currents.

(25)

(See Appendixes B and C for the derivations.)

The second source of noise is due to the presence of diode leakage current. Since the diode is common to both the input and output circuits and the shot noise has a flat power density spectrum, there is shot noise currents in both circuits. This shot noise has negligible effect at the output tank due to the presence of much larger thermal noise currents. However, it is an effect at the input circuit.

$$\overline{v_{eq}^2} = \frac{v_L^2}{\overline{\left|\frac{V_{2,3}}{V_1}\right|^2}} + 2q(\Delta f)i_s R_s^2$$

Equivalent input noise voltage.

$$\overline{v_{eq}^2} = \frac{1}{(\omega_0 C_1)^2} 4KT(\Delta f)g_T + 2q(\Delta f)I_s R_s^2 \quad . \tag{26}$$

There is reportedly no significant $1/f$ noise down as far as 1 cps.[6]

For purposes of this analysis, the noise bandwidth is defined as the bandwidth of the input signal. This is reasonable since noise at frequencies other than which overlaps with the signal frequency can be filtered out. Furthermore, this gives a standard by which the three different types of devices discussed in this paper can be compared.

$$F = 1 + \frac{1}{(\omega_0 C_1)^2} \quad 2\frac{g_T}{R_s} + \bar{\bar{2}}\,\frac{q}{KT}\, I_s R_s \tag{27}$$

Blackwell and Kotzebue[7] have computed the coefficients of the harmonics term of the time varying capacitance in terms of the time average capacitance for different types of diode junctions. Assuming an abrupt junction diode with $C_0 = 3 \times 10^{-10}$ fd.:

$$C_0 = 3 \times 10^{-10} \text{ fd.}; C_1 = 1.5 \times 10^{-10} \text{ fd.}$$

$$C_2 = 6 \times 10^{-11} \text{ fd.}; \ \omega_0 = 3 \times 10^6 \text{ rad/sec.}$$

Using the value of g_L in the chopper amplifier and extrapolating from the data on the Pacific Semiconductor standard varicap V_{100} for g_T and I_s:

$$g_L = 10^{-4} \quad ; \quad g_T = 10^{-5} \text{ mhos}$$

$$I_s = 10^{-11} \text{ AMPS}$$

(continued)

$$F = 1 + \frac{50}{R_s} + 2 \times 10^{-10} R_s$$

$$\left|\frac{V_{2,3}}{V_1}\right| \cong 40. \tag{28}$$

A further improvement in noise performance can be realized by eliminating the second frequency component of time varying capacitance. This can be partially done by determining the output tank from passive resonance. The best way is to pump with a square wave, since a square wave is devoid of a second harmonic component.

The result is that there are no second frequency component of capacitance at all and the first frequency component C_1 will be made larger. The gain due to the negative resistance effect will disappear but so will the same amount of noise. However, the regular up-conversion gain is increased $(\omega_0 C_1)$ without a proportionate increase in noise. The next result is an improvement in the output signal-to-noise ratio.

