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OSCILLATIONS IN THE PLASMA SHEATH

by

Simon H. Schwartz

JULY 1965

POLYTECHNIC INSTITUTE OF BROOKLYN

DEPARTMENT
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AEROSPACE ENGINEERING
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SUMMARY

The behavior of the ion plasma sheath next to a wall is investigated under a sinusoidal-in-time perturbation of the potential of the wall. A collisionless sheath is assumed, and the ion density distribution is considered to remain at the steady state value, with only the electron distribution being perturbed. The collisionless macroscopic plasma equations are solved by a perturbation procedure, using an exact equation for the steady state potential distribution. A second order differential equation which is linear, but which has variable coefficients is obtained for the perturbation. This equation is transformed into the form of a wave equation with variable propagation constant so that the analytical behavior can be deduced. One observes that above a certain range of frequencies, but below the plasma frequency of the uniform plasma, the perturbation may propagate over a finite region which may begin away from the wall.

A numerical integration of the equation is performed, using an asymptotic approximation in order to obtain the boundary conditions. The predicted oscillations

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[‡]Research Associate

are seen to occur, and two resonances in the sheath are found at frequencies below the plasma frequency.

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SECTION I

INTRODUCTION

The physics of plasma adjacent to a boundary, the so-called "plasma sheath," is not completely understood. Yet boundary phenomena play an essential role in numerous well-known situations, for example, those associated with Langmuir Probes, resonance probes and communication antennae.

A few of the numerous papers dealing with the steady-state behavior of the sheath are listed in the bibliography. With respect to time-dependent behavior, experimental studies of natural oscillations in the sheath have been performed by Gabor, Ash, and Dracott[1], by Ott, Gierke and Schwirzke[2], and by Harp and Kino[3] with conflicting results. The resonance probe employing appliedoscillationshas been studied by Wimmel[4], and the theory of the RF plasma sheath has been studied by Pavkovich and Kino[5] and Pavkovich[6]. The latter approach the problem theoretically by integrating the collisionless Boltzmann equation numerically, assuming a parabolic steady-state potential distribution, a harmonic time-wise perturbation in the plasma and in the consistent boundary conditions,and a time-independent perturbation amplitude. They reach the conclusion that no spatial oscillations about zero electric field are possible in the sheath when the electron density distribution is Maxwellian. The behavior of temporally amplified or damped perturbations is not included in their studies. Pavkovich[6] also investigates a theory similar to the one to be set forth in this paper, i.e., a fluid dynamical approach, but with a parabolic potential distribution. However, he discounts it because of the possibility of waves (spatial) near the walls. Moreover, on the basis on an examination of the so-called "sheath impedance", he decides that no oscillations are possible, at least in the numerical cases considered.

In the present paper, an ion plasma sheath next to a solid boundary is considered. This boundary can be considered to be insulated with respect to ground; such a boundary is termed "floating". It is well-known that the plasma region adjacent to a floating wall is characterized by a large potential drop. In particular, a large negative potential and negative surface charge is created, since the electrons of the plasma are more mobile than the ions and diffuse to the wall more readily. The resulting preponderance of ions and the associated positive space charge in this neighborhood suggest the designation "ion sheath".

The plasma sheath is assumed to be subjected to a temporally oscillatory perturbation of the surface potential. The amplitude of this oscillation is assumed to be time-wise invariant both at the boundary and in the plasma. In another view we may consider the oscillation to have appeared in the plasma, whereas the surface oscillation follows as a result. Finally, this type of problem may be interpreted as an investigation of the existence and spatial properties of a neutral disturbance (undamped and unamplified) consistent with the boundary conditions.

In this paper, it is assumed that collisions between charged particles within this sheath are negligible. The collisionless nonlinear macroscopic plasma equations are taken to be applicable here. The equations are linearized by a perturbation method and an equation for the perturbation derived. It is assumed that since the electrons are so much more mobile than the ions, only the electron density distribution will be perturbed. Under the conditions of this problem the electron energy equation can be integrated to give an adiabatic equation of state for the electron gas. However, the value of the adiabatic exponent γ is not specified at the outset, although it is clear that if the frequency is high enough, there will be insufficient time for equipartition of energy between the degrees of freedom so that $\gamma = 3$, while for low frequencies $\gamma = \frac{5}{3}$ is the correct value.

In this particular analysis a plane wall is treated and spatial variations in only one direction are considered, so that the problem is one-dimensional.

