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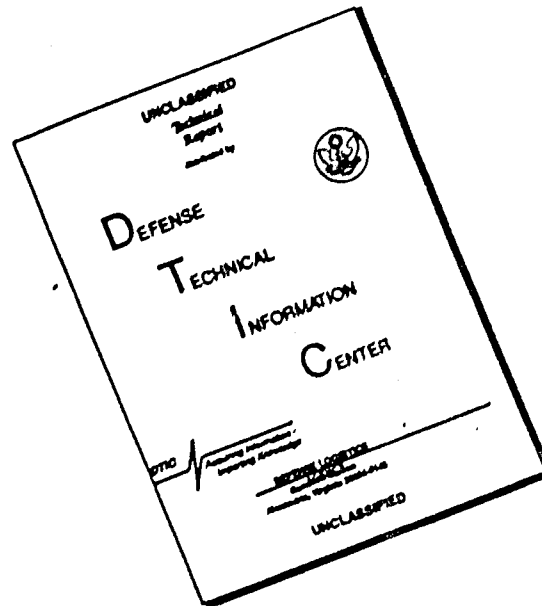
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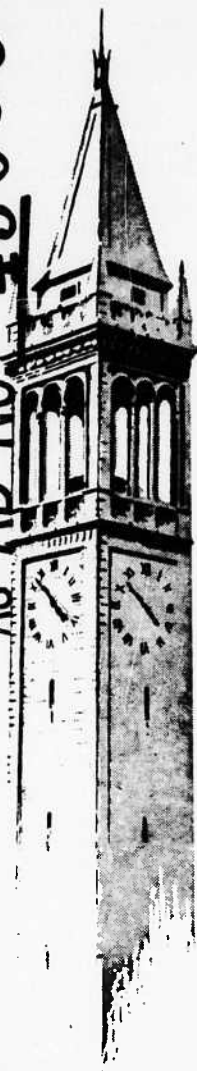


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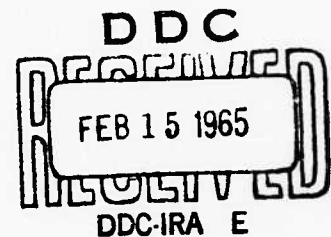


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NEARLY-FREE MOLECULAR FLOW: A
COMPARISON OF THEORY AND EXPERIMENT

by

F. S. Sherman
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F. S. Sherman
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ABSTRACT

Measured drag coefficients for cylinders, normal strips and spheres in nearly free-molecular flow at $M = 6, 8$ and 10 are correlated with a collision-rate parameter suggested by the modified Krook model, and compared with theoretical estimates based on a variety of kinetic models and methods of analysis.

The comparison of theory and experiment and the comparative effectiveness of various rarefaction parameters are discussed, and suggestions for further experimentation are made.

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1.0 INTRODUCTION

The principal piece of information which is currently needed to complete a description of the various regimes of rarefied gas dynamics, for purposes both of understanding and engineering prediction, concerns the behavior of typical flow patterns at their first departures from their free-molecular flow limits. Given this, together with the well-established theory of free-molecular flow and the theories and copious experimental data covering the slip flow regime, we can interpolate to describe the remaining transition flow regime with fair confidence.

Theoretical treatments of nearly free-molecular flows have run considerably ahead of experimental studies because of the difficulty of producing laboratory flows of known and suitably uniform condition with sufficiently large mean free paths. This latter difficulty is currently being overcome in a number of laboratories, by use of shock tunnels, or cryopumped steady-flow tunnels with cooled nozzles and boundary layer suction. At Berkeley, we are exploring new ranges of high Mach number, large-mean-free-path flow by use of the free jet testing technique.

Experimental and theoretical investigations of the free-jet flow field (Ashkenas and Sherman, 1964), and drag data for cylinders and strips (Maslach, et.al., 1964) have been presented earlier this year, at the Fourth International Symposium on Rarefied Gas Dynamics, held at Toronto. For the present paper we have made a critical review of this drag data, re-reducing it in terms of a rarefaction parameter α_0 , which is both more theoretically pertinent and more accurately measurable than the free stream Knudsen number. The sphere drag data of Kinslow and Potter (1962) are similarly re-expressed. The theories tested by comparison with these data are all those known (except those of a semi-empirical nature), and include some hitherto unpublished results.