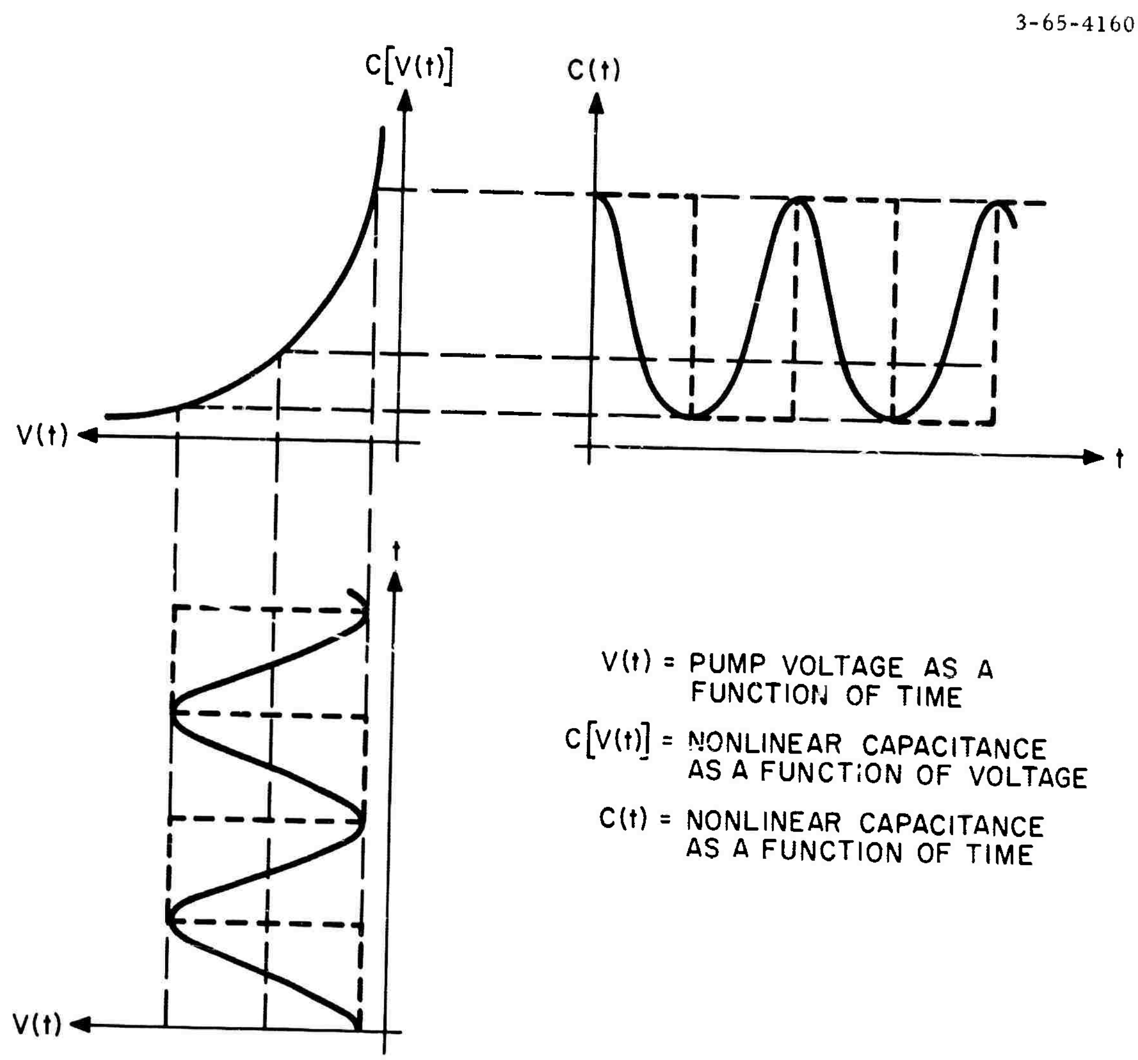


Fig. 15. Illustrating the effect of pumping with a square wave on the harmonic content of the time-varying capacitance.

APPENDIX A

DERIVATION OF THE VOLTAGE GAIN AND INPUT IMPEDANCE FOR PARAMETRIC CONVERTER

Consider the general class of four frequency reactor converters represented by the schematic diagram.