SECTION II

BASIC EQUATIONS

The basic equations of the problems are:

$$n_e \frac{\partial \underline{u}^*}{\partial t} = \frac{-q}{m} n_e \underline{E}^* - \frac{1}{m} \nabla p_e \tag{1}$$

$$\nabla \cdot \underline{E}^* = \frac{-q}{\epsilon_o} (n_e - n_i) \tag{2}$$

$$p_e n_e^{-\gamma} = \text{const.} \tag{3}$$

$$\nabla \times \overline{\underline{H}} = \epsilon_o \frac{\partial \overline{\underline{E}}}{\partial t} - n_{e_o} q \underline{u}^* \tag{4}$$

$$\nabla \times \overline{\underline{E}} = -\mu_o \frac{\partial \overline{\underline{H}}}{\partial t} \tag{5}$$

where $\underline{u}^*$ is the electron velocity, $\underline{E}^*$ is the total electric field, p_e is the total electron pressure, n_e and n_i are the total electron and ion number densities respectively, $\overline{\underline{E}}$ and $\overline{\underline{H}}$ are the perturbation electric and magnetic fields, γ is the adiabatic exponent, and n_{e_o} is the steady state electron number density. Equation (3) is used since one is concerned with high frequency oscillations and because in this case the energy equation can be solved to obtain the adiabatic equation of state. In Eq. (1) the nonlinear acceleration term $\underline{u}^* \cdot \nabla \underline{u}^*$ is left out since it is assumed to be small compared with the time derivative. In this set of equations $\underline{u}^*$ is a perturbation already, since the steady state velocity is zero for an ion sheath (the electrons are reflected back in the negative potential so that their average velocity is zero).

Now, let E^* and n_e be perturbed so that $\underline{E}^* = \underline{E}_o + \underline{\bar{E}}$ and $n_e = n_{e_o} + \bar{n}$ where $\left|\frac{\bar{E}}{E_o}\right| << 1$ and $\left|\frac{\bar{n}}{n_{e_o}}\right| << 1$ where $\underline{E}_o$ and n_{e_o} are the steady state solutions and $\underline{\bar{E}}$ and $\bar{n}$ are the perturbations. If this perturbation is substituted in Poisson's equation, Eq. (2), it becomes

$$\nabla \cdot (\underline{E}_o + \underline{\bar{E}}) = \frac{-q}{\varepsilon_o}(n_{e_o} + \bar{n} - n_i). \qquad (6)$$

The steady state Poisson's equation is

$$\nabla \cdot \underline{E}_o = \frac{-q}{\epsilon_o}(n_{e_o} - n_i) \qquad (7)$$

In the one-dimensional case this equation becomes (see Appendix)

$$\left(\frac{d\theta}{d\xi}\right)^2 = 2\sqrt{1-2\theta} + 2e^{\theta} - 4 \qquad (8)$$

where $\theta = \frac{q\varphi}{kT_o}$, $\xi = \frac{x}{\lambda_d}$, where $E_o = -\frac{d\varphi}{dx}$ and λ_d is the Debye length referred to undisturbed plasma, $\lambda_d^2 = \frac{\epsilon_o k T_o}{n_o q^2}$. Here, T_o is the electron temperature of the undisturbed plasma and k is the Boltzmann constant. Thus, Eq. (6) becomes

$$\frac{d\bar{E}}{dx} = \frac{-q}{\epsilon_o}\bar{n} \qquad (9)$$

Let the adiabatic equation of state Eq. (3) be examined more closely.

$$p_e n_e^{-\gamma} = p_{e_o} n_{e_o}^{-\gamma} = kT_o n_{e_o}^{1-\gamma} \tag{10}$$

where $p_{e_o} = n_{e_o} kT_o$ is the isothermal equation of state and holds for steady state.

$$p_e = kT_o n_{e_o}^{1-\gamma} n_e^{\gamma} \tag{10a}$$

Now, let one substitute the perturbation into (10a). Then (10a) becomes

$$p_e = kT_o n_{e_o}^{1-\gamma} n_{e_o}^{\gamma} \left(1 + \frac{\bar{n}}{n_{e_o}}\right)^{\gamma} = kT_o n_{e_o} \left(1 + \frac{\gamma \bar{n}}{n_{e_o}}\right) = kT_o n_{e_o} + \gamma kT_o \bar{n} \tag{11}$$

$$\nabla p_e = kT_o \nabla n_{e_o} + \gamma kT_o \nabla \bar{n} \tag{12}$$

$$\frac{1}{n_e} \nabla p_e = \frac{1}{(n_{e_o} + \bar{n})} (kT_o \nabla n_{e_o} + \gamma kT_o \nabla \bar{n}) = \frac{\gamma kT_o}{n_{e_o}} \nabla \bar{n} + \frac{kT_o}{n_{e_o}} \left(1 - \frac{\bar{n}}{n_{e_o}}\right) \nabla n_{e_o} \tag{13}$$