2.0 RESUME' OF THEORIES OF NEARLY FREE-MOLECULAR FLOW

We limit our discussion to those methods used to predict the drag of typical aerodynamic bodies placed in an infinite gaseous medium. (The same methods can often be applied to predict the heat transfer characteristics as well.) The semi-empirical methods of Rott and Whittenbury (1961), Brooks and Reis (1963), and Kinslow and Potter (1962), in which a free parameter in the theory is adjusted to give best agreement with experimental data, will not be discussed here. While they provide simple formulas for interpolation in the transition regime, they do not give a priori estimates independent of experiment.

The complexity of the general Maxwell-Boltzmann collision operator is so great that all results to date have been obtained by either assuming hard sphere molecules and calculating approximately the effect of collisions involving molecules emitted from the body ("first-collision" methods), or by replacing the collision operator by a relaxation type term and obtaining an approximate solution to the kinetic equation by iteration starting from the free molecular solution (iterative methods).

The first-collision methods have been restricted to the case where the molecules emitted from the body have an average speed very much less than the hypersonic free stream speed. The natural small parameter is $\beta = d n_{\infty} \sigma^2 S_b$ where d is a typical dimension of the body, n_{∞} the number density in the free stream, σ the diameter of the hard sphere molecule, and $S_b^2 = m u_{\infty}^2 / 2kT_b$. Assuming that the molecules are emitted from the body with a Maxwellian distribution specified by T_b , the results for the drag coefficient are of the form

$$C_D - C_{D_{fm}} = \beta \int \ln(\beta/S_b) G_1(S_b) + \beta G_2(S_b) + O(\beta^2 \ln \beta) \quad (1)$$

where

$j = 0$ for a three-dimensional body

$j = 1$ for a two-dimensional body

and G_1 and G_2 are of order unity for $S_b \gtrsim 1$.

The possible relationships between and the physical variables measured (or inferred) in the experiment are discussed in the next section.

The iterative methods fall into two classes. In the first class (Knudsen and integral iteration) the kinetic equation is rewritten as an integral equation and one iteration is performed starting from the free molecular solution (Willis, 1959). The collision term is represented by the "modified Krook" expression which was specifically designed for non-equilibrium situations (Maslach, et.al., 1964). The natural small parameter for this method is given by

$$\alpha = \delta n_\infty (d/2) (m/2kT_b)^{1/2}$$

where δ is a constant appearing in the collision model, such that the total number of collisions in the free stream is $\frac{1}{2}\delta n_\infty^2$ per unit time and volume. The calculations were restricted to supersonic Mach numbers, but T_b was allowed to vary in the range between (roughly) T_∞ and T_0 .

Typical drag results from this method are of the form

$$C_D - C_{D_{fm}} = \alpha j \ln(\alpha/S_b) H_1(S_b, S_\infty) + \alpha H_2(S_b, S_\infty) + O(\alpha^2 \ln \alpha) \quad (2)$$

where j is defined as above and H_1 and H_2 are functions of S_b and S_∞ which are of order unity for S_b and S_∞ both greater than one. For the two-dimensional geometries studied (cylinder and strip) only the H_1 term has been evaluated to date. Results using the simple Krook model (Bhatnagar, Gross and Krook, 1954) have also been obtained from the integral iteration method.

The second class of iterative methods uses a linearized form of the simple Krook model for the collision term and performs an iteration after taking a Fourier transform of the kinetic equation with respect to the spatial coordinate (Rose, 1964). The results for the drag of a finite length cylinder and for a sphere are of the same form as would be given by the integral iteration method.

It should be noted that only the terms of order α for the three-dimensional and order $\alpha \ln \alpha$ for the two-dimensional problems can be obtained by iterating on the formal free-molecular solution. To calculate higher order terms it would be necessary to take proper account of the non-uniform validity of the free-molecular solution far from the bodies (Pao, 1964).