3-65-4161

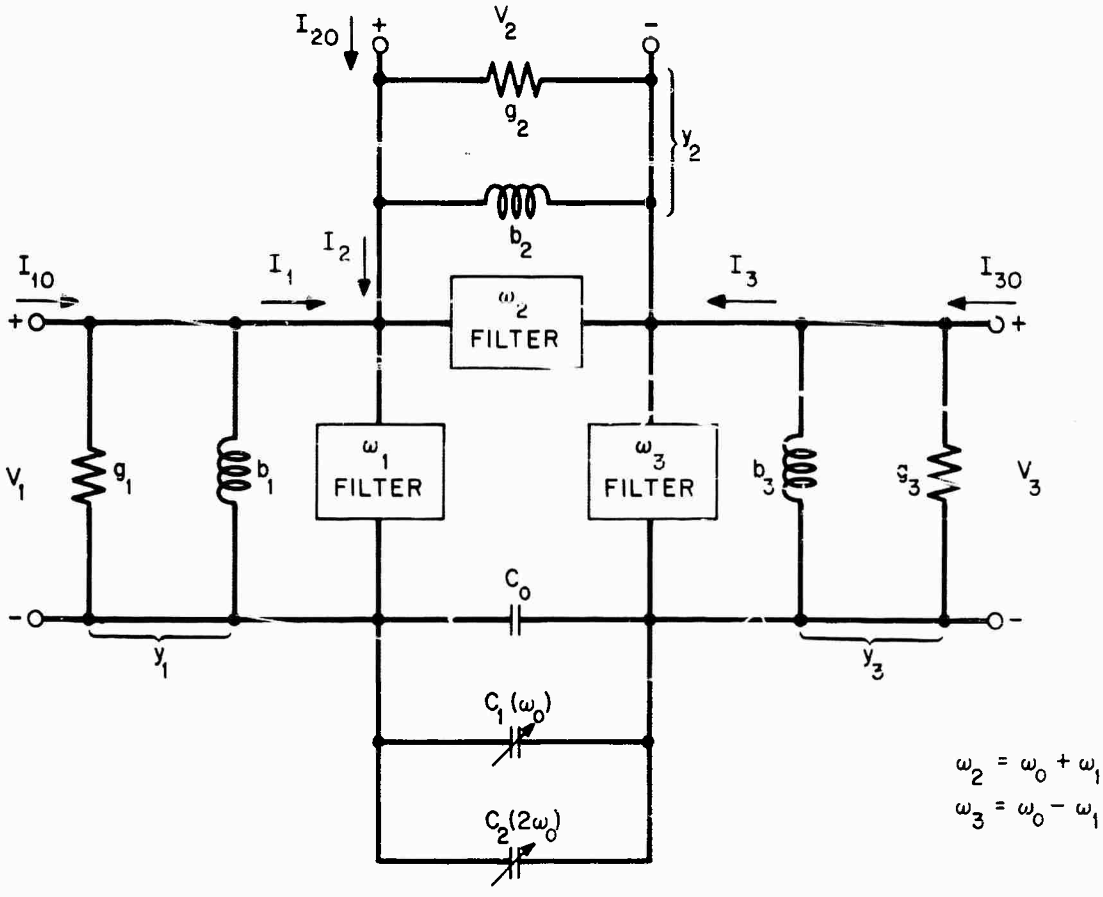


Fig. A-1. Four frequency reactance converters.

The three filters are short circuits to all frequencies outside their respective passbands. Therefore, voltages can exist only at three frequencies $(\omega_1, \omega_2, \omega_3)$.

The non-linear capacitor has a charge-voltage characteristic $q = f(v)$. The pump will bias the capacitor at some (q_1, v_1). The capacitor can be represented about this "operating point" by the relation $\delta q = f(v_1)\, \delta v$.

$$f'(v) = \frac{df(v)}{dv} \quad . \tag{A-1}$$

Now $f'(v_1)$ may be thought of as an equivalent linear time varying capacitance with a fundamental angular frequency ω_o .

$$f'(v_1) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o} \; ; \; C_{-n} = C_n^*$$

$$\delta v(t) = \sum_{n=1}^{3} (V_n e^{j\omega_n t} + V_n^* e^{-j\omega_n t})$$

$$\delta q(t) = \sum_{n=1}^{3} (Q_n e^{\delta\omega_n t} + Q_n^* e^{-j\omega_n t}) \quad . \tag{A-2}$$

The time varying charge $\delta q(t)$ exists at frequencies $n\omega_o \pm \omega_1$ $-\infty < n < \infty$. However, the voltage $\delta v(t)$ can exist only at three frequencies so although $\delta q(t)$ exists at many frequencies power can flow only at three frequencies. For this reason $\delta q(t)$ will be considered only at the three basic frequencies.

$$\delta q(t) = \delta v(t) \sum_{n=-\infty}^{\infty} C_n\, e^{jn\omega t}$$

$$C_{-n} = C_n^{*}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3^* \end{bmatrix} = \begin{bmatrix} C_1 & C_0 & C_{+1}^* \\ C_2 & C_1 & C_0 \\ C_0 & C_{+1}^* & C_{+2}^* \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3^* \end{bmatrix} \qquad \text{(A-3)}$$

$$I = j\omega Q \quad ; \quad I^* = -j\omega Q^*$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3^* \end{bmatrix} = \begin{bmatrix} j\omega_1 C_0 & +j\omega_1 C_1^* & j\omega_1 C_1 \\ j\omega_2 C_1 & j\omega_2 C_0 & j\omega_2 C_2 \\ -j\omega_3 C_1 & -j\omega_3 C_2^* & -j\omega_2 C_0 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3^* \end{bmatrix} \qquad \text{(A-4)}$$

Thus, we have characterized the non-linear capacitor in terms of an admittance matrix. H. E. Rowe[8] takes up the topic of three frequency reactor circuits where either the sum or difference frequency is shorted out. It is well worth reading prepertory to the four frequency analysis. D. K. Adams[9] starts with the above admittance matrix and does a general analysis of the four frequency reactance circuit. His analysis is much more general than which is to be presented here because he is not forced to consider the sum and difference frequencies as essentially the same frequency.

The capacitance of an abrupt junction varacter diode $C(t) =$

$$\frac{A}{\sqrt{V_o}} \frac{1}{\sqrt{1 + V_{ac}/V_o}} .$$

V_o = magnitude of reverse bias.

V_{ac} = alternating applied voltage.

