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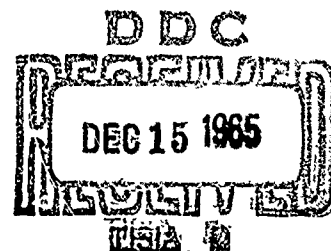
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ACCURACY AND DISPERSION OF UNGUIDED,
AIR-LAUNCHED ROCKETS

NOL

12 OCTOBER 1965



UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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Ballistics Research Report 103

ACCURACY AND DISPERSION OF UNGUIDED, AIR-LAUNCHED ROCKETS

Prepared by:

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ABSTRACT: Simplified non-dimensional equations of motion are derived. From these, 288 rocket trajectories are calculated, based upon a variety of choices for aerodynamic, inertial, and propulsive properties. The analysis includes off-axis thrust and response resulting from initial conditions of yaw angle and yaw rate. The most important results are presented in tables and graphs. Application of the results to calculation of dispersion is illustrated in an interpretation of the results. A conclusion reached is that high static stability is not necessary for useful accuracy.

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Captain, USN
Commander

A. E. Seigel
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By direction

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LIST OF SYMBOLS

- x = coordinate distance measured along the initial direction of the velocity vector of the rocket.*
- y = coordinate distance perpendicular to x in the yaw plane.*
- T = thrust force
- ϵ = ratio of the moment arm of the thrust force about the center of mass to the length of the rocket
- L = length of the rocket
- C_{M_α} = aerodynamic moment coefficient per radian angle of attack
- ρ_a = air density
- V = flight speed of rocket, or flight speed of aircraft
- A = reference area for aerodynamic forces. Usually this is taken as the maximum cross-sectional area of the rocket.
- D = reference moment arm for computing aerodynamic moments. For circular cross-sections, this length is customarily taken as the diameter.
- α = angle of attack, radians, measured from the velocity vector to the longitudinal axis of symmetry of the rocket. The relation between positive α and positive y is such that if the rocket rotates through positive α its nose moves toward positive y.
- θ = angle between x-axis and the longitudinal axis of symmetry. It is positive in the same sense as α .
- t = time

*In different parts of the analysis x and y are both dimensionless quantities. This change in meaning is either indicated by a prime or stated in the text wherever it occurs.

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- $C_{N\alpha}$ = aerodynamic lift coefficient per radian angle of attack
- R_e = distance from the center of mass of the rocket back to some point in the nozzle where it is assumed that the gas flow ceases to be guided by the nozzle walls
- R_B = radius of gyration of the burning surface of the grain of propellant about the center of mass of the rocket
- r_e^2 = $R_e^2 - R_B^2$
- μ = mass flow rate through the nozzle
- dm = infinitesimal mass element of burning propellant, distributed with uniform depth over the burning surface
- M = mass of rocket
- I = moment of inertia of the rocket about an axis perpendicular to the yaw plane and through the center of mass
- λ = yaw wavelength, or the approximate distance traveled by the rocket during one cycle of yaw oscillation, defined in equation (5)
- k = substitution for $\frac{C_{M\alpha} \rho_a A D}{2I} \frac{T^2}{M^2} = \left(\frac{2\pi T}{\lambda M}\right)^2$
- τ = dimensionless time coordinate, defined as $\tau = k^{1/4} t$
- K = dimensionless coupling parameter between y and θ equations, defined in equation (17)
- J = dimensionless jet damping parameter, defined in equation (18)
- B = dimensionless thrust moment bias parameter, defined in equation (19)
- l = subscript denoting an initial condition for a trajectory computation
- Ω = angular rolling velocity of rocket about its longitudinal axis

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- m = dispersion in mils
- α_A = angle of attack of the airplane, radians
- $\Delta\alpha_A$ = variation in angle of attack of the airplane away from some standard angle, radians
- W = gross weight of attacking aircraft
- δ = dive angle of attacking aircraft
- V_A = speed of the airplane
- s = reference area of airplane
- a = acceleration of the airplane
- g = acceleration of gravity
- v = lateral flow velocity of atmospheric gust
- G = distance through atmospheric gust

INTRODUCTION

The purpose of this report is to give an analysis of the motion of unguided rockets launched from aircraft, with the particular object of predicting the errors caused in the flight path of the rocket by the various disturbances which occur during the launching phase of flight, or by thrust misalignment. The analysis is not made for any specific existing weapon. Instead, the aerodynamic, inertial, and propulsive properties of the rocket are allowed wide latitude for variation. The results should therefore be of use for weapon feasibility studies, where one object of the work may be to decide upon the general features of a proposed rocket.

Because of the author's other duties, the time available for this work has not been sufficient to permit the academic thoroughness of a literature search or the compilation of a complete bibliography related to the subject. Reference (1) contains a bibliography. It is probable that some, if not all, of the present results are already available in existing literature. Nevertheless, the convenience and speed with which the problem could be investigated on the IBM 7090 computer prompted the decision to make the present independent approach.

AIMING AND LAUNCHING METHOD

Interpretation of the results of the present analysis is based upon the following method of carrying, aiming, and launching the rocket. The rocket is carried on the attacking aircraft within a launcher which is rigidly attached to the aircraft. The axis of the launcher is approximately parallel to the flight path of the airplane, which is in balanced flight, that is, not "crabbing" or sideslipping. The launcher can be aimed at a target only by aiming the entire airplane. The launcher constrains the longitudinal axis of the rocket to remain parallel to the launcher axis for a short distance after ignition of the rocket motor at the time of firing. Thereafter, the rocket is free to respond to any existing aerodynamic forces and moments, as well as to forces and moments associated with the rocket thrust.

The pilot views the target through a sight, in the center of which is a marker called a pip. When the line of sight is through the target, the apparent positions of target and pip coincide. To attack a surface target, the pilot, after maneuvering the aircraft into a suitable attacking position, dives the aircraft toward the target in balanced flight,

keeping the pip on the target. At the proper range, which for a given dive angle can be told approximately from the altimeter, the rocket is fired and the pilot begins his pull-out. Such dives are normally made with the power off and with dive brakes operating. Nevertheless, the aircraft speeds up considerably during the dive because of the forward component of its weight.

By means of a system of fiducial marks and fixtures, an axis is established running fore and aft through the airplane, which axis is called the armament datum line. Measurements from this line are used in orienting the rocket launcher and the axis of the sight with respect to the aerodynamic surfaces of the airplane. The line of sight is adjusted to compensate for the curvature of the rocket trajectory due to gravity, and for any other known systematic effects which would cause the rocket to fly in some direction other than that in which it is initially pointed. Among these are the aerodynamic effect of the initial flight of the rocket through the flow field of the airplane, and the aerodynamic effect known as "weathercocking," which causes an aerodynamically stable rocket fired out of the relative wind to turn into the wind by some fraction of the angle of misalignment with the relative wind. Some degree of weathercocking will generally be present, since the aerodynamic angle of attack of the airplane varies with speed, air density (thus altitude), total weight, dive angle, and flight path vertical curvature (pull-out or nose-over). Only for some chosen set or sets of these flight conditions, can a rocket launcher be mounted on the aircraft pointed into the relative wind. Variations in the conditions then necessitate weathercocking. If the pilot fails to maintain balanced flight, but flies in a sideslip, this is an additional source of weathercocking.

APPROACH TO THE MECHANICAL ANALYSIS

Three sources of disturbance to the trajectory of the rocket have been considered. They are initial yaw angle (weathercocking), initial angular rotation in yaw, and thrust misalignment. Equations of motion, simplified by linearization and other suitable approximations, have been transformed into dimensionless form and programmed for machine computation. Sets of numerical values were chosen for the dimensionless parameters which appeared in the derivation, covering a wide variety of possible rocket designs. Initial conditions corresponding to the above mentioned sources of disturbance were then used to compute a total of 288 representative trajectories. The results are presented in Tables 1 through 24 and graphically in figures 1 through 13.

The following assumptions have been made in deriving the equations of motion:

1. The yawing motion is planar.
2. Lift and static moment are the only aerodynamic forces considered. Drag and damping moment are ignored.
3. The aerodynamic forces are linear functions of the angle of attack, and are proportional to the square of the flight speed. The aerodynamic coefficients are true constants, independent of Mach number, Reynolds number, or other parameters.
4. The thrust of the rocket remains constant, but may be misaligned with the longitudinal axis of symmetry of the rocket.
5. The mass and moment of inertia of the rocket remain constant.
6. The mass flow rate through the rocket exhaust may be large enough to cause jet damping.
7. The thrust force continues to act throughout the entire flight, or at least for several yaw cycles.

The equations resulting from these assumptions yield fairly simple results. The obvious artificialities involved become worse the longer the flight time of the rocket. It turns out that the eventual destination of the rocket is essentially determined and becomes apparent before the elapse of the first yaw cycle after the rocket is launched. Thus, if judicious choice be made of the parameters used in actual computations, the present method can be expected to give results in good agreement with practical experience.

THE EQUATIONS OF MOTION

COORDINATE AXES

The positive x-axis is taken along the direction of the velocity vector of the rocket at the time of launching. The y-axis is perpendicular to the x-axis in the assumed yaw plane. Positive y is in the direction moved by the nose of the rocket in rotating about its center of mass toward positive angles of attack. These axes are taken as an inertial system. That is,

their direction is fixed in space, but they may have any arbitrary constant linear translational motion. This translational motion is taken to be such as to keep them fixed with respect to the air; thus, they are not necessarily fixed with respect to the ground.

THE FORCES ACTING

The thrust T, assumed constant, pushes the rocket forward along its body axis. It may be shifted laterally away from the center of mass of the rocket by a small amount eL, giving rise to a constant overturning moment.

The (static) aerodynamic moment is

$$\frac{C_{M\alpha} \rho_a V^2 A D \alpha}{2}$$

This moment tends to rotate the rocket toward its velocity vector. α , the angle of attack, is

$$\alpha = \theta - \frac{1}{V} \frac{dy}{dt}$$

The aerodynamic lift (normal force) is

$$\frac{C_{N\alpha} \rho_a V^2 A}{2} \left(\theta - \frac{1}{V} \frac{dy}{dt} \right)$$

The y-component of the thrust is $T\theta$.

The jet damping is associated with the rate of change of moment of momentum (angular momentum) which occurs because of the combined effect of the exhaust flow of the rocket and the yawing motion $\frac{d\theta}{dt}$. The moment of momentum of an element dm of propellant shortly before burning is

$$dm R^2 \frac{d\theta}{e dt}$$

The jet damping moment is therefore

$$\mu(R_e^2 - R_B^2) \frac{d\theta}{dt}, \text{ or since } r_e^2 = (R_e^2 - R_B^2), \text{ the jet}$$

damping moment is $\mu r_e^2 \frac{d\theta}{dt}$

This moment acts in such direction as to decrease $\frac{d\theta}{dt}$.

THE x - EQUATION

The deviations from the x-axis are assumed small enough that the x-equation is

$$M \frac{d^2x}{dt^2} = T, \text{ giving, by integration}$$

$$V = \frac{dx}{dt} = \frac{Tt}{M} \quad (1)$$

THE y - EQUATION

The y-equation consistent with the above stated forces is

$$M \frac{d^2y}{dt^2} = T\theta + \frac{C_{N\alpha} \rho_a V^2 A}{2} \left(\theta - \frac{1}{V} \frac{dy}{dt} \right) \quad (2)$$

THE θ - EQUATION

The θ -equation consistent with all previously stated conditions is

$$I \frac{d^2\theta}{dt^2} + \mu r_e^2 \frac{d\theta}{dt} + \frac{C_{M\alpha} \rho_a V^2 AD}{2} \left(\theta - \frac{1}{V} \frac{dy}{dt} \right) = T_e L \quad (3)$$

TRANSFORMATION OF EQUATIONS INTO DIMENSIONLESS FORM

THE YAW WAVELENGTH

The yaw wavelength, which determines a distance unit suitable for putting x and y into dimensionless form, is approximately the distance the rocket travels during one cycle of yaw oscillation. It is derived from a still further simplified equation of motion, as follows: In equation (3), the omission of the term for jet damping, the term in $\frac{dy}{dt}$, the term $T\epsilon L$ gives

$$I \frac{d^2\theta}{dt^2} + \frac{C_{M\alpha} \rho_a V^2 AD}{2} \theta = 0 \quad (4)$$

If the coefficients of equation (4) were actually constants, the solution would be in terms of sine and cosine functions of

the argument $\left[\frac{C_{M\alpha} \rho_a V^2 AD}{2I} \right]^{1/2} t$. During one period of this oscillation, the rocket would travel the distance

$$\lambda = 2\pi \left[\frac{2I}{C_{M\alpha} \rho_a AD} \right]^{1/2} \quad (5)$$

The distance $\lambda/2\pi$ is used as the distance unit mentioned above.