3.0 INTERPRETATION OF COLLISION-RATE PARAMETERS

Wind tunnel drag measurements have conventionally been reported as functions of free stream Mach number, Reynolds number, and the ratio of the body temperature to free stream temperature T_w/T_∞ . Often a Knudsen number is deduced from the formula^{*}

$$Kn_\infty = \lambda_\infty/d = (\gamma\pi/2)^{1/2} M_\infty/Re_d \quad (3)$$

and results presented in terms of this parameter. For a comparison between theory and experiment we have to postulate a value for T_b (in all cases to date it has been set equal to T_w) and relate β and α to the experimental parameters. One suggestion for obtaining a relation is to equate the actual viscosity of the gas, at some appropriate reference temperature, to the Chapman-Enskog viscosity corresponding to the collision model.

^{*}However, other formulas have also been used. A return to presenting results in terms of Re/M might prevent possible confusion.

This choice is motivated by the fact that μ is, except for very low temperatures, a known function of temperature and that like the drag perturbation it is an evidence of the transfer of momentum due to inter-molecular collisions. In previous work (Maslach, et.al., 1964) T_∞ was used as the reference temperature. This choice has the advantage of reducing the theoretical formulas naturally to a convenient form in terms of Kn_∞ . Based on this choice it was shown there that the first-collision results using hard sphere molecules are in serious disagreement with experiment, while fair to good agreement is given by the integral iteration results.

There are, however, two points in favor of a different choice of the reference temperature. In the first place, for the higher Mach numbers used in the unheated experiments at Berkeley, T_∞ is so low that the viscosity is not well known and has to be estimated. In the second place, intuitive arguments suggest that the important collisions are those between molecules emitted from the body and free stream molecules. Typical relative kinetic energies involved in such collisions correspond to temperatures in an equilibrium gas (for which the Chapman-Enskog results are valid) of the order of the stagnation temperature (in fact, it can be as much as $1.6 T_0$ for an insulated body). As the stagnation temperature is a well-determined quantity for wind tunnel tests, we will consider the effect of using it as the reference temperature.

The appropriate formulas relating the viscosity to the collisional parameters are, for hard sphere molecules

$$\sigma^2 = (m kT/\pi)^{1/2} / [\pi \mu(T)] \quad (4)$$

and for the simple Krook model

$$\delta = kT/\mu(T) \quad (5)$$

For the modified Krook model there are some questions as to the meaning of the viscosity, but we propose to use the same formula as for the simple Krook model.

Using the above formulas we find that

$$\frac{\beta}{(1/Kn_{\infty})} = \left(\frac{S_b}{2\pi^{1/2}} \right) C_1 \quad (6)$$

$$\frac{\alpha}{(1/Kn_{\infty})} = \frac{\pi^{1/2}}{4} \left(\frac{S_b}{S_{\infty}} \right) C_2 \quad (7)$$

where

$$C_1 = (T_{\text{ref}}/T_{\infty})^{1/2} [\mu(T_{\infty})/\mu(T_{\text{ref}})]$$

$$C_2 = (T_{\text{ref}}/T_{\infty}) [\mu(T_{\infty})/\mu(T_{\text{ref}})]$$

When T_{ref} is taken as T_{∞} , both C_1 and C_2 are unity. The effect of taking $T_{\text{ref}} = T_0$, the stagnation temperature, is shown below for typical conditions in the Berkeley and AEDC (Kinslow and Potter, 1962) facilities.

M_{∞}	$T_0 (^{\circ}\text{K})$	C_1	C_2
6	290	0.42	1.15
10	290	0.26	1.40
10	3350	0.53	2.40

It can be seen that there will be a marked effect of the choice of T_{ref} , particularly for the hard sphere molecules. (Indicative of the fact that $[\mu(T)/T^{1/2}]$ is by no means constant for appreciable ranges of temperature.)