If

$$V_{ac} = V \cos \omega_0 t$$

$$C(t) = \frac{A}{V_o} \frac{1}{\sqrt{1 + a \cos \omega_0 t}} \quad a = \frac{V}{V_o} . \tag{A-5}$$

Hence, $C(t)$ is an even function of time and all of the fourier coefficients will be real. Furthermore, since $\omega_0+\omega_1$ and $\omega_0-\omega_1$ are essentially the same frequency (ω_2) and (ω_3) in the admittance matrix will be replaced by (ω_0).

Also, including the circuit input admittances yields:

$$Y_2 = Y_3 \triangleq Y_0 .$$

$$\begin{bmatrix} I_{10} \\ I_{20} \\ I_{30}^* \end{bmatrix} = \begin{bmatrix} (j\omega_1 C_0+Y_1) & +j\omega_1 C_1 & j\omega_1 C_1 \\ j\omega_0 C_1 & (j\omega_0 C_0+Y_0) & j\omega_0 C_2 \\ -j\omega_0 C_1 & -j\omega_0 C_2 & -j\omega_0 C_0+Y_0 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3^* \end{bmatrix} \tag{A-6}$$

Now, setting $I_{20} = I_{30}^* = 0$ and solving for the input admittance and voltage gains:

$$0 = I_{20} = (Y_0^* - j\omega_0 C_0)[j\omega_0 C_1 V_1 + (Y_0 + j\omega_0 C_0)V_2 + j\omega_0 C_2 V_3^*]$$

$$0 = I_{30}^* = j\omega_0 C_2[-j\omega_0 C_1 V_1 - j\omega_0 C_2 V_2 + (Y_0^* - j\omega_0 C_0)V_3^*]$$

$$\frac{V_2}{V_1} = \frac{-j\omega_0 C_1 (Y_0^* - j\omega_0 C_0) + \omega_0^2 C_1 C_2}{|Y_0 + j\omega_0 C_0|^2 - \omega_0^2 C_2^2} \quad . \qquad \text{(A-7)}$$

$$0 = I_{20} = j\omega_0 C_2[j\omega_2 C_1 V_1 + (Y_0 + j\omega_0 C_0)V_2 + j\omega_0 C_2 V_3^*]$$

$$0 = I_{30}^* = (Y_0 + j\omega_0 C_0)[-j\omega_0 C_1 V_1 - j\omega_0 C_2 V_2 + (Y_0^* - j\omega_0 C_0)V_3^*]$$

$$\frac{V_3^*}{V_1} = \frac{j\omega_0 C_1 (Y_0 + j\omega_0 C_0) + \omega_0^2 C_1 C_2}{|Y_0 + j\omega_0 C_0|^2 - \omega_0^2 C_2^2} \quad . \qquad \text{(A-8)}$$

$$\frac{I_{10}}{V_1} = j\omega_1 C_0 + Y_1 + j\omega_1 C_1 \left[\frac{-j\omega_0 C_1 (Y_0^* - j\omega_0 C_0) + \omega_0^2 C_1 C_2}{|Y_0 + j\omega_0 C_0|^2 - \omega_0^2 C_2^2}\right]$$

$$+ j\omega_1 C_1 \left[\frac{j\omega_0 C_1 (Y_0 + j\omega_0 C_0) + \omega_0^2 C_1 C_2}{|Y_0 + j\omega_0 C_0|^2 - \omega_0^2 C_2^2}\right] \quad . \qquad \text{(A-9)}$$

$$Y_0 = -j \frac{1}{\omega_0 L_o} + g_o \quad ; \quad Y_1 = j\omega_1 C_G$$

Now setting $Y_0 + j\omega_0 C_0 = g_o$; $\frac{1}{\omega_0 L_o} = \omega_0 C_0$. That is, the output tank is being tuned to the pump frequency. This is a practical means of realizing the filters in Fig. A-1.

$$\frac{V_2}{V_1} = \frac{-j\omega_0 C_1 g_o + \omega_0^2 C_1 C_2}{g_o^2 - \omega_0^2 C_2^2}$$

$$\frac{V_3^*}{V_1} = \frac{j\omega_0 C_1 g_o + \omega_0^2 C_1 C_2}{g_o^2 - \omega_0^2 C_2^2}$$

$$\frac{I_{10}}{V_1} = Y_{in} = j\omega_1 (C_0 + C_G + \frac{2(\omega_0 C_1)^2 C_2}{g_o^2 - (\omega_0 C_2)^2} \quad . \qquad \text{(A-10)}$$