TRANSFORMATIONS

The following steps are taken:

1. The expression (1) is substituted for V in equations (2) and (3).
2. Equation (2) is divided by M.
3. Equation (3) is divided by I.

The results of these three steps are:

$$\frac{d^2y}{dt^2} = \frac{T}{M} \theta + \frac{C_N \rho_a A}{2M} \frac{T^2}{M^2} t^2 \left(\theta - \frac{M}{Tt} \frac{dy}{dt} \right) \quad (6)$$

$$\frac{d^2\theta}{dt^2} - \frac{\mu r_e^2}{I} \frac{d\theta}{dt} + \frac{C_M \rho_a AD}{2I} \frac{T^2}{M^2} t^2 \left(\theta - \frac{M}{Tt} \frac{dy}{dt} \right) = \frac{T\epsilon L}{I} \quad (7)$$

The third term of equation (7) is seen to be of the form $kt^2\theta$, where

$$k = \left(\frac{2\pi T}{\lambda M} \right)^2 \quad (8)$$

The dimensions of k are time^{-4} . It is therefore convenient to use k for defining a dimensionless time coordinate:

$$\tau = k^{1/4}t \quad (9)$$

The transformations of the derivatives with respect to time are

$$\frac{d}{dt} = \frac{d\tau d}{dt d\tau} = k^{1/4} \frac{d}{d\tau}, \text{ and } \frac{d^2}{dt^2} = k^{1/2} \frac{d^2}{d\tau^2} \quad (10), (11)$$

Dimensionless distances are defined as

$$y' = \frac{2\pi y}{\lambda}, \text{ and } x' = \frac{2\pi x}{\lambda} \quad (12), (13)$$

After these substitutions are made, and the resulting cancellations performed, equation (1) becomes

$$\frac{dx'}{d\tau} = \tau \quad (14)$$

$$\text{Equation (6) becomes } \frac{d^2 y'}{d\tau^2} = \theta + K(\tau^2 \theta - \tau \frac{dy'}{d\tau}) \quad (15)$$

$$\text{Equation (7) becomes } \frac{d^2 \theta}{d\tau^2} + J \frac{d\theta}{d\tau} = -\tau^2 \theta + \tau \frac{dy'}{d\tau} + B \quad (16)$$

The dimensionless parameters K, J, and B are respectively given by

$$K = \frac{C_{N\alpha}}{M} \left[\frac{\rho_a AI}{2 DC_{M\alpha}} \right]^{1/2}, \quad (17)$$

$$J = \frac{\mu r_e^2 k^{-1/4}}{I} = \frac{\mu r_e^2}{I} \left(\frac{\lambda M}{2\pi T} \right)^{1/2}, \text{ and} \quad (18)$$

$$B = \frac{T_e L k^{-1/2}}{I} = \frac{\epsilon L \lambda M}{2\pi I} \quad (19)$$

At this point the primes are dropped from y and x, and a derivative with respect to τ is denoted by a dot over the variable. Equation (14) is integrated directly to give

$$x = \tau^2/2. \quad (20)$$

A simple substitution between equations (15) and (16) gives

$$\ddot{\theta} + J\dot{\theta} = -\tau^2 \theta + \tau \dot{y} + B \quad (21)$$

$$\text{and } \ddot{y} = \theta - K(\ddot{\theta} + J\dot{\theta} - B) \quad (22)$$

COMPUTATIONS PERFORMED

Equations (20), (21), and (22) have been used with the

IBM 7090 computer to compute a variety of trajectories, covering selected numerical values for the parameters K, J, and B, and the necessary variations of initial conditions.

Initial conditions are imposed upon the above named equations by the use of equation (14), with reference to all the previous transformations of variables:

$$V_1 = \left. \frac{dx}{dt} \right|_1 = \frac{\lambda}{2\pi} \left. \frac{dx'}{dt} \right|_1 = \frac{\lambda}{2\pi} k^{1/4} \left. \frac{dx'}{d\tau} \right|_1 = \frac{\lambda}{2\pi} k^{1/4} \tau_1 \quad (23)$$

or, with reference to equation (8),

$$V_1 = \left(\frac{\lambda}{2\pi} \frac{T}{M} \right)^{1/2} \tau_1 \quad (24)$$

V_1 is the launching speed of the rocket, that is, the speed of its flight immediately after its launching, and after the assumed error producing disturbances have occurred.

The complete statement of initial conditions now has the form:

$$\text{When } \tau = \tau_1 = \left(\frac{2\pi M}{\lambda T} \right)^{1/2} V_1,$$

$$y_1, \dot{y}_1, \theta_1, \text{ and } \dot{\theta}_1 = \text{specified values.}$$

In specifying $\dot{\theta}_1$, it is necessary to remember that

$$\dot{\theta}_1 = \left. \frac{d\theta}{d\tau} \right|_1 = k^{-1/4} \left. \frac{d\theta}{dt} \right|_1 = \left(\frac{\lambda M}{2\pi T} \right)^{1/2} \left. \frac{d\theta}{dt} \right|_1 \quad (25)$$

Before the specification of initial conditions can be completed, it is necessary to consider the nature of equations (21) and (22). If the parameter B be chosen zero (no off-axis thrust moment), the remaining terms of these equations constitute a pair of linear, homogeneous equations. For these equations, any two solutions may be added in any linear combination to obtain another solution. Therefore it is sufficient to consider only two cases of each possible initial condition, namely, the given variable is either zero or unity, all other possibilities

and combinations being obtainable by superposition. Furthermore, initial values of y and \dot{y} other than zero have been ruled out by the specification of axes. The apparent necessity of providing for the possibility of launching the rocket with a lateral translational motion is thus covered by the choice of axes and by the possibility of using

$$\theta_1 = \left. \frac{-dy}{dt} \right|_1 / V_1$$

as an initial condition. The only necessary cases of initial conditions are therefore:

$$\theta_1 = 1$$

$$\dot{\theta}_1, \dot{y}_1, \text{ and } y_1 = 0 \quad (26)$$

and $\dot{\theta}_1 = 1$

$$\theta_1, \dot{y}_1, \text{ and } y_1 = 0 \quad (27)$$

The following cases have been computed from equations (21) and (22), with the parameter B set equal to zero:

$$K = 0, .05, .10, \text{ and } .20$$

$$J = 0, .10, .18, .25, .35, \text{ and } .50$$

$$\tau_1 = .5, 1.0, 2.0, \text{ and } 3.0$$

Initial condition sets: (26) and (27)

These comprise a total of 192 cases. The choice of the above parameters and launching conditions was made so as to bracket several known practical weapons, and should be found sufficient for the study of a wide variety of possible proposed rockets. In all cases, the computations were carried to $x - x_1 = 36$, corresponding to a travel of 5.73 yaw wavelengths.

If $B \neq 0$, the pair of equations (21) and (22) are no longer homogeneous. However, any solution to the homogeneous equations can be added to any solution of the nonhomogeneous equations to obtain another solution to the nonhomogeneous equations. Therefore, it is sufficient to use the initial condition

$$\dot{\theta}_1, \theta_1, \dot{y}_1, \text{ and } y = 0$$

in solving the nonhomogeneous cases. All others are then obtainable by adding the previous solutions. Further, owing to the linearity of equations (21) and (22), it is sufficient to consider only a single case of the parameter B , namely, $B = 1$. All other cases are then obtainable by multiplying the values of y and θ obtained for $B = 1.0$ by the actual value of B that it is desired to consider.

Computations for $B = 1$ were performed for the same values of K , J , and τ_1 as those for $B = 0$; these values, used with only the above single set of initial conditions, gave a total of 96 cases.

RESULTS OF COMPUTATIONS

The complete results of the above 288 trajectory computations are too voluminous for inclusion in this report. It is most important for accuracy considerations to know the lateral drift as a fraction of range distance (Here "lateral" refers to going off target in any direction). This number is therefore given for each of the 288 computations for a range of 5.73 yaw cycles (the arbitrarily chosen cut-off point) in Tables 1 through 24. Each table gives the results for a particular pair of K and J values. The first column of each table gives the value of τ_1 , which involves the launching speed of the rocket, its thrust-to-mass ratio (acceleration), and its yaw wavelength. The second and third columns give the results for trajectories without off-axis thrust. The second column gives the y/x value for the case of initial yaw angle (weathercocking effect), and the third column gives the y/x value for the case of an initial yaw rate disturbance. The fourth column is for the case of off-axis thrust.

Reference is made to equations (17), (18), (19), and (24), for the explicit definitions of K , J , B , and τ_1 , as well as to the list of symbols, pages iv - vi.

Samples of the complete behavior of θ and y/x for an initial yaw angle disturbance are given graphically for the case $K = .05$, $J = 0$, in figures 1 and 2. Also on the same graphs, the dashed lines or line segments show, for comparison, the effect of jet damping to the extent of $J = 0.5$ (a value perhaps twice as large as might be expected in practice). It is seen from the y/x curves in figure 2 that y/x does not change appreciably after the first yaw cycle. Hence, it was considered sufficient to tabulate only the single value obtained for the range of 5.73 yaw cycles.

Figures 3 and 4 give sample trajectories for the case of an initial yaw rate disturbance, again for $K = .05$, $J = 0$, and with some dashed line segments indicating the effect for $J = 0.5$. To interpret the results for initial yaw rate disturbances, it is essential to remember that the standard input yaw rate disturbance is based upon the dimensionless time variable defined earlier, not on physical time. The standard input, based on equation (25) is

$$\left(\frac{\lambda M}{2\pi I}\right)^{1/2} \frac{d\theta}{dt} \Big|_1 = 1.0$$

Figures 5 and 6 give sample trajectories with off-axis thrust. The parameter B (equation (19)) was taken as 1.0. This can be shown as equivalent to making the moment arm of the off-axis thrust the same fraction of the radius of gyration of the rocket as the radius of gyration is of the distance unit $\lambda/2\pi$. It is noted from equation (19) that the thrust does not appear in the expression for B , having been cancelled in combination with the dimensionless time variable. This does not mean that the behavior with off-axis thrust is independent of the acceleration of the rocket, however, since the acceleration is involved in the initial conditions through the influence of τ_1 . $B = 1.0$ is probably somewhat larger than is to be expected in practice, although it is of the correct order of magnitude. For a typical situation this may correspond to a thrust moment arm of .002 times the length of the rocket. Reference (2) gives information about the amount of thrust misalignment which has been experienced in practice.

Figure 7 shows graphically the information in the second column of Tables 1 through 4, giving the final y/x values for initial yaw angle and without jet damping. Figure 8 is the corresponding plot from Tables 21 through 24, which are the results for the maximum value of jet damping.

Figure 9 is a plot of the third column of Tables 1 through 4. It shows the cases of initial yaw rate disturbance without jet damping. Figure 10 is plotted from the third column of Tables 21 through 24, giving the same information with maximum jet damping.

Figure 11 shows the effect of off-axis thrust without jet damping, being a plot of the fourth column of Tables 1 through 4; and finally, figure 12, plotted from Tables 21 through 24, gives the same information as modified by maximum jet damping.

