FTD-TT- 63-397

TRANSLATION

DETERMINATION OF THE PROFILE OF LEAST RESISTANCE OF AN ANNULAR WING

By

B. I. Borisov

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE
OHIO

FTD-TT-63-397/1+2+4

UNEDITED ROUGH DRAFT TRANSLATION

DETERMINATION OF THE PROFILE OF LEAST RESISTANCE OF AN ANNULAR WING

BY: B. I. Borisov

English Pages: 5

SOURCE: Russian Periodical, Vestnik Moskovskogo Universiteta, No. 1, 1961, pp. 62-65.

T-96
S/55-61-0-1

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

DETERMINATION OF THE PROFILE OF LEAST RESISTANCE OF AN ANNULAR WING

B. I. Borisov

A thin axisymmetric wing in a supersonic flow at velocity V_0 is examined. We start from the linearized equation for the velocity potential:

$$(M^2-1)\frac{\partial^2\varphi}{\partial \bar{x}^2}=\frac{\partial^2\varphi}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial\varphi}{\partial \bar{r}}$$

We introduce dimensionless variables

$$x=\frac{\bar{x}}{r_0\sqrt{M^2-1}},\quad r=\frac{\bar{r}}{r_0} \qquad (1)$$

In these variables we will have

$$\frac{\partial^2\varphi}{\partial x^2}=\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}. \qquad (2)$$

The boundary conditions for this equation are the following:

when $r = 1$ we have

$$\frac{\partial\varphi}{\partial r}=r_0V_0\beta_e(x) \quad \text{on the outer surface of the body,}$$
$$\frac{\partial\varphi}{\partial r}=r_0V_0\beta_i(x) \quad \text{on the inner surface of the body;} \qquad (3)$$

when $r = 1 \pm x$ potential $\varphi = 0$.

Here $\beta_e(x)$, $\beta_i(x)$ are the angles between axis ox and the tangents to the outer and inner surface of the body at the given point.

The solution is sought in the form

$$\varphi = \bar{r}_0 V_0 \int_0^x \beta_e (x - \tau)\, \psi_1 (r, \tau)\, d\tau \qquad \text{(outer problem)}$$

$$\varphi = \bar{r}_0 V_0 \int_0^x \beta_i (x - \tau)\, \psi_2 (r, \tau)\, d\tau \qquad \text{(inner problem)} \tag{4}$$

Calculating using the method of characteristics, the functions ψ_1 and ψ_2 were found on the surface of the body

$$\psi_1 (1, x) = -1 + 0{,}4195x - 0{,}0795x^2,$$
$$\psi_2 (1, x) = 1 + 0{,}209x + 0{,}651x^2. \tag{5}$$

From the Bernoulli equation we have the following formula for pressure:

$$p = p_0 - V_0 \rho_0 \frac{\partial \varphi}{\partial x}. \tag{6}$$

We will examine the symmetrical profile of an annular wing of length l (Fig. 1);

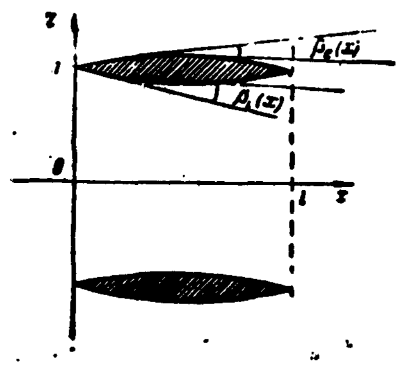

Fig. 1.

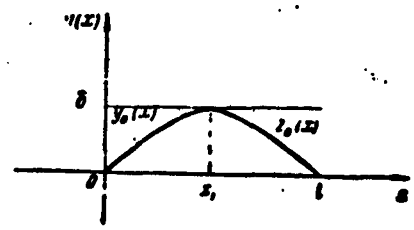

Fig. 2.

$$\beta_e (x) = -\beta_i (x) = \beta (x).$$

From Formula (6) it is easy to obtain the drag

$$Q = 2\pi \frac{V_0^2 \rho_0}{\sqrt{M^2 - 1}} \int_0^l \beta (x) \frac{d}{dx} \int_0^x \beta (x - \tau)\, \psi (\tau)\, d\tau\, dx.$$

where $\psi(\tau)$ designates the function $\psi(\tau) = \psi_2(1, \tau) - \psi_1(1, \tau)$.

Thus, the drag force is proportional to the integral

$$I=\int_0^l \beta(x)\frac{d}{dx}\int_0^x \beta(x-\tau)\psi(\tau)\,d\tau\,dx.$$

Let $\beta(x) = y'(x)$; then $y(x)$ will be the equation of the profile in the new coordinates (Fig. 2). On the strength of this, integral I can be reduced to the form

$$I=\int_0^l\left[y'^2(x)\psi_0+y'(x)\int_0^x y'(\tau)\psi'(x-\tau)\,d\tau\right]dx, \tag{7}$$

where $\psi_0 = \psi(0)$.