3-65-4162

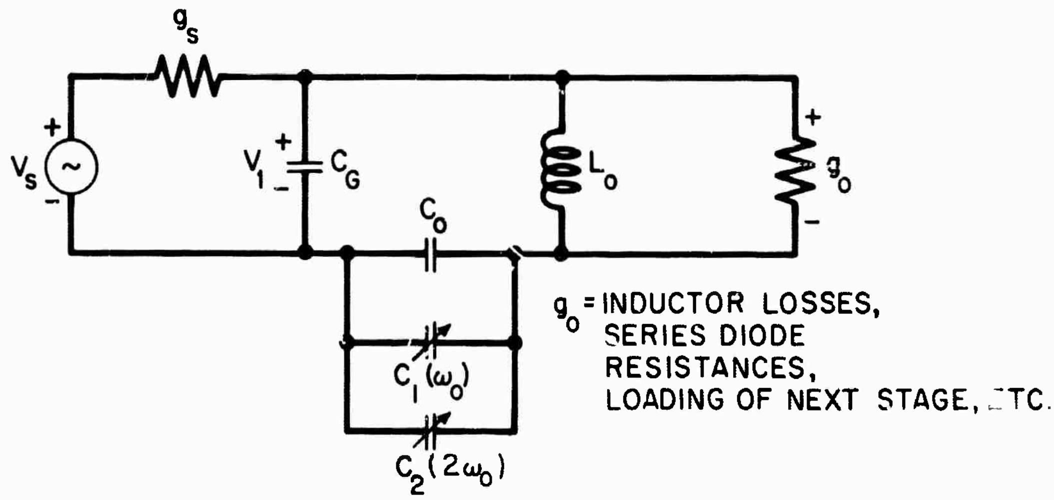


Fig. A-2. Small signal model.

Let $V(t) = v \cos \omega_1 t$. Also assume $g_s \gg Y_{in}$; $\therefore v(t) \cong v_1(t)$.

$$V_2 = V \omega_0 C_1 \frac{\sqrt{g_o^2 + (\omega_0 C_2)^2}}{g_o^2 - (\omega_0 C_2)^2} e^{j\varphi_2}$$

$$V_3 = V^* \omega_0 C_1 \frac{\sqrt{g_o^2 + (\omega_0 C_2)^2}}{g_o^2 - (\omega_0 C_2)^2} e^{j\varphi_2} \quad . \tag{A-11}$$

$$\varphi_2 = \mathrm{TAN}^{-1} \left(\frac{-g_o}{\omega_0 C_2}\right) = \frac{\pi}{2} + \mathrm{TAN}^{-1} \left(\frac{\omega_0 C_2}{g_o}\right) \quad .$$

$$v_o(t) = \mathrm{Re}\left[V \omega_0 C_1 \frac{\sqrt{g_o^2 + (\omega_0 C_2)^2}}{g_o^2 - (\omega_0 C_2)^2} \left(e^{j(\omega_0+\omega_1)t} + e^{j(\omega_0-\omega_1)t}\right)\right]$$

$$v_o(t) = 2V \omega_0 C_1 \frac{\sqrt{g_o + (\omega_0 C_2)^2}}{g_o^2 - (\omega_0 C_2)^2} \cos(\omega_0 t + \varphi_2) \cos \omega_1 (t) \quad . \tag{A-12}$$

It is interesting to note that the input admittance is completely imaginary. This indicates that the loading effect of the sum frequency was just counterbalanced by the negative resistance effect of the difference frequency: $W_{11} = W_{1-1}$.

For a conjugate match at the input, we would have the condition of infinte gain predicted by the Manley-Rowe equations.

It is also interesting to note the negative resistance effect caused by the second harmonic component of time varying capacitance. If g_o^2 is less than or equal to $\omega_0^2 C_2$ an unstable situation results.

APPENDIX B

NOISE ANALYSIS

Initially I would like to solve the voltages V_2 and V_3 that result when $\{ {I_{10} = 0 \atop I_{30} = 0} \}$ and I_{20} is considered as a source.

Again, going back to the matrix:

$$Y_1 \triangleq g_s + j\omega_1 C_G \quad ; \quad C \triangleq C_o + C_G$$

$$0 = I_{10} = j\omega_0 C_1 [(j\omega_1 C + g_s)V_1 + j\omega_1 C_1 V_2 + j\omega_1 C_1 V_3^*]$$

$$0 = I_{30} = (j\omega_1 C + g_s)[-j\omega_0 C_1 V_1 - j\omega_0 C_2 V_2 (-j\omega_0 C_0 + Y_0^*) V_3^*]$$

$$\frac{V_3^*}{V_2} = \frac{\omega_0 \omega_1 C_1^2 + j\omega_0 C_2 (j\omega_1 C + g_s)}{-\omega_0 \omega_1 C_1^2 + (j\omega_1 C + g_1)(Y_0^* - j\omega_0 C_0)} \quad . \tag{B-1}$$