An empirical equation which represents the data plotted in figure 7, for $\theta_1 = 1.0$, to within a few percent, is

$$y/x = (1.043 - .36K - .08K^2) \exp \left[(-1.143 + .90K + .11K^2) \tau_1 \right] \quad (28)$$

An empirical equation which represents figure 9, for $\theta_1 = 1.0$, is

$$y/x = (.795 - .24K + 1.86K^2) \exp \left[(-1.082 + 2.70 K - 5.02K^2) \tau_1 \right] \quad (29)$$

A rather coarse formula which could be useful for making quick estimates of the weathercocking effect is

$$y/x = \exp (-\tau_1) \quad (30)$$

INTERPRETATION OF RESULTS

OFF-AXIS THRUST

Potentially, off-axis thrust is the worst offender among the causes of errors in the trajectories of unguided rockets. References (1), (2), and (3) contain discussions of this problem. Figure 11 shows that a rocket with a B value of 1.0 launched at initial condition $\tau_1 = 1.0$, and having a K value of .05, would have a lateral error of .30 of the range, or 300 mils, after a flight of 5.7 yaw cycles. Figure 6 shows how this error varies with range; after one yaw cycle it is already 187 mils. Errors of this order are intolerable.

The means of reducing this error to a tolerable level are of three kinds. The first is careful manufacture, in order to avoid asymmetries in mass distribution, propellant burning rate, rocket nozzle contour, or aerodynamic surfaces. The second is to increase τ_1 by providing the rocket with large stabilizing fins in order to make the static stability as high as possible, and by launching the rocket at lowered acceleration from an airplane traveling with high air speed. It appears from figure 11 that a reduction of the error by a factor of 2 or 3 might be achieved in this way. This would leave an error of the order, depending upon range, of 50 to 150 mils, which is still too high for most uses.

The third method of reducing the error of off-axis thrust is to cause the rocket to roll at a moderate speed about its longitudinal axis. This allows the angular impulse of the thrust to work successively in all directions, causing a very large degree of cancellation in the integrated effect. A quantitative statement of this fact is in order.

For the simplified case of planar yawing motion, which has been assumed in the analysis, the thrust moment which corresponds to a rocket rolling with angular velocity Ω would be a sine or cosine function of the argument Ωt . Since the total angular impulse given to the rocket in a specified time interval is the time integral of this sinusoidal moment function over the time interval, it follows that the maximum possible value of the integral is $2/\Omega$ times the magnitude of the thrust moment, and that the value can range from zero to this maximum, with a most probable value, if the length of the time interval is random, of $1/\Omega$ times the thrust moment.

This approach to the problem assumes that gyroscopic effects are small, as they will be unless the roll rate used is so large as to approach those used for spin-stabilized projectiles. The roll rates contemplated here should be sufficient to cause an otherwise planar yawing motion to precess slowly about the longitudinal axis of the rocket, without appreciable effect upon the magnitudes and frequencies of the oscillations.

It is thus apparent that a reduction of the error of off-axis thrust of the order of a factor of 50 to 100 could be achieved by rolling the rocket at speeds from 8 to 16 revolutions per second (50 to 100 radians per second). It is important, in a specific case, to avoid rolling the rocket at a speed approximately corresponding to 1 revolution per yaw

cycle, which leads to a catastrophic type of instability called roll-yaw resonance. A roll rate either less than half or at least twice the resonance value should avoid this problem.

Inspection of figure 6 reveals that at least half the error of off-axis thrust occurs at short range, within the first yaw cycle. It is obvious that, to be fully effective as a means of reducing error, the rolling of the rocket at the desired speed should begin before the error builds up, that is, very early in the trajectory, preferably before the rocket leaves the constraint of its launcher. Slow acceleration in roll through the resonance speed at the beginning of the trajectory could in some cases completely undo any benefit to be gained by the roll.

The foregoing discussion suggests that the practice of using large fins on the type of rocket being considered probably provides its benefit because of the rolling motion induced by the fins (be it intentional or accidental), and not because of the marginal improvement pertaining to the increase of the τ_1 value.

Conclusions summarizing this discussion are given in a later section.

WEATHERCOCKING ERRORS

Weathercocking, as mentioned earlier, is the tendency of a rocket to fly into the relative wind instead of the direction it is pointed at the time of firing, if these two directions be different. The part of the present analysis relevant to weathercocking is that part having the initial condition $\theta_1 = 1.0$. A rocket pointed in one direction, but launched in another, travels to a destination which is a weighted average of the two directions. The y/x value obtained for the corresponding trajectory is this average, being the ratio of the angle between the initial velocity vector and the impact point to the total angle between the initial velocity vector and the direction in which the rocket was pointed.

Whether it is desirable to design a rocket with a high or low y/x value depends upon whether the attacking pilot is better able to control accurately the direction the rocket points, or the direction in which the rocket initially travels, respectively. A high value binds the impact point tightly to the pointing direction and loosely to the relative wind, while a low value puts the stronger control with the relative wind

at the time of launching.

Ideally, both of these directions would coincide with the direction of the desired target, in which case the actual value of y/x would be immaterial. In practice, it has been found (reference (3)) that a good pilot can aim the aircraft and its rockets to within about a 2 or 3 mil accuracy. This determines the pointing direction of the rockets to the same accuracy. The aerodynamic angle of attack of the airplane, as discussed earlier, varies with a rather large number of variable flight conditions, some either difficult to control exactly or difficult to observe. The pilot is thus faced with the problem of aiming the airplane, which he can see, and at the same time trying to fly in some standard pattern so that the velocity vector, which he cannot see, will also be properly aimed. It can reasonably be assumed that the aiming of the rockets is carried out with somewhat better precision than is the aiming of the velocity vector of the aircraft.

If the y/x value for the rocket be known, any systematic deviation between the rocket pointing direction and the initial velocity vector can be compensated by the adjustment of the sight. Dispersion at the point of impact can then be expected according to the following equation:

$$m = 1,000(1 - \frac{y}{x})\Delta\alpha_A \quad (31)$$

An equation for α_A is

$$\alpha_A = \frac{2W'}{\rho_a V_A^2 C_{N\alpha_A} S} \quad (32)$$

$$\text{with } W' = W(\cos\delta + \frac{a}{g}) \quad (33)$$

Logarithmic differentiation of equation (32) gives:

$$\frac{\Delta\alpha_A}{\alpha_A} = \frac{\Delta W'}{W'} - \frac{\Delta\rho_a}{\rho_a} - \frac{2\Delta V}{V} \quad (34)$$

showing the multiplicity of influences affecting the angle of attack, and hence the dispersion of the rocket.

The value of $\Delta\alpha_A$ from equation (34) may be substituted into equation (31), giving

$$m = 1,000(1 - \frac{y}{x})\alpha_A(\frac{\Delta W'}{W'} - \frac{\Delta\rho\alpha}{\rho\alpha} - \frac{2\Delta V_A}{V_A}) \quad (35)$$

For a given rocket, y/x is controlled by the value of τ_1 , which is controlled by the launching speed, all other factors influencing τ_1 having been fixed by the design of the rocket. Also for a given airplane, the angle of attack is inversely proportional to the flight speed, as shown in equation (32). From this point of view, the most desirable launching speed is that

which minimizes the value of $\tau_1^{-2}(1 - y/x)$. It can be shown

by numerical substitution of the y/x values from figure 7 into

this product that it is dominated by τ_1^{-2} , so that the effect of high launching speed in decreasing variations in the angle of attack of the airplane is stronger than its effect in increasing the tendency of the rocket to follow these variations. It is concluded that with a given rocket and a given airplane, accuracy is enhanced by firing at the highest practicable flying speed.

Nevertheless, in the design of the rocket, it is advantageous to minimize $(1 - y/x)$, which will be accomplished by designing the rocket for high acceleration and long yaw wavelength.

In the choice of an airplane, or in the design of one for the purpose, the airplane capable of flying during the attack with the least value of α_A , regardless of its speed, would tend to deliver the rocket with the least weathercocking dispersion due to angle of attack variations.

As a reservation on the above statements, it should be emphasized that they depend on the previously implied premise that the aiming of the aircraft is more accurate than the control of the angle of attack. If the aim point, due to causes within the airplane, "jitters" around during the attack this may not be true.

In such case, the inherent time lag of the velocity vector of the airplane in following such motions of the airplane would tend to perform a smoothing function. The velocity vector of the airplane would then follow a time average, over a time

interval of the same order as the lag mentioned, of the pointing direction of the airplane, and this average would probably be more accurate than the instantaneous pointing direction. If the "jitter" of the aim point, however, is caused by rough air encountered by the airplane, the time lag is the other way around, and the smoothing acts in favor of the superior accuracy of the aim point.

For a numerical example, it may be supposed that the angle of attack of the airplane is of the order of .04 radian. Equation (34) would then predict that a 10 percent change in W' , or a 5 percent change in the speed, would cause a 10 percent change in the angle of attack, giving $\Delta\alpha_A = 4$ mils. If a τ_1 value of 1.0 be assumed, the dispersion would be $m = 2.5$ mils.

ERRORS ARISING FROM INITIAL YAW RATE

Initial yaw rates are caused by any form of angular impulse which happens to operate on the rocket momentarily at the time of firing. The chief aerodynamic source of such an impulse would be the cross flow components encountered by the rocket before it clears the aerodynamic flow field of the attacking aircraft. A region of cross flow might in a particular case be either larger than the length of the rocket, or smaller. If it be larger, the angular velocity imparted to the rocket can be computed by means of equation (4).

$$\left. \frac{d^2\theta}{dt^2} \right|_1 = - \frac{C_{M_\alpha} \rho_\alpha V_1^2 AD}{2I} \theta_F, \quad (36)$$

where θ_F is an effective angle of cross flow. Substitution of equation (5) gives (without regard to sign)

$$\left. \frac{d^2\theta}{dt^2} \right|_1 = \left(\frac{2\pi}{\lambda} \right)^2 V_1^2 \theta_F \quad (37)$$

If the rocket must gain the distance C (perhaps about one wing chord length) on the aircraft in order to escape from the cross flow, the time required is

$$\Delta t = \left(\frac{2CM}{T}\right)^{1/2} \quad (38)$$

Analogous to equation (37), $\frac{d\theta}{dt}_1 = \left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{2CM}{T}\right)^{1/2} V_1^2 \theta_F$

Now, by use of equations (24) and (25), it can be shown that

$$\dot{\theta}_1 = \left(\frac{4\pi C}{\lambda}\right)^{1/2} \theta_F \tau_1^2 \quad (40)$$

The deviation of the rocket caused by the cross flow is now found by multiplying $\dot{\theta}_1$ from equation (40) by the appropriate y/x value from figure 9:

$$\text{deviation (mils)} = 1,000(y/x) \left(\frac{4\pi C}{\lambda}\right)^{1/2} \theta_F \tau_1^2 \quad (41)$$

θ_F may be thought of as composed of three superimposed effects: (a) the angle of attack of the aircraft, (b) the flow field arising from the passage of the bulk of the aircraft, and (c) the circulation flow, or other modifications to these effects caused by the action of the lifting surfaces. It can be shown from the line vortex analogy to the lifting airfoil that the flow inclination angles caused by effect (c) are proportional to the lift, and therefore to the angle of attack of the airfoil. Consequently, effects (a) and (c) may be lumped together, and their sum may be assumed to be proportional to the angle of attack. Further, it can be seen that in a region very near the airfoil surface effects (a) and (c) must indeed cancel because the flow is constrained to remain parallel to the surface. This fact, together with the fact that the angle of attack of the airplane remains of the same order at various flying conditions, leads to the strong likelihood that θ_F is dominated by the effect (b), and that variations in θ_F may be neglected. At some distance from the airfoil, where (a) and (c) do not cancel, the effect (a) is merely the weathercocking effect discussed previously.

Dispersion caused by the cross flow would result from any variations of the product $\tau_1^2 y/x$, appearing in the right side of equation (41). Figure 13 gives a plot of this product for various values of K . For fixed designs of the rocket and the airplane, this plot may be regarded qualitatively as showing the deviation due to cross flow as a function of launching speed. The slope of the curves is therefore an

indication of the dispersion to be expected. In the fortunate circumstance that the operating point could be placed near a maximum in its K-curve, there should be little dispersion, but merely a constant correction to the sight.