If we decide to use T_0 as the reference temperature it is in fact more convenient to work directly with α_0 or β_0 which are related to the physical parameters by

$$\beta_o = \text{Re}^*(T_o/T_b)^{1/2} / [\pi(2\pi)^{1/2}]$$

$$\alpha_o = \text{Re}^*(T_o/T_b)^{1/2} / 4 S_o \quad (8)$$

where

$$\text{Re}^* = \rho_\infty u_\infty d / \mu(T_o)$$

and

$$S_o^2 = m u_\infty^2 / 2kT_o$$

As S_o is of order unity for hypersonic conditions [approaching $(\gamma/\gamma-1)^{1/2}$ for infinite free stream Mach number] the two parameters are seen to have essentially the same meaning. Re^* is a convenient parameter depending only upon $\mu(T_o)$ which is well known, and ρ_∞ , the free stream density, which is relatively easy to infer from experimental measurements.

4.0 RESUME' OF EXPERIMENTAL RESULTS

4.1 Cylinder and Strip Drag

The basic references for these data are the reports of Maslach (1963), Tang (1964), and Ko (1964), which explain in detail the microbalance techniques, the procedures used to convert drag data from the radially diverging free jet flows into drag coefficients for uniform flow, and a number of special tests performed to verify the independence of the final results from the flow property gradients of the free jets.

Aside from probable random errors of 1 to 2 percent in the force readings and in the model diameters, the principal uncertainty in the computed quantities C_D and α_o comes from residual non-uniformities of 2 to 3 percent in density and dynamic pressure in the model vicinity in the case of Maslach's tests, which employed a set of well-designed but not quite perfect converging-diverging nozzles. In the free jet tests, a possibility of systematic error in density and dynamic pressure arises when we use the inviscid-flow theory to predict these quantities as a function of position,

given the stagnation chamber pressure and temperature. It seems natural to assume that viscous boundary-layer growth in the converging nozzle may decrease the effective nozzle diameter and correspondingly compress the scale of the entire jet flow pattern, but a definitive determination of this effect over the present range of nozzle Reynolds numbers ($2350 < \rho_* u_* D / \mu_* < 25600$) has not yet been made. In our re-reduction of the data of Tang and Ko we have chosen to ignore any such effect, using the formulas

$$\left. \begin{aligned} q/p_o &= 0.309 (x/D - 0.13)^{-2} \\ \text{and} \\ \rho/\rho_o &= 0.0887 (x/D - 0.40)^{-2} \end{aligned} \right\} x \geq 3D \quad (9)$$

which are accurate fits to theoretical data for air, obtained by the method of characteristics.

The principal experimental results for cylinder and strip drag are the following:

1) For $M = 6, 8$, and 10 , and for $0.004 \leq \alpha_o \leq 0.285$, the measured drag coefficients varied linearly with $\alpha_o \ln \alpha_o$, with 76 percent of the cylinder drag coefficients and 94 percent of the strip drag coefficients falling within ± 2 percent of a "best fit" straight line.

2) Extrapolation of these straight lines to $\alpha_o = 0$ gave free-molecular drag coefficients which were within 1 percent of values computed with an assumption of fully diffuse and accommodated molecular reemission from the model surface.

3) A plot of $C_{D_{fm}} - C_D$ versus $-\alpha_o \ln \alpha_o$ collapsed all data for each model geometry onto a single straight line independent of Mach number. The slope of this line is 2.6 ± 0.1 for the strips, and 1.8 ± 0.1 for the cylinders. (See Fig. 1.)

4.2 Sphere Drag

These data were taken directly from the paper of Kinslow and Potter (1962), which contains all information necessary for the calculation of α_o . All of Kinslow and Potter's data for which $\alpha_o \leq 2.5$ are shown in Fig. 2, which has been prepared with the theoretical value of $C_{D_{fm}}$ for fully diffuse and accommodated reemission.