We will seek the minimum of this integral under the condition $y(x_1) = \delta$, that is, with a given thickness of the profile 2δ. We will write the equation of line $y(x)$ as

$$y(x)=\begin{matrix} y_0(x) \text{ when } 0<x\leqslant x_1 \\ z_0(x) \text{ when } x_1\leqslant x\leqslant l. \end{matrix} \tag{8}$$

After simplifications the first variation of integral δI will have the form

$$\begin{aligned}\delta I &= \left[-y_0'^2\psi_0+z_0'^2\psi_0\right]_{x_1}\delta x_1-\int_0^{x_1}\Bigg[2\psi_0 y_0''+\frac{d}{dx}\int_0^x y_0'\psi'(x-\tau)\,d\tau+\\ &+\frac{d}{dx}\int_x^{x_1} y_0'\psi'(x-\tau)\,d\tau+\frac{d}{dx}\int_{x_1}^l z_0'\psi'(\tau-x)\,d\tau\Bigg]\delta y\,dx-\\ &-\int_{x_1}^l\Bigg[2\psi_0 z_0''+\frac{d}{dx}\int_{x_1}^x z_0'\psi'(x-\tau)\,d\tau+\frac{d}{dx}\int_x^l z_0'\psi'(\tau-x)\,d\tau+\\ &+\frac{d}{dx}\int_0^{x_1} y_0'\psi'(x-\tau)\,d\tau\Bigg]\delta y\,dx=0.\end{aligned} \tag{9}$$

The necessary condition of the extremum is $\delta I = 0$. We will narrow the class of permissible curves, having fixed point (x_1, y_1) and line $z_0(x)$. Then condition $\delta I = 0$ will be the same as equating the first integral in Formula (9) to zero. Similarly, the necessary condition of the extremum with variation only of $z_0(x)$ will be the equating of the second integral to zero.

Hence,

$$\left[-y_0'^2\psi_0 + z_0'^2\psi_0\right]_{x_1}\delta x_1 = 0.$$

Thus we have the necessary condition of the extremum

$$\begin{gathered}
2\psi_0 y_0'' + \frac{d}{dx}\int_0^x y_0'\psi'(x-\tau)\,d\tau + \frac{d}{dx}\int_x^{x_1} y_0'\psi'(\tau-x)\,d\tau + \\
+ \frac{d}{dx}\int_{x_1}^{l} z_0'\psi'(\tau-x)\,d\tau = 0, \\
2\psi_0 z_0'' + \frac{d}{dx}\int_{x_1}^{x} z_0'\psi'(x-\tau)\,d\tau + \frac{d}{dx}\int_x^{l} z_0'\psi'(\tau-x)\,d\tau + \\
+ \frac{d}{dx}\int_0^{x_1} y_0'\psi'(x-\tau)\,d\tau = 0, \\
y_0'^2|_{x_1} = z_0'^2|_{x_1}
\end{gathered} \tag{10}$$

and boundary conditions

$$y_0(0) = 0, \quad y_0(x_1) = \delta, \quad z_0(x_1) = \delta, \quad z_0(l) = 0. \tag{11}$$

We substitute in (10) the well-known function

$$\psi(x) = 0{,}73\,x^2 - 0{,}21\,x + 2.$$

The solution of System (10) with Boundary Conditions (11) is as follows:

$$y_0 = \frac{\delta}{\sin\frac{kl}{2}}\sin kx, \quad z_0 = \frac{\delta}{\sin\frac{kl}{2}}\sin k(l-x), \quad x_1 = \frac{l}{2},$$

where k = 0.85.

Converting to dimensional coordinates by Formulas (1) and causing the radius of the annular wing r_0 to tend toward infinity we obtain at the limit

$$y_0 = \frac{\delta}{\frac{\bar{l}}{2}}\bar{x}, \quad z_0 = \frac{\delta}{\frac{\bar{l}}{2}}(\bar{l}-\bar{x}),$$

that is, a rhomboid profile which coincides with the well-known solution in the plane problem.

Submitted September 9, 1959 Department of Wave- and Gasdynamics

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies	MAJOR AIR COMMANDS	Nr. Copies
		AFSC	
		SCFDD	1
		DDC	25
		TDBTL	5
HEADQUARTERS USAF		TDBDP	3
		AEDC (AEY)	1
AFCIN-3D2	1	AFFTC (FTY)	1
ARL (ARB)	1	AFWL (WLF)	1
		ASD (ASYIM)	2
		SSD (SSF)	2
OTHER AGENCIES			
CIA	1		
NSA	6		
DIA	9		
AID	2		
OTS	2		
AEC	2		
PWS	1		
NASA	1		
ARMY (FSTC)	3		
NAVY	3		
NAFEC	1		
RAND	1		
AFCRL (CRXLR)	1		