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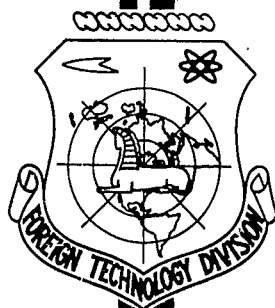
# TRANSLATION

DETERMINATION OF THE PROFILE OF LEAST RESISTANCE  
OF AN ANNULAR WING

By

B. I. Borisov

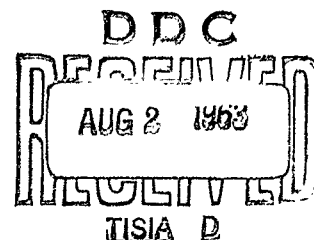
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## UNEDITED ROUGH DRAFT TRANSLATION

DETERMINATION OF THE PROFILE OF LEAST RESISTANCE  
OF AN ANNULAR WING

BY: B. I. Borisov

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# DETERMINATION OF THE PROFILE OF LEAST RESISTANCE OF AN ANNULAR WING

B. I. Borisov

A thin axisymmetric wing in a supersonic flow at velocity  $V_0$  is examined. We start from the linearized equation for the velocity potential:

$$(M^2 - 1) \frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r}$$

We introduce dimensionless variables

$$x = \frac{\bar{x}}{r_0 \sqrt{M^2 - 1}}, \quad r = \frac{\bar{r}}{r_0} \quad (1)$$

In these variables we will have

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} \quad (2)$$

The boundary conditions for this equation are the following:

when  $r = 1$  we have

$$\begin{aligned} \frac{\partial \varphi}{\partial r} &= r_0 V_0 \beta_e(x) \quad \text{on the outer surface of the body,} \\ \frac{\partial \varphi}{\partial r} &= r_0 V_0 \beta_1(x) \quad \text{on the inner surface of the body;} \end{aligned} \quad (3)$$

when  $r = 1 \pm x$  potential  $\varphi = 0$ .

Here  $\beta_e(x)$ ,  $\beta_1(x)$  are the angles between axis  $ox$  and the tangents to the outer and inner surface of the body at the given point.

The solution is sought in the form

$$\psi = \bar{r}_0 V_\infty \int_0^x \beta_r(x-\tau) \psi_1(r, \tau) d\tau \quad (\text{outer problem}) \quad (4)$$

$$\psi = \bar{r}_0 V_\infty \int_0^x \beta_i(x-\tau) \psi_2(r, \tau) d\tau \quad (\text{inner problem})$$

Calculating using the method of characteristics, the functions  $\psi_1$  and  $\psi_2$  were found on the surface of the body

$$\psi_1(1, x) = -1 + 0.4195x - 0.0795x^2, \quad (5)$$

$$\psi_2(1, x) = 1 + 0.209x + 0.651x^2.$$

From the Bernoulli equation we have the following formula for pressure:

$$p = p_0 - V_\infty^2 \rho_\infty \frac{\partial \tau}{\partial x}. \quad (6)$$

We will examine the symmetrical profile of an annular wing of length  $l$  (Fig. 1);

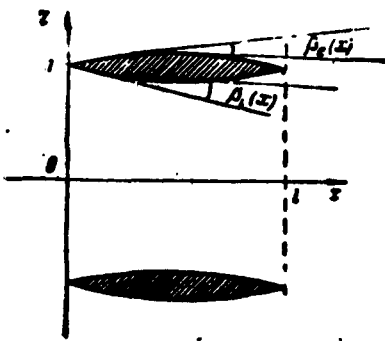


Fig. 1.

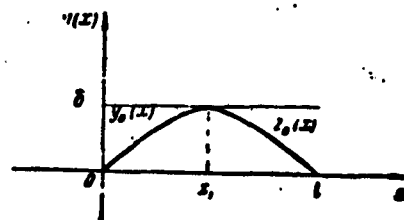


Fig. 2.

$$\beta_r(x) = \beta_i(x) = \beta(x).$$

From Formula (6) it is easy to obtain the drag

$$Q = 2\pi \frac{V_\infty^2 \rho_\infty}{\lambda^2} \int_0^l \beta(x) \frac{d}{dx} \int_0^x \beta(x-\tau) \psi(\tau) d\tau dx,$$

where  $\psi(\tau)$  designates the function  $\psi(\tau) = \psi_2(1, \tau) - \psi_1(1, \tau)$ .