A safer assumption, based on the steepest slopes in figure 13, would be that the dispersion from cross flow is about the same fraction of the total deviation due to cross flow as the uncertainty in launching speed is of the total launching speed. This assumption leads to the following equation for the dispersion:

$$m = 1,000 \theta_F \tau_1^2 (y/x) \left(\frac{4\pi C}{\lambda} \right)^{1/2} \frac{\Delta V_1}{V_1} \quad (42)$$

If, for a numerical example, C be taken as 12 ft., λ as 1200 ft., θ_F as .1 radian, τ_1 as 2.0, K as .05, and $\Delta V_1/V_1$ as .10, equation (42) and figure 9 give $m = 1.5$ mils, which may be regarded as a typical dispersion due to cross flow. The deviation (sight correction) due to cross flow would be expected to be perhaps ten times this amount, or 15 mils. The dispersion, as well as the constant deviation, can be reduced by long yaw wavelength. It is not very sensitive to T/M.

A reservation on the foregoing statement is that the dispersion would be only slightly sensitive to yaw wavelength in case the operating point be near a maximum of one of the curves of figure 13.

A second reservation is that the statement is based upon the premise that the rocket has a fixed configuration. If a rocket should be provided with folding fins, which remain out of action until the cross flow has been traversed, the yaw wavelength (with fins unfolded) would have the reverse effect, since decreasing the yaw wavelength would decrease the y/x factor.

If an atmospheric gust of length G and lateral component of velocity v be traversed by the rocket, similar considerations to those given for cross flow lead to the equation

$$m = 1,000 \frac{y}{x} \left(\frac{2\pi}{\lambda} \right)^{3/2} \left(\frac{M}{T} \right)^{1/2} vG \quad (43)$$

In this case, an increase in λ increases y/x but decreases the

factor $(\frac{2\pi}{\lambda})^{3/2}$. The latter factor predominates the dependence on λ . An increase in T/M decreases the factor $(\frac{M}{T})^{1/2}$, and thus decreases τ_1 and increases y/x . The result is that the dispersion is not very sensitive to T/M .

Other than aerodynamic causes of yaw rate disturbance are conceivable. If an angular impulse of fixed magnitude, perhaps arising from a mechanical vibration in the carrying airframe, be imparted to the rocket, it is necessary to examine specific cases numerically in order to determine the effect. Examination of figure 9, consideration of the influence of the moment of inertia of the rocket upon τ_1 , and consideration of its influence upon the initial angular velocity point strongly toward the generalization that in most cases it will be found beneficial to have a large moment of inertia, that is, a large rocket, compared to a small one.

If a constant angular velocity be imparted to the rocket, perhaps by a yawing motion of the airplane itself at the time of firing, it is safe to say that the effect on a larger rocket will be worse than that on a smaller one, since both the dimensionless yaw rate, from equation (25), and the y/x factor, from figure 9, are adversely affected by large λ .

CONCLUSIONS

It must first be emphasized that all conclusions, including those already stated in the previous chapter, must be regarded in the light of the assumptions and approximations made. To reach firm conclusions about the desirability of various possible design features, each specific case must be evaluated individually, and the properties of the attacking aircraft, as well as those of the rocket, must be taken into account. If a very accurate estimate of dispersion is essential, more accurate equations of motion (see references (1) and (2)) accounting for mass, thrust, and moment of inertia variations, as well as for aerodynamic drag and damping, should be used, and trajectory computations carried out.

Nevertheless it seems worthwhile to draw a few general conclusions on the basis of the present analysis. They are as follows:

1. Off-axis thrust is potentially the worst source of error to be expected. The error may be lessened to a degree by careful manufacture, by providing high static stability, and by launching the rocket at a high initial speed with low acceleration. These measures alone will normally be insufficient. The most effective method of reducing the error is to provide the rocket with a suitable rate of roll, as indicated earlier.

2. For the reduction of errors other than those caused by off-axis thrust, it will usually be found that low static stability, high launching speed, large moment of inertia, and a considerable amount of damping (jet or aerodynamic) are beneficial. There appears to be no strong influence of the thrust-to-mass ratio upon these errors.

3. In view of the above, and in view of the fact that large fins are a nuisance to manufacture, as well as a nuisance in packaging and handling the weapons, it seems likely that some improvement could be realized by reducing the fin size to a minimum for static stability and by providing a suitable spin rate by some other method, such as rifling the launcher or inserting canted vanes in the rocket exhaust. In either case, best accuracy will be achieved if the rocket comes out of the launcher spinning.

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TABLE 1

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	B = 0	B = 0	B = 1.0
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	K = 0		J = 0
0.5	.593	.482	.411
1.0	.324	.263	.234
2.0	.0935	.0840	.0860
3.0	.0306	.0322	.0375

TABLE 2

	K = .05		J = 0
0.5	.599	.514	.496
1.0	.332	.291	.305
2.0	.1011	.1050	.136
3.0	.0362	.0478	.0733

TABLE 3

	K = .10		J = 0
0.5	.604	.544	.579
1.0	.340	.318	.375
2.0	.1085	.126	.185
3.0	.0417	.0633	.1088

TABLE 4

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = .20$		$J = 0$
0.5	.613	.603	.742
1.0	.354	.372	.512
2.0	.123	.166	.281
3.0	.0523	.0936	.179

TABLE 5

	$K = 0$	$J = .10$
0.5	.600	.460
1.0	.332	.253
2.0	.0989	.0824
3.0	.0332	.0318

TABLE 6

	$K = .05$	$J = .10$
0.5	.606	.489
1.0	.342	.280
2.0	.1081	.1029
3.0	.0402	.0473

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TABLE 7

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = .10$		$J = .10$
0.5	.612	.517	.562
1.0	.351	.306	.367
2.0	.117	.123	.183
3.0	.0471	.0626	.1081

TABLE 8

	$K = .20$		$J = .10$
0.5	.623	.572	.721
1.0	.368	.357	.501
2.0	.134	.162	.278
3.0	.0603	.0924	.177

TABLE 9

	$K = 0$		$J = .18$
0.5	.606	.443	.389
1.0	.339	.246	.225
2.0	.1030	.0811	.0842
3.0	.0353	.0316	.0371

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TABLE 10

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = .05$		$J = .18$
0.5	.612	.471	.470
1.0	.349	.272	.293
2.0	.1135	.1013	.133
3.0	.0434	.0469	.0725

TABLE 11

	$K = .10$		$J = .18$
0.5	.619	.498	.550
1.0	.359	.297	.361
2.0	.124	.121	.181
3.0	.0513	.0620	.1075

TABLE 12

	$K = .20$		$J = .18$
0.5	.631	.549	.704
1.0	.378	.346	.492
2.0	.143	.159	.275
3.0	.0665	.0915	.176

TABLE 13

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = 0$		$J = .25$
0.5	.610	.429	.381
1.0	.344	.240	.221
2.0	.107	.0800	.0835
3.0	.0371	.0314	.0369

TABLE 14

	$K = .05$		$J = .25$
0.5	.617	.456	.461
1.0	.356	.266	.289
2.0	.118	.1000	.132
3.0	.0462	.0466	.0722

TABLE 15

	$K = .10$		$J = .25$
0.5	.624	.481	.539
1.0	.366	.290	.356
2.0	.129	.119	.180
3.0	.0549	.0615	.107

TABLE 16

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = .20$		$J = .25$
0.5	.637	.530	.691
1.0	.387	.337	.485
2.0	.150	.157	.273
3.0	.0719	.0907	.175

TABLE 17

	$K = 0$		$J = .35$
0.5	.616	.411	.370
1.0	.352	.232	.217
2.0	.112	.0785	.0826
3.0	.0397	.0310	.0367

TABLE 18

	$K = .05$		$J = .35$
0.5	.624	.436	.449
1.0	.364	.257	.284
2.0	.125	.0981	.131
3.0	.0500	.0461	.0718

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TABLE 19

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = .10$		$J = .35$
0.5	.631	.460	.525
1.0	.376	.280	.349
2.0	.137	.117	.178
3.0	.0601	.0609	.106

TABLE 20

	$K = .20$	$J = .35$
0.5	.646	.505
1.0	.399	.325
2.0	.161	.154
3.0	.0794	.0895

TABLE 21

	$K = 0$	$J = .50$
0.5	.624	.386
1.0	.363	.221
2.0	.119	.0764
3.0	.0434	.0306

TABLE 22

τ_1 y/x at $x = 36$ (5.73 yaw cycles)

	$B = 0$	$B = 0$	$B = 1.0$
	$\theta_1 = 1.0$	$\dot{\theta}_1 = 1.0$	$\theta_1 = \dot{\theta}_1 = 0$
	$K = .05$		$J = .50$
0.5	.633	.409	.431
1.0	.377	.244	.276
2.0	.134	.0955	.129
3.0	.0557	.0454	.071

TABLE 23

	$K = .10$		$J = .50$
0.5	.642	.431	.505
1.0	.390	.266	.340
2.0	.148	.114	.175
3.0	.0676	.0599	.105

TABLE 24

	$K = .20$		$J = .50$
0.5	.657	.472	.647
1.0	.415	.308	.462
2.0	.176	.149	.265
3.0	.0904	.0879	.172

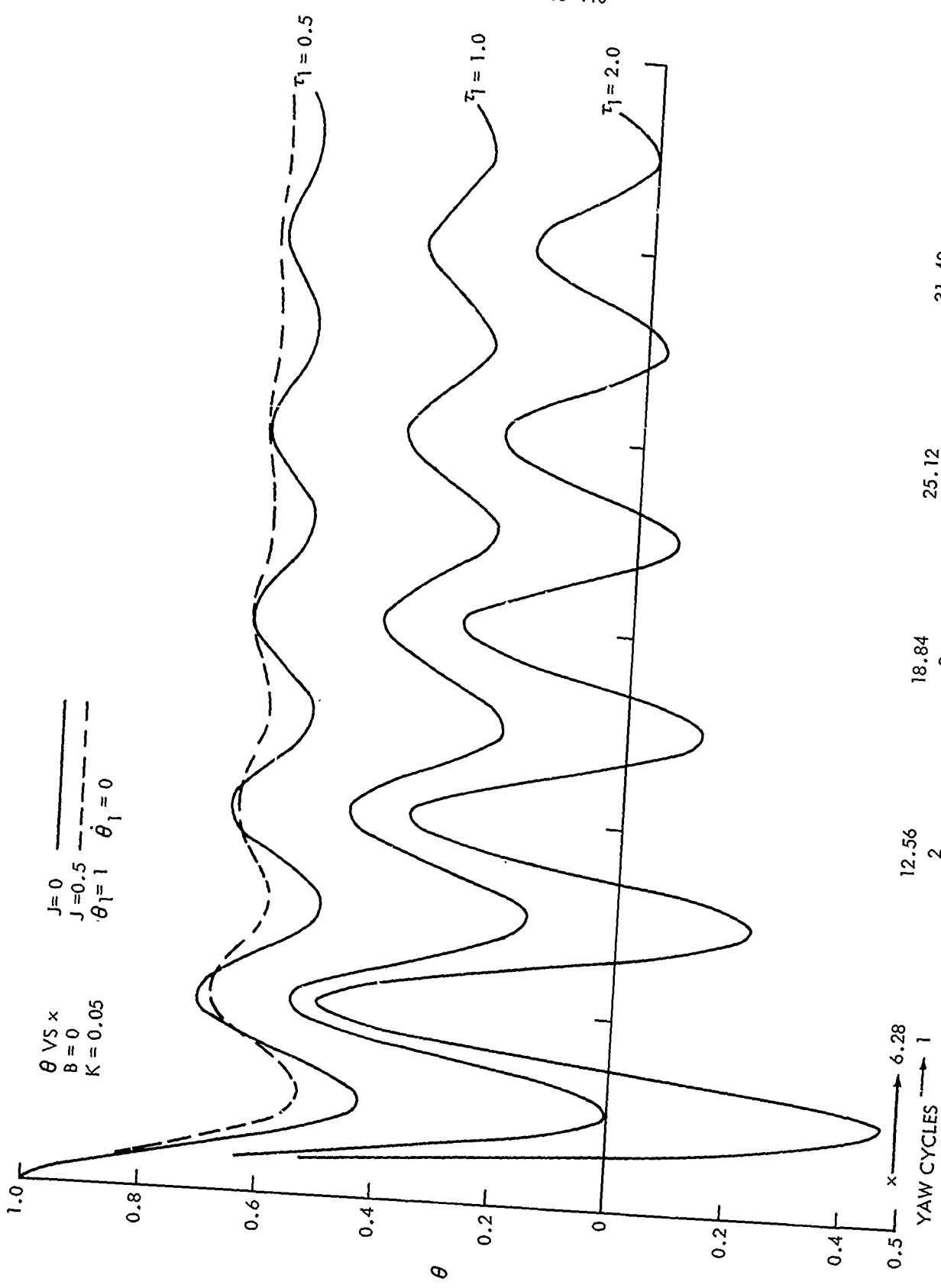


FIG. 1 YAWING MOTION ($\theta_1 = 1.0$)