With Eq. (13), Eq. (1) becomes

$$m \frac{\partial \underline{u}^*}{\partial t} = -q(\underline{E}_o + \underline{\bar{E}}) - \frac{\gamma kT_o}{n_{e_o}} \nabla \bar{n} - \frac{kT_o}{n_{e_o}} \left(1 - \frac{\bar{n}}{n_{e_o}}\right) \nabla n_{e_o} \tag{14}$$

$$n_{e_o} \frac{\partial \underline{u}^*}{\partial t} = \frac{-q n_{e_o}}{m} \underline{E}_o - \frac{q n_{e_o} \underline{\bar{E}}}{m} - \frac{\gamma kT_o}{m} \nabla \bar{n} - \frac{kT_o}{m} \nabla n_{e_o} + \frac{kT_o}{m} \frac{\nabla n_{e_o}}{n_{e_o}} \bar{n} \tag{15}$$

The steady state momentum equation is

$$\frac{-q n_{e_o} \underline{E}_o}{m} - \frac{kT_o}{m} \nabla n_{e_o} = 0 \tag{16}$$

which has a solution

$$n_e = n_o e^{\frac{q\varphi}{kT_o}} \tag{17}$$

where $\underline{E}_o = -\nabla\varphi$ and n_o is the value of the electron density at $\varphi = 0$.

Thus, Eq. (15) becomes

$$n_{e_o} \frac{\partial \underline{u}^*}{\partial t} = -\frac{q}{m} n_{e_o} \underline{\bar{E}} - \frac{\gamma k T_o}{m} \nabla \bar{n} + \frac{kT_o}{m} \frac{\nabla n_{e_o}}{n_{e_o}} \bar{n} \tag{18}$$

which with the $e^{j\omega t}$ time dependence becomes

$$j\omega n_{e_o} \underline{u} = \frac{-q}{m} n_{e_o} \underline{E} - \frac{\gamma k T_o}{m} \nabla n + \frac{kT_o}{m} \frac{\nabla n_{e_o}}{n_{e_o}} n \tag{18a}$$

SECTION III

DERIVATION OF PERTURBATION EQUATION FOR E

Let one substitute Eq. (14a) into Maxwell Eq. (4).

$$\nabla \times \underline{H} = j\omega\epsilon_o \underline{E} + \frac{q^2}{j\omega m} n_{e_o} \underline{E} + \frac{q}{j\omega} \frac{\gamma k T_o}{m} \nabla n - \frac{q}{j\omega} \frac{kT_o}{m} \frac{\nabla n_{e_o}}{n_{e_o}} n \tag{19}$$

$$= j\omega\epsilon_o \underline{E} + \epsilon_o \frac{\omega_p^2}{j\omega} \underline{E} + \frac{a^2 q}{j\omega} \nabla n - \frac{a^2 q}{\gamma j\omega} \frac{\nabla n_{e_o}}{n_{e_o}} n \tag{19a}$$

where $a^2 = \frac{\gamma k T_o}{m}$ and $\omega_p^2 = \frac{n_{e_o} q^2}{\epsilon_o m}$.

$$j\omega \nabla \times \underline{H} = -\varepsilon_o(\omega^2 - \omega_p^2)\underline{E} + a^2 q \nabla n - \frac{a^2 q}{\gamma}\left(\frac{\nabla n_{e_o}}{n_{e_o}}\right) n \tag{19b}$$

Making use of Eq. (18), this becomes

$$j\omega \nabla \times \underline{H} = -\varepsilon_o(\omega^2 - \omega_p^2)\underline{E} - \varepsilon_o a^2 \nabla(\nabla \cdot \underline{E}) + \frac{\varepsilon_o a^2}{\gamma}\left(\frac{\nabla n_{e_o}}{n_{e_o}}\right) \nabla \cdot \underline{E} \tag{19c}$$

Now, we can make use of the vector identity

$$\nabla(\nabla \cdot \underline{E}) = \nabla \times (\nabla \times \underline{E}) + \nabla \cdot (\nabla \underline{E}) \tag{19d}$$

$$j\omega \nabla \times \underline{H} = -\varepsilon_o(\omega^2 - \omega_p^2)\underline{E} - \varepsilon_o a^2 \nabla \times (\nabla \times \underline{E}) - \varepsilon_o a^2 \nabla . (\nabla \underline{E}) + \frac{\varepsilon_o a^2}{\gamma}\left(\frac{\nabla n_{e_o}}{n_{e_o}}\right) \nabla \cdot \underline{E} \tag{19e}$$

which by Eq. (5) becomes

$$j\omega \nabla \times \underline{H} = -\varepsilon_o(\omega^2 - \omega_p^2)\underline{E} + \varepsilon_o a^2 j\omega \mu_o \nabla \times \underline{H} - \varepsilon_o a^2 \nabla \cdot (\nabla \underline{E}) + \frac{\varepsilon_o a^2}{\gamma}\left(\frac{\nabla n_{e_o}}{n_{e_o}}\right) \nabla \cdot \underline{E} \tag{19f}$$