In like manner

$$\frac{V_1}{V_2} = \frac{\omega_0 \omega_1 C_1 C_2 - j\omega_1 C_1 (Y_0^* - j\omega_0 C_0)}{(j\omega_1 C + g_s)(Y_0^* - j\omega_0 C_0) - \omega_0 \omega_1 C_1^2}$$

Setting $Y_0 + j\omega_0 C_0 = g_o$

$$\frac{I_2}{V_2} = \frac{2j\omega_0^2 \omega_1 C_1^2 C_2 + (g_o^2 - (\omega_0 C_2)^2)(j\omega_1 C + g_s)}{(j\omega_1 C + g_s) g_o - \omega_0 \omega_1 C_1^2} \tag{B-2}$$

$$\frac{V_2}{I_2} = \frac{(j\omega_1 C+g_s)g_o-\omega_0\omega_1 C_1^{\,2}}{2j\omega_0^{\,2}\omega_1 C_1^{\,2}C_2+(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})(j\omega_1 C+g_s)}$$

$$\frac{V_3^{\,*}}{I_2} = \frac{\omega_0\omega_1 C_1^{\,2}+j\omega_0 C_2(j\omega_1 C+g_s)}{2j\omega_0^{\,2}\omega_1 C_1^{\,2}C_2+(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2}(j\omega_1 C+g_s)} \quad . \qquad \text{(B-3)}$$

Now making several assumptions regarding the relative importance of the parameters in the above two equations:

(1) $|j\omega_1 C_1| \ll g_1$

(2) $g_o g_s \gg \omega_0\omega_1 C_1^{\,2}$

(3) $C_2 g_s \gg \omega_1 C_1^{\,2}-\omega_1 C_2 C$

(4) $g_s(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2}) \gg 2\omega_0^{\,2}\omega_1 C_1^{\,2}C_2-\omega_0^{\,2}\omega_1 C_2^{\,2}C \quad .$ (B-4)

We have

$$\frac{V_2}{I_2} = \frac{g_o}{g_o^{\,2}-\omega_0^{\,2}C_2^{\,2}}$$

$$\frac{V_3^{\,*}}{I_2} = \frac{j\omega_0 C_2}{g_o^{\,2}-\omega_0^{\,2}C_2^{\,2}} \quad . \qquad \text{(B-5)}$$

Now, if $I_{10} = 0$ and $I_{20} = 0$ and I_{30}^* is considered as the source:

$$\frac{V_2}{I_3^*} = \frac{-j\omega_0 C_2}{g_o^2 - \omega_0^2 C_2^2}$$

$$\frac{V_3^*}{I_3^*} = \frac{g_o}{g_o^2 - \omega_0^2 C_0^2} \quad . \qquad \text{(B-6)}$$

Now assume the existance of noise currents in both the sum and difference sides of the output tank pass band.

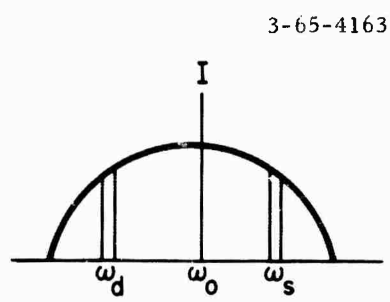


Fig. B-1. Output tank.

I will use the relations just derived for finding the voltages created by the differential noise currents and integrate over the output tank to find the auto-correlation functions of the noise voltage.

$$\langle v_{2s}(t)v_{2s}(t+\tau)\rangle = \varphi_{2s2s}(\tau) = \int \frac{g_o^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2} |I_2(\omega_s)|^2 e^{j\omega_s\tau} d\omega_s$$

over output tank

$$\langle v_{3s}(t)v_{3s}(t+\tau)\rangle = \varphi_{3s3s}(\tau) = \int \frac{\omega_0^{\,2}C_2^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2} |I_2(\omega_s)|^2 e^{j\omega_s\tau} d\omega_s$$

over output tank

$|I_2(\omega_s)|^2$ is the sum noise current power density spectrum. (B-7)

However, we are essentially only interested in a narrow region about ω_0 corresponding to the input signal pass band.