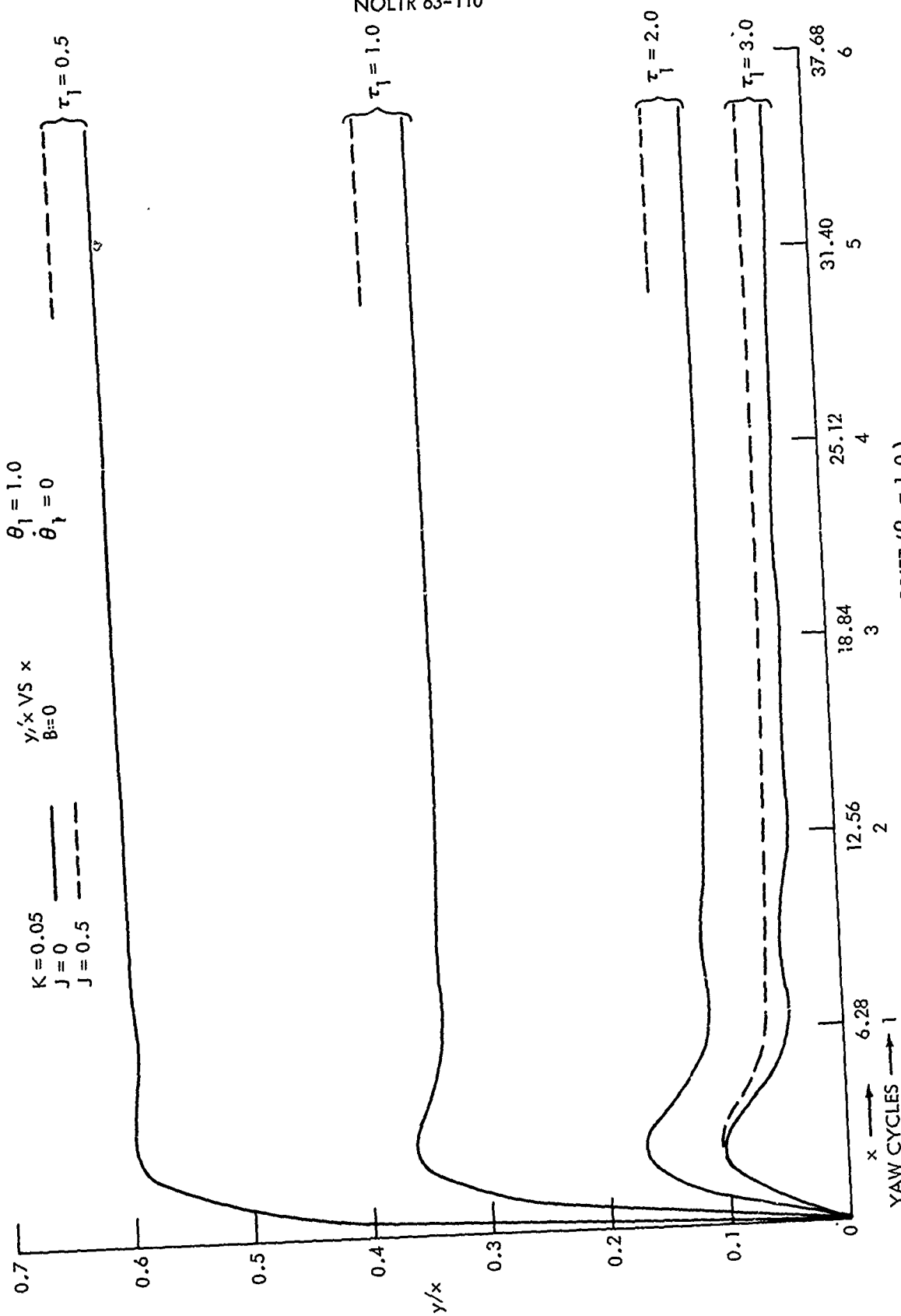


FIG. 2 LATERAL DRIFT ($\theta_1 = 1.0$)

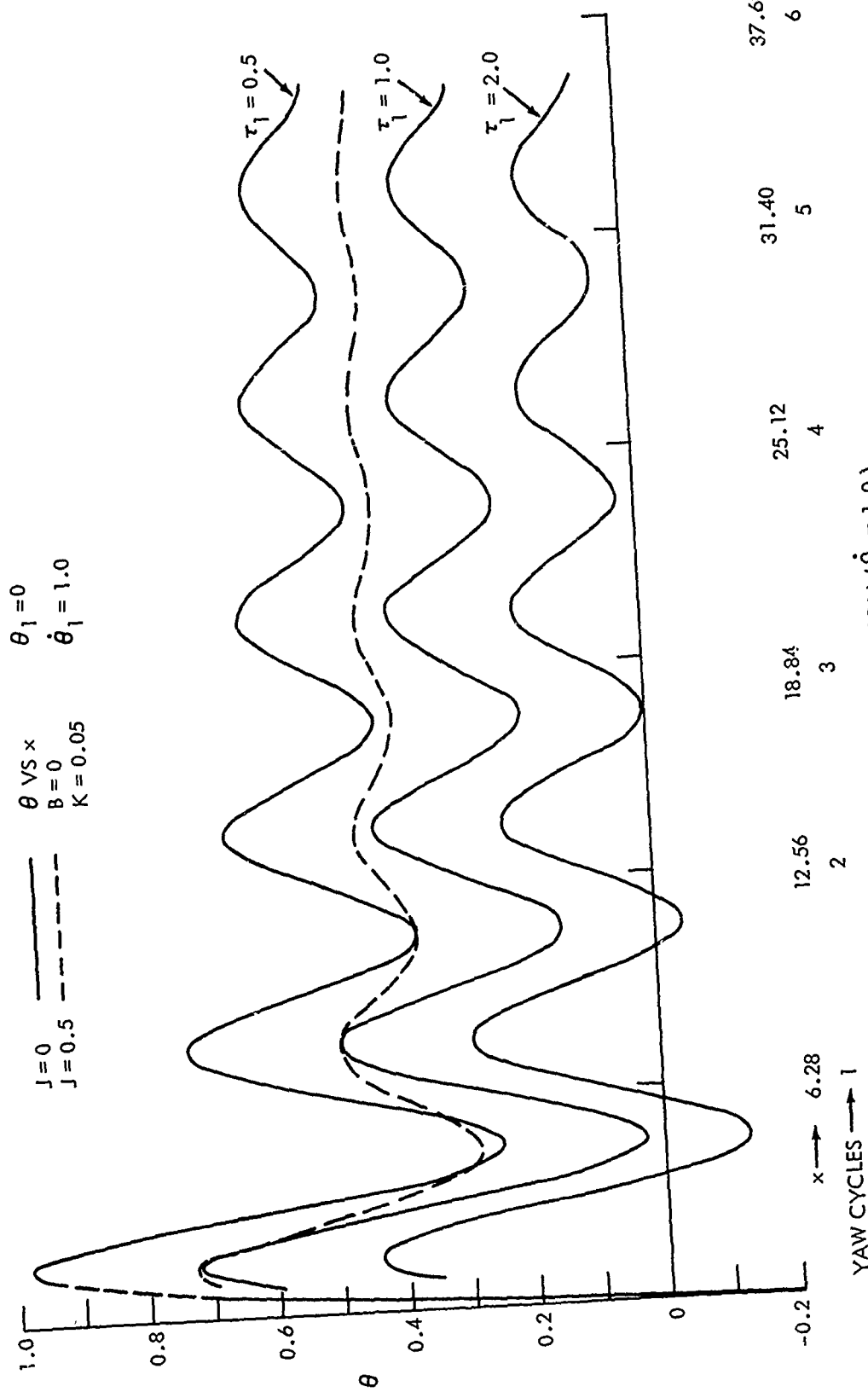


FIG. 3 YAWING MOTION ($\dot{\theta}_1 = 1.0$)

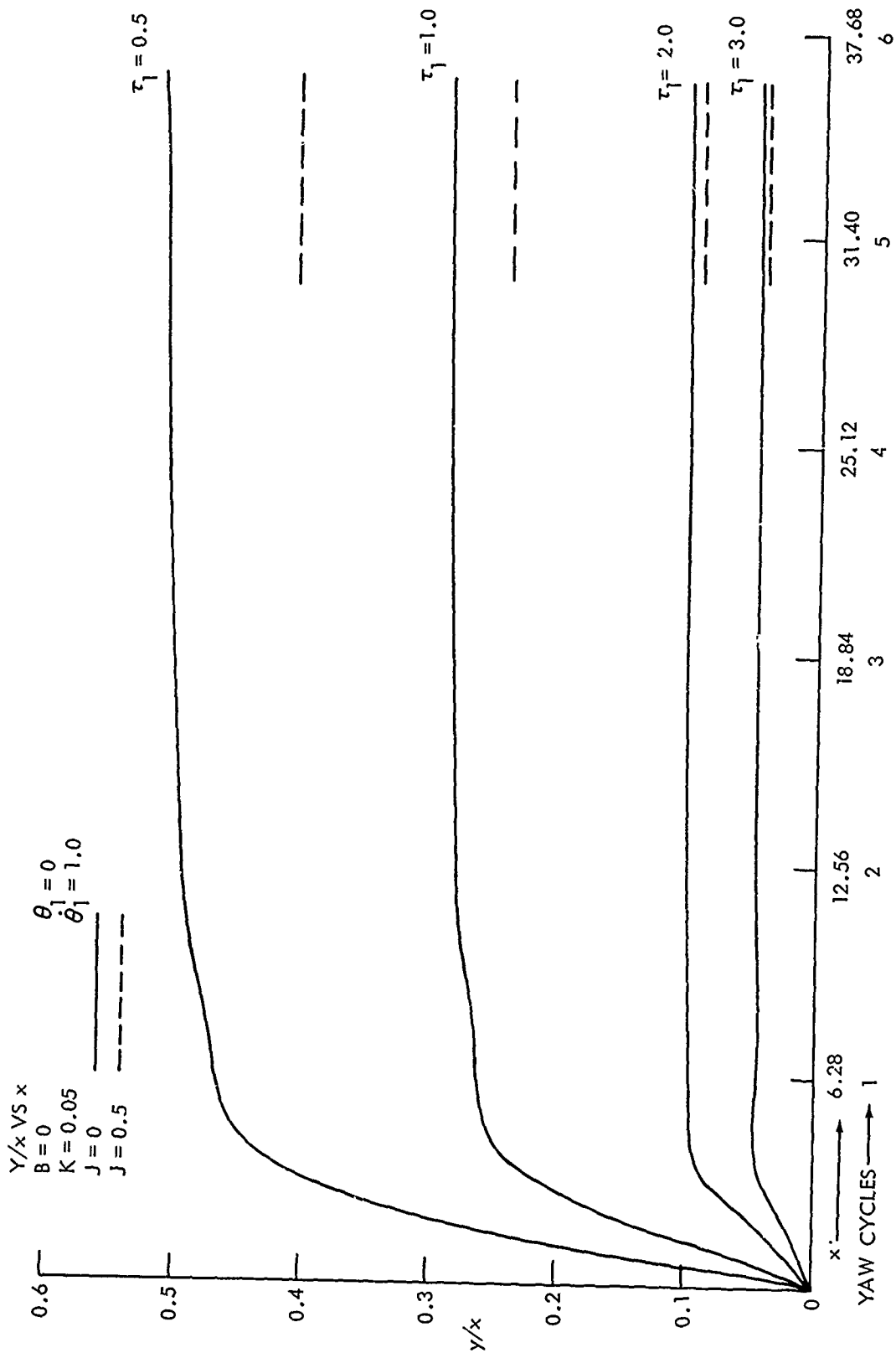


FIG. 4 LATERAL DRIFT ($\dot{\theta}_1 = 1.0$)