If one assumes that H = 0, this becomes

$$-\varepsilon_o(\omega^2 - \omega_p^2)\underline{E} - \varepsilon_o a^2 \nabla \cdot (\nabla \underline{E}) + \frac{\varepsilon_o a^2}{\gamma}\left(\frac{\nabla n_{e_o}}{n_{e_o}}\right) \nabla \cdot \underline{E} = 0 \tag{20}$$

which can be written in the form

$$\nabla \cdot (\nabla \underline{E}) - \frac{1}{\gamma}\frac{\nabla n_{e_o}}{n_{e_o}} \nabla \cdot \underline{E} + \left(\frac{\omega^2 - \omega_p^2}{a^2}\right)\underline{E} = 0 \tag{20a}$$

In the one-dimensional case this becomes

$$\frac{d^2 E}{dx^2} - \frac{1}{\gamma}\frac{\nabla n_{e_o}}{n_{e_o}}\frac{dE}{dx} + \left(\frac{\omega^2 - \omega_p^2}{a^2}\right) E = 0 \tag{21}$$

In terms of the nondimensional variable ξ this becomes

$$\gamma \frac{d^2 E}{d\xi^2} - \frac{\nabla n_{e_o}}{n_{e_o}} \frac{dE}{d\xi} + \left(\frac{\omega^2 - \omega_p^2}{\omega_o^2}\right) E = 0 \tag{22}$$

where $\frac{\lambda_d^2}{a^2} = \frac{\epsilon_o m}{\gamma n_o q^2} = \frac{1}{\gamma \omega_o^2}$ and ω_o is the plasma frequency of the uniform plasma. Equations (21) and (22) are the perturbation equations for E. Eqs. (21) and (22) imply that the electrons move in a field of force which is due to the steady state electric field caused by the non-uniform density distribution.

SECTION IV

TRANSFORMATION OF THE PERTURBATION EQUATION

Equation (21) can be written in the form

$$\frac{d^2 E}{dx^2} - 2 f(x) \frac{dE}{dx} + g(x) E = 0 \tag{23}$$

where f(x) and g(x) are given by

$$f(x) = \frac{1}{2\gamma} \frac{\nabla n_{e_o}}{n_{e_o}}, \quad g(x) = \frac{\omega^2 - \omega_p^2}{a^2} .$$

Assume a solution to (23) of the form

$$E(x) = y(x)\, h(x) \tag{24}$$

where h(x) is given by

$$h(x) = e^{\int_{x_0}^{x} f(x)\, dx} . \tag{24a}$$

If (24) is substituted into (23), the following differential equation is obtained

$$\frac{d^2y}{dx^2} + \Psi(x)\, y = 0 \tag{25}$$

where $\Psi(x)$ is given by

$$\Psi(x) = \frac{df}{dx} + g - f^2 \quad . \tag{26}$$

When the definitions of f and g are substituted into Eq. (26), $\Psi(x)$ becomes

$$\Psi(x) = \frac{\omega^2 - \omega_p^2}{a^2} + \frac{1}{2\gamma}\frac{d^2}{dx^2}\, \ell n \frac{\omega_p^2}{\omega_o^2} - \frac{1}{4\gamma^2}\left(\frac{d\ell n \frac{\omega_p^2}{\omega_o^2}}{dx}\right)^2 \tag{27}$$

where

$$\frac{\nabla n_{e_o}}{n_{e_o}} = \frac{\nabla \omega_p^2}{\omega_p^2} = \frac{\nabla \frac{\omega_p^2}{\omega_o^2}}{\frac{\omega_p^2}{\omega_o^2}} = \nabla \ell n \frac{\omega_p^2}{\omega_o^2} .$$

Now let one nondimensionalize $\Psi(x)$ and Eq. (25) . Equation (25) becomes

$$\frac{d^2y}{d\xi^2} + \lambda_d^2\, \Psi(x)\, y = 0 \tag{28}$$

and if one defines $\Psi(\xi) = \lambda_d^2\, \Psi(x)$, then this becomes

$$\frac{d^2y}{d\xi^2} + \Psi(\xi)\, y = 0 \tag{28a}$$

$$\Psi(\xi) = \frac{\omega^2 - \omega_p^2}{\gamma\omega_o^2} + \frac{1}{2\gamma}\frac{d^2}{d\xi^2}\, \ell n \frac{\omega_p^2}{\omega_o^2} - \frac{1}{4\gamma^2}\left(\frac{d\ell n \frac{\omega_p^2}{\omega_o^2}}{d\xi}\right)^2 \tag{29}$$