Unfortunately for our present aims, these data do not extend to values of α_o much below 0.5, and do not suffice either to define directly the slope of an initially linear dependence of C_D upon α_o , or to permit extrapolation to $\alpha_o = 0$ to provide a check on the assumption of diffuse reemission. We note, however, that

- 1) In a correlation of $C_D - C_{D_{fm}}$ versus α_o , the data show no systematic dependence upon S_b in the range $0.5 \leq \alpha_o \leq 2.5$, $4.5 \leq S_b \leq 6.3$.
- 2) Three-fourths of the measured values of C_D are predicted to ± 2 percent accuracy or better by the simple formula

$$C_D = C_{D_{fm}} - 0.50 \alpha_o + 0.09 \alpha_o^2 \quad (10)$$

5.0 SUMMARY OF THEORETICAL RESULTS AND COMPARISON WITH EXPERIMENT

A summary of the theoretical formulas for drag are given in Tables I - III. For uniformity of comparison all results are expressed in terms of α_o (using Eq. 8 to convert the hard sphere results), and restricted to the case $S_\infty \rightarrow \infty$. The only parameter now entering the expressions is S_b , and it is further assumed that this dependence can be represented by a power series in $1/S_b$. Some of the expressions represent asymptotic expansions in $1/S_b$ (i.e., "cold wall" analyses); others represent best fits to numerical data.

5.1 Cylinder and Strip Drag

For the same collision model the differences between the cylinder and strip results are quite modest, and are in the direction suggested by intuition and confirmed by experiment. There are marked differences between the results predicted from the different collision models.

For Berkeley experiments $S_b \approx 1.68$, and only the integral iteration results are applicable. From Fig. 1 we can judge that the theories are qualitatively very successful when α is interpreted as α_0 . For both cylinders and strips the modified Krook model underestimates the measured $C_{D_{fm}} - C_D$ in the approximate ratio 2:3, the simple Krook model overestimates in the ratios 2.3:1 for cylinders, 2.1:1 for strips. A much better comparison could be given if the theoretical contribution of order α_0 were also calculated. (This is by no means a simple matter, unfortunately.) The presently available data are too scattered to provide any experimental estimate of the coefficient of a term of order α_0 , except to suggest from the effectiveness of a simple linear fit of $C_{D_{fm}} - C_D$ versus $\alpha_0 \ln \alpha_0$, that this coefficient cannot be much larger than unity.

5.2 Sphere Drag

Results are shown in Table III and Fig. 2. When we interpret the theories in terms of α_0 , the most successful appears to be that based on the simple Krook model, analyzed by the Fourier transform method (Rose, 1964). This successfully predicts the observed lack of marked dependence upon S_b and overestimates the apparent initial slope (inferred from the fitting formula 10) by a ratio of 1.4:1. The modified-Krook model appears next best, predicting a much larger effect of S_b (but one which might still well be masked by the scatter of the data), and overestimating the apparent initial slope by ratios of 1.6:1 ($S_b = 6.3$) to 1.9:1 ($S_b = 4.5$). The results for a rigid sphere collision model seem worst off, showing about

the same S_b -dependence as the modified Krook model, but overestimating the apparent initial slope about twice as badly.

6.0 CONCLUSIONS AND SUGGESTED FURTHER WORK

1. The present study indicates that analyses of nearly free molecular flow, based on either the simple Krook model or the modified Krook model, are qualitatively successful in predicting the drag perturbation ($C_{D_{fm}} - C_D$) for two- and three-dimensional bodies, and that quantitative discrepancies are modest (not exceeding a factor of 2 in the present range of investigation) when the collision rate parameter is evaluated at stagnation temperature.

2. Analyses based upon rigid sphere collision models appear, at the present stage of their development, to be less successful than those based on the Krook models.

3. The modified-Krook model, which has been worked out for all the body geometries under discussion, underestimates the leading term of the drag perturbation in the two-dimensional case, and overestimates it for the sphere, when $\alpha = \alpha_0$.

4. Experimental data are needed, to extend the range of S_b for two-dimensional bodies, and to get closer to free-molecular conditions, for wide ranges of S_∞ and S_b , for three-dimensional bodies. To be of any use in guiding theoretical developments, these data must not contain random or systematic errors exceeding ± 2 percent of C_D , and if the data are to provide any estimates of the coefficients of higher-order terms in the theoretical perturbation expansions, the restriction on error becomes about $\pm 1/2$ percent.