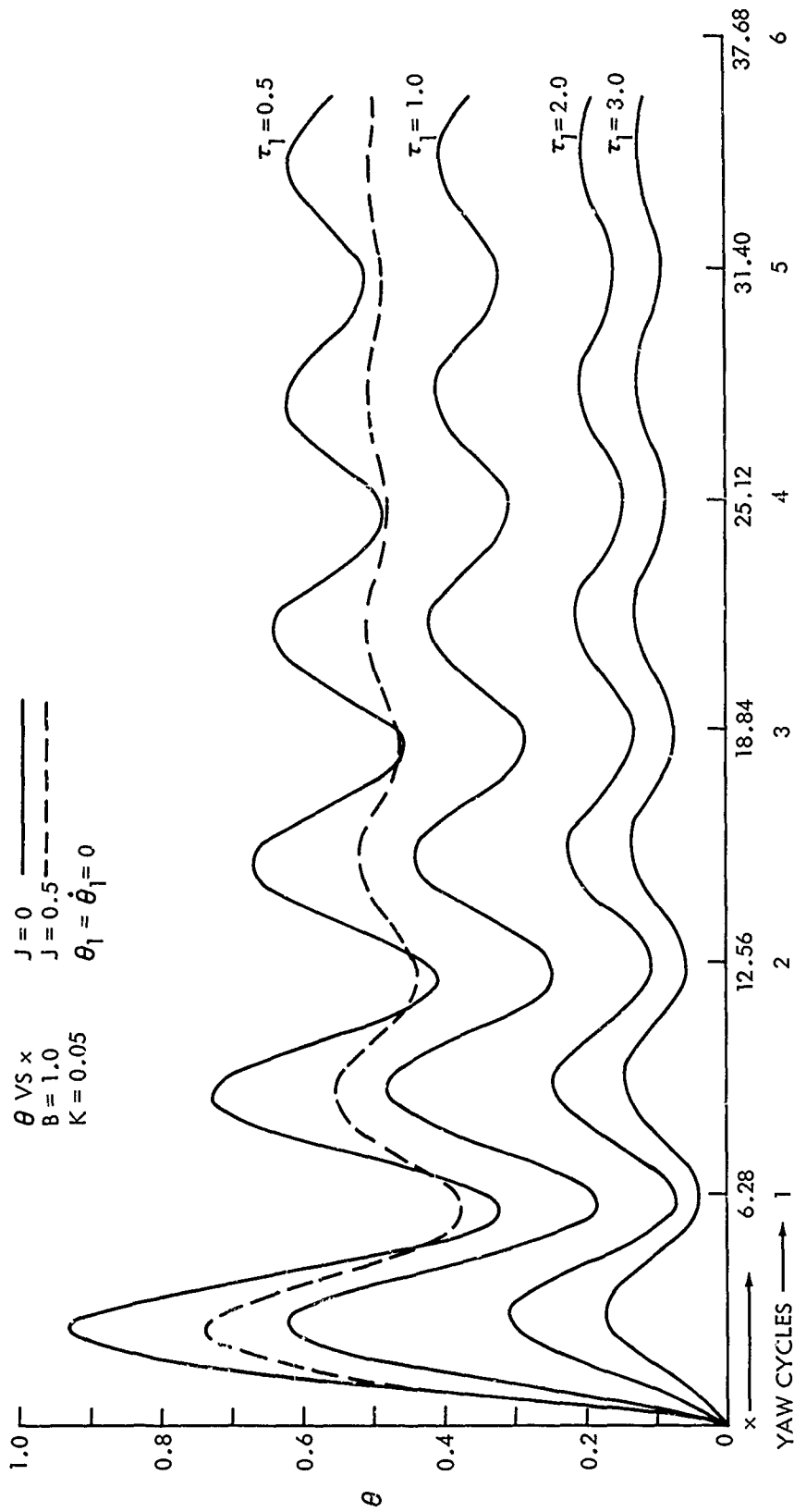


FIG. 5 YAWING MOTION WITH OFF-AXIS THRUST

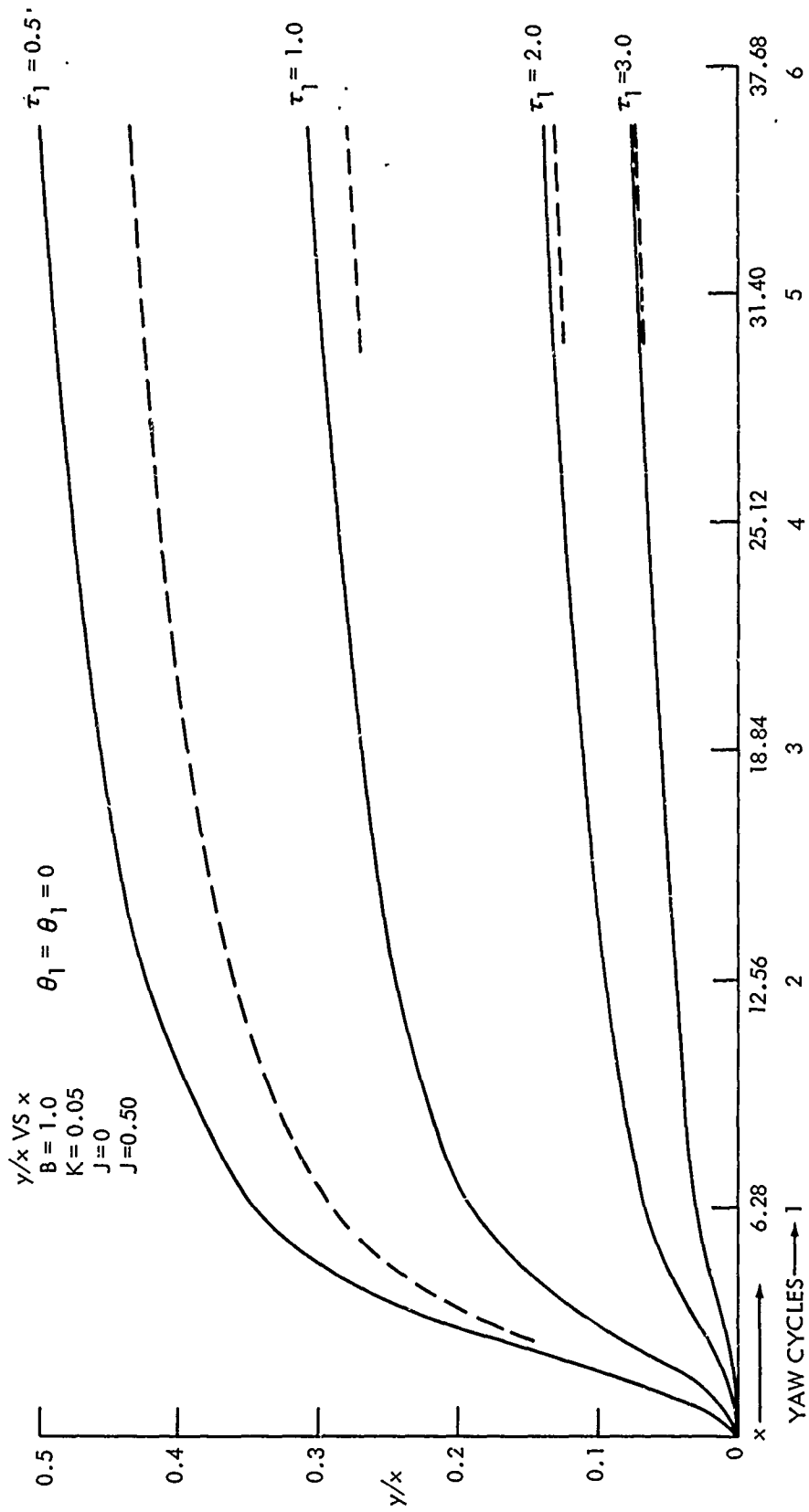


FIG. 6 LATERAL DRIFT WITH OFF-AXIS THRUST

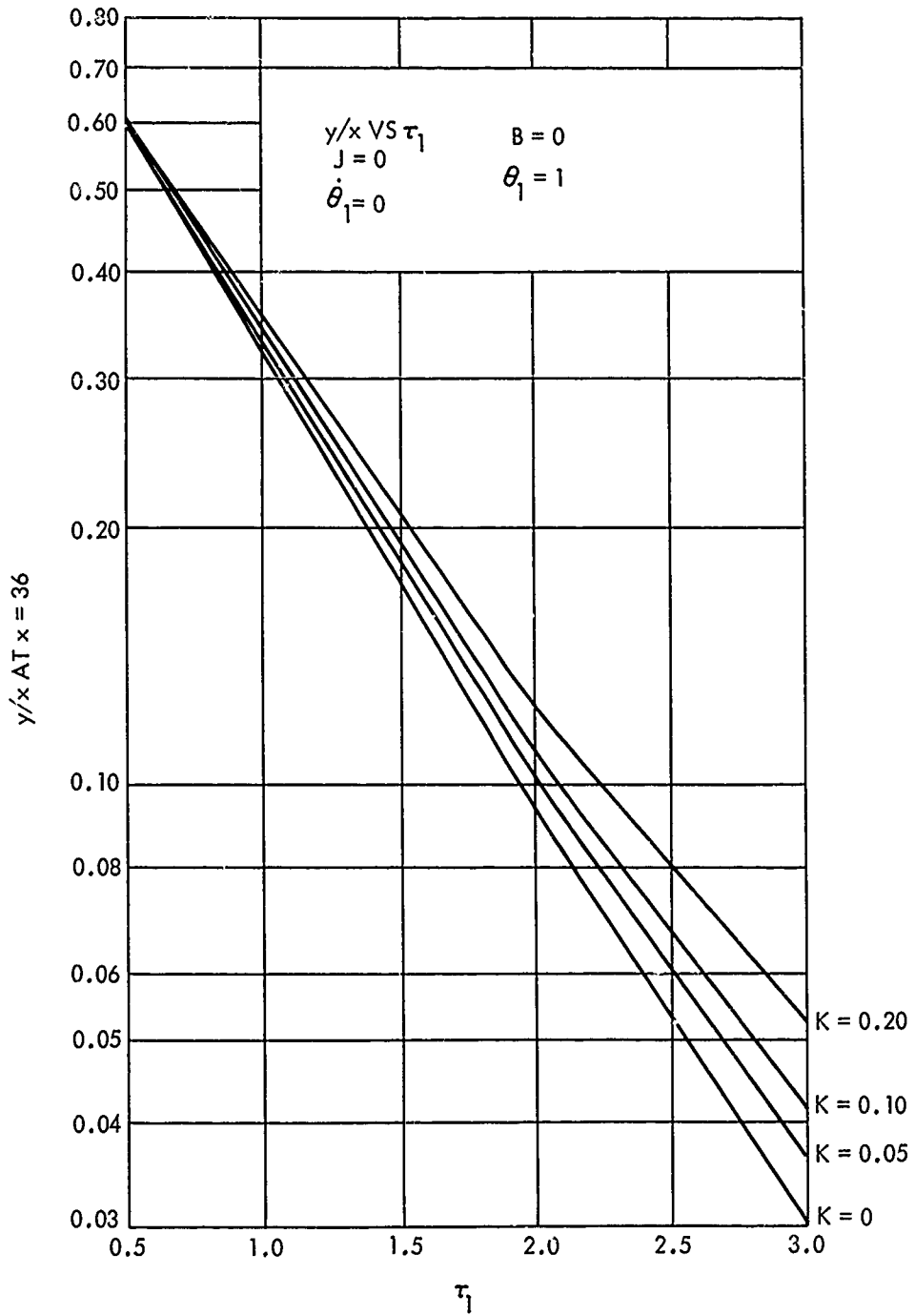


FIG. 7 INITIAL YAW ANGLE

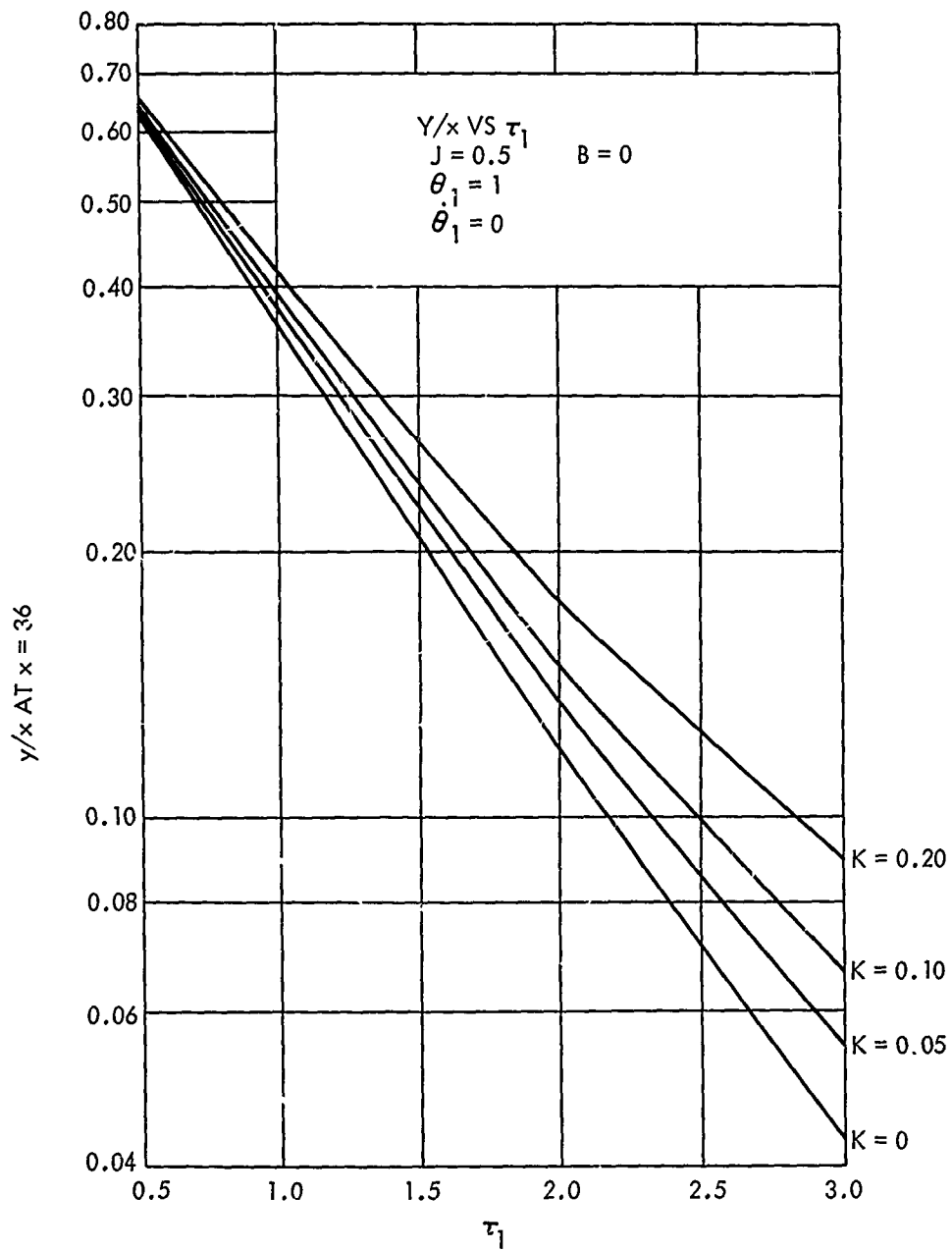


FIG. 8 INITIAL YAW ANGLE WITH JET DAMPING

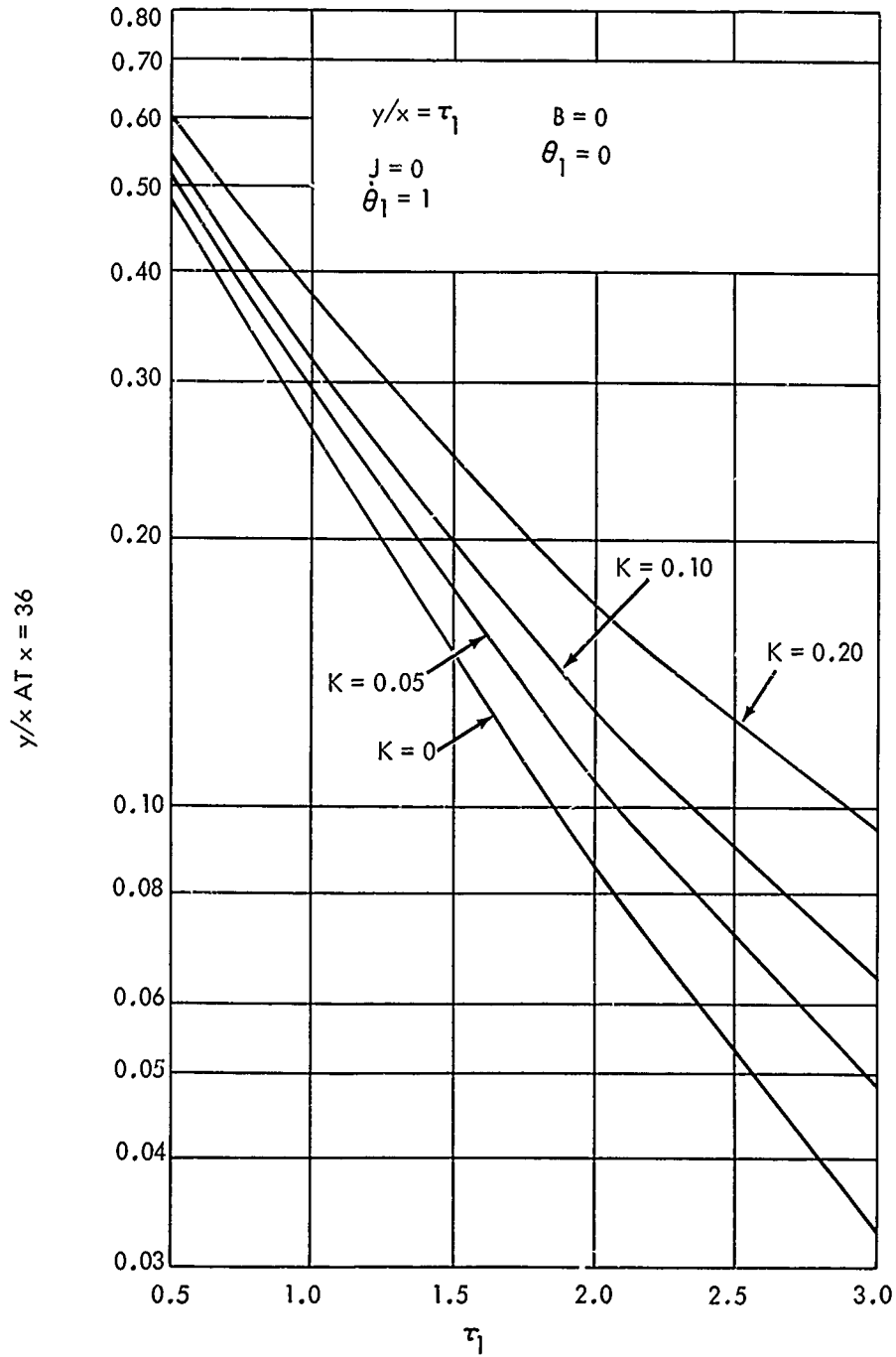


FIG. 9 INITIAL YAW RATE

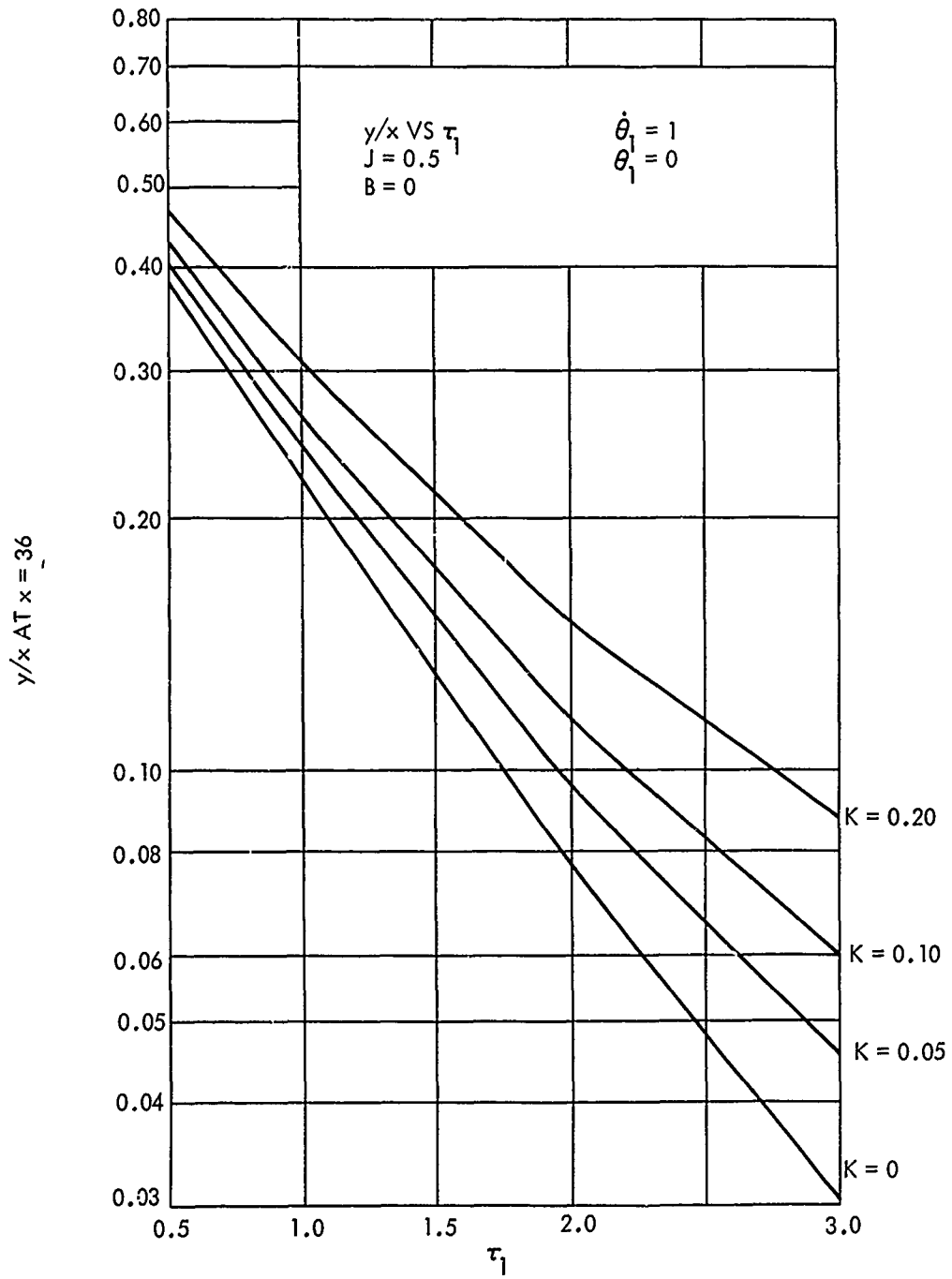


FIG. 10 INITIAL YAW RATE WITH JET DAMPING