Since $\frac{\omega_p^2}{\omega_o^2} = e^{\theta}$, $\Psi(\xi)$ can be written as

$$\Psi(\xi) = \frac{\omega^2 - \omega_p^2}{\gamma \omega_o^2} + \frac{1}{2\gamma} \frac{d^2\theta}{d\xi^2} - \frac{1}{4\gamma^2} \left(\frac{d\theta}{d\xi}\right)^2 \qquad (29a)$$

The integration of Eq. (17) giving θ vs ξ, is shown in Fig. 1. The plot of Ψ vs ξ for values of γ of $\frac{5}{3}$ and 3 is shown in Fig. 2

If the value of f(x) is substituted into

$$e^{\int_{x_o}^{x} f(x)\,dx} = h(x) ,$$

it becomes

$$h(x) = e^{\int_{x_o}^{x} f(x)\,dx} = e^{\frac{\theta - \theta_o}{2\gamma}} . \qquad (30)$$

$e^{-\frac{\theta_o}{2\gamma}}$ is just an arbitrary multiplicative constant, so that the solution for E can be given by

$$E = yh = ye^{\frac{\theta}{2\gamma}} \qquad (31)$$

SECTION V

RESULTS

Since h(x) is a monotonically increasing function from the wall (negative θ) to the sheath edge ($\theta = 0$), one sees that the behavior of E is determined by the behavior of y. The equation for y is given by (28a), which is recognized as a type of wave equation. If $\Psi(\xi)$ were constant and positive, wave solutions and hence propagation of the perturbation would be possible. If Ψ were constant and negative, the solution would be a damped or growing exponential. The same type of behavior is expected, although modified slightly, by the solutions if Ψ is a variable. Fig. 2 makes it clear that Ψ indeed does become positive for a finite range of ξ.

One particularly interesting effect due to this behavior of Ψ is that the propagation is cut off near the wall where the local plasma frequency is very small. If Eq. (29a) is considered under the assumption that θ is a large negative value, then $\frac{\omega_p^2}{\omega_o^2} \approx 0$ and $\frac{d^2\theta}{d\xi^2} \approx 0$, so that $\Psi(\xi)$ becomes

$$\Psi(\xi) \approx \frac{\omega^2}{\gamma\omega_o^2} - \frac{1}{4\gamma^2}\left(\frac{d\theta}{d\xi}\right)^2 \qquad (32)$$

and $\Psi(\xi)$ can then become negative for an appropriately large $\frac{d\theta}{d\xi}$ and a cutoff can take place. Since $\frac{d\theta}{d\xi}$ is related to the electric field, this result is not surprising in view of the fact that the electron gas is in equilibrium with the electric field E_o, and this effect is analogous to what is found in an atmosphere which is in equilibrium with a gravitational field.

In view of the complexity of Eq. (28a), this equation had to be solved numerically. In order to do this, $\Psi(\xi)$ was approximated for large values of $\xi(\theta \rightarrow 0)$ by a constant and the damped exponential solution for y was chosen, so that the perturbation would go to zero at infinity as it should. Then the equation was integrated backwards from this starting point at a large value of ξ ($\xi = 36$ to be exact) to the wall. This was done for the range of frequencies of $\frac{\omega}{\omega_o}$ from 0 to 1 and for both values of γ. The results for $\gamma = 3$ are shown in Fig. 3. It is seen that an oscillation does indeed occur, and also that two resonances occur.

SECTION VI

CONCLUSIONS

The results of this analysis show that the resonances in the sheath occur at frequencies below the asymptotic plasma frequency. It is of interest to observe that they occur in a relatively narrow range below ω_o. Only two resonances are found, one at about $\frac{\omega}{\omega_o} \approx 0.75$ and another at $\frac{\omega}{\omega_o} \approx 0.95$. In the limit of $\frac{\omega}{\omega_o} \approx 1$, the unperturbed plasma appears, and for larger values of $\frac{\omega}{\omega_o}$ only a continuous spectrum of frequencies may occur. These results appear to be in agreement with the experimental behavior of the Resonance Probe.

A further examination, including the effect of collisions, is in the process of being carried out.

SECTION VII

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APPENDIX

Poisson's equation for the steady state distribution is given as

$$\frac{dE_o}{dx} = \frac{-q}{\epsilon_o}(n_{e_o} - n_i) \qquad \text{(A-1)}$$

or in terms of φ this is

$$\frac{d^2\varphi}{dx^2} = \frac{q}{\epsilon_o}(n_{e_o} - n_i) \qquad \text{(A-2)}$$