Assuming thermal noise due to inductor losses and series diode resistance =>

$$|I_2(\omega_s)|^2 = 4KT(\Delta f)g_T = \overline{i_s^{\,2}}$$

$$|I_3(\omega_d)|^2 = 4KT(\Delta f)g_T = \overline{i_d^{\,2}}$$

(continued)

$$\langle v_{2s}(t)^2 \rangle = \varphi_{2s2s}(0) = \frac{g_o^{\,2}}{(g_0^{\,2}-\omega_0^{\,2}C_2^{\,2})^2}\ \overline{i_s^{\,2}}$$

$$\langle v_{3s}(t)^2 \rangle = \varphi_{3s3s}(0) = \frac{\omega_0^{\,2}C_2^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2}\ \overline{i_s^{\,2}}$$

$$\langle v_{2d}(t)^2 \rangle = \varphi_{2d2d}(0) = \frac{\omega_0^{\,2}C_2^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2}\ \overline{i_d^{\,2}}$$

$$\langle v_{3d}(t)^2 \rangle = \varphi_{3d3d}(0) = \frac{g_o^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2}\ \overline{i_d^{\,2}} \quad . \qquad \text{(B-8)}$$

$$\overline{v_s^{\,2}} = \overline{v_{2s}^{\,2}(t)} + \overline{v_{3s}^{\,2}(t)} = \frac{g_o^{\,2}+\omega_0^{\,2}C_2^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2}\ \overline{i_s^{\,2}}$$

$$\overline{v_d^{\,2}} = \overline{v_{2d}^{\,2}(t)} + \overline{v_{3d}^{\,2}(t)} = \frac{g_o^{\,2}+\omega_0^{\,2}C_2^{\,2}}{(g_o^{\,2}-\omega_0^{\,2}C_2^{\,2})^2}\ \overline{i_d^{\,2}} \quad . \qquad \text{(B-9)}$$

APPENDIX C

NOISE ANALYSIS OF NON-LINEAR CAPACITOR

I. INPUT CIRCUIT

3-65-4164

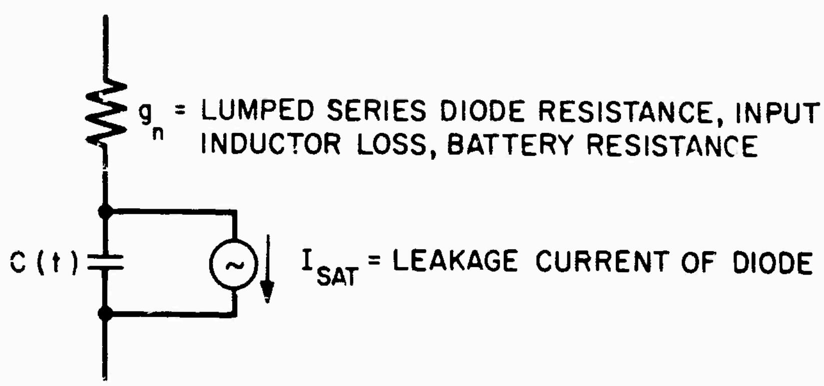
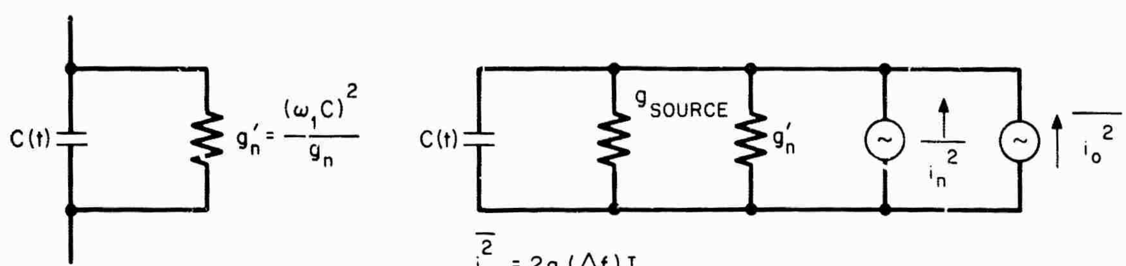


Fig. C-1. Diode model.

At signal frequency $g_n >> \omega_1 C$ and the series resistor can be placed in parallel with the diode.

3-65-4165



$$\overline{i_o^2} = 2q(\Delta f)I$$

$$\overline{i_n^2} = 4KT(\Delta f)g_n$$

$$\overline{i_o^2} \gg \overline{i_n^2}$$

Fig. C-2. Noise model of input circuit.

The principal noise source in the input circuit other than the source resistor is the shot noise due to the diode leakage current. The thermal noise current is negligible compared with the shot noise.