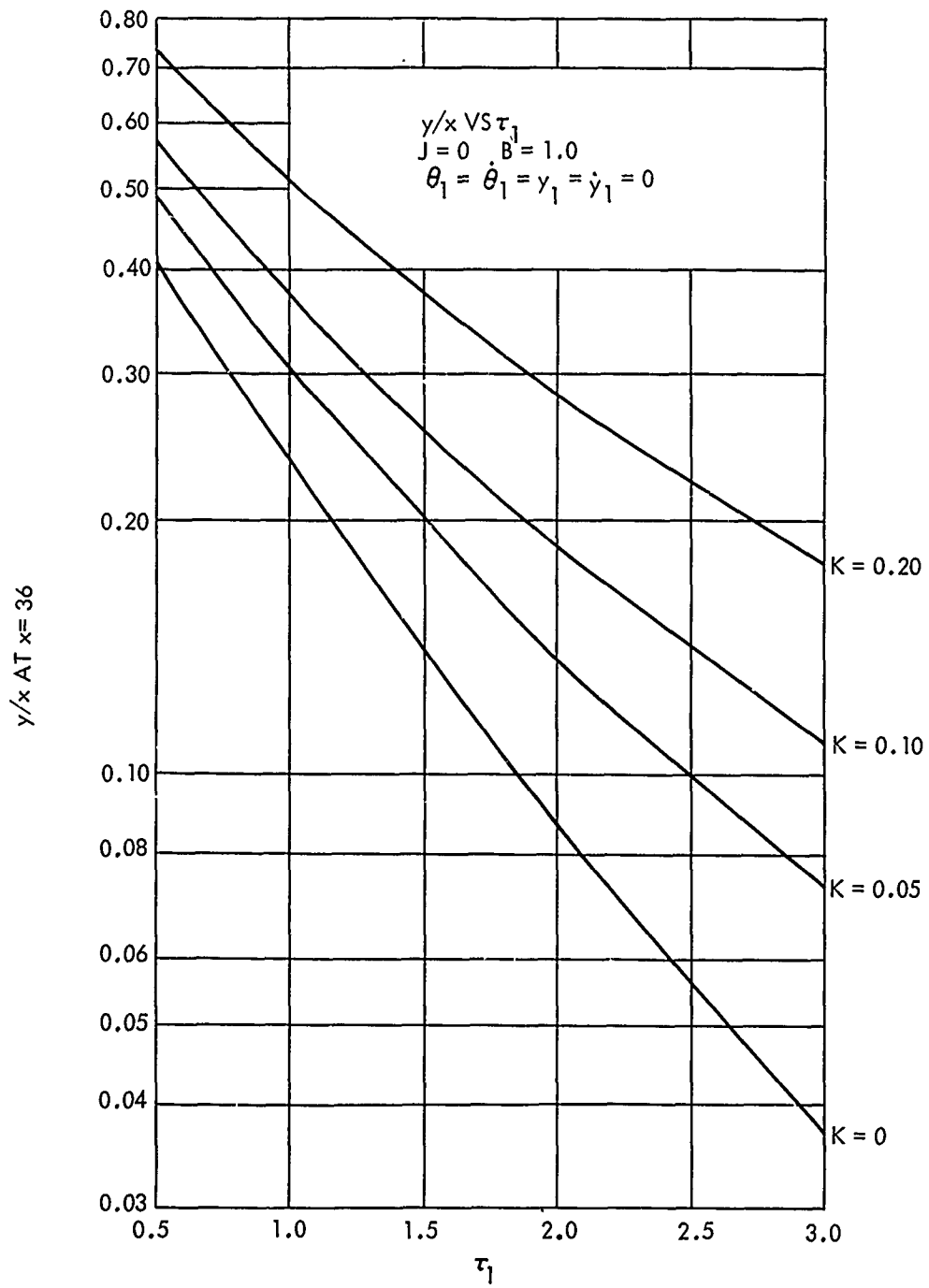


FIG. 11 OFF-AXIS THRUST

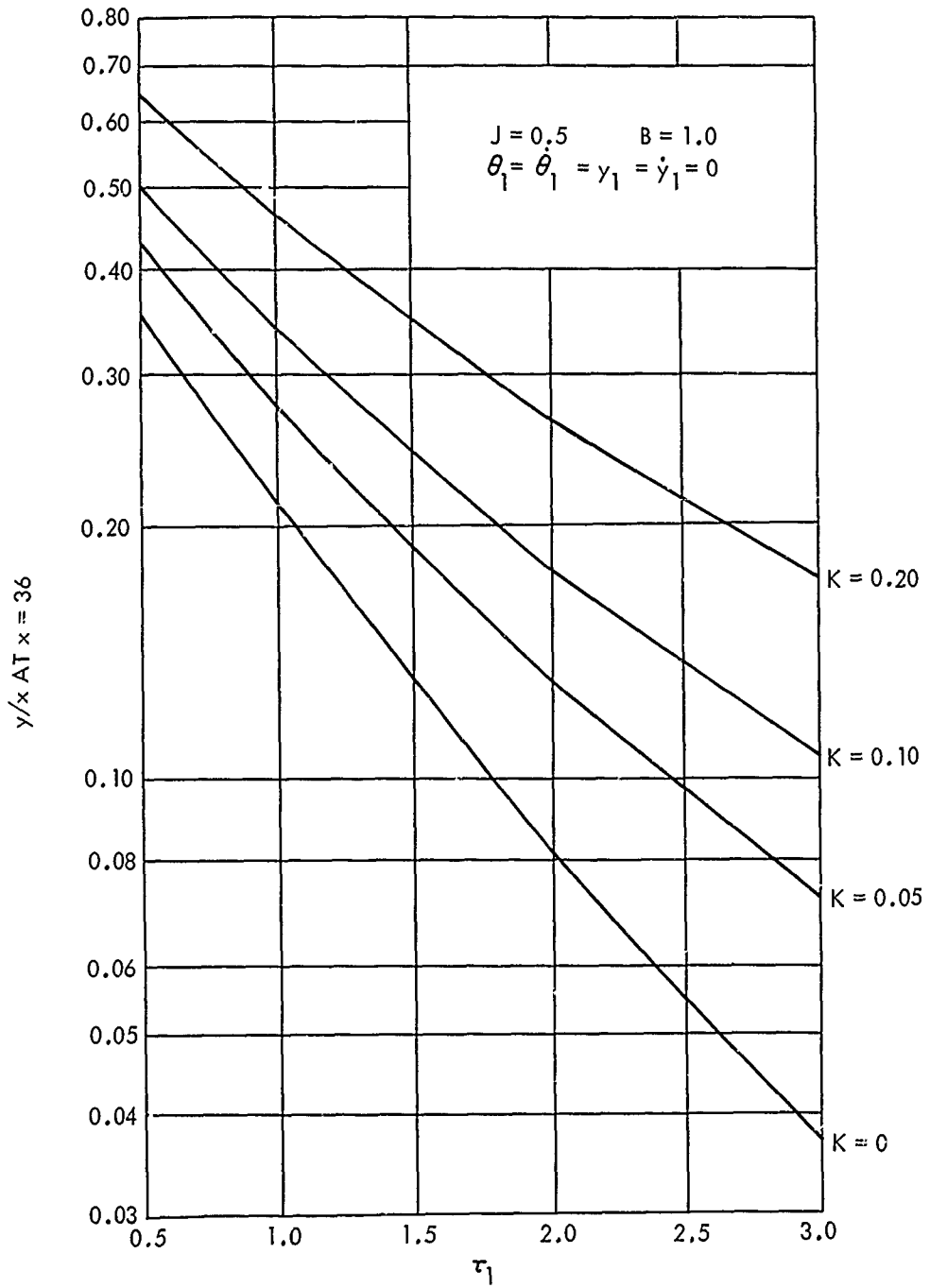


FIG.12 OFF-AXIS THRUST WITH JET DAMPING

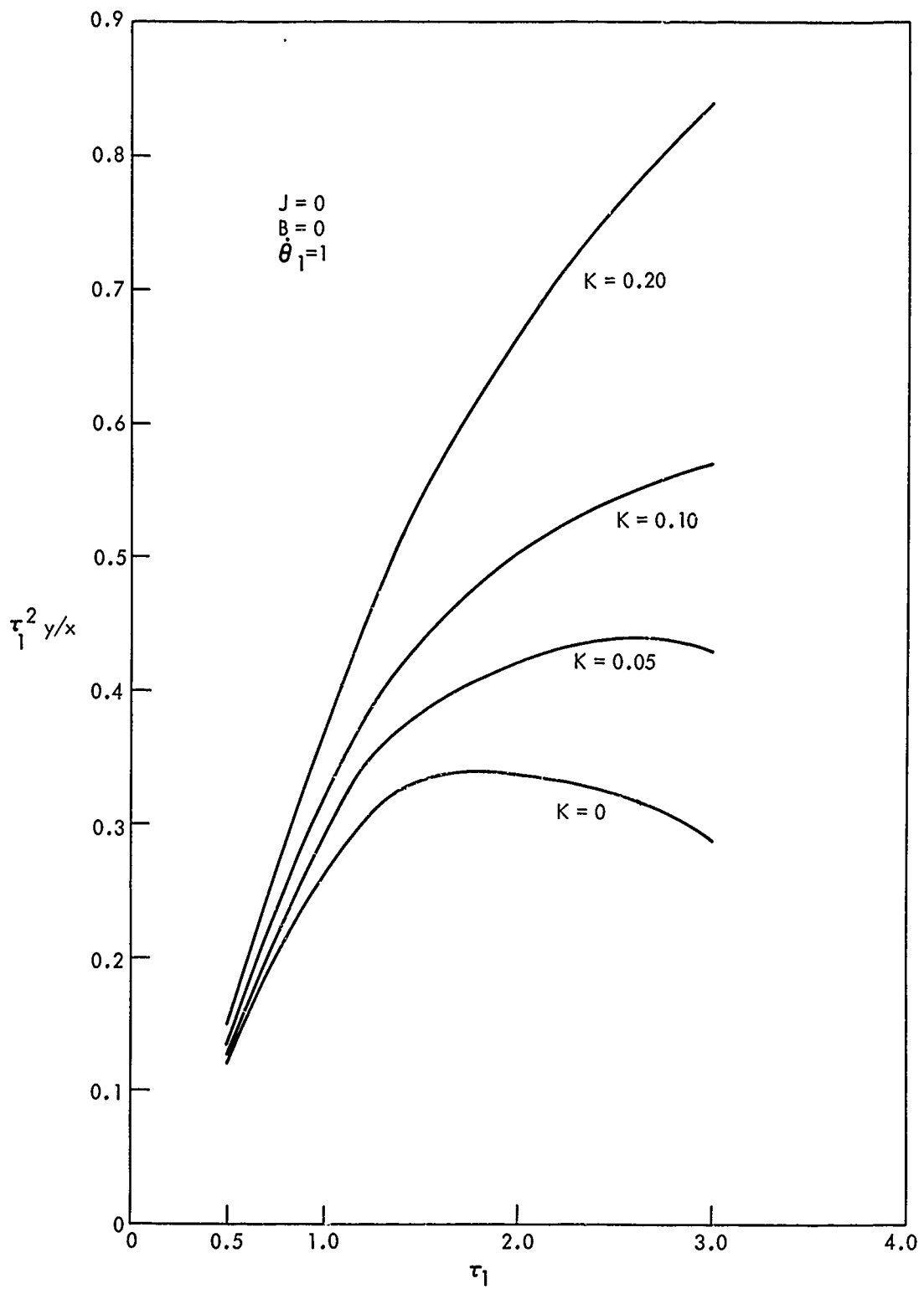


FIG. 13 $\tau_1^2 y/x$ VERSUS τ_1

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13. ABSTRACT <p>Simplified non-dimensional equations of motion are derived. From these, 288 rocket trajectories are calculated, based upon a variety of choices for aerodynamic, inertial, and propulsive properties. The analysis includes off-axis thrust and response resulting from initial conditions of yaw angle and yaw rate. The most important results are presented in tables and graphs. Application of the results to calculation of dispersion is illustrated in an interpretation of the results. A conclusion reached is that high static stability is not necessary for useful accuracy.</p>		

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14. KEY WORDS	LINK A		LINK B		LINK C	
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