We know that $n_{e_o} = n_o e^{\frac{q\varphi}{kT}}$ and we can say that $n_i = \frac{j_i}{qu_i}$. u_i is given by

$$u_i = \sqrt{u_{i_o}^2 - \frac{2q}{m}\varphi}$$

where we assume that $\varphi = 0$ at the sheath edge and u_{i_o} is the ion velocity at the sheath edge. By the Bohm criterion for a stable sheath, u_{i_o} can be replaced by $u_{i_o} = \sqrt{\frac{kT_o}{m}}$. Then Eq. (A-2) becomes

$$\frac{d^2\varphi}{dx^2} = \frac{q}{\epsilon_o} n_o (e^{\theta} - \frac{1}{\sqrt{1-2\theta}}) \qquad \text{(A-3)}$$

or in terms of the nondimensional distance ξ

$$\frac{d^2\theta}{d\xi^2} = e^{\theta} - \frac{1}{\sqrt{1-2\theta}} \qquad \text{(A-4)}$$

If we integrate(A -4)and apply the boundary condition that $\theta = 0$ at $\frac{d\theta}{d\xi} = 0$, then this becomes

$$(\frac{d\partial}{d\xi})^2 = 2e^{\theta} + 2\sqrt{1-2\theta} - 4 \; . \qquad \text{(A-5)}$$

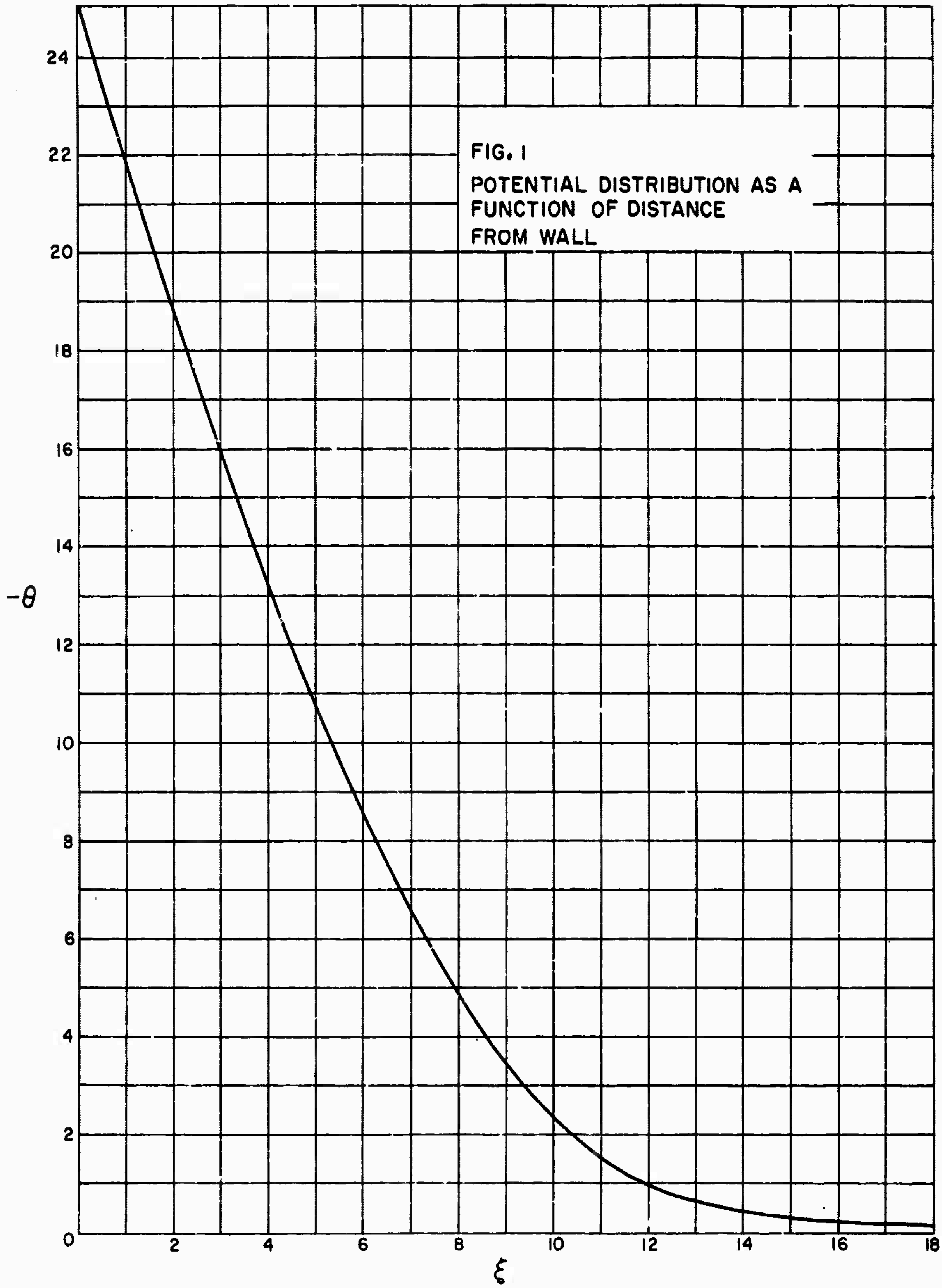
FIG. 1
POTENTIAL DISTRIBUTION AS A FUNCTION OF DISTANCE FROM WALL
-θ
24
22
20
18
16
14
12
10
8
6
4
2
0
2
4
6
8
10
12
14
16
18
ξ