5. Most of the data correlations we have shown in terms of α_0 look just about as good in terms of α_∞ , but in two-dimensional flow the theories fit the data better when α_0 is used. For the sphere data, the Krook models look about equally good with either α_0 or α_∞ , α_0 leading to an over-

estimate of initial slope, and α_{∞} to an approximately equal underestimate. The rigid sphere theory for sphere drag looks much better in terms of α_0 than α_{∞} . The intuitive attractiveness of α_0 and the fact that use of it bypasses the nasty question of evaluating viscosities or collision cross sections at very low temperatures also contribute to our commendation of it as a most useful rarefaction parameter for the nearly-free molecular flow regime.

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TABLE I. CYLINDER DRAG $S_\infty \rightarrow \infty$

$$C_D - C_{D_{fm}} = (-\alpha_o \ln \alpha_o) F(S_b)$$

$$F(S_b) = d_o + d_1(1/S_b) + d_2(1/S_b)^2 \dots$$

Collision Model	Method	d_o	d_1	d_2	d_3	$F(1.68)$
Modified Krook	Integral Iteration	-0.21	-1.7	-	-	-1.15
Simple Krook	Integral Iteration	-0.59	-3.9	-3.1	-0.6	-4.1
Simple Krook	Transform (Rose)	-0.56*	-	-	-	-

BEST FIT EXPERIMENTAL DATA						-1.8 \pm 0.1

* Assuming that Rose's expression which is given as $\alpha_o \ln$ (aspect ratio) goes over to $-\alpha_o \ln \alpha_o$ as the aspect ratio of the cylinder becomes very much larger than the Knudsen number.

TABLE II. NORMAL STRIP DRAG $S_\infty \rightarrow \infty$

$$C_D - C_{D_{fm}} = -(\alpha_o \ln \alpha_o) F(S_b)$$

$$F(S_b) = d_o + d_1(1/S_b) + d_2(1/S_b)^2$$

Collision Model	Method	d_o	d_1	d_2	d_3	$F(1.68)$
Modified Krook	Integral Iteration	-0.27	-1.86	-	-	-1.66
Simple Krook	Integral Iteration	-0.75	-4.7	-4.7	-1.0	-5.4
Hard Spheres	First Collision (Lunc & Lubonski)	-0.56*	-	-	-	-