Thus, the drag force is proportional to the integral

$$I = \int_0^l \psi(x) \frac{d}{dx} \int_0^x \psi(x-z) \psi(z) dz dx.$$

Let  $\beta(x) = y'(x)$ ; then  $y(x)$  will be the equation of the profile in the new coordinates (Fig. 2). On the strength of this, integral I can be reduced to the form

$$I = \int_0^l \left[ y'^2(x) \psi_0 + y'(x) \int_0^x y'(z) \psi'(x-z) dz \right] dx, \quad (7)$$

where  $\psi_0 = \psi(0)$ .

We will seek the minimum of this integral under the condition  $y(x_1) = \delta$ , that is, with a given thickness of the profile  $2\delta$ . We will write the equation of line  $y(x)$  as

$$y(x) = \begin{cases} y_0(x) & \text{when } 0 < x \leq x_1 \\ z_0(x) & \text{when } x_1 < x \leq l. \end{cases} \quad (8)$$

After simplifications the first variation of integral  $\delta I$  will have the form

$$\begin{aligned} \delta I = & [-y_0'^2 \psi_0 + z_0'^2 \psi_0]_{x_1} + \int_0^{x_1} \left[ 2\psi_0 y_0' + \frac{d}{dx} \int_0^x y_0' \psi'(x-z) dz + \right. \\ & \left. + \frac{d}{dx} \int_x^{x_1} y_0' \psi'(x-z) dz + \frac{d}{dx} \int_{x_1}^l z_0' \psi'(z-x) dz \right] \delta y dx - \\ & - \int_{x_1}^l \left[ 2\psi_0 z_0' + \frac{d}{dx} \int_{x_1}^x z_0' \psi'(x-z) dz + \frac{d}{dx} \int_x^l z_0' \psi'(z-x) dz + \right. \\ & \left. + \frac{d}{dx} \int_0^{x_1} y_0' \psi'(x-z) dz \right] \delta y dx = 0. \end{aligned} \quad (9)$$

The necessary condition of the extremum is  $\delta I = 0$ . We will narrow the class of permissible curves, having fixed point  $(x_1, y_1)$  and line  $z_0(x)$ . Then condition  $\delta I = 0$  will be the same as equating the first integral in Formula (9) to zero. Similarly, the necessary condition of the extremum with variation only of  $z_0(x)$  will be the

equating of the second integral to zero.

Hence,

$$| -y_0''\psi_0 + z_0''\psi_0 |_{x_1} = 0.$$

Thus we have the necessary condition of the extremum

$$\begin{aligned} 2\psi_0 y_0'' + \frac{d}{dx} \int_0^x y_0' \psi' (x-z) dz + \frac{d}{dx} \int_x^{x_1} y_0' \psi' (z-x) dz + \\ + \frac{d}{dx} \int_{x_1}^l z_0' \psi' (z-x) dz = 0, \\ 2\psi_0 z_0'' + \frac{d}{dx} \int_{x_1}^x z_0' \psi' (x-z) dz + \frac{d}{dx} \int_x^l z_0' \psi' (z-x) dz + \\ + \frac{d}{dx} \int_0^x y_0' \psi' (x-z) dz = 0, \\ y_0''|_{x_1} = z_0''|_{x_1}, \end{aligned} \quad (10)$$

and boundary conditions

$$y_0(0) = 0, \quad y_0(x_1) = \psi, \quad z_0(x_1) = \psi, \quad z_0(l) = 0. \quad (11)$$

We substitute in (10) the well-known function

$$\psi(x) = 0.73x^2 - 0.21x + 2.$$

The solution of System (10) with Boundary Conditions (11) is as follows:

$$y_0 = \frac{\psi}{\sin \frac{kl}{2}} \sin kx, \quad z_0 = \frac{\psi}{\sin \frac{kl}{2}} \sin k(l-x), \quad x_1 = \frac{l}{2},$$

where  $k = 0.85$ .

Converting to dimensional coordinates by Formulas (1) and causing the radius of the annular wing  $r_0$  to tend toward infinity we obtain at the limit

$$y_0 = \frac{\psi}{\frac{l}{2}} \bar{x}, \quad z_0 = \frac{\psi}{\frac{l}{2}} (\bar{l} - \bar{x}),$$

that is, a rhomboid profile which coincides with the well-known



solution in the plane problem.

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Department of Wave- and Gasdynamics

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