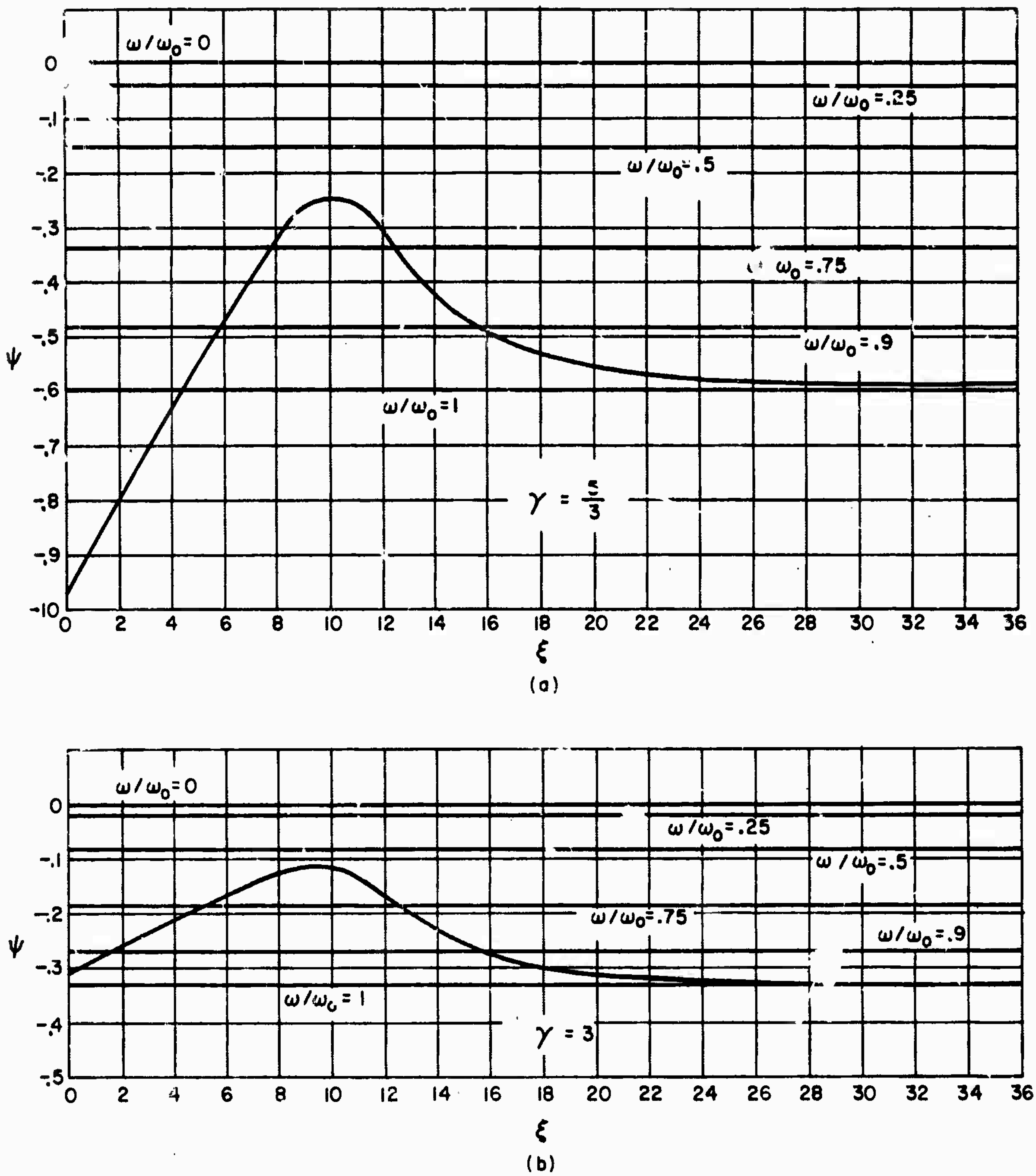

FIG. 2 PROPAGATION FACTOR ψ vs DISTANCE FROM WALL

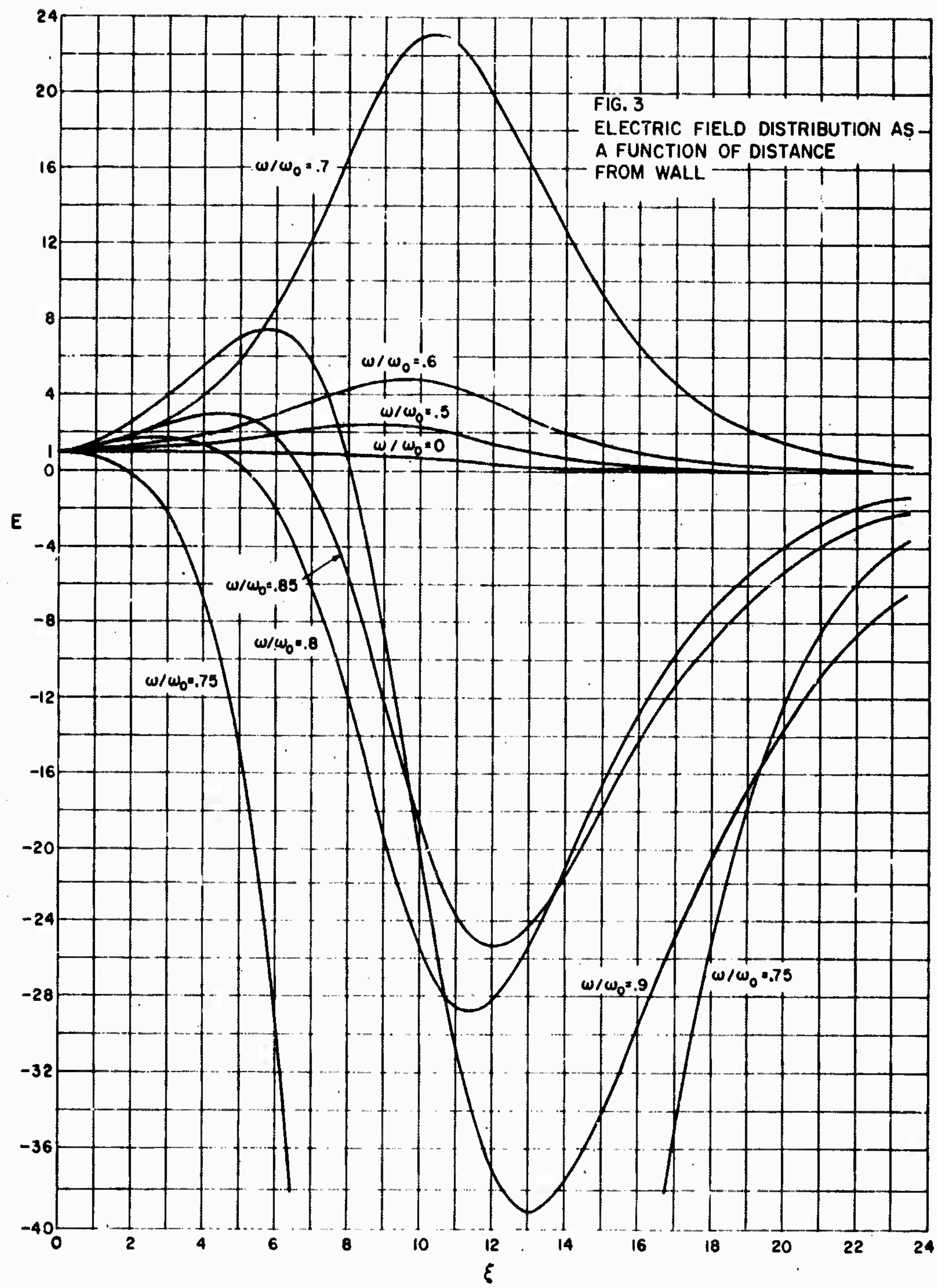
FIG. 3
ELECTRIC FIELD DISTRIBUTION AS A FUNCTION OF DISTANCE FROM WALL
ω/ω₀ = .7
ω/ω₀ = .6
ω/ω₀ = .5
ω/ω₀ = 0
ω/ω₀ = .85
ω/ω₀ = .8
ω/ω₀ = .75
ω/ω₀ = .9
ω/ω₀ = .75
E
ξ
24
20
16
12
8
4
1
0
-4
-8
-12
-16
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13. ABSTRACT

The behavior of the ion plasma sheath next to a wall is investigated under a sinusoidal-in-time perturbation of the potential of the wall. A collisionless sheath is assumed, and the ion density distribution is considered to remain at the steady state value, with only the electron distribution being perturbed. The collisionless macroscopic plasma equations are solved by a perturbation procedure, using an exact equation for the steady state potential distribution. A second order differential equation which is linear, but which has variable coefficients is obtained for the perturbation. This equation is transformed into the form of a wave equation with variable propagation constant so that the analytical behavior can be deduced. One observes that above a certain range of frequencies, but below the plasma frequency of the uniform plasma, the perturbation may propagate over a finite region which may begin away from the wall. A numerical integration of the equation is performed, using an asymptotic approximation in order to obtain the boundary conditions. The predicted oscillations are seen to occur, and two resonances in the sheath are found at frequencies below the plasma frequency.

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