BEST FIT EXPERIMENTAL DATA						-2.6 \pm 0.1

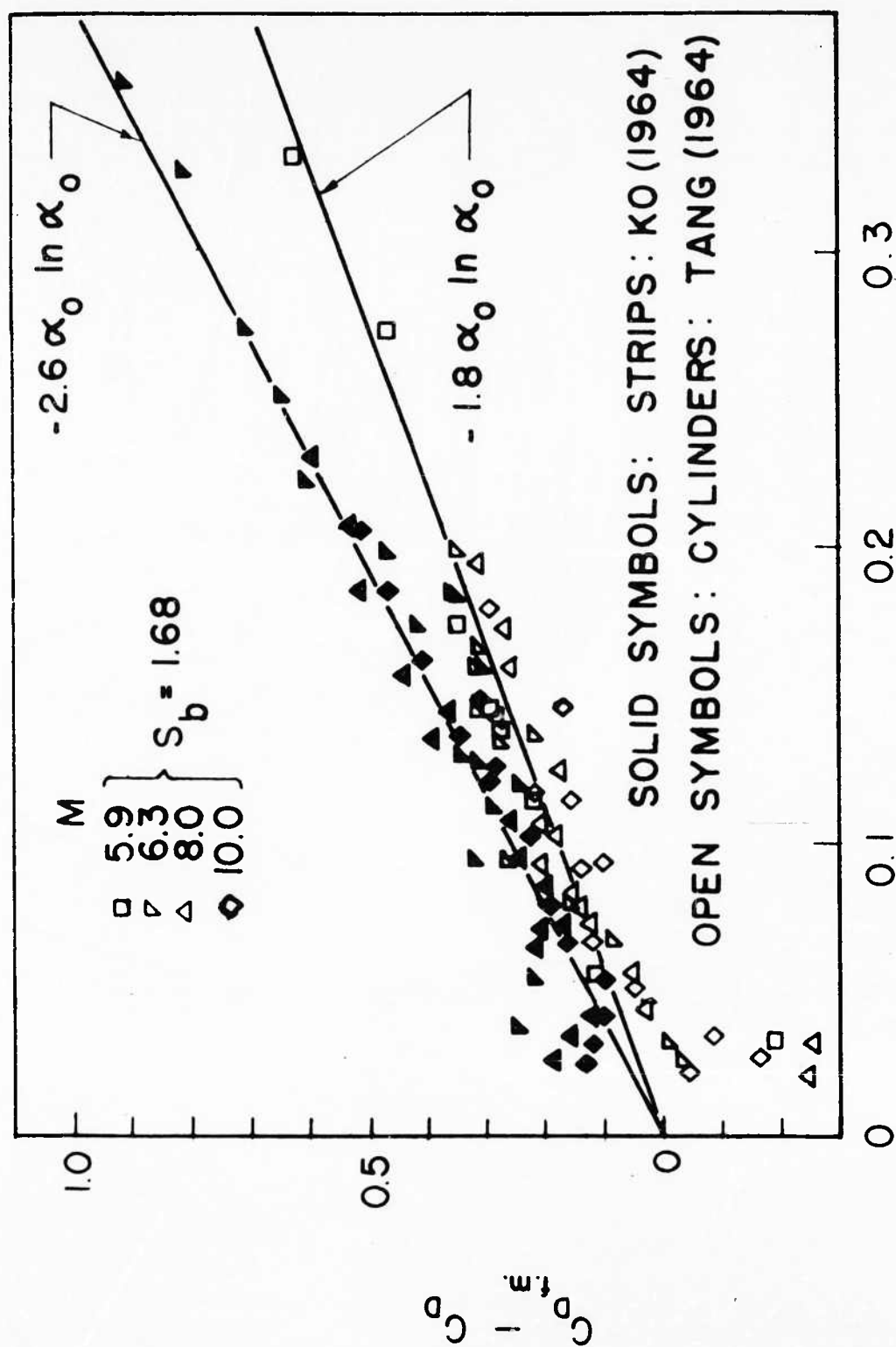
*Lunc and Lubonski's formula is given in terms of a mean speed $\bar{\xi}_b$ for molecules leaving the strip. We have taken $\bar{\xi}_b = (9\pi kT_b/8)^{1/2}$ which gives the same free molecular drag as a Maxwellian emission at temperature T_b .

TABLE III. SPHERE DRAG $S_{\infty} \rightarrow \infty$

$$C_D - C_{D_{fm}} = -\alpha_o [d_o + d_1(1/s_b) + d_2(1/s_b)^2 + \dots]$$

Collision Model	Method	d_o	d_1	d_2
Modified Krook	Integral Iteration	-0.37	-3.32	+2.57
Simple Krook	Transform (Rose)	-0.75	+0.26	-
Hard Spheres	First Collision (Baker & Charwat)	-1.0*	-4.36*	-
BEST FIT EXPERIMENTAL DATA, $C_D - C_{D_{fm}} = -0.50 \alpha_o + 0.09 \alpha_o^2$				

*As in Table II, assuming an expression for the average speed of the molecules leaving the sphere which gives the same free molecular drag as Maxwellian emission at temperature T_b .