II. OUTPUT CIRCUIT

3-65-4166

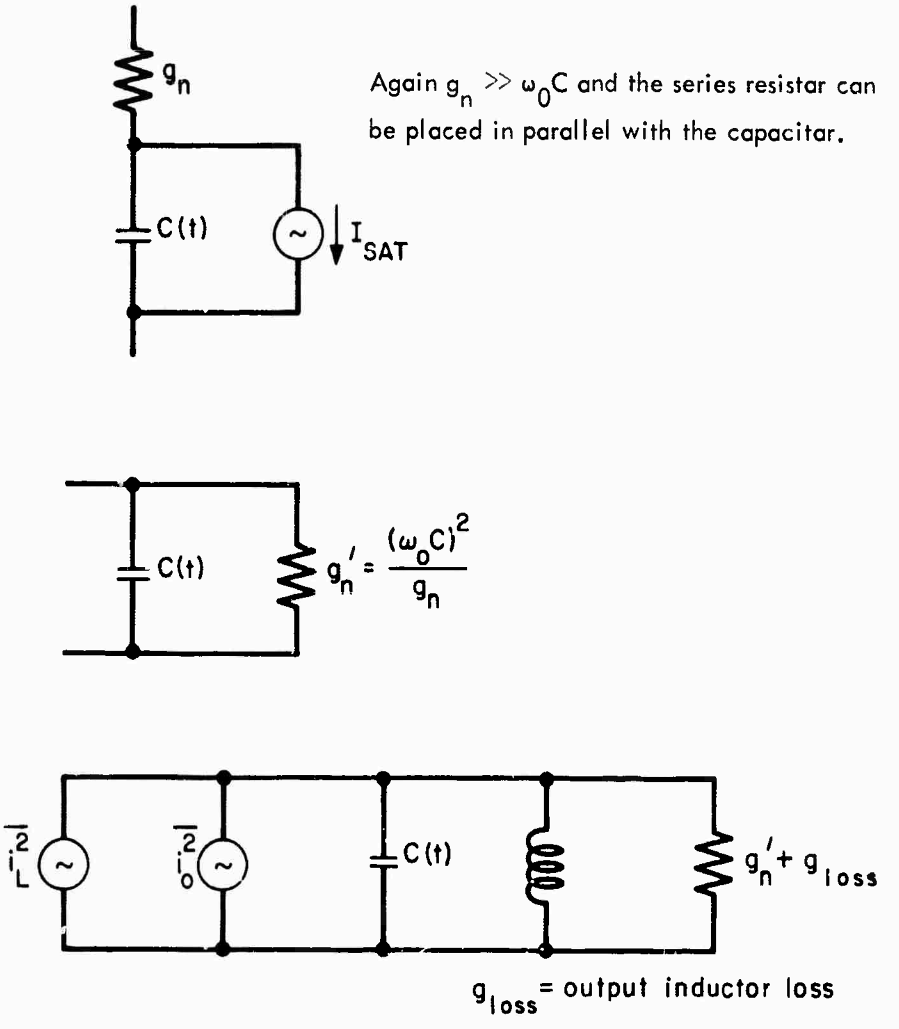


Fig. C-3. Noise model at output circuit.

$$\overline{i_L^{\,2}} = 4KT(\Delta f)(g_n' + g\ \text{loss})$$

$$\overline{i_o^{\,2}} = 2q(\Delta f)\,I_{SAT} \qquad \overline{i_L^{\,2}} >> \overline{i_o^{\,2}} \quad . \tag{C-1}$$

Now the shot noise is much less than the thermal noise and can be neglected. The diode loss resistor can be lumped with the inductor loss resistor. This sum resistance is the principal source of the aforementioned noise currents $\overline{i_s^{\,2}}$ and $\overline{i_d^{\,2}}$.

$$g_T = g_n' + g_{loss}$$

$$\overline{i_L^{\,2}} = \overline{i_s^{\,2}} = \overline{i_d^{\,2}} = 4KT(\Delta f)\ g_T \quad . \tag{C-2}$$

REFERENCES

1. H. A. Haus, R. B. Adler, *Circuit Theory of Linear Noisy Networks* (John Wiley & Sons, Inc., New York, 1958).
2. A. Van der Ziel, Proc. IRE, *50*, 1808 (August 1962).
3. H. Heffner, G. Wade, J. Appl. Phys., *29*, 1321 (September 1958).
4. J. M. Manley, H. E. Rowe, Proc. IRE, *44*, 904 (July 1956).
5. J. B. Biard, Proc. IRE, *51*, 298 (February 1963).
6. *Ibid*. (February 1963).
7. L. A. Blackwell, K. L. Kotzebue, *Semiconductor-Diode Parametric Amplifiers* (Prentice-Hall, New Jersey, 1961).
8. H. E. Rowe, Proc. IRE, *46*, 850 (May 1958).
9. D. K. Adams, Proc. IRE (Trans. on Microwave Theory) *274* (May 1960).