$$-\alpha_0 \ln \alpha_0$$

FIGURE 1. CYLINDER & STRIP DRAG

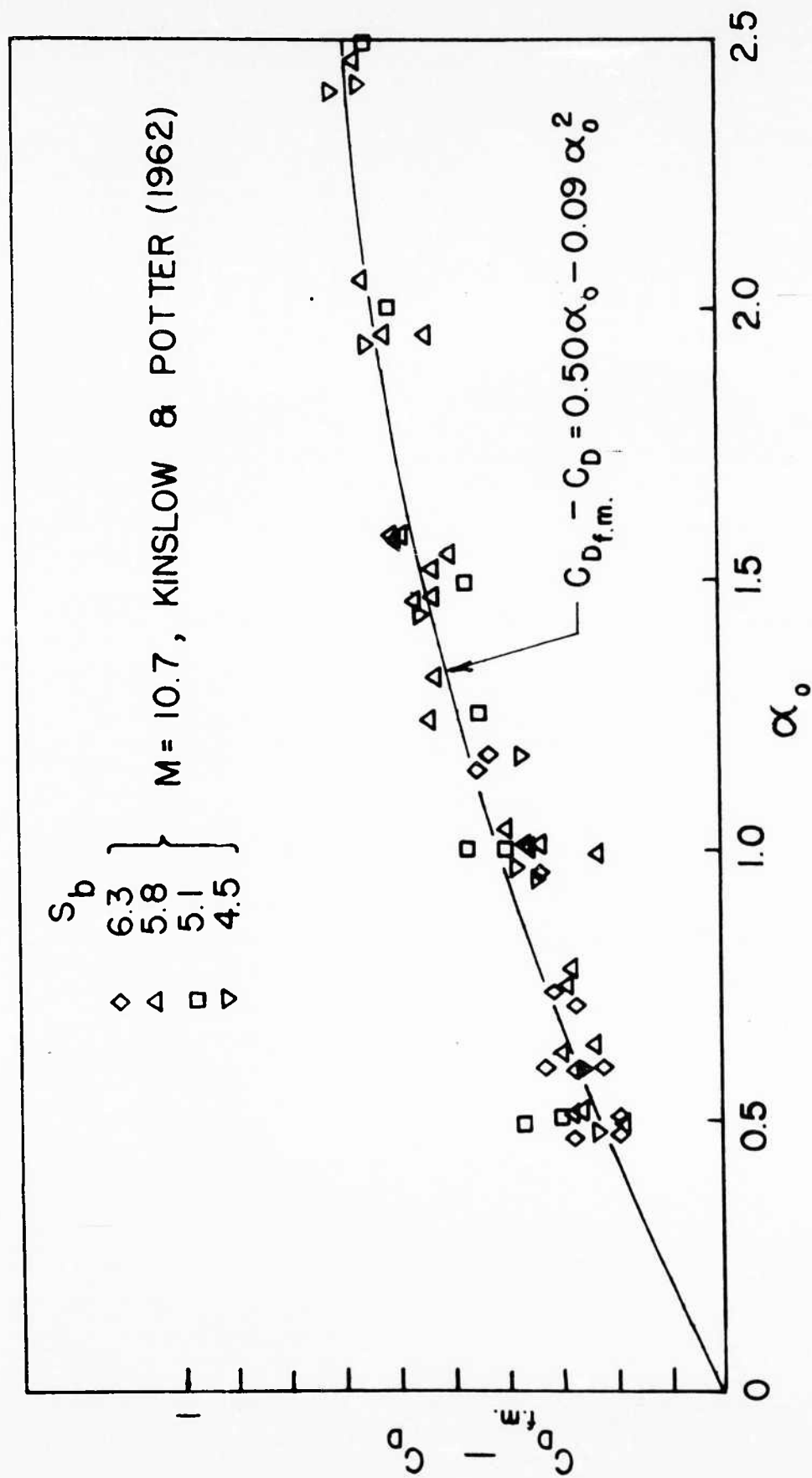


FIGURE 2. SPHERE DRAG

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<p>Measured drag coefficients for cylinders, normal strips and spheres in nearly free-molecular flow at $M = 6, 8$ and 10 are correlated with a collision-rate parameter suggested by the modified Krook model, and compared with theoretical estimates based on a variety of kinetic models and methods of analysis.</p> <p>The comparison of theory and experiment and the comparative effectiveness of various rarefaction parameters are discussed, and suggestions for further experimentation are made.</p>			

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Near-free-molecule flow Kinetic theory Blunt body drag						

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